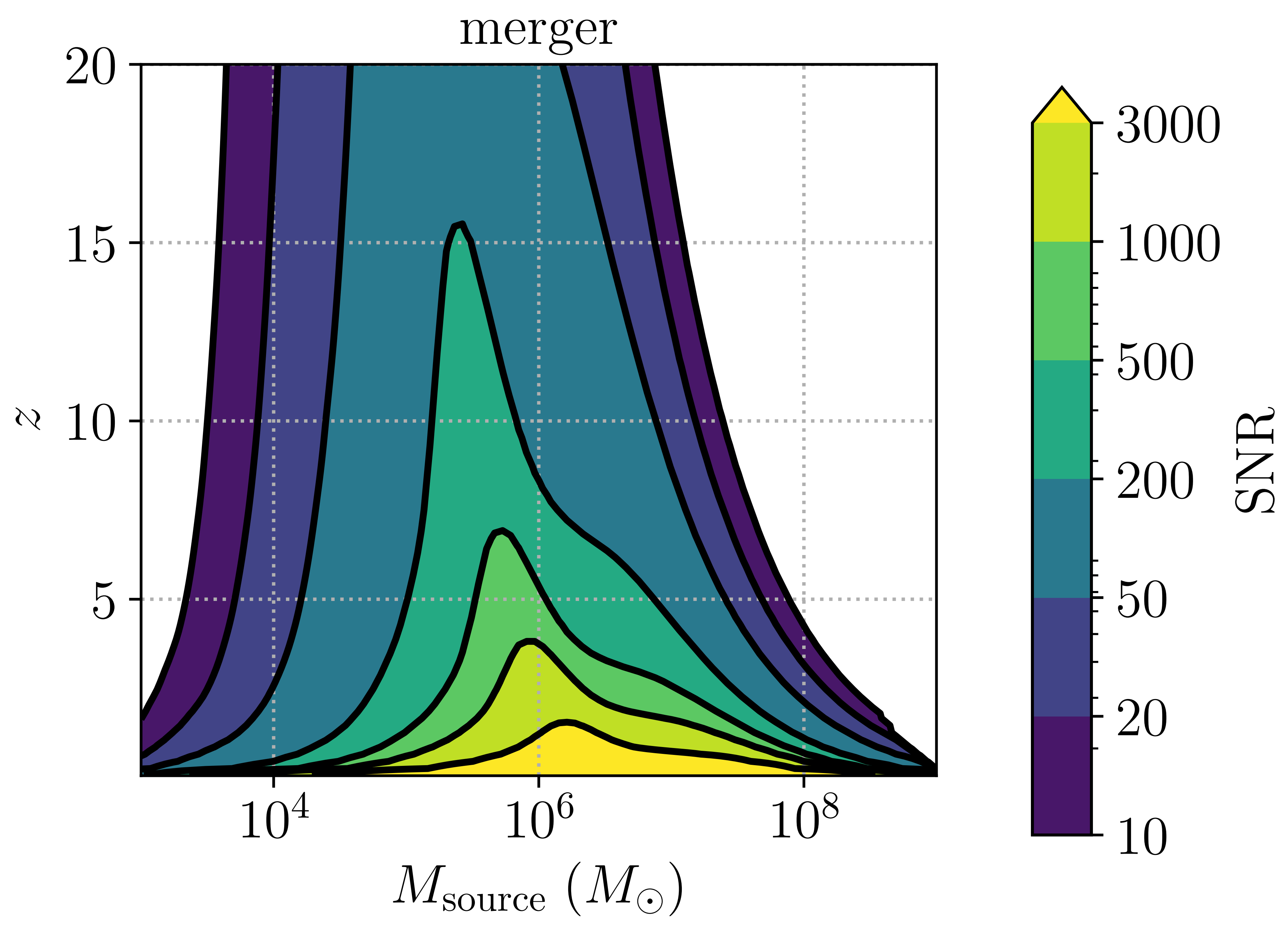
Waveform systematics for massive black hole binaries in LISA

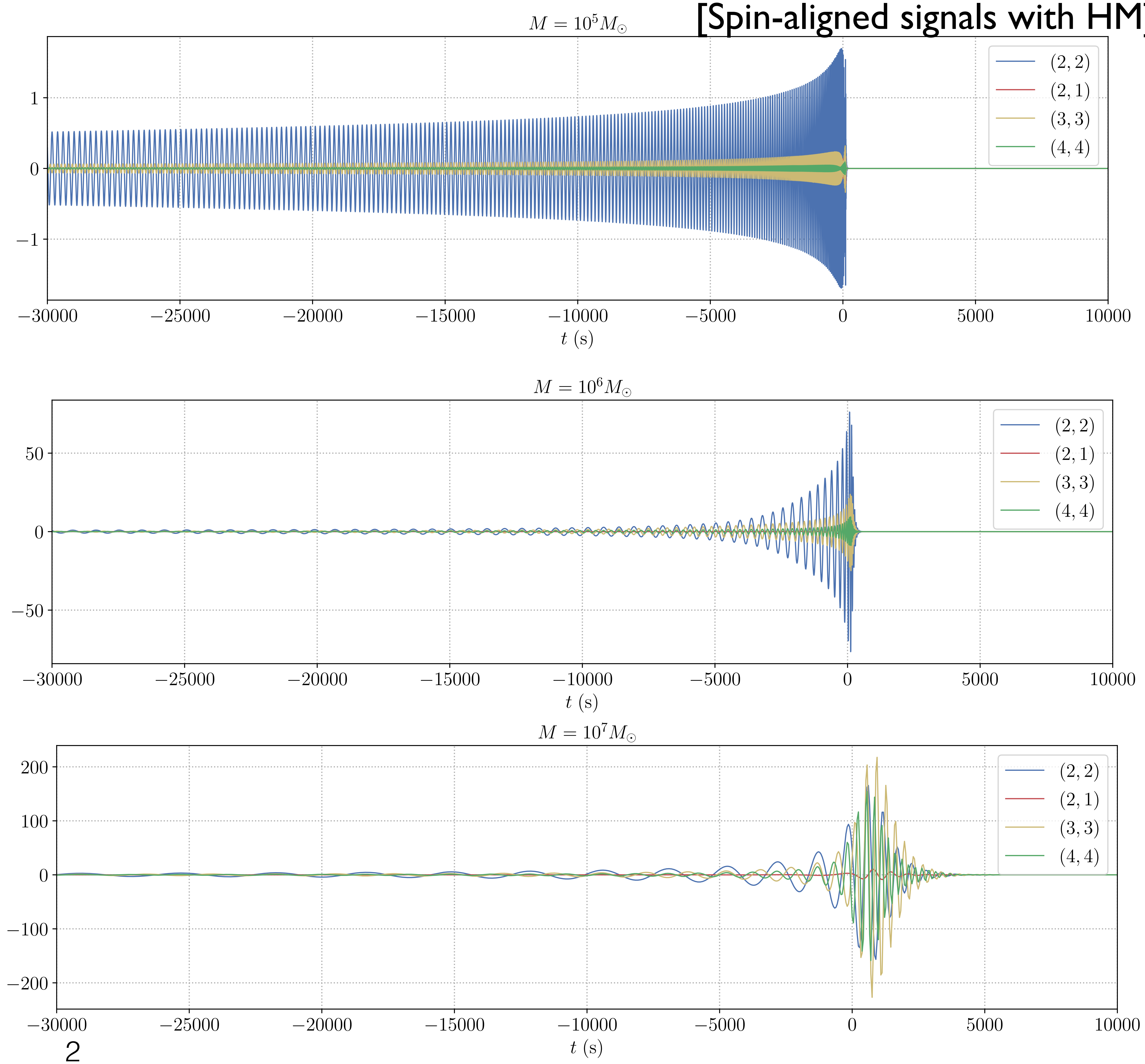
Sylvain Marsat (L2IT, Toulouse)

MBHB signals in LISA



Whitened time-domain signals

[Spin-aligned signals with HM]



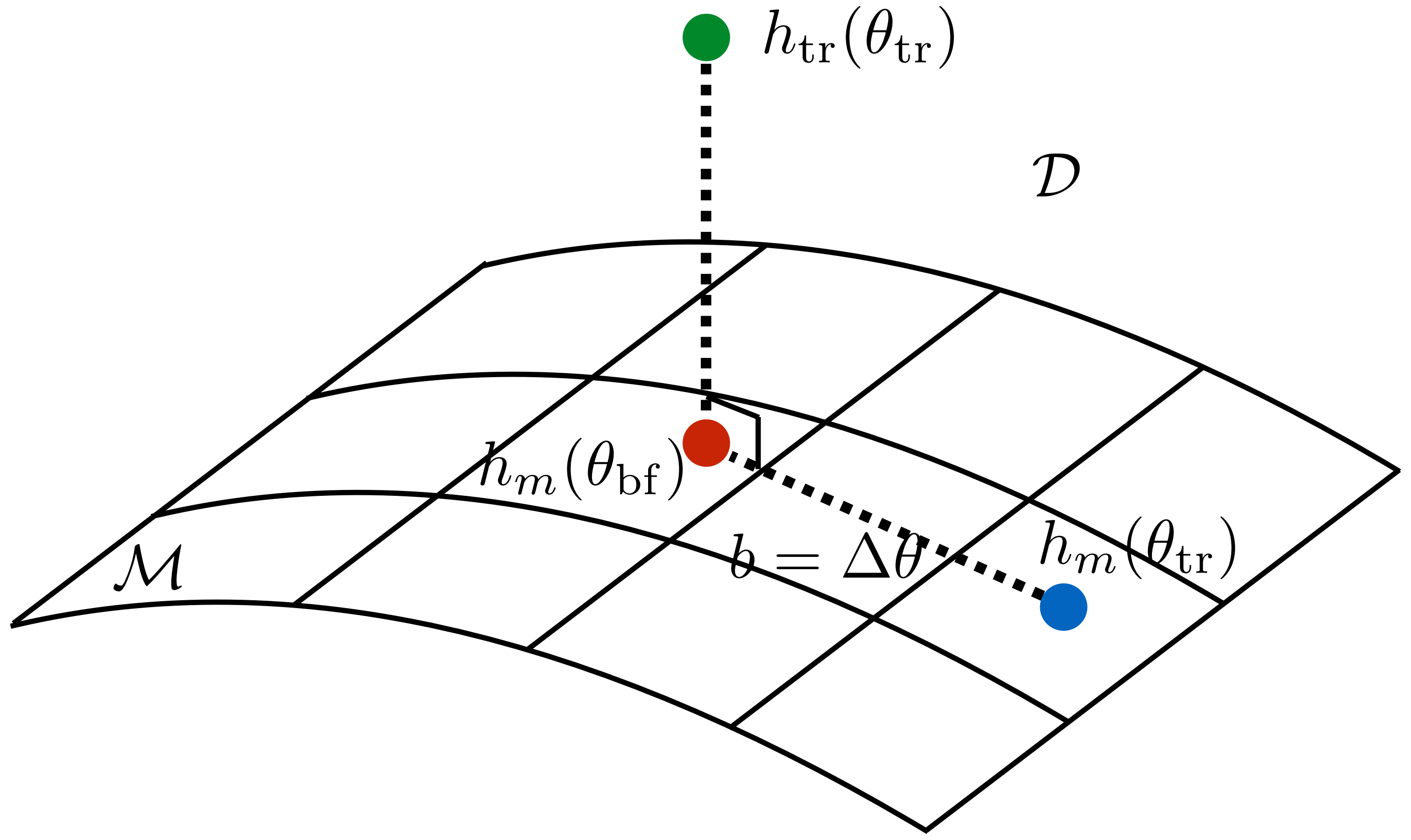
Science case for MBHB systematics

MBHBs high-SNR sources for LISA, waveform systematics crucial for:

- golden events for EM counterparts
- golden events for TGR
- population inference and cosmology
- global fit and residuals

Broader parameter space than LVK: high-q, high-spins, precession, eccentricity

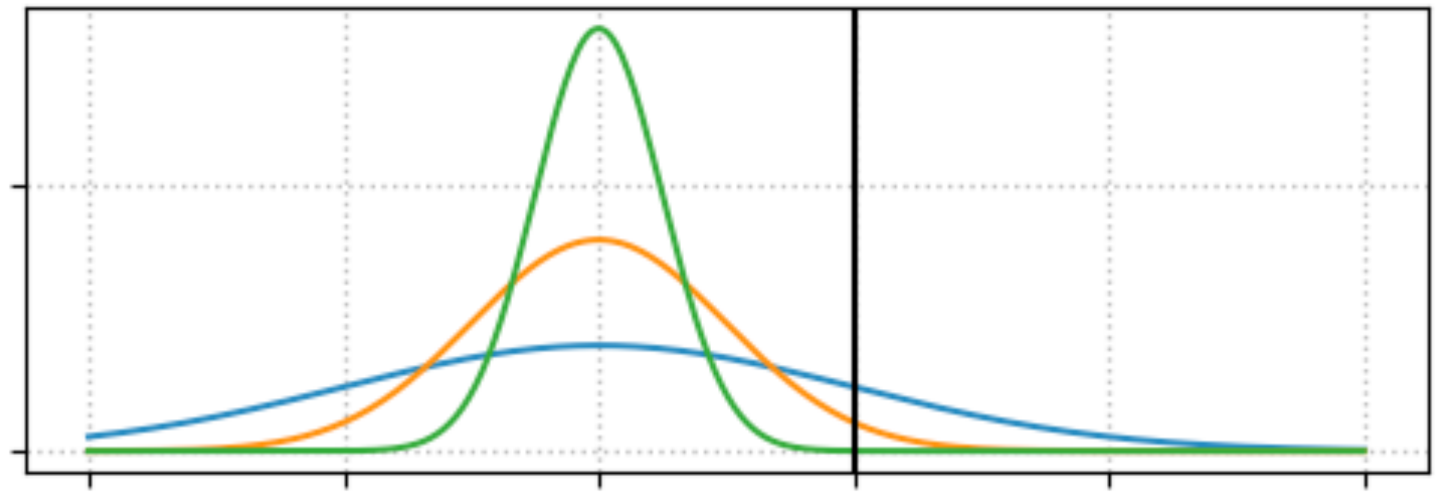
Waveform systematics and parameter estimation



Systematic biases:

Ignoring the effect of the noise, bias given by the **best-fit** parameters on the model signal manifold: $\Delta\theta = \theta_{\rm bf} - \theta_{\rm tr}$

- the **bias** is SNR-independent, requires to explore the full parameter space **[expensive]**
- the statistical errors scale with SNR



Mismatch, used in waveform modelling. Optimization over time/phase/polarization:

$$\mathrm{MM} = 1 - \max_{t,\varphi,\psi,\dots} \frac{(h_m|h_{\rm tr})}{\sqrt{(h_m|h_m)}\sqrt{(h_{\rm tr}|h_{\rm tr})}}$$

- Computed locally **[fast]**
- SNR-independent
- Different versions: single-detector optimized over sky, combining $h_+, h_\times$

Indistinguishability criterion:

[Lindblom&al 2008]
[Chatziioannou&al 2019]
[Toubiana-Gair 2024]

$$\ln \mathcal{L}(\theta) = -\frac{1}{2}(h(\theta) - h_{\rm tr}|h(\theta) - h_{\rm tr})$$

$$\ln \mathcal{L}(\theta_{\rm bf}) \sim \ln \mathcal{L}(\theta_{1-\sigma})$$

$$\mathrm{MM} < \frac{D}{2}\frac{1}{\mathrm{SNR}^2}$$

$\mathrm{SNR} \sim 10^3$ for LISA !

- Constant D: dimension, approximate
- Scaling SNR^2 robust

Is this criterion a good representation of requirements ?

Linearized biases (Cutler-Vallisneri):

[Flanagan-Hughes 1997]
[Cutler-Vallisneri 2007]

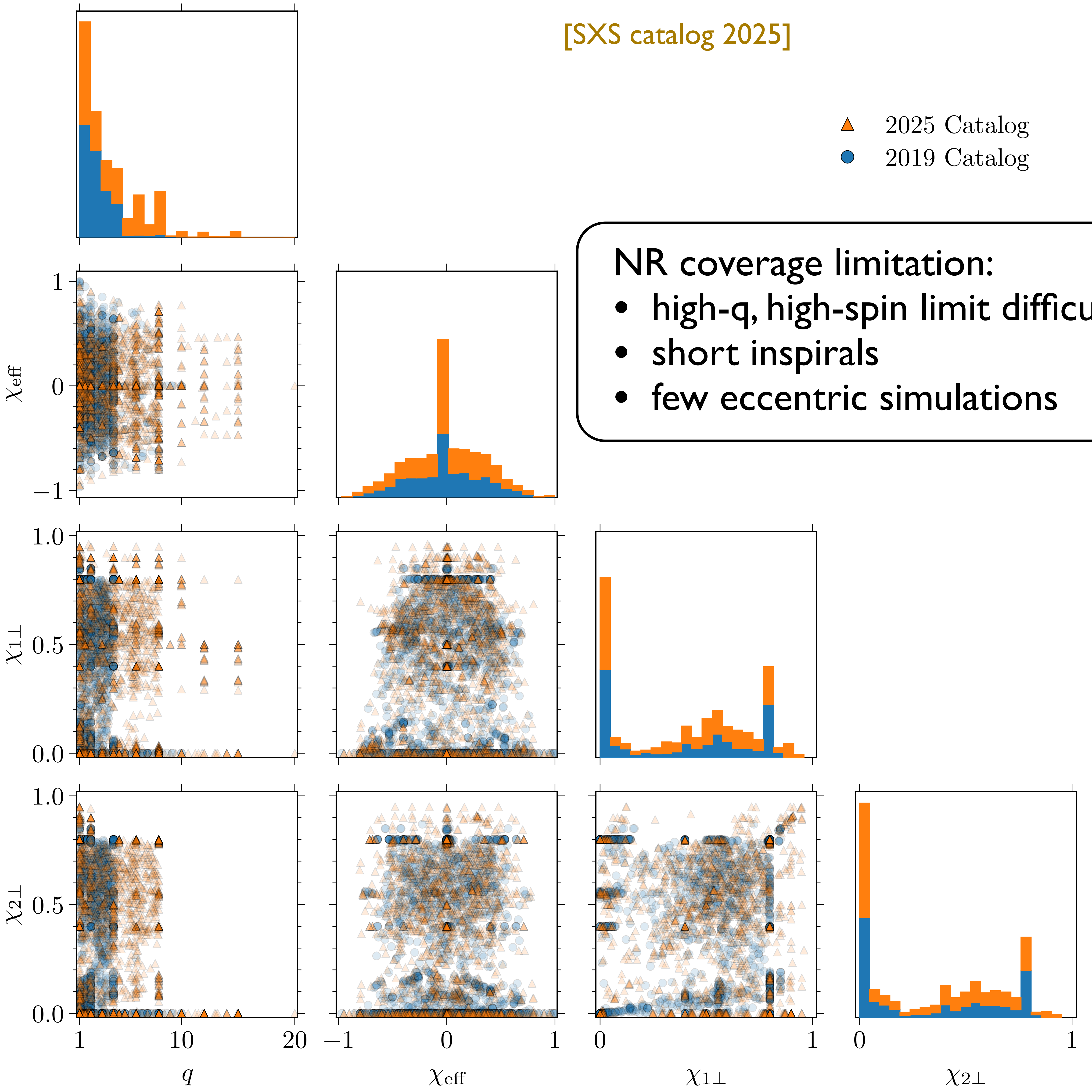
In the linear signal approximation, estimation of bias **[fast]**:

$$F_{ij} = (\partial_i h|\partial_j h)$$
$$\Delta\theta_i = F_{ij}^{-1}(\partial_j h|\delta h)$$

Can we assess biases with efficient tools ?

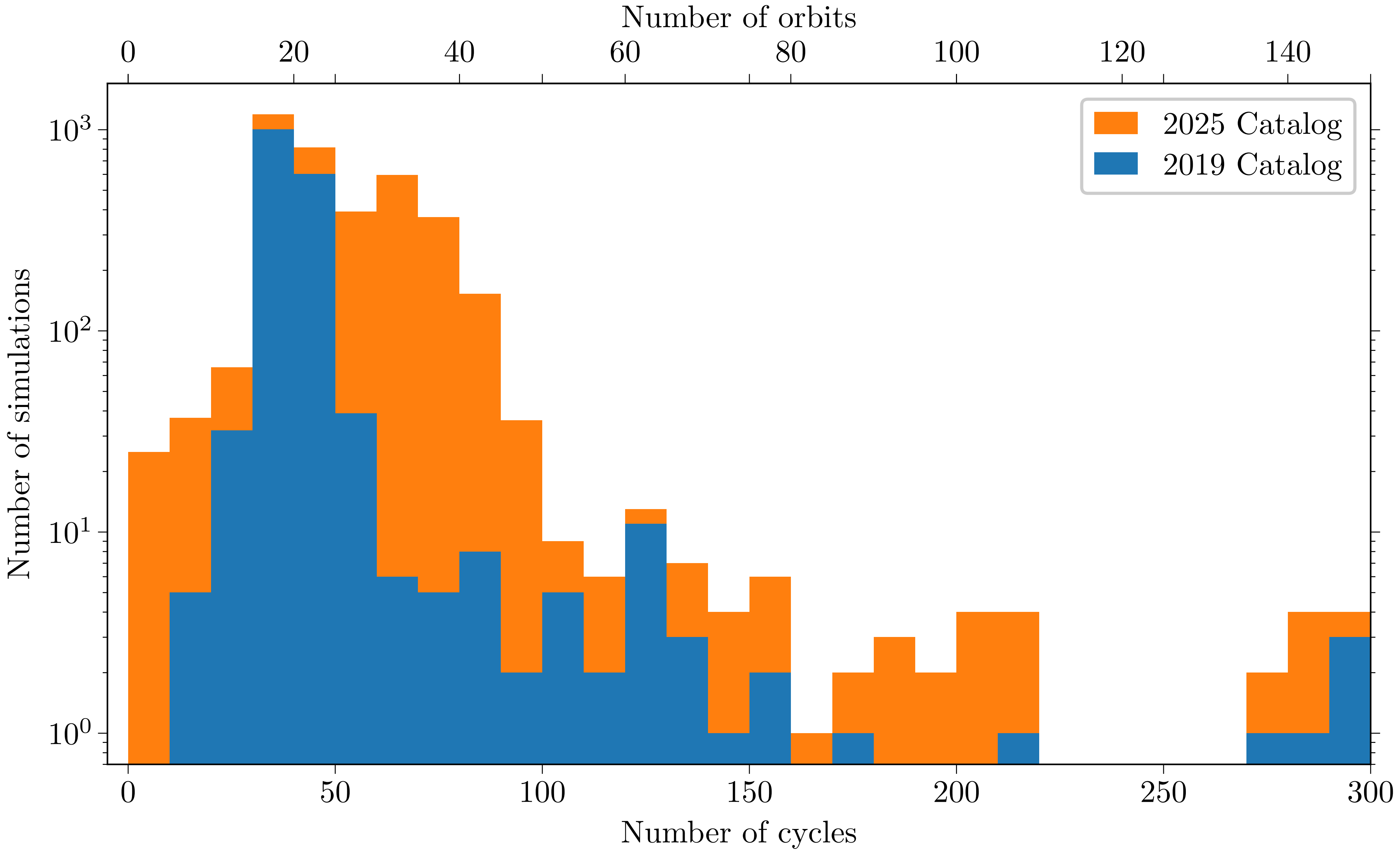
Overview of waveform models: NR

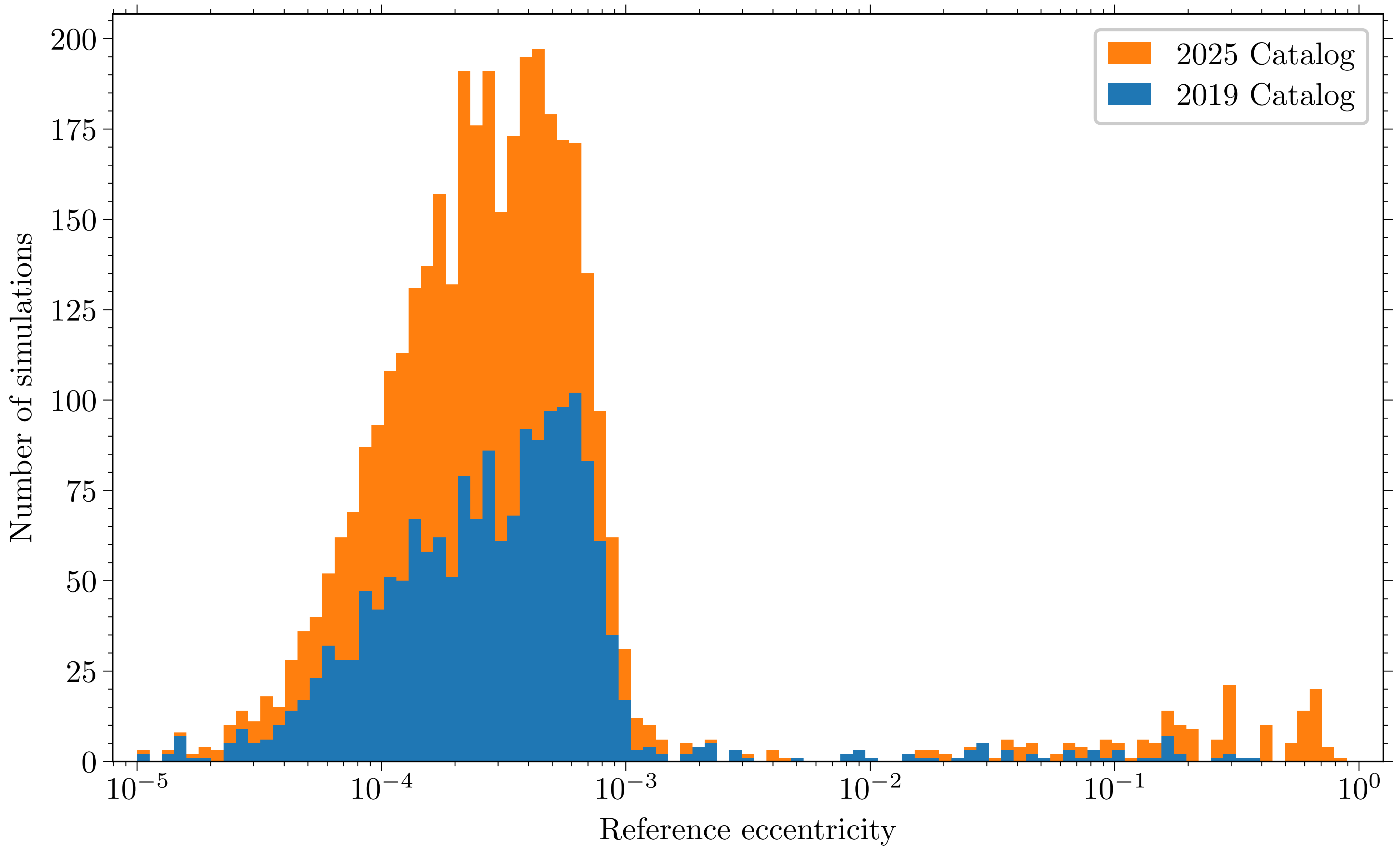
[SXS catalog 2025]



NR coverage limitation:

- high-q, high-spin limit difficult
- short inspirals
- few eccentric simulations





Overview of waveform models: Phenom / SEOB / NR Surrogates

Phenom:

IMRPhenomXPNR [Hamilton+ 2025]
- Fourier-domain, precessing

IMRPhenomTPHM [Estelles+ 2021]
- Time-domain, precessing
- GPU acceleration

IMRPhenomTEHM [Planas+ 2025]
- Time-domain, eccentric

EOB:

SEOBNRv5PHM [Ramos-Buades+ 2023]
- Time-domain, precessing

SEOBNRv5EHM [Gamboa+ 2024]
- Eccentric, spin-aligned

TEOBResumS-Dali [Nagar+ 2024]
- Spin-precessing, Eccentric

+ _ROM
- accelerated reduced order models (spin aligned)

NR surrogates:

NRHybSur3dq8 [Varma+ 2018]
- SXS NR simulations hybridized with long EOB inspirals (covers ~6 months for $M = 10^5 M_\odot$)
- Surrogate interpolant, time-domain

NRSur7dq4 [Varma+ 2019]
- SXS NR simulations, short
- 7d, fully precessing, with mode asymmetries

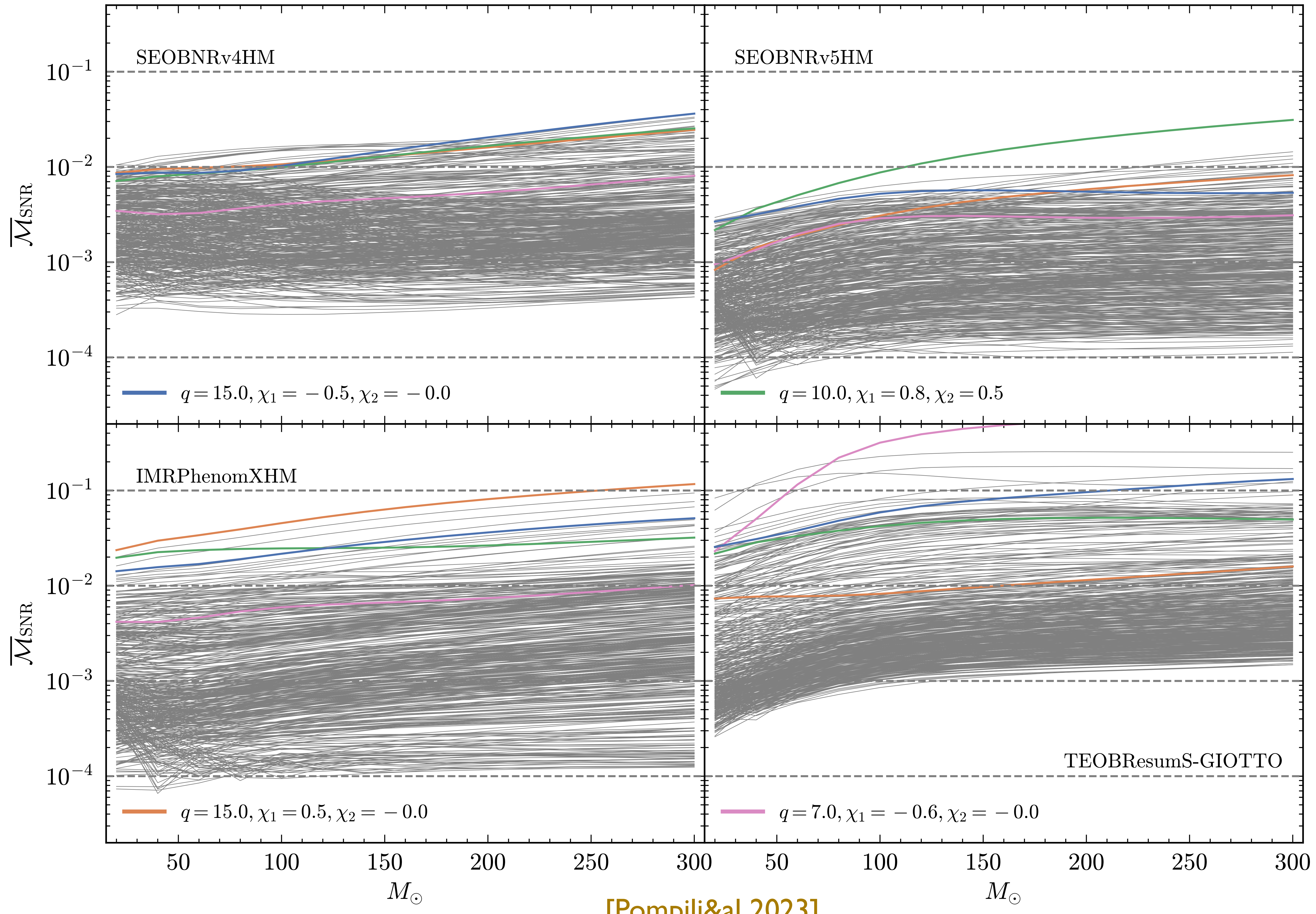
+ high-q surrogates
- BH pert. + NR

+ eccentric surrogates
- non-spinning

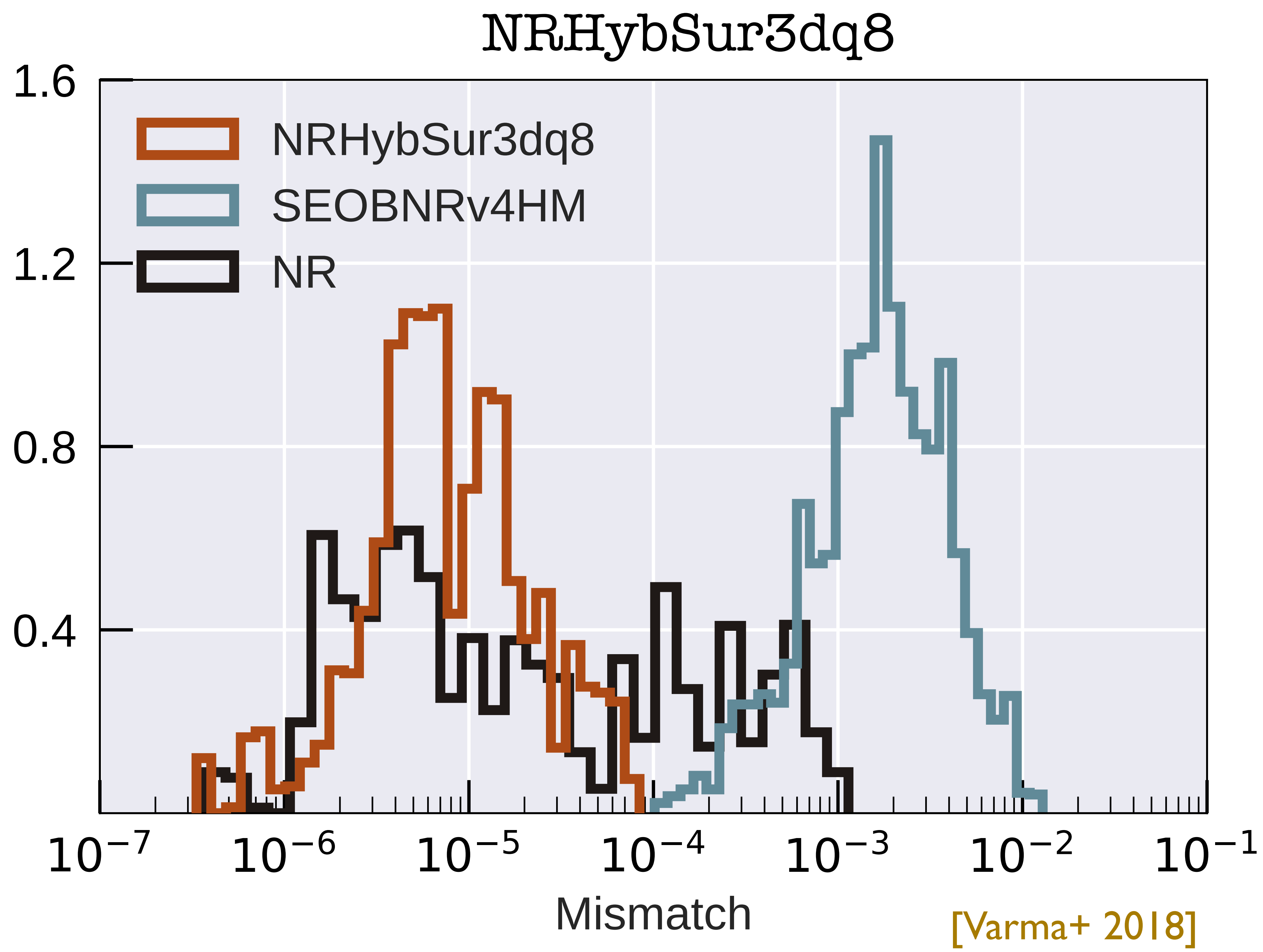
Packages:

Moving towards modular codes:
- phenomxpy
- pyseobnr
- gwsurrogate

Overview of waveform models: mismatches for the circularized non-precessing case



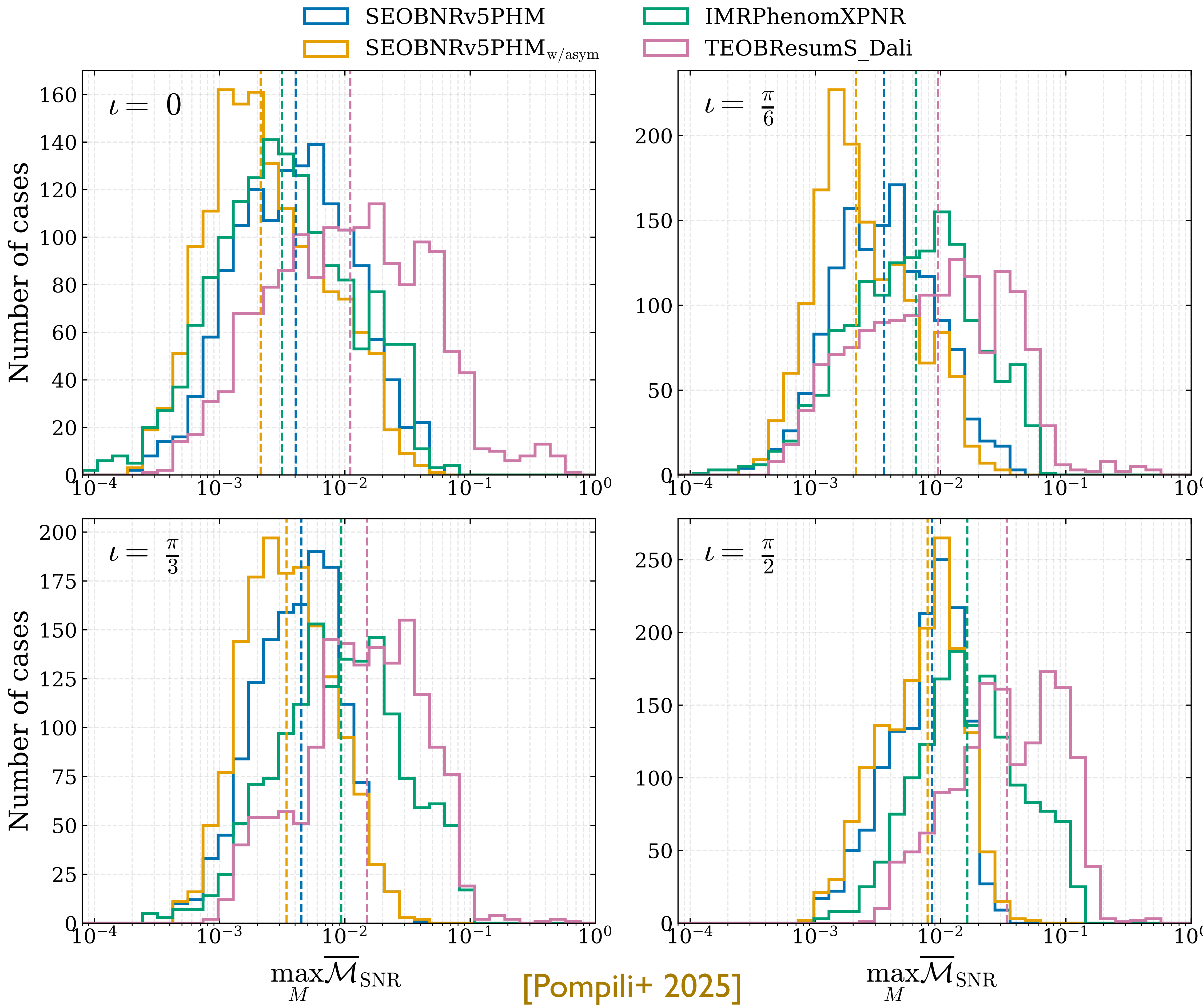
[Pompili&al 2023]



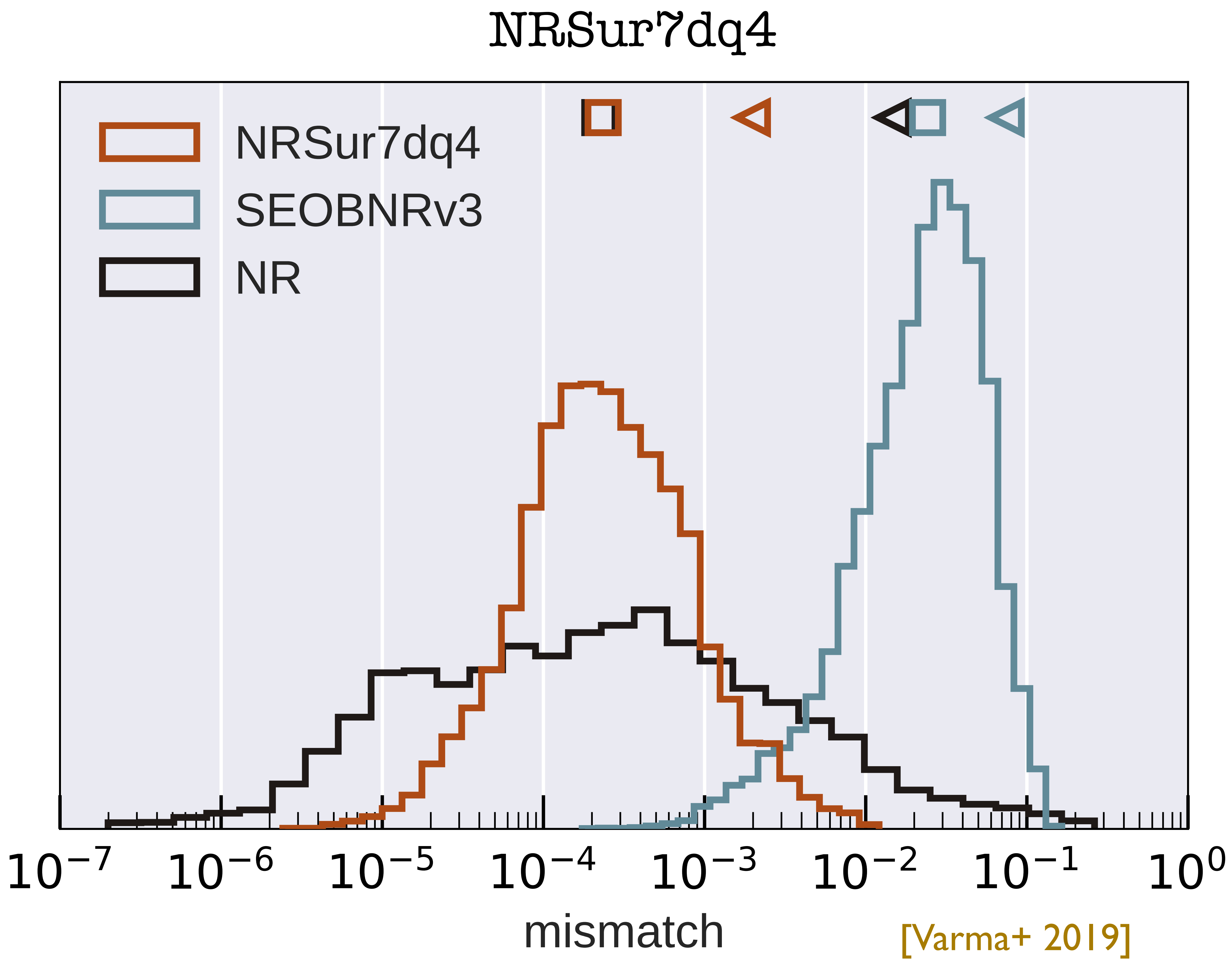
[Varma+ 2018]

Aligned spin case: mismatch with NR ~ $10^{-4} - 10^{-2}$

Overview of waveform models: mismatches for precession



[Pompili+ 2025]

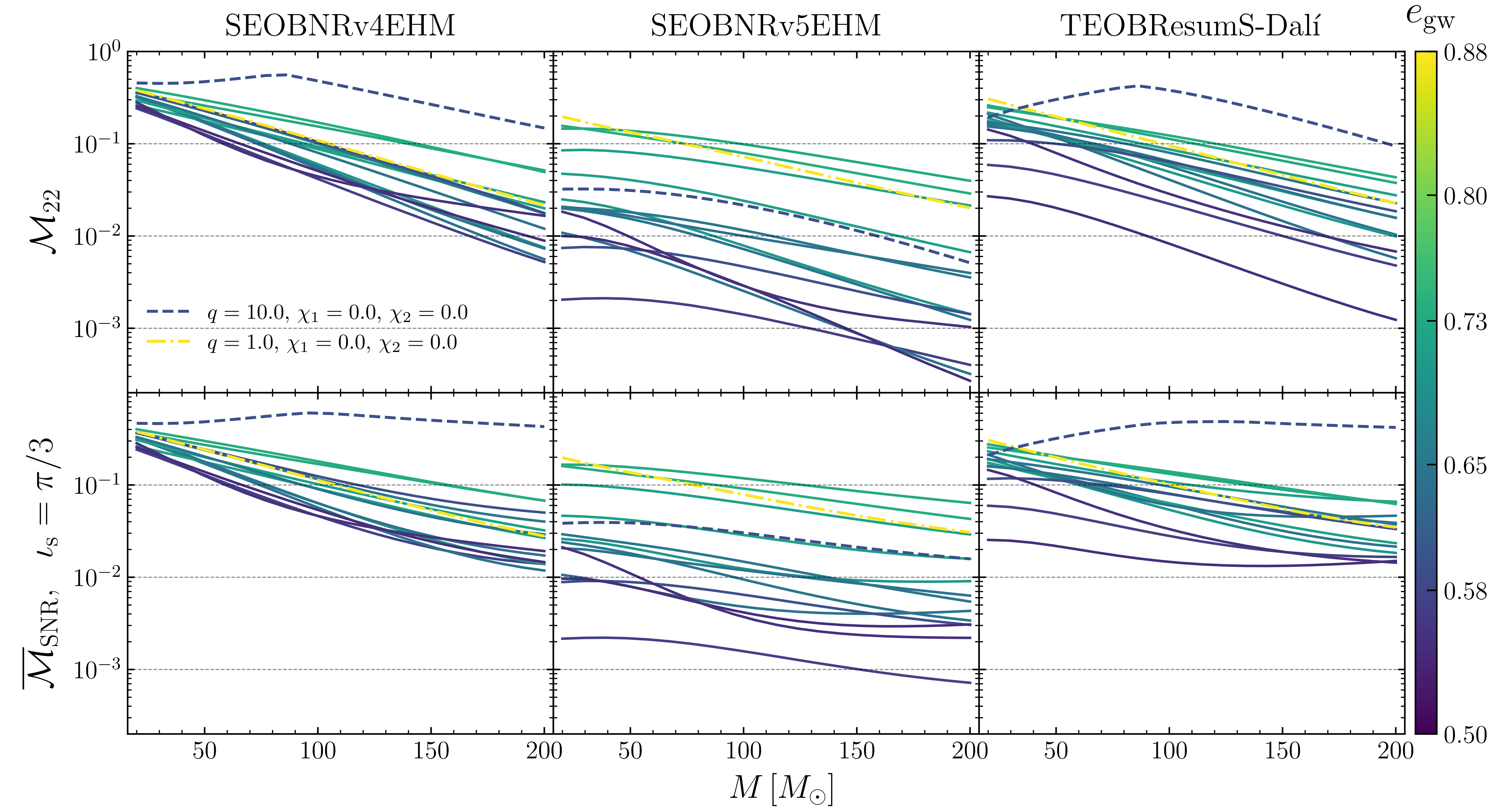


[Varma+ 2019]

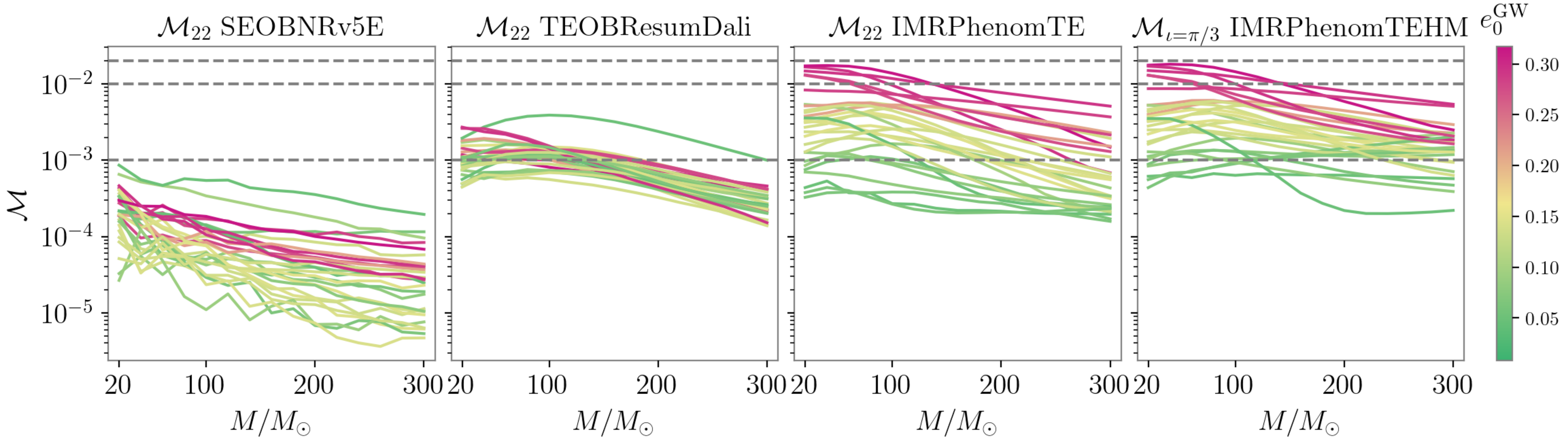
Aligned spin case: mismatch with NR ~ 10^{-3} – few 10^{-2}

Overview of waveform models: mismatches for eccentricity

[Gamboa+ 2024]



Eccentric case: mismatch with NR ~ $10^{-2} - 10^{-1}$



[Planas+ 2025]

See also:

NRSurE_q4NoSpin_22

[Nee+ 2026]

Analysis settings

Injections:

NRHybSur3dq8 [Varma&al 2018]

- SXS NR simulations hybridized with long EOB inspirals (covers ~6 months for $M = 10^5 M_\odot$)
- Surrogate interpolant, time-domain

Templates:

Efficient Fourier-domain models from 2 families:

- PhenomHM [London&al 2017]
- PhenomXHM [García-Quirós&al 2020]
- SEOBNRv4HM_ROM [Cotesta&al 2018]
- SEOBNRv5HM_ROM [Pompili&al 2023]

Mode content for all: 22, 21, 33, 44

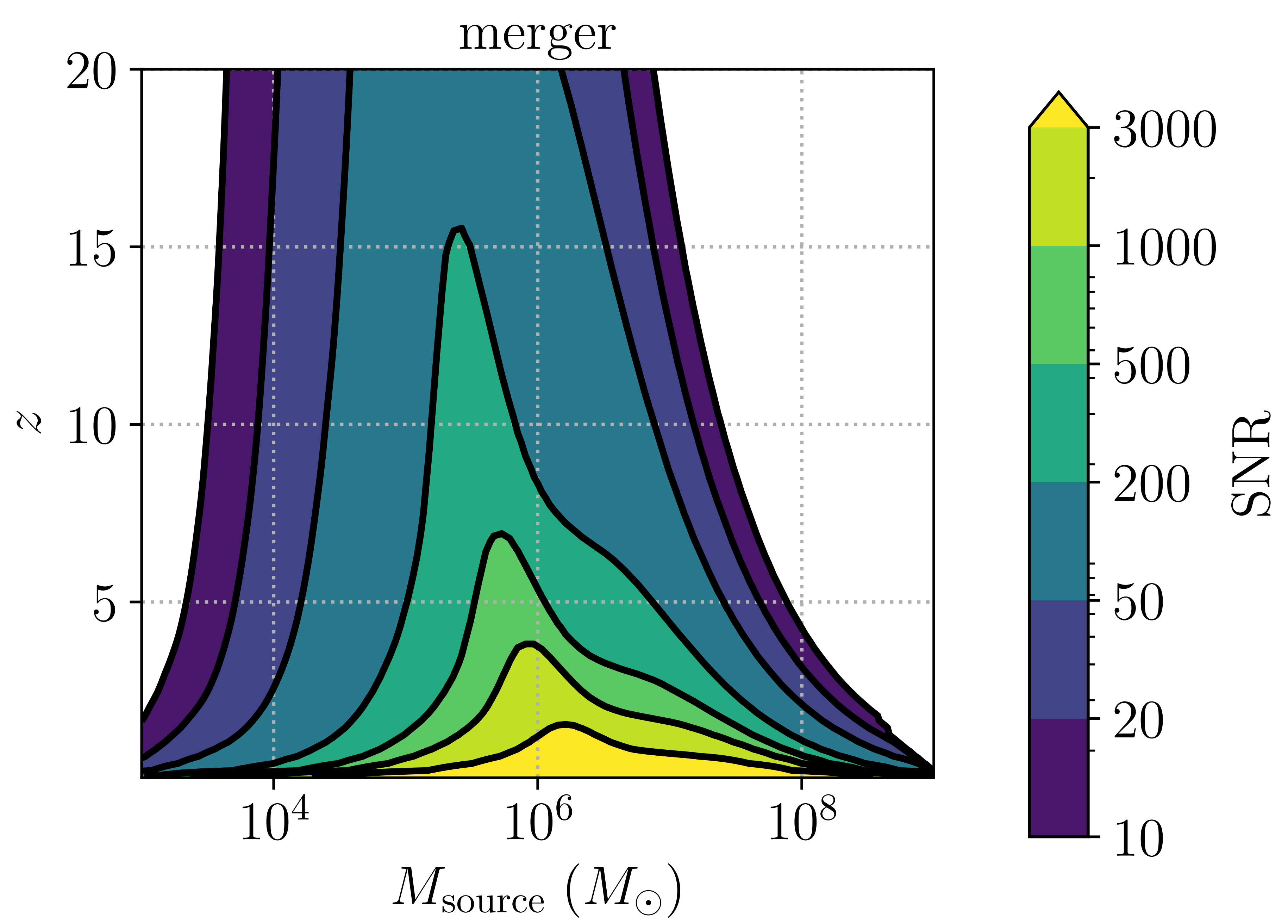
Limitations:

- **aligned spins only**
- in the inspiral, all based on PN/EOB

Parameter space exploration:

- $M_z = [10^5, 10^6, 10^7] M_\odot$
- $z_{\rm min} = 1$

- $N = 240$ simulations
- uniform $q \in [1,8]$
- uniform $\chi_1 \in [-0.8, 0.8]$
- uniform $\chi_2 \in [-0.8, 0.8]$
- randomize orientations



Analysis:

- Bayesian PE: lisabeta
- Posterior gives statistical uncertainty, $\max \ln \mathcal{L}$ gives the bias
- Multi-stage tempering to ensure convergence
- Fourier-domain response equal-armlength (same for injection and templates)
- Direct FD Whittle likelihood (no approx.)

SNRs and mismatches

Signal-to-noise ratios:

$$\mathrm{SNR}^2 = (h_0|h_0)$$

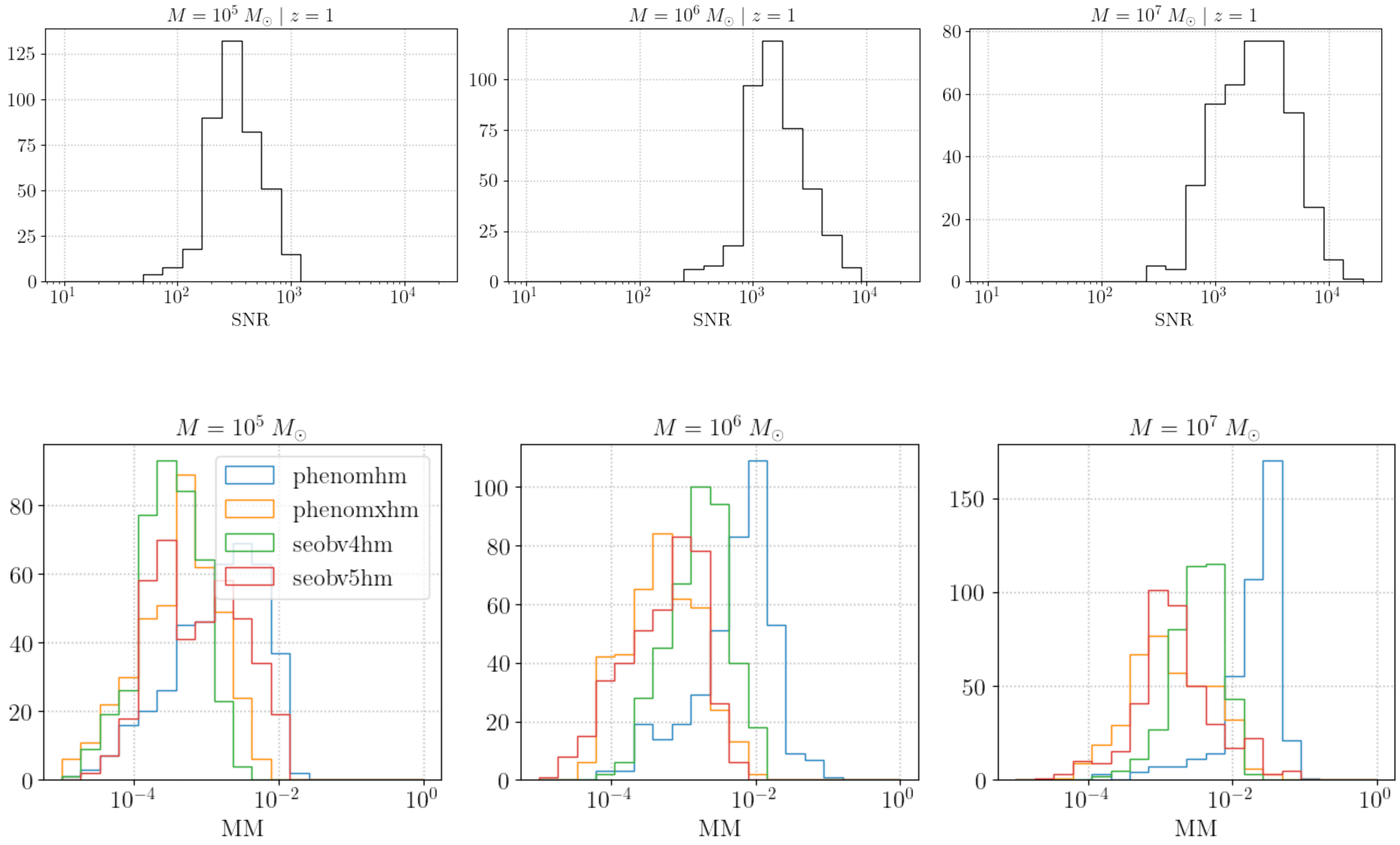
SNRs up to thousands at $z = 1$

Mismatch:

- averaged response
- single-detector
- optimized: time, phase, polarization, sky

$$\mathrm{MM} = 1 - \max_{t,\varphi,\psi,\ldots} \frac{(h_m|h_0)}{\sqrt{(h_m|h_m)}\sqrt{h_0|h_0}}$$

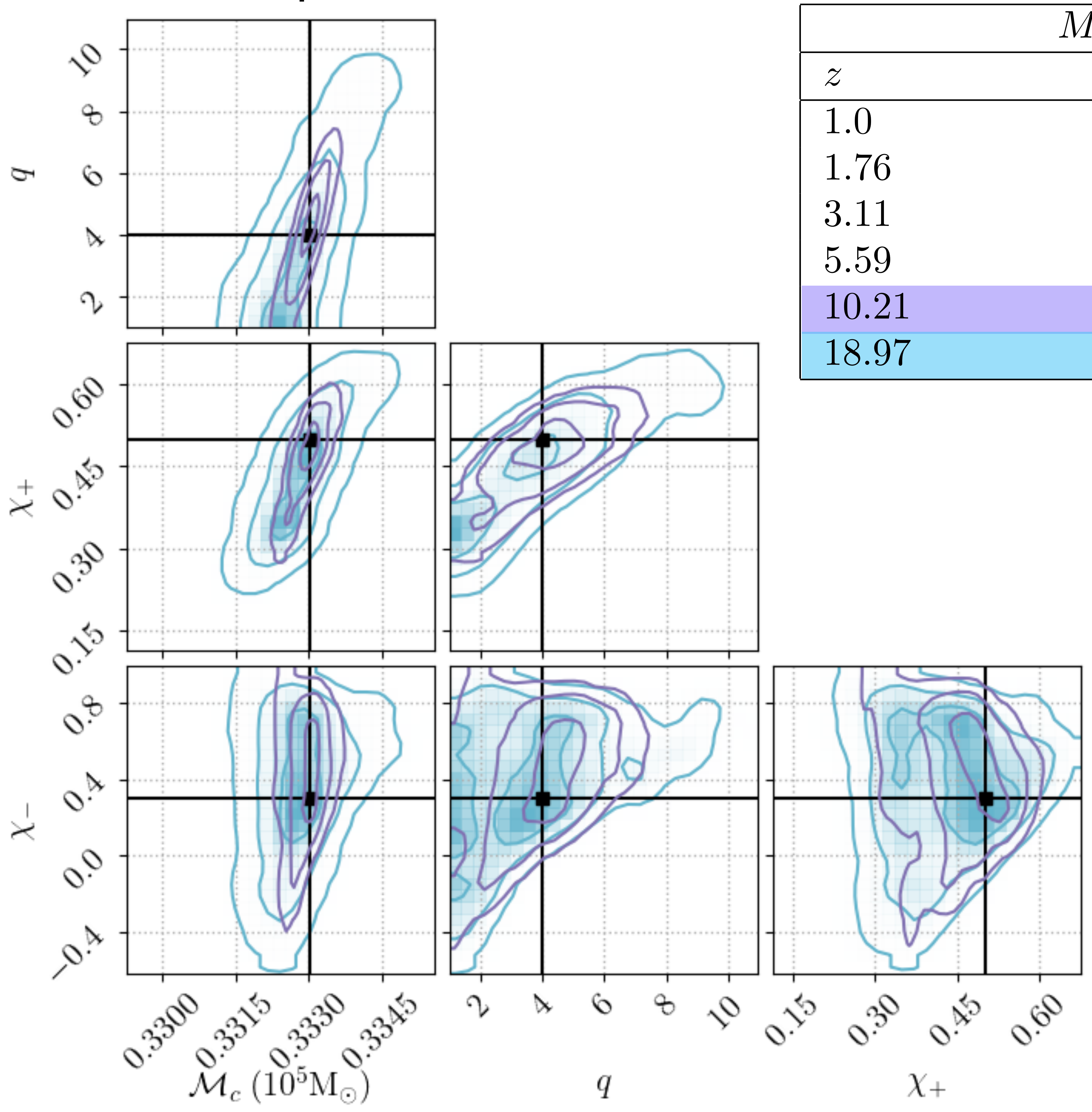
- Mismatches order-of-magnitude similar to LVK
- Improvement for more recent models



Example Parameter estimation with systematics I

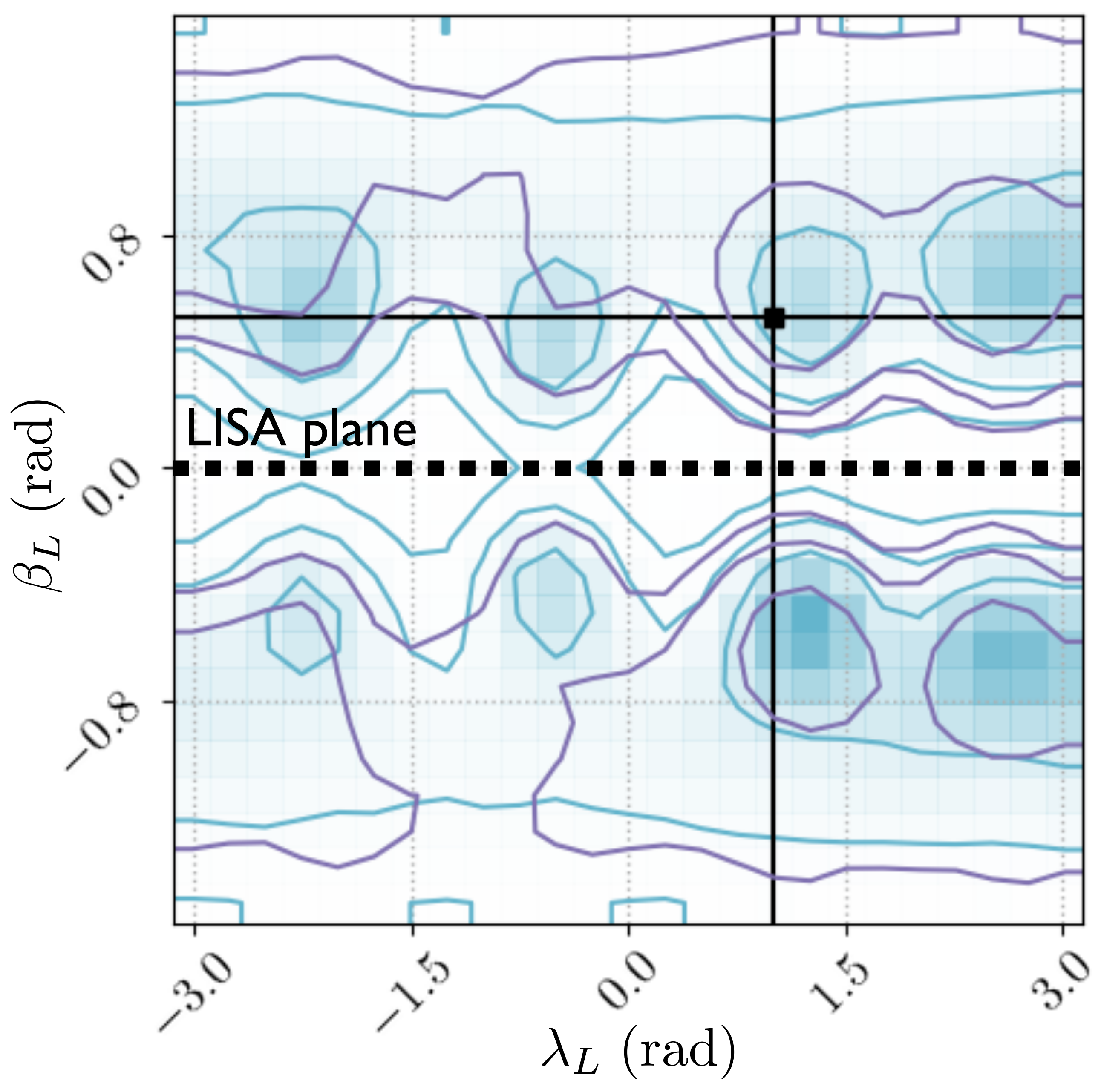
- **Injection:** NRHybSur3dq8 $\{M = 10^5 M_\odot, q = 4, \chi_1 = 0.5, \chi_2 = 0.3\}$
- **Template:** PhenomXHM

Intrinsic params.



$M_z = 10^5\ M_\odot$	
z	SNR
1.0	317
1.76	158
3.11	79
5.59	40
10.21	20
18.97	10

Sky localisation



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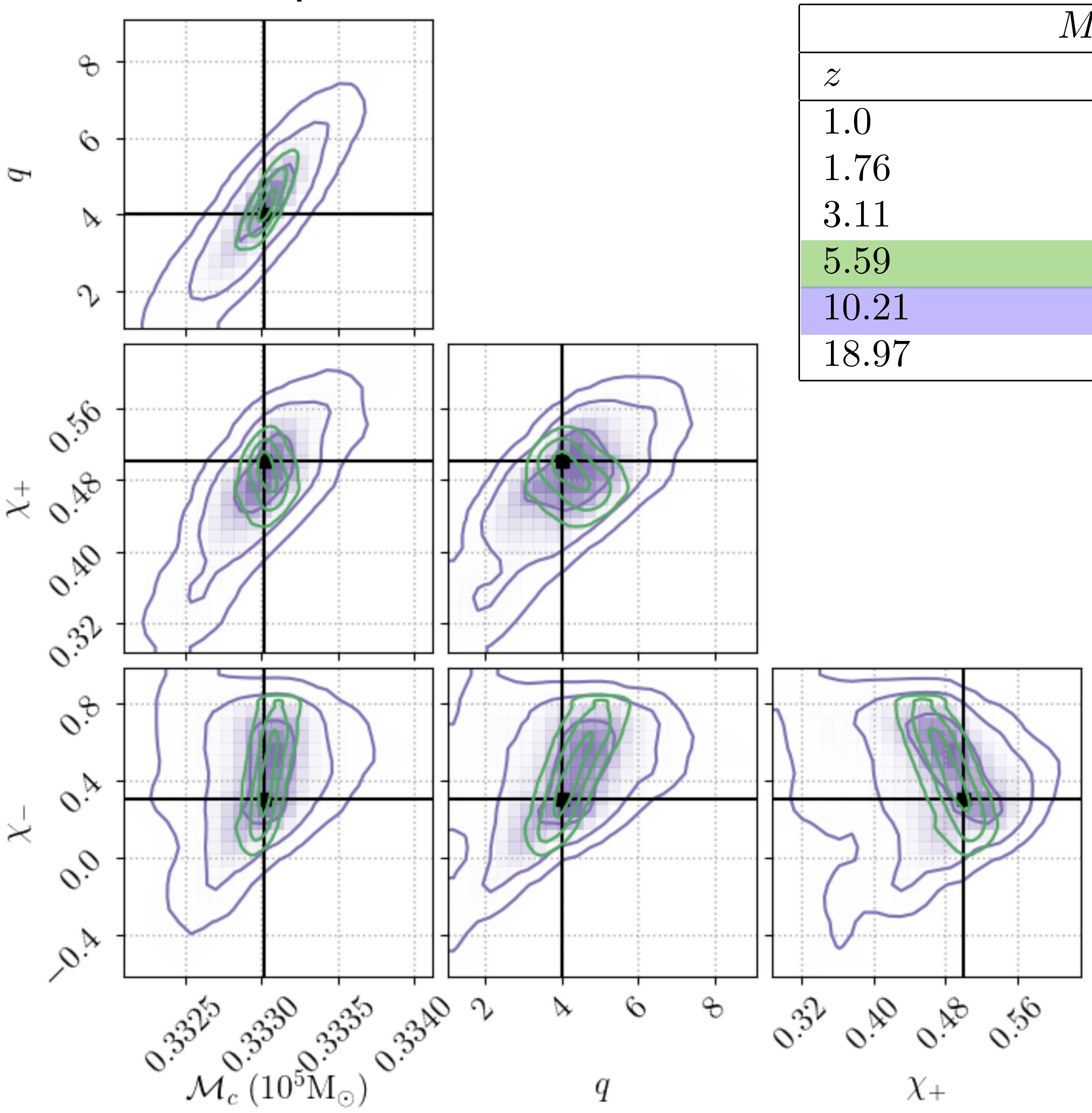

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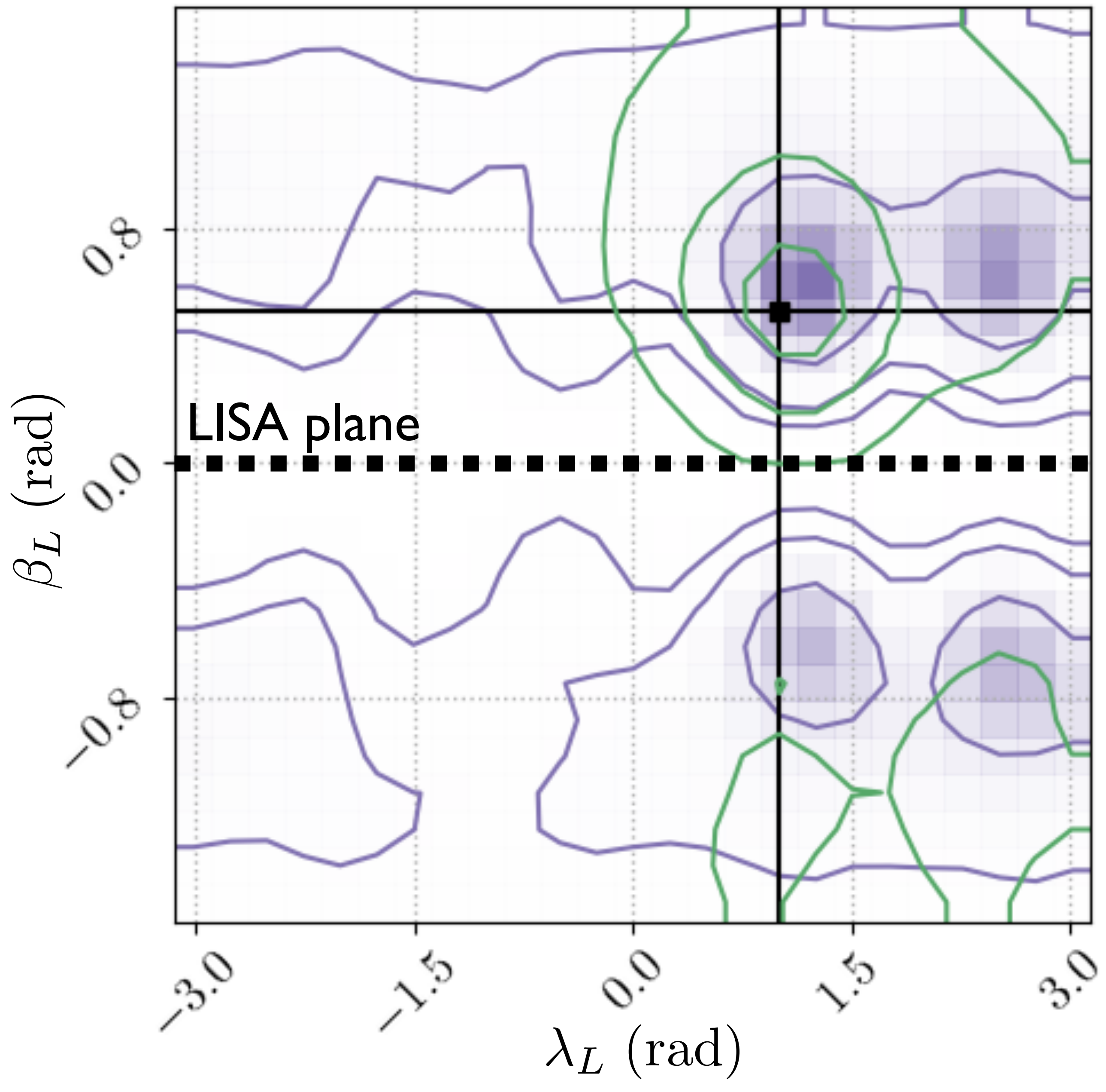

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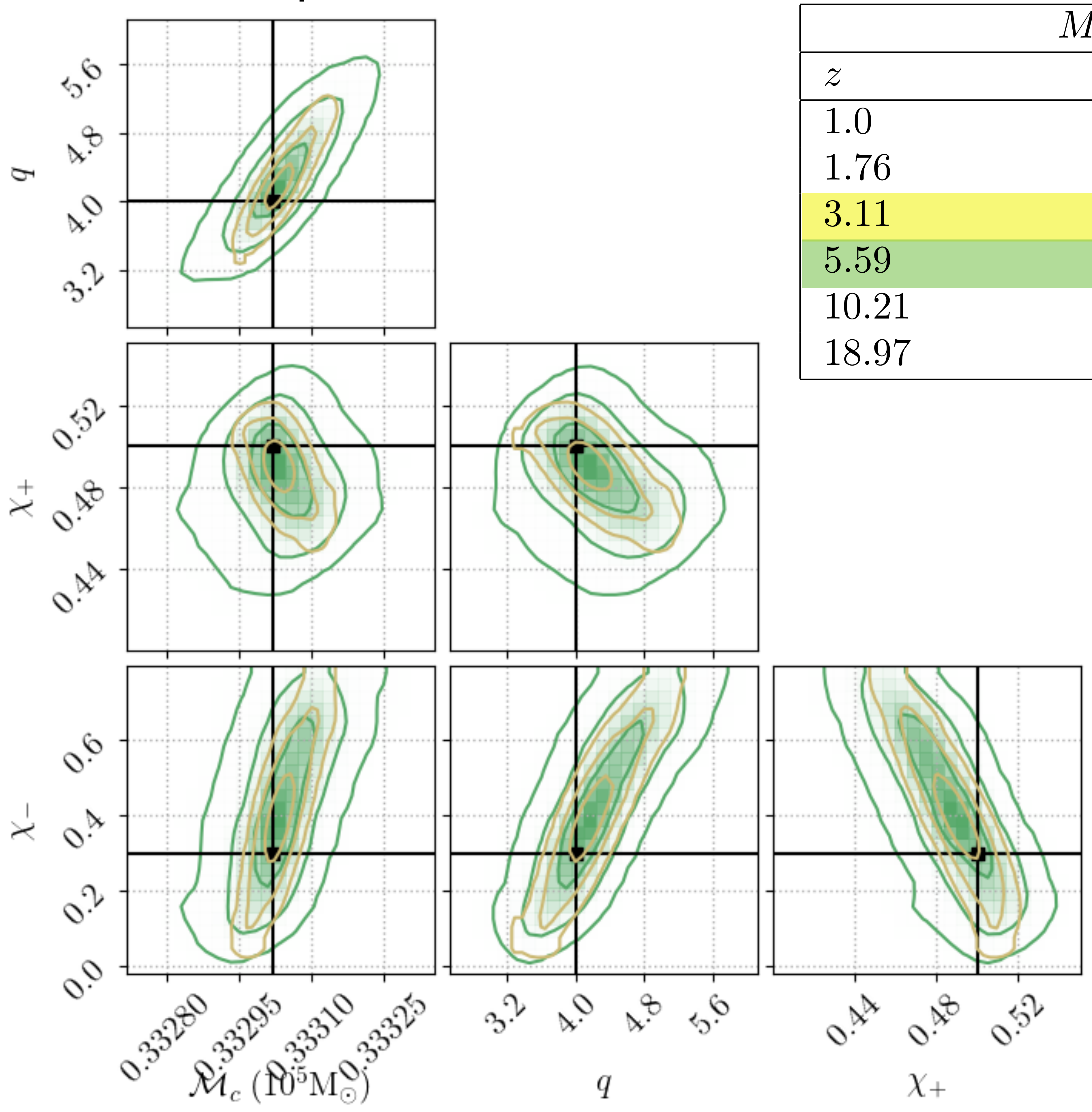

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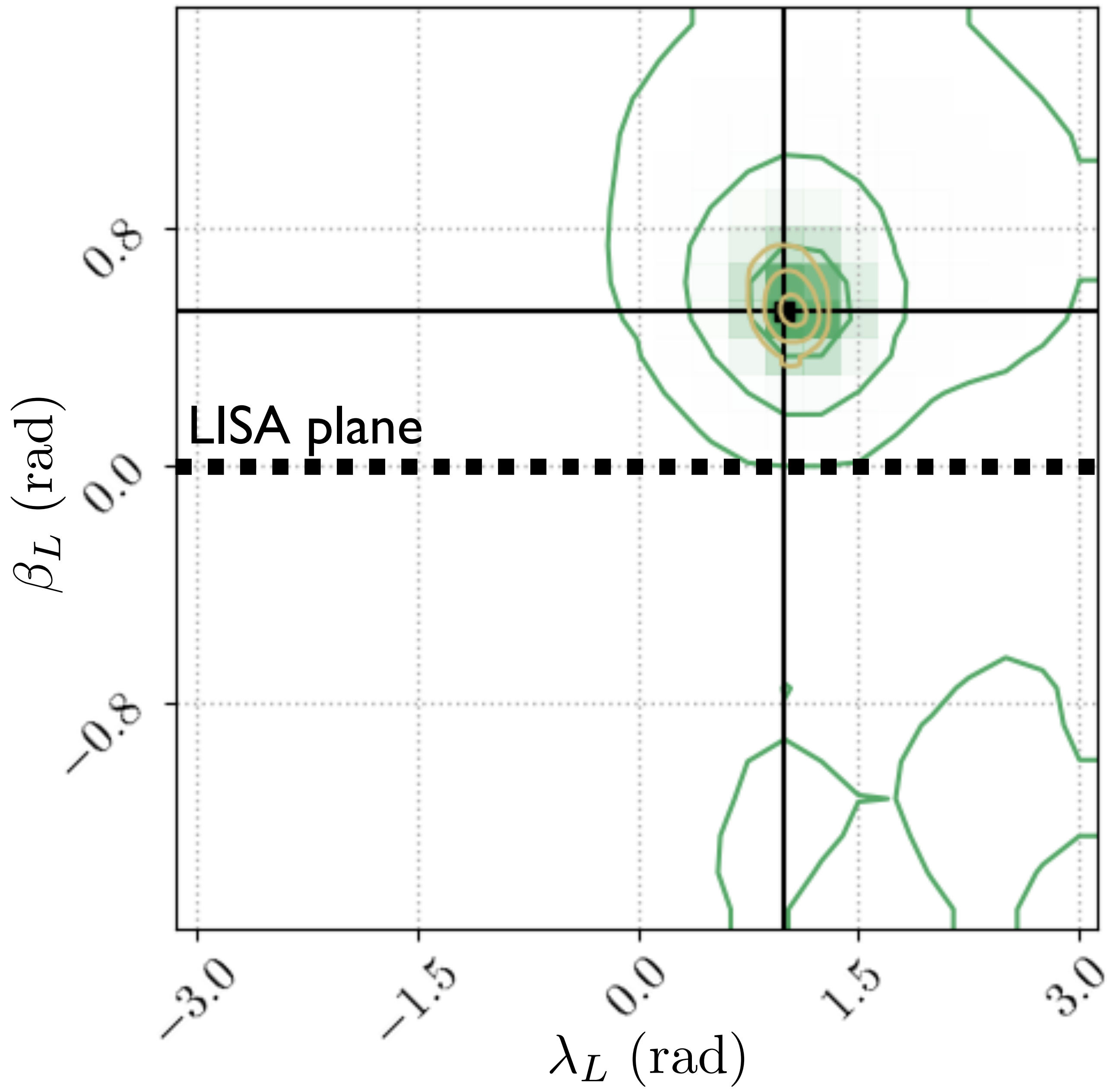

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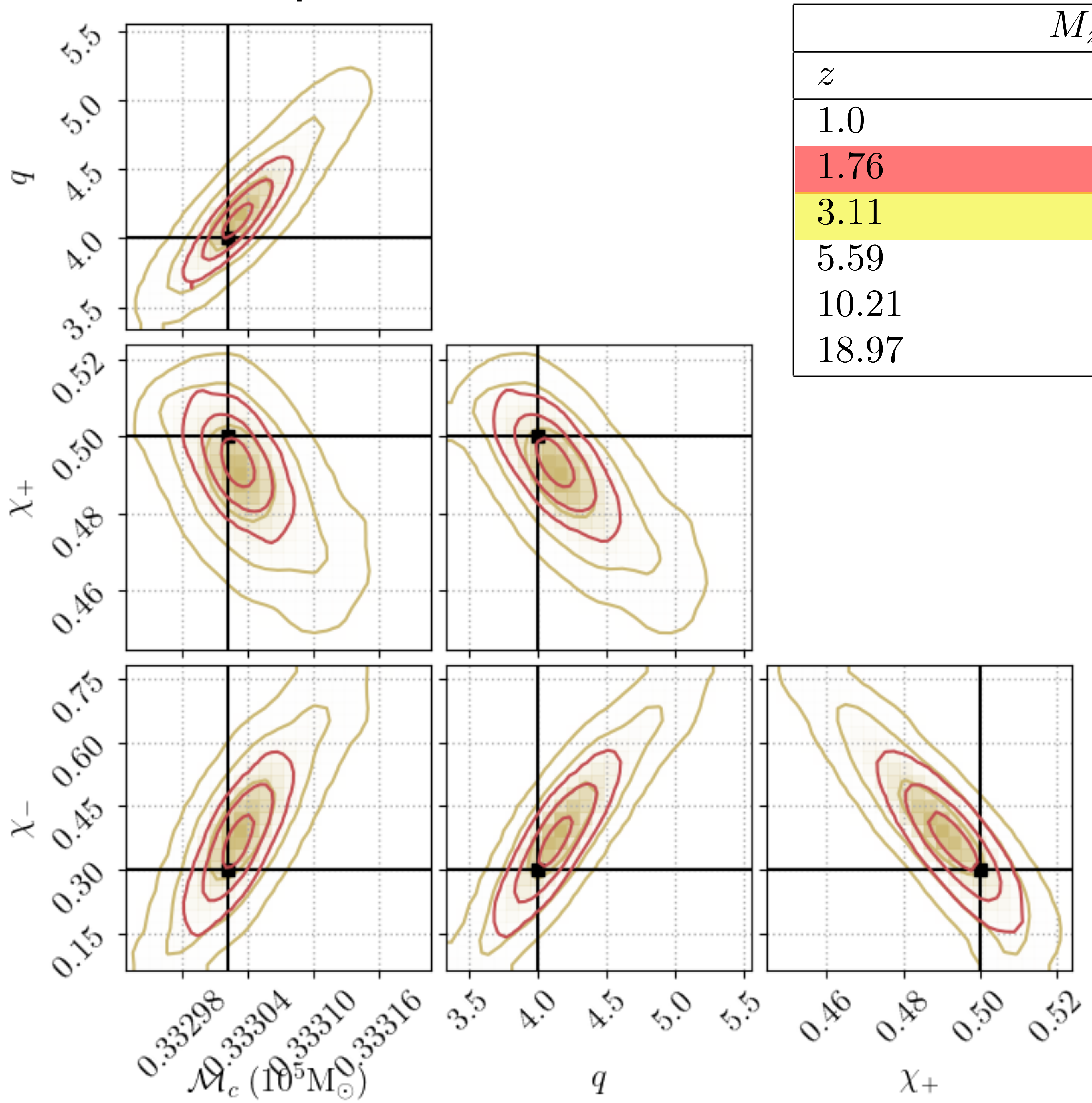

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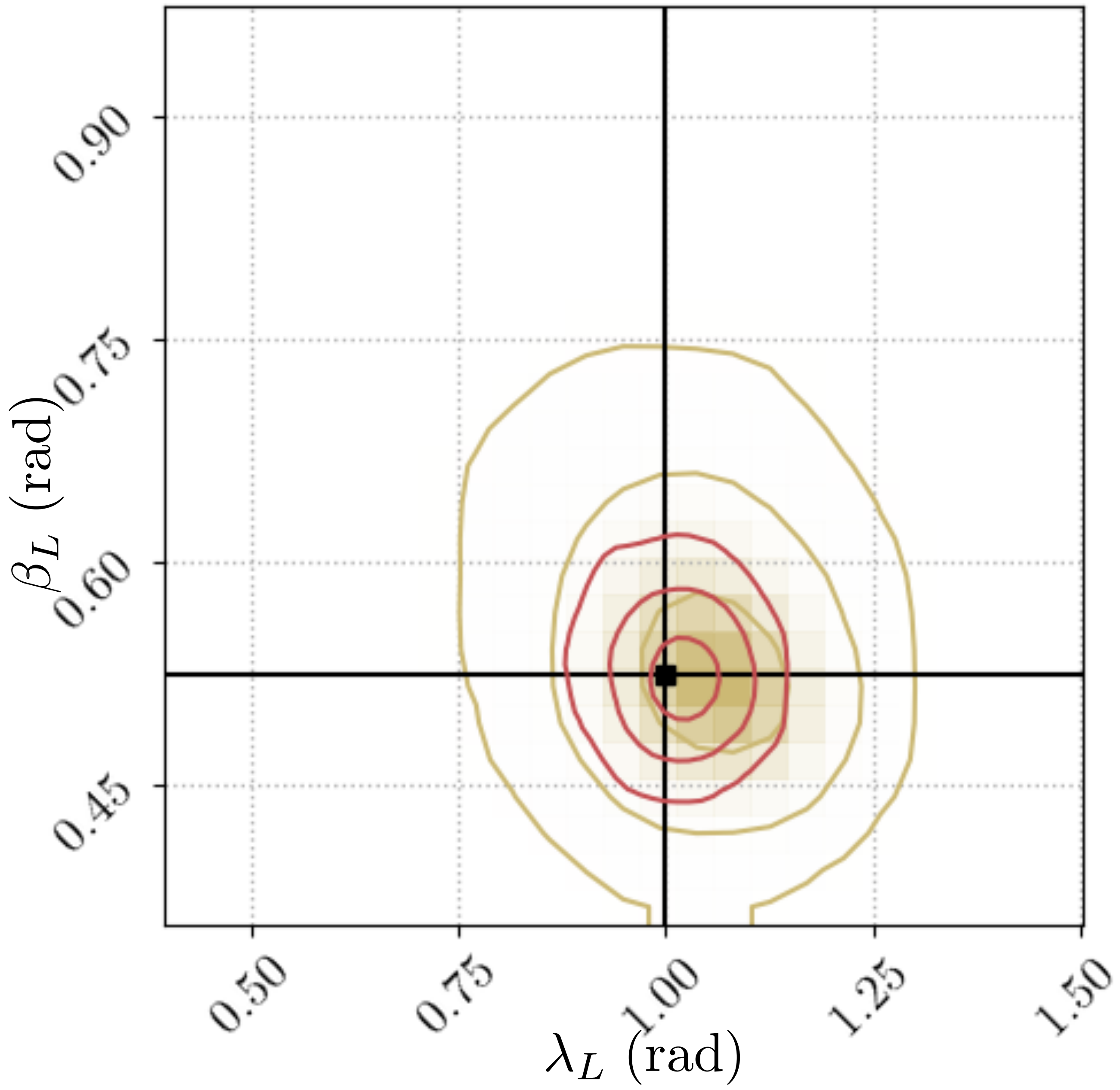

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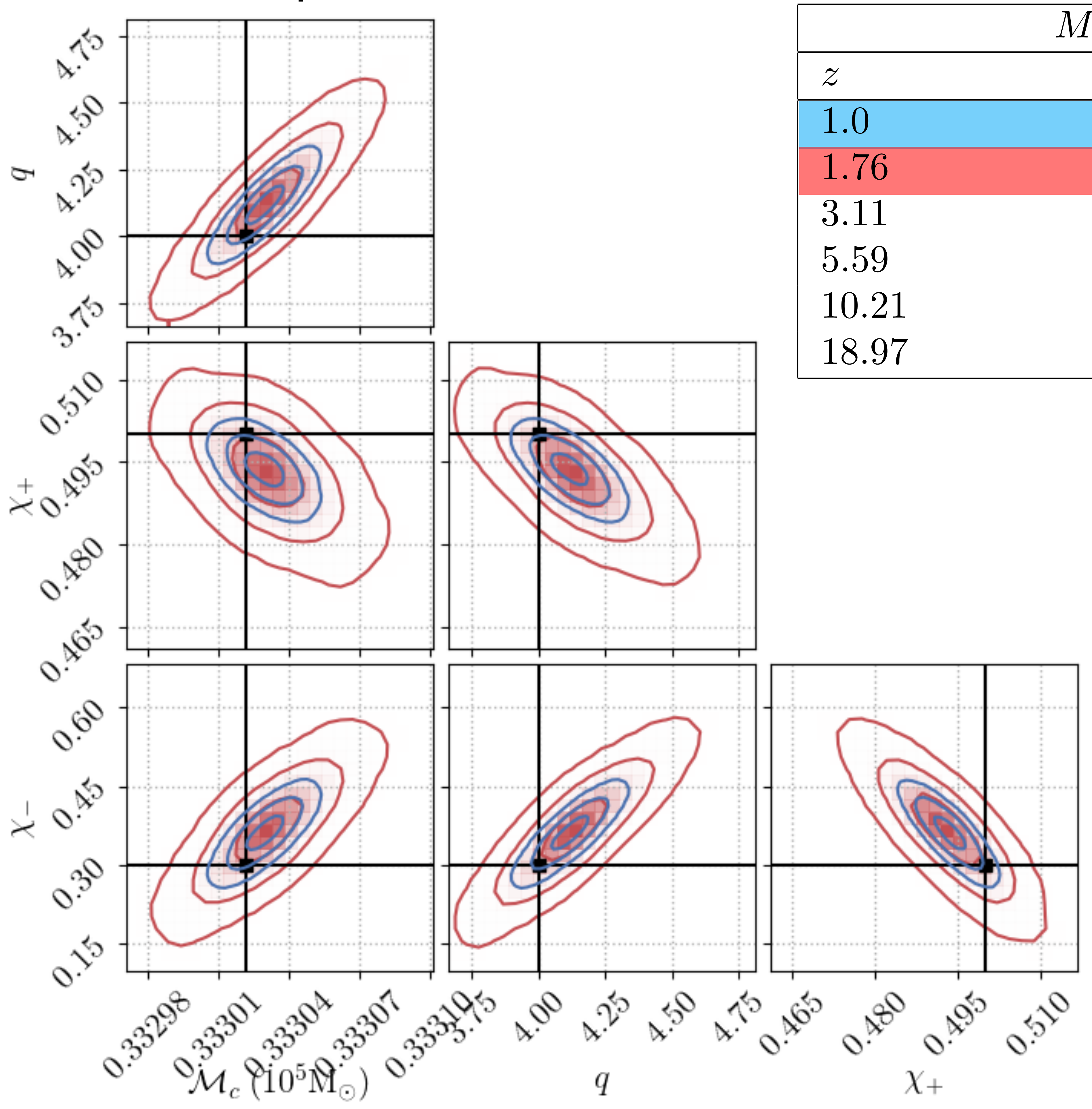

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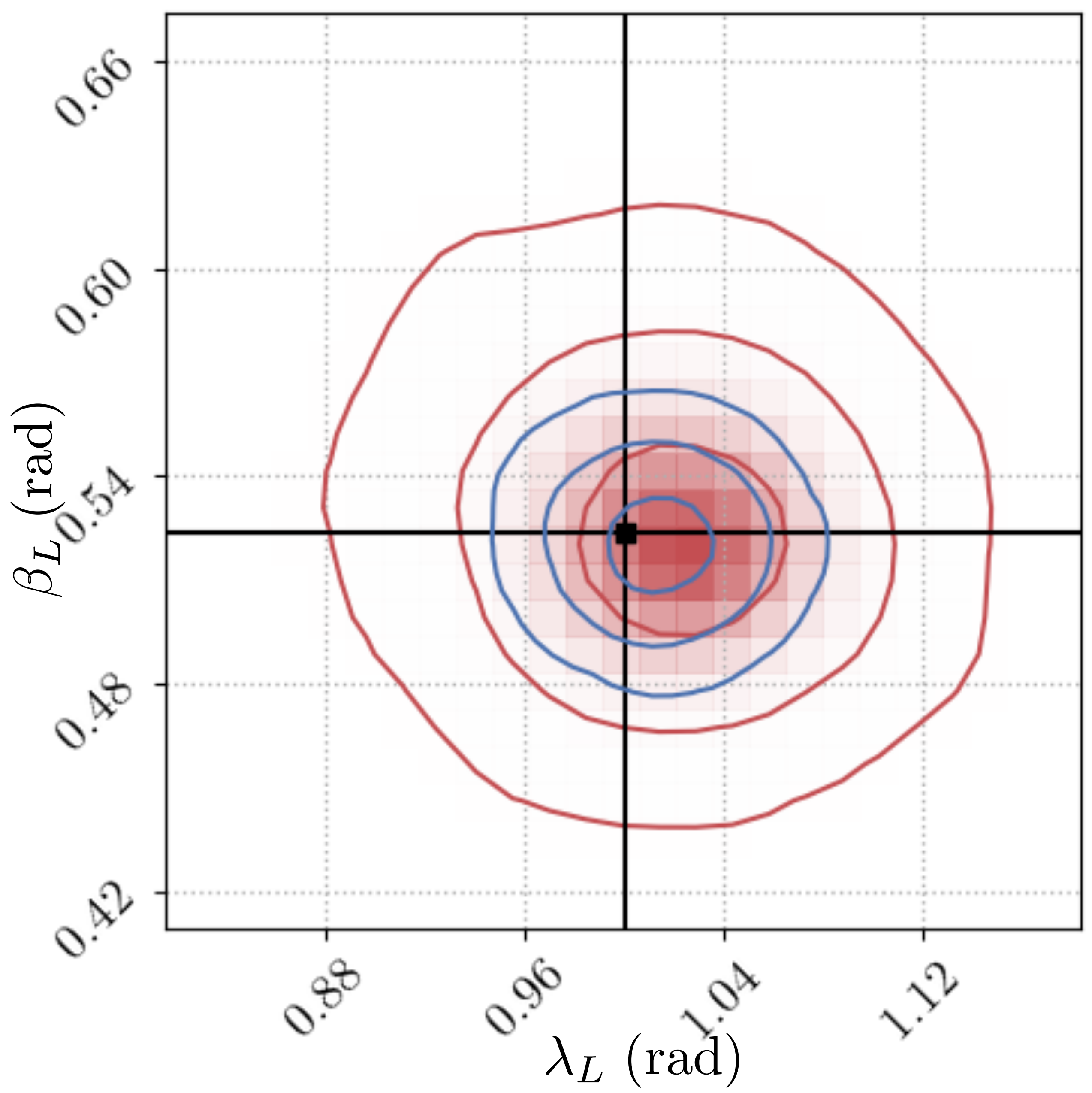

The good:
- converges on the true parameters
- mild bias at $z = 1$, SNR $= 317$

Example Parameter estimation with systematics II

- **Injection:** NRHybSur3dq8 $\{M = 10^6 M_\odot, q = 4, \chi_1 = 0.5, \chi_2 = 0.3\}$
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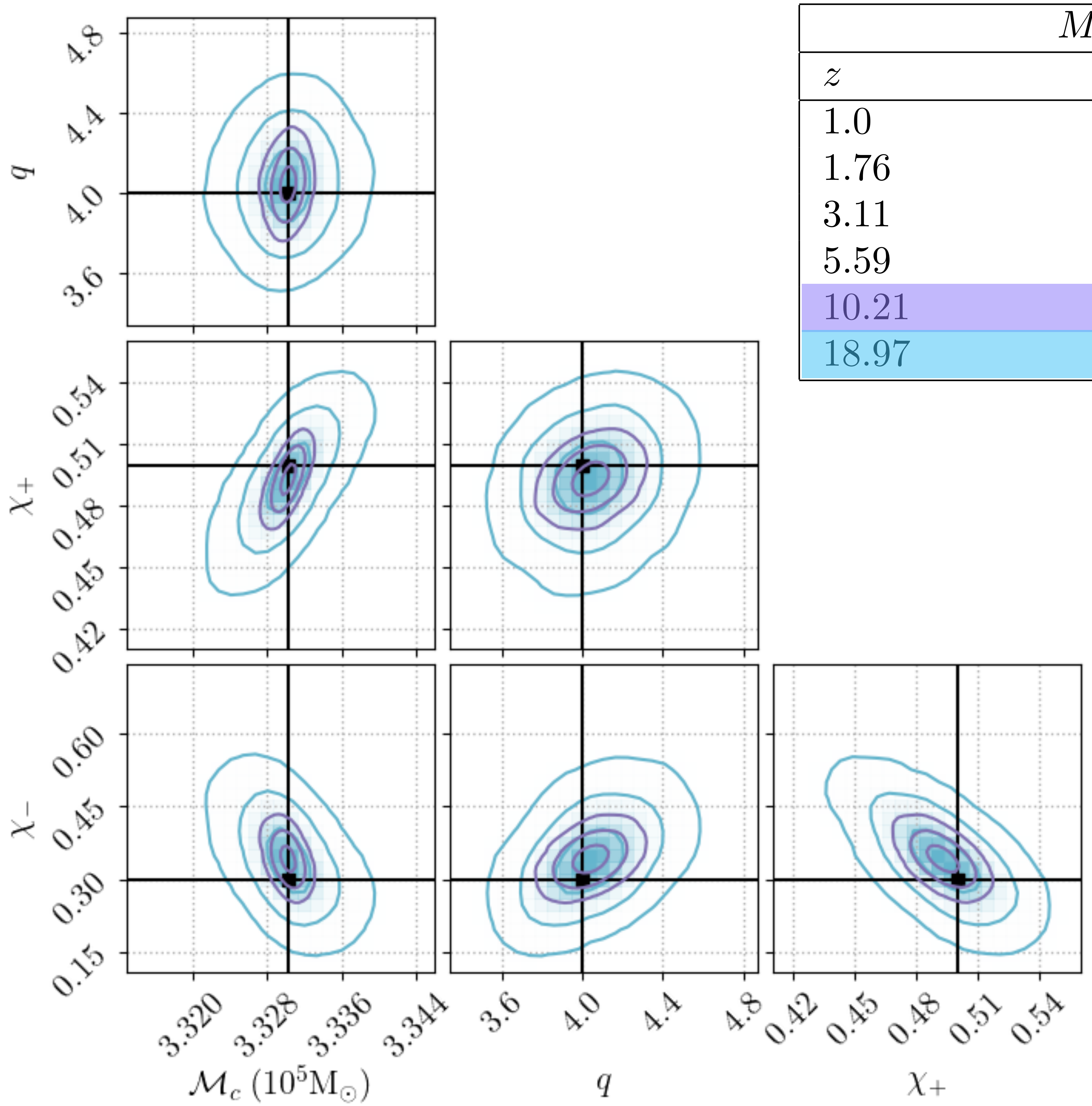
Intrinsic params.



$M_z = 10^6\ M_\odot$	
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1.0	1907
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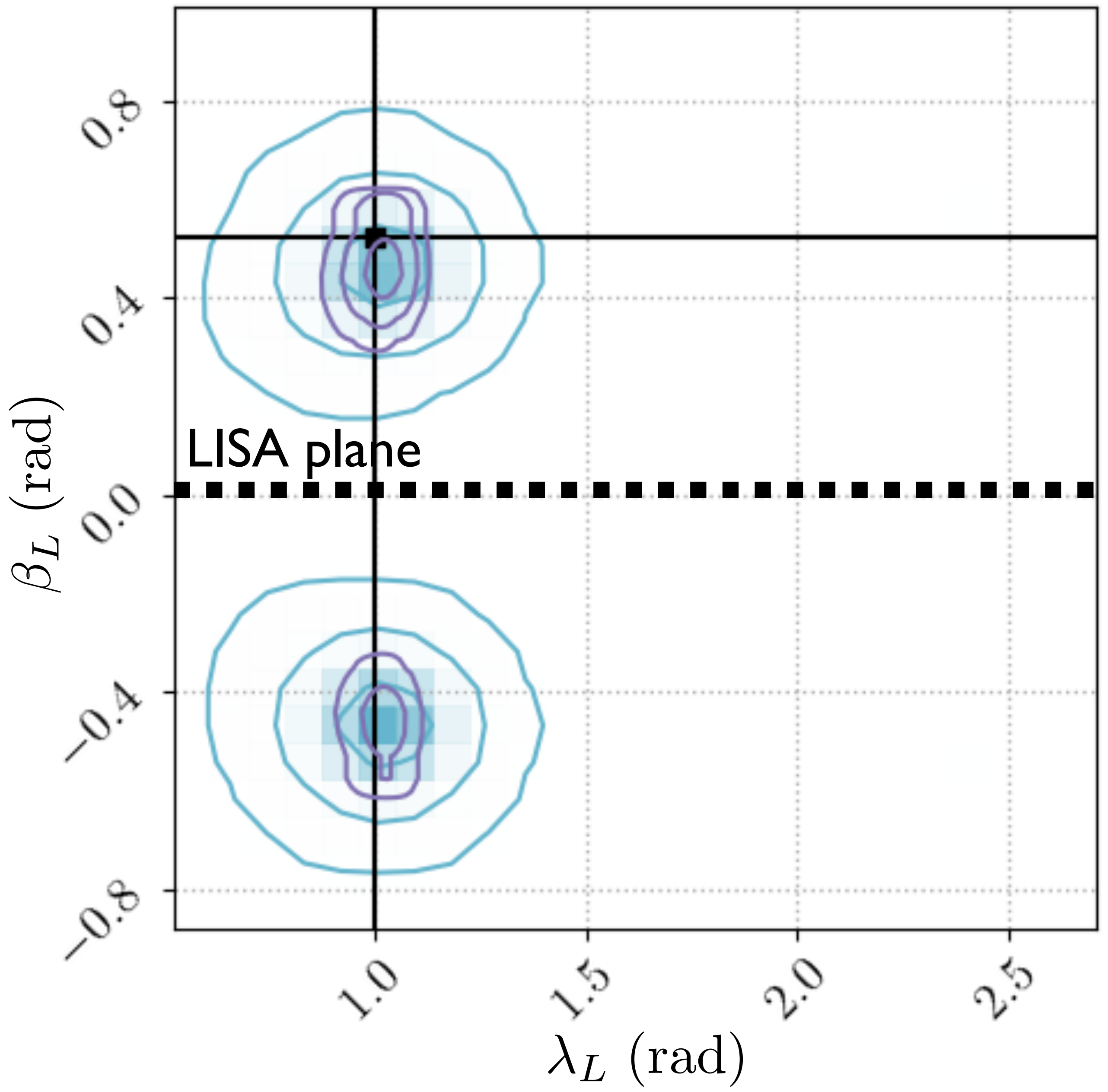
Sky localisation



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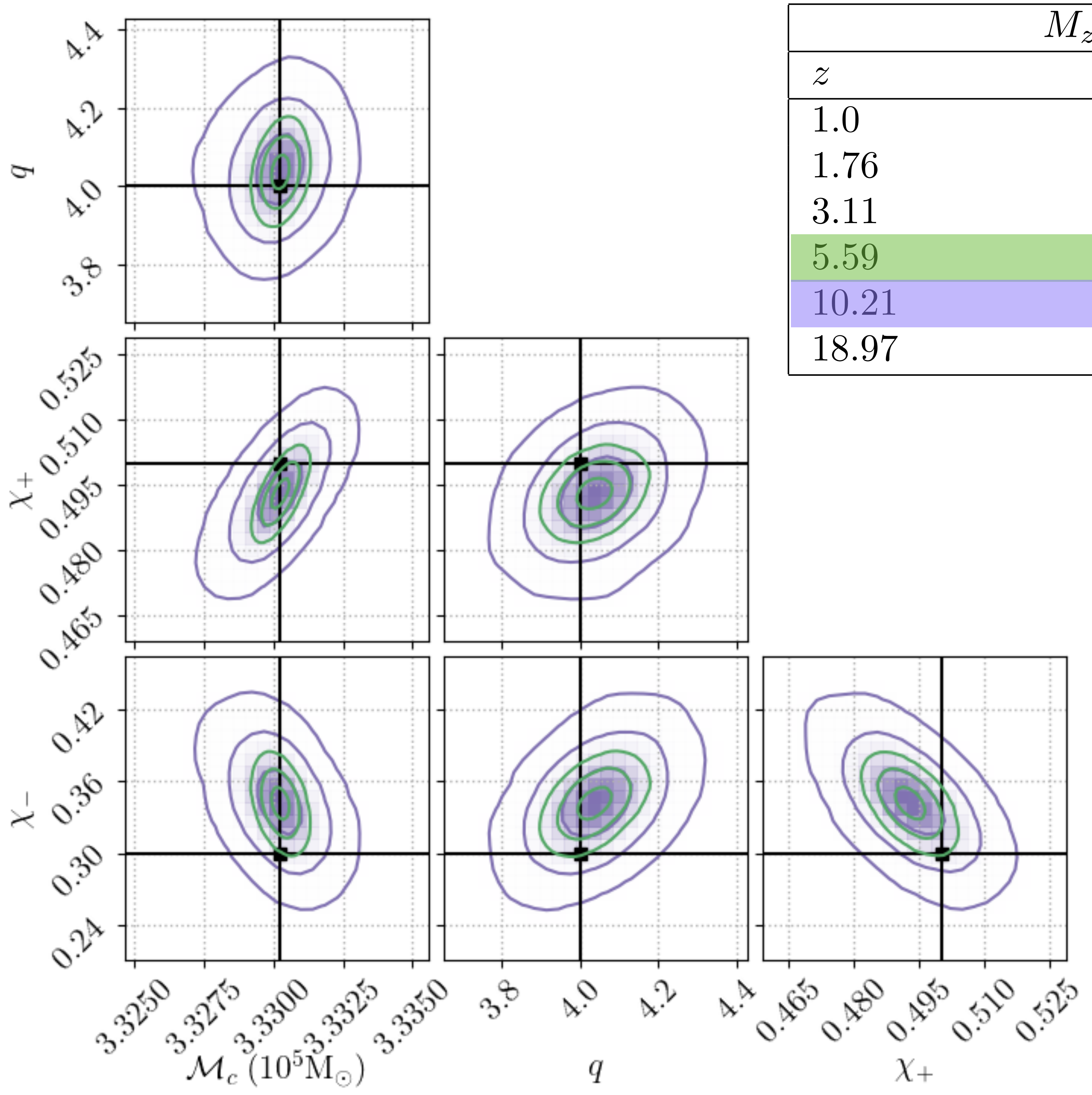

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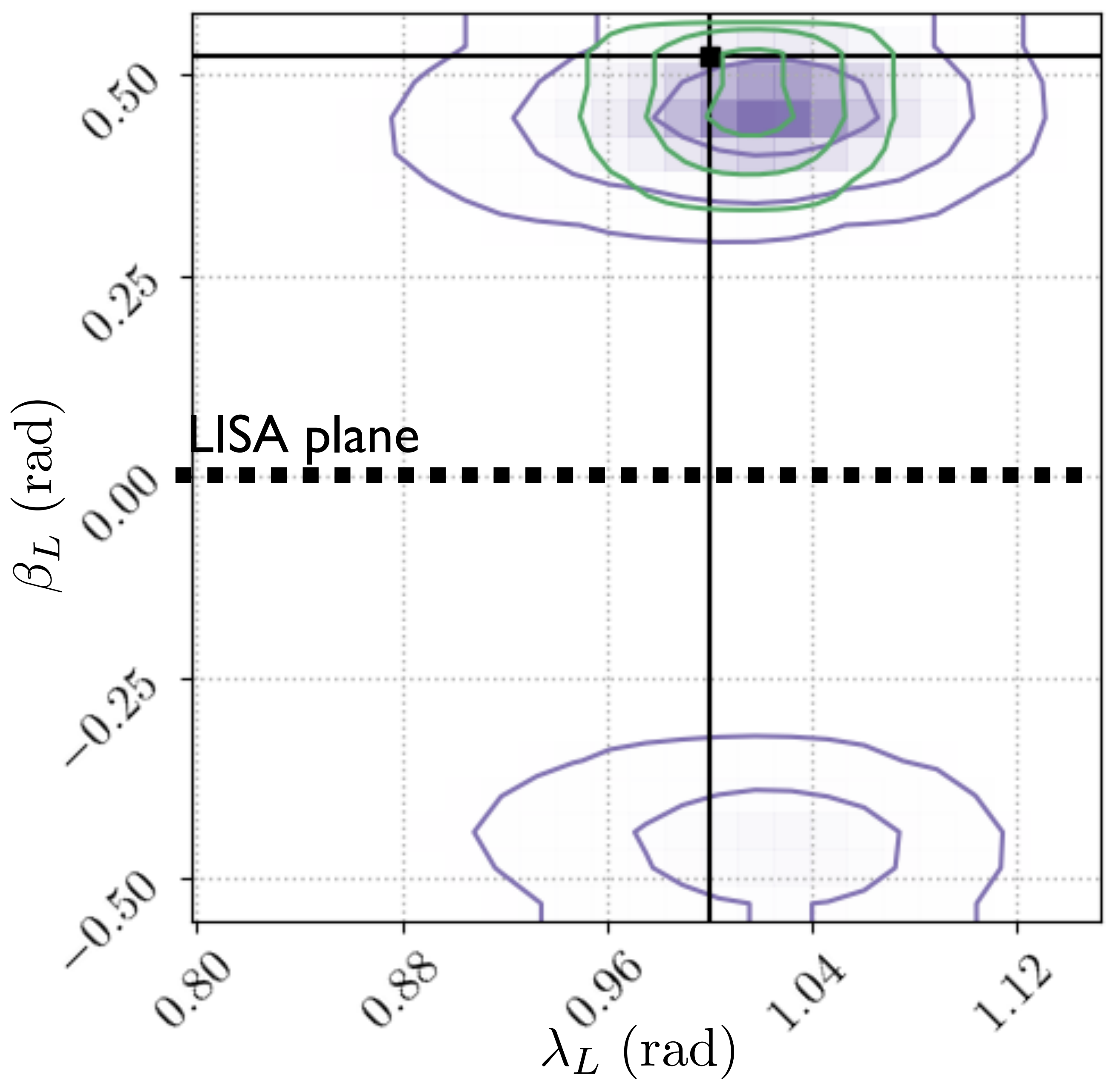

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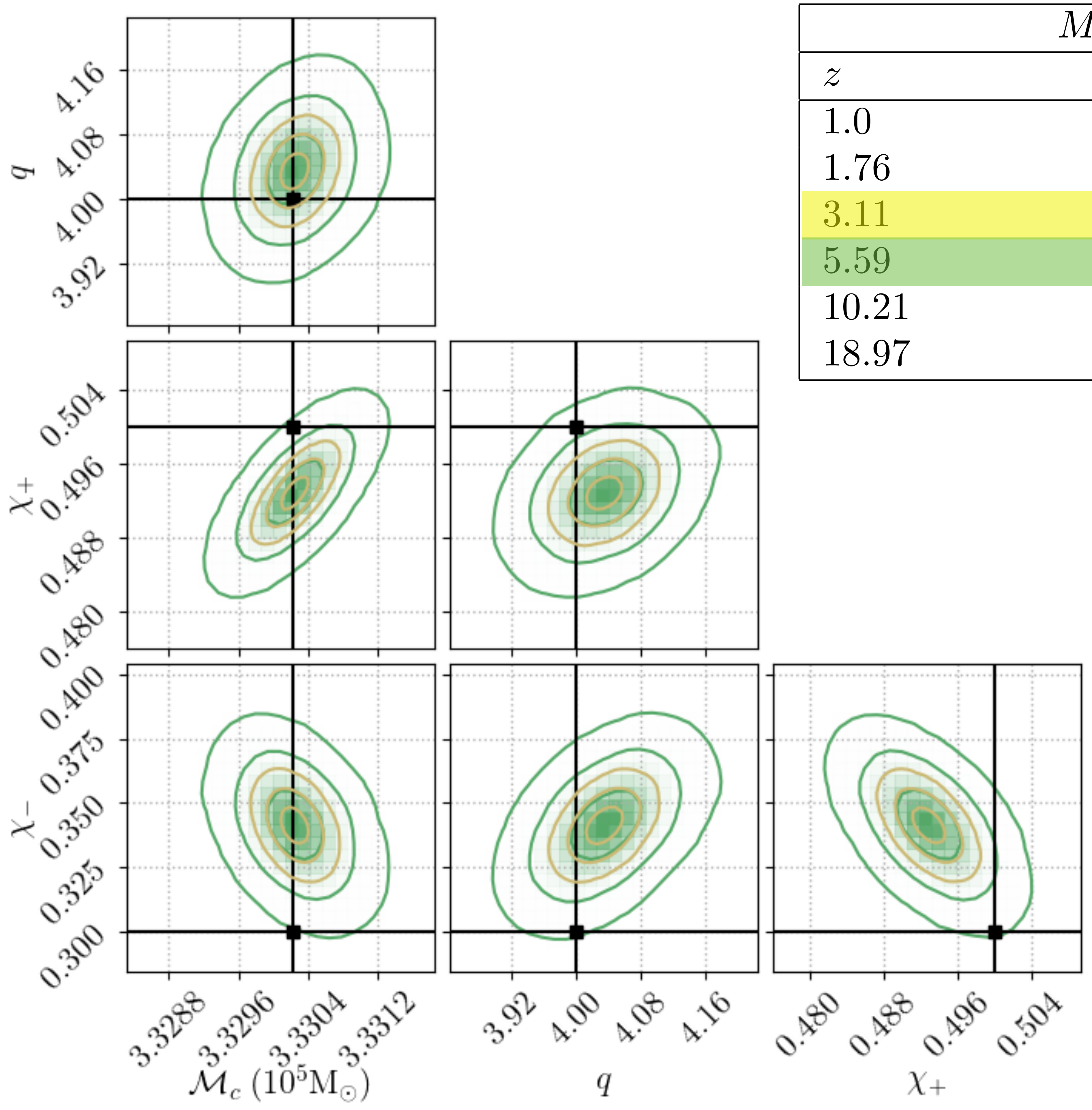

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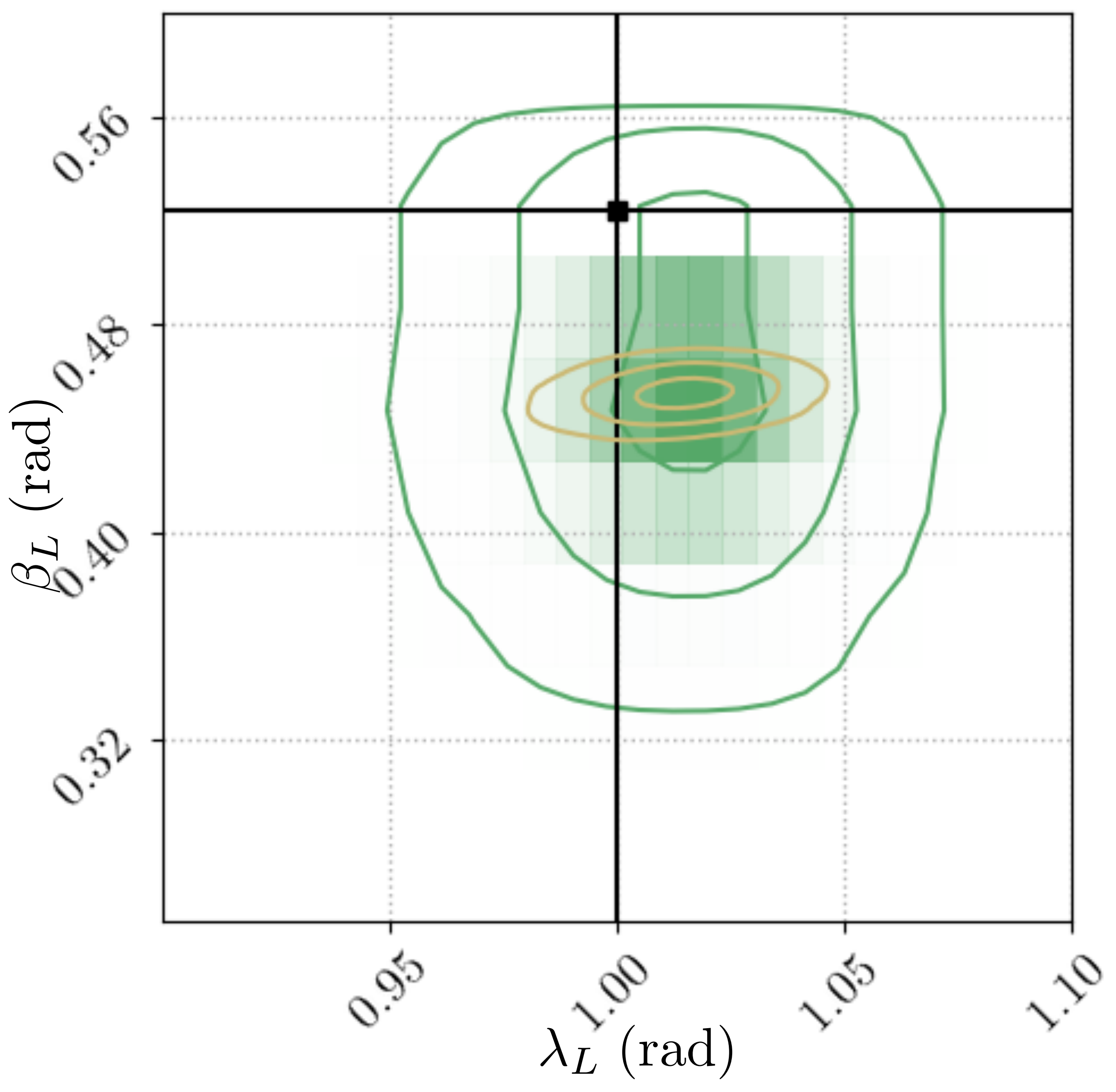

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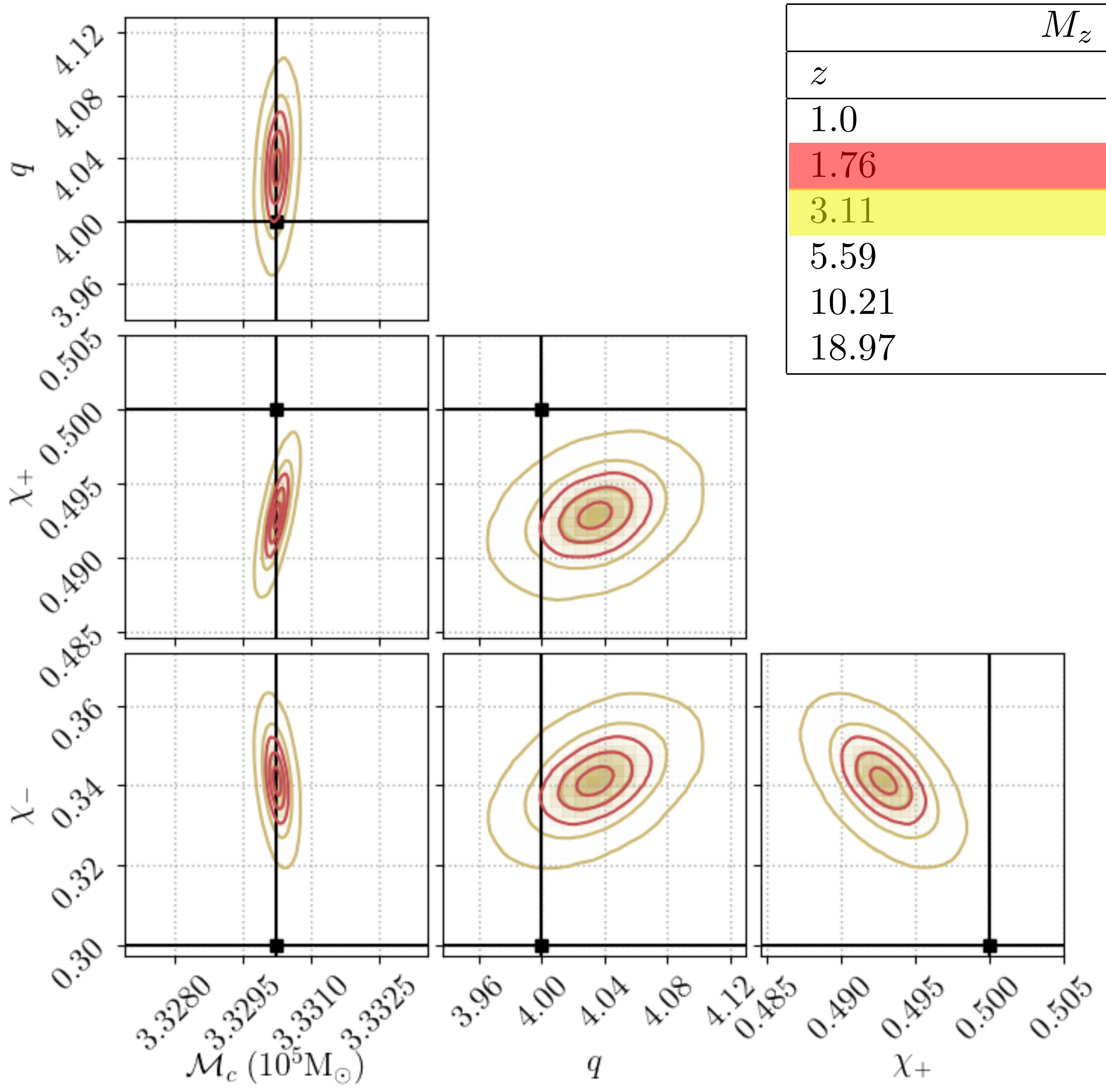

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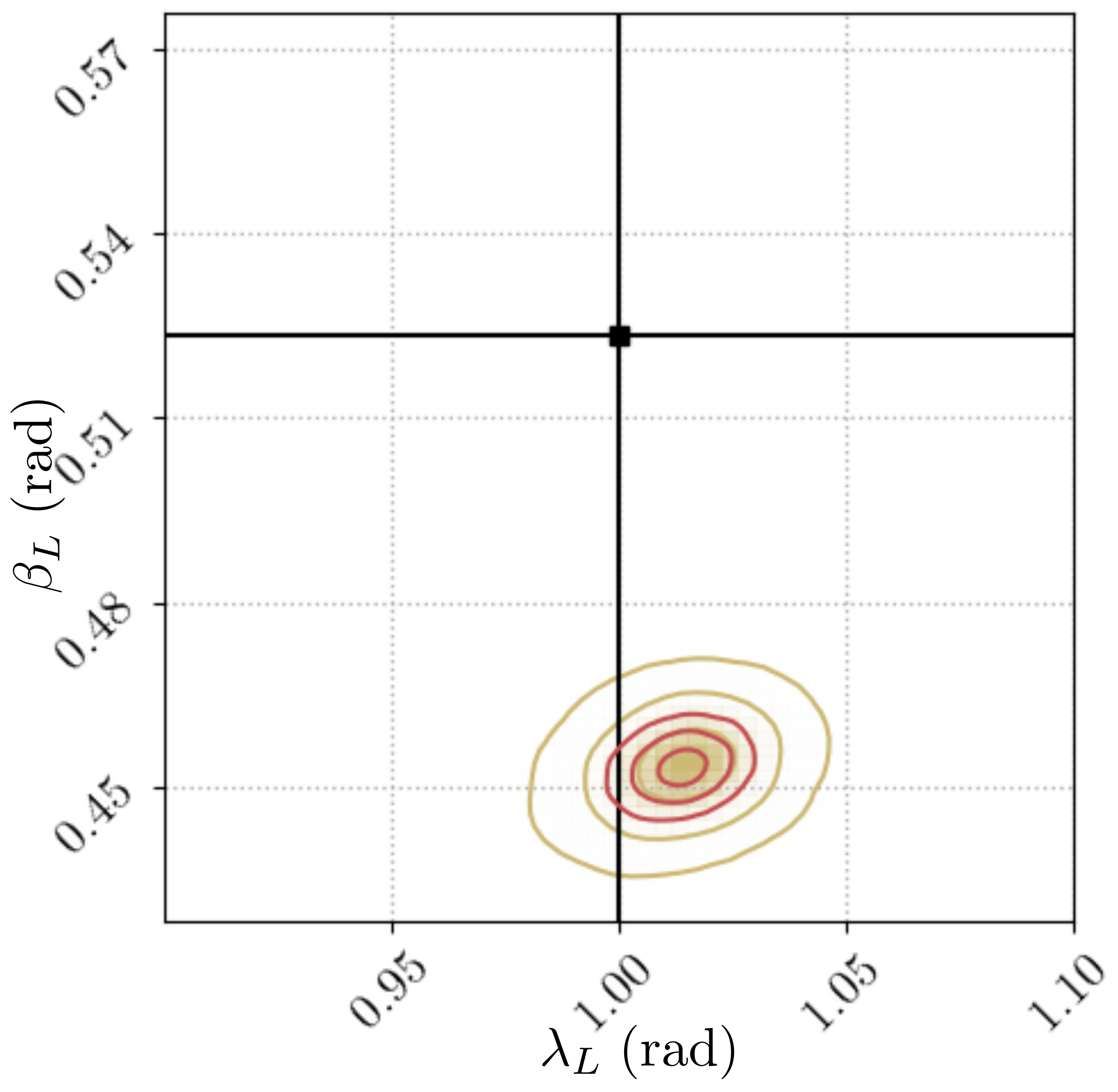

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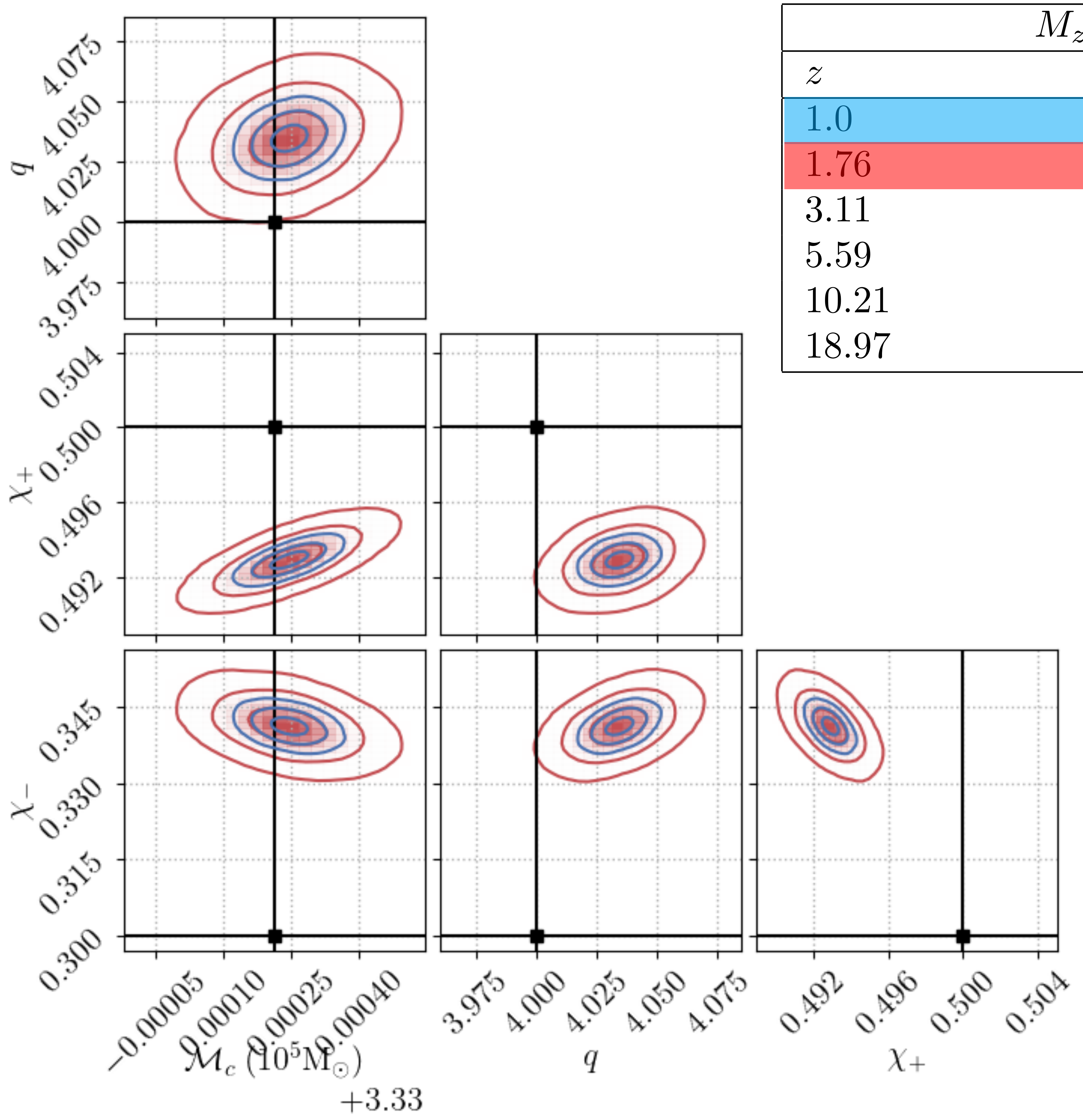

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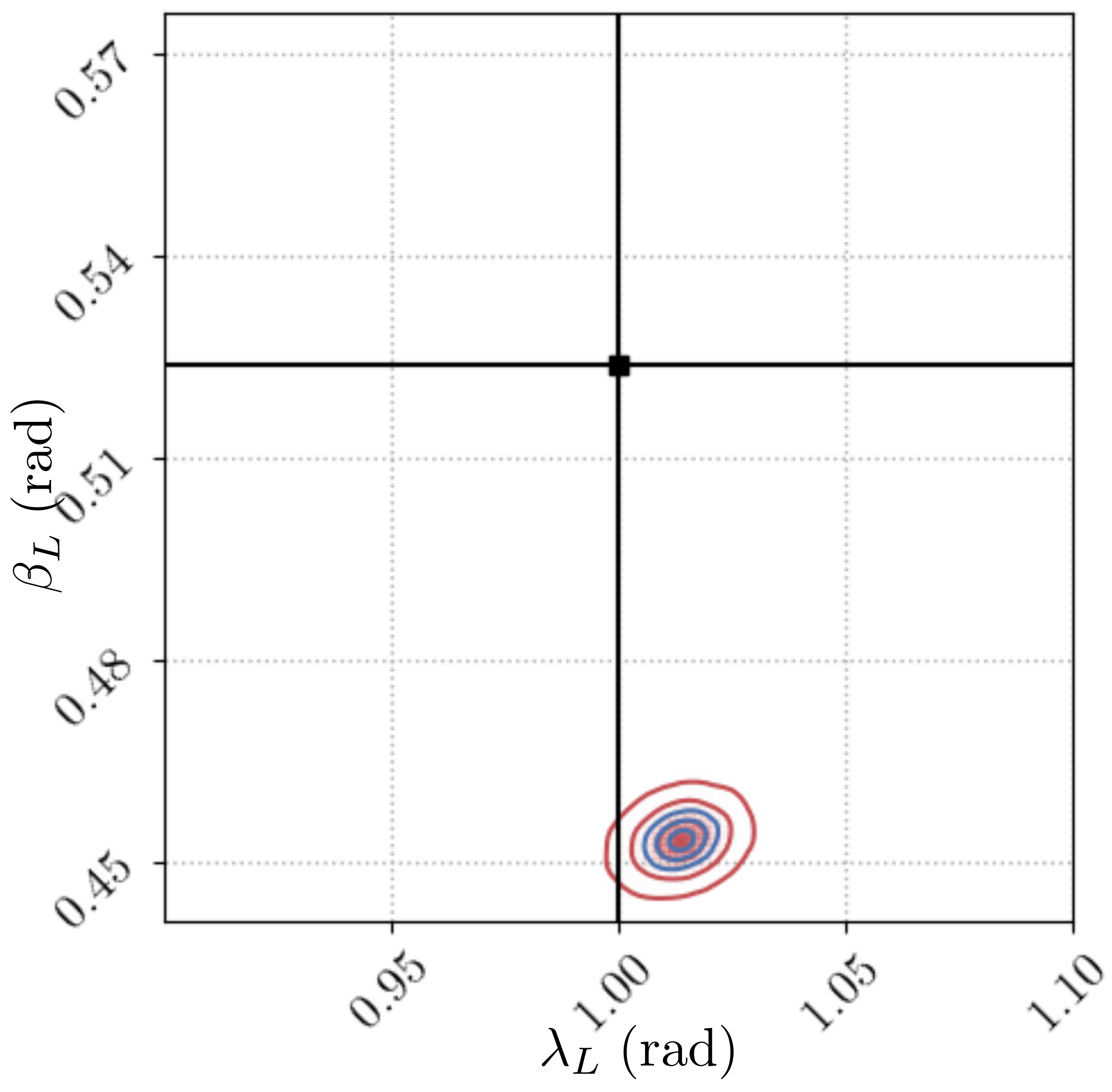
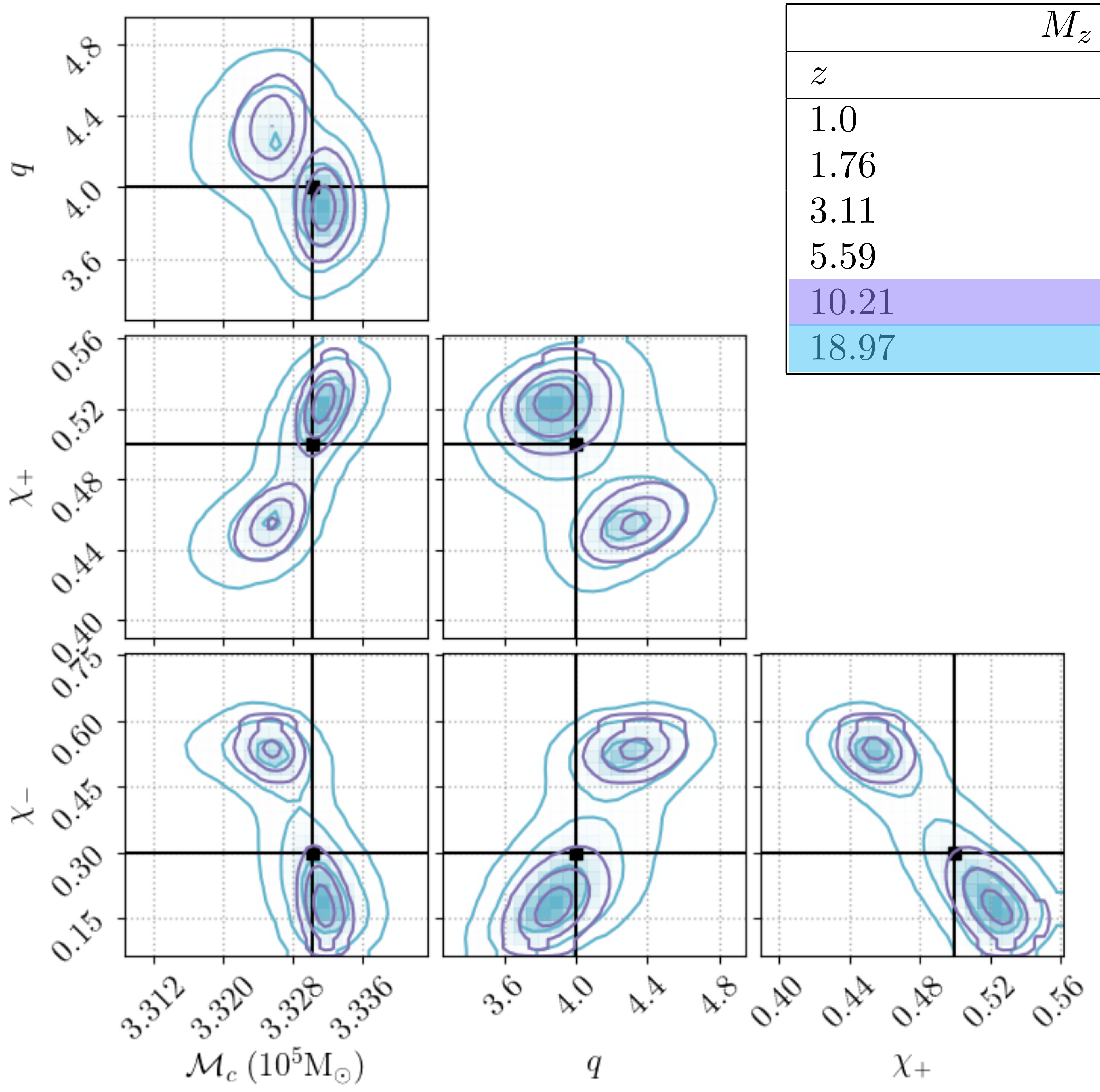


The bad:
- converges ‘near’ the true parameters
- significant bias at $z = 1$, SNR $= 1907$

Example Parameter estimation with systematics III

- **Injection:** NRHybSur3dq8 $\{M = 10^6 M_\odot, q = 4, \chi_1 = 0.5, \chi_2 = 0.3\}$
- **Template:** SEOBNRv5HM_ROM

Intrinsic params.



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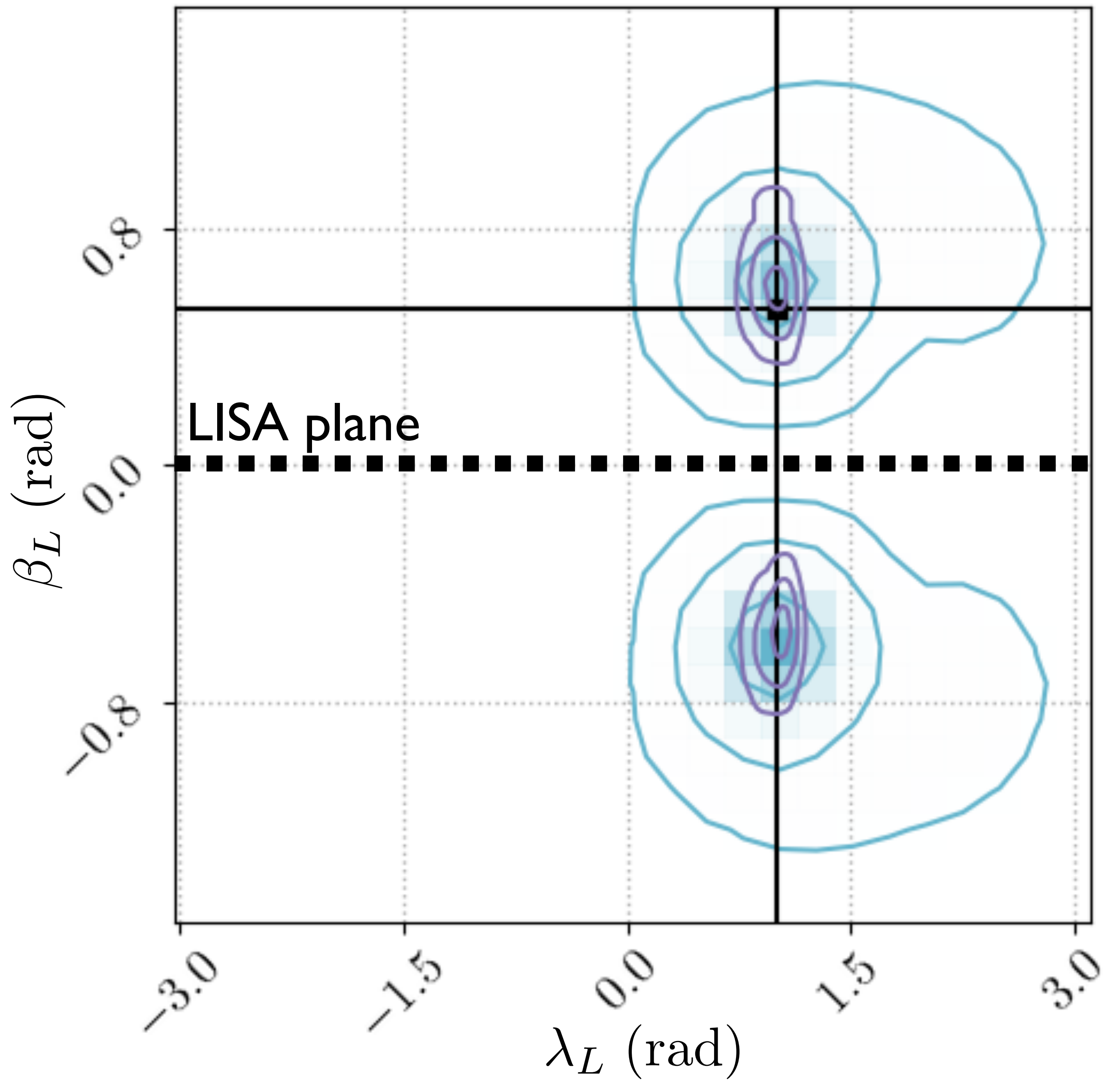

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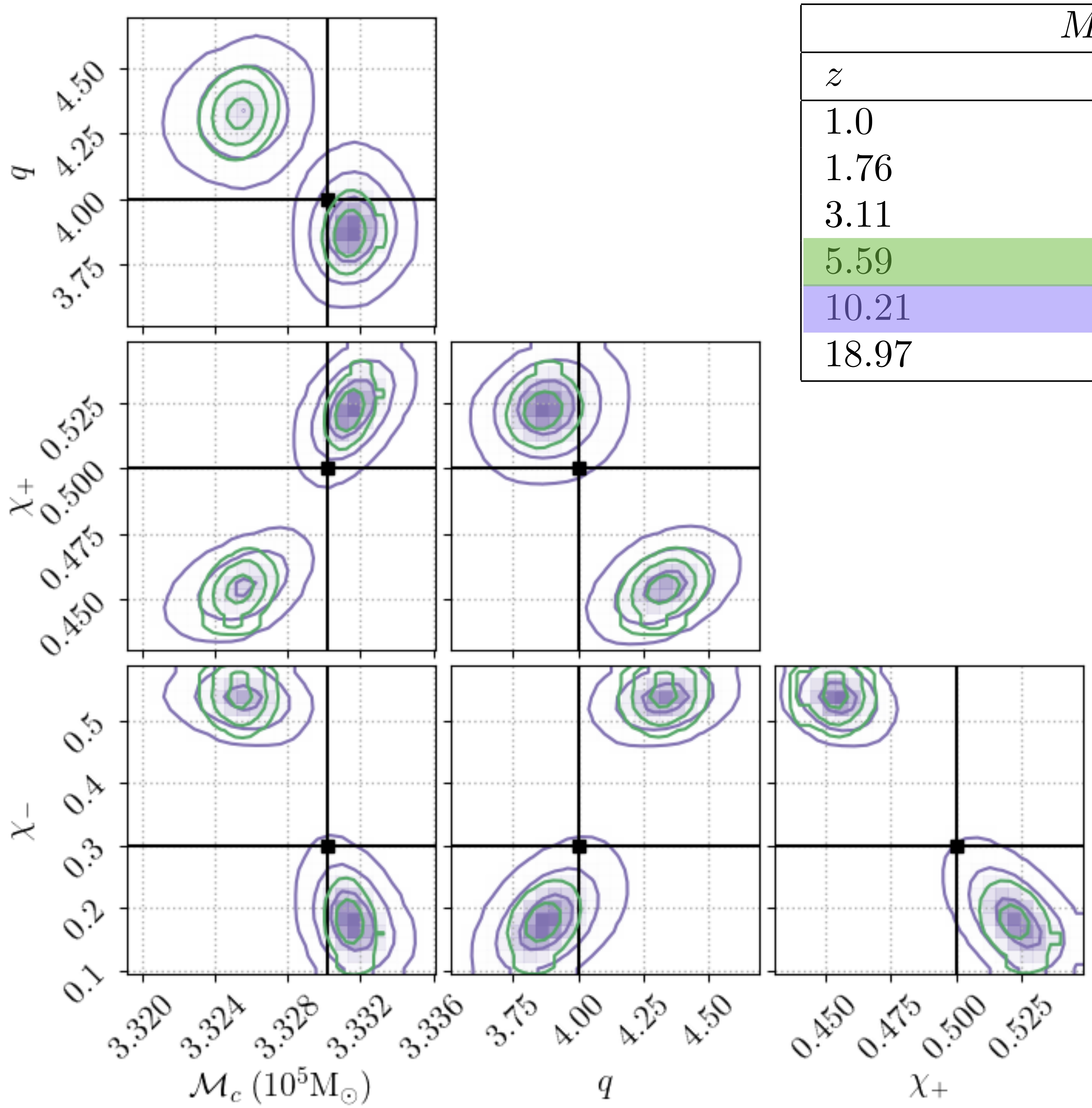

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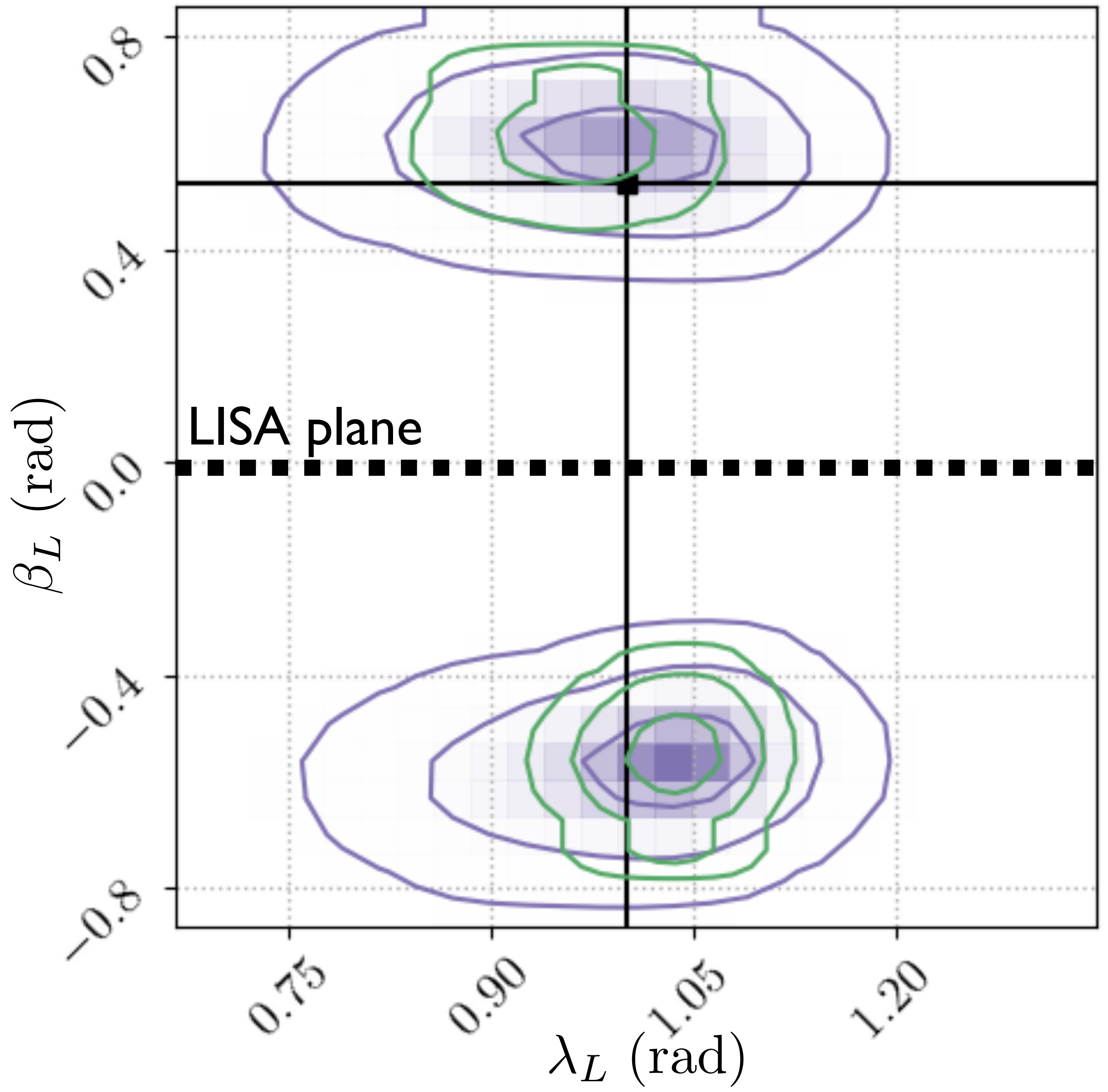

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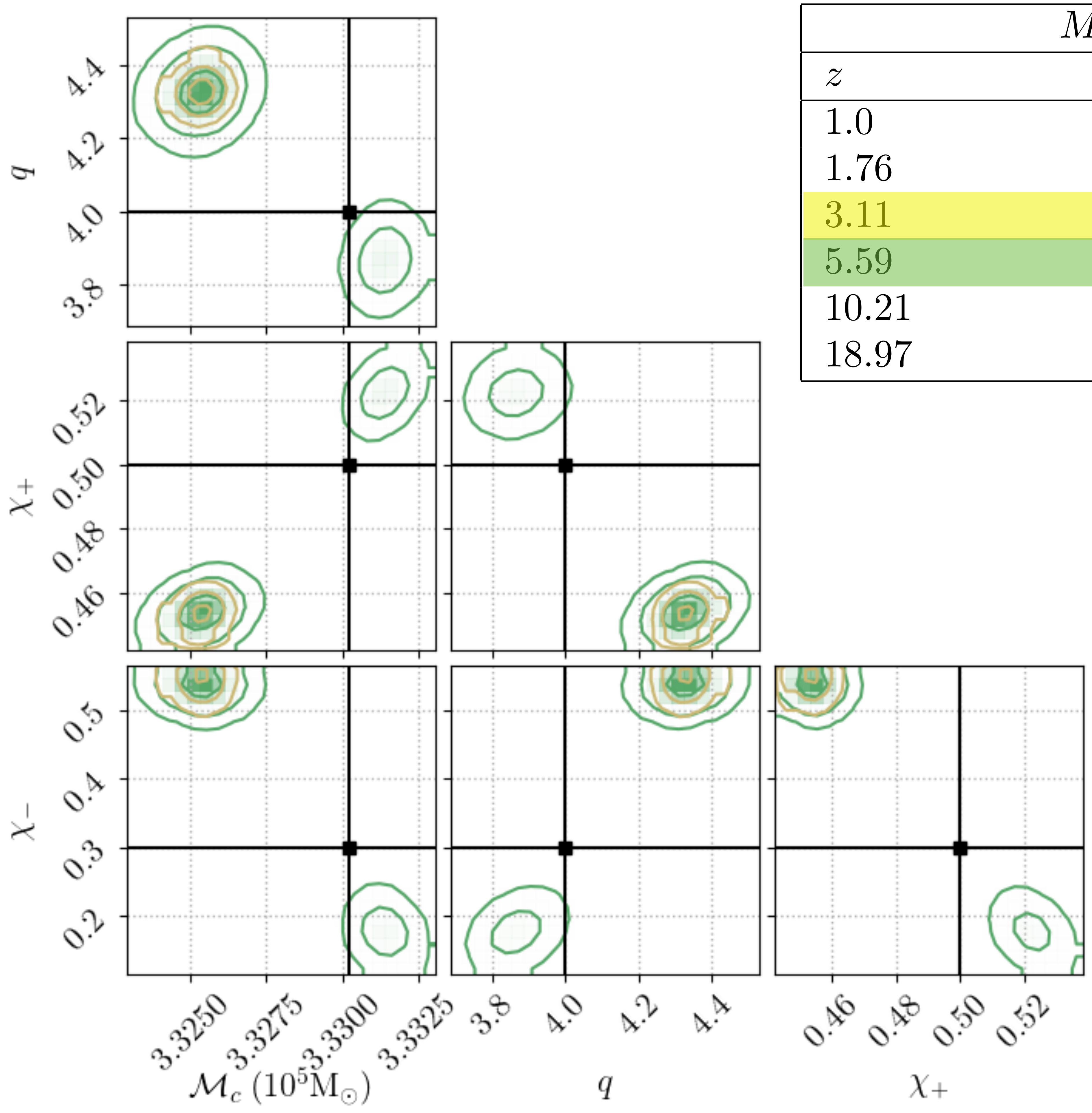

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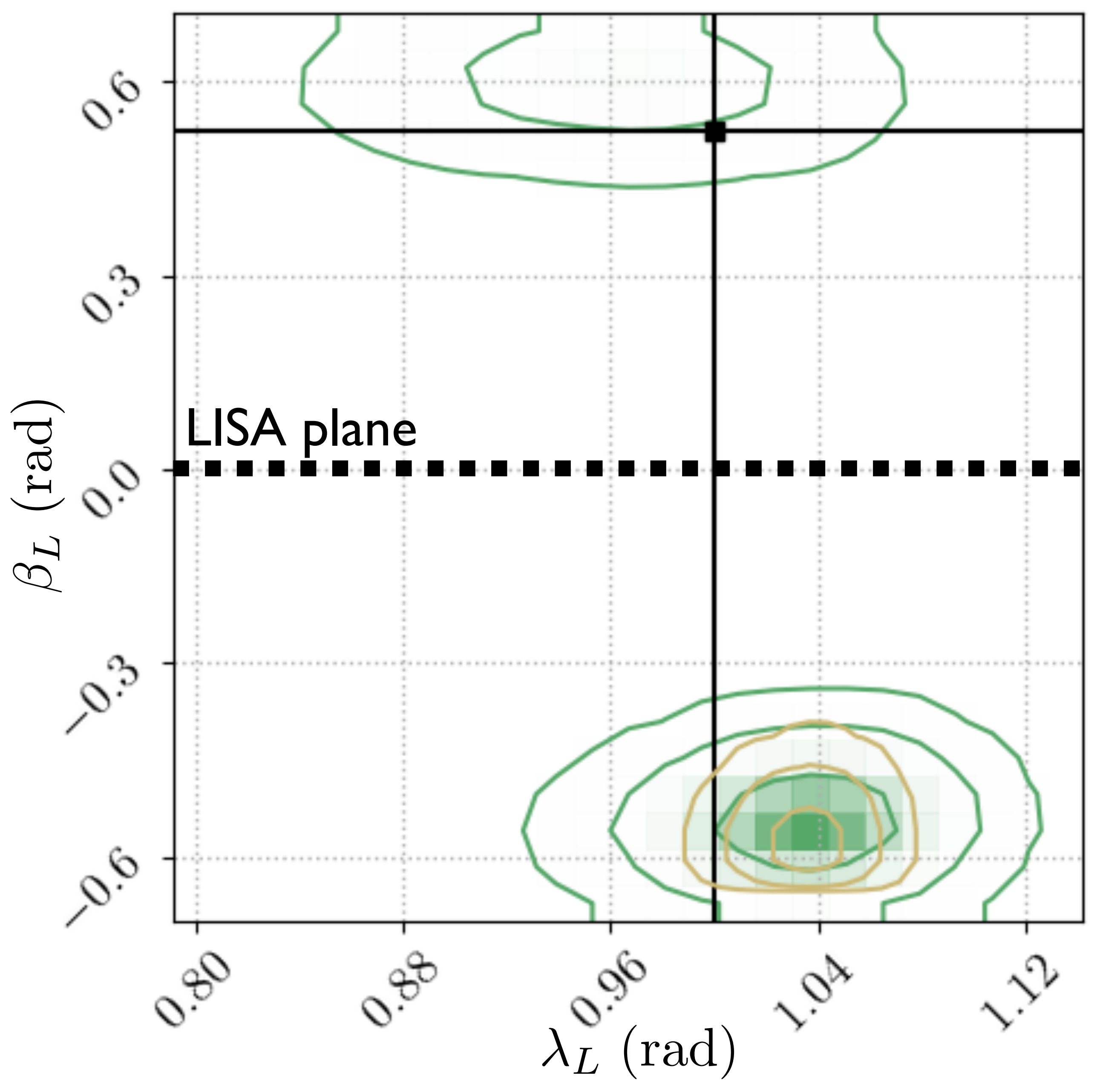

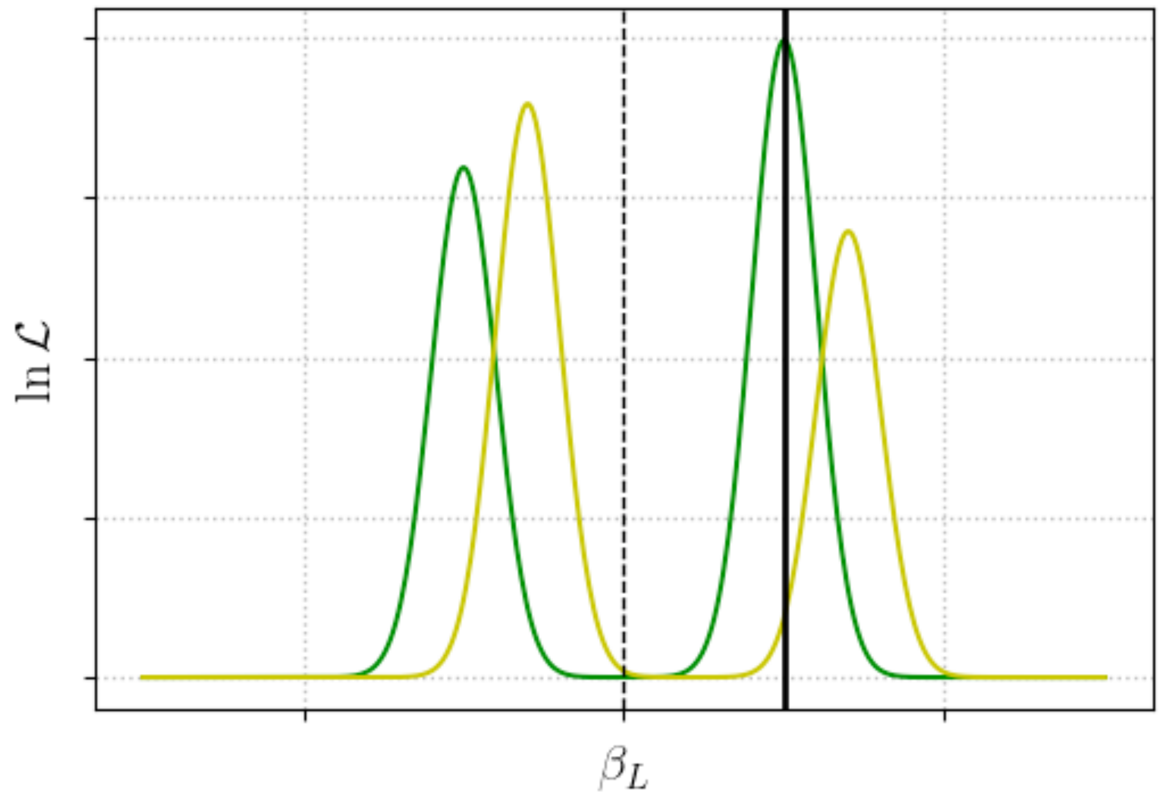

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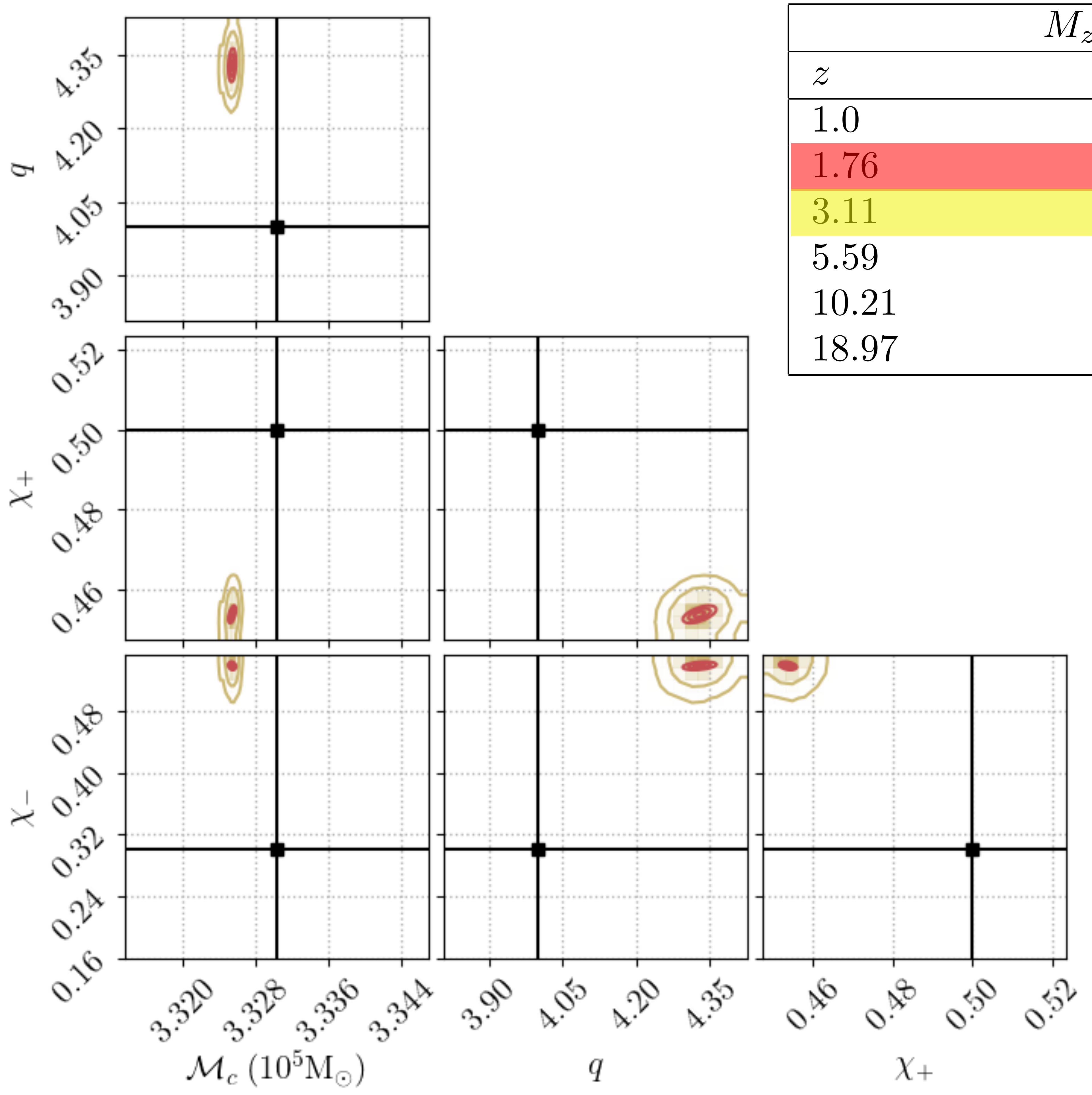
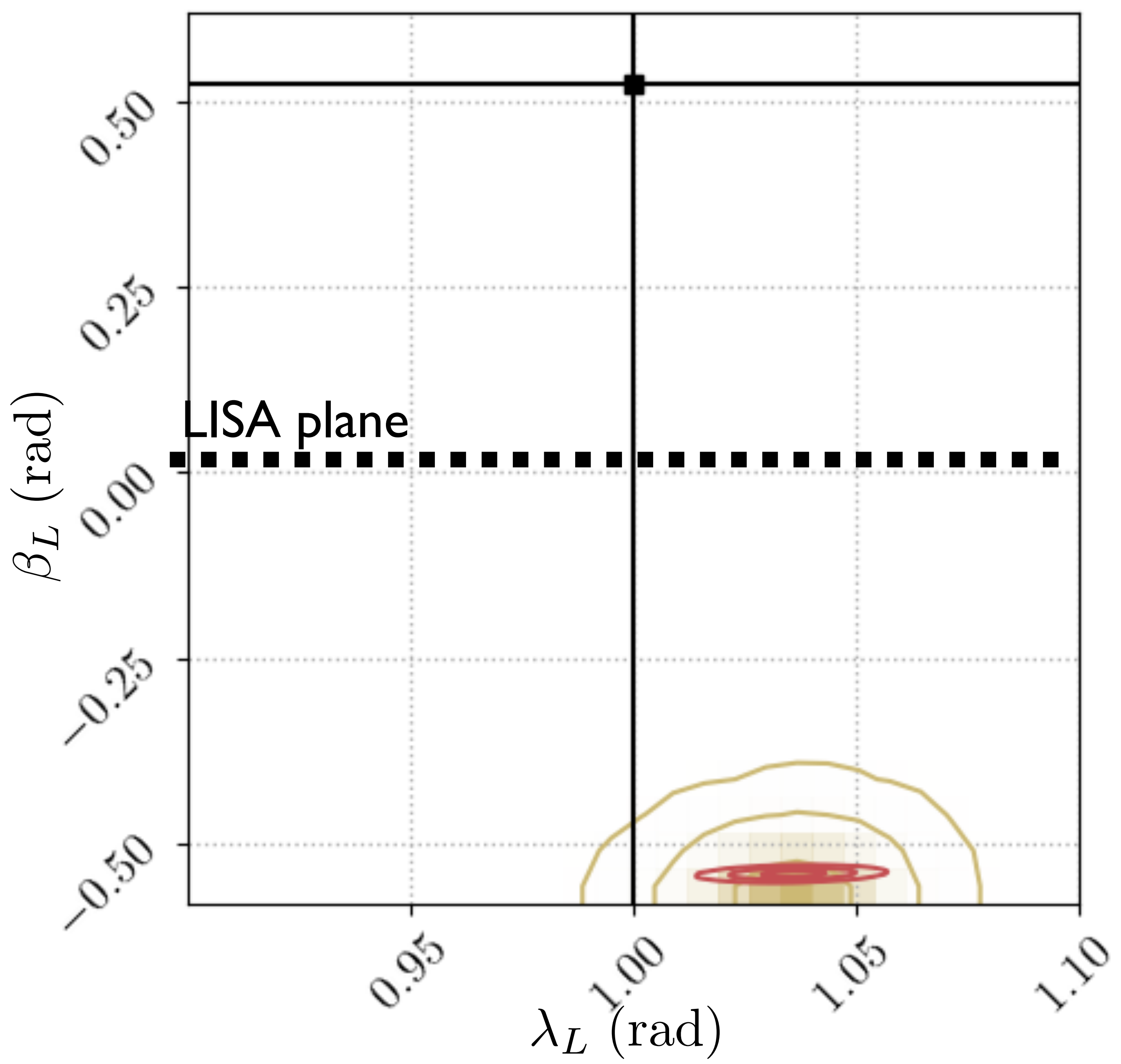
- **Template:** SEOBNRv5HM_ROM

Intrinsic params.



$M_z = 10^6\ M_\odot$	
z	SNR
1.0	1907
1.76	954
3.11	477
5.59	238
10.21	119
18.97	59

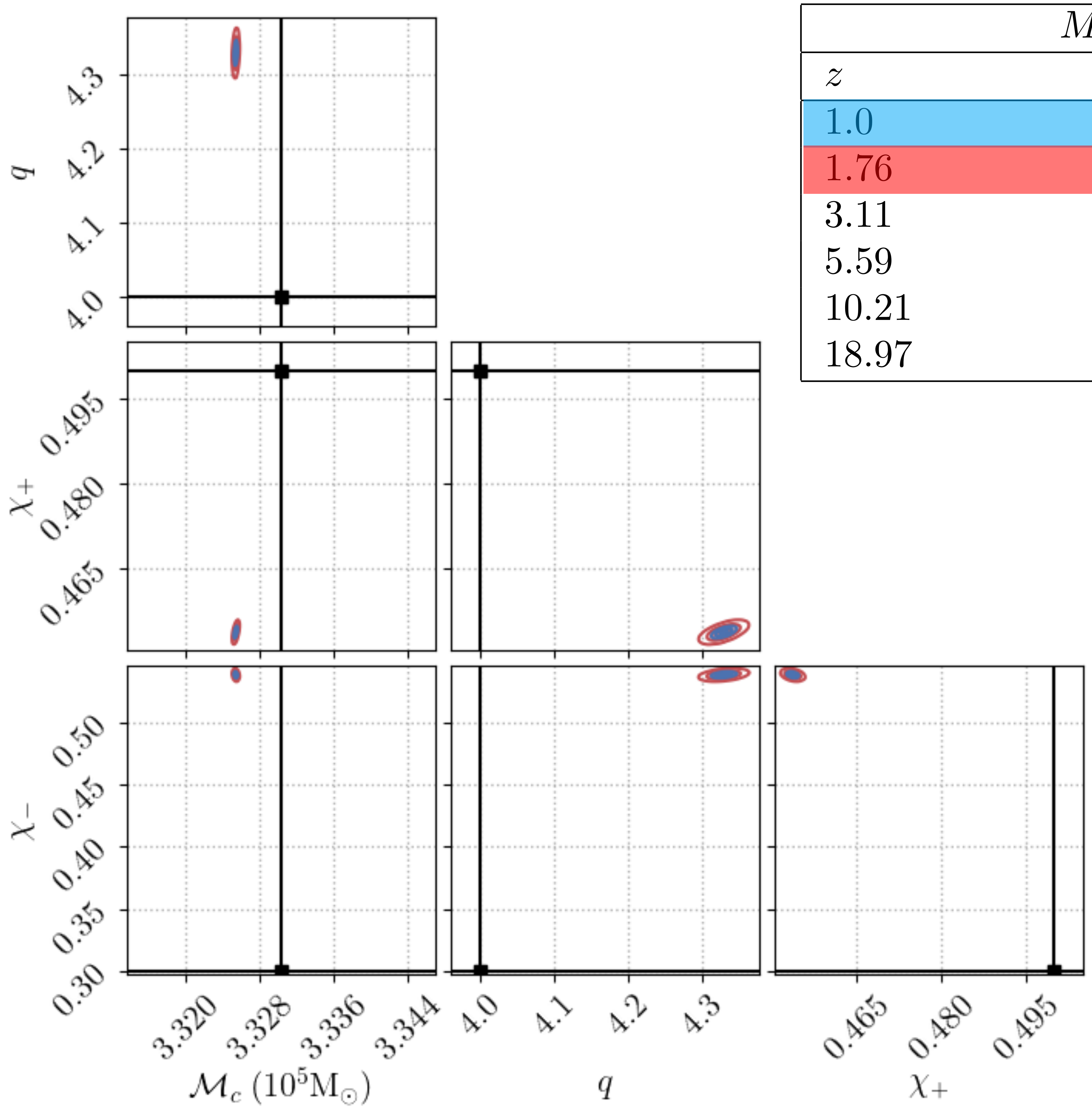
Sky localisation



Example Parameter estimation with systematics III

- **Injection:** NRHybSur3dq8 $\{M = 10^6 M_\odot, q = 4, \chi_1 = 0.5, \chi_2 = 0.3\}$
- **Template:** SEOBNRv5HM_ROM

Intrinsic params.



$M_z = 10^6\ M_\odot$	
z	SNR
1.0	1907
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3.11	477
5.59	238
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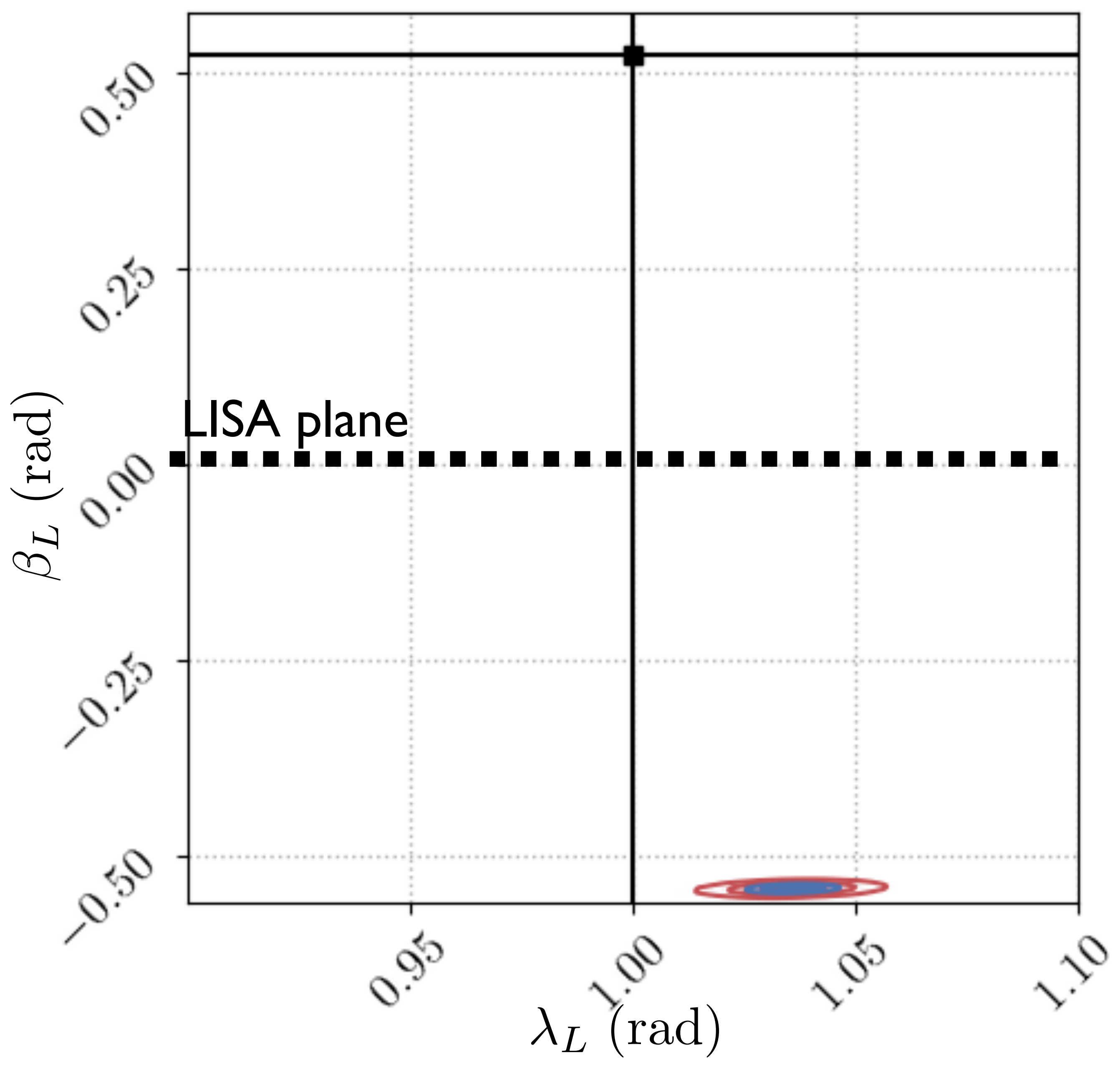
18.97	59

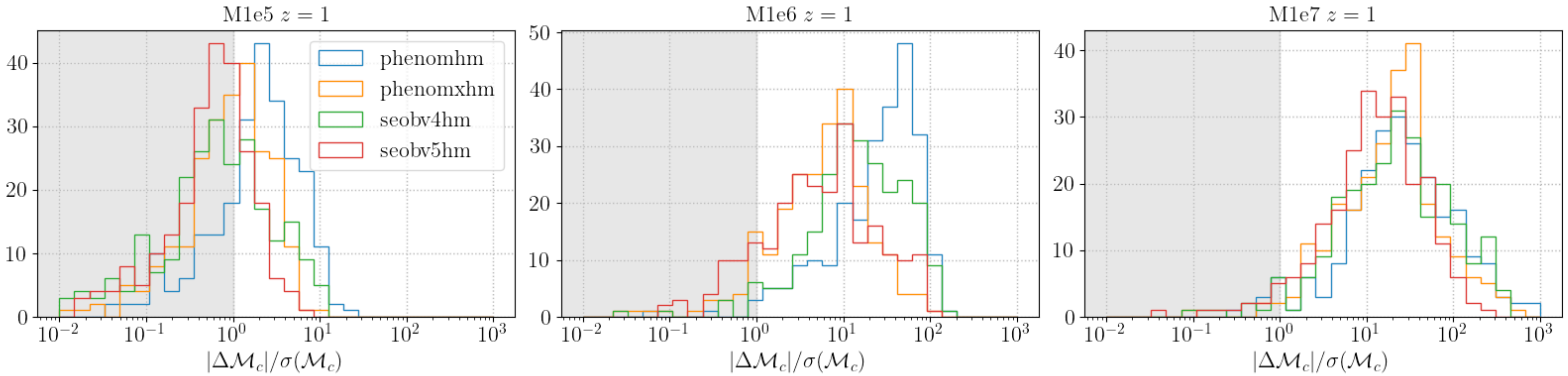
Sky localisation



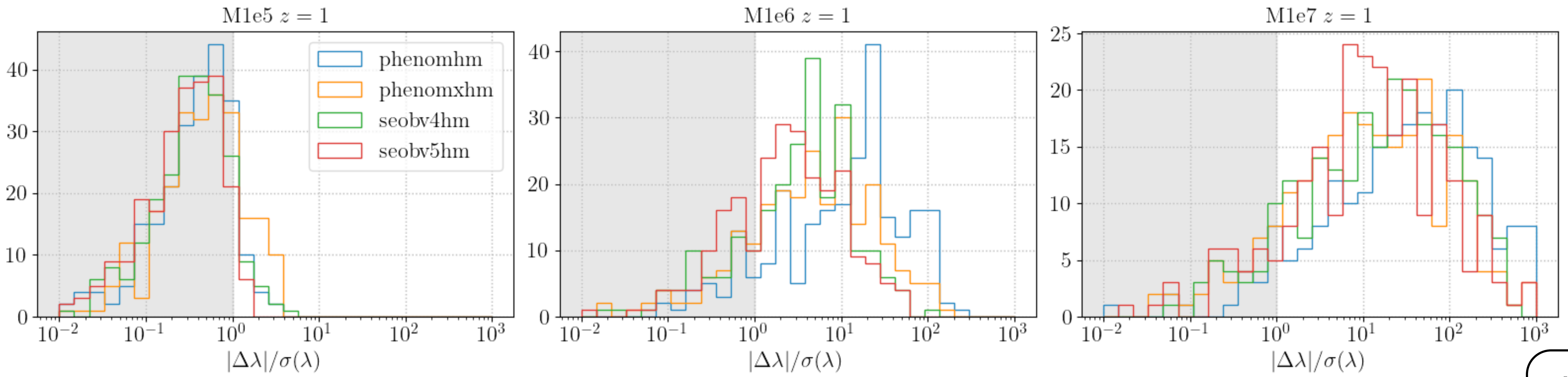
The ugly:
- converges to the **wrong sky mode**
- important bias at $z = 1$, SNR = 1907

Statistical significance of biases

Bias in chirp mass:



Bias in longitude (on corrected skymode):

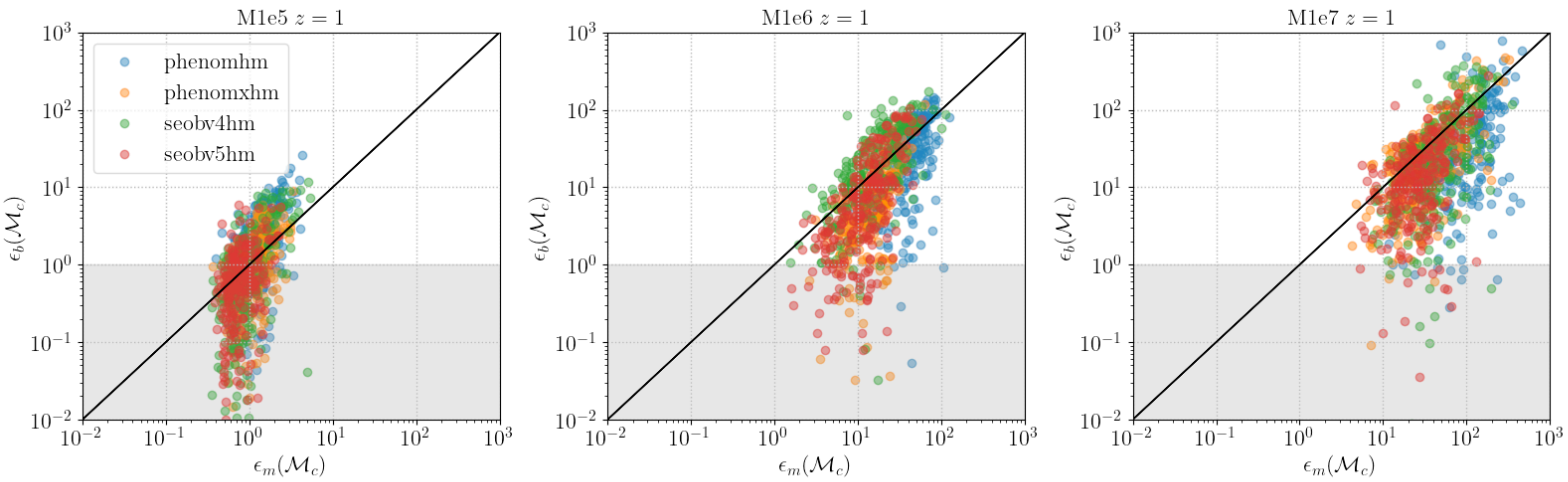


Wrong skymode recovered:

phenomxhm	
$M_z(M_\odot)$	% wrong
10^5	0 %
10^6	10 %
10^7	54 %

Large biases at high mass
Wrong skymodes common

Linking mismatches and biases



From indistinguishability criterion:

$$\mathrm{MM} < \frac{D}{2}\frac{1}{\mathrm{SNR}^2}$$

$$\epsilon_m = \sqrt{\frac{2}{D}\mathrm{SNR}^2\mathrm{MM}}$$

$\epsilon_m > 1$ means that the mismatch is large enough to indicate a significant bias

From bias measured in PE:

$$\epsilon_b = \frac{\Delta\theta}{\sigma(\theta)}$$

$\epsilon_b > 1$ indicates means that PE measures a significant bias

Both $\epsilon_b,\ \epsilon_m \propto \mathrm{SNR}$

Relation between mismatch and bias not straightforward

Complement: NR vs NR surrogate

Injections:

NR from SXS catalog [SXS 2025]

- SXS NR simulations hybridized with long EOB inspirals (covers ~6 months for $M = 10^5 M_\odot$)
- Surrogate interpolant, time-domain

Templates:

NRHybSur3dq8 [Varma&al 2018]

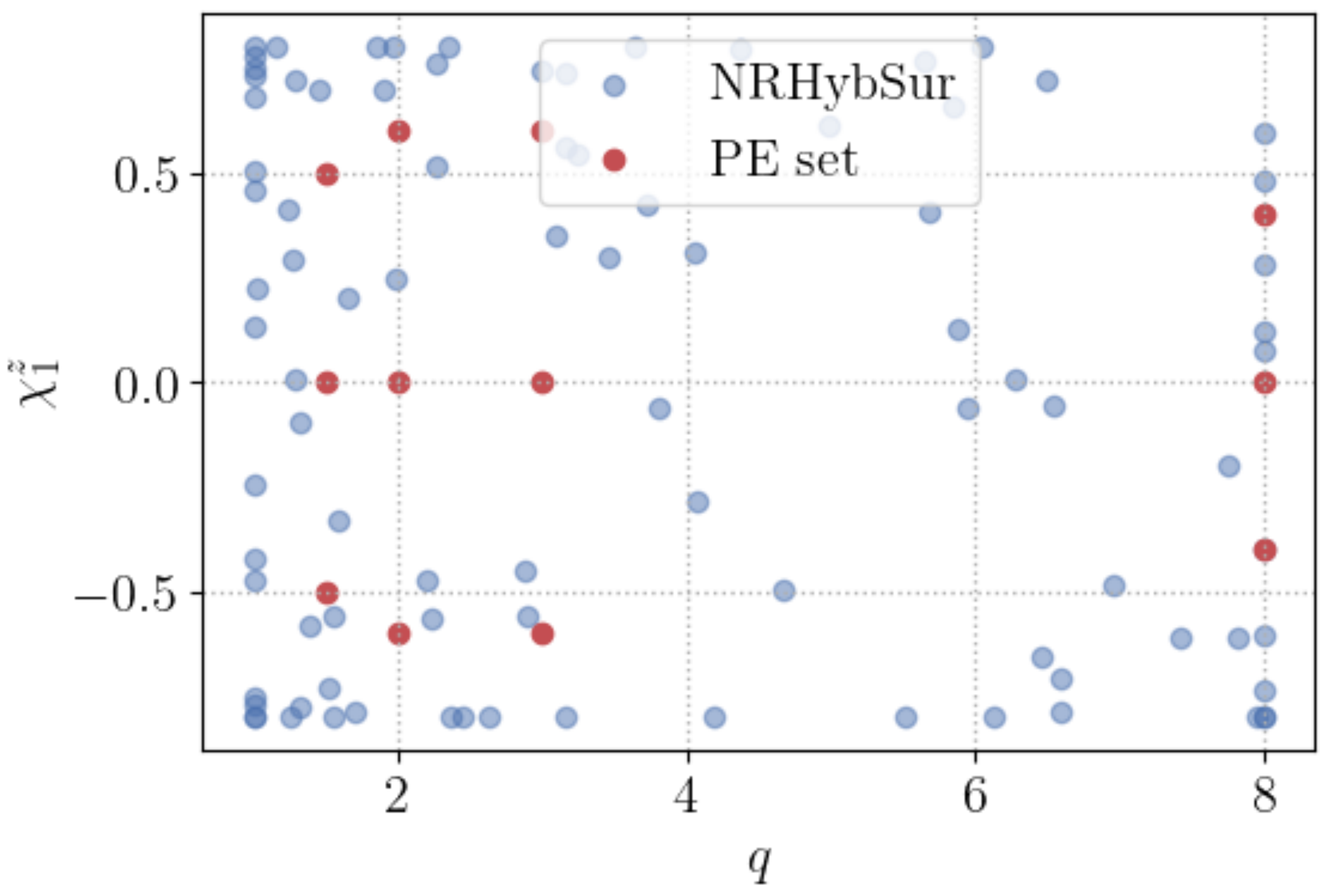
- SXS NR simulations hybridized with long EOB inspirals (covers ~6 months for $M = 10^5 M_\odot$)
- Surrogate interpolant, time-domain

Mode content for all: 22, 21, 33, 44

Time-domain LISA response

Limitations:
- aligned spins only
- M=1e7 only, 2 days of signal

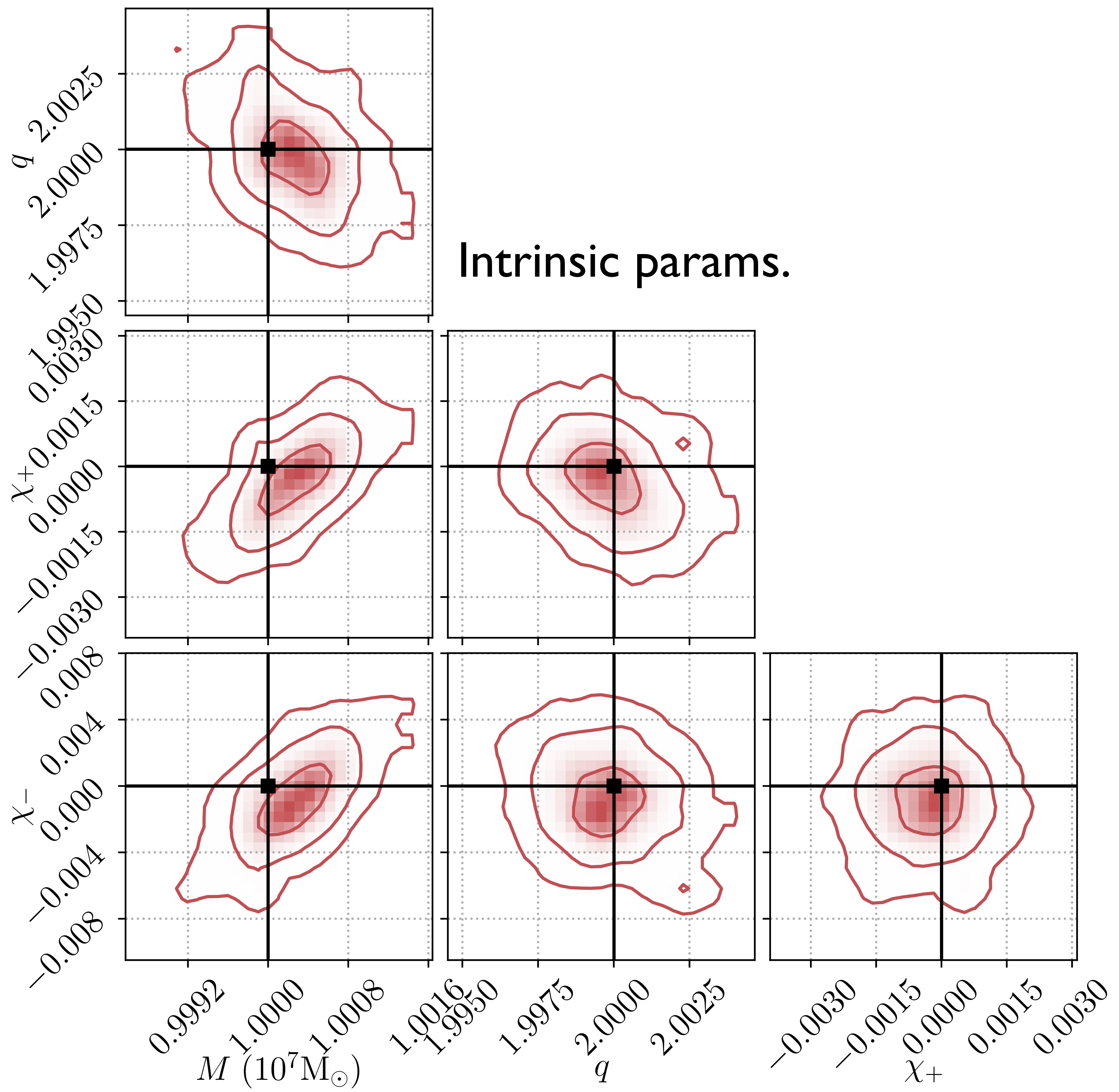
[Preliminary]

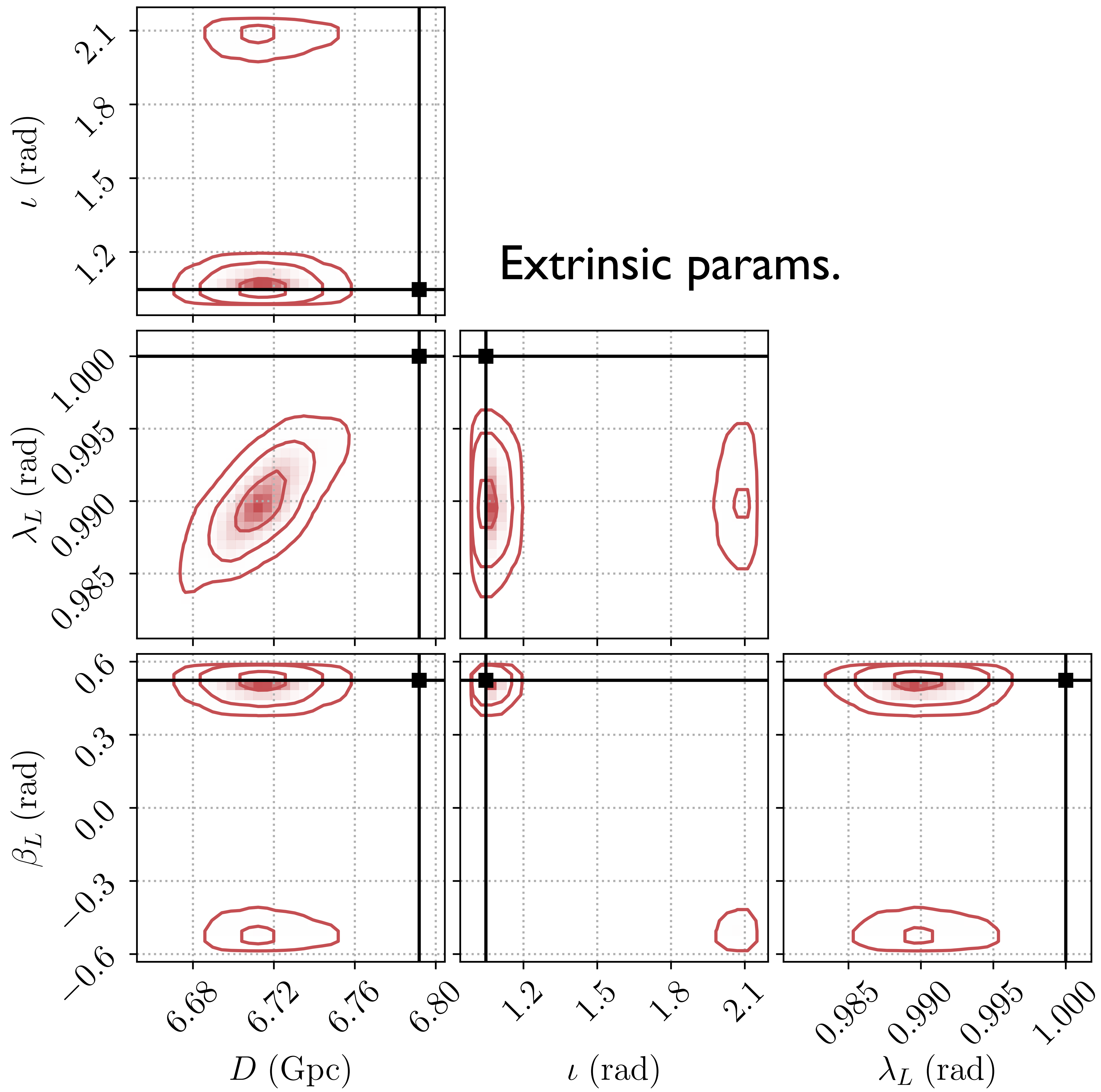


Complement: NR vs NR surrogate

SXS:BBH:2497

$M = 10^7 M_\odot,\ q = 2,\ \chi_1 = 0,\ \chi_2 = 0,\ z = 1$

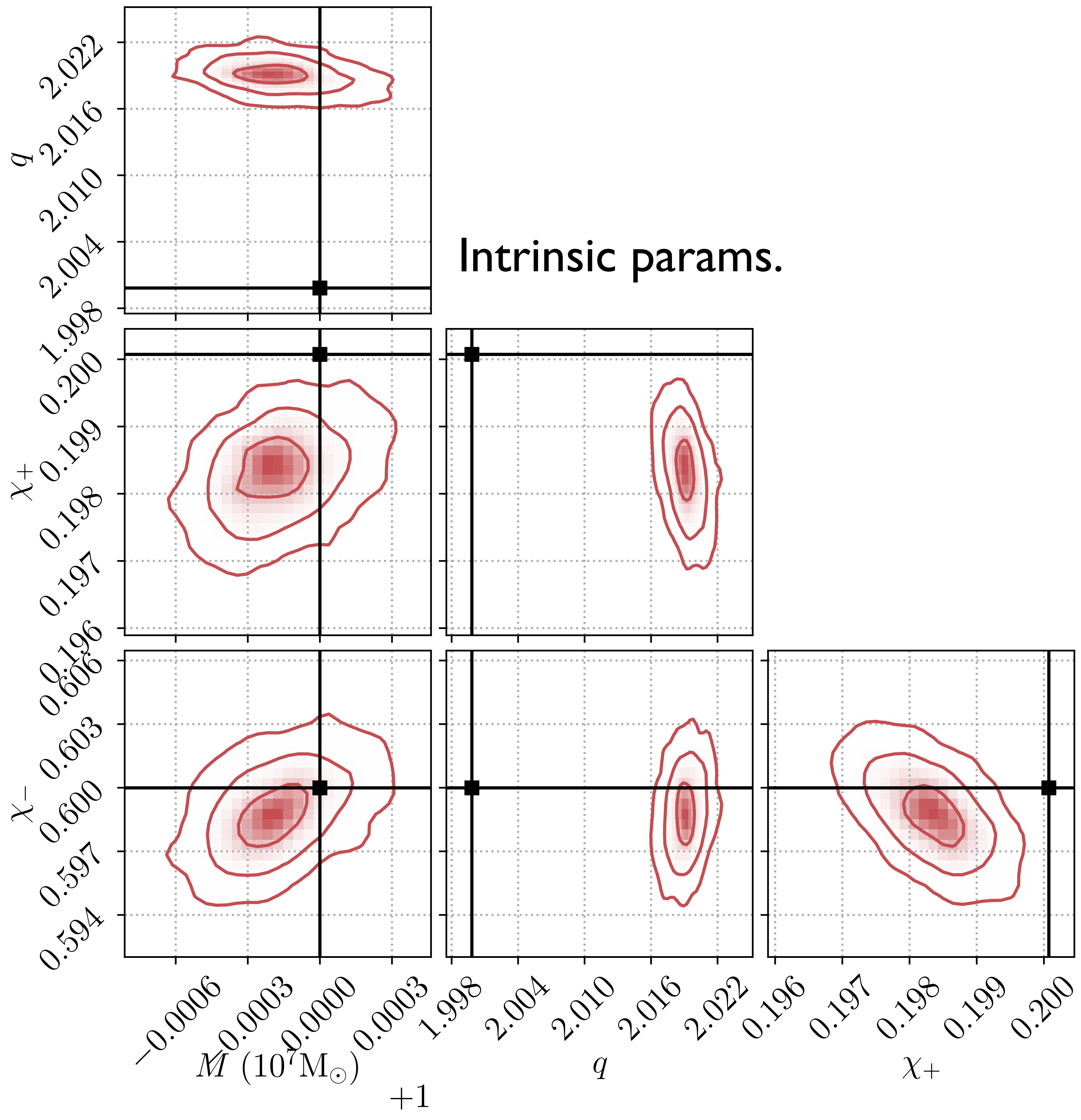


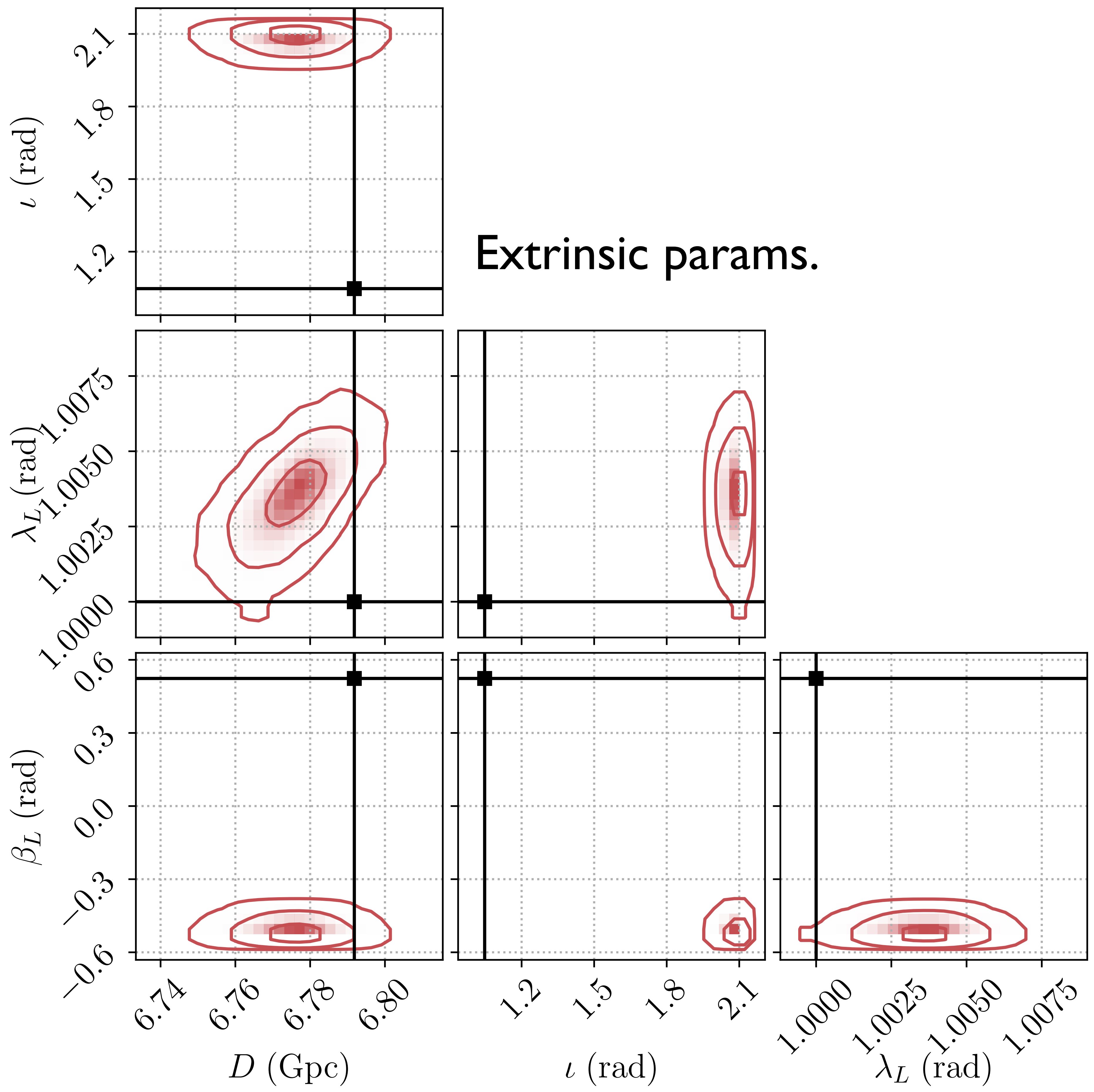


Complement: NR vs NR surrogate

SXS:BBH:2128

$M = 10^7 M_\odot,\ q = 2,\ \chi_1 = 0.6,\ \chi_2 = -0.6,\ z = 1$

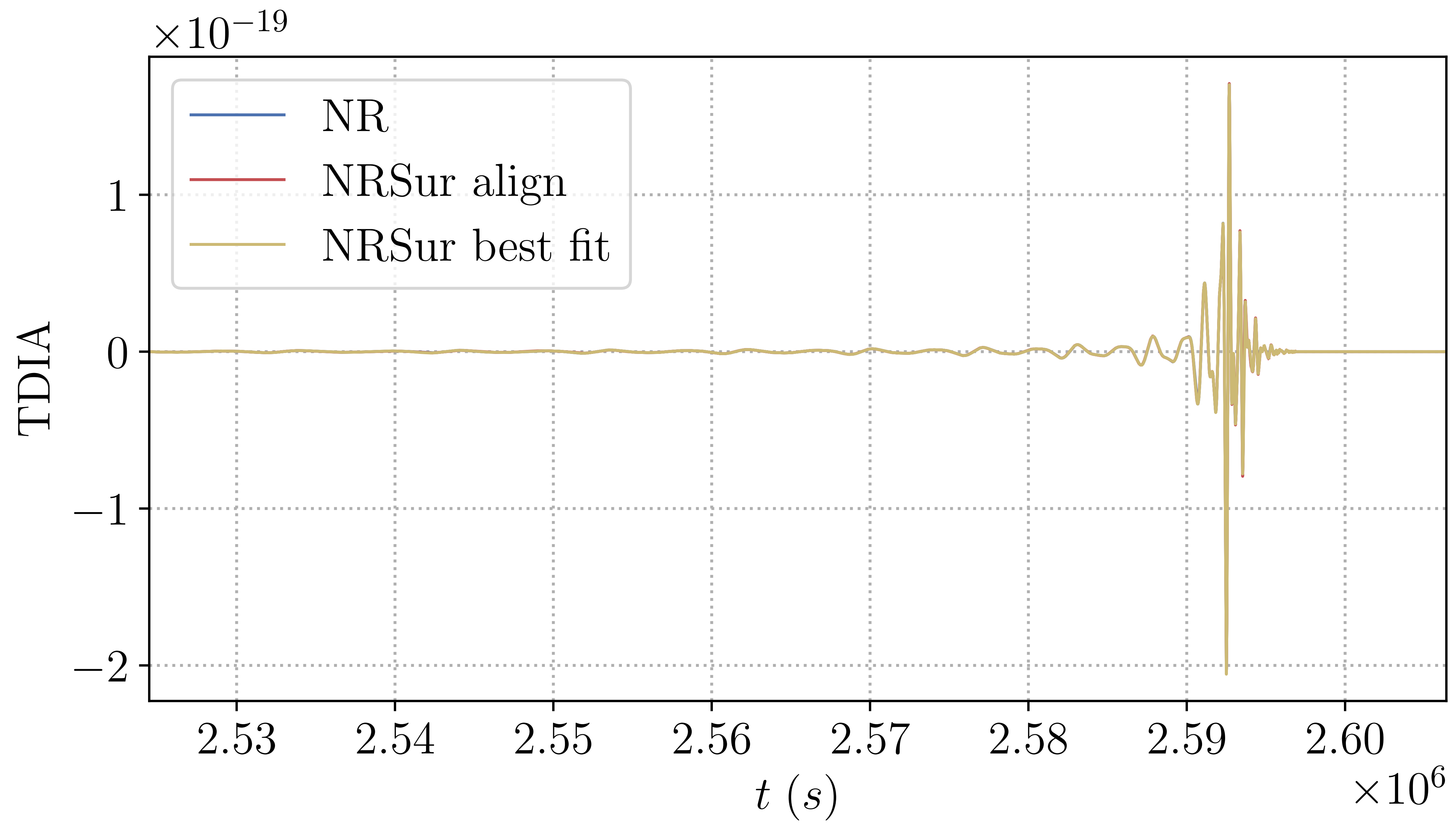


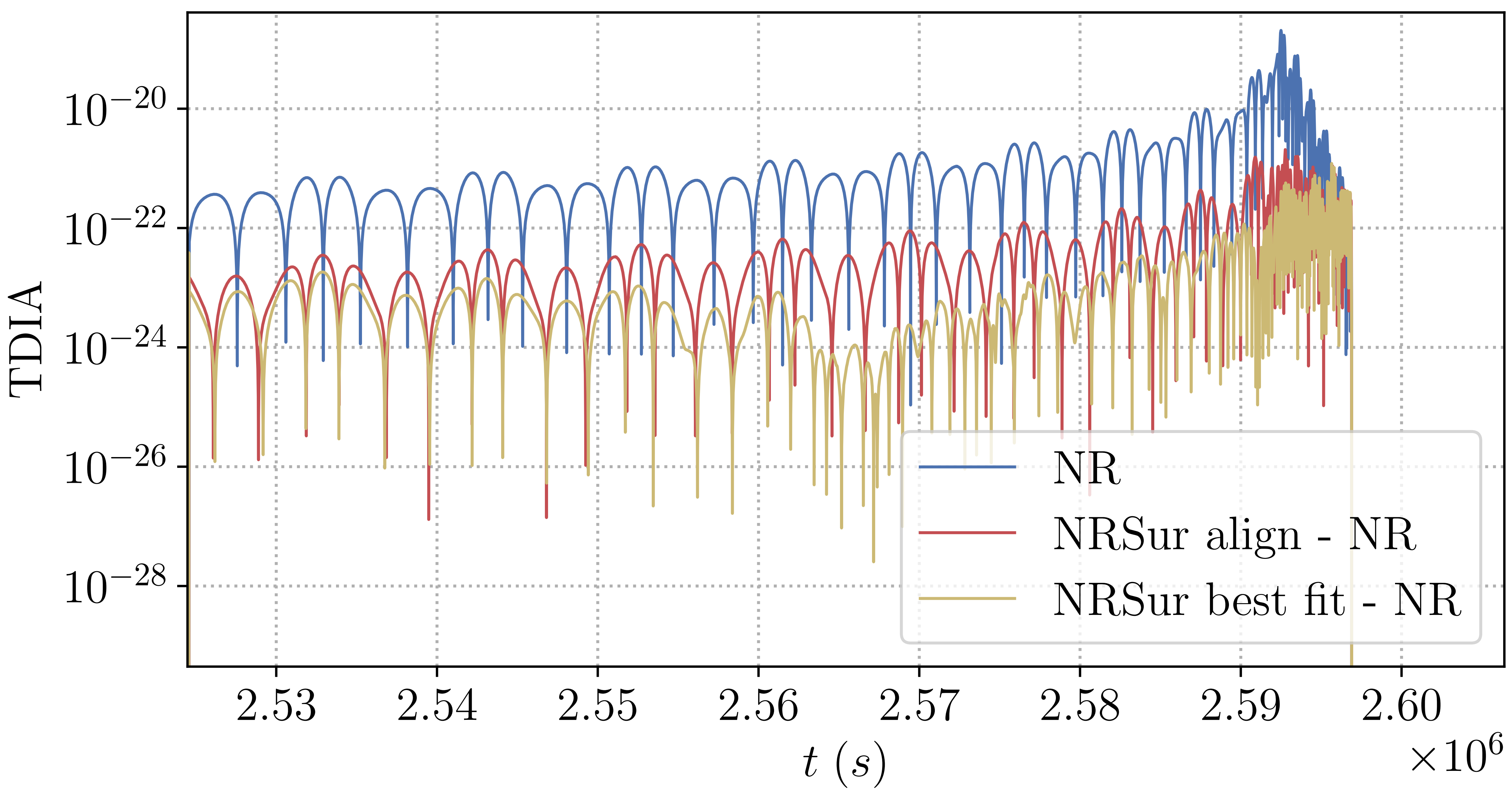


Complement: NR vs NR surrogate

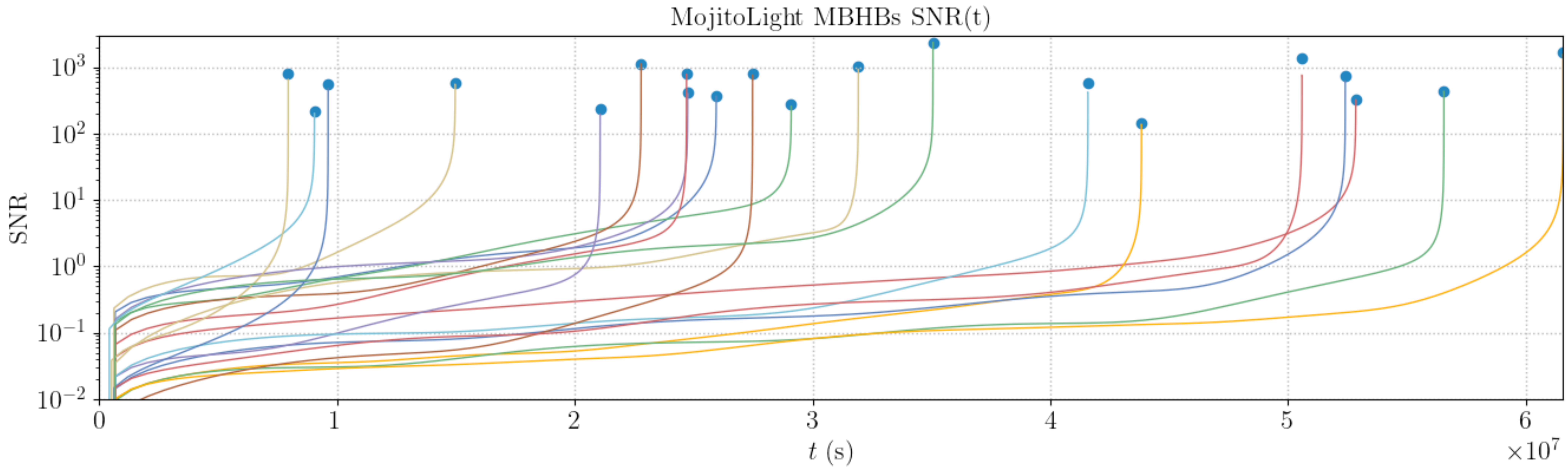
SXS:BBH:2160

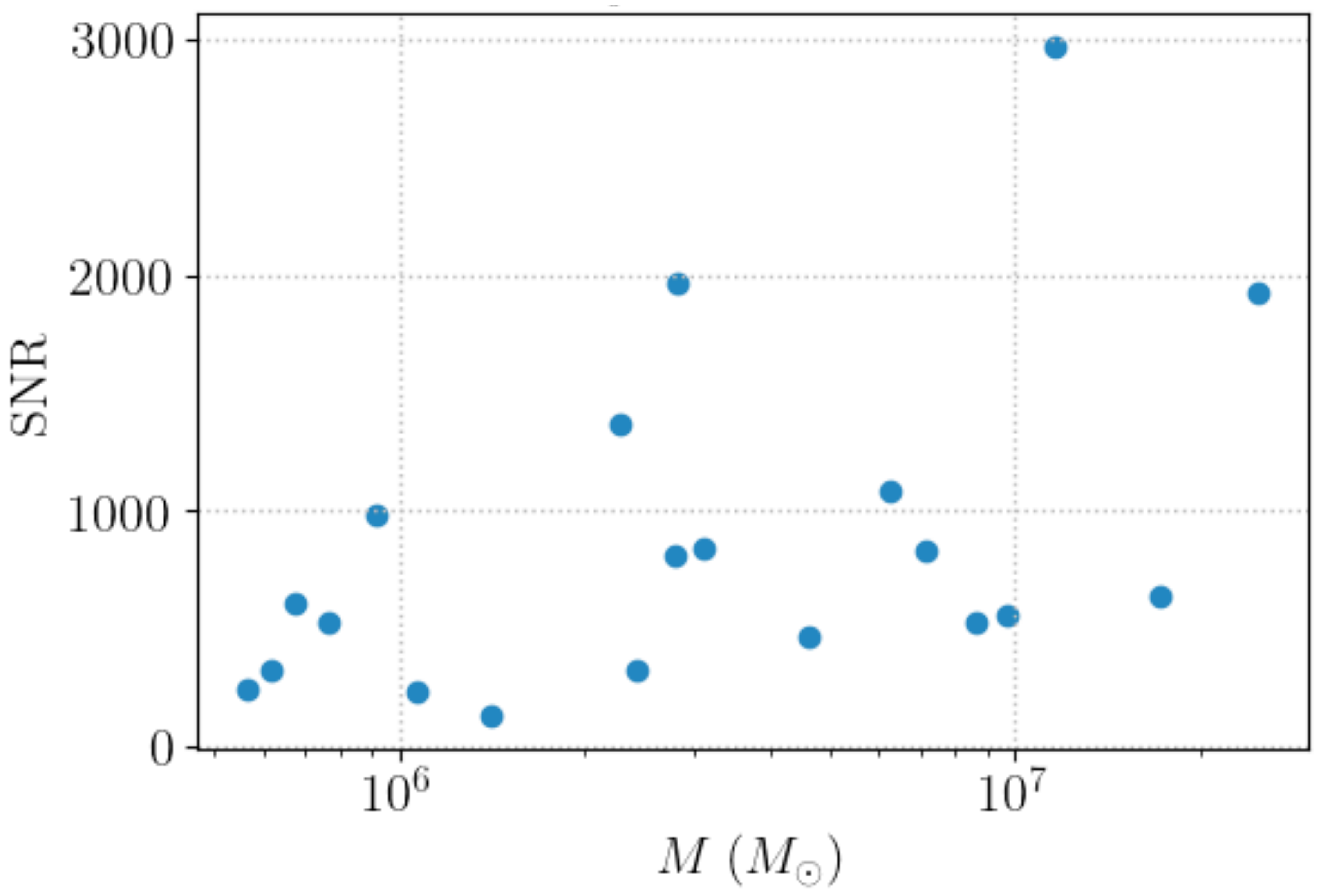
$M = 10^7 M_\odot, q = 3, \chi_1 = 0.6, \chi_2 = -0.4, z = 1$





MojitoLight MBHBs: a first look





MojitoLight sources

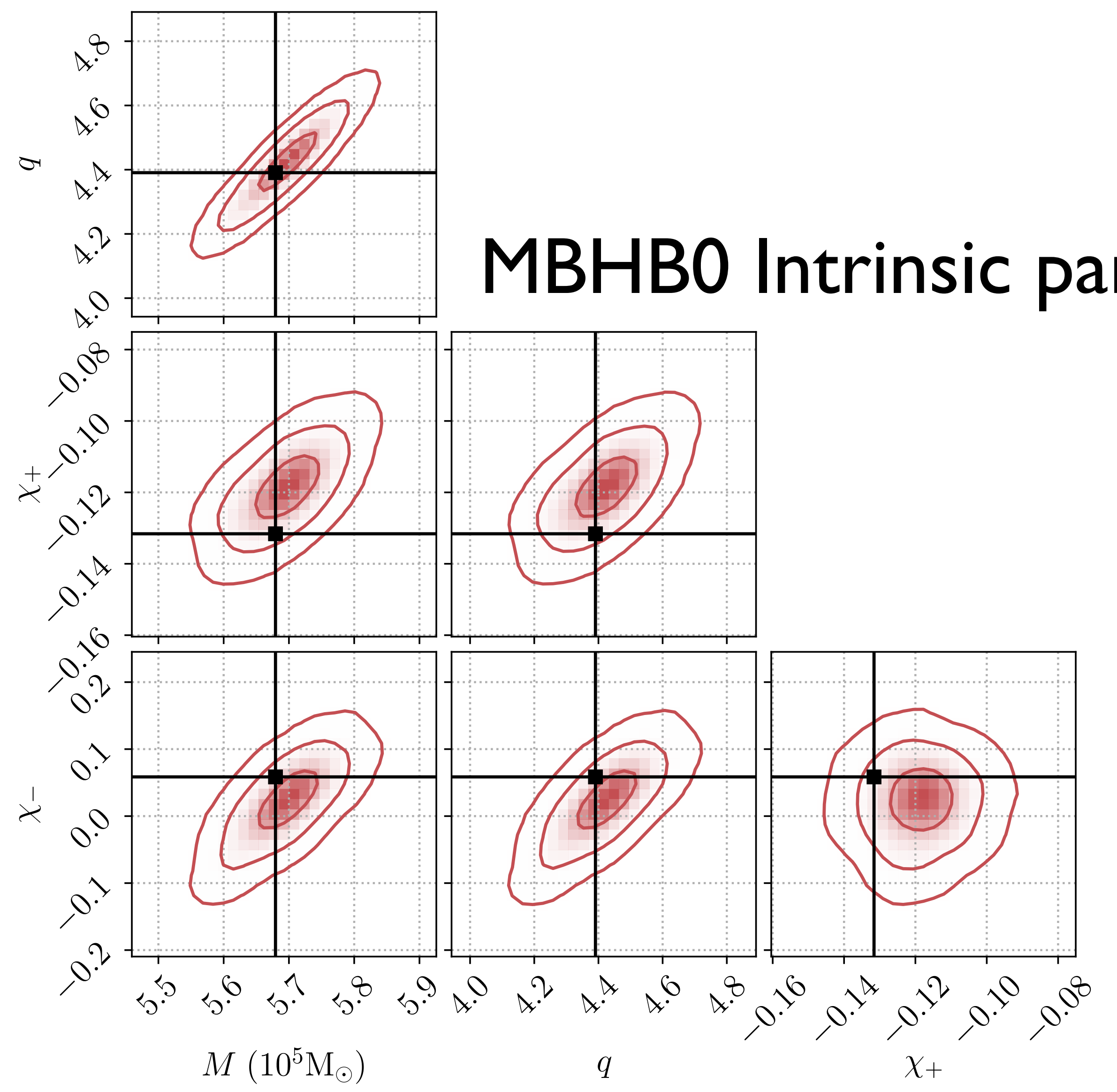
- 20 MBHBs ranging from $M \sim 5.10^5 M_{\odot}$ to $M \sim 2.10^7 M_{\odot}$
- Reaching SNR=10 from <1day to ~1 month before merger (highest mass happens to be long !)
- Longer signals if looking back in time to SNR=1 threshold
- IMRPhenomTHM waveform model

Analysis

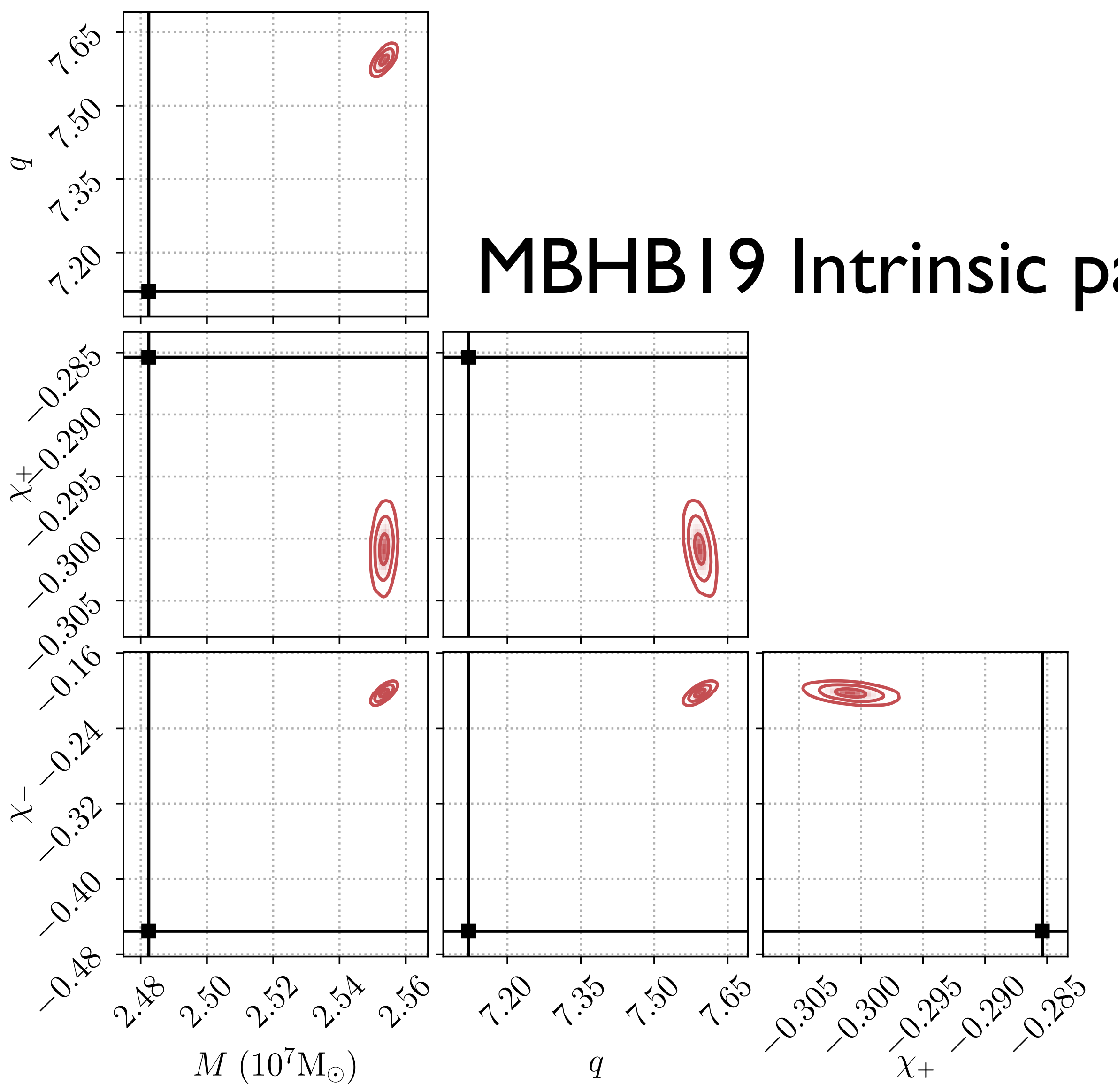
[Preliminary]

- IMRPhenomXHM templates
- FD response vs TD response, both for equal-armlength orbits

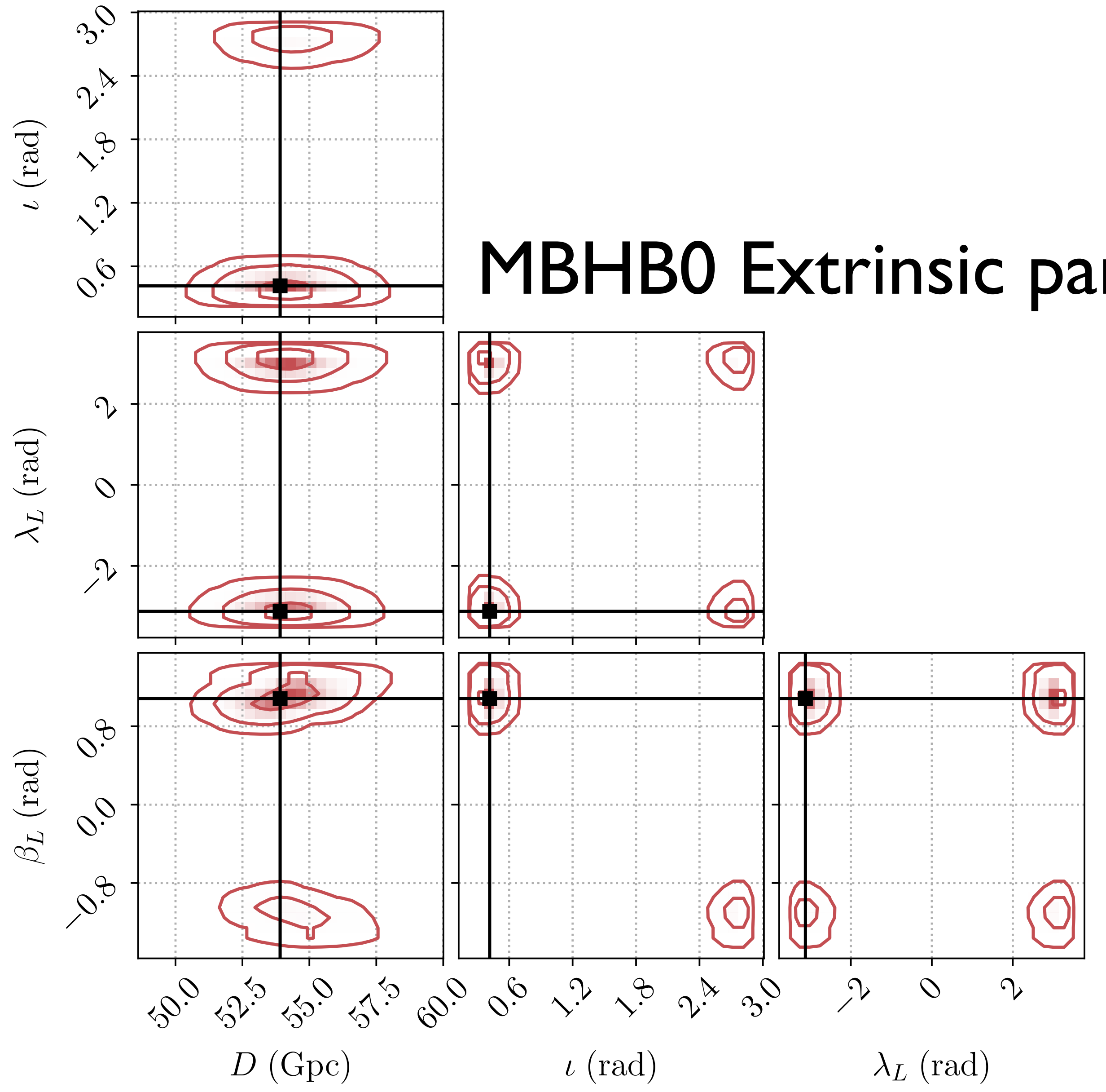
MojitoLight MBHBs: a first look



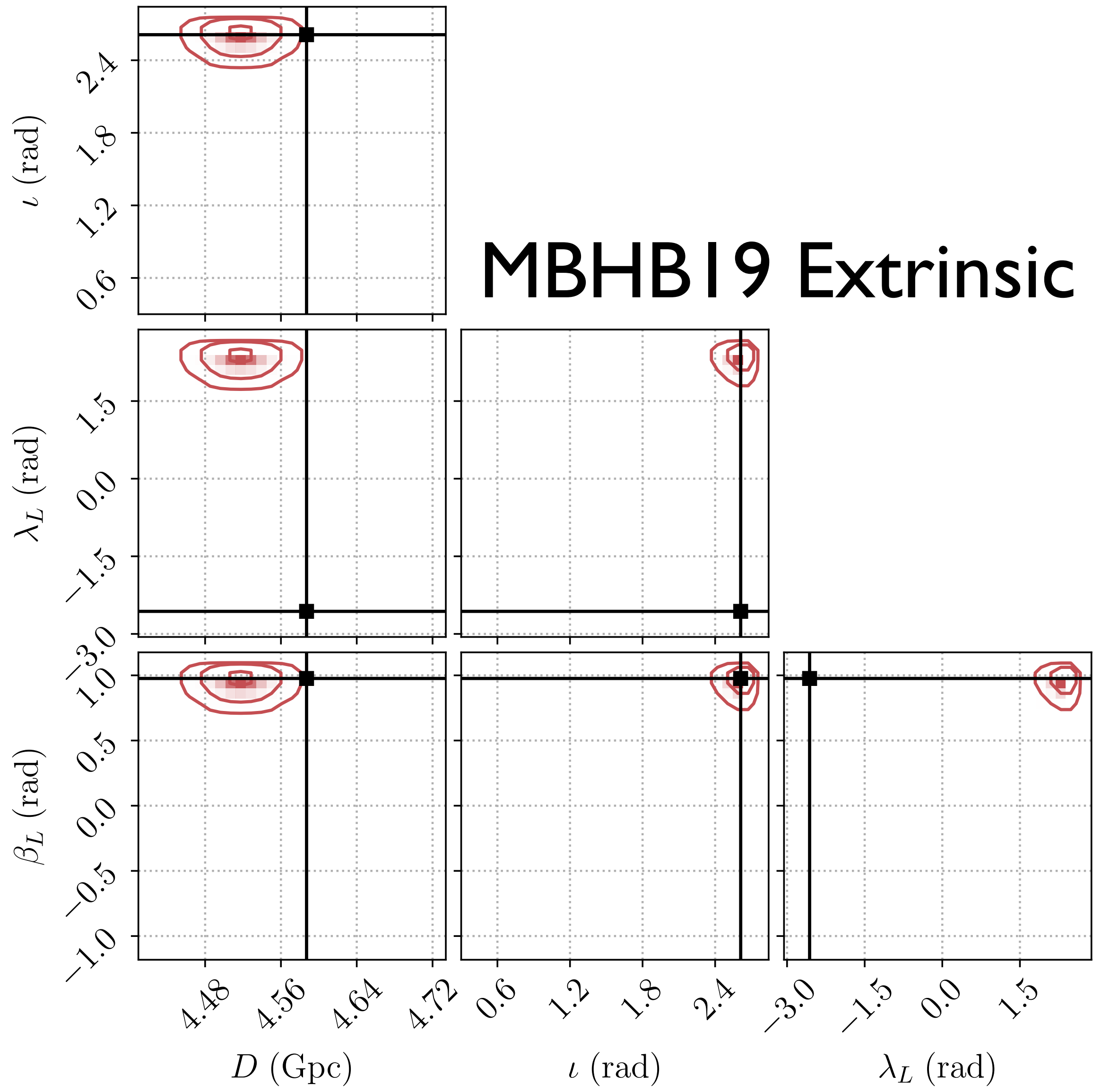
MBHB0 Intrinsic params.



MBHB19 Intrinsic params.



MBHB0 Extrinsic params.



MBHB19 Extrinsic params.

MBHB biases caused by missing higher harmonics

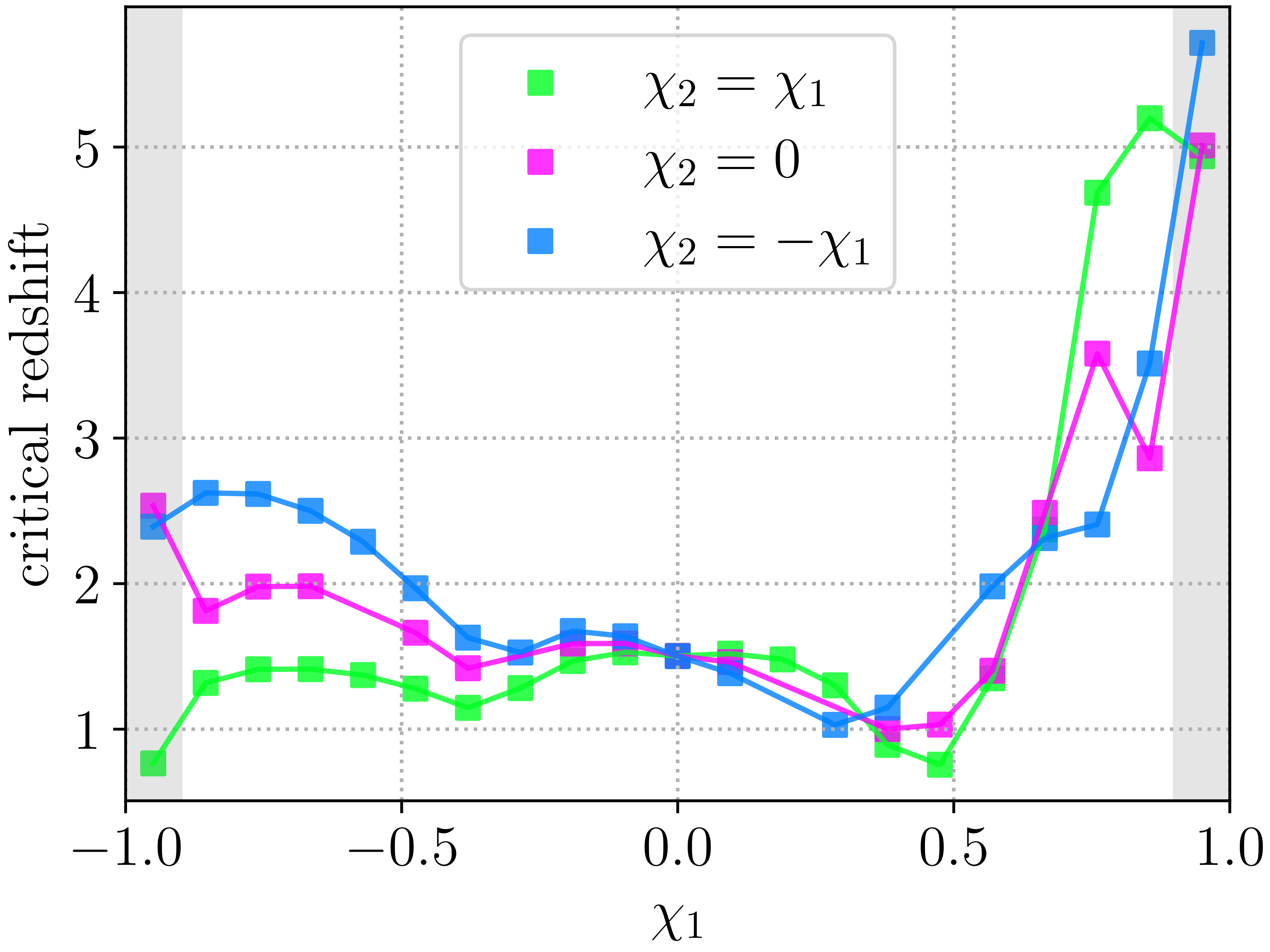
Question: how many $h_{\ell m}$ should be included in waveform templates ? Current models go up to h_{55} at most.

[Pitte+ 2023]
[Yi+ 2025]
[Yi+ 2026]

LISA consortium project [Waveform Working Group] coordinator: Sophia Yi (Johns Hopkins U.)

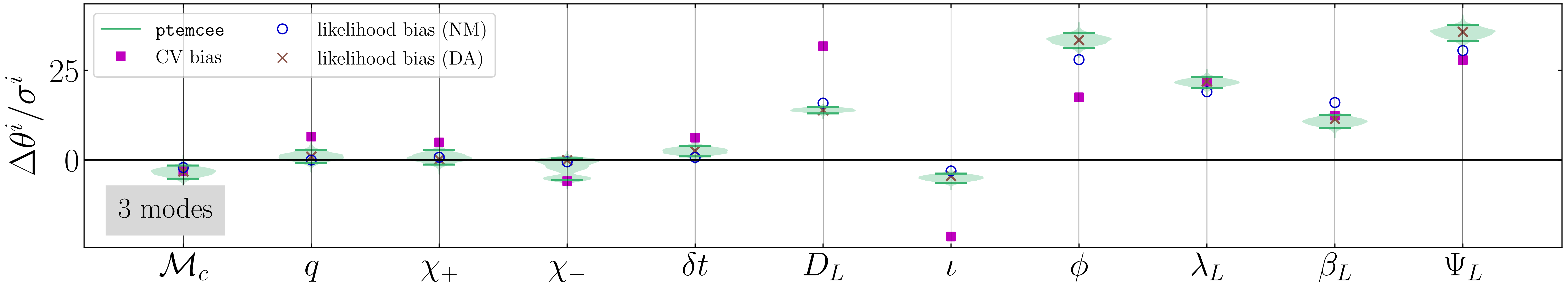
Min. redshift unbiased when ignoring (3,2) mode:

$M = 10^6 M_\odot,\ q = 4,\ \iota = \pi/3$ [Yi+ 2026]



Estimating biases: Cutler-Vallisneri (can fail), direct Nelder-Mead

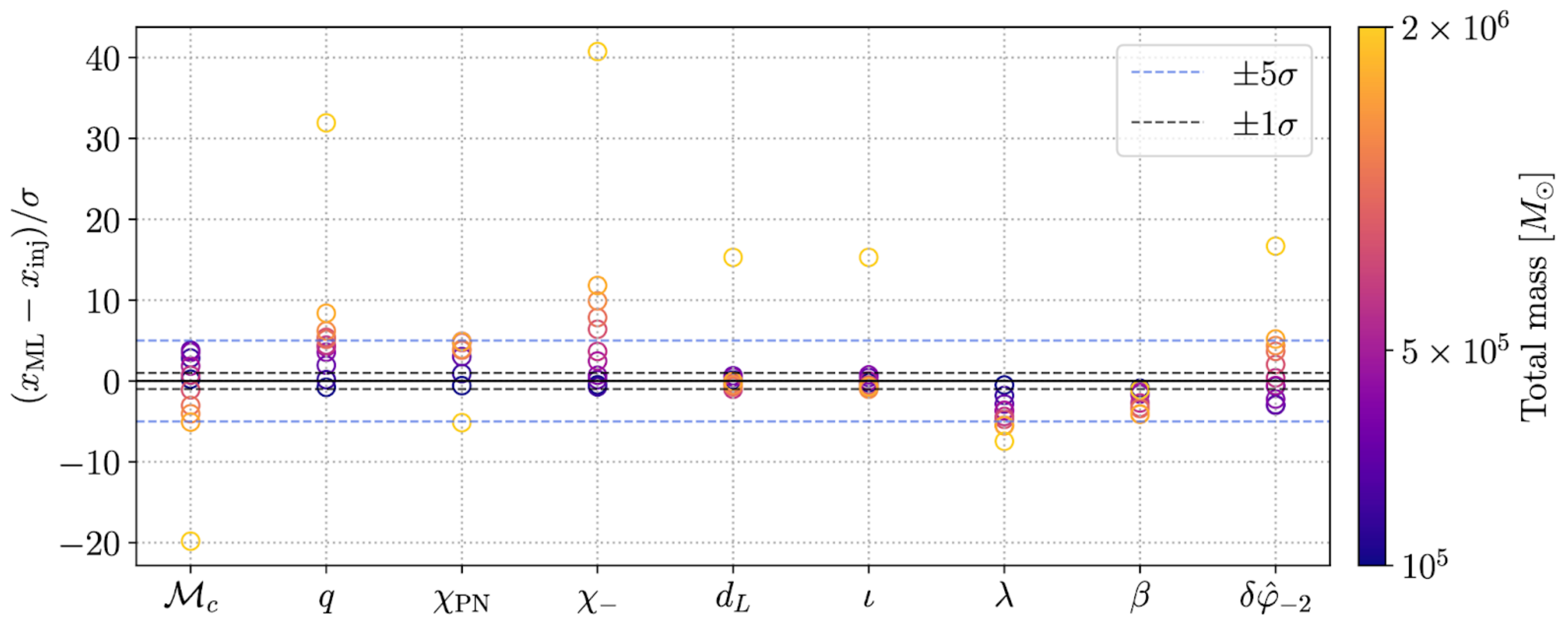
$M = 10^6 M_\odot,\ q = 1.1,\ \iota = \pi/3$ [Yi+ 2026]



MBHB biases: impact on tests of GR

Question: how does waveform inaccuracy affect tests of GR, and can waveform errors be mistaken for false deviations from GR ?

LISA consortium project [Fundamental Physics Working Group] coordinators: Manuel Piarulli (L2IT), Elisa Maggio (La Sapienza, Roma)



[Piarulli+ in preparation]

Conclusion and outlook

Highlights

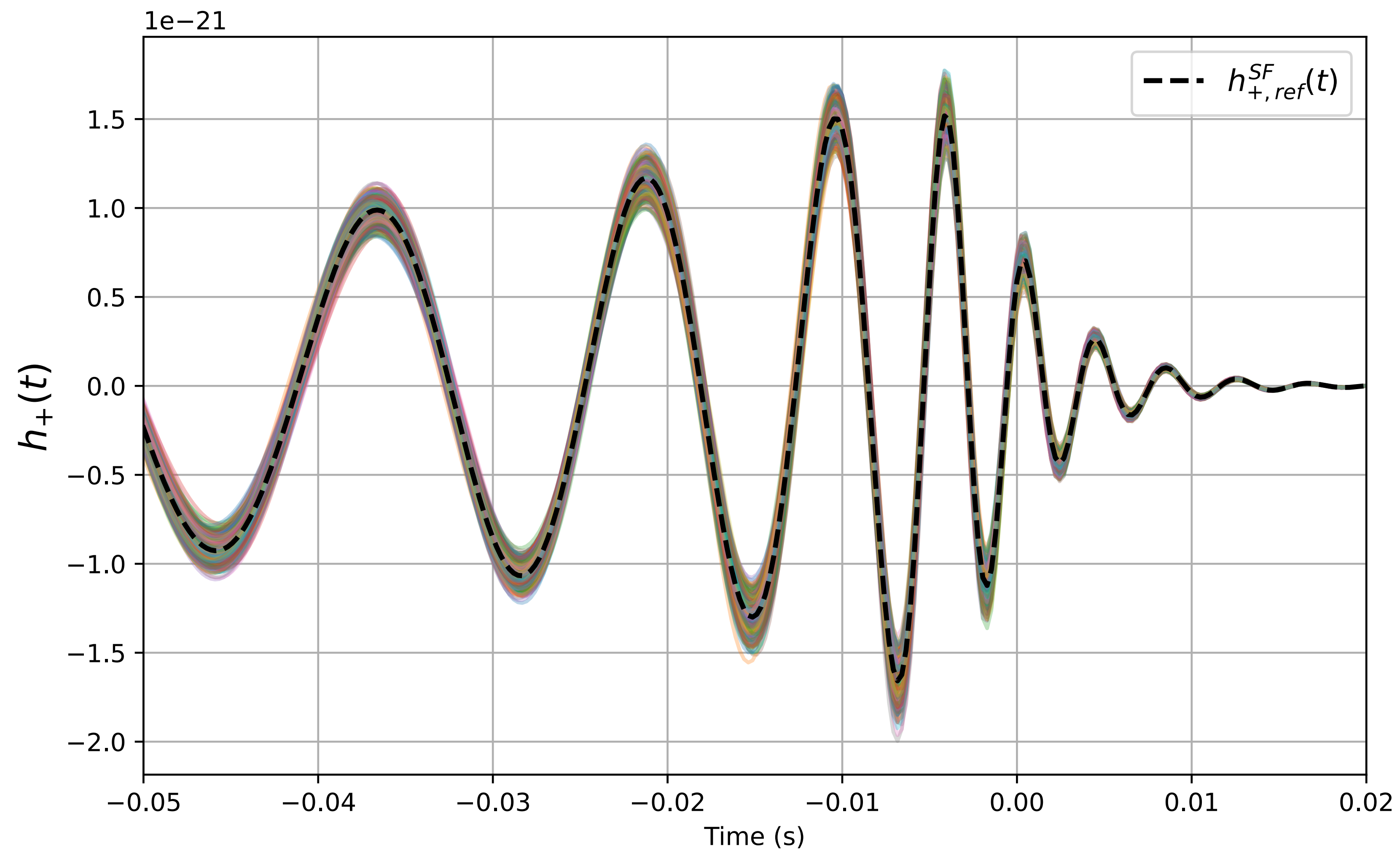
- Even for the simpler case of aligned spins + HM, significant systematic biases at high masses
- Strong progressivity: worse at higher masses, for merger-dominated signals
- Systematics can also mislead us towards the wrong sky mode

Outlook

- Expectation: systematics could be even worse in presence of precession and eccentricity...
- Mitigation: waveforms should come with uncertainties ! How to determine the error envelopes ? The LISA science case could require revisiting with a marginalization over waveform uncertainty.
- Mitigation: targeted a posteriori improvement of waveform models for golden events ?

Dealing with waveform errors

Marginalization over waveform errors:



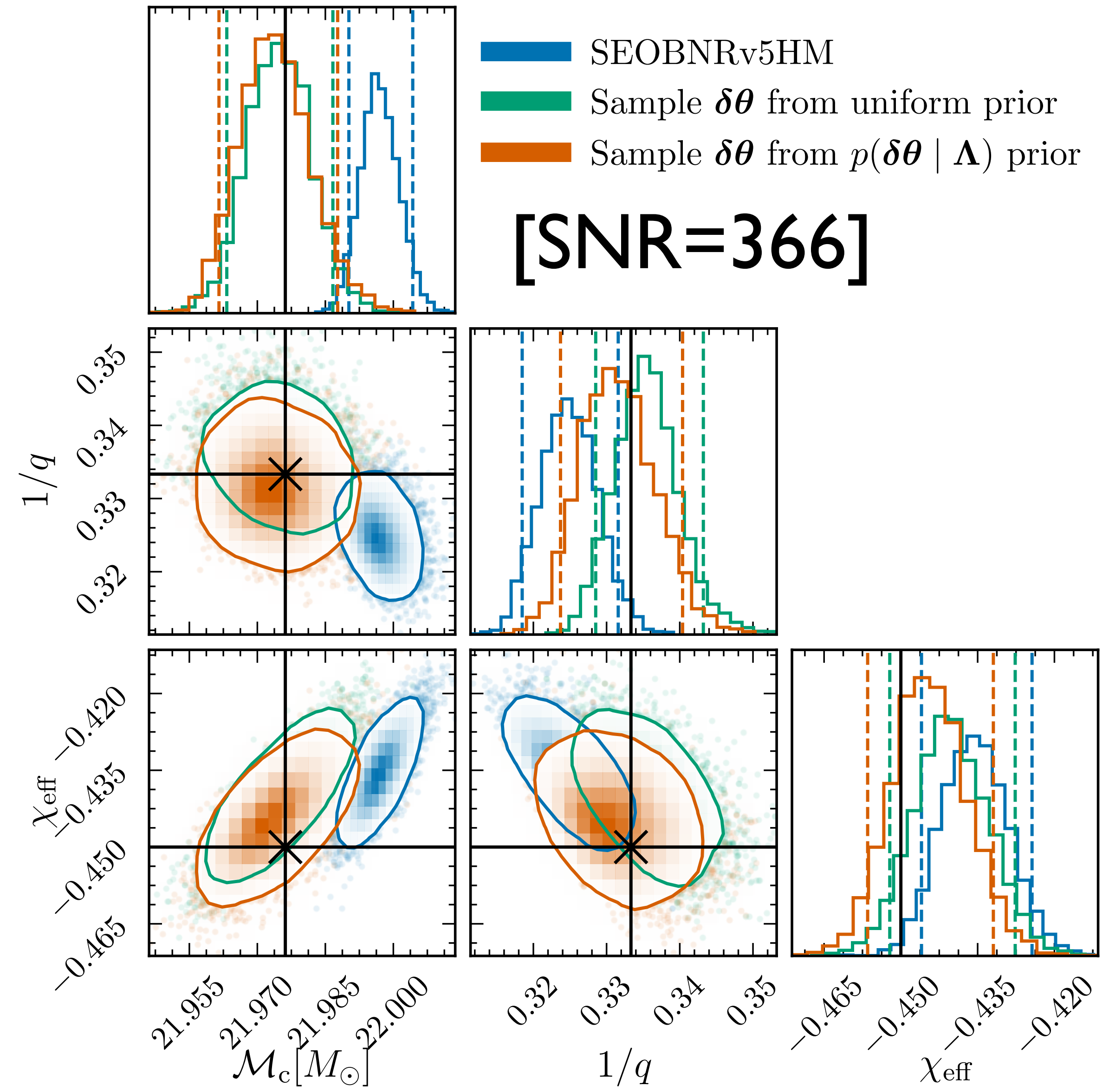
Probabilistic waveforms:

- amplitude/phase envelopes, as in detector calibration
$h(f) = (A + \delta A)e^{i(\phi+\delta\phi)}$ [Kumar+ 2025]

- constructed as Gaussian Process Regression (also guides next simulations)

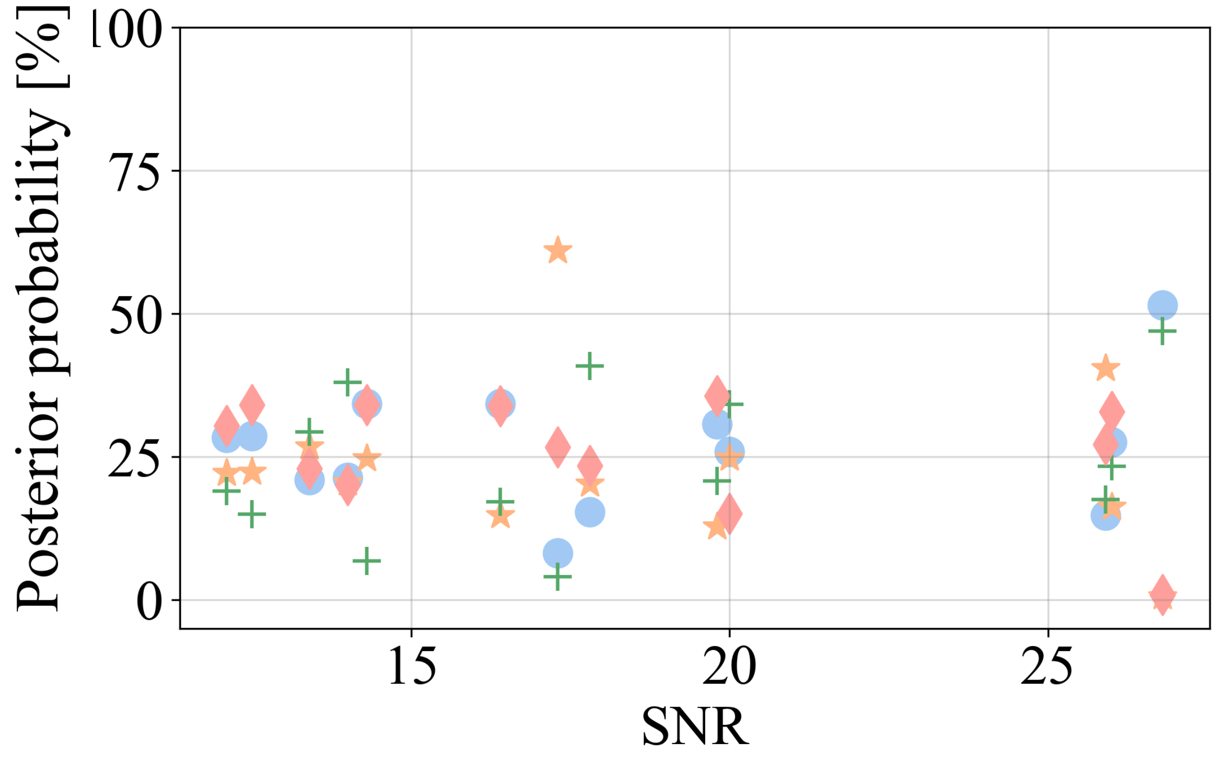
[Moore-Gair 2014]
[Williams+2019]
[Bachhar+ 2024]

Marginalization over SEOB NR calibration:



Calibration params
$\theta \sim p(\theta|\mathrm{NR})$
[Pompili+ 2024]

Hypermodels:



$$\Omega = \{\Omega_0, \Omega_1, ..., \Omega_{n-1}\}$$

$$\mathcal{O}^A_B = \frac{p_A}{p_B} = \frac{n_A}{n_B}$$

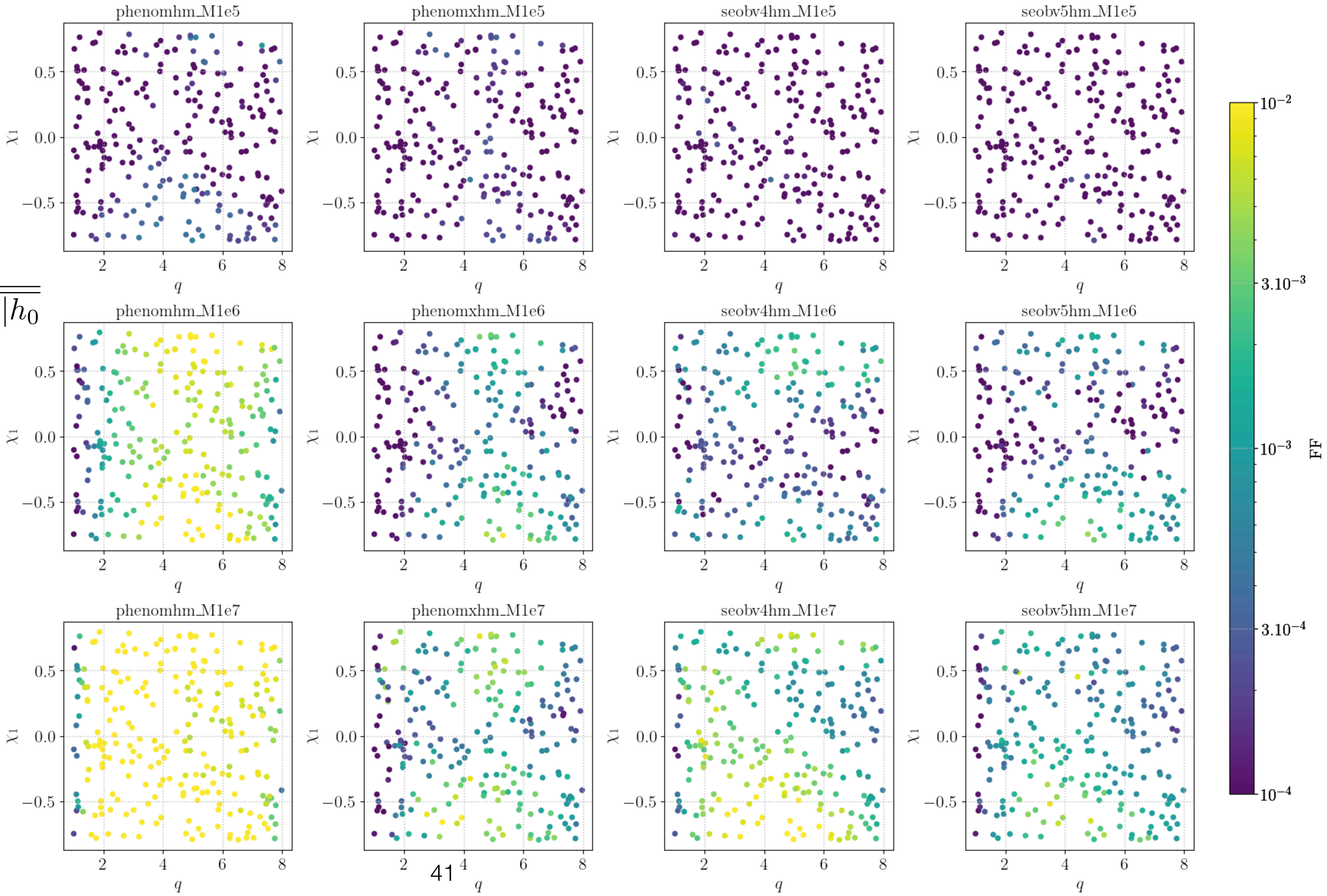
[Puecher+ 2023]
[Hoy+ 2024]
[Ashton+ 2019]

Fitting factors in parameter space

Fitting factor, computed at best-fit params (optimization over all parameters):

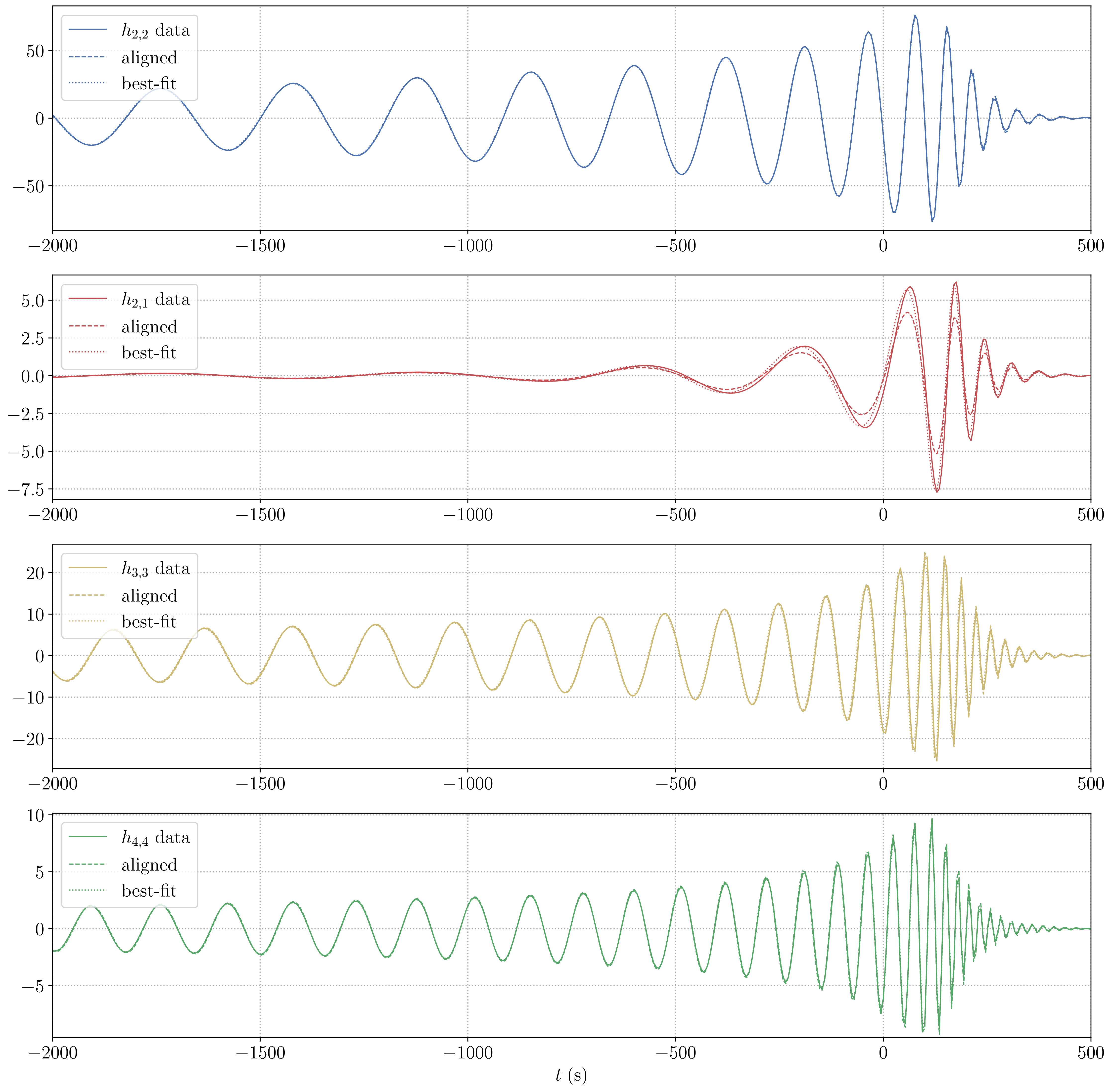
$$\mathrm{FF} = 1 - \max_{\boldsymbol{\theta}} \frac{(h_m|h_0)}{\sqrt{(h_m|h_m)}\sqrt{h_0|h_0}}$$

$$\ln \mathcal{L}_{\max} = -\mathrm{SNR}^2 \times \mathrm{FF}$$

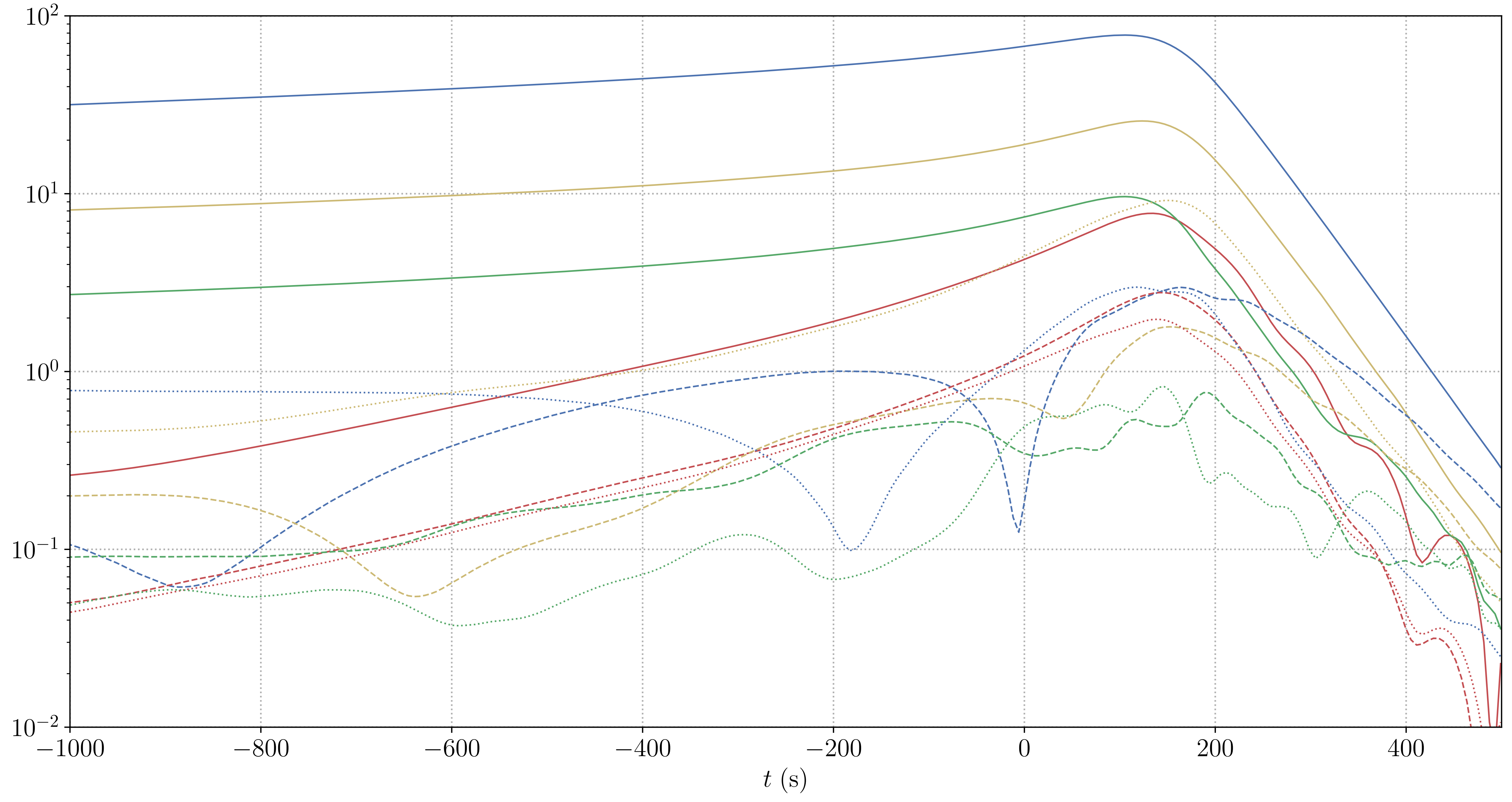


Trend: strong dependence on M
Trend: larger errors at large spins
Trend in q ?

Example Parameter estimation with systematics III: TD signals and residuals



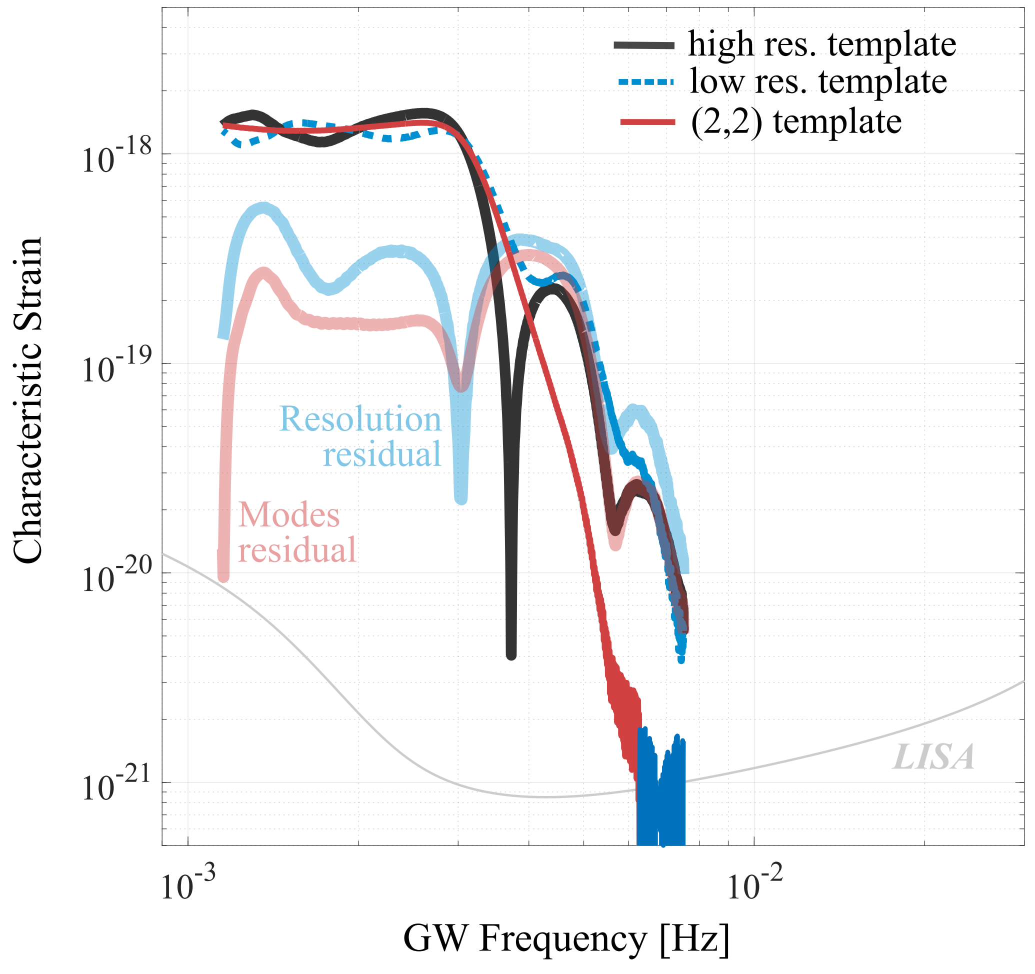
Residuals for the 'ugly' case



- Residuals are 'visually' small…
- Details of HM at merger are important

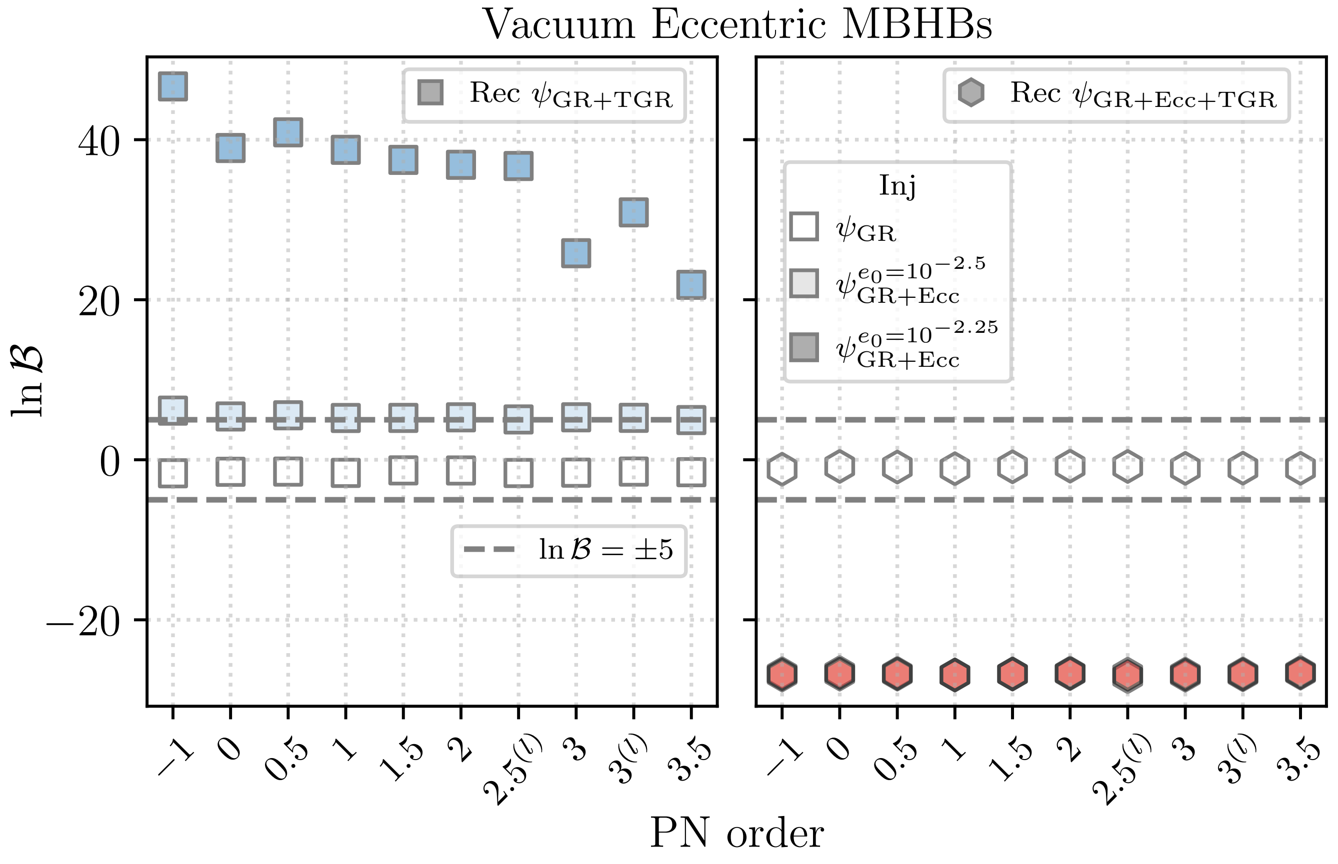
Waveform systematics studies for LISA MBHBs

NR residuals (Maya code): [Ferguson+ 2020]

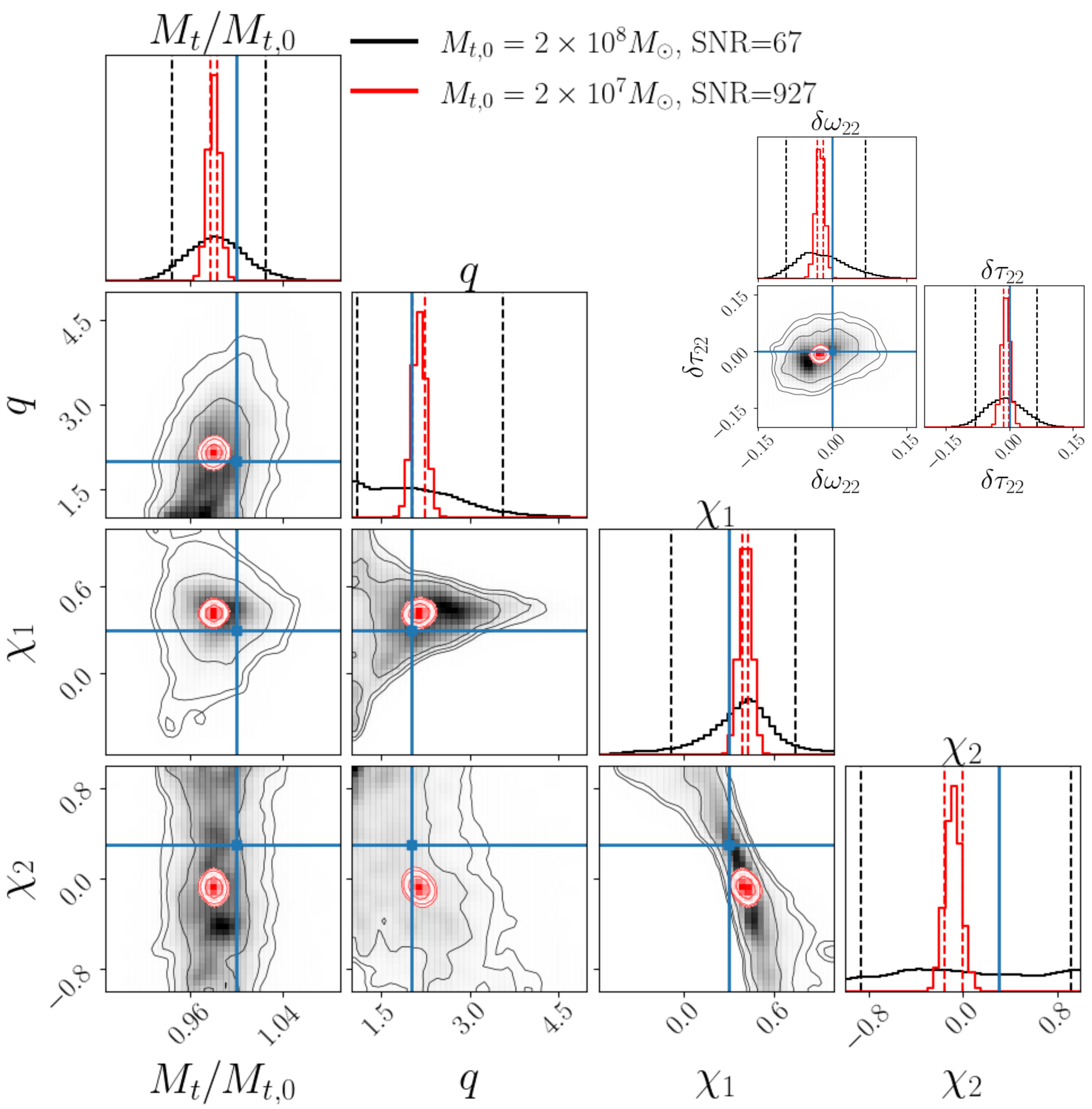


TGR systematics when ignoring ecc/env.:

[Garg+ 2024]

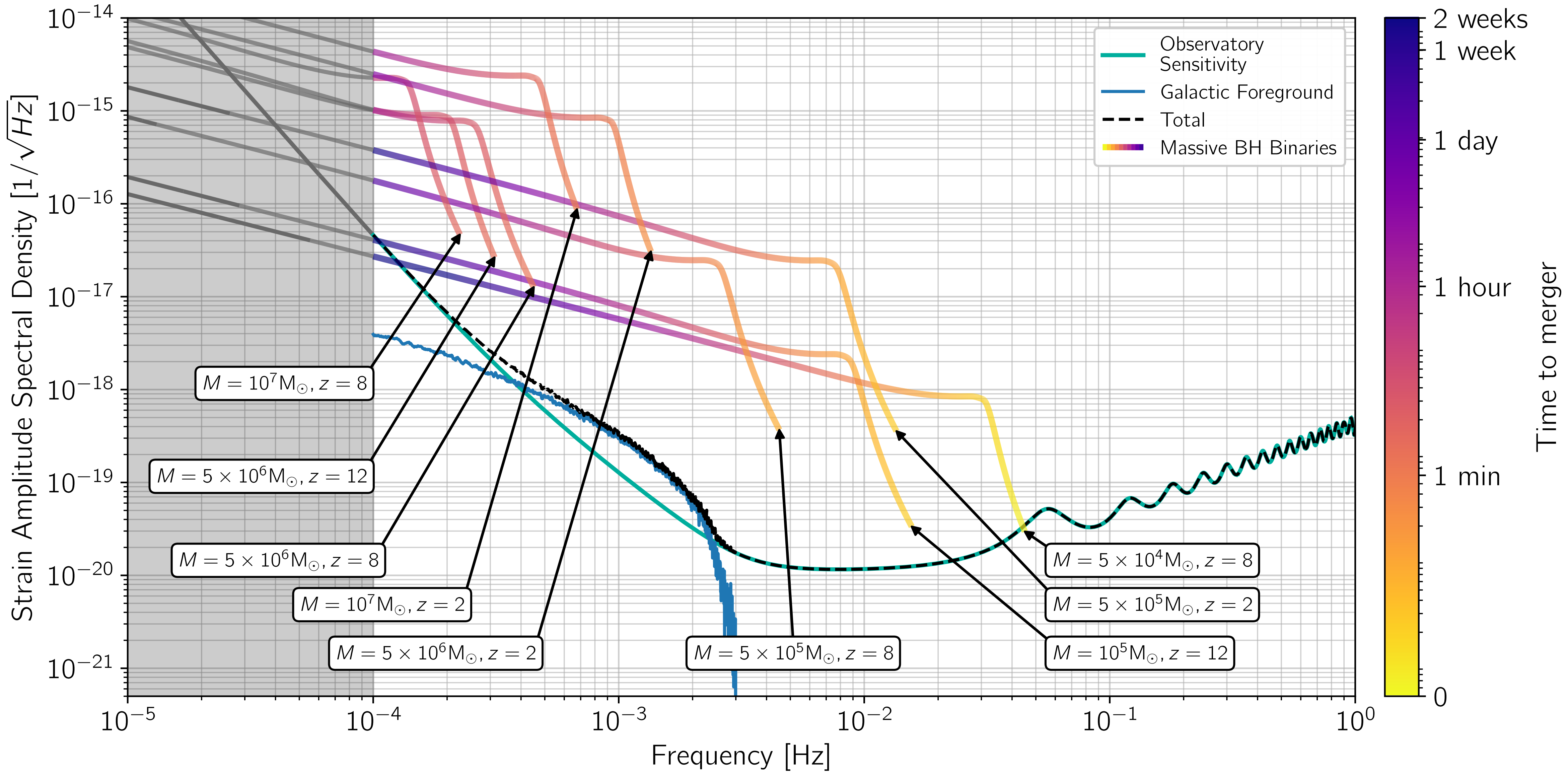


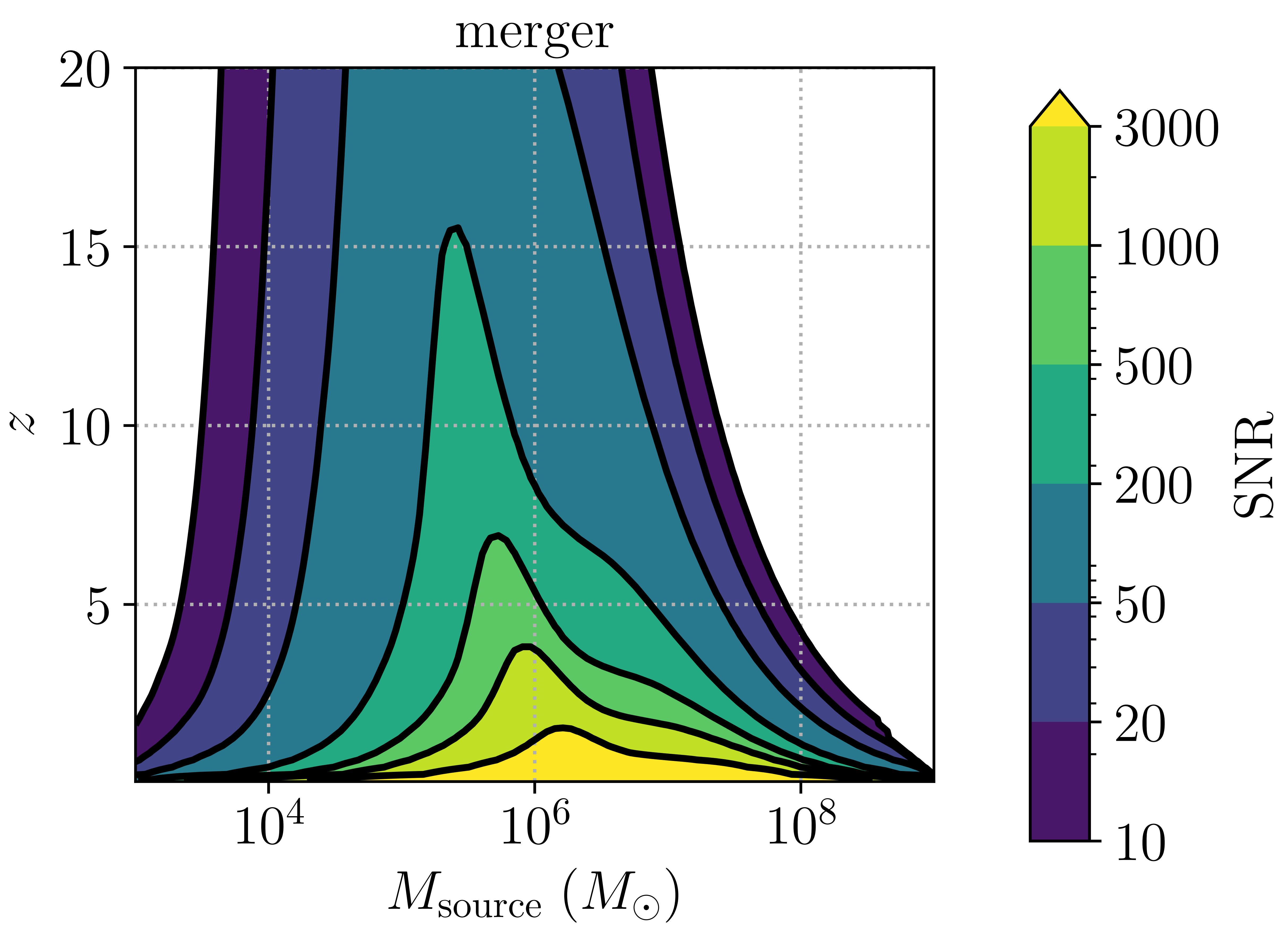
pSEOBNR vs NR:



[Toubiana+ 2023]

Massive black hole binaries for LISA





Extremely loud mergers…

Broader parameter space than LVK: high-q, high-spins, precession, eccentricity

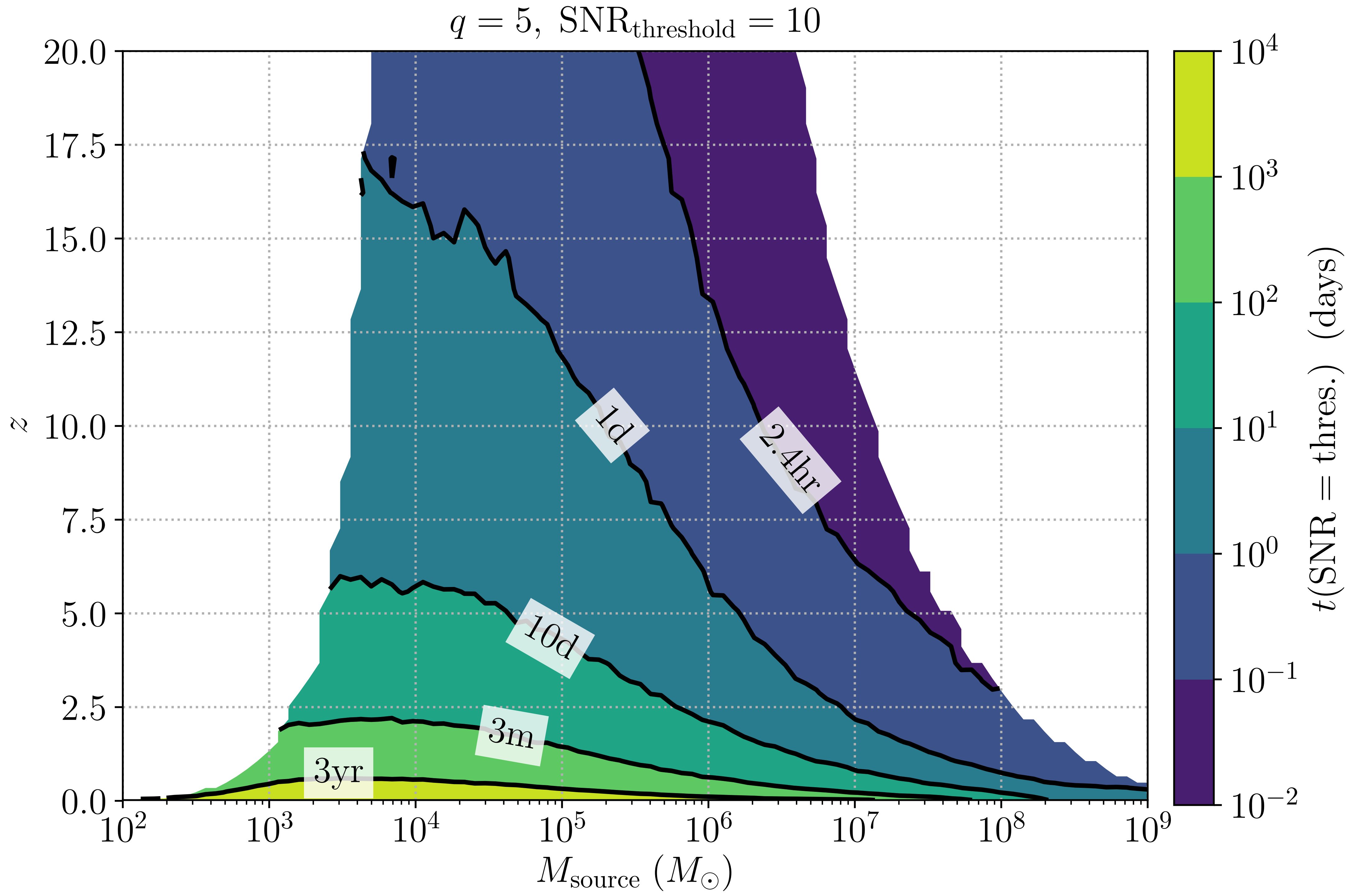
Science case

MBHBs cardinal sources for LISA, waveform systematics crucial for:

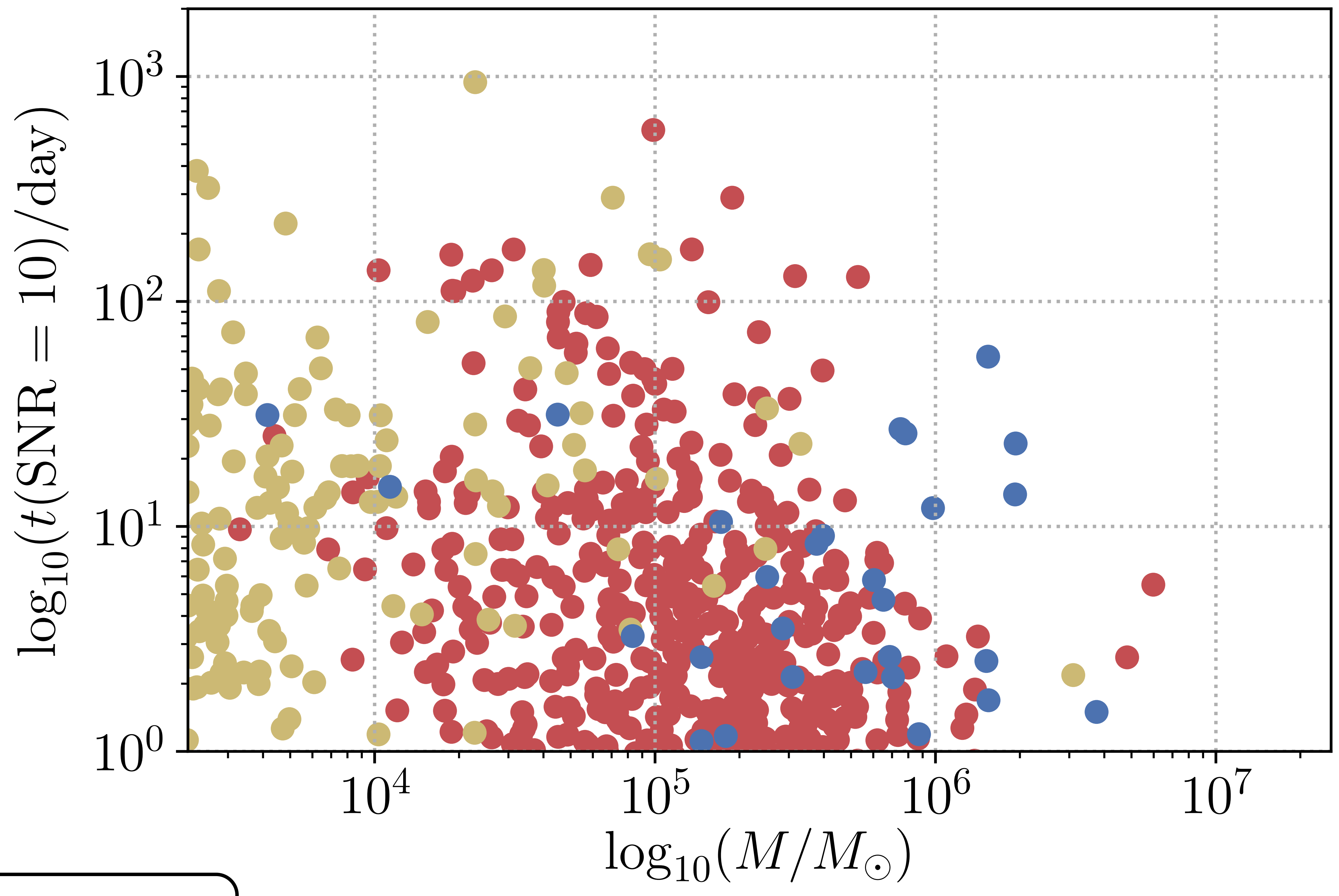
- golden events for EM counterparts
- golden events for TGR
- population inference and cosmology
- global fit and residuals

The length of MBHB signals

- How long before merger can we detect the signal ?
- SNR=10 to claim detection



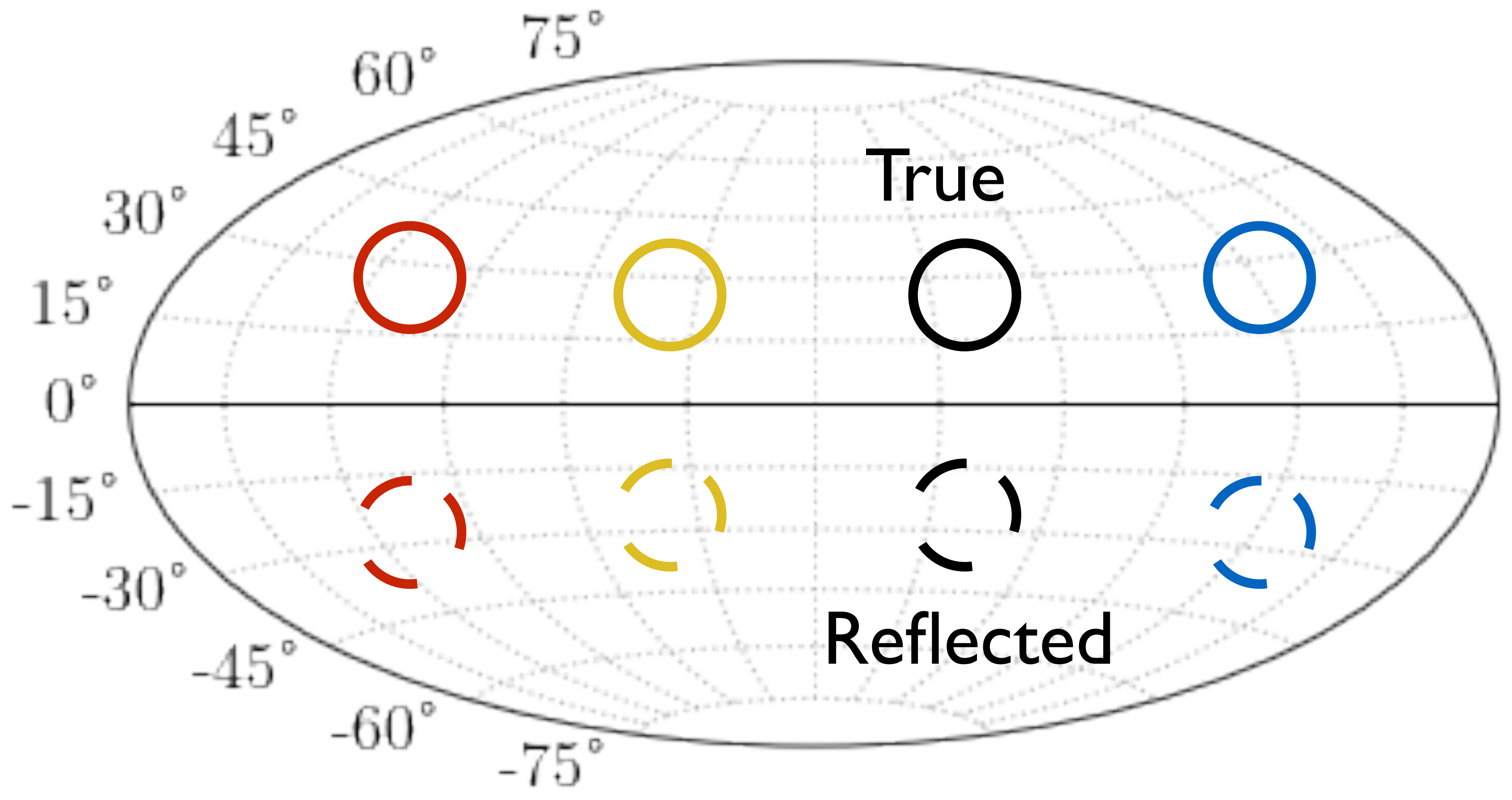
Astrophysical models [Barausse 2012]:
- Heavy seeds - delay
- Heavy seeds - no delay
- PopIII seeds - delay

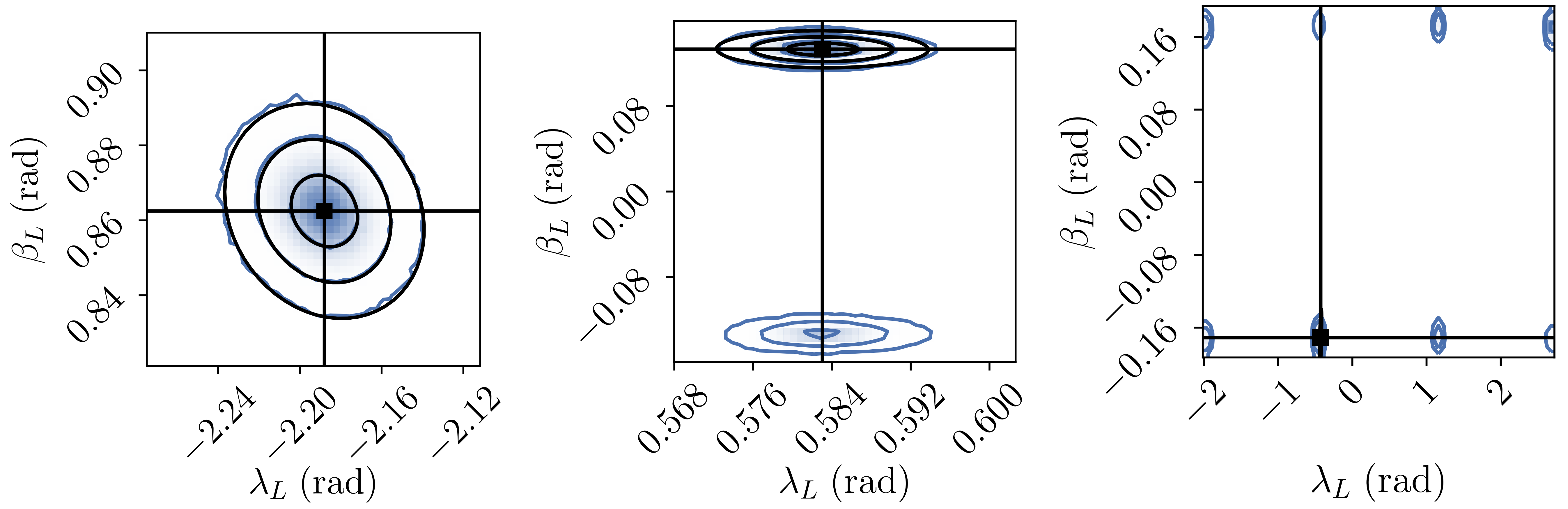


MBHB detected signals:
Bulk shorter than ~10days
Tail extending to ~3months

LISA response and multimodality in the sky

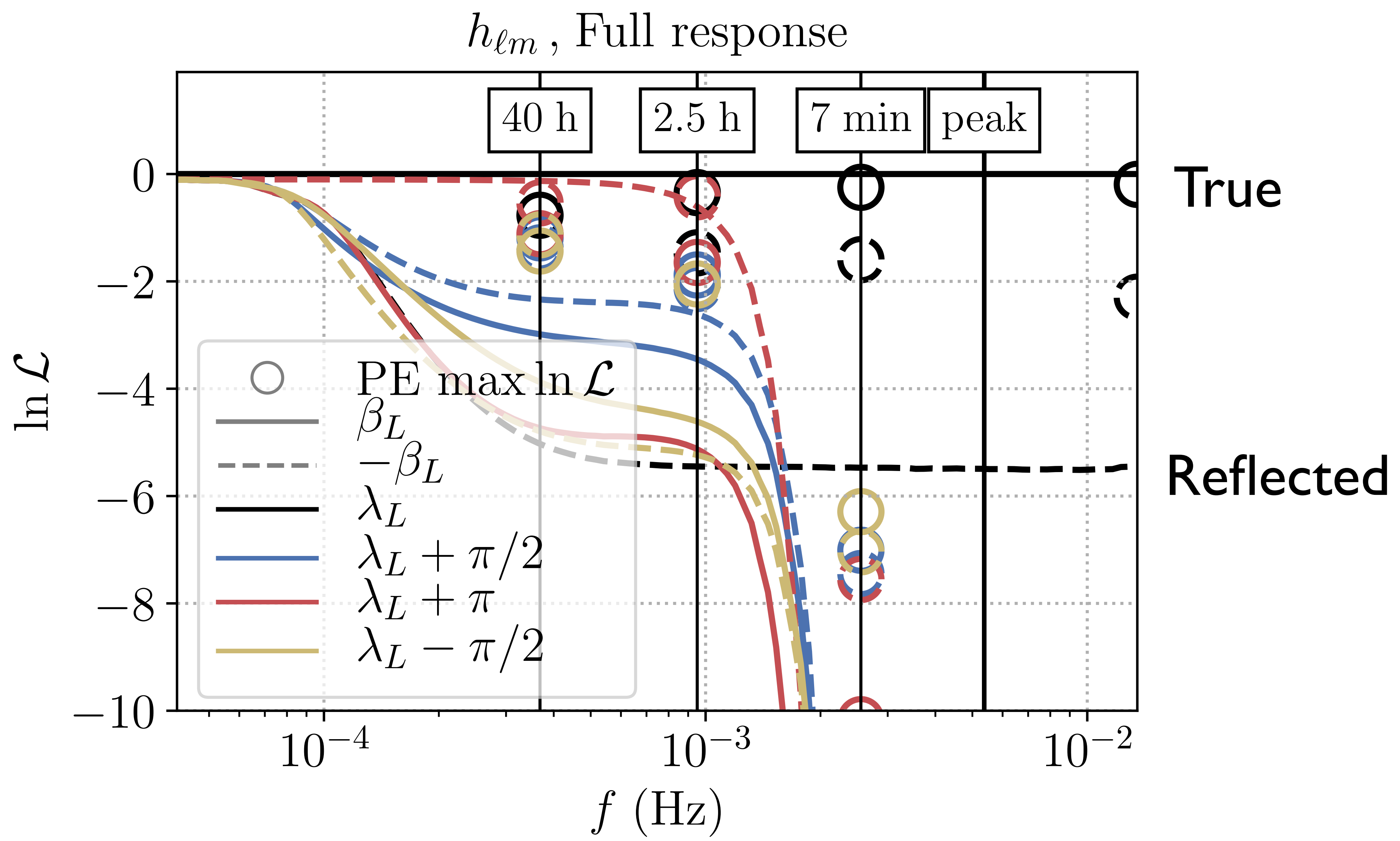
Multimodality pattern:





Degeneracy breaking:

- motion of LISA: eliminates all modes but the antipodal, weak for short high-mass signals
- high-frequency effects in the response: eliminates all modes but the reflected, only at high frequencies



- Multimodality broken by subdominant effects in response (motion, high-f)

Sky multimodalities for LISA MBHBs

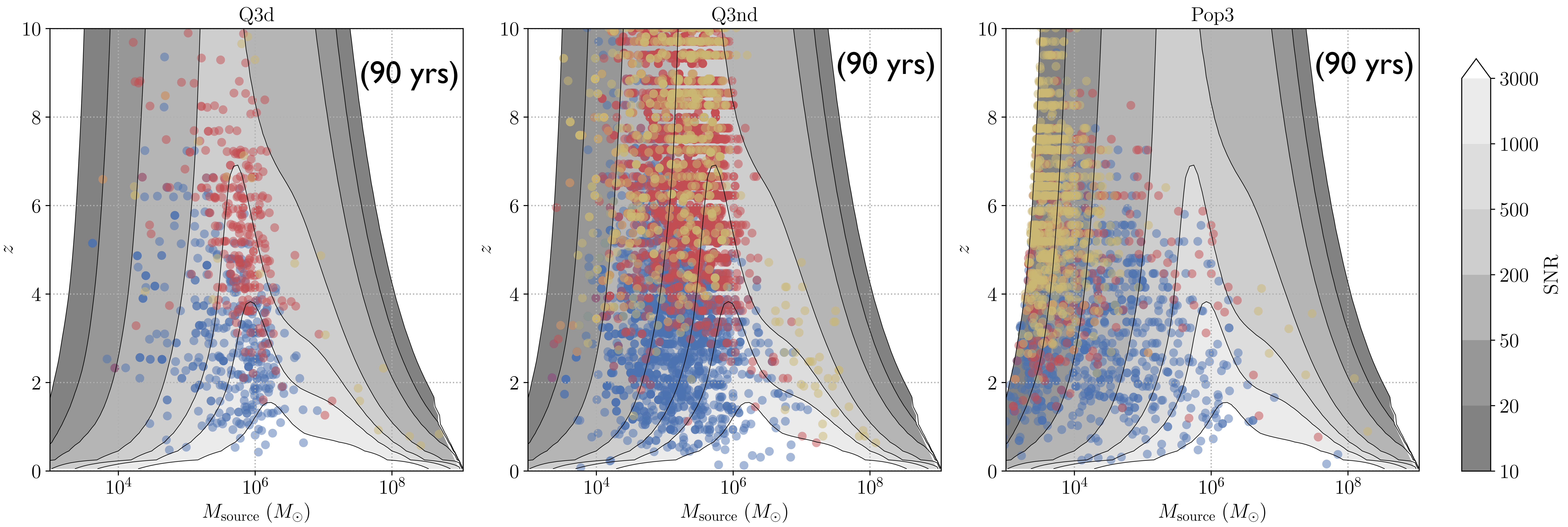
- **Bayesian PE** required to explore multimodal posteriors
- Simulation of 90yrs catalogs
- Custom proposals for degeneracies

Astrophysical models [Barausse 2012]:

- Heavy seeds - delay (Q3d)
- Heavy seeds - no delay (Q3nd)
- PopIII seeds - delay (Pop3)

o 1 mode
o 2 modes
o >2 modes

Threshold: 5% prob.



Applications: EM counterparts and cosmological inference [Mangiagli&al 2022, Mangiagli&al 2023]

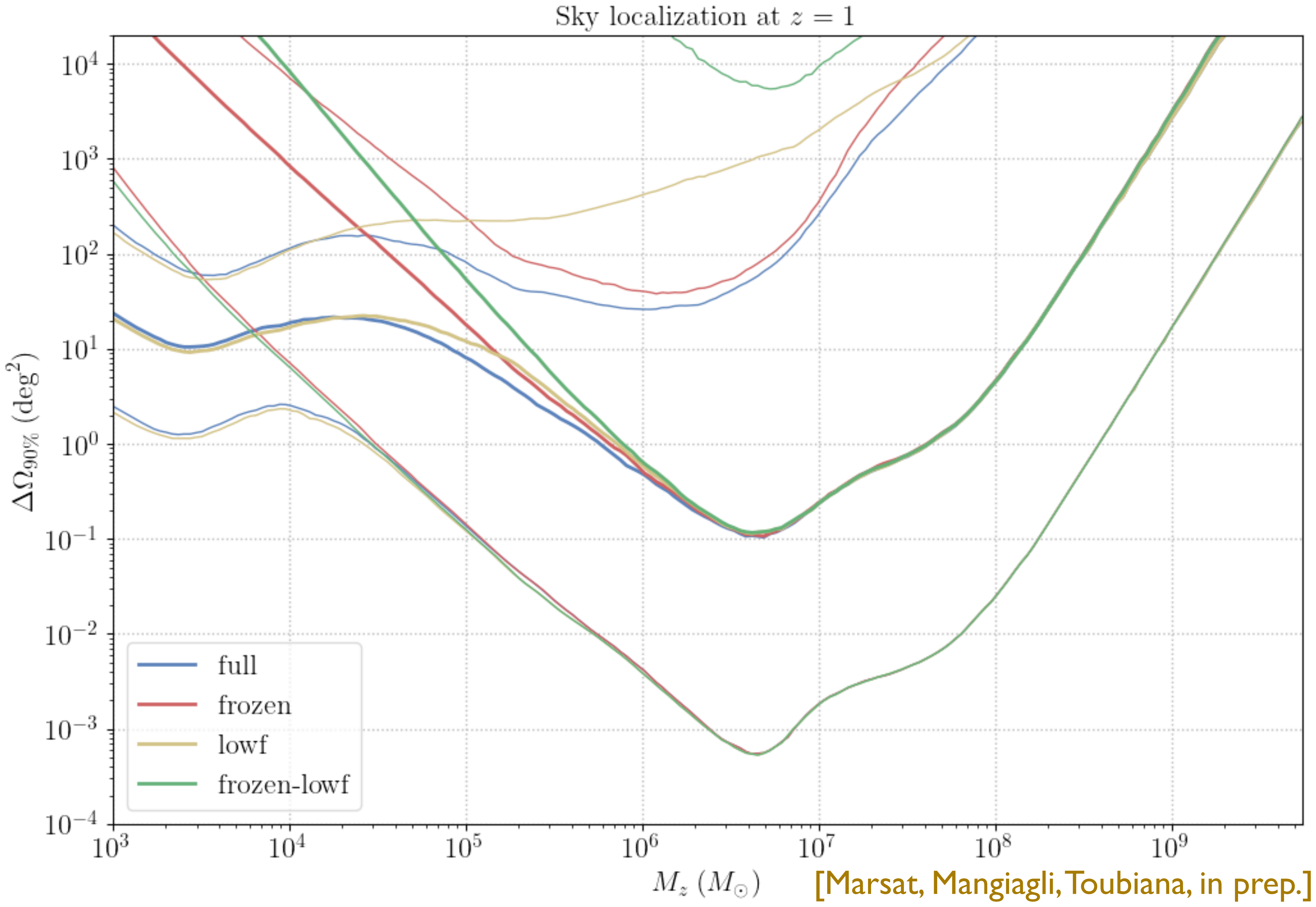
Multimodality in the sky present, but rare for counterpart candidates post-merger

Fisher localization: which part of the response is important ?

Analysis settings:

- Fisher matrix localization: sky area of the main mode of the posterior
- Randomization over 1000 orientations, mass ratios, spins
- Change the response model: keep or ignore the motion and high-f effects

- 'Pattern function' response is the main source of main-mode localization at high mass, from subdominant HM
- Multimodality broken in turn by subdominant effects in response (motion, high-f)



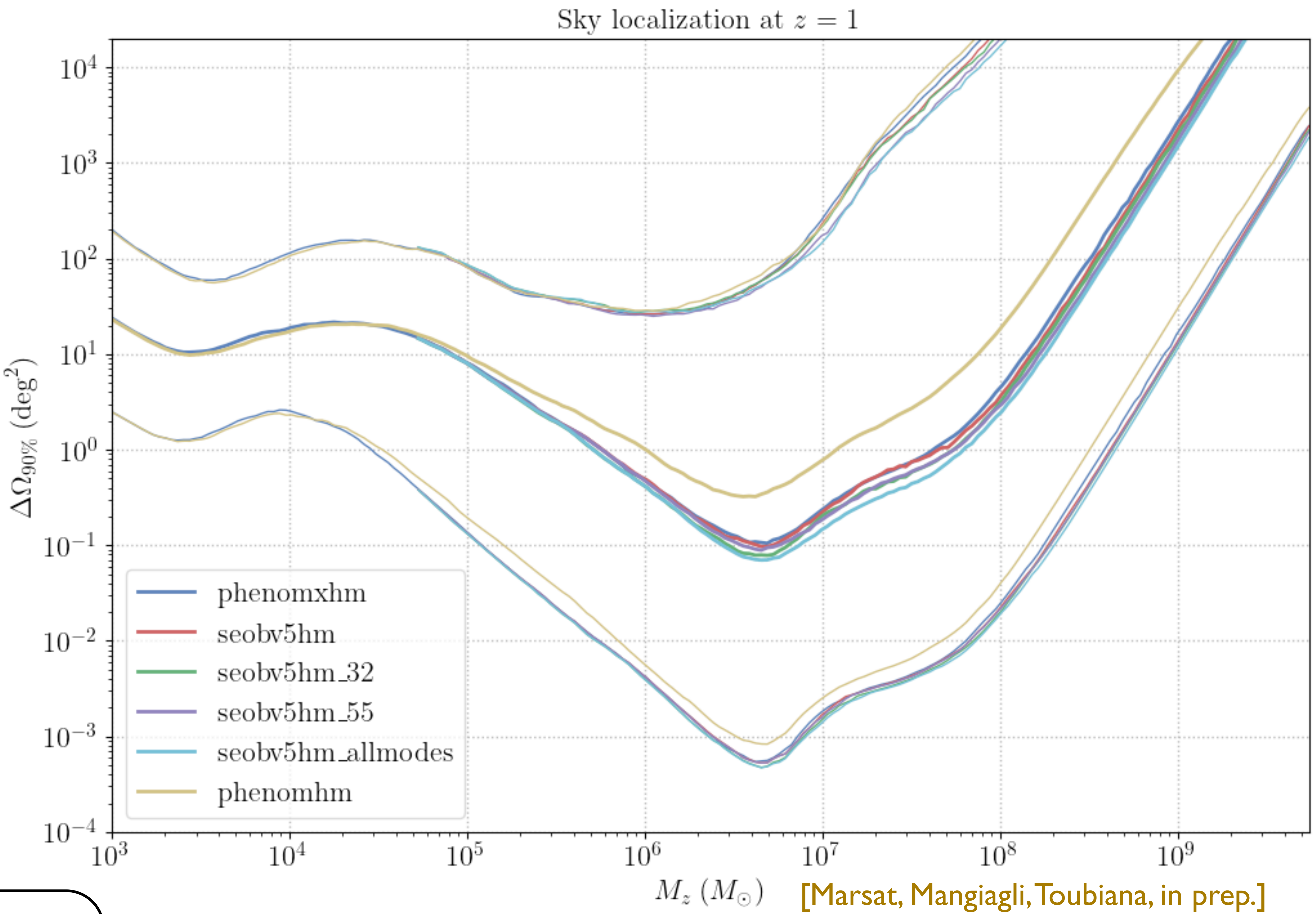
[Marsat, Mangiagli, Toubiana, in prep.]

- Sky localization at high mass: weak effects, high SNR
- Unlike LVK localization from triangulation, LISA localization potentially vulnerable to systematics

Fisher localization: do waveform models agree on prospective ?

Analysis settings:

- Fisher matrix localization: sky area of the main mode of the posterior
- Randomization over 1000 orientations, mass ratios, spins
- Change the waveform model: PhenomHM, PhenomXHM, SEOBNRv5HM_ROM



[Marsat, Mangiagli, Toubiana, in prep.]

- In the high-mass range (HM important), older waveform models can be inaccurate also for prospective PE
- Modern waveform models agree well for prospective

Example Parameter estimation with systematics III: Cutler-Vallisneri bias

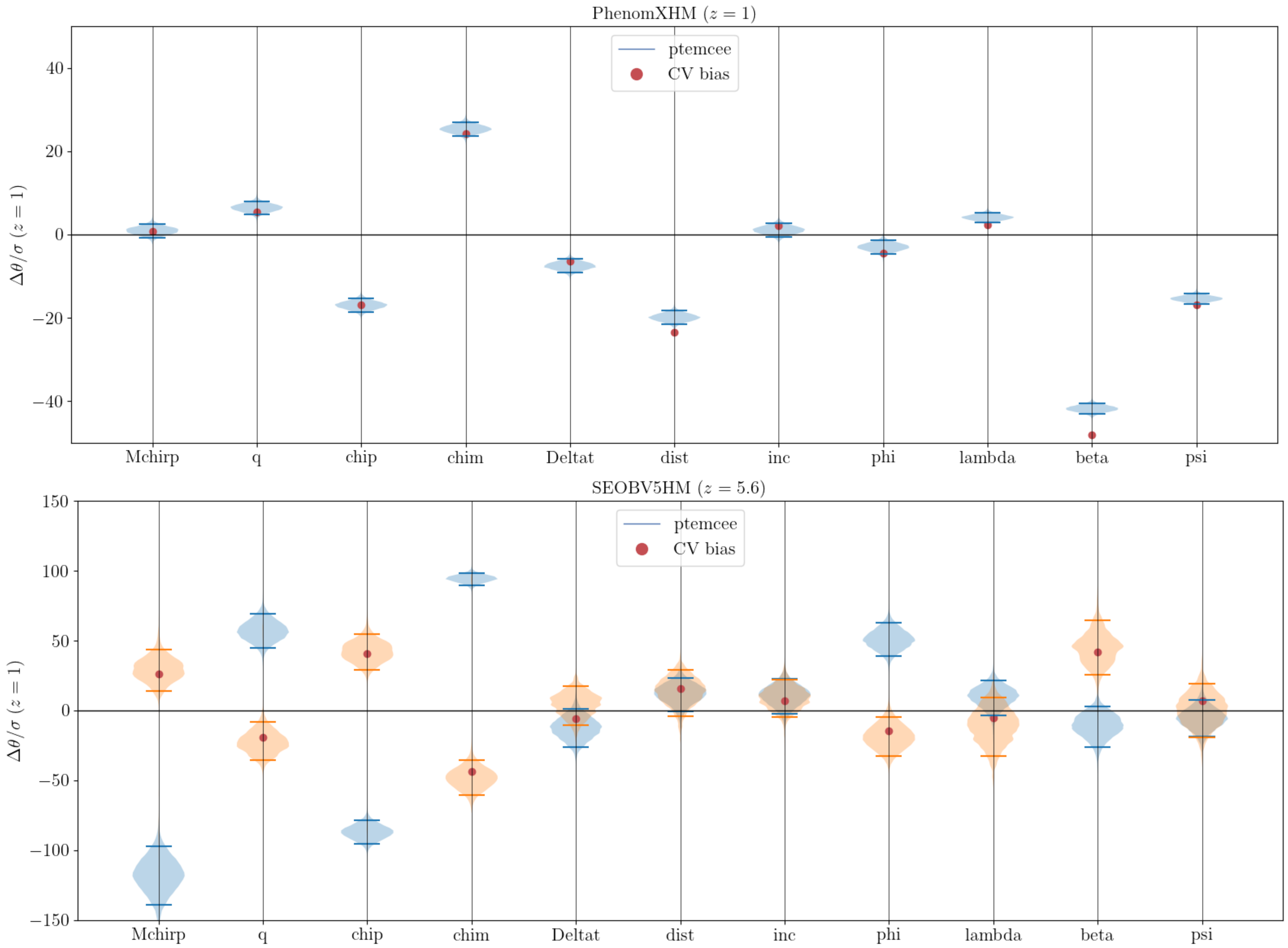
Linearized biases (Cutler-Vallisneri):

[Flanagan-Hughes 1997]
[Cutler-Vallisneri 2007]

In the linear signal approximation, estimation of bias:

$$F_{ij} = (\partial_i h | \partial_j h)$$
$$\Delta\theta_i = F_{ij}^{-1} (\partial_j h | \delta h)$$

- Cutler-Vallisneri biases give reasonable estimates in mild cases
- Fail to capture distant secondary mode — needs to be adapted
- Can fail for severe biases



Biases caused by missing higher harmonics

Estimating the bias is an optimization problem

[Preliminary]

- Cutler Vallisneri is one step of Newton's gradient descent, approximating the Hessian with the Fisher matrix

$$\Delta\theta = H^{-1} \cdot \nabla \ln \mathcal{L}$$
$$H_{ij} = \partial_i \partial_j \ln \mathcal{L} \sim (\partial_i h | \partial_j h)$$

- Are there better optimization algorithms ? (e.g. simplex method)

- One simple idea is to repeat CV as iterated gradient descent

There should be better bias estimators than CV - robustness ?

[Sophia Yi &al, in prep.]