Mitigation of the Flexing Filtering Effect in TDI for LISA

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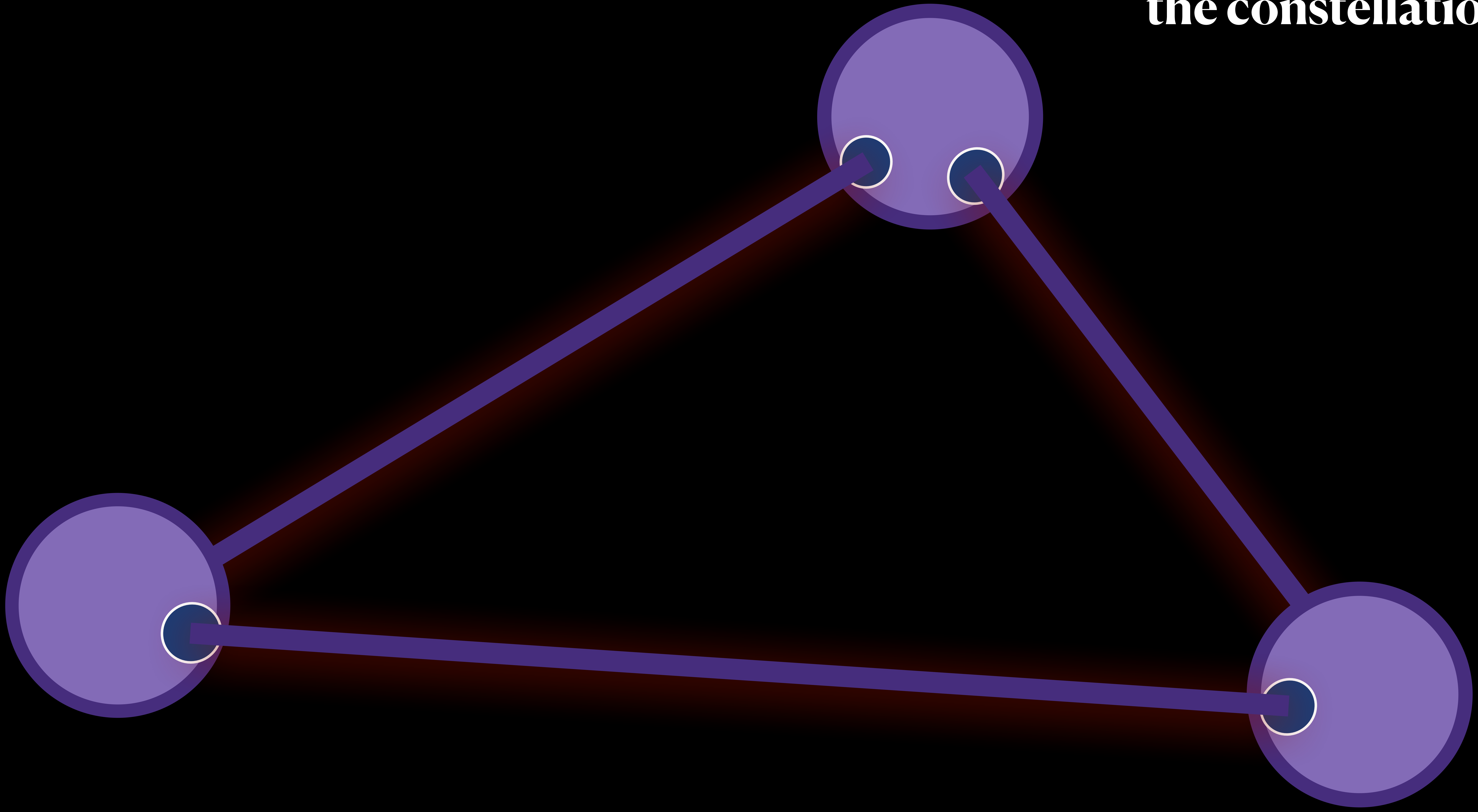
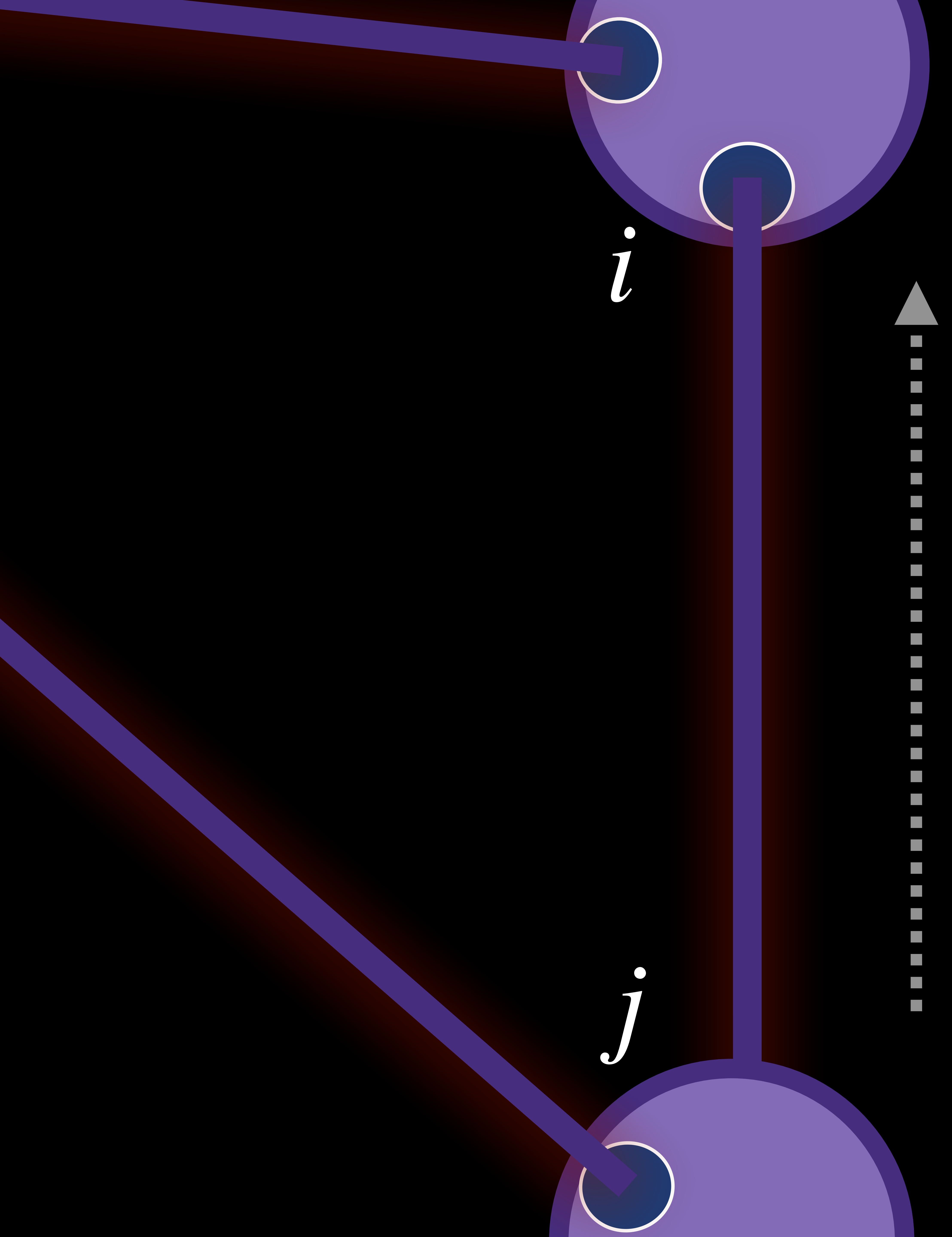
the constellation



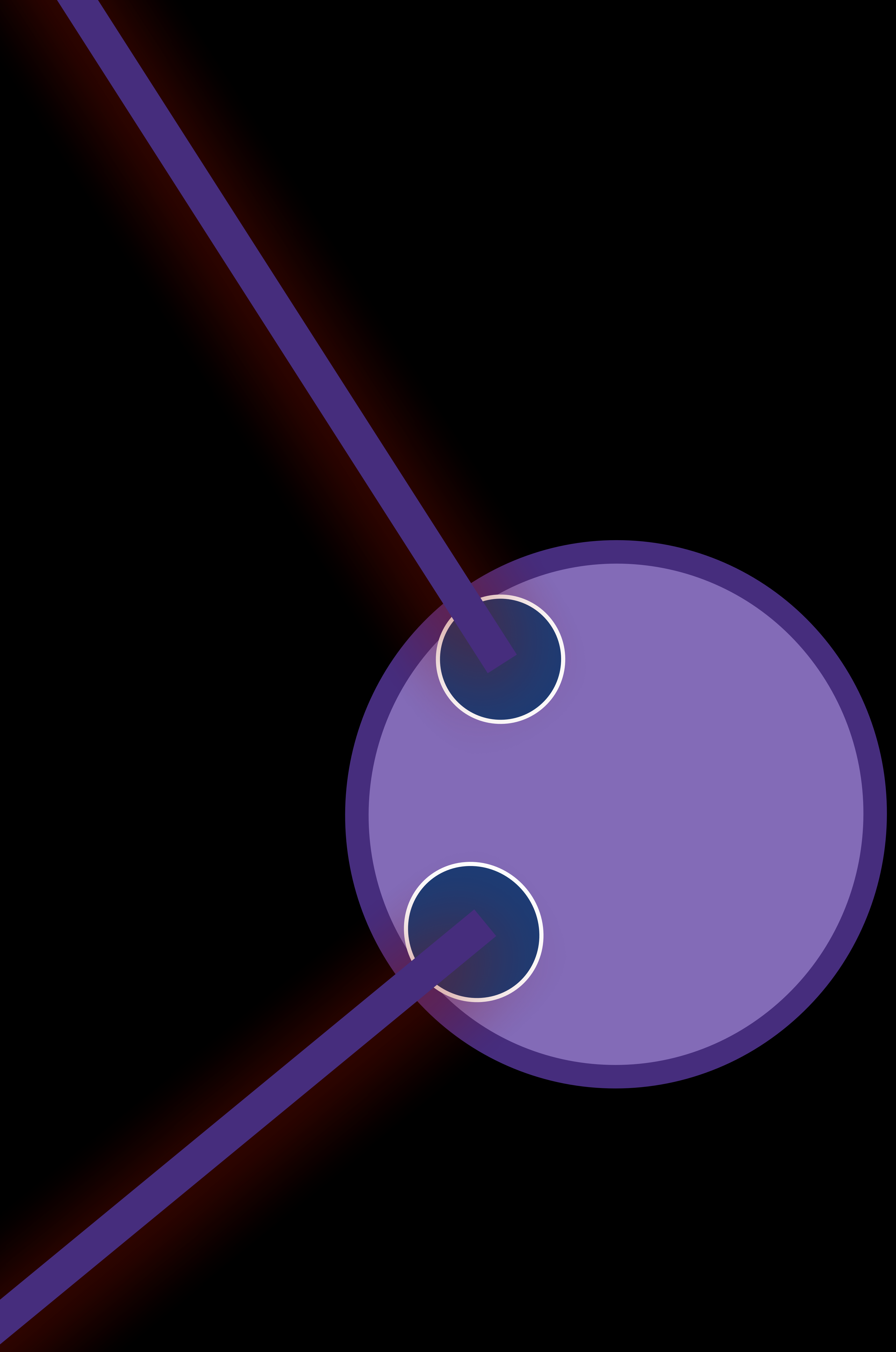
the measurement

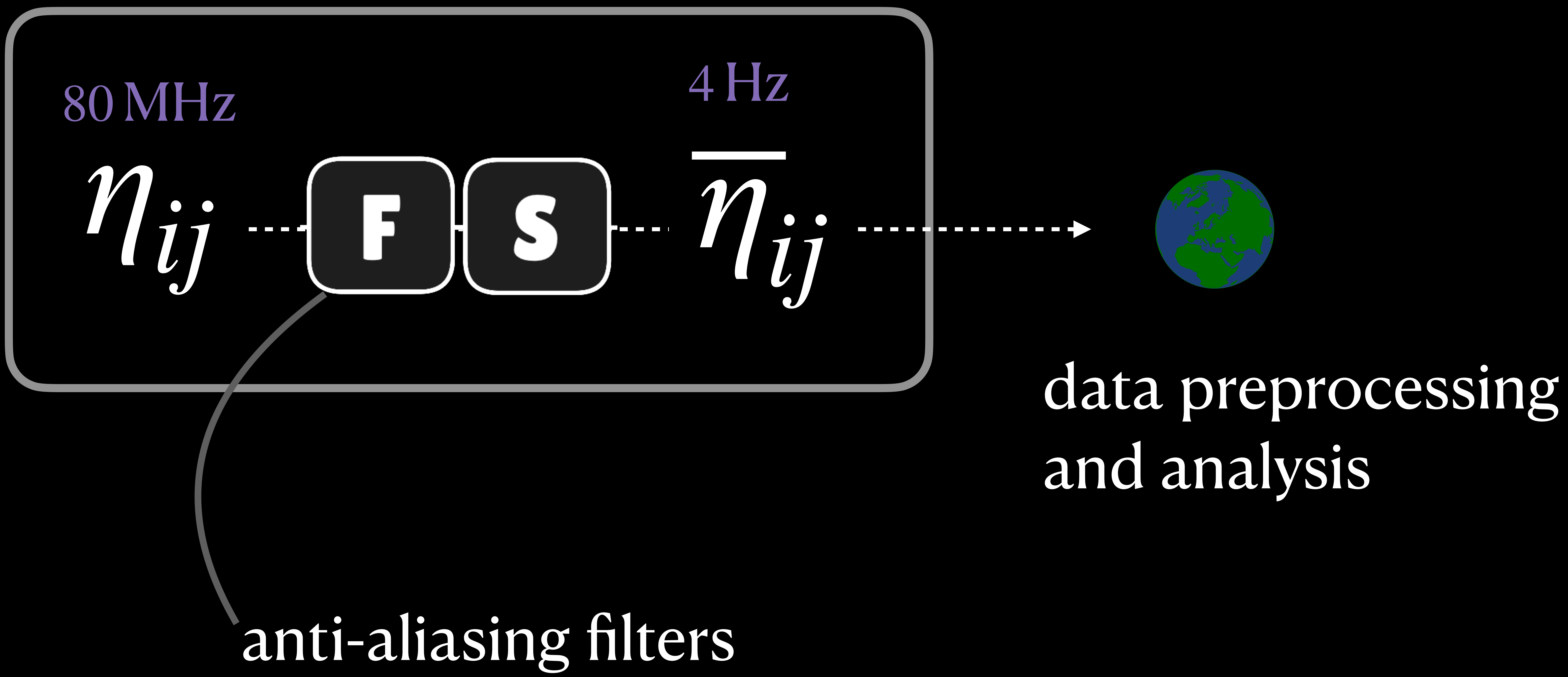


transmitted to Earth

$$\eta_{ij} = \mathbf{D}_{ij} p_j - p_i$$

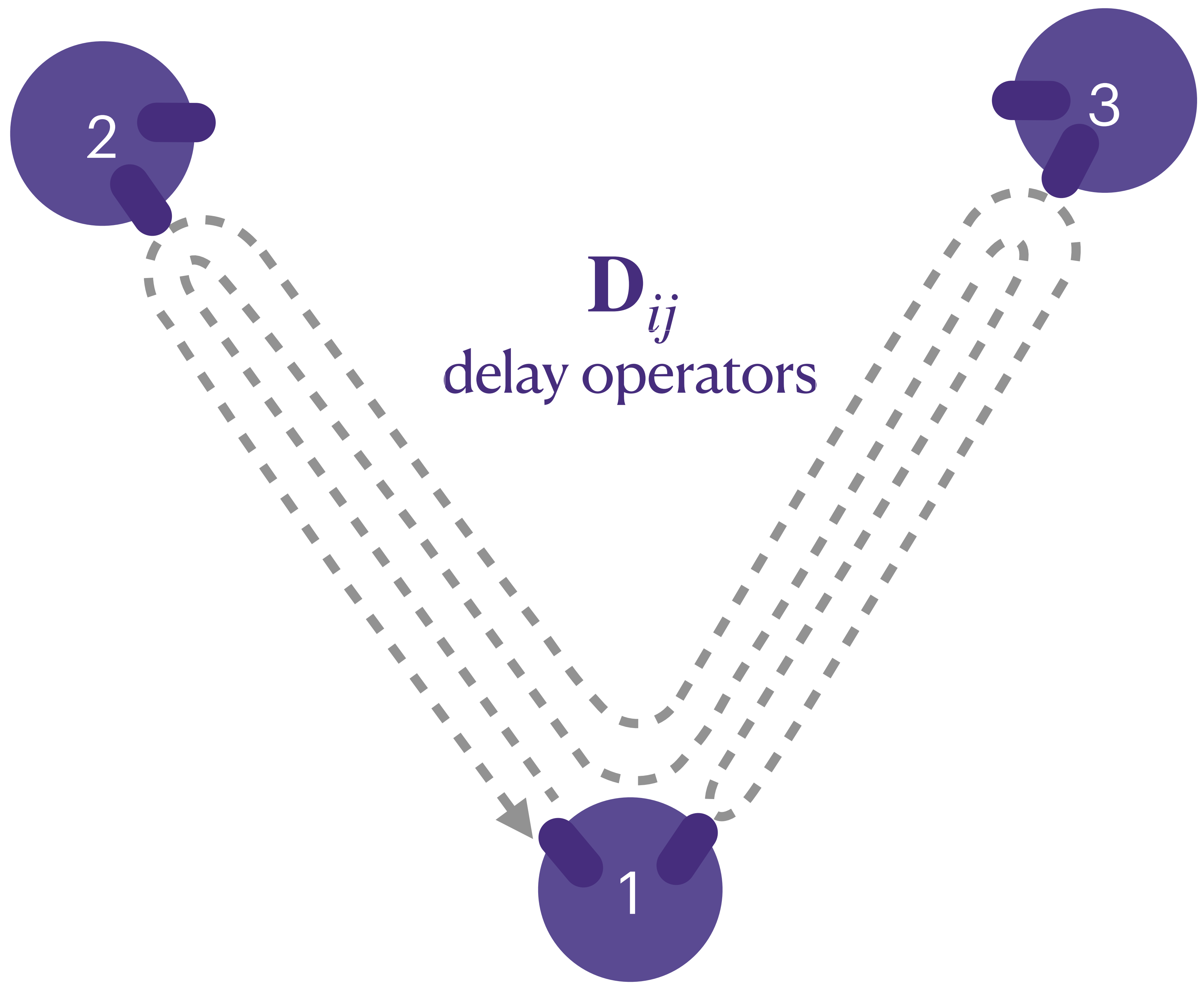
we need to reduce the sampling rate of the data before transmission





why do we need to do preprocessing?

- largest source of noise in LISA is laser frequency noise
- does not get automatically suppressed as in ground-based detectors because of varying arm length
- we apply a technique : **Time-Delay Interferometry (TDI)** on ground to suppress laser frequency noise.



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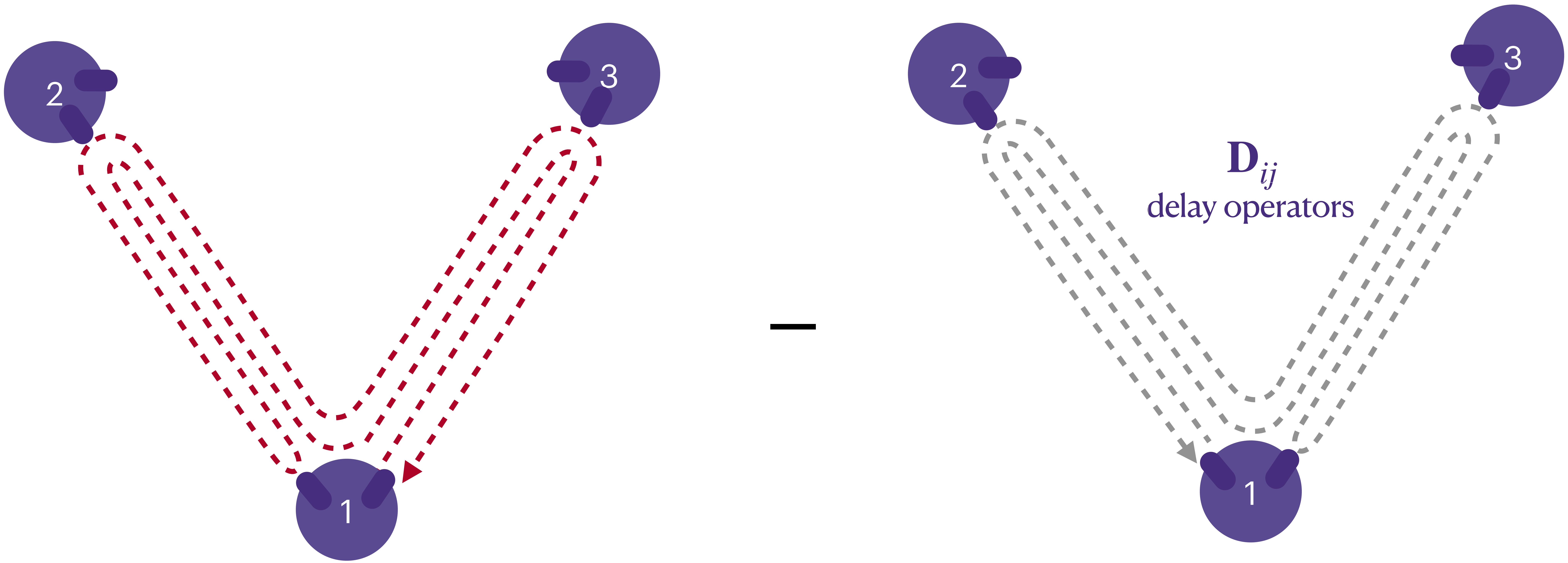


$$X = \sigma_{13} - \sigma_{12}$$

TDI variable (Y, Z centered on spacecraft 2,3)

However, the filter on the spacecraft and delay operator of TDI do not commute.

$$\bar{\eta}_{ij} = \mathbf{F}\mathbf{D}_{ij} p_j - \mathbf{F}\, p_i$$

$$\bar{\eta}_{ij} = \mathbf{F}\,\mathbf{D}_{ij}\, p_j - \mathbf{F}\, p_i + \underbrace{[\mathbf{F}, \mathbf{D}_{ij}] p_j}$$

flexing filtering[1] residual

[1] Bayle J B, Lilley M, Petiteau A and Halloin H 2019 Phys. Rev. D 99 084023

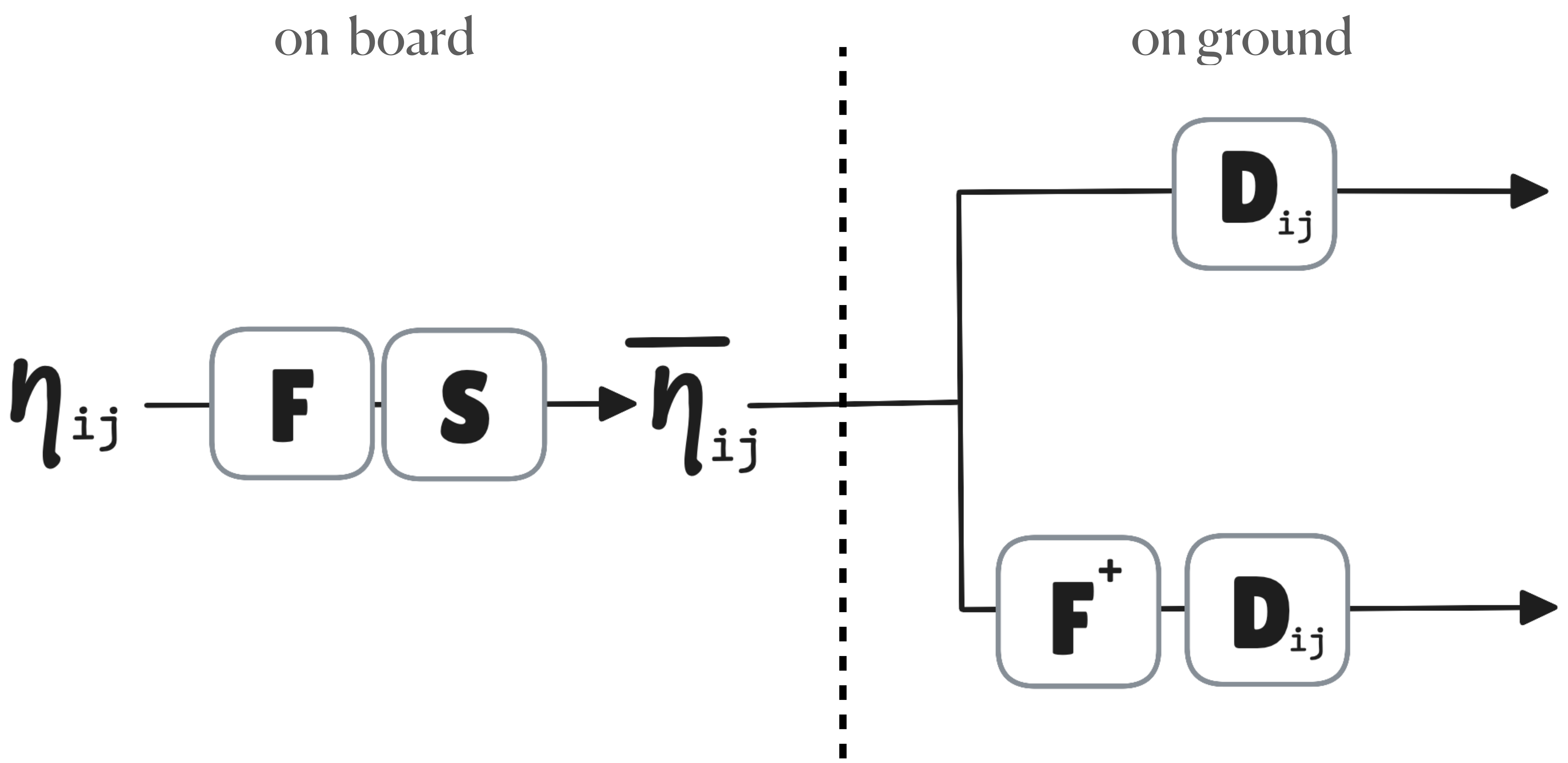
The amplitude of flexing-filtering residual*
depends on:

1. the delay derivative $\dot{d}$
2. the "flatness" of the filter in-band

$$\frac{\dot{d}}{2\pi} \cdot \frac{\mathrm{d}\tilde{h}_{\mathbf{F}}(f)}{\mathrm{d}f}$$

* to leading order

how we currently keep the "flatness" of the filter (in-band)



1. **Standard TDI** : using long, flat kaiser filters (145 taps).

2. **TDI with Compensation[2]** : using a quasi-inverse filter $\mathbf{F}^{+}$ to "lift" non-unity frequency response in-band.

$$\mathbf{F}^{+}\bar{\eta}_{ij} = \mathbf{F}^{+}\mathbf{S}\mathbf{F}(\mathbf{D}_{ij}\phi_j - \phi_i) \approx \eta_{ij}$$

[2] Staab M 2023 Ph.D. thesis Gottfried Wilhelm Leibniz University Hannover xiv

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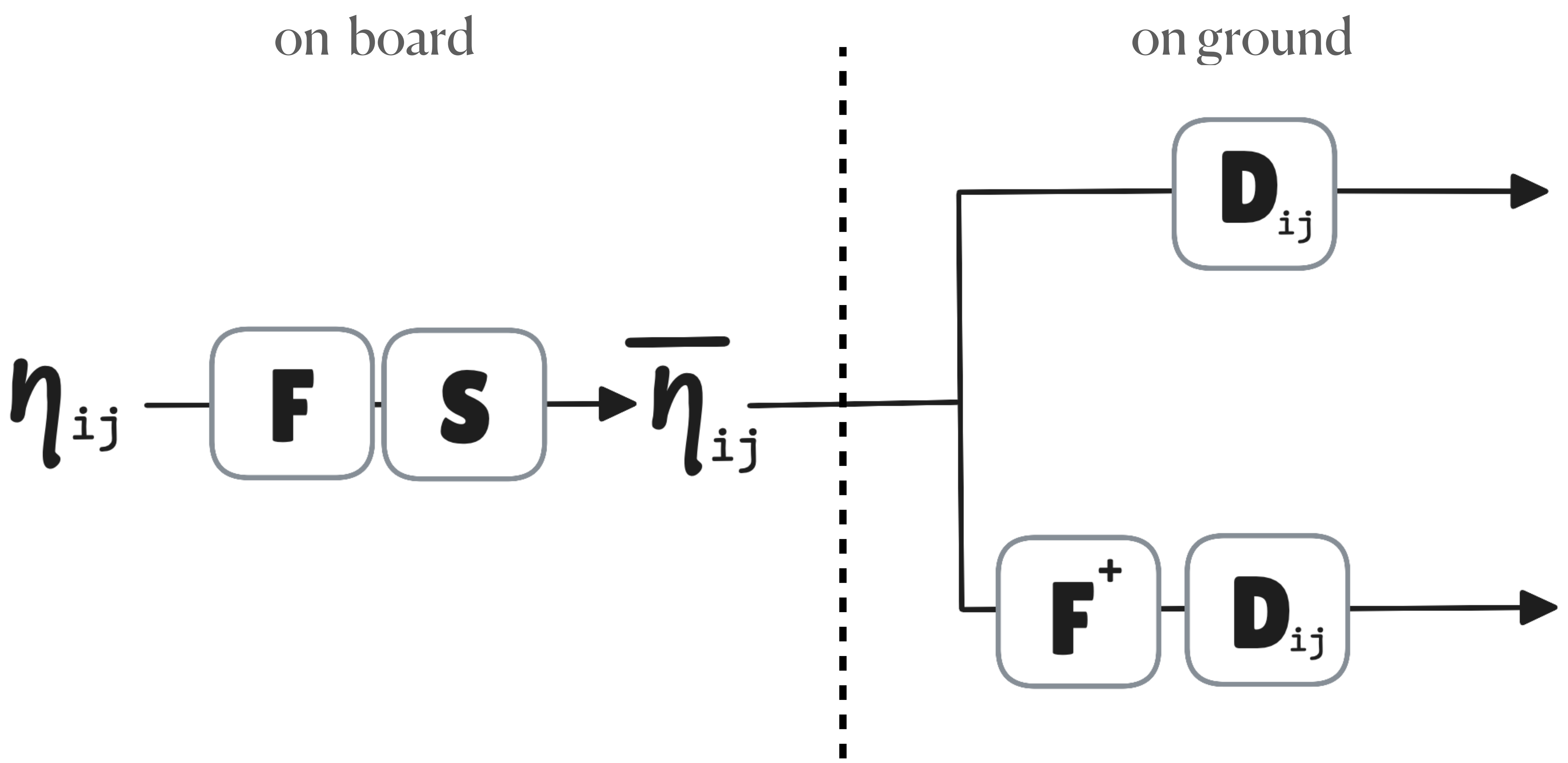


1. **the gap :** A flat filter chain on the spacecraft is computationally expensive and causes additional group delay.

2. **the gap :** Needs accurate inverse filter, adds to filter chain and causes additional group delay.

[2] Staab M 2023 Ph.D. thesis Gottfried Wilhelm Leibniz University Hannover xiv

Q. Can we reduce flexing filtering noise without compromising on data quality or overburdening the spacecraft's computational resources?

modify the delay operator : modified TDI

$$\bar{\eta}_{ij} = \mathbf{F}\mathbf{D}_{ij}\mathbf{F}^{-1}\mathbf{F}\,p_j - \mathbf{F}\,p_i$$

modified delay operator $\widehat{\mathbf{D}}_{ij}$

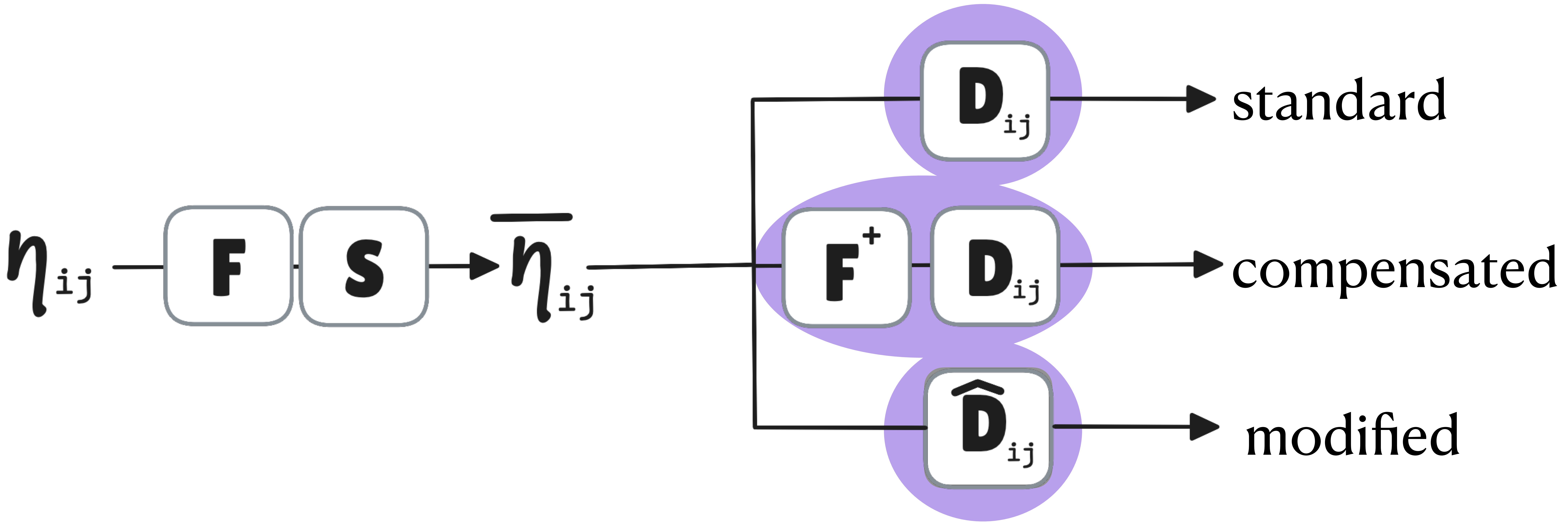
modify the delay operator : modified TDI

The modified delay operator is approximated as a sum of the normal delay operator and a small correction scaled by $\dot{d}$

$$\widehat{\mathbf{D}} = \mathbf{F}\mathbf{D}_{ij}\mathbf{F}^{-1}$$

$$\approx \mathbf{D} + \dot{d} \cdot \underbrace{\mathbf{D}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{G}\mathbf{F}^{-1}}_{\mathbf{H}}$$

$h_{\mathbf{G}}(\tau) = \tau \cdot h_{\mathbf{F}}(\tau)$

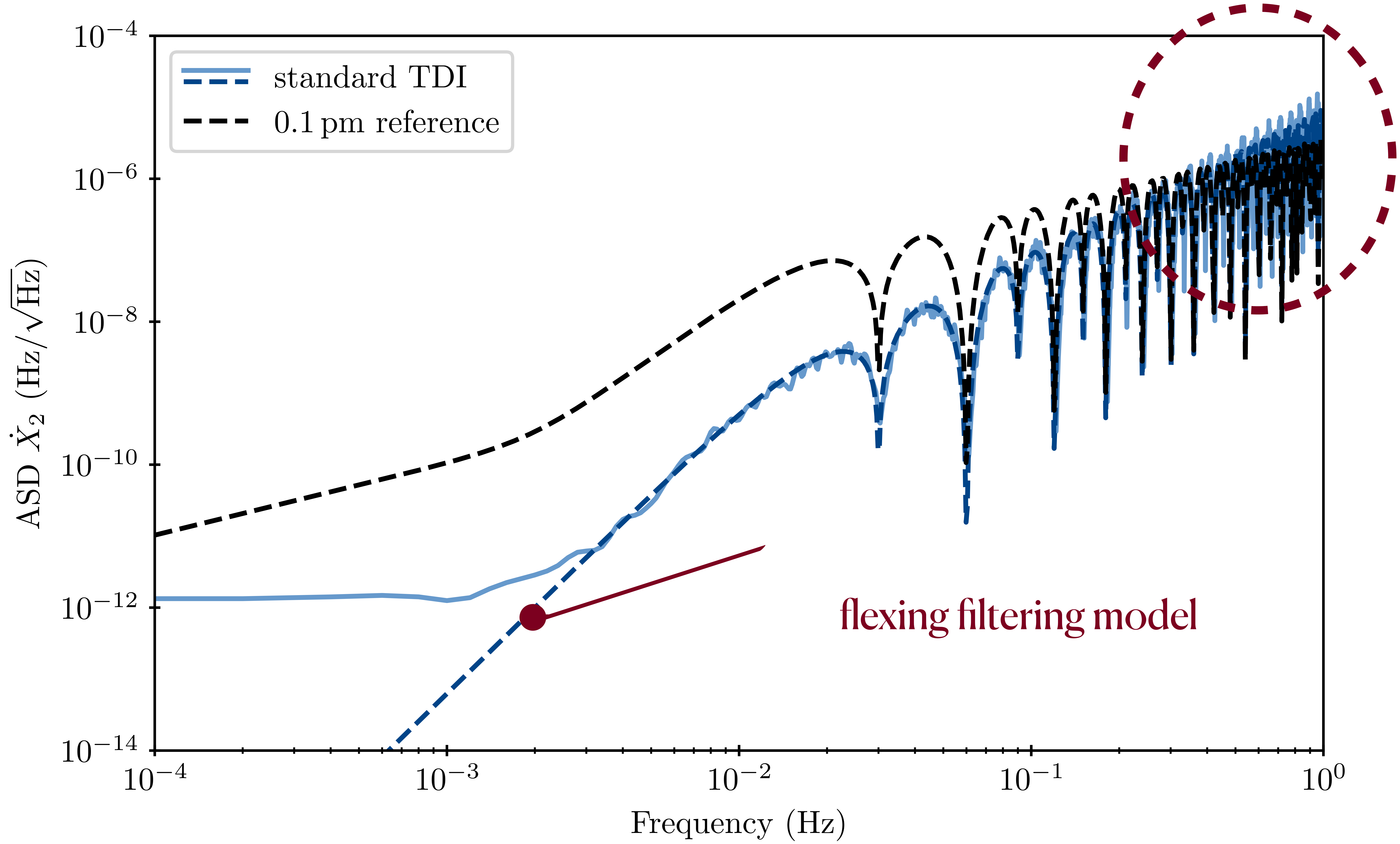


η_{ij}
F
S
$\bar{\eta}_{ij}$
D_{ij}
standard
F^+
D_{ij}
compensated
$\hat{D}_{ij}$
modified

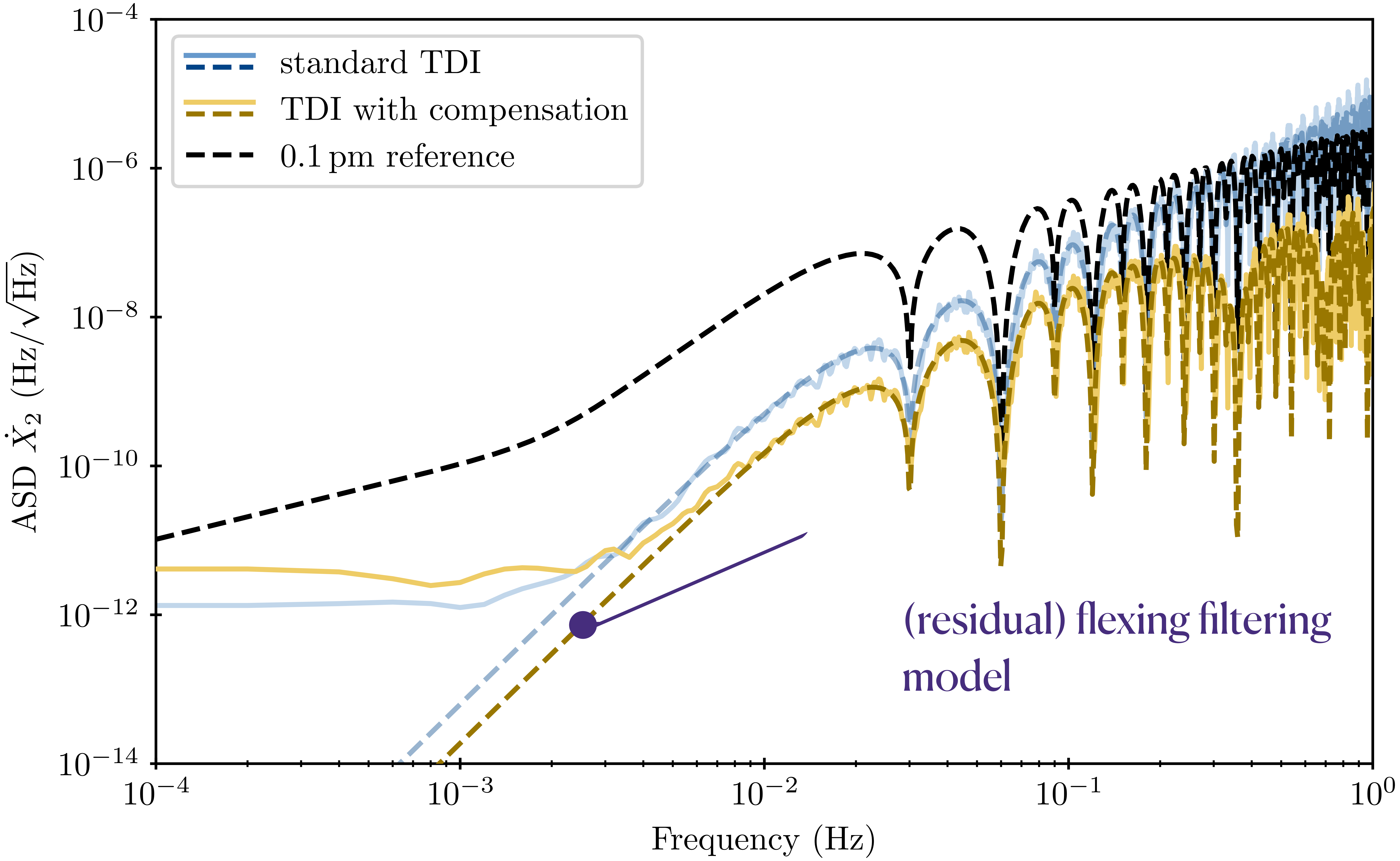
performance review

comparing the three topologies while using a non-flat filter on board

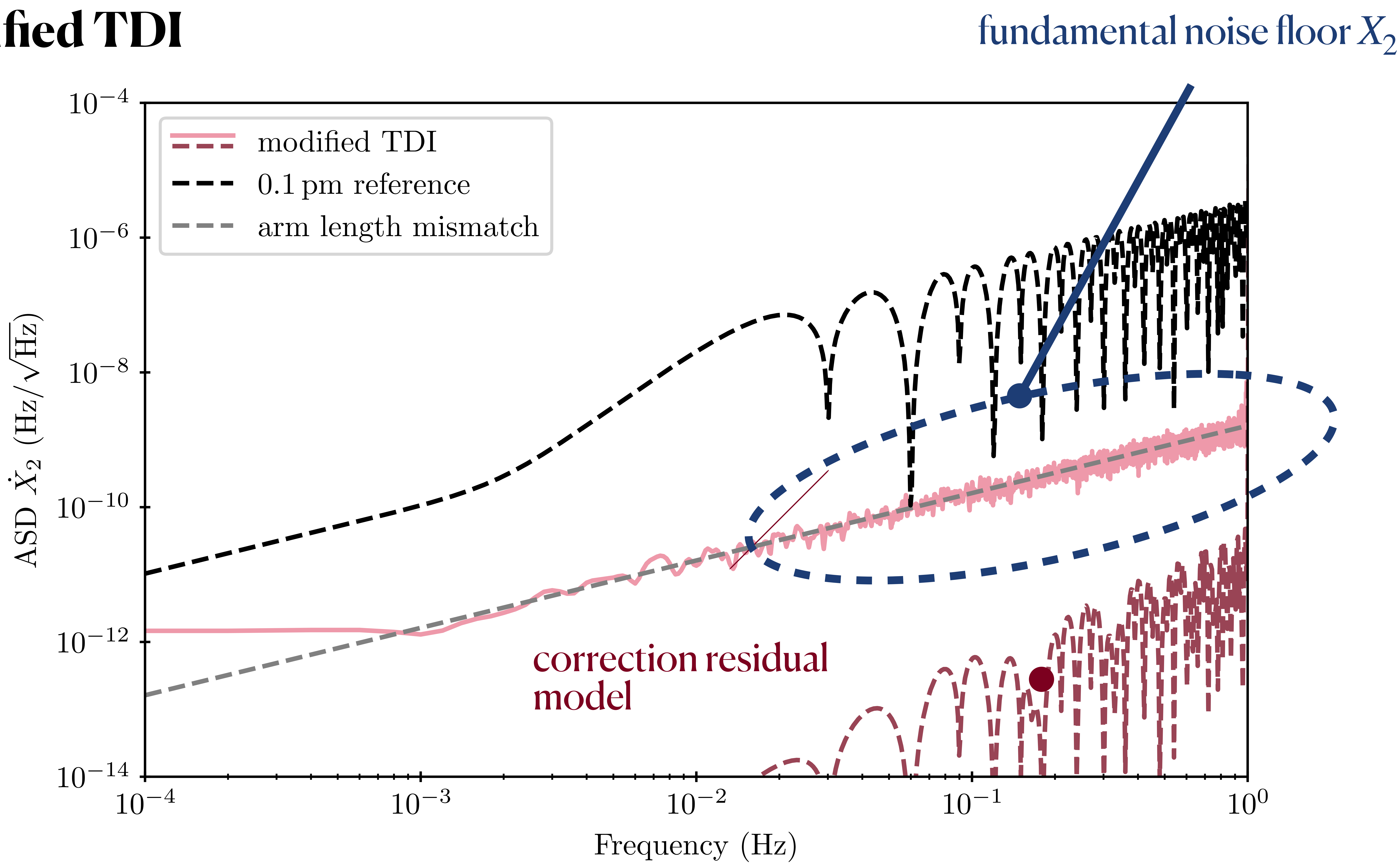
Standard TDI



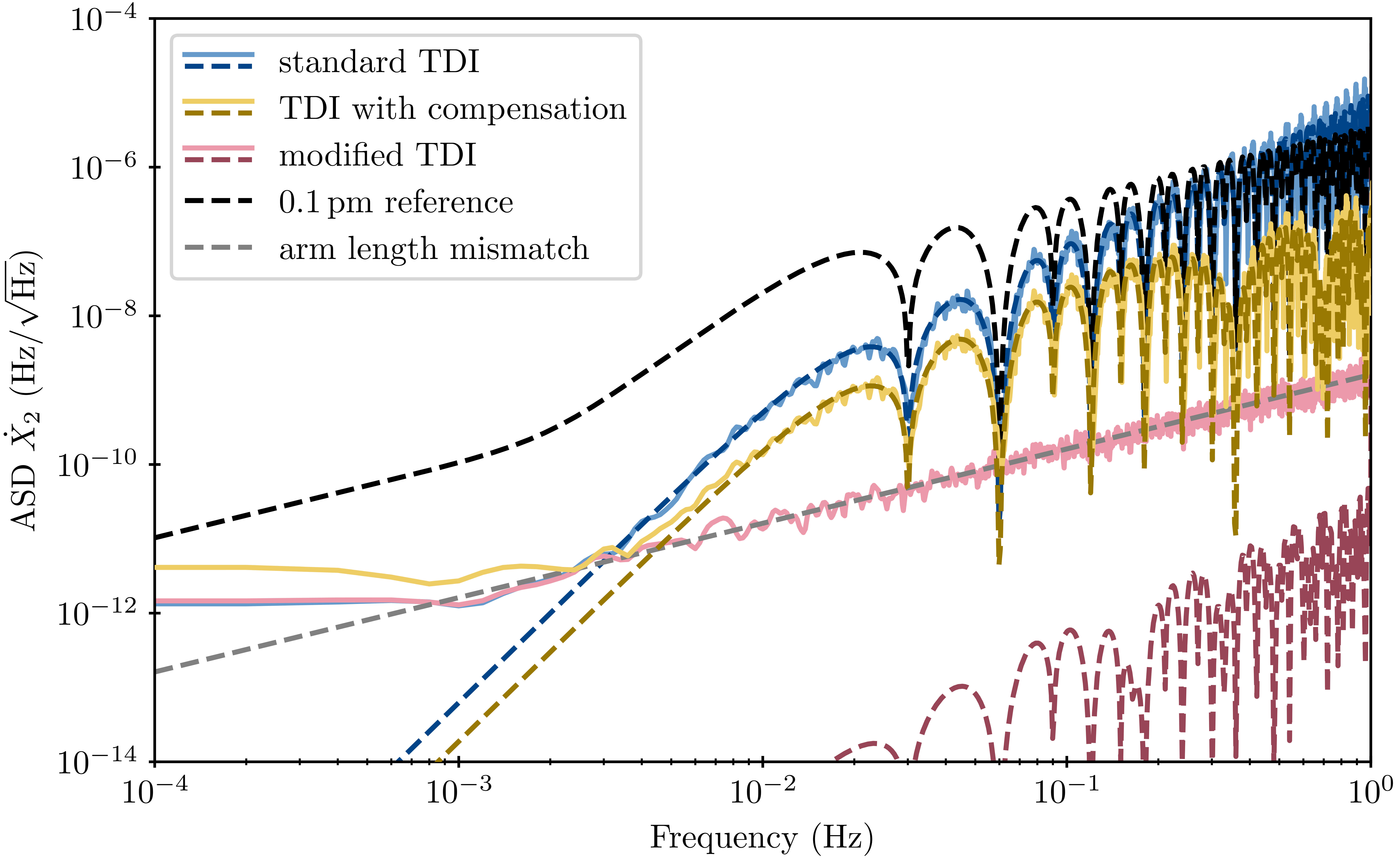
TDI with Compensation



Modified TDI



Modified TDI outperforms the other two topologies



summary and conclusion

Q. Can we reduce flexing filtering noise without compromising on data quality or overburdening the spacecraft's computational resources?

Yes.
Modified TDI allows for 6 order of noise reduction , contributes no additional group delay and keeps on-board computational cost low via an inexpensive anti-aliasing filter.

Follow up studies : other locking configurations, include primary noise sources, effect on other secondary noises (clock noise)

thanks :)

If you're interested, you can find our paper on CQG and arXiv:2506.04316

backup slides

We need to **approximate the correction H** to apply it on discrete data.

Similar to the pure delay operation
i.e. a discrete convolution between the data and this approximate correction $\mathscr{H}$.

$$y(nT_s) = \sum_{m=-\infty}^{\infty} x\left((n-m)T_s\right) \cdot h_{\mathscr{H}}\left(mT_s - d\right)$$

The kernel $h_{\mathscr{H}}$ is designed using cosine-sum kernels [3]

$$h_{\mathscr{H}}(t) = \mathrm{rect}\left(\frac{t}{NT_\mathrm{s}}\right)\sum_{n=0}^{N-1} a_n \cdot \cos\left(2\pi f_\mathrm{s}\frac{n}{N}t\right)$$

where the coefficients a_n are approximated using the Parks-McClellan algorithm.

[3] Staab M, Bayle J B, Hartwig O, Hees A, Lilley M, Woan G and Wolf P 2024 Optimal design of interpolation methods for time-delay interferometry (Preprint 2412.14884)

Residual Noise in Modified TDI

Errors between the exact operator $\widehat{\mathbf{D}}$ and the approximate design $\widehat{\mathscr{D}}$ result in noise residues.

$$\underbrace{(\mathbf{D} - \mathscr{D})\phi}_{\text{interpolation error}} + \underbrace{\dot{d} \cdot (\mathbf{H} - \mathscr{H})\phi}_{\text{correction residual}}$$

LISA Instrument[3] parameters

i. three laser lock
ii. white laser noise, ASD of 30 $\mathrm{Hz}/\sqrt{\mathrm{Hz}}$
iii. sampling: 4 Hz for 25000 s
iv. anti-aliasing filter: filter at 4 Hz with **9 taps**
v. interpolation: Lagrange (N = 62)
vi. ESA Trailing Orbits
vii. $t_0 = 2.0813 \times 10^9$ s ; to maximize $\dot{d}$

[3] Bayle, Jean-Baptiste, Hartwig, Olaf and Staab, Martin — 10.5281/zenodo.13809621

What contributes to the residual noise?

Topology	Noise source
Standard TDI	flexing filtering $[\mathbf{F}, \mathbf{D}]$
TDI with compensation	(residual) flexing filtering $[\mathbf{F}^{+}\mathbf{F}, \mathbf{D}]$
Modified TDI	correction $(\mathbf{H} - \mathscr{H})$