Exact Metric solutions from a cubic action

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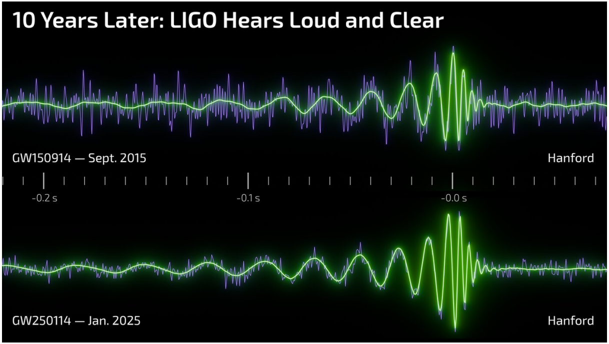
LICHNEROWICZ CONFERENCE:
Journées relativistes, Tours

Based on work with

Stavros Mougiakakos, Ludovic Planté, Emil Bjerrum-Bohr, Poul Damgaard

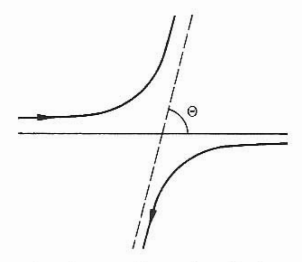
Метод важнее открытия, ибо правильный метод исследования приведет к новым, еще более ценным открытиям. — Л. Д. Ландау

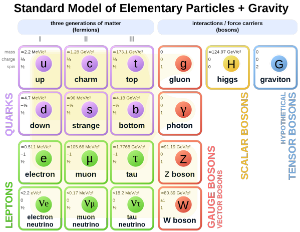
A method is better than a discovery, because a good method can lead to new results, and much more valuable discoveries. — L. D. Landau

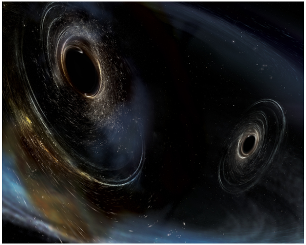


Ten years ago LIGO detected the first-ever detection of gravitational waves signal from black hole merger GW150914.

In the mean time amplitude based methods for computing the wave-forms have been developed.
This approach started before the first detection [Bjerrum-Bohr, Donoghue, Vanhove; Porto; Goldberger-Roshtein] and is a now a very power approach with the recent tour de force computation of the 5PM 1SF computation by the [Plefka et al.]





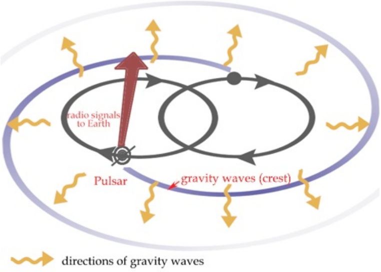


In this talk, I will review how to extract classical general relativity from quantum S-matrix

- **Black-holes:** This will be used to derive **exact** black-hole metric in various dimensions
- **Classical scattering:** Self-force expansion for small mass ratio $\nu = \frac{m}{M} \ll 1$ for all orders in G_N.
- **Quantum scattering:** applies to any EFT of gravity in various dimensions

[Damour; Veneziano et al.; Porto et al.; Bern et al.; Goldberger, Rothstein; Damgaard, Bjerrum-Bohr, Vanhove, ...]

Post-Minkowskian Hamiltonian



A relativistic Hamiltonian for the two-body dynamics in the center-of-mass frame:

$$\widehat{\mathcal{H}}_{\rm PM}(\gamma, r) = \sqrt{\widehat{p}^2 + m_1^2} + \sqrt{\widehat{p}^2 + m_2^2} + \widehat{\mathcal{V}}_{\rm PM}(\gamma, r)$$

with a relativistic potential organized in a series of Newton's constant G_N:

$$\mathcal{V}_{\rm PM}(\gamma, r) = \sum_{L\geq 0} \frac{G_N^{L+1} m_1^2 m_2^2}{r^{L+1}} \sum_{r_1+r_2=L} v_{r_1,r_2}(\gamma) m_1^{r_1} m_2^{r_2}$$

which is the general relativity correction to Newton's potential for $L = 0$:

$$\mathcal{V}_1(\gamma, r) = -\frac{G_N}{E_1 E_2} \frac{m_1^2 m_2^2}{r} (2\gamma^2 - 1), \quad \gamma = \frac{p_1 \cdot p_2}{m_1 m_2} = \frac{1}{\sqrt{1 - \frac{\vec{v}^2}{c^2}}} \geq 1.$$

From Amplitudes to Post-Minkowskian Potential

In an operatorial form

$$\widehat{\mathcal{H}}_{\rm PM}(\gamma, r) = \sqrt{\widehat{p}^2 + m_1^2} + \sqrt{\widehat{p}^2 + m_2^2} + \widehat{\mathcal{V}}_{\rm PM},$$
$$\widehat{\mathcal{T}} = \widehat{\mathcal{V}}_{\rm PM} + \widehat{\mathcal{V}}_{\rm PM} \times (E_p - \widehat{\mathcal{H}}_{\rm PM} + i\epsilon)^{-1} \times \widehat{\mathcal{T}}.$$

The Lippmann-Schwinger equations relate the matrix elements of the transition operator $\widehat{\mathcal{T}}$ with the ones of potential $\widehat{\mathcal{V}}$:

$$\langle p|\widehat{\mathcal{T}}|p'\rangle = \langle p|\widehat{\mathcal{V}}_{\rm PM}|p'\rangle + \int \frac{d^3\ell}{(2\pi)^3} \frac{\langle p|\widehat{\mathcal{V}}_{\rm PM}|\ell\rangle\langle \ell|\widehat{\mathcal{T}}|p'\rangle}{E_p - E_\ell + i\epsilon}.$$

Expanding $\widehat{\mathcal{V}}_{\rm PM}$ in perturbation in G_N, we can evaluate the potential as an series in the matrix elements of $\mathcal{T}$

$$\langle p|\widehat{\mathcal{V}}_{\rm PM}|p'\rangle = \langle p|\widehat{\mathcal{T}}|p'\rangle - \int \frac{d^3\ell}{(2\pi)^3} \frac{\langle p|\widehat{\mathcal{T}}|\ell\rangle\langle \ell|\widehat{\mathcal{T}}|p'\rangle}{E_p - E_\ell + i\epsilon} + \dots$$

When doing a scattering process, the scattering amplitude is:

$$\frac{i}{\hbar}\langle p|\widehat{\mathcal{T}}|p'\rangle = \mathcal{M}(p, p').$$

We then have a fully relativistic potential order by order in perturbation in G_N.

Gravity Effective Field Theories

We want to develop a formalism that allows precise classical post-Minkowskian results and is suited for effective field theory extensions of Einstein gravity.

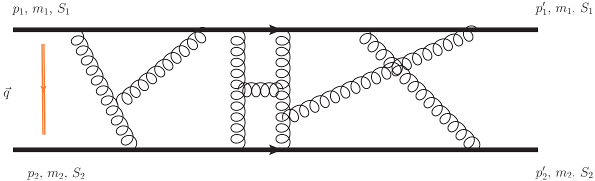
- Standard QFT (local, unitary, Lorentz invariant, ...)
- The low-energy DOF: graviton, usual matter fields
- Standard symmetries: General relativity as we know it

$$\mathcal{S}_{\rm eff} = \mathcal{S}_{\rm eff}^{\rm gravity} + \mathcal{S}_{\rm eff}^{\rm matter}(\psi_i, g_{\mu\nu})$$

$$\mathcal{S}_{\rm eff}^{\rm gravity} = \frac{1}{16\pi G_N} \int d^4x \sqrt{g}\mathcal{R} + \cdots$$

- This setup is important to treat both black holes and neutron stars
- Higher derivative terms enter the evaluation of tidal response (e.g. Love numbers) and will be important when LISA will be in service
- Some couplings to higher derivative terms like R^2 or $R_{\mu\nu}F_{\rho\sigma}$ terms can be bounded from observations the shadow of the black-hole

Perturbative Gravity



We consider the pure gravitational interaction between massive and massless matter of various spins:

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G_N} h_{\mu\nu}$$

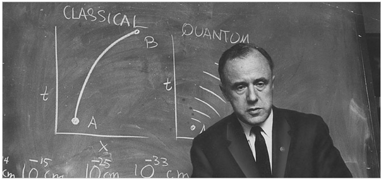
$$\mathcal{S} = \int d^4x \sqrt{-g} \left(-\frac{\mathcal{R}}{16\pi G_N} + \frac{1}{2} \sum_{a=1}^{2} \left(g^{\mu\nu} \partial_\mu \phi_a \partial_\nu \phi_a - m_a^2 \phi_a^2 \right) \right)$$

Evaluating the quantum scattering S-matrix scaling $q = p_1' - p_1 = \hbar \underline{q}$:

$$\mathfrak{M}^{\rm GR}(p_1 \cdot p_2, \underline{q}, \hbar) = \sum_{L \geq 0} G_N^{L+1} \mathfrak{M}_L(p_1 \cdot p_2, \underline{q}, \hbar)$$

We take the $\hbar \to 0$ limit.

Classical Physics from Loops: $\hbar$ Counting (All PM)



The classical limit $\hbar \to 0$ fixed $q \ll m_1, m_2$ of the amplitude [Bjerrum-Bohr, Damgaard, Vanhove, Planté]:

$$\mathfrak{M}_L(\gamma, \underline{q}^2, \hbar) = \frac{\mathcal{M}_L^{(L+1)}(\gamma)}{\hbar^{L+1}|\underline{q}|^{2+\frac{L(4-D)}{2}}} + \cdots + \frac{\mathcal{M}_L^{(-1)}(\gamma)}{\hbar|\underline{q}|^{2-L+\frac{L(4-D)}{2}}} + O(\hbar^0)$$

- A classical contribution of order $1/\hbar$ from all loop orders.
- The dimensional regularization scheme gives control of the IR divergences from radiation [Di Vecchia, Heissenberg, Russo, Veneziano; Parra-Martinez et al.; Bjerrum-Bohr et al.].
- The computation is explicitly relativistic invariant.

Classical Physics from Loops: $\hbar$ Counting

The connection between quantum scattering and classical gravitational physics has forced a rethinking of the S-matrix for dealing with the $\hbar$ expansion [Damgaard, Planté, Vanhove]:

$$\hat{S} = \mathbb{I} + \frac{i}{\hbar}\hat{T} =: \exp\left(\frac{i\hat{N}}{\hbar}\right)$$

Doing the Dyson expansion with the conservative and radiation part:

$$\hat{T} = G_N \sum_{L\geq 0} G_N^L \hat{T}_L + G_N^{\frac{1}{2}} \sum_{L\geq 0} G_N^L \hat{T}_L^{\rm rad}, \quad \hat{N} = G_N \sum_{L\geq 0} G_N^L \hat{N}_L + G_N^{\frac{1}{2}} \sum_{L\geq 0} G_N^L \hat{N}_L^{\rm rad}$$

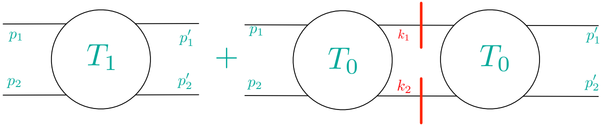
Solving the relation between the operator $\hat{N}$ and $\hat{T}$:

$$\begin{aligned}
\hat{N}_0 &= \hat{T}_0, \quad \hat{N}_0^{\rm rad} = \hat{T}_0^{\rm rad},\\
\hat{N}_1 &= \hat{T}_1 - \frac{i}{2\hbar}\hat{T}_0^2, \quad \hat{N}_1^{\rm rad} = \hat{T}_1^{\rm rad} - \frac{i}{2\hbar}\left(\hat{T}_0\hat{T}_0^{\rm rad} + \hat{T}_0^{\rm rad}\hat{T}_0\right),\\
\hat{N}_2 &= \hat{T}_2 - \frac{i}{2\hbar}(\hat{T}_0^{\rm rad})^2 - \frac{i}{2\hbar}(\hat{T}_0\hat{T}_1 + \hat{T}_1\hat{T}_0) - \frac{1}{3\hbar^2}\hat{T}_0^3.
\end{aligned}$$

Unitarity of the S-matrix $SS^\dagger = \mathbb{I}$ implies that $\hat{N} = \hat{N}^\dagger$ is Hermitian.

Exponential Formalism: One-Loop

$$\langle p_1, p_2|\hat{N}_1|p_1', p_2'\rangle = \langle p_1, p_2|\hat{T}_1|p_1', p_2'\rangle - \frac{i}{2\hbar}\langle p_1, p_2|\hat{T}_0 \mathbb{I} \hat{T}_0|p_1', p_2'\rangle$$



Completness relation

$$\mathbb{I} = \sum_{n=0}^{\infty} \frac{1}{n!} \int \frac{d^D k_1}{(2\pi\hbar)^{D-1}} \delta^+(k_1^2 - m_1^2) \frac{d^D k_2}{(2\pi\hbar)^{D-1}} \delta^+(k_2^2 - m_2^2)$$
$$\times \frac{d^{D-1}\ell_1}{(2\pi\hbar)^{D-1}} \frac{1}{2E_{\ell_1}} \cdots \frac{d^{D-1}\ell_n}{(2\pi\hbar)^{D-1}} \frac{1}{2E_{\ell_n}} \times |k_1, k_2; \ell_1, \ldots, \ell_n\rangle\langle \ell_1, \ldots, \ell_n; k_1, k_2|$$

At one-loop, the textbook amplitude has the small $\hbar$ expansion:

$$\langle p_1, p_2|\hat{T}_1|p_1', p_2'\rangle = \frac{\hbar}{i}\mathfrak{M}_1(\gamma, \underline{q}^2, \hbar) = -i\frac{\mathcal{M}_L^{(2)}(\gamma)}{\hbar|\underline{q}|^{2+\frac{4-D}{2}}} - i\frac{\mathcal{M}_L^{(-1)}(\gamma)}{|\underline{q}|^{2-L+\frac{4-D}{2}}} + O(\hbar)$$

The $1/\hbar$ piece is automatically subtracted:

$$\langle p_1, p_2|\hat{N}_1|p_1', p_2'\rangle = \frac{\mathcal{M}_L^{(-1)}(\gamma)}{|\underline{q}|^{2-L+\frac{4-D}{2}}} + O(\hbar)$$

Classical Observables

The change in an observable $\hat{O}$ is given by the [Kosower, Maybee, O'Connell] expression:

$$\langle\Delta\hat{O}\rangle := \langle\text{out}|\hat{O}|\text{out}\rangle - \langle\text{in}|\hat{O}|\text{in}\rangle$$

with

$$\langle\Delta\hat{O}\rangle(p_1,p_2,r) = \int \frac{d^D(\hbar\underline{q})}{(2\pi)^{D-2}}\delta(2\hbar p_1\cdot\underline{q} - \hbar^2\underline{q}^2)\delta(2\hbar p_2\cdot\underline{q} + \hbar^2\underline{q}^2)e^{ir\cdot\underline{q}}\langle p_1', p_2'|\hat{O}|p_1,p_2\rangle$$

Which is naturally expanded using the $\hat{N}$-operator:

$$\langle\Delta\hat{O}\rangle = \langle\text{in}|\hat{S}^\dagger\hat{O}\hat{S} - \hat{O}|\text{in}\rangle = \sum_{n\geq 1}\frac{(-i)^n}{\hbar^n n!}\langle\text{in}|\underbrace{[\hat{N},[\hat{N},\ldots,[\hat{N},\hat{O}],\ldots,]]}_{n\text{ times}}|\text{in}\rangle$$

Because the v.e.v is expressed in terms of nested commutators involving the $\hat{N}$ operator, we have that the $\hbar\to 0$ limit gives directly the classical answer [Damgaard, Planté, Vanhove]:

$$\langle\Delta\hat{O}\rangle = \Delta O^{\text{classical}}(p_1,p_2,r) + O(\hbar)$$

With the exponential representation, all superclassical pieces cancel automatically.
Notice that the same formalism allows to computation of quantum corrections to GR observables and applies to any EFT of GR (including higher derivative corrections, ...)

The Radial Action

Applying the previous formalism to the momentum kick $\hat{O}^\mu = \hat{P}_1^\mu$ gives in the conservative sector [Bjerrum-Bohr, Damgaard, Planté, Vanhove]:

$$\Delta\tilde{P}_1^\mu(\gamma, r)\Big|_{\rm cons} = -\frac{p_\infty r^\mu}{|r|}\sin\left(-\frac{\partial\tilde{N}(\gamma, J)}{\partial J}\right) + p_\infty^2 L^\mu\left(\cos\left(-\frac{\partial\tilde{N}(\gamma, J)}{\partial J}\right) - 1\right)$$

With the angular momentum:

$$L^\mu := \frac{(m_1\gamma + m_2)m_2 p_1^\mu - m_1(m_1 + m_2\gamma)p_2^\mu}{(m_1 m_2)^2(\gamma^2 - 1)}, \quad p_\infty^2 L^2 = 1$$

We then see that in the conservative sector, $\tilde{N}(\gamma, J)$ is the radial action used by [Landau, Lifshitz; Damour] for computing the scattering angle in classical GR:

$$\chi(\gamma, J) = -\frac{\partial\tilde{N}(\gamma, J)}{\partial J} = \sum_{L=0}^{\infty}\left(\frac{G_N m_1 m_2}{J}\right)^{L+1}\chi_{\rm cons}^{(L+1)}(\gamma)$$

Classical Physics from Loops

**THE GENERATION OF GRAVITATIONAL WAVES.
IV. BREMSSTRAHLUNG*†‡**

Sándor J. Kovács, Jr.

and

Kip S. Thorne

Received 1977 October 21; accepted 1978 February 28

g) The Feynman-Diagram Approach

Any classical problem can be solved quantum-mechanically; and sometimes the quantum solution is easier than the classical. There is an extensive literature on the Feynman-diagram, quantum-mechanical treatment of gravitational bremsstrahlung radiation (e.g., Feynman 1961, 1963; Barker, Gupta, and Kaskas 1969; Barker and Gupta

We have realized what Kovacs and Thorne suggested by using quantum methods for extracting the classical contribution:

$$\mathfrak{M}_L(\gamma, \underline{q}^2, \hbar) = \frac{\mathcal{M}_L^{(L+1)}(\gamma)}{\hbar^{L+1}|\underline{q}|^{2+\frac{L(4-D)}{2}}} + \cdots + \frac{\mathcal{M}_L^{(-1)}(\gamma)}{\hbar|\underline{q}|^{2-L+\frac{L(4-D)}{2}}} + O(\hbar^0)$$

How to efficiently extract the classical contribution for GR computations?

Classical solution from quantum field

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Quantum Tree Graphs and the Schwarzschild Solution

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(Received 7 July 1972)

I. INTRODUCTION

In an attempt to find quantum corrections to solutions of Einstein's equations, the question naturally arises as to whether the $\hbar \to 0$ limit of the quantum theory correctly reproduces the classical results. Formally, at least, the correspondence between the tree-graph approximation to quantum field theory and the classical solution of the field equations is well known,[1] i.e., the classical field produced by an external source serves as the generating functional for the connected Green's functions in the tree approximation, the closed-loop contributions vanishing in the limit $\hbar \to 0$. The purpose of this paper is to present an explicit calculation of the vacuum expectation value (VEV) of the gravitational field in the presence of a spherically symmetric source and verify, to second order in perturbation theory, that the result is in agreement with the classical Schwarzschild solution of the Einstein equations. This would appear to be the first step towards tackling the much more ambitious program of including the radiative quantum corrections.

In 1973 Duff asked the question about the classical limit of quantum gravity. He showed how to reproduce the Schwarzschild back hole metric from quantum tree graphs to G_N^3 order

Since then the relation between quantum and classical gravity in amplitude have been rethought with new insights [Donoghue, Holstein], [Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove], [Kosower, Maybee, O'Connell] , [Mougiakakos, Vanhove], [Damgaard et al.]

The Cubic Action

We consider a binary system of a heavy body of mass M and a light body of mass m interacting gravitationally:

$$\mathcal{S} = \mathcal{S}_{EH} + \mathcal{S}_l + \mathcal{S}_H$$

with the gothic inverse metric of the Landau-Lifshitz formulation:

$$\mathfrak{g}^{ab} = \sqrt{-g}g^{ab} = \eta^{ab} - \sqrt{32\pi G_N}h^{ab}$$

and the cubic formulation introduced by [Cheung, Remmen]:

$$16\pi G_N \mathcal{S}_{EH} = -\int d^Dx \left(\left(A^a_{bc}A^b_{ad} - \frac{1}{D-1}A^a_{ac}A^b_{bd} \right) \mathfrak{g}^{cd} + A^a_{bc}\partial_a \mathfrak{g}^{bc} \right)$$

and the worldline actions for the light and heavy body:

$$\mathcal{S}_l = -\frac{m}{2}\int d\tau_l \left(\frac{\mathfrak{g}^{\mu\nu}v_\mu v_\nu}{(\sqrt{-\mathfrak{g}})^{\frac{2}{D-2}}} + 1 \right),$$

$$\mathcal{S}_H = -\frac{M}{2}\int d\tau_H \left(\frac{\mathfrak{g}^{\mu\nu}v_{H\mu} v_{H\nu}}{(\sqrt{-\mathfrak{g}})^{\frac{2}{D-2}}} + 1 \right).$$

Notice that the auxiliary field A does not couple to the matter field.

Deriving Exact Black-Hole Metric

The off-shell currents for graviton emission from the heavy source M:

$$\sqrt{32\pi G_N} h^{(n)}_{\mu\nu}(\mathbf{x}) = \int d^{D-1}x\, e^{i\mathbf{k}\cdot\mathbf{x}} J^{(n)}_{\mu\nu}(\mathbf{k})$$

$$J^{(n)}_{\mu\nu}(\mathbf{k}) = \rho(|\mathbf{k}|, D, n) \left(\chi_1^{(n)} \delta^0_\mu \delta^0_\nu + \chi_2^{(n)} \left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{\mathbf{k}^2} \right) \right)$$

which leads to the parametrization of the waveform:

$$h^{(n)}_{\mu\nu}(\mathbf{x}) = \rho(r, D)^n \left[\left(\chi_1^{(n)} - \chi_2^{(n)} \right) \delta^0_\mu \delta^0_\nu + \chi_2^{(n)} \frac{1 - (n-1)(D-3)}{2 - (n-1)(D-3)} \delta_{ij} + \chi_2^{(n)} \frac{n(D-3)}{2 - (n-1)(D-3)} n_\mu n_\nu \right]$$

with a similar parametrization for the auxiliary field A^a_{bc}.
The order parameter in direct space:

$$\int \frac{d^{D-1}k}{(2\pi)^{D-1}} e^{i\mathbf{k}\cdot\mathbf{r}} \rho(|\mathbf{k}|, D, n) = \rho(r, D)^n, \quad \rho(r, D) = \frac{\Gamma\left(\frac{D-3}{2}\right)}{\pi^{\frac{D-3}{2}}} \frac{G_N m}{r^{D-3}}$$

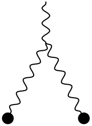
Deriving Exact Black-Hole Metric (cont.)

We compute the emission of the graviton (and the auxiliary field) and identify the form factors.

- At tree-level, i.e., order G_N:

$$J^{(1)}_{\mu\nu} =$$  $$= \rho(|\mathbf{k}|, D, 1) 4\delta^0_\mu \delta^0_\nu \Longrightarrow \chi^{(1)}(D) = (4, 0, 0, 0, 0, 0, 0, 0)$$

- At one-loop, i.e., order G_N^2:

$$J^{(2)}_{\mu\nu} =$$ 

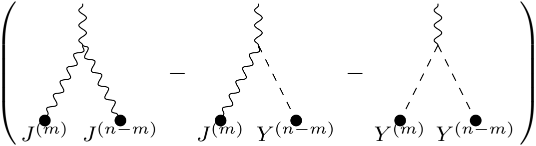
$$= \int \frac{d^{D-1}q}{(2\pi)^{D-1}} \frac{8\pi^2 \left(G_N m \chi^{(1)}_1\right)^2}{(\mathbf{q})^2 (\mathbf{q}-\mathbf{k})^2} \times \left(\left(1 - \frac{(D-3)}{4(D-2)^2}\right) \delta^0_\mu \delta^0_\nu + \frac{(D-3)^2}{4(D-2)^2} \left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{|\mathbf{k}|^2}\right)\right)$$

$$= \rho(|\mathbf{k}|, D, 2) \frac{\left(\chi^{(1)}_1\right)^2}{2} \left(\left(1 - \frac{(D-3)}{4(D-2)^2}\right) \delta^0_\mu \delta^0_\nu + \frac{(D-3)^2}{4(D-2)^2} \left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{|\mathbf{k}|^2}\right)\right)$$

Self-Force Expansion: Exact Metric

At higher orders, the off-shell current is given by sums of cubic graphs only, presenting a recursive nature:

$$J^{(n)} = \sum_{m=1}^{n-1}$$



allowing the sum over **all loop orders** and leading to a non-perturbative resummation given by the exact Schwarzschild metric in D dimensions:

$$\chi_k^{(n)}(D) = \sum_{i,j=1}^{8} \sum_{m=1}^{n-1} \chi_i^{(m)}(D)\chi_j^{(n-m)}(D) M_k^{ij}(D)$$

Black Hole Metric

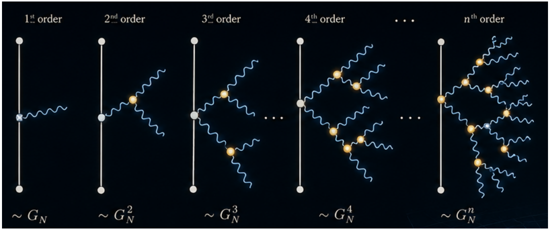
- All integrals are finite, and there is no need to use the non-minimal coupling from the EFT approach to cancel divergences from finite size effects.
- The result matches exactly the Schwarzschild metric in the "usual" coordinate system because the computation is done in the harmonic gauge:

$$\mathfrak{g}^{\mu\nu}\Gamma^{\lambda}_{\mu\nu} = 0 \quad \Longleftrightarrow \quad \partial_\mu \mathfrak{g}^{\mu\nu} = 0$$

$$\mathfrak{g}^{\mu\nu} = \begin{pmatrix} -\frac{(1+\rho)^4}{1-\rho^2} & 0 & 0 & 0 \\ 0 & -1+\frac{\rho^2 x^2}{R^2} & \frac{\rho^2 xy}{R^2} & \frac{\rho^2 xy}{R^2} \\ 0 & \frac{\rho^2 xy}{R^2} & -1+\frac{\rho^2 y^2}{R^2} & \frac{\rho^2 yz}{R^2} \\ 0 & \frac{\rho^2 xy}{R^2} & \frac{\rho^2 yz}{R^2} & -1+\frac{\rho^2 z^2}{R^2} \end{pmatrix}$$

- This gives an all-order in G_N derivation of the Schwarzschild-Tangherlini metric in general dimensions D by summing an infinite number of perturbative contributions.

Infinite resummation



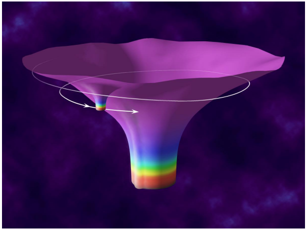
The derivation of the black-hole Schwarzschild-Tanghelini metric is the result of an infinite summation of perturbative computations

It is suprising that this can be achieved in particular in a theory of gravity.

A few points are worth stressing

- The Schwarzschild metric is an exact result of general relativity. We have claimed to reproduce GR from the classical limit, then one must have rederived it
- The sum of triangle graphs is a subsector of the full quantum gravity theory. We do not expect to be able to resum perturbative quantum gravity.
- In particular there is no known instantons contributions that would signal that the perturbative series is asymptotic
- The classical limit projects on solving the classical Einstein equation in a form equivalent to the perturbiner method developped by [Damgaard et al.]

Strong Field Regime: Self-Force Expansion



A principal objective of LISA is to investigate **extreme mass ratio inspiral** which are out of reach for ground based detectors.

We reorganize the double summation according to the mass-ratio order:

$$N(\gamma, J) = \frac{M^3\nu^2}{|q|^3} \sum_{r\geq 0} \nu^r \sum_{L\geq 2r} (G_N M|q|)^{L+1} c_r^L(\gamma); \quad \nu = \frac{m_1 m_2}{M^2}; \quad M = m_1 + m_2$$

The self-force expansion is an expansion in $\nu \sim m/M \ll 1$ for $m_1 = m \ll m_2 = M$ but to all orders in G_N.

Self-Force Expansion: Effective Action

Integrating out the graviton and the auxiliary field, we have the effective action:

$$e^{iS_{\rm eff}[x_l,x_H]}=\int \mathcal{D}h\,\mathcal{D}A\,e^{iS_{EH}[h,A]+iS_{GF}[h]+iS_l[x_l,h]+iS_H[x_H,h]}$$

The self-force effective action has an expansion in powers $m/M \ll 1$:

$$\mathcal{S}_{\rm eff}=-\frac{M}{2}\int d\tau_H\,\eta^{\mu\nu}v_{H\mu}v_{H\nu}+M\sum_{n=0}^{\infty}\int d\tau_l\left(\frac{m}{M}\right)^{n+1}\mathcal{L}_n[x_l(\tau_l),x_H(\tau_H)]$$

where the leading term is the worldline action for the heavy body.

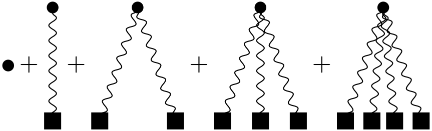
We parametrize the trajectory of the light body as:

$$x_l^\mu(\tau_l)\equiv x^\mu(\tau)=\sum_{n=0}^{\infty}\left(\frac{m}{M}\right)^n\delta x^{(n)\mu}(\tau),\quad x_H^\mu(\tau_H)=u_H^\mu\tau_H+\sum_{n=1}^{\infty}\left(\frac{m}{M}\right)^n\delta x_H^{(n)\mu}(\tau_H)$$

Self-Force Expansion: Geodesic (oSF)

Now that we have obtained a non trivial curved space geometry one can look at the mouvement of a light particle in this background. Again everything is derived from multiple summation, no input the classical geometry.

The geodesic equation for the light mass m is obtained by a double infinite resummation.

$\mathcal{L}_0[x^\mu(\tau), u^\mu_H \tau_H] =$  $+ \cdots$

$$\mathcal{L}_0[x^\mu(\tau), u^\mu_H \tau_H] = \frac{1}{2} v_\mu(\tau) v_\nu(\tau) \left(-\eta^{\mu\nu} + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \mathcal{I}^{(L)}_{\alpha_1\beta_1,\ldots,\alpha_L\beta_L} \mathcal{T}^{\mu\nu\alpha_1\beta_1,\ldots,\alpha_L\beta_L}_{(L)} \right)$$

At each loop order, the $\mathcal{I}^{(L)}_{\alpha_1\beta_1,\ldots,\alpha_L\beta_L}$ tensors are given by infinite sums building the Schwarzschild metric generated by the heavy body:

$$\mathcal{I}^{(L)}_{\alpha_1\beta_1,\ldots,\alpha_L\beta_L} = \int_{\mathbb{R}^3} \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}(\tau)} \int_{\mathbb{R}^{3L}} \prod_{i=1}^{L} \frac{d^3\mathbf{q}_i}{(2\pi)^3} \delta^{(3)}\left(\sum_{i=1}^{L} \mathbf{q}_i - \mathbf{k}\right)$$
$$\times \sum_{n_1=1}^{\infty} \cdots \sum_{n_L=1}^{\infty} \rho(|\mathbf{q}_i|, n_i) \left(\chi_1^{(n_i)} \delta^0_{\alpha_i} \delta^0_{\beta_i} + \chi_2^{(n_i)} \left(\eta_{\alpha_i\beta_i} - \frac{q_{i,\alpha_i} q_{i,\beta_i}}{\mathbf{q}_i^2} \right) \right)$$

$\mathcal{T}^{\mu\nu\alpha_1\beta_1,\ldots,\alpha_L\beta_L}_{(L)}$ are projectors, and $\rho(|\mathbf{k}_i|, n_i) \propto (G_N M)^n / \mathbf{k}^{3-n}$.

Self-Force Expansion: Geodesic (0SF) (cont.)

Computing the triangle integral is not difficult. Resumming, we get:

$$\mathcal{L}_0[x^\mu(\tau), u_H^\mu \tau_H] = -\frac{1}{2} v_\mu(\tau) v_\nu(\tau) \mathfrak{g}^{\mu\nu}(\mathbf{x})$$

which matches the inverse gothic metric for Schwarzschild:

$$\mathfrak{g}^{\mu\nu}(\rho) = -\frac{1}{(1+\rho)^2}\left(\delta_0^\mu \delta_0^\nu \left(\frac{4\rho(1+\rho)}{(1-\rho)} - \rho^2\right) - \eta^{\mu\nu} + \rho^2 n^\mu n^\nu\right)$$

Then the Lagrangian for the light body reads, at the rest frame of the heavy body:

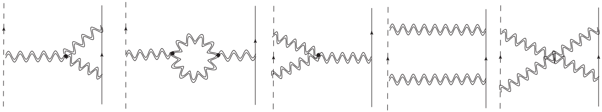
$$\mathcal{L}_l = m\mathcal{L}_0 + \mathcal{O}(m/M)$$

so that the equations-of-motion for the light body derived in the self-force EFT formalism are exactly equivalent to the geodesic equation for a massive probe in a Schwarzschild spacetime.

This gives a totally consistent self-force formalism without any need to introduce an external metric.

We haven't specified the kinematics of the system, so the formalism is suitable for both bound and scattering problems.

Quantum Correction to the Bending Angle



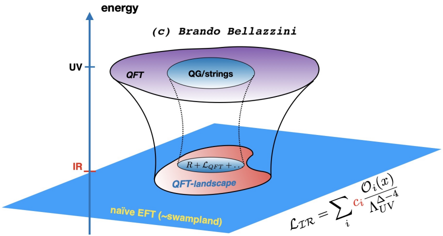
TheEFT formalism presented allows as well to keep the quantum correction. For instance the quantum corrections to the star light bending is obtained from the scattering of a massless particle of spin S against a massive scalar one obtains from the one-loop computation [Bjerrum-Bohr, Donoghue, Holstein, Planté, Vanhove]:

$$\theta_S \simeq \frac{4GM}{b} + \frac{15}{4}\frac{G^2M^2\pi}{b^2} + \frac{8bu^S + 9 - 48\log\frac{b}{2b_0}}{\pi}\frac{G^2\hbar M}{b^3}$$

The difference between the bending angles shows an intriguing dependence on the spin induced by quantum effects:

$$\theta_\gamma - \theta_\varphi = \frac{8(bu^\gamma - bu^\varphi)}{\pi}\frac{G^2\hbar M}{b^3}$$

Constraining Beyond Einstein Gravity



This provides a way of constraining possible corrections to Einstein's gravity:

- Quantum gravity corrections to star light bending.
- Quantum gravity correction effects to the metric of black hole solutions.
- Quantum contributions to the causal cone.

[Bellazzini, Isabella, Riva; Maldacena, Zhiboedov; Caron-Huot, Parra Martinez, ...]

The multi-messenger detections improve the constraints on various modified gravity models [Gubitosi, Piazza, Vernizzi]:

- Gravitational waves propagate with $|c_{GW} - c| < 10^{-15}c$.
- strongly restrict higher derivative (Hornedsky) operators

LISA will be senstive to small (solar) mass black hole where tidal effects are more important. (see talk of [Platania; Vernizzi; Parra Martinez] at this meeting)