How to detect and construct Integrable Quantum Circuits?

Chiara Paletta

University of Ljubljana

Based on:

2503.04673 with U. Duh, B. Pozsgay, L. Zadnik || 2603.25424 with T. Prosen || 2607.02093 with A. L. Retore, M. G. Fernández

In progress with R. Frassek, V. Popkov, T. Prosen || (orthogonal) In progress with A. Torrielli

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RAQIS, Annecy

Overview:

- Yang-Baxter Integrability
- Quantum circuit
- How can we determine if a given quantum circuit is integrable?

Integrable Quantum Circuits

Yang-Baxter

Periodic boundary conditions

Open boundary conditions

Beyond Yang-Baxter

Periodic boundary conditions - (deformed) Rule 54

Open boundary conditions - XXX + reset gates; (deformed) Rule 54 + boundaries

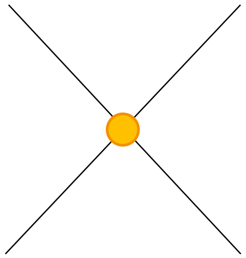
Towards a general algorithm to detect Yang-Baxter (and beyond) integrability in quantum circuits!

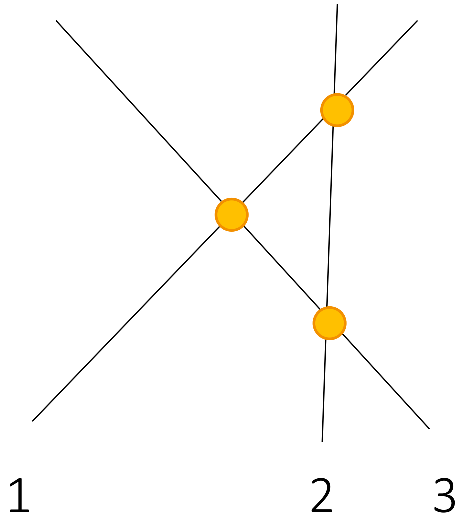
What is a quantum integrable model?

Models with a high amount of symmetry and a tower of conserved charges

$$[Q_i, Q_j] = [Q_i, H] = 0$$

Yang-Baxter quantum integrable model

R-matrix  ,



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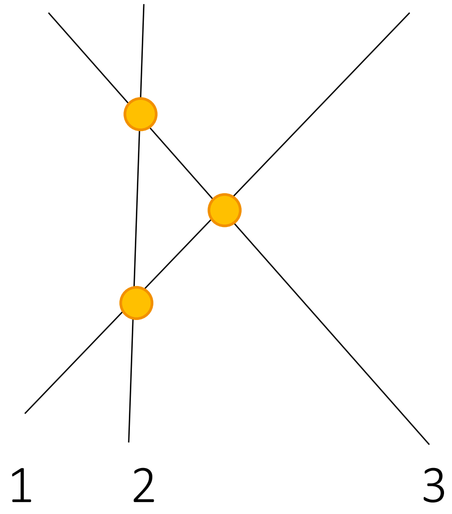
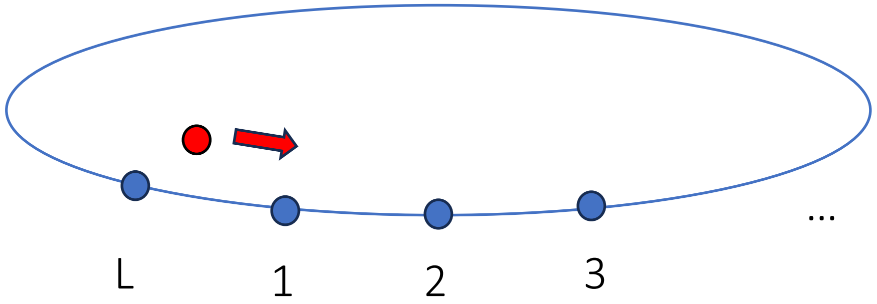


Yang-Baxter equation

$$R_{23}R_{13}R_{12} = R_{12}R_{13}R_{23}$$

Quantum integrability

Connection R-matrix and charges



Periodic boundary condition

$$t(u) = Tr_0 R_{0L}(u-u_L) R_{0L-1}(u-u_{L-1}) \ldots R_{01}(u-u_1)$$

$$[t(u), t(v)] = 0$$

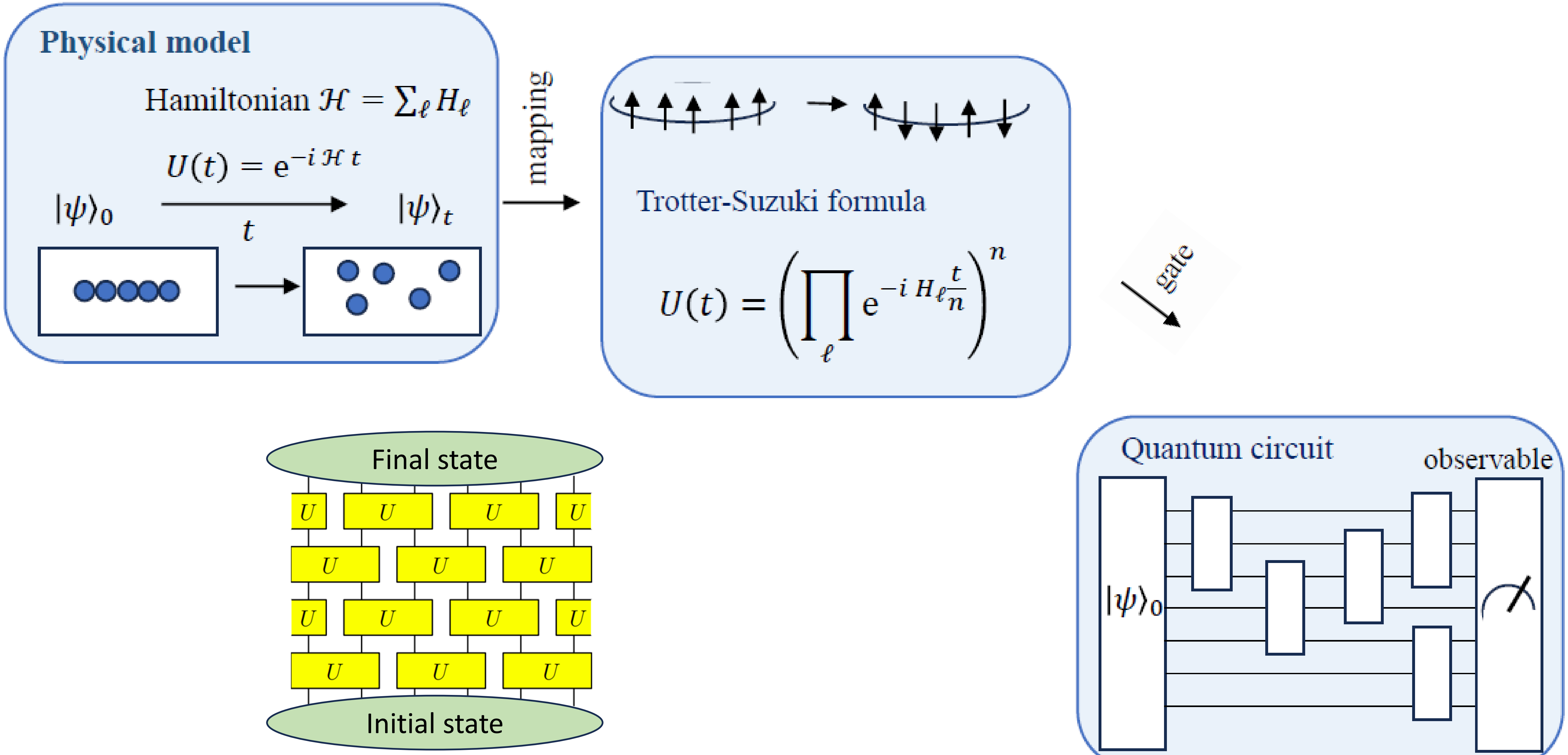
$$\log t(u) = Q_1 + u\, Q_2 + \frac{u^2}{2} Q_3 + \cdots$$

$$Q_2 = \sum_{i=1}^{L} q_{i,i+1}^{(2)}$$

$$Q_i = \partial_u^{i-1} \log t(u) \Big|_{u\to 0}$$

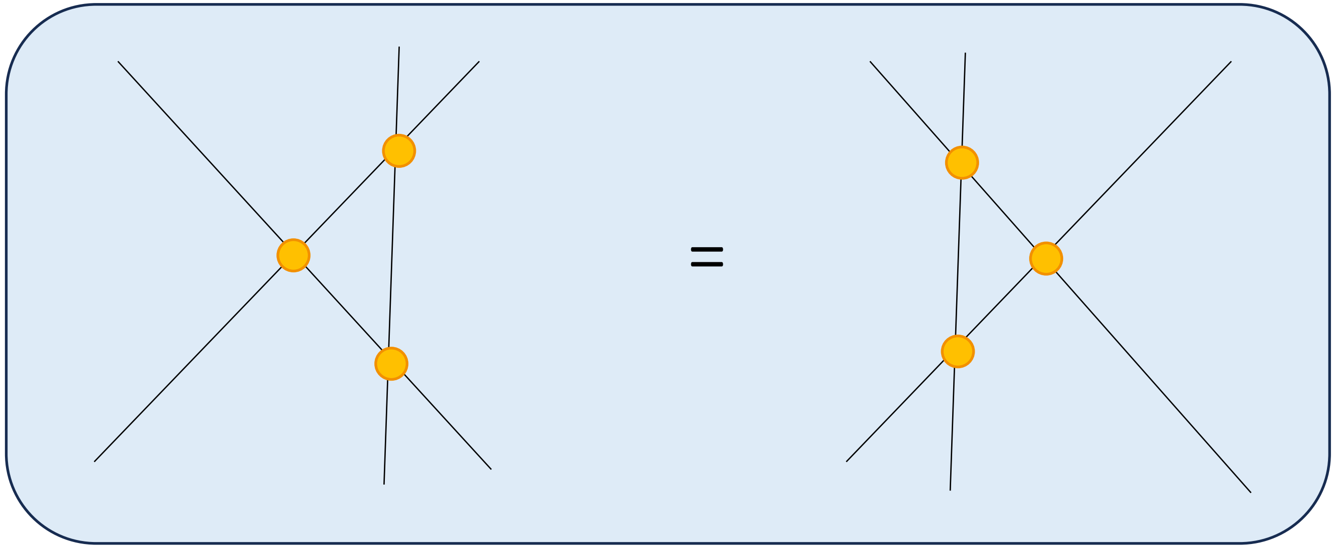
$$q_{i,i+1}^{(2)} = \partial_u P_{i,i+1} R_{i,i+1}(u) \Big|_{u\to 0}$$

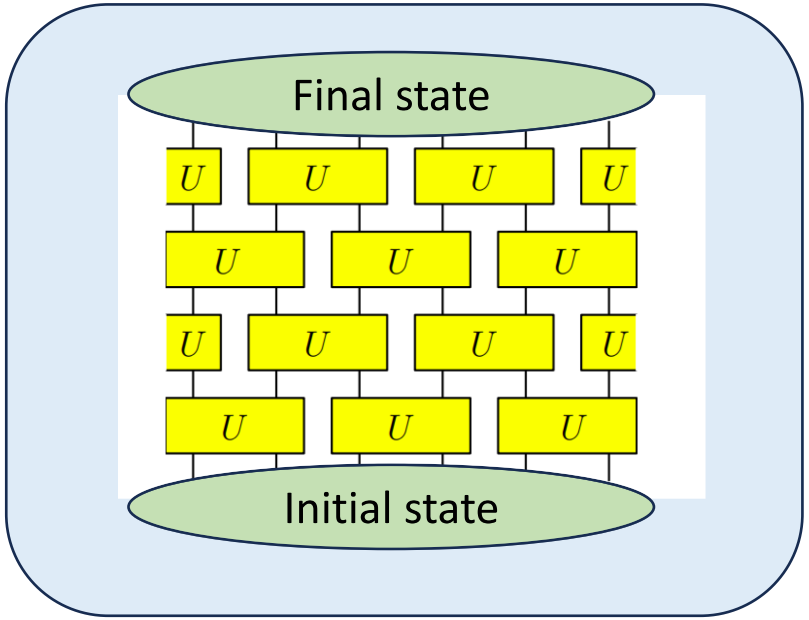
Quantum circuit

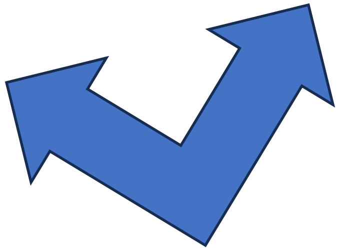


Connections:

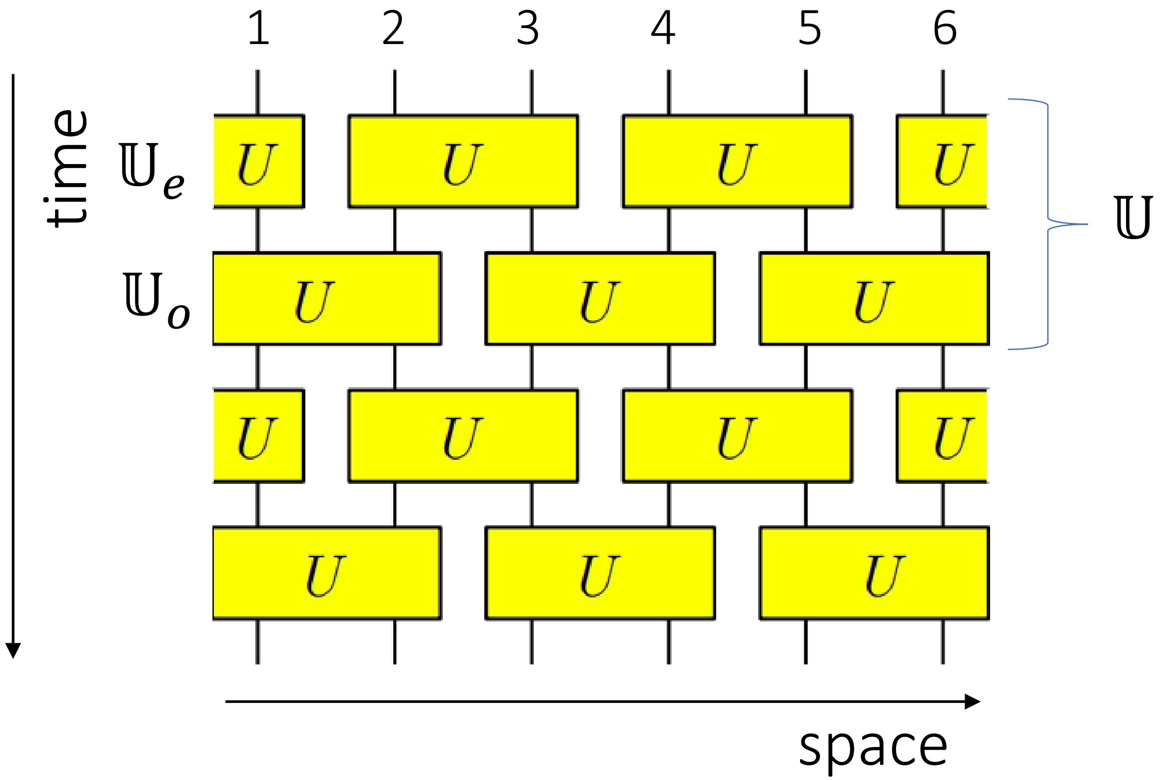
[Prosen, Zadnik, Vanicat PRL ’18,
Vanicat NPhys B ‘18]







Let us consider a **Brickwork type circuit**



Write the dynamical evolution:

$$\mathbb{U} = \mathbb{U}_o \mathbb{U}_e = U_{12}\, U_{34} U_{56} U_{23}\, U_{45}\, U_{61}$$

$$\mathbb{U}_e = U_{23}\, U_{45}\, U_{61}$$

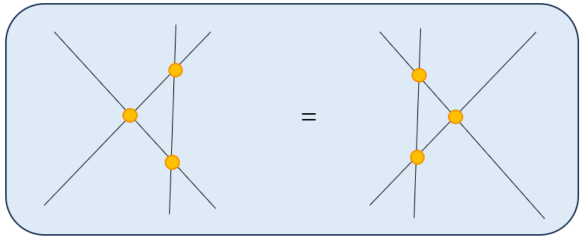
$$\mathbb{U}_o = U_{12}\, U_{34}\, U_{56}$$

Integrable Quantum Circuits

The circuit is integrable if the dynamical evolution generator $\mathbb{U}$ commutes with a tower of conserved charges.

Integrability for quantum circuits

One possible way to realize it is:



[Prosen, Zadnik, Vanicat PRL '18, Vanicat NPhys B '18]

The gate U is connected to the solution of the Yang-Baxter equation

$$U = PR(2\kappa)$$

For a model characterized by an R-matrix solution of the YBE:

$$t(u) = \mathrm{Tr}_0 R_{06}(u-u_6)R_{05}(u-u_5)R_{04}(u-u_4)R_{03}(u-u_3)R_{02}(u-u_2)R_{01}(u-u_1)$$

$$[t(u), t(v)] = 0, \qquad \text{for all } u \text{ and } v$$

If

$$u_1 = u_3 = u_5 = -\kappa, \qquad u_2 = u_4 = u_6 = \kappa \quad \Rightarrow \quad [\mathbb{U}, t(u)] = 0$$

After some manipulation

$$\boxed{\mathbb{U} = t^{-1}(-\kappa)t(\kappa)}$$

and since $[t(v), t(u)] = 0, \ \forall\, u, v$ $\quad$ $t(u)$ generates the infinite tower of conserved charges commuting with $\mathbb{U}$

Given a quantum circuit with a given geometry, can we understand if it is integrable?

Numerical techniques to detect integrability:

- From the energy level distributions
- To obtain the conserved charges by using a **brute-force algorithm**

Disadvantages:

- Numerical errors
- We cannot obtain all the charges!

Can we use analytical techniques to answer the question?

Advantages

- We can use analytical techniques, for example: Bethe ansatz techniques, QQ-system, ODE/IM, …

  to compute correlation functions, transport properties, dynamics, non-equilibrium steady states

- **Benchmark** quantum devices and algorithms

[K. Maruyoshi, T. Okuda, J. W. Pedersen, R. Suzuki, M. Yamazaki, Y. Yoshida, ‘23]

- Understanding **universal** aspects of quantum dynamics. For example:

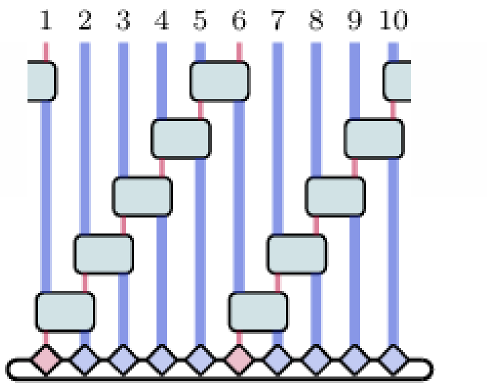
[E. Rosenberg, T. Prosen et al., ‘24]

Detecting Yang-Baxter integrability in Quantum circuits (periodic BC)

If we start from a gate U solution of the YBE and a circuit with a given geometry, can we understand:

whether it is Yang–Baxter integrable and how to place the inhomogeneities?

For example, for $L = 10$



$$\mathbb{U} = U_{56}\, U_{10,1} U_{45}\, U_{9,10} U_{34}\, U_{89} U_{23} U_{78}\, U_{12} U_{67} \quad ,$$

$$t(u) = Tr_0 \overleftarrow{\prod_{i=1}^{10}} R_{0i}(u - u_i)$$

Solve

$$[\mathbb{U}, t(u)] = 0 \qquad \text{for} \qquad u_1, u_2, \dots, u_{10}$$

$$\vec{u} = \{\kappa, -\kappa, -\kappa, -\kappa, -\kappa, \kappa, -\kappa, -\kappa, -\kappa, -\kappa\}$$

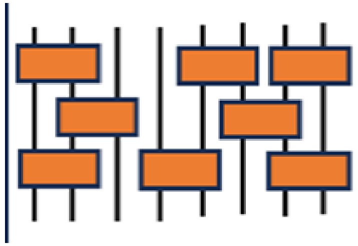
$$\mathbb{U} = t^{-1}(-\kappa) t(\kappa)$$

Equivalent to *cutting the lattice*
[2503.04673 CP., Duh, Pozsgay, Zadnik]

Given a quantum circuit with a given geometry, can we understand if it is integrable?

[2503.04673 CP., Duh, Pozsgay, Zadnik]

<u>For periodic boundary conditions:</u>

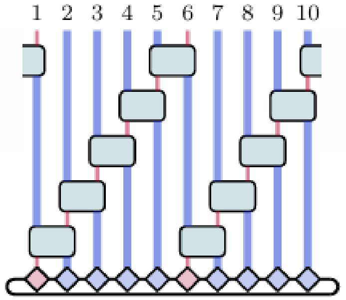
Step 1: Draw the circuit	**Step 3: Insert the expressions of the quantum gate U**	**Step 4: Does the gate satisfy the YBE?**	**Step 5: Is the quantum circuits integrable?**
 $M = U_{12}U_{45}U_{78}$ $U_{23}U_{67}U_{12}U_{34}U_{78}$	$U = \begin{pmatrix} 1.76041 & -0.991837 & 0.422449 & -0.55102 \\ 0.16898 & 0.779592 & 0.323878 & -0.422449 \\ -0.396735 & 1.78531 & 0.779592 & 0.991837 \\ -0.0881633 & 0.396735 & -0.16898 & 1.76041 \end{pmatrix}$	YES	Yes! (or NO) The transfer matrix is: …

Classification of nearest neighbour **integrable** circuits

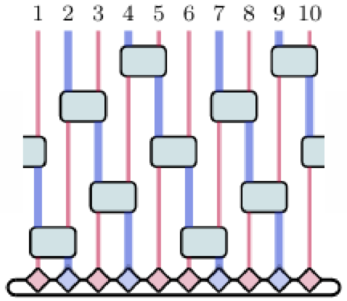
[Duh, CP, Pozsgay, Zadnik , 2503.04673]
[Miao, Gritsev, Kurlov, 2206.15142]

All quantum unitary circuits in which the gate (solution of the YBE) is applied to each pair of neighbouring qubits exactly once per period are integrable.

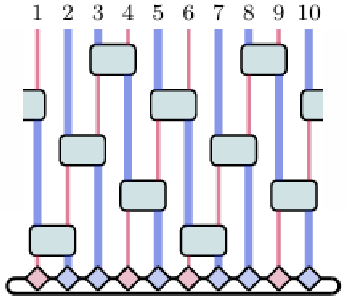
For example, for $L = 10$



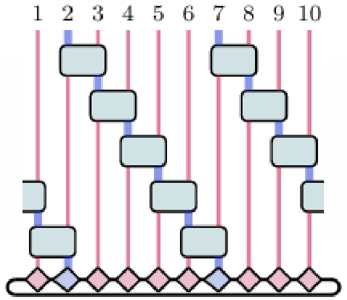
(a) $(q, r) = (5, 1)$, $p = 2$



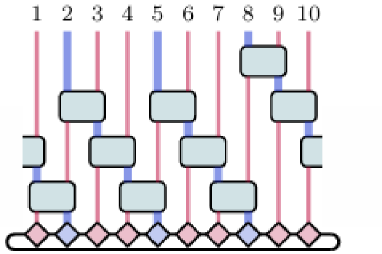
(b) $(q, r) = (5, 2)$, $p = 6$



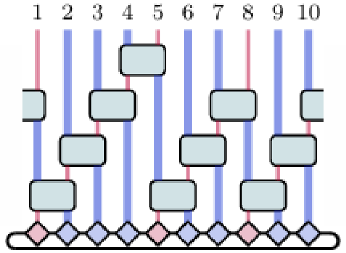
(c) $(q, r) = (5, 3)$, $p = 4$



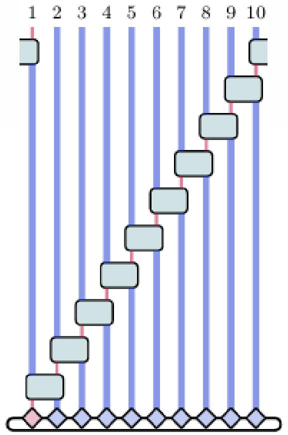
(d) $(q, r) = (5, 4)$, $p = 8$



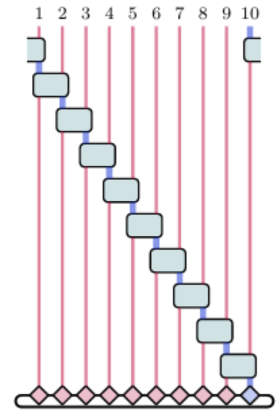
(e) $(q, r) = (10, 3)$, $p = 7$



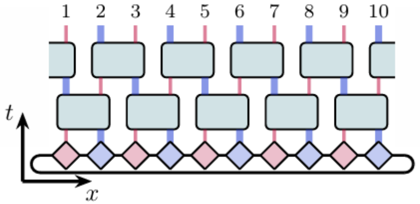
(f) $(q, r) = (10, 7)$, $p = 3$



(g) $(q, r) = (10, 1)$, $p = 1$



(h) $(q, r) = (10, 9)$, $p = 9$

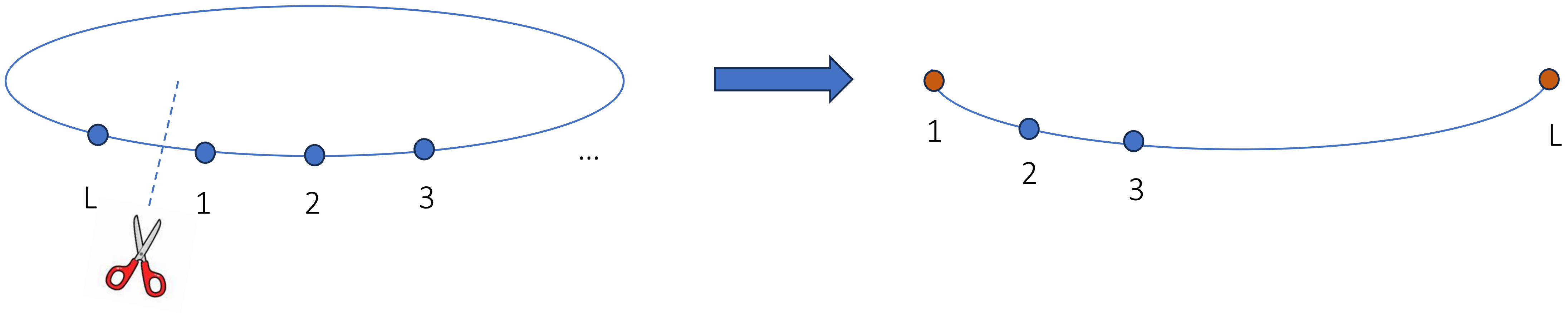


Detect Yang-Baxter integrable quantum circuits

Integrability \ Boundary Condition	Periodic	Open
Yang-Baxter type	✓	
Beyond Yang-Baxter type		

[Sklyanin JPHYS A '88]

Open boundary conditions



Integrable open spin chain:

Yang-Baxter equation + boundary Yang-Baxter equation

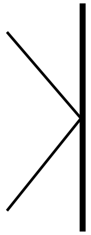
Open boundary condition

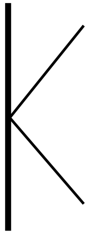
[Sklyanin JPHYS A '88]

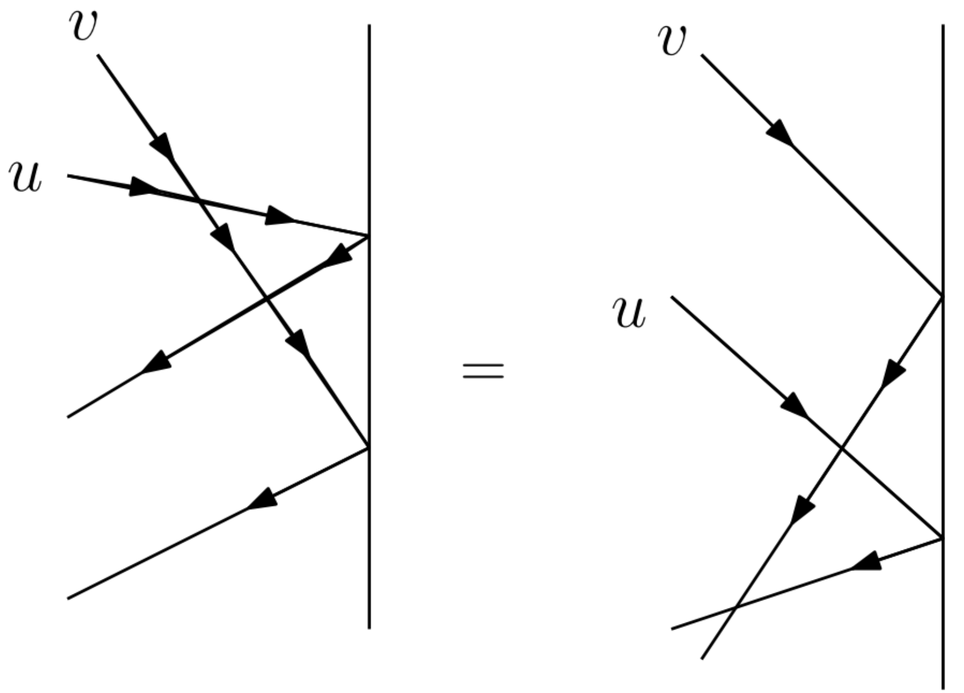
right

boundary Yang-Baxter equation

Two extra equations for the left and right boundaries

K^R = 

K^L = 



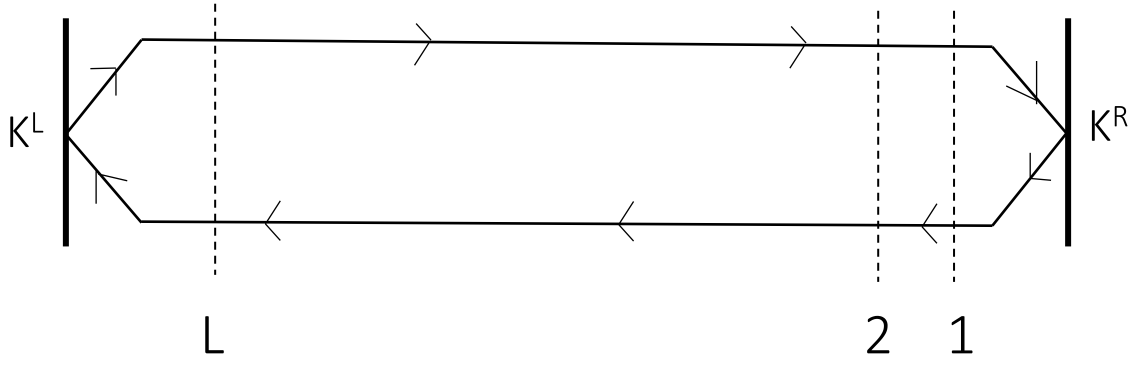
$$K_2 R_{21} K_1 R_{12} = R_{21} K_1 R_{12} K_2$$

$R_{12}=R_{21}$

Tower of conserved charges?

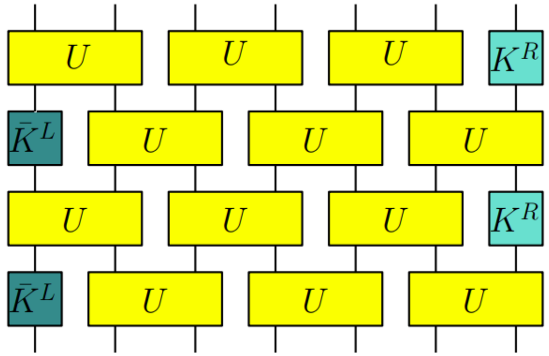
[Sklyanin JPHYS A '88]

$$t(u) = Tr_0\, K_0^L\,(u)\underbrace{T_0(u,\vec{u}\,)}_{R_{0L}(u-u_1)R_{0L-1}(u-u_2)\ldots R_{01}(u-u_L)}\,K_0^R(u)\underbrace{\hat{T}_0(u,\vec{u}\,)}_{R_{01}(u-u_L)R_{0,2}(u-u_{L-1})\ldots R_{0L}(u-u_1)}$$



$$Q_2 = \partial_u t(u)\Big|_{u\to 0} = Q_{bulk} + Q_1^R + Q_L^L$$

Quantum circuits

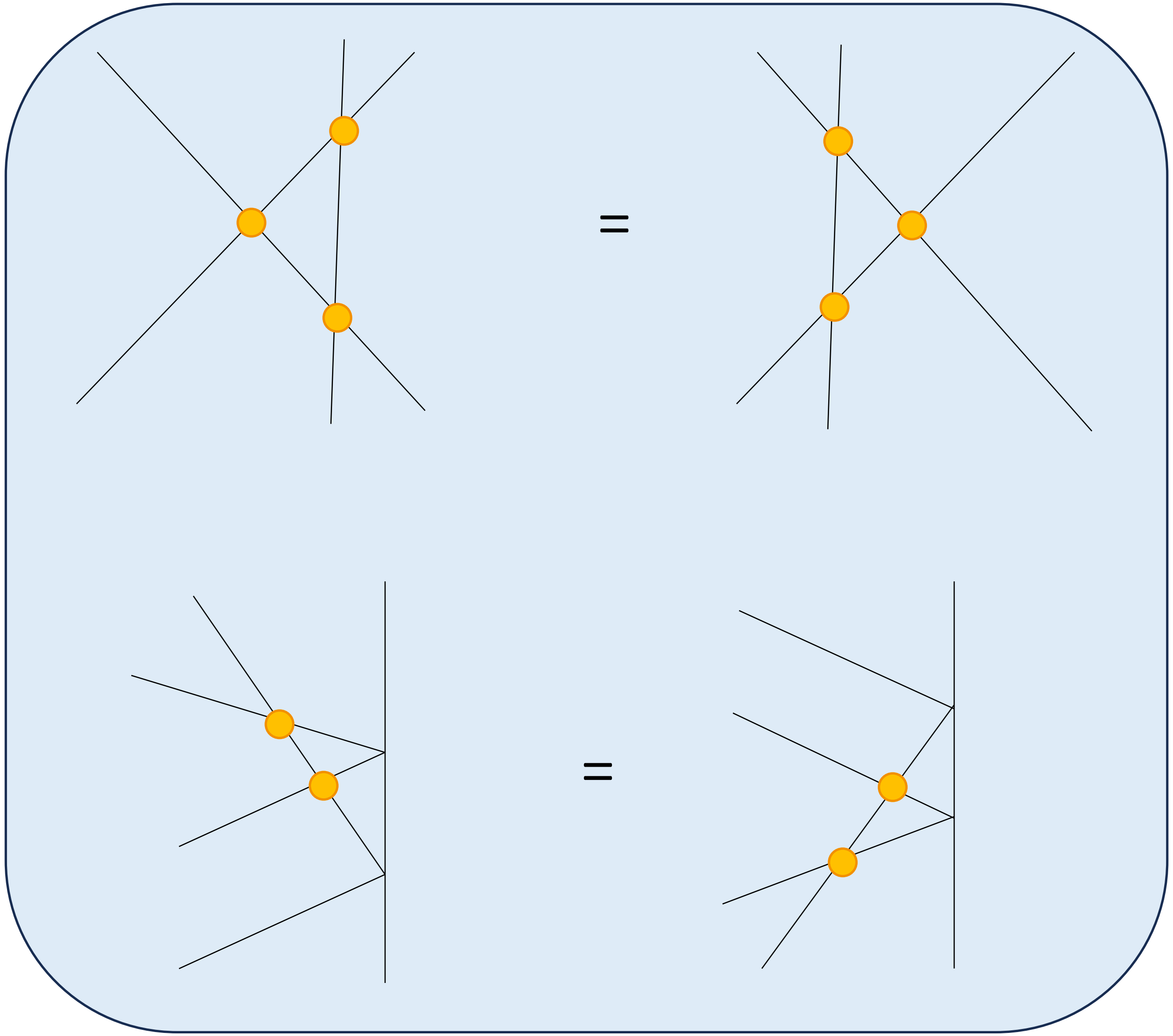


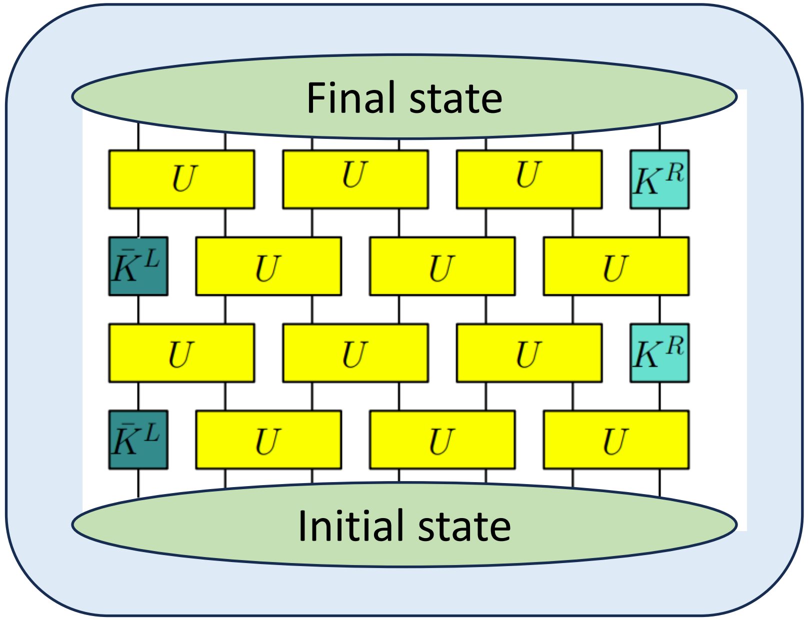
Discrete-time version of the Hamiltonian Q_2

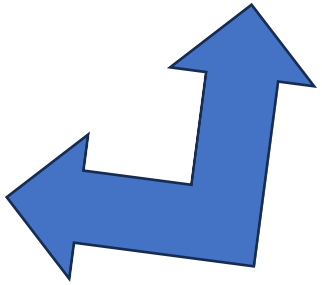
$$\mathbb{U} = K_1^R U_{23}\, U_{45}\, U_{67} U_{12}\, U_{34}\, U_{56} \bar{K}_7^L$$

Connections:

[Prosen, Zadnik, Vanicat PRL ’18, Vanicat NPhys B ‘18]





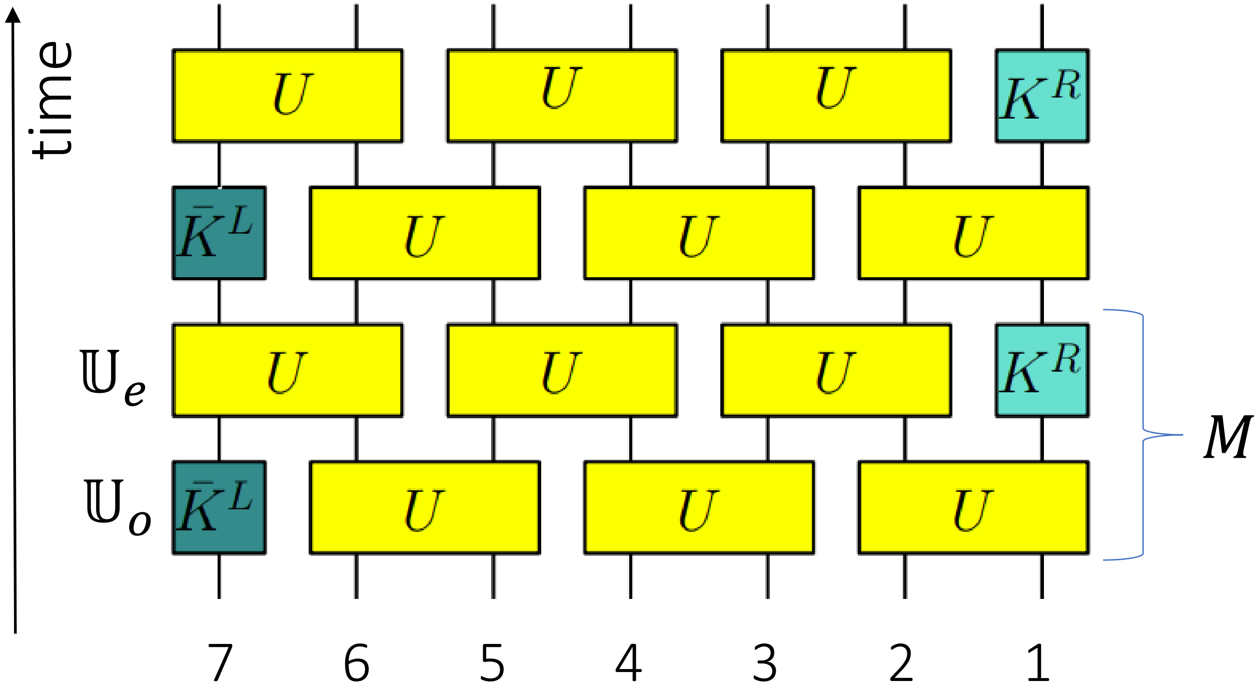


[2607.02093 M. G. Fernández, CP, A. L. Retore]

We generalized this to circuits with non difference form R-matrix

Connections: When can we say that a quantum circuit is integrable?

As before, the dynamical evolution generator M should commute with infinitely many conserved charges!



Write the dynamical evolution:

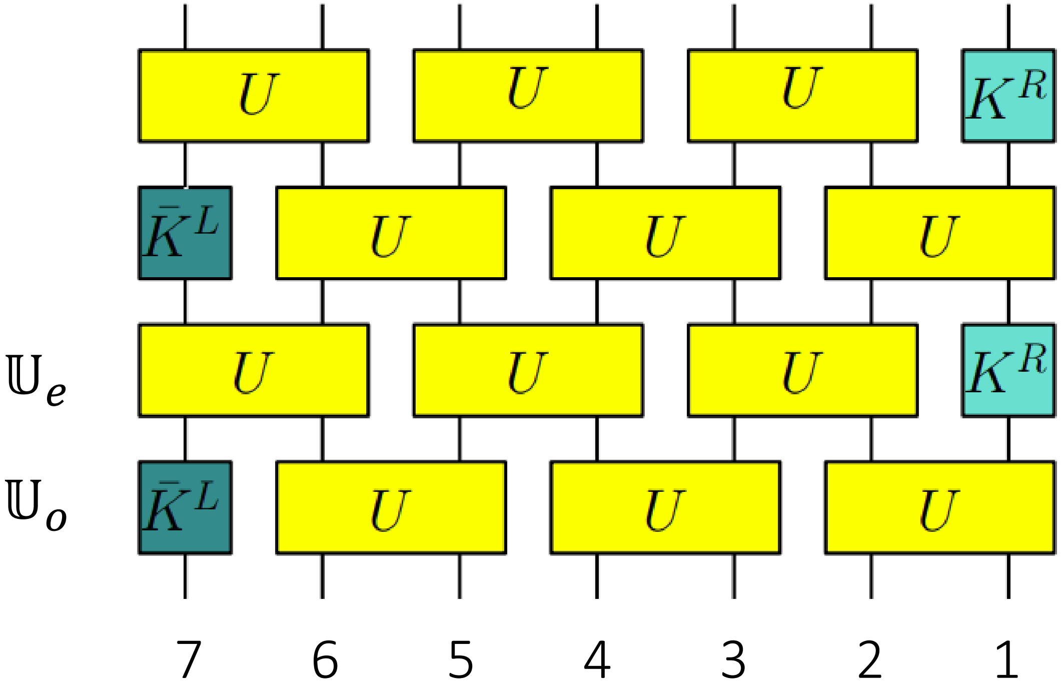
$$M = \mathbb{U}_e \mathbb{U}_o$$

$$\mathbb{U}_e = K_1^R U_{23}\, U_{45}\, U_{67}$$

$$\mathbb{U}_o = U_{12}\, U_{34}\, U_{56} \bar{K}_7^L$$

$$\mathrm{U} = \check{R} = P\ R$$

The dynamical evolution generator M should commute with infinitely many conserved charge!



$$M = \mathbb{U}_e \mathbb{U}_o = K_1^R U_{23}\, U_{45}\, U_{67} U_{12}\, U_{34}\, U_{56} \mathrm{K}_7^{\mathrm{L}}$$

M

$$\mathrm{U} = \check{R} = P\, R$$

$$t(u) = Tr_0 K_0^R(u) R_{01}(u-\theta_1) R_{02}(u-\theta_2) \ldots R_{0L}(u-\theta_L) K_0^L(u) R_{0L}(u-\theta_L) \ldots R_{01}(u-\theta_1)$$

Can we fix the values of the inhomogeneities $\theta_1, \ldots, \theta_L$ such that:

$[M, t(u)] = 0$, for some θ_i?

$$t(u) = Tr_0 K_0^R(u) R_{01}(u-\theta_1) R_{02}(u-\theta_2) \dots R_{0L}(u-\theta_L) \mathrm{K}_0^{\mathrm{L}}(\mathrm{u}) \mathrm{R}_{0\mathrm{L}}(\mathrm{u}-\theta_{\mathrm{L}}) \dots R_{01}(u-\theta_1)$$

Can we fix the values of the inhomogeneities $\theta_1, \dots, \theta_L$ such that:

$[M, t(u)] = 0$, for some θ_i?

YES! If

$$\theta_1 = \theta_3 = \theta_5 = \cdots = \theta_{2i+1} = -\kappa$$
$$\theta_2 = \theta_4 = \theta_6 = \cdots = \theta_{2i} = \kappa$$

We have to find a way to write M as a function of $t(u)$,

And since $[t(v), t(u)] = 0, \quad \forall\, u, v$

$M = t(\kappa)$ $\Longrightarrow$ The circuit is integrable!

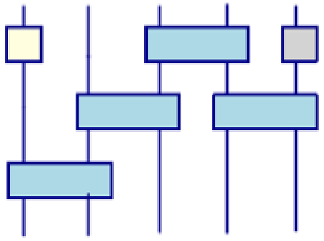
$$[M, t(u)] = 0$$

$t(u)$ generates the infinite tower of conserved charges commuting with M

Given a quantum circuit with a given geometry, can we understand if it is integrable?

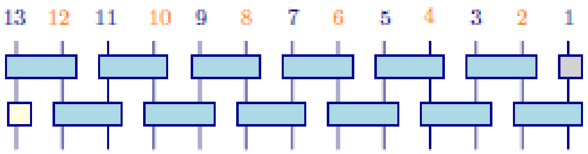
[2607.02093 M. G. Fernández, CP, A. L. Retore]

<u>For open boundary conditions:</u>

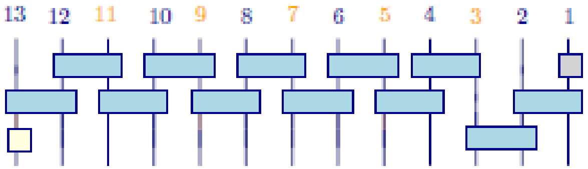
Step 1: Draw the circuit	**Step 3: Insert the expressions of the quantum gate U**	**Step 4: Does the gate satisfy the YBE?**	**Step 5: Is the quantum circuits integrable?**
 $M = U_{45}U_{12}U_{34}$ $K_1^R U_{23} K_5^L$	$U = \begin{pmatrix} 1.76041 & -0.991837 & 0.422449 & -0.55102 \\ 0.16898 & 0.779592 & 0.323878 & -0.422449 \\ -0.396735 & 1.78531 & 0.779592 & 0.991837 \\ -0.0881633 & 0.396735 & -0.16898 & 1.76041 \end{pmatrix}$ $K^R = \begin{pmatrix} 0.142762 & 0.21696 \\ 0.168167 & 0.964382 \end{pmatrix}$ $K^L = \begin{pmatrix} 0.405221 & 0.032988 \\ 0.576967 & 0.405913 \end{pmatrix}$	Does U satisfy the YBE? Yes! (or NO) Does U and K satisfy the YBE? Yes! (or NO)	Yes! (or NO) The transfer matrix is: …

Classification of nearest neighbour **integrable** circuits with open boundary conditions

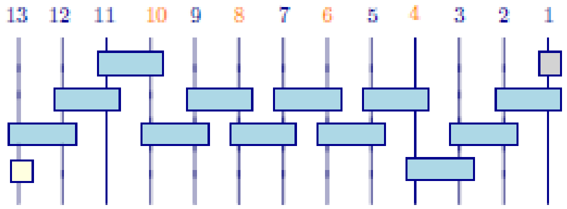
$\vec{n}$ = position of the inhomogeneities $-\kappa$



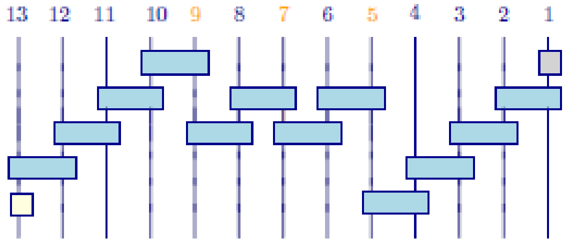
(a) $\kappa_- = 6$, depth $= 2$ and $\vec{n} = (12, 10, 8, 6, 4, 2)$



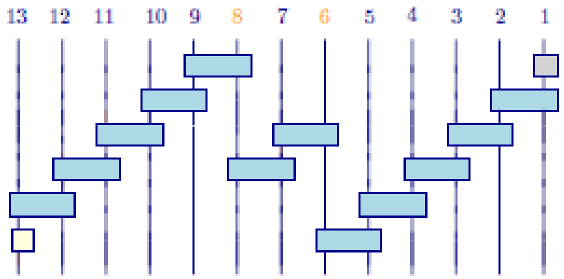
(b) $\kappa_- = 5$, depth $= 3$ and $\vec{n} = (11, 9, 7, 5, 3)$



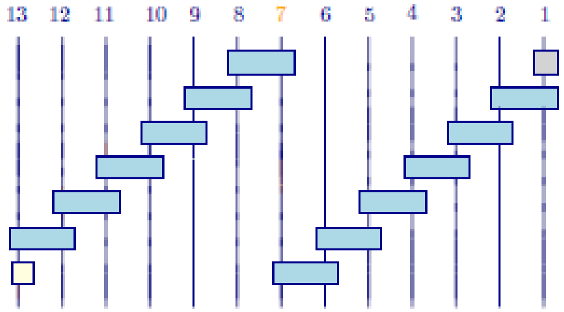
(c) $\kappa_- = 4$, depth $= 4$ and $\vec{n} = (10, 8, 6, 4)$



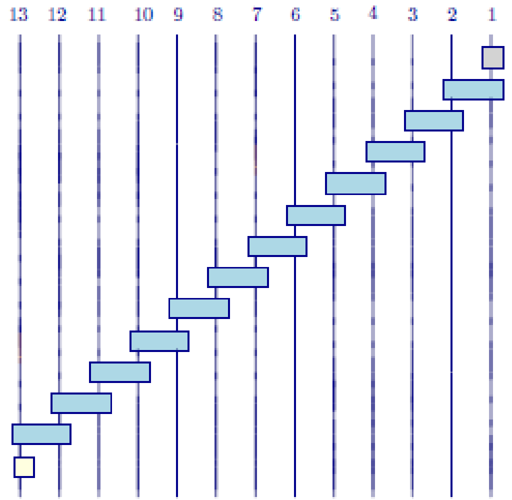
(d) $\kappa_- = 3$, depth $= 5$ and $\vec{n} = (9, 7, 5)$



(e) $\kappa_- = 2$, depth $= 6$ and $\vec{n} = (8, 6)$



(f) $\kappa_- = 1$, depth $= 7$ and $\vec{n} = (7)$



(g) $\kappa_- = 0$, depth $= 14$ and $\vec{n} = (\emptyset)$

All quantum unitary circuits in which the gate (solution of the YBE) is applied to each pair of neighbouring qubits exactly once per period are integrable and the boundaries are solutions of the bYBE.

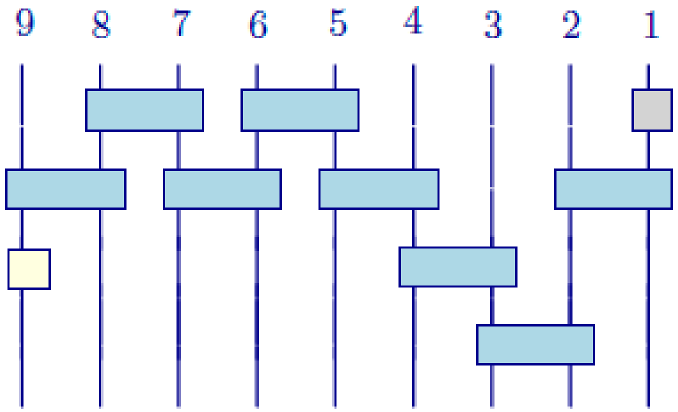
[2607.02093 M. G. Fernández, CP, A.L. Retore]

How to quickly go from the position of the inhomogeneities to the circuit?

[M. G. Fernández, CP, A.L. Retore]
Zenodo repository 20763606

```
ClearAll[listϰm, listρ];
listϰm = {3, 4, 5, 7};
listρ = {};
listγ = {};
M[9]
```

$$U_{7,8} \otimes U_{5,6} \otimes U_{4,5} \otimes U_{3,4} \otimes KR_1[\kappa] \otimes U_{1,2} \otimes U_{2,3} \otimes U_{6,7} \otimes U_{8,9} \otimes KL_9[\kappa]$$



Detect Yang-Baxter integrable quantum circuits

Integrability \ Boundary Condition	Periodic	Open
Yang-Baxter type	✓	✓
Beyond Yang-Baxter type		

Up to now, we have worked with the R-matrix of the gate (solution of the Yang-Baxter equation).

What to do if U does not solve the YB equation?

Is there some hope that the quantum circuit is still Yang-Baxter integrable?

YES!

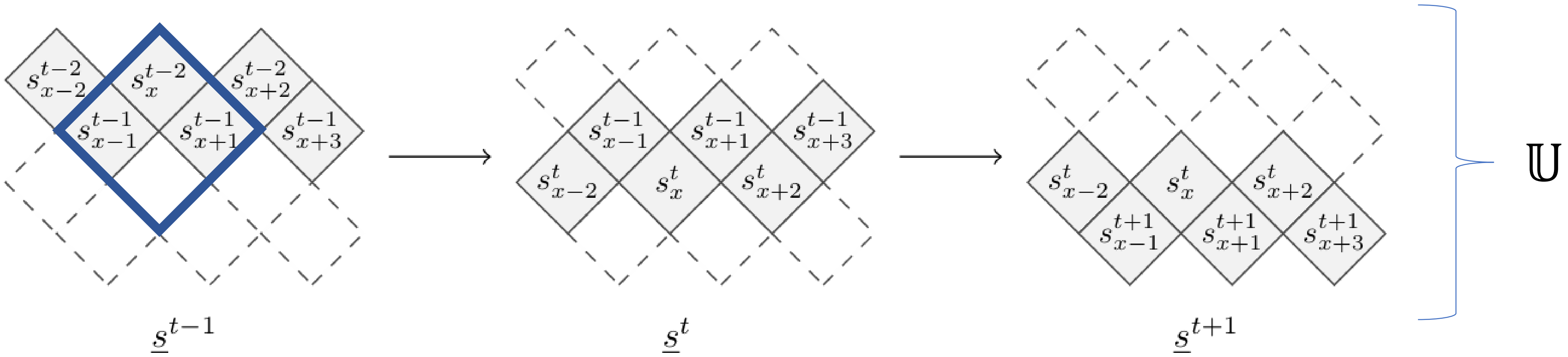
Example: Deformed Rule 54 model «is» Yang-Baxter integrable!

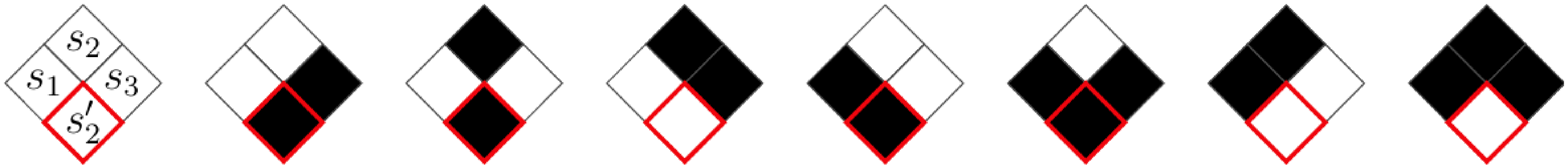
2603.25424 CP, T. Prosen

Model: Cellular automata – Rule 54

Simplest deterministic dynamics

[B. Buča, K. Klobas, T. Prosen 2021]

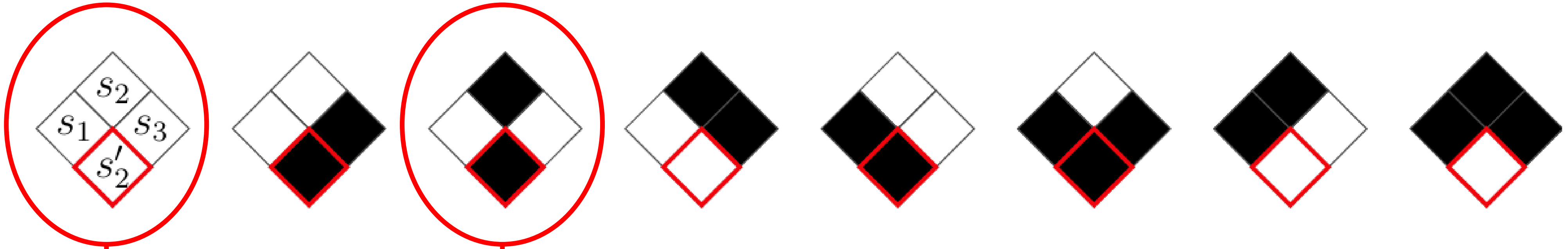




Model: Cellular automata – Rule 54

+ Quantum deformation

[New model introduced by T. Prosen '21]



$|000\rangle \to \alpha\,|000\rangle + \beta|010\rangle$

$|010\rangle \to \delta|000\rangle + \gamma|010\rangle$

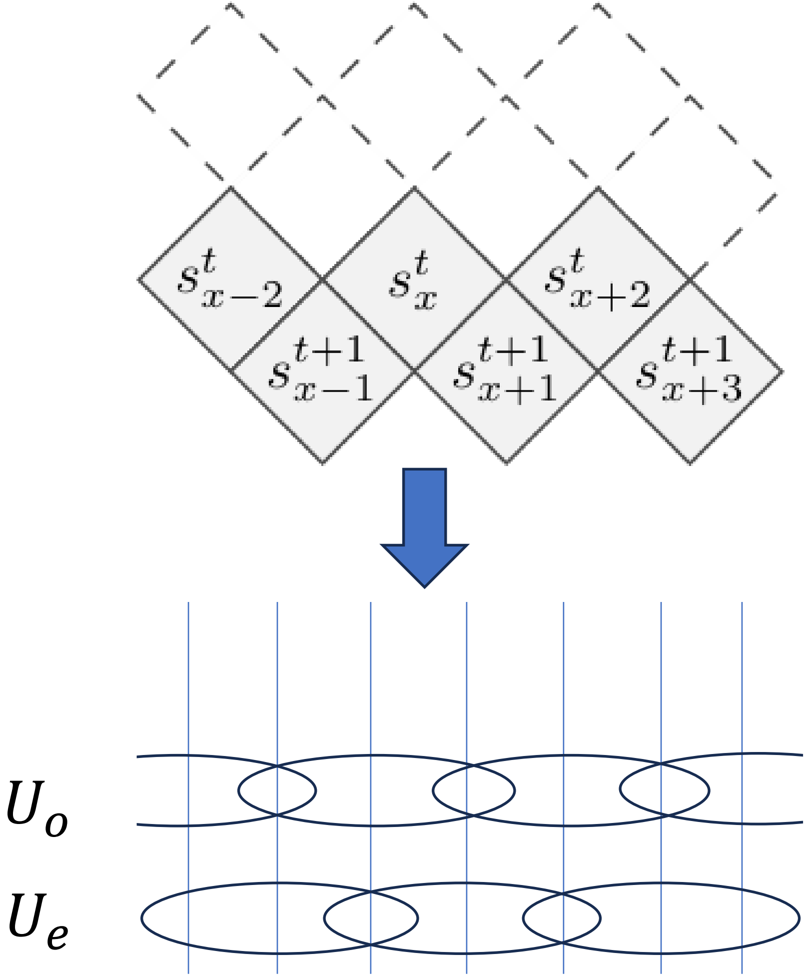
By naively counting the charges and using numerical techniques, it seems that the deformed Rule 54 is integrable.

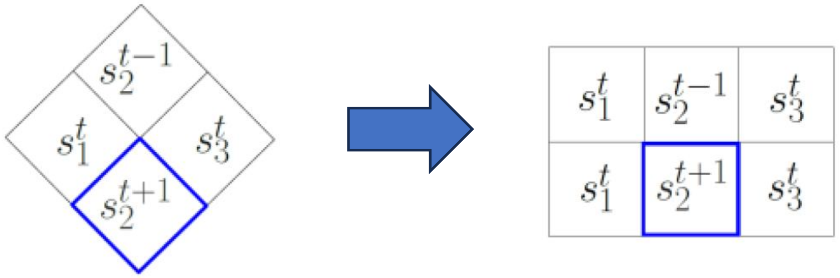
$$U = \begin{pmatrix} \alpha & 0 & \beta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \gamma & 0 & \delta & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

How to modify our algorithm?

Rule 54 (and the deformed version) as a quantum circuit

[undeformed: T. Gombor, B. Pozsgay 2021]





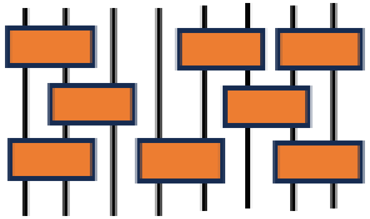
$$U = \begin{pmatrix} \alpha & 0 & \beta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \gamma & 0 & \delta & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$U_e = U_{L-1,L,1} \dots U_{3,4,5} U_{1,2,3}$$
$$U_o = U_{L,1,2} \dots U_{4,5,6} U_{2,3,4}.$$

Dynamical evolution described by: $U_{i,i+1,i+2}$

Can we use the algorithm for U nearest-neighbor gate modified to next to NN?

[medium range integrability T. Gombor, B. Pozsgay]
[de Leeuw, Retore]

Step 1: Draw the circuit	Step 2: Insert D	Step 3: Insert the expression of the quantum gate U	Step 4: Does the gate satisfy the YBE?	Step 5: Is the quantum circuits integrable?
	$D = 2$	$U = \frac{1}{\sqrt{1-u^2}} \begin{pmatrix} 1+u & 0 & 0 & 0 \\ 0 & 1 & u & 0 \\ 0 & u & 1 & 0 \\ 0 & 0 & 0 & 1+u \end{pmatrix}$	YES	YES! (or NO) $M = U_{12}U_{56}U_{78}U_{23}U_{67}U_{12}U_{45}U_{78}$ The transfer matrix is: …

Does the gate satisfy the YBE?

Does U solve the RLL relation?

$$R_{A,B}(u,v)\mathcal{L}_{A,j}(u)\,\mathcal{L}_{B,j}(v) = \mathcal{L}_{B,j}(v)\mathcal{L}_{A,j}(u)R_{A,B}(u,v)$$

No!

$$R_{ab,cd}(u,v)\mathcal{L}_{ab,j}(u)\,\mathcal{L}_{cd,j}(v) = \mathcal{L}_{cd,j}(v)\mathcal{L}_{ab,j}(u)R_{ab,cd}(u,v)$$

Is there some hope that deformed Rule 54 is still Yang-Baxter integrable?

YES!

How to modify our algorithm?

We can use a numerical algorithm to detect the range of the lowest charge:

$$[\mathbb{U}, Q_i] = 0\,, \qquad Q_i = \sum_{j=1} q_{[j,j+i-1]}$$

[medium range integrability T. Gombor, B. Pozsgay]

[de Leeuw, Retore]

We obtain that, for deformed Rule 54: $i = 6$

Open question:
Why?

Is deformed Rule 54 YB integrable?

$[\mathbb{U}, Q_6] = 0\,,$

Does Q_6 belong to a tower of conserved charges of an integrable model?

How to modify our algorithm?

Analytical algorithm:

We use the boost operator approach:

$$\partial_u \ln t(u)\,|_{u\to 0} = Q_6 \ ,$$

$$[Q_6, B[Q_6]] = 0 \quad \Longrightarrow \quad [Q_i, Q_j] = 0$$

Reshetikhin condition implies integrability [Hokkyo, '25]

This gives a hint that the model may be Yang-Baxter integrable:

$$[Q_i, Q_j] = 0$$

$$[\mathbb{U}, Q_i] = 0$$

Proof

For a choice of deformation parameters:

$$\alpha = \frac{1}{7}, \beta = \frac{1}{2}, \gamma = \frac{1}{8}, \delta = \frac{3}{11}$$

By using the boost operator we obtain the analytical expression of the first three conserved charges

$$q^{(10)} = [q_i, q_{i+1} + q_{i+2}] + w_i$$

$$q^{(14)} = \left[q_{i+4} + q_{i+3} + \frac{1}{2} q_{i+2}, [q_i + q_{i+1}, q_{i+2}]\right] - [q_{i+4}, w_{i+2} + w_{i+3}] + \frac{1}{2}[q_{i+2} + q_{i+3}, w_{i+4}] + y_i$$

We can check:

$$[Q_6, \mathbb{U}] = [Q_{10}, \mathbb{U}] = [Q_{14}, \mathbb{U}] = 0$$

$$[Q_6, Q_{10}] = [Q_{10}, Q_{14}] = [Q_6, Q_{14}] = 0$$

$$\check{\mathcal{L}}(u) = 1 + u\, q_i + \frac{u^2}{2}(w_i + q_i^2) + \frac{u^3}{3!}\left(q_i^3 + 3q_i(-q_i^2 + w_i) + y_i\right) + O\,(u^3)$$

We obtain the expression of the Lax operator in perturbation theory and we resum it!

Proof

For a choice of deformation parameters:

$$\alpha = \frac{1}{7}, \beta = \frac{1}{2}, \gamma = \frac{1}{8}, \delta = \frac{3}{11}$$

We obtain a Lax operator such that:

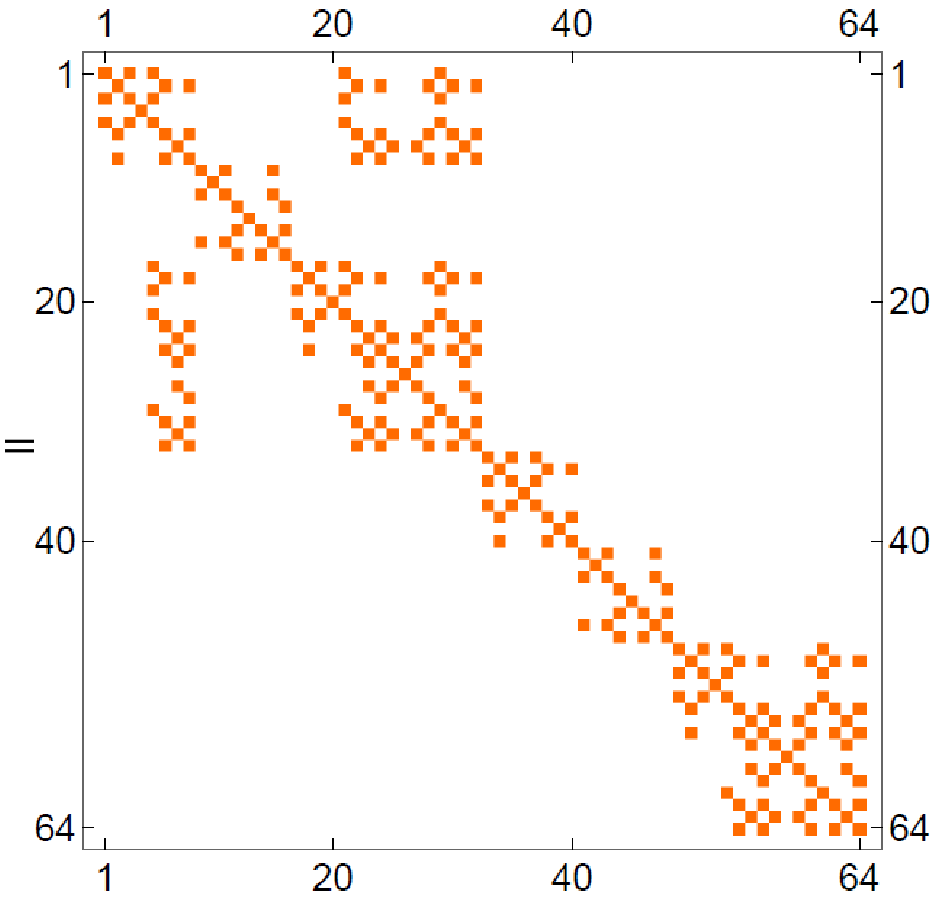
$$t(u) = \mathrm{Tr}_A \mathcal{L}_{a,b,L}(u)\, \mathcal{L}_{a,b,L-1}(u) \ldots \mathcal{L}_{a,b,1}(u)$$

$$[t(u), t(v)] = 0 \qquad Q_6 = \partial_u \log t(u)$$

Derivation is technical (and long) Feel free to ask if you are interested!

The Lax operator is:

$$\mathcal{L}_{a,b,1}(u) =$$



Some of the entries:

$$f_i(u) = \frac{P_4(u)}{P_2(u)},$$

$$f_{120}(u) = \frac{W_4(u)\left(W_2(u) + \sqrt{G_4(u)}\right)}{F_4(u)},$$

Full Expression on Zenodo.19170399

2603.25424 CP, T. Prosen

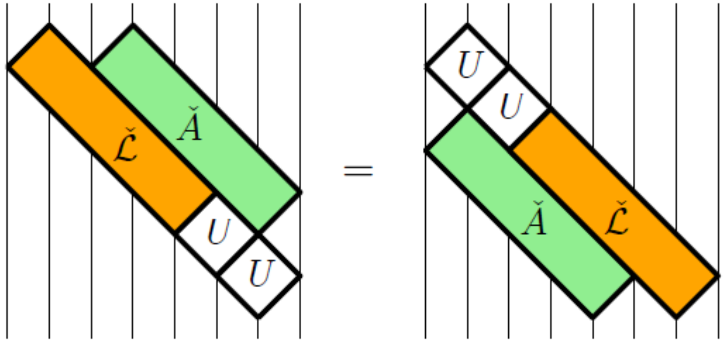
Proof

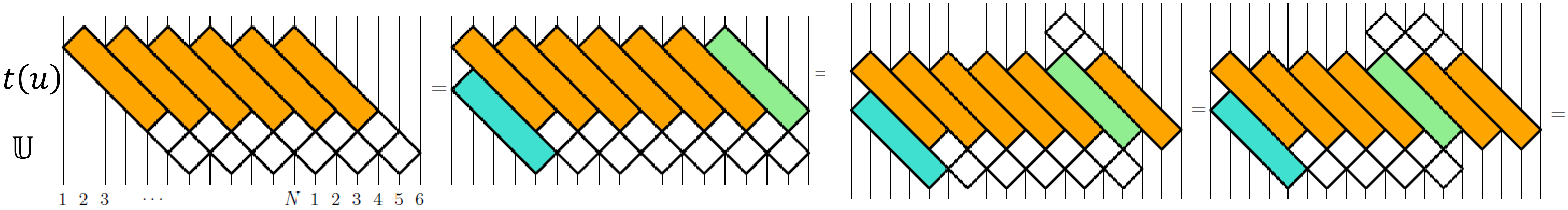
For a choice of deformation parameters:

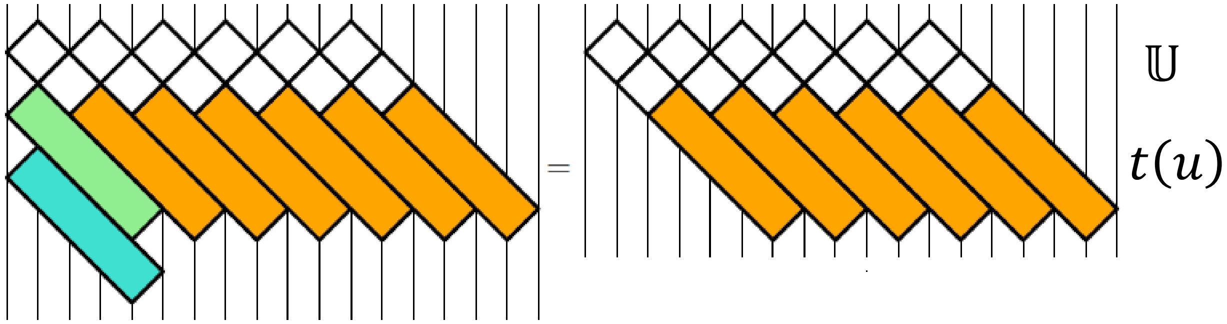
$$\alpha = \frac{1}{7}, \beta = \frac{1}{2}, \gamma = \frac{1}{8}, \delta = \frac{3}{11}$$

Proof that the charges are conserved

$$[t(u), \mathbb{U}] = 0 \Rightarrow [Q_i, \mathbb{U}] = 0$$



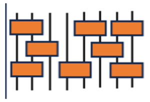


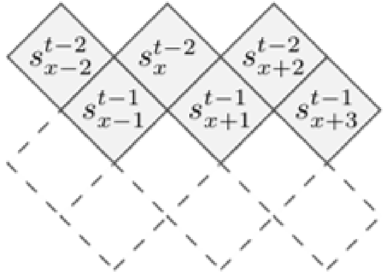


$$[\mathbb{U}, Q_i] = 0$$ ✓

What to do if the gate does not satisfy the YBE?

For periodic boundary conditions:

Step 1: Draw the circuit	Step 3: Insert the expressions of the quantum gate U	Step 4: Does the gate satisfy the YBE?	Step 5: Is the quantum circuits integrable?
 $M = U_{12}U_{45}U_{78}$ $U_{23}U_{67}U_{12}U_{34}U_{78}$	$U = \begin{pmatrix} 1.76041 & -0.991837 & 0.422449 & -0.55102 \\ 0.16898 & 0.779592 & 0.323878 & -0.422449 \\ -0.396735 & 1.78531 & 0.779592 & 0.991837 \\ -0.0881633 & 0.396735 & -0.16898 & 1.76041 \end{pmatrix}$	YES	Yes! (or NO) The transfer matrix is: ...

Step 1: Draw the circuit	Step 3: Insert the expressions of the quantum gate U	Step 4: Does the gate satisfy the YBE?	Step 5: Check the conserved charges
	$U = \begin{pmatrix} \alpha & 0 & \beta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \gamma & 0 & \delta & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$	NO	Does the first conserved charges belong to a tower of conserved charges? Yes → Lax operator

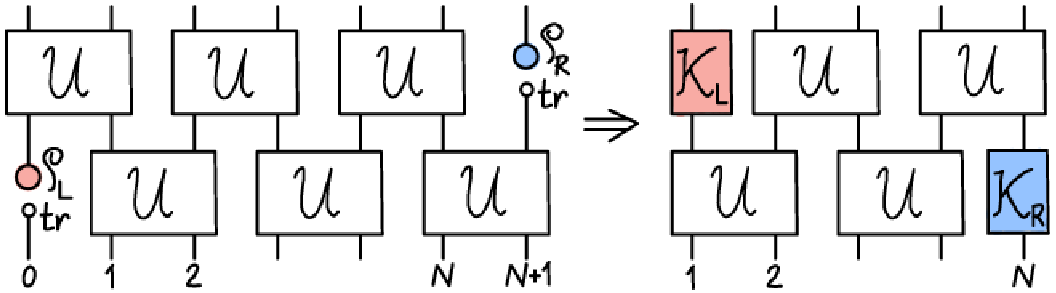
Detect Yang-Baxter integrable quantum circuits

Integrability \ Boundary Condition	Periodic	Open
Yang-Baxter type	✓	✓
Beyond Yang-Baxter type	✓	

Last bit to complete the story: What to do for the open boundary case beyond Yang Baxter?

[Popkov, Presilla, Prosen, Zhang, '25, '26]

Example



U=XXX, XXZ, XYZ gates in the folded pictures

They satisfy the Yang-Baxter equation

K – reset gate

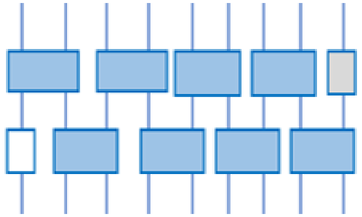
Does not satisfy the Yang-Baxter equation

Numerical evidence of integrability for U = XXX, XXZ, XYZ and K reset gate!

Analytics?!

In progress with R. Frassek, V. Popkov, T. Prosen

Last bit to complete the story: What to do for the open boundary case beyond Yang Baxter?

Step 1: Draw the circuit	Step 3: Insert the expressions of the quantum gate U	Step 4: Does the gate satisfy the YBE?	Step 5: Is the quantum circuits integrable?
 $M = U_{89}U_{67}U_{45}\,U_{23}$ $K_1^R U_{78}U_{56}U_{34}U_{12}K_9^L$	$U = \begin{pmatrix} 1.76041 & -0.991837 & 0.422449 & -0.55102 \\ 0.16898 & 0.779592 & 0.323878 & -0.422449 \\ -0.396735 & 1.78531 & 0.779592 & 0.991837 \\ -0.0881633 & 0.396735 & -0.16898 & 1.76041 \end{pmatrix}$ $K^R = \begin{pmatrix} 0.142762 & 0.21696 \\ 0.168167 & 0.964382 \end{pmatrix}$ $K^L = \begin{pmatrix} 0.405221 & 0.032988 \\ 0.576967 & 0.405913 \end{pmatrix}$	Does U satisfy the YBE? Yes! Does U and K satisfy the YBE? No	

We think that one should construct a transfer matrix $t(u) = \mathrm{Tr}_A \ldots$

Infinite-dimensional auxiliary space

[Popkov, Presilla, Prosen, Zhang, ‘25, ’26]

In progress with R. Frassek, V. Popkov, T. Prosen

Detect Yang-Baxter integrable quantum circuits

Missing bit

Integrability \ Boundary Condition	Periodic	Open
Yang-Baxter type	✓	✓
Beyond Yang-Baxter type	✓	

[K. K. Kozlowski, E. Sklyanin and A. Torrielli, 1607.07685]

(orthogonal) Setting: Can the boost strategy work beyond quantum lattices?

The quantum KP equation

Some of the conserved charges

$$H_0 = \sum_{m \in \mathbb{N}} m \int_{-\infty}^{+\infty} dx\, \Psi_m^\dagger(x) \Psi_m(x)$$

Total mass

$$H_1 = -i \sum_{m \in \mathbb{N}} \int_{-\infty}^{+\infty} dx\, \Psi_m^\dagger(x) \partial_x \Psi_m(x)$$

Generator of translation in x

For example the mass M=2 sector

- 2 particles of mass 1
- 1 particle of mass 2

Hamiltonian

$$H_2 = -\sum_{m \geq 1} \frac{1}{m} \int dx\, \Psi_m^\dagger(x) \partial_x^2 \Psi_m(x) + \sum_{m \geq 1} \gamma_m \int dx\, \Psi_m^\dagger(x) \Psi_m(x)$$
$$+ \sum_{m_1, m_2 \geq 1} \beta_{m_1, m_2} \int dx\, \left(\Psi_M^\dagger(x) \Psi_{m_1}(x) \Psi_{m_2}(x) + \Psi_{m_1}^\dagger(x) \Psi_{m_2}^\dagger(x) \Psi_M(x) \right)$$

$$[\Psi_m(x), \Psi_n^\dagger(y)] = \delta_{mn} \delta(x - y)$$

[CP, A. Torrielli in progress]

(orthogonal) Setting

The quantum KP equation

Hamiltonian

$$H_2 = -\sum_{m\geq 1}\frac{1}{m}\int dx\,\Psi_m^\dagger(x)\partial_x^2\Psi_m(x) + \sum_{m\geq 1}\gamma_m\int dx\,\Psi_m^\dagger(x)\Psi_m(x) + \sum_{m_1,m_2\geq 1}\beta_{m_1,m_2}\int dx\,\left(\Psi_M^\dagger(x)\Psi_{m_1}(x)\Psi_{m_2}(x) + \Psi_{m_1}^\dagger(x)\Psi_{m_2}^\dagger(x)\Psi_M(x)\right)$$

Next conserved charge

$$\begin{aligned}H_3 = i\sum_{m\geq 1}\frac{1}{m^2}\int_{-\infty}^{+\infty}dx\,\Psi_m^\dagger(x)\,\partial_x^3\Psi_m(x) \\ + i\sum_{m_1,m_2\geq 1}\beta_{m_1m_2}\int_{-\infty}^{+\infty}dx\,\Bigg[&\frac{1}{m_1+m_2}(\partial_x\Psi_{m_1+m_2}^\dagger)\Psi_{m_1}\Psi_{m_2} \\ &-\frac{2}{m_1}\Psi_{m_1+m_2}^\dagger(\partial_x\Psi_{m_1})\Psi_{m_2} - \frac{2}{m_2}\Psi_{m_1+m_2}^\dagger\Psi_{m_1}(\partial_x\Psi_{m_2}) \\ &-\frac{1}{m_1+m_2}\Psi_{m_1}^\dagger\Psi_{m_2}^\dagger(\partial_x\Psi_{m_1+m_2}) \\ &+\frac{2}{m_1}(\partial_x\Psi_{m_1}^\dagger)\Psi_{m_2}^\dagger\Psi_{m_1+m_2} + \frac{2}{m_2}\Psi_{m_1}^\dagger(\partial_x\Psi_{m_2}^\dagger)\Psi_{m_1+m_2}\Bigg] \\ -3i\gamma\sum_{m\geq 1}m^2\int_{-\infty}^{+\infty}dx\,\Psi_m^\dagger(x)\,\partial_x\Psi_m(x).\end{aligned}$$

(orthogonal) Setting

The quantum KP equation

[CP, A. Torrielli in progress]

We proved that $$[H_2, H_3] = 0$$

Open question:

Does there exists a quantum boost operator such that $H_3 = B[H_2]$ that we can use to detect quantum integrability starting only from the charge H_2?

Conclusions

Efficient algorithm to determine YB integrability in quantum circuits

Integrability \ Boundary Condition	Periodic	Open
Yang-Baxter type	Using transfer matrix inhomogeneities	Using transfer matrix inhomogeneities
Beyond Yang-Baxter type	Using Boost operator approach	

Open Questions:

Circuits

Open boundary conditions

- Gate does not solve the YB equation

Broader

Can we construct a quantum boost operator also for quantum field theory (for example KP model) and use it as an efficient algorithm to detect integrability?

Conclusions

Efficient algorithm to determine YB integrability in quantum circuits

Integrability \ Boundary Condition	Periodic	Open
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Open Questions:

Circuits

Open boundary conditions

- Gate does not solve the YB equation

Broader

Can we construct a quantum boost operator also for quantum field theory (for example KP model) and use it as an efficient algorithm to detect integrability?

Thanks!