Conformal geodesics and timelike infinity

Revisiting the semiglobal stability of the Minkowski spacetime

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Summary

Key messages…

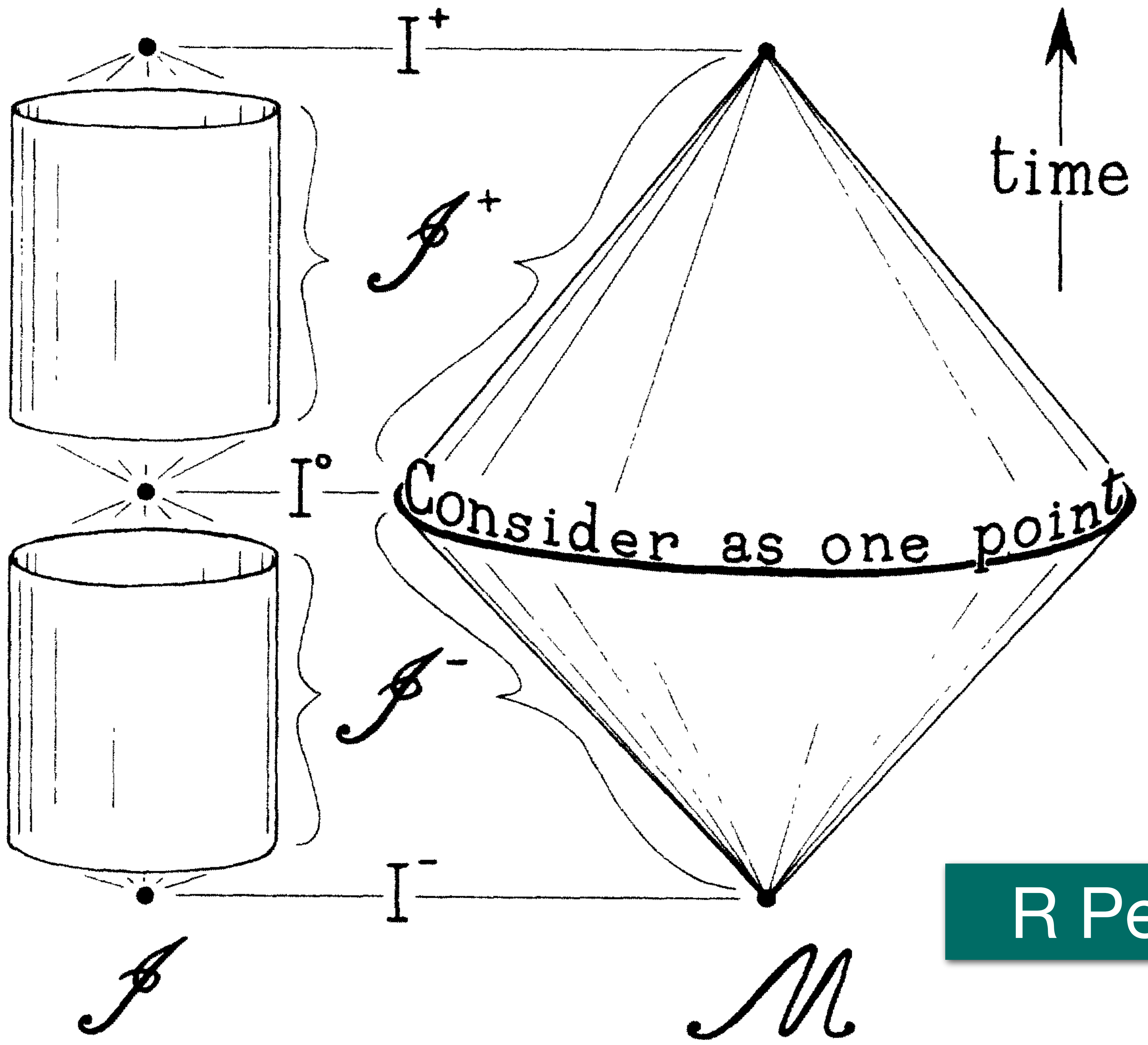
- ***Conformal geodesics (CGs)*** are a powerful tool to study the conformal structure of spacetimes
- CGs provide a ***natural link*** between various approaches to i^+
- In particular, CGs allow to ***blow-up*** i^+ in Friedrich's ***semiglobal perturbations of Minkowski*** spacetime to a hyperboloid $\mathscr{H}^+$
- The strategy can be generalised to more complicated (blackhole) spacetimes
- The geometric set-up is the starting point for the use of tools of ***microlocal analysis (b-geometry/analysis)***

Motivation

How do the various representations of timelike infinity arise/relate?

Penrose's original idea:

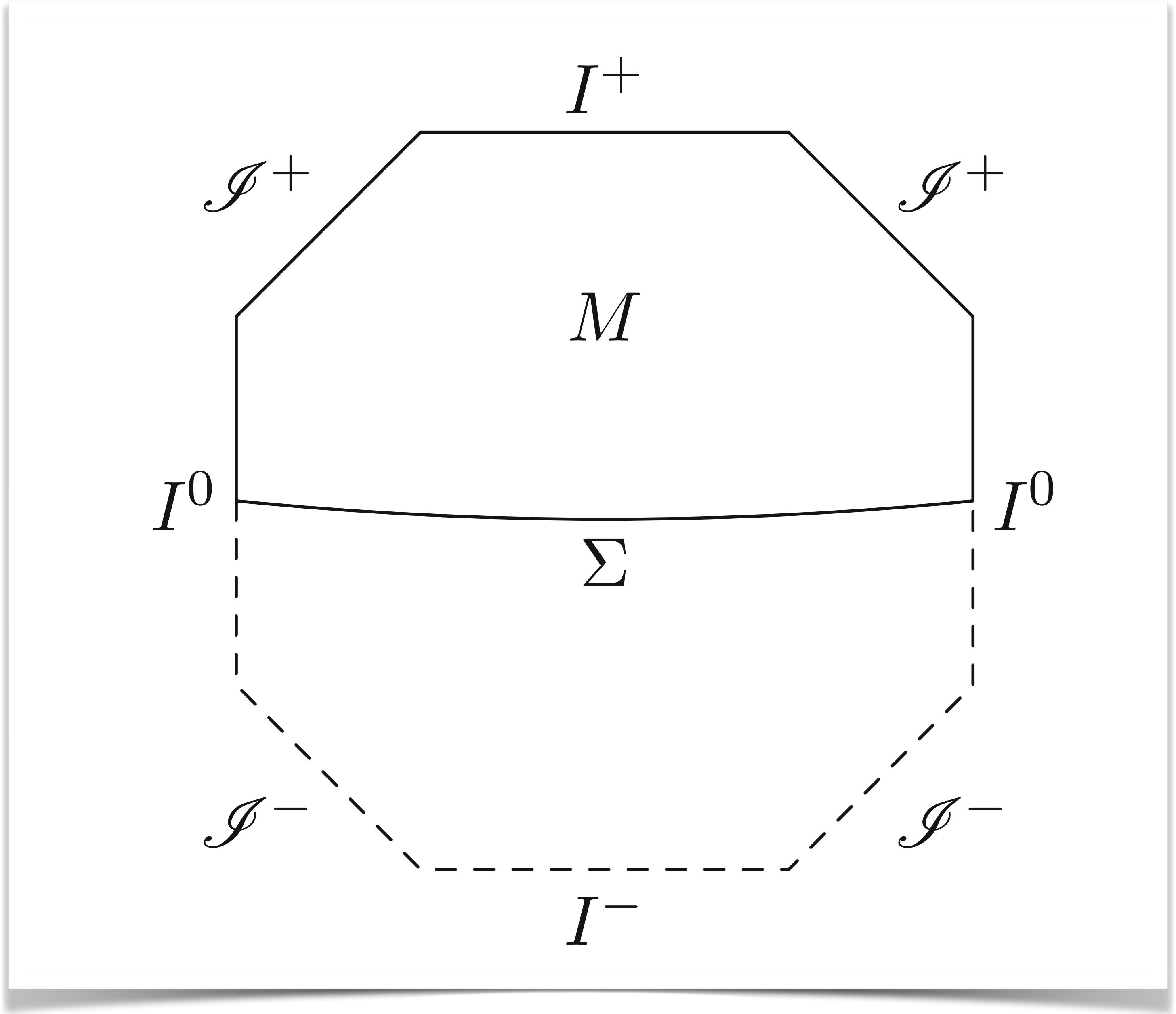
treat i^0, i^- and i^+ as points

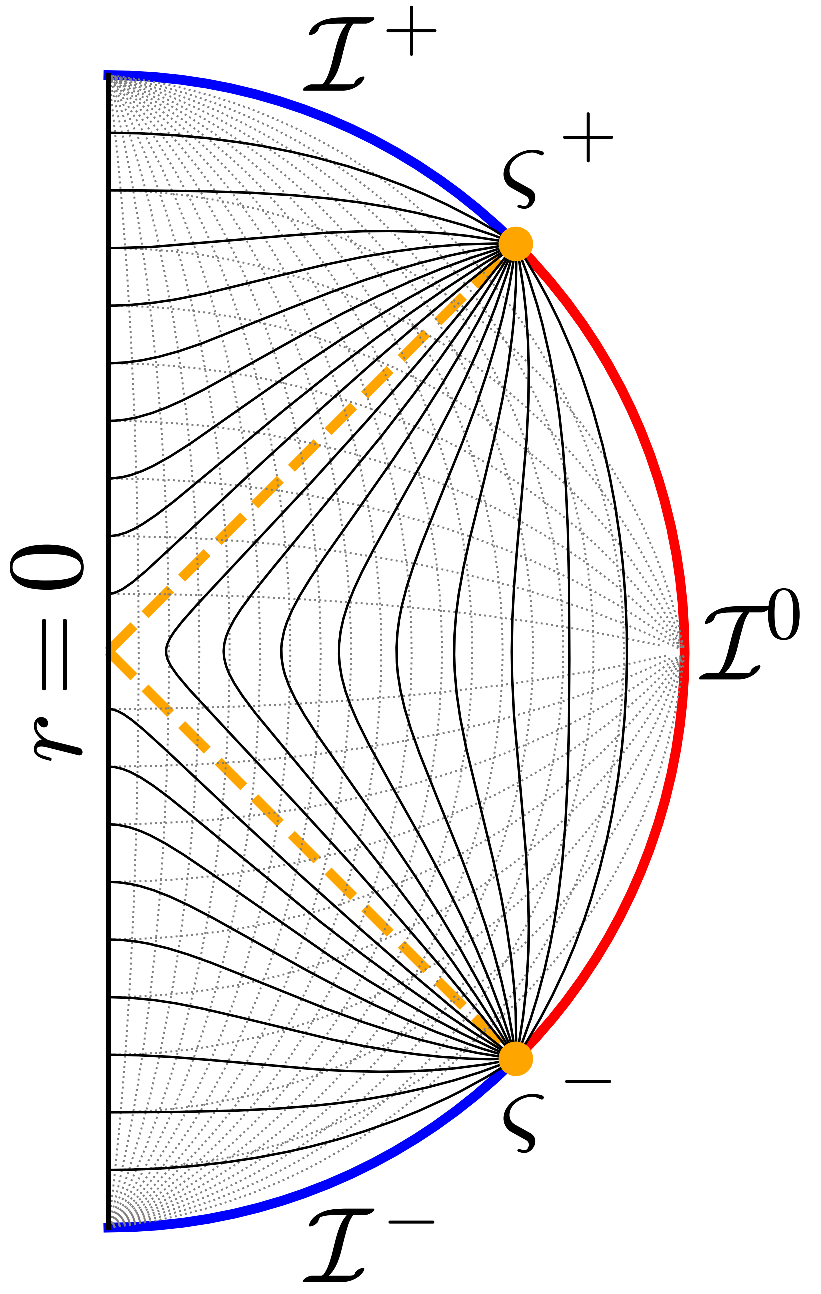


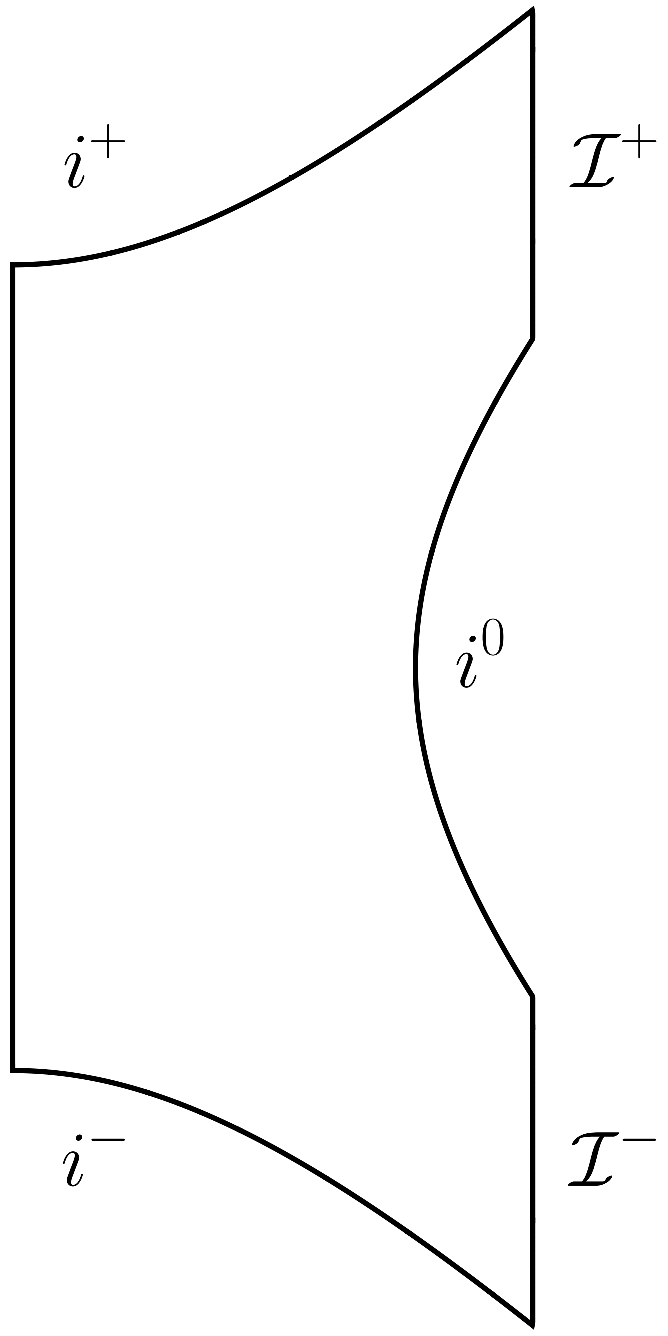
R Penrose, PRL (1963)

Motivation

How do the various representations of timelike infinity arise/relate?







P Hintz & A Vasy (2020)

Microlocal analysis

Bortwick & Herfray (2024)

Affine geometry

Compère, Gralla & Wei (2024)

Scattering

The big question:

Can one show that these structures arise naturally as a consequence of the evolution dictates by the Einstein equations?

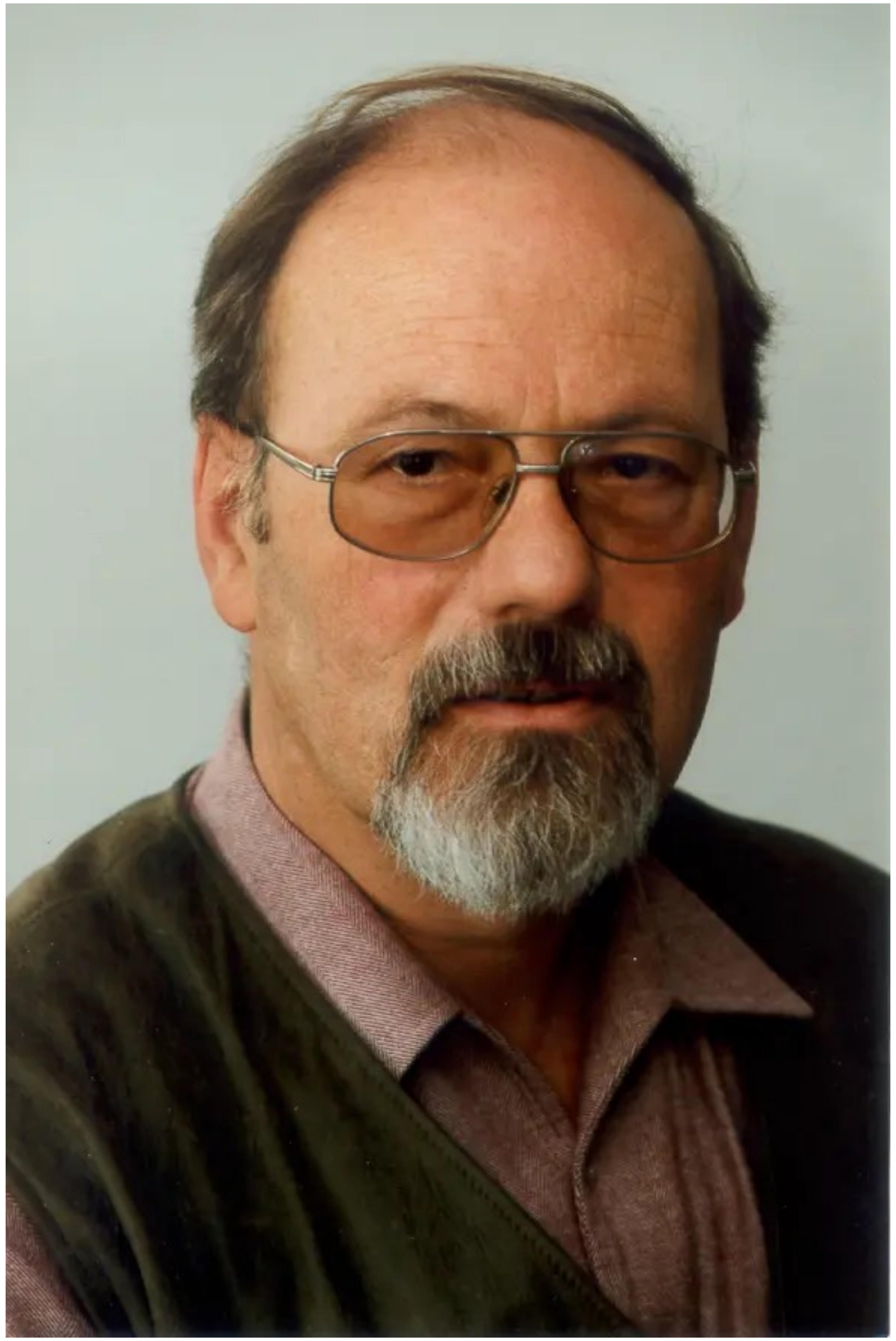
Prologue

Compactifying spacetimes

What is the best way to conformally compactly a spacetime?

“*Ask Bernd Schmidt!*”

(heard from Alan Rendall)



(1921-2023)

What would Bernd Schmidt suggest?

Use conformal geodesics!

Conformal geodesics provide a canonical way to obtain conformal completions of spacetime.

CONFORMAL GEODESICS

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A conformal structure $\mathcal{C}$ on a n-dimensional manifold M is determined by the following equivalence relation between nondegenerate metrics of any signature:

(1) $\bar{g} \sim g \Leftrightarrow \bar{g} = e^{2\sigma} g \quad , \quad \sigma: M \to R$

In the case of Lorentz metrics, one can visualize $\mathcal{C}$ as the distribution of the null cones on M. $\mathcal{C}$ determines uniquely a reduction P(M) of L(M), the frame bundle over M. P(M) consists of all frames which are orthonormal for some metric $g \in \mathcal{C}$. The structure group of P is the direct product of the generalized orthogonal group O(p,q) and the 1-dimensional group of dilatations.

A conformal structure $\mathcal{C}$ determines further a collection $\Gamma(\mathcal{C})$ of symmetric linear connections over M, by the condition that parallel propagation does not lead out of P(M).

The coordinate components of the difference tensor S_{ik}^{ℓ} of two connections $\nabla, \bar{\nabla}$ in $\Gamma(\mathcal{C})$ are determined by a 1-form $b = b_i dx^i$ as follows [1]:

(2) $S_{ik}^{\ell}(b) = \delta_i^{\ell} b_k + \delta_k^{\ell} b_i - g_{ik} g^{\ell s} b_s$

$(g_{ik} g^{\ell s} = \bar{g}_{ik} \bar{g}^{\ell s} \text{ if } \bar{g} = e^{2\sigma} g)$

Hence, at a fixed point $x_o \in M$, there is a 1-1-relation between elements of $\Gamma(\mathcal{C})$ and 1-forms $b \in T_{xo}(M)$.

The purpose of this note is to draw attention to the following property of a conformal structure* (n > 2):

*I don't know where these curves have been defined for the first time. They are mentioned in Yano's book about the Lie derivative [4].

BG Schmidt (1986)

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Conformal geodesics in general relativity

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(*Communicated by R. Penrose, F.R.S. – Received* 27 *February* 1987)

Conformal geodesics, space-time curves which are related to conformal structures in a similar way as geodesics are related to metric structures, are discussed. 'Conformal normal coordinates', 'conformal Gauss systems' and their associated 'normal connections', 'normal frames' and 'normal metrics' are introduced and used to study:
(i) asymptotically simple solutions of $\mathrm{Ric}(\tilde{g}) = \Lambda \tilde{g}$ near conformal infinity,
(ii) asymptotically simple solutions of $\mathrm{Ric}(\tilde{g}) = 0$ with a past null infinity, which can be represented as the future null cone of a point i^-, past time-like infinity.
In the first case we define an ∞-parameter family of (physical) Gauss systems near conformal infinity, in the second case a ten-parameter family of (physical) Gauss systems covering a neighbourhood of i^-. The behaviour of physical geodesics can be analysed in a particularly simple way in these coordinate systems. Each of these systems allows an extremely simple transition from the conformal analysis to the physical description of space-time. For $\Lambda \eta_{00} < 0$ (De-Sitter type solutions) all solutions are characterized in terms of the physical space-time by their data on past time-like infinity. For $\Lambda = 0$ the conserved quantities of Newman and Penrose are characterized as the first non-trivial coefficient, given by the value of the rescaled Weyl tensor at i^-, in an expansion of the physical field in a Gauss system of the type considered before.

1. INTRODUCTION

Because null geodesics considered as point sets are the same for two conformally related metrics, these curves are of fundamental importance in the analysis of the conformal structure of space-time. Though in general they provide a convenient tool, there are situations, two of them will be described below, where for geometrical reasons it is more appropriate to study the conformal structure in terms of space- or time-like curves. But the use of, say, time-like geodesics in such investigations leads to new problems. There is no general relation between the two sets of time-like geodesics belonging to two different conformally related metrics. This makes it difficult to express a result obtained by analysing the behaviour of time-like geodesics in one metric of the conformal class in terms of the metric one is really interested in.

BG Schmidt & H Friedrich (1987)

ELSEVIER
Journal of Geometry and Physics 17 (1995) 125–184
JOURNAL OF GEOMETRY AND PHYSICS

Einstein equations and conformal structure: Existence of Anti-de Sitter-type space–times

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Received 23 August 1994

Abstract

We discuss Einstein's equations in the context of normal conformal Cartan connections, derive a new conformal representation of the equations, and express the equations in a conformally invariant gauge. The resulting formulation of the equations is used to show the existence of asymptotically simple solutions to Einstein's equations with a positive cosmological constant. The solutions are characterized by Cauchy data on a space-like slice and by the intrinsic conformal structure on the conformal boundary at space-like and null infinity.

Keywords: Einstein equations; Conformal structure
1991 MSC: 53A30; 83C05; 83C15

1. Introduction

Einstein's field equations show a very peculiar transformation behaviour under conformal rescalings of the metric field. In a suitable representation the equations for the rescaled metric retain their hyperbolicity even under conformal rescalings with conformal factors which vanish on subsets of space–time. The consequences of this property for the long-time behaviour of gravitational fields have been worked out in detail for de Sitter-type space–times [5,7], i.e. for solutions to Einstein's field equations

$$Ric(\tilde{g}) = \lambda \tilde{g} \tag{1.1}$$

with negative cosmological constant λ (the signature $(+, -, -, -)$ of the metric is assumed here, cf. Section 2 for our convention concerning the Ricci tensor). Beside the global non-linear stability of the de Sitter space–time and other asymptotically simple solutions it has been shown that the conformal properties of the equations allow to deduce sharp and complete information on the fall-off behaviour of the gravitational field.

H Friedrich (1995)

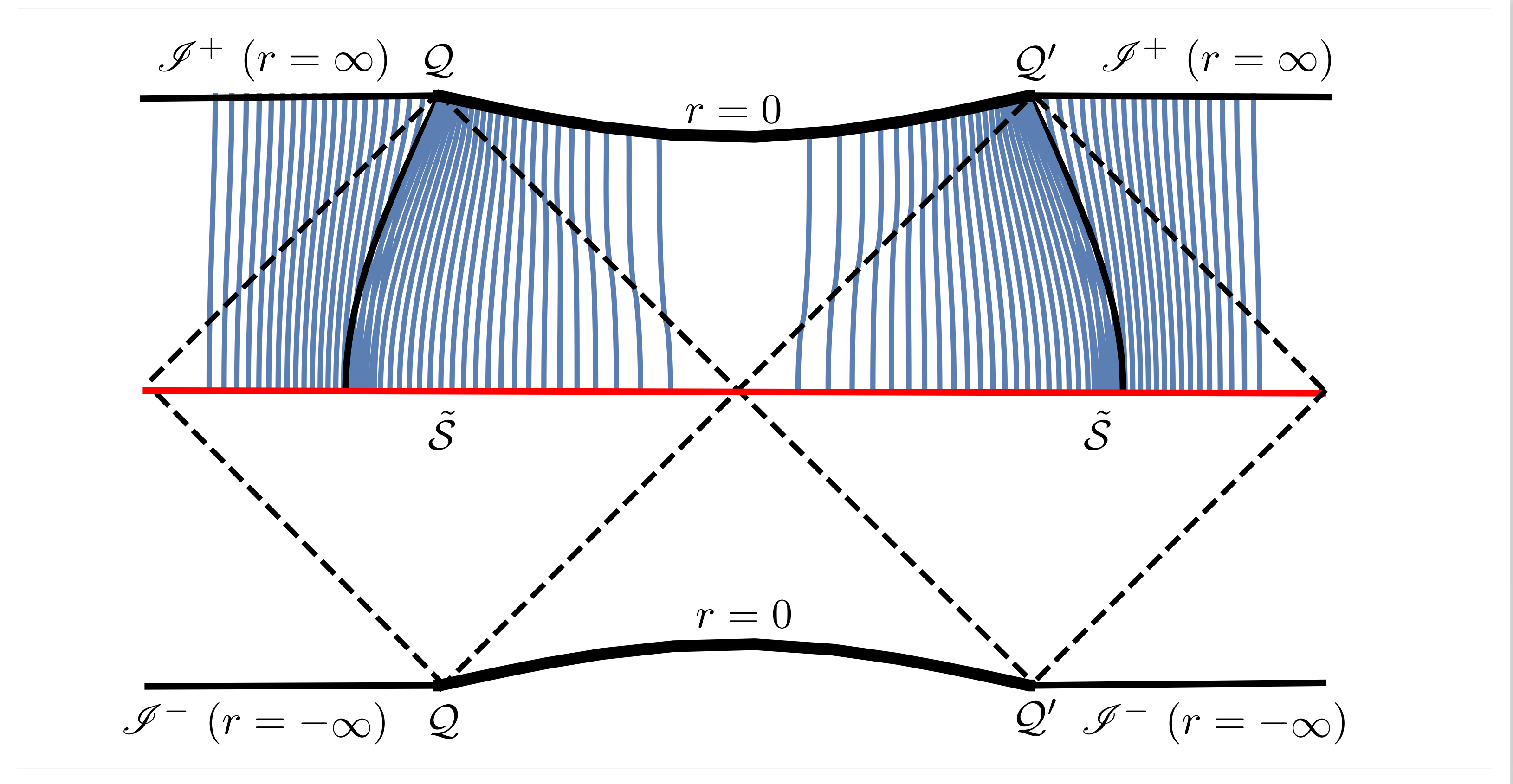
Conformal geodesics

Using conformal structures to fix the gauge

$$\tilde{\nabla}_{\dot{\boldsymbol{x}}}\dot{\boldsymbol{x}} = -2\langle \boldsymbol{\beta}, \dot{\boldsymbol{x}}\rangle \dot{\boldsymbol{x}} + \tilde{\boldsymbol{g}}(\dot{\boldsymbol{x}}, \dot{\boldsymbol{x}})\boldsymbol{\beta}^{\sharp},$$

$$\tilde{\nabla}_{\dot{\boldsymbol{x}}}\boldsymbol{\beta} = \langle \boldsymbol{\beta}, \dot{\boldsymbol{x}}\rangle \boldsymbol{\beta} - \frac{1}{2}\tilde{\boldsymbol{g}}^{\sharp}(\boldsymbol{\beta}, \boldsymbol{\beta})\dot{\boldsymbol{x}}^{\flat} + \tilde{\boldsymbol{L}}(\dot{\boldsymbol{x}}, \cdot),$$

- General properties: Friedrich & Schmidt (1987). Connection to the value NP constants at i^+.
- **Construction of adS-like spacetimes**: (HF, 1995)
- Conformal Gaussian systems in Schwarzschild (2003)
- Schwarzschild-(a)dS: García-Parrado, Gasperín, JAVK (2018)



Proposition 5.1 (*the canonical conformal factor associated to a conformal geodesic*) *Let* $(\tilde{\mathcal{M}}, \tilde{\boldsymbol{g}})$ *denote an Einstein spacetime. Suppose that* $(x(\tau), \boldsymbol{\beta}(\tau))$ *is a solution to the conformal geodesic equations (5.35a) and (5.35b) and that* $\{\boldsymbol{e}_{\boldsymbol{a}}\}$ *is a* $\boldsymbol{g}$*-orthonormal frame propagated along the curve according to Equation (5.36). If* $\boldsymbol{g} = \Theta^2\tilde{\boldsymbol{g}}$ *is such that* $\boldsymbol{g}(\dot{\boldsymbol{x}}, \dot{\boldsymbol{x}}) = 1$*, then the conformal factor* Θ *satisfies*

where the coeffici

along the conform

$\dot{\Theta}_{\star}$

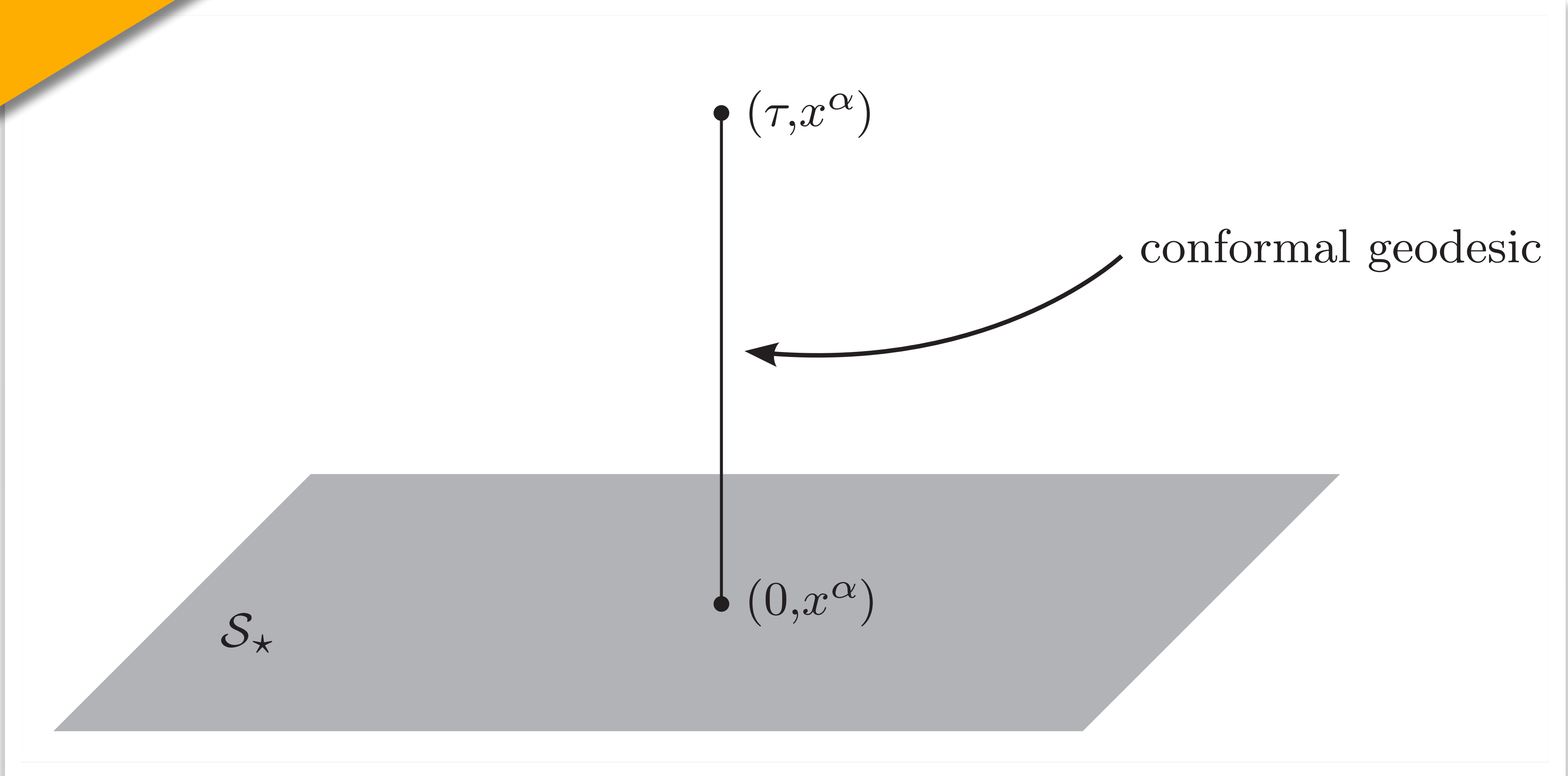
An explicitly known canonical conformal factor associated to the conformal structure!

Conformal Gaussian gauges

A canonical way of describing the spacetime

Connection to affine geometry!

- The one form β defines a Weyl connection (torsion free, nonmetric) $\hat{\nabla}$ along the congruence
- Make use of the canonical conformal factor selected by the congruence of GCs
- Propagate the coordinates along the curves (Gaussian) from some fiduciary hypersurface
- Use Weyl propagation to evolve the frame



$$\boldsymbol{e_0} = \boldsymbol{\partial_\tau}, \qquad \text{so that} \qquad \boldsymbol{e_0}^\mu = \delta_0{}^\mu.$$

$$\hat{\Gamma}_{\boldsymbol{0}}{}^{\boldsymbol{a}}{}_{\boldsymbol{b}} = 0, \qquad \hat{L}_{\boldsymbol{0a}} = 0.$$

$$f_{\boldsymbol{0}} = 0.$$

The extended conformal Einstein field equations

A conformal representation of the vacuum Einstein equations

- Expressed in terms of geometric fields in the conformally rescaled spacetime:
    - Frame, connection, Schouten tensor, ***Weyl tensor***.
- The equations are ***formally regular*** at the conformal boundary.

•A hyperbolic reduction implies a ***conformal evolution***

$$\partial_\tau \hat{\boldsymbol{\upsilon}} = \mathbf{K}\hat{\boldsymbol{\upsilon}} + \mathbf{Q}(\hat{\boldsymbol{\Gamma}})\hat{\boldsymbol{\upsilon}} + \mathbf{L}(x)\boldsymbol{\phi},$$

$$\big(\mathbf{I} + \mathbf{A}^0(\boldsymbol{e})\big)\partial_\tau\boldsymbol{\phi} + \mathbf{A}^\alpha(\boldsymbol{e})\partial_\alpha\boldsymbol{\phi} = \mathbf{B}(\hat{\boldsymbol{\Gamma}})\boldsymbol{\phi}$$

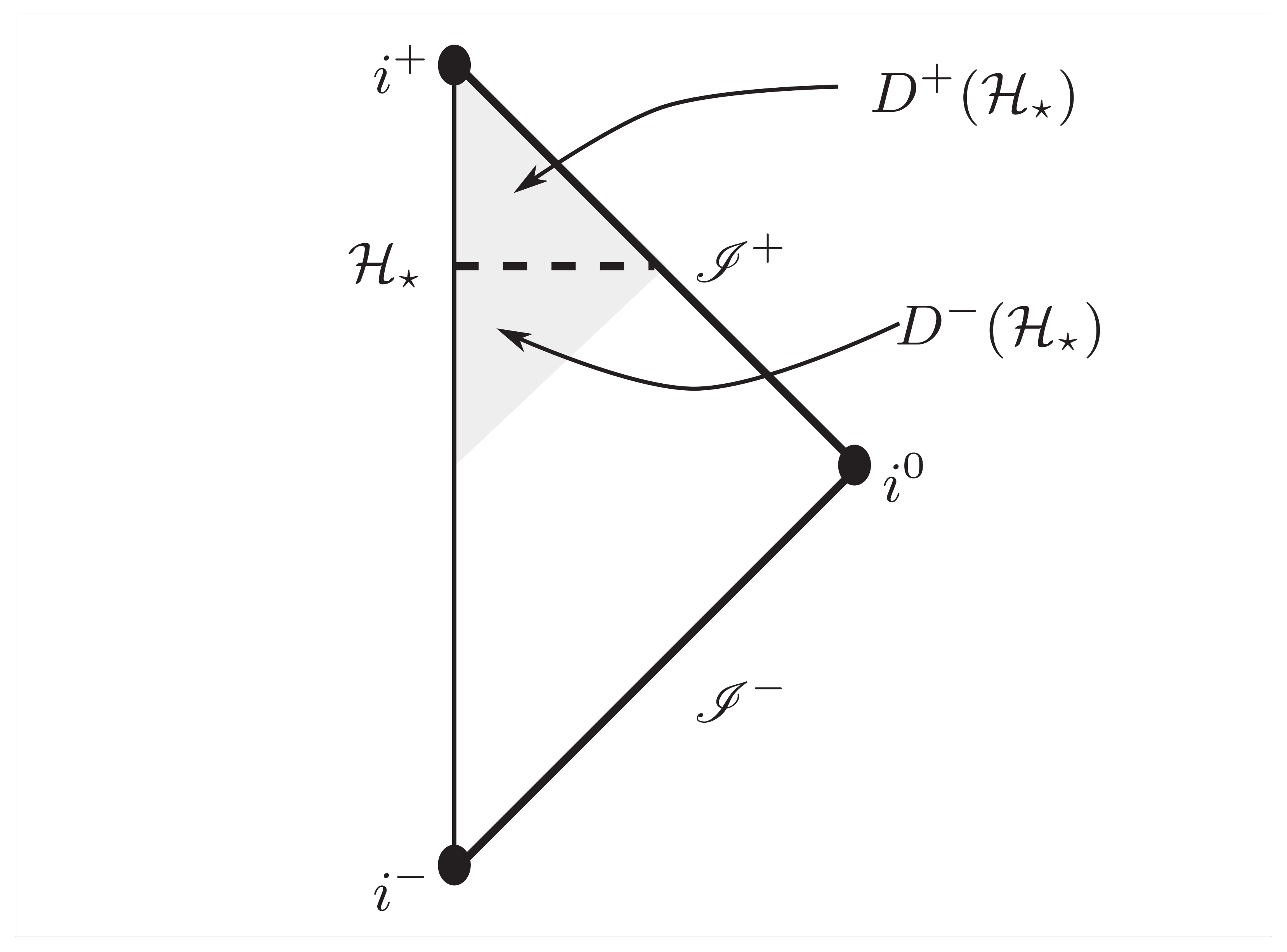
Key idea: the conformal factor is known *a priori*

Friedrich’s semiglobal stability theorem of the Minkowski spacetime

Semiglobal stability of the Minkowski spacetime

(and global stability of de Sitter)

Theorem (Friedrich, 1986). *Perturbations of hyperboloidal data for the Minkowski spacetime which are regular at $\mathscr{I}^+$ give rise to a development with the same global properties of the Minkowski spacetime. In particular, the solutions admit a smooth conformal extension.*



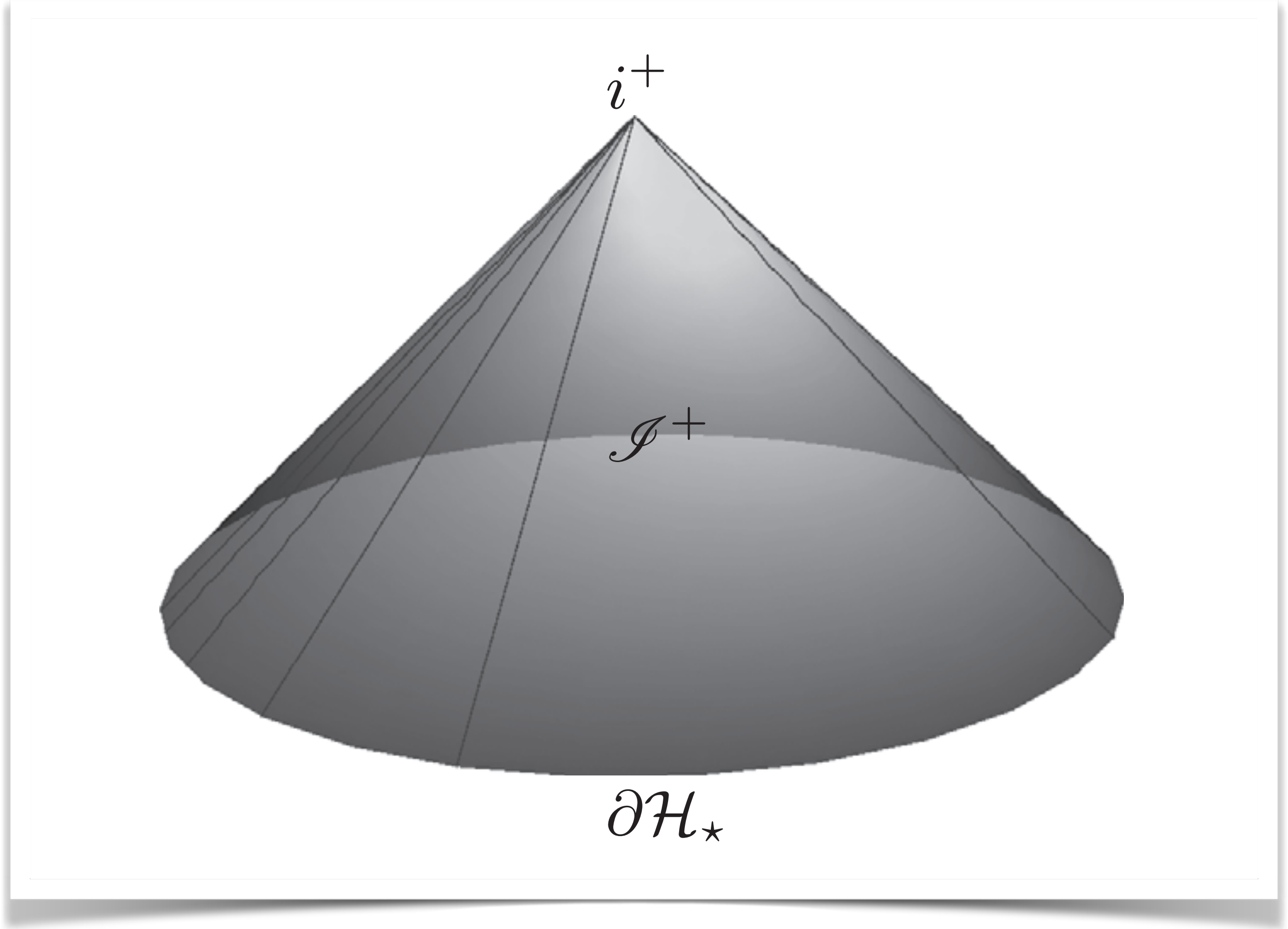
- Existence of solutions to the constraint equations with the required properties has been analysis by Anderson, Chrusciel & Friedrich (1990) and Anderson & Chrusciel (1991)

The structure of timelike infinity

A couple of corollaries

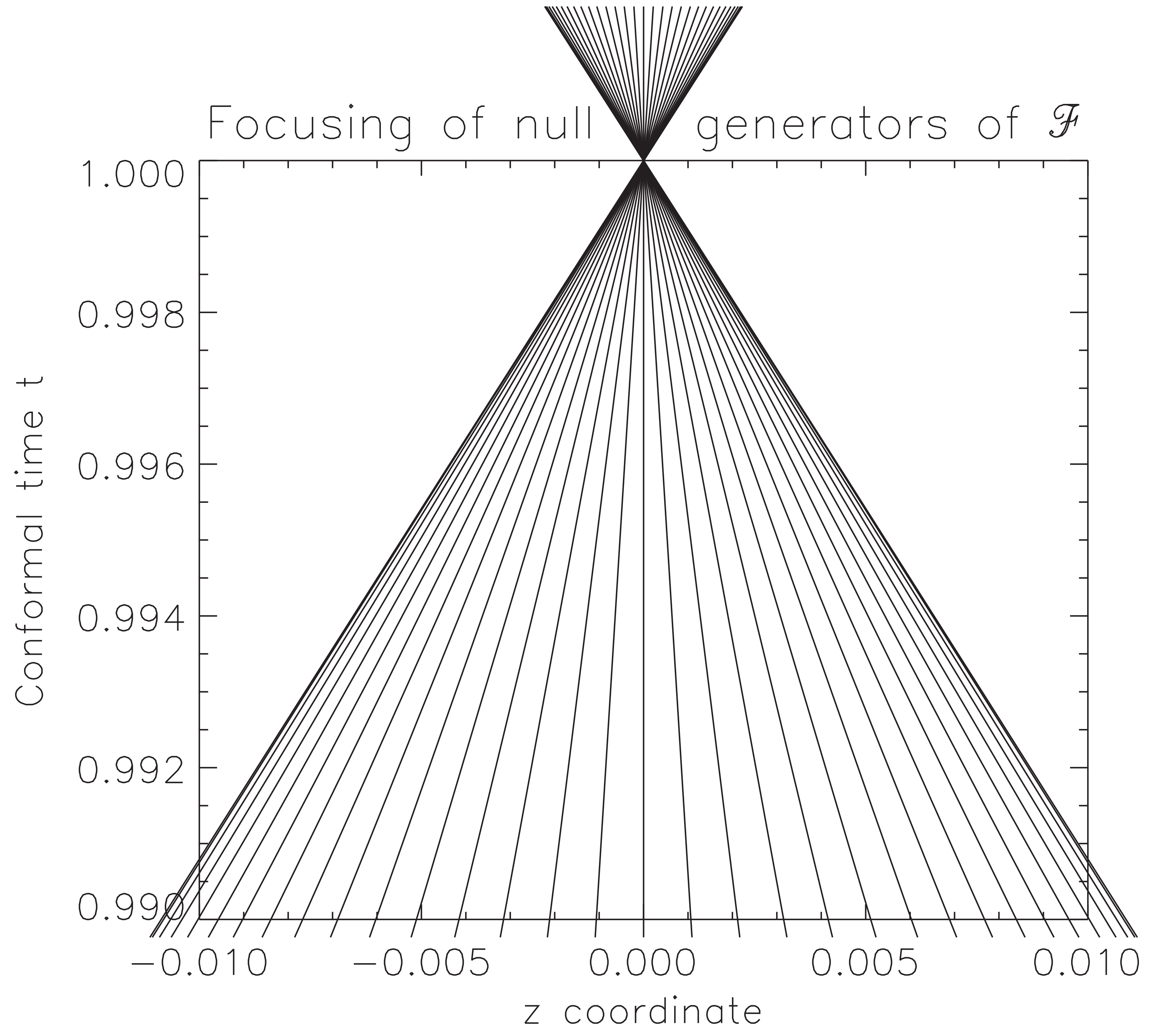
Corollary 1. *The spacetimes obtained in the previous theorem are future timelike geodesically complete.*

Corollary 2. *The null generators of* $\mathscr{I}^+$ *intersect at single point* i^+ *corresponding to the endpoints of (physical) timelike geodesics.*



Numerics

Using the conformal field equations for numerical simulations



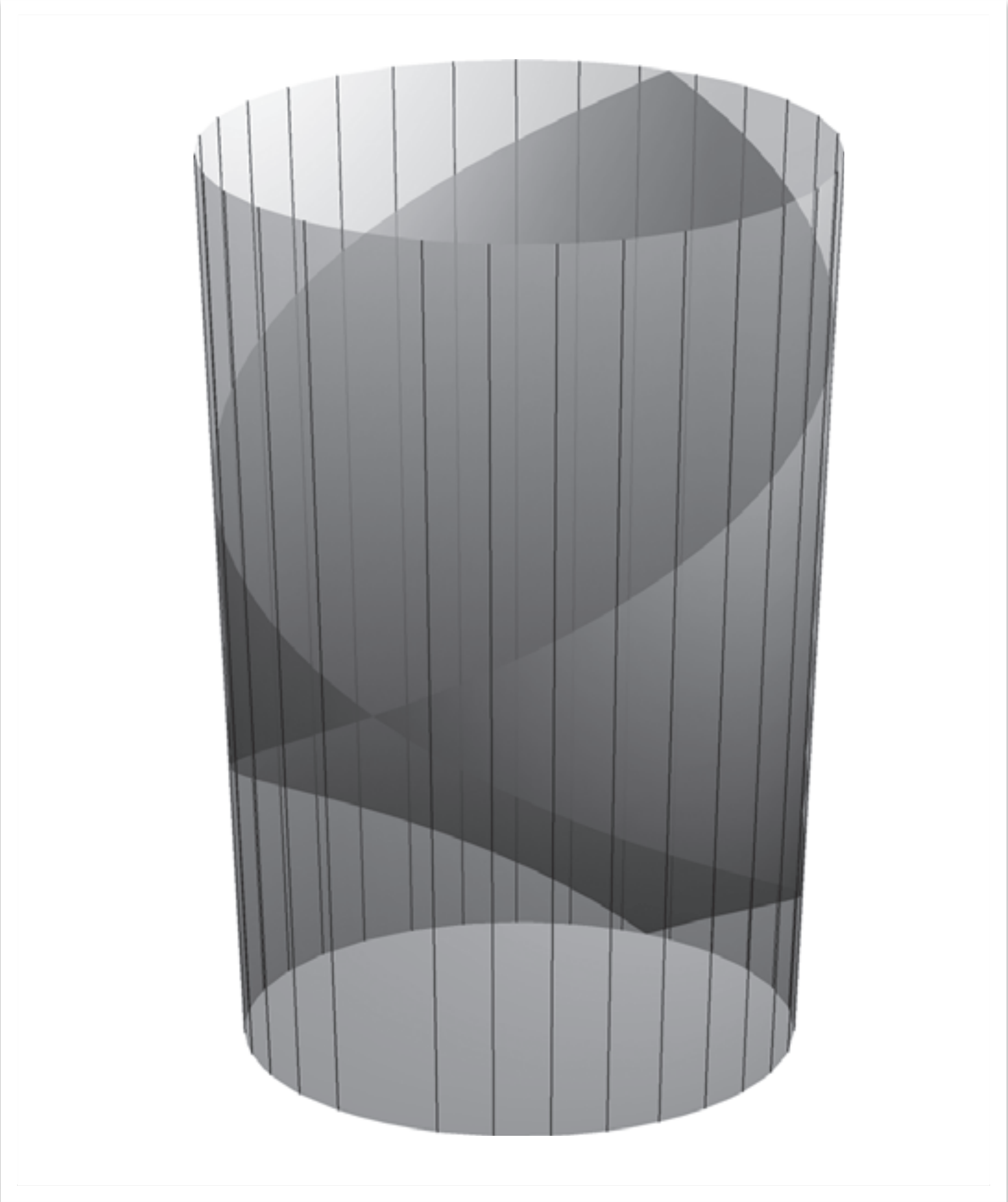
Credit: P Hübner (2001)

- The semiglobal stability of Minkowski spacetime has been *numerically illustrated* by P. Hübner (2001)
- New developments regarding scattering problems by Frauendiener, Stevens et al.

- Renewed interest by the community working on self-force problems —eg Zenginoglu,…

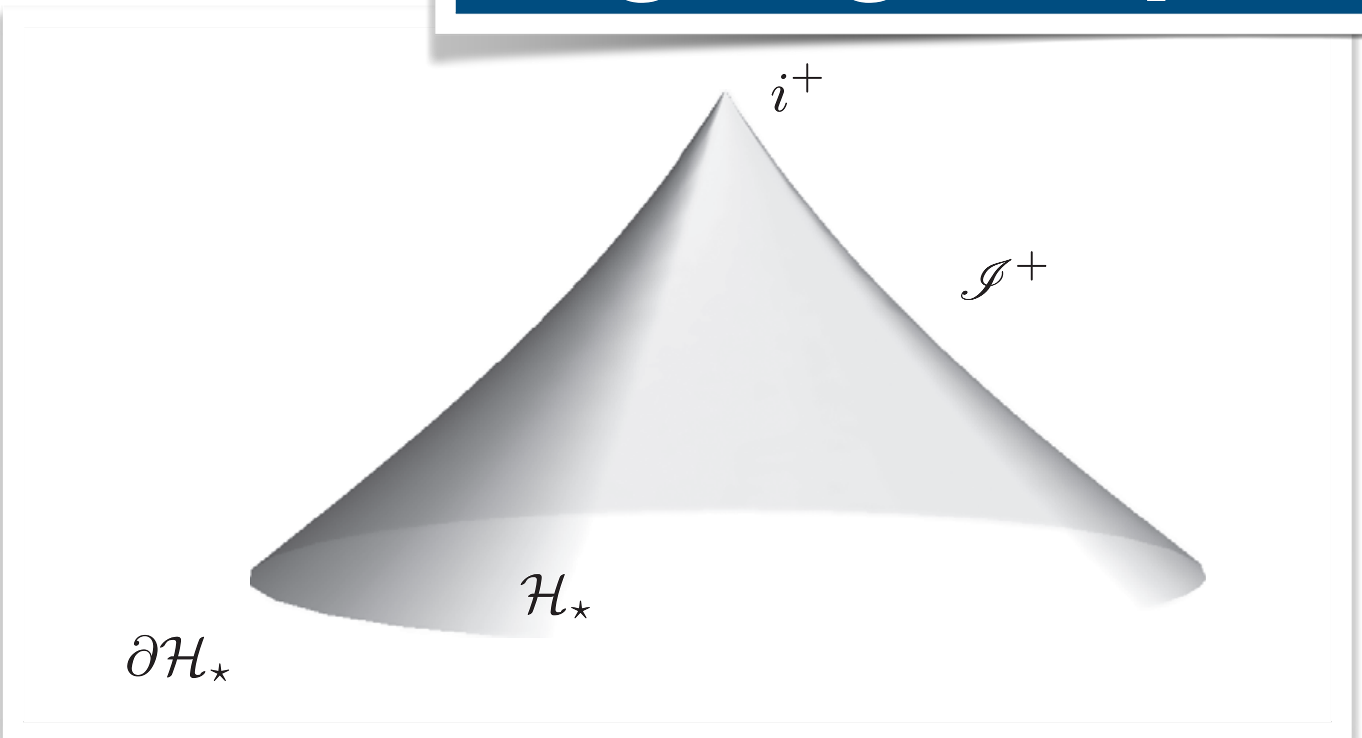
A proof using conformal geodesics

Revisiting Friedrich's original proof…



Key idea 1: use the CGs ruling the Einstein cylinder

Conformal Gaussian gauge representation



Key idea 2: apply Cauchy stability to the conformal evolution equations

$$\partial_\tau \hat{\boldsymbol{v}} = \mathbf{K}\hat{\boldsymbol{v}} + \mathbf{Q}(\hat{\boldsymbol{\Gamma}})\hat{\boldsymbol{v}} + \mathbf{L}(x)\boldsymbol{\phi},$$

$$\big(\mathbf{I} + \mathbf{A}^0(\boldsymbol{e})\big)\partial_\tau\boldsymbol{\phi} + \mathbf{A}^\alpha(\boldsymbol{e})\partial_\alpha\boldsymbol{\phi} = \mathbf{B}(\hat{\boldsymbol{\Gamma}})\boldsymbol{\phi}$$

A conformal Gaussian system for timelike infinity

A key observation

Timelike geodesics as conformal geodesics

Proposition. *Any non-null metric geodesic in an Einstein spacetime is, up to a reparametrisation, a non-null conformal geodesic.*

CG equations

Solving…

$$\tilde{\nabla}_{\boldsymbol{x}'}\boldsymbol{x}' = 0$$

$$s = s(\tau)$$

$$\boldsymbol{\beta} = \alpha(\tau)\boldsymbol{x}'^{\flat}$$

Hypothesis and Ansatz

$$\ddot{s} + \alpha\dot{s}^2\tilde{\boldsymbol{g}}(\boldsymbol{x}', \boldsymbol{x}') = 0$$

$$\dot{\alpha} = \frac{1}{2}\alpha^2\dot{s}\tilde{\boldsymbol{g}}(\boldsymbol{x}', \boldsymbol{x}')$$

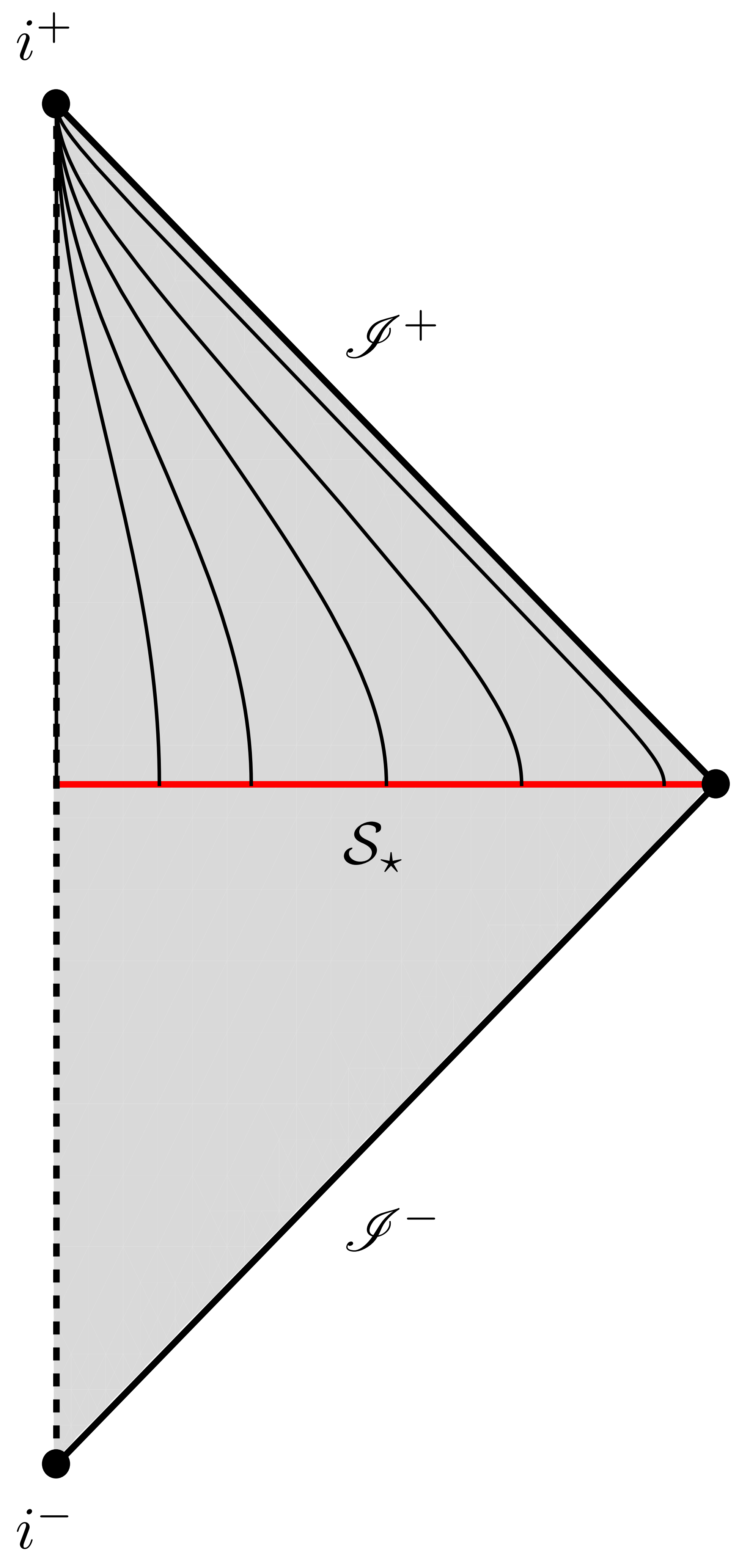
$$s - s_\star = \frac{2(\tau - \tau_\star)}{2\tau'_\star + \alpha_\star(\tau - \tau_\star)}$$

$$\alpha - \alpha_\star = \frac{\alpha_\star^2}{2\tau'_\star}(\tau - \tau_\star)$$

Universal solution if proper time is used!

Which timelike geodesics?

Blow-up i^+ by looking at the integral curves of ∂_t



- A natural congruence to blow-up timelike infinity is given by the integral curves of ∂_t (time translation Killing vector)
- In the Minkowski spacetime these curves are geodesics and, thus, can be reparametrised as conformal geodesics
- The reparametrised curves reach i^+ for a finite value of the new parameter τ
- The curves can be parametrised by their location at the fiduciary hyper surface

A first example

Reaching i^+ from a Minkowski Cauchy hypersurface

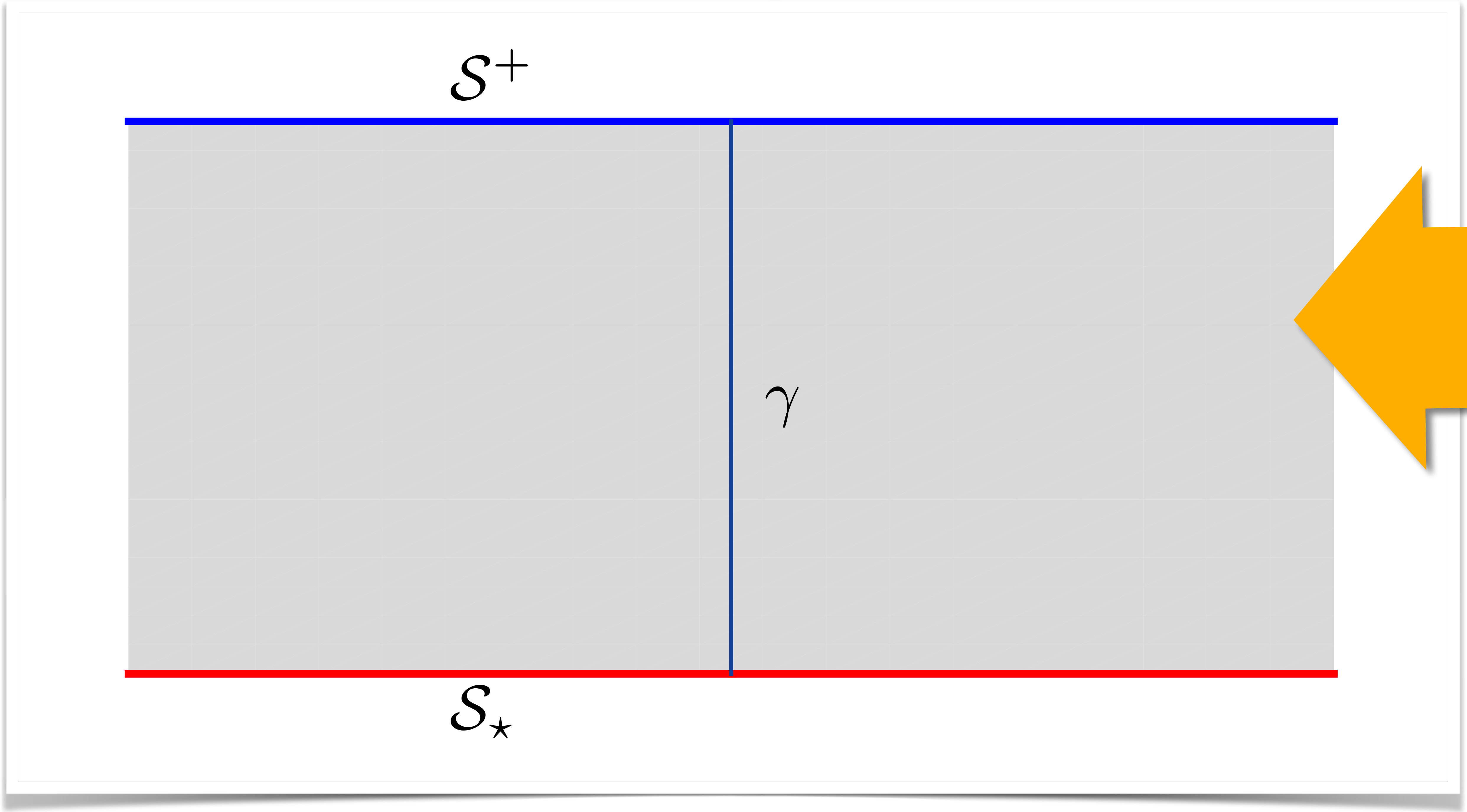
$$\tilde{\boldsymbol{\eta}} = \mathbf{d}t \otimes \mathbf{d}t - \mathbf{d}r \otimes \mathbf{d}r - r^2 \boldsymbol{\sigma}$$

$$\gamma_{(r_\star, \theta_\star, \varphi_\theta)} \equiv (s, r_\star, \theta_\star, \varphi_\star), \qquad s \in \mathbb{R}$$

$$\boldsymbol{x}'(s) = \boldsymbol{\partial}_t$$

$$s(\tau) = \frac{\tau}{1-\tau} \qquad \tau \in [0,1]$$

$$\Theta = (1-\tau)^2$$



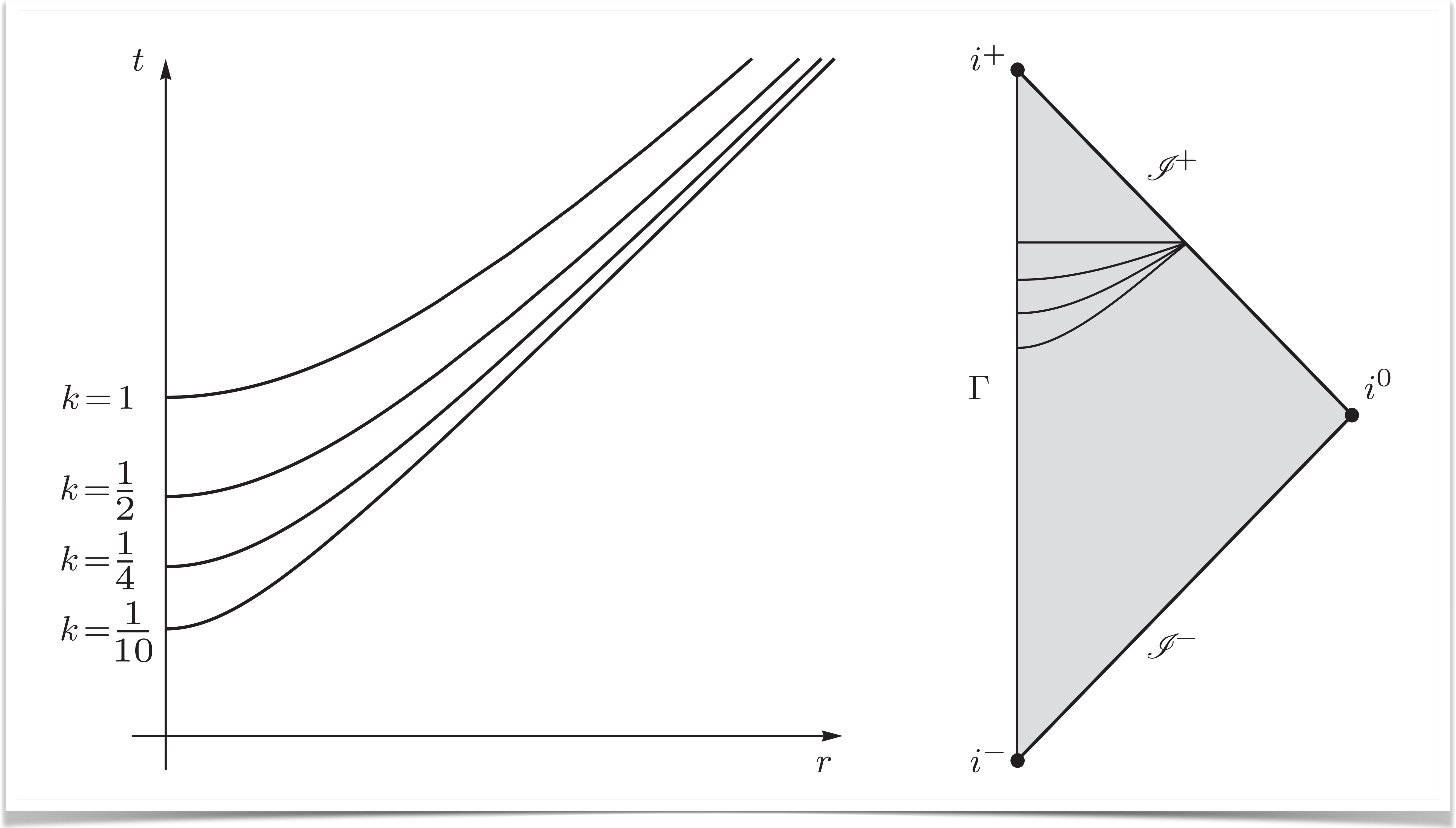
Key observations:

$$\boldsymbol{\eta} = \mathbf{d}\tau \otimes \mathbf{d}\tau - (1-\tau)^2 \boldsymbol{\delta}.$$

$$\Theta|_{\tau=1} = 0, \qquad \mathbf{d}\Theta|_{\tau=1} = 0.$$

CMC hyperboloids in Minkowski spacetime

A more convenient starting point for an investigation of i^+



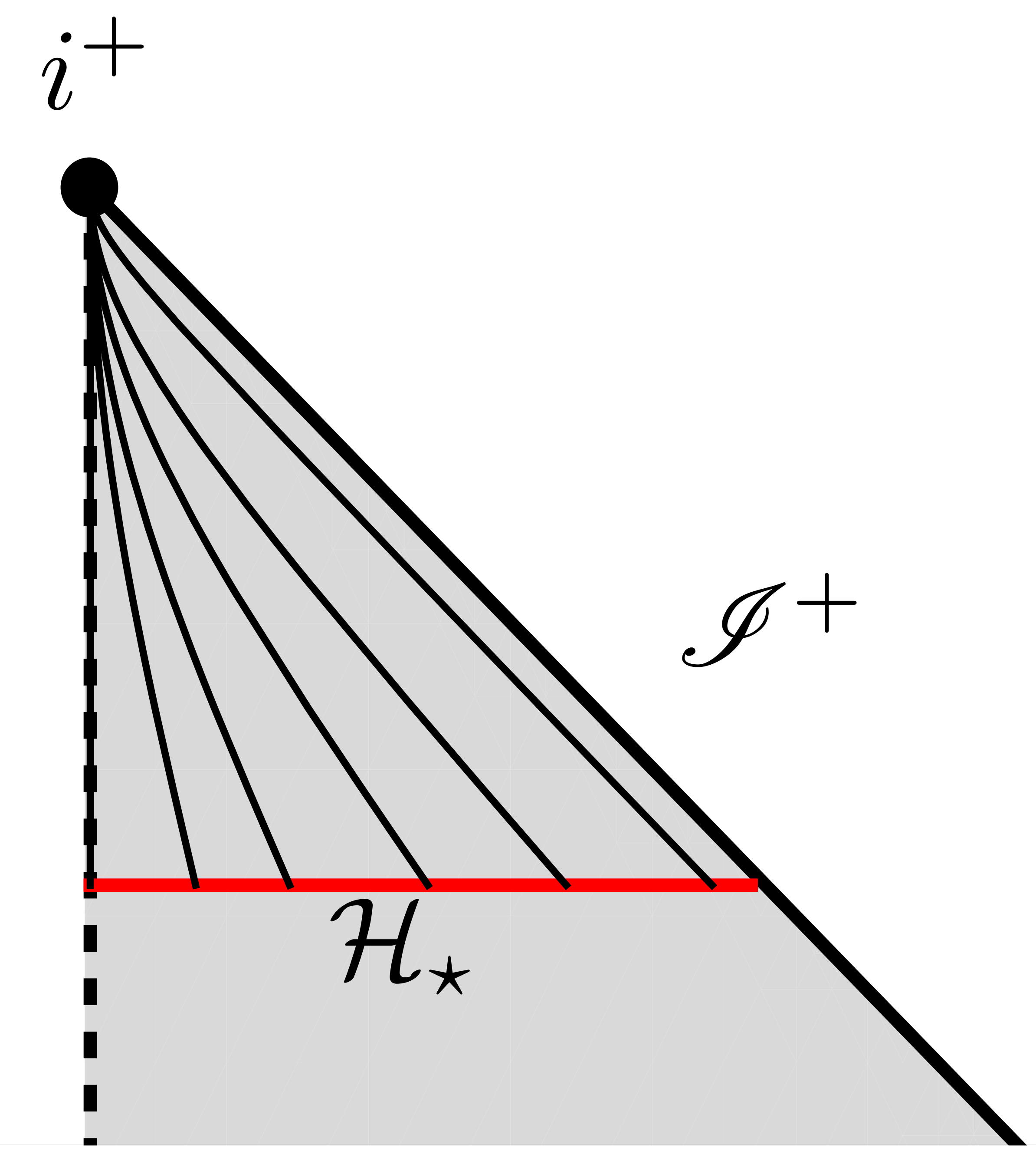
$$\mathcal{H}_k = \{p \in \mathbb{R}^4 \,|\, t^2(p) - r^2(p) = k\},$$

$$k > 0$$

$$\tilde{K} = \frac{3}{\sqrt{k}}$$

Using the standard hyperboloid

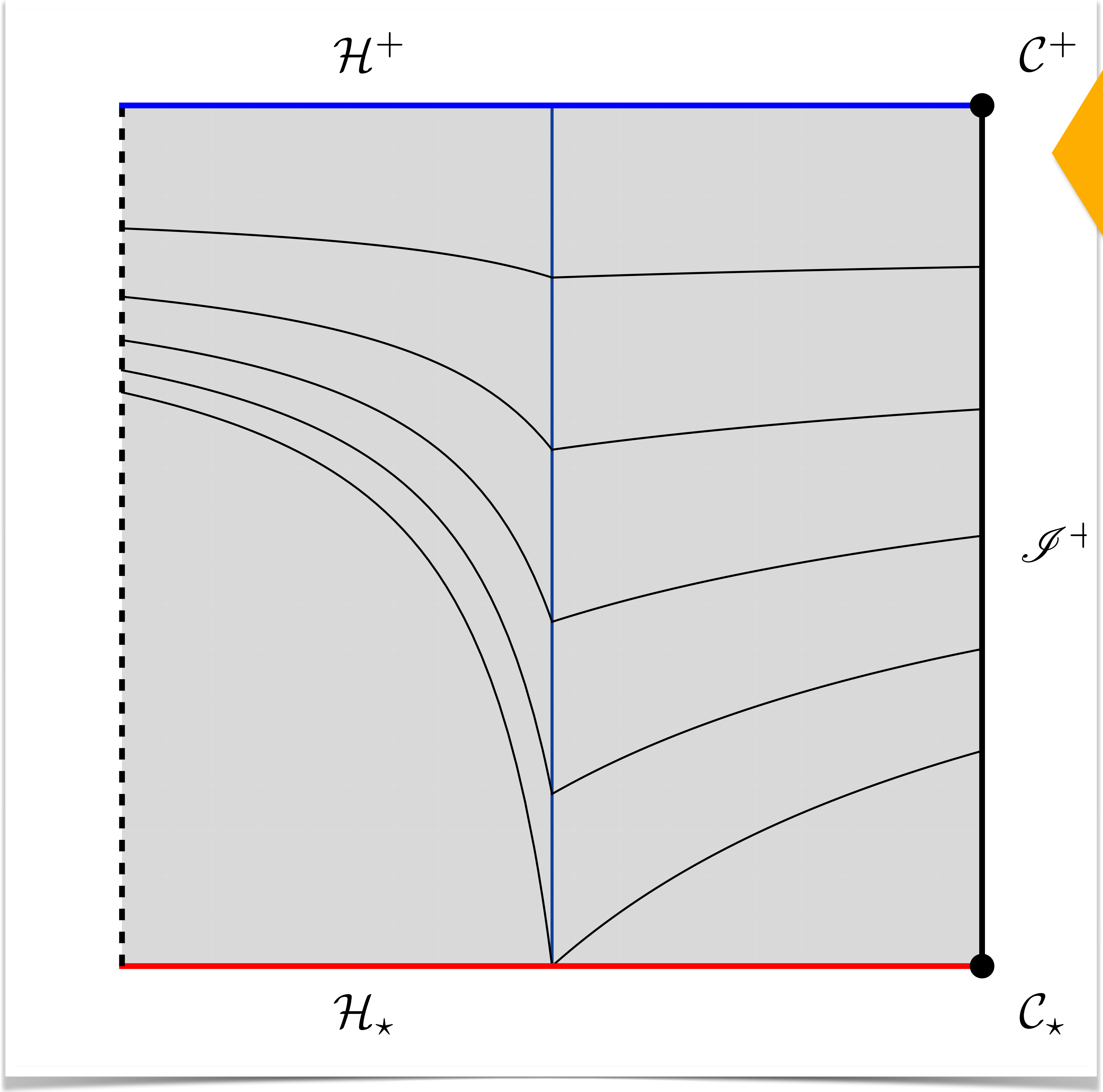
Encoding the information on $\mathscr{H}_\star$

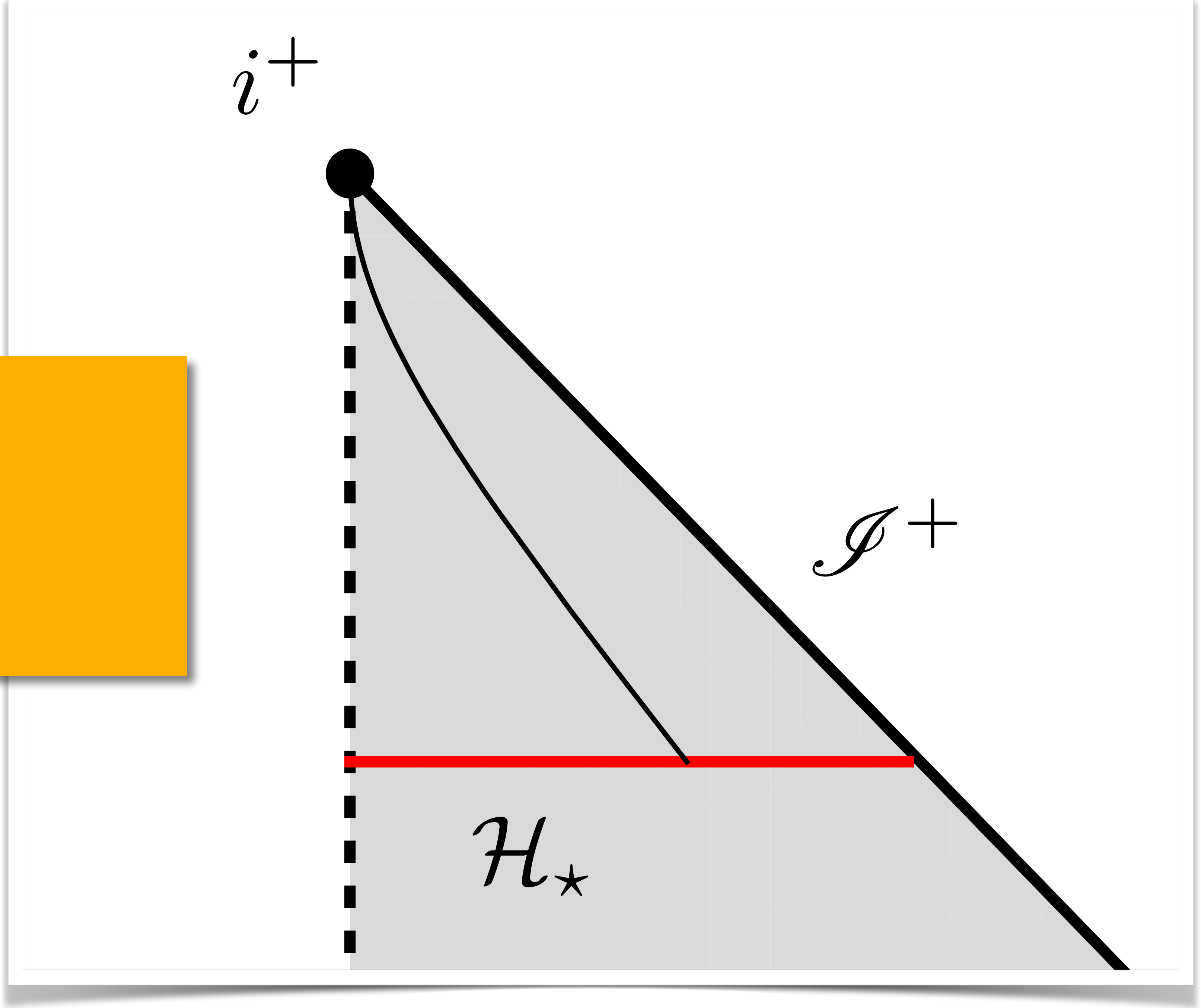


- Starting from $\mathscr{H}_\star$ allows to naturally include $\mathscr{I}^+$ in the discussion
- $\mathscr{I}^+$ is generated by limiting CGs
- Conformally compactify $\mathscr{H}_\star$ to the interior of $\mathbb{S}^2$ (Beltrami-Klein model of hyperbolic space)
- The CGs are not orthonormal to $\mathscr{H}_\star$

The null cones near $\mathscr{H}^+$

Implementing $\mathscr{I}$-fixing





- $\mathscr{I}^+$ is described by a fixed coordinate location ($\chi = 1$)
- $\mathscr{H}^+$ is given by the condition $\tau = 1$
- Lightcones open as $\mathscr{H}^+$ is approached along a conformal geodesic

Side remark: i^0 vs i^+

First attempts at formalising spatial infinity

Geroch (1972), Ashtekar & Hansen (1978) —see also Wald (1984)

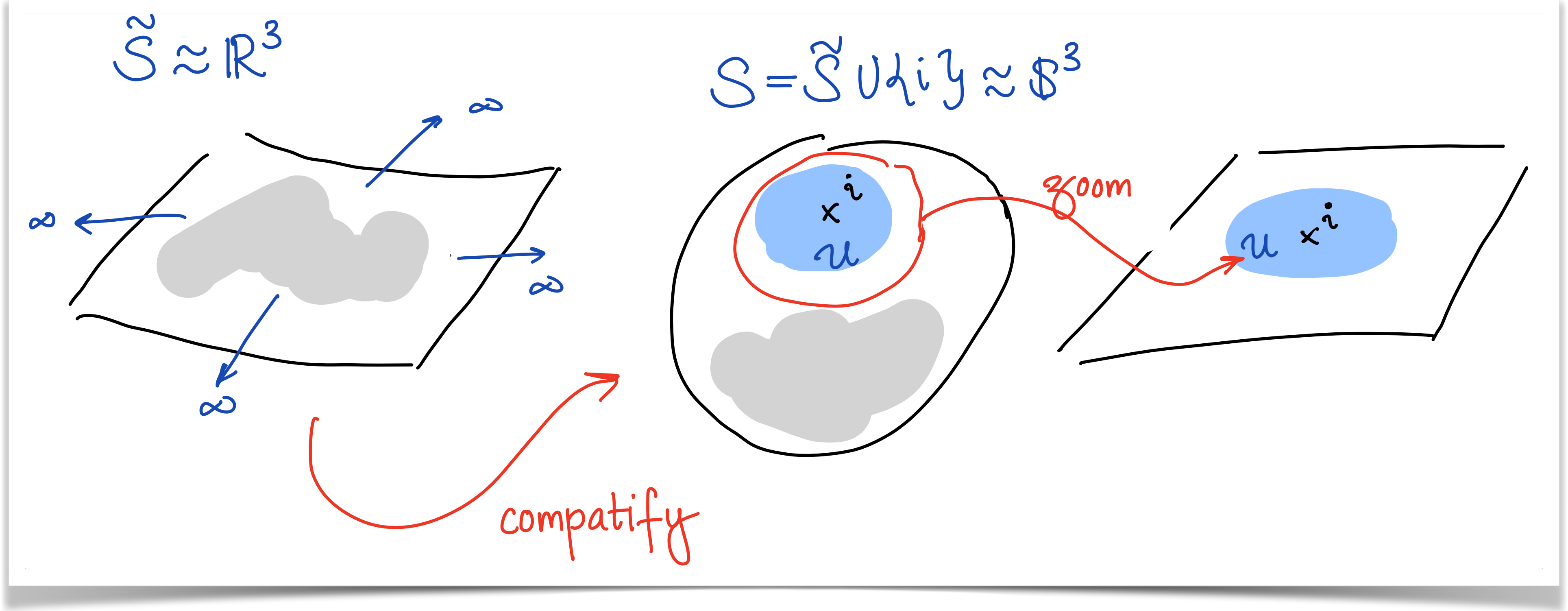
Definition 11.2 (***asymptotically Euclidean and regular manifolds***) *A three-dimensional Riemannian manifold* $(\tilde{\mathcal{S}}, \tilde{\boldsymbol{h}})$ *will be said to be* ***asymptotically Euclidean and regular*** *if there exists a three-dimensional, orientable, compact manifold* $(\mathcal{S}, \boldsymbol{h})$ *with points* $i_k \in \mathcal{S}$, $k = 1, \ldots, N$ *with* N *some integer, a diffeomorphism* $\varphi : \mathcal{S} \setminus \{i_1, \ldots, i_N\} \to \tilde{\mathcal{S}}$ *and a function* $\Omega \in C^2$ *such that:*

(i) $\Omega(i_k) = 0,\ \mathbf{d}\Omega(i_k) = 0,\ \mathbf{Hess}\,\Omega(i_k) = -2\boldsymbol{h}(i_k)$.

(ii) $\Omega > 0$ *on* $\mathcal{S} \setminus \{i_1, \ldots, i_N\}$.

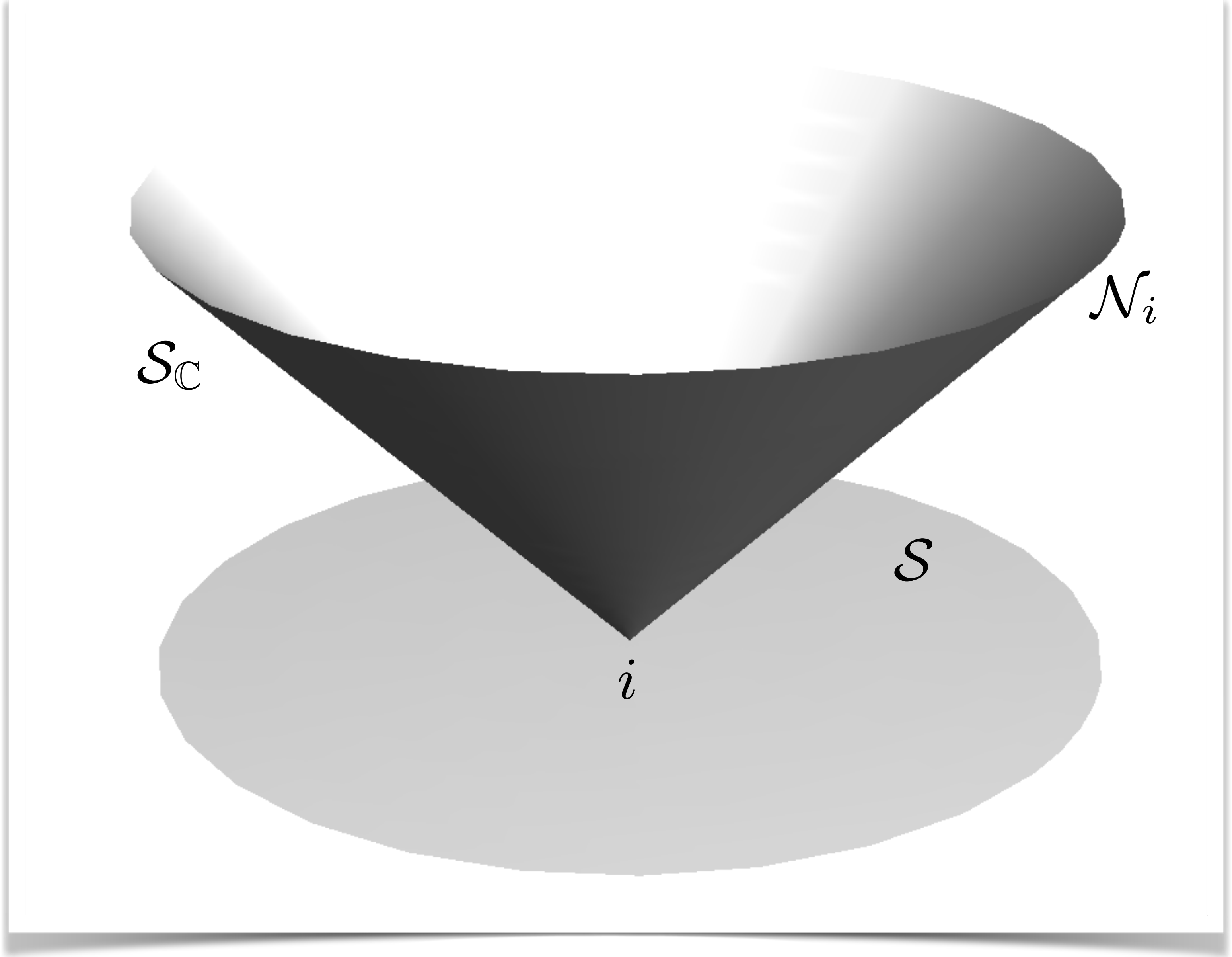
(iii) $\boldsymbol{h} = \Omega^2 \varphi^* \tilde{\boldsymbol{h}}$ *on* $\mathcal{S} \setminus \{i_1, \ldots, i_N\}$ *with* $\boldsymbol{h} \in C^2(\mathcal{S}) \cap C^\infty(\mathcal{S} \setminus \{i_1, \ldots, i_N\})$.

- Point compactification of spatial infinity using ***stereographic projection***.
- Later used to construct solutions to the constraints by the ***puncture method*** —Beig & O'Murchadha (1990), Friedrich (1998).



Duality between i^0 and i^+

On static and radiative spacetimes (HF, CMP 1986)



Constructing *radiative spacetimes near* i^+ from *static solutions near* i^0

Identifies the *radiativity condition* for the Cotton tensor

$$D_{\{i_p}\cdots D_{i_1}b_{jk\}}(i) = 0, \quad p = 0,1,2,\ldots$$

ensuring that

$$d_{ij} = \frac{1}{\Omega^2}\left(D_{\{i}D_{j\}}\Omega + \Omega s_{ij}\right)$$

is *analytic at* i.

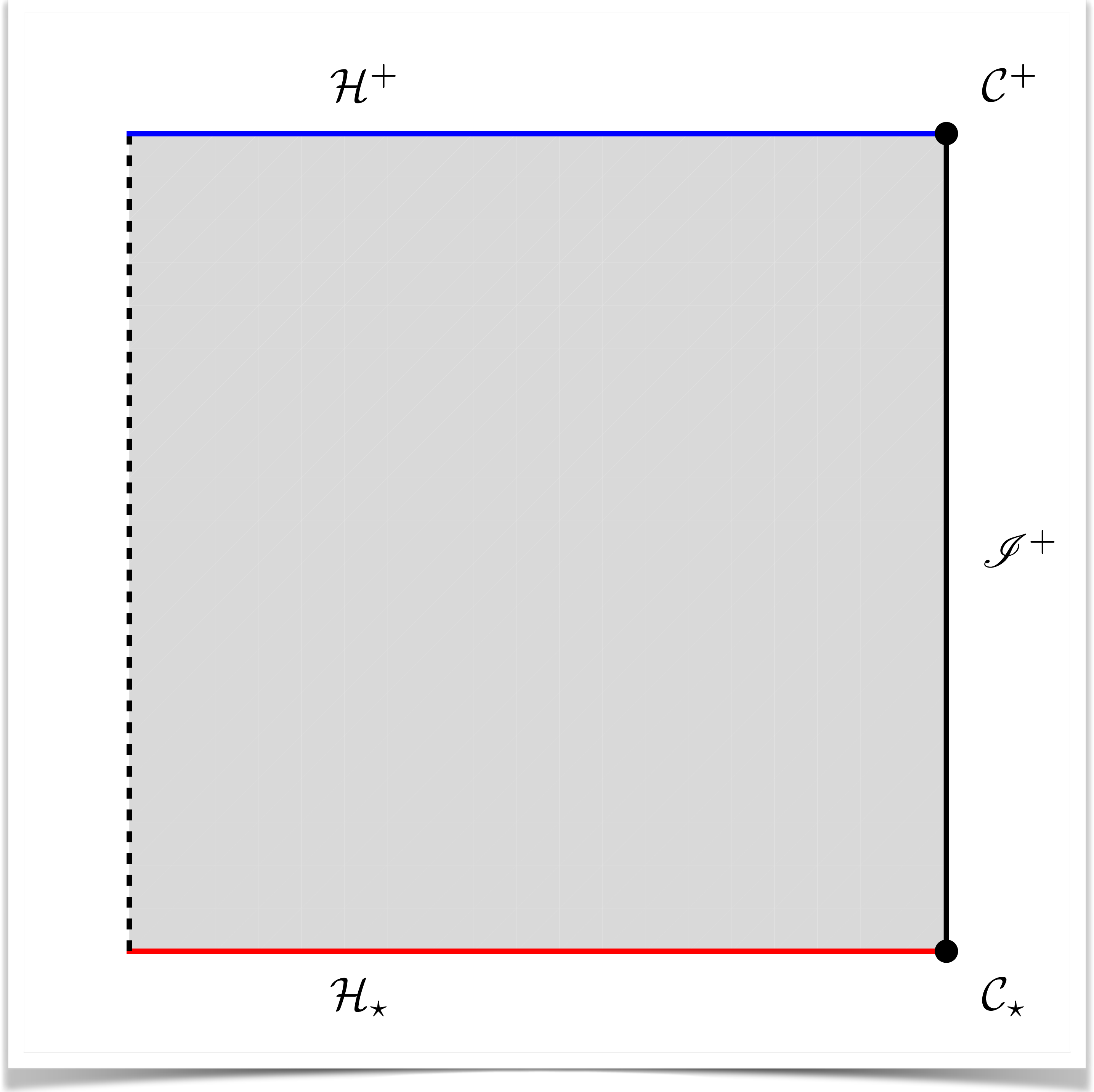
Proposition (Beig 1990). *Static solutions satisfy the radiativity condition.*

Stability of the gauge

Stability of the gauge

The structures are preserved for a larger class of spacetimes

Theorem (MMagdy & JA Valiente Kroon, 2026). *The semiglobal perturbations of Minkowski of Friedrich's theorem admit a hyperboloidal representation of i^+ similar to that of the Minkowski spacetime.*



- This follows from the timelike geodesic completeness of the spacetimes and the fact that the reparametrisation formulae are universal!

Evolution equations near $\mathscr{H}^+$

A toy model: the wave equation

Degeneracy of the principal part near $\mathscr{H}^+$

$$\Box\phi = 0$$

A useful toy model!

$$\begin{aligned}
&\frac{4\mathfrak{a}^2(1-\tau)^4}{(1+\chi^2)^2}\partial_\tau^2\phi + \frac{8\mathfrak{a}(1-\tau)^2\chi}{(1+\chi^2)^2}\partial_\tau\partial_\chi\phi - \frac{(1-\chi^2)^2}{(1+\chi^2)^2}\partial_\chi^2 \\
&\quad + \frac{4\mathfrak{a}}{(1+\chi^2)^2}(1-\tau)^2\Big(3+\chi^2+6\mathfrak{a}(\tau-1)(1+\chi^2)\Big)\partial_\tau \\
&\quad - \frac{1}{\chi(1+\chi^2)^3}\Big(2\big(1-(4+8\mathfrak{a}(\tau-1))\big)\chi^2 + \big(1+8\mathfrak{a}(1-\tau)\big)\chi^4 + 2\chi^6\Big)\partial_\chi\phi \\
&\quad - \frac{1}{\chi^2}\not{\Delta}\phi = 0.
\end{aligned}$$

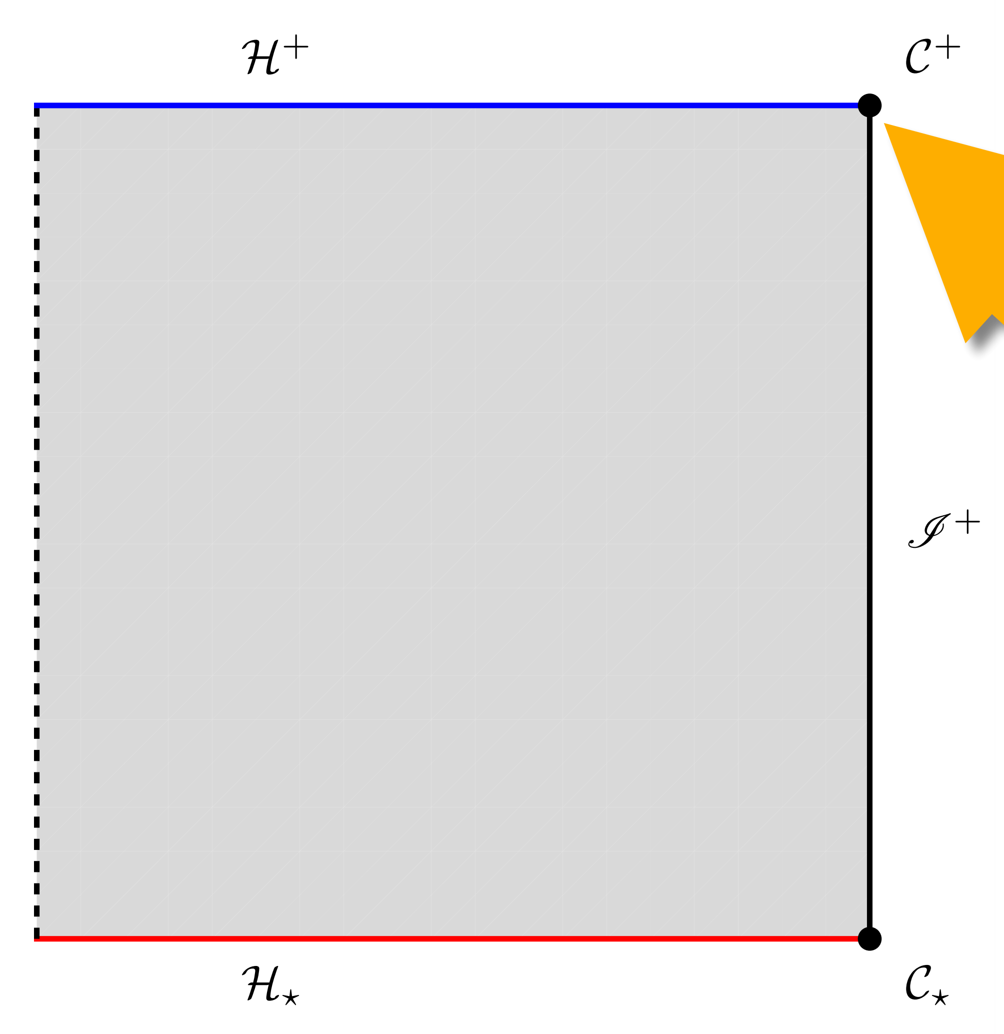
Key observations:

- The principal part degenerates at $\mathscr{H}^+$
- However, it is regular from the point of view of b-geometry

The constraint at $\mathscr{H}^+$

The values of ϕ are not independent from each other

$$\frac{(1-\chi^2)^2}{(1+\chi^2)^2}\partial_\chi^2\phi + \frac{2(1-\chi^2)}{\chi(1+\chi^2)^3}(2\chi^4+3\chi^2-1)\partial_\chi\phi + \frac{1}{\chi^2}\Delta\!\!\!/\,\phi = 0$$



- This is a degenerate elliptic equation
- Regular from the point of view of ***b-geometry***
- Solutions, in principle, determined by the value of ϕ at the corner $\mathscr{C}^+$
- Analogous behaviour to that of the cylinder at spatial infinity

The transport equations at $\mathscr{I}^+$

A first investigation of the degeneracy at $\mathscr{C}^+$

The restriction of the wave equation at $\mathscr{I}^+$:

$$\mathfrak{a}^2(1-\tau)^4\partial_\tau^2\phi + \frac{\mathfrak{a}}{2}(1-\tau)^2\big(4-12\mathfrak{a}(1-\tau)\big)\partial_\tau\phi - \Delta\!\!\!/\,\phi = 4\mathfrak{a}(1-\tau)\partial_\chi\phi - 2\mathfrak{a}(1-\tau)^2\partial_\tau\partial_\chi\phi.$$

Transverse derivatives: regard as sources

$$\Delta\!\!\!/\, Y_{\ell m} = -\ell(\ell+1)Y_{\ell m}$$

Decompose in harmonics

$$(1-\tau)^4\ddot{\varphi} + \frac{1}{2\mathfrak{a}}(1-\tau)^2\big(4-12\mathfrak{a}(1-\tau)\big)\dot{\varphi} + \frac{1}{\mathfrak{a}^2}\ell(\ell+1)\varphi = 0$$

Solutions in terms of Whittaker functions. Can be used to ***study the behaviour towards*** $\tau \to 1^-$

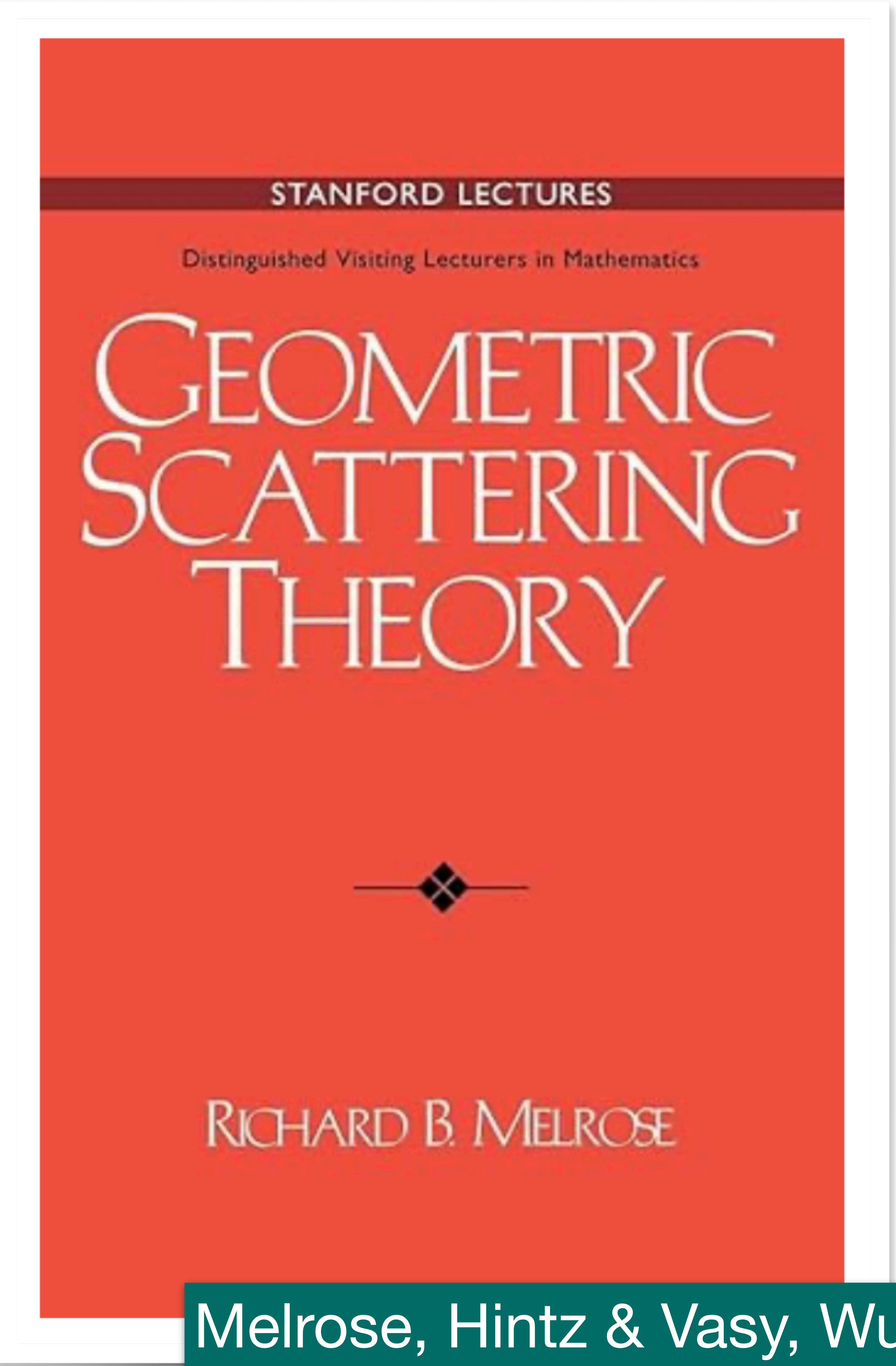
Conclusions

Geometric scattering theory

A fresh perspective

- Melrose's framework of geometric scattering theory provides a modular framework to study PDEs on ***compact manifolds***.
- Building on ***microlocal analysis***, allows to understand the ***singularities*** of solutions to PDEs.

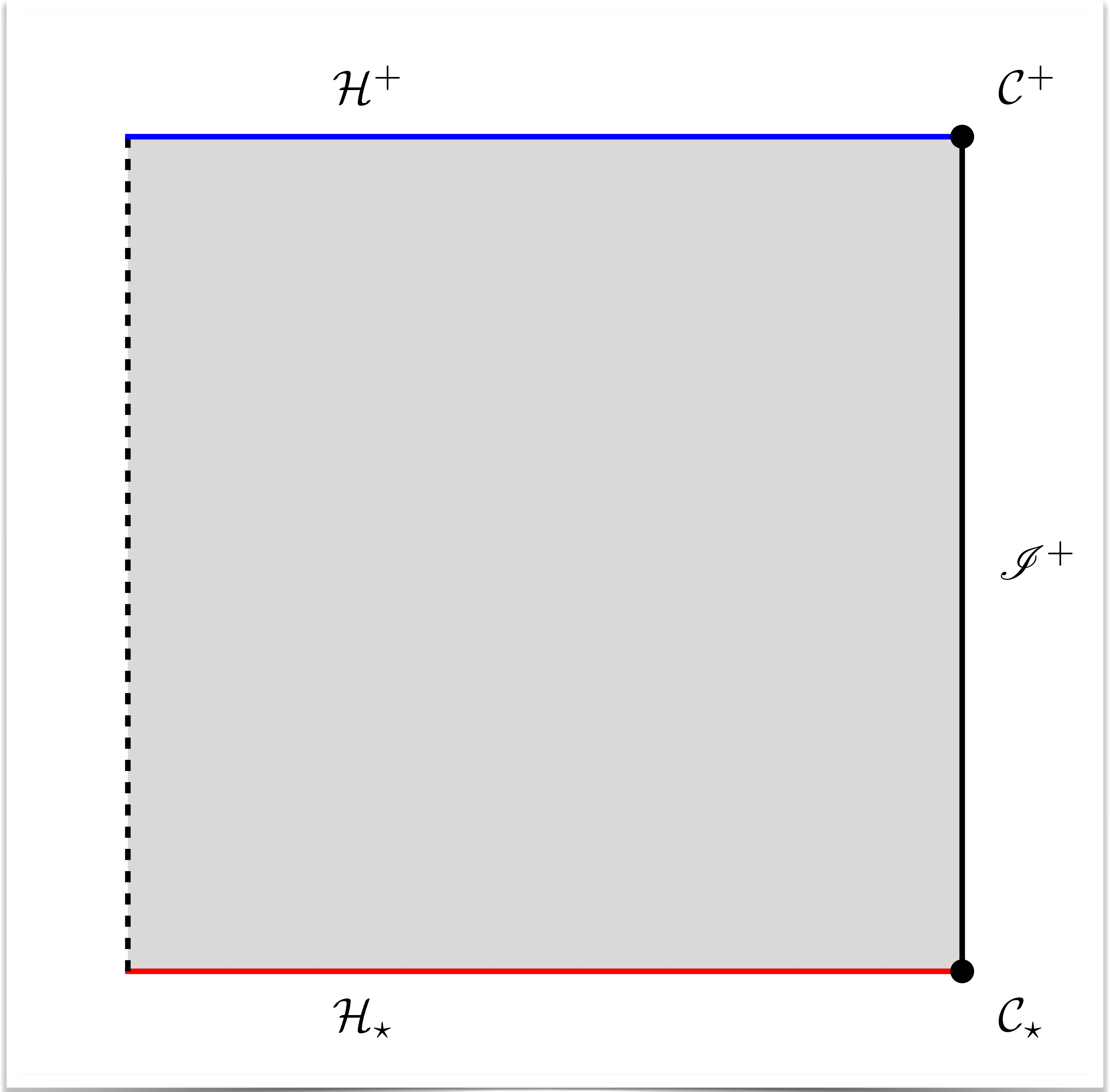
First applied to study stability of the Minkowski spacetime by Hintz & Vasy (2020).



Melrose, Hintz & Vasy, Wunsch, Kadar & Kehrberger

Back to the semi global stability of Minkowski

A fresh perspective



Beheshti, Koutras & JAVK
(in preparation)

$$\partial_\tau \hat{\boldsymbol{\upsilon}} = \mathbf{K}\hat{\boldsymbol{\upsilon}} + \mathbf{Q}(\hat{\boldsymbol{\Gamma}})\hat{\boldsymbol{\upsilon}} + \mathbf{L}(x)\boldsymbol{\phi},$$

$$\big(\mathbf{I} + \mathbf{A}^0(\boldsymbol{e})\big)\partial_\tau\boldsymbol{\phi} + \mathbf{A}^\alpha(\boldsymbol{e})\partial_\alpha\boldsymbol{\phi} = \mathbf{B}(\hat{\boldsymbol{\Gamma}})\boldsymbol{\phi}$$

Obtain a ***polyhomogeneous*** version of the semiglobal stability of Minkowski:

- Exploit the conformal Gaussian gauge and the structure equations
- Construct weighted estimates *à la* Hintz & Vasy

Thank you for your attention!