IRN Terascale @ IJCLab Orsay
20-22 avril 2026

Observable gravitational waves from TeV-scale horizontal SU(2) phase transitions

Anna Chrysostomou (LPTHE), Alan S. Cornell (UJ), Aldo Deandrea (IP2I), Luc Darmé (IP2I), Thibault Demartini (CEA)

arXiv: 2512.02148

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Les bulles de savon
Édouard Manet (1867)

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Bubble nucleation, plasma dynamics, and gravitational-wave production in the early universe

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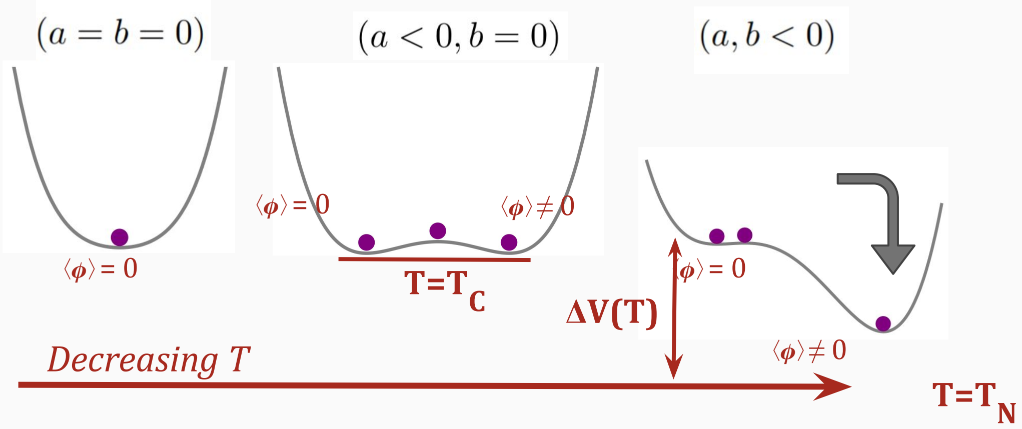
SGWB sourced from TeV-scale symmetry breaking

BSM phase transition → bubble expansion in plasma → collisions + **sound waves** + turbulence → gravitational waves

$$V(\phi) = a\phi^2 + b\phi^3 + c\phi^4$$



$$\alpha = \frac{\Delta\rho_{\rm vac}}{\rho_{\rm rad}} = \frac{1}{\rho_{\rm rad}}\left[\Delta V - \frac{T}{4}\frac{d(\Delta V)}{dT}\right]$$

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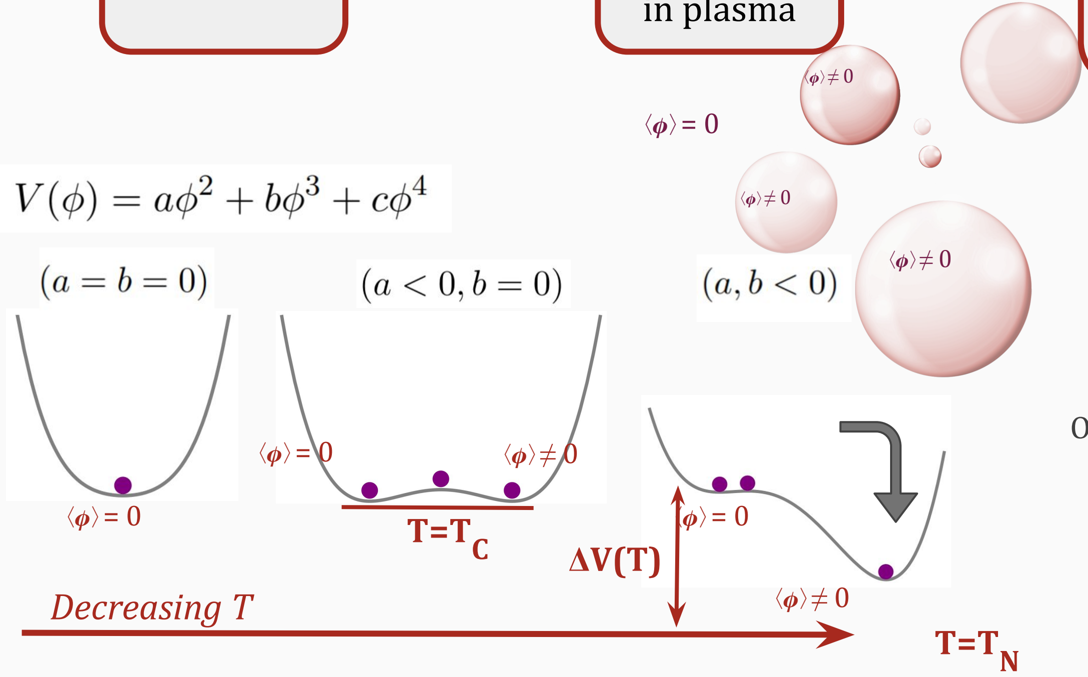

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O(3)-*symmetric tunnelling (“bounce” solution)*

$$S_3(T) = \int_0^\infty dr\, 4\pi r^2 \left[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + \Delta V(T)\right]$$

$$\Gamma(T) \simeq T^4 \left(\frac{S_3(T)}{2\pi T}\right)^{3/2} \exp\left(-\frac{S_3(T)}{T}\right)$$

SGWB sourced from TeV-scale symmetry breaking

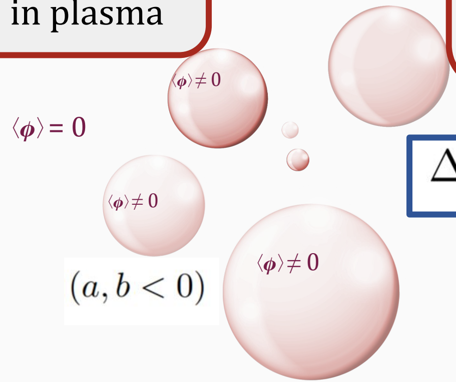
BSM phase transition → bubble expansion in plasma → collisions + sound waves + turbulence → gravitational waves

$$\Omega_{\rm GW} \propto \left(\frac{\kappa_i \alpha}{1+\alpha}\right)^2$$

wall *plasma*

$$\Delta\rho_{\rm vac} \;\rightarrow\; \kappa_\phi\,\Delta\rho_{\rm vac} + \kappa_v\,\Delta\rho_{\rm vac} \;\rightarrow\; \Omega_{\rm GW}$$

$$\alpha = \frac{\Delta\rho_{\rm vac}}{\rho_{\rm rad}} = \frac{1}{\rho_{\rm rad}}\left[\Delta V - \frac{T}{4}\frac{d(\Delta V)}{dT}\right]$$



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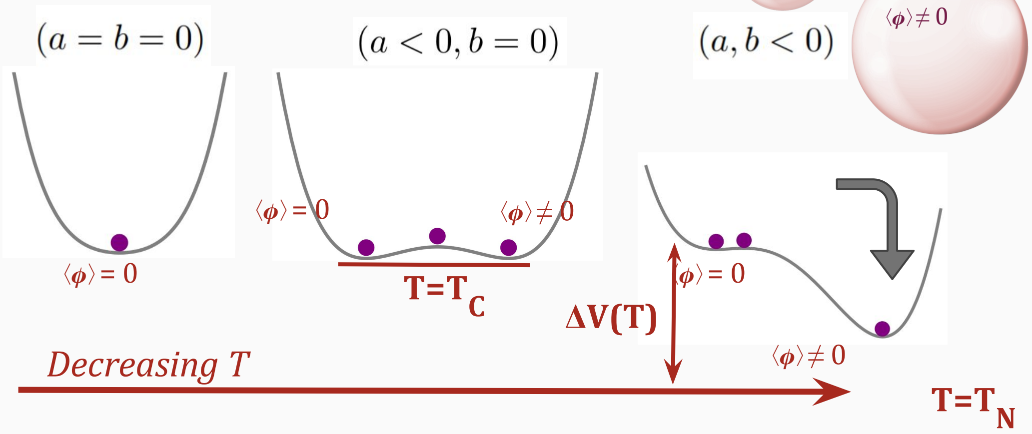

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Flavoured horizontal gauge extension: SM+SU$(2)_f$

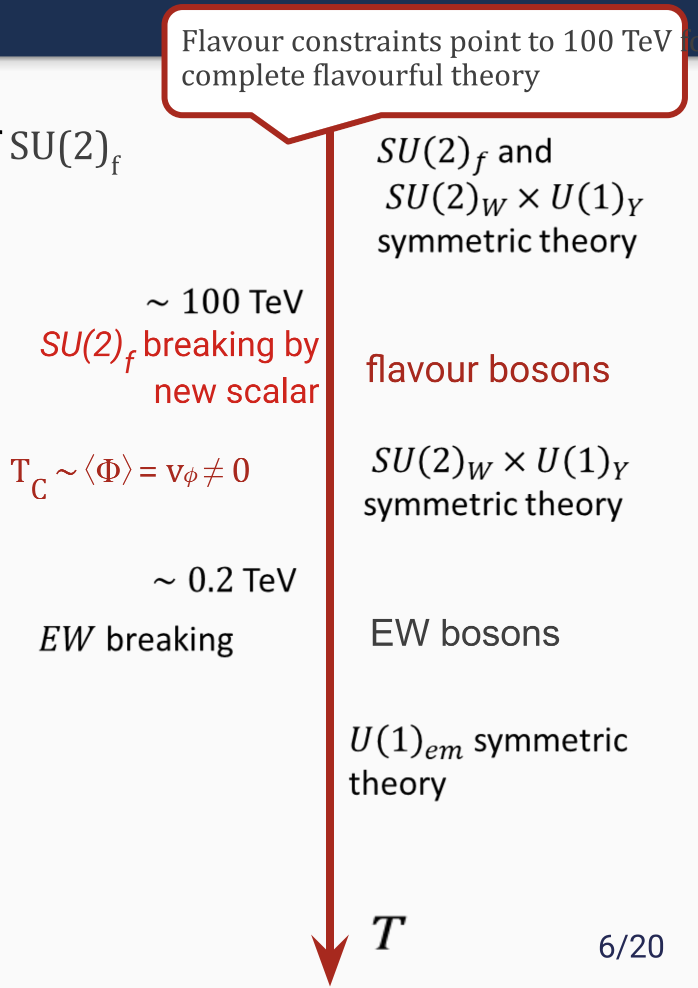
Add a new SU(2) gauge group in the SM, acting on flavour space

→ a new scalar doublet Φ in the fundamental representation of SU$(2)_f$

SSB: 3 new **flavour gauge bosons**

$$M_{V_1}^2 = M_{V_2}^2 = M_{V_3}^2 = \frac{g_f^2}{4} \sum v_\phi^2$$

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$SU(2)_f$	*DoF*
Φ	1	1	0	2	1
V	1	1	0	3	$3 \times 3 = 9$



L. Darmé, A. Deandrea, and F. Mahmoudi, JHEP 05 (2024)
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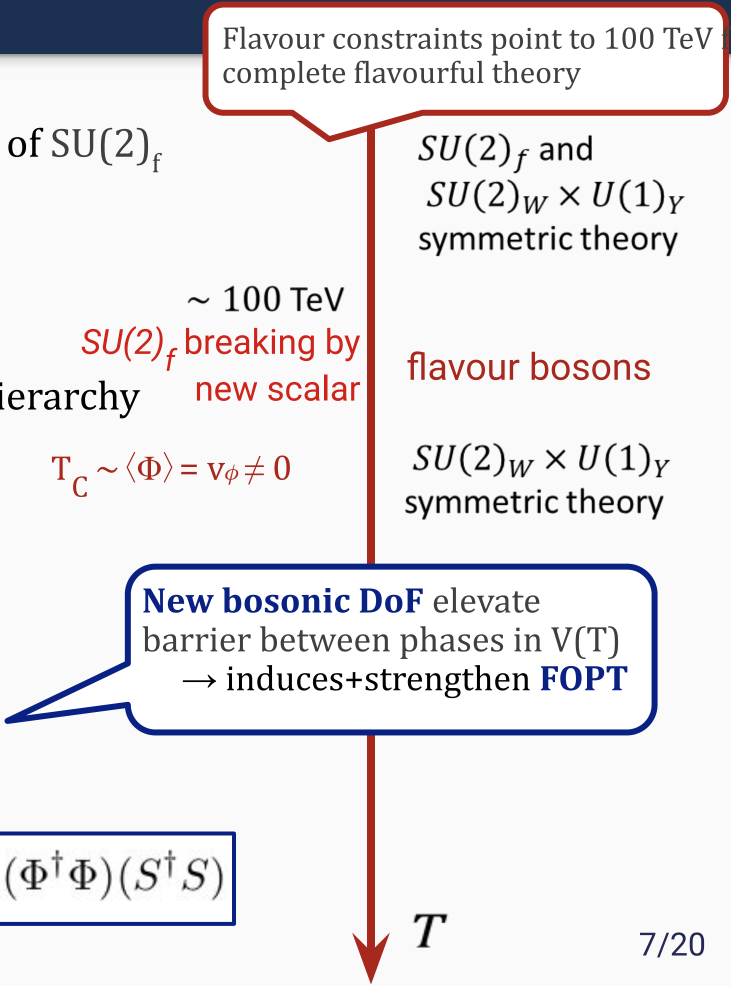
Why flavour models?

New scalar or fermionic fields, creating the Yukawa couplings hierarchy

→ cannot be completely decoupled from the spectrum,

even in cases where $m_S \gg v_\phi$

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$SU(2)_f$	*DoF*
Φ	1	1	0	2	1
S	3	1	2/3	2	$3\times 2\times 2 = 12$
V	1	1	0	3	$3\times 3 = 9$



The scalar sector:

$$V(\Phi, S) \supset -\mu_\phi^2\Phi^\dagger\Phi + \lambda_\phi(\Phi^\dagger\Phi)^2 + \mu_s^2 S^\dagger S + \lambda_s(S^\dagger S)^2 + \lambda_{\phi s}(\Phi^\dagger\Phi)(S^\dagger S)$$

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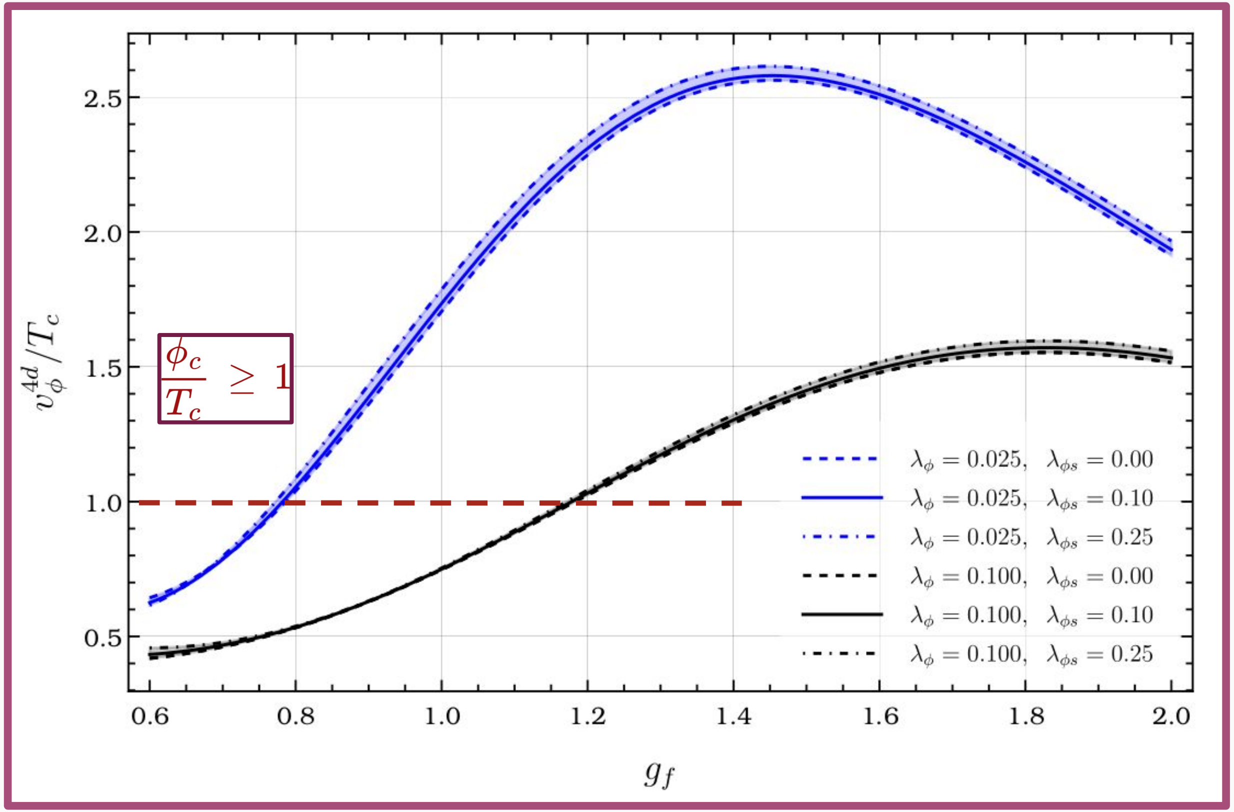
Heuristic indicator for FOPT strength

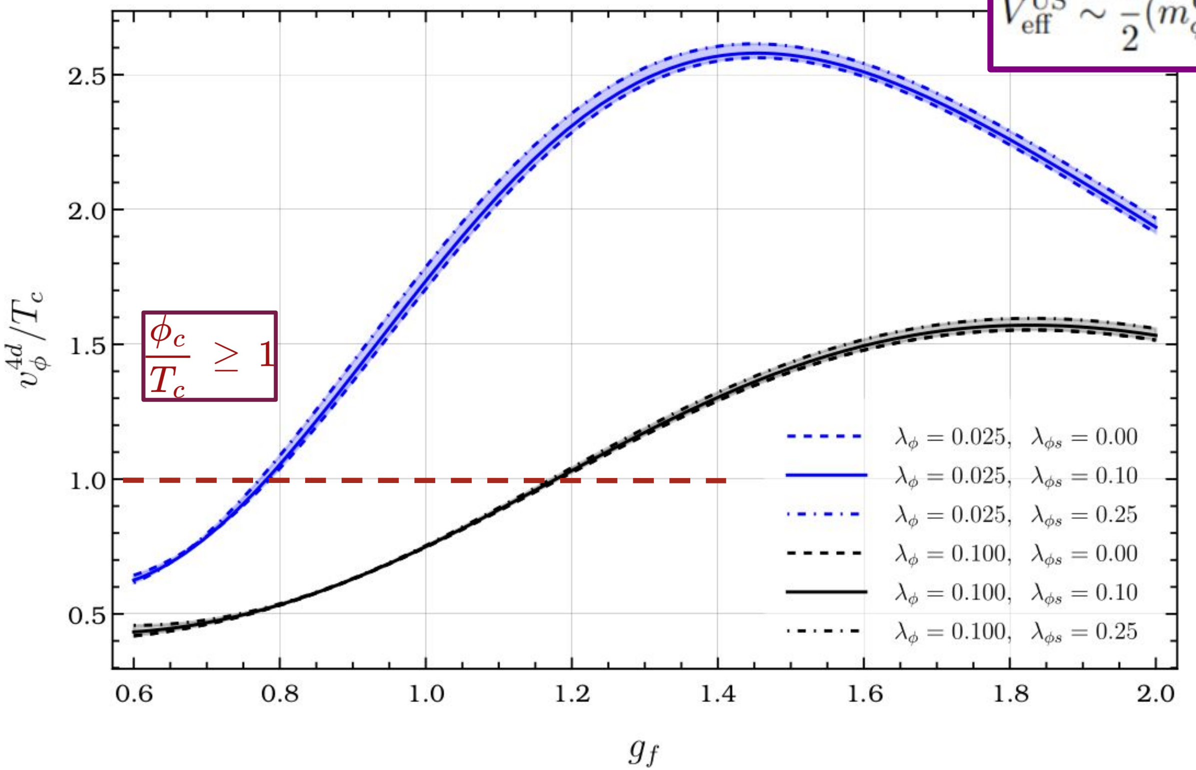


★ *larger* v_ϕ / T_c $\Rightarrow$ *more strongly developed broken phase at* T_c *(order-parameter diagnostic)*

★ *Quartic:* small ~0.005
Portal: larger ~ 0.25
→ *weak FOPT w/o portal coupling*

(sphaleron decoupling)

Heuristic indicator for FOPT strength



$$V_{\rm eff}^{\rm US} \sim \frac{1}{2}(m_\phi^{\rm US})^2\phi^2 + \frac{1}{4}\lambda_\phi^{\rm US}\phi^4 - \frac{3\phi^3(g_f^{\rm US})^3 + \ldots}{48\pi} + \ldots$$

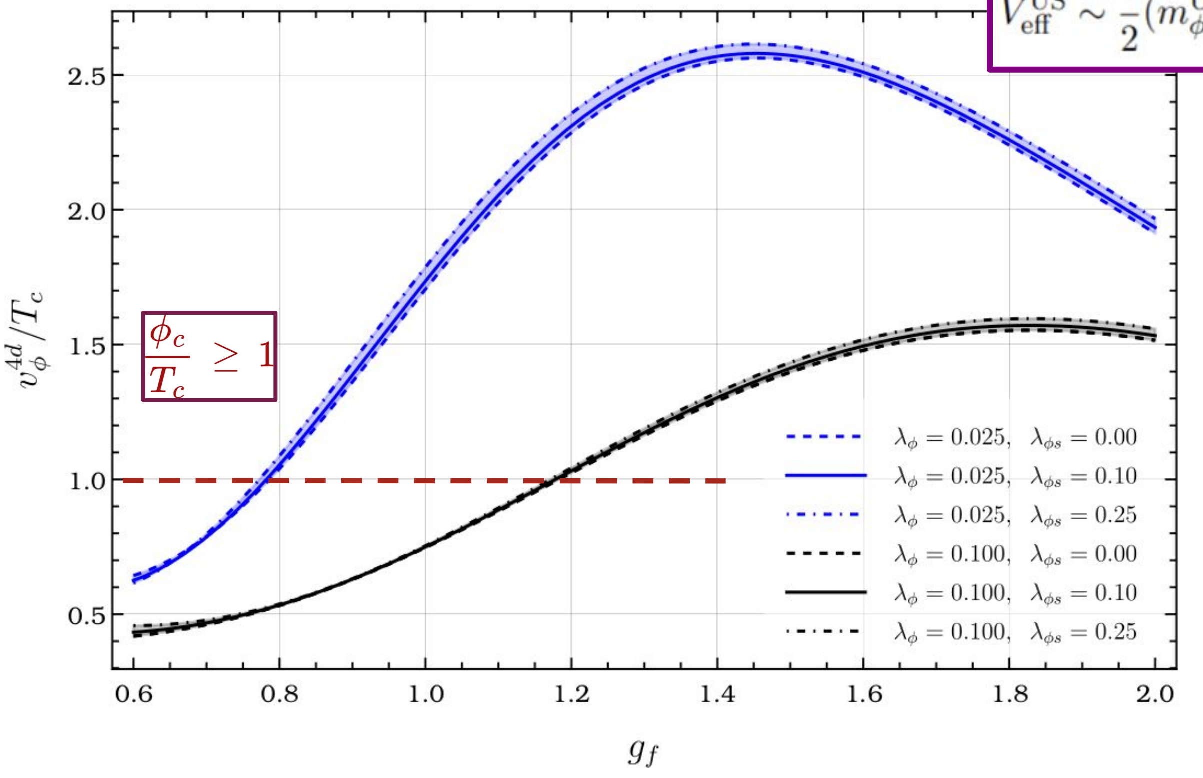
$$v_\phi \equiv \langle\phi\rangle_{T_c} \sim \frac{\kappa_3}{\lambda_\phi^{\rm US}}\,, \qquad \kappa_3 \sim \frac{(g_f^{\rm US})^3}{16\pi}$$

$$\lambda_\phi^{\rm US} = \lambda_\phi^{\rm 3d} + \ldots$$

$$\lambda_\phi^{\rm 3d} = T\left[\lambda_\phi + \frac{1}{(16\pi)^2}\left(g_f^4\,(6 - 9L_b) + \ldots\right.\right.$$

- ★ *to enhance barrier,* $\downarrow \lambda^{US}$ *or* $\uparrow g^{US}$
- ★ g^4 *matching corrections to* λ^{US} *begin to dominate, such that* $v_\phi \propto 1/g^{US}$

Heuristic indicator for FOPT strength



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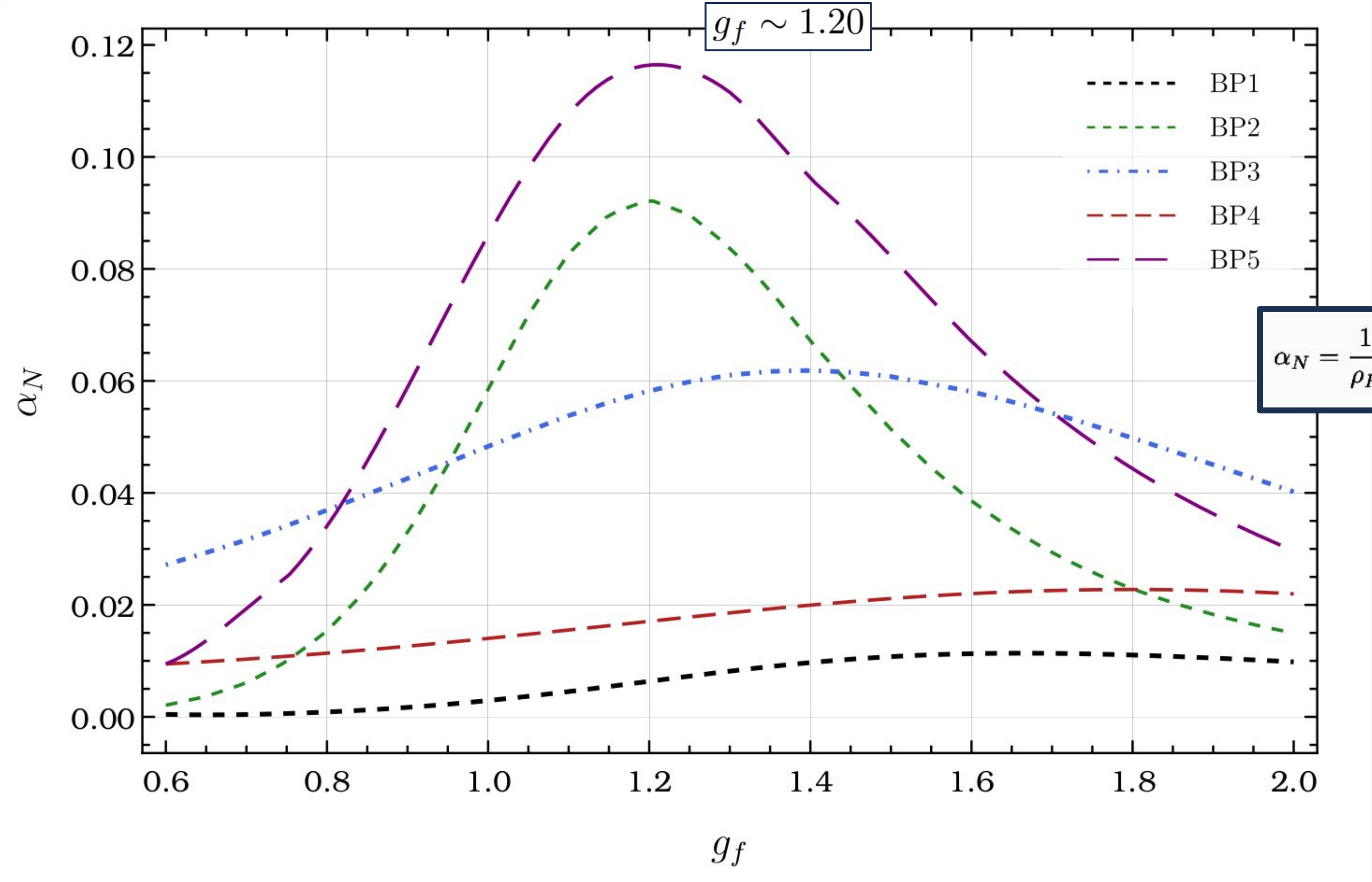
★ g^4 *matching corrections to* λ^{US} *begin to dominate, such that* $v_\phi \propto 1/g^{US}$

→ **g ~ O(1)** *needed for FOPT*

→ **BUT** *increasing the gauge coupling eventually weakens the FOPT !*

Phase transition strength = energy of the vacuum (normalised)

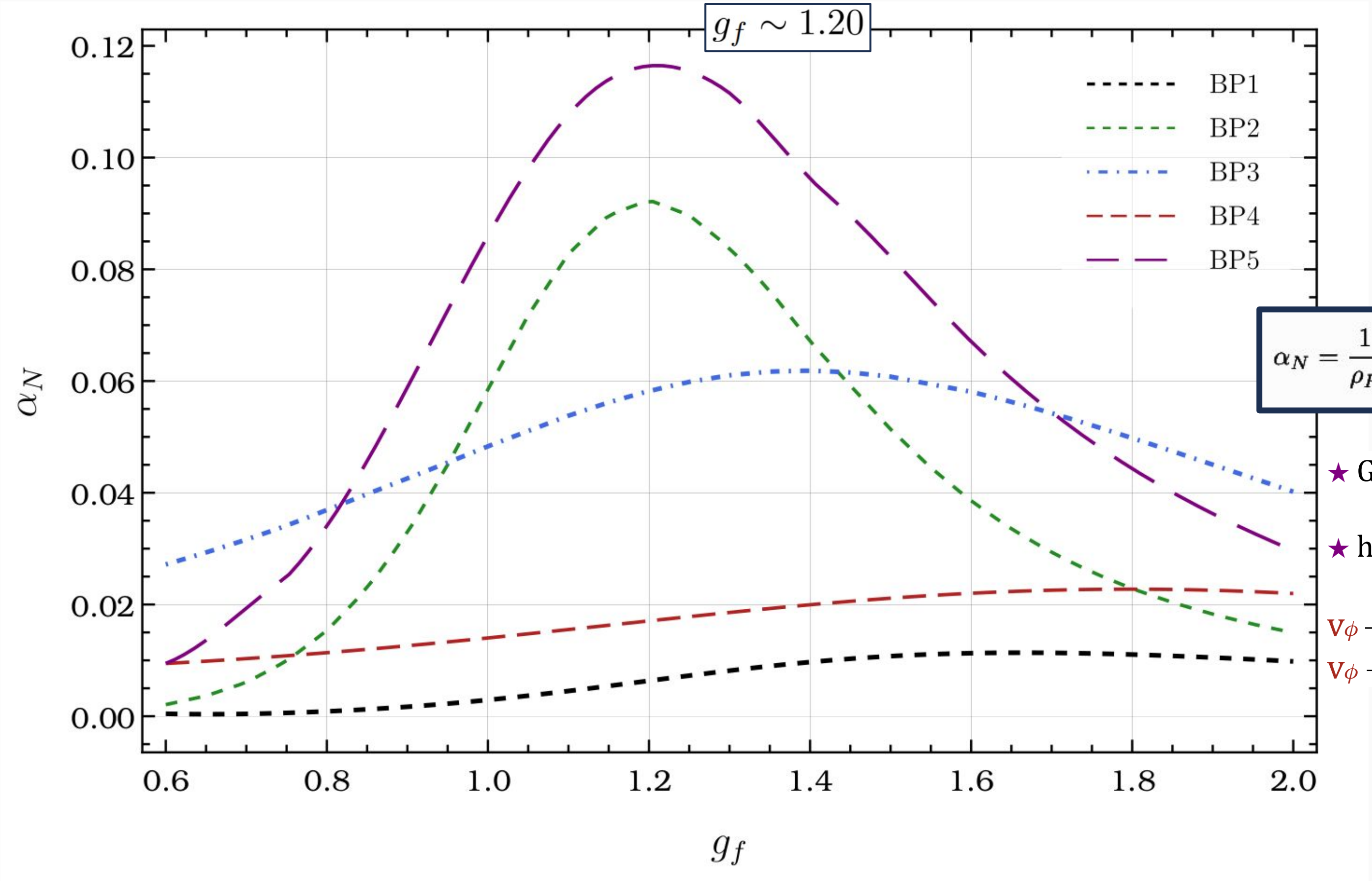


	m_S/v_0	λ_ϕ	$\lambda_{\phi s}$	g_f
BP1	1.00	0.025	0.00	1.5
BP2	1.00	0.005	0.01	1.0
BP3	0.25	0.025	1.00	1.0
BP4	0.10	0.100	1.50	1.5
BP5	0.50	0.005	0.25	1.5

$$\alpha_N = \frac{1}{\rho_R}\left[\Delta V_{\text{eff}}(\phi, T) - \frac{T}{4}\Delta\frac{dV_{\text{eff}}(\phi, T)}{d\ln T}\right]_{T=T_N}$$

$$\rho_R = \frac{\pi^2}{30} g_* T_N^4$$

Phase transition strength = energy of the vacuum (normalised)



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- ★ GW prediction depends on wall velocity & fluid profile
- ★ hydrodynamics determines how energy is distributed:

$v_\phi \to$ wall: rapid bubble expansion

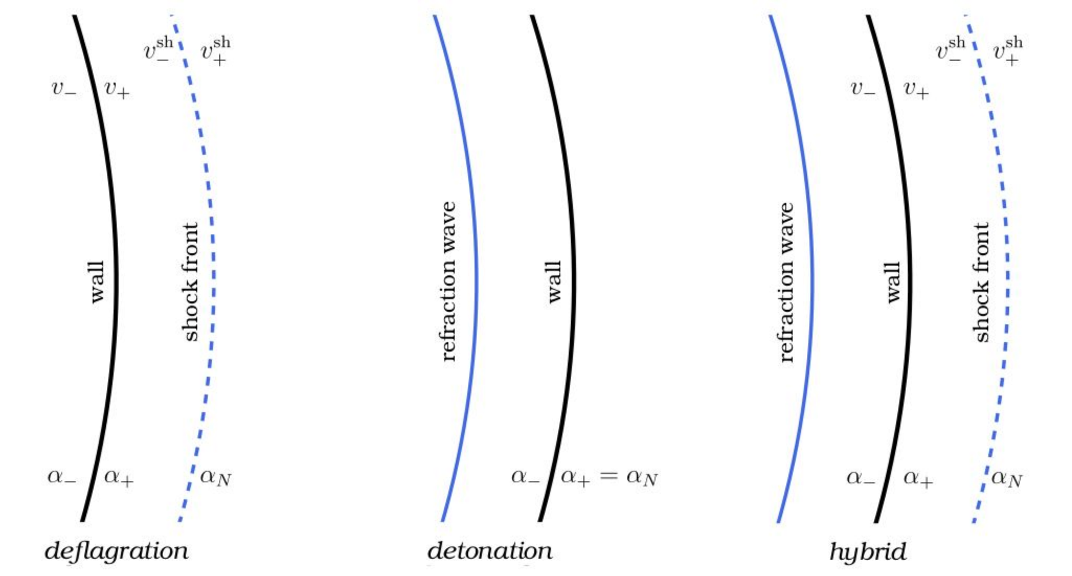
$v_\phi \to$ plasma: sound wave

$$\Omega_{\text{GW}} \propto \left(\frac{\kappa_i \alpha}{1+\alpha}\right)^2$$

Hydrodynamics – at equilibrium!

ONLY when the system around the wall has reached local thermal equilibrium
=> capture the hydrodynamics of the bubble wall velocity and the plasma fluid velocity:

★ *steady-state (constant v)*
★ *runaway (accelerating v) solutions*



$$\partial^\mu T_{\mu\nu} = \partial^\mu T^\phi_{\mu\nu} + \partial^\mu T^{\rm eq}_{\mu\nu} = 0$$

$$T^\phi_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}\partial_\rho\phi\partial^\rho\phi - V_{\rm eff}(\phi, T)\right]$$

$$T^{\rm eq}_{\mu\nu} = w u_\mu u_\nu - p g_{\mu\nu}$$

$$w_+ v_+^2\gamma_+^2 + p_+ = w_- v_-^2\gamma_-^2 + p_-, \qquad w_+ v_+\gamma_+^2 = w_- v_-\gamma_-^2$$

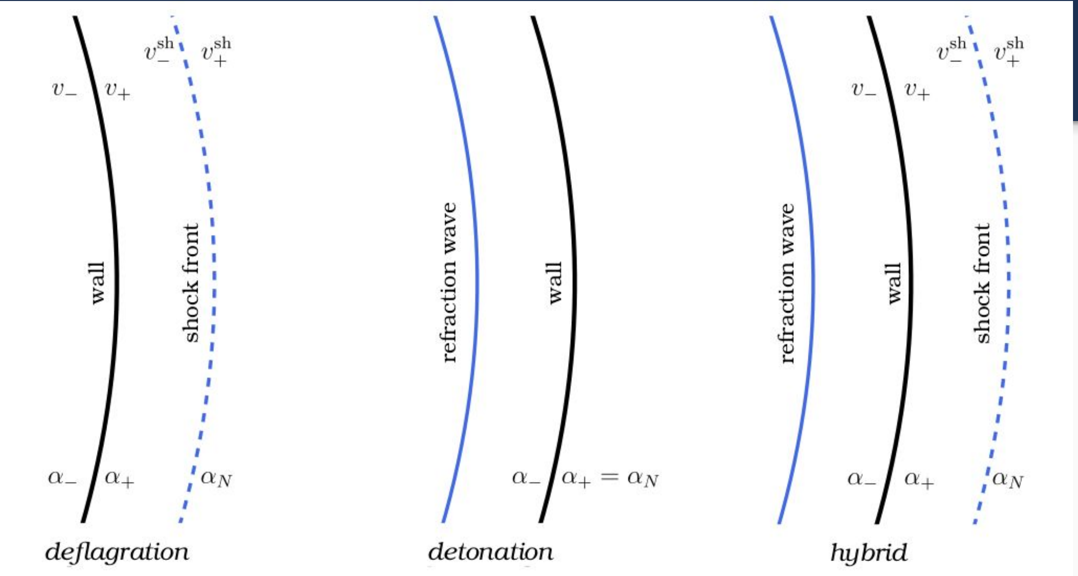
Regime	v_w	v_+	v_-	Features of the expansion
Detonation	$v_w > c_s$	$v_+ = v_w$	$v_- < v_w$	Supersonic: rarefaction wave behind wall
Deflagration	$v_w < c_s$	$v_+ > 0$	$v_- = v_w$	Subsonic: shock wave in front of the wall
Hybrid	$v_w > c_s$	$c_s > v_+ > 0$	$v_- = c_s$	Sonic: rarefaction and shock wave
Runaway	$v_w \to 1$	$v_+ \approx 0$	$v_- \approx 0$	No steady-state, walls accelerate indefinitely

Hydrodynamics – at equilibrium!

Each transition is matched to a steady-state hydrodynamical solution, with an effective friction parameter.

Friction from plasma interactions can prevent runaway

Soft flavour gauge boson interactions with the plasma provide the dominant friction.



$$w_+ v_+^2 \gamma_+^2 + p_+ = w_- v_-^2 \gamma_-^2 + p_-, \qquad w_+ v_+ \gamma_+^2 = w_- v_- \gamma_-^2$$

Regime	v_w	v_+	v_-
Detonation	$v_w > c_s$	$v_+ = v_w$	$v_- < v_w$
Deflagration	$v_w < c_s$	$v_+ > 0$	$v_- = v_w$
Hybrid	$v_w > c_s$	$c_s > v_+ > 0$	$v_- = c_s$
Runaway	$v_w \to 1$	$v_+ \approx 0$	$v_- \approx 0$

Solve numerically for wall velocity & efficiency parameter

Input: η and thermal parameters from FOPT
Hydrodynamics: solve for fluid profile $v(\xi)$ & enthalpy $w(\xi)$
Matching: determine v_+, v_-, and α_+ across the wall
Regime selection: η determines expansion regime
Output: extract v_w & compute κ from the fluid profile

Considering the energy budget: efficiency vs friction

thermal input

$V_{\text{eff}}(\phi, T) \quad \Rightarrow \quad T_N, \ \phi_N, \ \alpha_N$

wall profile

planar-wall approximation,

$$\phi(z) \quad = \frac{1}{2}\phi_N \left[\tanh\left(\frac{z}{L_w}\right) + 1\right] \quad ,$$

wall thickness L_w fixed from moment conditions

plasma backreaction

relativistic fluid + effective friction η

dominant source: soft horizontal gauge bosons

driving pressure = friction + backreaction

hydrodynamic matching

determine the expansion regime:

deflagration, detonation, hybrid; runaway

Extract v_w and κ

GW output

$V_{\text{eff}}, \ \alpha_n, \ \eta \quad \rightarrow \quad v_w, \ \kappa \quad \rightarrow \quad \Omega_{\text{GW}}$

$$\kappa = \frac{3}{\alpha_N \rho_R v_w^3} \int_{c_s}^{v_w} w \, \xi^2 \frac{v^2}{1 - v^2} \, d\xi$$

$\xi = r/t.$

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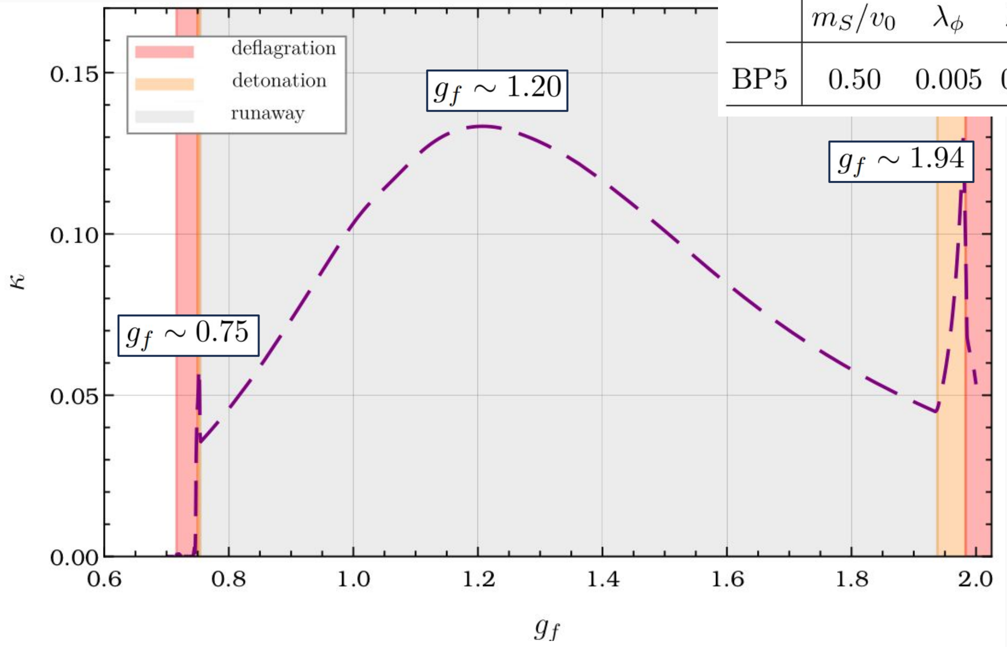

	m_S/v_0	λ_ϕ	$\lambda_{\phi s}$	g_f
BP5	0.50	0.005	0.25	1.5

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GW output

$V_{\rm eff},\ \alpha_n,\ \eta \quad \rightarrow \quad v_w,\ \kappa \quad \rightarrow \quad \Omega_{\rm GW}$

GW spectra from FOPTs

Caprini *et al.* JCAP 03 (2020)

$$h^2\Omega_{\rm sw}(f) = 2.59\times10^{-6}\left[\left(\frac{g_*}{100}\right)^{-1/3}\right]\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2\left(\frac{\beta}{H_N}\right)^{-1}\max(v_w,c_s)\; S_{\rm sw}\left(f,v_w\right)$$

$$S_{\rm sw}(f) = \left(\frac{f}{f_{\rm sw}^{peak}}\right)^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2}$$

$$f_{\rm sw}^{\rm peak} = 8.9\times10^{-6}\ {\rm Hz}\left[\left(\frac{g_*}{100}\right)^{1/6}\left(\frac{T_N}{100\ {\rm GeV}}\right)\right]\frac{1}{\max(v_w,c_s)}\left(\frac{\beta}{H_N}\right)$$

★ *sound waves dominate (we checked)*
★ *for large amplitude, we want strong phase transition with a swift onset*
★ *small quartic and coupling between symmetry-breaking scalar ϕ & scalar LQ*

	m_S/v_0	λ_ϕ	$\lambda_{\phi s}$	g_f	T_c [TeV]	T_N [TeV]	α_N	β/H_N
BP1	1.00	0.025	0.00	1.5	21.28	19.06	0.011	1886
BP2	1.00	0.005	0.01	1.0	16.95	12.34	0.055	1123
BP3	0.25	0.025	1.00	1.0	23.61	19.50	0.048	1339
BP4	0.10	0.100	1.50	1.5	25.76	24.09	0.021	2850
BP5	0.50	0.005	0.25	1.5	17.37	13.76	0.085	884

GW spectra from FOPTs

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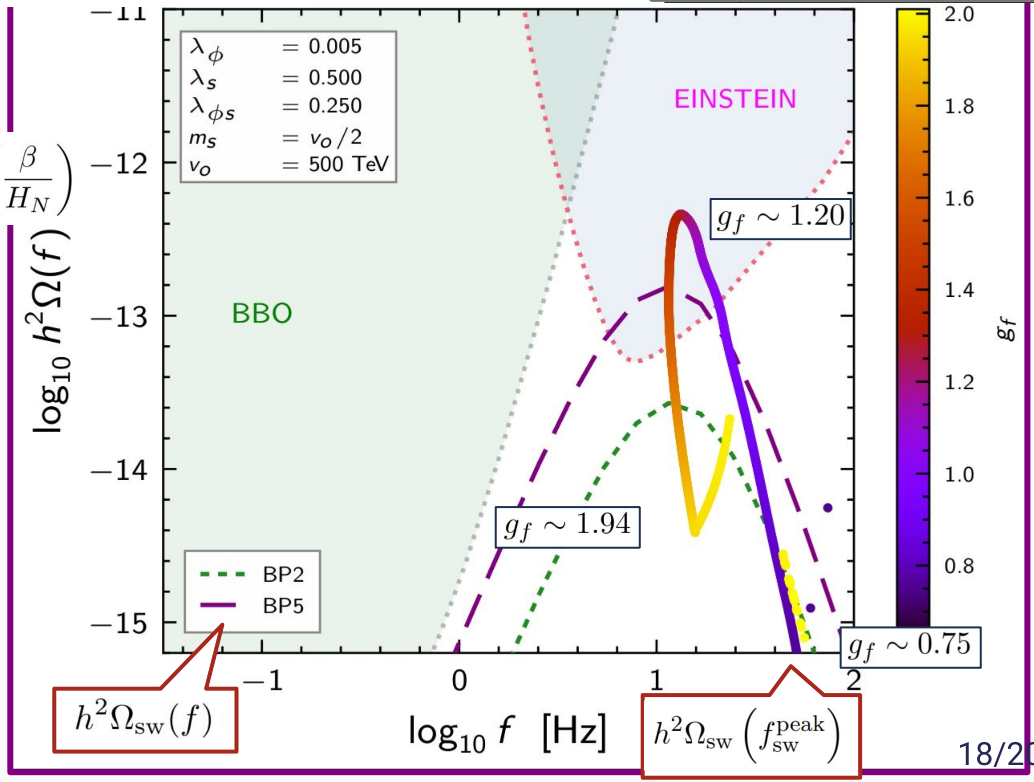
★ *GW spectrum peaks in BBO when vev = 50 TeV*
★ *setting the reference scale to 500 TeV, we plot the peak frequency and amplitude for BP5, varying g_f => Einstein predicted to be sensitive to the signal*



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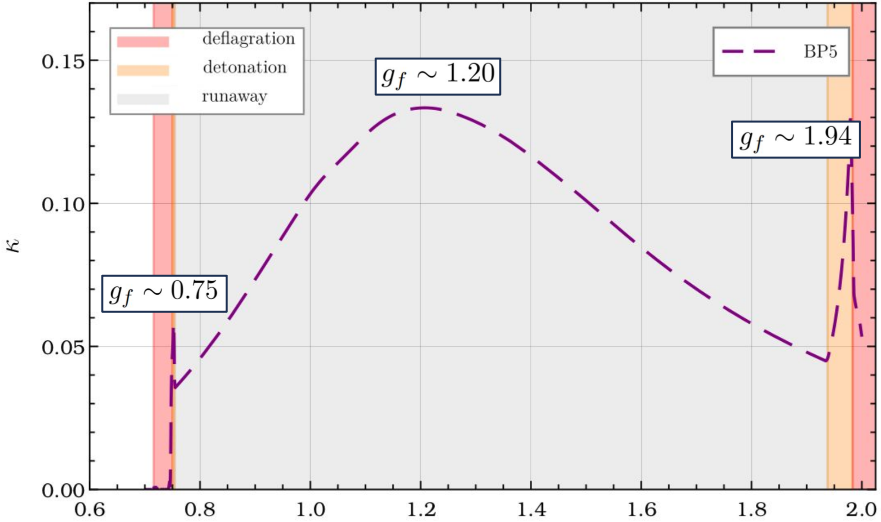

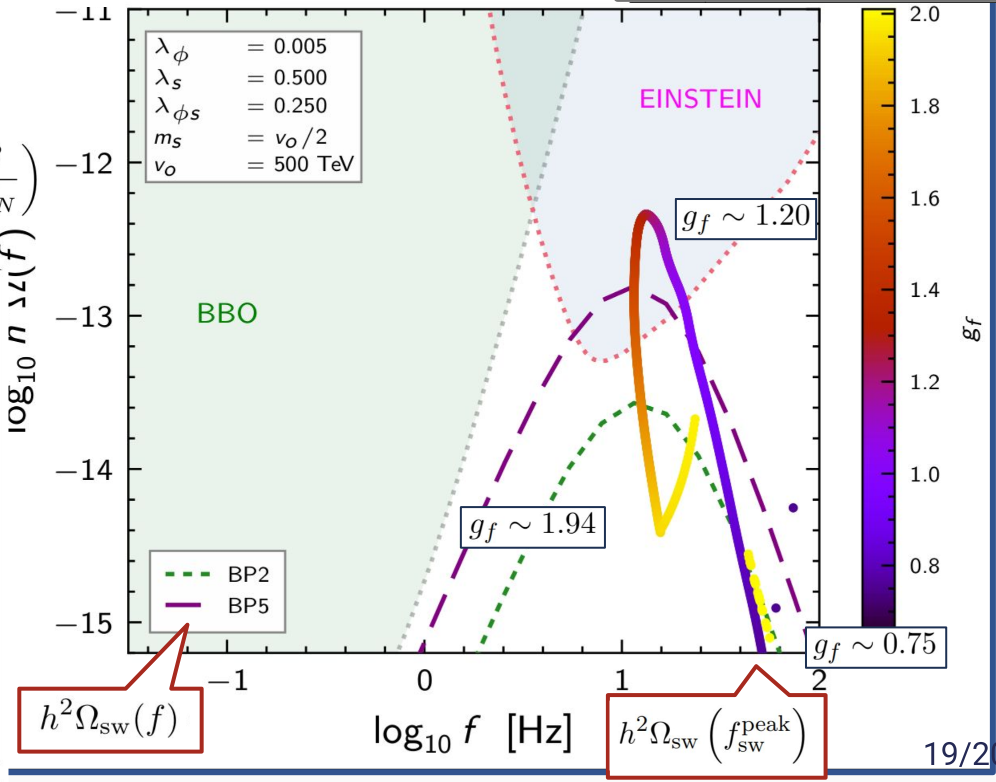

Conclusions

- ⋆ Here: state-of-the-art thermal calculations (DRalgo, FindBounce) + plasma friction
- ⋆ TeV-scale SU(2) horizontal breaking can trigger observable FOPTs
- ⋆ Scalar + LQ degrees of freedom significantly enlarge the viable FOPT region
- ⋆ The key regime is $g_f \sim O(1)$; below $g_f \approx 0.7$, FOPTs are typically lost
- ⋆ Stronger FOPTs occur for small quartic couplings; more generic with additional DoF (LQs)
- ⋆ Strong transitions ($\alpha \gtrsim 0.1$) favour runaway bubbles
- ⋆ ET and BBO probe well beyond collider reach, up to $\sim 10^7$ GeV [discovery potential]
- ⋆ GWs open a new window on flavour-sector BSM physics
- ⋆ Ongoing work: full flavour completion & improved wall–plasma modelling

Thanks for your attention!

For more, see our paper,
arXiv: *2512.02148* [JHEP],

the $SU(2)_f$ model and its realisations,
arXiv: *2307.09595* ,

or write to us at
chrysostomou@lpthe.jussieu.fr

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Les bulles de savon
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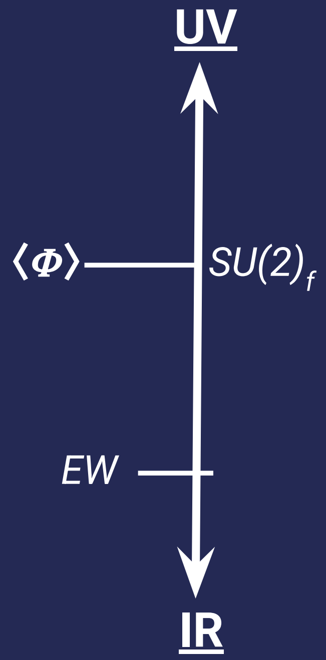
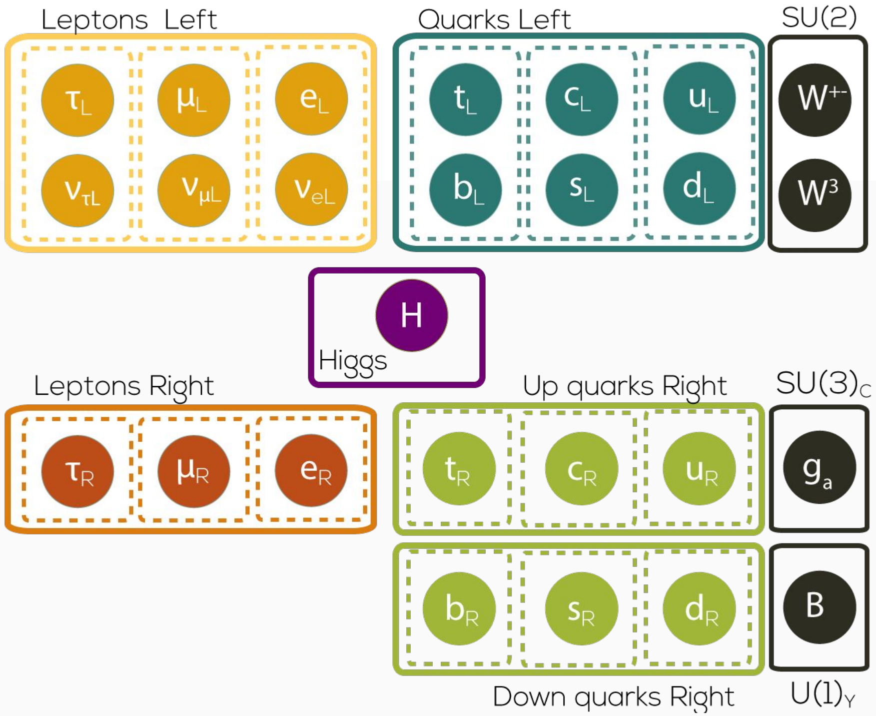
Backup slides



Horizontal flavour gauge group



The SM has a large global $U(3)^5$ symmetry group

→broken by the Yukawa interactions

$$\mathcal{L}_Y = -Y^d_{ij}\,\overline{Q^I_{Li}}\,\phi\,d^I_{Rj} - Y^u_{ij}\,\overline{Q^I_{Li}}\,\epsilon\,\phi^*u^I_{Rj} + \text{h.c.},$$

We can gauge a subset of this group ?

→U(1) case: Frogatt-Nielsen constructions, $L_\mu - L_\tau$, flavons, etc…

→The non-abelian case has been sparsely studied.

→In any case the new gauge coupling is a free parameter

Horizontal flavour gauge group

L. Darmé, A. Deandrea, and F. Mahmoudi, JHEP 05 (2024)
arXiv: 2307.09595

Add a new SU(2) gauge group in the SM, acting on flavour space

- The « charged » SM fermion is part of a doublet
- Only the $SU(2)_f{}^2 \times U(1)_Y$ mixed anomaly is non-zero

$$\mathcal{A} = ([C(Q_i) - C(L_i)] - [2C(u_{R,i}) - C(d_{R,i}) - C(e_{Ri})])$$

Gauge boson masses are free parameters!

- We push to multi-TeV scale, with upper bound on g_f enforced by perturbativity limit
- Even with a large VEV, small gauge couplings (required by flavour constraints) imply light new states
  - For instance: left-handed scenario with $(12)_\ell(12)_{Q_L}$ interactions
    - → Reduce the number of fundamental fermions
    - → Couples both to LH leptons and LH quarks

3 new « W-like » gauge bosons carrying a « flavour-charge »

$$M_{V_1}^2 = M_{V_2}^2 = M_{V_3}^2 = \frac{g_f}{2}\sum_i v_\phi^2$$

\+ *rotation matrices to mass basis: V_{uL}, V_{dL}, ...*

Masses and textures

The presence of $SU(2)_f$ implies that the fermion mass matrices have a structure: let us focus on a left-handed model with Q_i, L_i

→We introduce δY_i , a $SU(2)_f$ spurion

→In the most generic case, this does not distinguish first and second generation

$$\delta Y_i = (\delta Y, 0)$$

$$L \supset y_d^\alpha \delta Y_i \, \bar{Q}^i \cdot H \, d_{R,\alpha} + \tilde{y}_d^\alpha \delta Y^{\dagger,i} \epsilon_{ij} \bar{Q}^j \cdot H \, d_{R,\alpha} + Y_{3,d} \bar{Q}_3 \cdot H \, b_R$$

$$L \supset \delta Y(\bar{Q}^1 \cdot H \, (y_d^\alpha d_{R,\alpha}) - \delta Y \, (\bar{Q}^2 \cdot H \, (\tilde{y}_d^\alpha d_{R,\alpha}) + Y_{3,d} \bar{Q}_3 \cdot H \, b_R$$

α are generation indices but NOT gauge indices

i,j are $SU(2)_f$ gauge indices

We use the $U(3)_f$ global reparametrisation for $d_{R,\alpha}$

Arranging $\delta Y \ll Y_3$ still leads to the same mass scale for first and second generation

Masses and textures

How can we generate a hierarchy between 1st and 2nd generation ?

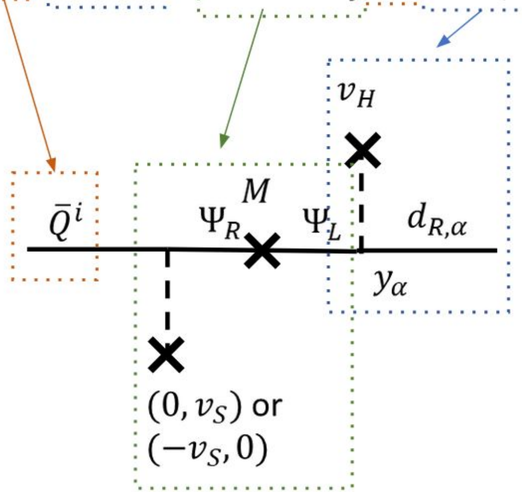
→ Standard approach: add another U(1) factor distinguishing 1st and 2nd

and therefore a new spurion…

→ We take a step back and realise that y_d^α and $\tilde{y}_d^\alpha$ are not necessarily independent parameters

→ Let's consider a simple model with a $SU(2)_f$ breaking scalar S_i and a VL quark

$$y_d^\alpha \delta Y_i \bar{Q}^i \cdot H\, d_{R,\alpha} + \tilde{y}_d^\alpha \delta Y^{\dagger,i} \epsilon_{ij} \bar{Q}^j \cdot H\, d_{R,\alpha}$$



A simple VL-driven example

$$L \supset \delta Y(\bar{Q}^1 \cdot H\,(y_d^\alpha d_{R,\alpha}) - \delta Y\,(\bar{Q}^2 \cdot H\,(\tilde{y}_d^\alpha d_{R,\alpha}) + Y_{3,d} \bar{Q}_3 \cdot H\, b_R$$

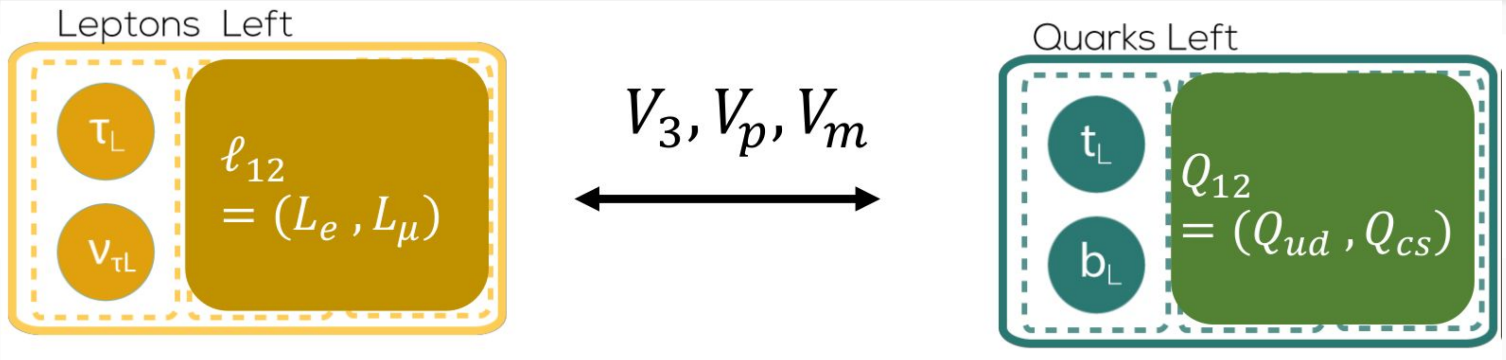
Leads to $y_d^\alpha \propto \tilde{y}_d^\alpha$

→ The down-quark mass matrix is only rank 2

$$L \supset \delta Y(\bar{\tilde{Q}}^2 \cdot H\,(y_d^\alpha d_{R,\alpha}) + Y_{3,d} \bar{Q}_3 \cdot H\, b_R$$

□ Repeat for the third generation

The « flavour-transfer » mechanism



rather than break flavour, the new gauge bosons transfer flavour from one fermionic sector to another

A flavour-violating transition ΔF_f in one fermionic sector is pairwise related to $\Delta F'_f$ in another
Four-fermion operators arising from flavour gauge boson exchanges satisfy $\Delta F_f + \Delta F'_f = 0$

➔ Ensures overall balance in the flavour structure.

➔ Only the $SU(2)_f \times SU(2)_f \times U(1)_Y$ mixed anomaly is non-zero

$$V_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad V_p = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad V_m = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The corresponding generators in flavour space

Thermal corrections : TFD vs DR

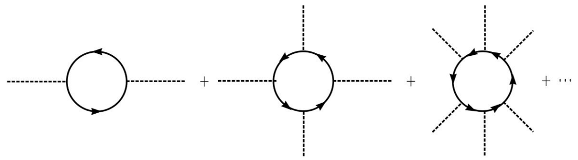
M. Quiros, ICTP HEPAC Summer School (1998)
arXiv: hep-ph/9901312
D. Curtin, P. Meade, and H. Ramani, EPJC 78 (2018)
arXiv: 1612.00466

How to compute the effective thermal potential ?

➔ Describe the correlation functions a QFT in a thermal bath, Green's functions can be computed by compactifying time along the imaginary time direction

➔ Stability of the vacuum be estimated from this quantity (equivalent to free energy in thermodynamics)

Stay in 4d, every loop comes with an infinite sum from the modes in along the imaginary time direction



Standard approach - TFD

Compactify 4d theory onto circle of radius $\beta = T^{-1}$. Phenomena on length scales $L >> \beta$ do not "feel" compact dimension, and thus can be described by a purely 3d EFT. Integrate out the heavy & $n > 0$ modes and match the 4d theory to a 3d theory.

Modern "EFT-like" approach - DR
partially automated through DRalgo

Effective potential: Truncated Full Dressing (Parwani)

D. Croon , O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, JHEP 04 (2021)
arXiv: 2009.10080

Multiple sources of theoretical uncertainty :

- ➔ Nonperturbativity (IR modes at high T) [Linde 1980]
- ➔ Inconsistencies (non-negligible Im{V}) [Weinberg & Wu 1987; Weinberg 1992]
- ➔ higher-order perturbative corrections [Arnold & Espinosa 1992]
- ➔ gauge dependence [Laine 1994]
- ➔ renormalisation scale dependence [Farakos *et al.* 1994]

Uncertainties

D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, G. White, JHEP 04 (2021)
arXiv: 2009.10080

$\Delta\Omega_{\rm GW}/\Omega_{\rm GW}$	4d approach	3d approach
RG scale dependence	$\mathcal{O}(10^2 - 10^3)$	$\mathcal{O}(10^0 - 10^1)$
Gauge dependence	$\mathcal{O}(10^1)$	$\mathcal{O}(10^{-3})$
High-T approximation	$\mathcal{O}(10^{-1} - 10^0)$	$\mathcal{O}(10^0 - 10^2)$
Higher loop orders	unknown	$\mathcal{O}(10^0 - 10^1)$
Nucleation corrections	unknown	$\mathcal{O}(10^{-1} - 10^0)$
Nonperturbative corrections	unknown	unknown

Sources of theoretical uncertainty and relative importance quantified by the parameter ΔΩGW /ΩGW over the range M = {580 − 700} GeV in the SMEFT. Although we do not have reliable estimates for the uncertainties of the 4d approach due to higher loop orders and nucleation corrections, they are expected to be much larger than the corresponding uncertainties of the 3d approach

Thermal resummation: DR approach to IR sensitivities of light bosons

L. Gould and T.V. I. Tenkanen, JHEP 06 (2021)
arXiv: 2104.04399

How to compute the effective thermal potential ?

- ➔ Describe the correlation functions of a QFT in a thermal bath, Green's functions can be computed by compactifying time along the imaginary direction: $t \to i\beta$ for $\beta = T^{-1}$
- ➔ Phenomena on length scales $L \gg \beta$ do not "feel" compact dimension, and thus can be described by a purely 3d EFT

$$\underbrace{\left(\frac{g}{4\pi}\right)^2 \pi T}_{\text{"ultrasoft"}} \ll \underbrace{\left(\frac{g}{4\pi}\right)^{3/2}}_{\text{supersoft}} \ll \underbrace{\left(\frac{g}{4\pi}\right)\pi T}_{\text{"soft"}} \ll \underbrace{\left(\frac{g}{4\pi}\right)^{1/2}}_{\text{semisoft}} \pi T \ll \underbrace{\pi T}_{\text{"hard"}}$$

- ➔ At sufficiently high T, the infinite tower of non-zero modes can then be integrated out, leaving only the purely spatial (static) zero modes which live at the soft scale.
- ➔ DR approach resums the leading IR-divergent contributions captured by daisy resummation, while additionally correctly resumming large logs that spoil the perturbation theory

$\Rightarrow V_{\text{eff}}$ remains perturbative at high T

Calculation of propagators depends on the chosen path C in the complex plane going from an initial arbitrary time t to t - $i\beta$.
Momentum of propagators replaced with Euclidean momentum or Matsubara frequencies,

$$p_\mu = (i\omega_n, \vec{k}),$$

$$\omega_n = \begin{cases} 2\pi n T & \text{bosons} \\ 2\pi(n+1)T & \text{fermions} \end{cases}$$

$$n_B(E_p, T) \equiv \left(e^{E_p/T} - 1\right)^{-1}$$
$$n_F(E_p, T) \equiv \left(e^{E_p/T} + 1\right)^{-1}$$
$$E_p = \sqrt{p^2 + m^2}$$

Replace integrals over p^0 with Matsubara sums:

$$\int \frac{dp^0}{2\pi} \to \frac{1}{\beta}\sum_n$$

Step-by-step approach to decouple all thermal degrees of freedom

1. RGE from μ_{ini} to μ_{hard}
2. Match 4d to 3d at « hard scale »

   $\mu_{hard} \sim \pi T$ (thermal mass of fermions + transverse gauge bosons)
3. Run gT in the 3d theory
4. Decouple remaining bosonic modes, except scalar field ϕ triggering the PT

Implement using DRalgo

Up to NNLO matching in some cases !

4D theory – μini = 50 TeV

"hard" μ4d $\sim \pi T$

Decoupling of the towers of thermal modes

3D theory

"soft" μ3d $\sim gT$

Scalars + temporal (longitudinal) components of gauge bosons

Symmetry breaking scale

"ultrasoft" μlow $\sim \frac{g^2}{\pi} T$

Only the lightest scalar → corresponds to the effective potential

T

L. Gould and T.V. I. Tenkanen, JHEP 06 (2021) arXiv: 2104.04399
A. Ekstedt, P. Schicho, T. V. I. Tenkanen, CPC 288 (2023) arXiv: 2205.08815

Thermal resummation: DR approach to IR sensitivities of light bosons

$$\lambda_\phi^{3\mathrm{d}} = T\left[\lambda_\phi + \frac{1}{(16\pi)^2}\left(g_f^4\left(6-9L_b\right) + 72 g_f^2 \lambda_\phi L_b - 48 L_b\left(\lambda_{\phi s}^2 + 4\lambda_\phi^4\right)\right)\right]$$

$$L_b = 2\gamma + \log\frac{\mu^2}{(4\pi T)^2}$$

$$\lambda_\phi^{\mathrm{US}} = \lambda_\phi^{3\mathrm{d}} - \frac{3}{32\pi}\left[\frac{\left(\lambda_{\phi A_0}^{3\mathrm{d}}\right)^2}{\sqrt{m_{D,f}}}\right]$$

$$v_\phi \equiv \langle\phi\rangle_{T_c} \sim \frac{\kappa_3}{\lambda_\phi^{\mathrm{US}}}\,, \qquad \kappa_3 \sim \frac{(g_f^{\mathrm{US}})^3}{16\pi}$$

★ *to enhance the barrier, decrease λ^{US} or increase g^{US}*

★ *but at some point, g^4 matching corrections to λ^{US} begin to dominate, such that $\langle\phi\rangle_{Tc} \propto 1/g^{US}$*

4D theory – μini = 50 TeV

Decoupling of the towers of thermal modes

"hard" μ**4d** $\sim \pi T$

3D theory

"soft" μ**3d** $\sim gT$

integrated out hard scale

Symmetry breaking scale

Scalars + temporal (longitudinal) components of gauge bosons

"ultrasoft" μ**low** $\sim \frac{g^2}{\pi}T$

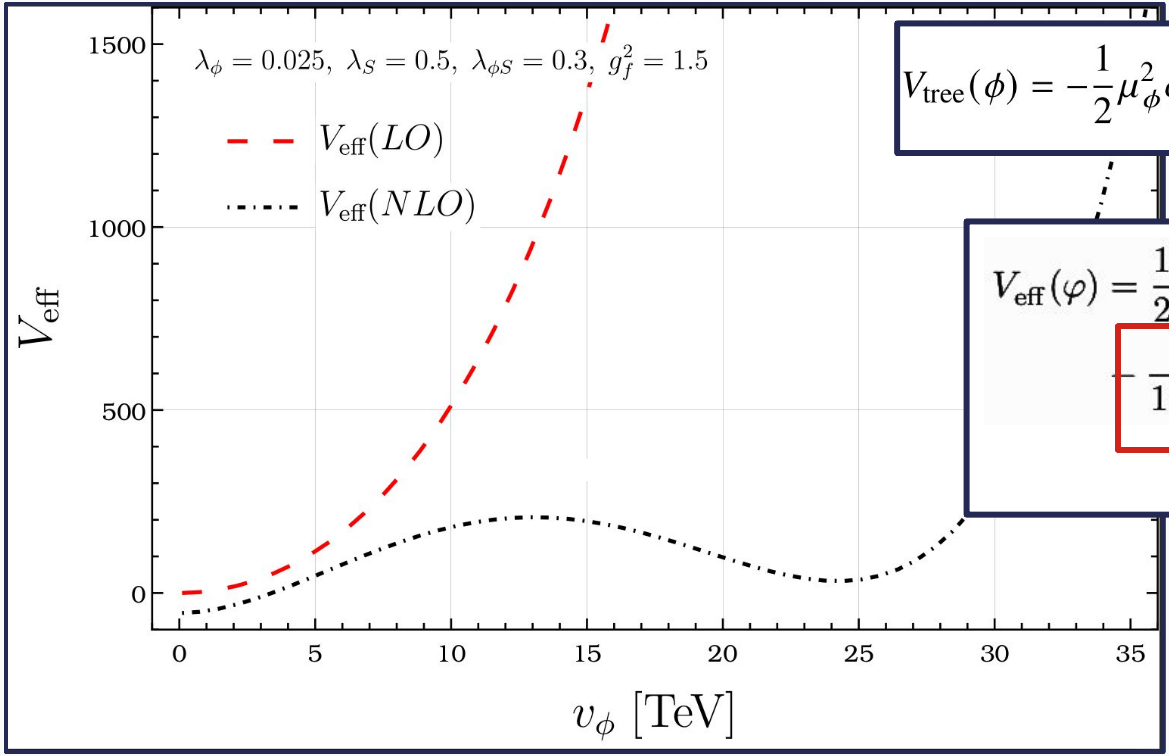
integrated out soft scale

Only the lightest scalar → corresponds to the effective potential

T

Effective potential: LO vs NLO

$$\text{BM:}\ \ \lambda_\phi = 0.025,\ \lambda_S = 0.5,\ \lambda_{\phi S} = 0.3,\ M_S = 50\ \text{TeV}$$



$$V_{\rm tree}(\phi) = -\frac{1}{2}\mu_\phi^2\phi^2 + \frac{1}{4}\lambda_\phi\phi^4 + \frac{1}{2}\mu_s^2|s|^2 + \frac{1}{4}\lambda_s|s|^4 + \frac{1}{2}\lambda_{\phi s}\phi^2|s|^2$$

$$V_{\rm eff}(\varphi) = \frac{1}{2}m_{\phi,3}^2\varphi^2 + \frac{1}{4}\lambda_{\phi,3}\varphi^4 - \frac{1}{16\pi}g_3^3\varphi^3 - \frac{1}{12\pi}\left(3(m_{\rm D}^2 + \frac{1}{2}h_3\varphi^2)^{\frac{3}{2}}\right)$$

vector boson term

temporal scalar term

$$V_{\rm eff}^{\rm 3d}\,T \simeq V_{\rm eff}^{\rm 4d} \quad \text{to } \mathcal{O}(g^3)$$
$$\phi^{\rm 3d}\sqrt{T} \simeq \phi^{\rm 4d} \quad \text{to } \mathcal{O}(g^2)$$

JHEP 06 (2021)

From Lagrangia … pic phenomena

latent heat released by PT, normalised against the radiation density

$$\alpha_N = \frac{1}{\rho_R}\left[\Delta V_{\rm eff}(\phi,T) - \frac{T}{4}\Delta\frac{dV_{\rm eff}(\phi,T)}{d\ln T}\right]_{T=T_N} \qquad \rho_R = \frac{\pi^2}{30}g_* T_N^4$$

inverse phase transition duration relative to the Hubble rate at T_N

$$\frac{\beta}{\mathcal{H}*} = T\,\frac{dS_3(T)}{dT}\bigg|_{T=T_N}$$

$$h^2\Omega_{\rm GW}(f; H_*, \alpha, \beta, v_w)$$

Ω_{peak}, f_{peak}

model → **action** → **bubble dynamics** → **GW vs SNR**

Build $V_{eff}(\mu,T)$

Determine field content, dof, etc.
Potential: zero-T + finite-T
Find degenerate min{$V_{eff}(\mu,T)$}

Is the PT first order ?

$$\frac{\phi_c}{T_c} \geq 1$$

Compute PT parameters

Compute Euclidean / 3d action
Extract phase transition parameters:
> Transition temperature $T_* \approx T_N$
> PT strength α_N
> Inverse of PT duration β/H_N

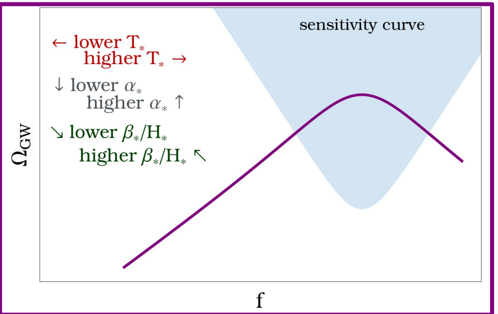
Energy budget & hydrodynamics

Model wall-plasma interactions &
plasma hydrodynamics around w
Compute energy budget paramete
> Efficiency parameter κ
> Scalar field energy density ρ_φ
> Friction/out-of-equilibrium
> Bubble wall speed v_w

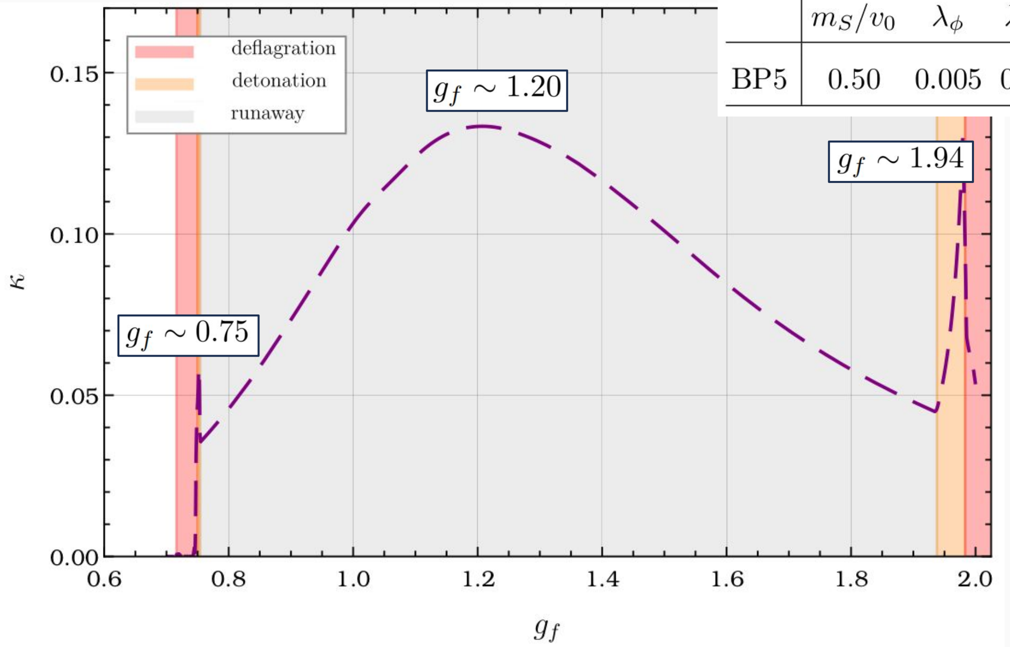
flavour gauge bosons interacting with plasma

GW spectrum vs sensitivity of detectors

Compute energy density of GWs



Considering the energy budget: efficiency vs friction



	m_S/v_0	λ_ϕ	$\lambda_{\phi s}$	g_f
BP5	0.50	0.005	0.25	1.5

$$T^{\rm pl}_{\mu\nu} = T^{\rm eq}_{\mu\nu} + T^{\rm out}_{\mu\nu}\,,$$
$$T^{\rm eq}_{\mu\nu} = \sum_i \int \frac{d^3p}{(2\pi)^3 E_i} p_\mu p_\nu f_i^{\rm eq}(p_\mu, x)\,,$$
$$T^{\rm out}_{\mu\nu} = \sum_i \int \frac{d^3p}{(2\pi)^3 E_i} p_\mu p_\nu \delta f_i(p_\mu, x)\,.$$

pressure from the out-of-equilibrium contributions,

$$P^{m_f}_{\rm out} = \gamma_w v_w \frac{9 m_{\rm D}^2 T_N}{32\pi L_w} \int_0^1 \frac{1-x}{x} dx$$

$$x \equiv \phi(z)/\phi_N$$

treatment of gauge bosons as classical fields breaks down when the particle wavelength approaches that of the wall thickness => IR cutoff

$$\lambda_f \ll L_w \quad \Rightarrow \quad m_f(v_\phi) L_w \gg 1$$

$$x_{\rm IR} = \frac{1}{m_f(v_\phi) L_w}$$

$$\kappa = \frac{3}{\alpha_N \rho_R v_w^3} \int_{c_s}^{v_w} w\, \xi^2 \frac{v^2}{1-v^2}\, d\xi$$

$\xi = r/t.$

$$\alpha_N = \frac{1}{\rho_R}\left[\Delta V_{\rm eff}(\phi,T) - \frac{T}{4}\Delta\frac{dV_{\rm eff}(\phi,T)}{d\ln T}\right]_{T=T_N}$$

$$\phi(z) = \frac{1}{2}\phi_N\left[\tanh\left(\frac{z}{L_w}\right)+1\right]$$