Modified rational six vertex model on a rectangular lattice

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In collaboration with Samuel Belliard

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The origin story

Model introduced by Pauling (1935) to describe crystalline structure of ice.

Figure: Semnoz ski resort (not this week...)

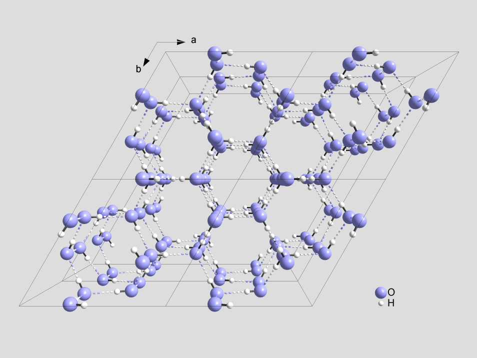

Figure: Crystalline structure of ice.

Construction of the six-vertex model and partition function

2D lattice with 6 different vertices, coming from the ice rule.

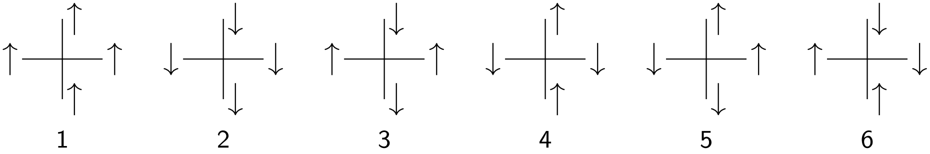

Figure: The six different vertices with spins.

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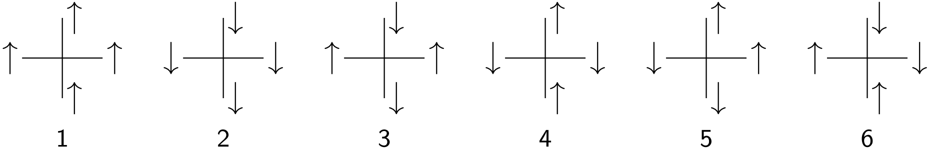

Figure: The six different vertices with spins.

In the zero-field model (inversion symmetry of all spins), three different Boltzmann weights :

$$a(u,v), \quad b(u,v) \quad \text{and} \quad c(u,v).$$

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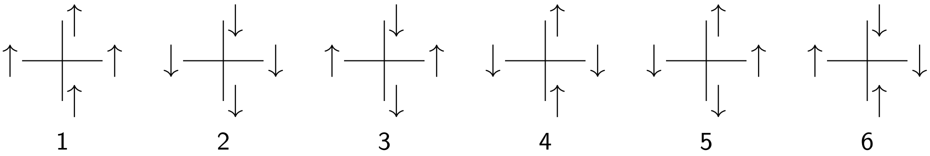

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In the zero-field model (inversion symmetry of all spins), three different Boltzmann weights :

$$a(u,v), \quad b(u,v) \quad \text{and} \quad c(u,v).$$

The partition function is the sum over all the states $\{l\}$ of the Boltzmann weights :

$$Z = \sum_{\{l\}} a^{n^l_{1,2}} \, b^{n^l_{3,4}} \, c^{n^l_{5,6}}$$

Inhomogeneous lattice and Quantum Inverse Scattering Method

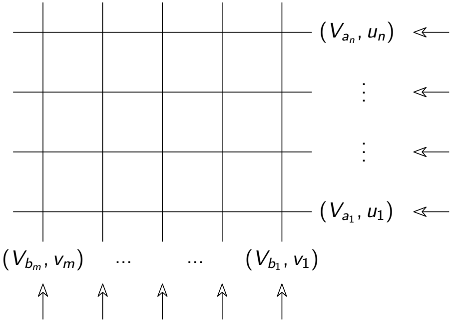

Figure: Inhomogeneous model on the $n \times m$ rectangular lattice. $V_k \sim \mathbb{C}^2$ and $u_i, v_j \in \mathbb{C}$. *The hollow arrows indicate the direction of reading for the lattice.*

Equivalence between spins and vectors of canonical basis of $\mathbb{C}^2$:

$$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

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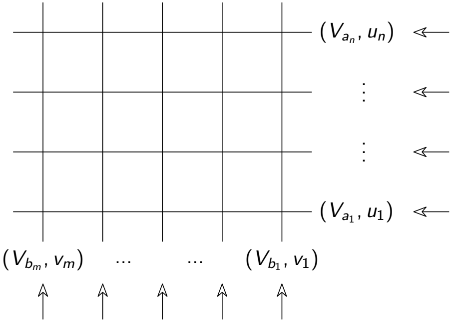

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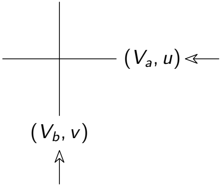

Figure: R-matrix associated to one vertex, $R_{ab}(u, v)$, acting on $V_a \otimes V_b$ (*line* $\otimes$ *column*).

Yang-Baxter equation

The R-matrix satisfies the Yang-Baxter equation (equation acting on $V_a \otimes V_b \otimes V_c$) to respect the symmetries of the system :

$$R_{ab}(u_a, u_b)R_{ac}(u_a, u_c)R_{bc}(u_b, u_c) = R_{bc}(u_b, u_c)R_{ac}(u_a, u_c)R_{ab}(u_a, u_b).$$

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Searching $R(u, v)$ in the following form,

$$R(u,v) = \begin{pmatrix} a(u,v) & 0 & 0 & 0 \\ 0 & b(u,v) & c(u,v) & 0 \\ 0 & c(u,v) & b(u,v) & 0 \\ 0 & 0 & 0 & a(u,v) \end{pmatrix}$$

Simplest solution (one nomalization) : XXX model

$$a(u,v) = \frac{u - v + c}{c},$$
$$b(u,v) = \frac{u - v}{c},$$
$$c(u,v) = 1.$$

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Domain Wall Boundary Conditions (DWBC) and Izergin determinant

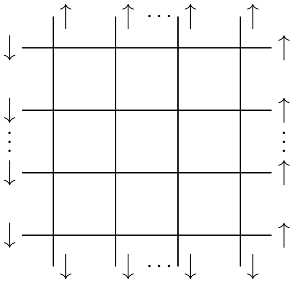

Figure: Square lattice ($m = n$) with DWBC, Korepin (1982)

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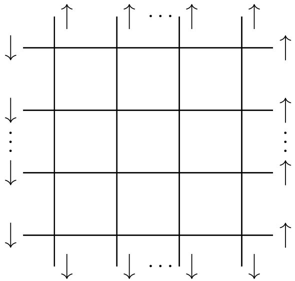

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Definition

$$Z_n(\bar{u}|\bar{v}) = \langle W| \otimes \langle N| \left(\prod_{i,j=1}^{n} R_{a_i b_j}(u_i, v_j) \right) |E\rangle \otimes |S\rangle$$

$$\langle N| = \bigotimes_{j=1}^{n} \langle \uparrow|_{b_j}\,, \qquad \langle W| = \bigotimes_{i=1}^{n} \langle \downarrow|_{a_i}\,,$$

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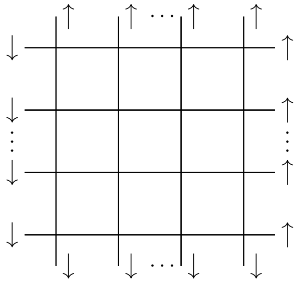

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Following Korepin's works, Izergin (1987) obtained a formula in terms of a determinant, called the **Izergin determinant**

$$Z_n(\bar{u}|\bar{v}) = \frac{1}{g(\bar{u}, \bar{v})} K_n(\bar{u}|\bar{v}).$$

Partial DWBC

Foda-Wheeler (2012) : generalization of DWBC to the $n \times m$ rectangular lattice, $n \leqslant m$.

West and east unchanged. Place $k \in [\![0, m-n]\!]$ spins on the north and $m-n-k$ on the south in the opposite direction, compared to DWBC.

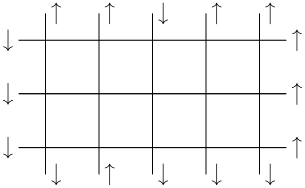

Figure: One configuration for $n = 3$, $m = 5$ and $k = 1$.

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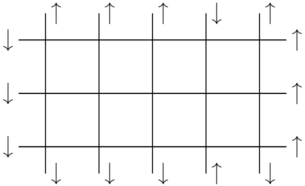

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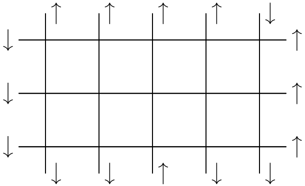

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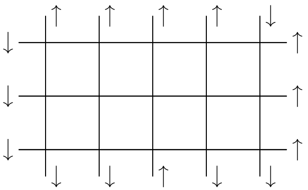

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Construction of $Z_{nm}(\bar{u}|\bar{v}|k)$:

- Start from a m square lattice.
- Remove successively extra rows to obtain the rectangular lattice, doing tend extra parameters towards infinity $(\frac{c}{u}R(u,v) \xrightarrow[u\to\infty]{} I)$.

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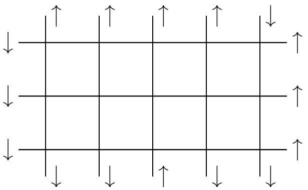

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Obtained an **Izergin-Vandermonde determinant**

$$Z_{nm}(\bar{u}|\bar{v}|k) = \binom{m-n}{k} \frac{1}{g(\bar{u},\bar{v})} K_{nm}(\bar{u}|\bar{v})$$

where the n first rows are Izergin type and the $m-n$ others are Vandermonde type.

Homogeneous and thermodynamic limits

Homogeneous limit

Izergin-Coker-Korepin (1992) : starting from the Izergin determinant, $u_i \to u$ and $v_j \to v$.

Homogeneous and thermodynamic limits

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Doing successively for each row and each column, using the Taylor expansion of the function inside the determinant,

$$Z_n(x) = \frac{\alpha_{nn}}{\phi^{n^2}(x)} \underbrace{\det_{1 \leqslant i,j \leqslant n} \left(\phi^{(i+j-2)}(x) \right)}_{\tau_n(x)}$$

with $x = u - v$ and $\phi(x) = \frac{c}{x} - \frac{c}{x+c}$.

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Thermodynamic limit (infinite lattice : $n \to \infty$)

Korepin and Zinn-Justin (2000) for XXZ.
$\tau_n(x)$ is a Hankël determinant and satisifies the Toda equation (in Hirota form)

$$\tau_{n+1}\tau_{n-1} = \tau_n \tau_n'' - (\tau_n')^2.$$

Expectation :

$$\ln(Z_n(x)) = -n^2 \frac{F(x)}{k_B T} + o(n^2)$$

where $F(x)$ is the bulk free energy.
Computed some physical characteristics.

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General Boundary Conditions (GBC)

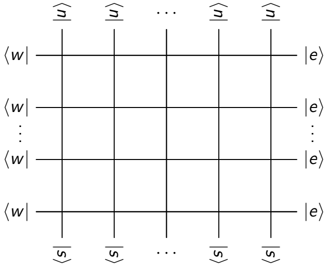

Figure: The $n \times m$ rectangular lattice with GBC.

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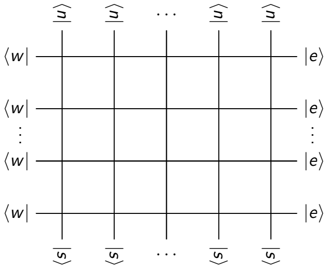

Figure: The $n \times m$ rectangular lattice with GBC.

Definition (Belliard Pimenta Slavnov 2024)

$$Z_{nm}(\bar{u}|\bar{v}|B|\hat{C}) = \operatorname*{tr}_{\bar{a},\bar{b}} \left(\left(\prod_{j=1}^{m} \hat{C}_{b_j} \right) \left(\prod_{i=1}^{n} B_{a_i} \right) \left(\prod_{i=1}^{n} \prod_{j=1}^{m} R_{a_i b_j}(u_i, v_j) \right) \right)$$

Twist (boundary) operators :

$$B = |e\rangle \otimes \langle w| \quad \text{and} \quad \hat{C} = |s\rangle \otimes \langle n| \,.$$

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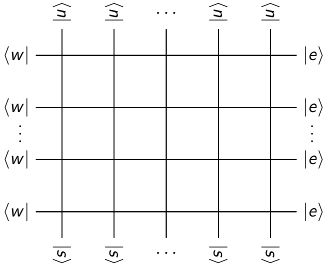

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Modified Izergin determinant

Using tools of the Modified Algebraic Bethe Ansatz and solving the obtained linear system, formula in terms of the **modified Izergin determinant**

$$Z_{nm}(\bar{u}|\bar{v}|B|\hat{C}) = \frac{\mathrm{tr}(B)^n \mathrm{tr}(\hat{C})^m}{(1-\beta)^m} \frac{1}{g(\bar{u},\bar{v})} K_{nm}^{(\beta)}(\bar{u}|\bar{v})$$

$$\beta = 1 - \frac{\mathrm{tr}(B)\mathrm{tr}(\hat{C})}{\mathrm{tr}(B\hat{C})}, \quad \mathrm{tr}(B) = \langle w|e\rangle\,, \quad \mathrm{tr}(\hat{C}) = \langle n|s\rangle \quad \text{and} \quad \mathrm{tr}(B\hat{C}) = \langle n|e\rangle\,\langle w|s\rangle\,.$$

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From previous works, two known formulae for rectangular lattice and one specificly for square lattice, having same form as the known formula of the Izergin determinant

$$K_{nn}^{(1)}(\bar{u}|\bar{v}) = K_n(\bar{u}|\bar{v}).$$

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$$K_{nn}^{(1)}(\bar{u}|\bar{v}) = K_n(\bar{u}|\bar{v}).$$

Last one more practical to compute homogeneous limit $\longrightarrow$ Want to generalize this formula to rectangle.

New formula and equivalence with the other two ones

Using the same method as Foda-Wheeler, new formula for $Z_{nm}(\bar{u}|\bar{v}|B|\hat{C})$ (and so for $K_{nm}^{(\beta)}(\bar{u}|\bar{v})$) with same form as the Izergin-Vandermonde determinant : $K_{nm}^{(1)}(\bar{u}|\bar{v}) = K_{nm}(\bar{u}|\bar{v})$.

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$$\frac{h(\bar{u},\bar{v})}{\Delta(\bar{u})\Delta'(\bar{v})} \times \begin{cases} \det\limits_{1\leqslant i,j\leqslant m}\left(\begin{cases} \phi_\beta(u_i - v_j) & \text{if } i \leqslant n \\ \psi_{m-i}(-v_j) & \text{if } i \geqslant n+1 \end{cases}\right) & \text{if } n \leqslant m \\ \det\limits_{1\leqslant i,j\leqslant n}\left(\begin{cases} \phi_\beta(u_i - v_j) & \text{if } j \leqslant m \\ \psi_{n-j}(u_i) & \text{if } j \geqslant m+1 \end{cases}\right) & \text{if } n \geqslant m \end{cases}$$

proved for $n \geqslant m$

proved for $n \leqslant m$

$$\det_{1\leqslant k,l\leqslant m}\left(-\beta\delta_{kl} + \frac{f(\bar{u},v_k)f(v_k,\bar{v}_k)}{h(v_k,v_l)}\right) \longleftrightarrow (1-\beta)^{m-n}\det_{1\leqslant k,l\leqslant n}\left(\delta_{kl}f(u_k,\bar{v}) - \beta\frac{f(u_k,\bar{u}_k)}{h(u_k,u_l)}\right)$$

proved by Gordsky, Zabrodin, Zotov 2014

Figure: Ways of proofs between the three formulae for $K^{(\beta)}_{nm}(\bar{u}|\bar{v})$.

New formula and equivalence with the other two ones

Using the same method as Foda-Wheeler, new formula for $Z_{nm}(\bar{u}|\bar{v}|B|\hat{C})$ (and so for $K_{nm}^{(\beta)}(\bar{u}|\bar{v})$) with same form as the Izergin-Vandermonde determinant : $K_{nm}^{(1)}(\bar{u}|\bar{v}) = K_{nm}(\bar{u}|\bar{v})$.

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Figure: Ways of proofs between the three formulae for $K_{nm}^{(\beta)}(\bar{u}|\bar{v})$.

Last equality : rational version of source identity. Exist trigonometric and elliptic versions (see Motegi-Ohkawa 2024) $\rightarrow Z_{nm}$ for XXZ six-vertex and eight-vertex model with GBC ?

Homogeneous limit and thermodynamic limit (infinite lattice)

Homogeneous limit : Using this new formula and same method as Izergin-Coker-Korepin, computation of the homogeneous limit ($u_i \to u$, $v_j \to v$ and $x = u - v$) :

$$Z_{nm}(x|B|\hat{C}) = \frac{\alpha_{nm}}{\phi^{nm}(x)} \underbrace{\det_{1\leqslant i,j \leqslant \min(n,m)} \left(\phi_\beta^{(d+i+j-2)}(x)\right)}_{\tau_{nm}(x)}$$

with $\phi_\beta(x) = \frac{c}{x} - \beta\frac{c}{x+c}$ ($\phi_{\beta=1} = \phi$) and $d = |n-m|$.

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Infinite lattice : Using same method as Korepin and Zinn-Justin, computation of the thermodynamic limit : $n, m \to \infty$ with d constant. Indeed,

$$\tau_{n+1,m+1}\tau_{n-1,m-1} = \tau_{n,m}\tau''_{n,m} - (\tau'_{n,m})^2.$$

Expectation :

$$\ln(Z_{nm}(x|B|\hat{C})) = -nm\frac{F(x)}{k_B T} + o(nm)$$

where $F(x)$ is the bulk free energy.

Thermodynamic limit (infinite lattice)

Solving a non-linear 2th order ODE and using values of $F(0)$, $F'(0)$ and $F''(0)$,

$$\frac{F(x)}{k_B T} = \begin{cases} -\ln(1+x/c) + \ln\left(\dfrac{\sinh\left(\sqrt{\tilde{\beta}}x/c\right)}{\sqrt{\tilde{\beta}}x/c}\right) & \text{if } \tilde{\beta} > 0 \\ -\ln(1+x/c) & \text{if } \tilde{\beta} = 0 \\ -\ln(1+x/c) + \ln\left(\dfrac{\sin\left(\sqrt{-\tilde{\beta}}x/c\right)}{\sqrt{-\tilde{\beta}}x/c}\right) & \text{if } \tilde{\beta} < 0 \end{cases}$$

with

$$\tilde{\beta} = \begin{cases} 0 & \text{if } d > 1 \\ 3\beta & \text{if } d = 1 \\ 3\beta(\beta-2) & \text{if } d = 0 \end{cases} .$$

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Boundary effects only for $d \leqslant 1$ and for $\tilde{\beta} \neq 0$, with an **additional term.** $\tilde{\beta} = 0$ seems to be a **phase transition**.
Otherwise, it is like the lattice contains only up spins or only down spins.
Other orders of the asymptotic development should be considered.

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Thermodynamic limit (infinite lattice) : a little physics

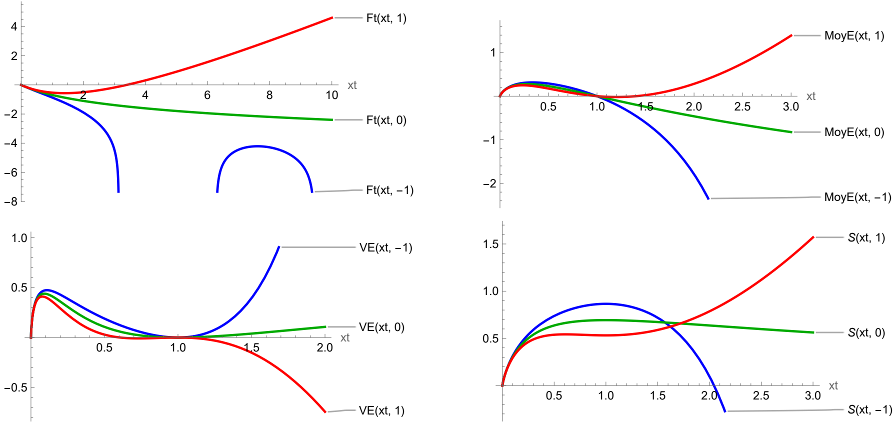

Figure: Graphs of $\frac{F}{k_B T}$, $\frac{\bar{E}}{k_B T}$, $\left(\frac{\Delta E}{k_B T}\right)^2 = \frac{C_V}{k_B}$ and $\frac{S}{k_B}$ as a function of $\tilde{x} = \frac{x}{c}$, for $\tilde{\beta} = -1, 0, 1$.

Outlooks

- Partition function for trigonometric six-vertex model (or eight-vertex model) with GBC. Physics may be more interesting, with interaction anisotropy between spins..
  $\longrightarrow$ Currently, work on the MABA for XXZ.
- Partition function for XXX six-vertex model with general integrable reflective end (work in progress - Belliard and Pimenta).

The end

Thank you for your attention !

Feel free to ask questions !

Expressions of Izergin and Izergin-Vandermonde determinants

Basic functions :

$$g(u,v)=\frac{c}{u-v},\quad f(u,v)=g(u,v)+1=\frac{u-v+c}{u-v},\quad h(u,v)=\frac{f(u,v)}{g(u,v)}=\frac{u-v+c}{c},$$

Vandermonde determinant : $\quad \Delta(\bar{u})=\prod_{i<j} g(u_j,u_i)^{-1} \quad$ and $\quad \Delta'(\bar{u})=\Delta(-\bar{u}).$

Izergin determinant :

$$K_n(\bar{u}|\bar{v})=\frac{h(\bar{u},\bar{v})}{\Delta(\bar{u})\Delta'(\bar{v})}\det_{1\leqslant i,j\leqslant n}\big(\phi(u_i-v_j)\big)\quad \text{with}\quad \phi(x)=\frac{c}{x}-\frac{c}{x+c}.$$

Izergin-Vandermonde determinant :

$$K_{nm}(\bar{u}|\bar{v})=\frac{h(\bar{u},\bar{v})}{\Delta(\bar{u})\Delta'(\bar{v})}\det_{1\leqslant i,j\leqslant m}\left(\begin{cases}\phi(u_i-v_j) & \text{if } \; i\leqslant n\\ \psi_{m-i}(-v_j) & \text{if } \; i\geqslant n+1\end{cases}\right)\quad \text{with}\quad \psi_k(x)=\left(-\frac{x}{c}\right)^k.$$