Spi and symmetries at spatial infinity

Maël Chantreau

June 4, 2026

Institut Denis Poisson, Tours

Outline

Based on "Ti and Spi, Carrollian extended boundaries at timelike and spatial infinity" [Herfray, Borthwick, MC, 2025] and work in progress.

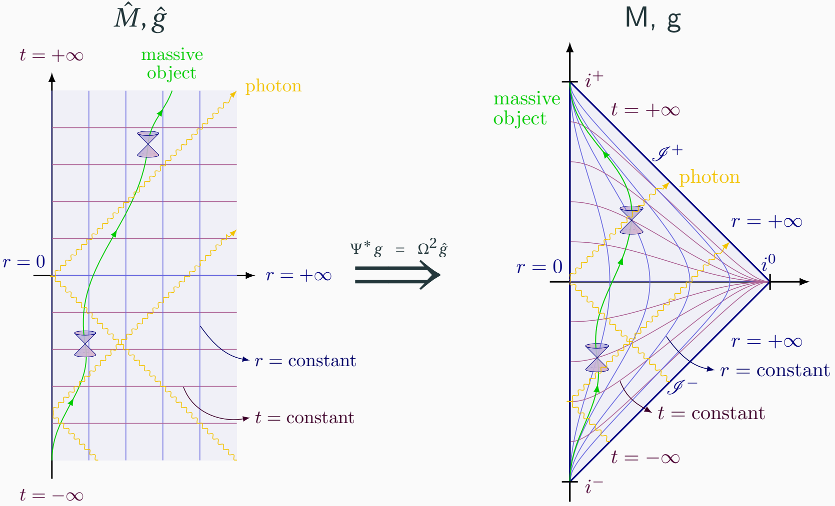
Asymptotically flat space-times and motivations

Spi and Carrollian geometry

Parity condition and link with regularity

Asymptotically flat space-times and motivations

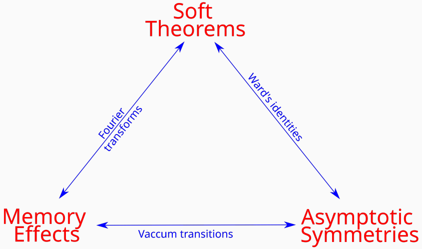
Conformal compactification



Minkoswki space

Penrose diagram

Global BMS group and symmetries of the S matrix



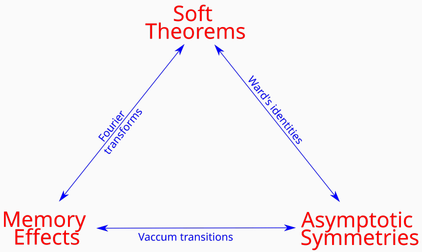
Infrared triangle

Weinberg soft graviton theorem

$\Leftrightarrow \quad \langle \Psi_{out} | [S, Q_{BMS}] | \Psi_{in} \rangle = 0$

[Strominger, 2014]

Global BMS group and symmetries of the S matrix



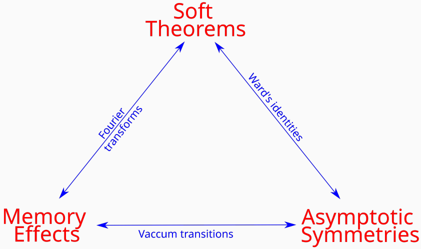
Infrared triangle

Weinberg soft graviton theorem

$$\Leftrightarrow \quad \langle \Psi_{out} | [S, Q_{BMS}] | \Psi_{in} \rangle = 0$$

[Strominger, 2014]

Global BMS group and symmetries of the S matrix

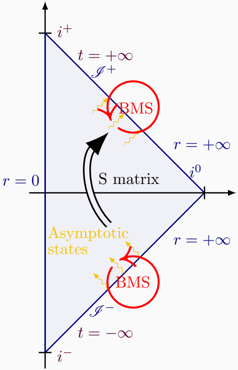


Infrared triangle

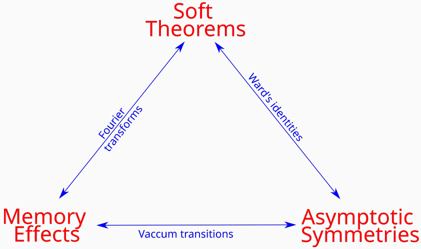
Weinberg soft graviton theorem

$\Leftrightarrow \quad \langle \Psi_{out} | [S, Q_{BMS}] | \Psi_{in} \rangle = 0$

[Strominger, 2014]



Global BMS group and symmetries of the S matrix

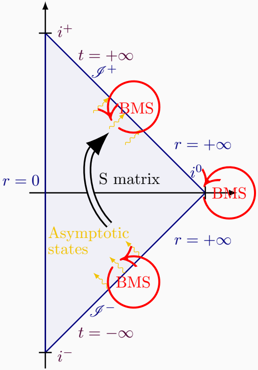


Infrared triangle

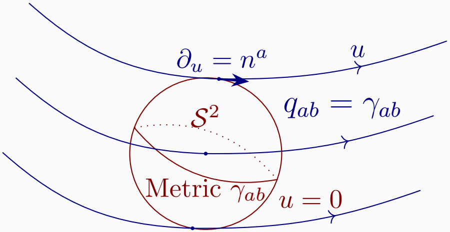
Weinberg soft graviton theorem

$$\Leftrightarrow \quad \langle \Psi_{out} | [S, Q_{BMS}] | \Psi_{in} \rangle = 0$$

[Strominger, 2014]



Structure of $\mathscr{I}$



Representation of $\mathscr{I}$

$(\mathscr{I},\ n^a,\ q_{ab})$

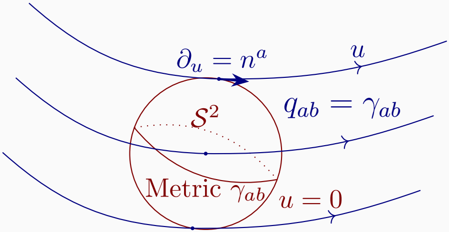
$\left.\begin{aligned} n^a q_{ab} &= 0 \\ \mathcal{L}_n q_{ab} &= 0 \end{aligned}\right\}$ has a (weak) carrollian structure

Conformal automorphisms = BMS group: $\begin{cases} \mathcal{L}_\xi n^a = \Omega^{-1} n^a \\ \mathcal{L}_\xi q_{ab} = \Omega^2 q_{ab} \end{cases}$

The BMS group is $SO(3,1) \ltimes \mathcal{C}^\infty(\mathcal{S}^2)$

Compare with the Poincaré group, $SO(3,1) \ltimes \mathbb{R}^4$

Structure of $\mathscr{I}$



Representation of $\mathscr{I}$

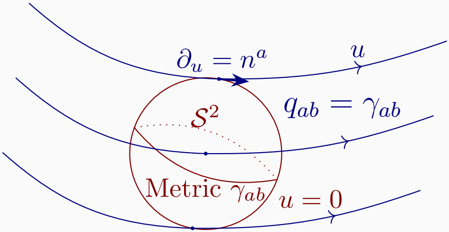
$n^a q_{ab} = 0$
$\mathcal{L}_n q_{ab} = 0$
} $(\mathscr{I},\ n^a,\ q_{ab})$ has a (weak) carrollian structure

Conformal automorphisms = BMS group: $\begin{cases} \mathcal{L}_\xi n^a = \Omega^{-1} n^a \\ \mathcal{L}_\xi q_{ab} = \Omega^2 q_{ab} \end{cases}$

The BMS group is $SO(3,1) \ltimes \mathcal{C}^\infty(\mathcal{S}^2)$

Compare with the Poincaré group, $SO(3,1) \ltimes \mathbb{R}^4$

Structure of $\mathscr{I}$



Representation of $\mathscr{I}$

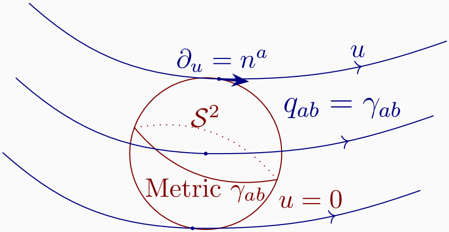
$$\left.\begin{aligned} n^a q_{ab} &= 0 \\ \mathcal{L}_n q_{ab} &= 0 \end{aligned}\right\}$$ $(\mathscr{I},\ n^a,\ q_{ab})$ has a (weak) carrollian structure

Conformal automorphisms = BMS group: $\begin{cases} \mathcal{L}_\xi n^a = \Omega^{-1} n^a \\ \mathcal{L}_\xi q_{ab} = \Omega^2 q_{ab} \end{cases}$

The BMS group is $SO(3,1) \ltimes \mathcal{C}^\infty(\mathcal{S}^2)$

Compare with the Poincaré group, $SO(3,1) \ltimes \mathbb{R}^4$

Structure of $\mathscr{I}$



Representation of $\mathscr{I}$

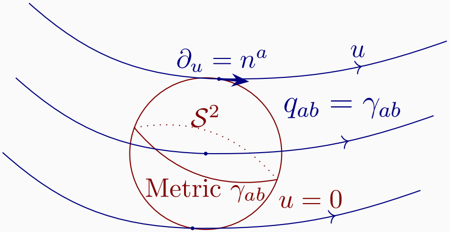
$$\left.\begin{aligned} n^a q_{ab} &= 0 \\ \mathcal{L}_n q_{ab} &= 0 \end{aligned}\right\} \; (\mathscr{I},\ n^a,\ q_{ab}) \text{ has a (weak) carrollian structure}$$

Conformal automorphisms = BMS group: $\begin{cases} \mathcal{L}_\xi n^a = \Omega^{-1} n^a \\ \mathcal{L}_\xi q_{ab} = \Omega^2 q_{ab} \end{cases}$

The BMS group is $SO(3,1) \ltimes \mathcal{C}^\infty(\mathcal{S}^2)$

Compare with the Poincaré group, $SO(3,1) \ltimes \mathbb{R}^4$

Structure of $\mathscr{I}$



Representation of $\mathscr{I}$

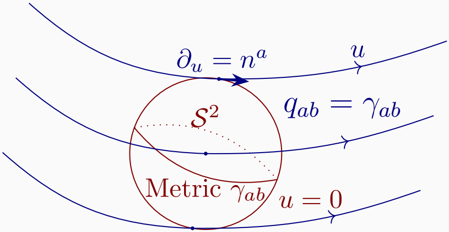
$$\left.\begin{aligned} n^a q_{ab} &= 0 \\ \mathcal{L}_n q_{ab} &= 0 \end{aligned}\right\} (\mathscr{I},\ n^a,\ q_{ab}) \text{ has a (weak) carrollian structure}$$

Conformal automorphisms = BMS group:
$$\begin{cases} \mathcal{L}_\xi n^a = \Omega^{-1} n^a \\ \mathcal{L}_\xi q_{ab} = \Omega^2 q_{ab} \end{cases}$$

The BMS group is $SO(3,1) \ltimes \mathcal{C}^\infty(\mathcal{S}^2)$

Compare with the Poincaré group, $SO(3,1) \ltimes \mathbb{R}^4$

Structure of $\mathscr{I}$



Representation of $\mathscr{I}$

$(\mathscr{I},\ n^a,\ q_{ab})$

$$\left.\begin{array}{r} n^a q_{ab} = 0 \\ \mathcal{L}_n q_{ab} = 0 \end{array}\right\}$$ has a (weak) carrollian structure

Conformal automorphisms = BMS group: $\begin{cases} \mathcal{L}_\xi n^a = \Omega^{-1} n^a \\ \mathcal{L}_\xi q_{ab} = \Omega^2 q_{ab} \end{cases}$

The BMS group is $SO(3,1) \ltimes \mathcal{C}^\infty(\mathcal{S}^2)$

Compare with the Poincaré group, $SO(3,1) \ltimes \mathbb{R}^4$

Asymptotically flat space-time at spatial infinity

Axiomatic definition based on the inverse metric, equivalent to [Ashtekar, Romano, 1991].

Metric close to the boundary [Beig, Schmidt, 1982]:

$$g_{ab} = \rho^{-2}\Big(1 + 2\rho\sigma + \rho^2\sigma^2 + O(\rho^3 \ln\rho)\Big) d\rho^2 + \Big(h_{\alpha\beta} + \rho\left(k_{\alpha\beta} - 2\sigma h_{\alpha\beta}\right) + O(\rho^2 \ln\rho)\Big) dy^\alpha dy^\beta;$$

- ρ boundary defining function $(\{\rho = 0\} = \mathcal{B},\ \nabla\rho\big|_{\mathcal{B}} \neq 0)$
- y^α coordinates on $\mathcal{B}$
- σ mass aspect
- $h_{\alpha\beta}$ metric on $\mathcal{B}$
- $k_{\alpha\beta}$ very important in what follows

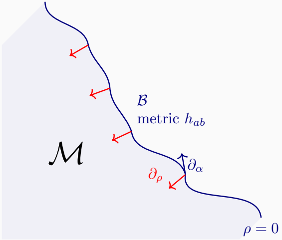
Asymptotically flat space-time at spatial infinity

Axiomatic definition based on the inverse metric, equivalent to [Ashtekar, Romano, 1991].

Metric close to the boundary [Beig, Schmidt, 1982]:

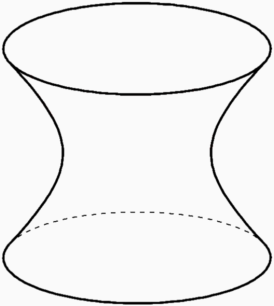
$$
\begin{aligned}
g_{ab} = & \rho^{-2}\left(1 + 2\rho\sigma + \rho^2\sigma^2 + O(\rho^3 \ln\rho)\right) d\rho^2 \\
& + \left(h_{\alpha\beta} + \rho\left(k_{\alpha\beta} - 2\sigma h_{\alpha\beta}\right) + O(\rho^2 \ln\rho)\right) dy^\alpha dy^\beta;
\end{aligned}
$$

- ρ boundary defining function $\left(\{\rho = 0\} = \mathcal{B},\ \nabla\rho\big|_{\mathcal{B}} \neq 0\right)$
- y^α coordinates on $\mathcal{B}$
- σ mass aspect
- $h_{\alpha\beta}$ metric on $\mathcal{B}$
- $k_{\alpha\beta}$ very important in what follows



Spi and Carrollian geometry

Spi, extended boundary



Representation of dS_3

$$\left.\begin{aligned} n^a h_{ab} &= 0 \\ \mathcal{L}_n h_{ab} &= 0 \end{aligned}\right\}$$ weak carrollian structure

$\rho' = \rho + u(y^\alpha)\rho^2 + \mathrm{O}(\rho^3)$

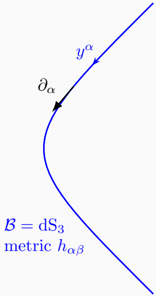
Rigorous definition of Spi:

Let

$$\phi : \mathcal{B} \xrightarrow{J^2\rho} \frac{i^*_{\mathcal{B}}\left(J^2M\right)}{i^*_{\mathcal{B}}\left(J^2\rho^2\right)}, \text{then}$$

$$\begin{array}{ccc} \mathrm{Spi} = \phi^* L & & L = i^*_{\mathcal{B}}\left(J^2M\right) \\ \downarrow & & \downarrow \\ \mathcal{B} & \xrightarrow{\phi} & \dfrac{i^*_{\mathcal{B}}(J^2M)}{i^*_{\mathcal{B}}(J^2\rho^2)} \end{array}$$

Spi, extended boundary



Representation of dS_3

$$\left.\begin{aligned} n^a h_{ab} &= 0 \\ \mathcal{L}_n h_{ab} &= 0 \end{aligned}\right\}$$ weak carrollian structure

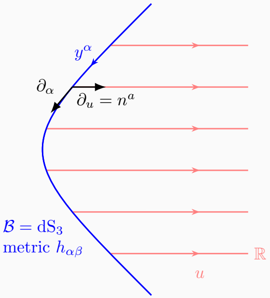
$\rho' = \rho + u(y^\alpha)\rho^2 + \mathrm{O}(\rho^3)$

Rigorous definition of Spi:

Let

$$\phi : \mathcal{B} \xrightarrow{J^2\rho} \frac{i^*_{\mathcal{B}}\left(J^2 M\right)}{i^*_{\mathcal{B}}\left(J^2 \rho^2\right)}, \text{then}$$

$$\begin{array}{ccc} \mathrm{Spi} = \phi^* L & & L = i^*_{\mathcal{B}}\left(J^2 M\right) \\ \downarrow & & \downarrow \\ \mathcal{B} & \xrightarrow{\ \phi\ } & \frac{i^*_{\mathcal{B}}(J^2 M)}{i^*_{\mathcal{B}}(J^2 \rho^2)} \end{array}$$



Representation of Spi

Generalized from

[Figueroa-O'Farrill et al, 2022]

$$\left.\begin{array}{r} n^a h_{ab} = 0 \\ \mathcal{L}_n h_{ab} = 0 \end{array}\right\}$$ weak carrollian structure

$\rho' = \rho + u(y^\alpha)\rho^2 + \mathrm{O}(\rho^3)$

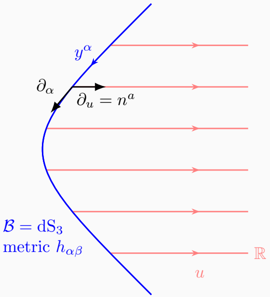
Rigorous definition of Spi:

Let

$$\phi : \mathcal{B} \xrightarrow{J^2\rho} \frac{i^*_{\mathcal{B}}\left(J^2 M\right)}{i^*_{\mathcal{B}}\left(J^2 \rho^2\right)}, \text{then}$$

$$\begin{array}{ccc} \mathrm{Spi} = \phi^* L & & L = i^*_{\mathcal{B}}\left(J^2 M\right) \\ \downarrow & & \downarrow \\ \mathcal{B} & \xrightarrow{\quad\phi\quad} & \frac{i^*_{\mathcal{B}}(J^2 M)}{i^*_{\mathcal{B}}(J^2 \rho^2)} \end{array}$$

Spi, extended boundary



Representation of Spi

Generalized from

[Figueroa-O'Farrill et al, 2022]

$$\left.\begin{aligned} n^a h_{ab} &= 0 \\ \mathcal{L}_n h_{ab} &= 0 \end{aligned}\right\}$$ weak carrollian structure

$$\rho' = \rho + u(y^\alpha)\rho^2 + \mathrm{O}(\rho^3)$$

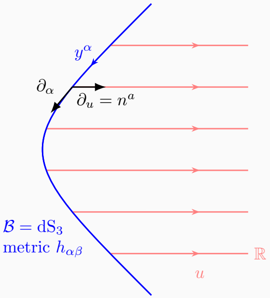
Rigorous definition of Spi:

Let

$$\phi : \mathcal{B} \xrightarrow{J^2\rho} \frac{i^*_{\mathcal{B}}(J^2 M)}{i^*_{\mathcal{B}}(J^2\rho^2)}, \text{then}$$

$$\begin{array}{ccc} \mathrm{Spi} = \phi^* L & & L = i^*_{\mathcal{B}}(J^2 M) \\ \downarrow & & \downarrow \\ \mathcal{B} & \xrightarrow{\phi} & \frac{i^*_{\mathcal{B}}(J^2 M)}{i^*_{\mathcal{B}}(J^2\rho^2)} \end{array}$$

Spi, extended boundary



Representation of Spi

Generalized from

[Figueroa-O'Farrill et al, 2022]

$$\left.\begin{aligned} n^a h_{ab} &= 0 \\ \mathcal{L}_n h_{ab} &= 0 \end{aligned}\right\}$$ weak carrollian structure

$\rho' = \rho + u(y^\alpha)\rho^2 + O(\rho^3)$

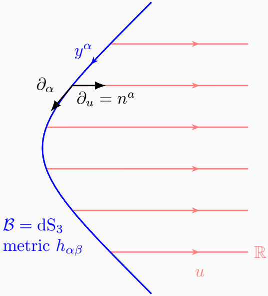
Rigorous definition of Spi:

Let

$$\phi : \mathcal{B} \xrightarrow{J^2\rho} \frac{i^*_{\mathcal{B}}\left(J^2 M\right)}{i^*_{\mathcal{B}}\left(J^2 \rho^2\right)}, \text{then}$$

$$\begin{array}{ccc} \mathrm{Spi} = \phi^* L & & L = i^*_{\mathcal{B}}\left(J^2 M\right) \\ \big\downarrow & & \big\downarrow \\ \mathcal{B} & \xrightarrow{\quad\phi\quad} & \dfrac{i^*_{\mathcal{B}}(J^2 M)}{i^*_{\mathcal{B}}(J^2 \rho^2)} \end{array}$$

Spi, extended boundary



Representation of Spi

Generalized from

[Figueroa-O'Farrill et al, 2022]

$$\left.\begin{aligned} n^a h_{ab} &= 0 \\ \mathcal{L}_n h_{ab} &= 0 \end{aligned}\right\}$$ weak carrollian structure

$\rho' = \rho + u(y^\alpha)\rho^2 + \mathrm{O}(\rho^3)$

Rigorous definition of Spi:

Let

$$\phi : \mathcal{B} \xrightarrow{J^2\rho} \frac{i^*_{\mathcal{B}}\left(J^2 M\right)}{i^*_{\mathcal{B}}\left(J^2\rho^2\right)}, \text{then}$$

$$\begin{array}{ccc} \mathrm{Spi} = \phi^* L & & L = i^*_{\mathcal{B}}\left(J^2 M\right) \\ \downarrow & & \downarrow \\ \mathcal{B} & \xrightarrow{\quad\phi\quad} & \frac{i^*_{\mathcal{B}}(J^2 M)}{i^*_{\mathcal{B}}(J^2\rho^2)} \end{array}$$

SPI group

SPI group: automorphisms of the carrollian structure:

- $\mathcal{L}_\xi n^a = n^a$
- $\mathcal{L}_\xi h_{ab} = h_{ab}$

SPI group: $SO(3,1) \ltimes \mathcal{C}^\infty(\mathrm{dS}_3)$

BMS group: $SO(3,1) \ltimes \mathcal{C}^\infty(\mathcal{S}^2)$

BMS-supertranslations $\subset$ SPI-supertranslations

However, no canonical way to include BMS into SPI

SPI group

SPI group: automorphisms of the carrollian structure:

- $\mathcal{L}_\xi n^a = n^a$
- $\mathcal{L}_\xi h_{ab} = h_{ab}$

SPI group:
$SO(3,1) \ltimes \mathcal{C}^\infty(\mathrm{dS}_3)$
BMS group:
$SO(3,1) \ltimes \mathcal{C}^\infty(\mathcal{S}^2)$

BMS-supertranslations $\subset$ SPI-supertranslations
However, no canonical way to include BMS into SPI

SPI group

SPI group: automorphisms of the carrollian structure:

- $\mathcal{L}_\xi n^a = n^a$
- $\mathcal{L}_\xi h_{ab} = h_{ab}$

SPI group:
$SO(3,1) \ltimes \mathcal{C}^\infty(\mathrm{dS}_3)$
BMS group:
$SO(3,1) \ltimes \mathcal{C}^\infty(\mathcal{S}^2)$

BMS-supertranslations $\subset$ SPI-supertranslations
However, no canonical way to include BMS into SPI

Strong Carrollian structure

Carrollian connection ∇:

$\nabla_c n^a = 0 \qquad \nabla_c h_{ab} = 0$

General form:

$$(\Gamma_c)^a{}_b = \begin{pmatrix} 0 & C_{\gamma\beta}\, dy^\gamma \\ 0 & \Gamma^\alpha_{\gamma\beta}\, dy^\gamma \end{pmatrix}$$

$C_{\alpha\beta} = u h_{\alpha\beta} + \frac{1}{2} k_{\alpha\beta}$

Under supertranslation:

$u' = u + \omega,\ k'_{\alpha\beta} = k_{\alpha\beta} - 2(h_{\alpha\beta} + D_\alpha D_\beta)\omega$

$\implies C'_{\alpha\beta} = C_{\alpha\beta} - D_\alpha D_\beta \omega.$

Strong Carrollian structure

Carrollian connection ∇:

$\nabla_c n^a = 0 \qquad \nabla_c h_{ab} = 0$

General form:

$$(\Gamma_c)^a{}_b = \begin{pmatrix} 0 & C_{\gamma\beta} dy^\gamma \\ 0 & \Gamma^\alpha_{\gamma\beta} dy^\gamma \end{pmatrix}$$

$C_{\alpha\beta} = u h_{\alpha\beta} + \frac{1}{2} k_{\alpha\beta}$

Under supertranslation:

$u' = u + \omega,\ k'_{\alpha\beta} = k_{\alpha\beta} - 2(h_{\alpha\beta} + D_\alpha D_\beta)\omega$

$\implies C'_{\alpha\beta} = C_{\alpha\beta} - D_\alpha D_\beta \omega.$

Strong Carrollian structure

Carrollian connection ∇:

$\nabla_c n^a = 0 \qquad \nabla_c h_{ab} = 0$

General form:

$$(\Gamma_c)^a{}_b = \begin{pmatrix} 0 & C_{\gamma\beta} dy^\gamma \\ 0 & \Gamma^\alpha_{\gamma\beta} dy^\gamma \end{pmatrix}$$

$C_{\alpha\beta} = u h_{\alpha\beta} + \frac{1}{2} k_{\alpha\beta}$

Under supertranslation:

$u' = u + \omega,\ k'_{\alpha\beta} = k_{\alpha\beta} - 2(h_{\alpha\beta} + D_\alpha D_\beta)\omega$

$\implies C'_{\alpha\beta} = C_{\alpha\beta} - D_\alpha D_\beta \omega.$

Strong Carrollian structure

Carrollian connection ∇:

$\nabla_c n^a = 0 \qquad \nabla_c h_{ab} = 0$

General form:

$$(\Gamma_c)^a{}_b = \begin{pmatrix} 0 & C_{\gamma\beta} dy^\gamma \\ 0 & \Gamma^\alpha_{\gamma\beta} dy^\gamma \end{pmatrix}$$

$C_{\alpha\beta} = u h_{\alpha\beta} + \frac{1}{2} k_{\alpha\beta}$

Under supertranslation:

$u' = u + \omega,\ k'_{\alpha\beta} = k_{\alpha\beta} - 2(h_{\alpha\beta} + D_\alpha D_\beta)\omega$

$\implies C'_{\alpha\beta} = C_{\alpha\beta} - D_\alpha D_\beta \omega.$

Strong Carrollian structure

Carrollian connection ∇:

$\nabla_c n^a = 0 \qquad \nabla_c h_{ab} = 0$

General form:

$$(\Gamma_c)^a{}_b = \begin{pmatrix} 0 & C_{\gamma\beta}\, dy^\gamma \\ 0 & \Gamma^\alpha_{\gamma\beta}\, dy^\gamma \end{pmatrix}$$

$C_{\alpha\beta} = u h_{\alpha\beta} + \frac{1}{2} k_{\alpha\beta}$

Under supertranslation:

$u' = u + \omega,\ k'_{\alpha\beta} = k_{\alpha\beta} - 2(h_{\alpha\beta} + D_\alpha D_\beta)\omega$

$\implies C'_{\alpha\beta} = C_{\alpha\beta} - D_\alpha D_\beta \omega.$

Conservation of the strong structure

Naively:

$$\mathcal{L}_\xi h_{ab} = 0, \qquad \mathcal{L}_\xi n^a = 0, \qquad (\mathcal{L}_\xi \nabla_c)^a{}_b = 0$$

Equivalent to

$$\xi^a = \omega(y^\alpha)\partial_u + \chi^\alpha(y^\beta)\partial_\alpha, \qquad \Big(D_\beta D_\gamma + h_{\beta\gamma}\Big)\omega = -\frac{1}{2}\mathcal{L}_\chi k_{\beta\gamma}$$

Only has solutions if Spi is flat ($k_{ab} \sim 0$). In that case, we obtain the *Poincaré* group!

We always have $n^c(\mathcal{L}_\xi \nabla_c)^a{}_b = 0$

Impose $h^{bc}(\mathcal{L}_\xi \nabla_c)^a{}_b = 0 \implies (D^2 + 3)\omega = -\frac{1}{2}\mathcal{L}_\chi k.$

$\implies$ BMS group at spatial infinity?

Conservation of the strong structure

Naively:

$$\mathcal{L}_\xi h_{ab} = 0, \qquad \mathcal{L}_\xi n^a = 0, \qquad (\mathcal{L}_\xi \nabla_c)^a{}_b = 0$$

Equivalent to

$$\xi^a = \omega(y^\alpha)\partial_u + \chi^\alpha(y^\beta)\partial_\alpha, \qquad \left(D_\beta D_\gamma + h_{\beta\gamma}\right)\omega = -\frac{1}{2}\mathcal{L}_\chi k_{\beta\gamma}$$

Only has solutions if Spi is flat ($k_{ab} \sim 0$). In that case, we obtain the *Poincaré* group!

We always have $n^c(\mathcal{L}_\xi \nabla_c)^a{}_b = 0$

Impose $h^{bc}(\mathcal{L}_\xi \nabla_c)^a{}_b = 0 \implies (D^2 + 3)\omega = -\frac{1}{2}\mathcal{L}_\chi k$.

$\implies$ BMS group at spatial infinity?

Conservation of the strong structure

Naively:

$$\mathcal{L}_\xi h_{ab} = 0, \qquad \mathcal{L}_\xi n^a = 0, \qquad (\mathcal{L}_\xi \nabla_c)^a{}_b = 0$$

Equivalent to

$$\xi^a = \omega(y^\alpha)\partial_u + \chi^\alpha(y^\beta)\partial_\alpha, \qquad \left(D_\beta D_\gamma + h_{\beta\gamma}\right)\omega = -\frac{1}{2}\mathcal{L}_\chi k_{\beta\gamma}$$

Only has solutions if Spi is flat ($k_{ab} \sim 0$). In that case, we obtain the *Poincaré* group!

We always have $n^c(\mathcal{L}_\xi \nabla_c)^a{}_b = 0$

Impose $h^{bc}(\mathcal{L}_\xi \nabla_c)^a{}_b = 0 \implies (D^2+3)\omega = -\frac{1}{2}\mathcal{L}_\chi k.$

$\implies$ BMS group at spatial infinity?

Conservation of the strong structure

Naively:

$$\mathcal{L}_\xi h_{ab} = 0, \qquad \mathcal{L}_\xi n^a = 0, \qquad (\mathcal{L}_\xi \nabla_c)^a{}_b = 0$$

Equivalent to

$$\xi^a = \omega(y^\alpha)\partial_u + \chi^\alpha(y^\beta)\partial_\alpha, \qquad \left(D_\beta D_\gamma + h_{\beta\gamma}\right)\omega = -\frac{1}{2}\mathcal{L}_\chi k_{\beta\gamma}$$

Only has solutions if Spi is flat ($k_{ab} \sim 0$). In that case, we obtain the *Poincaré* group!

We always have $n^c(\mathcal{L}_\xi \nabla_c)^a{}_b = 0$

Impose $h^{bc}(\mathcal{L}_\xi \nabla_c)^a{}_b = 0 \implies (D^2+3)\omega = -\frac{1}{2}\mathcal{L}_\chi k.$

$\implies$ BMS group at spatial infinity?

Parity condition and link with regularity

Parity conditions

General solution: $\omega(y^\alpha) = \varpi(y^\alpha) + \omega^O(y^\alpha) + \omega^E(y^\alpha)$

Metric:

$h_{\mathrm{dS}_3} = -d\psi^2 + \cosh^2\psi\, d\Omega^{n-1}$

Parity transformation on dS_3:

$P_{\mathrm{dS}_3} := (\psi, \vartheta) \mapsto (-\psi, a^*\vartheta)$

Solution: parity condition! [Troessaert, 2018]

Extend to Spi: $P_{\mathrm{Spi}} := (u, \psi, \vartheta) \mapsto (-u, -\psi, a^*\vartheta)$

$P_{\mathrm{Spi}*} n^a = -n^a$

$P_{\mathrm{Spi}}{}^* h_{ab} = h_{ab}$

Even strong carrollian geometry: $P_{\mathrm{Spi}}{}^* \nabla = \nabla$

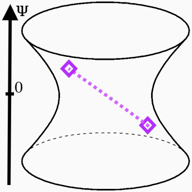
Consequences:

$P_{\mathrm{Spi}}{}^* k_{ab} = -k_{ab}$

$P_{\mathrm{Spi}}{}^* \omega = -\omega$ because $k'_{\alpha\beta} = k_{\alpha\beta} - 2(h_{\alpha\beta} + D_\alpha D_\beta)\omega$

Parity conditions

General solution: $\omega(y^\alpha) = \varpi(y^\alpha) + \omega^O(y^\alpha) + \omega^E(y^\alpha)$



Metric:

$h_{\mathrm{dS}_3} = -d\psi^2 + \cosh^2\psi\, d\Omega^{n-1}$

Parity transformation on dS_3:

$P_{\mathrm{dS}_3} := (\psi, \vartheta) \mapsto (-\psi, a^*\vartheta)$

Solution: parity condition! [Troessaert, 2018]

Extend to Spi: $P_{\mathrm{Spi}} := (u, \psi, \vartheta) \mapsto (-u, -\psi, a^*\vartheta)$

$P_{\mathrm{Spi}*}\, n^a = -n^a$

$P_{\mathrm{Spi}}{}^*\, h_{ab} = h_{ab}$

Even strong carrollian geometry: $P_{\mathrm{Spi}}{}^*\nabla = \nabla$

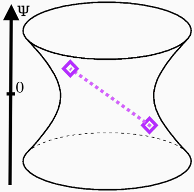
Consequences:

$P_{\mathrm{Spi}}{}^*\, k_{ab} = -k_{ab}$

$P_{\mathrm{Spi}}{}^*\,\omega = -\omega$ because $k'_{\alpha\beta} = k_{\alpha\beta} - 2(h_{\alpha\beta} + D_\alpha D_\beta)\omega$

Parity conditions

General solution: $\omega(y^\alpha) = \varpi(y^\alpha) + \omega^O(y^\alpha) + \omega^E(y^\alpha)$



Metric:

$h_{\mathrm{dS}_3} = -d\psi^2 + \cosh^2\psi \, d\Omega^{n-1}$

Parity transformation on dS_3:

$P_{\mathrm{dS}_3} := (\psi, \vartheta) \mapsto (-\psi, a^* \vartheta)$

Solution: parity condition! [Troessaert, 2018]

Extend to Spi: $P_{\mathrm{Spi}} := (u, \psi, \vartheta) \mapsto (-u, -\psi, a^* \vartheta)$

$P_{\mathrm{Spi}*} n^a = -n^a$

$P_{\mathrm{Spi}}{}^* h_{ab} = h_{ab}$

Even strong carrollian geometry: $P_{\mathrm{Spi}}{}^* \nabla = \nabla$

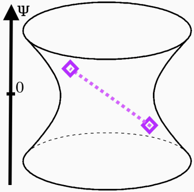
Consequences:

$P_{\mathrm{Spi}}{}^* k_{ab} = -k_{ab}$

$P_{\mathrm{Spi}}{}^* \omega = -\omega$ because $k'_{\alpha\beta} = k_{\alpha\beta} - 2(h_{\alpha\beta} + D_\alpha D_\beta)\omega$

Parity conditions

General solution: $\omega(y^\alpha) = \varpi(y^\alpha) + \omega^O(y^\alpha) + \omega^E(y^\alpha)$



Metric:

$h_{\mathrm{dS}_3} = -d\psi^2 + \cosh^2\psi\, d\Omega^{n-1}$

Parity transformation on dS_3:

$P_{\mathrm{dS}_3} := (\psi, \vartheta) \mapsto (-\psi, a^*\vartheta)$

Solution: parity condition! [Troessaert, 2018]

Extend to Spi: $P_{\mathrm{Spi}} := (u, \psi, \vartheta) \mapsto (-u, -\psi, a^*\vartheta)$

$P_{\mathrm{Spi}*} n^a = -n^a$

$P_{\mathrm{Spi}}{}^* h_{ab} = h_{ab}$

Even strong carrollian geometry: $P_{\mathrm{Spi}}{}^* \nabla = \nabla$

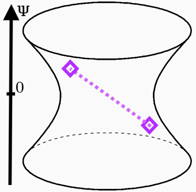
Consequences:

$P_{\mathrm{Spi}}{}^* k_{ab} = -k_{ab}$

$P_{\mathrm{Spi}}{}^* \omega = -\omega$ because $k'_{\alpha\beta} = k_{\alpha\beta} - 2(h_{\alpha\beta} + D_\alpha D_\beta)\omega$

Parity conditions

General solution: $\omega(y^\alpha) = \varpi(y^\alpha) + \omega^O(y^\alpha) + \omega^E(y^\alpha)$



Metric:

$h_{\mathrm{dS}_3} = -d\psi^2 + \cosh^2\psi\, d\Omega^{n-1}$

Parity transformation on dS_3:

$P_{\mathrm{dS}_3} := (\psi, \vartheta) \mapsto (-\psi, a^* \vartheta)$

Solution: parity condition! [Troessaert, 2018]

Extend to Spi: $P_{\mathrm{Spi}} := (u, \psi, \vartheta) \mapsto (-u, -\psi, a^* \vartheta)$

$P_{\mathrm{Spi}*} n^a = -n^a$

$P_{\mathrm{Spi}}{}^* h_{ab} = h_{ab}$

Even strong carrollian geometry: $P_{\mathrm{Spi}}{}^* \nabla = \nabla$

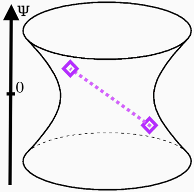
Consequences:

$P_{\mathrm{Spi}}{}^* k_{ab} = -k_{ab}$

$P_{\mathrm{Spi}}{}^* \omega = -\omega$ because $k'_{\alpha\beta} = k_{\alpha\beta} - 2(h_{\alpha\beta} + D_\alpha D_\beta)\omega$

Parity conditions

General solution: $\omega(y^\alpha) = \varpi(y^\alpha) + \omega^O(y^\alpha) + \omega^E(y^\alpha)$



Metric:

$h_{dS_3} = -d\psi^2 + \cosh^2\psi \, d\Omega^{n-1}$

Parity transformation on dS_3:

$P_{dS_3} := (\psi, \vartheta) \mapsto (-\psi, a^* \vartheta)$

Solution: parity condition! [Troessaert, 2018]

Extend to Spi: $P_{Spi} := (u, \psi, \vartheta) \mapsto (-u, -\psi, a^* \vartheta)$

$P_{Spi*} n^a = -n^a$

$P_{Spi}{}^* h_{ab} = h_{ab}$

Even strong carrollian geometry: $P_{Spi}{}^* \nabla = \nabla$

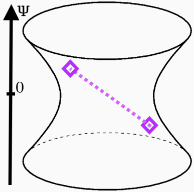
Consequences:

$P_{Spi}{}^* k_{ab} = -k_{ab}$

$P_{Spi}{}^* \omega = -\omega$ because $k'_{\alpha\beta} = k_{\alpha\beta} - 2(h_{\alpha\beta} + D_\alpha D_\beta)\omega$

Parity conditions

General solution: $\omega(y^\alpha) = \varpi(y^\alpha) + \omega^O(y^\alpha) + \omega^E(y^\alpha)$



Metric:

$h_{dS_3} = -d\psi^2 + \cosh^2\psi \, d\Omega^{n-1}$

Parity transformation on dS_3:

$P_{dS_3} := (\psi, \vartheta) \mapsto (-\psi, a^* \vartheta)$

Solution: parity condition! [Troessaert, 2018]

Extend to Spi: $P_{Spi} := (u, \psi, \vartheta) \mapsto (-u, -\psi, a^* \vartheta)$

$P_{Spi*} n^a = -n^a$

$P_{Spi}{}^* h_{ab} = h_{ab}$

Even strong carrollian geometry: $P_{Spi}{}^* \nabla = \nabla$

Consequences:

$P_{Spi}{}^* k_{ab} = -k_{ab}$

$P_{Spi}{}^* \omega = -\omega$ because $k'_{\alpha\beta} = k_{\alpha\beta} - 2(h_{\alpha\beta} + D_\alpha D_\beta)\omega$

General solution of k_{ab}

Einstein's equations: $h^{\gamma\beta} D_{[\beta} k_{\alpha]\gamma} = 0, \quad h^{\gamma\delta} D_\gamma D_{[\delta} k_{\alpha]\beta} = 0$

General solution [Borthwick, Herfray, MC, 2026?]:

$$\begin{cases} \bar{k}_{ss} = 2\partial_s^2 \bar{\omega}, \\ \bar{k}_{is} = 2\nabla_i \partial_s \bar{\omega} + \nabla^k [\Delta_{S^2} + 2] \varphi_{ik}, \\ \bar{k}_{ij} = \left(2\nabla_i \nabla_j|_{\text{tf}} - \gamma_{ij}(1-s^2)\partial_s^2\right)\bar{\omega} + \left(s + (1-s^2)\partial_s\right)\left(2\nabla_{(i}\nabla^k \varphi_{j)k}\right), \end{cases}$$

$\bar{k}_{ab} = \sqrt{1-s^2}\, k_{ab}$, $\bar{\omega} = \sqrt{1-s^2}\,\omega$

$\varphi_{ij} = \varphi \epsilon_{ij}$

φ obeys Legendre equation

(See [Compère, Robert, 2025] for an alternative presentation)

General solution of k_{ab}

Einstein's equations: $h^{\gamma\beta} D_{[\beta} k_{\alpha]\gamma} = 0, \quad h^{\gamma\delta} D_\gamma D_{[\delta} k_{\alpha]\beta} = 0$

General solution [Borthwick, Herfray, MC, 2026?]:

$$\begin{cases} \bar{k}_{ss} = 2\partial_s^2 \bar{\omega}, \\ \bar{k}_{is} = 2\nabla_i \partial_s \bar{\omega} + \nabla^k [\Delta_{S^2} + 2]\varphi_{ik}, \\ \bar{k}_{ij} = \left(2\nabla_i \nabla_j|_{\text{tf}} - \gamma_{ij}(1-s^2)\partial_s^2\right)\bar{\omega} + \left(s + (1-s^2)\partial_s\right)\left(2\nabla_{(i}\nabla^k \varphi_{j)k}\right), \end{cases}$$

$\bar{k}_{ab} = \sqrt{1-s^2}\, k_{ab}$, $\bar{\omega} = \sqrt{1-s^2}\,\omega$

$\varphi_{ij} = \varphi \epsilon_{ij}$

φ obeys Legendre equation

(See [Compère, Robert, 2025] for an alternative presentation)

General solution of k_{ab}

Einstein's equations: $h^{\gamma\beta}D_{[\beta}k_{\alpha]\gamma}=0, \quad h^{\gamma\delta}D_\gamma D_{[\delta}k_{\alpha]\beta}=0$

General solution [Borthwick, Herfray, MC, 2026?]:

$$\begin{cases} \bar{k}_{ss} = 2\partial_s^2\bar{\omega}, \\ \bar{k}_{is} = 2\nabla_i\partial_s\bar{\omega} + \nabla^k[\Delta_{S^2}+2]\varphi_{ik}, \\ \bar{k}_{ij} = \left(2\nabla_i\nabla_j|_{\text{tf}} - \gamma_{ij}(1-s^2)\partial_s^2\right)\bar{\omega} + \left(s+(1-s^2)\partial_s\right)\left(2\nabla_{(i}\nabla^k\varphi_{j)k}\right), \end{cases}$$

$\bar{k}_{ab} = \sqrt{1-s^2}\,k_{ab},\ \bar{\omega} = \sqrt{1-s^2}\,\omega$

$\varphi_{ij} = \varphi\epsilon_{ij}$

φ obeys Legendre equation

(See [Compère, Robert, 2025] for an alternative presentation)

Link with regularity at $\mathscr{I}$

φ obeys Legendre equation

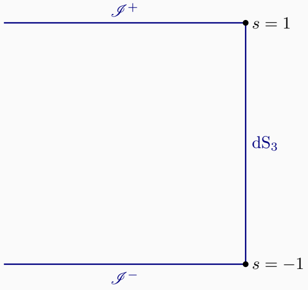
Two branches:

- φ even, k_{ab} odd, completely regular
- φ odd, k_{ab} even, asymptotically goes in $\ln(1-s^2)$ (additional terms in BMS?)

BMS expansion at $\mathscr{I}$:

$$g_{ab} = V\Omega^3 e^{2\beta} d^2u + 2e^{2\beta} du d\Omega + \Gamma_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

Link with regularity at $\mathscr{I}$



Cylinder at spatial infinity [Valiente-Kroon, Magdy, 2021]

φ obeys Legendre equation

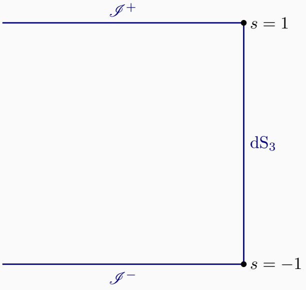
Two branches:

- φ even, k_{ab} odd, completely regular
- φ odd, k_{ab} even, asymptotically goes in $\ln(1-s^2)$ (additional terms in BMS?)

BMS expansion at $\mathscr{I}$:

$$g_{ab} = V\Omega^3 e^{2\beta} d^2u + 2e^{2\beta} du d\Omega + \Gamma_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

Link with regularity at $\mathscr{I}$



Cylinder at spatial infinity [Valiente-Kroon, Magdy, 2021]

φ obeys Legendre equation

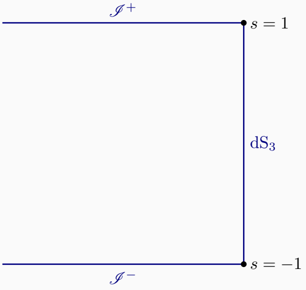
Two branches:

- φ even, k_{ab} odd, completely regular
- φ odd, k_{ab} even, asymptotically goes in $\ln(1-s^2)$ (additional terms in BMS?)

BMS expansion at $\mathscr{I}$:

$$g_{ab} = V\Omega^3 e^{2\beta} d^2u + 2e^{2\beta} du d\Omega + \Gamma_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

Link with regularity at $\mathscr{I}$



Cylinder at spatial infinity [Valiente-Kroon, Magdy, 2021]

φ obeys Legendre equation

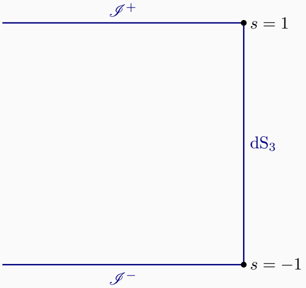
Two branches:

- φ even, k_{ab} odd, completely regular
- φ odd, k_{ab} even, asymptotically goes in $\ln(1-s^2)$ (additional terms in BMS?)

BMS expansion at $\mathscr{I}$:

$$g_{ab} = V\Omega^3 e^{2\beta} d^2u + 2e^{2\beta} du d\Omega + \Gamma_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

Link with regularity at $\mathscr{I}$



Cylinder at spatial infinity [Valiente-Kroon, Magdy, 2021]

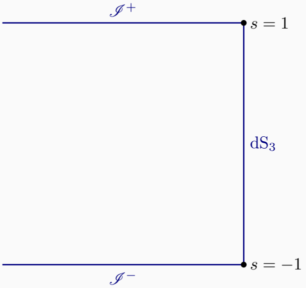
φ obeys Legendre equation

Two branches:

- φ even, k_{ab} odd, completely regular
- φ odd, k_{ab} even, asymptotically goes in $\ln(1-s^2)$ (additional terms in BMS?)

BMS expansion at $\mathscr{I}$:

$$g_{ab} = V\Omega^3 e^{2\beta} d^2u + 2e^{2\beta} du d\Omega + \Gamma_{AB}(dx^A - U^A du)(dx^B - U^B du)$$



Cylinder at spatial infinity [Valiente-Kroon, Magdy, 2021]

φ obeys Legendre equation

Two branches:

- φ even, k_{ab} odd, completely regular
- φ odd, k_{ab} even, asymptotically goes in $\ln(1-s^2)$ (additional terms in BMS?)

BMS expansion at $\mathscr{I}$:

$$g_{ab} = V\Omega^3 e^{2\beta} d^2u + 2e^{2\beta} du d\Omega + \Gamma_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

Conclusion

- Spi, an extended boundary at spatial infinity, can be constructed without any additionnal axiom.
- It possesses some strongly carrollian structure.
- A BMS group at spatial infinity appears as symmetries preserving some of this structure.
- The necessary parity condition which appears along the way is equivalent to regularity conditions on $\mathscr{I}$.

Conclusion

- Spi, an extended boundary at spatial infinity, can be constructed without any additionnal axiom.
- It possesses some strongly carrollian structure.
- A BMS group at spatial infinity appears as symmetries preserving some of this structure.
- The necessary parity condition which appears along the way is equivalent to regularity conditions on $\mathscr{I}$.

Conclusion

- Spi, an extended boundary at spatial infinity, can be constructed without any additionnal axiom.
- It possesses some strongly carrollian structure.
- A BMS group at spatial infinity appears as symmetries preserving some of this structure.
- The necessary parity condition which appears along the way is equivalent to regularity conditions on $\mathscr{I}$.

Conclusion

- Spi, an extended boundary at spatial infinity, can be constructed without any additionnal axiom.
- It possesses some strongly carrollian structure.
- A BMS group at spatial infinity appears as symmetries preserving some of this structure.
- The necessary parity condition which appears along the way is equivalent to regularity conditions on $\mathscr{I}$.

Conclusion

- Spi, an extended boundary at spatial infinity, can be constructed without any additionnal axiom.
- It possesses some strongly carrollian structure.
- A BMS group at spatial infinity appears as symmetries preserving some of this structure.
- The necessary parity condition which appears along the way is equivalent to regularity conditions on $\mathscr{I}$.

End slide

Thanks for listening