Integrable chiral circuits and spin ladders:

a playground for space-time symmetries in charge transport

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Outline

Based on

Superdiffusion and anomalous fluctuations in chiral integrable dynamics 2026
C Muzzi, DS Bhakuni, M Dalmonte, L Zadnik, HB Xavier
Physical Review B 113 (18), 184305

Quantum many-body spin ratchets 2024
L Zadnik, M Ljubotina, Ž Krajnik, E Ilievski, T Prosen
PRX quantum 5 (3), 030356

A. transport universality

B. the role of spacetime symmetries

C. transport properties of (some) chiral models

D. origins of fluctuation symmetries and the “KPZ / not KPZ” question

Charge transport

1. conserved charge (1D example)

$$Q = \int \mathrm{d}x q(x) \qquad Q(t) = Q$$

2. structure factor (**two-point** correlations only)

$$S(x,t) = \langle q(x,t) q(0,0) \rangle$$

moments determine the scaling function

$$m_n(t) = \int \mathrm{d}x\, x^n S(x,t)$$

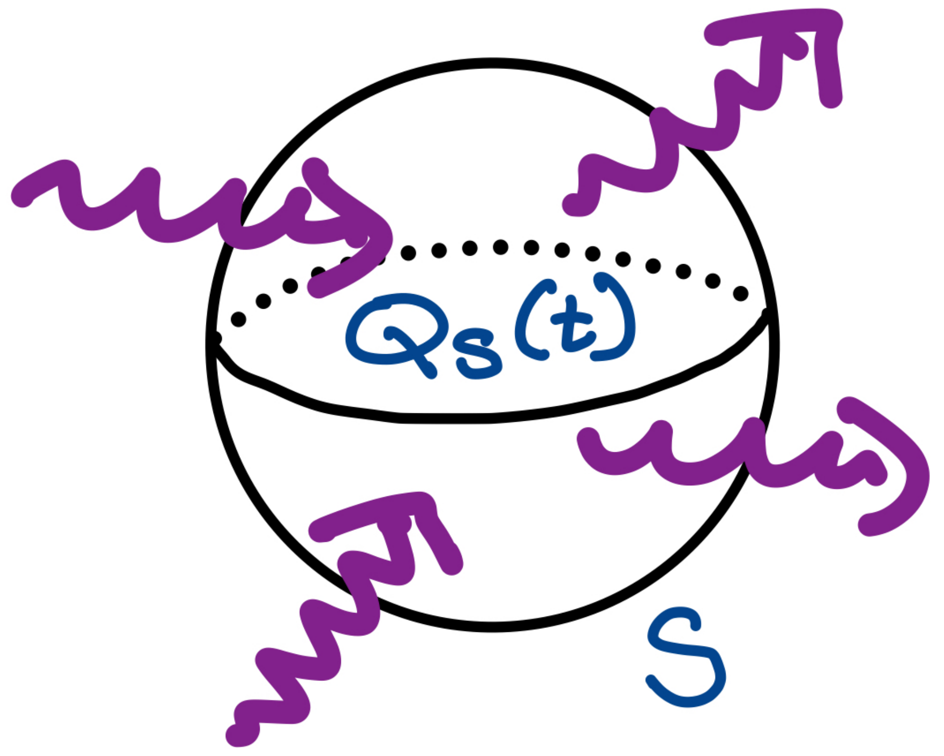
3. higher-order correlations: **net charge transfer**

$$\mu_n(t) := \langle [\Delta Q_S(t)]^n \rangle \qquad \Delta Q_S(t) = Q_S(t) - Q_S(0)$$

II. LINEAR-RESPONSE THEORY

In most studies of transport in interacting 1d lattice quantum systems, the linear response is the dominant approach. In the context of this review, one reason is that much of the focus has been on ballistic transport in integrable models which can be characterized by the so-called Drude weight, naturally appearing in linear response theory. One appealing aspect of linear-response theory is that correlation functions, in terms of which transport coefficients are expressed, and specifically their Fourier transformations (i.e., spectral functions) are readily accessible in various scattering experiments.

// Bertini et al., Rev. Mod. Phys. 93, 25003 (2021)



Charge transport

Net charge transfer from region $\mathcal{S}$ is related to accumulated current through the boundary $\partial\mathcal{S}$

$$\Delta Q_{\mathcal{S}}(t) = Q_{\mathcal{S}}(t) - Q_{\mathcal{S}}(0) = \boxed{\int_0^t \mathrm{d}\tau j(x=0,\tau) =: \mathfrak{J}_t} \quad \text{for } \mathcal{S} = (-\infty, 0) \text{ in 1D}$$

The “higher-order correlations” mean cumulants that determine the distribution $\mathbb{P}(\mathfrak{J}|t)$ of values of $\mathfrak{J}_t$.

For rare events (e.g., ballistic scale current), $\mathbb{P}(\mathfrak{J}|t)$ is expected to obey the **large-deviation principle**

$$\boxed{\mathbb{P}(\mathfrak{J} = \mathfrak{j}t|t) \asymp e^{-tI(\mathfrak{j})}}$$

Gallavotti-Cohen (fluctuation) symmetry: in the absence of a drive the fluctuations in both directions are equally probable $I(\mathfrak{j}) = I(-\mathfrak{j})$. In terms of cumulants / cumulant-generating function:

$$c_n^{(\mathrm{SC})} \equiv \lim_{t\to\infty} \frac{1}{t}\langle \mathfrak{J}_t^n \rangle^c \qquad F(\zeta) = \lim_{t\to\infty} \frac{1}{t} \log\langle e^{\zeta \mathfrak{J}_t}\rangle \longrightarrow \boxed{F(\zeta) = F(-\zeta)}$$

What is transport universality?

1. ChatGPT:

> What does it meant that some phenomenon is in the KPZ universality class?

When we say a **phenomenon belongs to the KPZ universality class**, we mean that its *large-scale, long-time fluctuations* behave like those predicted by the **Kardar–Parisi–Zhang (KPZ) equation**, regardless of the microscopic details of the system.

2. charge transport: **all** fluctuations (= multi-point correlation functions)

Dynamical exponent is not universality

1. isotropic Heisenberg magnet (integrable & non-Abelian rotation symmetry)

Kardar-Parisi-Zhang Physics in the Quantum Heisenberg Magnet

Marko Ljubotina, Marko Žnidarič, and Tomaž Prosen
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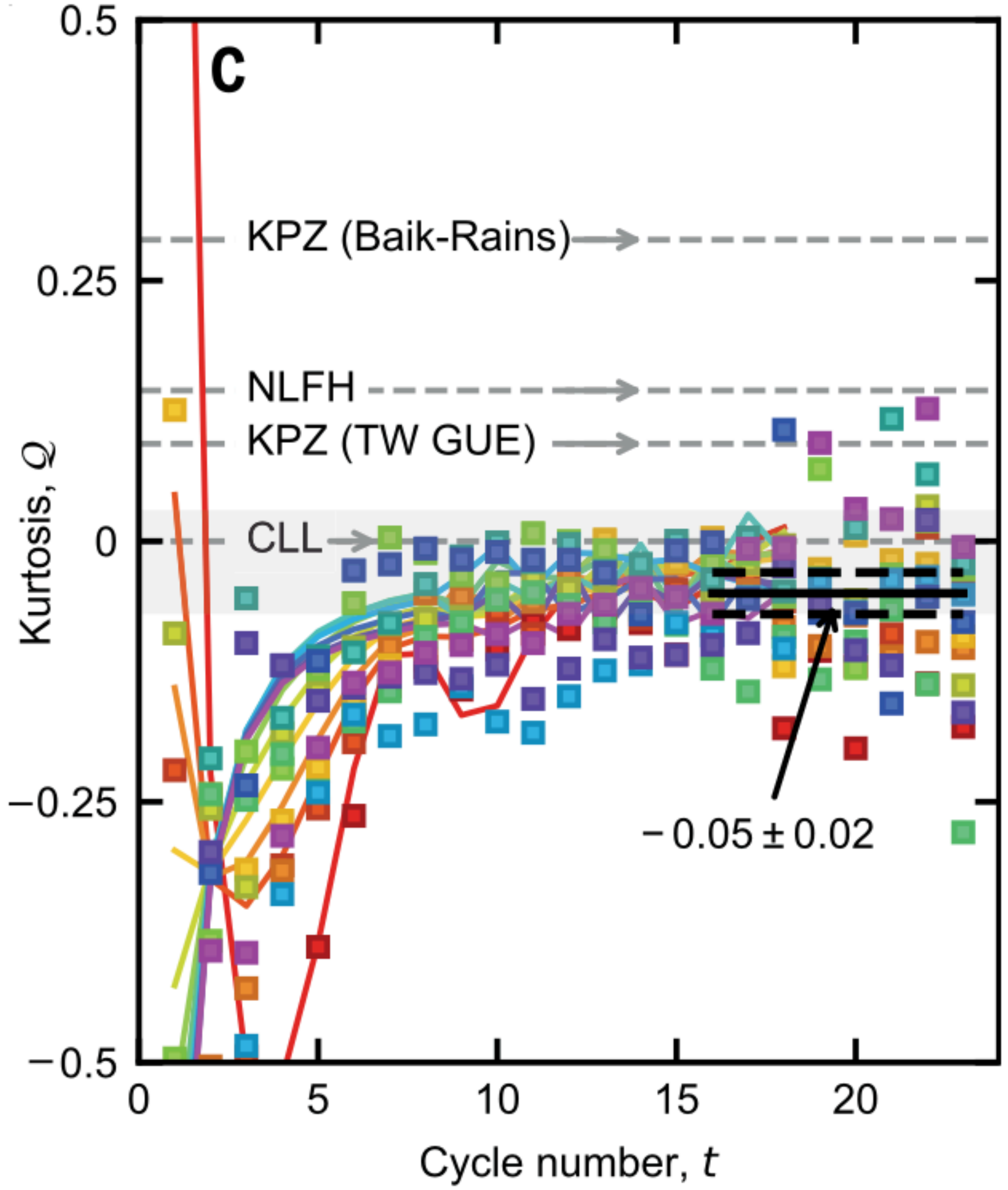
(Received 7 March 2019; published 31 May 2019)

Equilibrium spatiotemporal correlation functions are central to understanding weak nonequilibrium physics. In certain local one-dimensional classical systems with three conservation laws they show universal features. Namely, fluctuations around ballistically propagating sound modes can be described by the celebrated Kardar-Parisi-Zhang (KPZ) universality class. Can such a universality class be found also in quantum systems? By unambiguously demonstrating that the KPZ scaling function describes magnetization dynamics in the $SU(2)$ symmetric Heisenberg spin chain we show, for the first time, that this is so.

Discussion.—We have shown that the infinite temperature spin-spin correlation function in the isotropic Heisenberg spin-$\frac{1}{2}$ model obeys the Kardar-Parisi-Zhang scaling. This is the first such observation in a deterministic quantum model. We stress that in order to reliably show the KPZ physics one has to look at the full distribution function of fluctuations and not, e.g., just the dynamical scaling exponent being $z = \frac{3}{2}$. For instance, a related spreading exponent $\frac{1}{3}$ generically appears in free or dilute models; see, e.g., Refs. [32,33].

Dynamical exponent is not universality

1. isotropic Heisenberg magnet: integrability + SU(2)



Properties of net charge transfer statistics, e.g., kurtosis

$$\gamma_4(t) = \frac{\mu_4(t)}{\mu_2^2(t)} - 3$$

kurtosis inconsistent with KPZ

// E. Rosenberg et al., Science 384, 48 (2024)

The role of space-time symmetries

1. Can lack of time-reversal stabilize KPZ universality class?

However, integrable spin chains have time-reversal and parity symmetries that are absent from the KPZ (Kardar-Parisi-Zhang) or stochastic Burgers equation, which force higher-order spin fluctuations to deviate from standard KPZ predictions.

// De Nardis, Gopalakrishnan, Vasseur, PRL 131, 197102 (2023)

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// De Nardis, Gopalakrishnan, Vasseur, PRL 131, 197102 (2023)

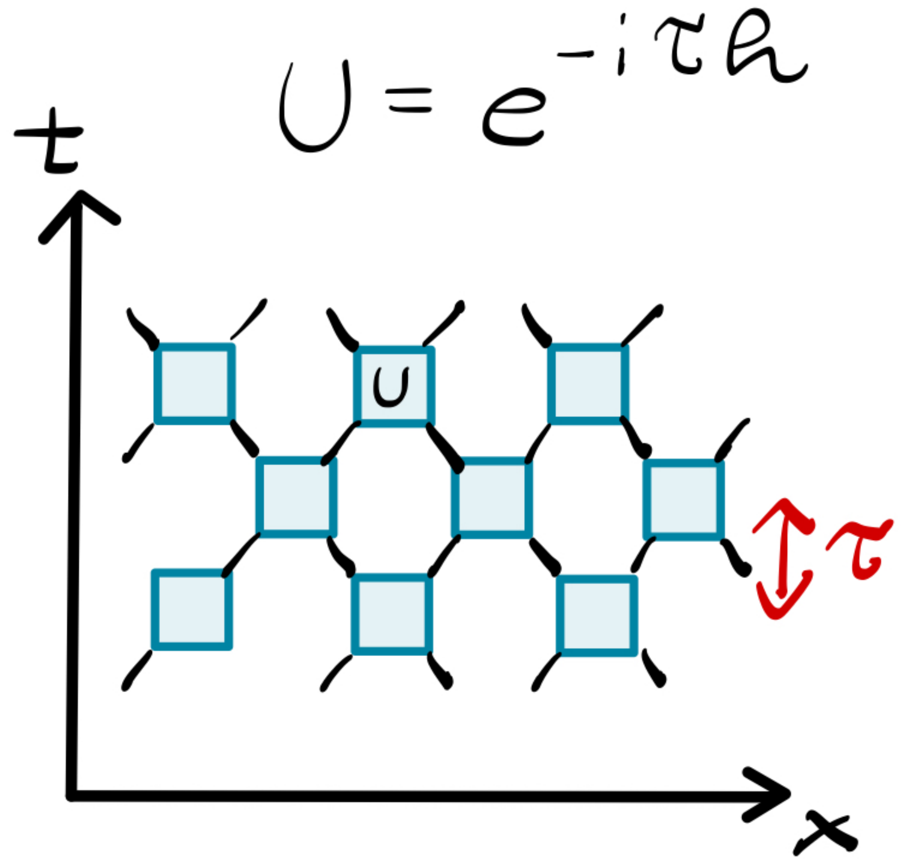
2. Is fluctuation symmetry in integrable models always due to time-reversal symmetry?

> We will show, using the BMFT, that the GCFT holds solely as a consequence of a *time-reversal symmetry of the Euler hydrodynamics*.
> In particular, by combining with generalised hydrodynamics, this provides, as far as we are aware, the first proof of the GCFT in all integrable many-body systems.

// Doyon et al., SciPost 15, 136 (2023)

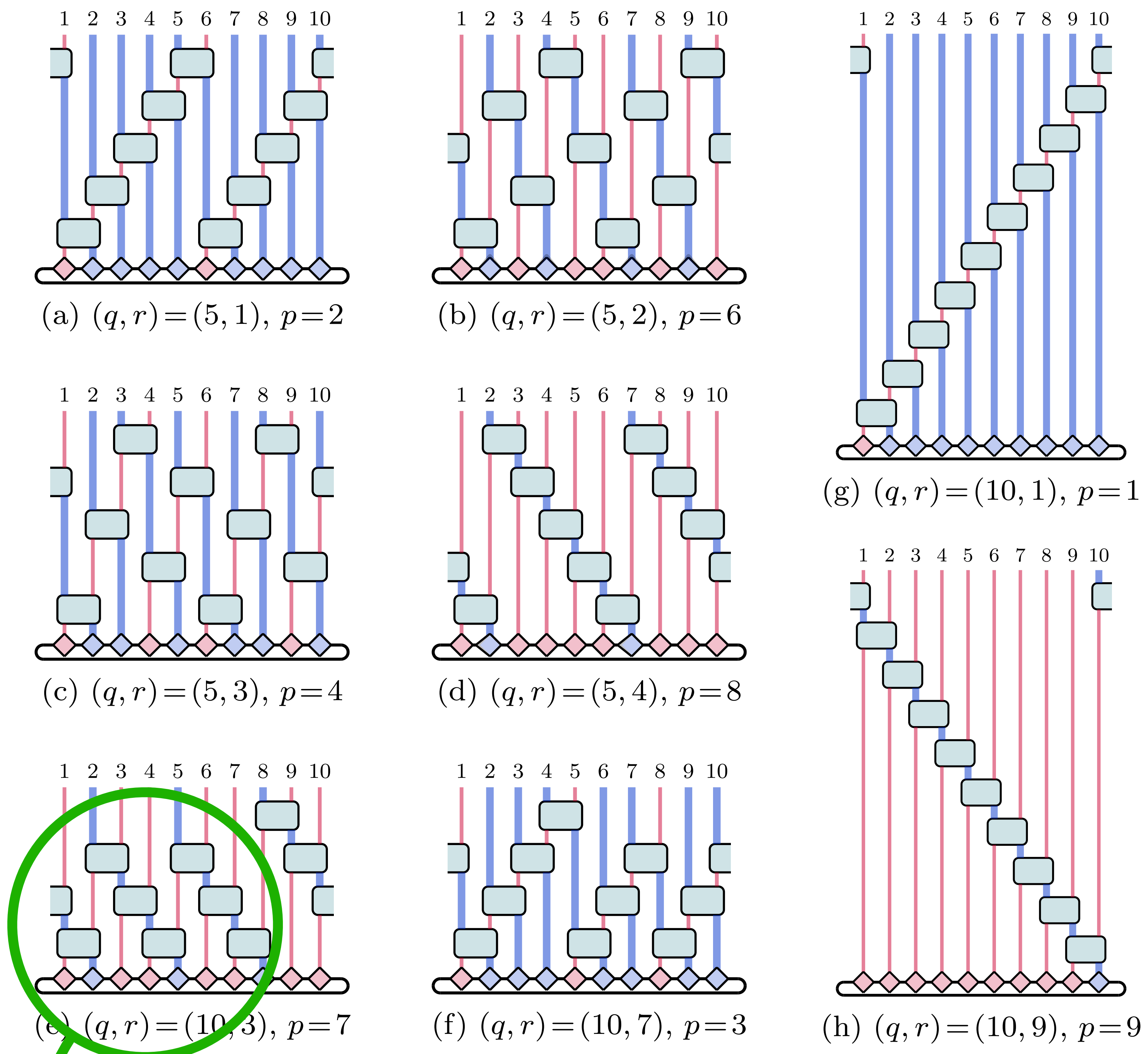
Models with (some sort of) chirality

1. integrable circuits with asymmetric geometry or asymmetric gates



Premise: U(1)-conserving Yang-Baxter unitary gate

// Vanicat, LZ, Prosen, PRL 121, 030606 (2018)
// Žnidarič, Duh, LZ, PRB 112, L020302 (2025)

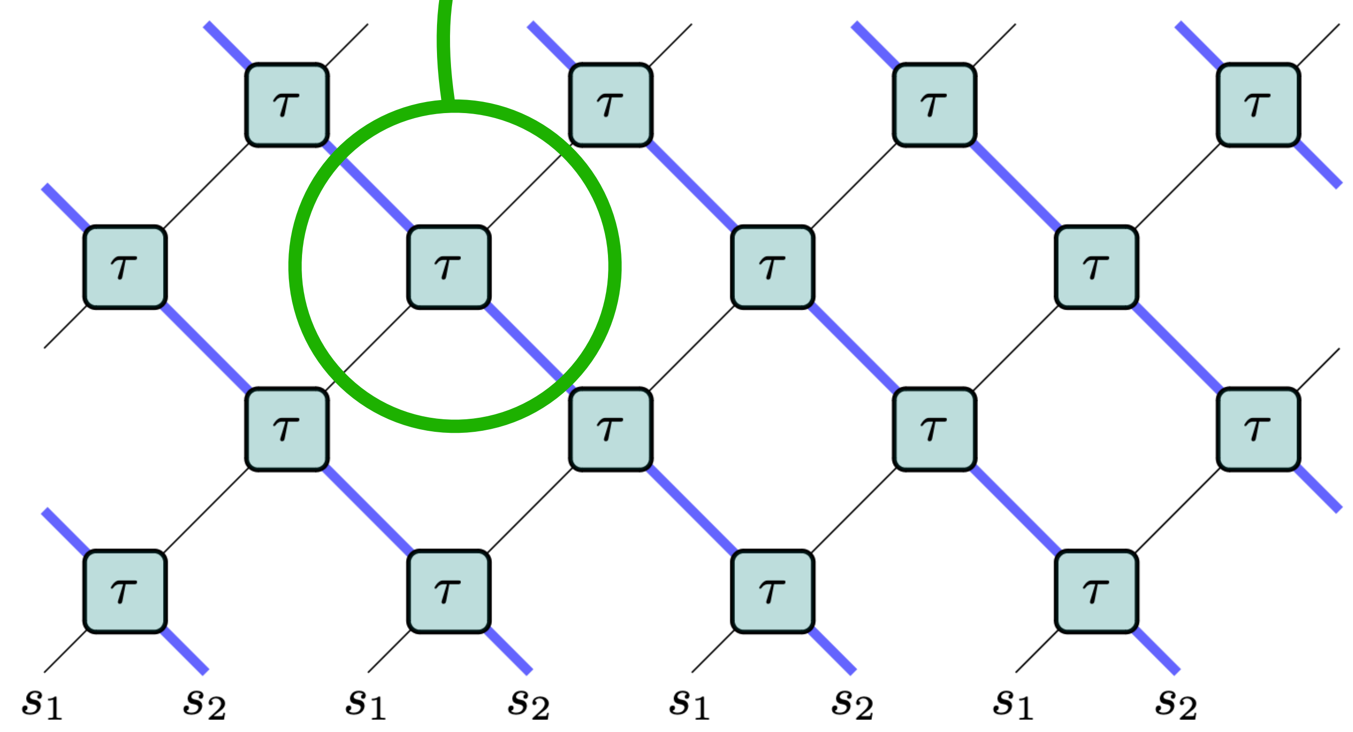


(a) $(q,r)=(5,1)$, $p=2$ (b) $(q,r)=(5,2)$, $p=6$ (g) $(q,r)=(10,1)$, $p=1$
(c) $(q,r)=(5,3)$, $p=4$ (d) $(q,r)=(5,4)$, $p=8$
(e) $(q,r)=(10,3)$, $p=7$ (f) $(q,r)=(10,7)$, $p=3$ (h) $(q,r)=(10,9)$, $p=9$

// Miao, Gritsev, Kurlov, SciPost Phys. 16, 078 (2024)
// Paletta, Duh, Pozsgay, LZ, JPA 58, 275001 (2025)

chirality from asymmetric arrangement

chirality from asymmetric legs



"quantum spin ratchet"

// LZ, Krajnik, Ilievski, Prosen, PRX quant. 5, 030356 (2024)

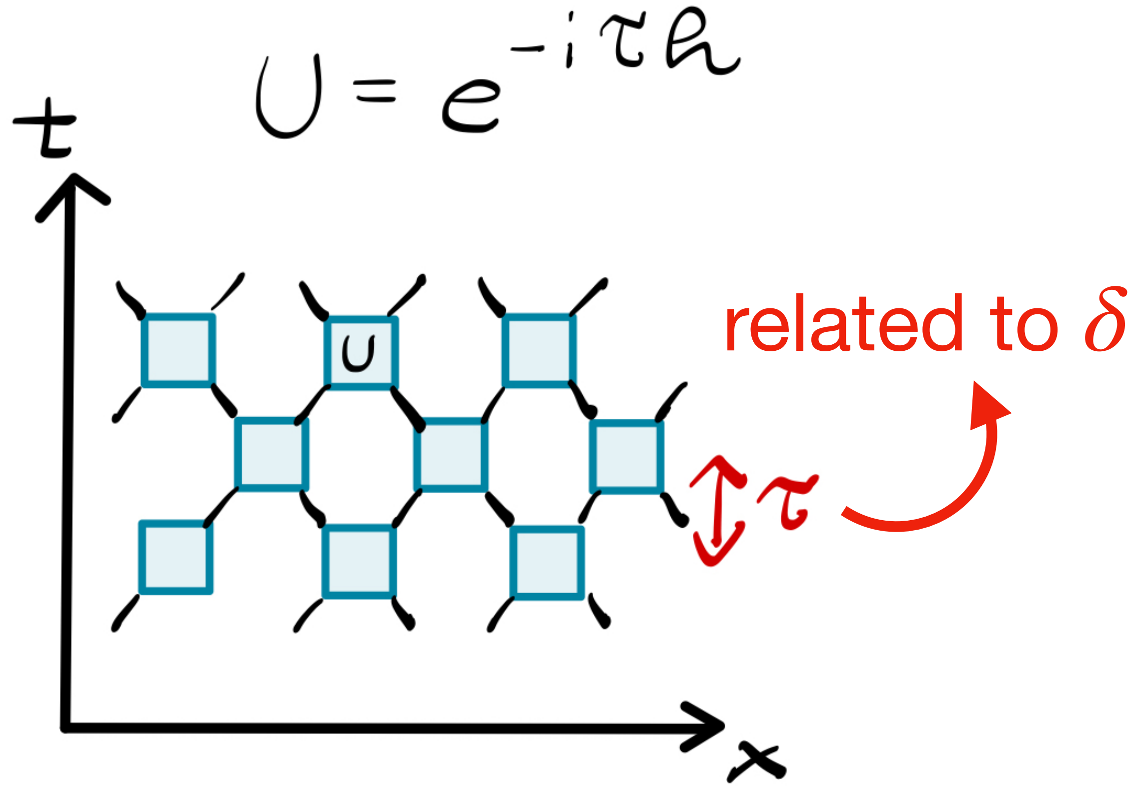
$$J(J+\mathbb{1}) = (\boldsymbol{S}_1+\boldsymbol{S}_2)^2$$

$$R^{s_1,s_2}(\lambda) = (-1)^{J+j}\frac{\Gamma(j+1+i\lambda)\Gamma(J+[1-i\lambda]\mathbb{1})}{\Gamma(j+1-i\lambda)\Gamma(J+[1+i\lambda]\mathbb{1})}$$

$$U = P^{s_1,s_2}R^{s_1,s_2}(\tau)$$

Models with (some sort of) chirality

2. chiral integrable SU(2)-invariant ladder: obtained from the integrable XXX circuit



$$U_{1,2} = \frac{1+\mathrm{i}\delta P_{1,2}}{1+\mathrm{i}\delta}$$

$$P_{1,2} = \tfrac{1}{2}(1 + \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$$

// Vanicat, LZ, Prosen, PRL 121, 030606 (2018)

$$\longrightarrow$$

$$Q_1^+ = \sum_{n=1}^{N/2} q^{[1,+]}_{2n-2,2n-1,2n}, \quad Q_1^- = \sum_{n=1}^{N/2} q^{[1,-]}_{2n-1,2n,2n+1}$$

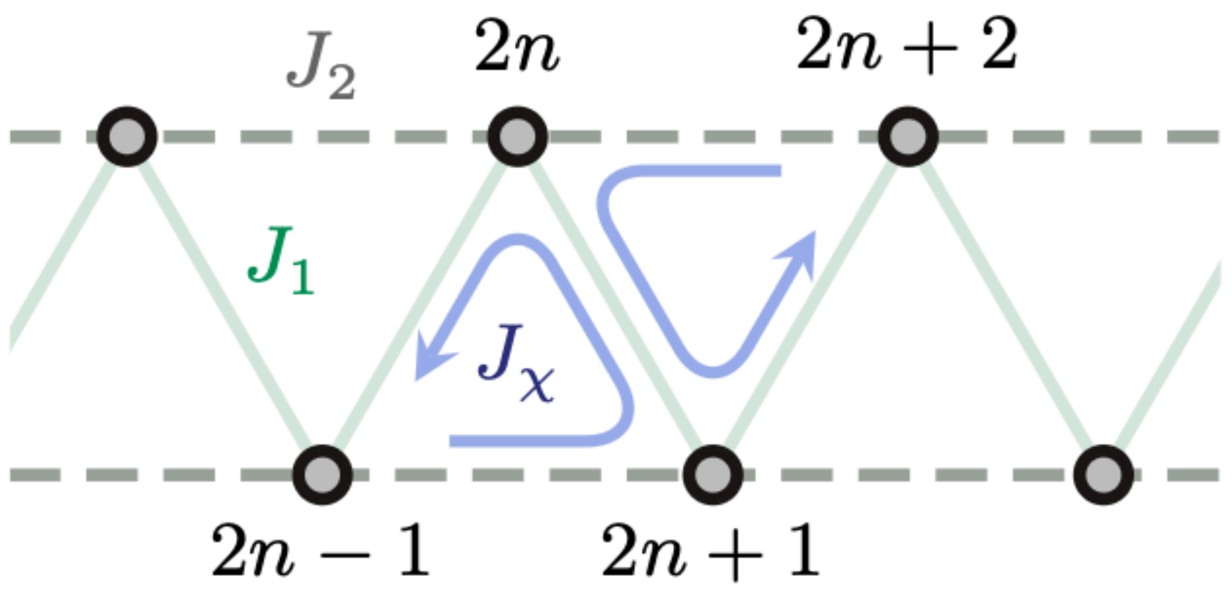
$$q^{[1,\pm]}_{1,2,3} = \frac{i}{2(1+\delta^2)}\left[\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3 + \delta^2\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3 \mp \mp\,\delta\,\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3)\right]$$

Hamiltonian dynamics generated by

$$H = H_0 + H_\chi,$$

$$H_0 = \sum_{n=1}^{L} J_1 \mathbf{S}_n\cdot\mathbf{S}_{n+1} + J_2 \mathbf{S}_n\cdot\mathbf{S}_{n+2},$$

$$H_\chi = J_\chi \sum_{n=1}^{L} (-1)^n \mathbf{S}_n\cdot(\mathbf{S}_{n+1}\times\mathbf{S}_{n+2}),$$



// Popkov, Zvyagin, Phys. Lett. A (1993)
// Frahm, Rödenbeck, Europhys. Lett. (1996)

$$\boxed{\begin{aligned} J_1 &= 1-\eta \\ J_2 &= \frac{\eta}{2} \\ J_\chi &= \sqrt{\eta(1-\eta)}, \\ \delta &= -\sqrt{\eta/(1-\eta)} \end{aligned}}$$

integrable line

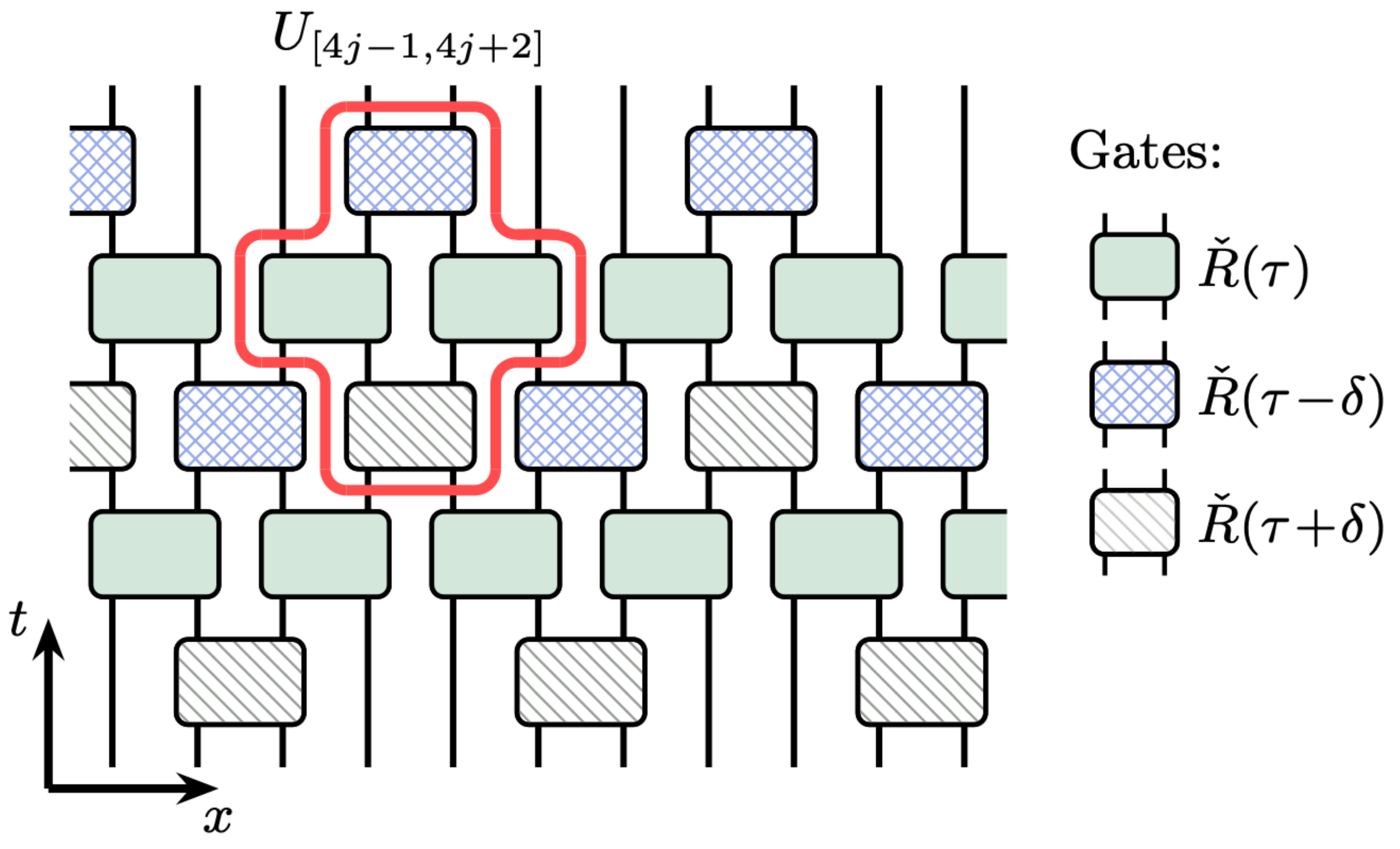
Models with (some sort of) chirality

3. integrable Trotterization of the chiral SU(2)-invariant ladder

$$\check{R}(\tau) = (\mathbb{1} + \mathrm{i}\tau P)/(1 + \mathrm{i}\tau)$$

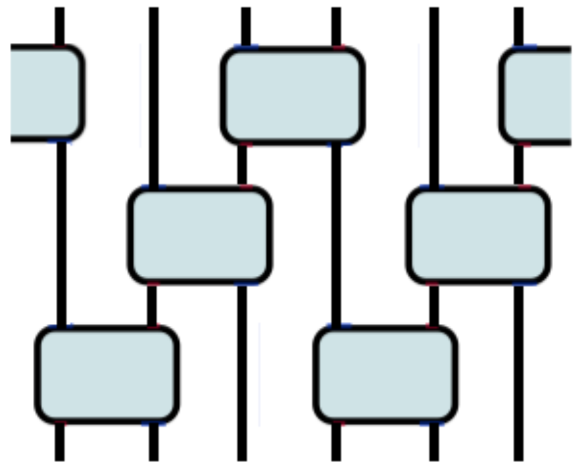
$$h_{[1,4]} = J_1 \mathbf{S}_1 \cdot \mathbf{S}_2 + J_1 \mathbf{S}_2 \cdot \mathbf{S}_3 + J_2 \mathbf{S}_1 \cdot \mathbf{S}_3 + J_2 \mathbf{S}_2 \cdot \mathbf{S}_4 + J_\chi \mathbf{S}_1 \cdot (\mathbf{S}_2 \times \mathbf{S}_3) - J_\chi \mathbf{S}_2 \cdot (\mathbf{S}_3 \times \mathbf{S}_4),$$

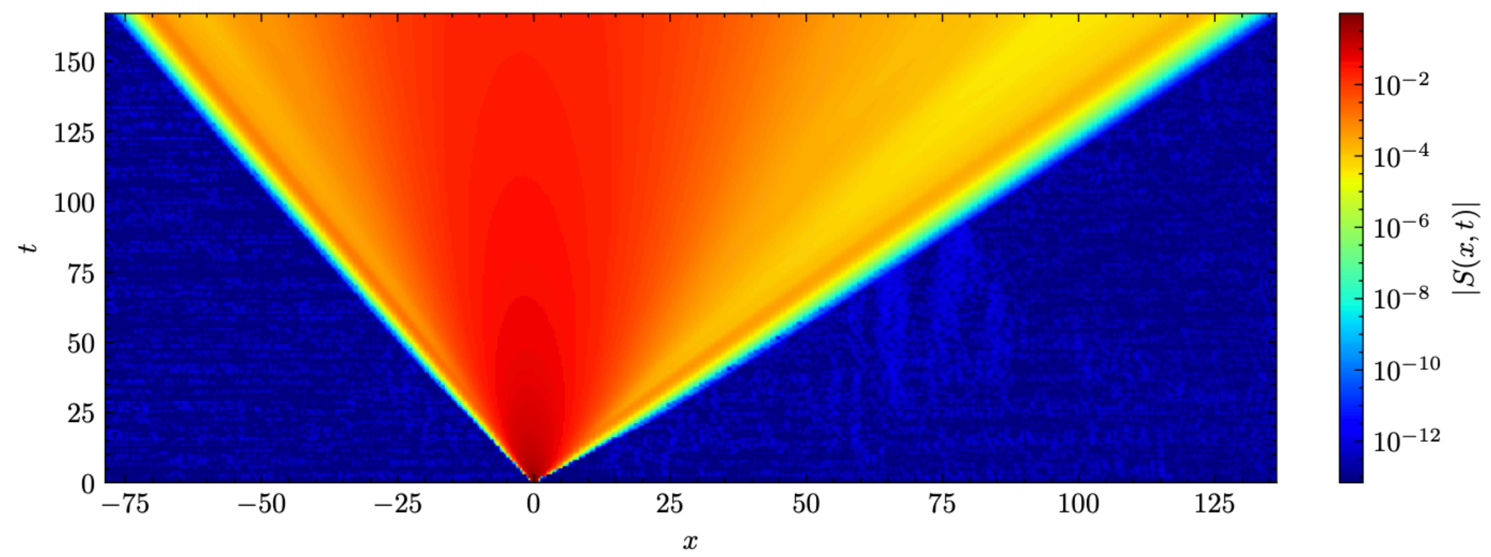
$$U_{[4j-1,4j+2]} = \mathrm{e}^{\mathrm{i}\tau h_{[4j-1,4j+2]}} + O(\tau^3)$$



What we observe

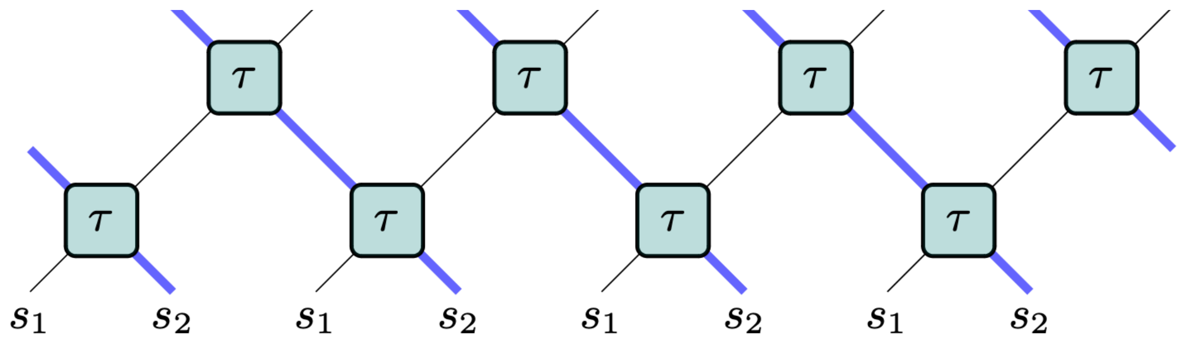
asymmetric arrangement

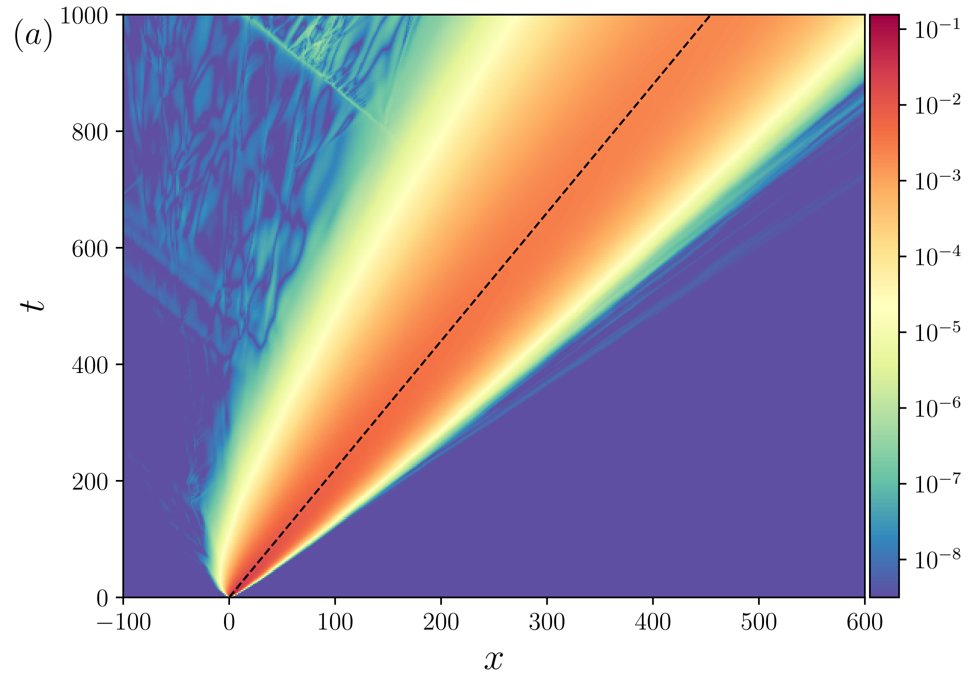




1. no drift
2. asymmetric *S(x,t)*
3. fluctuation symmetry

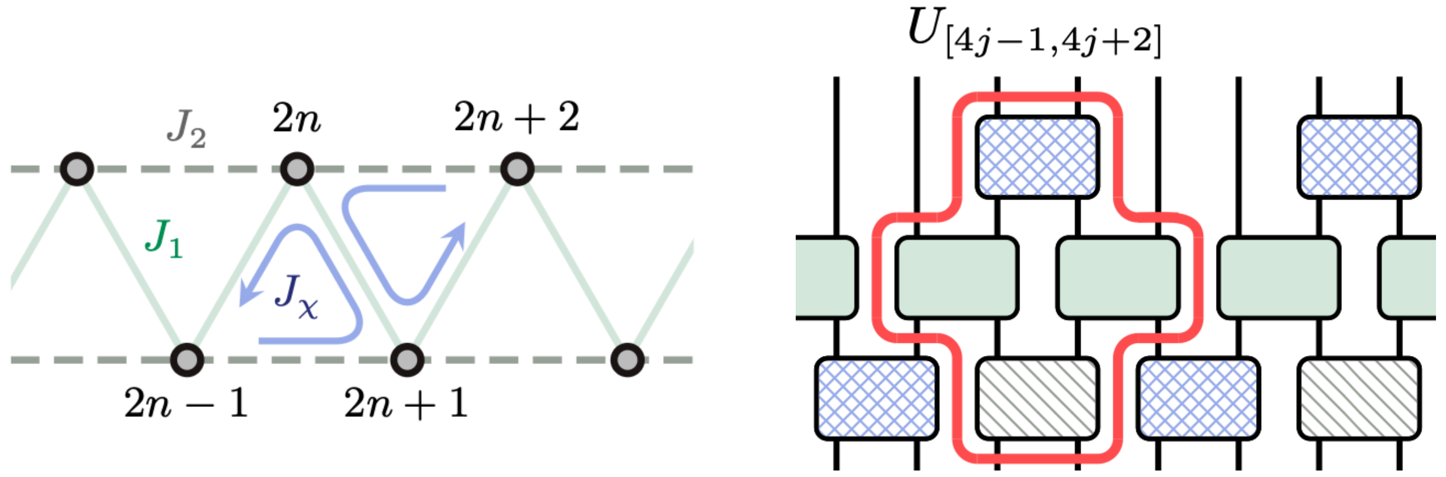
asymmetric gate (ratchet)





1. drift
2. asymmetric *S(x,t)*
3. broken fluctuation symmetry

chiral ladder



1. no drift
2. symmetric *S(x,t)*
3. fluctuation symmetry

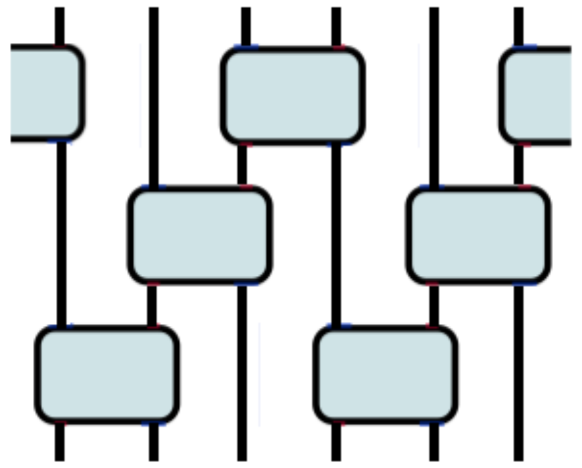
Fluctuation symmetry due to a crystallographic symmetry of *H*, not due to *PT*, *P*, *T* symmetries!

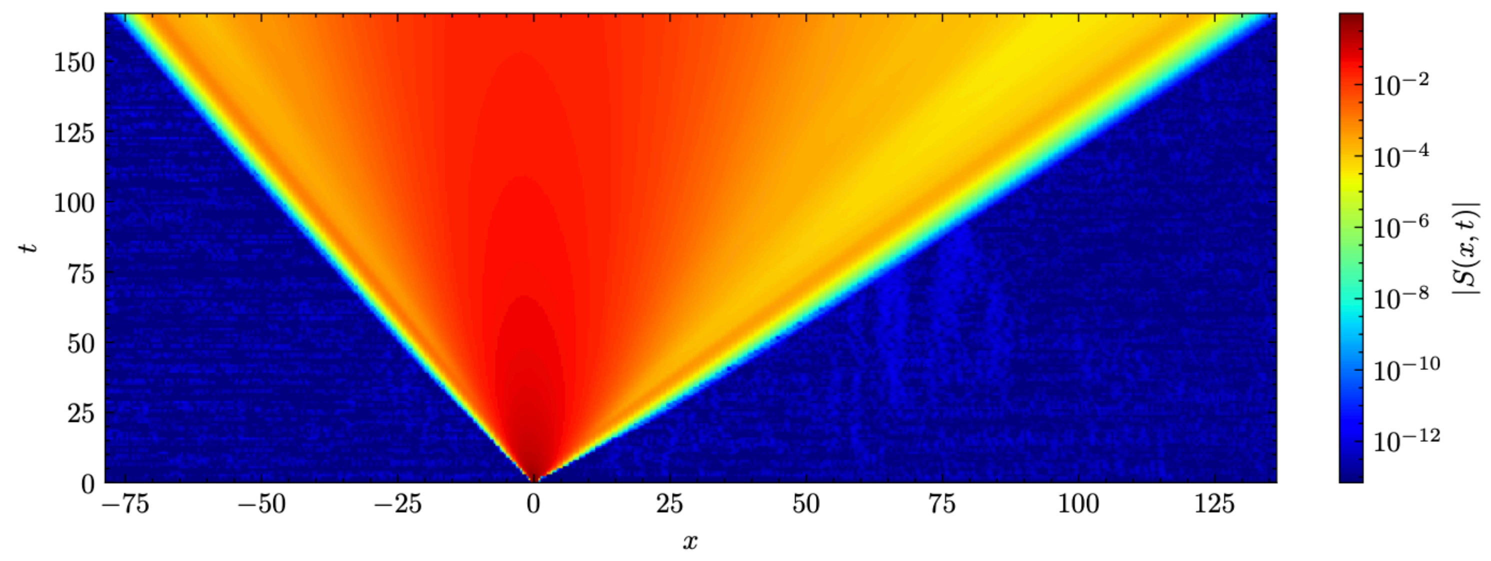
All cases: z = 2/3 non-KPZ superdiffusion

What we observe

Note: the relevant spacetime symmetries are those of the gate not of the circuit

asymmetric arrangement





1. no drift
2. asymmetric $S(x,t)$
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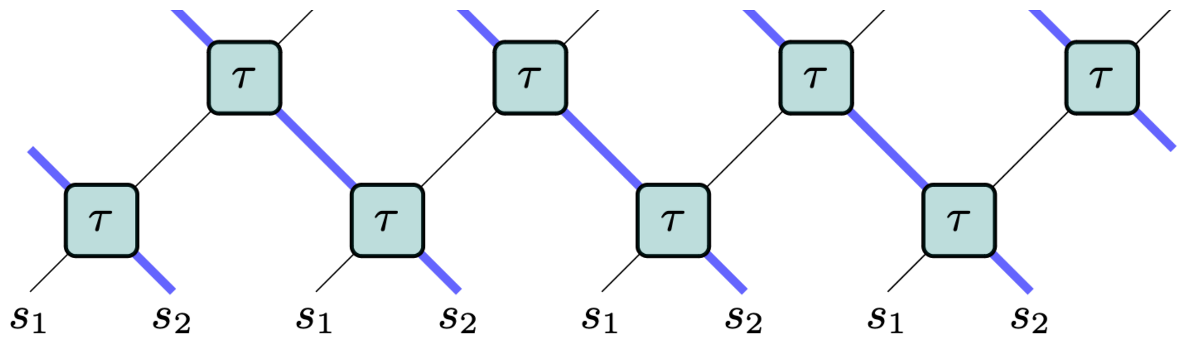

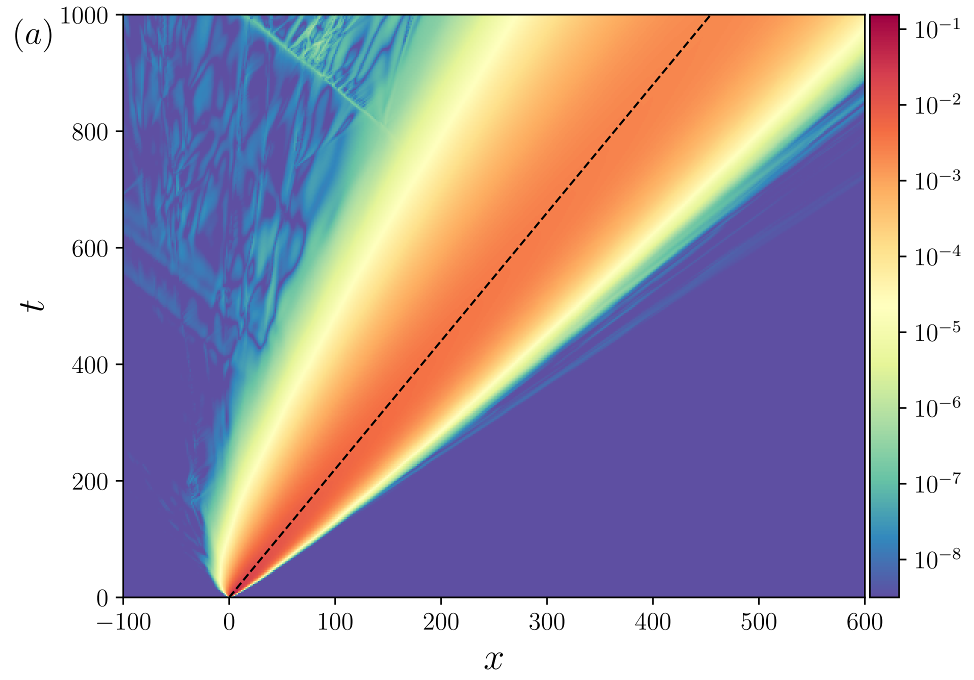

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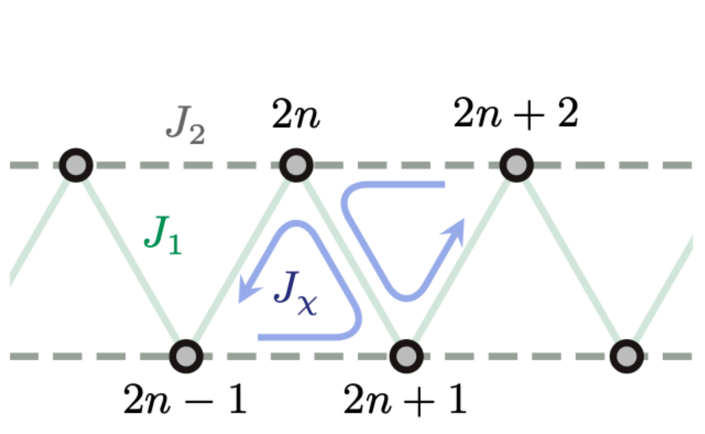

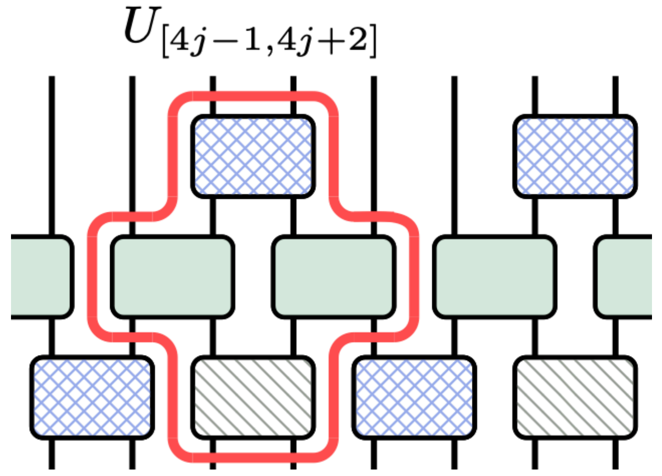

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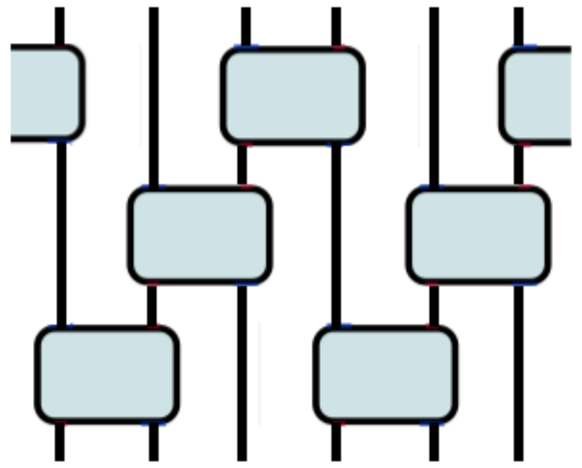

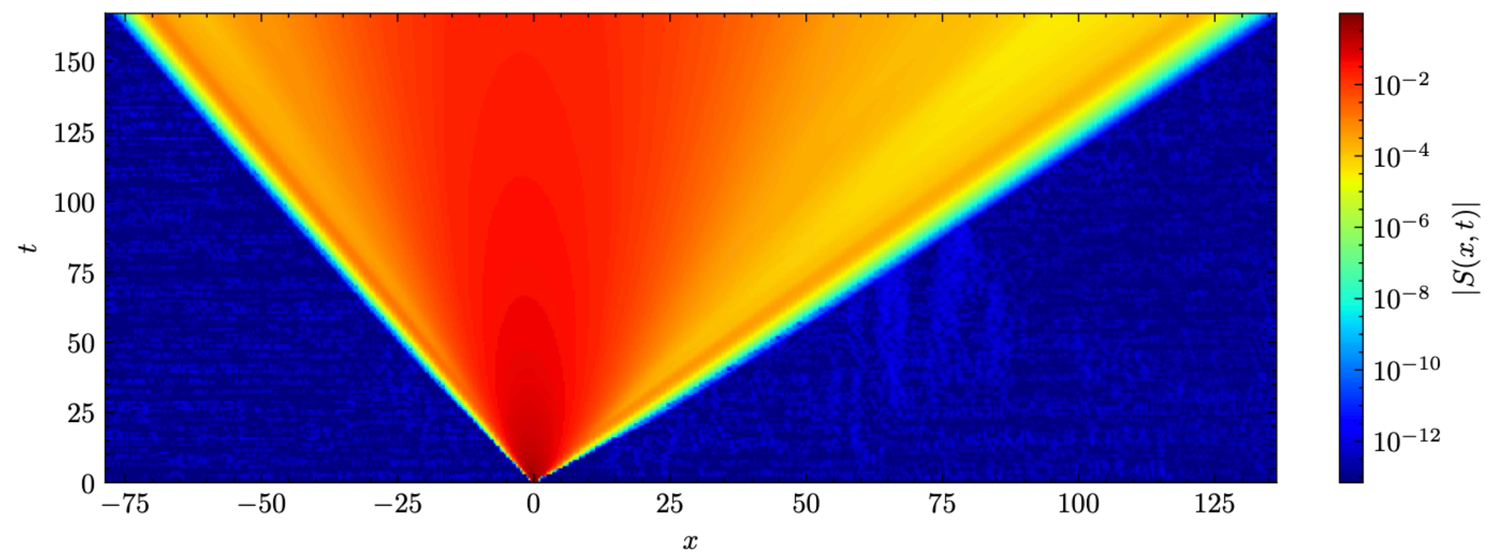

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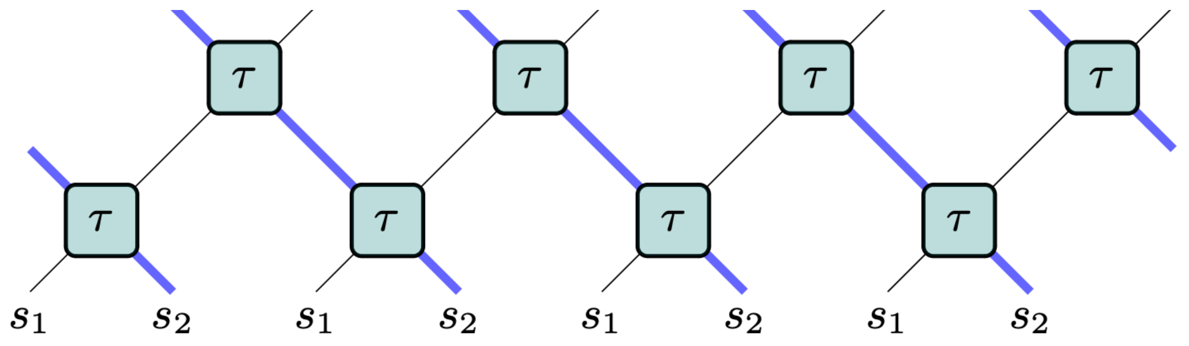

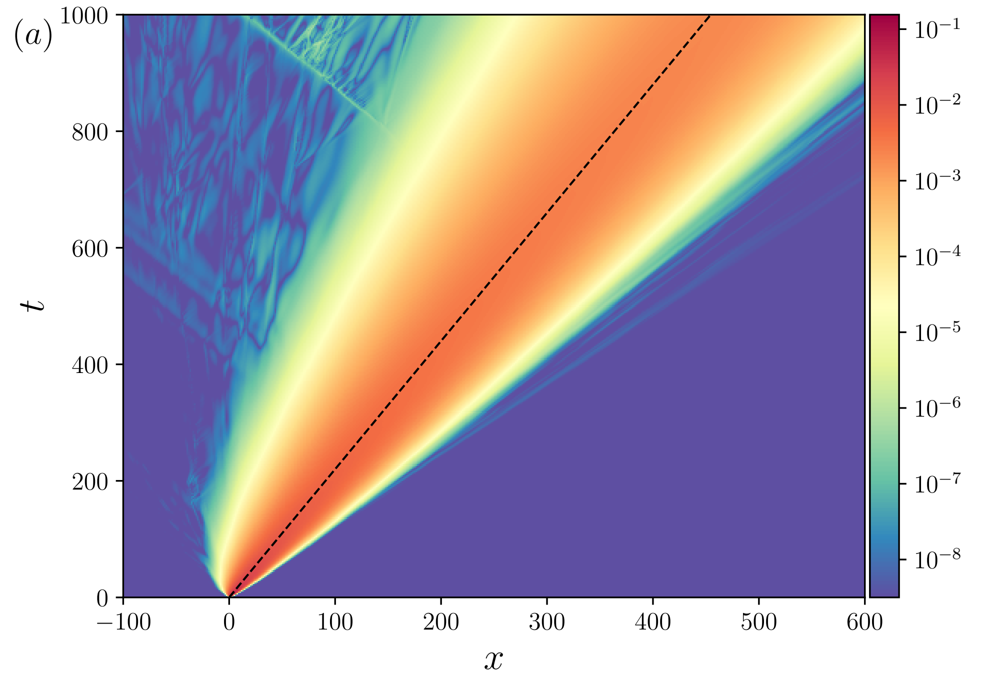

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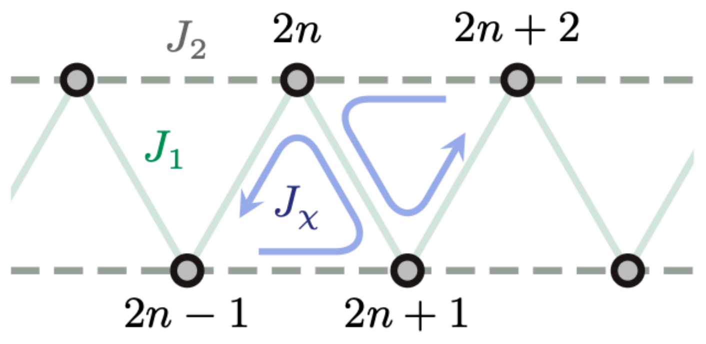

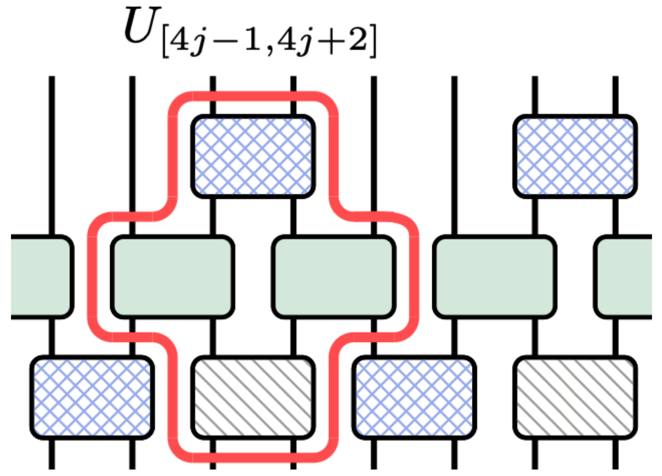

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All cases: z = 2/3 non-KPZ superdiffusion

Integrable structure → propagators → momenta & energies

1. integrable quantum spin ratchet (asymmetric gate, two spin species)

$$T_{s_3}(\lambda) = \mathrm{Tr}_a \left(\prod_{1\leq j \leq L/2}^{\rightarrow} R^{s_1,s_3}_{2j-1,a}(\lambda_+) R^{s_2,s_3}_{2j,a}(\lambda_-) \right) \quad \Rightarrow \quad \begin{array}{l} \mathbb{U} = T_{s_2}(\tfrac{\tau}{2}) \left[T_{s_1}(-\tfrac{\tau}{2}) \right]^{-1} \\ \mathbb{T} = T_{s_2}(\tfrac{\tau}{2}) T_{s_1}(-\tfrac{\tau}{2}). \end{array} \quad \Rightarrow \quad \begin{array}{l} \mathbb{U}\,|\{\lambda_j\}\rangle = e^{i2\sum_{j=1}^{N}\varepsilon(\lambda_j)}\,|\{\lambda_j\}\rangle \\ \mathbb{T}\,|\{\lambda_j\}\rangle = e^{-2i\sum_{j=1}^{N}p(\lambda_j)}\,|\{\lambda_j\}\rangle \end{array}$$

$(\lambda_\pm = \lambda \pm \tau/2)$

$$p(\lambda) = \frac{1}{2}\left[p^{(2s_1)}(\lambda_+) + p^{(2s_2)}(\lambda_-) \right]$$

$$\varepsilon(\lambda) = \frac{1}{2}\left[p^{(2s_1)}(\lambda_+) - p^{(2s_2)}(\lambda_-) \right]$$

$$p^{(2s)}(\lambda) \equiv i \log\left(\frac{\lambda + is}{\lambda - is} \right)$$

contributions from two sublattices s_1 and s_2

2. chiral SU(2)-invariant ladder is obtained from the first two charges of the XXX circuit $\boxed{\Rightarrow s_1 = s_2 = s}$

Generalized fluctuation relation in spin ratchets

1. probability to observe ballistic scale accumulated current through the middle of the system

$$\mathcal{J}_t = \int_0^t \mathrm{d}\tau\, [j(x{=}0,\tau) - \langle j \rangle]$$

$$\mathbb{P}(\mathcal{J} = \mathrm{j}t|t) \asymp e^{-tI(\mathrm{j})} \longleftrightarrow \boxed{c_k^{(\mathrm{sc})} \equiv \lim_{t\to\infty} \frac{\langle \mathcal{J}_t^k \rangle^c}{t}}$$

2. hydrodynamic mode decomposition for the scaled cumulants admits a tree-graph expansion

// D.-L. Vu, arXiv:2008.06901
// B. Doyon et al., SciPost Phys. 15, 136 (2023)

$$c_1^{(\mathrm{sc})} = 0$$

$$c_2^{(\mathrm{sc})} = \sum_{m=1}^{\infty} w_m^{(2)} \left(q_m^{\mathrm{dr}}\right)^2 \int \mathrm{d}\lambda\, \underline{\sigma_m(\lambda)(\varepsilon_m')^{\mathrm{dr}}(\lambda)}$$

$$w_m^{(k)} = \sum_{r\geq 1} \frac{(-1)^{r-1} r^{k-1}}{2\pi} \left(\frac{n_m}{1-n_m}\right)^r$$ statistical weight

$$c_{3;(4^3)}^{(\mathrm{sc})} = \sum_{m=1}^{\infty} w_m^{(3)} \left(q_m^{\mathrm{dr}}\right)^3 \int \mathrm{d}\lambda \underline{(\varepsilon_m')^{\mathrm{dr}}(\lambda)}$$

$$\sigma_m(\lambda) = \mathrm{sgn}\, \varepsilon_m'(\lambda)$$ sign of quasiparticle velocity

$$c_{3;(3^1,3^2)}^{(\mathrm{sc})} = 3 \sum_{m=1}^{\infty} w_m^{(2)} q_m^{\mathrm{dr}} \int \mathrm{d}\lambda \underline{\sigma_m(\lambda)(\varepsilon_m')^{\mathrm{dr}}(\lambda)} \left\{ \left[(1-n_m)\underline{\sigma_m} \left(q_m^{\mathrm{dr}}\right)^2\right]^{\mathrm{dr}}(\lambda) - \left[(1-n_m)\underline{\sigma_m} \left(q_m^{\mathrm{dr}}\right)^2\right] \right\}$$

Generalized fluctuation relation in spin ratchets

Observations: $\boxed{\partial_\lambda \varepsilon_m^{(s_1,s_2)}(\lambda) = -\partial_\lambda \varepsilon_m^{(s_2,s_1)}(-\lambda)}$

(1) The hydrodynamic formulae for all scaled cumulants involve one dressed derivative of the dispersion under the integral over the rapidity λ.
(2) The odd (even) cumulants additionally involve an even (respectively, odd) number of signs $\sigma_m^{(s_1,s_2)}(\lambda)$

As a result, the generating function of the scaled cumulants, $F(\zeta) = \lim_{t\to\infty} \frac{1}{t} \log\langle e^{\zeta \mathfrak{J}_t}\rangle$, satisfies

$$\boxed{F^{(s_1,s_2)}(\zeta) = F^{(s_2,s_1)}(-\zeta)}$$

$\Longrightarrow$ via Legendre transform

$$\boxed{\frac{\mathbb{P}_{s_1,s_2}(\mathfrak{J} = \mathfrak{j}t|t)}{\mathbb{P}_{s_2,s_1}(\mathfrak{J} = -\mathfrak{j}t|t)} \asymp 1}$$

Generalized fluctuation relation in spin ratchets

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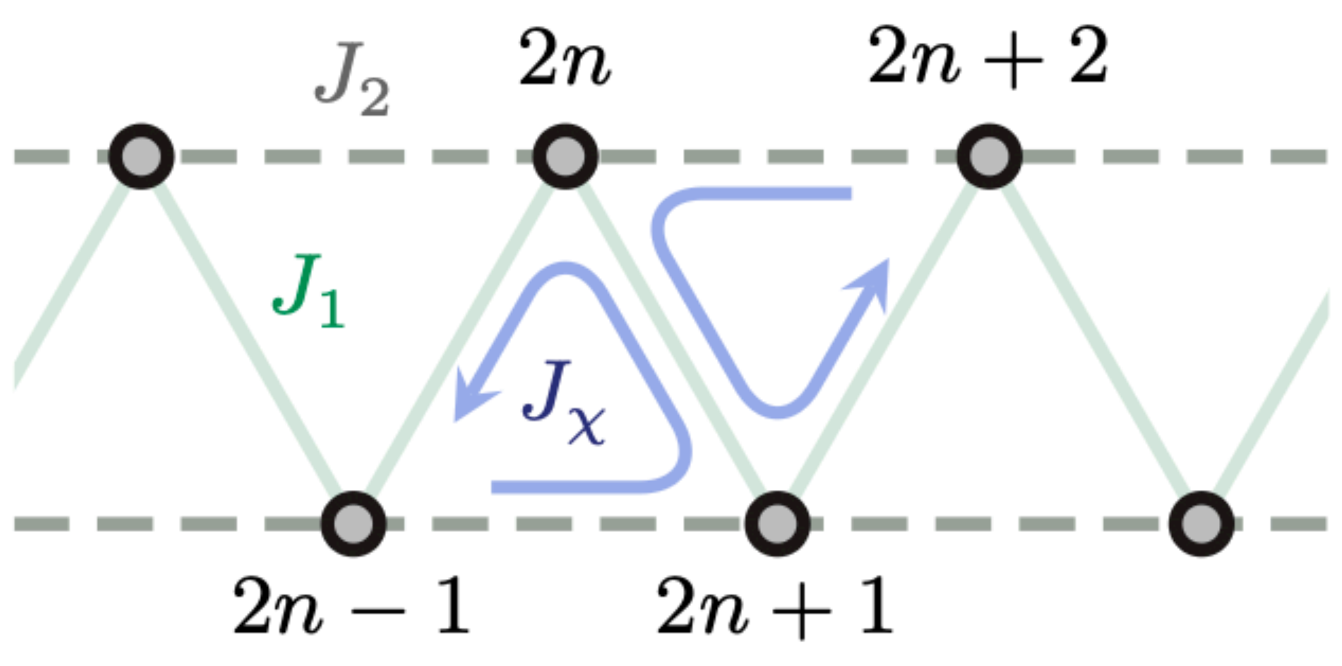
integrable model w/o fluctuation symmetry
OR
with a generalized fluctuation symmetry and no T symmetry

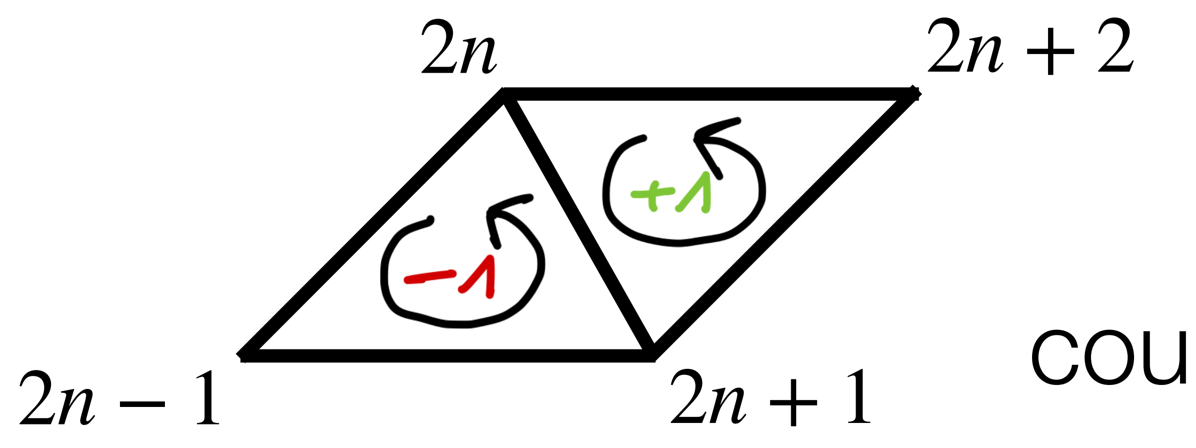
Fluctuation relation from a “crystallographic” symmetry

1. chiral SU(2)-invariant spin ladder

$$
\begin{aligned}
H &= H_0 + H_\chi, \\
H_0 &= \sum_{n=1}^{L} J_1 \mathbf{S}_n \cdot \mathbf{S}_{n+1} + J_2 \mathbf{S}_n \cdot \mathbf{S}_{n+2}, \\
H_\chi &= J_\chi \sum_{n=1}^{L} (-1)^n \mathbf{S}_n \cdot (\mathbf{S}_{n+1} \times \mathbf{S}_{n+2}),
\end{aligned}
$$



2. link parity $\mathcal{P}_\ell$ (C_2 symmetry)



counter-clockwise!

Fluctuation relation from a “crystallographic” symmetry

1. chiral SU(2)-invariant spin ladder

$$H = H_0 + H_\chi,$$

$$H_0 = \sum_{n=1}^{L} J_1 \mathbf{S}_n \cdot \mathbf{S}_{n+1} + J_2 \mathbf{S}_n \cdot \mathbf{S}_{n+2},$$

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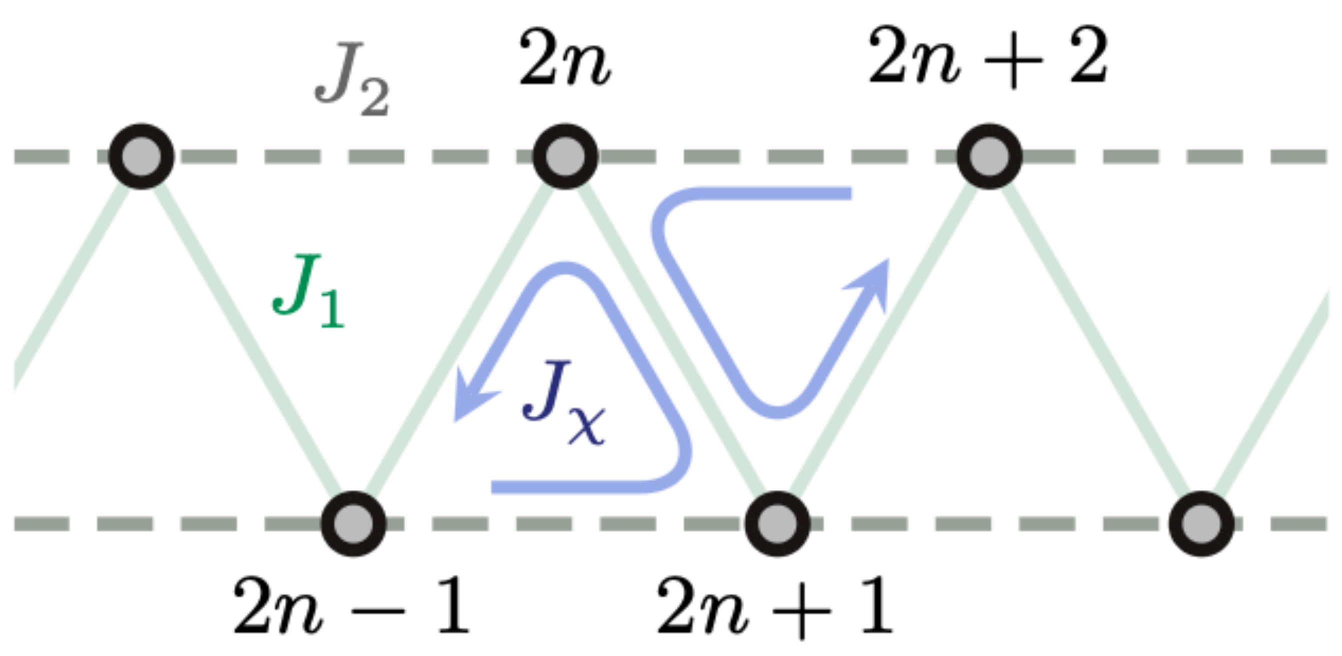

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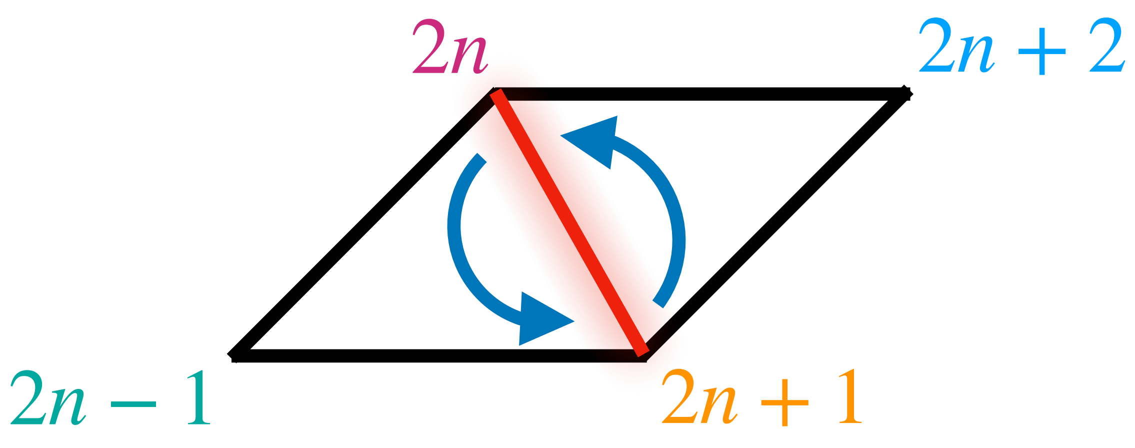

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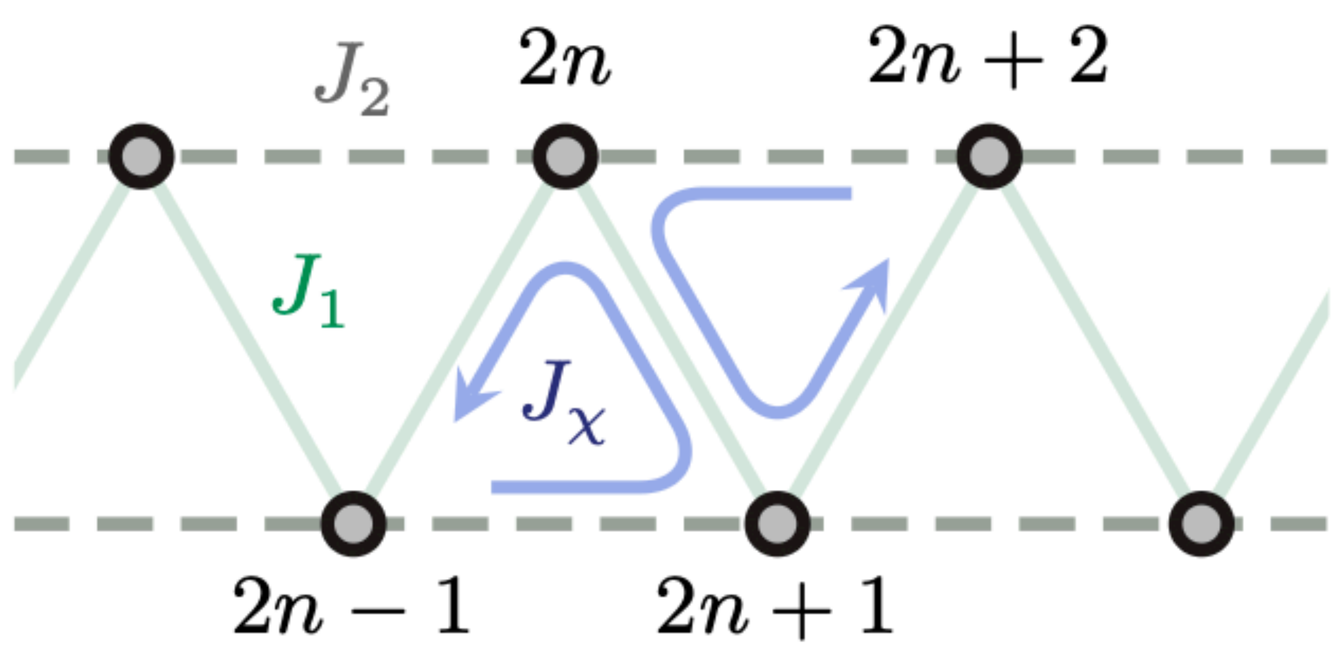

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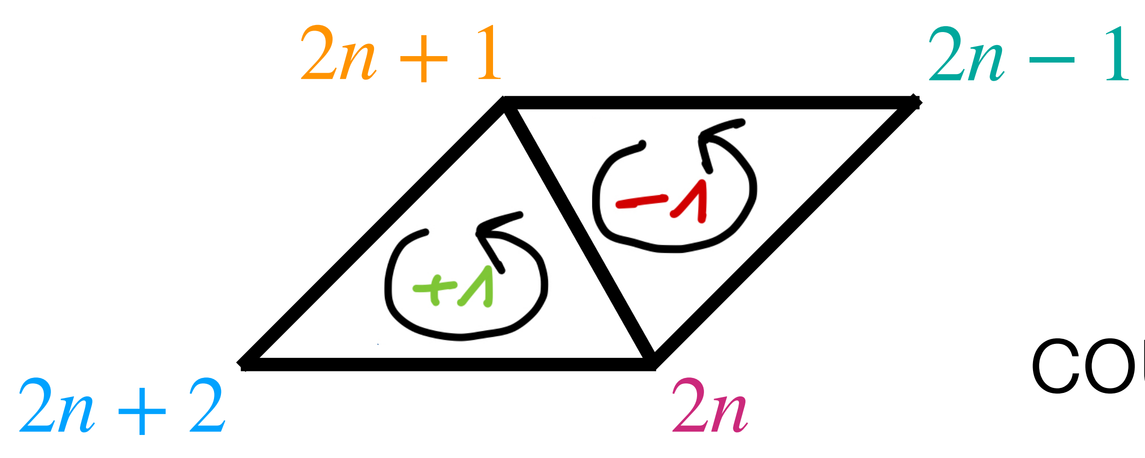

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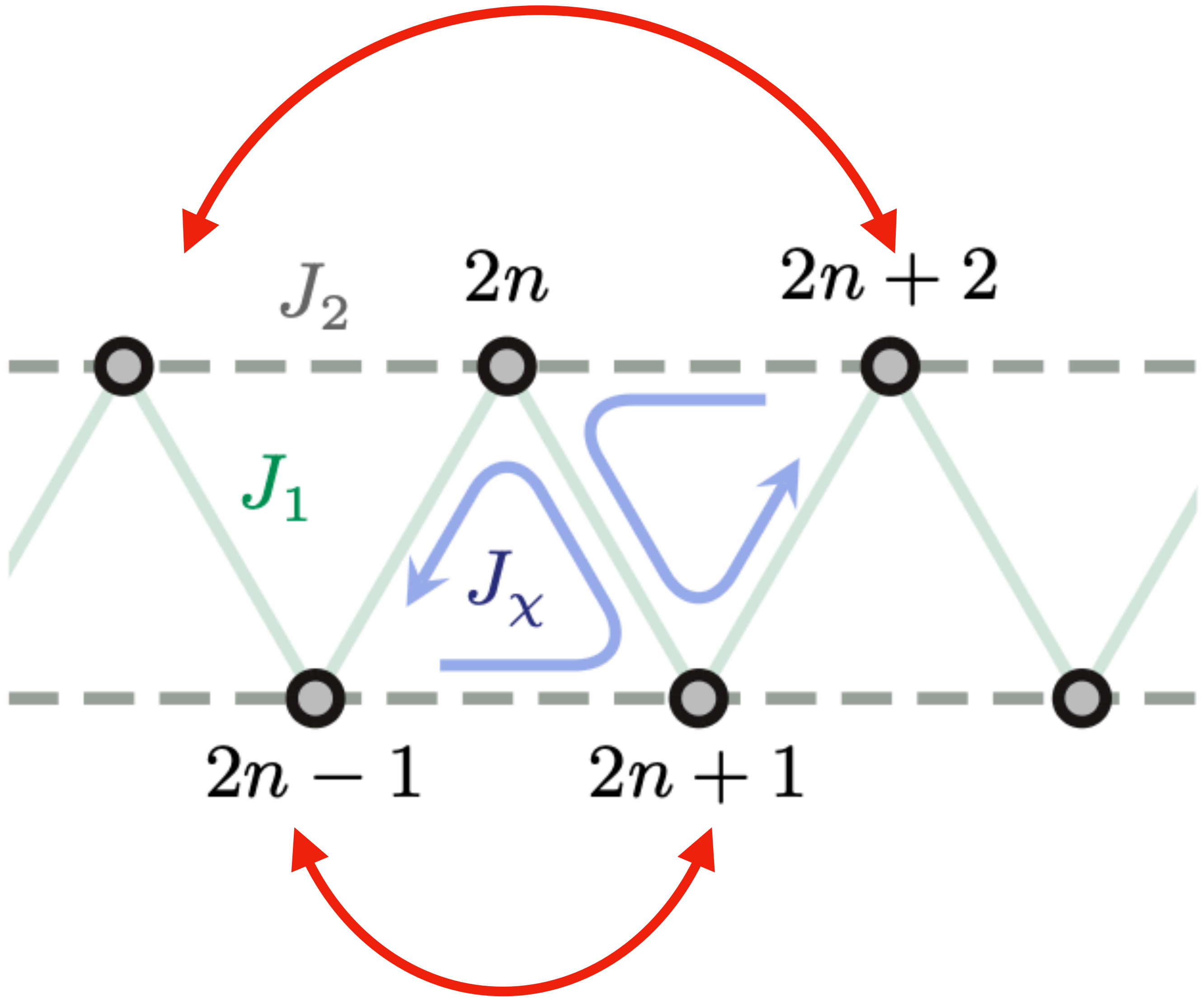


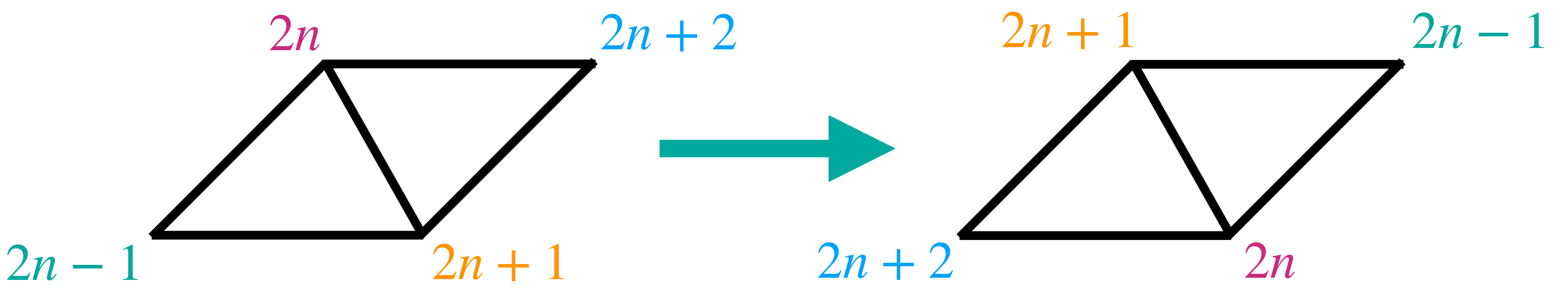
2. link parity $\mathcal{P}_\ell$ (C_2 symmetry)

3. no site parity *P* or time-reversal symmetry *T* (however, there is a combined *PT* symmetry)

Fluctuation relation from a “crystallographic” symmetry

C_2 invariant state $\langle \mathcal{P}_\ell[\bullet] \rangle = \langle \bullet \rangle$

$$\mathcal{P}_\ell \left[S^z_{\text{left}}(t) - S^z_{\text{left}}(0) \right] = S^z_{\text{right}}(t) - S^z_{\text{right}}(0)$$



conservation of total magnetization:

$$S^z_{\text{left}}(t) - S^z_{\text{left}}(0) = -S^z_{\text{right}}(t) + S^z_{\text{right}}(0)$$

$$\Longrightarrow \boxed{\mu_n(t) = \langle [S^z_{\text{left}}(t) - S^z_{\text{left}}(0)]^n \rangle = 0 \quad \text{for odd } n}$$

odd cumulants contain odd moments in each term → they also vanish!

Fluctuation relation from a “crystallographic” symmetry

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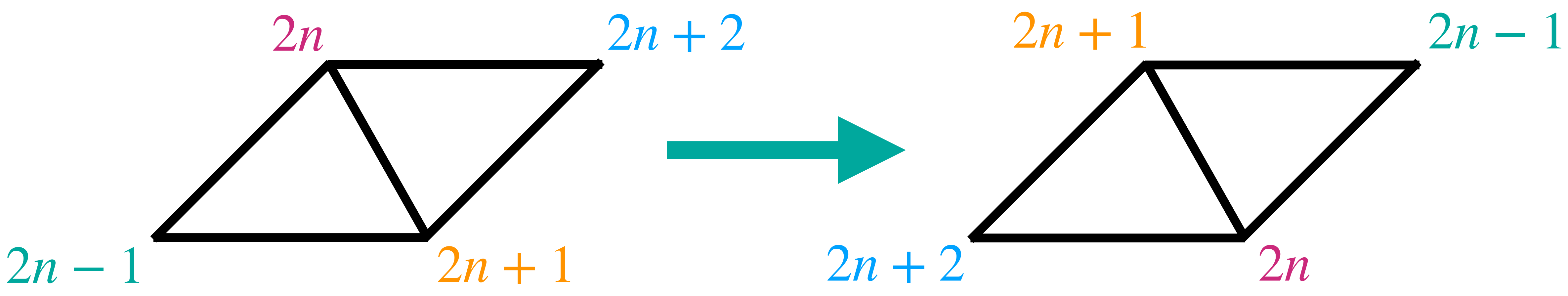

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integrable model with fluctuation symmetry from a C_2 symmetry (not T)

odd cumulants contain odd moments in each term → they also vanish!

Recap

1. **Example 1:** integrable model without fluctuation symmetry / with a generalized fluctuation symmetry without time-reversal invariance

2. **Example 2:** integrable model with fluctuation symmetry following from a "crystallographic" symmetry rather than time-reversal invariance

Both models are integrable SU(2) invariant spin models that generalize (or are related to) the XXX integrable structure → expect transport with **some KPZ-like properties**.

Q: Is full KPZ universality recovered simply due to broken time-reversal invariance?

Structure factor

1. Structure-factor moments → basic transport properties

$$m_n(t) := \int \mathrm{d}x\, x^n S(x,t)$$

$$\boxed{m_2(t) \sim t^{2/z}}$$

variance of the structure factor

→ superdiffusive $(1 < z < 2)$

→ diffusive $(z = 2)$

→ subdiffusive $(z > 2)$

→ ballistic $(z = 1)$

2. Chiral spin ladder (hydro mode decomposition arguments & TBA): $\boxed{z = 3/2}$

// Ilievski et al. PRX 11, 031023 (2021)
// Gopalakrishnan, Vasseur, PRL (2019)

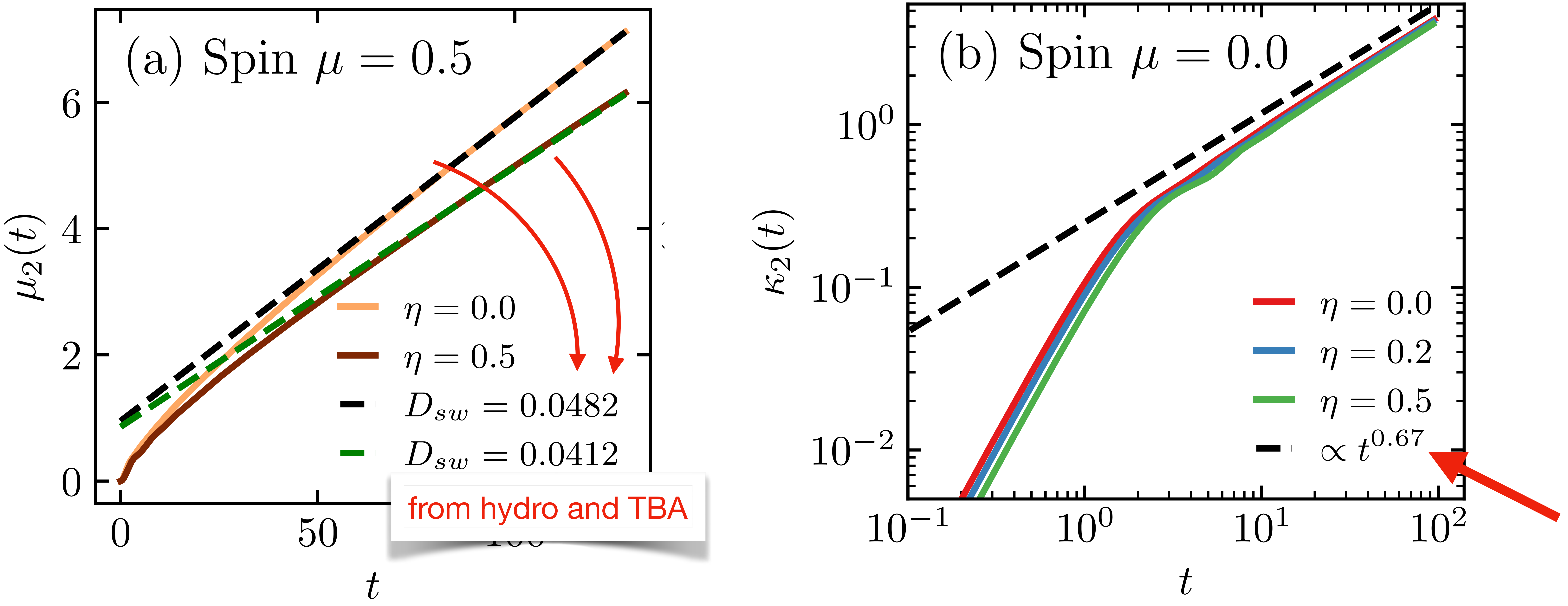
3. Universality class requires distribution of charge fluctuations at large scales

Comparison with numerics: 1st absolute moment of *S(x,t)*

$$\boxed{\mu_2(t) = \kappa_2(t) = \int \mathrm{d}x|x|S(x,t) \sim t^{1/z}}$$

$L = 256, \quad \chi = 512$

SU(2) point: $z = 3/2$



Access to fluctuations: QGF approach

1. Quantum generating function

$$G(\lambda, t) = \langle \mathrm{e}^{\mathrm{i}\lambda \Delta Q_{\mathcal{S}}(t)} \rangle \longrightarrow G_{\mathrm{QGF}}(\lambda, t) = \langle \mathrm{e}^{\mathrm{i}\lambda Q_{\mathcal{S}}(t)} \mathrm{e}^{-\mathrm{i}\lambda Q_{\mathcal{S}}(0)} \rangle$$

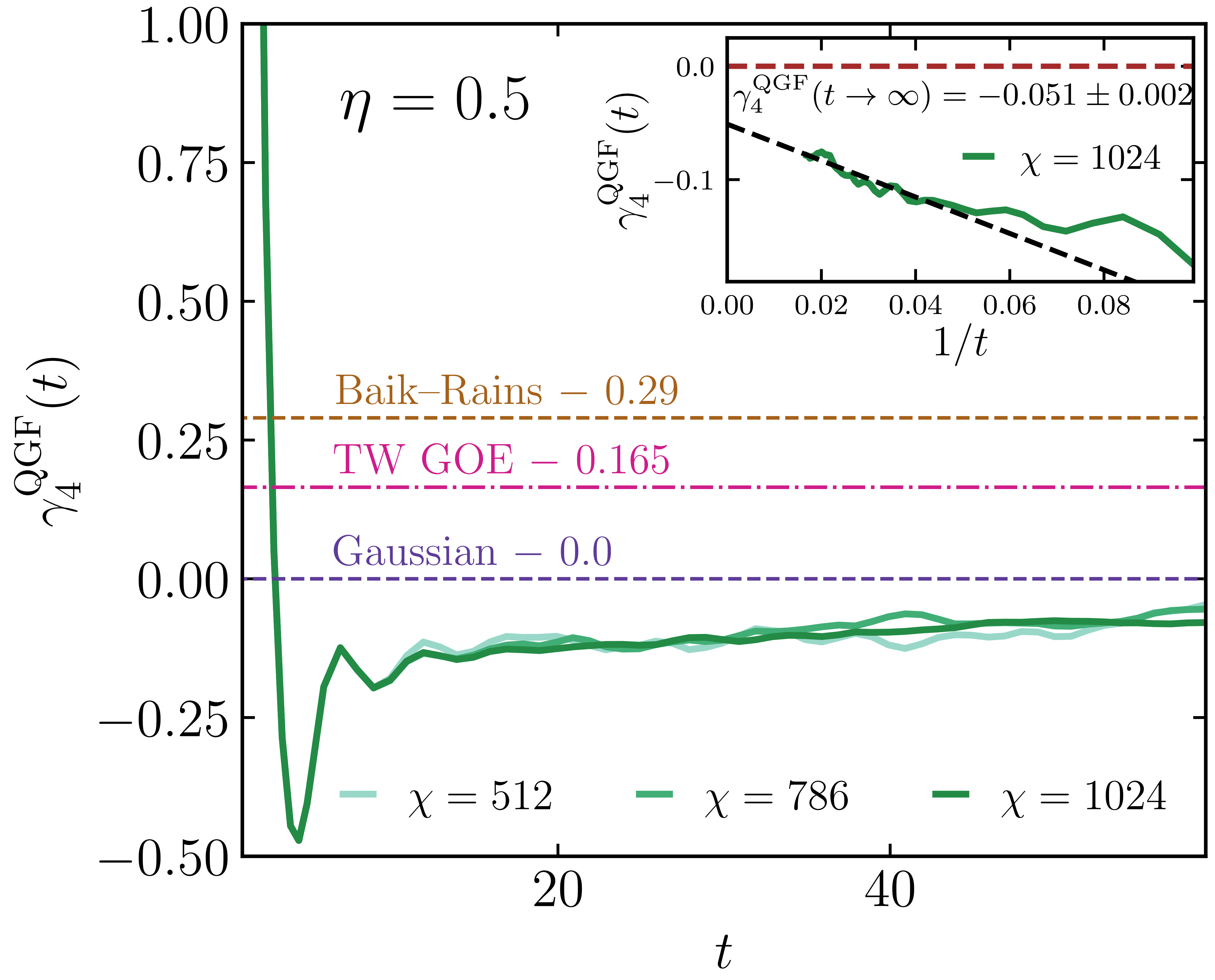
// A. Valli et al., PRL 135, 100401 (2025)

2. Charge = total spin: $Q_{\mathcal{S}} = \sum_j S_j^z \longrightarrow \bigotimes_j \mathrm{e}^{\mathrm{i}\lambda S_j^z}$

Low bond dimension → efficient tensor-network simulation

Access to fluctuations: QGF approach

Similar results as in XXX model, despite broken space-time symmetries (**no KPZ universality**)



$$\gamma_4(t) = \frac{\mu_4(t)}{\mu_2^2(t)} - 3$$

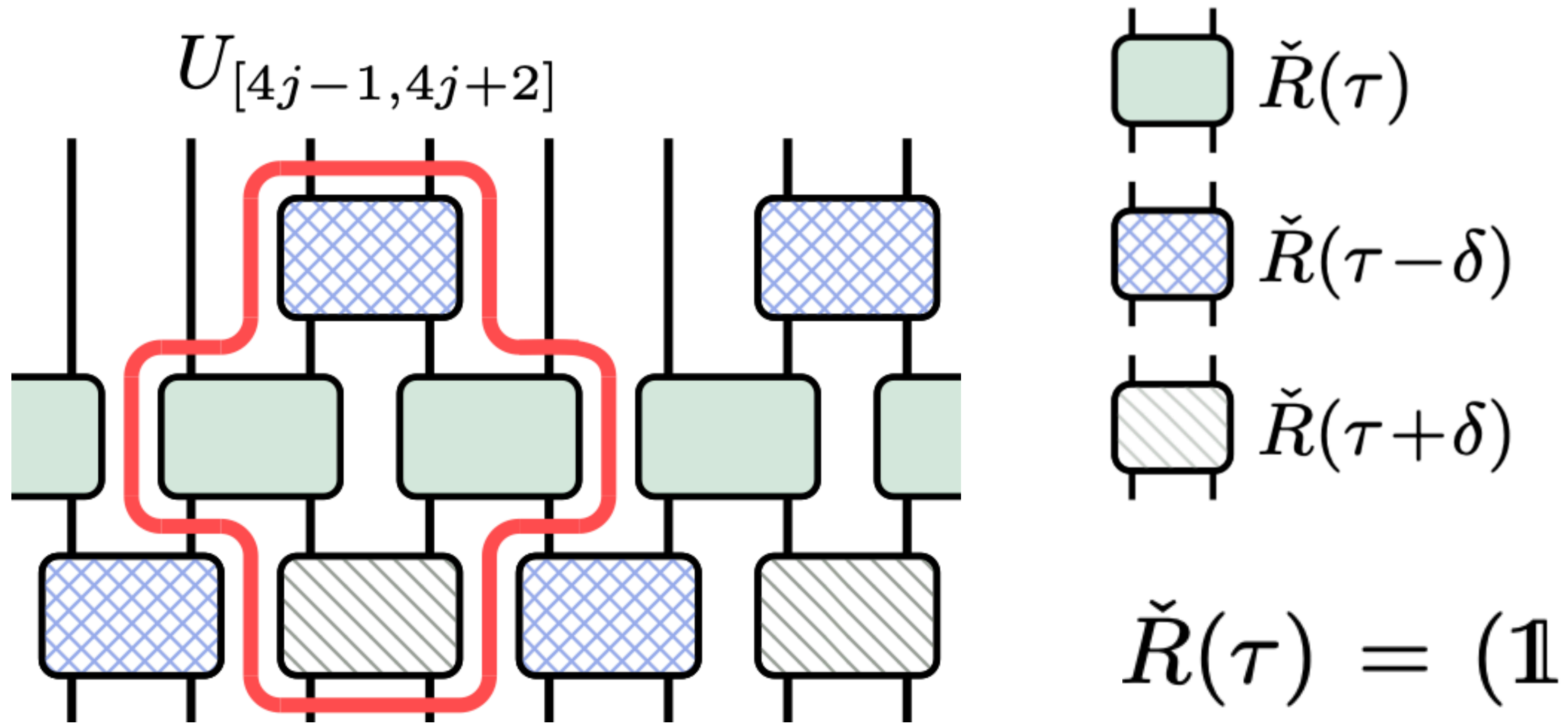
kurtosis inconsistent with KPZ

Broken time-translation symmetry

The integrable gate is not “faithful”

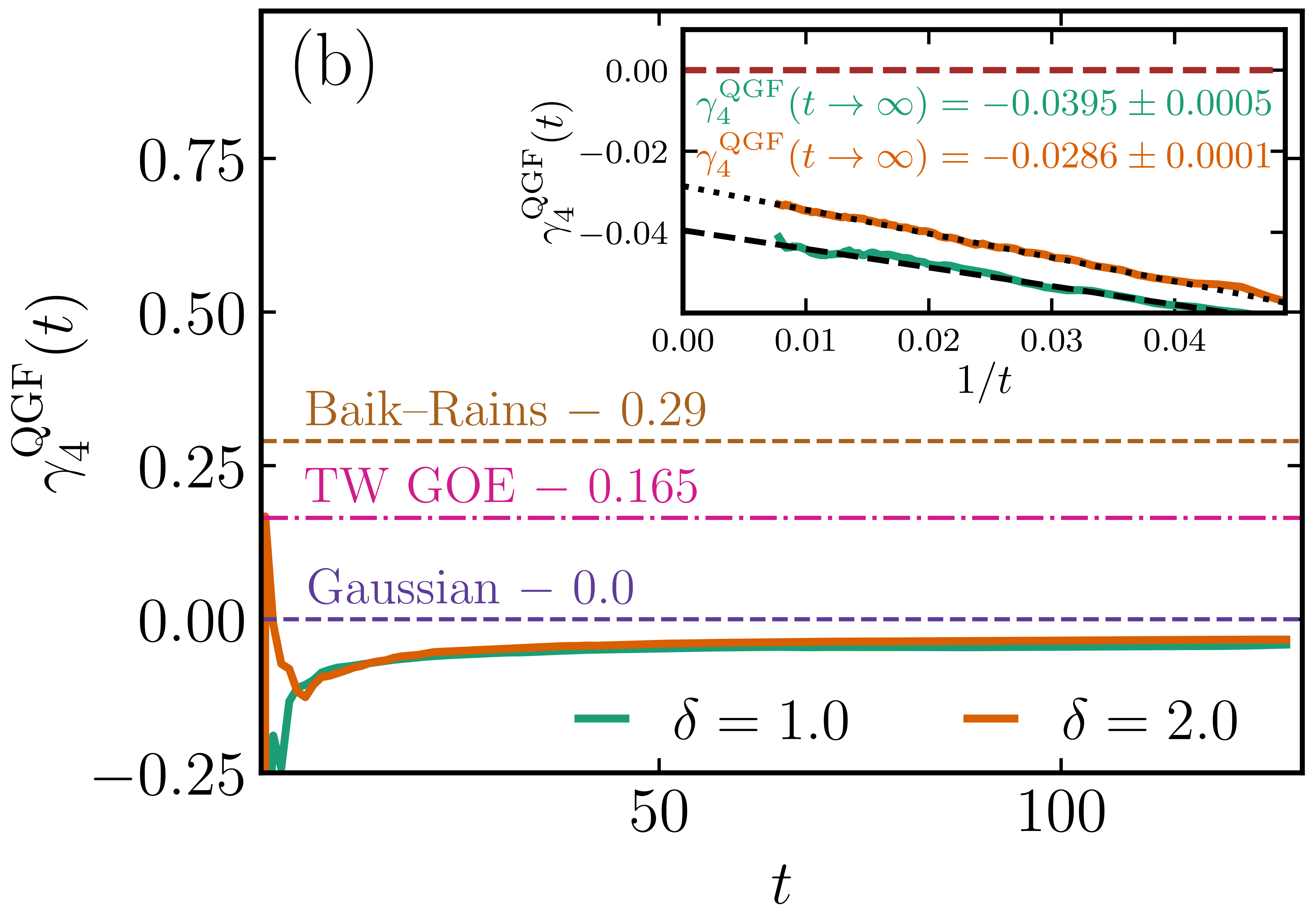
$$h_{[1,4]} = J_1 \mathbf{S}_1 \cdot \mathbf{S}_2 + J_1 \mathbf{S}_2 \cdot \mathbf{S}_3 + J_2 \mathbf{S}_1 \cdot \mathbf{S}_3 + J_2 \mathbf{S}_2 \cdot \mathbf{S}_4 + J_\chi \mathbf{S}_1 \cdot (\mathbf{S}_2 \times \mathbf{S}_3) - J_\chi \mathbf{S}_2 \cdot (\mathbf{S}_3 \times \mathbf{S}_4),$$

$$U_{[4j-1,4j+2]} = \mathrm{e}^{\mathrm{i}\tau h_{[4j-1,4j+2]}} + O(\tau^3)$$



$$\check{R}(\tau) = (\mathbb{1} + \mathrm{i}\tau P)/(1 + \mathrm{i}\tau)$$

integrable XXX gate



kurtosis still not KPZ

Summary

1. XXX circuit $\longrightarrow$ chiral spin models with broken ***P*** and ***T*** symmetries
2. fluctuation symmetry may arise from space (“crystallographic”) symmetries
3. integrable models need not have “internal” fluctuation symmetry (this does not preclude a more general external one, e.g., $s_1 s_2 \leftrightarrow s_2 s_1$)

(One?) message: relevant space-time symmetries are microscopic ones, not macroscopic ones

Outlook / work in progress: transport in spin ladders $\longrightarrow$ transport in spin ribbons

Thank you!