SORBONNE UNIVERSITÉ
CRÉATEURS DE FUTURS DEPUIS 1257

cnrs

PATRICK PETER

GRεCO

Vides cosmologiques : défauts et structures

Institut d'astrophysique de Paris

Chacun a son défaut, où toujours il revient
J. de la Fontaine, Fables, 1668

Overview

- **Topological defects**
  - **Symmetry breaking: Higgs mechanism**
  - **Phase transition: early Universe**
  - **Monopoles, strings, walls… constraints**
    - **String network formation and evolution**
    - **Gravitational radiation**
    - **PBHs**

- **Quantum vacuum fluctuations and large scale structures**
  - **Large scale structure formation = gravitational collapse**
  - **Inflation**
    - **Early Universe initial conditions: vacuum!**
  - **Quantizing gravity (perturbations)**
    - **Varying mass scalar fields**
    - **Quantum vacuum fluctuations → squeezed states…**
      - **Power spectrum, CMB and all that.**

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Symmetry breaking: $G \rightarrow H$

$$\mathcal{L} = -D_\mu \Phi \left(D^\mu \Phi\right)^\dagger - \frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} - V(\Phi)$$

Covariant derivative

$$\mathrm{D}_\mu \Phi = (\partial_\mu + \mathrm{i} g \boldsymbol{A}_\mu \cdot \boldsymbol{T})\Phi$$

Field strength tensor

$$\mathrm{F}^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^a{}_{bc} A^b_\mu A^c_\nu$$

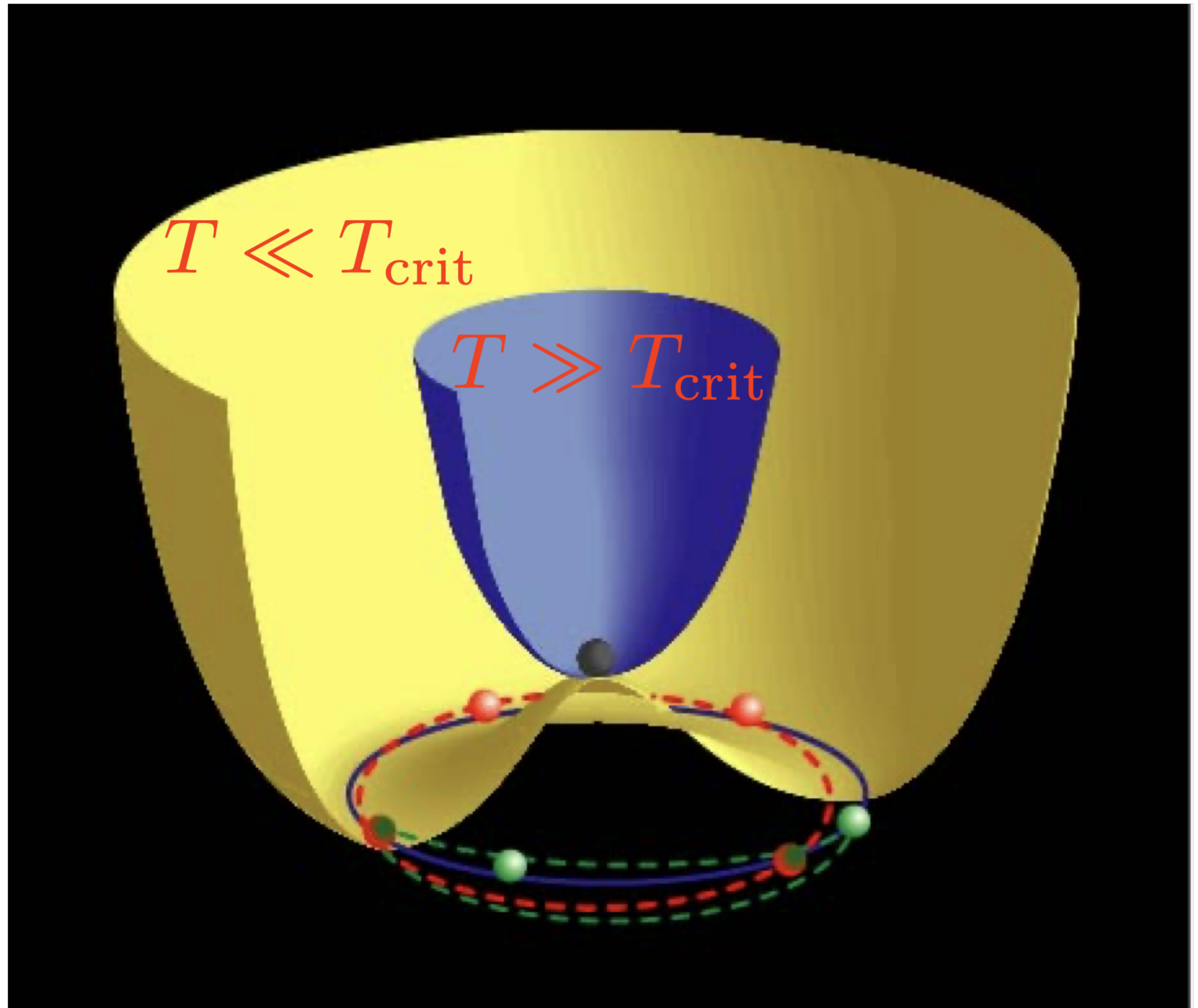
G structure constants

$$\left[T^a, T^b\right] = i f_{ab}{}^c T_c$$

Temperature dependent potential $V(\Phi, T)$

Higgs field and potential

$V(\phi)$



ϕ

Simplest possible Model: real scalar field $G = Z_2$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - V(\varphi)$$

Potential $V(\varphi) = \lambda\left(\varphi^2 - \eta^2\right)^2$

Next-to-simplest possible Model: complex scalar field: Abelian Higgs mechanism

$G = U(1)$

$$\mathcal{L} = -D_\mu\phi\,(D^\mu\phi)^\star - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi)$$

Covariant derivative

$$\mathrm{D}_\mu\phi = (\partial_\mu + \mathrm{i}qA_\mu)\phi$$

Field strength tensor

$$\mathrm{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Potential $V(\phi) = \lambda\left(|\phi|^2 - \eta^2\right)^2$

Simplest non Abelian Model: $SU(2) \to \{\mathrm{Id}\}$

$$\mathcal{L} = -D_\mu\Phi\,(D^\mu\Phi)^\dagger - \frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu} - V(\Phi)$$

Covariant derivative

$$\mathrm{D}_\mu\Phi = \left(\partial_\mu + \mathrm{i}\frac{g}{2}\boldsymbol{A}_\mu\cdot\boldsymbol{\sigma}\right)\Phi$$

$$\left[\frac{\sigma^a}{2},\frac{\sigma^b}{2}\right] = i\varepsilon_{ab}{}^{c}\frac{\sigma_c}{2}$$

Field strength tensor

$$\mathrm{F}^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon^a{}_{bc}A^b_\mu A^c_\nu$$

$SU(2)$ structure constants

Potential $V(\Phi) = \lambda(\Phi^\dagger\Phi - \eta^2)^2$

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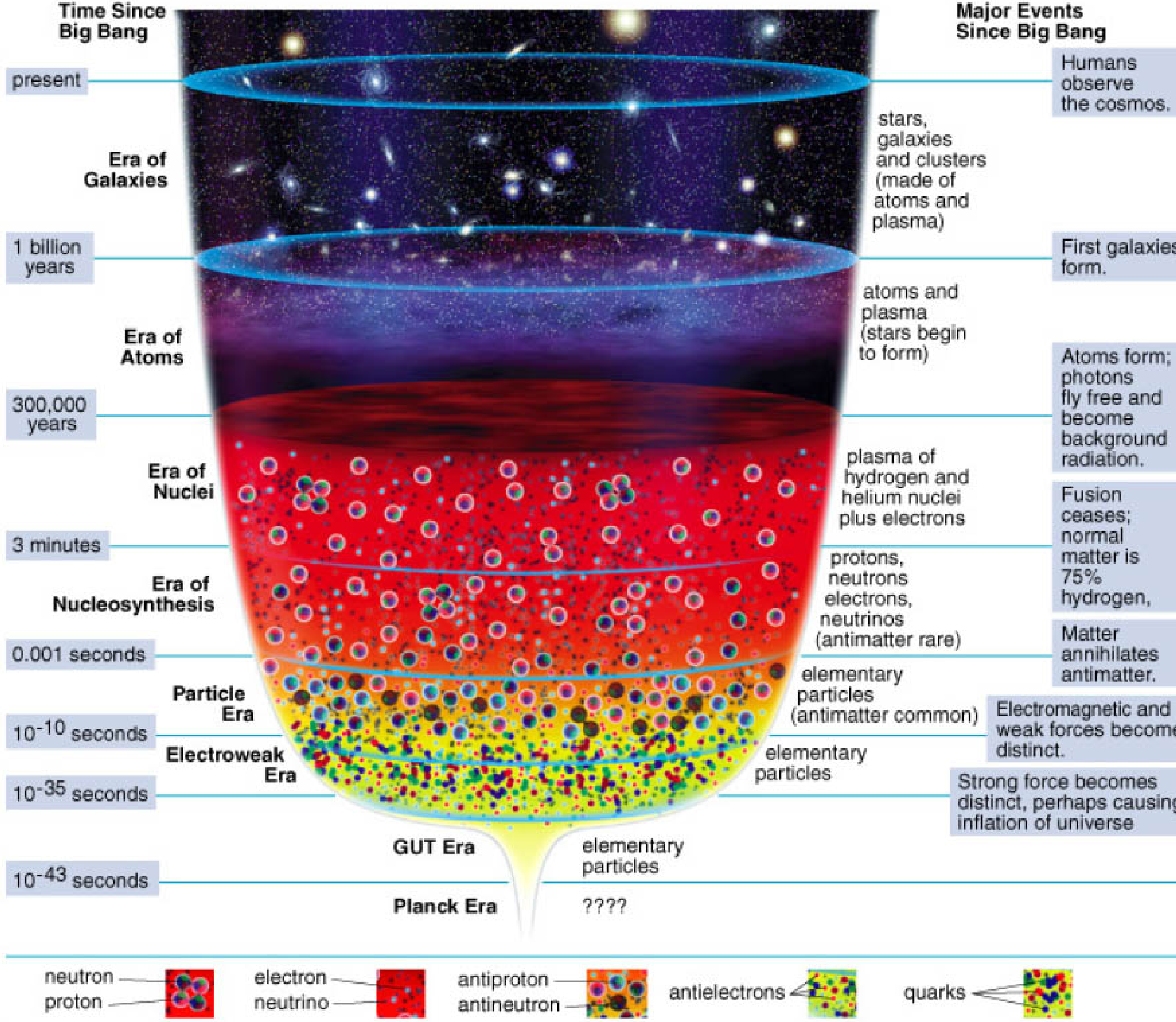
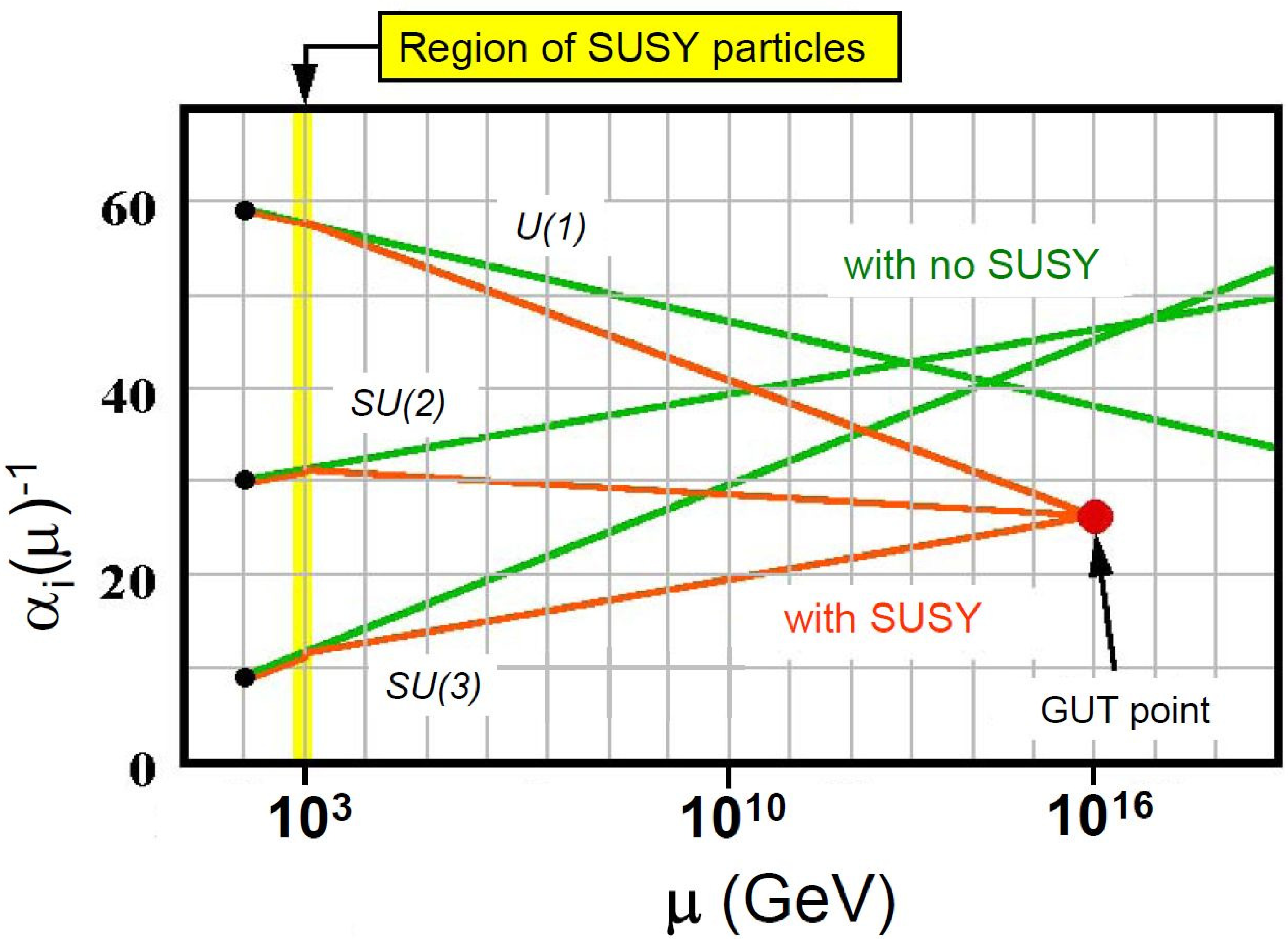
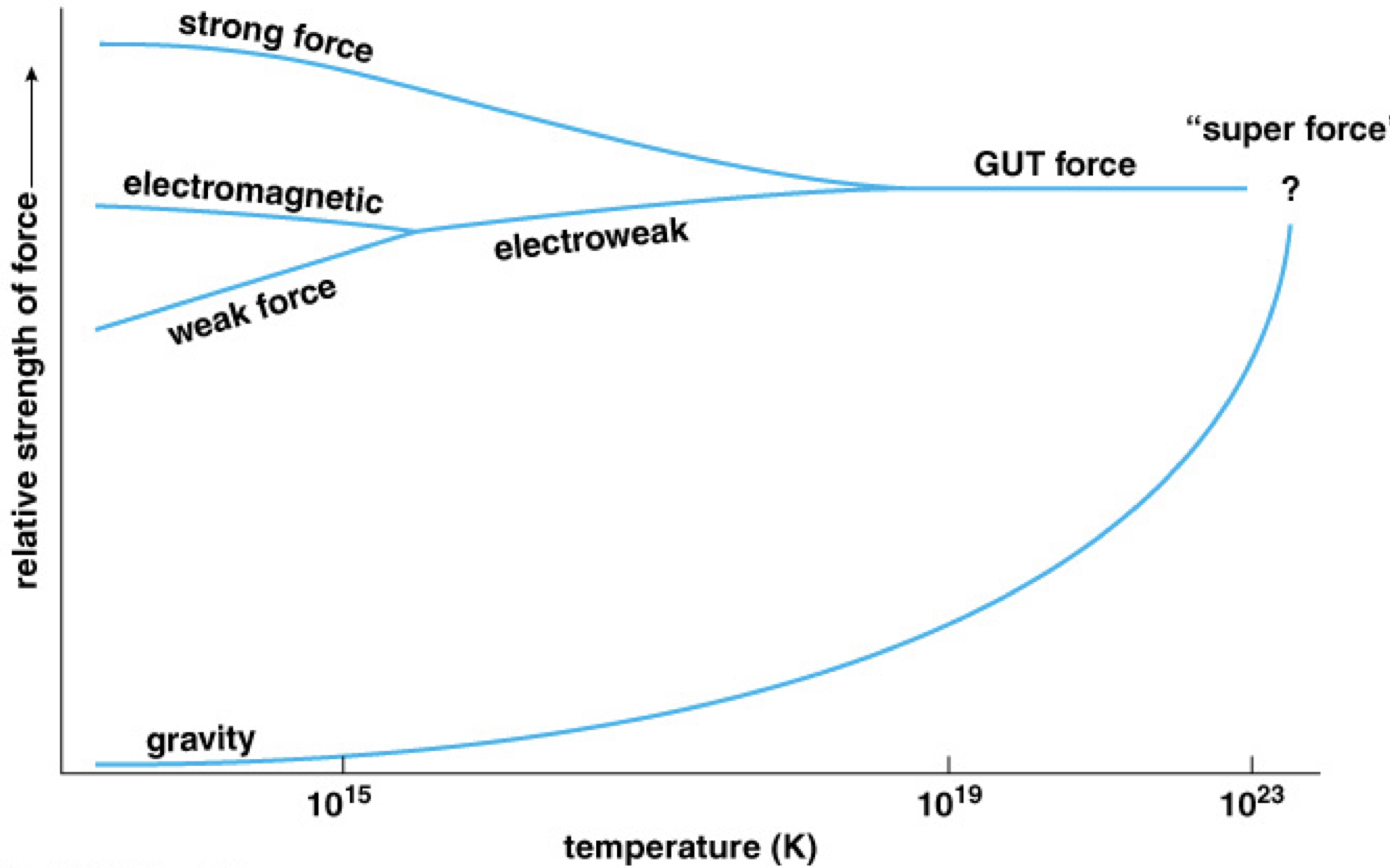


Time Since Big Bang
Major Events Since Big Bang
present
Humans observe the cosmos.
Era of Galaxies
stars, galaxies and clusters (made of atoms and plasma)
1 billion years
First galaxies form.
Era of Atoms
atoms and plasma (stars begin to form)
300,000 years
Atoms form; photons fly free and become background radiation.
Era of Nuclei
plasma of hydrogen and helium nuclei plus electrons
3 minutes
Fusion ceases; normal matter is 75% hydrogen,
Era of Nucleosynthesis
protons, neutrons electrons, neutrinos (antimatter rare)
0.001 seconds
Matter annihilates antimatter.
Particle Era
elementary particles (antimatter common)
10^{-10} seconds
Electromagnetic and weak forces become distinct.
Electroweak Era
elementary particles
10^{-35} seconds
Strong force becomes distinct, perhaps causing inflation of universe
GUT Era
elementary particles
10^{-43} seconds
Planck Era
????
neutron
proton
electron
neutrino
antiproton
antineutron
antielectrons
quarks

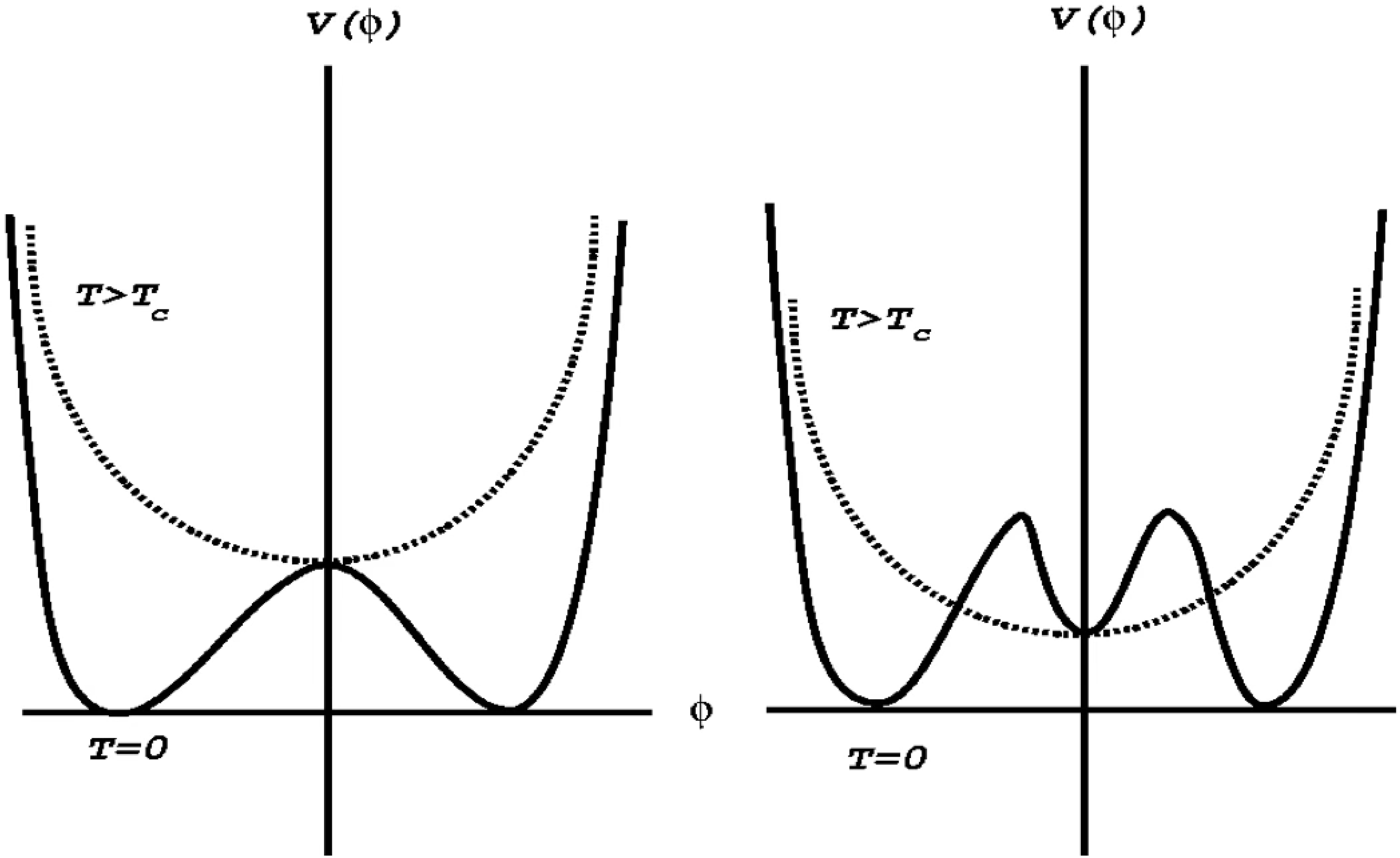


Region of SUSY particles
60
40
20
0
U(1)
with no SUSY
SU(2)
with SUSY
SU(3)
GUT point
$\alpha_i(\mu)^{-1}$
10^3
10^{10}
10^{16}
μ (GeV)



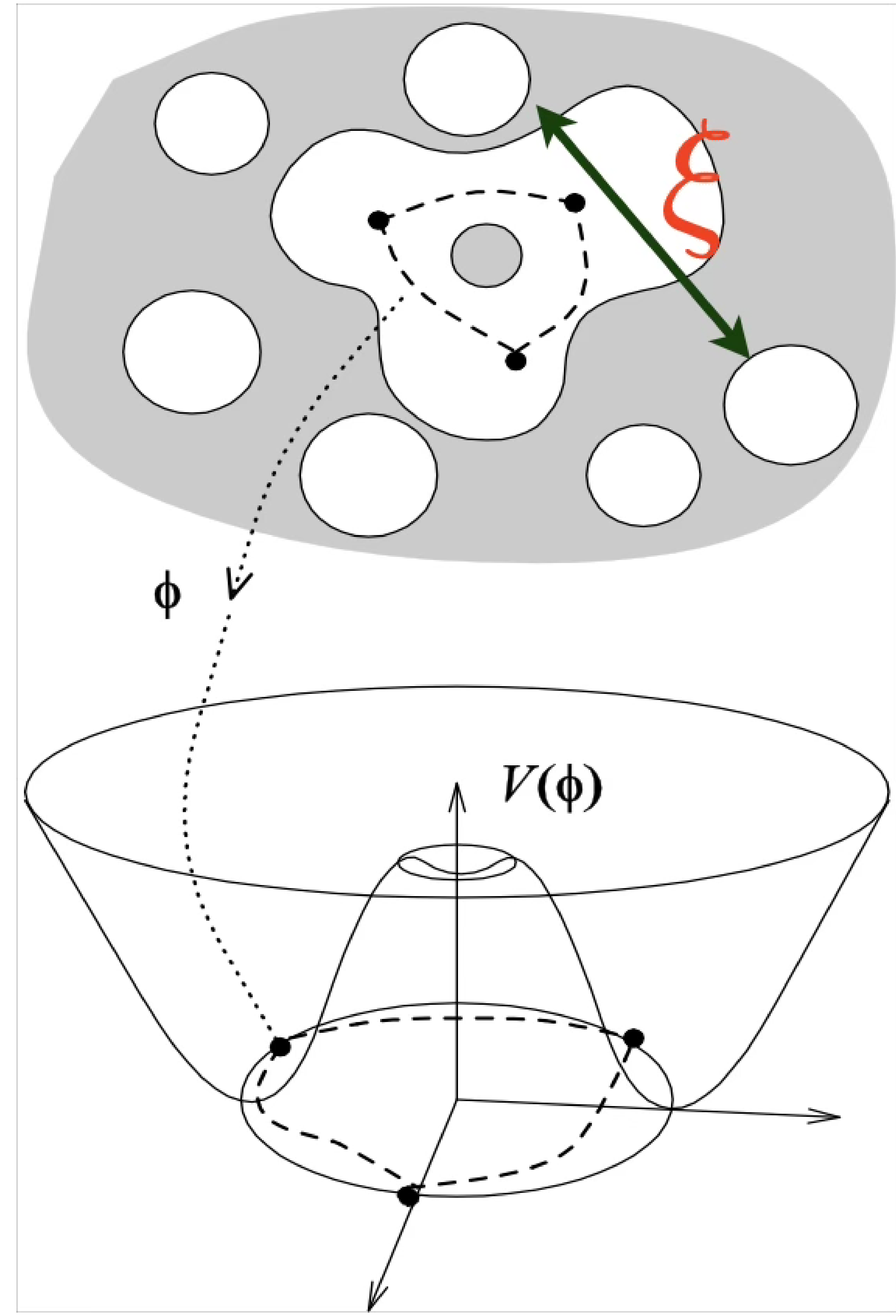
strong force
electromagnetic
weak force
electroweak
GUT force
"super force"
?
gravity
relative strength of force
10^15
10^19
10^23
temperature (K)

Symmetry breaking $\Longrightarrow$ phase transition



2nd order **1st order**

Phase transition



Correlation length

$$\langle 0|\phi(\vec{x})\phi(\vec{x}+\vec{r})|0\rangle \propto \mathrm{e}^{-|\vec{r}|/\xi}$$

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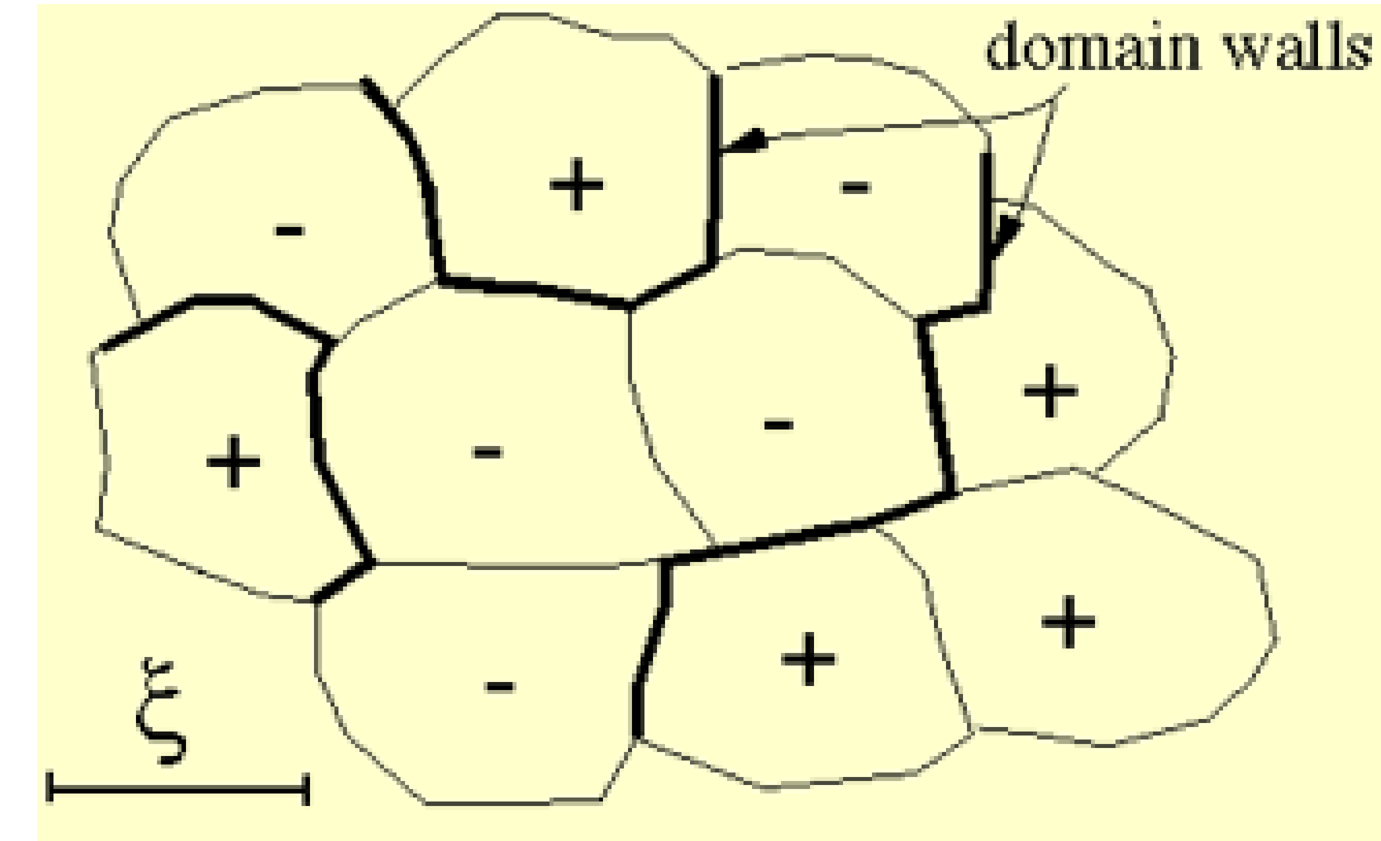
Simplest possible Model: real scalar field $G = Z_2$

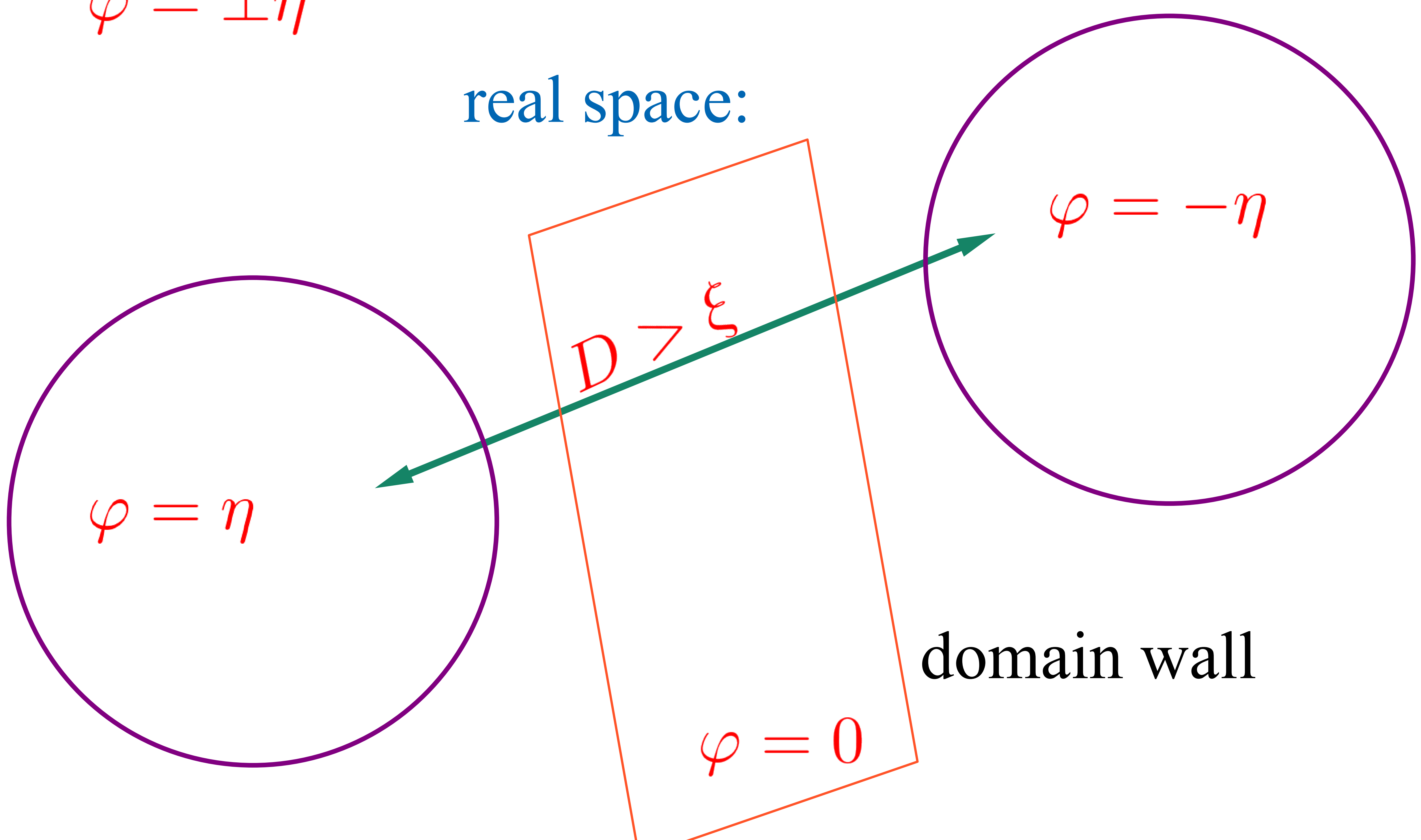


$$\mathcal{L} = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - V(\varphi)$$

Potential $V(\varphi) = \lambda\left(\varphi^2 - \eta^2\right)^2$

minimum: $\varphi = \pm\eta$

real space:



1 DW @ 100 GeV:

$\Omega_{\mathrm{DW}} \sim 10^8$

network = perturbations

$$\frac{\delta\rho}{\rho} \sim 10^{12} \frac{\sigma}{\mathrm{GeV}^3}$$

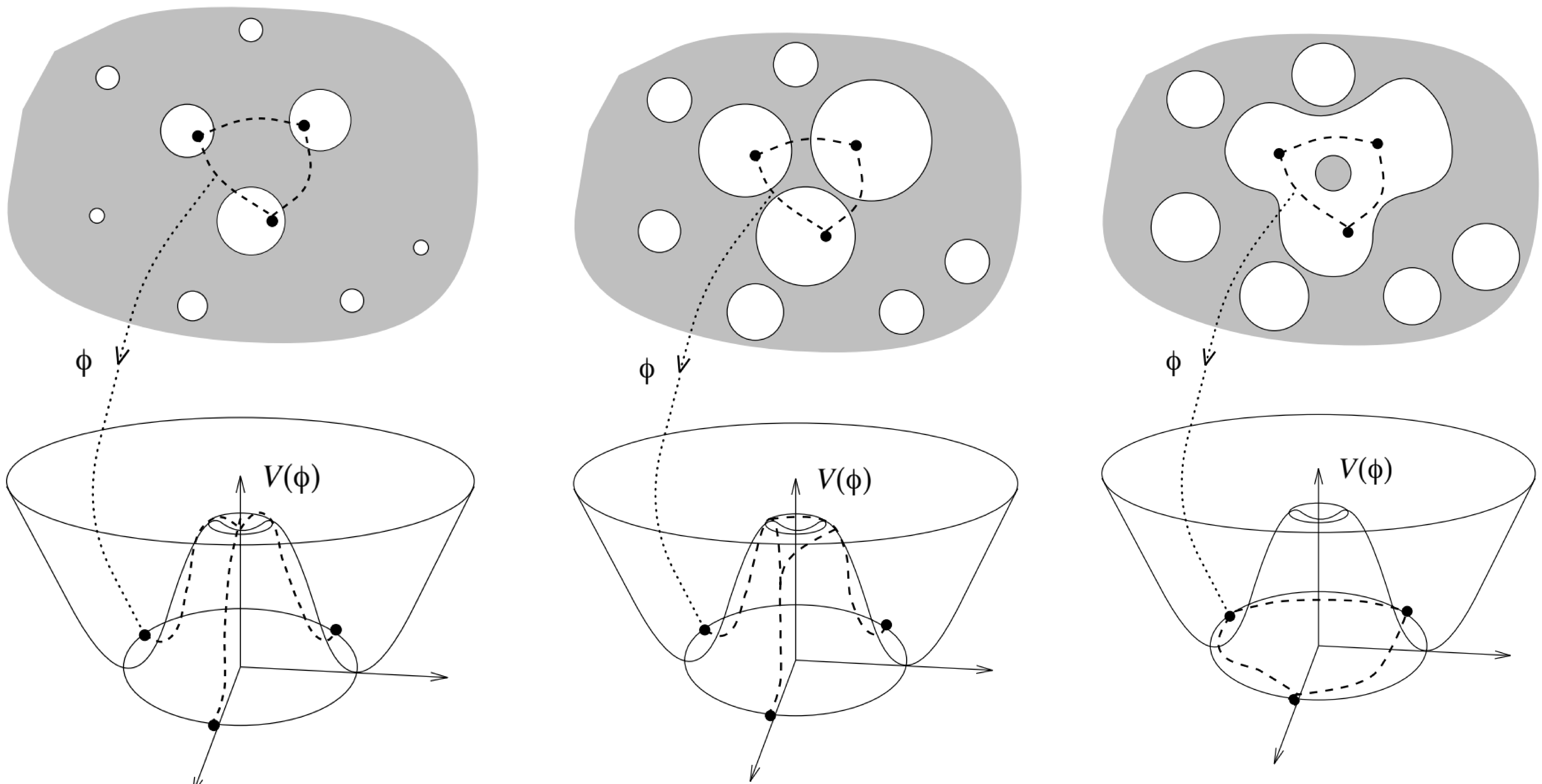
$$\Longrightarrow \sigma^{1/3} \lesssim \mathrm{MeV}$$

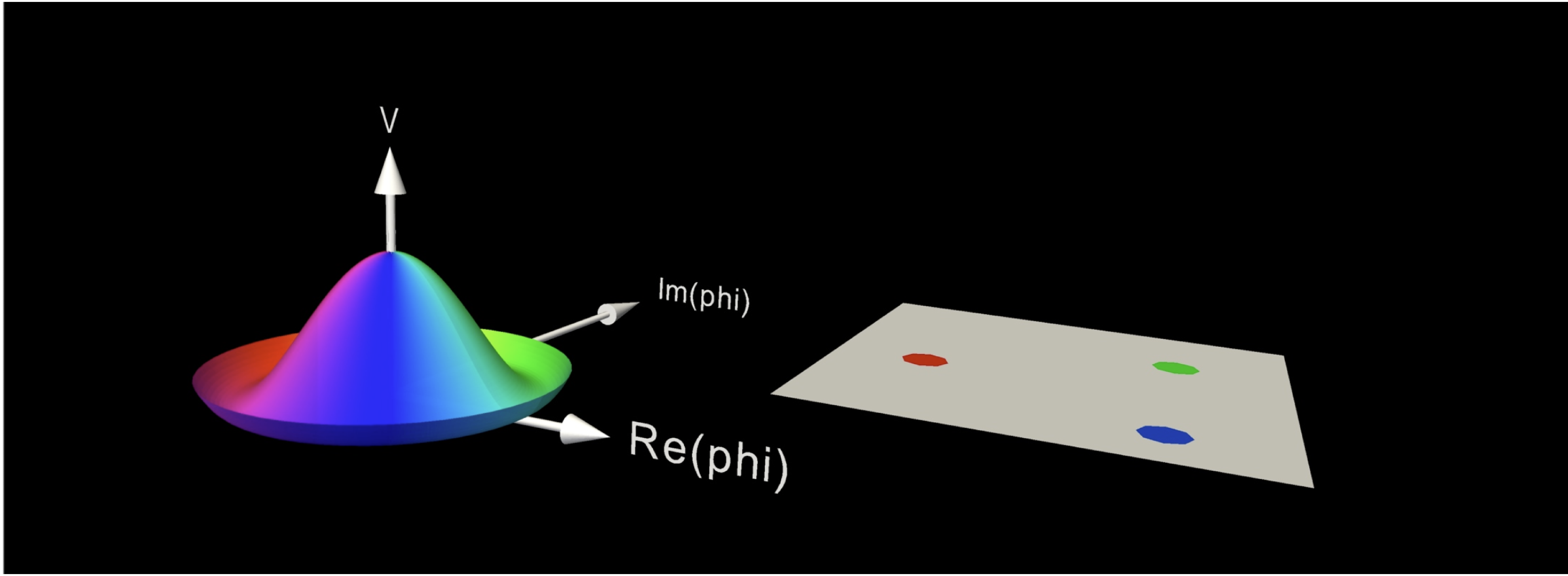
Next-to-simplest possible Model: complex scalar field: Abelian Higgs mechanism

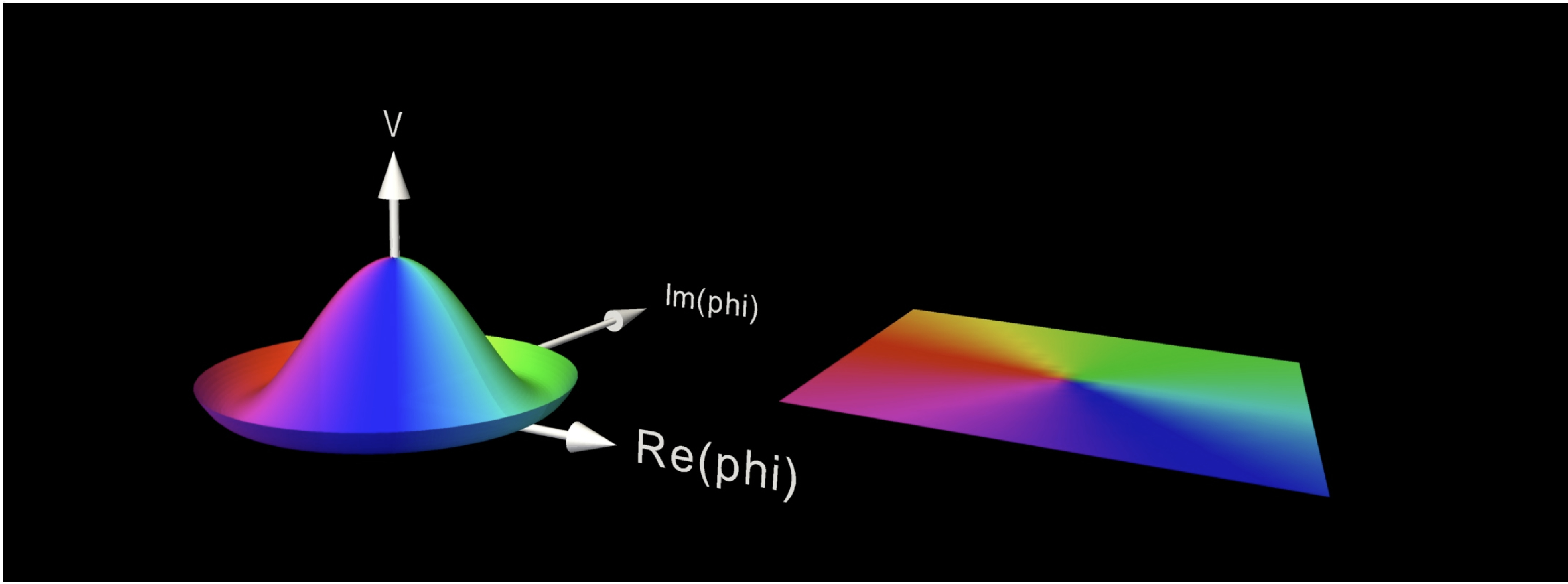
T.W.B. Kibble, *J. Phys.* **A9**, 1387 (1976)
M.B. Hindmarsh and T.W.B. Kibble, *Rep. Prog. Phys.* **58**, 477 (1995)

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}D_\mu\Phi\left(D^\mu\Phi\right)^* - V(\Phi)$$

cosmic string




V
Im(phi)
Re(phi)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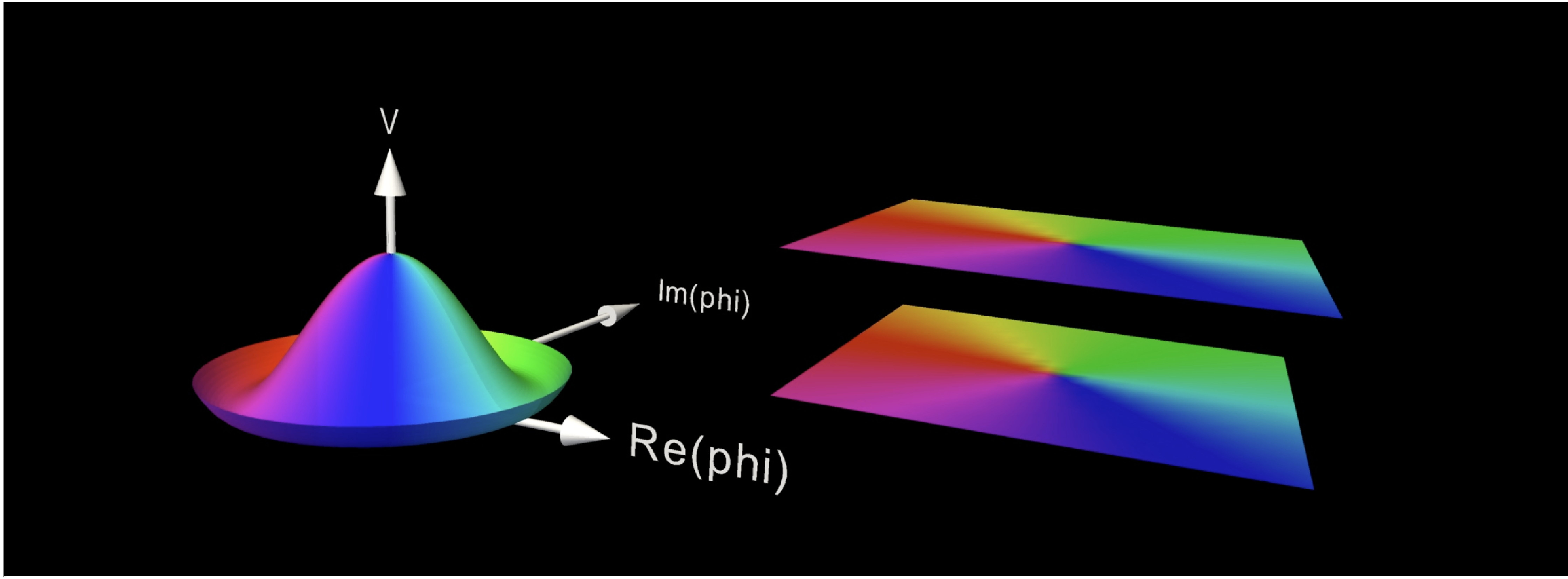
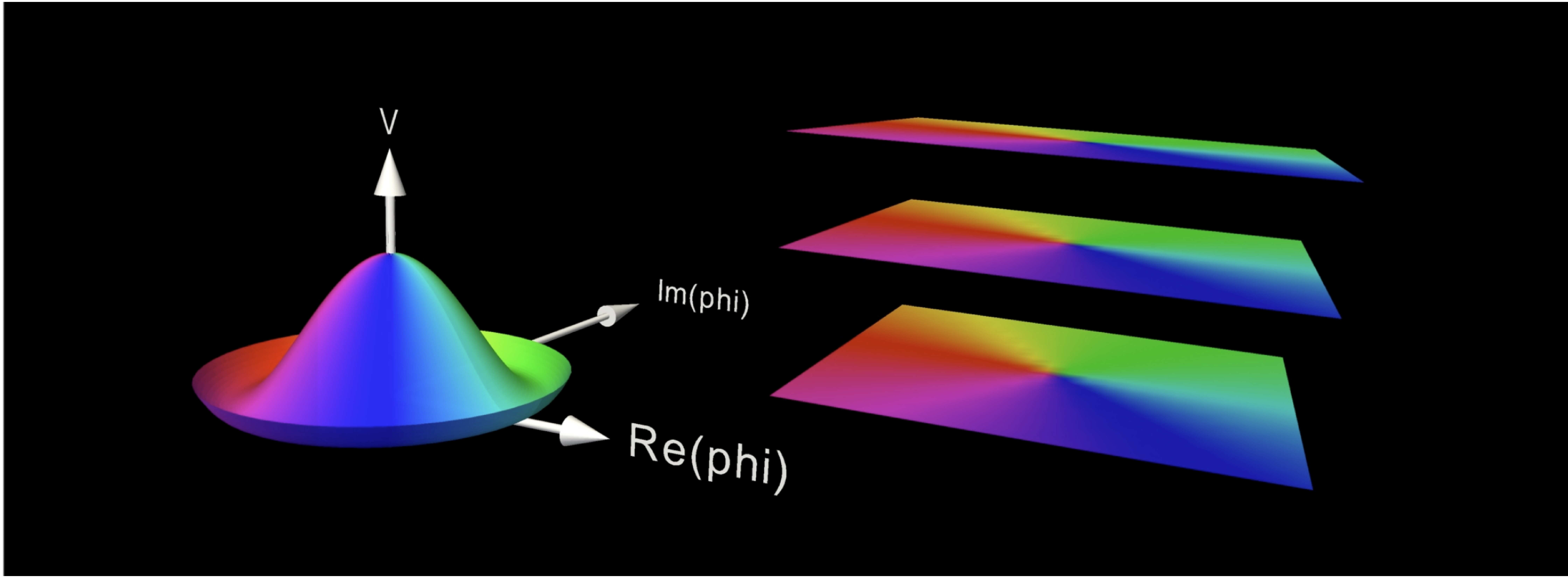
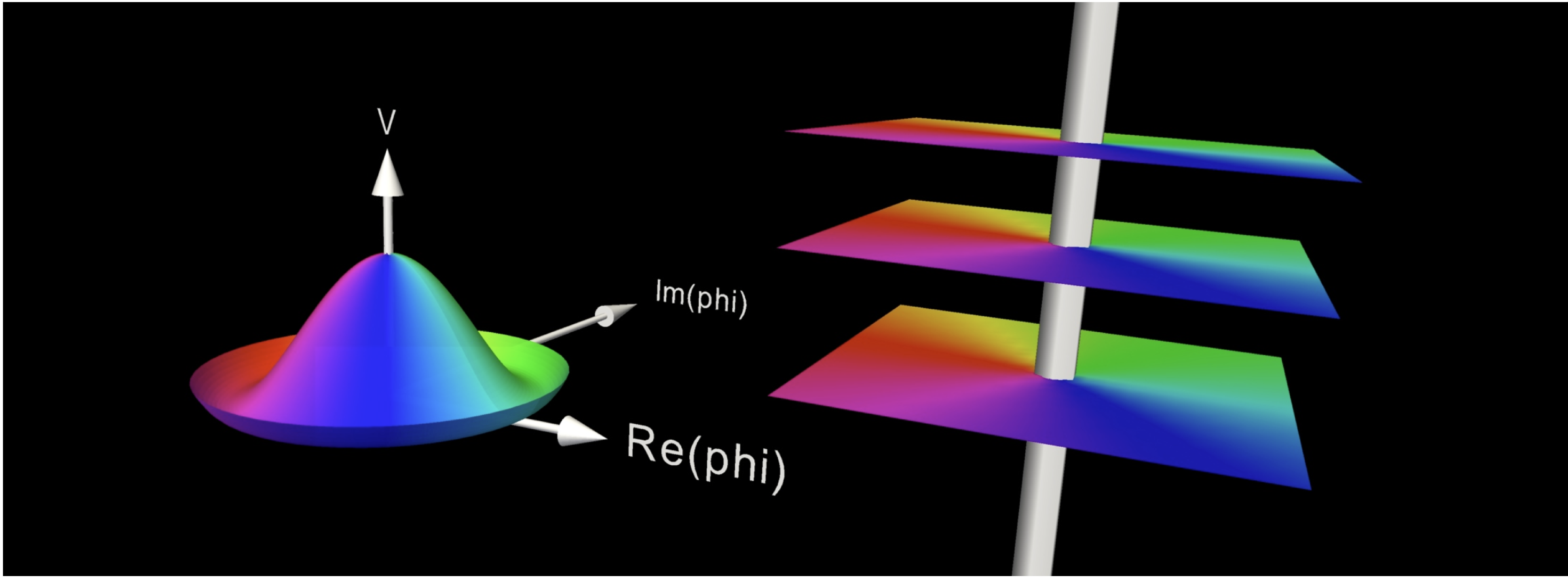
Im(phi)
Re(phi)


V
Im(phi)
Re(phi)


V
Im(phi)
Re(phi)



Thx Jose J. Blanco-Pillado

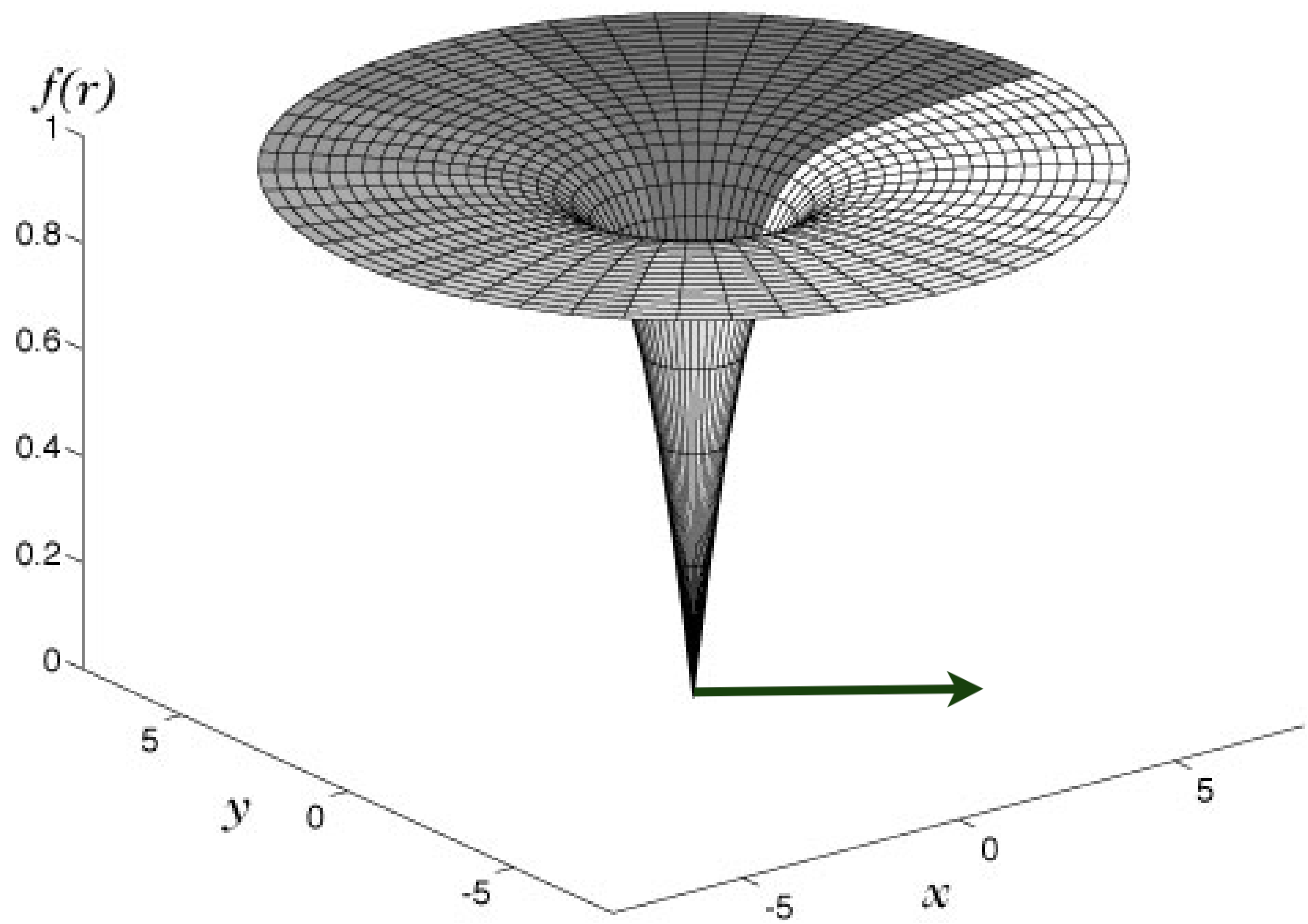
Next-to-simplest possible Model: cosmic string

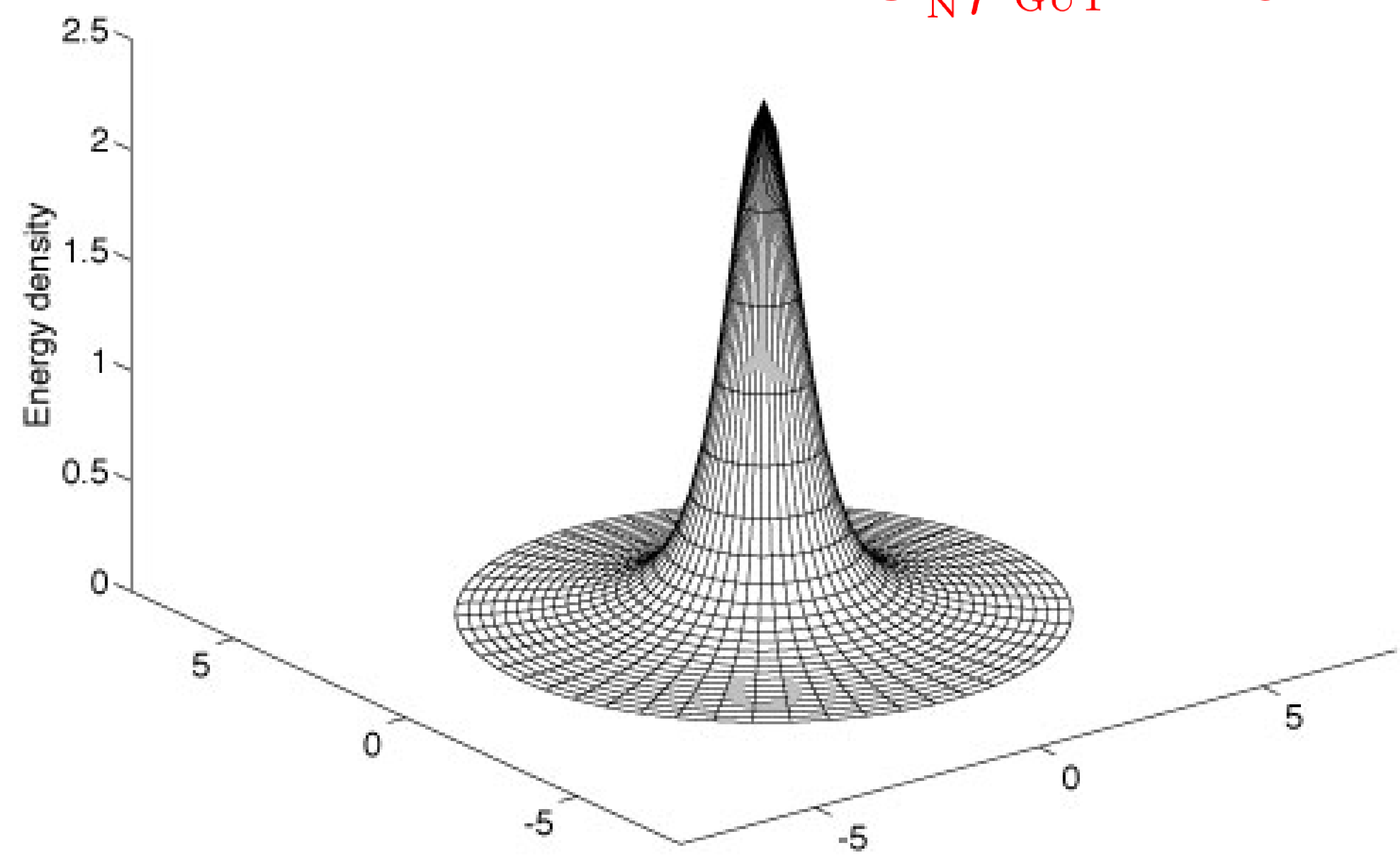
T.W.B. Kibble, *J. Phys.* **A9**, 1387 (1976)
M.B. Hindmarsh and T.W.B. Kibble, *Rep. Prog. Phys.* **58**, 477 (1995)

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}D_\mu\Phi\left(D^\mu\Phi\right)^* - V(\Phi)$$

$$\eta \sim \eta_{\mathrm{GUT}} \quad \Longrightarrow \quad \mu_{\mathrm{GUT}} \sim 10^{15}\mathrm{tons/cm}$$

$$G_{\mathrm{N}}\mu_{\mathrm{GUT}} \sim 10^{-6}$$





$\Phi \to f(r)\mathrm{e}^{in\theta}$ Nielsen-Olesen ansatz

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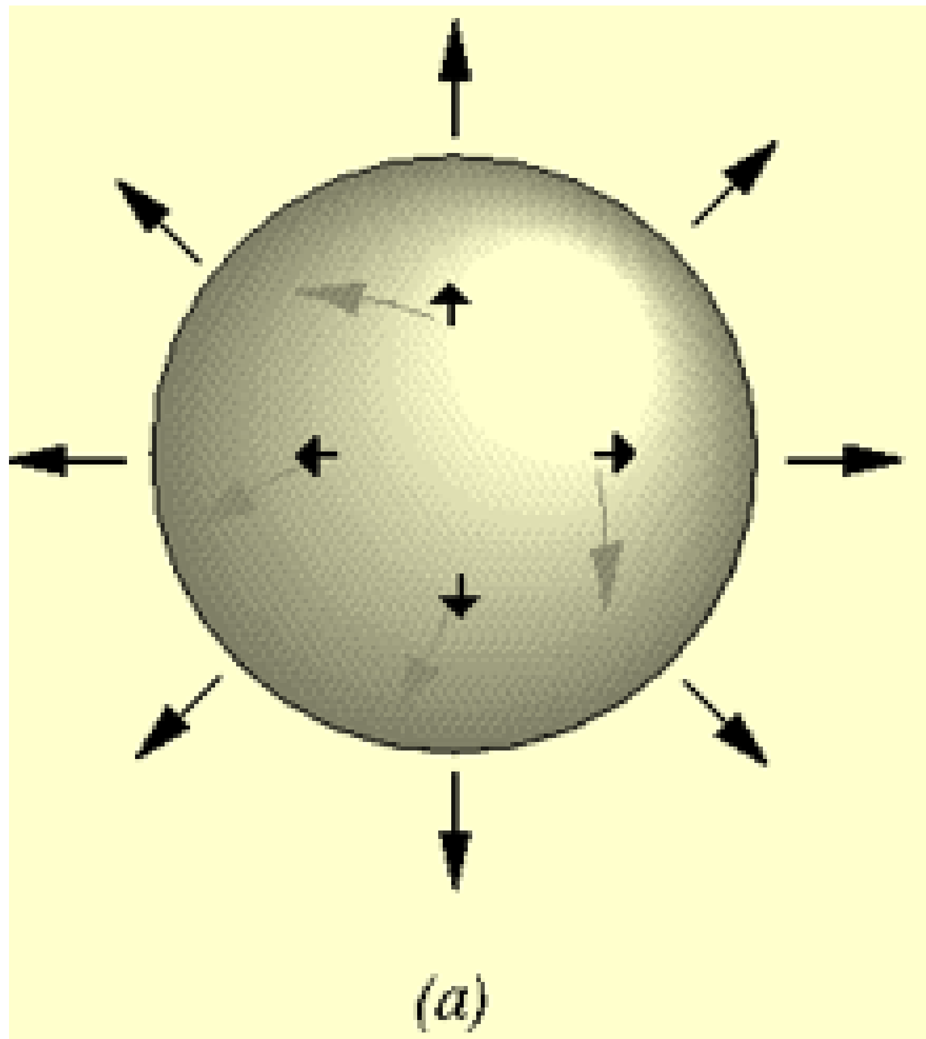
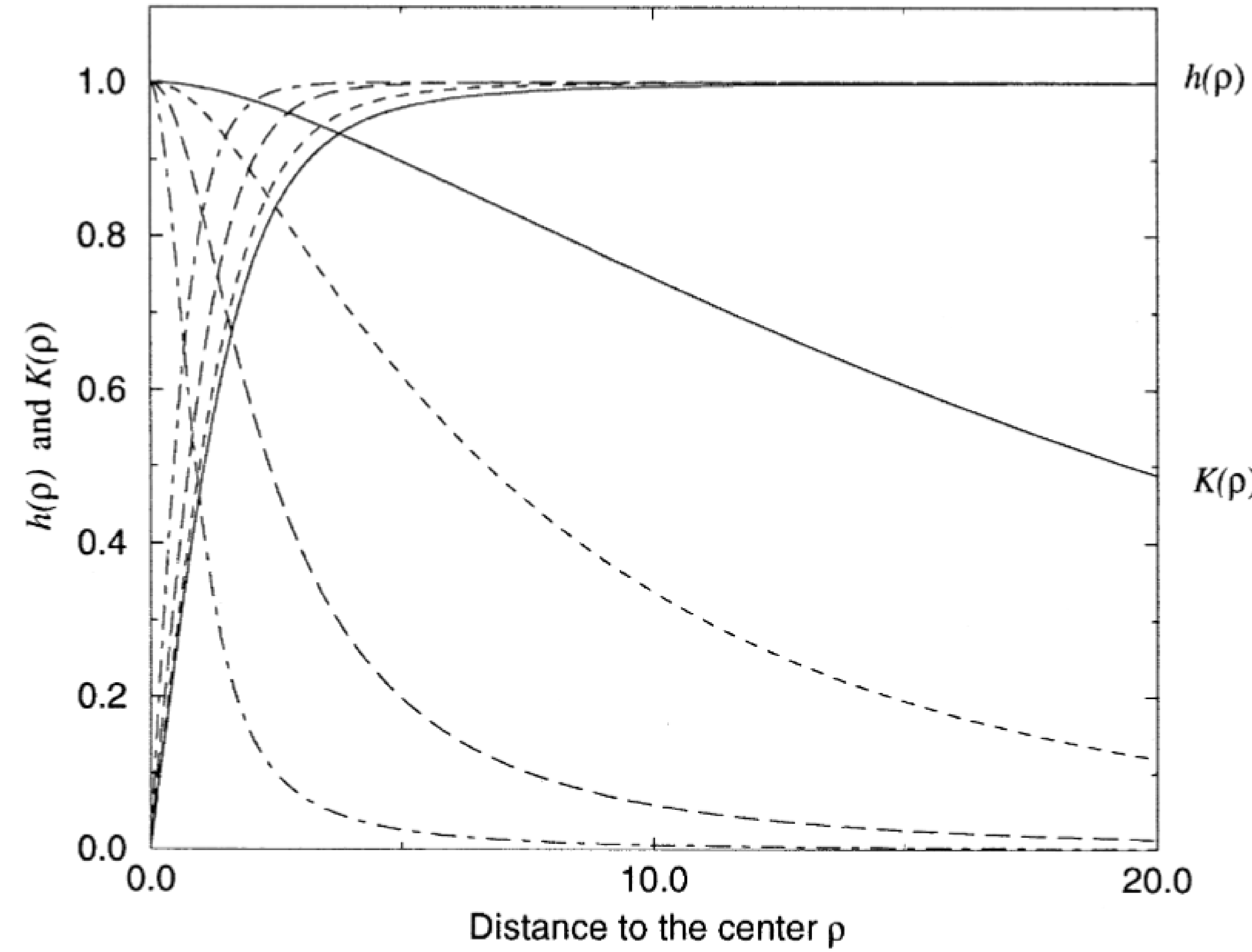


monopole

$$\Phi^a = \eta h(r)\frac{x^a}{r}$$

$$A^i_a = \frac{K(r)-1}{qr^2}\varepsilon^a{}_{ij}x^j$$



- **Unavoidable:**

$$G \to SU_{\mathrm{c}}(3)\times SU_{\mathrm{EW}}(2)\times U_{\mathrm{Y}}(1) \to SU(3)_{\mathrm{c}}\times U_{\mathrm{em}}(1)$$

$$\Longrightarrow G/H \supset U(1) \Longrightarrow \pi_2(G/H) \sim \pi_1(H) \supset \mathbb{Z}$$

- $\Omega_{\mathrm{mon}} \gg 1$

The monopole problem: inflation!

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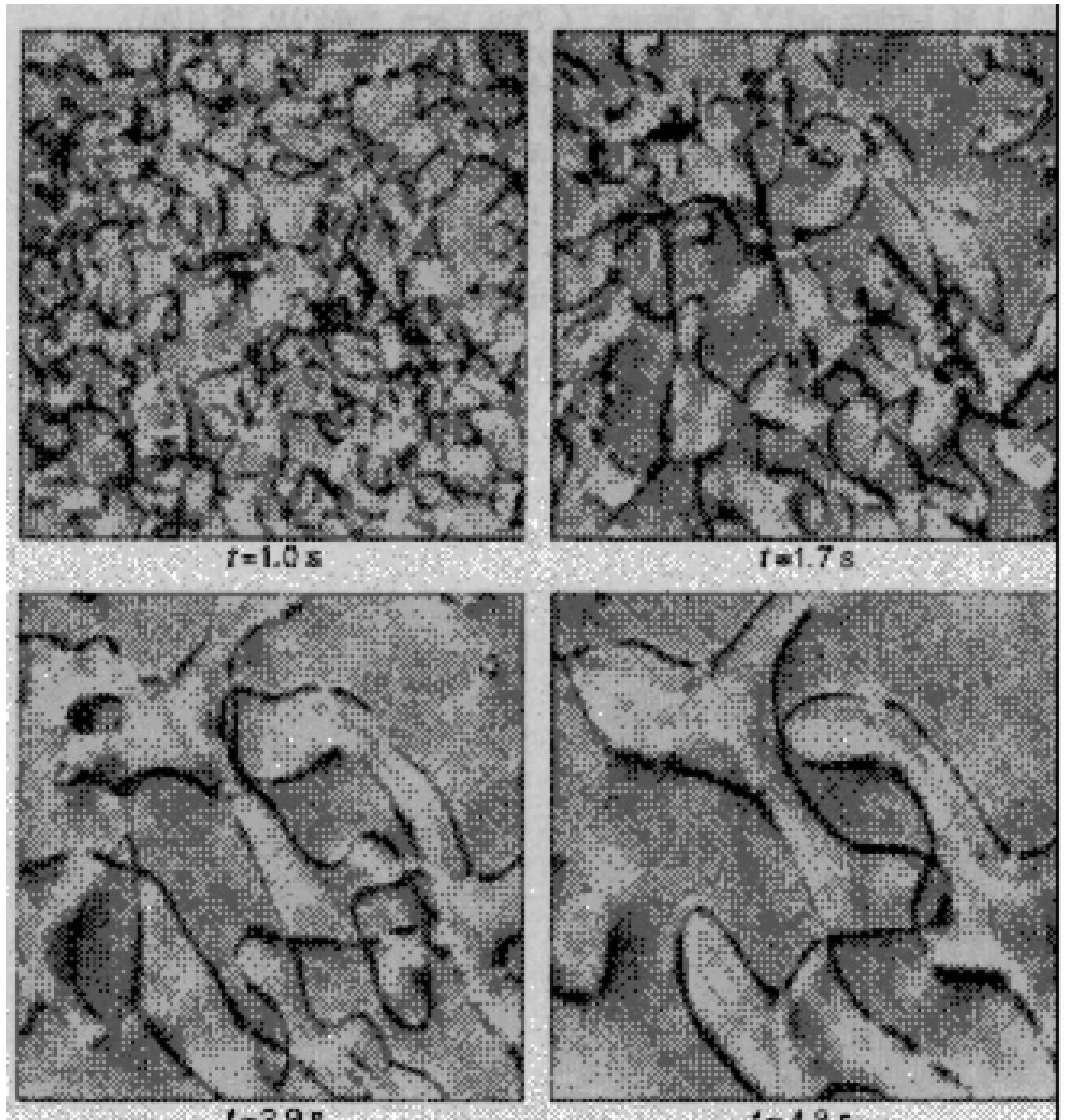
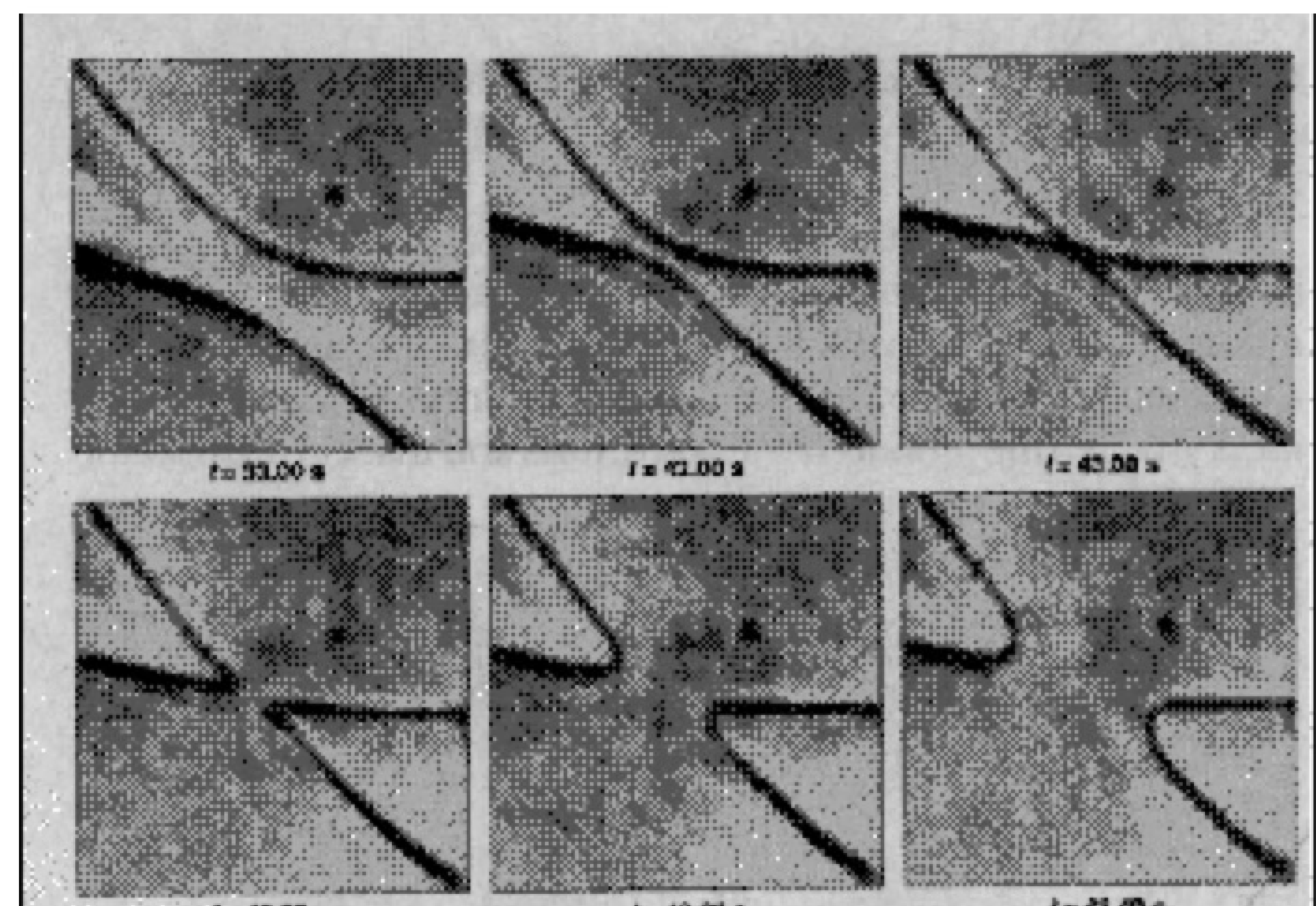
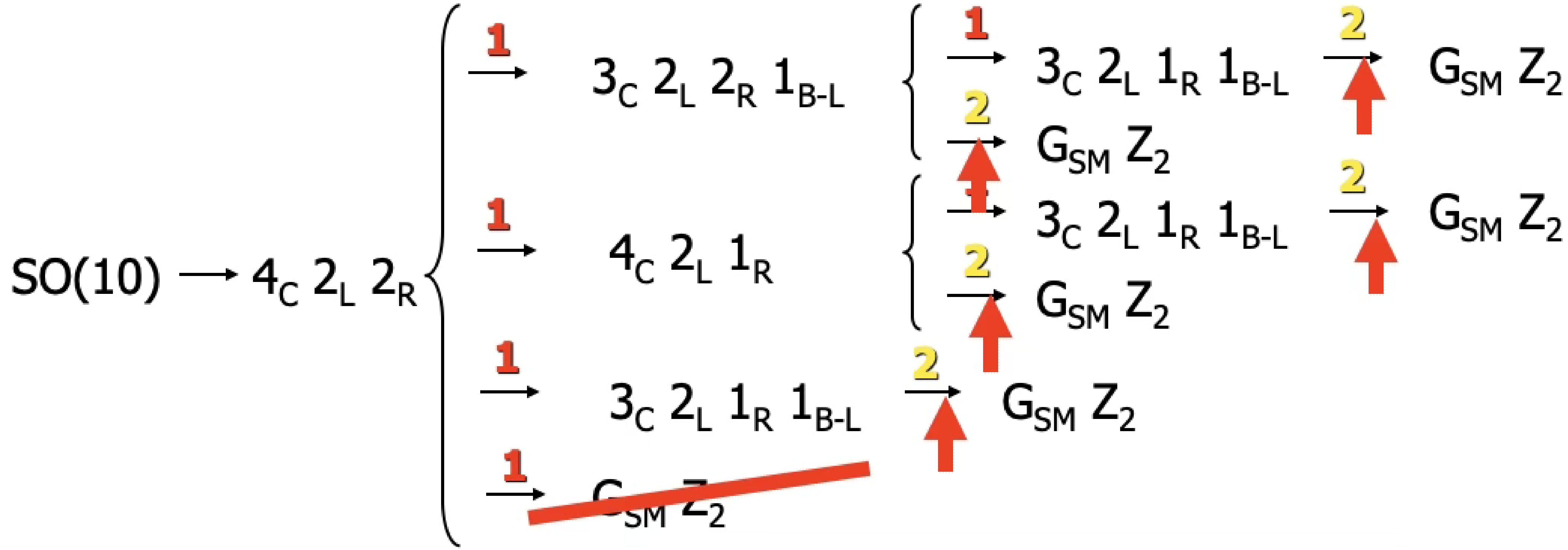
Not so exotic objects ...





SUSY GUT example: *SO(10)*



1: Monopoles **2**: Cosmic strings ↑INFLATION

SO(10) : 34 possible schemes

E_6 : 1024 …

Hybrid Inflation ...

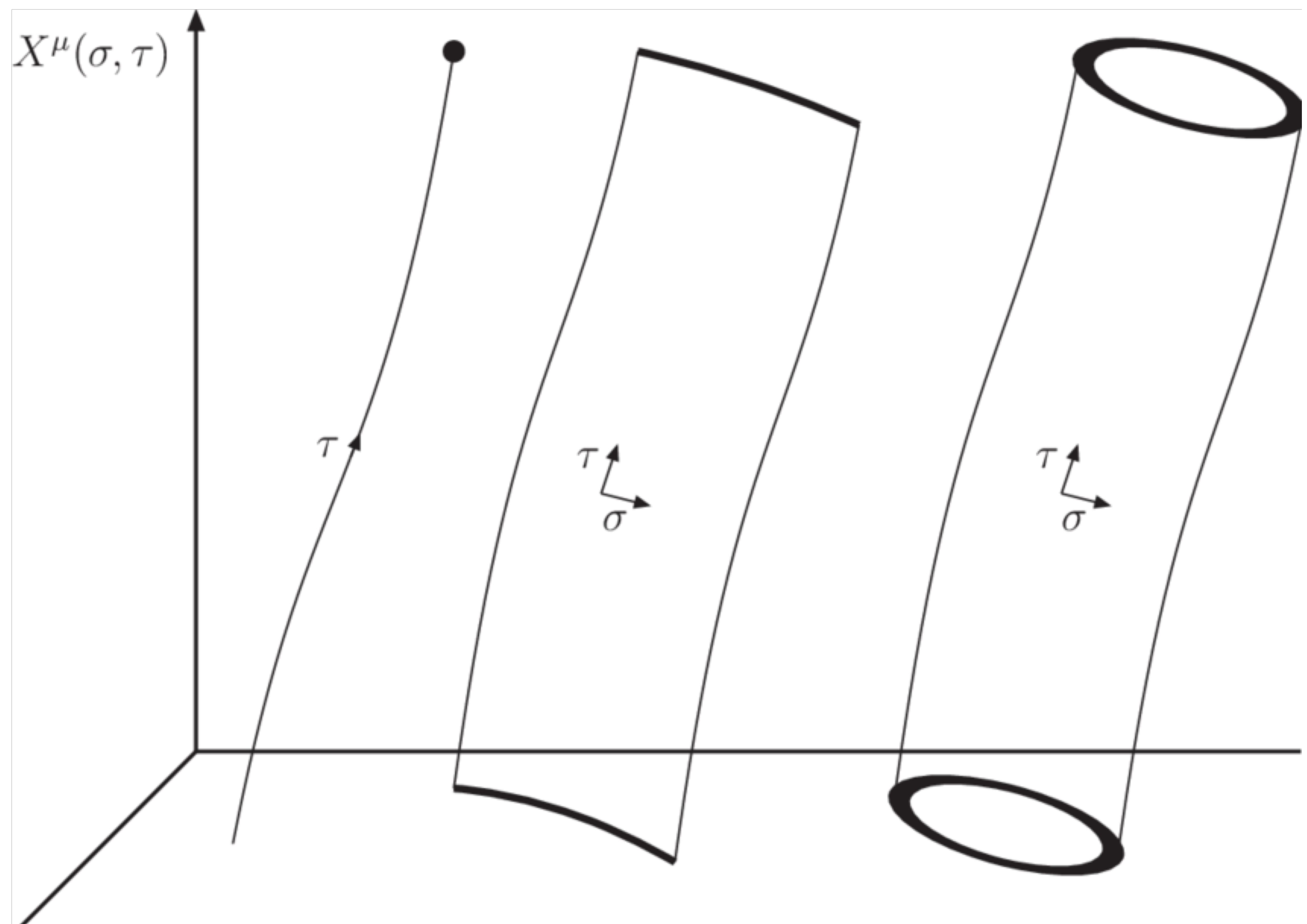
+ SUSY breaking and R-parity

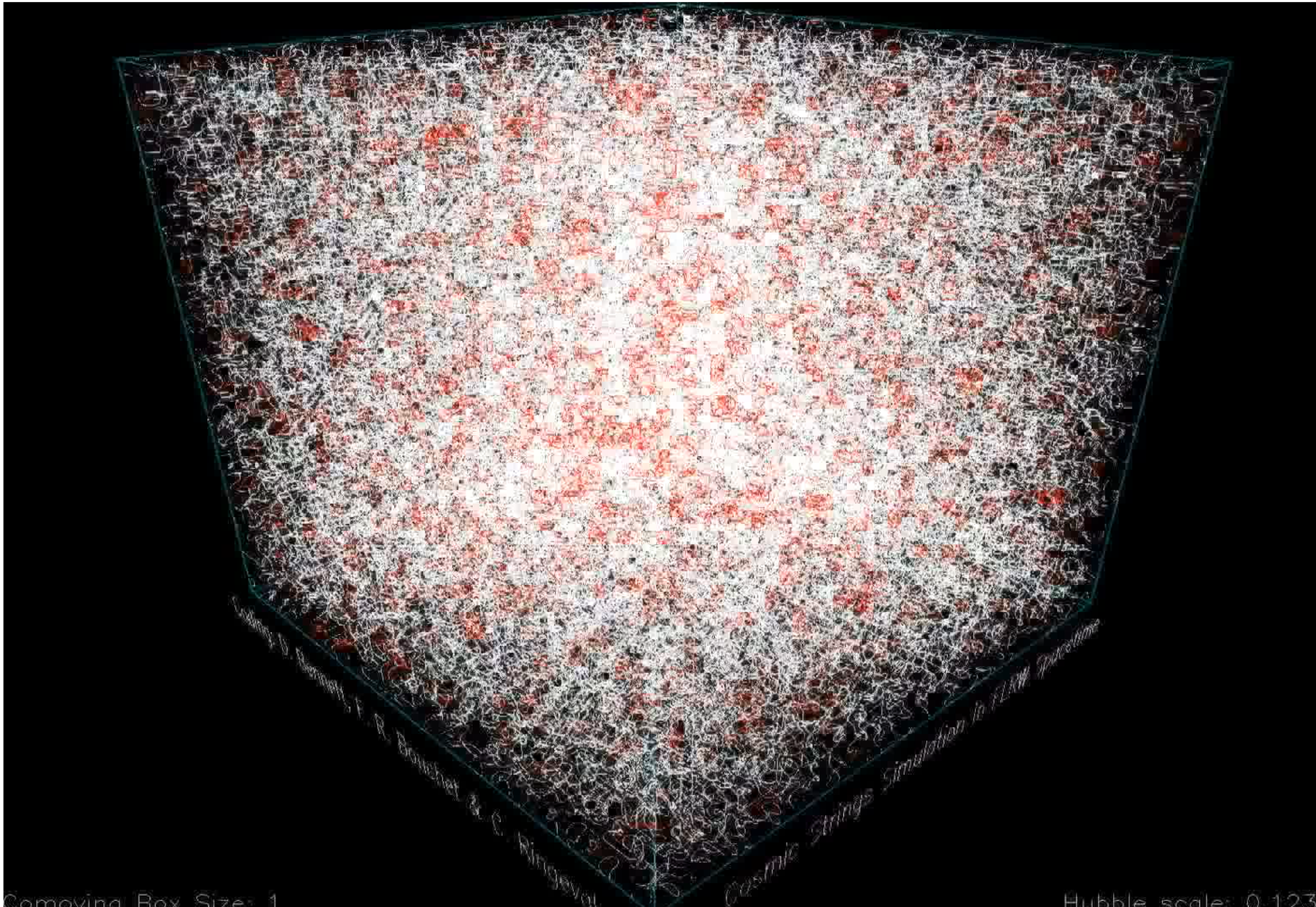
Many fields ⟹ **many** possible couplings

R. Jeannerot, J. Rocher & M. Sakellariadou, PR**D 68**, 104514 (2003)

Nambu-Goto strings

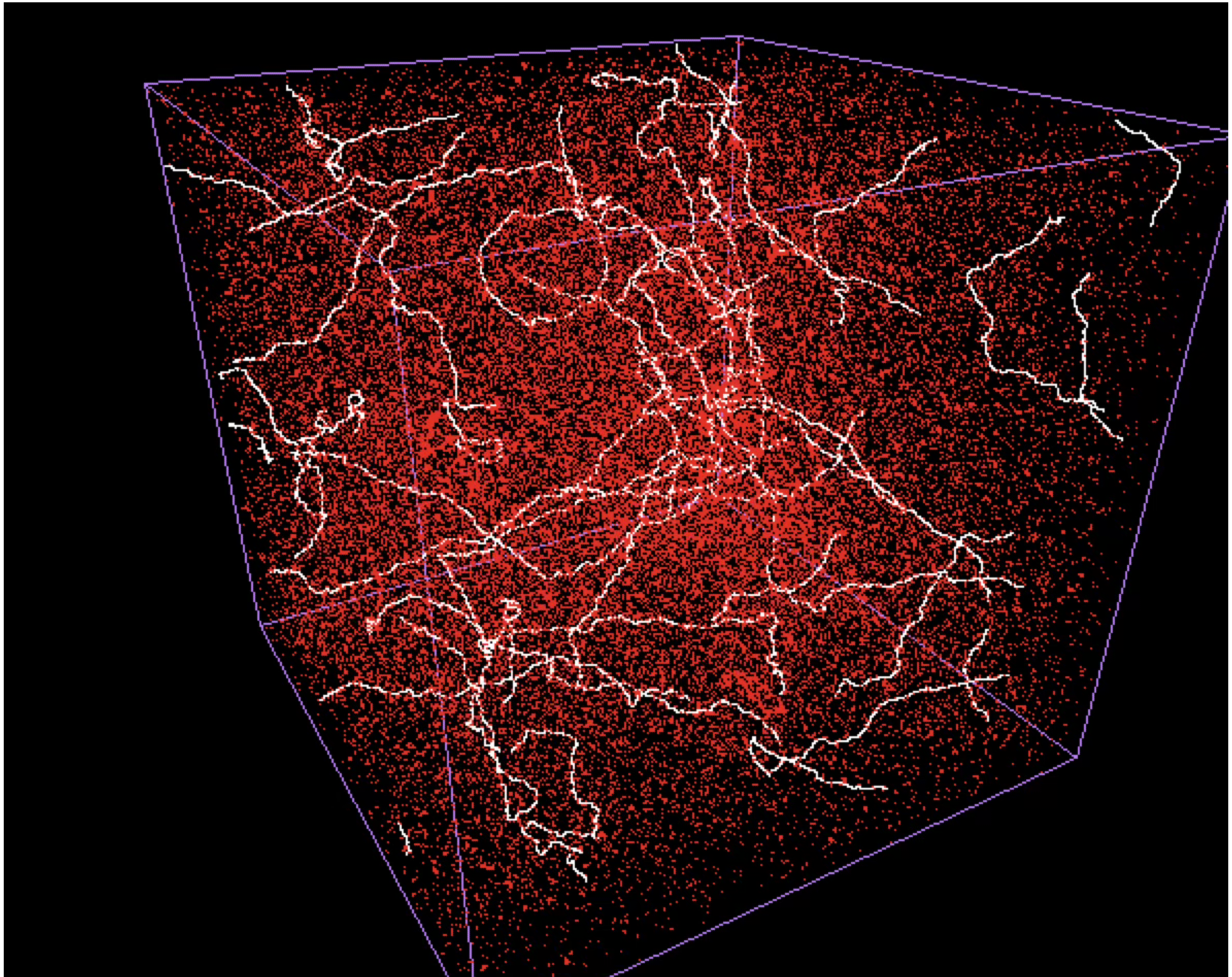
$$\mathcal{S} = \int \sqrt{-\gamma}\,\mathrm{d}\sigma\mathrm{d}\tau \quad \Longrightarrow \quad \ddot{\boldsymbol{x}} = \boldsymbol{x}''$$



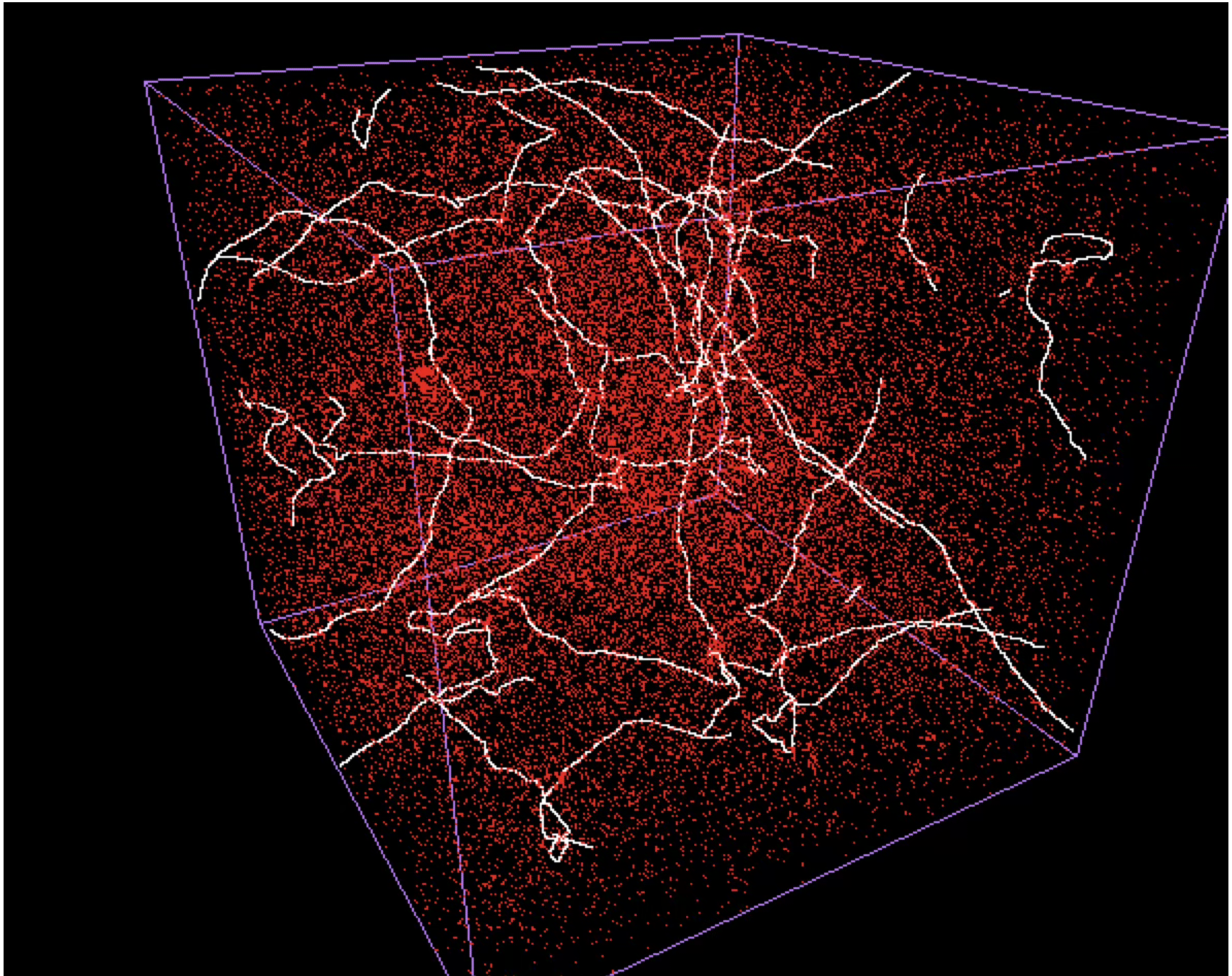


https://commons.wikimedia.org/wiki/File:Cosmic_strings_evolution_during_the_expansion_of_the_Universe.webm

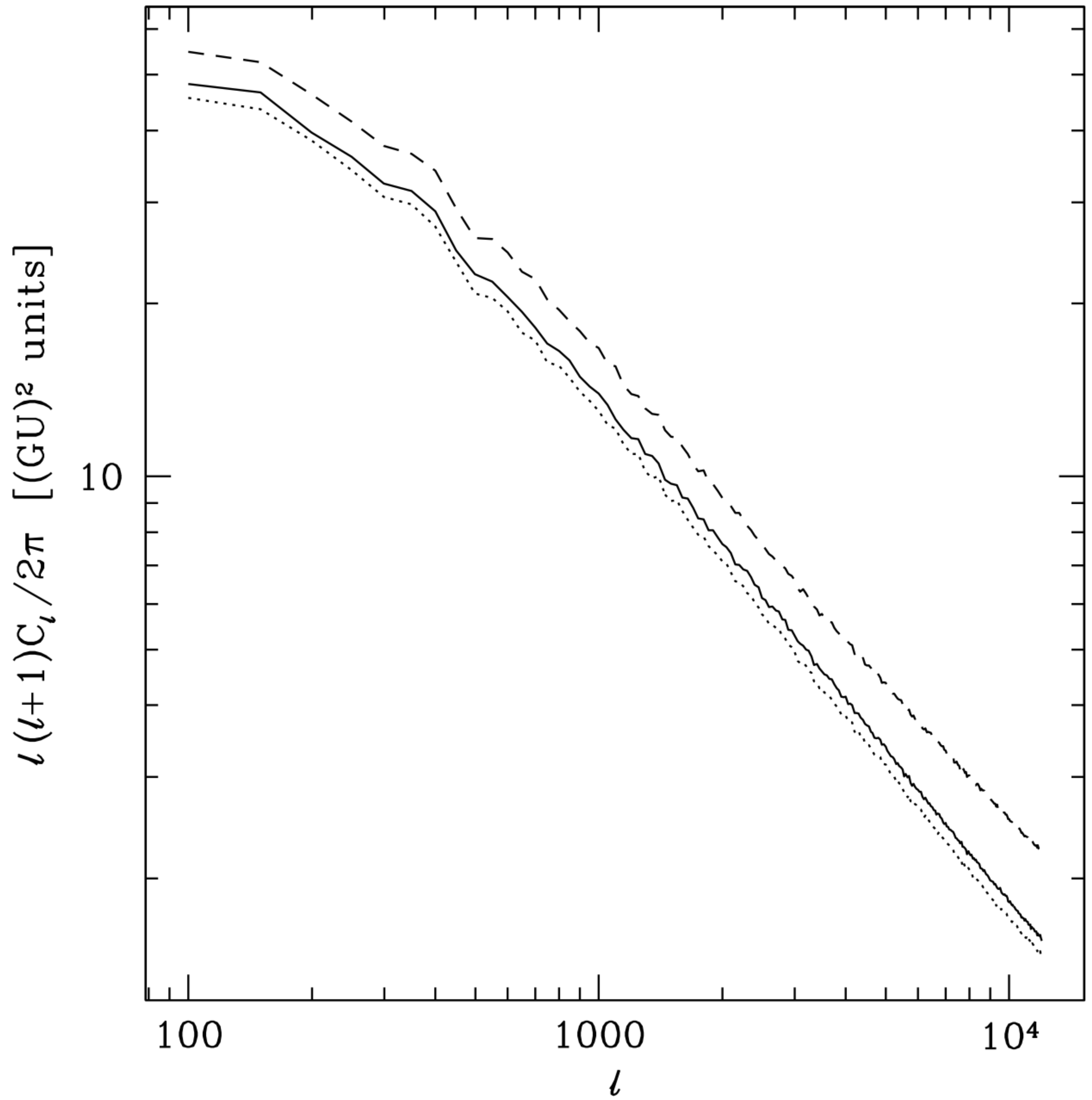
End of radiation dominated era

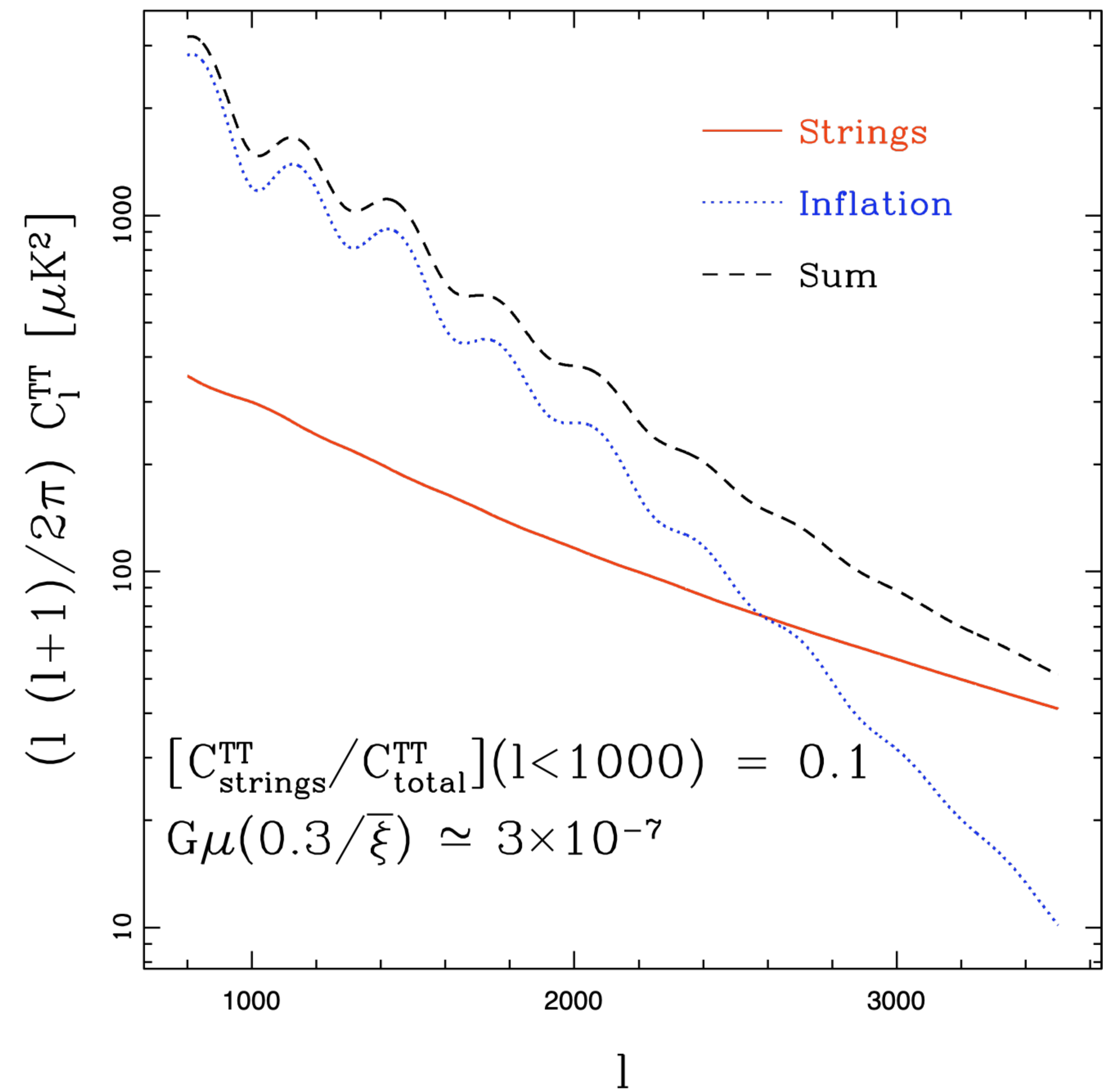


End of matter dominated era



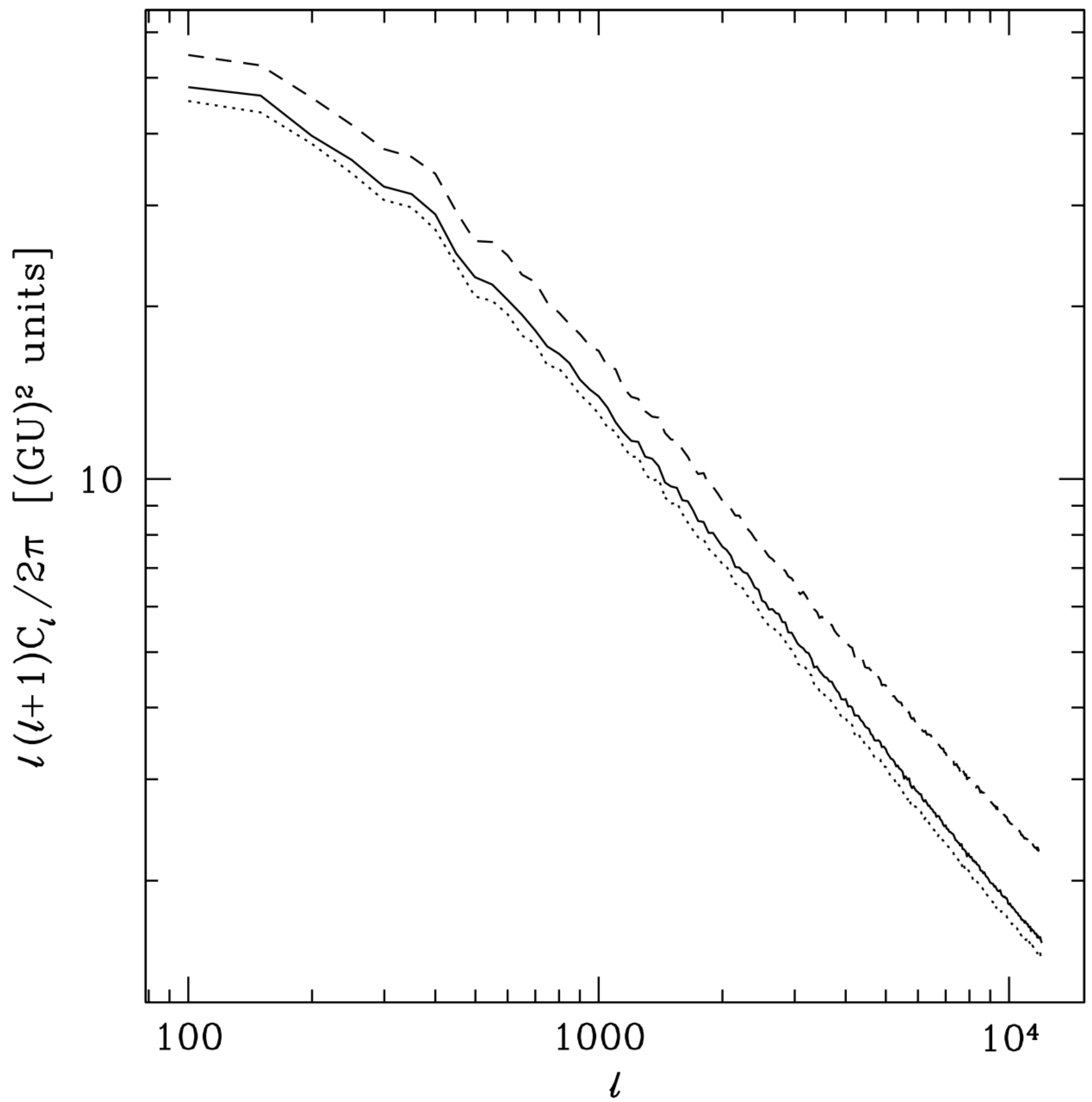
Fraisse, Ringeval, Spergel & Bouchet (2008)

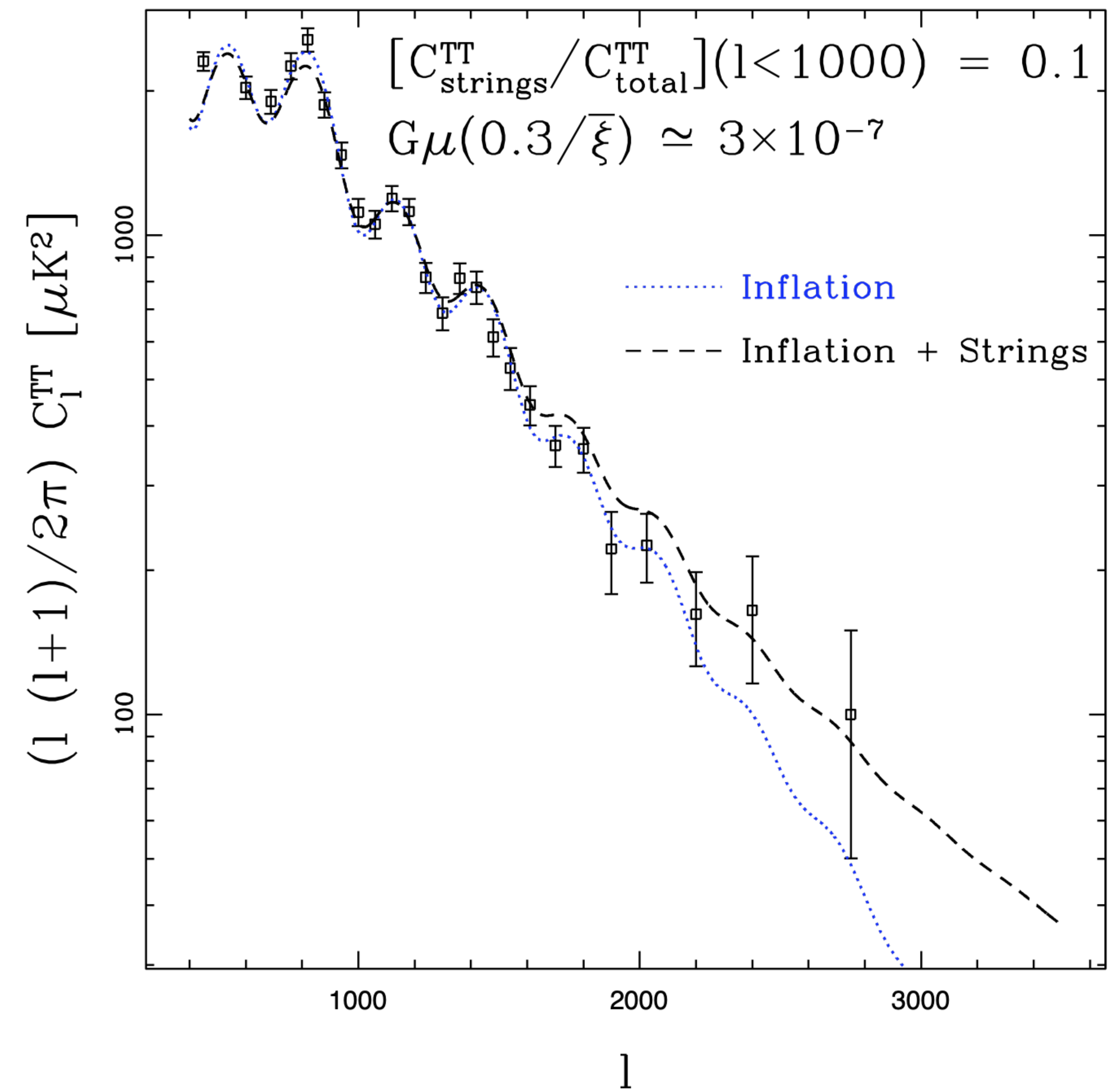




Pogosian et al. (2008)

Fraisse, Ringeval, Spergel & Bouchet (2008)





Pogosian et al. (2008)

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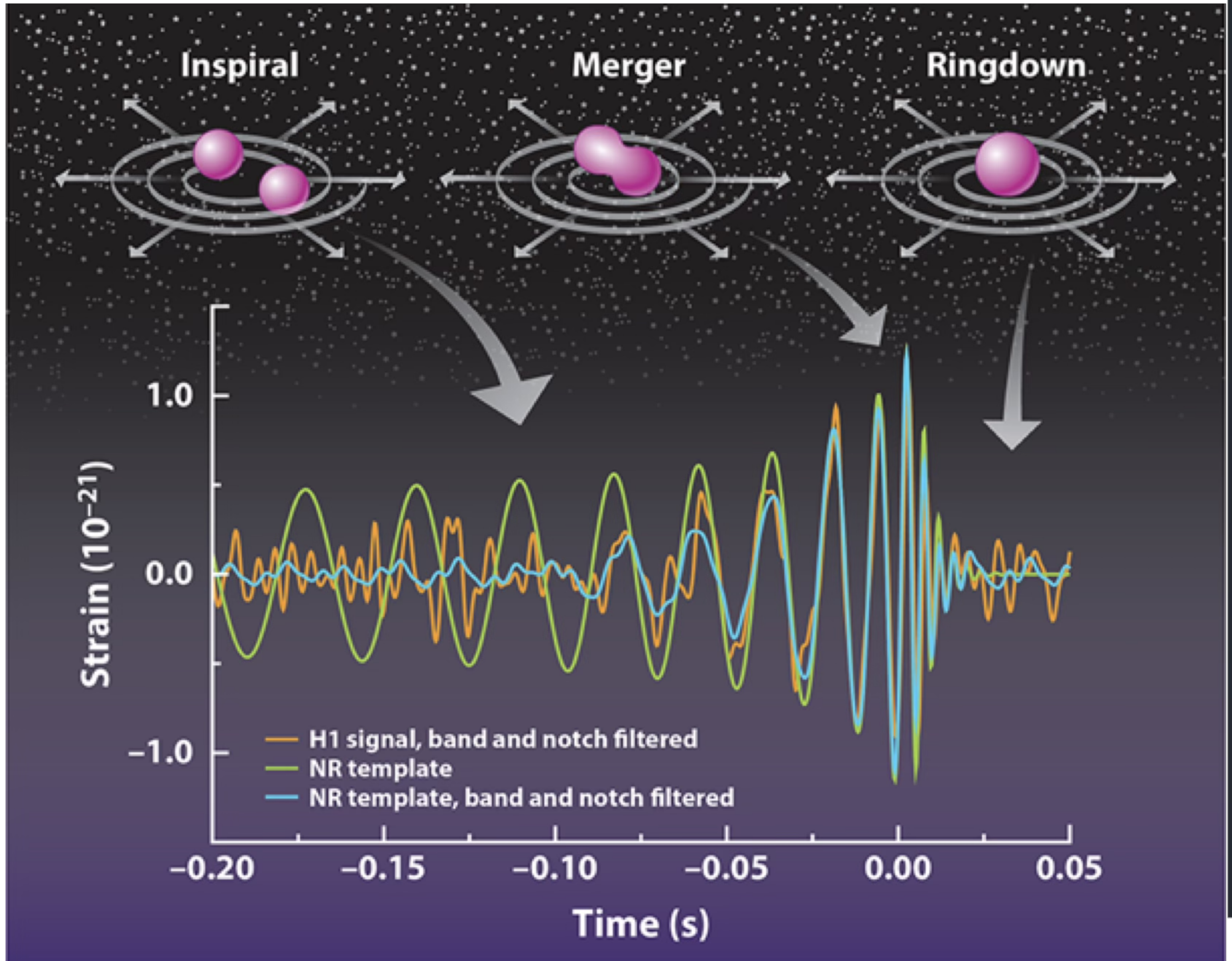
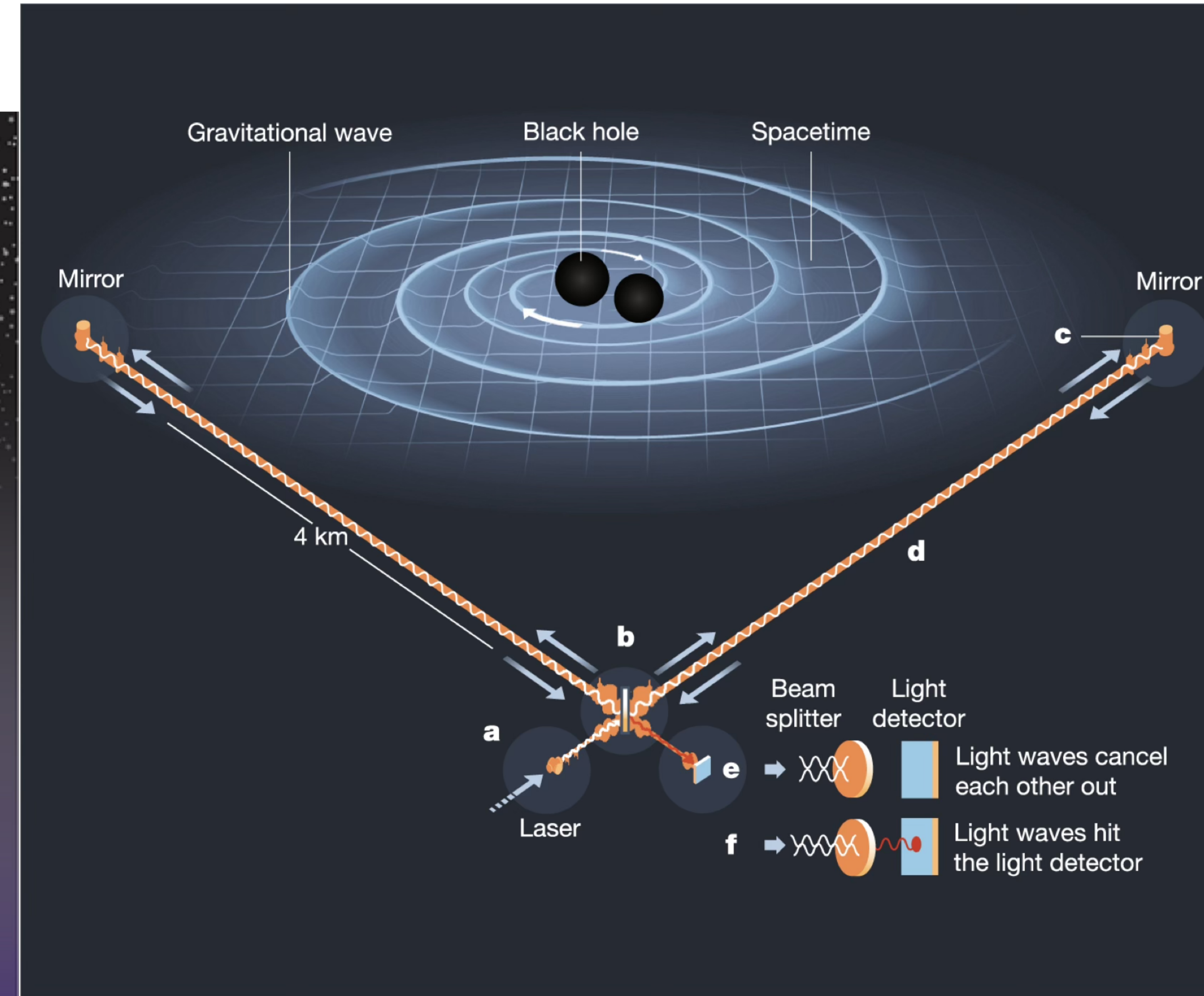
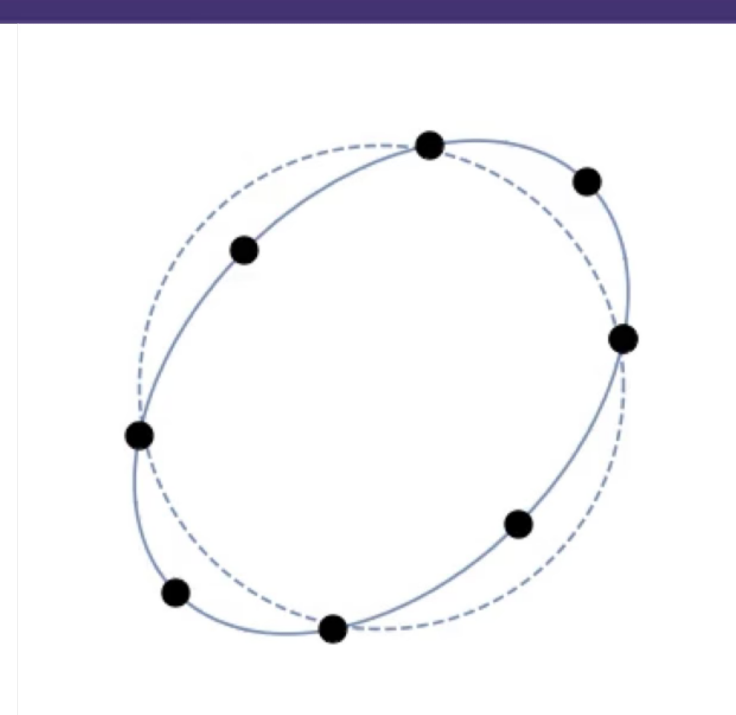
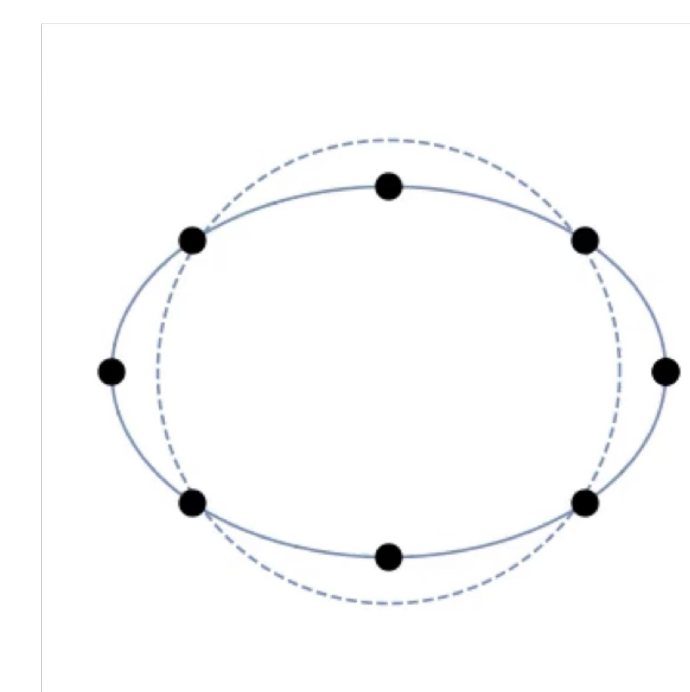
Gravitational wave



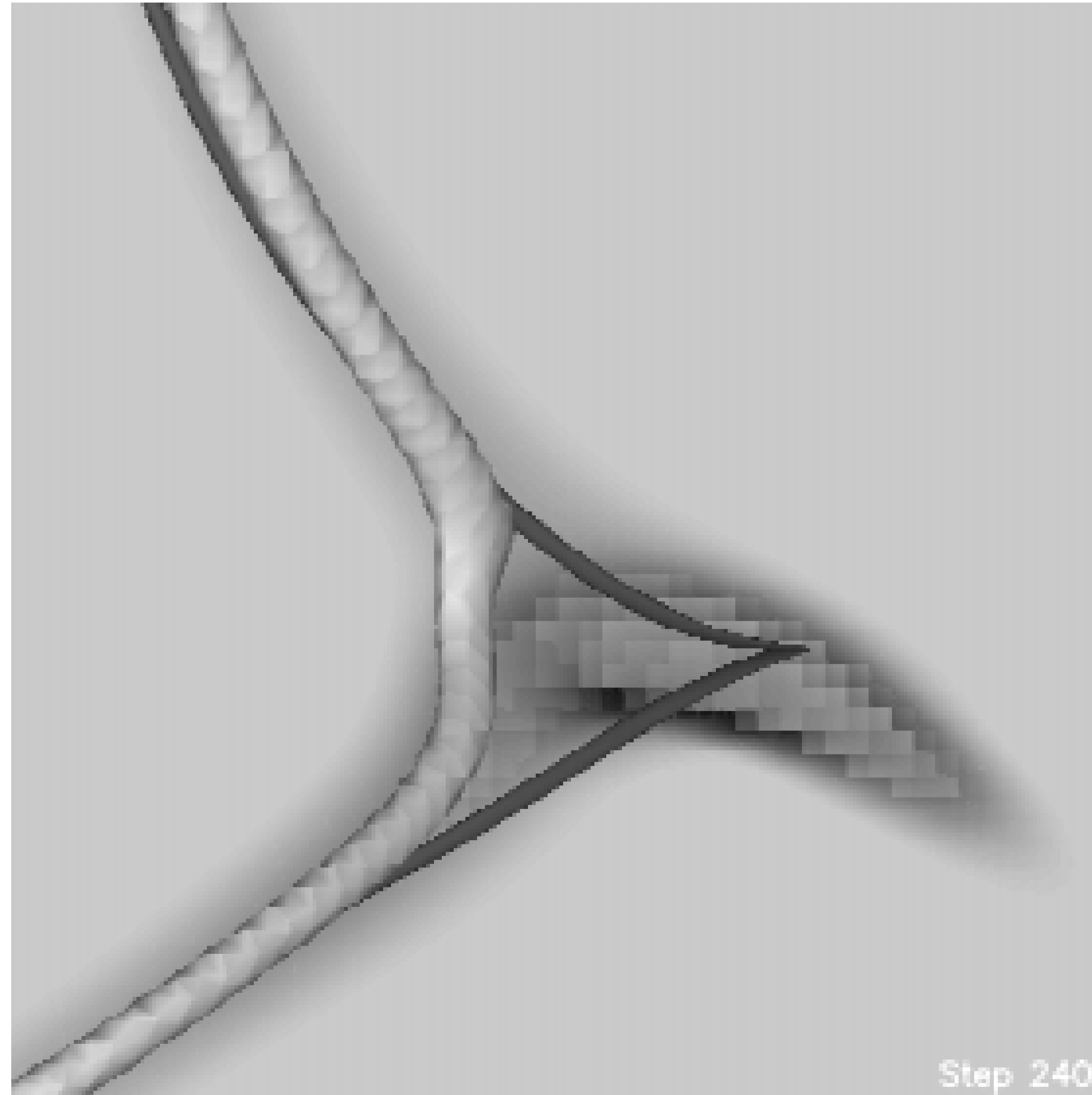






GW stochastic background

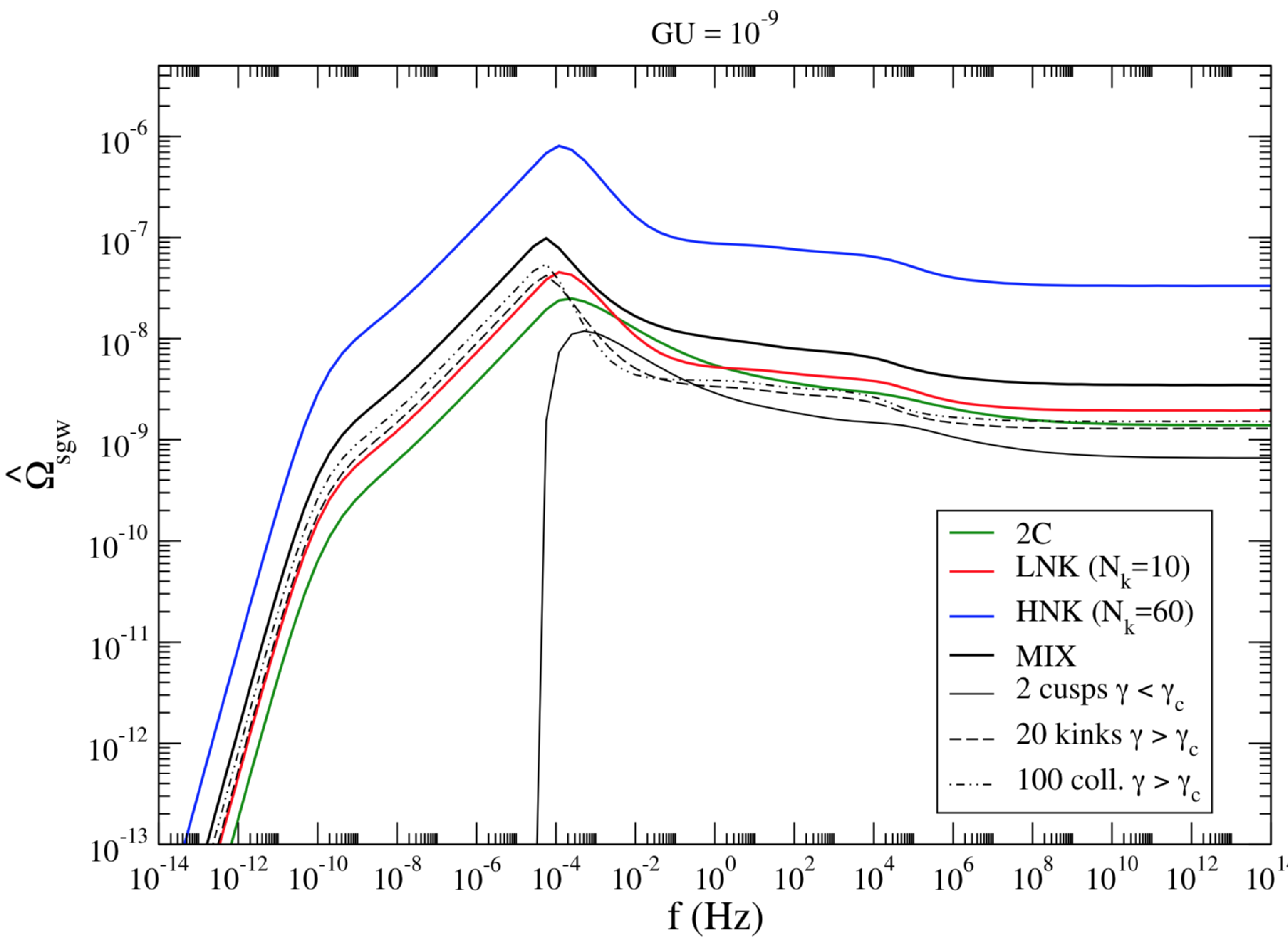
Mostly based on existence of cusps…

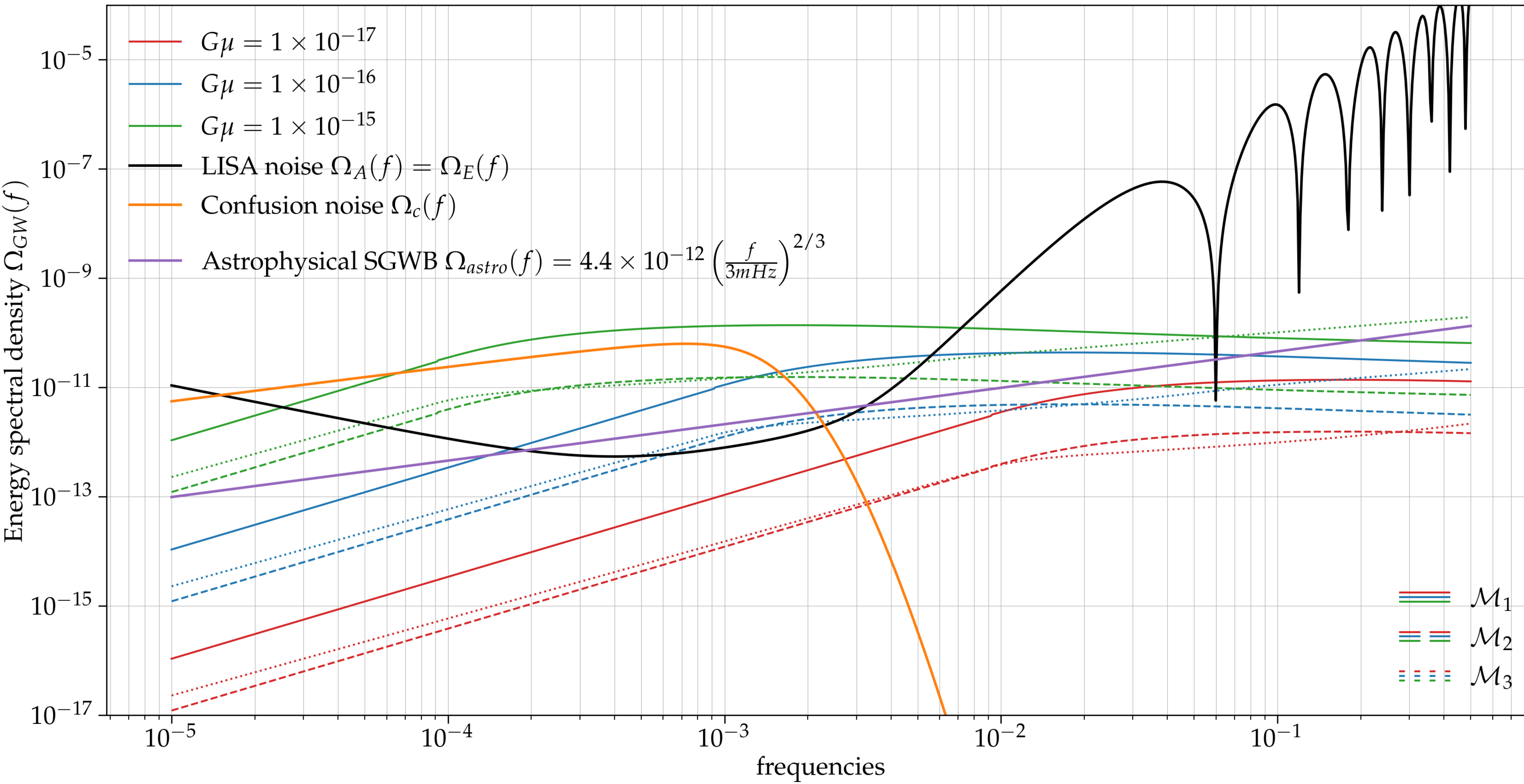


K. D. Olum & J. J. Blanco-Pillado, *Phys. Rev.* **D60,** 023503 (1999)

GW stochastic background

C. Ringeval & T. Suyama, *JCAP* **17,** 12 (2017)
J. J. Blanco-Pillado & K. D. Olum, *Phys. Rev.* **D96**, 104046 (2017)
J. J. Blanco-Pillado K. D. Olum & X. Siemens, *Phys. Lett.* **B778,** 392 (2018)




$G\mu = 1 \times 10^{-17}$
$G\mu = 1 \times 10^{-16}$
$G\mu = 1 \times 10^{-15}$
LISA noise $\Omega_A(f) = \Omega_E(f)$
Confusion noise $\Omega_c(f)$
Astrophysical SGWB $\Omega_{astro}(f) = 4.4 \times 10^{-12} \left(\frac{f}{3mHz}\right)^{2/3}$
$\mathcal{M}_1$
$\mathcal{M}_2$
$\mathcal{M}_3$
Energy spectral density $\Omega_{GW}(f)$
frequencies
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}

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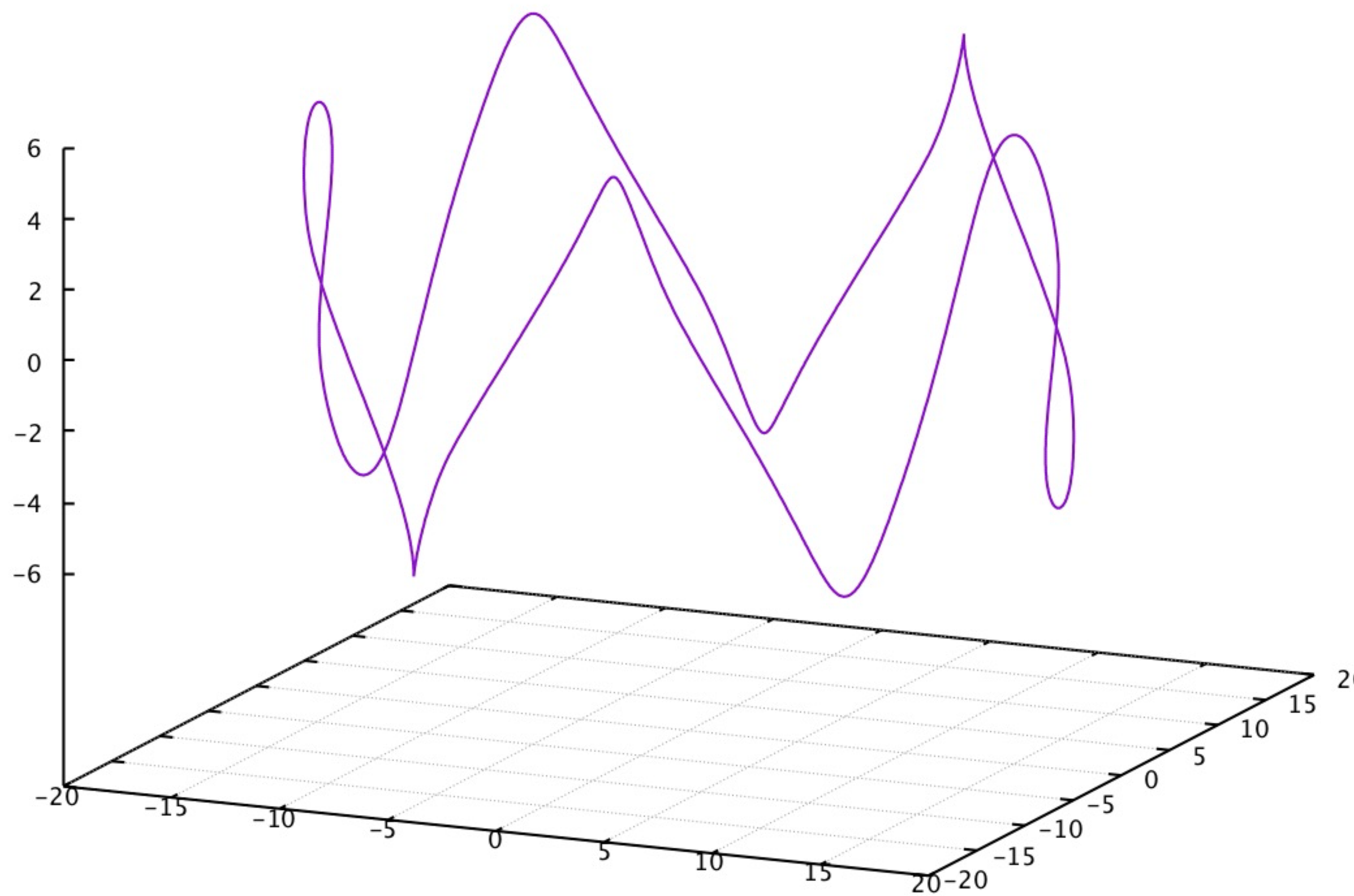
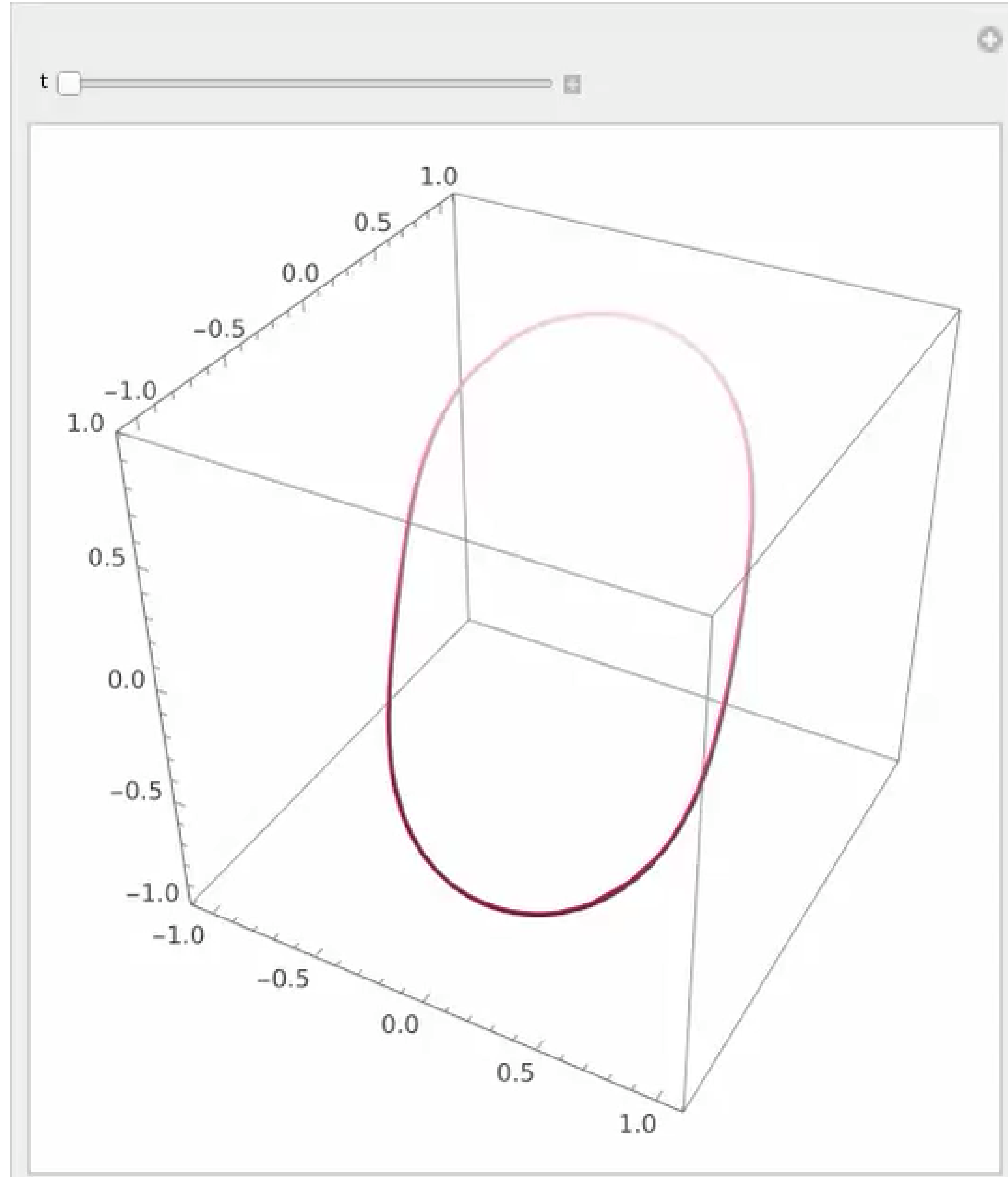


6
4
2
0
−2
−4
−6
−20
−15
−10
−5
0
5
10
15
20
−20
−15
−10
−5
0
5
10
15
20



Thx Maria Eduarda da Silva Inacio
UNESP - Universidade Estadual Paulista

Overview

- Topological defects
  - Symmetry breaking: Higgs mechanism
  - Phase transition: early Universe
  - Monopoles, strings, walls… constraints
    - String network formation and evolution
    - Gravitational radiation
    - PBHs
- **Quantum vacuum fluctuations and large scale structures**
  - Large scale structure formation = gravitational collapse
  - Inflation
    - Early Universe initial conditions: vacuum!
  - Quantizing gravity (perturbations)
    - Varying mass scalar fields
    - Quantum vacuum fluctuations → squeezed states…
      - Power spectrum, CMB and all that.

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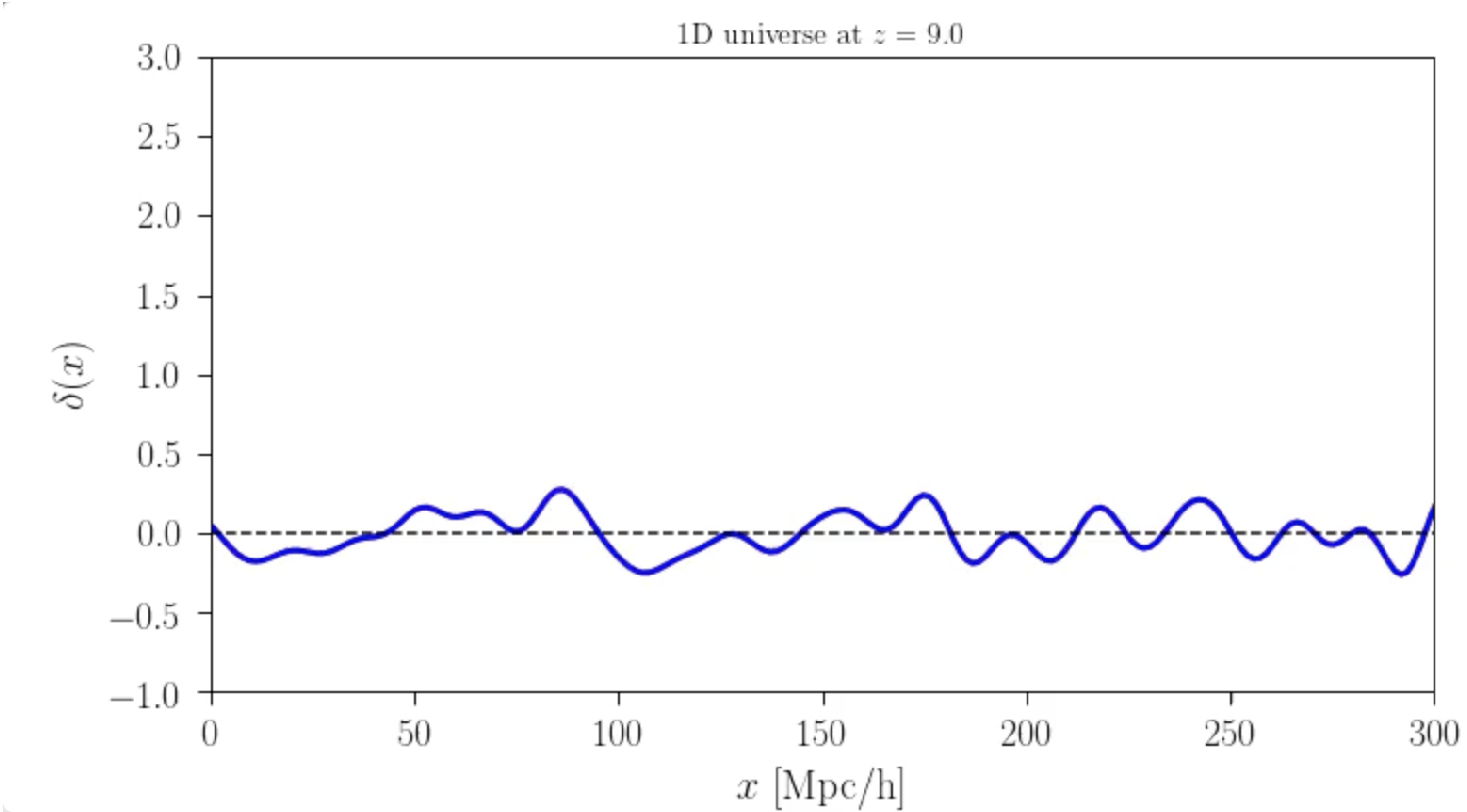
small perturbations (geometry & matter):

$$\delta(x) := \frac{\rho(x) - \bar{\rho}}{\bar{\rho}}$$



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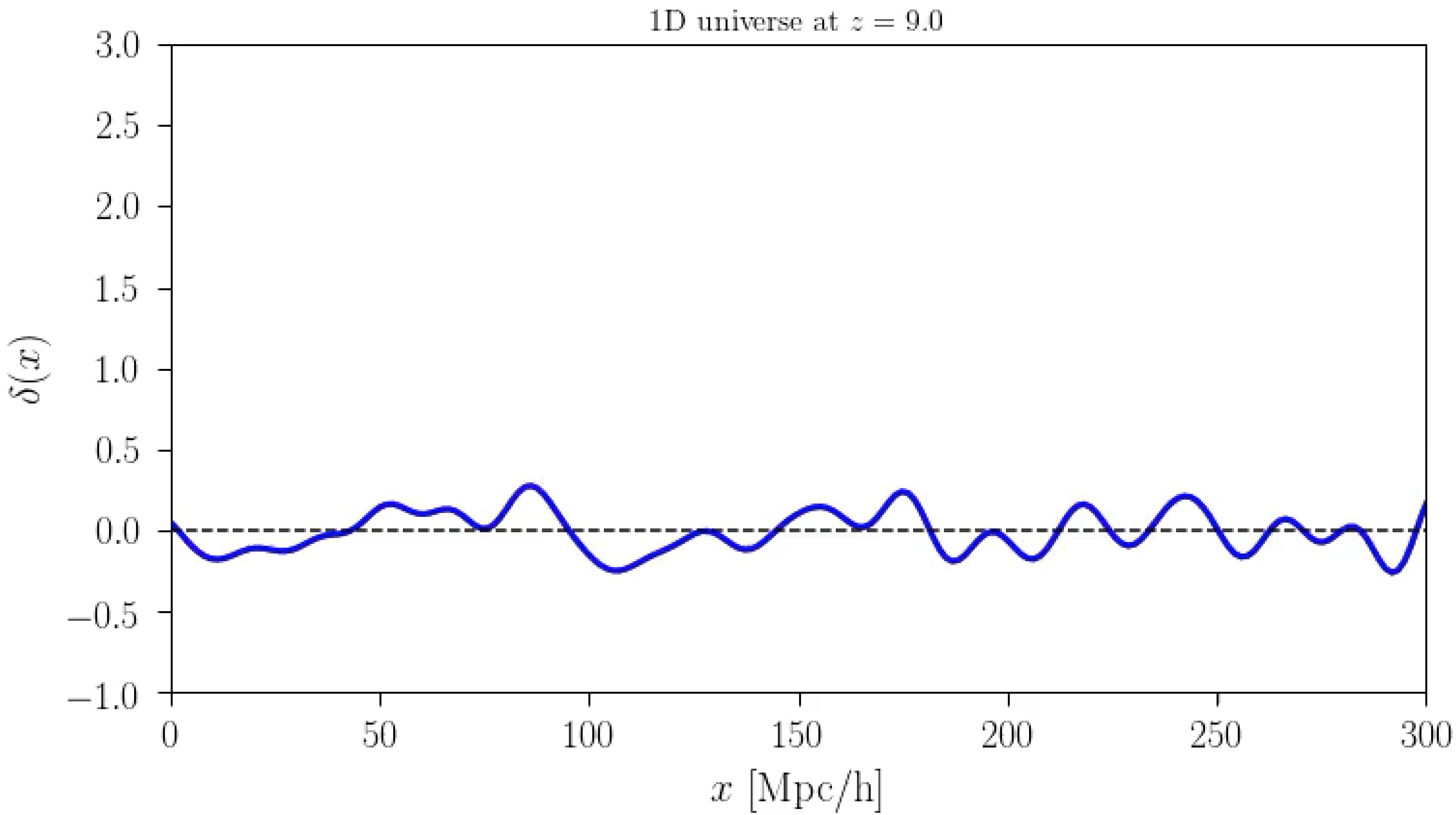

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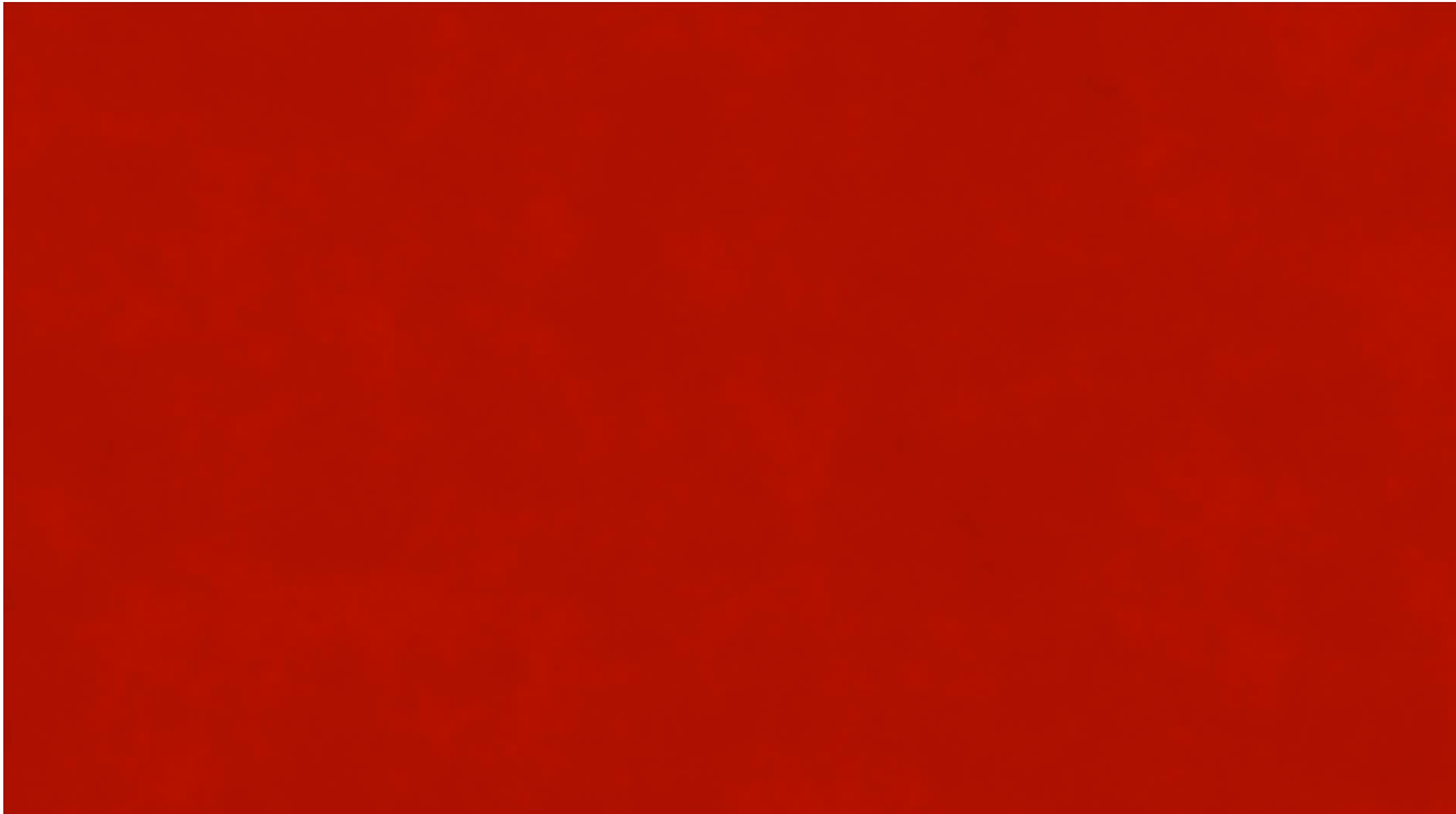
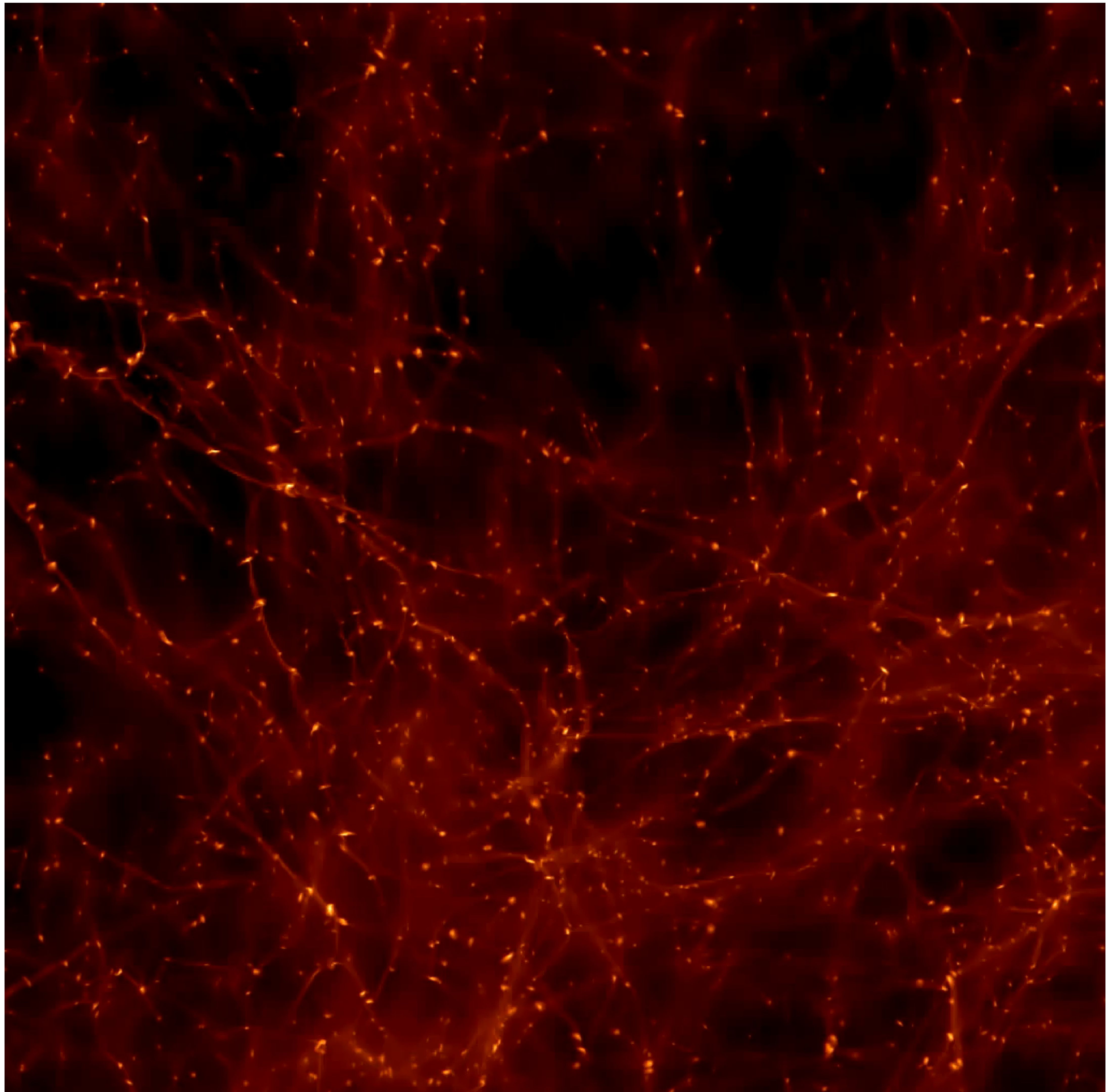
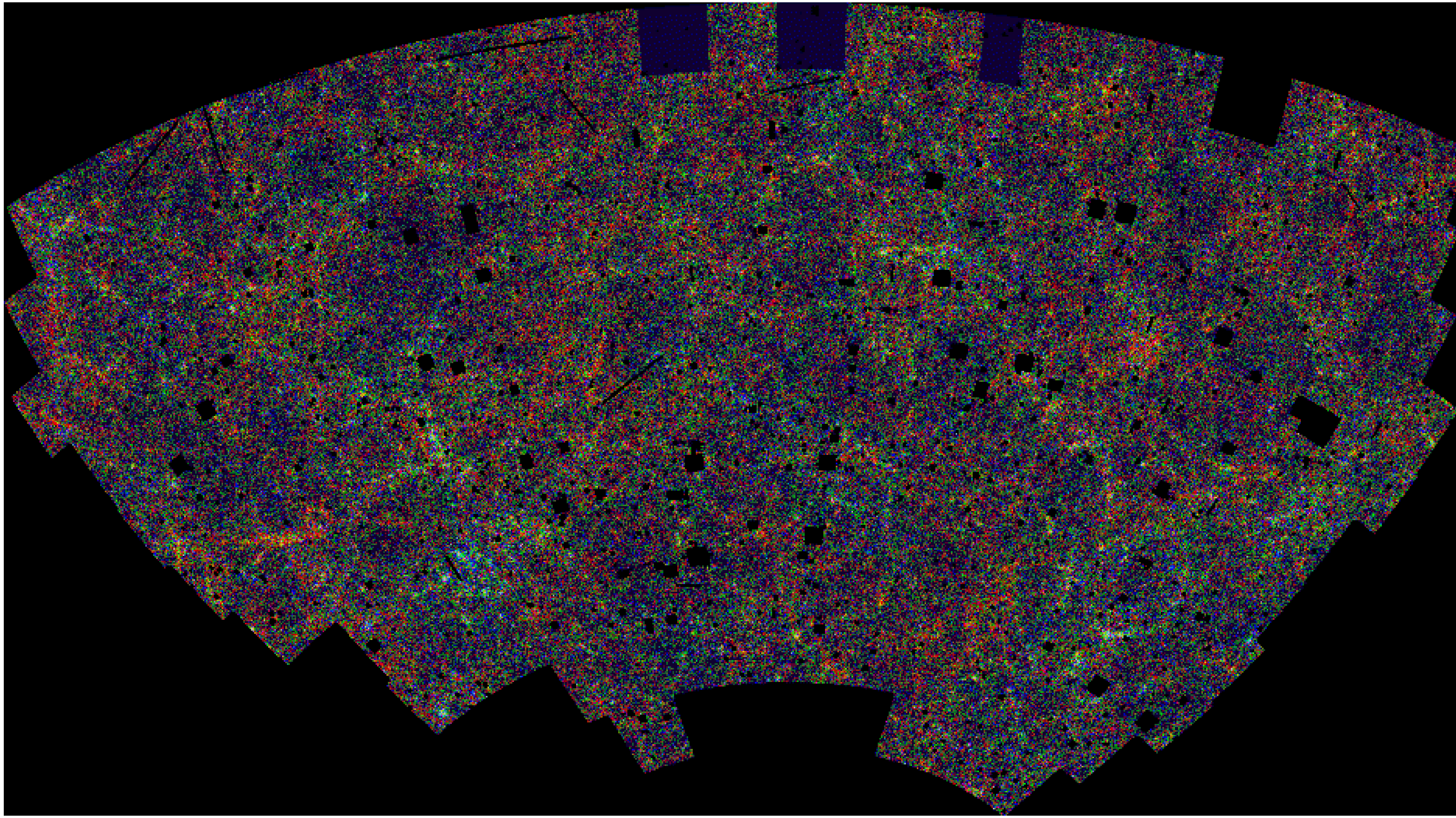


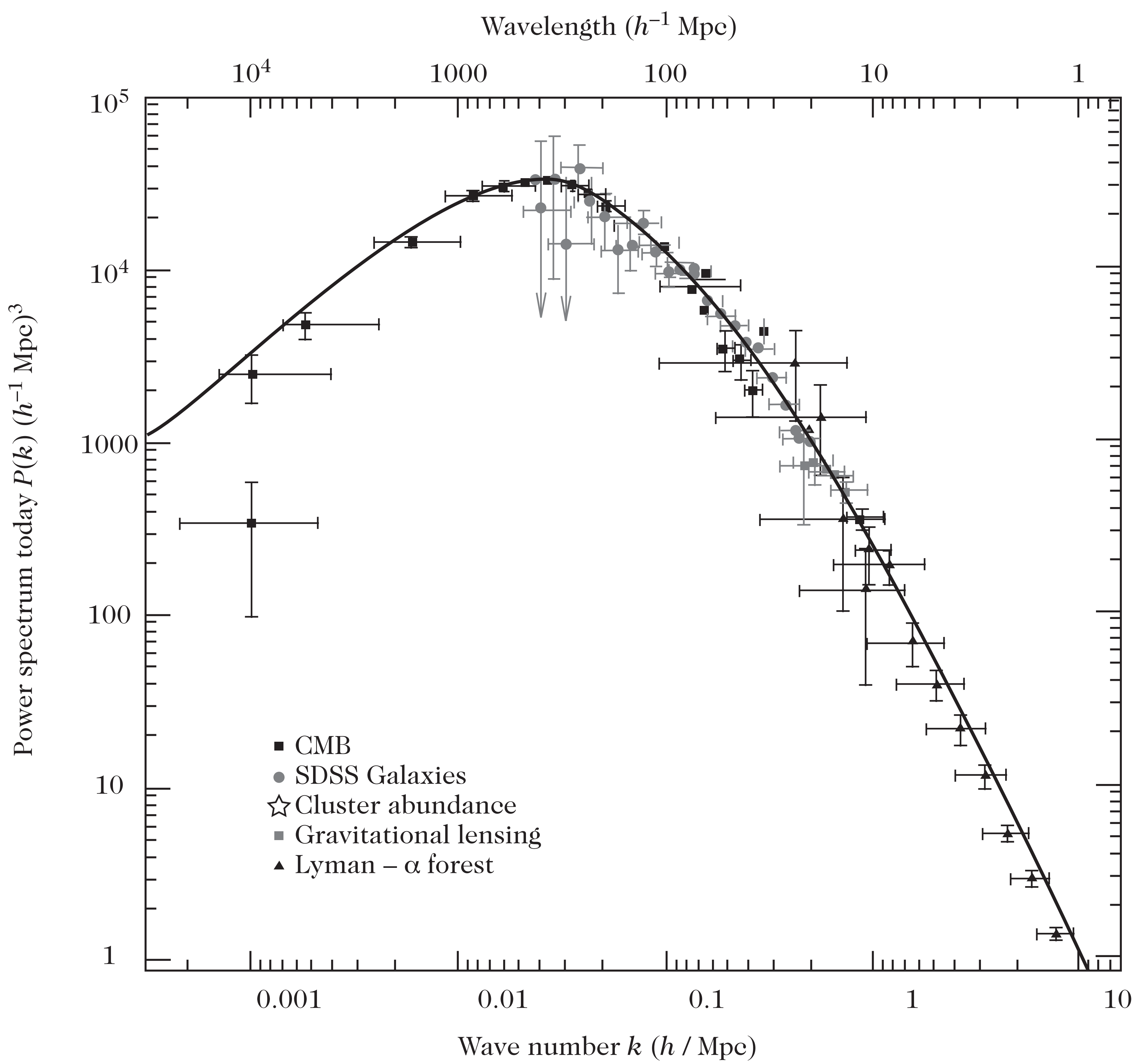
Christophe Pichon (IAP)



Predictions…

… to compare with actual observations!




Wavelength (h^{-1} Mpc)
10^4
1000
100
10
1
10^5
10^4
1000
100
10
1
Power spectrum today $P(k)$ (h^{-1} Mpc)^3
CMB
SDSS Galaxies
Cluster abundance
Gravitational lensing
Lyman – α forest
0.001
0.01
0.1
1
10
Wave number k (h / Mpc)

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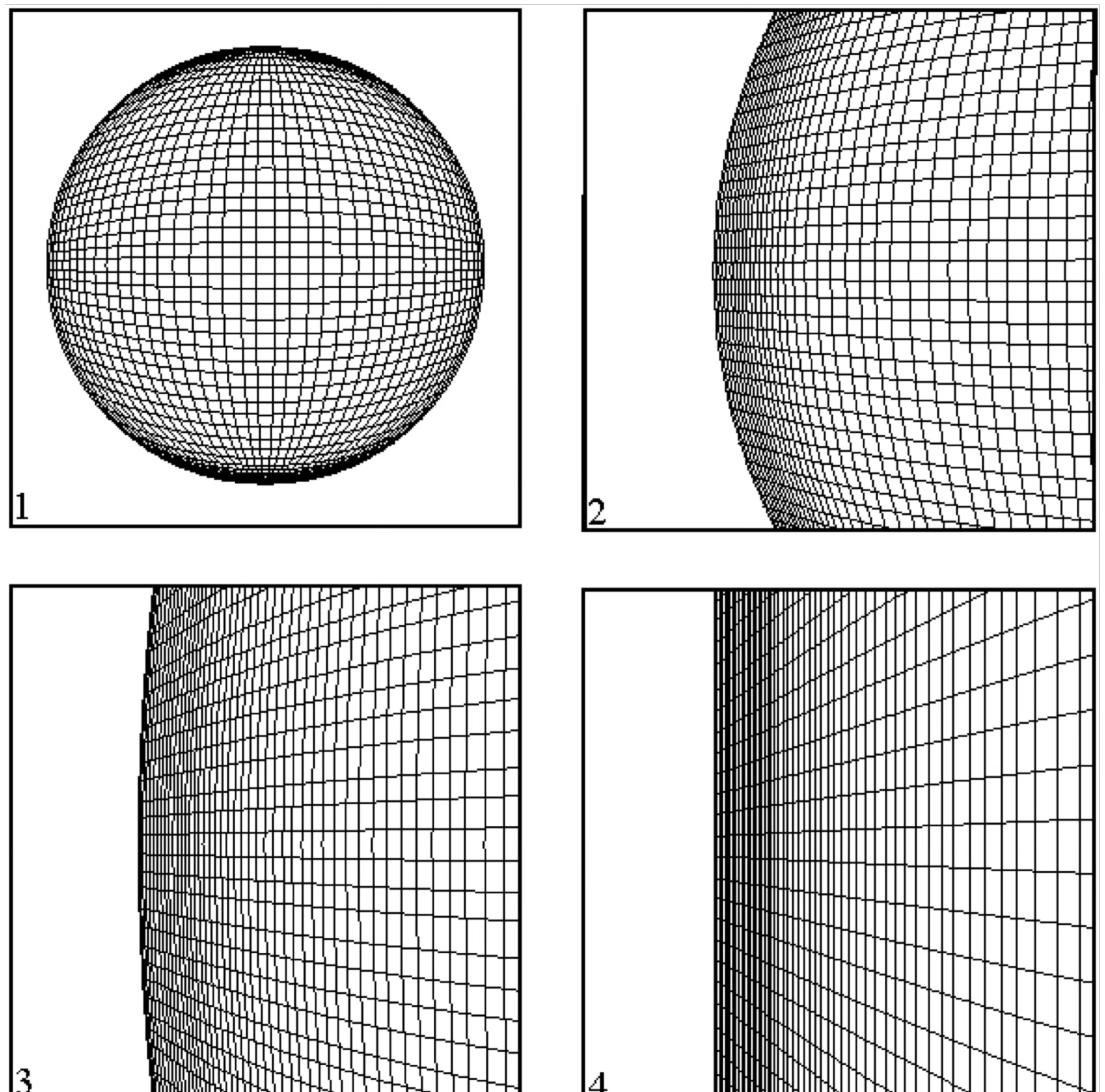
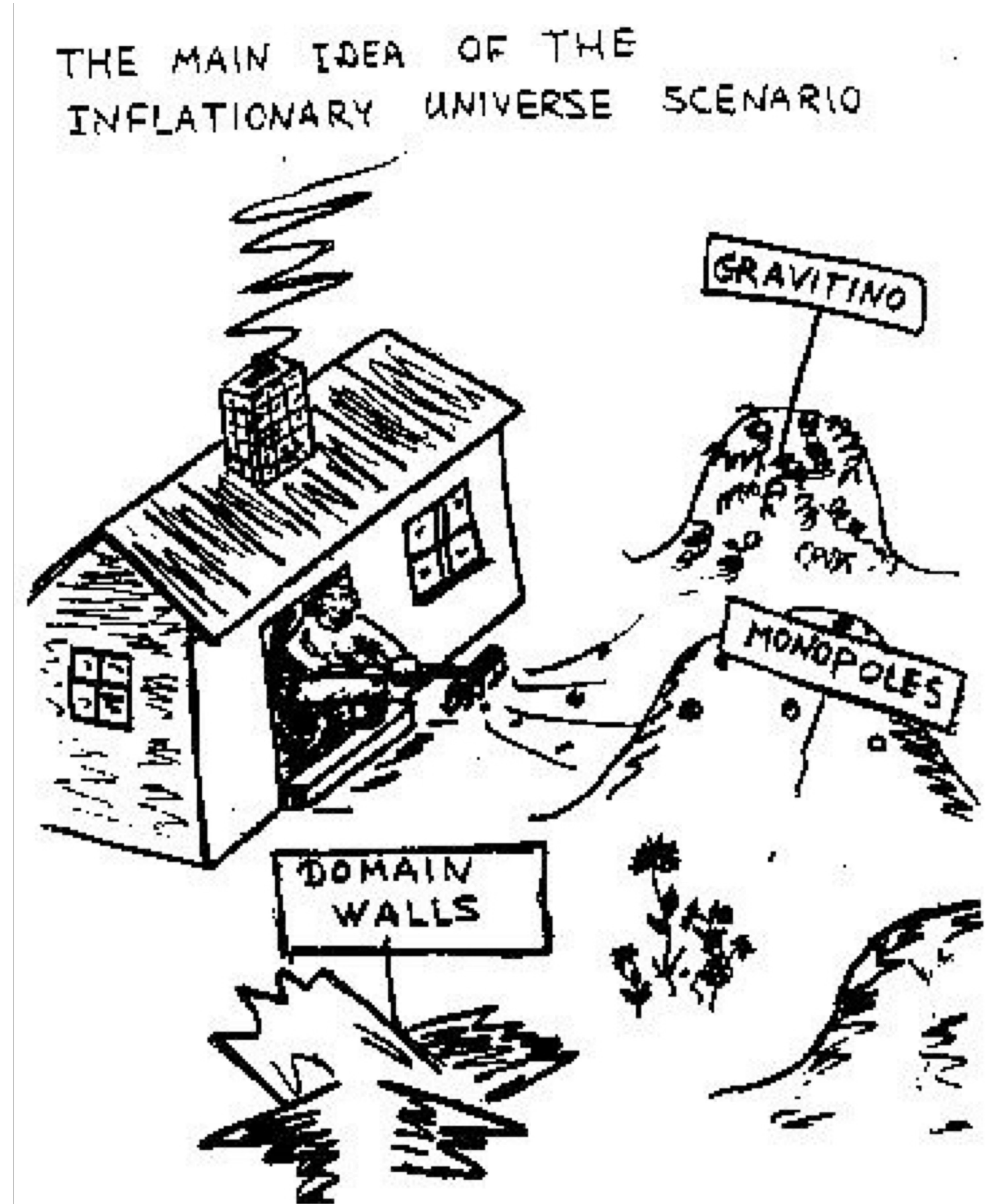
Standard paradigm: inflation!

Phase of accelerated expansion





Implementing inflation

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right]$$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} \Longrightarrow T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right] \longrightarrow \begin{cases} \rho = \frac{1}{2}\dot{\phi}^2 + V \\ P = \frac{1}{2}\dot{\phi}^2 - V \end{cases}$$

inflation $\Longleftrightarrow \dfrac{\mathrm{d}^2a}{\mathrm{d}t^2} > 0$

$$\rho + 3P = 2\left(\dot{\phi}^2 - V\right)$$

Friedmann $\dfrac{\ddot{a}}{a} = \dot{H} + H^2 = -\dfrac{\kappa}{3}\left(\dot{\phi}^2 - V\right)$

Potential energy must dominate over kinetic energy

$\longrightarrow V > \dot{\phi}^2$ … in practice: $V \gg \dot{\phi}^2$

Limit $\dot{\phi}^2 \to 0 \Longrightarrow V \to V_0 \Longrightarrow H^2 = \dfrac{\kappa V_0}{3}$ & $\dfrac{\ddot{a}}{a} = \dfrac{\kappa V_0}{3}$

$$a \propto \exp\left(\sqrt{\frac{\kappa V_0}{3}}t\right)$$

Friedmann $\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{\kappa}{3}\left(\dot{\phi}^2 - V\right)$

Potential energy must dominate over kinetic energy

formally define

$$\epsilon := -\frac{\dot{H}}{H^2} = 3\frac{\frac{1}{2}\dot{\phi}^2}{\frac{1}{2}\dot{\phi}^2 + V} \ll 1$$

Klein-Gordon $\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{dV}}{\mathrm{d}\phi} = 0$

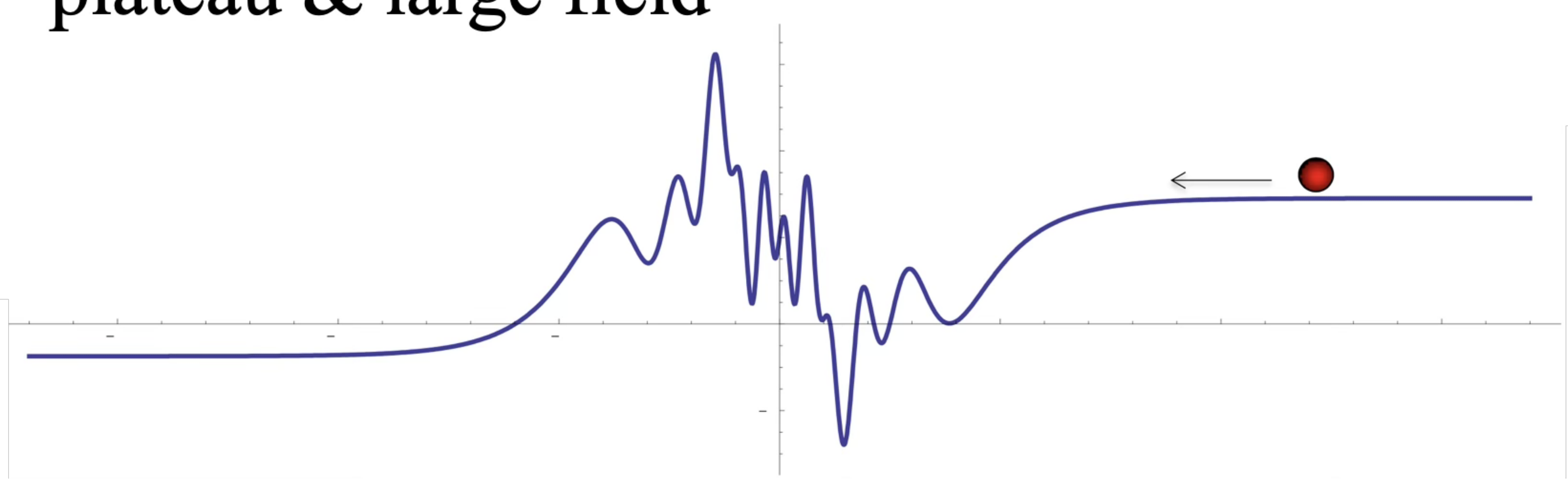
slow roll parameters

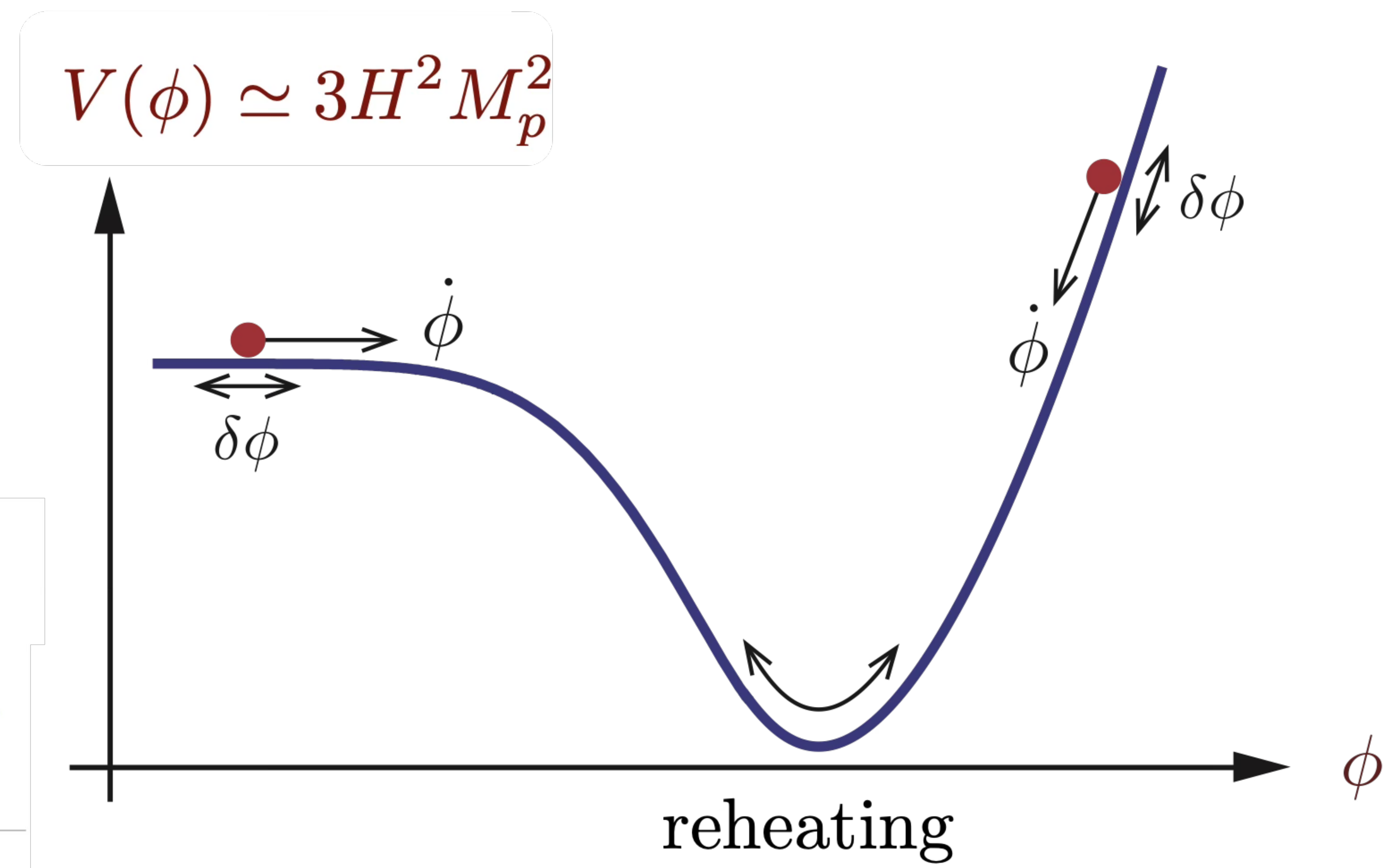
$$\delta := -\frac{\ddot{\phi}}{H\dot{\phi}} = -\frac{\dot{\epsilon}}{2H\epsilon} + \epsilon$$

$$\epsilon \approx \frac{1}{16\pi G}\left(\frac{V_{,\phi}}{V}\right)^2$$

$$\delta \approx -\frac{1}{16\pi G}\left(\frac{V_{,\phi}}{V}\right)^2 + \frac{1}{8\pi G}\frac{V_{,\phi\phi}}{V}$$

plateau & large field





Background equations $H^2 = \frac{\kappa V}{3-\epsilon} \approx \frac{\kappa}{3} V(\phi) + \mathcal{O}(\epsilon)$ & $\frac{\mathrm{d}\phi}{\mathrm{d}t} = -\frac{V_{,\phi}}{(3-\delta)H} \approx -\frac{V_{,\phi}}{3H} + \mathcal{O}(\delta)$

concrete example $V(\phi) = \alpha \left(\frac{\phi}{M_\mathrm{p}}\right)^n$

$\Longrightarrow$ $\epsilon = \frac{n^2 M_\mathrm{P}^2}{16\pi\phi^2}$ slow-roll $\epsilon \ll 1$ demands $\phi > \phi_\mathrm{end} = \frac{n}{4\sqrt{\pi}} M_\mathrm{P}$

solve eom: $\phi(t) \propto (A + Bt)^{2/(4-n)}$

and $a(t) \propto \exp\left\{\frac{4\pi}{nM_\mathrm{P}^2}\left[\phi_\mathrm{ini}^2 - \phi^2(t)\right]\right\}$

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Standard Failures and inflationary solutions

Singularity Not solved... actually not addressed!

Horizon $d_{\mathrm{H}} \equiv a(t) \int_{t_{\mathrm{i}}}^{t} \frac{\mathrm{d}\tau}{a(\tau)}$ can be made as big as one wishes

Flatness $\frac{\mathrm{d}}{\mathrm{d}t} |\Omega - 1| = -2 \frac{\ddot{a}}{\dot{a}^3}$

$\ddot{a} > 0 \quad \& \quad \dot{a} > 0$

accelerated expansion (inflation)

Homogeneity & Isotropy

Initial Universe = very small patch
Accelerated expansion drives the shear to zero... $\Longrightarrow$ vacuum state!

+ attractor

Perturbations Bonus of the theory: superb predictions!!!

Others dark matter/energy, baryogenesis, ...

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Origin of perturbations: quantum fluctuations / time-varying background

Classical temperature fluctuations promoted to quantum operators

$$\frac{\widehat{\Delta T}}{T} \propto \hat{v} \sim \Phi \sim \delta g_{00}$$

second order perturbed Einstein action $^{(2)}\delta S = \frac{1}{2}\int \mathrm{d}^4x \left[(v')^2 - \delta^{ij}\partial_i v \partial_j v + \frac{(a\sqrt{\epsilon_1})''}{a\sqrt{\epsilon_1}} v^2\right]$

variable-mass scalar field in Minkowski spacetime

$\epsilon_1 = 1 - \mathcal{H}'/\mathcal{H}^2$

slow-roll parameter

\+ Fourier transform $v(\eta, \boldsymbol{x}) = \frac{1}{(2\pi)^{3/2}} \int_{\mathbb{R}^3} \mathrm{d}^3\boldsymbol{k}\, v_{\boldsymbol{k}}(\eta) \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}}$

$$\Rightarrow \quad {}^{(2)}\delta S = \int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{k} \left\{ v_{\boldsymbol{k}}' v_{\boldsymbol{k}}^{*\prime} + v_{\boldsymbol{k}} v_{\boldsymbol{k}}^* \left[\frac{(a\sqrt{\epsilon_1})''}{a\sqrt{\epsilon_1}} - k^2\right]\right\}$$

Lagrangian formulation...

Canonical quantisation

$$v(\boldsymbol{x},\eta) = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^{3/2}} \left[v_{\boldsymbol{k}}(\eta)\,\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}} a_{\boldsymbol{k}} + v_{\boldsymbol{k}}^{\star}(\eta)\,\mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}} a_{\boldsymbol{k}}^{\dagger} \right]$$

$$\left[\hat{a}_{\boldsymbol{k}}, \hat{a}_{\boldsymbol{q}}^{\dagger}\right] = \delta^{(3)}(\boldsymbol{k}-\boldsymbol{q})$$

$$[\hat{v}(\boldsymbol{x},\eta), \hat{v}(\boldsymbol{y},\eta)] = 0 = [\hat{\pi}(\boldsymbol{x},\eta), \hat{\pi}(\boldsymbol{y},\eta)] \quad \text{and} \quad [\hat{v}(\boldsymbol{x},\eta), \hat{\pi}(\boldsymbol{y},\eta)] = i\delta^{(3)}(\boldsymbol{x}-\boldsymbol{y})$$

Vacuum state $\hat{a}_{\boldsymbol{k}}|0\rangle = 0$ for all $\boldsymbol{k}$ ➡ $v_{\boldsymbol{k}} \underset{|k\eta|\to\infty}{\longrightarrow} \frac{\mathrm{e}^{-ik\eta}}{\sqrt{2k}}$

Hamiltonian

$$H = \int \mathrm{d}^3\boldsymbol{k} \left\{ p_{\boldsymbol{k}} p_{\boldsymbol{k}}^* + v_{\boldsymbol{k}} v_{\boldsymbol{k}}^* \left[\overbrace{k^2 - \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}}}^{\omega^2(\eta, \boldsymbol{k})} \right] \right\}$$

collection of parametric oscillators with time dependent frequency

factorization of the full wave function

real and imaginary parts

$$\Psi\left[v(\eta, \boldsymbol{x})\right] = \prod_{\boldsymbol{k}} \Psi_{\boldsymbol{k}}\left(v_{\boldsymbol{k}}^{\mathrm{R}}, v_{\boldsymbol{k}}^{\mathrm{I}}\right) = \prod_{\boldsymbol{k}} \Psi_{\boldsymbol{k}}^{\mathrm{R}}\left(v_{\boldsymbol{k}}^{\mathrm{R}}\right) \Psi_{\boldsymbol{k}}^{\mathrm{I}}\left(v_{\boldsymbol{k}}^{\mathrm{I}}\right)$$

$$i \frac{\Psi_{\boldsymbol{k}}^{\mathrm{R,I}}}{\partial \eta} = \hat{\mathcal{H}}_{\boldsymbol{k}}^{\mathrm{R,I}} \Psi_{\boldsymbol{k}}^{\mathrm{R,I}}$$

$$\hat{\mathcal{H}}_{\boldsymbol{k}}^{\mathrm{R,I}} = -\frac{1}{2} \frac{\partial^2}{\partial \left(v_{\boldsymbol{k}}^{\mathrm{R,I}}\right)^2} + \frac{1}{2} \omega^2(\eta, \boldsymbol{k}) \left(\hat{v}_{\boldsymbol{k}}^{\mathrm{R,I}}\right)^2$$

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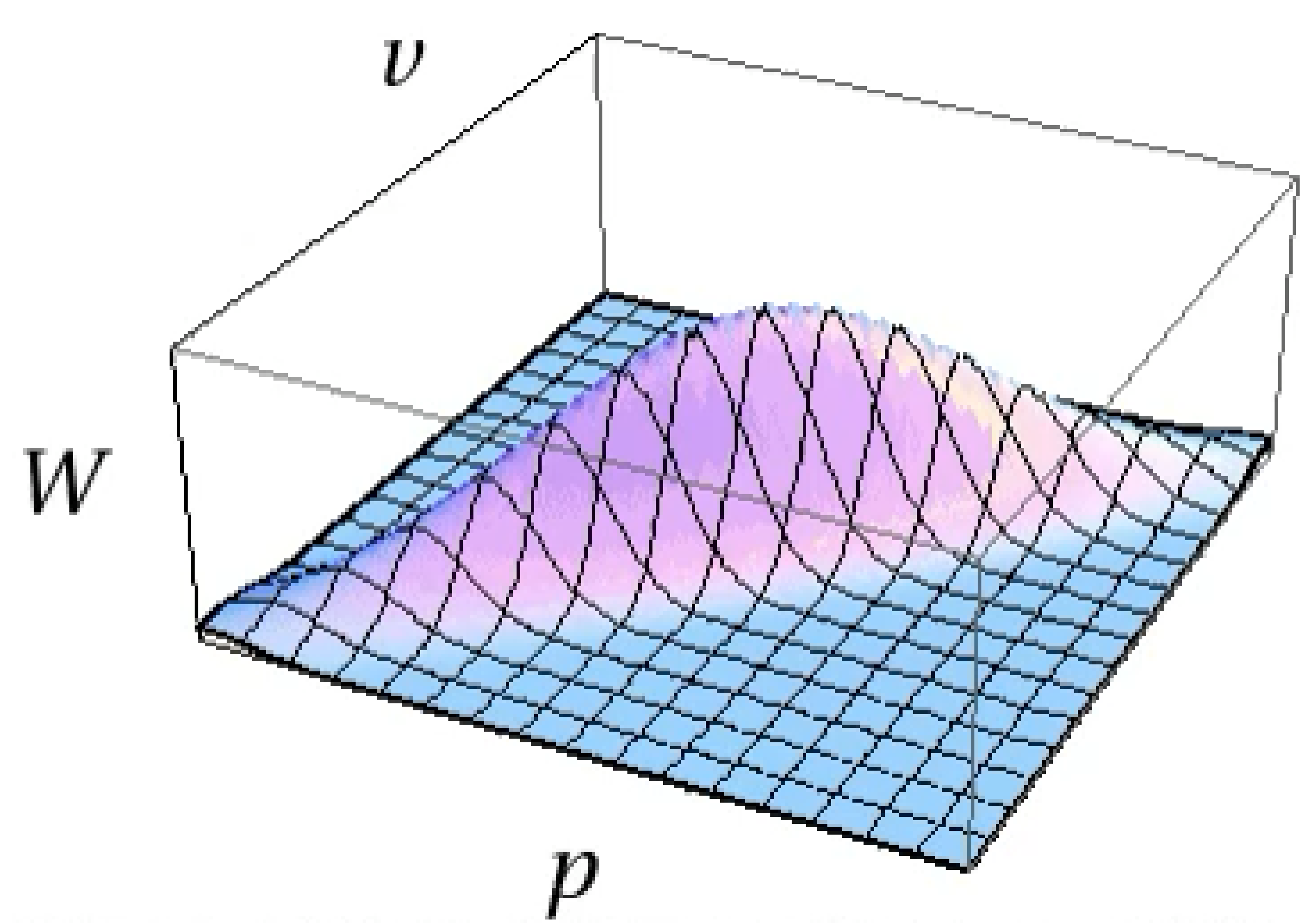
Gaussian state solution $\Psi\left(\eta, v_{\boldsymbol{k}}\right)=\left[\frac{2 \Re \mathrm{e}\, \Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1 / 4} \mathrm{e}^{-\Omega_{\boldsymbol{k}}(\eta) v_{\boldsymbol{k}}^2}$

Wigner function $W\left(v_{\boldsymbol{k}}, p_{\boldsymbol{k}}\right)=\int \frac{\mathrm{d} x}{2 \pi^2} \Psi^*\left(v_{\boldsymbol{k}}-\frac{x}{2}\right) \mathrm{e}^{-i p_{\boldsymbol{k}} x} \Psi\left(v_{\boldsymbol{k}}+\frac{x}{2}\right)$

large squeezing limit ⟹ $W \propto \delta\left(p_{\boldsymbol{k}}+k \tan \phi_{\boldsymbol{k}} v_{\boldsymbol{k}}\right)$



Stochastic distribution of classical processes

realization, spatial direction

Ergodicity $\left\langle\frac{\Delta T(\xi, \boldsymbol{e})}{T}\right\rangle_{\xi} \simeq\left\langle\frac{\Delta T(\xi, \boldsymbol{e})}{T}\right\rangle_{\boldsymbol{e}}$

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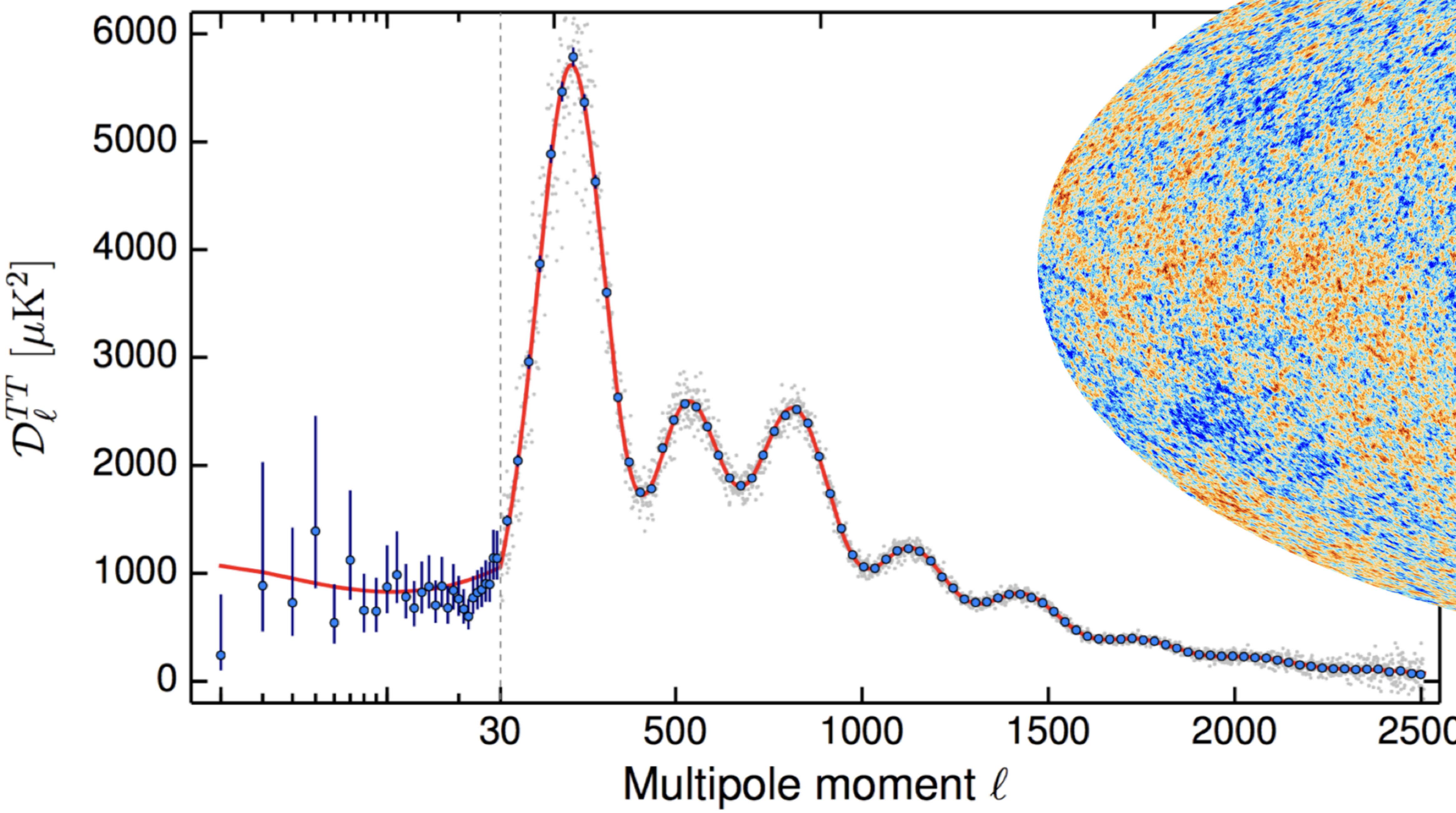
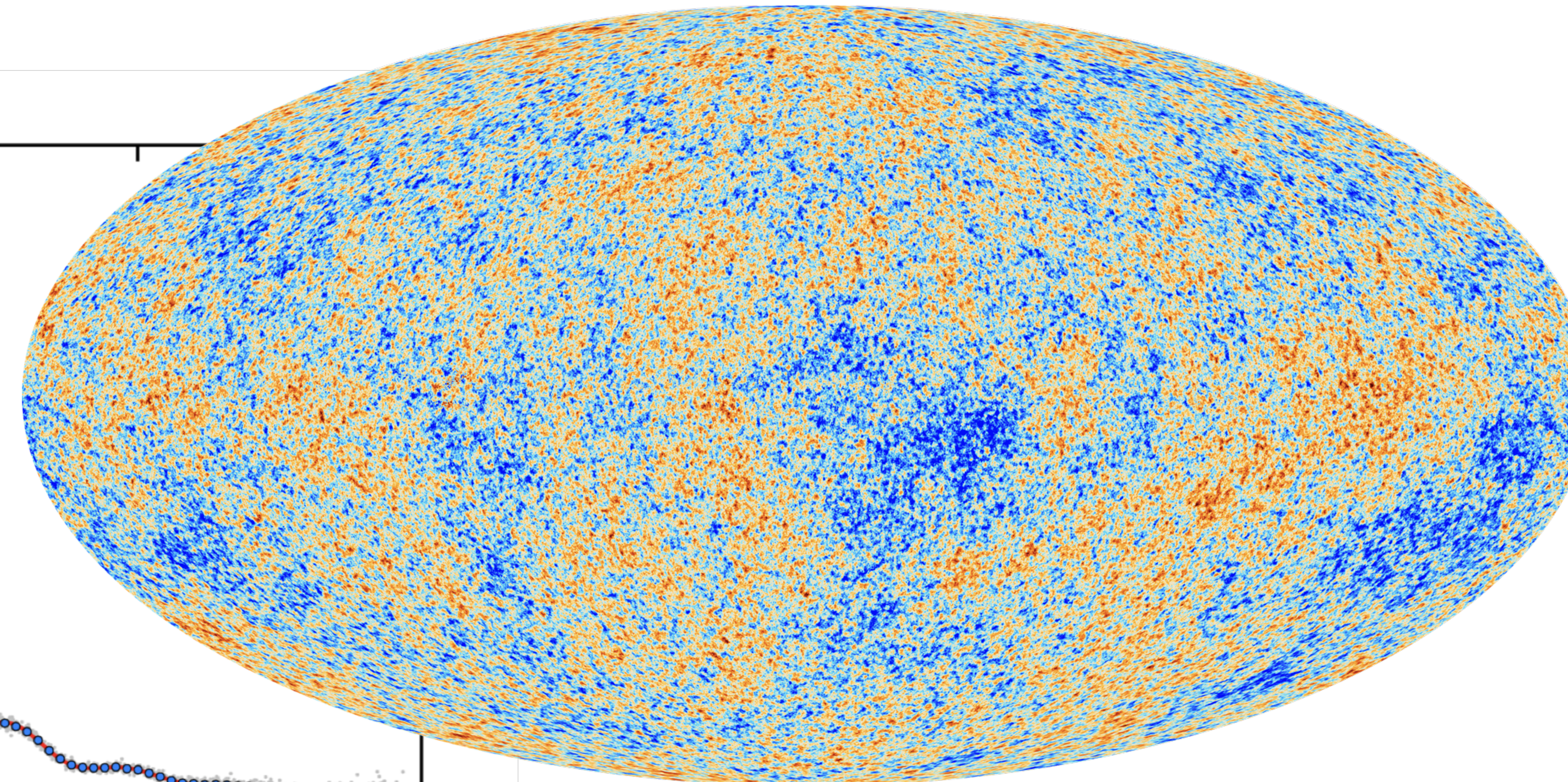
Planck 2018





$\Omega_{\mathcal{K}} = 0.000 \pm 0.005$

$n_s = 0.9639 \pm 0.0047$ almost scale invariant

excluded

$f_{NL}^{loc} = 0.8 \pm 5$

$f_{NL}^{eq} = -4 \pm 43$

$f_{NL}^{ort} = -26 \pm 21$

Gaussian signal

Isocurvature $\lesssim 1\,\%$

quantum vacuum fluctuations of a single scalar d.o.f

compatible with INFLATION

$r < 0.08$

<u>THANK YOU FOR YOUR ATTENTION</u>