GDR 2026

AN ALTERNATING APPROACH TO RECONSTRUCT PROMPT GAMMA DISTRIBUTION AND HADRON VELOCITY FROM TIME-OF-FLIGHT MEASUREMENTS

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Outline

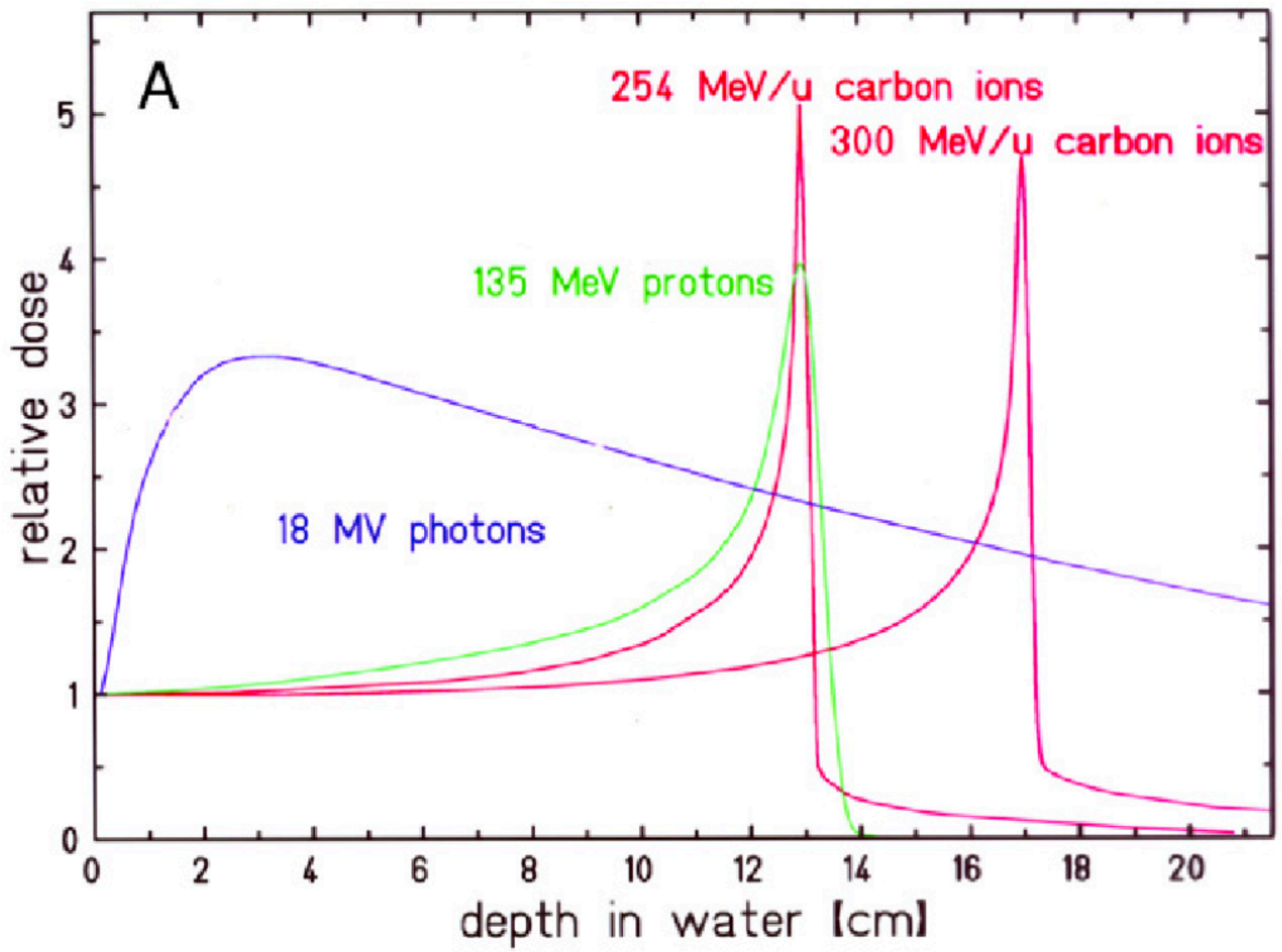
1. Context
2. Setting of the problem
3. An alternating minimisation approach
4. Numerical results
5. Perspectives

Hadrontherapy

Advantage: Protons deposit a maximum dose at a specific depth (Bragg Peak), sparing healthy tissues behind tumor

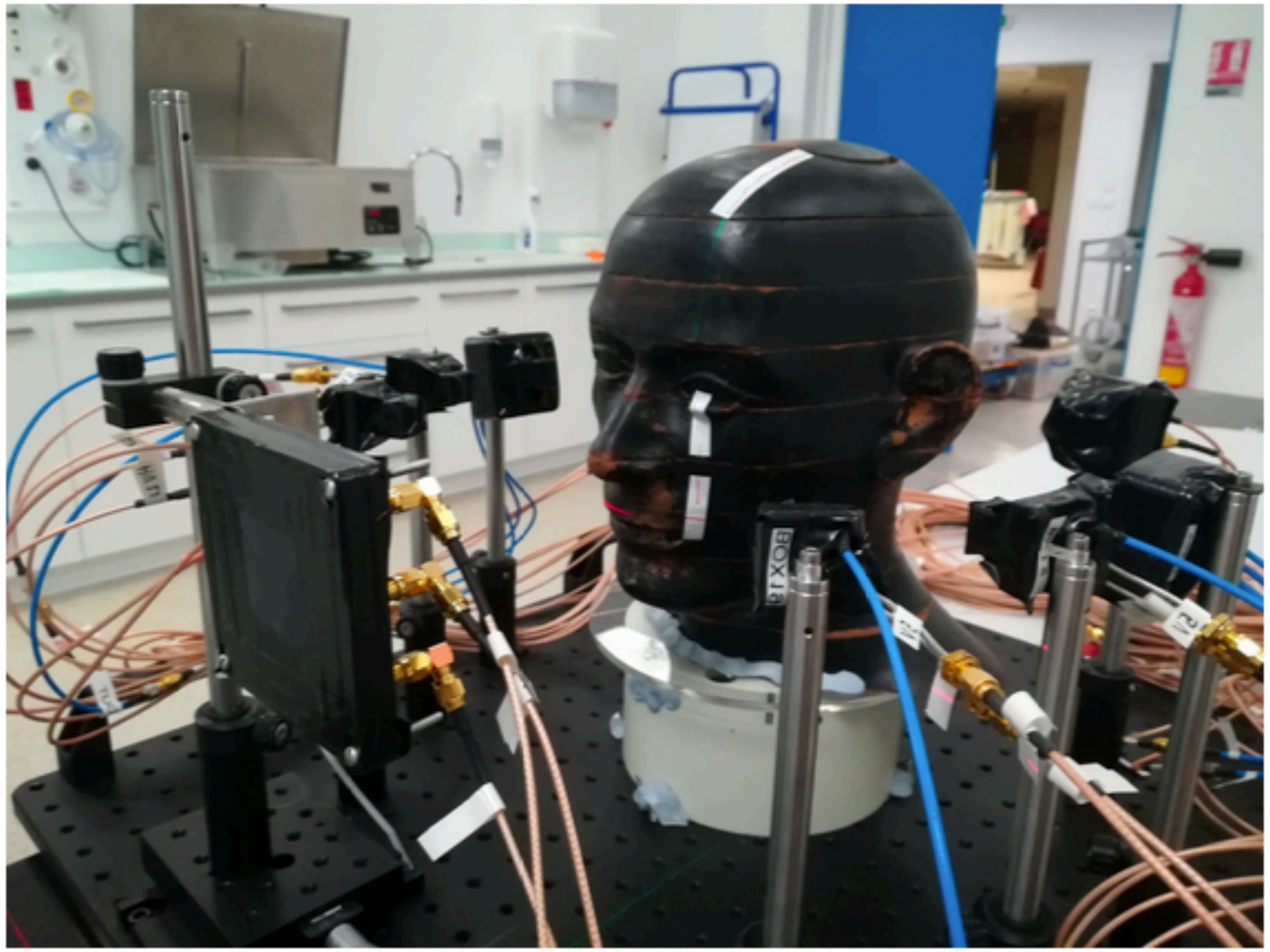
Range sensitivity: Small changes in patient anatomy significantly shift the Bragg Peak position

Challenge: Develop real-time range verification methods to ensure accurate dose administration



Comparison of the energy loss of protons, carbons and X-rays in water. The Bragg peak is clearly visible for protons and carbons, while X-rays show a continuous energy loss (Durante et al. 2016).

The Prompt Gamma Time Imaging (PGTI) project

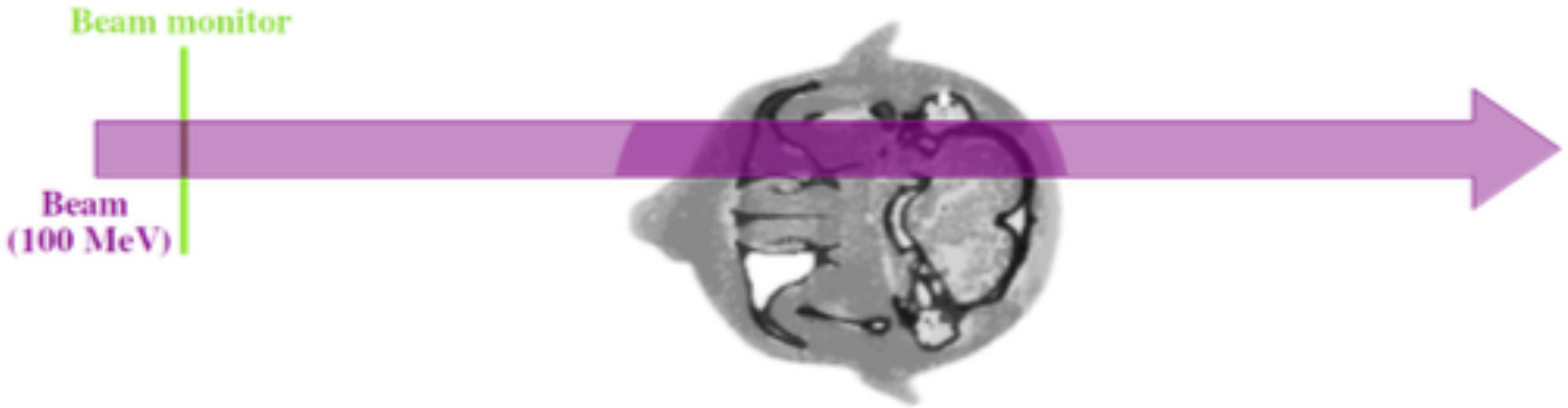


TIARA detector: Beam monitor + Prompt gamma detectors

Physics Principle: Protons interact with patient nuclei, emitting **Prompt Gammas (PG)** along their path

Mechanism: PGTI project develops the TIARA detector to measure Time-Of-Flights (TOFs)

Encoded Information: TOF depends on two unknowns: the **emission position** and the **proton velocity**



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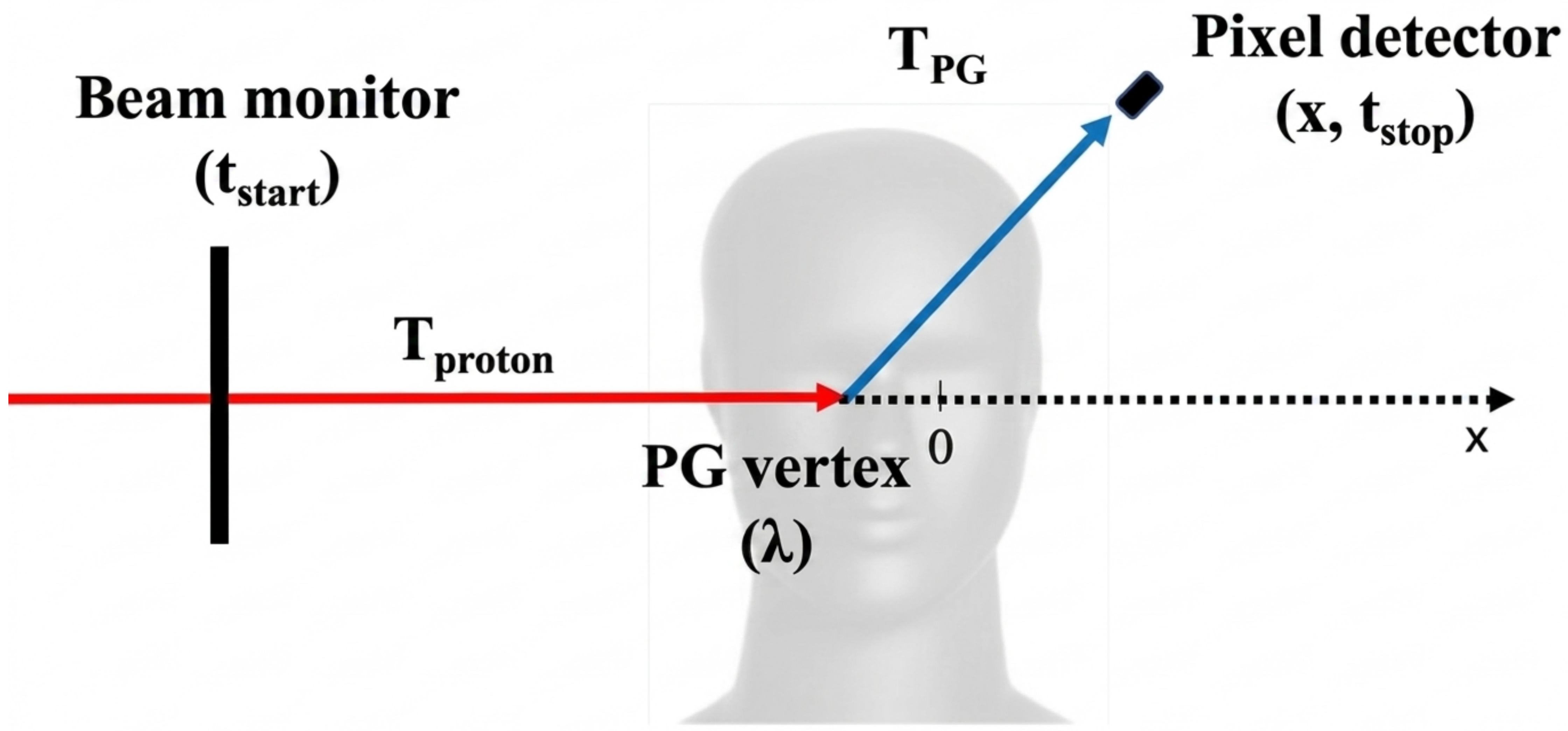
Setting of the problem



For one prompt Gamma emitted λ, the total Time of flight reads:

$$\begin{cases} T(\lambda, v) = T_{proton}(\lambda, v) + T_{PG}(\lambda) \\ \qquad\qquad = \int_0^{\lambda} \frac{1}{v(s)} ds + \frac{\|\lambda - x\|}{c} \end{cases}$$

Where:

c: speed of light

v: the velocity of the proton

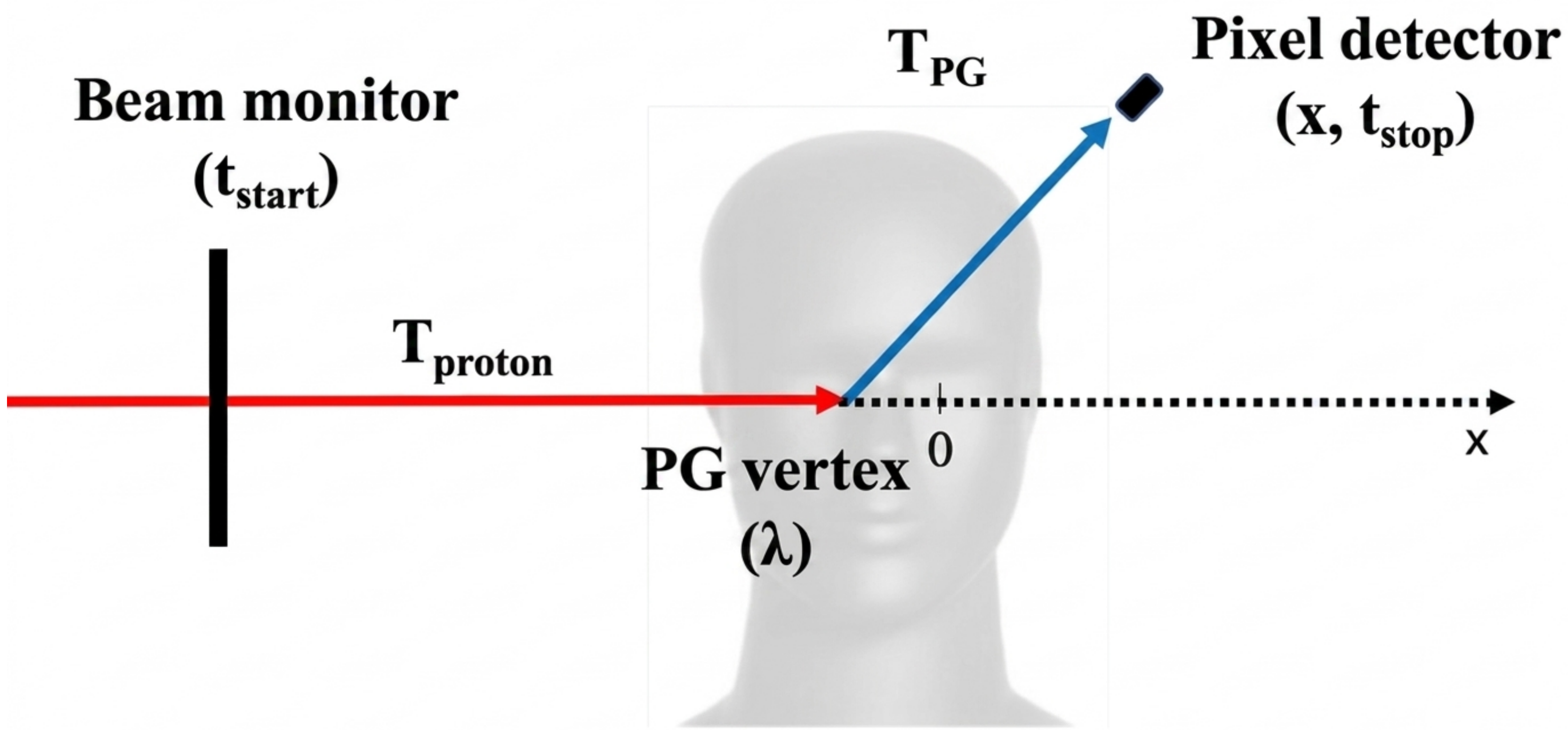
For n prompt gammas, the emission positions are denoted by $\boldsymbol{\lambda} = [\lambda_1, .., \lambda_n] \in \mathbb{R}^n$

$$T_i(\lambda_i, v) = \int_0^{\lambda_i} \frac{1}{v(s)} ds + \frac{\| \lambda_i - x_i \|}{c}, \qquad \forall 0 < i < n$$

The proton mean velocity, discretized on spatial dimension, is given by $\boldsymbol{v} = [v_1, .., v_d] \in \mathbb{R}^d$. Let's take $\boldsymbol{\gamma} = [1/v_1, .., 1/v_d] \in \mathbb{R}^d$

$$\boldsymbol{T}(\boldsymbol{\lambda}, \boldsymbol{\gamma}) = \begin{pmatrix} T_1(\lambda_1, \boldsymbol{\gamma}) \\ \vdots \\ T_n(\lambda_n, \boldsymbol{\gamma}) \end{pmatrix} = A(\boldsymbol{\lambda})\boldsymbol{\gamma} + \frac{1}{c} \begin{pmatrix} \| \lambda_1 - x_1 \| \\ \vdots \\ \| \lambda_n - x_n \| \end{pmatrix} \qquad where \quad A(\lambda) = \begin{pmatrix} a_1(\lambda_1)^t \\ a_2(\lambda_2)^t \\ \vdots \\ a_n(\lambda_2)^t \end{pmatrix}$$

Setting of the problem



For n prompt Gammas emitted $\lambda = [\lambda_1, \dots, \lambda_n] \in \mathbb{R}^n$, the total Time of flight reads:

$$\begin{cases} T(\lambda, \gamma) = T_{proton}(\lambda, \gamma) + T_{PG}(\lambda) \\ \qquad\qquad = A(\lambda)\gamma + \frac{\|\lambda - x\|}{c} \end{cases}$$

Where:

c: speed of light

$x = [x_1, \dots x_n] \in \mathbb{R}^n$ the detected position of prompt gammas

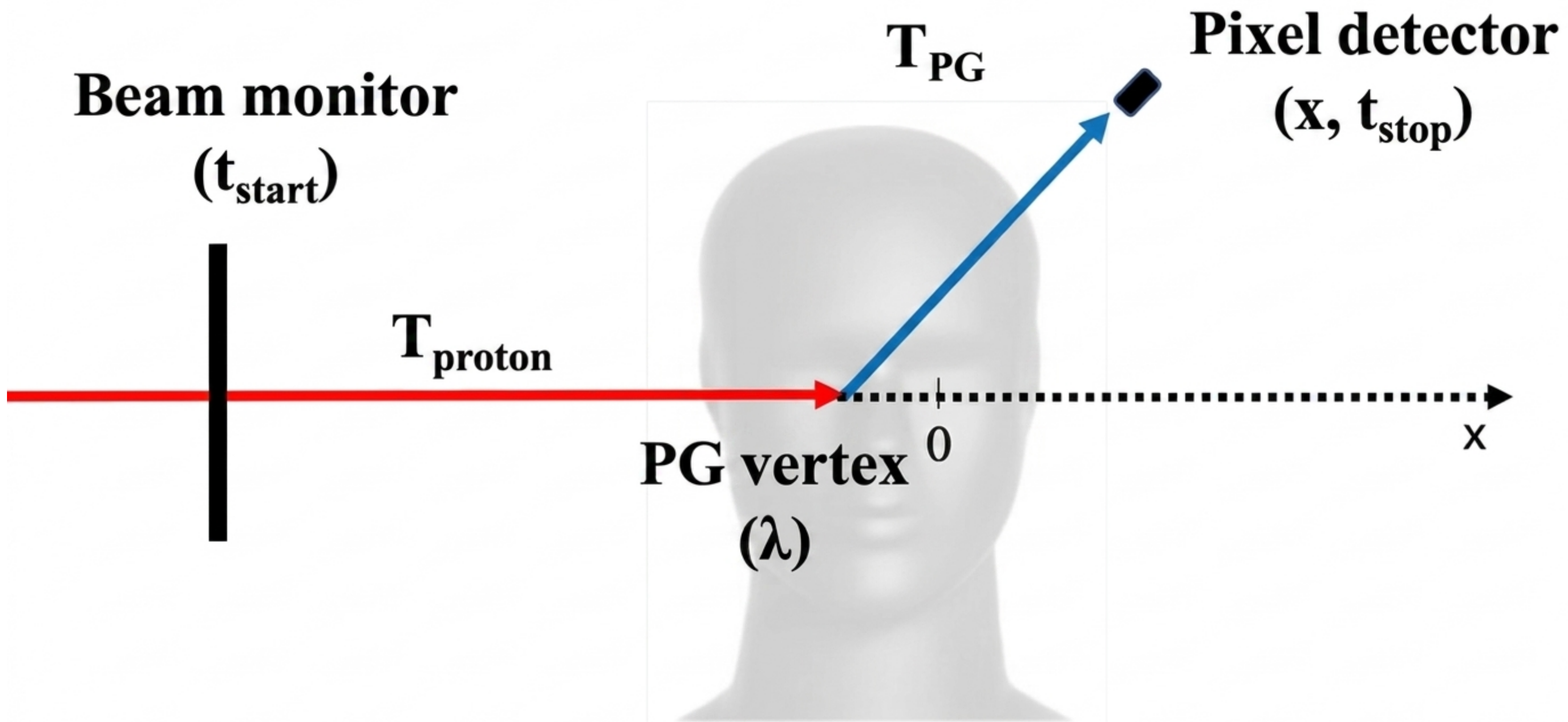
$\gamma = [\gamma_1, \dots, \gamma_d] \in \mathbb{R}^d$ the velocity of the proton

Objective: Find $\lambda \in \mathbb{R}^n, \gamma \in \mathbb{R}^d$

$$\min_{\lambda, \gamma} J(\lambda, \gamma) = \frac{1}{2\sigma^2} \| t - T(\lambda, \gamma) \|^2$$

Where: $t = t_{stop} - t_{start}$

Setting of the problem



For n prompt Gammas emitted $\lambda = [\lambda_1, .., \lambda_n] \in \mathbb{R}^n$, the total Time of flight reads:

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ill-posed problem !!!

NON-CONVEXITY

Chaima Abid

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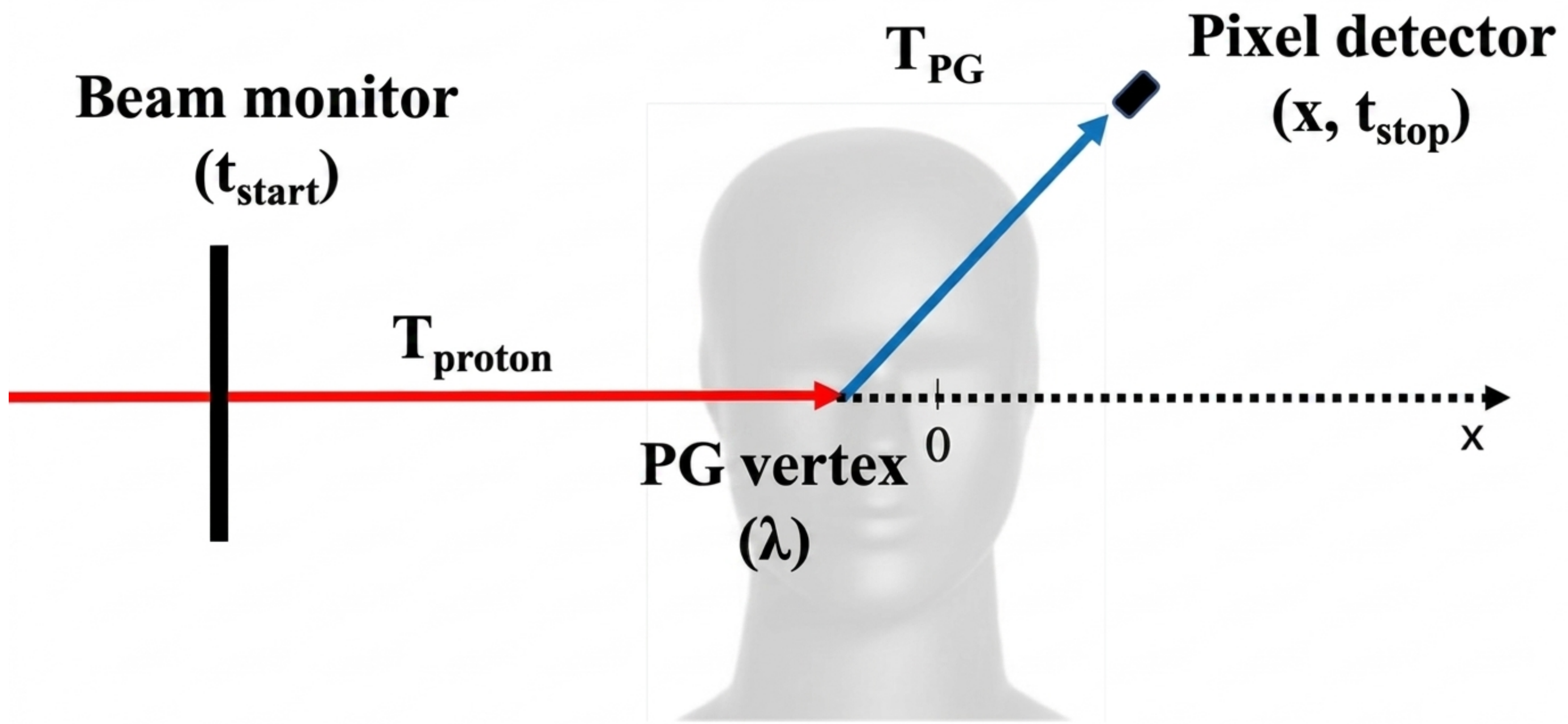
An alternating minimization approach ?

Requires Bi-Convexity!

$$\min_{\lambda,\gamma} J(\lambda,\gamma) = \frac{1}{2\sigma^2} \parallel t - T(\lambda,\gamma) \parallel^2$$

$$t = t_{stop} - t_{start}$$



$$\begin{cases} T(\lambda,\gamma) = T_{proton}(\lambda,\gamma) + T_{PG}(\lambda) \\ \qquad = A(\lambda)\gamma + \frac{\|\lambda - x\|}{c} \end{cases}$$

$$t = [t_1, .., t_n] \in \mathbb{R}^n$$

$$\lambda = [\lambda_1, .., \lambda_n] \in \mathbb{R}^n$$

$$\gamma = [\gamma_1, .., \gamma_d] \in \mathbb{R}^d$$

An alternating minimization approach ?

Requires Bi-Convexity!

$$\min_{\lambda,\gamma} J(\lambda,\gamma) = \frac{1}{2\sigma^2} \| \boldsymbol{t} - \boldsymbol{T}(\lambda,\gamma) \|^2$$

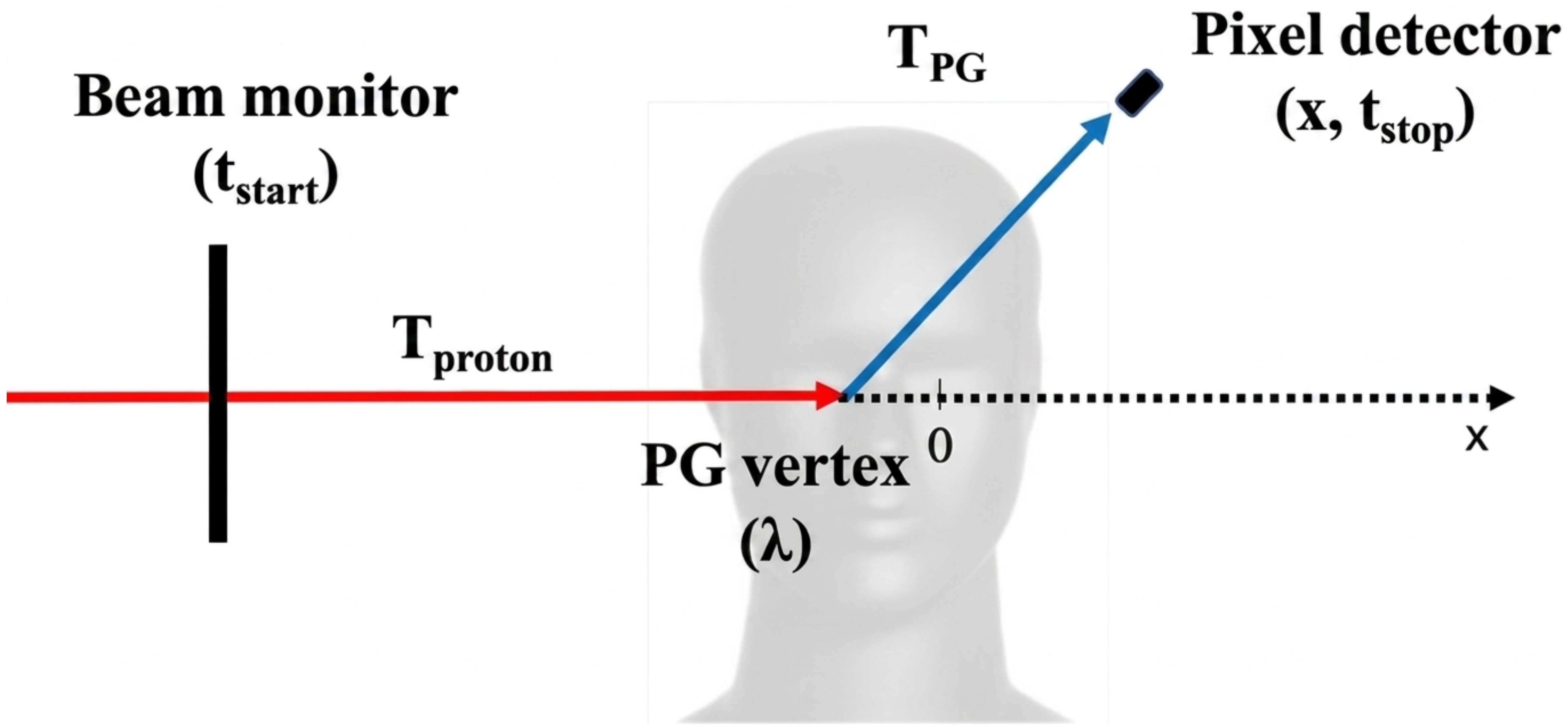
$$\boldsymbol{t} = \boldsymbol{t}_{stop} - \boldsymbol{t}_{start}$$

Step 1: Fix γ

$$\min_{\lambda} J(\lambda, .\,) = \frac{1}{2\sigma^2} \| \boldsymbol{t} - \boldsymbol{T}(\lambda, .\,) \|^2$$

Step 2: Fix λ

$$\min_{\gamma} J(\,.\,,\gamma) = \frac{1}{2\sigma^2} \| \boldsymbol{t} - \boldsymbol{T}(\,.\,,\gamma) \|^2$$



$$\begin{cases} \boldsymbol{T}(\lambda,\gamma) = \boldsymbol{T}_{proton}(\lambda,\gamma) + \boldsymbol{T}_{PG}(\lambda) \\ \qquad\qquad = A(\lambda)\gamma + \frac{\|\lambda - \boldsymbol{x}\|}{c} \end{cases}$$

$$\boldsymbol{t} = [t_1, \ldots, t_n] \in \mathbb{R}^n$$

$$\lambda = [\lambda_1, \ldots, \lambda_n] \in \mathbb{R}^n$$

$$\gamma = [\gamma_1, \ldots, \gamma_d] \in \mathbb{R}^d$$

An alternating minimization approach ?

Requires Bi-Convexity!

$$\min_{\lambda,\gamma} J(\lambda,\gamma) = \frac{1}{2\sigma^2} \| t - T(\lambda,\gamma) \|^2$$

- **Assume that the proton mean velocity is known**

$$\min_{\lambda\in\mathbb{R}^n} J_1(\lambda,\gamma) = \frac{1}{2\sigma^2} \| \boldsymbol{t} - \boldsymbol{T}(\lambda,\gamma) \|^2$$

Proposition: $\lambda \to J_1(\lambda, .)$ is quasi-convex on $[\lambda_{min}, \lambda_{max}]$

- $J_1'(\lambda) = -\dfrac{1}{\sigma^2} T'(\lambda)(t - T(\lambda))$
- $J_1''(\lambda) = \dfrac{1}{\sigma^2}(T'(\lambda)^2 - T''(\lambda))(t - T(\lambda))$

J_1 has global minimum at $\lambda^* = T^{-1}(t)$

$J_1 \searrow on\ [\lambda_{min}, \lambda^*] \quad J_1 \nearrow on\ [\lambda^*, \lambda_{max}]$

An alternating minimization approach ?

Requires Bi-Convexity!

$$\min_{\lambda,\gamma} J(\lambda,\gamma) = \frac{1}{2\sigma^2} \| t - T(\lambda,\gamma) \|^2$$

- **Assume that the proton mean velocity is known**

$$\min_{\lambda \in \mathbb{R}^n} J_1(\lambda,\gamma) = \frac{1}{2\sigma^2} \| t - T(\lambda,\gamma) \|^2$$

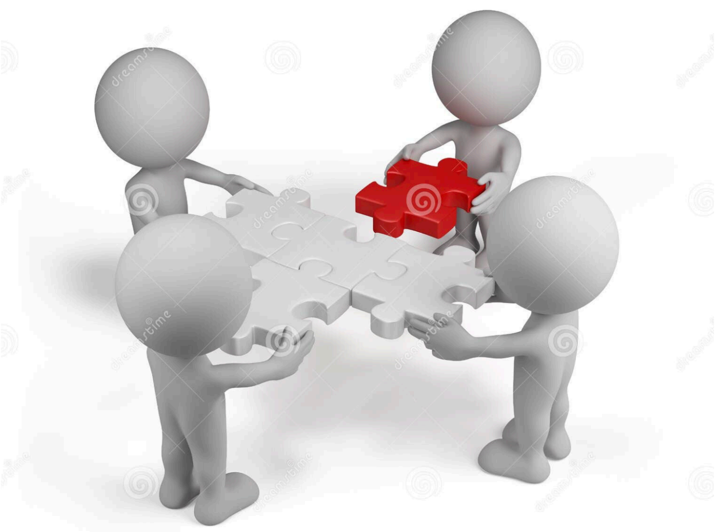
Proposition: $\lambda \to J_1(\lambda, .)$ is quasi-convex on $[\lambda_{min}, \lambda_{max}]$

- **Assume that the distribution of prompt gammas is known**

$$\min_{\gamma \in \mathbb{R}^d} J_2(\lambda,\gamma) = \frac{1}{2\sigma^2} \| b - A\gamma \|^2$$

Proposition: $\gamma \to J_2(. , \gamma)$ is convex $\forall \gamma$

- $J_2'(\gamma) = A^t(b - A\gamma)$
- $J_2''(\gamma) = A^t A$
- $J_2''(\gamma) \geq 0$



An alternating minimization approach ?

- Alternating algorithm

$$
\begin{cases}
\text{Initialisation} \\
\boldsymbol{\lambda_0}, \boldsymbol{\gamma_0} \\
\text{For } i = 0,\dots,N_{max} \\
\begin{cases}
\boldsymbol{\lambda}_{i+1} = \underset{\boldsymbol{\lambda} \in \mathbb{R}^n}{\arg\min}\, J_1(\boldsymbol{\lambda}_i, \boldsymbol{\gamma}_i) \\
\boldsymbol{\gamma}_{i+1} = \underset{\boldsymbol{\gamma} \in \mathbb{R}^d}{\arg\min}\, \tilde{J}_2(\boldsymbol{\lambda}_{i+1}, \boldsymbol{\gamma}_i)
\end{cases} \\
\text{Update} \\
\boldsymbol{\gamma}_i = \boldsymbol{\gamma}_{i+1} \\
\boldsymbol{\lambda}_i = \boldsymbol{\lambda}_{i+1}
\end{cases}
$$

$$J_1(\boldsymbol{\lambda}, \boldsymbol{\gamma}) = \frac{1}{2\sigma^2} \parallel \boldsymbol{t} - \boldsymbol{T}(\boldsymbol{\lambda}, \boldsymbol{\gamma}) \parallel^2$$

$$\tilde{J}_2(\boldsymbol{\lambda}, \boldsymbol{\gamma}) = \frac{1}{2\sigma^2} \parallel A^t\boldsymbol{b} - A^tA\boldsymbol{\gamma} \parallel^2 + \underbrace{\chi \, \|I_{adp}\boldsymbol{\gamma}\|^2}_{\text{Regularization term}}$$

- $I_{adp} = diag(A^tA)$
- Accounts for local sensitivity
- Improves stability of the inverse problem

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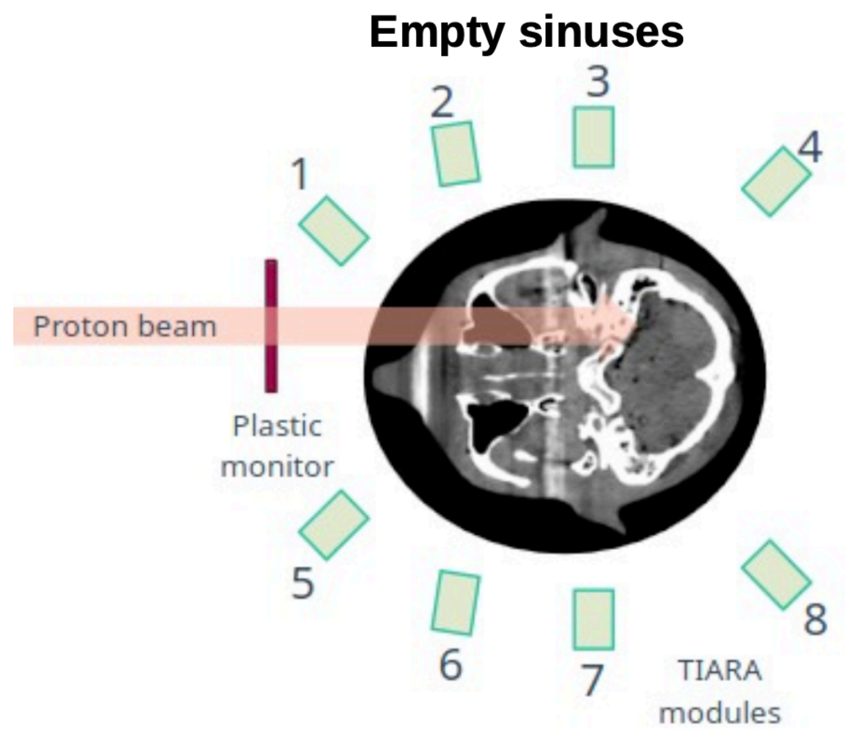
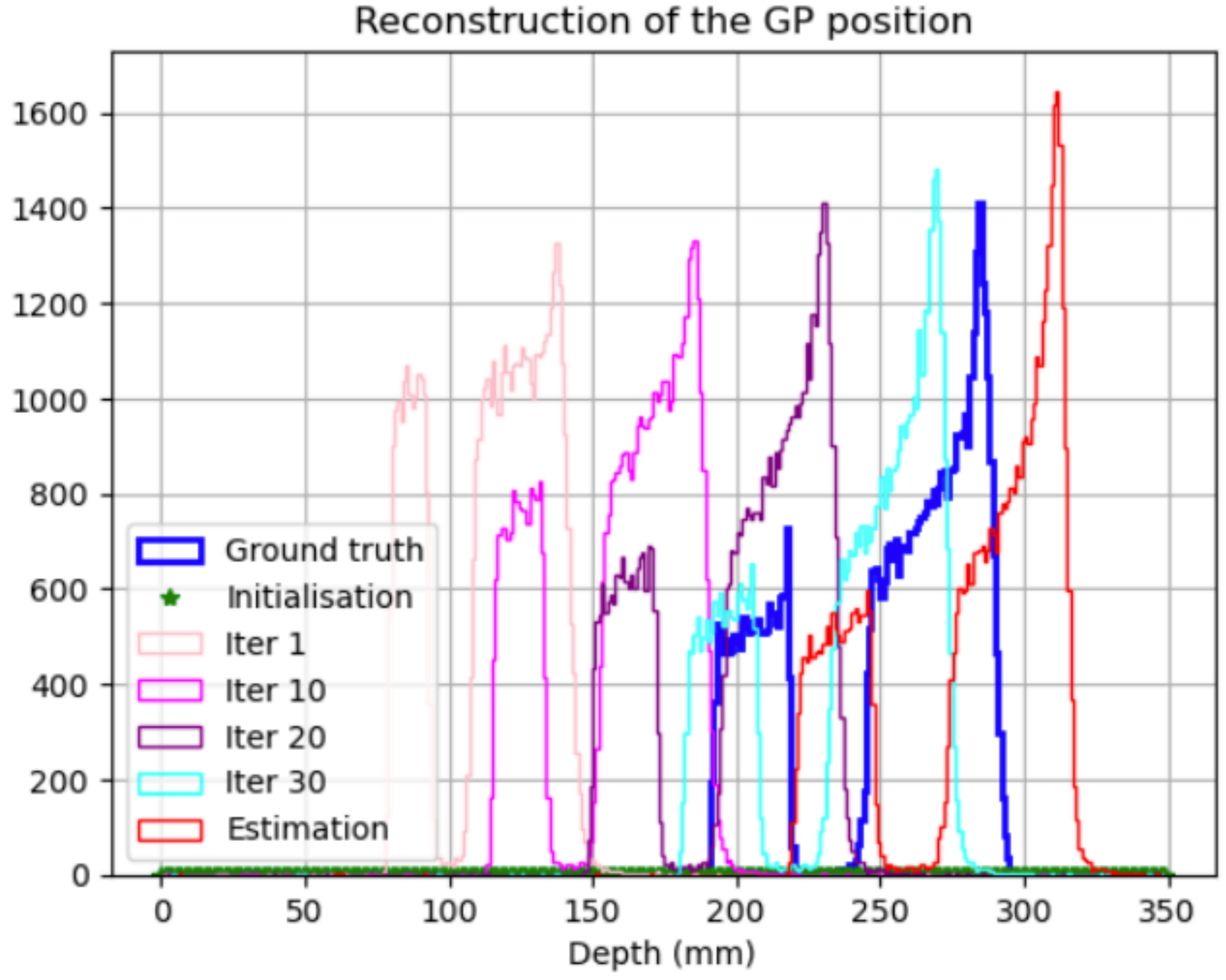
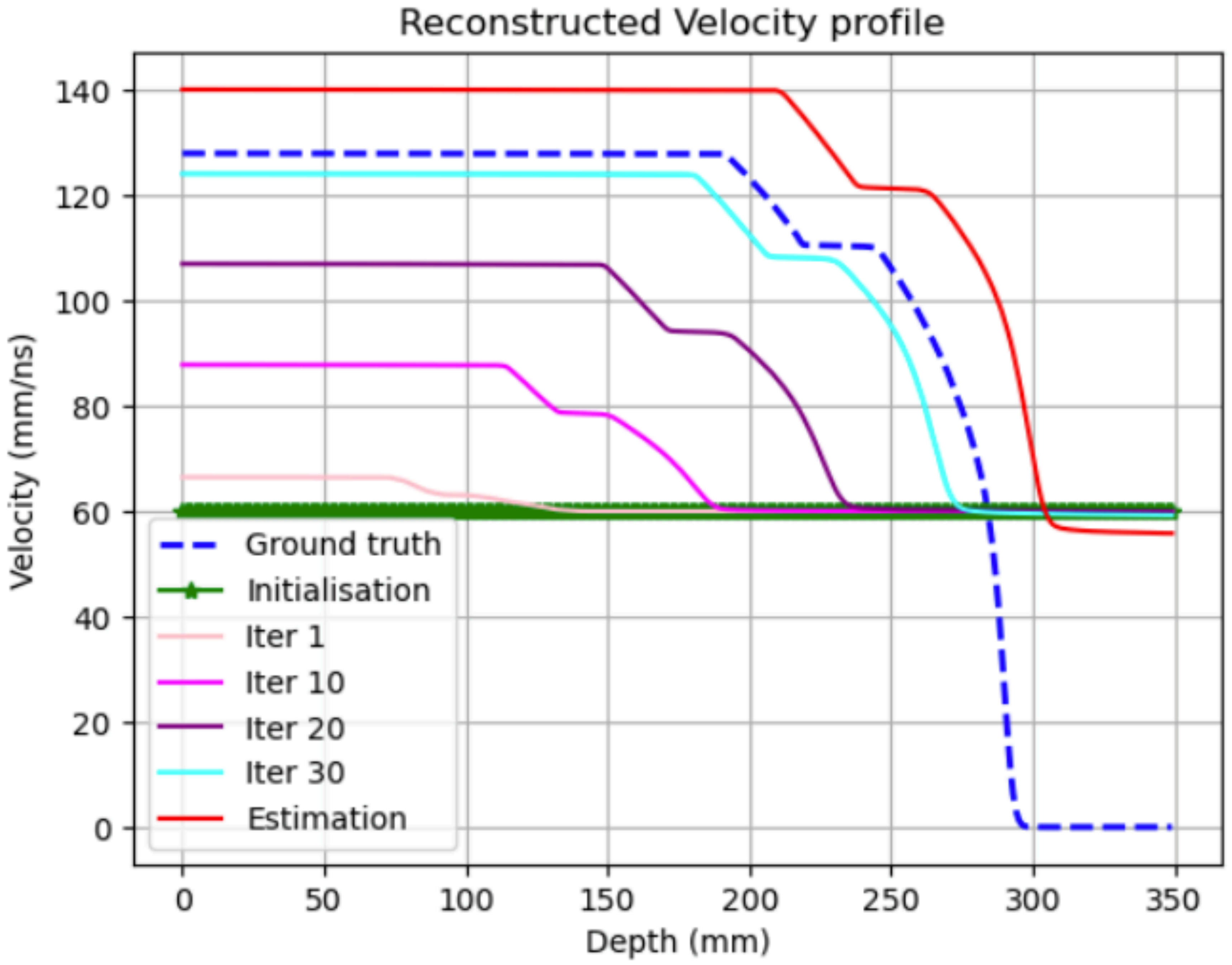
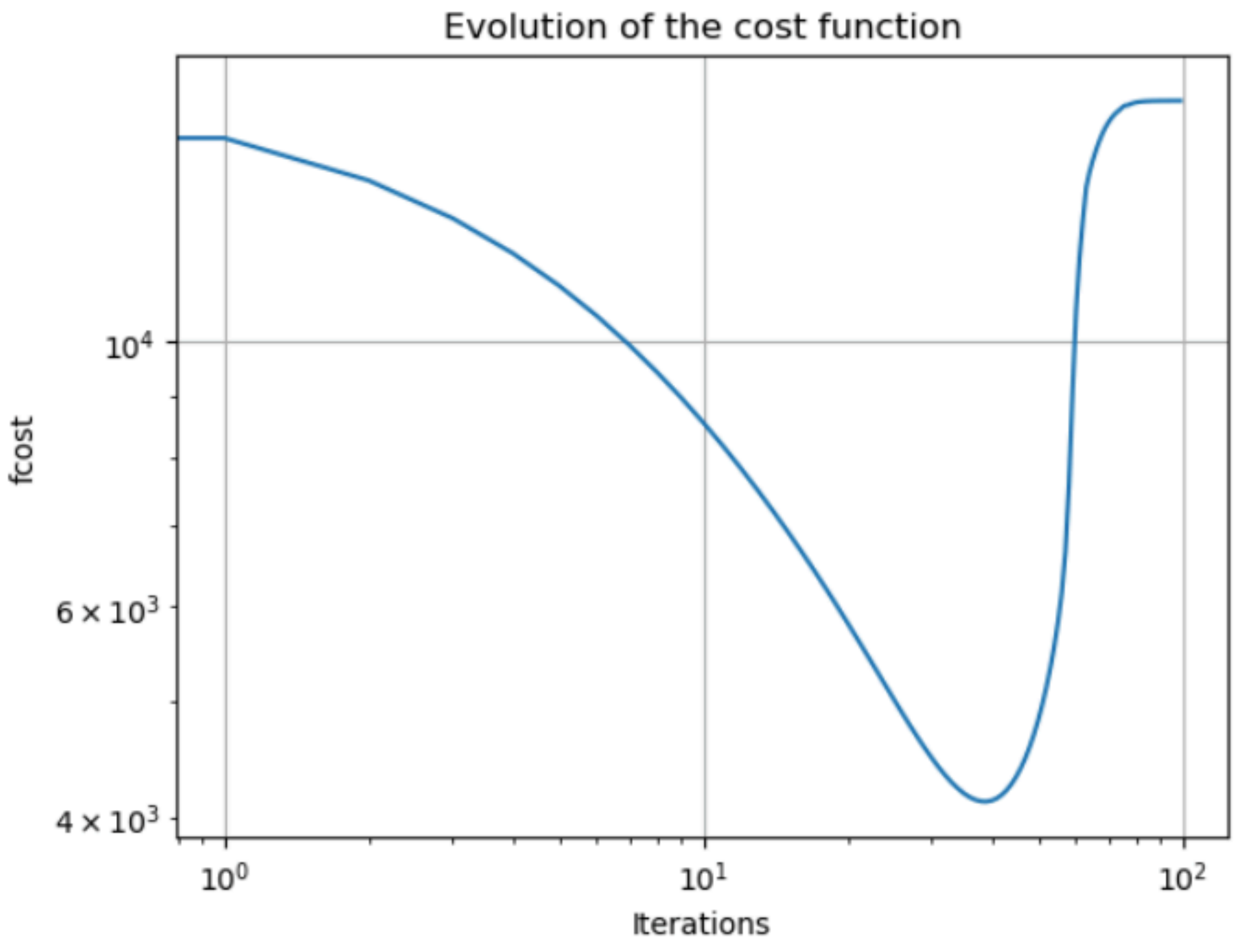
Estimating Both Unknowns via an Alternating Algorithm

Empty sinus case



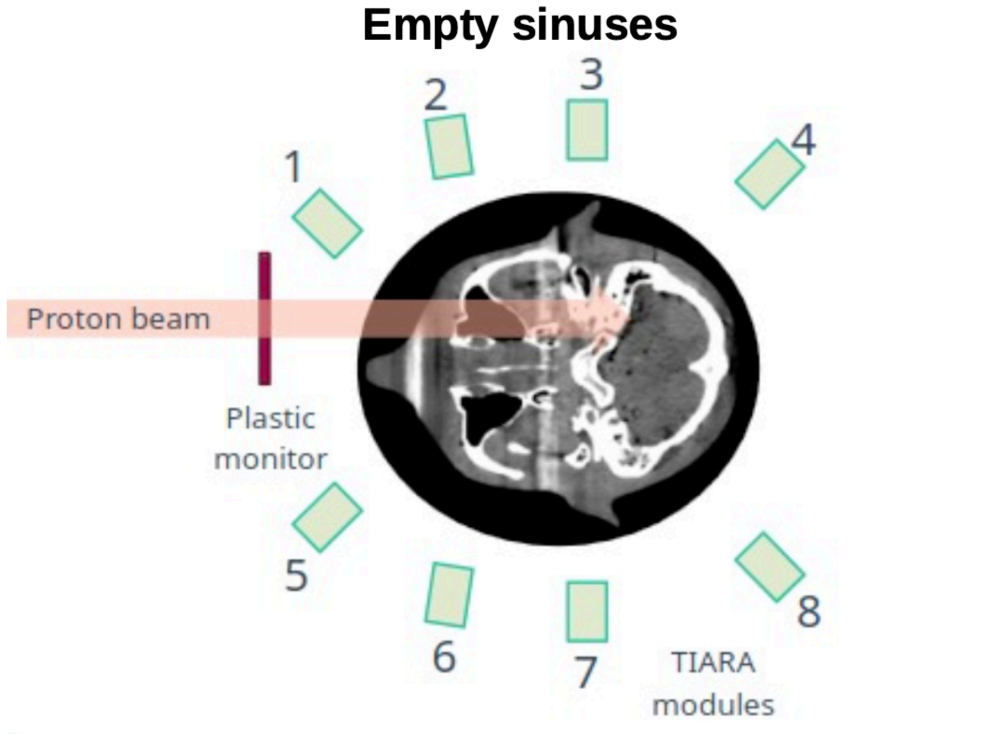


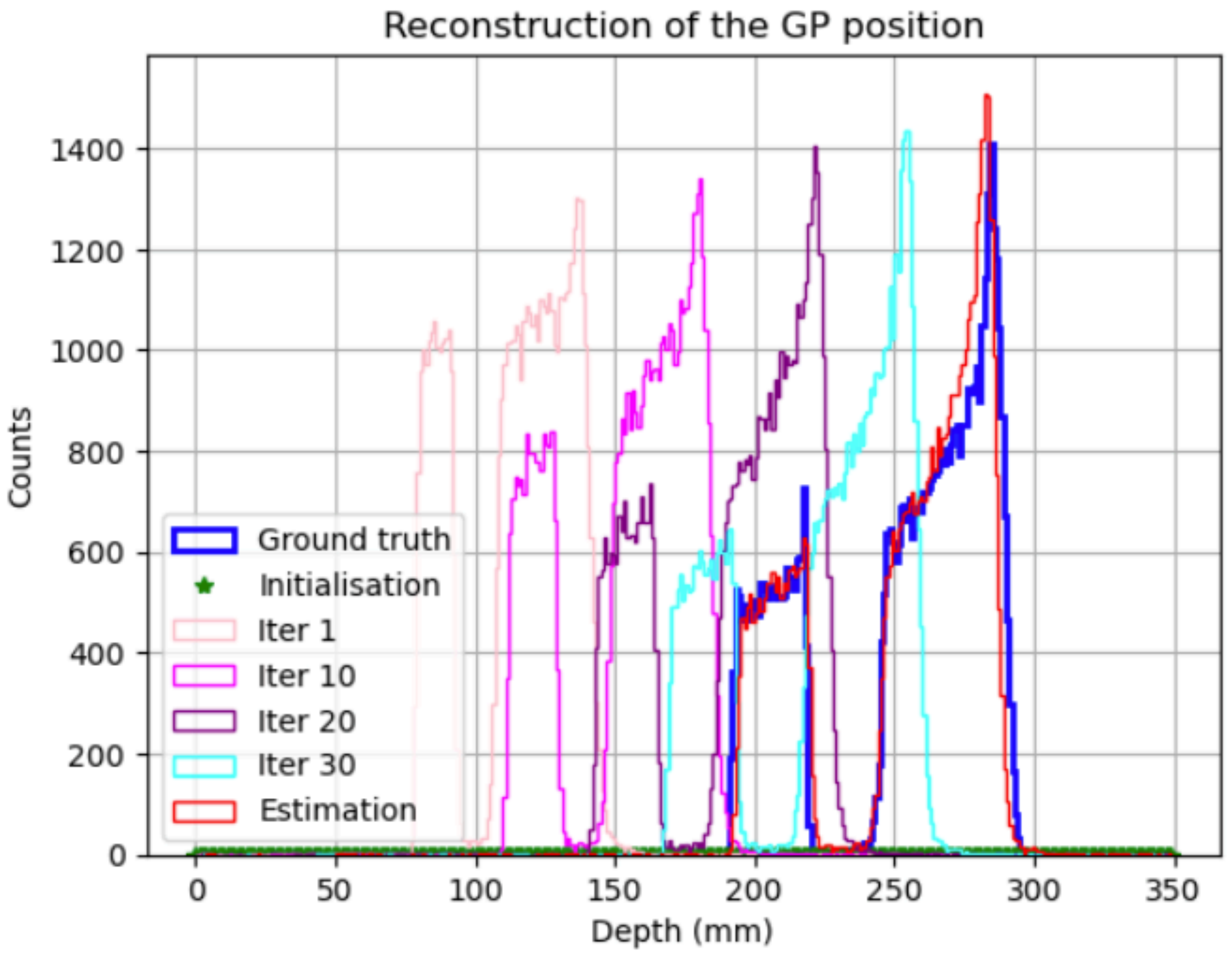


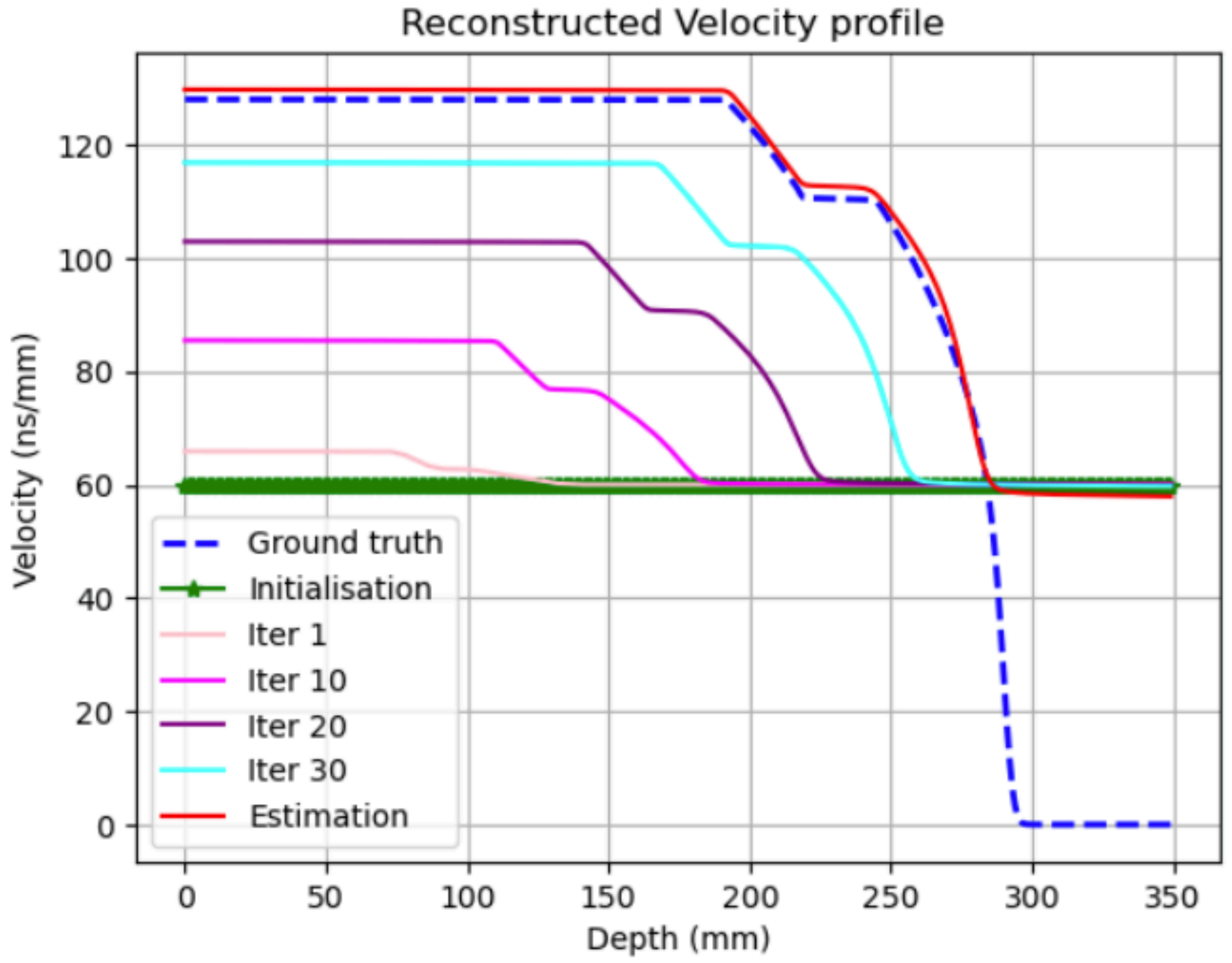


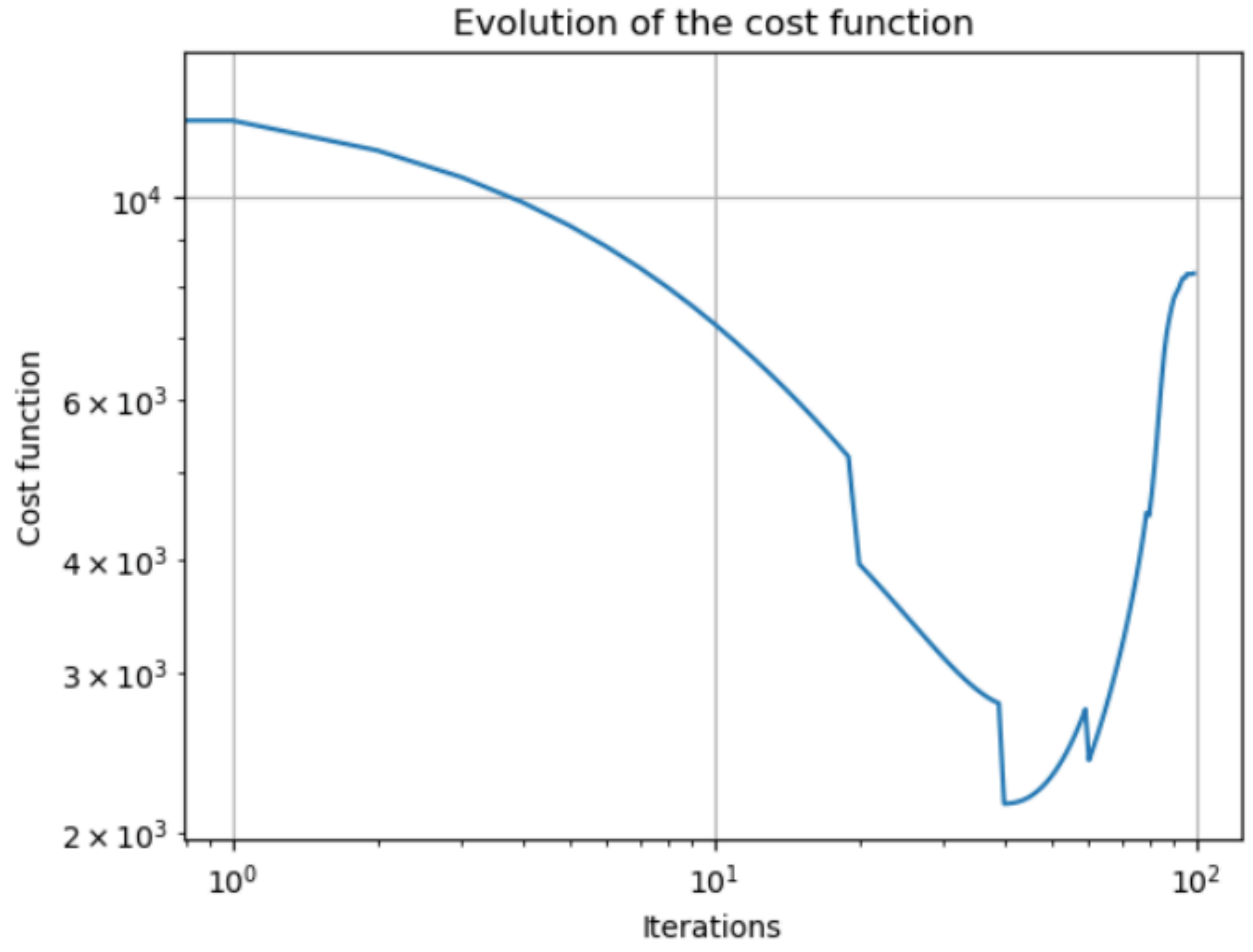
Estimating Both Unknowns via an Alternating Algorithm - Empty sinus

Decaying strategy





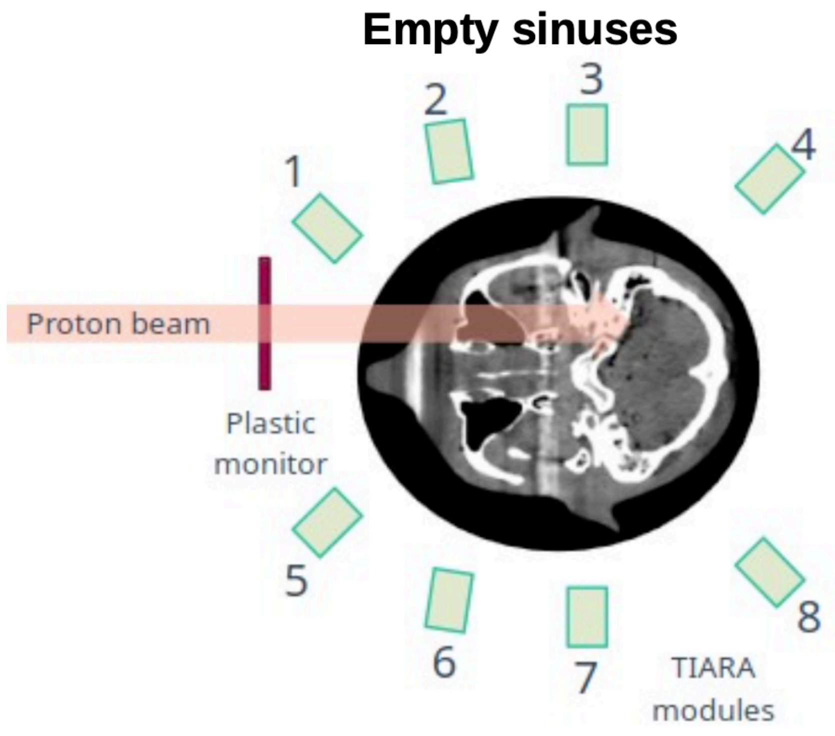


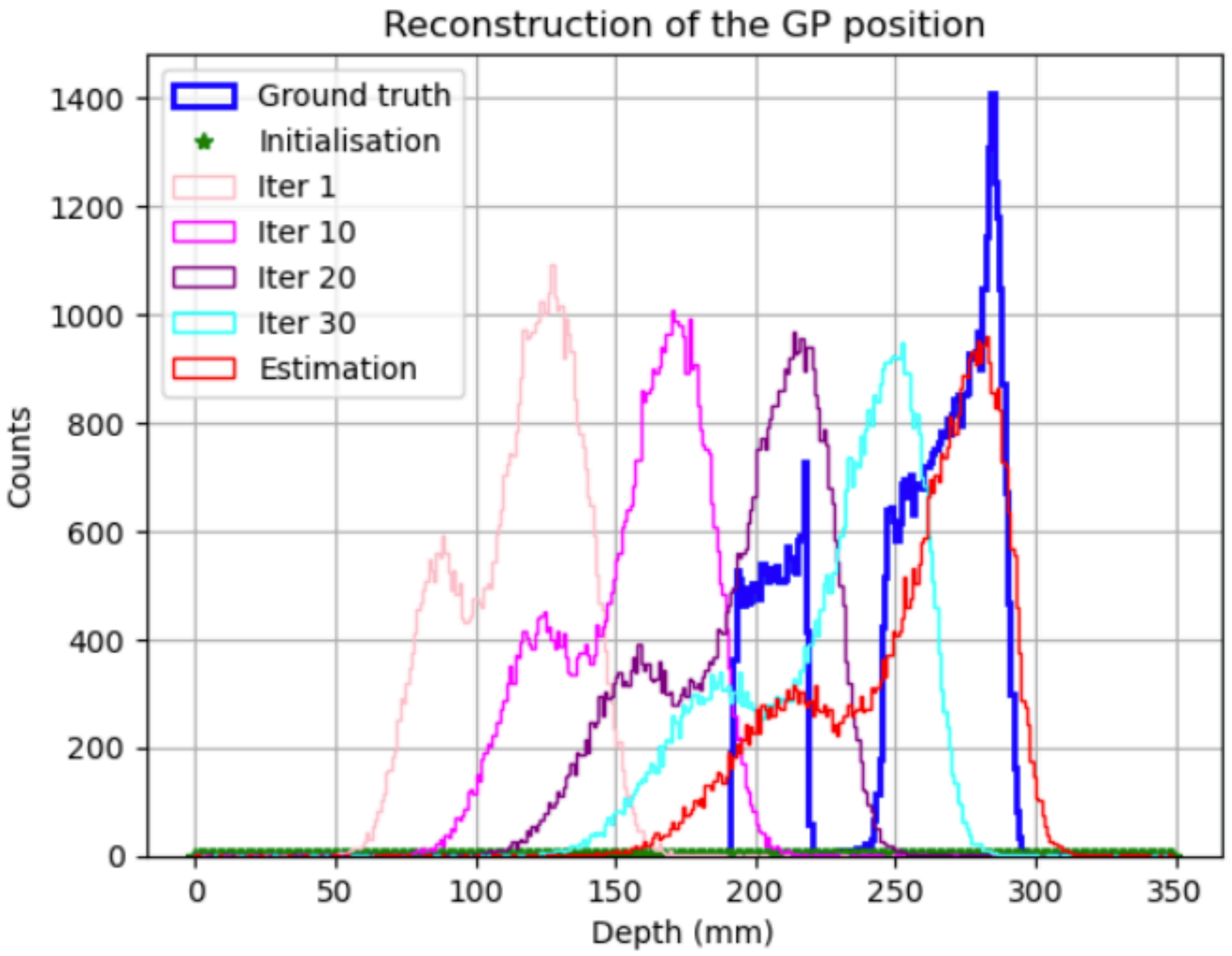


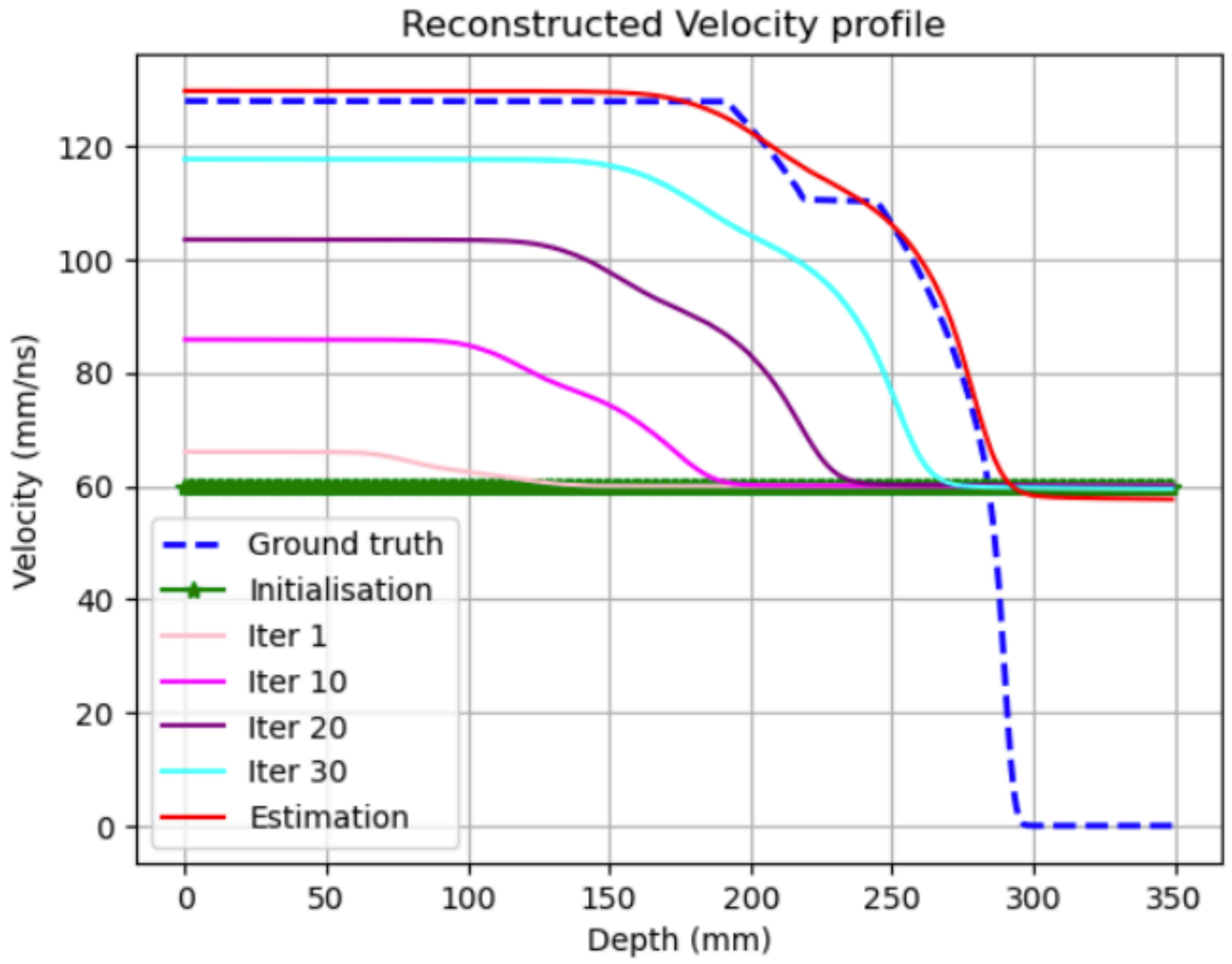
Estimating Both Unknowns via an Alternating Algorithm - Empty sinus

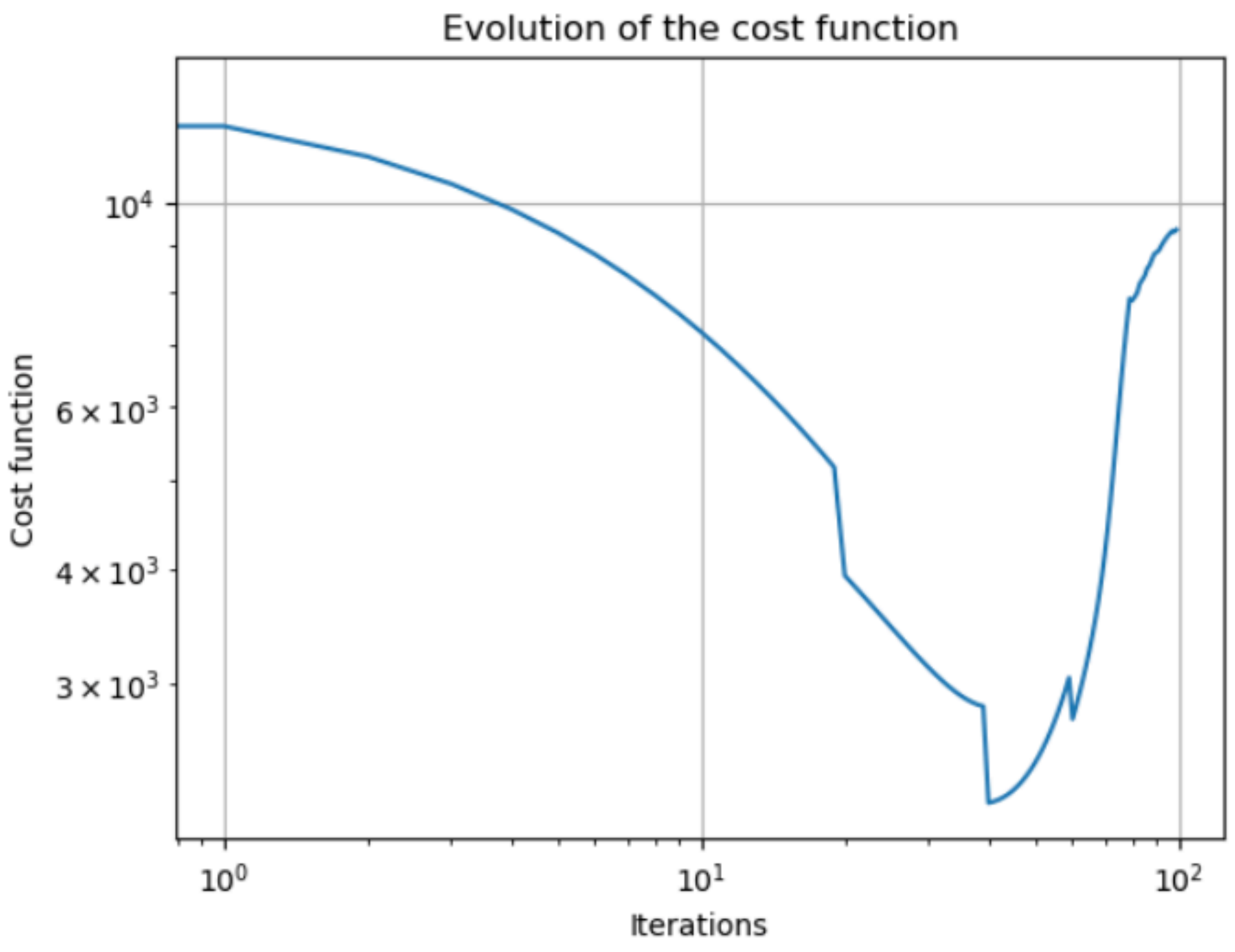
Noisy Data

Noise: $\sigma = 100$ ps



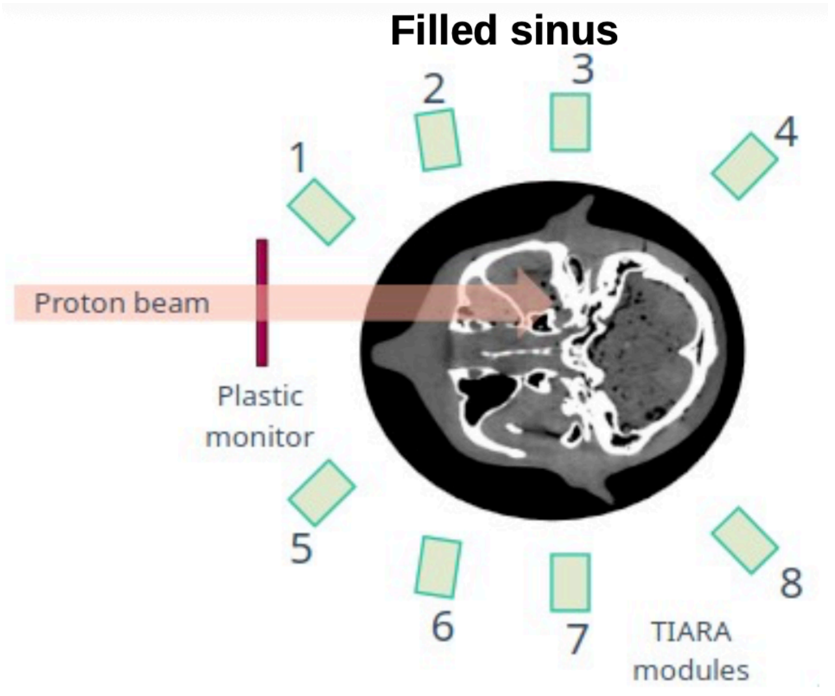


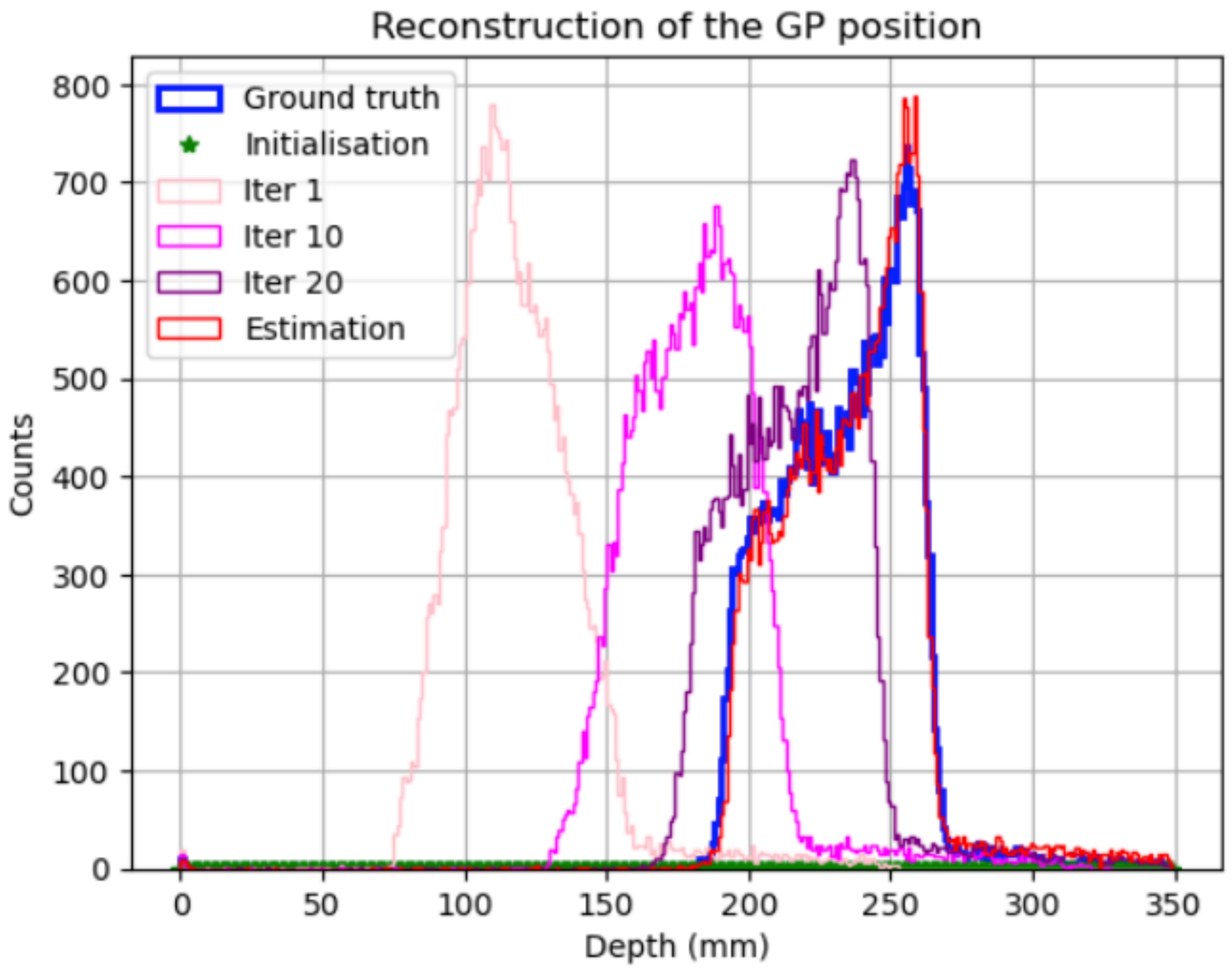


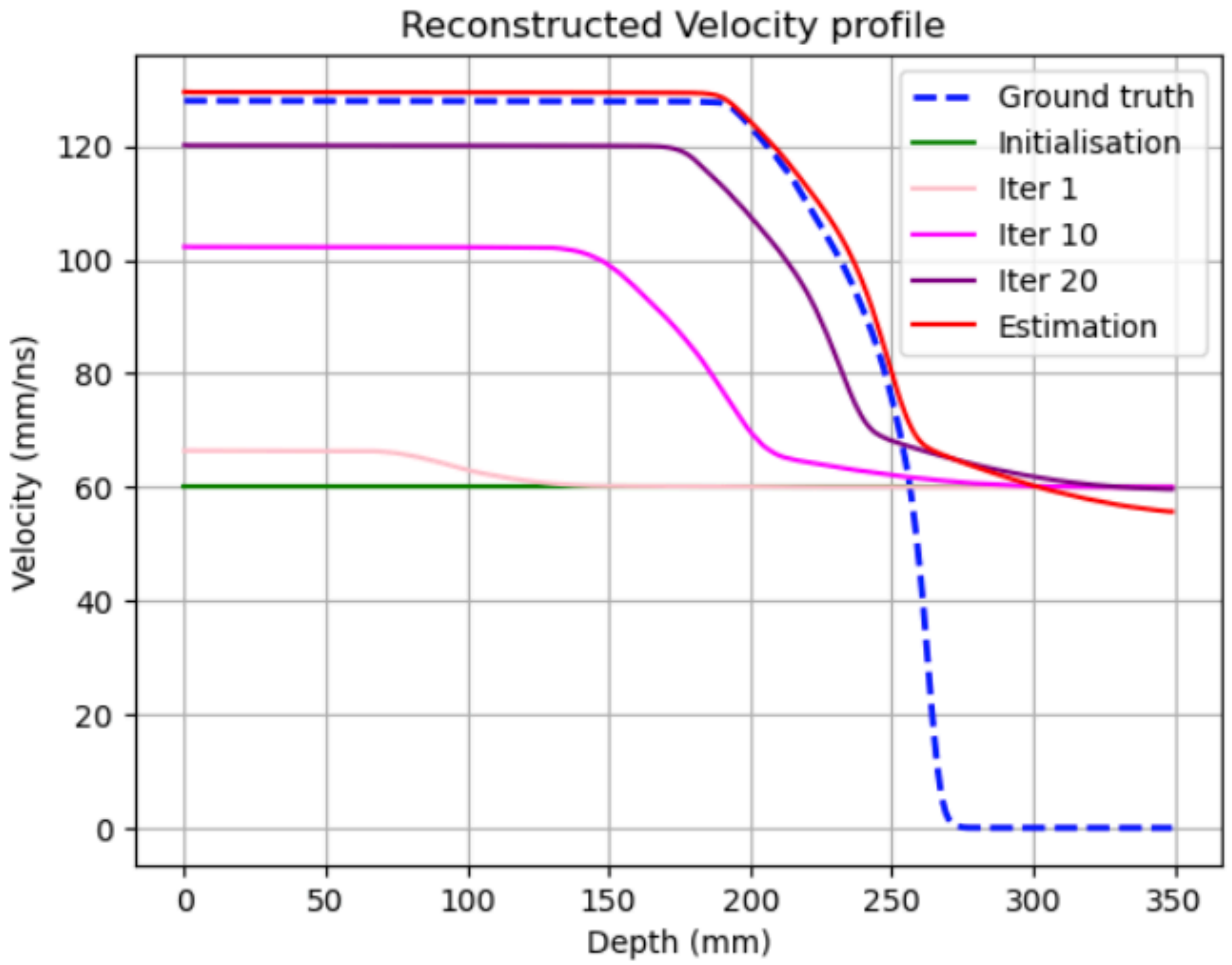


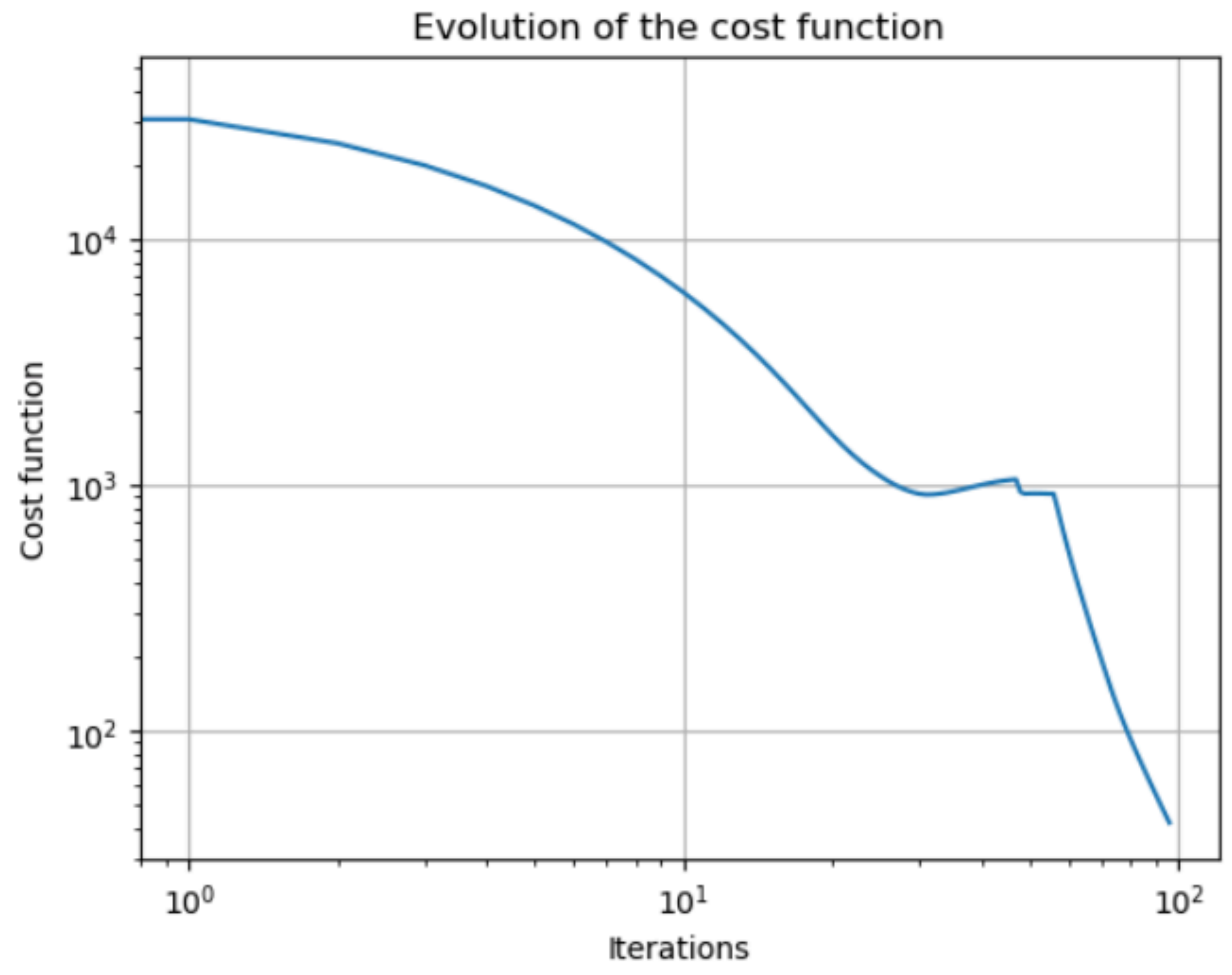
Estimating Both Unknowns via an Alternating Algorithm

Full sinus case



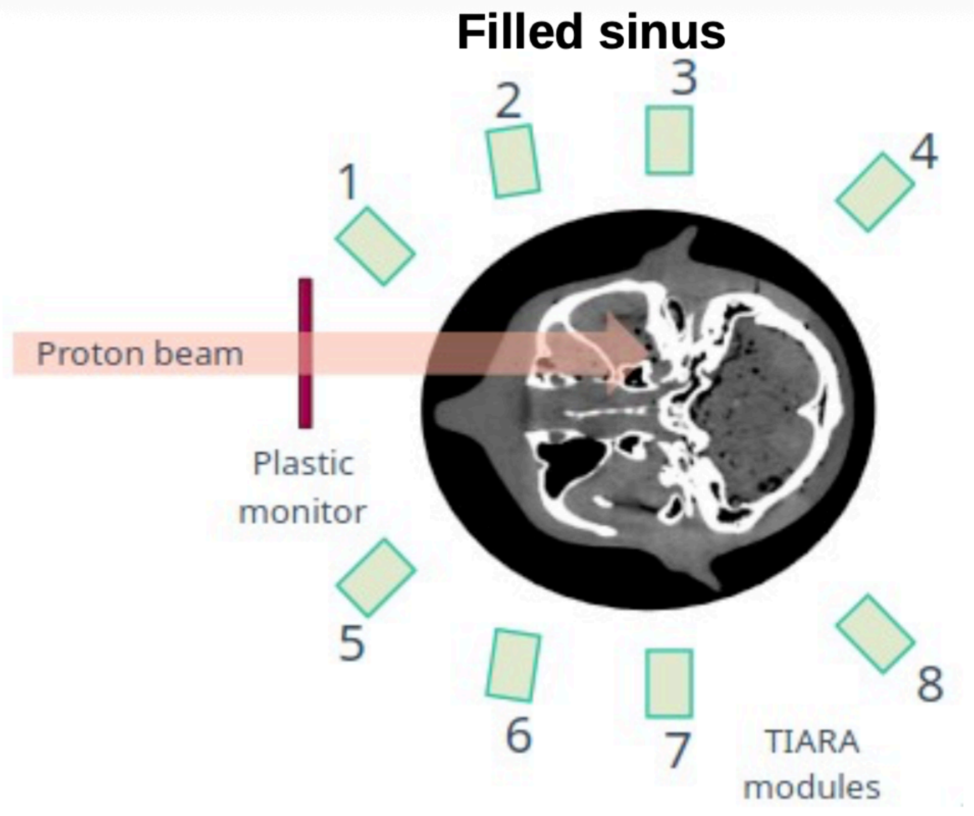


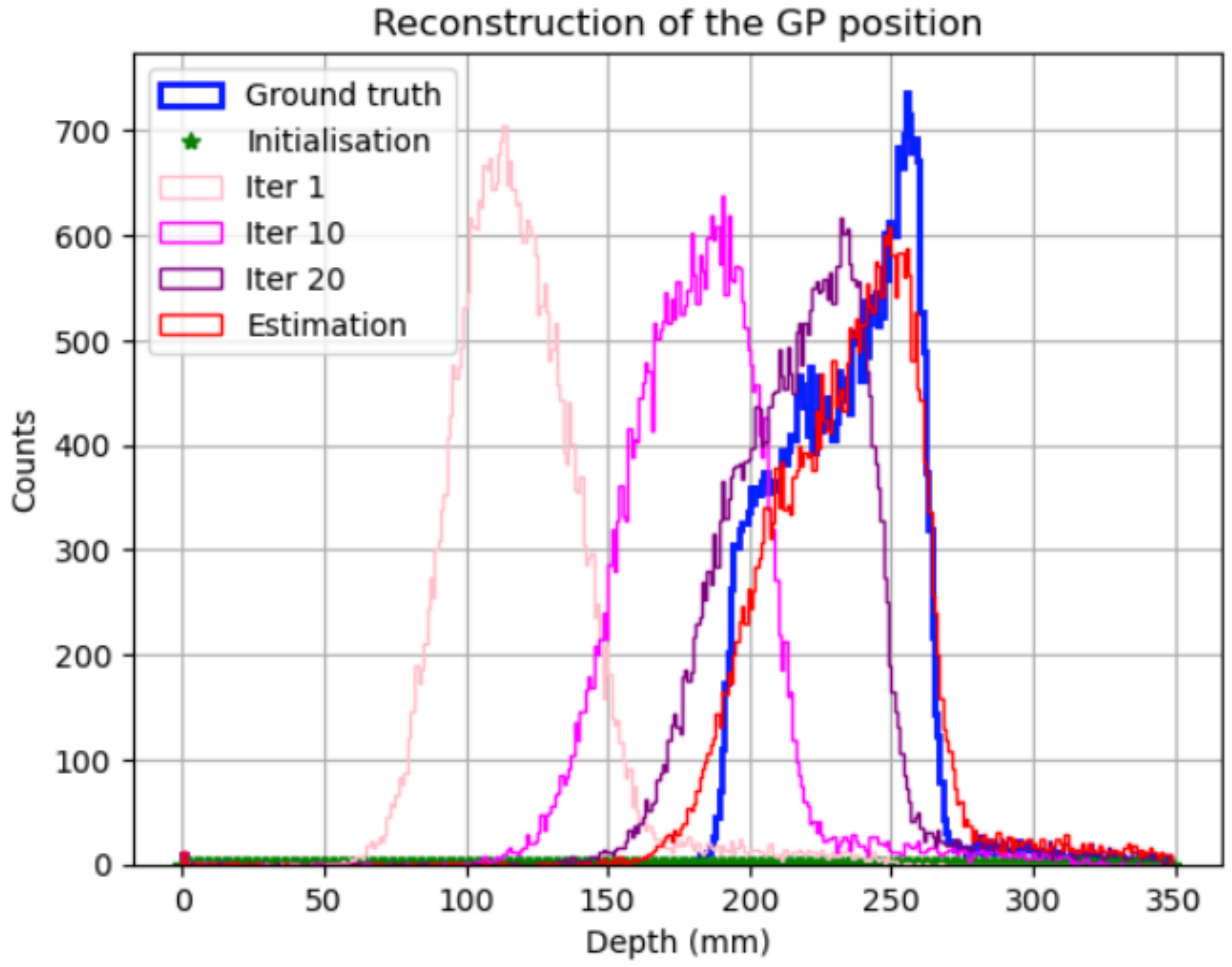


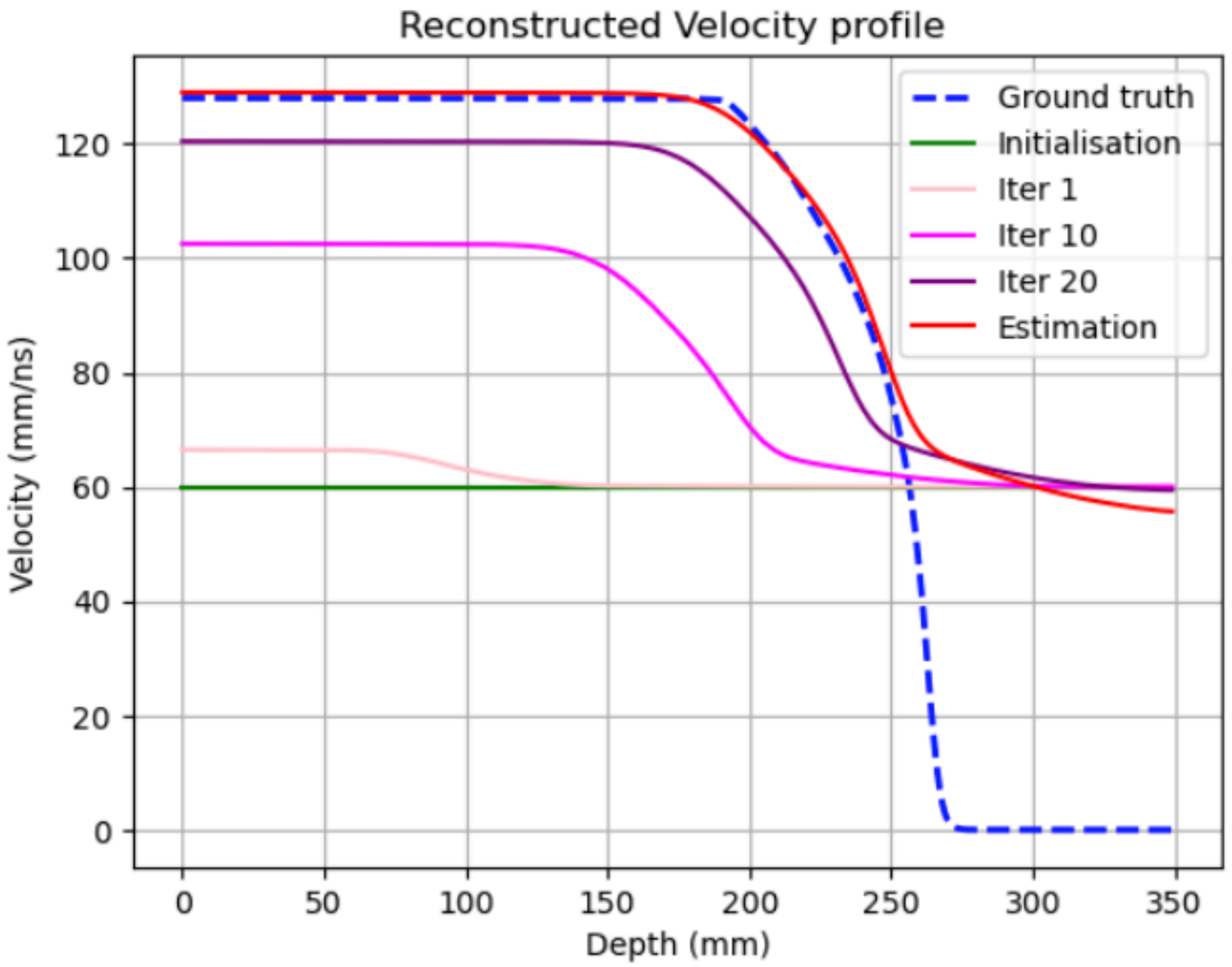


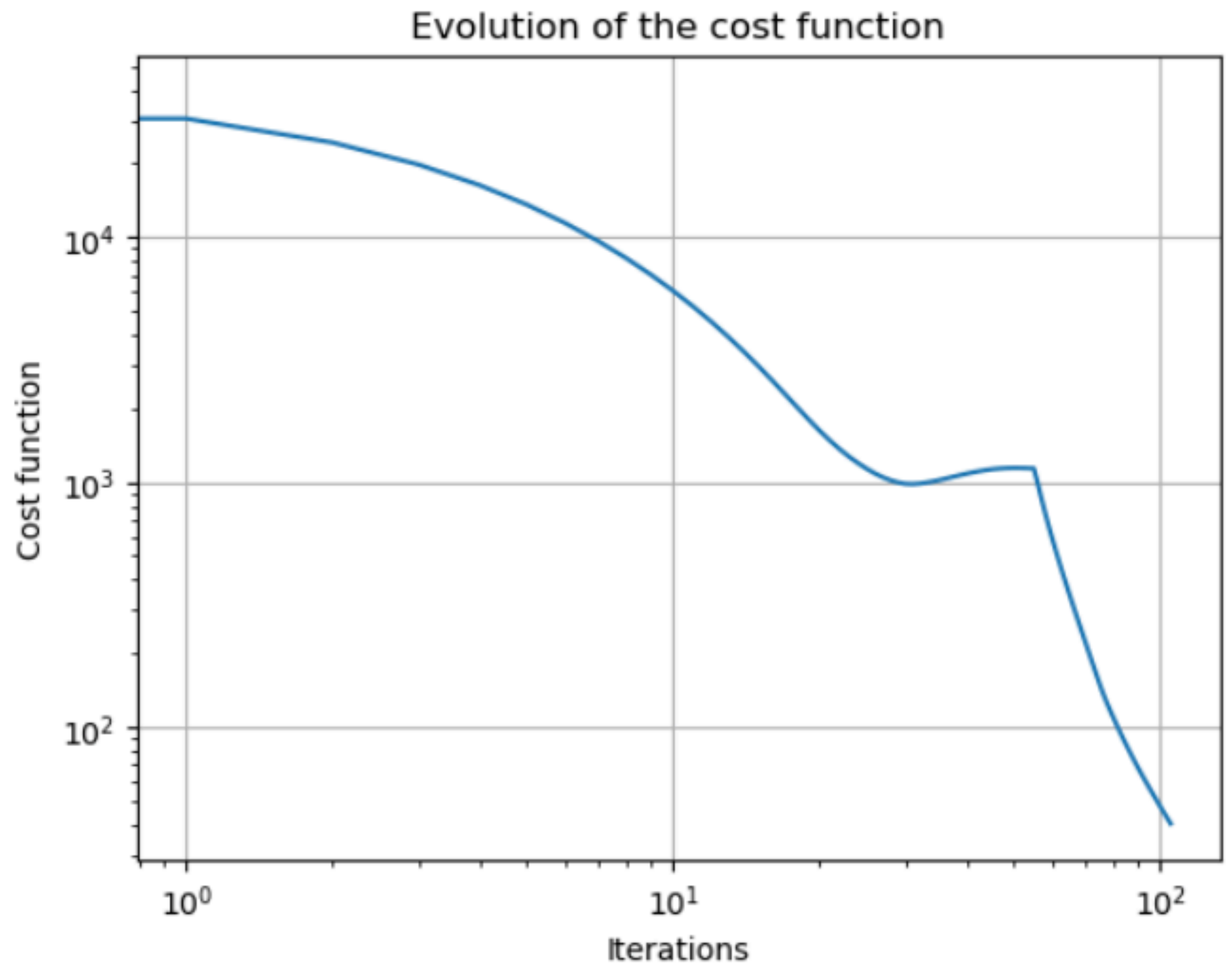
Estimating Both Unknowns via an Alternating Algorithm - Full sinus

Noisy Data



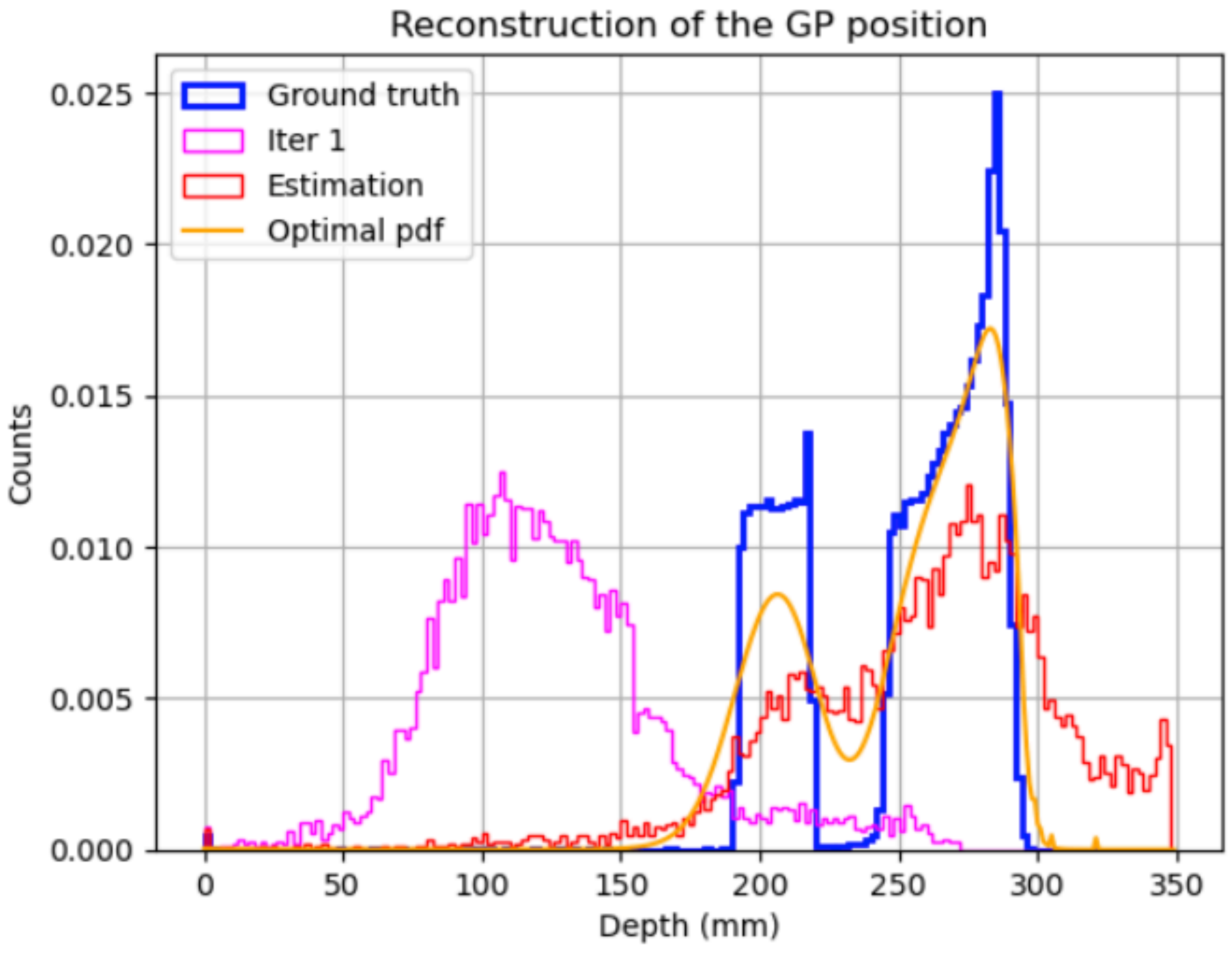


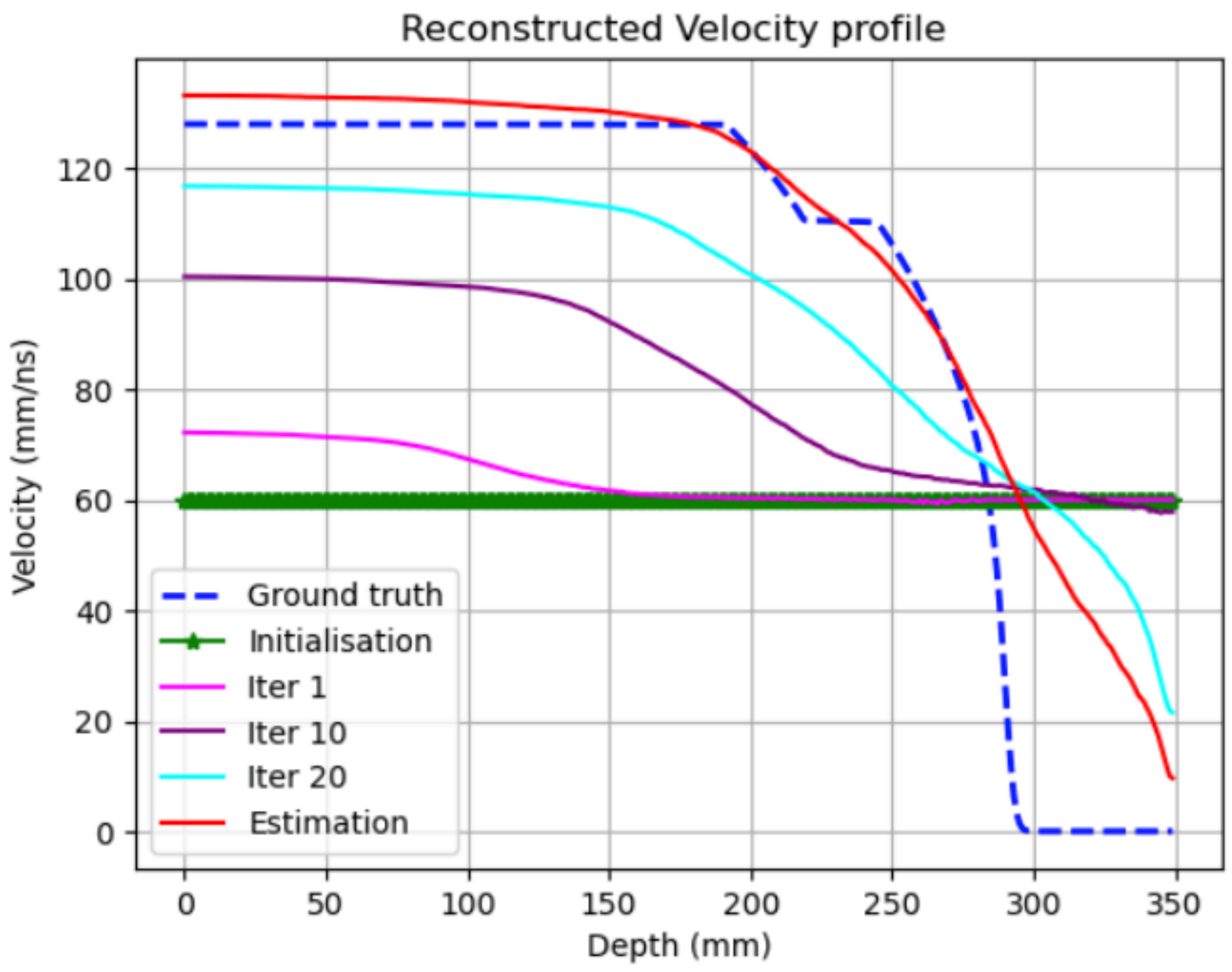


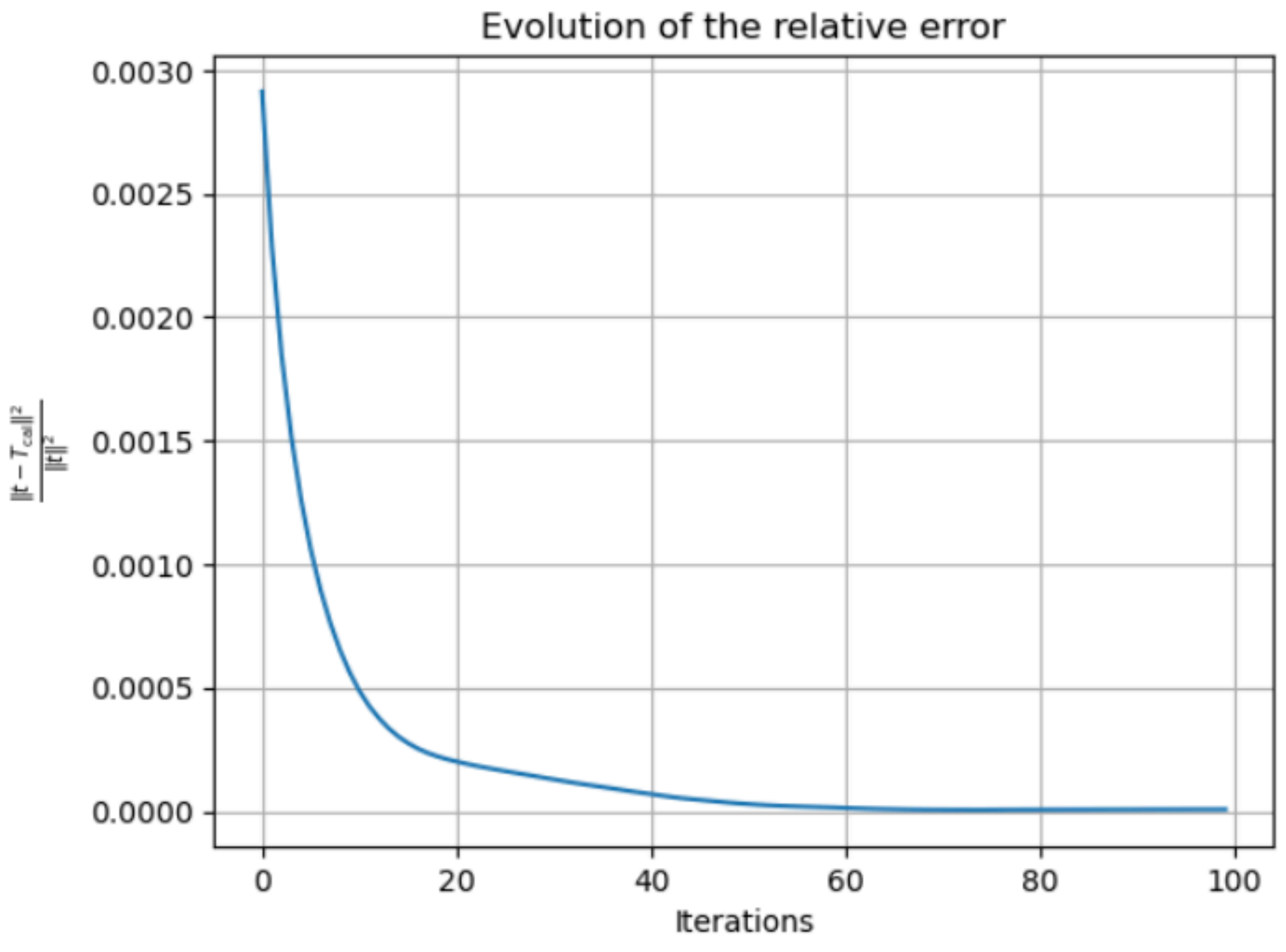


Preliminary Results

Estimating Both Unknowns via an Alternating Algorithm
(Experimental Data- ES case)

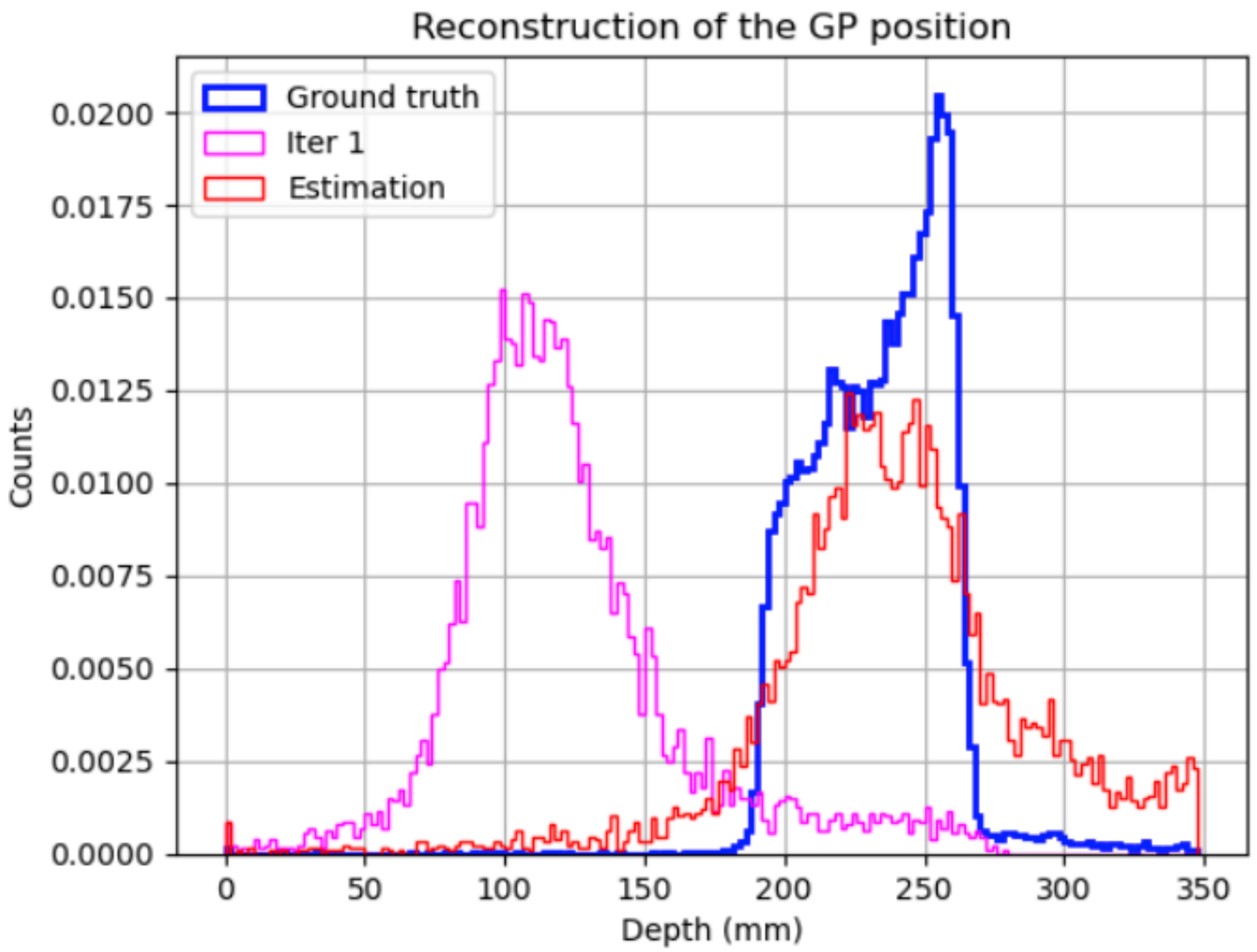


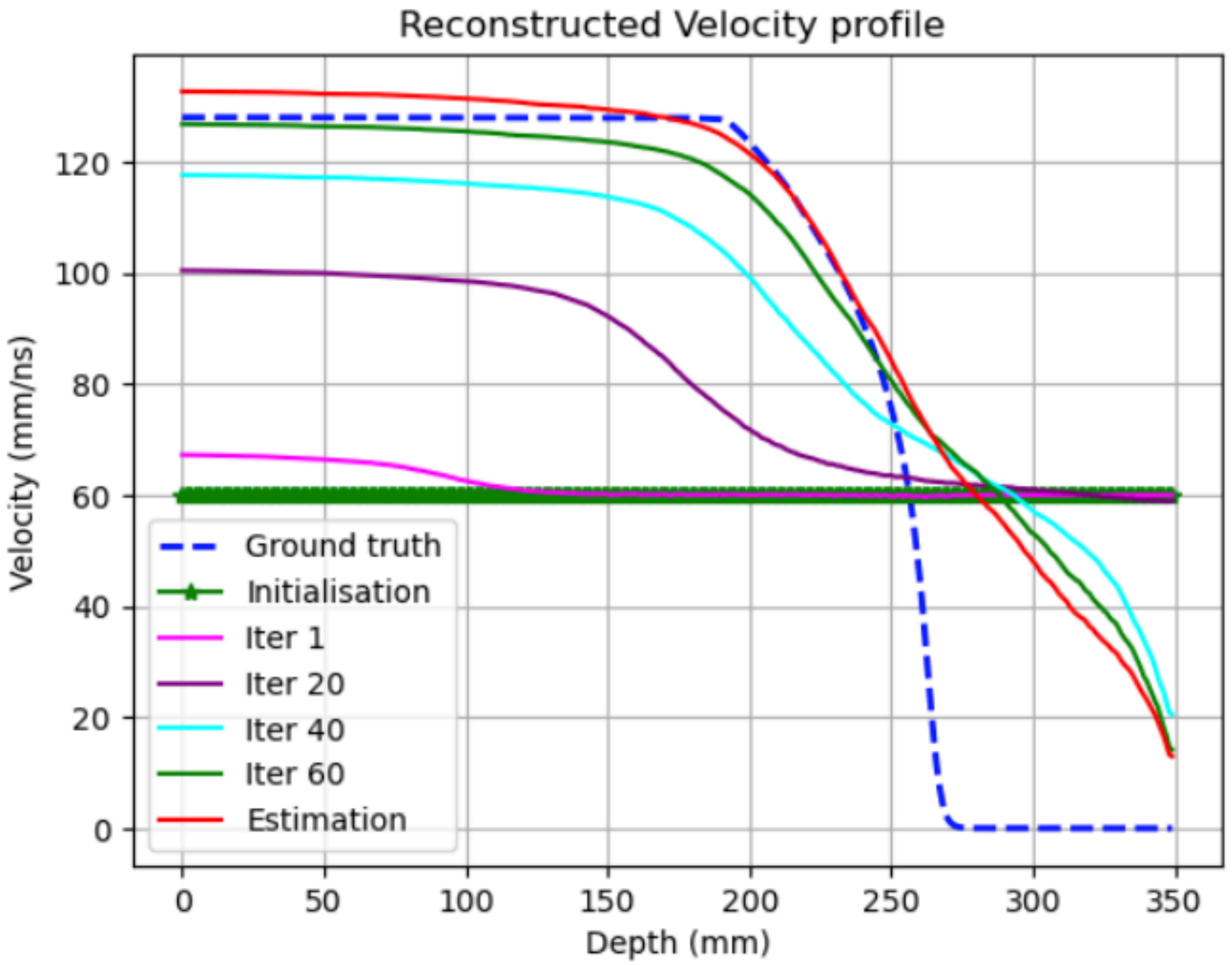


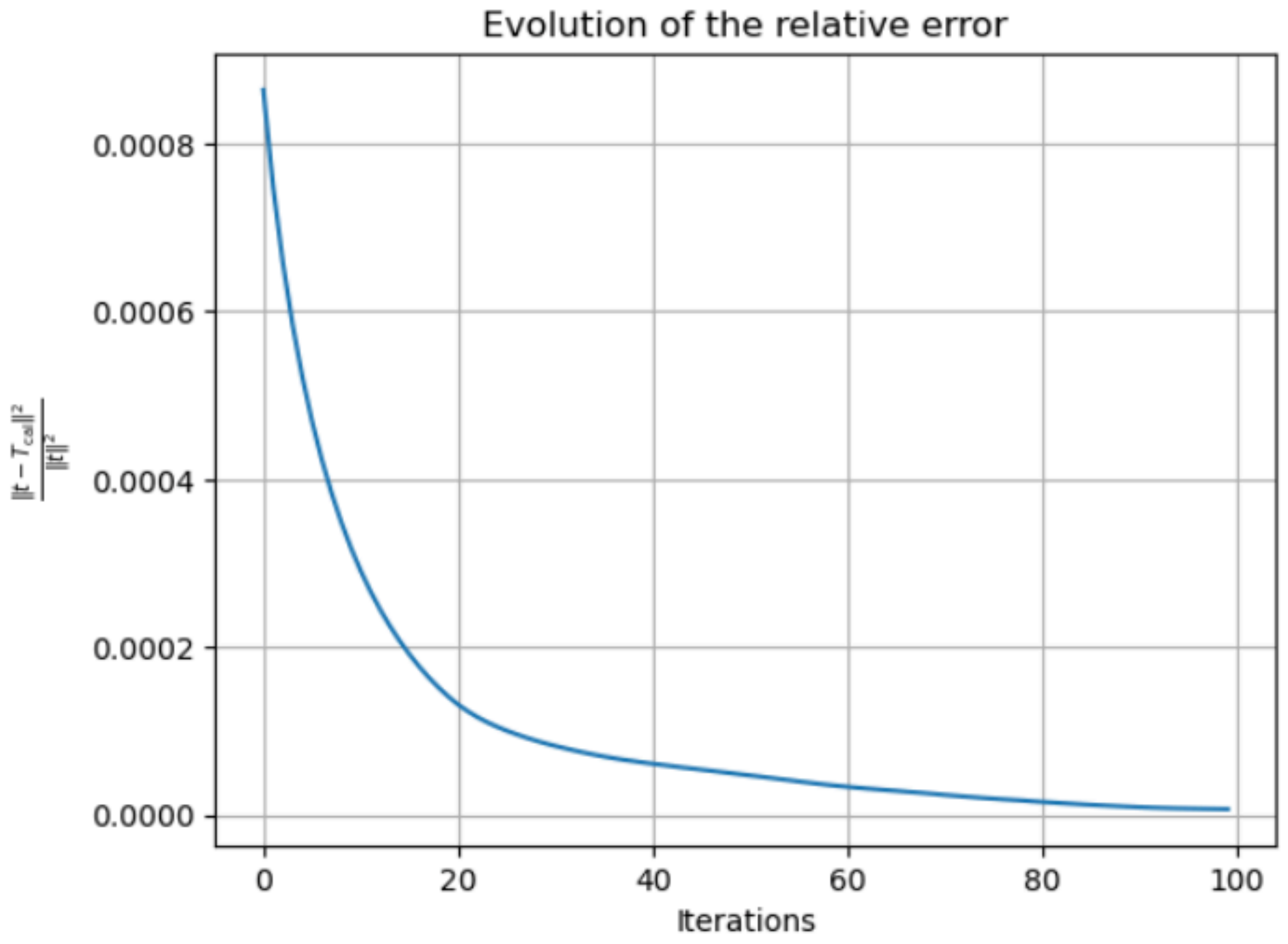


Estimating Both Unknowns via an Alternating Algorithm

(Experimental Data- FS case)







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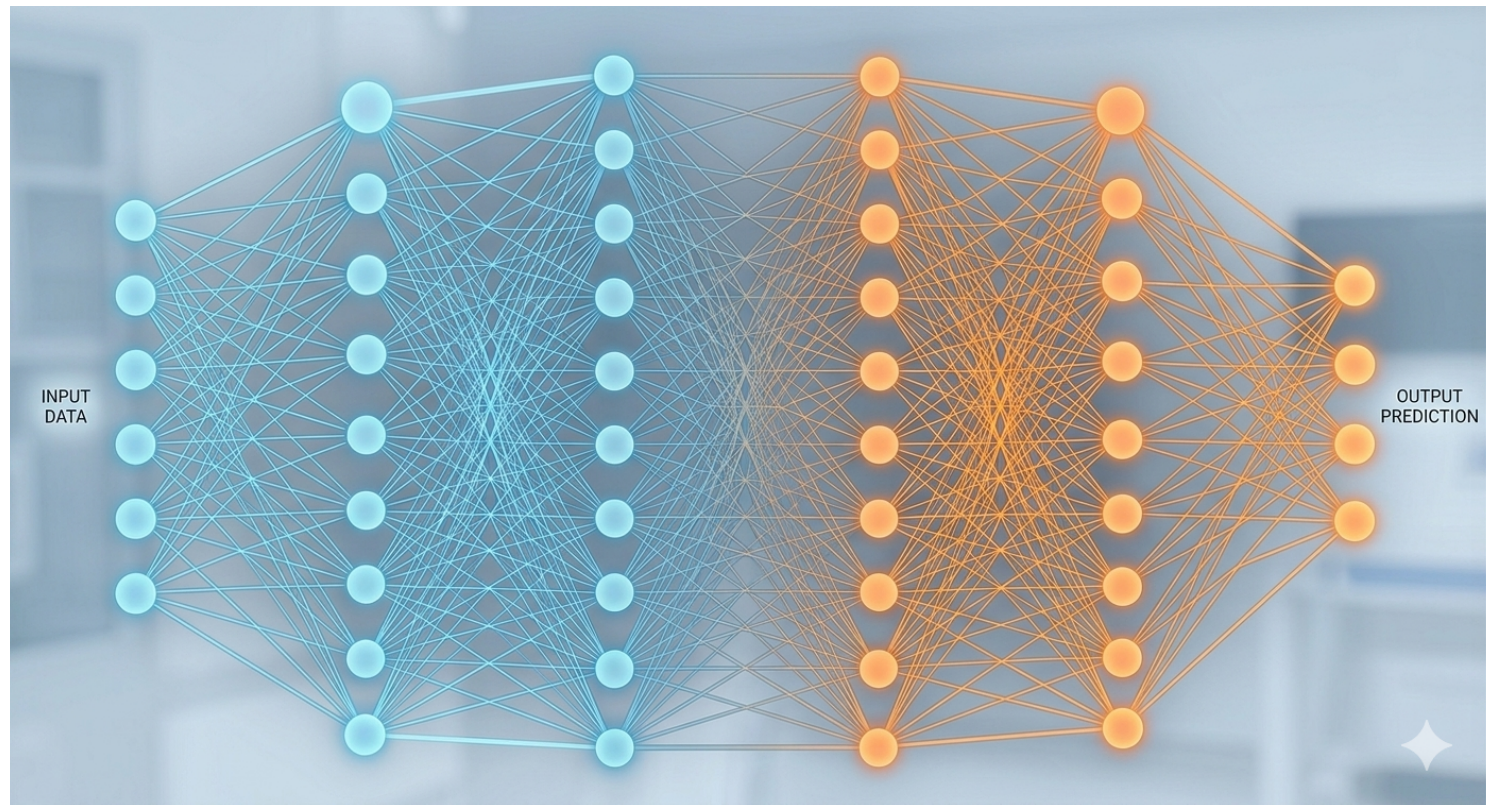
Perspectives

- Using AI in combination with the alternating algorithm to improve result accuracy

- Extending the reconstruction approach to full 3D geometries



Thank you for your attention

The PGTI Collaboration

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PGTI PROMPT GAMMA TIME IMAGING

cnrs IN2P3 Les deux infinis