Null infinity and the black hole horizon

- new conserved quantities from a geometric duality -

with Shreyansh **AGRAWAL**, Panagiotis **CHARALAMBOUS**
arXiv:2506.15526

Laura DONNAY

Lichnerowicz Conference - Journées Relativistes
Tours
June 3, 2026

SISSA

European Research Council
Established by the European Commission

Context / Motivations

The holographic principle

René Magritte (1960)

(quantum) gravity in
Anti-de Sitter spacetimes
$\Lambda < 0$

= lower-dimensional QFT

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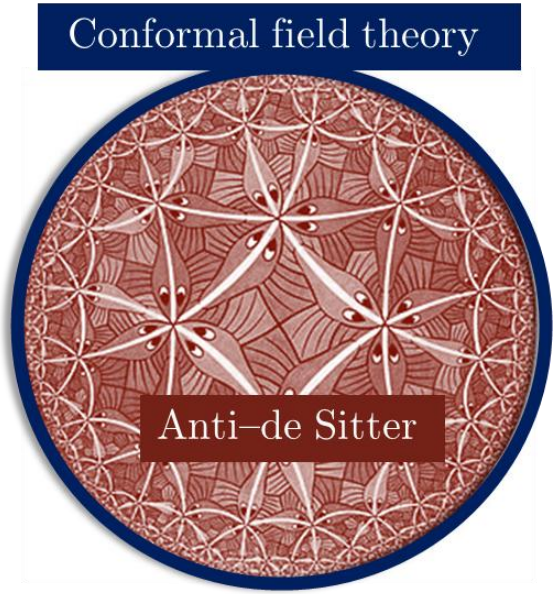

AdS/CFT correspondence

Flat space
holography

Goal : establish a holographic correspondence for

quantum gravity in
4D asymptotically flat spacetimes

Flat space
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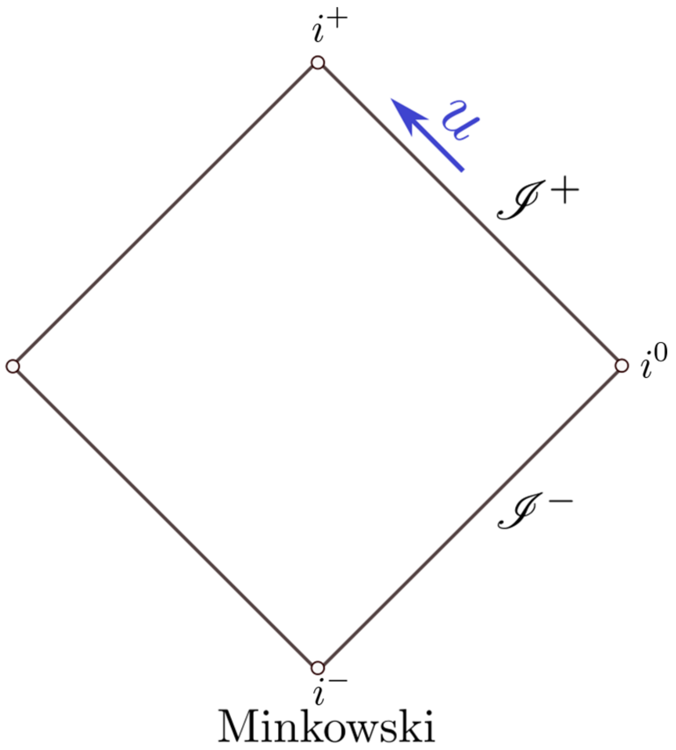

quantum gravity in
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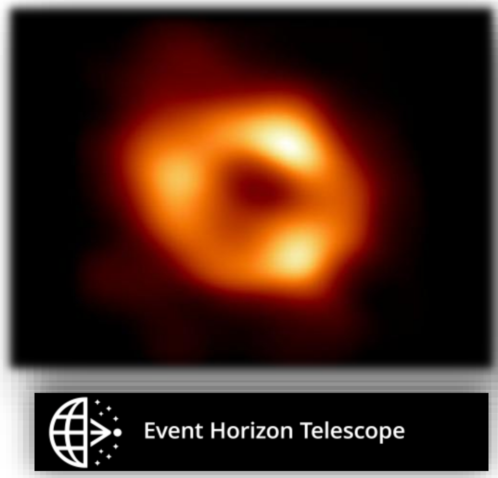
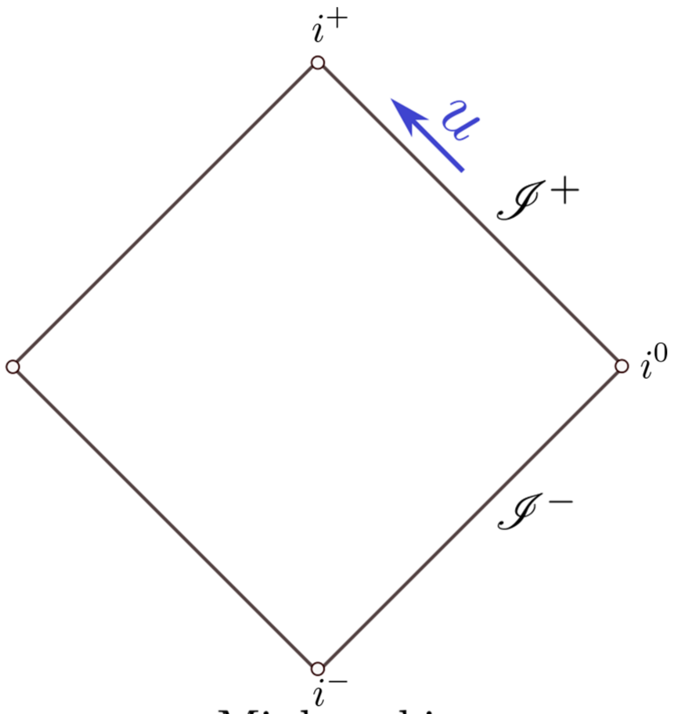




quantum gravity in
4D asymptotically flat spacetimes

Not a new idea:
[Susskind ‘99][Polchinski ‘99][Giddings ’99]
[de Boer, Solodukhin ‘03][Arcioni, Dappiaggi ’03 ’04]
[Dappiaggi, Moretti, Pinamonti ’06][Mann, Marolf ’06]…

→ much harder to formulate / differ from AdS/CFT

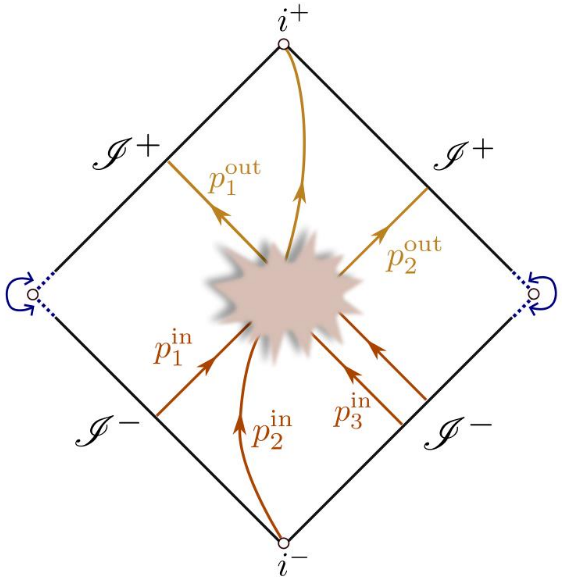
Flat space
holography

Recent new input [Strominger '14]

On BMS Invariance of Gravitational Scattering

Andrew Strominger

Center for the Fundamental Laws of Nature, Harvard University, Cambridge, MA 02138, USA



$$\langle \mathrm{out}|S|\mathrm{in}\rangle$$

Key point: BMS acts *globally*

Flat space holography

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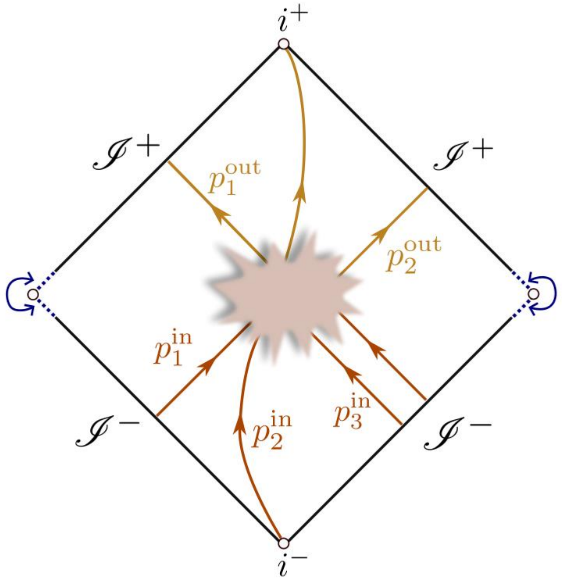


$$\langle \text{out}|S|\text{in}\rangle$$

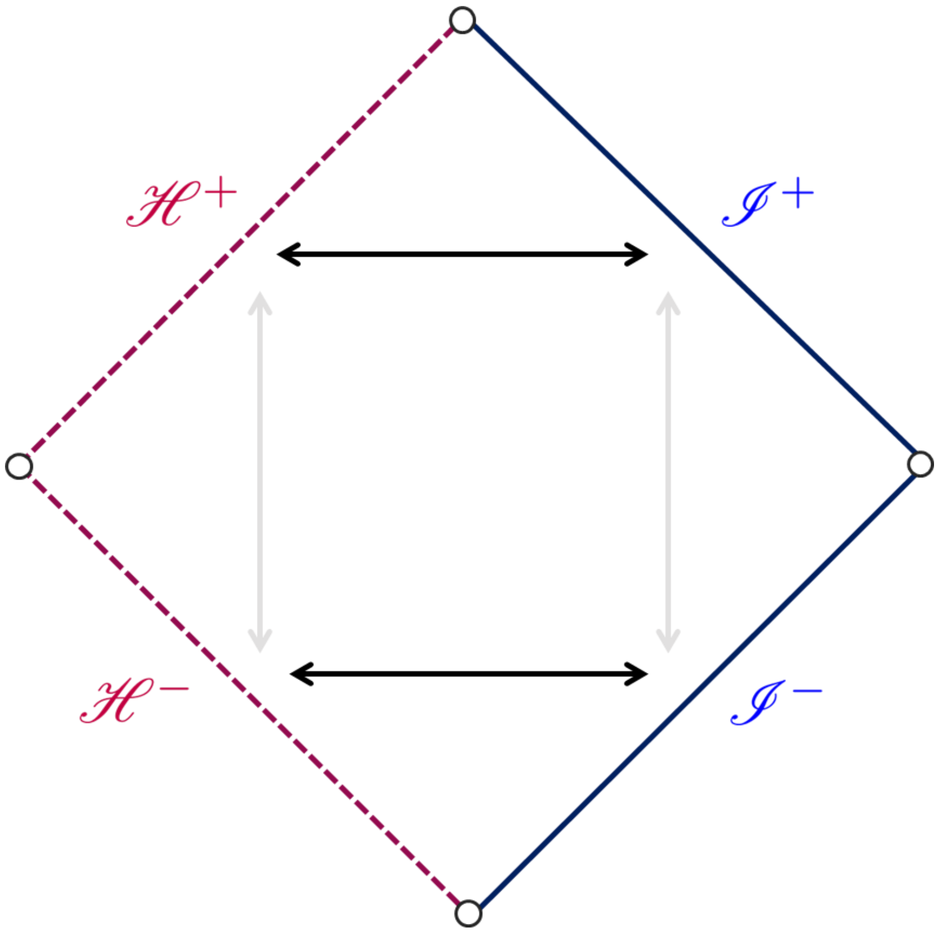
→ **The S-matrix is BMS invariant**

Provided a (new) starting point for flat space holography

↓

Celestial / Carrollian holography

Flat space holography with horizons?

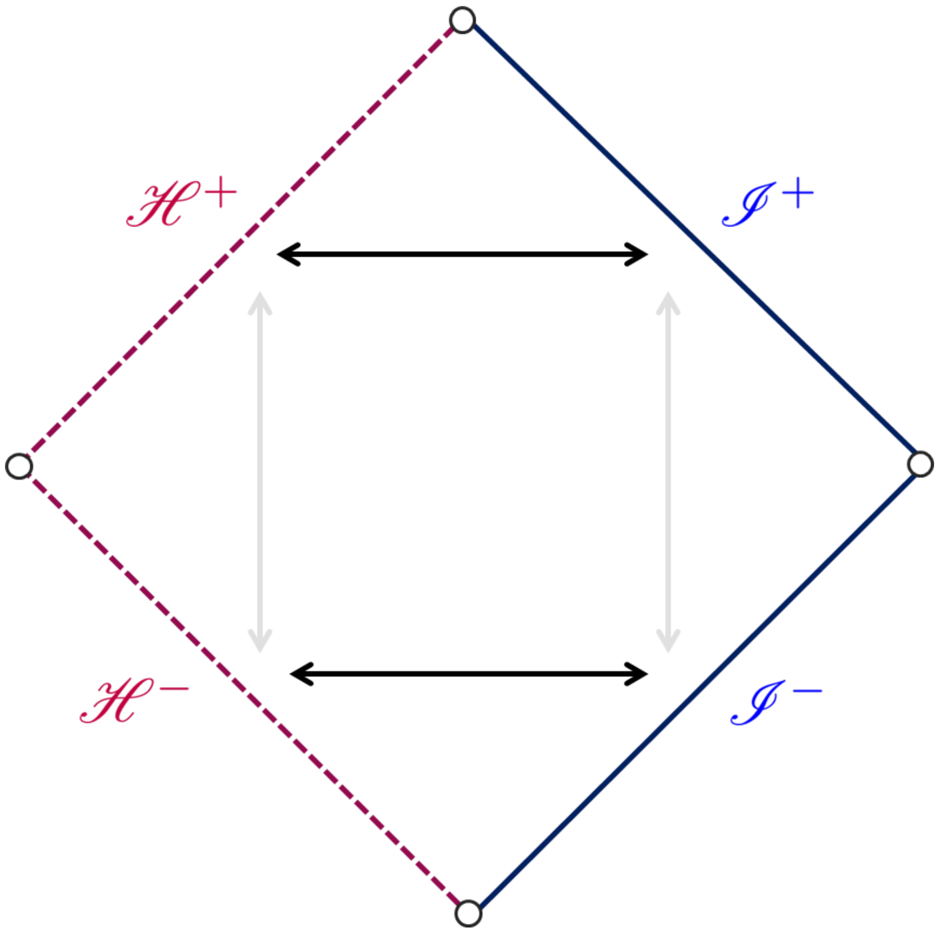


The role of asymptotic symmetries was key for the flat space holography program.

Remarkably, BMS-like symmetries also appear in the *near-horizon region* of black holes!

[LD, Giribet, Gonzalez, Pino , PRL '16]

Flat space holography with horizons?



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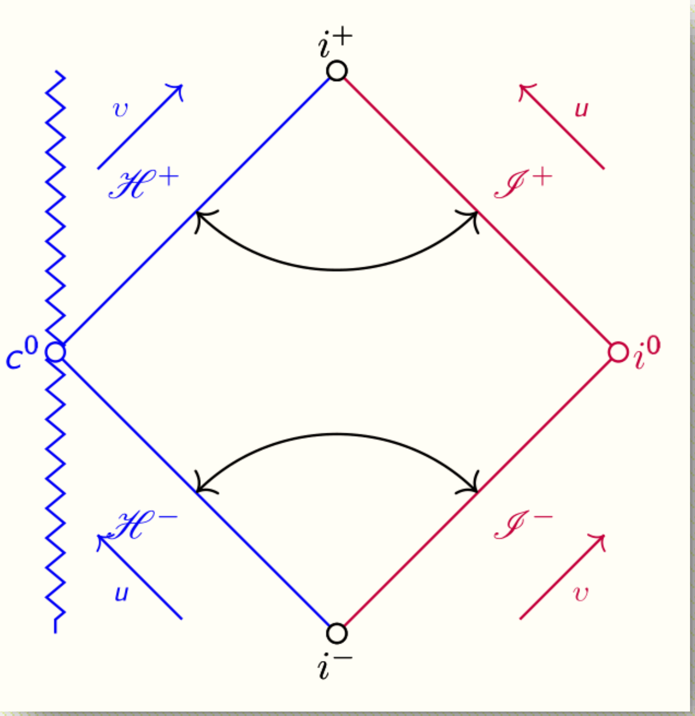
↓

What are the physical constraints implied by the infinite conservation laws for an asymptotically flat spacetime with a horizon?

→ can we interpolate between $\mathscr{H} \leftrightarrow \mathscr{I}$?

Outline

- Black hole horizon vs null infinity
- $\mathscr{H} \leftrightarrow \mathscr{I}$ geometric duality
- Matching an infinite number of conserved quantities



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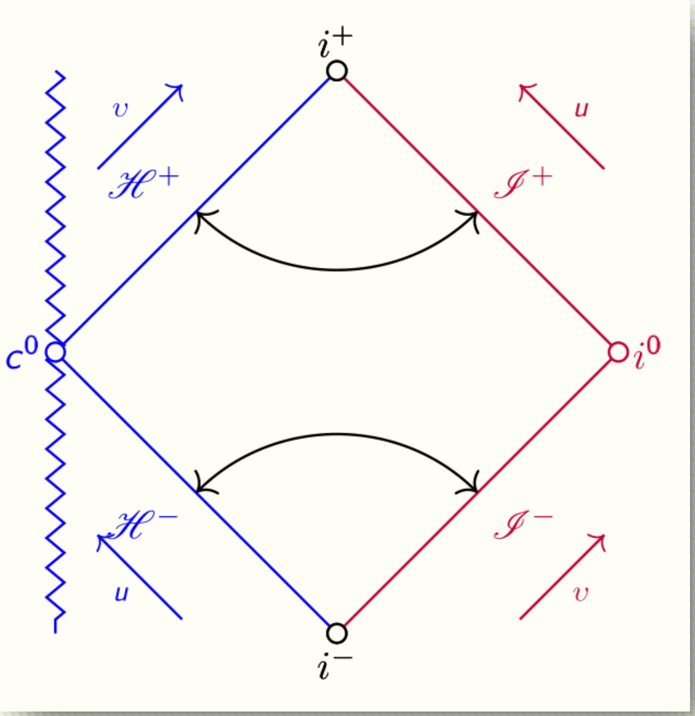
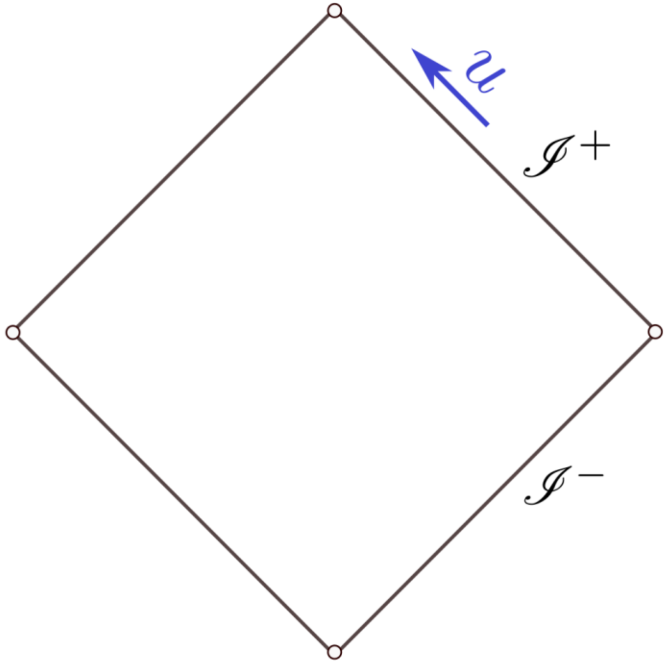


Geometry of null infinity

- **Asymptotically flat** spacetime
  [Bondi, van der Burg, Metzner '62] [Sachs '62] [Newman, Unti '62]
  (u, r, x^A)

$$ds^2 = ds^2_{\text{flat}} + \mathcal{O}(\tfrac{1}{r})$$



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BONDI METZNER SACHS

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$r \to \infty$

$$ds^2_{\mathscr{I}^+} = -F du^2 - 2\, du dr + r^2 \mathcal{H}_{AB} \left(dx^A - \frac{U^A}{r^2} du\right)\left(dx^B - \frac{U^B}{r^2} du\right)$$

$$F\left(u, r, x^A\right) = \frac{R\,[q]}{2} - \frac{1}{2}\partial_u C_A{}^A - \frac{2G m_{\rm B}}{r} + \dots$$

$$\mathcal{H}_{AB}\left(u, r, x^C\right) = q_{AB}\left(x^C\right) + \frac{1}{r} C_{AB} + \dots$$

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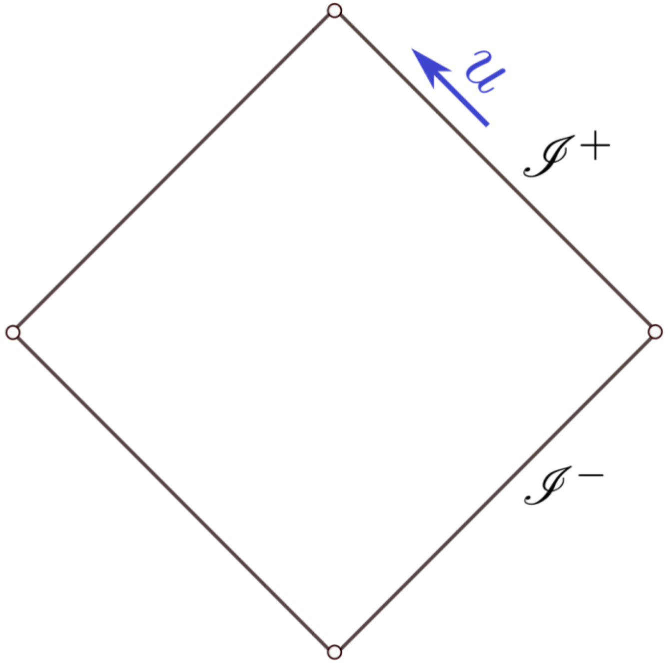

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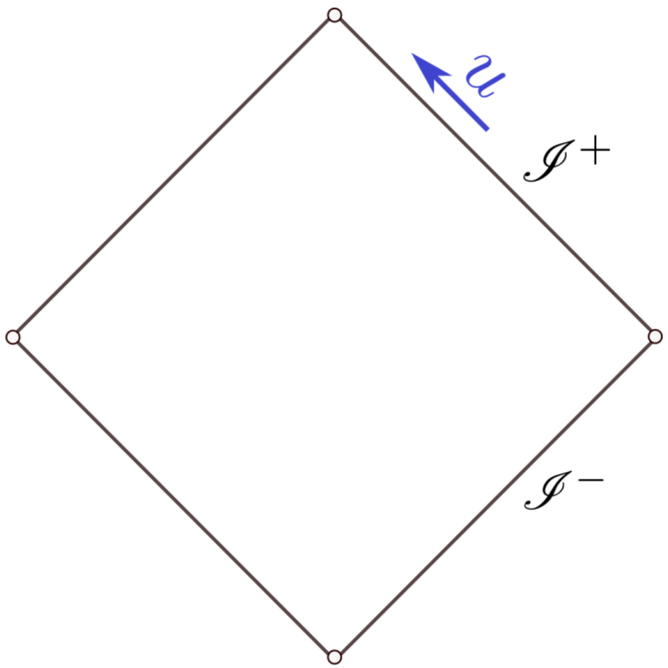

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$m_B(u, x^A)$: "Bondi mass aspect"

$N^A(u, x^A)$: angular momentum aspect

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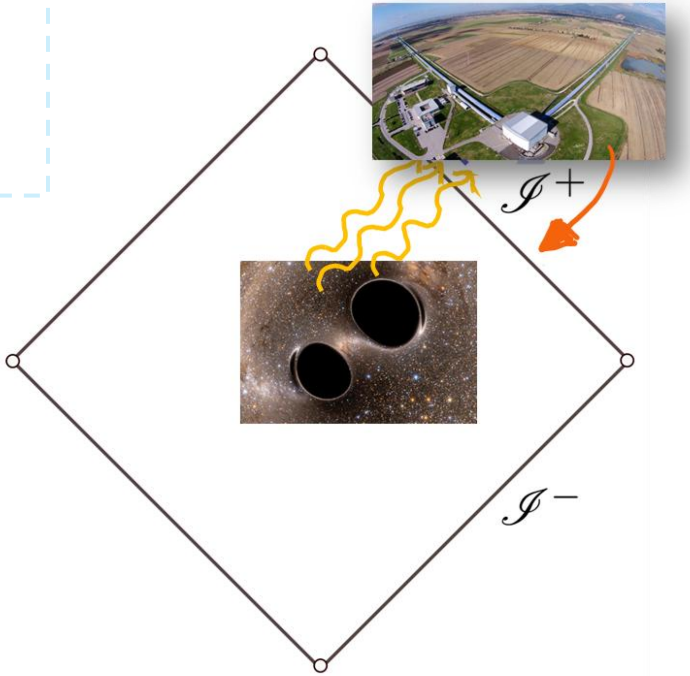

$m_B(u, x^A)$: “Bondi mass aspect”

$N^A(u, x^A)$: angular momentum aspect

$C_{AB}(u, x^A)$: asymptotic shear

$N_{AB}(u, x^A) = \partial_u C_{AB}$: “news”

Geometry of a black hole horizon

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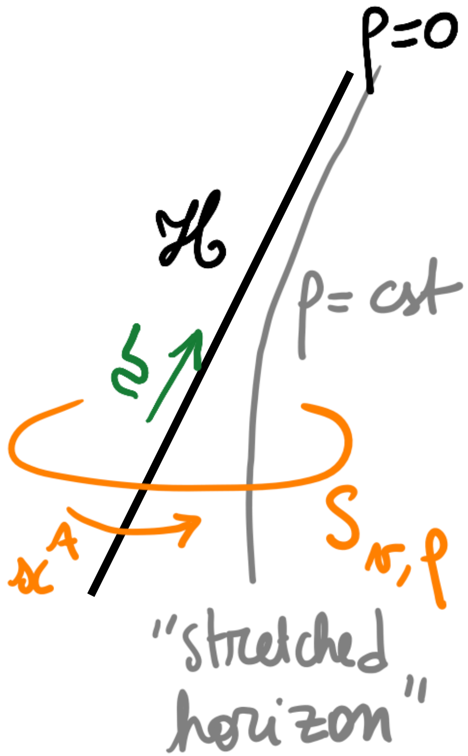


Coordinates : (v, ρ, x^A)

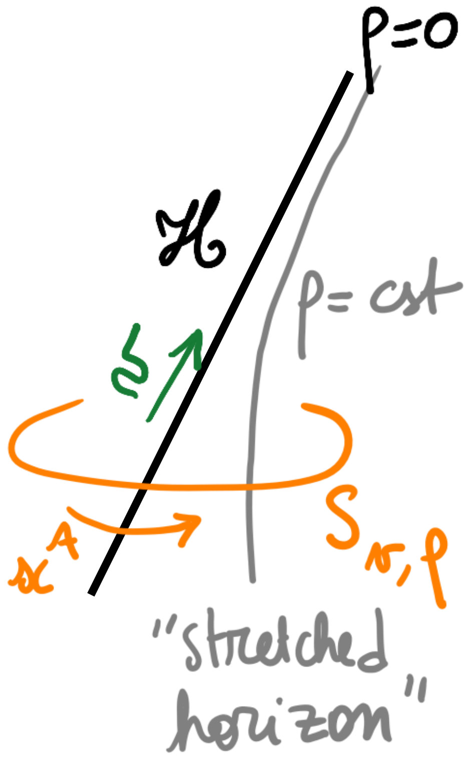
advanced null time

radial coordinate

(d-2) sphere coordinates

In these coordinates, the black hole horizon is located at $\rho = 0$.

Geometry of a black hole horizon



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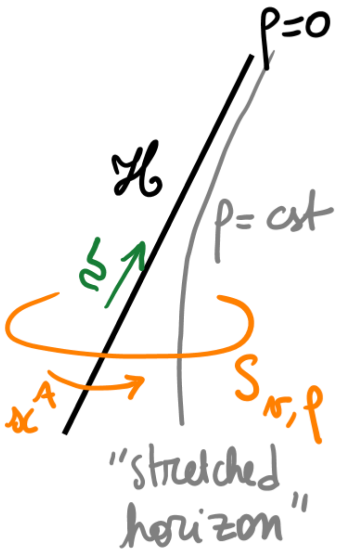
Near-horizon geometry:

$$ds^2_{\mathscr{H}^+} \underset{\rho \sim 0}{=} -\rho\,\kappa\, dv^2 + 2\, dv d\rho + 2\rho\,\theta_A dv dx^A + \Big(\Omega_{AB} + \rho\lambda_{AB}\Big) dx^A dx^B + \dots$$

κ , θ_A , Ω_{AB} : functions of (v, x^A)

Geometry of a black hole horizon



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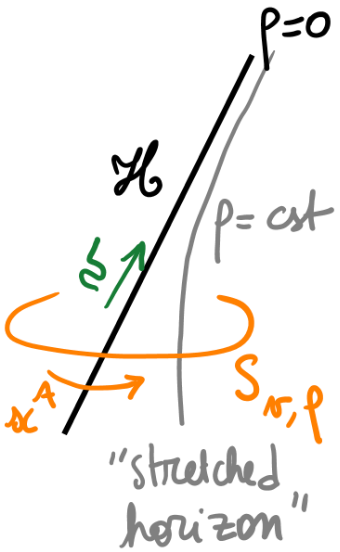

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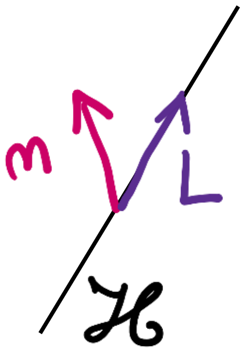
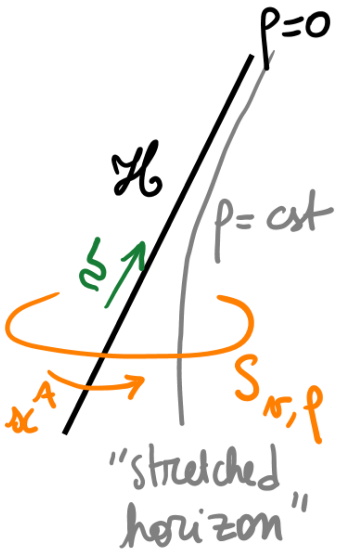
surface gravity
(for a Killing horizon)
$$L^b D_b L^a = \kappa L^a$$

"twist"



$$\vec{L} = \partial_v$$

Geometry of a black hole horizon



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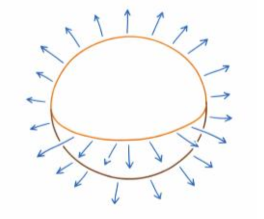
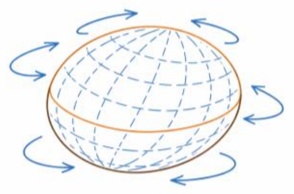
$$\Sigma_{AB} = \frac{1}{2}\partial_v \Omega_{AB} \xrightarrow{\text{trace}} \Theta = \Omega^{AB}\Sigma_{AB}$$

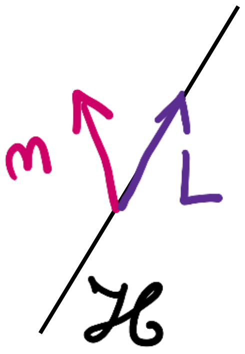
« expansion »



$$\xrightarrow{\text{traceless part}} \sigma_{AB} = \frac{1}{2}\partial_v \Omega_{AB} - \frac{\Theta}{d-2}\Omega_{AB}$$

« shear »





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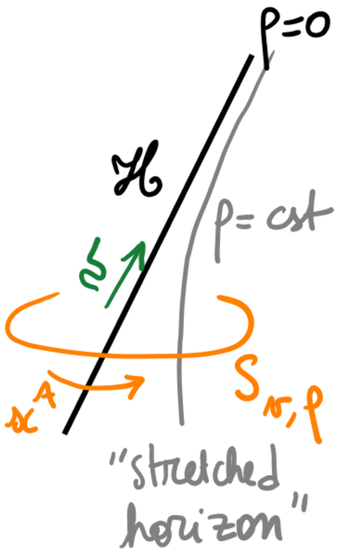

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"twist"

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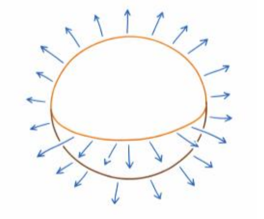
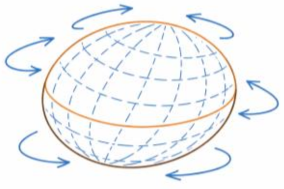
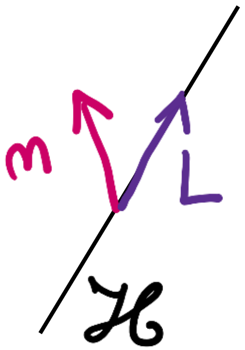
trace $\rightarrow$ $\Theta = \Omega^{AB}\Sigma_{AB}$ « expansion »



traceless part $\rightarrow$ $\sigma_{AB} = \frac{1}{2}\partial_v \Omega_{AB} - \frac{\Theta}{d-2}\Omega_{AB}$

« shear »





$$\Xi_{AB} = -\frac{1}{2}\lambda_{AB}$$

trace $\rightarrow$ $\Theta^{(m)}$

traceless $\rightarrow$ $\sigma^{(m)}_{AB}$: transversal shear

Horizon dynamics

- **Null Raychaudhuri** equation: $(\partial_v - \kappa)\,\Theta + \frac{1}{d-2}\Theta^2 + \sigma_{AB}\sigma^{AB} = 0$

--> describes how the **expansion** evolves along the null geodesic congruence
Key in the proof of singularity theorems (+ energy conditions)

Horizon dynamics

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- **Damour** equation: $(\partial_v + \Theta)\,\theta_A + 2D_A\left(\kappa + \frac{d-3}{d-2}\Theta\right) - 2D^B\sigma_{AB} = 0$

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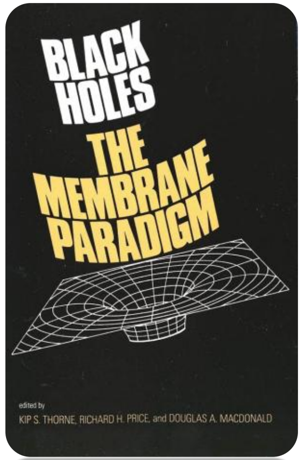
originally thought of as the Navier-Stokes equation for a viscous **fluid** [Damour '79][Price,Thorne '86]

In fact, it is a conservation equation of a ***Carrollian*** (not a Galilean) fluid [LD, Marteau '19]



Horizon dynamics

EINSTEIN EQUATIONS

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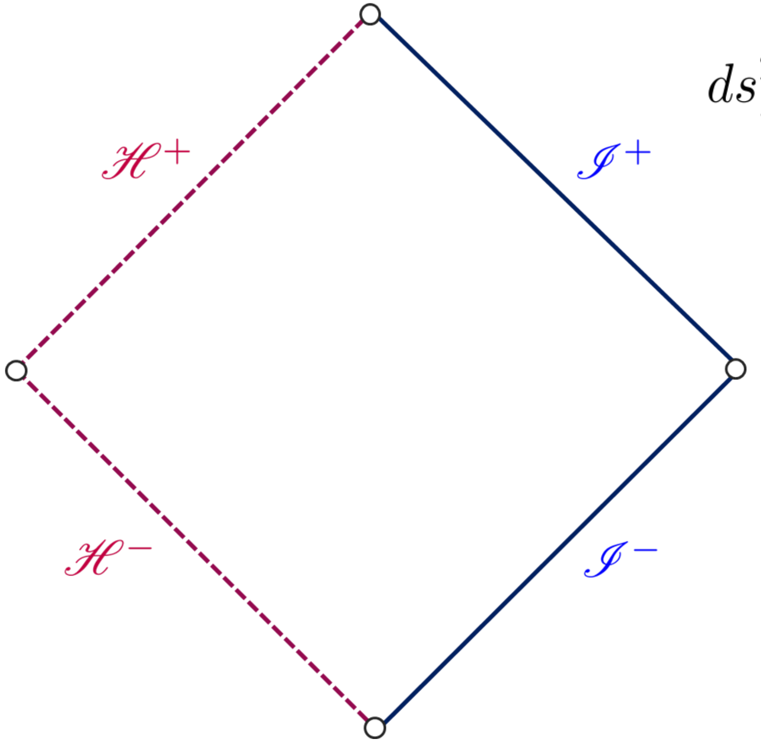
- **Transverse shear** evolution equation:

$$
\begin{aligned}
&R_{AB}\left[\Omega\right] - (\partial_v + \kappa)\, \lambda_{AB} - 2D_{(A}\omega_{B)} - 2\omega_A\omega_B \\
&+ 2\sigma^C{}_{(A}\left[\lambda_{B)C} - \frac{1}{4}\Omega_{B)C}\lambda^D{}_D\right] - \frac{d-6}{2\,(d-2)}\Theta\left[\lambda_{AB} + \frac{1}{d-6}\Omega_{AB}\lambda^C{}_C\right] = 0
\end{aligned}
$$

- **Tidal force** equation:

$$
(\partial_v - \kappa)\, \sigma_{AB} - \sigma_{AC}\sigma_B{}^C - \frac{1}{d-2}\Omega_{AB}\sigma_{CD}\sigma^{CD} = 0
$$

$\mathscr{I}$ vs $\mathscr{H}$



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$$ds^2_{\mathscr{H}^+} = -\rho \mathcal{F}\, dv^2 + 2\, dv d\rho + g_{AB}\left(dx^A + \rho\, \vartheta^A dv\right)\left(dx^B + \rho\, \vartheta^B dv\right) ,$$

$$\mathcal{F}\left(v, \rho, x^A\right) = 2\kappa\left(v, x^A\right) + \rho \mathcal{F}_0\left(v, x^A\right) + \dots$$

$$\vartheta^A\left(v, \rho, x^B\right) = \theta^A\left(v, x^B\right) + \dots$$

$$g_{AB}\left(v, \rho, x^C\right) = \Omega_{AB}\left(v, x^C\right) + \rho\, \lambda_{AB}\left(v, x^C\right) + \dots$$

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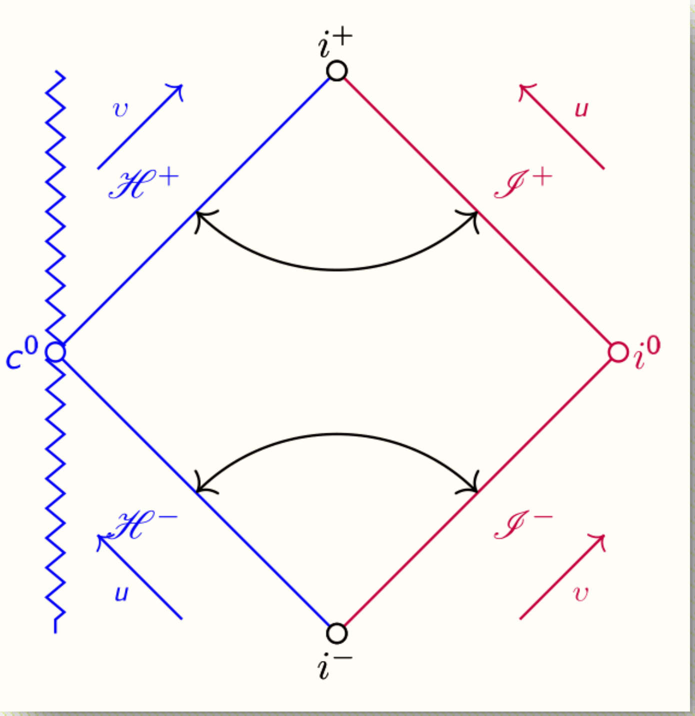

Null infinity as an extremal horizon

$\mathscr{I}$ is an **extremal non-expanding horizon** for the (unphysical) conformally completed spacetime

$$d\tilde{s}^2_{\mathscr{I}^+} = \Omega^2 ds^2_{\mathscr{I}^+}\,, \qquad \Omega = \frac{\alpha}{r}$$

[Ashtekar, Khera, Kolanowski, Lewandowski ‘22] [Ashtekar, Speziale ‘24] [Agrawal, Charalambous, LD ‘25]

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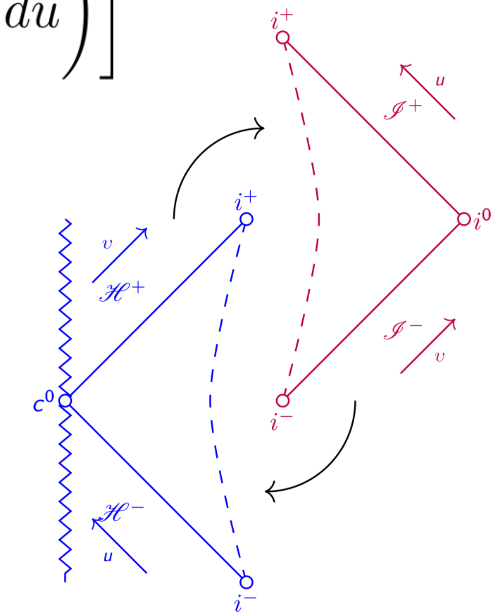
spatial inversion

$$r = \frac{\alpha^2}{\rho}\,, \quad u = v$$



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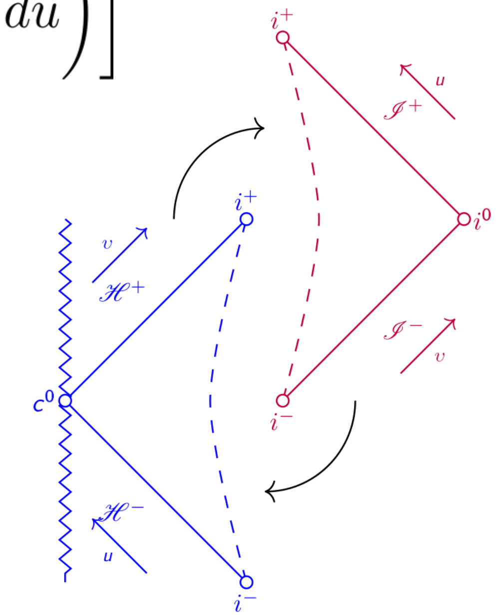
$$d\tilde{s}^2_{\mathscr{I}^+} = \Omega^2 ds^2_{\mathscr{I}^+} = \Omega^2 \left[-F du^2 - 2\, du dr + r^2 \mathcal{H}_{AB} \left(dx^A - \frac{U^A}{r^2} du \right) \left(dx^B - \frac{U^B}{r^2} du \right) \right]$$

spatial inversion

$$r = \frac{\alpha^2}{\rho}, \quad u = v$$

$$\boxed{d\tilde{s}^2_{\mathscr{I}^+} = ds^2_{\mathscr{H}^+}} = -\rho \mathcal{F}\, dv^2 + 2\, dv d\rho + g_{AB} \left(dx^A + \rho\, \vartheta^A dv \right) \left(dx^B + \rho\, \vartheta^B dv \right) \,,$$



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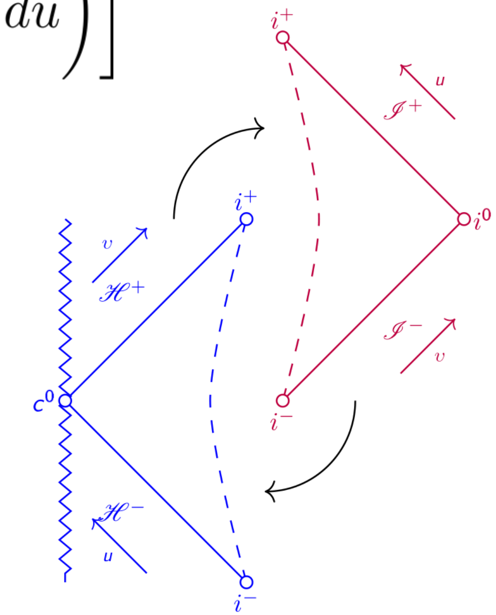
with $\mathcal{F} = \alpha^{-2} F\,, \quad g_{AB} = \alpha^2 \mathcal{H}_{AB}\,, \quad \vartheta^A = -\rho \alpha^{-4} U^A$



$\longrightarrow$ $\kappa = 0$ — **extremal**, $\Theta = 0$ — **non-expanding**, $\theta_A = 0$ — **non-twisting**

Null infinity as an extremal horizon

$\mathscr{H}$ / $\mathscr{I}$ dictionary

$\mathscr{H}$	Name	Evolution equation	$\mathscr{I}$
κ	surface gravity	-	0
Θ	expansion	Null Raychaudhuri eq.	0
ω_A	twist	Damour eq.	0
σ_{AB}	longitudinal shear	Tidal force eq.	0
Ω_{AB}	horizon metric	-	q_{AB} (fixed)
λ_{AB}	transversal shear	Trans. deform. rate ev. eq.	C_{AB} (free data)

[Agrawal, Charalambous, LD '25]

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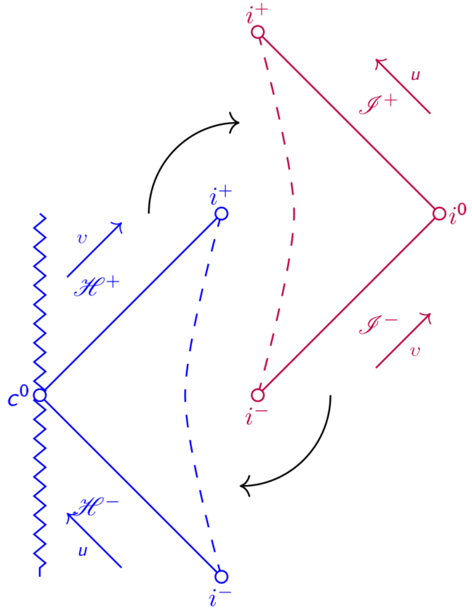

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If the spacetime contains an **extremal non-rotating horizon**, then

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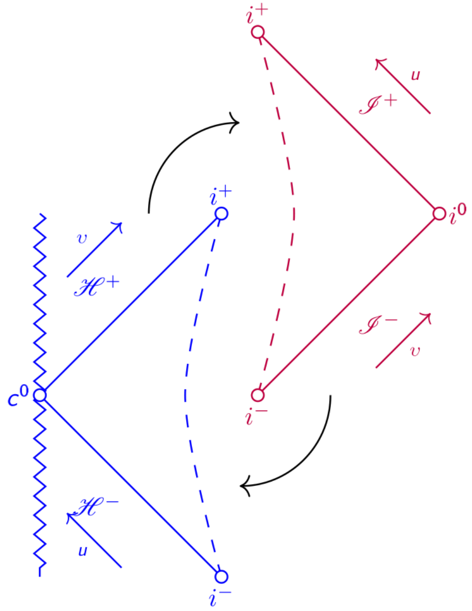

[Agrawal, Charalambous, LD '25]

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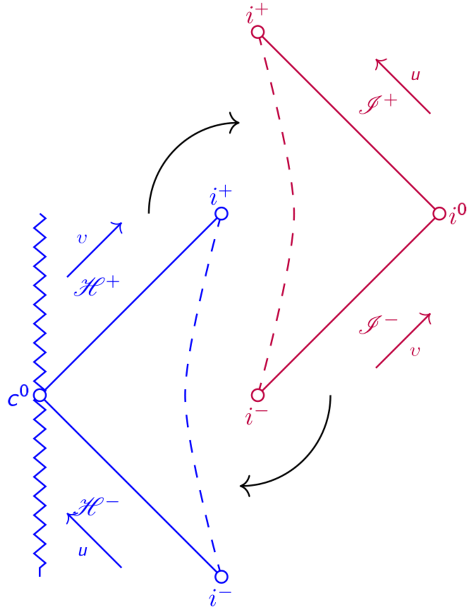
→ This 'explains' the **Couch-Torrence symmetry** of **extreme Reissner-Nordstrom (ERN)** black holes



[Agrawal, Charalambous, LD '25]

A self-dual example:

the 4D extremal Reissner-Nordström geometry

ERN as a self-dual example: the Couch-Torrence inversion symmetry

Extremal Reissner-Nordström (ERN) black hole

$M = |Q|$

$$ds^2_{\mathrm{ERN}} = -\frac{\Delta(r)}{r^2}dt^2 + \frac{r^2}{\Delta(r)}dr^2 + r^2 d\Omega_2^2$$

$$\Delta(r) = (r - M)^2$$

$$r_H = M$$

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Couch-Torrence (CT) discrete conformal **symmetry** [Couch, Torrence '84]

$$r \xrightarrow{\text{CT}} \tilde{r} = \frac{Mr}{r - M} \quad \Rightarrow \quad ds^2_{\text{ERN}} \xrightarrow{\text{CT}} d\tilde{s}^2_{\text{ERN}} = \Omega^2\, ds^2_{\text{ERN}}$$

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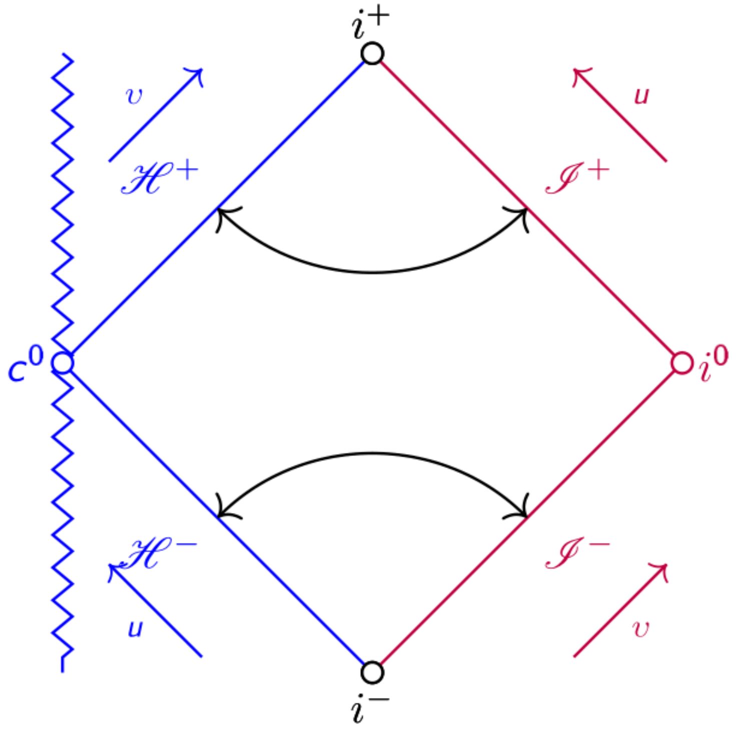
effect: exchanges null infinity and the horizon!

$$r_* \xrightarrow{\mathrm{CT}} -r_* \qquad r_* = r - M - \frac{M^2}{r-M} + 2M\ln\left|\frac{r-M}{M}\right|$$

$$\Rightarrow \quad r = M \overset{\mathrm{CT}}{\longleftrightarrow} r = \infty \quad \Leftrightarrow \quad \boxed{\mathscr{H}^\pm \overset{\mathrm{CT}}{\longleftrightarrow} \mathscr{I}^\pm}$$



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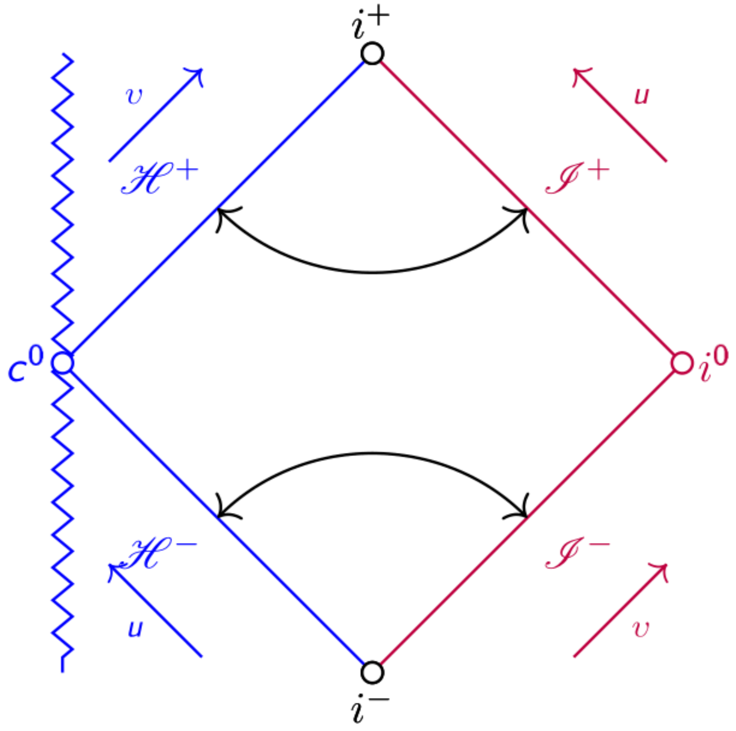
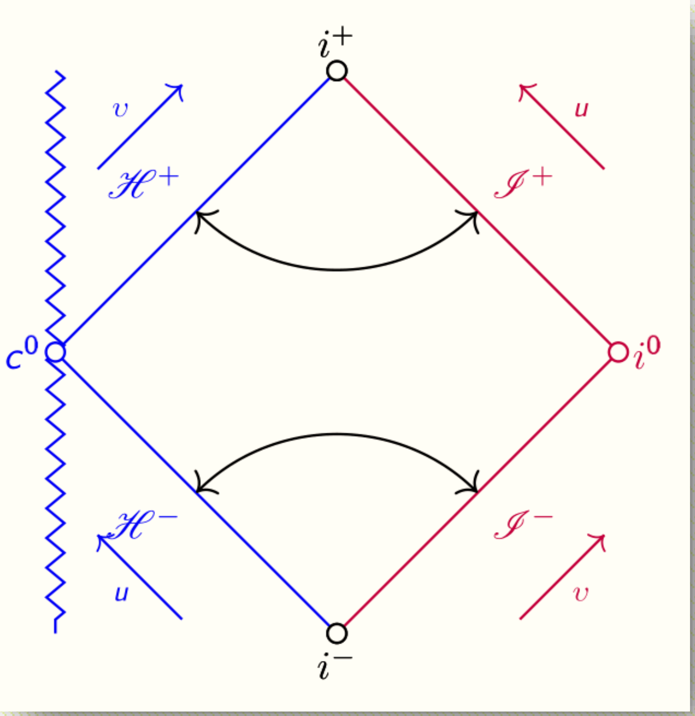
$$r = 2M \xleftrightarrow{\mathrm{CT}} r = 2M$$ fixed point: photon sphere!



Outline

- Black hole horizon vs null infinity
- $\mathscr{H} \leftrightarrow \mathscr{I}$ geometric duality
- Matching an infinite number of conserved quantities



Matching of near-$\mathscr{I}$ charges to near-$\mathscr{H}$ charges

Massless **scalar perturbations** on ERN black hole

$$\Box_{\mathrm{ERN}}\Phi = 0$$

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$$\Phi \sim \frac{1}{(r-M)}\sum_{n=0}^{\infty}\frac{\Phi^{(n)}\left(u,x^A\right)}{(r-M)^n} \qquad\qquad \Phi^{(n)}\left(u,x^A\right) = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\Phi^{(n)}_{\ell m}\left(u\right){}_0Y_{\ell m}\left(x^A\right)$$

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infinite tower of conserved quantities

Newman-Penrose conserved quantities [Newman, Penrose '65 '68]

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Aretakis charges [Aretakis '11]

key in the proof of **linear instability** of ERN black hole

Aretakis instability

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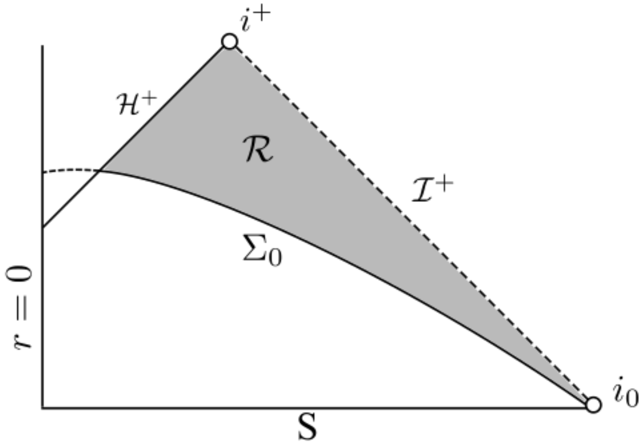
(+ decay results)

The Wave Equation on Extreme Reissner-Nordström Black Hole Spacetimes: Stability and Instability Results



$$\partial_r^{k+1}\Phi\Big|_{r=M} \sim v^k$$

transverse derivatives **blow up** as $v \to \infty$

linear instability of ERN black hole

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Conserved quantities near $\mathscr{H}^+$

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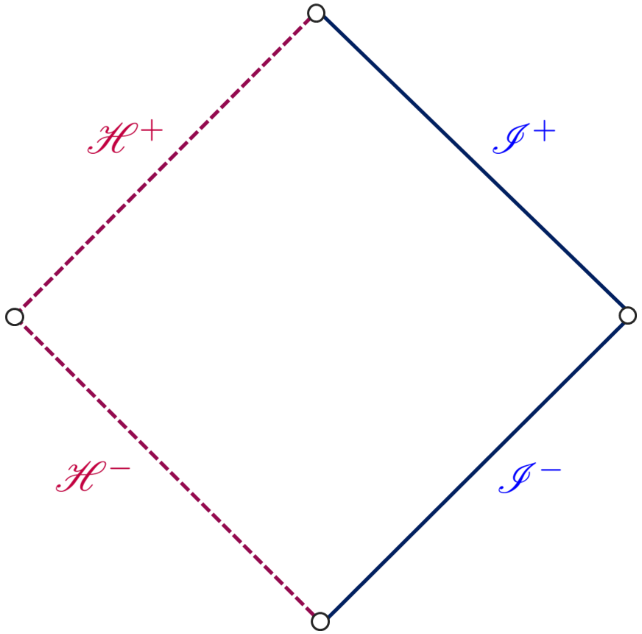

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$$\boxed{M^{\ell+2}A_{\ell m} = N_{\ell m}} \qquad \ell \geq 0$$

Map between Aretakis and Newman-Penrose
conserved quantities

[Bizon, Friedrich '12][Lucietti, Murata, Reall, Tanahashi '12] [Agrawal, Charalambous, LD '24]

Linearized gravitational perturbations on ERN

$$\psi_s \leftrightarrow \begin{cases} \Phi & \text{for } s = 0 \\ \phi_{1-s} & \text{for } s = \pm 1 \\ \Psi_{2-s} & \text{for } s = \pm 2 \end{cases}$$

- Near $\mathscr{I}^+$ expansion:

$$-2r^2 \Box_{\text{ERN}} = (r-M)^{-2s}\, \partial_r \,(r-M)^{2(s+1)}\, \partial_r + 2\eth'_{\mathbb{S}^2}\eth_{\mathbb{S}^2} - 2\left(r^2\partial_r + (2s+1)\, r\right)\partial_u$$

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NP conserved quantities [Newman, Penrose ‘65 ‘68]

infinite tower of conserved quantities

$2(2s+1)$
quantities remain conserved at nonlinear level!

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New infinite tower of near-horizon **conserved quantities** [Agrawal, Charalambous, LD ‘24]

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New infinite tower of near-horizon **conserved quantities** [Agrawal, Charalambous, LD '24]

$$\boxed{M^{\ell+s+2}\, {}_sA_{\ell m} = {}_sN_{\ell m}}$$

1:1 match with horizon conservation laws

Summary and outlook

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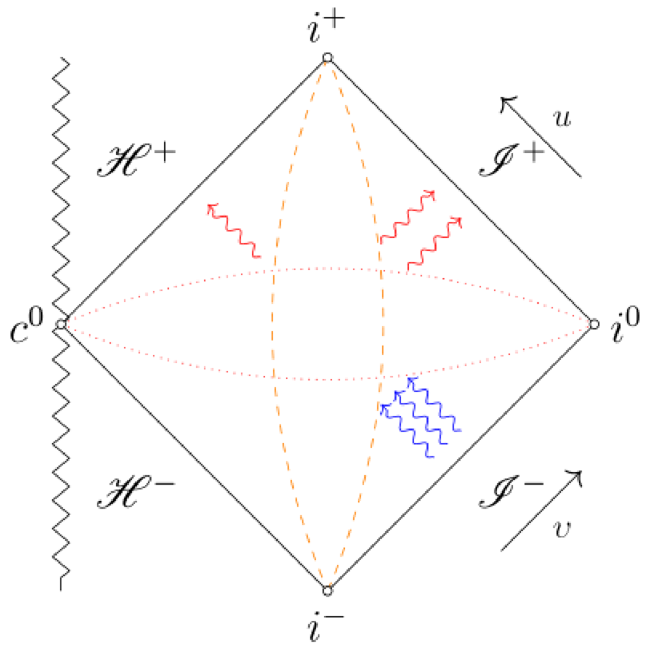
- We explored a **geometric duality** between null infinity and black hole horizons

- $\mathscr{I}$ and its 'dual' black hole horizon are generically not part of the same spacetime



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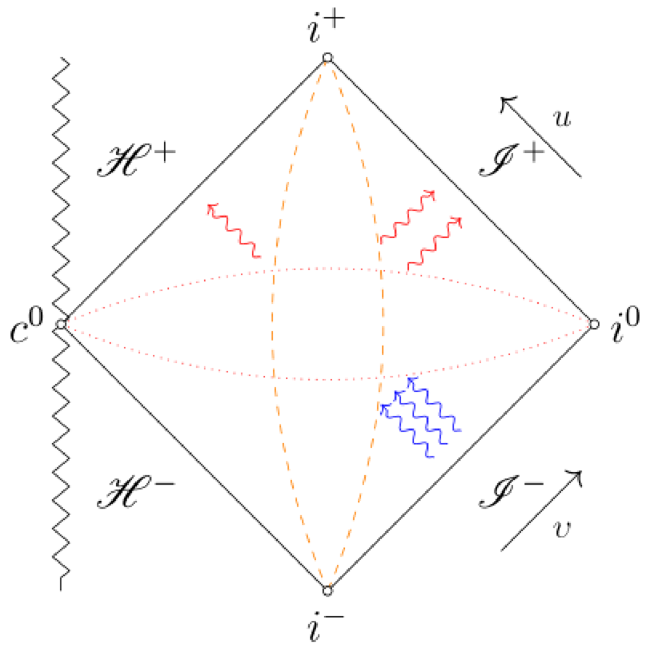

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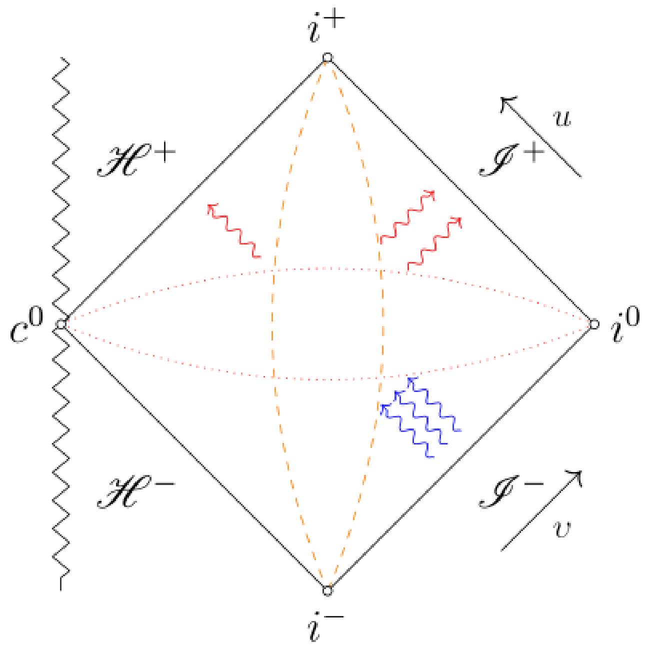

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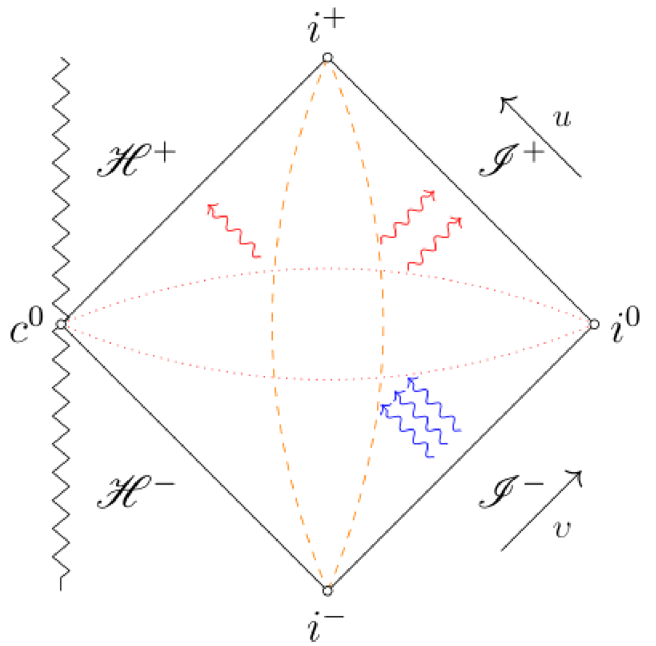

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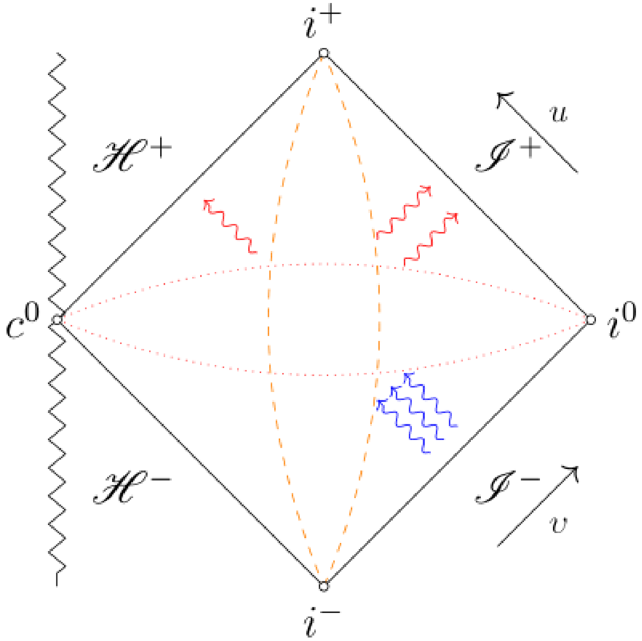
- Beyond self-dual example? **extreme Kerr-Newman** black holes

  Remarkably, the matching of conserved quantities can also be extended to this case for axisymmetric perturbations of any spin!

Summary and outlook

Future directions



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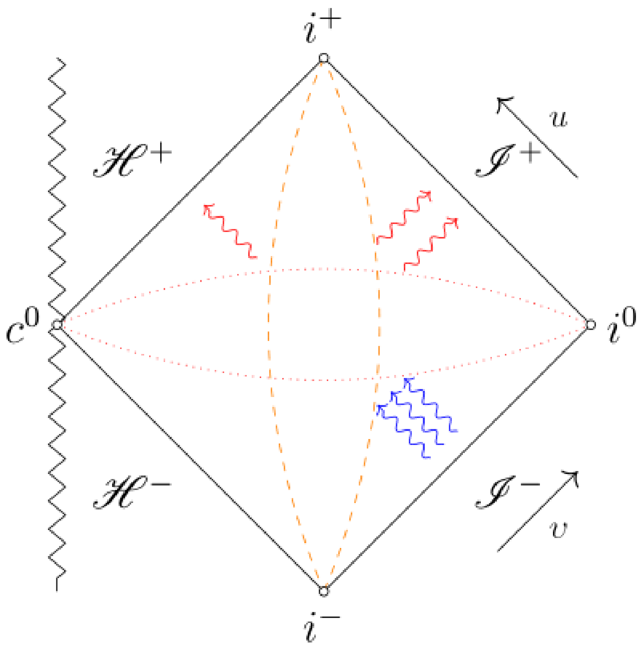

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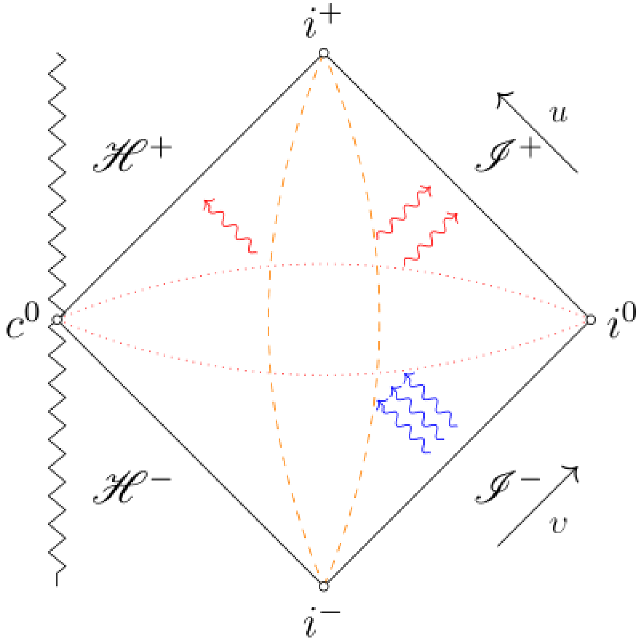

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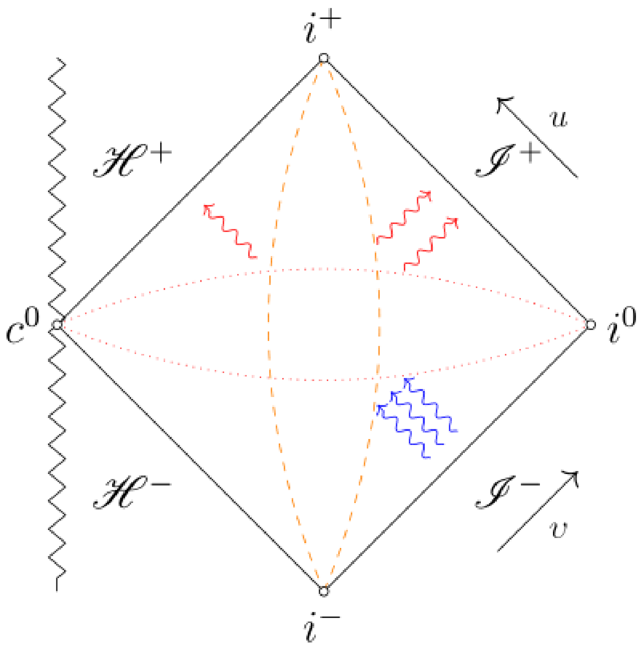

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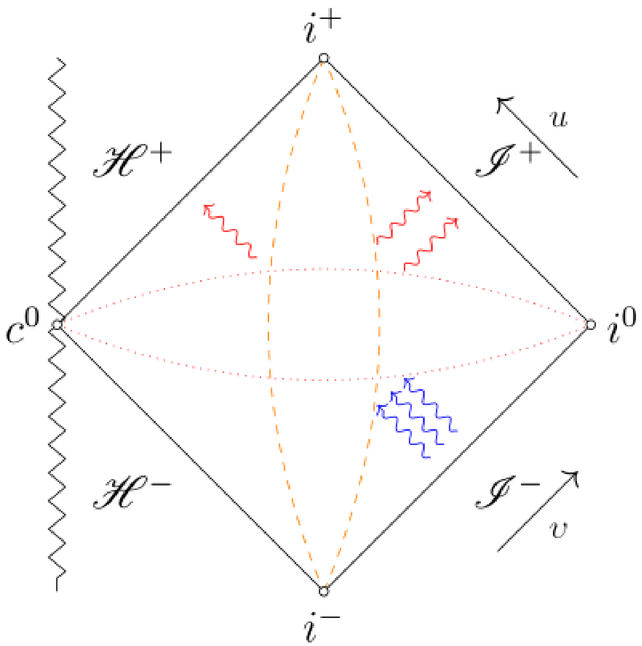

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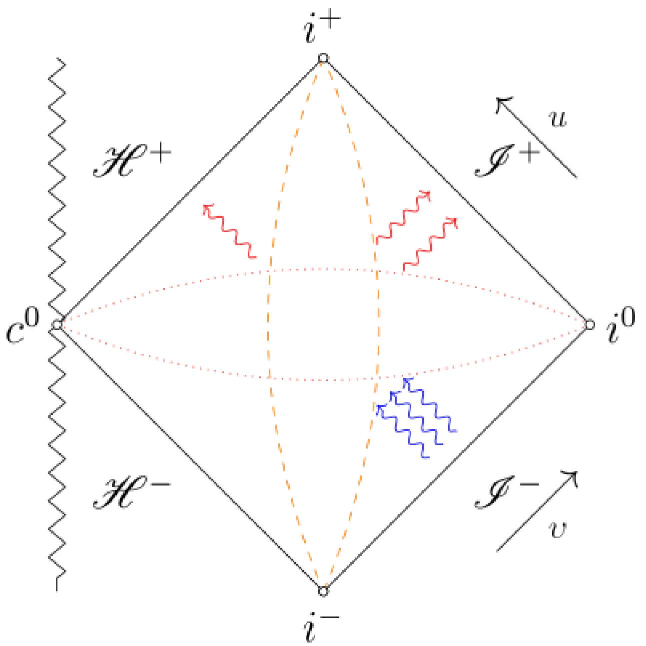

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  → view this from the photon sphere perspective! [Bianchi, di Russo ‘22]

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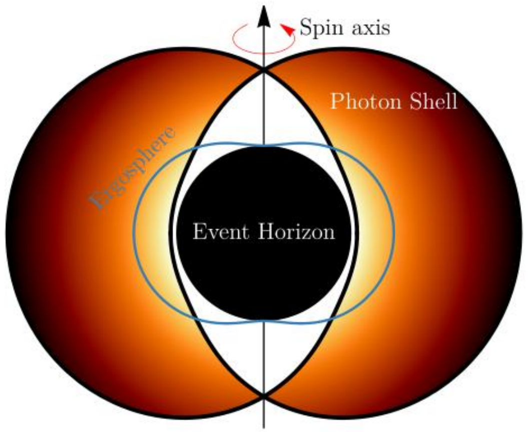

[Lupsasca et al.]

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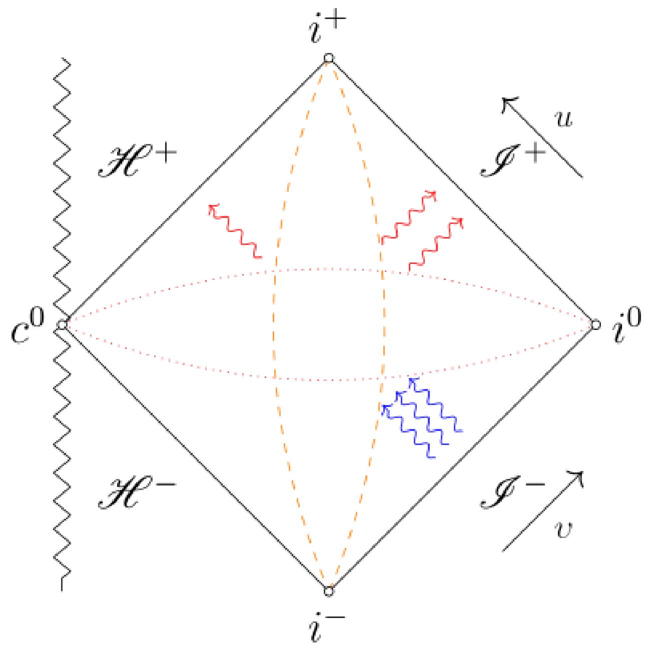

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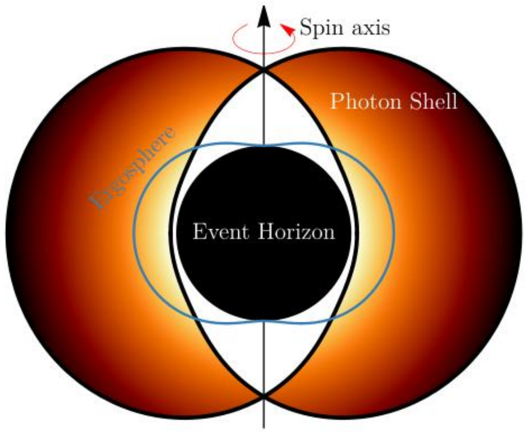
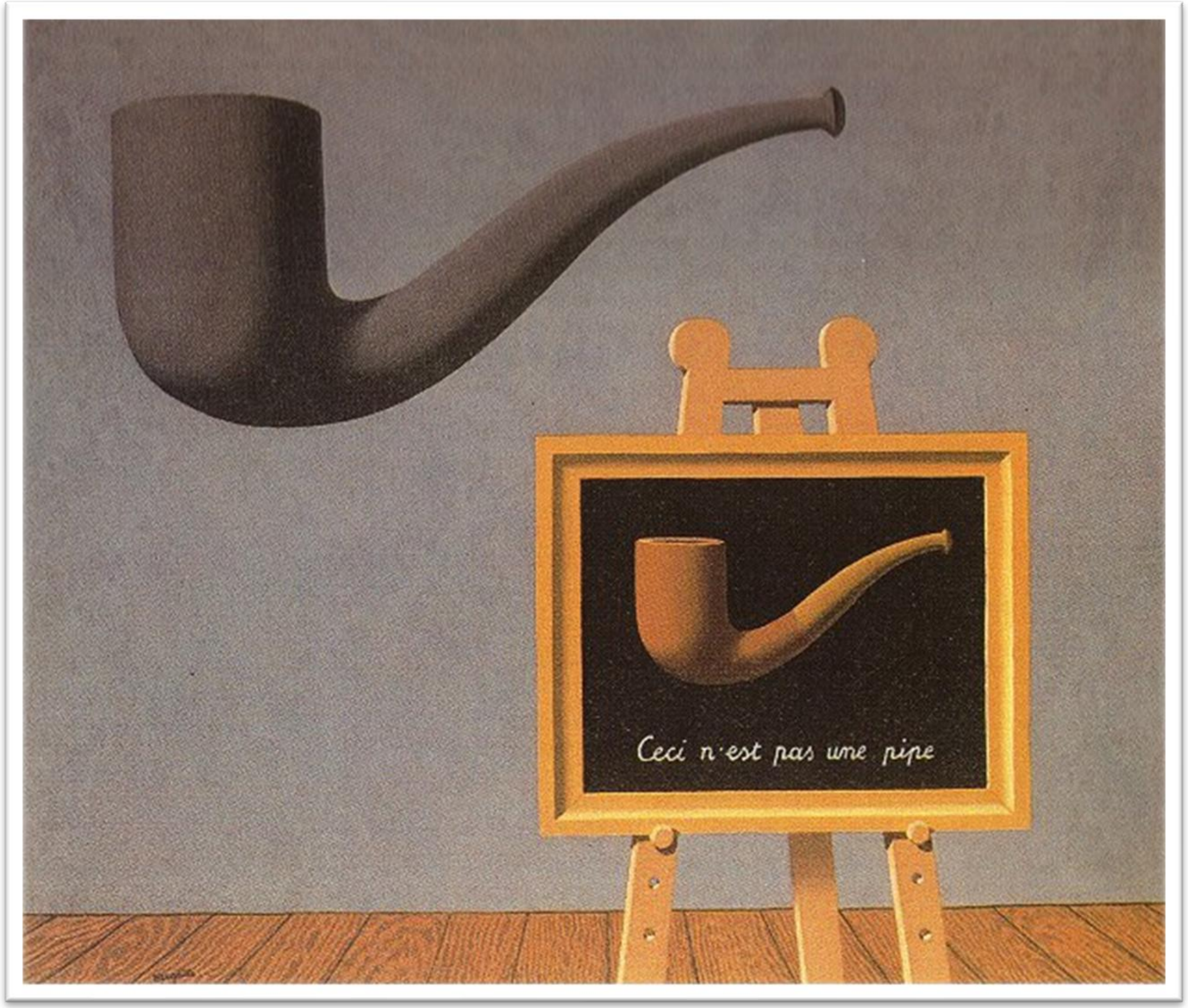


- **Quantum** fluctuations of near-extremal black holes?
  [Iliesiu, Turiaci ’21]



René Magritte, *Les deux mystères* (1966)

Thank you.