Wormholes as the black hole mimickers and their scalar Love numbers

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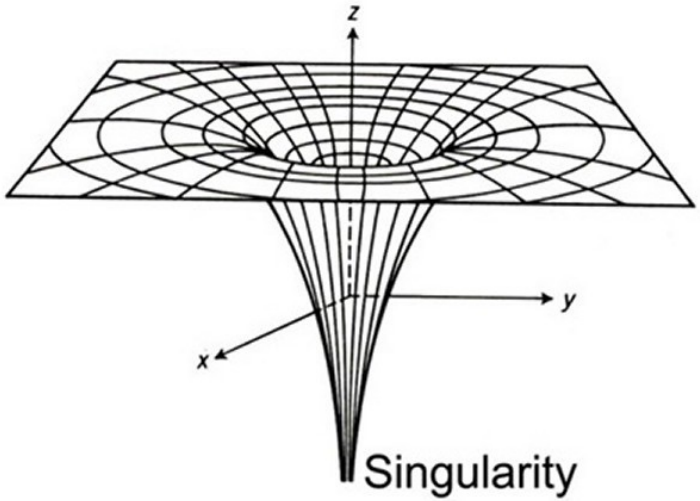
Motivation

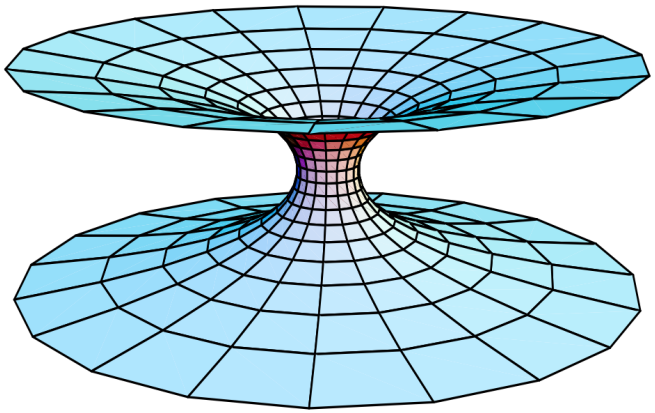
Black hole

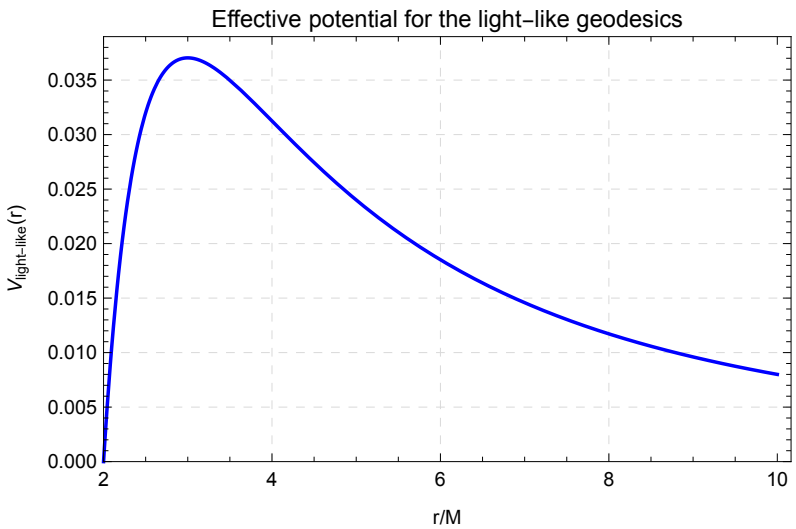


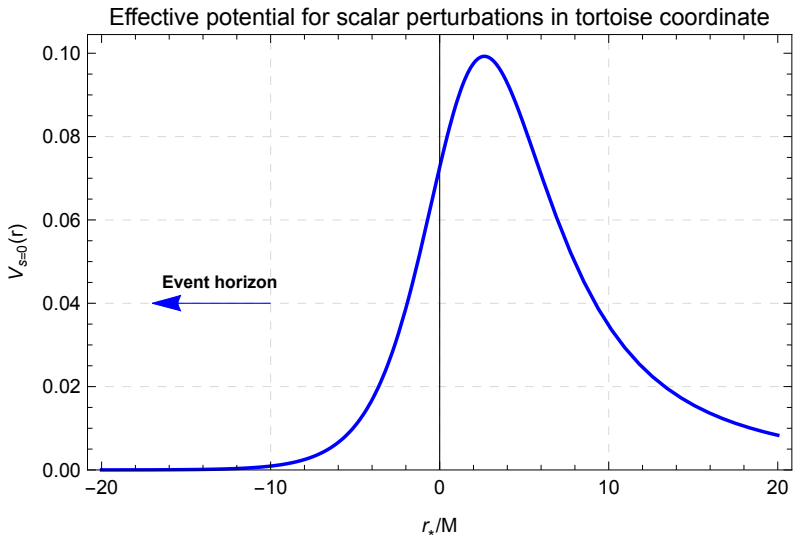
$\Longrightarrow$

quantum modifications near the horizon

Wormhole





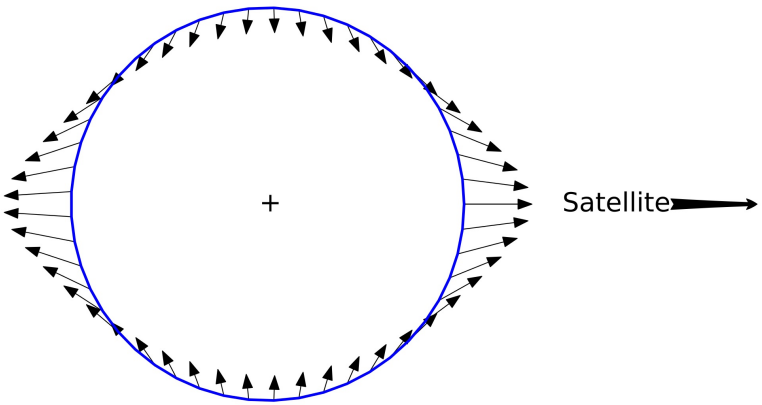


We can not directly examine an event horizon of a black hole by optical and gravitational wave experiments [Cardoso et al. Phys. Rev. Lett. **117** (2016)]

Love numbers

Tidal and response

$$\varphi^{(s)}(x) = \sum_{l=s} \underbrace{\mathcal{E}_{i_1 \dots i_l} x^{i_1} \dots x^{i_l}}_{\sim r^l} \Bigg[\underbrace{\left(1 + a_1 \left(\frac{R}{r}\right) + a_2 \left(\frac{R}{r}\right)^2 + \dots\right)}_{\text{tidal field}} + k_l \left(\frac{R}{r}\right)^{2l+1} \underbrace{\left(1 + b_1 \left(\frac{R}{r}\right) + b_2 \left(\frac{R}{r}\right)^2 + \dots\right)}_{\text{response}} \Bigg]$$



Love numbers

$$k_l = \operatorname{Re} k_l + i \operatorname{Im} k_l$$

Gravitational Corrections

$$a_{2l+1} + k_l \Rightarrow k_l^{\text{Love}} = \bar{k}_l + \hat{k}_l \ln \frac{r}{R}$$

Schwarzschild black hole

Love numbers for spin-s field

$$k_{l,s}^{\mathrm{Sch}}(\omega) = \underbrace{\left(\underbrace{0}_{\text{static}} + \mathcal{O}(\omega r_s)^2\right)}_{\text{conservative}} + \underbrace{\left(i\omega r_s \frac{[(l+s)!(l-s)!]^2}{(2l)!(2l+1)!} + \mathcal{O}(i\omega^3 r_s^3)\right)}_{\text{dissipative}}$$

- Static conservative Love numbers vanish for 4D Schwarzschild black holes.
- This vanishing is related to a hidden Love symmetry of the perturbation equations.[Charalambous, Dubovsky, Ivanov, JHEP 10, 175 (2022)]
- The dynamical response starts at linear order in ωr_s and is dissipative.
- At order $(\omega r_s)^2$, logarithmic terms appear and lead to running Love numbers
  [Saketh, Zhou, Ivanov, PRD 109, 064058 (2024)], [Chakraborty et al., PRD 112, 104015 (2025)]

Wormhole backgrounds

Damour-Solodukhin wormhole [Damour, Solodukhin, PRD 76, 024016 (2007)]

$$ds^2 = -\left(1 - \frac{r_s}{r} + \lambda^2\right) dt^2 + \frac{dr^2}{1 - \frac{r_s}{r}} + r^2 d\Omega_2^2 \quad \xrightarrow{t \to \sqrt{1+\lambda^2}t}$$

$$ds^2 = -\left(1 - \frac{r_s}{(1+\lambda^2)r}\right) dt^2 + \frac{dr^2}{1 - \frac{r_s}{r}} + r^2 d\Omega_2^2, \quad \lambda \ll 1$$

- No event horizon.
- Two asymptotically flat regions (universes) connected by a throat $r_{\text{th}} = r_s$.
- Requires exotic matter in the full spacetime within GR.

Surgical Schwarzschild wormhole [Visser, Nucl. Phys. B 328, 203 (1989)]

$$\mathcal{M} = \mathcal{M}^+_{\text{Sch}}\left(r > r_{\text{th}}\right) \cup \mathcal{M}^-_{\text{Sch}}\left(r > r_{\text{th}}\right) \text{d}, \quad ds^2_{\text{Sch}} = -\left(1 - \frac{r_s}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{r_s}{r}} + r^2 d\Omega_2^2$$

- No event horizon.
- Two asymptotically flat regions (universes) connected by a throat $r_{\text{th}} = r_s\,(1+\delta)$, $\delta \ll 1$.
- Exotic matter is localized only on thin shell. Outside the shell, the geometry is vacuum Schwarzschild.

Damour-Solodukhin wormhole

$$\nabla_\mu \nabla^\mu \varphi(x) = 0 \xrightarrow{\varphi(x)=e^{-i\omega t}R(r)Y_{lm}(\theta,\phi)} \sqrt{\frac{\Delta}{\bar{\Delta}}}\partial_r\left(\sqrt{\Delta\bar{\Delta}}\partial_r R(r)\right) + \left(\frac{r^4\omega^2}{\bar{\Delta}} - l(l+1)\right)R(r) = 0, \quad \Delta = r^2 - r_s r,\ \bar{\Delta} = r^2 - \frac{r_s}{1+\lambda^2}r$$

Equation

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}R(z) + \left(\frac{1}{2z} + \frac{1}{z-1} + \frac{1}{2(z-t)}\right)\frac{\mathrm{d}}{\mathrm{d}z}R(z) + \frac{(\omega r_s)^2(1-z)^4 - l(l+1)(z-1)(z-t)}{z(z-1)^2(z-t)}R(z) = 0$$

where $t = \frac{\lambda^2}{1+\lambda^2}$ and $z = \frac{r_s - r}{r_s}$.

- Three regular singular points: $z = 0$ ($r = r_{\text{th}} = r_s$), $z = t$ ($r = \frac{r_s}{1+\lambda^2}$) and $z = 1$ ($r = 0$).
- One irregular singular point $z = \infty$ ($r = \infty$)

Small frequency regime $\omega r_s \ll 1$ + near zone/far zone approximations to replace the irregular singularity by a regular one.

Equation in near zone approximation $r_{\text{th}} < r \ll \frac{1}{\omega}$

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}R(z) + \left(\frac{1}{2z} + \frac{1}{z-1} + \frac{1}{2(z-t)}\right)\frac{\mathrm{d}}{\mathrm{d}z}R(z) + \frac{(\omega r_s)^2 - l(l+1)(z-1)(z-t)}{z(z-1)^2(z-t)}R(z) = 0$$

Solution

$$R_{(0)}^{(\text{right,left})}(z) = (1-z)^{i\omega r_s}\left[A_1^{(\text{right,left})}u_+^{(0)}(z) + A_2^{(\text{right,left})}u_-^{(0)}(z)\right], \quad \hat{l} = \underbrace{l}_{\in\mathbb{Z}_{\geq 0}} - \epsilon, \underbrace{\left(\hat{l} = \frac{l}{D-3}\right)}_{\text{EFT}}$$

Solutions at $z = 0$ ($r = r_{\text{th}}$)

$$u_-^{(0)}(z) = \text{HeunG}\left(t, q \middle| -\hat{l} + i\omega r_s, \hat{l} + 1 + i\omega r_s, \frac{1}{2}, 1 + 2i\omega r_s \middle| z\right), \quad q = -t\hat{l}(\hat{l}+1) + i\omega r_s \frac{t}{2} + \omega^2 r_s^2$$

$$u_+^{(0)}(z) = z^{1/2}\text{HeunG}\left(t, q + \frac{1}{2}\left(t(1+2i\omega r_s) + \frac{1}{2}\right) \middle| -\hat{l} + i\omega r_s + \frac{1}{2}, \hat{l} + \frac{3}{2} + i\omega r_s, \frac{3}{2}, 1 + 2i\omega r_s \middle| z\right)$$

Solution at $z = \infty$ ($r = \infty$)

$$u_+^{(\infty)}(z) = z^{\hat{l} - i\omega r_s}\text{HeunG}\left(t, \hat{l}^2 - \frac{\hat{l}}{2}(t + 4i\omega r_s) \middle| -\hat{l} + i\omega r_s, -\hat{l} + \frac{1}{2} + i\omega r_s, -2\hat{l}, \frac{1}{2} \middle| \frac{t}{z}\right)$$

$$u_-^{(\infty)}(z) = z^{-\hat{l} - 1 - i\omega r_s}\text{HeunG}\left(t, (\hat{l}+1)^2 + \frac{\hat{l}+1}{2}(t + 4i\omega r_s) \middle| \hat{l} + 1 + i\omega r_s, \hat{l} + \frac{3}{2} + i\omega r_s, 2\hat{l} + 2, \frac{1}{2} \middle| \frac{t}{z}\right)$$

$$\text{HeunG}(t, q|\alpha, \beta, \gamma, \delta|z) = \sum_{k=0}^{\infty} \frac{\Gamma(\gamma)}{t^k \Gamma(\gamma + k) k!} \Delta_k z^k, \quad \Delta_k = (q + Q_{k-1})\,\Delta_{k-1} - P_{k-1}R_{k-2}\Delta_{k-2}, \quad \Delta_0 = 1, \quad \Delta_{-1} = 0$$

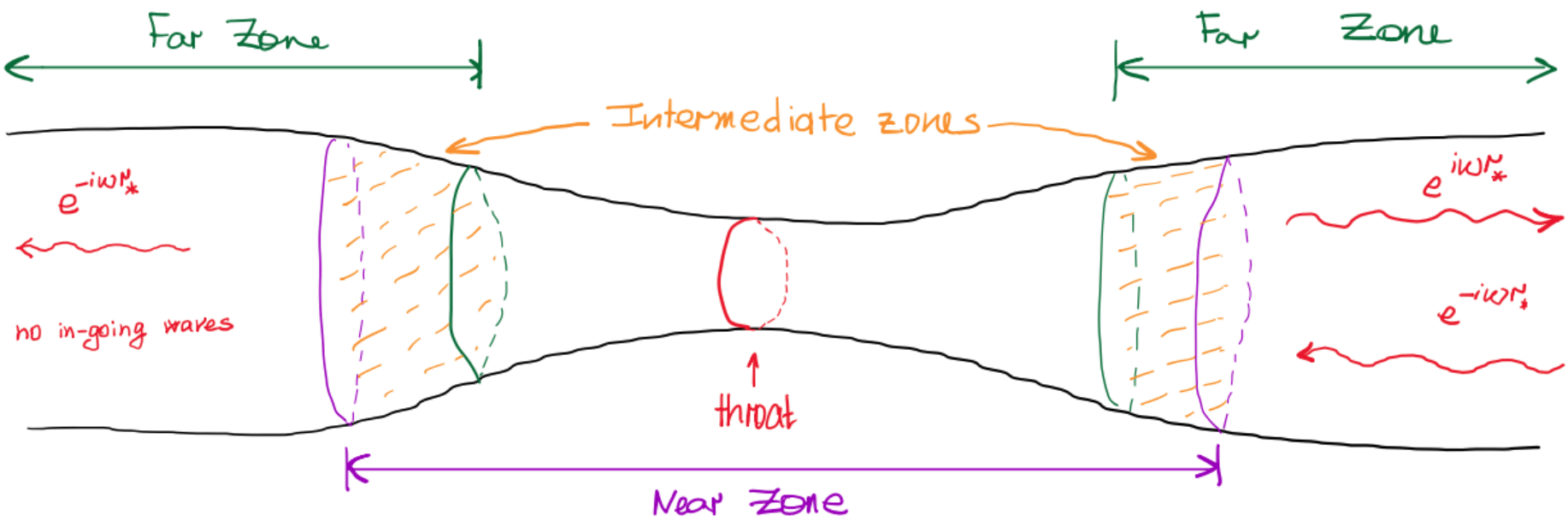
$$R_k = t(k+1)(k+\gamma), \quad Q_k = k\left((k-1+\gamma)(1+t) + t\delta + 1 + \alpha + \beta - \gamma - \delta\right), \quad P_k = (k-1+\alpha)(k-1+\beta)$$

Matching at the throat

Matching at the throat: conservation of the energy flux through the throat

$$\mathcal{F}^{(\text{left})}(\omega, r_{\text{th}}) + \mathcal{F}^{(\text{right})}(\omega, r_{\text{th}}) = 0, \quad \mathcal{F}(\omega) = \frac{1}{2i}\left(\bar{\psi}(r_*)\partial_{r_*}\psi(r_*) - \psi(r_*)\partial_{r_*}\bar{\psi}(r_*)\right) \Rightarrow$$

$$A_1^{(\text{left})} = A_1^{(\text{right})} = A_1, \quad A_2^{(\text{left})} = -A_2^{(\text{right})} = -A_2$$



Solutions in the intermediate zones $r_{\text{th}} \ll r \ll \frac{1}{\omega}$

$$R^{(\text{right,left})}(r) = \left(\frac{r}{r-r_s}\right)^{i\omega r_s}\left\{\left(A_1 C^{(0)+}_{-(\infty)} \pm A_2 C^{(0)+}_{+(\infty)}\right)\left(\frac{r}{r_s}\right)^{l} Z^{\text{source}}(r) + \left(A_1 C^{(0)-}_{-(\infty)} \pm A_2 C^{(0)-}_{+(\infty)}\right)\left(\frac{r}{r_s}\right)^{-l-1} Z^{\text{response}}(r)\right\}$$

$C^{(0)\pm}_{\pm(\infty)}$ are the connection coefficients for Heun functions, relating the solutions around $z = 0$ and $z = \infty$ [Bonelli et al., Commun. Math. Phys. 397, 635 (2023)].

Far Zone approximation

Source/Response functions

$$Z^{\text{source}}(r) = \left(1-\frac{r_s}{r}\right)^{\hat{l}} \text{HeunG}\left(t, \hat{l}^2 - \frac{\hat{l}}{2}(t+4i\omega r_s)\middle| -\hat{l}+i\omega r_s, -\hat{l}+i\omega r_s+\frac{1}{2}, -2\hat{l}, \frac{1}{2}\middle| \frac{tr_s}{r_s-r}\right)$$

$$Z^{\text{response}}(r) = \left(1-\frac{r_s}{r}\right)^{-\hat{l}-1} \text{HeunG}\left(t, (\hat{l}+1)^2 + \frac{\hat{l}+1}{2}(t+4i\omega r_s)\middle| \hat{l}+1+i\omega r_s, \hat{l}+i\omega r_s+\frac{3}{2}, 2\hat{l}+2, \frac{1}{2}\middle| \frac{tr_s}{r_s-r}\right)$$

Far Zone approximation $r \gg r_{\text{th}}$

$$\partial_r\left(r^2\partial_r R(r)\right) + \left(\omega^2 r^2 - \hat{l}(\hat{l}+1)\right)R(r) = 0 \Rightarrow R^{(\text{right,left})}(r) = \sqrt{\frac{\pi}{2\omega r}}\left[B_1^{(\text{right,left})} J_{\hat{l}+\frac{1}{2}}(\omega r) + B_2^{(\text{right,left})}(-1)^{\hat{l}+1} J_{-\hat{l}-\frac{1}{2}}(\omega r)\right]$$

Vanishing of the left in-going solution

$$B_2^{(\text{left})} = iB_1^{(\text{left})}$$

Matching solutions in the left universe in the intermediate zone + vanishing of the left in-going solution

$$\frac{A_1}{A_2} = \frac{C^{(0)+}_{+(\infty)}}{C^{(0)+}_{-(\infty)}} \underbrace{\frac{1 - i\,(\omega r_s)^{2\hat{l}+1}\left(C^{(0)-}_{+(\infty)}/C^{(0)+}_{+(\infty)}\right)\frac{2^{2\hat{l}}\Gamma^2(\hat{l}+1)}{\Gamma(2\hat{l}+1)\Gamma(2\hat{l}+2)}}{1 - i\,(\omega r_s)^{2\hat{l}+1}\left(C^{(0)-}_{-(\infty)}/C^{(0)+}_{-(\infty)}\right)\frac{2^{2\hat{l}}\Gamma^2(\hat{l}+1)}{\Gamma(2\hat{l}+1)\Gamma(2\hat{l}+2)}}}_{\kappa}$$

Solution in the Intermediate Zone

Intermediate zone in right universe

$$R^{(\text{right})}(r) \sim \left(\frac{r}{r_s}\right)^{\hat{l}} \left(\frac{r}{r-r_s}\right)^{i\omega r_s} \left[Z^{\text{source}}(r) + k\left(\frac{r_s}{r}\right)^{2\hat{l}+1} Z^{\text{response}}(r)\right], \quad k = \frac{C^{(0)-}_{-(\infty)}}{C^{(0)+}_{-(\infty)}} \frac{\kappa}{1+\kappa} + \frac{C^{(0)-}_{+(\infty)}}{C^{(0)+}_{+(\infty)}} \frac{1}{1+\kappa}$$

Expansion of k in ϵ ($\hat{l} \to \mathbb{Z}_{\geq 0} - \epsilon$)

$$k = \frac{(l!)^4}{(2l)!(2l+1)!} \left\{ -t\frac{l(l+1)}{4\epsilon} + t\frac{2l+1}{4} + \frac{1}{2\left(\ln\frac{t}{4} + 2H_l\right)} \left(1 - \frac{t}{2}(1+2l) - \frac{t}{2}\frac{1}{\ln\frac{t}{4} + 2H_l}\right) + \right.$$

$$+ t\frac{l(l+1)}{4\left(\ln\frac{t}{4} + 2H_l\right)} \left[\frac{1}{3}\left(\ln\frac{t}{4} + 2H_l\right)^2 + \left(\ln\frac{t}{4} + 2H_l\right)\left(\ln\frac{t}{4} + 2H_l + 4\psi(l+1) - \right.\right.$$

$$\left.\left. -2\psi(2l+1) - 2\psi(2l+2)\right) - \frac{\pi^2}{3} - 2H_l^{(2)} - \frac{6\zeta(3) + 2H_l^{(3)}}{3\left(\ln\frac{t}{4} + 2H_l\right)}\right] \left.\right\} + \mathcal{O}\left((\omega r_s)^2\right), \quad \kappa = 1 + \mathcal{O}\left((\omega r_s)^{2l+1}\right)$$

$\psi(x)$ is a digamma function, $H_l^{(r)} = \sum_{k=1}^{l} \frac{1}{k^r}$ are the generalized harmonic numbers. The function $Z^{\text{source}}(r)$ is singular when $l \in \mathbb{Z}_{\geq 0}$. Therefore,

Gravitational "corrections" to the tidal field

$$R^{(\text{right})}(r) \sim \left(\frac{r}{r_s}\right)^{l} \left[\underbrace{\left(1 + a_1\left(\frac{r_s}{r}\right) + \cdots + \underbrace{a_{2l+1}}_{\neq 0}\left(\frac{r_s}{r}\right)^{2l+1} + \ldots\right)}_{\text{tidal field}} + k_l\left(\frac{r_s}{r}\right)^{2l+1} \underbrace{(1+\ldots)}_{\text{response}}\right]$$

Scalar Love numbers of the DS wormhole

Combining together $a_{2l+1} + k_l$

$$k_{\text{Love}}^{(\omega\neq 0)} = \frac{(l!)^4}{2(2l)!(2l+1)!}\frac{1}{\ln\frac{t}{4}+2H_l}\left(1-\frac{t}{2}\frac{1+(1+2l)\left(\ln\frac{t}{4}+2H_l\right)}{\ln\frac{t}{4}+2H_l}\right)-$$

$$-t\frac{(l!)^4 l(l+1)}{2(2l)!(2l+1)!}\ln\left(\frac{r-r_s}{r_s}\right)+t\frac{(l!)^4 l(l+1)}{4(2l)!(2l+1)!\left(\ln\frac{t}{4}+2H_l\right)}\left[\frac{1}{3}\left(\ln\frac{t}{4}+2H_l\right)^2+\right.$$

$$\left.+\left(\ln\frac{t}{4}+2H_l\right)\left(\ln\frac{t}{4}+6H_l-2H_{2l+1}\right)-\frac{\pi^2}{3}-2H_l^{(2)}-\frac{6\zeta(3)+2H_l^{(3)}}{3\left(\ln\frac{t}{4}+2H_l\right)}\right]-$$

$$-t\frac{l(l+1)(l!)^3}{2(2l)!}\sum_{n=0}^{l-1}\frac{n!(l-1-n)!}{(l+1+n)!(l-n)!}-\frac{(l!)^4}{(2l)!(2l+1)!}i\omega r_s\left\{1+tl(l+1)\left[\frac{(l-1)!(2l+1)!}{2(l+1)!(l!)^2}+\right.\right.$$

$$\left.\left.+\frac{(2l+1)!}{l!}\sum_{n=0}^{l-1}\frac{n!(l-1-n)!}{(l+1+n)!(l-n)!}(H_n-H_l)\right]\right\}+\mathcal{O}\left((\omega r_s)^2,t^2\right)$$

$$k_{\text{Love}}^{(\omega\neq 0)} = \text{Near Zone} + \text{Intermediate Zone} + \text{running Love numbers}$$

Running Love numbers

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}k_{\text{Love}}(\mu) = t\frac{(l!)^4 l(l+1)}{2(2l)!(2l+1)!},\quad \mu=(r-r_s)^{-1},\quad k_{\text{Love}}^{(\omega\neq 0)}\sim\frac{1}{\ln\frac{t}{4}+2H_l}$$ [Cardoso et al., PRD 95, 084014 (2017)]

Surgical Schwarzschild wormhole

$$\nabla_\mu \nabla^\mu \varphi(x) = 0 \xrightarrow{\varphi(x)=e^{-i\omega t}R(r)Y_{lm}(\theta,\phi)} \partial_r\left(\Delta \partial_r R(r)\right) + \left(\frac{r^4\omega^2}{\Delta} - l(l+1)\right) R(r) = 0, \quad \Delta = r^2 - r_s r$$

Equation

$$\frac{\mathrm{d}^2}{\mathrm{d}r^2}R(r) + \left(\frac{1}{r} + \frac{1}{r-r_s}\right)\frac{\mathrm{d}}{\mathrm{d}r}R(r) + \frac{\omega^2 r^4 - l(l+1)r(r-r_s)}{r^2(r-r_s)^2}R(r) = 0$$

- Two regular singular points: $r = r_s$, $r = 0$.
- One irregular singular point $r = \infty$.

Small frequency regime $\omega r_s \ll 1$ + near zone/far zone approximations to replace the irregular singularity by a regular one.

Equation in near zone approximation $r_{\text{th}} = r_s(1+\delta) < r \ll \frac{1}{\omega}$

$$\frac{\mathrm{d}^2}{\mathrm{d}r^2}R(z) + \left(\frac{1}{r} + \frac{1}{r-r_s}\right)\frac{\mathrm{d}}{\mathrm{d}r}R(r) + \frac{(\omega r_s)^2 - \hat{l}(\hat{l}+1)r(r-r_s)}{r^2(r-r_s)^2}R(r) = 0, \quad \hat{l} = \underbrace{l}_{\in\mathbb{Z}_{\geq 0}} - \epsilon, \quad \left(\text{EFT: } \hat{l} = \frac{l}{D-3}\right)$$

Solution in the near zone approximation

$$R^{(\text{right,left})}(z) = \left(\frac{r}{r_s}\right)^{\hat{l}}\left[A_1^{(\text{right,left})}\left(1-\frac{r_s}{r}\right)^{-i\omega r_s}{}_2F_1\left(-\hat{l}, -\hat{l}-2i\omega r_s, 1-2i\omega r_s; \frac{r-r_s}{r}\right) + \right.$$
$$\left. + A_2^{(\text{right,left})}\left(1-\frac{r_s}{r}\right)^{i\omega r_s}{}_2F_1\left(-\hat{l}, -\hat{l}+2i\omega r_s, 1+2i\omega r_s; \frac{r-r_s}{r}\right)\right]$$

Matching at the throat

Matching at the throat: conservation of the energy flux through the throat $r_{\rm th} = r_s(1+\delta)$, $\delta \ll 1$

$$\mathcal{F}^{(\rm left)}(\omega, r_{\rm th}) + \mathcal{F}^{(\rm right)}(\omega, r_{\rm th}) = 0, \quad \mathcal{F}(\omega) = \frac{1}{2i}\left(\bar{\psi}(r_*)\partial_{r_*}\psi(r_*) - \psi(r_*)\partial_{r_*}\bar{\psi}(r_*)\right) \Rightarrow$$

$$A_1^{(\rm right)} = \delta^{2i\omega r_s} A_2^{(\rm left)} \left(1 - i\delta \frac{\hat{l}(\hat{l}+1)+1+2\omega^2 r_s^2}{\omega r_s}\right) - iA_1^{(\rm left)} \delta \frac{\hat{l}(\hat{l}+1)+4\omega^2 r_s^2}{\omega r_s + 4\omega^3 r_s^3}$$

$$A_2^{(\rm right)} = \delta^{-2i\omega r_s} A_1^{(\rm left)} \left(1 + i\delta \frac{\hat{l}(\hat{l}+1)+1+2\omega^2 r_s^2}{\omega r_s}\right) + iA_2^{(\rm left)} \delta \frac{\hat{l}(\hat{l}+1)+4\omega^2 r_s^2}{\omega r_s + 4\omega^3 r_s^3}$$

Solutions in the intermediate zones $r_{\rm th} \ll r \ll \frac{1}{\omega}$

$$R^{(\rm right,left)}(r) = \left(1-\frac{r_s}{r}\right)^{-i\omega r_s}\left[\left(A_1^{(\rm right,left)}\frac{\Gamma(1-2i\omega r_s)}{\Gamma(\hat{l}+1-2i\omega r_s)} + A_2^{(\rm right,left)}\frac{\Gamma(1+2i\omega r_s)}{\Gamma(\hat{l}+1+2i\omega r_s)}\right)\frac{\Gamma(2\hat{l}+1)}{\Gamma(\hat{l}+1)}\left(\frac{r}{r_s}\right)^{\hat{l}} Z^{\rm source}(r) + \right.$$

$$\left. + \left(A_1^{(\rm right,left)}\frac{\Gamma(1-2i\omega r_s)}{\Gamma(-\hat{l}-2i\omega r_s)} + A_2^{(\rm right,left)}\frac{\Gamma(1+2i\omega r_s)}{\Gamma(-\hat{l}+2i\omega r_s)}\right)\frac{\Gamma(-2\hat{l}-1)}{\Gamma(-\hat{l})}\left(\frac{r}{r_s}\right)^{-\hat{l}-1} Z^{\rm response}(r)\right]$$

$$Z^{\rm source}(r) = \left(1-\frac{r_s}{r}\right)^{\hat{l}} {}_2F_1\left(-\hat{l}, -\hat{l}+2i\omega r_s, -2\hat{l}; \frac{r_s}{r_s - r}\right)$$

$$Z^{\rm response}(r) = \left(1-\frac{r_s}{r}\right)^{-\hat{l}-1} {}_2F_1\left(\hat{l}+1, \hat{l}+1+2i\omega r_s, 2\hat{l}+2; \frac{r_s}{r_s - r}\right)$$

Far Zone approximation/Solution in the Intermediate Zone

Far Zone approximation $r \gg r_{\text{th}}$

$$\partial_r\left(r^2\partial_r R(r)\right)+\left(\omega^2 r^2-\hat{l}(\hat{l}+1)\right)R(r)=0 \Rightarrow R^{(\text{right,left})}(r)=\sqrt{\frac{\pi}{2\omega r}}\left[B_1^{(\text{right,left})}J_{\hat{l}+\frac{1}{2}}(\omega r)+B_2^{(\text{right,left})}(-1)^{\hat{l}+1}J_{-\hat{l}-\frac{1}{2}}(\omega r)\right]$$

Vanishing of the left in-going solution

$$B_2^{(\text{left})}=iB_1^{(\text{left})}$$

Matching solutions in the left universe in the intermediate zone + vanishing of the left in-going solution

$$\frac{A_1^{(\text{left})}}{A_2^{(\text{left})}}=-\frac{\Gamma(1+2i\omega r_s)\Gamma(\hat{l}+1-2i\omega r_s)}{\Gamma(1-2i\omega r_s)\Gamma(\hat{l}+1+2i\omega r_s)}\underbrace{\frac{1-i2^{2\hat{l}}(\omega r_s)^{2\hat{l}+1}\frac{\Gamma^3(\hat{l}+1)}{\Gamma(2\hat{l}+2)\Gamma^2(2\hat{l}+1)}\frac{\Gamma(-2\hat{l}-1)\Gamma(\hat{l}+1+2i\omega r_s)}{\Gamma(-\hat{l})\Gamma(-\hat{l}+2i\omega r_s)}}{1-i2^{2\hat{l}}(\omega r_s)^{2\hat{l}+1}\frac{\Gamma^3(\hat{l}+1)}{\Gamma(2\hat{l}+2)\Gamma^2(2\hat{l}+1)}\frac{\Gamma(-2\hat{l}-1)\Gamma(\hat{l}+1-2i\omega r_s)}{\Gamma(-\hat{l})\Gamma(-\hat{l}-2i\omega r_s)}}}_{\kappa}$$

Intermediate zone in right universe

$$R^{(\text{right})}(r)\sim\left(\frac{r}{r_s}\right)^{\hat{l}}\left(\frac{r}{r-r_s}\right)^{i\omega r_s}\left[Z^{\text{source}}(r)+k\left(\frac{r_s}{r}\right)^{2\hat{l}+1}Z^{\text{response}}(r)\right]$$

$$k=\frac{\Gamma(-2\hat{l}-1)\Gamma(\hat{l}+1)\Gamma(\hat{l}+1-2i\omega r_s)}{\Gamma(2\hat{l}+1)\Gamma(-\hat{l})\Gamma(-\hat{l}-2i\omega r_s)}\frac{1-\left(A_2^{(\text{right})}/A_1^{(\text{right})}\right)\frac{\Gamma(1+2i\omega r_s)\Gamma(\hat{l}+1-2i\omega r_s)}{\Gamma(1-2i\omega r_s)\Gamma(\hat{l}+1+2i\omega r_s)}}{1+\left(A_2^{(\text{right})}/A_1^{(\text{right})}\right)\frac{\Gamma(1+2i\omega r_s)\Gamma(\hat{l}+1-2i\omega r_s)}{\Gamma(1-2i\omega r_s)\Gamma(\hat{l}+1+2i\omega r_s)}}$$

Scalar Love numbers of surgical Schwarzschild wormhole

Gravitational "corrections" to the tidal field

$$R^{(\text{right})}(r) \sim \left(\frac{r}{r_s}\right)^l \left[\underbrace{\left(1 + a_1\left(\frac{r_s}{r}\right) + \cdots + a_l\left(\frac{r_s}{r}\right)^l\right)}_{\text{tidal field}} + k_l\left(\frac{r_s}{r}\right)^{2l+1} \underbrace{(1+\ldots)}_{\text{response}}\right], \quad \Rightarrow \quad a_{2l+1} = 0$$

Love numbers

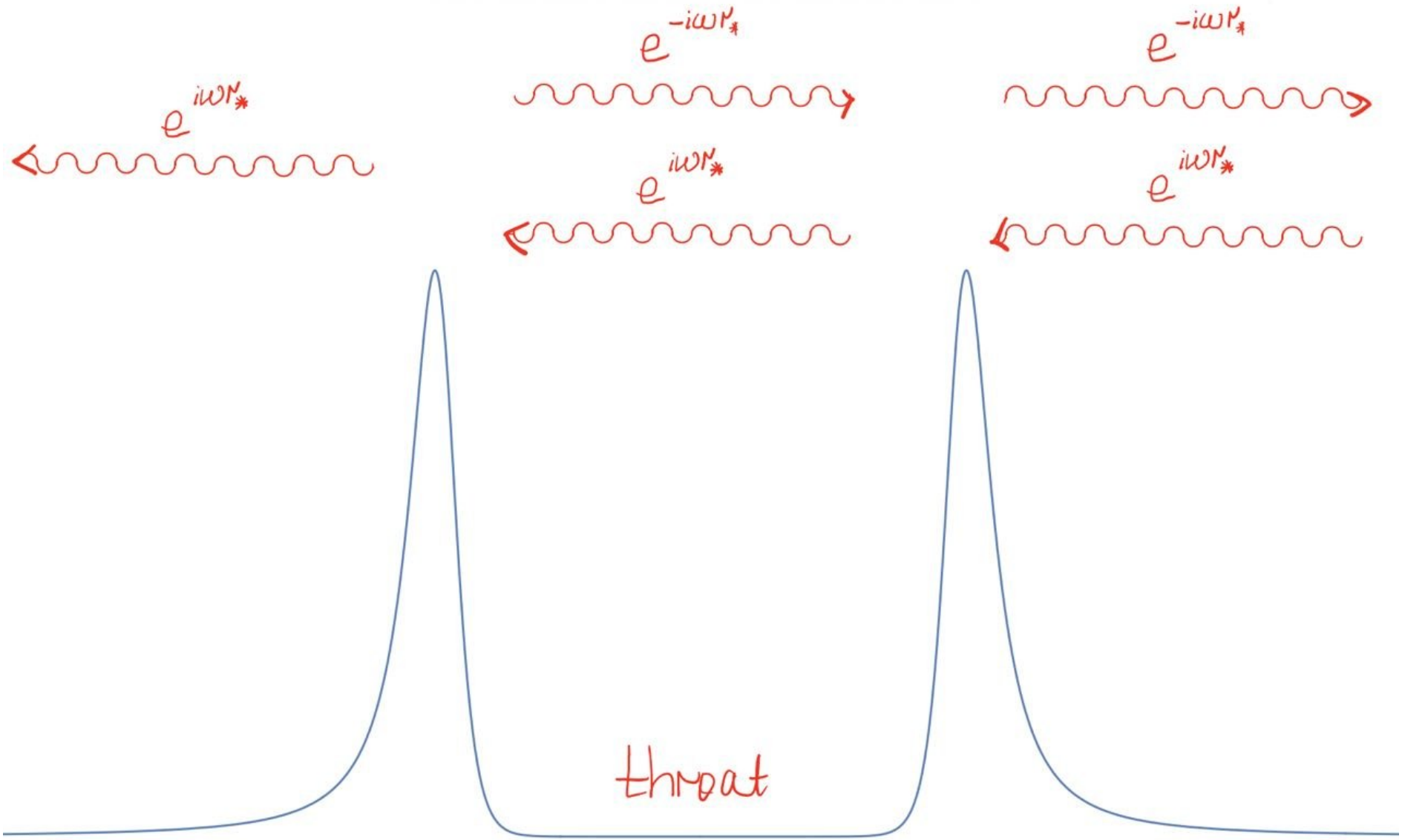
$$k_{\text{Love}} = \frac{(l!)^4}{2(2l+1)!(2l)!}\left[\frac{1}{\ln\delta + 2H_l} + \frac{\delta}{\ln\delta + 2H_l}\left((1+l(l+1))(\ln\delta + 2H_l) + \frac{1+2l(l+1)}{\ln\delta + 2H_l}\right)\right] + \mathcal{O}(\omega^2 r_s^2)$$

$k_{\text{Love}} \sim \frac{1}{\ln\delta + 2H_l}$ [Cardoso et al., PRD 95, 084014 (2017)]

- No running Love numbers.
- The dynamical Love numbers start at order $(\omega r_s)^2$.
- In the limit $\delta \to 0$, the Schwarzschild black hole Love numbers are not reproduced.
- The Love numbers are determined entirely by the near zone behavior.
- Actually, based on the dependence on κ, one expects Im $k_{\text{Love}} \sim (\omega r_s)^{2l+3}$

Conclusion

- Two static wormhole geometries were studied
- Static scalar Love numbers were computed. Their behavior is consistent with the previously known logarithmic behavior at leading order [Cardoso et al., PRD 95, 084014 (2017)].
- DS wormhole: dynamical Love numbers start at ωr_s, are dissipative and are running. They are arise at first order from gravitational corrections to the tidal-field contribution in the intermediate zone. In the limit $t \approx \lambda^2 \to 0$, the Schwarzschild black hole Love numbers are reproduced.
- Surgical Schwarzschild wormhole: dynamical Love numbers start at $(\omega r_s)^2$, dissipative appears only at $(\omega r_s)^{2l+3}$. In the limit $\delta \to 0$, the Schwarzschild black hole Love numbers are NOT reproduced.
- Love numbers can distinguish not only black holes from wormholes, but also different wormhole geometries, that may seem "topologically" equivalent at first sight.

Thank you for your attention!

Dissipative properties of a wormhole