Agnostic Localization of EMRI Sources with the TDI Variable κ

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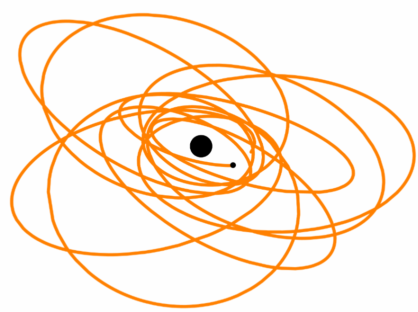
EMRIs

What is an EMRI?



Representation of an EMRI (*P. Ramond*)

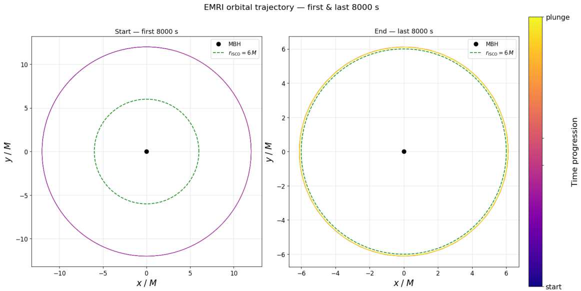
Parameter	Description
M	Mass of central black hole
μ	Mass of compact object
a	Spin of the black hole
x_0	Inclination
e	Orbital eccentricity
p	Semi-latus rectum
d_L	Luminosity distance

EMRIs are systems where a compact object inspirals into a massive black hole, producing gravitational waves observable by LISA.

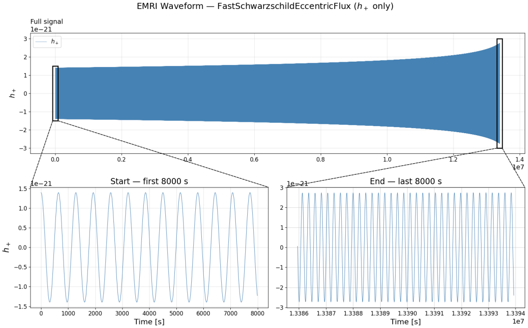
They are characterized by a mass ratio $10^{-6} < q < 10^{-4}$.

Complexity of EMRI (1/2)

Schwarzschild (static) Black Hole.



EMRI trajectory for the first and last 8000s in the Schwarzschild formalism



EMRI waveform generated with the *FEW* package

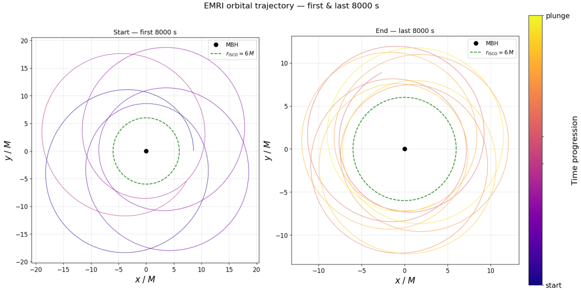
Parameters:

$$M = 10^6\, M_\odot,\ \mu = 10^2\, M_\odot$$
$$\iota = 0$$
$$p_0 = 12,\ e_0 = 0$$
$$d_L = 1\ \text{Gpc}$$

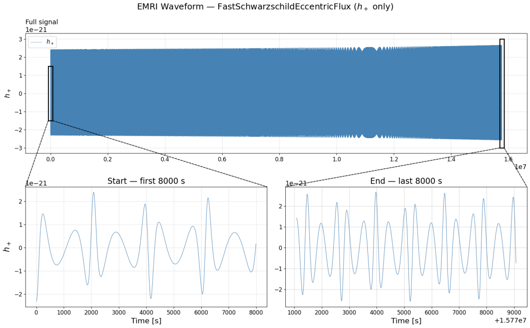
Katz et al., Phys. Rev. D 104, 064047 (2021)

Complexity of EMRI (2/2)

Kerr (spinning) Black Hole.



EMRI trajectory for the first and last 8000 s in the Kerr formalism



EMRI waveform generated with the *FEW* package

Parameters:

$$M = 10^6\ M_\odot,\ \mu = 10^2\ M_\odot$$
$$\iota = 0,\ a = 0.9$$
$$p_0 = 12,\ e_0 = 0.6$$

$$d_L = 1\ \text{Gpc}$$

Why Localization Matters

Wang, Y. Y., Wang, F. Y., Zou, Y. C., & Dai, Z. G. (2019)

In addition to MBHBs, EMRIs may also produce electromagnetic counterparts.

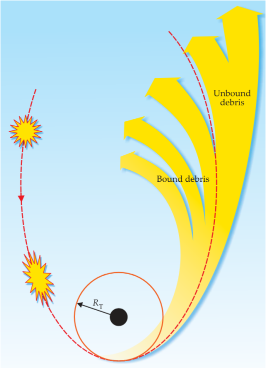


Illustration of a Tidal Disruption Event (*S. Gezari, Physics Today*)

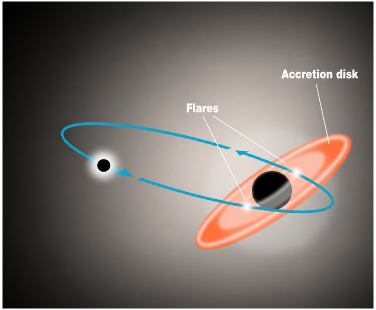
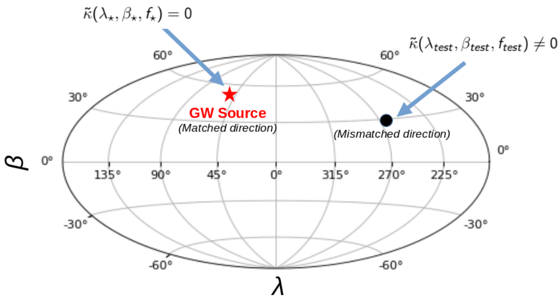


Illustration of a flare in a binary black hole system (*A. Rojas*)

Nullstream: Principle



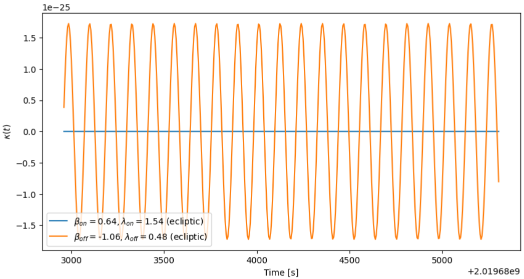
Sky map of the TDI variable : $\kappa(\lambda, \beta)$

$$\tilde{\kappa}(\lambda, \beta, f) = \vec{A}(\lambda, \beta, f) \begin{pmatrix} \tilde{\alpha}(f) \\ \tilde{\beta}(f) \\ \tilde{\gamma}(f) \end{pmatrix}$$

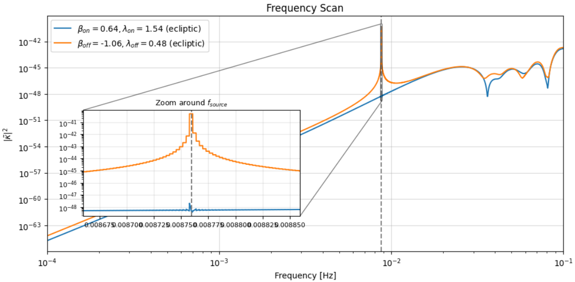
- $\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}$: Sagnac TDI variables
- $\tilde{\kappa} \to 0$ when $(\lambda, \beta) \to (\lambda_\star, \beta_\star)$
- **Objective:** construct a sky map $\kappa(\lambda, \beta)$ and estimate the source location by minimizing κ

Example: Galactic Binary (Noiseless)

Correct localization requires both direction and frequency matching (GB generated during 1 day)



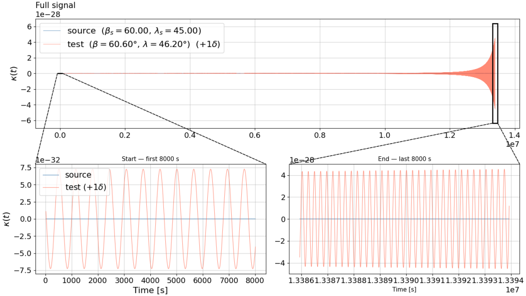
Time-domain coronagraphic TDI



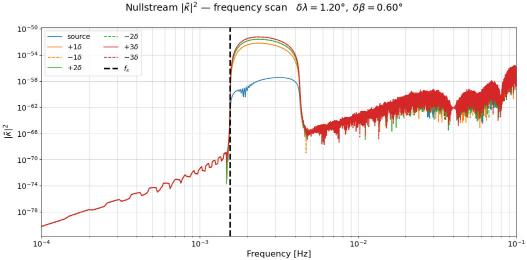
Frequency-domain representation

$$\text{When } (\lambda, \beta) \to (\lambda_\star, \beta_\star),\ \tilde{\kappa}(f) = \varepsilon$$

Example: EMRI (Noiseless)



Time-domain nullstream

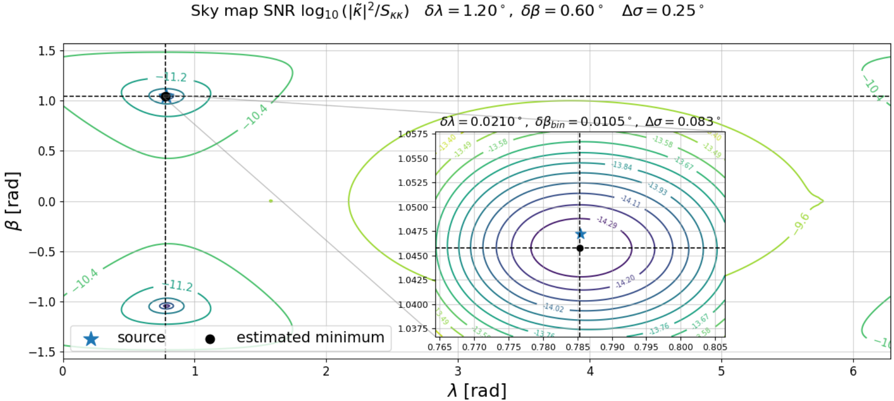


Frequency-domain nullstream

Unlike Galactic binaries (quasi-monochromatic), EMRIs show a frequency drift, leading to multiple characteristic frequencies.

Signal not fully cancelled but strongly attenuated → residual ∼ 6 orders of magnitude below sky-position bin signals.

EMRI Sky Localization from Nullstream (Noiseless)

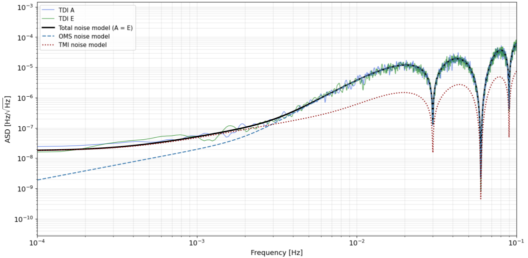


Nullstream sky map obtained in the noiseless case where $f_{source} = 1.55 \times 10^{-3}$ Hz

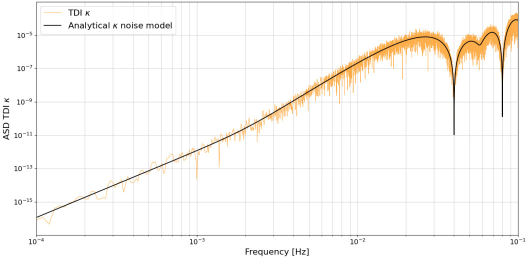
A degeneracy in latitude is observed, reflecting the detector geometry.

Noise Impact

Presence of Instrumental Noise



Instrumental noise propagation in TDI A & E (ASD)

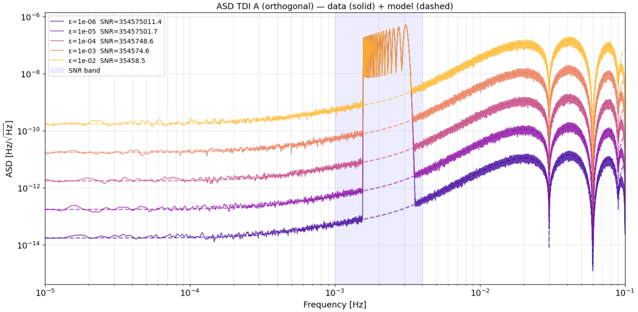


Instrumental noise propagation in TDI κ (ASD)

Two main noise sources:

- $\rightarrow$ Optical Metrology System (OMS): laser measurement fluctuations between spacecraft optical benches and test masses
- $\rightarrow$ Test mass acceleration noise: non-gravitational forces on free-falling test masses

Different Noise Levels



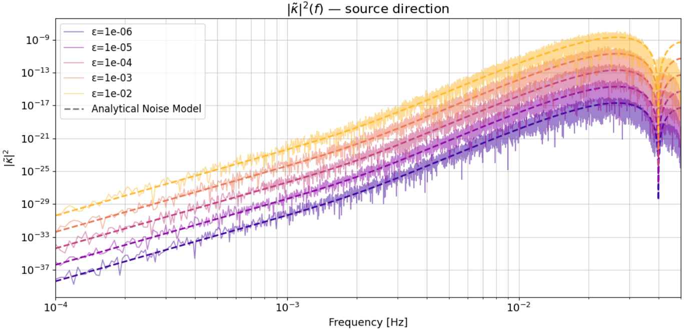
ASD of the noise in the TDI A channel, generated using the *LISA Suite* pipeline.

SNR scaling via noise attenuation

ϵ	SNR
10^{-6}	354575011.4
10^{-5}	35457501.7
10^{-4}	3545748.6
10^{-3}	354574.6
10^{-2}	35458.5

SNR scaling strategies:
- Astrophysical scaling: $\{M, D_L\} \Rightarrow$ changes source physics (incl. redshift)
- Noise scaling: $\epsilon \Rightarrow$ preserves the underlying physics

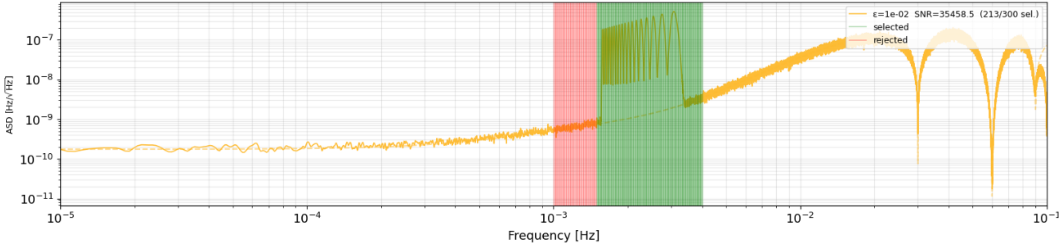
Frequency Scan of $\kappa^2(f)$



Frequency scan of $\kappa^2(f)$ at the true sky location (λ, β)

- κ signal entirely buried in noise
- Noise dominates over the residual signal for each noise level

Frequency Selection for Nullstream Estimation



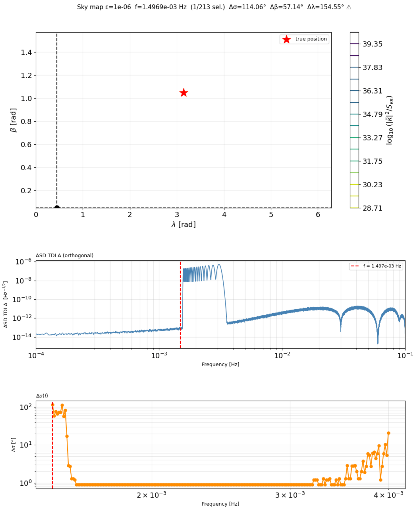
Frequency selection with local SNR estimation. Green: selected frequencies — Red: rejected frequencies

- EMRIs are multi-frequency source → multiple frequencies must be selected
- Which ones? Those contributing to the signal power:

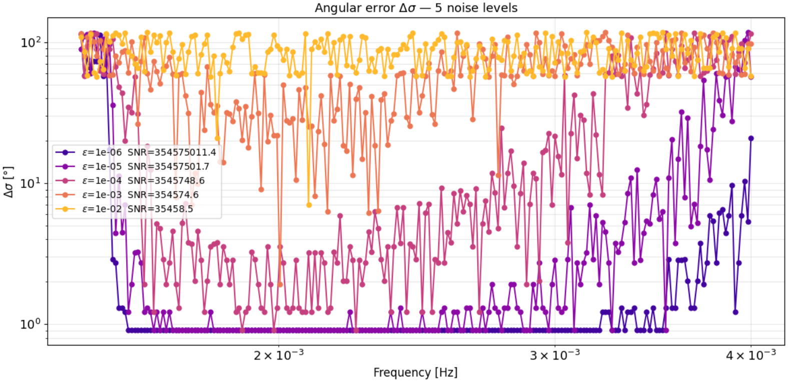
$$\mathrm{SNR}_{local}(f_i) \geq 8$$

- → Green: retained (significant contribution)
- → Red: rejected (negligible contribution)

Animated sky map — $\varepsilon = 10^{-6}$

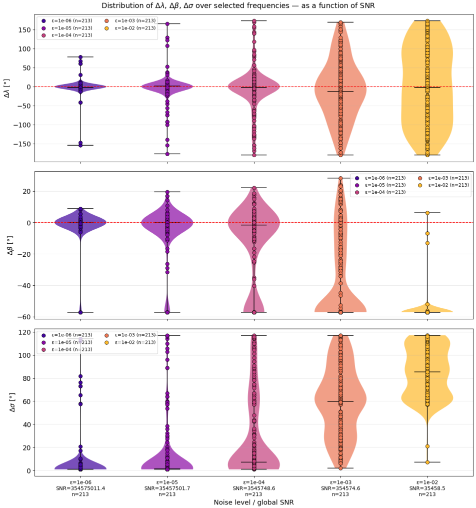


Impact of Frequency Selection on Localization Accuracy



- Frequencies near the selection threshold $\rightarrow$ poor localization
- Lower SNR leads to poorer accuracy in source localization

Localization Accuracy vs Noise Level



Distribution of localization errors: $\Delta\lambda$, $\Delta\beta$, $\Delta\sigma$.

- Increasing noise $\rightarrow$ degradation of localization accuracy
- $\rightarrow$ larger angular separation from the true position
- $\Delta\lambda$: distribution tends toward uniformity at high noise
- $\Delta\beta$: localization collapses toward the detector blind region

Conclusion & outlook

Conclusion & Outlook

Conclusion

- In the noiseless case, the nullstream provides an efficient way to localize EMRIs
- In the presence of noise, localization remains possible for sufficiently high SNR
- $\rightarrow$ Lower SNR leads to poorer localization accuracy

Outlook

- Detect MBHBs with overlapping EMRIs as an astrophysical noise using the nullstream

Thank you for your attention

Backup: Signal-to-Noise Ratio (SNR)

The signal-to-noise ratio (SNR) quantifies the detectability of a signal with respect to detector noise.

$$\mathrm{SNR}^2 = \int_{f_{\mathrm{min}}}^{f_{\mathrm{max}}} \frac{|\tilde{d}(f)|^2}{S_n(f)}\, df$$

with:

$$d(t) = h(t) + n(t)$$

- $\rightarrow$ Measures signal power relative to noise power
- $\rightarrow$ Detection threshold typically set at:

$$\mathrm{SNR} \geq 8$$

Colpi et al. (2024), LISA Science Requirements

Backup: Angular Localization Error

The sky localization error is defined as the angular separation between the true and estimated source positions:

$$\Delta\sigma = \arccos\Big[\sin(\beta_{\text{test}})\sin(\beta_\star) + \cos(\beta_{\text{test}})\cos(\beta_\star)\cos(\lambda_{\text{test}} - \lambda_\star)\Big]$$

- $\rightarrow$ Measures angular distance on the celestial sphere

Backup: Robustness on Complex EMRIs

The same analysis has been performed for more complex EMRI, including:

- orbital eccentricity
- high black hole spin
- frequency modulation and harmonics

- $\rightarrow$ Localization still degrades as SNR decreases

Conclusion: EMRI complexity does not affect the main results.