Flat from AdS: In Any Dimension, for Any Spin

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The Clash: AdS v/s Flat

The goal

Fronsdal's equations in AdS_{d+2}

$$\mathcal{F}_{\mu(s)} := \Box\varphi_{\mu(s)} - s\,\nabla^{\alpha}\nabla_{\mu}\varphi_{\mu(s-1)\alpha} - \frac{2(s-1)(d+s-1)}{R^2}\,\varphi_{\mu(s)} = 0$$

Anti De Sitter

Solutions split rigidly into two **independent** branches near the boundary:

- **Source** (Δ_- branch)
- **Vev** (Δ_+ branch)

Minkowski

Solutions form a **coupled** structure near null infinity $\mathcal{I}^+$:

- **Radiation** (Shear)
- **Aspects** (Mass, Angular momentum)
- **Chthonians** (Subleading tower)
- **And more...**

Question: How does uncoupled AdS data dynamically reorganize into coupled flat-space asymptotic data?

The setup

Flat Bondi Coordinates

- Smooth limit as $R \to \infty$ from AdS to Minkowski

$$\mathrm{d}s^2_{AdS_{d+2}} = \frac{1}{z^2}\left[2\,\mathrm{d}u\mathrm{d}z - \frac{1}{R^2}\,\mathrm{d}u^2 + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right]$$

$$\Big\downarrow{\scriptstyle R\to\infty}$$

$$\mathrm{d}s^2_{Mink_{d+2}} = \frac{1}{z^2}\left[2\,\mathrm{d}u\mathrm{d}z + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right]$$

Bondi Like gauge

$$\varphi_{z\mu_1\ldots\mu_{s-1}} = 0, \quad g^{\alpha\beta}\varphi_{\alpha\beta\mu_1\ldots\mu_{s-2}} = 0$$

- **Maxwell:** Reduces to standard radial gauge ($A_z = 0$).
- **Linearised gravity:** Reduces to linearised Bondi gauge $h_{zz} = h_{zi} = 0$ and $\delta^{ij}h_{ij} = 0$, and on shell $h_{uz} = 0$ using residual gauge freedom.

Solution space in AdS

Solution space of Maxwell fields on AdS_{d+2}: Source

Asymptotic expansion:

$$A_\mu = z^0 A_\mu^{(0)} + z A_\mu^{(1)} \cdots + z^{d-2} A_\mu^{(d-2)} + z^{d-1} A_\mu^{(d-1)} + \ldots$$

1. The source (Δ_-) at order z^0

- $A_u^{(0)}(u, \mathbf{x})$ can be killed by residual gauge freedom
- $A_i^{(0)}(u, \mathbf{x})$ is free data source.

The recursion

$$A_i^{(n)} = R^2 \left(f^{(n)} A_i^{(n-1)} + g_{ij}^{(n)} A_j^{(n-2)} \right)$$

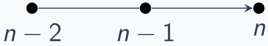
with

$$f^{(n)} = -(2n-d) b_n \, \partial_u \ , \qquad g_{ij}^{(n)} = -b_n \Big(\delta_{ij} \boldsymbol{\Delta} + (d-2) b_{n-1} \, \partial_i \partial_j \Big)$$

Diagrammatic representation of the recursion

Assign the weight $w_n := R^2 f^{(n)}$ to the single path

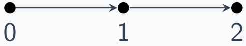
and the weight $w_n := R^2 g^{(n)}$ to the double path



$A_i^{(n)}$ is given by sum over distinct diagrams from 0 to n. For instance $A_i^{(2)}$

$$A_i^{(2)} = \left(R^4 f^{(2)} f^{(1)} \delta_{ij} + R^2 g_{ij}^{(2)} \right) A_j^{(0)}$$

The two terms correspond to

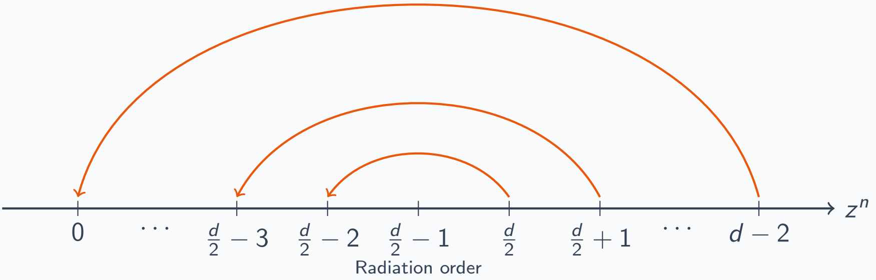


and

The mirror relation for Maxwell Fields in AdS_{d+2}

The mirror relation: for $1 \leqslant k \leqslant \frac{d}{2} - 1$

$$A_i^{\left(\frac{d}{2}-1+k\right)} = \frac{R^{2k}}{\left(\frac{d}{2}-k\right)_{2k}} \boldsymbol{\Delta}^{k-1} \left[\delta_i{}^j \boldsymbol{\Delta} - \frac{2k}{\frac{d}{2}+k-1} \partial_i \partial^j\right] A_j^{\left(\frac{d}{2}-1-k\right)}$$



Solution space of Maxwell fields on AdS_{d+2}: VeV

Asymptotic expansion:

$$A_\mu = z^0 A_\mu^{(0)} + z A_\mu^{(1)} \dots + z^{d-2} A_\mu^{(d-2)} + z^{d-1} A_\mu^{(d-1)} + \dots$$

2. The Vev (Δ_+) at order z^{d-1}

- $A_u^{(d-1)}(u, \mathbf{x})$ and $A_i^{(d-1)}(u, \mathbf{x})$ obeying a conservation equation:

$$\partial_u A_u^{(d-1)} - \frac{1}{R^2}\,\partial \cdot A^{(d-1)} = -\frac{d-3}{(d-1)(d-2)}\,\mathbf{\Delta}\partial \cdot A^{(d-3)}$$

Free asymptotic data in AdS_{d+2}:

$A_i^{(0)}(u, \mathbf{x})$, $A_i^{(d-1)}(u, \mathbf{x})$ and $A_u^{(d-1)}(\mathbf{x})$

Solution space of spin-s fields on AdS_{d+2}

Asymptotic expansion:

$$\phi_{\mu_1\dots\mu_s} = z^{2-2s}\phi^{(2-2s)}_{\mu_1\dots\mu_s} + z^{3-2s}\phi^{(3-2s)}_{\mu_1\dots\mu_s}\dots + z^{d-1}\phi^{(d-1)}_{\mu_1\dots\mu_s} + \dots$$

Free asymptotic data for linearised gravity:

$$h^{(-2)}_{ij}(u,\mathbf{x}),\ h^{(d-1)}_{ij}(u,\mathbf{x}),\ h^{(d-1)}_{ui}(\mathbf{x}),\ h^{(d-1)}_{uu}(\mathbf{x})$$

Free asymptotic data for spin s:

$$\phi^{(2-2s)}_{i_1\dots i_s}(u,\mathbf{x}),\ \phi^{(d-1)}_{i_1\dots i_s}(u,\mathbf{x}),\ \underbrace{\phi^{(d-1)}_{ui_1\dots i_{s-1}}(\mathbf{x}),\ \phi^{(d-1)}_{u(2)i_1\dots i_{s-2}}(\mathbf{x}),\ \dots,\ \phi^{(d-1)}_{u(s)}(\mathbf{x})}_{s \text{ integration functions of } \mathbf{x}}$$

Solution space of Minkowski

Solution space of Maxwell fields on $Mink_{d+2}$

Asymptotic expansion:

$$A_\mu = z^0 A_\mu^{(0)} + z^{\frac{d}{2}-1} A_\mu^{(\frac{d}{2}-1)} + z^{\frac{d}{2}} A_\mu^{(\frac{d}{2})} + z^{\frac{d}{2}+1} A_\mu^{(\frac{d}{2}+1)} + \dots$$

1. **Overleading mode at order z^0**
   - Soft mode $A_i^{(0)}(\mathbf{x}) = \partial_i \lambda(\mathbf{x})$
2. **Radiation at order $z^{\frac{d}{2}-1}$**
   - $A_i^{(\frac{d}{2}-1)}(u, \mathbf{x})$ is the free data at null infinity.
3. **Subleading modes: Chthonians from order $z^{\frac{d}{2}}$**
   - Chthonians obey evolution equation:

     $$\partial_u A_i^{(n)} = -\frac{1}{2\left(n - \frac{d}{2} + 1\right)} \left(\mathbf{\Delta} A_i^{(n-1)} + \frac{d-2}{n(n-d+1)} \partial_i \partial \cdot A^{(n-1)} \right)$$

   - Free function is

     $$A_i^{(n)}(u = 0, \mathbf{x})$$

Solution space of Maxwell fields on $Mink_{d+2}$ (Contd.)

4. **Celestial conserved current at order $\mathbf{z}^{d-2}$**
   - $\partial_i A_i^{(d-2)} = 0$
5. **Charge aspect at order $\mathbf{z}^{d-1}$**
   - $A_u^{(d-1)}$ Obeys an evolution equation

$$\partial_u A_u^{(d-1)} = -\frac{d-3}{(d-1)(d-2)} \boldsymbol{\Delta} \partial \cdot A^{(d-3)}$$

Free asymptotic data:

$$A_i^{\left(\frac{d}{2}-1\right)}(u, \mathbf{x}) \ , A_u^{(d-1)}(\mathbf{x}) \text{ and } \underbrace{A_i^{\left(\frac{d}{2}\right)}(\mathbf{x}) \ , A_i^{\left(\frac{d}{2}+1\right)}(\mathbf{x}) \ , A_i^{\left(\frac{d}{2}+2\right)}(\mathbf{x}) \ , \cdots}_{\text{Chthonians can be re-summed to one function on } u=0\text{: } A_i(z,\mathbf{x})}$$

Solution space of Spin-s fields on $Mink_{d+2}$

Asymptotic expansion:

$$\phi_{\mu_1\dots\mu_s} = \dots + z^{\frac{d}{2}-s}\phi^{\left(\frac{d}{2}-s\right)}_{\mu_1\dots\mu_s} + z^{\frac{d}{2}-s+1}\phi^{\left(\frac{d}{2}-s+1\right)}_{\mu_1\dots\mu_s} + \dots$$

Free asymptotic data for linearised gravity:

$$h^{\left(\frac{d}{2}-2\right)}_{ij}(u,\mathbf{x}), \quad \underbrace{h^{(d-1)}_{ui}(\mathbf{x}),\ h^{(d-1)}_{uu}(\mathbf{x})}_{\text{Angular momentum and mass aspect}}, \underbrace{h^{\left(\frac{d}{2}-1\right)}_{ij}(\mathbf{x}),\ h^{\left(\frac{d}{2}\right)}_{ij}(\mathbf{x}),\ \cdots}_{\text{Chthonians}}$$

Free asymptotic data for spin s:

$$\phi^{\left(\frac{d}{2}-s\right)}_{i_1\dots i_s}(u,\mathbf{x}), \underbrace{\phi^{(d-1)}_{ui_1\dots i_{s-1}}(\mathbf{x}),\ \phi^{(d-1)}_{u(2)i_1\dots i_{s-2}}(\mathbf{x}),\ \dots,\ \phi^{(d-1)}_{u(s)}(\mathbf{x})}_{s\text{ aspects}},$$

$$\underbrace{\phi^{\left(\frac{d}{2}-s+1\right)}_{i_1\dots i_s}(\mathbf{x}),\ \phi^{\left(\frac{d}{2}-s+2\right)}_{i_1\dots i_s}(\mathbf{x}),\ \cdots}_{\text{Chthonians (which can be re-summed)}}$$

How to flatten AdS (without losing your data)

Prescription

- Expand the free data in powers of the cosmological constant:

$$A_i^{(0)}(u,\mathbf{x}) = \sum_{k=0}^{\frac{d}{2}-1} \frac{1}{R^{2k}} A_i^{(0,k)}(u,\mathbf{x})\,, \quad A_i^{(d-1)}(u,\mathbf{x}) = \sum_{k=0}^{\infty} \frac{1}{R^{2k}} A_i^{(d-1,k)}(u,\mathbf{x})\,,$$

$$A_u^{(d-1)}(\mathbf{x}) = A_u^{(d-1,0)}(\mathbf{x})$$

- Treat all divergences in R^2 as constraint equations.

Question: What survives in the Flat limit $R \to \infty$?

The idea

The recursion

$$A_i^{(n)} = R^2 \left(f^{(n)} A_i^{(n-1)} + g_{ij}^{(n)} A_j^{(n-2)} \right)$$

with

$$f^{(n)} = -(2n-d) b_n \, \partial_u \,, \quad g_{ij}^{(n)} = -b_n \Big(\delta_{ij} \mathbf{\Delta} + (d-2) b_{n-1} \, \partial_i \partial_j \Big) \,.$$

$$A_i^{(0)} = A_i^{(0,0)} + \mathcal{O}(R^{-2})$$

$$A_i^{(1)} = R^2 f^{(1)} A_i^{(0,0)} + f^{(1)} A_i^{(0,1)} + \mathcal{O}(R^{-2})$$

$$A_i^{(2)} = R^4 f^{(2)} f^{(1)} A_i^{(0,0)} + R^2 \Big(f^{(2)} f^{(1)} A_i^{(0,1)} + g_{ij}^{(2)} A_j^{(0,0)} \Big) + R^0 \left(f^{(2)} f^{(1)} A_i^{(0,2)} + g_{ij}^{(2)} A_j^{(0,1)} \right) + \mathcal{O}(R^{-2})$$

Imposing the constraints: R^2 divergent terms must vanish

- From $\mathcal{O}(R^2)$ in $A_i^{(1)}$:
  $f^{(1)} A_i^{(0,0)} = 0 \implies \partial_u A_i^{(0,0)} = 0 \implies A_i^{(0,0)}(\mathbf{x})$
- ...

Flat limit: Source contribution

$$A_i(z,u,\mathbf{x}) = A_i^{(0,0)}(\mathbf{x}) + z^{\frac{d}{2}-1}\frac{(-1)^{\frac{d}{2}-1}}{(d-3)!!}\,\partial_u^{\frac{d}{2}-1}A_i^{\left(0,\frac{d}{2}-1\right)}$$

$$+\sum_{k=1}^{\frac{d}{2}-1}\frac{z^{\frac{d}{2}-1+k}(-1)^{\frac{d}{2}-1+k}}{(2k)!!(d-3)!!}\left(\delta_i{}^j\boldsymbol{\Delta}-\frac{2k}{\frac{d}{2}+k-1}\,\partial_i\partial^j\right)\boldsymbol{\Delta}^{k-1}\partial_u^{\frac{d}{2}-1-k}A_j^{\left(0,\frac{d}{2}-1\right)}$$

$$+z^{d-1}A_i^{(d-1,0)}+\mathcal{O}(z^d)$$

- $A_i^{(0,0)}(\mathbf{x}) = \partial_i\lambda(\mathbf{x})$ is a pure gauge mode (soft)
- $A_i^{(0,n)}$ for $0 < n < \frac{d}{2}-1$ are set to zero by $\mathcal{O}(R^2)$ divergent-constraint equations.
- $A_i^{\left(0,\frac{d}{2}-1\right)}$ is unconstrained: $\partial_u^{\frac{d}{2}-1}A_i^{\left(0,\frac{d}{2}-1\right)}$ identified with radiation.
- Source Chthonians arise from $\partial_u^{\frac{d}{2}-1-k}A_j^{\left(0,\frac{d}{2}-1\right)}$ which appears with less time derivatives at each order – corresponding to one integration function.

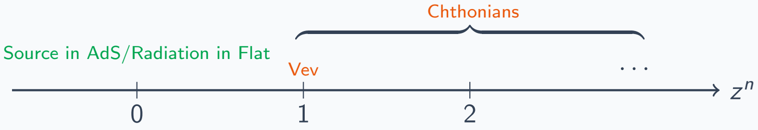
Flat limit: Vev contribution

$$A_i(z,u,\mathbf{x}) \sim \cdots + z^{d-1} A_i^{(d-1,0)} + \cdots + z^{d-1+n}\, \partial_u^n A_i^{(d-1,n)} + \cdots$$

- With constraints of the form: $\partial_u \left(\partial_u^n A_i^{(d-1,n)} \right) \sim$ lower order

4 Dimensional spacetime
- A happy accident

Maxwell fields in four dimensional spacetime



Conclusion

- We have successfully recovered Minkowski solution space from AdS by expanding the free data in AdS powers of the cosmological constant.

Future directions:

1. Odd spacetime dimensions
2. Logs?

Thank You!

Questions?