Integrability of Floquet PXP model at the deterministic point

Katja Klobas

School of Physics and Astronomy
University of Birmingham

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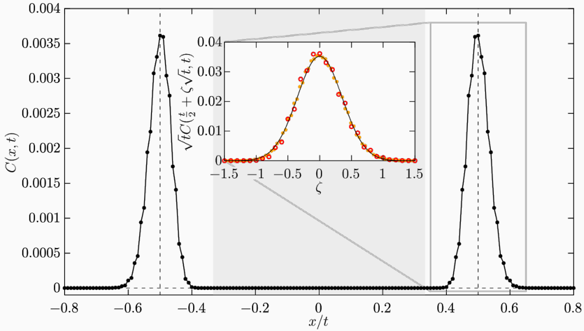
Many-body dynamics is hard, even in the presence of integrability.

Sometimes not the case!



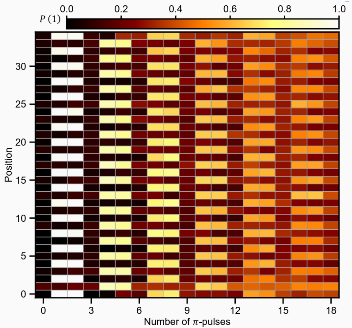
B. Buča, KK, T. Prosen, J. Stat. Mech. **2021**, 074001 (2021)

Is there a notion of dynamical integrability?

Implemented in dual-species Rydberg arrays

F. Cesa et al., 2026, arXiv:2601.16961

R. White et al., 2026, arXiv:2601.16257



Plan

The model: deterministic cellular automaton with PXP constraint

J. W. Wilkinson, KK, T. Prosen, J. P. Garrahan, Phys. Rev. E **102**, 062107 (2020)

Binary strings of length $L \in 2\mathbb{Z}$:

$$\underline{s}^t = (s_1^t, s_2^t, \ldots, s_L^t) \mapsto \underline{s}^{t+1} \qquad s_{L+1}^t \equiv s_1^t$$

$$s_j^{t+1} = \begin{cases} s_j^t, & j+t \equiv 0 \pmod 2 \\ \chi(s_{j-1}^t, s_j^t, s_{j+1}^t), & j+t \equiv 1 \pmod 2 \end{cases}$$

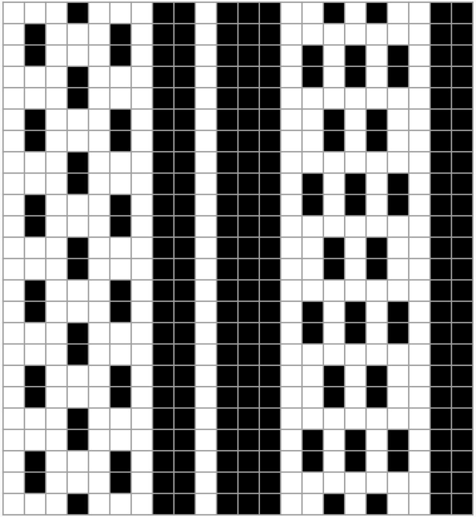
Local rule: deterministic flip if both neighbours are 0

$$\chi(s_1, s_2, s_3) = \delta_{s_1,0}\delta_{s_3,0}(1-s_2) + (1-\delta_{s_1,0}\delta_{s_3,0})s_2$$

$\rightarrow$ no evolution for $(s_j^t, s_{j+1}^t) = (1,1)$

From now: sector with Rydberg constraint

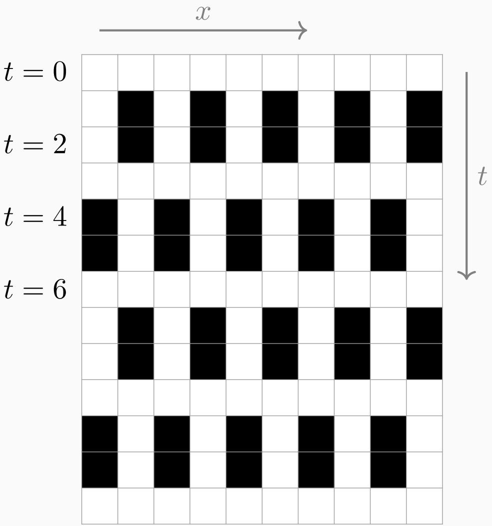
$$\mathcal{K}_L = \{\underline{s} \in \mathbb{Z}_2^{\times L}: \quad s_j + s_{j+1} \neq 2\}$$



Dynamics in the largest sector: Threefold vacuum

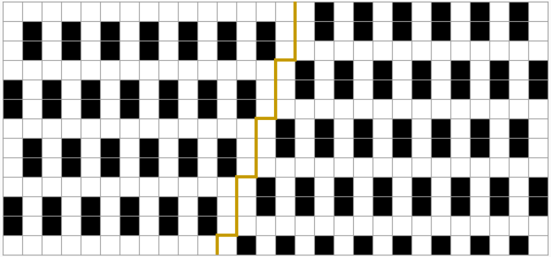
We have a short orbit:

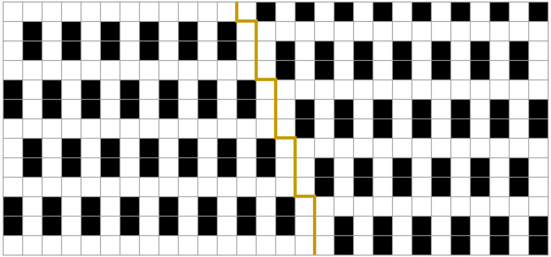
$$\underline{s}^0 = (0,0,0,0,0,0,\ldots,0,0)$$
$$\downarrow$$
$$\vdots$$
$$\downarrow$$
$$\underline{s}^2 = (0,1,0,1,0,1,\ldots,0,1)$$
$$\downarrow$$
$$\vdots$$
$$\downarrow$$
$$\underline{s}^4 = (1,0,1,0,1,0,\ldots,1,0)$$
$$\downarrow$$
$$\vdots$$
$$\downarrow$$
$$\underline{s}^6 = (0,0,0,0,0,0,\ldots,0,0)$$



Dynamics in the largest sector: Quasiparticles

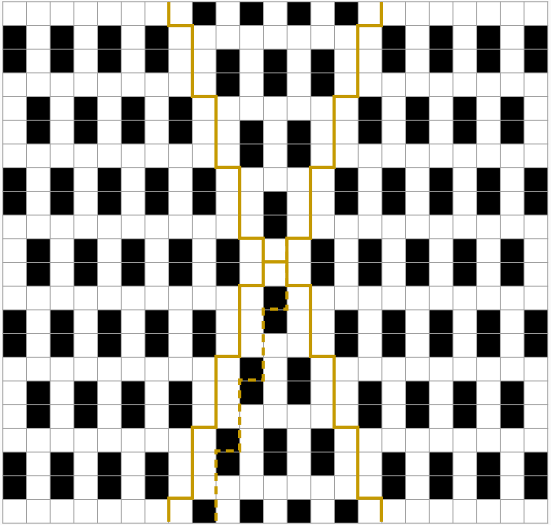
Domain walls: quasiparticles with $v = \pm\frac{1}{3}$





s^t_{j-2}	s^t_{j-1}	s^t_j	s^t_{j+1}	$\mathrm{sgn}(v)$
0	0	0	1	$(-1)^{t+j+1}$
1	0	0	0	$(-1)^{t+j+1}$
1	0	0	1	$(-1)^{t+j}$

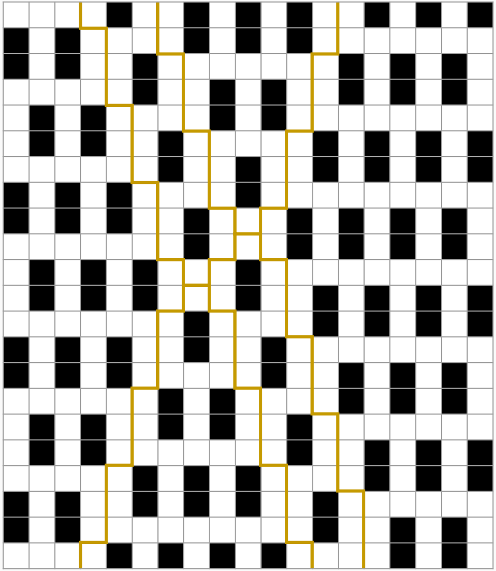
Dynamics in the largest sector: Quasiparticle interactions I



Nontrivial *attractive* interaction

Displacement after collision: $\pm\frac{4}{3}$ in the direction of movement

Dynamics in the largest sector: Quasiparticle interactions II



Generalises to all states in $\mathcal{K}_L$

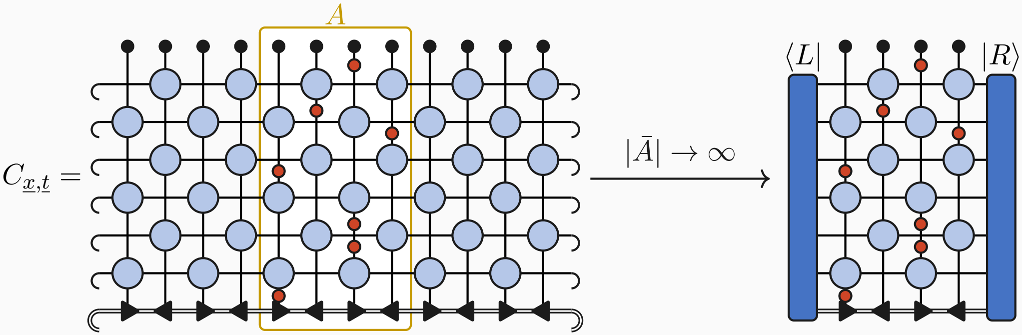
- stable quasi-particles with $v = \pm\frac{1}{3}$
- pairwise interactions
- displacements after collision: $\pm\frac{4}{3}$
- conserved asymptotic distances between particles with same velocity

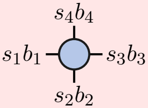
Looks integrable! Can we find Bethe equations?

Can we study finite subsystem dynamics?

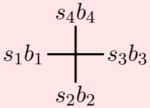
Finite subsystem dynamics

Consider: $C_{\underline{x},\underline{t}} = \mathrm{tr}\left[a_1(x_1,t_1)a_2(x_2,t_2)\cdots a_l(x_l,t_l)\rho\right]$ $\quad x_j \in A$ finite subsystem

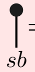




$= \delta_{s_4,\chi(s_1,s_2,s_3)}\delta_{b_4,\chi(b_1,b_2,b_3)}$



$= \prod_{j=1}^{3} \delta_{s_j,s_{j+1}}\delta_{b_j,b_{j+1}}$



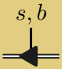
$= \delta_{s,b}$

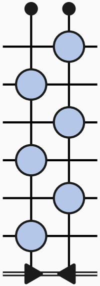
Gibbs state and corresponding transfer matrix

Family of stationary states: $z_\pm = \mathrm{e}^{-\mu_\pm}$

$\rho_{\mu_+,\mu_-} \propto$ 

$$= \delta_{s,b} \begin{bmatrix} \delta_{s,0} & 0 & 0 & z_+\delta_{s,0} \\ z_+\delta_{s,1} & 0 & \delta_{s,1} & z_-\delta_{s,1} \\ 0 & \delta_{s,0} & 0 & 0 \\ 0 & 0 & \delta_{s,0} & 0 \end{bmatrix} \xleftrightarrow{z_- \leftrightarrow z_+}$$



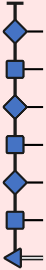
$\mathbb{W} =$ 

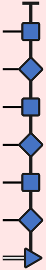
$$\mathbb{W}\,|R\rangle = \Lambda\,|R\rangle$$

$$\langle L|\,\mathbb{W} = \Lambda\,\langle L|$$

$$\lim_{L\to\infty} \frac{\mathrm{tr}\left[\rho_{\mu_+,\mu_-}\right]}{\Lambda^{L/2}} = 1$$

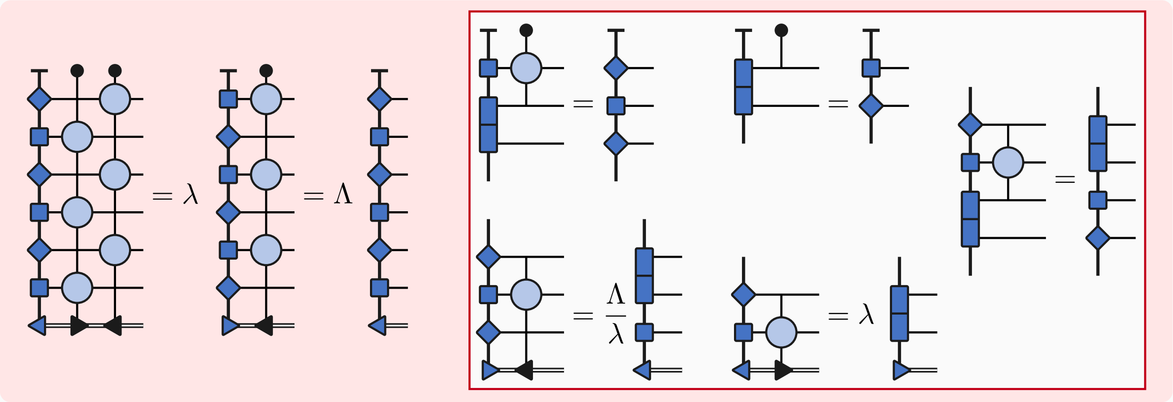
Ansatz

$\langle L| =$ 

$|R\rangle =$ 

Exact fixed points

KK, 2026, arXiv:2606.27337



Solution exists!

12×12 matrices $A_{sb}(\vartheta)$, $B_{sb}(\vartheta)$

Re-parametrisation of Gibbs state

- $\vartheta_\pm$ parameters of fixed points with $\lim_{L\to\infty} \mathrm{tr}\left[\rho_{\mu_+,\mu_-}\right]\Lambda^{-\frac{L}{2}} = 1$

$$\langle L| = \langle L(\vartheta_-)| \qquad |R\rangle = |R(\vartheta_+)\rangle \qquad \vartheta_- = \frac{z_-\left[z_-(\Lambda-1)+z_+^2\right]}{\Lambda\left[(\Lambda-1)z_+ + z_-^2\right]} \xleftrightarrow{z_+\leftrightarrow z_-} \vartheta_+$$

- Mapping can be inverted

$$z_- = \frac{\vartheta_-}{(1-\vartheta_-)^{\frac{4}{3}}(1-\vartheta_+)^{\frac{4}{3}}} \xleftrightarrow{\vartheta_+\leftrightarrow\vartheta_-} z_+ \qquad \Lambda = \frac{1}{(1-\vartheta_+)(1-\vartheta_-)}$$

Equivalent rewriting

$$\boxed{\log\eta_\nu = \mu_\nu + \sum_{\nu'\in\{+,-\}} T_{\nu\nu'}\log[1+\eta_{\nu'}^{-1}]} \qquad \eta_\nu = \frac{1-\vartheta_\nu}{\vartheta_\nu} \quad T_{++} = T_{--} = \frac{1}{3} \quad T_{+-} = T_{-+} = \frac{2}{3}$$

Thermal TBA for 2 modes $\nu\in\{-,+\}$ with filling functions ϑ_ν and scattering kernel $T_{\nu\nu'}$.

Coordinate Bethe ansatz

T. M. Edge, KK, in preparation, 2026

Goal: find eigenstates of the time-evolution operator U $\leftarrow U\,|\underline{s}^t\rangle = |\underline{s}^{t+2}\rangle$

$$|\Psi(\underline{k},\underline{\nu})\rangle = \sum_{0\le x_1<x_2<\ldots<x_n<L} \psi(\underline{x},\underline{k},\underline{\nu})\,|\underline{x},\underline{\nu}\rangle \qquad \psi(\underline{x},\underline{k},\underline{\nu}) = \sum_{\pi\in S_n} A_\pi(\underline{k},\underline{\nu})\mathrm{e}^{i\sum_{j=1}^n k_{\pi_j} x_j}$$

pairwise scattering $\to A_\pi$ factorised into *two-body* contributions

Problems:

- 2 left(right) movers never interact
- degeneracy due to no $\underline{k}$ dependence

Rule54 has the same issue

A. Friedman, S. Gopalakrishnan, R. Vasseur, Phys. Rev. Lett. **123**, 170603 (2019)

$$F(\lambda) = \mathrm{e}^{-i\lambda H} U, \qquad [H,U]=0$$

H moves quasiparticles around

1. Find H for our model
2. Find Bethe equations for $F(\lambda)$
3. Take limit $\lambda \to 0$

Operator H

$[U,H]P_{\mathcal{K}_L}=0:\ F(\lambda)=\mathrm{e}^{-i\lambda H}U$ dispersive $\leftarrow P_{\mathcal{K}}$ projector to $\mathcal{K}_L$

$H\,|\underline{x},\underline{\nu}\rangle$ superposition of states with the **same** quasiparticles shifted by ± 2

$$\left.\begin{aligned}
|\cdots 000001\cdots\rangle &\leftrightarrow |\cdots 000101\cdots\rangle\\
|\cdots 100000\cdots\rangle &\leftrightarrow |\cdots 101000\cdots\rangle\\
|\cdots 100101\cdots\rangle &\leftrightarrow |\cdots 101001\cdots\rangle
\end{aligned}\right\}\text{ one particle}$$

Extensive H with local support 8:

$$\left.\begin{aligned}
|\cdots 1010001\cdots\rangle &\leftrightarrow |\cdots 1001001\cdots\rangle\\
|\cdots 1000101\cdots\rangle &\leftrightarrow |\cdots 1001001\cdots\rangle\\
|\cdots 00001000\cdots\rangle &\leftrightarrow |\cdots 00010000\cdots\rangle
\end{aligned}\right\}\text{ two particles}$$

$$|\cdots 10001001\cdots\rangle \leftrightarrow |\cdots 10010001\cdots\rangle\}\text{ three particles}$$

Quasi-particle positions

What are the states $|\underline{x}, \underline{\nu}\rangle$?

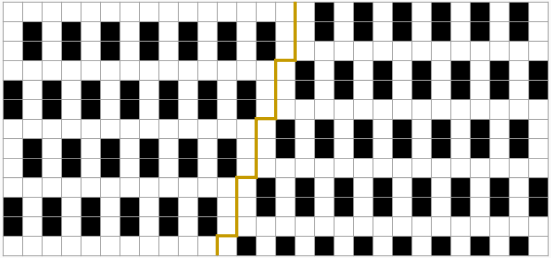
s^t_{j-2}	s^t_{j-1}	s^t_j	s^t_{j+1}	$\mathrm{sgn}(v)$
0	0	0	1	$(-1)^{t+j+1}$
1	0	0	0	$(-1)^{t+j+1}$
1	0	0	1	$(-1)^{t+j}$

$|\underline{x}^+, \underline{x}^-\rangle$: $\underline{x}^\nu = (x^\nu_1, x^\nu_2, \ldots, x^\nu_{n_\nu})$ $\quad 0 \leq x^\nu_1 < \ldots < x^\nu_{n_\nu} < \frac{L}{2}$ $\quad x^\nu_j \in \frac{\nu}{12} + \frac{1}{3}\mathbb{Z}$

Well-separated particles:

$$U\,|\ldots x^\nu_j \ldots\rangle = |\ldots x^\nu_j + \frac{\nu}{3} \ldots\rangle$$

$$H\,|\ldots x^\nu_j \ldots\rangle = \sum_{\delta^\nu_j \in \{-1,1\}} |\ldots x^\nu_j + \delta^\nu_j \ldots\rangle$$



Bethe equations

1. Solve 2-particle sector only 2 of the opposite kind
2. Solve 3-particle sector only 3 of the same kind
3. Extract A_π and check whether Bethe ansatz works for general quasi-particle sectors

$$\left|\underline{k}^+,\ \underline{k}^-\right\rangle = \sum_{\underline{x}^+,\ \underline{x}^-} \sum_{\sigma^+,\sigma^-} \mathrm{e}^{i\sum_\nu \sum_j k^\nu_{\sigma^\nu(j)} x^\nu_j} (-1)^{|\sigma^+|+|\sigma^-|} A_{\sigma^+,\sigma^-} \left|\underline{x}^-,\ \underline{x}^+\right\rangle$$

$$A_{\sigma^+,\ \sigma^-} = \left(\prod_\nu \prod_{\substack{\sigma^\nu(l)>\sigma^\nu(j)\\ l<j}} \exp\left[\frac{i}{3}(k^\nu_{\sigma^\nu(l)} - k^\nu_{\sigma^\nu(j)})\right] \right) \left(\prod_{x^+_l<x^-_j} \exp\left[\frac{2i}{3}(k^+_{\sigma^+(l)} - k^-_{\sigma^-(j)})\right] \right)$$

Quantisation condition:

$$\exp\left[ik^\nu_j \frac{L}{2}\right] = \exp\left[\frac{i}{3}\sum_l (k^\nu_j - k^\nu_l) + \frac{2i}{3}\sum_l (k^\nu_j - k^{-\nu}_l)\right]$$

compatible with “TBA” expression from before

Outlook

Bethe ansatz

- Warning: work in progress, might be wrong
- Bethe equations very similar to Rule 54

  A. Friedman, S. Gopalakrishnan, R. Vasseur, Phys. Rev. Lett. **123**, 170603 (2019)
- Can this be used to explore ABA and integrable deformations?

  T. Gombor, B. Pozsgay, SciPost Phys. **16**, 114 (2024)

  C. Paletta, T. Prosen, 2026, arXiv:2603.25424

Finite-subsystem dynamics

- Exact fixed points with bond-dimension 12
- Link to quantum transfer matrix?

  A. Klümper, in *Quantum magnetism* (Springer, 2008), pp. 349–379
- Solvable initial states?
- Other models?