EXCALIBUR : Relativistic raytracing for weak & strong lensing analysis

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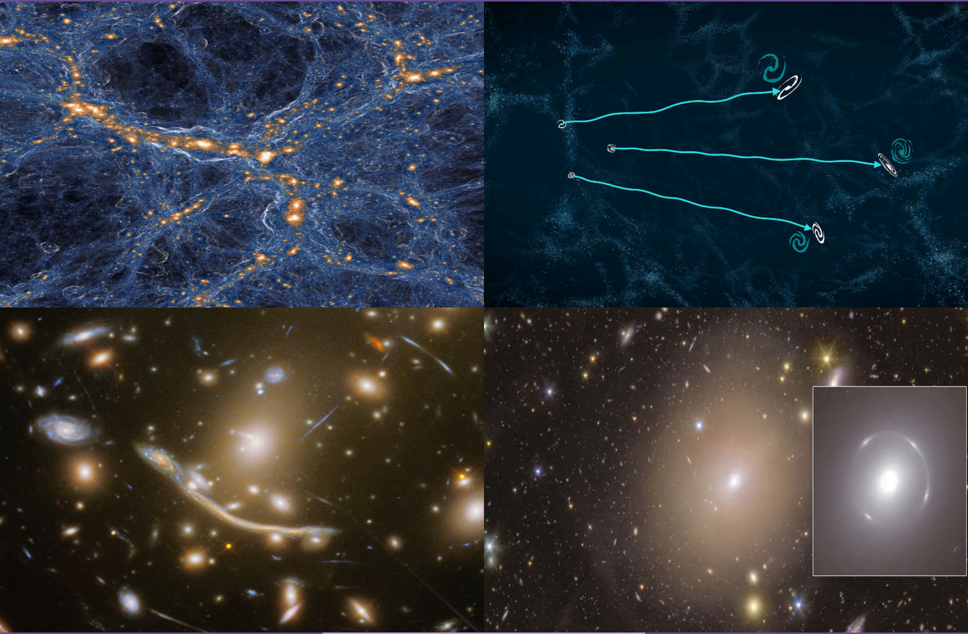
Galaxy clusters as a key cosmological probe



The upcoming surveys

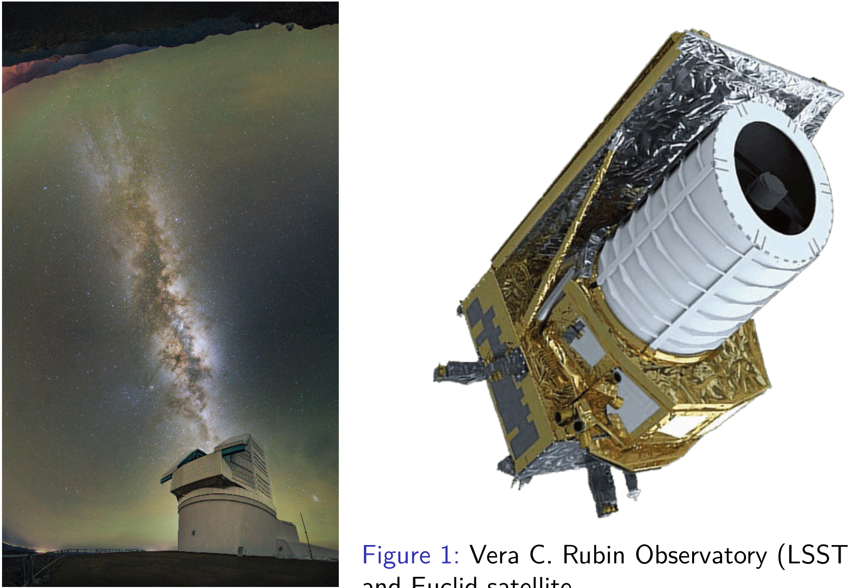
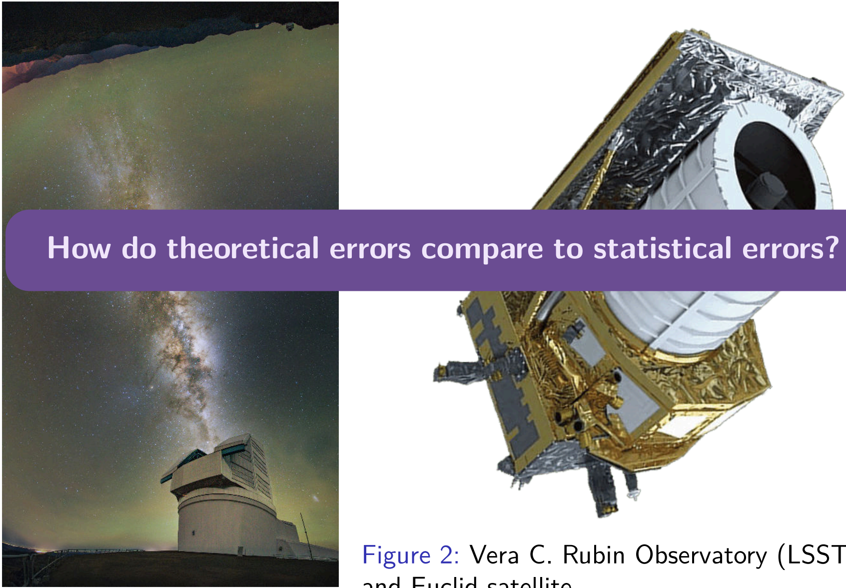


Figure 1: Vera C. Rubin Observatory (LSST) and Euclid satellite.

An important question



How do theoretical errors compare to statistical errors?

Figure 2: Vera C. Rubin Observatory (LSST) and Euclid satellite.

Weak lensing: a brief overview

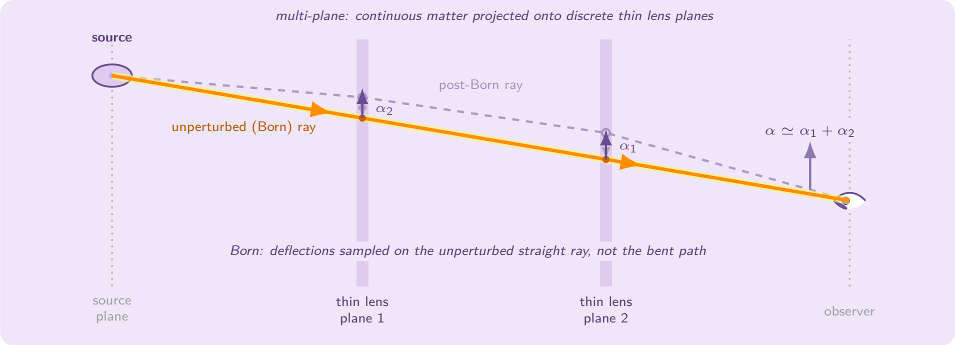


Figure 3: Current approximations in weak lensing frameworks: 1/ Born approximation 2/ thin lens planes. The dashed ray represents the post Born approach, which starts to be more commonly used in simulations, has yet to be implemented in analysis pipelines.

Lens equation and magnification matrix

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta}) \qquad \frac{\partial \beta_i}{\partial \theta_j} \approx \delta_{ij} - \frac{\partial \alpha_i}{\partial \theta_j} = A_{ij}$$

Weak lensing: convergence, shear, and flexion

First order approximation of the magnification matrix

$$A = \begin{pmatrix} 1-\kappa & 0 \\ 0 & 1-\kappa \end{pmatrix} - \gamma \begin{pmatrix} \cos(2\phi) & \sin(2\phi) \\ \sin(2\phi) & -\cos(2\phi) \end{pmatrix}$$

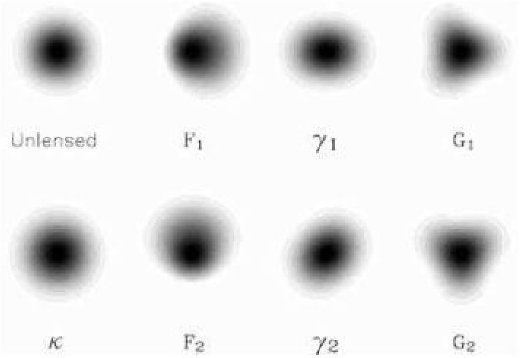


Figure 4: Effects of the different lensing fields on a Gaussian galaxy of radius 1 arcsec. 10% convergence/shear and 0.28 arcsec^{-1} flexion (which is a very high value for this quantity, chosen only to visualize) are applied. From Bacon et al. 2006

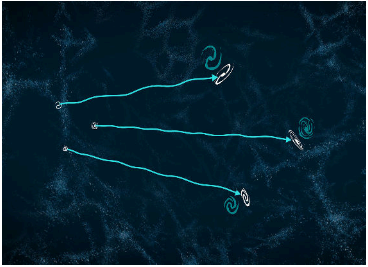
A relativistic description of light propagation

Goal

Describe light propagation directly in space-time, beyond the standard weak-lensing approximations.

Three ingredients

1. a background metric describing the geometry;
2. the geodesic equation for photon trajectories;
3. the Sachs formalism for beam deformations.



Why this drops the approximations

- Born → geodesic integrated along the true light path.
- Thin lens → continuous fields sampled in 4D.

1/ A background metric describing the geometry

The Friedmann-Lemaitre-Robertson-Walker metric

$$g_{\mu\nu} = \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & \dfrac{a^2(t)}{1-kr^2} & 0 & 0 \\ 0 & 0 & a^2(t)r^2 & 0 \\ 0 & 0 & 0 & a^2(t)r^2\sin^2\theta \end{pmatrix}$$

where:

- $a(t)$: scale factor
- k: curvature parameter $(k = 0, +1, -1)$

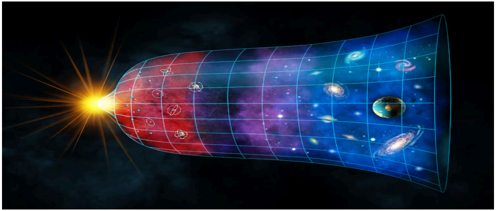


Figure 5: The story of the Universe: from the Big Bang to the present day.
Credit: Natalie Mayer

The perturbed Friedmann-Lemaitre-Robertson-Walker metric

$$g_{\mu\nu} = \begin{pmatrix} -c^2(1+2\Phi) & 0 & 0 & 0 \\ 0 & a^2(t)(1-2\Psi) & 0 & 0 \\ 0 & 0 & a^2(t)(1-2\Psi) & 0 \\ 0 & 0 & 0 & a^2(t)(1-2\Psi) \end{pmatrix}$$

in

- cartesian coordinates;
- flat space ($k = 0$).

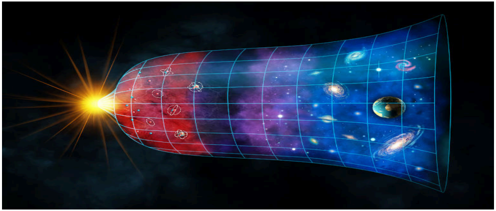
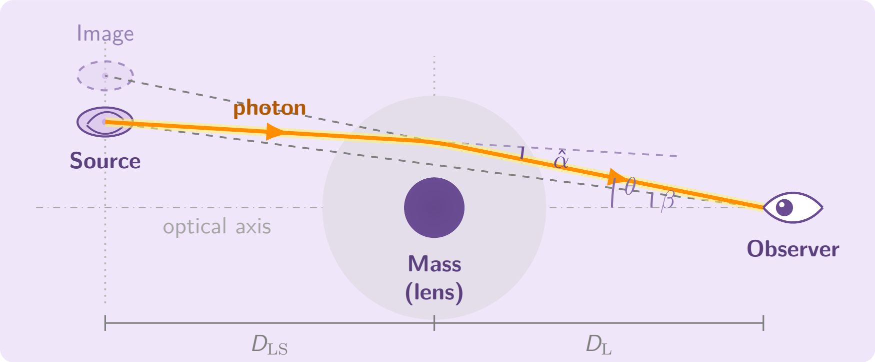


Figure 6: The story of the Universe: from the Big Bang to the present day.
Credit: Natalie Mayer

2/ The geodesic equation for photon trajectories

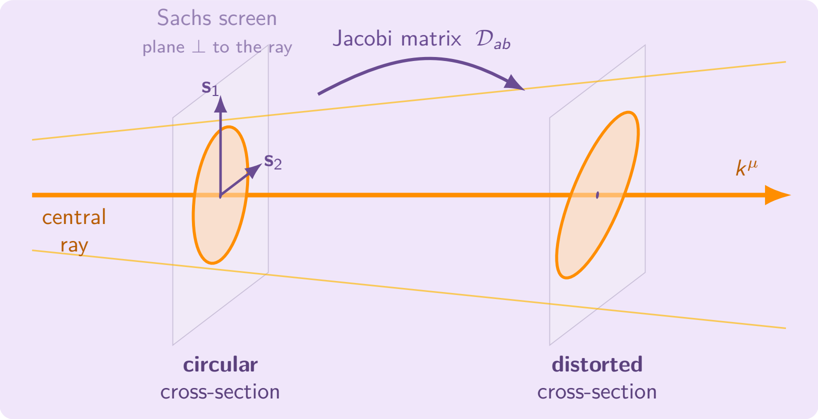


Geodesic equation

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\lambda^2} + \Gamma^\mu_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\lambda}\frac{\mathrm{d}x^\beta}{\mathrm{d}\lambda} = 0 \qquad g_{\mu\nu}k^\mu k^\nu = 0$$

The metric fixes the Christoffel symbols, which determine how photon trajectories curve through inhomogeneous matter distributions.

3/ The Sachs formalism for beam deformations



Optical tidal matrix

$$\mathcal{R}_{AB} = R_{\mu\nu\rho\sigma} k^\mu s_A^\nu s_B^\rho k^\sigma$$

$$\mathcal{R}_{AB} = -\frac{1}{2} R_{\mu\nu} k^\mu k^\nu \delta_{AB} + C_{\mu\nu\rho\sigma} s_A^\mu k^\nu k^\rho s_B^\sigma$$

Focusing equation and A_{ij}

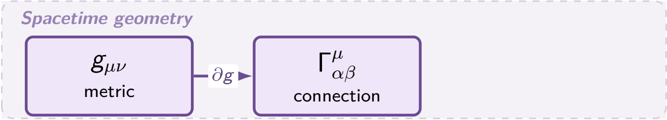
$$\frac{\mathrm{d}^2 \mathcal{D}_{AB}}{\mathrm{d}v^2} = \mathcal{R}_{AC}\, \mathcal{D}_{CB}$$

$$\mathcal{D} = A\bar{\mathcal{D}} + \dots$$

The tensor chain: from the metric to the Jacobi matrix

The **metric alone** fixes the whole chain: each tensor follows from derivatives of the previous one.

The tensor chain: from the metric to the Jacobi matrix



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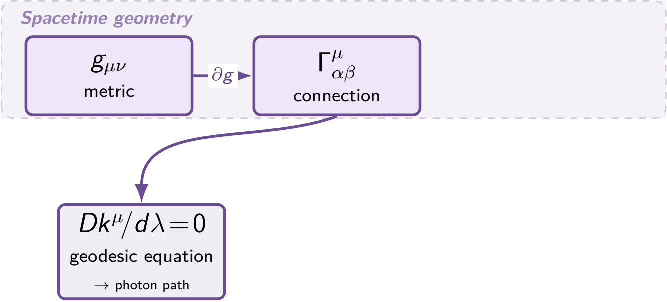

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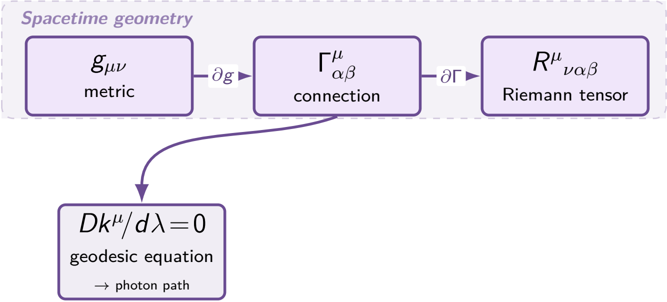

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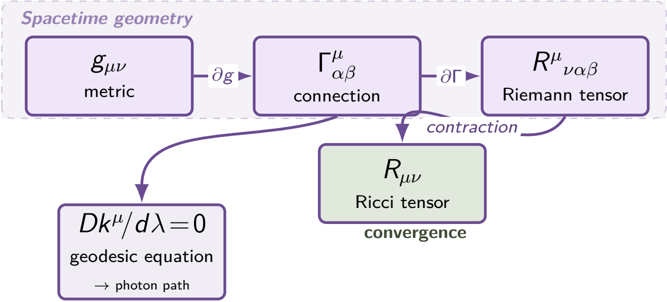

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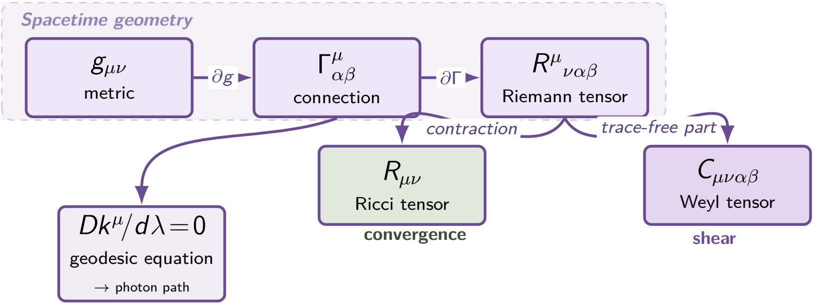

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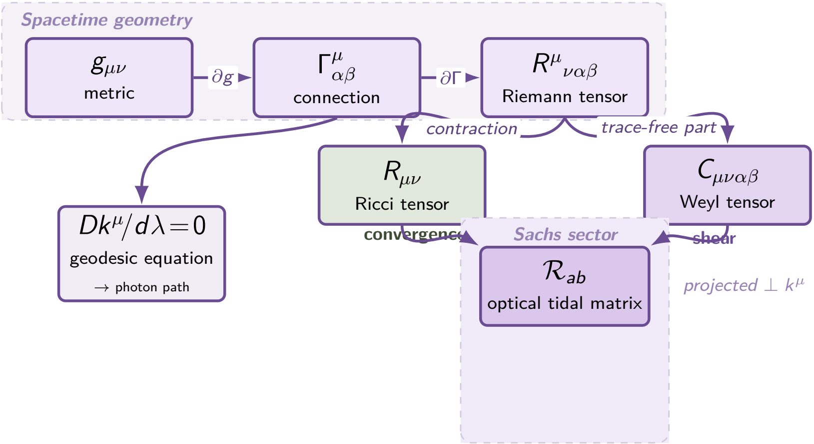

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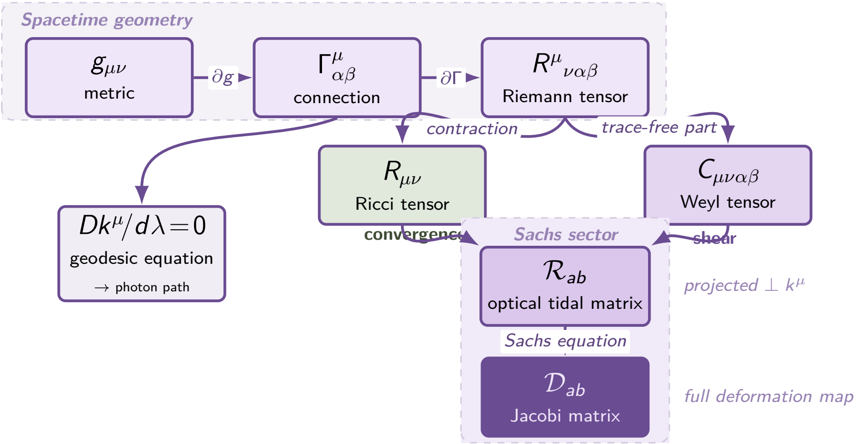

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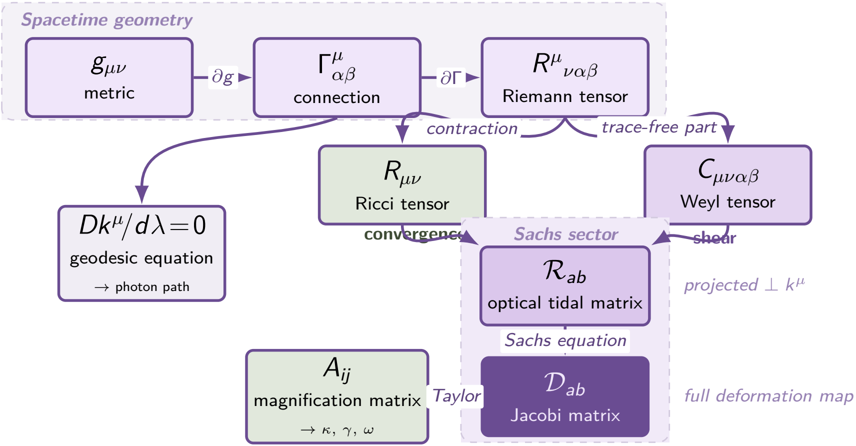

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Introducing EXCALIBUR

E X C A L I B U R

EX	**EX**act
CA	**CA**lculation (of)
LI	**LI**ght
B	**B**eams
U	**U**sing
R	**R**elativity

EXCALIBUR aims to be a modular and performant tool to compute light propagation in curved space-times.

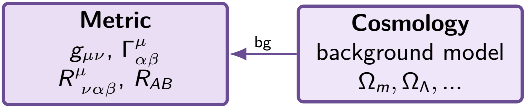
EXCALIBUR: numerical pipeline

Metric

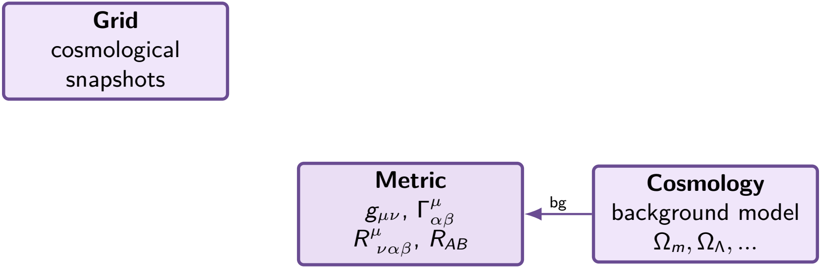
$g_{\mu\nu}$, $\Gamma^{\mu}_{\alpha\beta}$

$R^{\mu}{}_{\nu\alpha\beta}$, R_{AB}

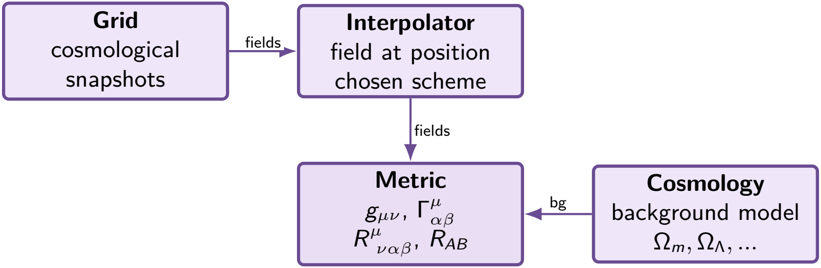
EXCALIBUR: numerical pipeline



EXCALIBUR: numerical pipeline



EXCALIBUR: numerical pipeline



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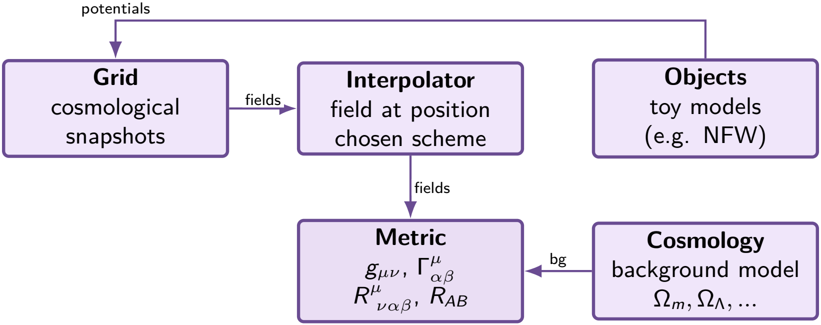

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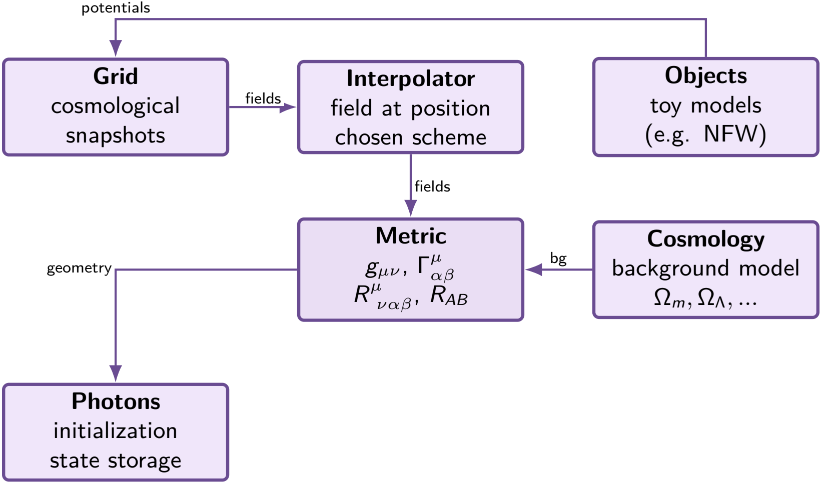

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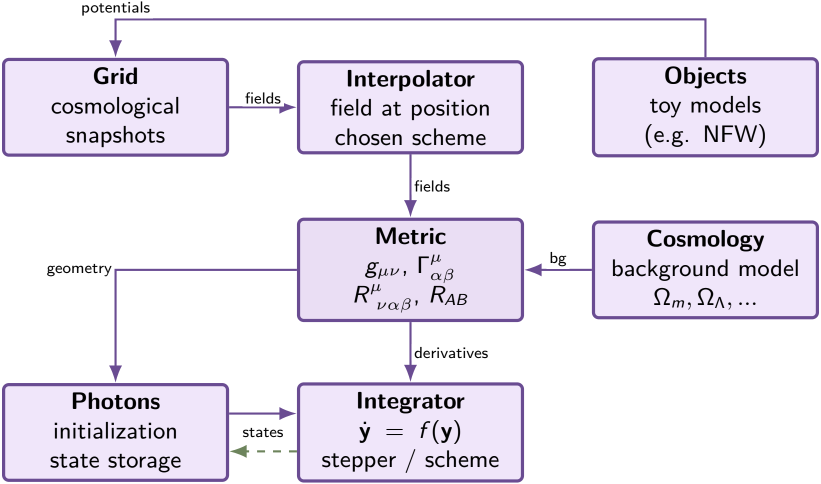

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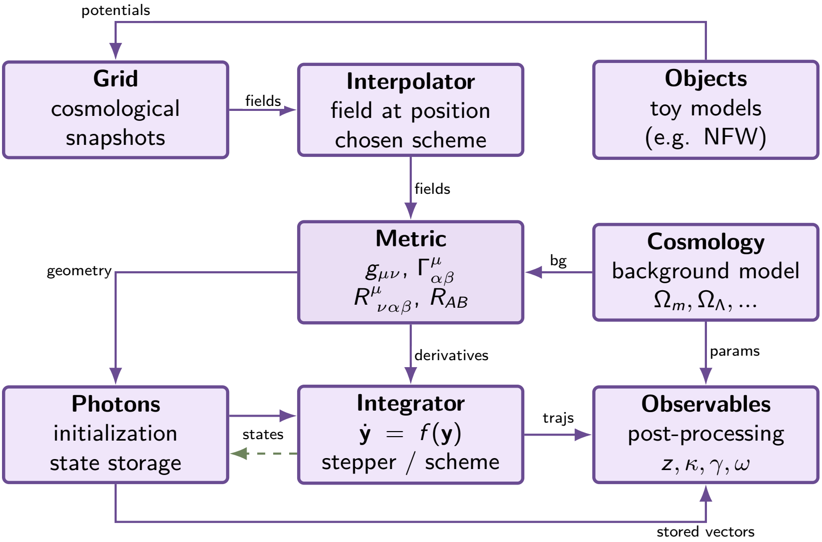
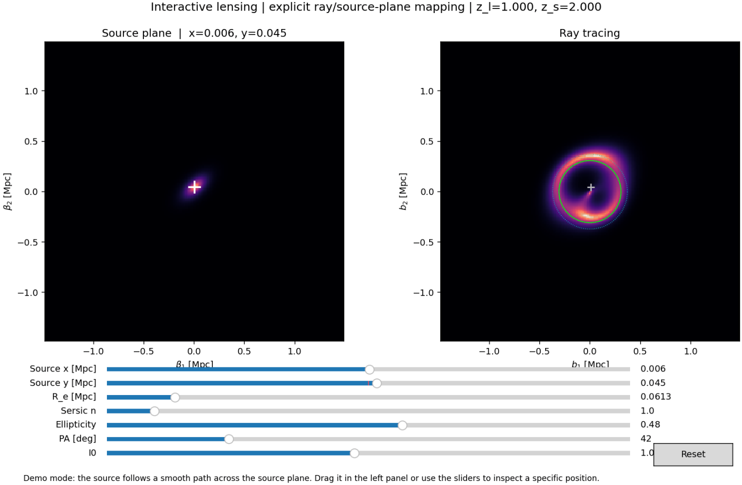


What EXCALIBUR enables



Code validation

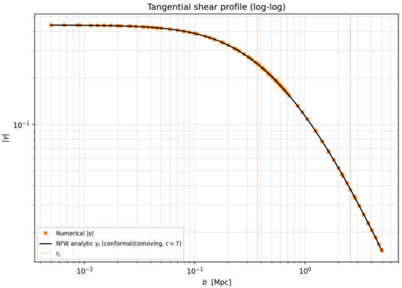


Figure 7: Shear profile of an NFW halo: numerical vs analytical.

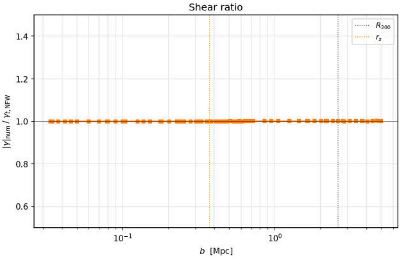
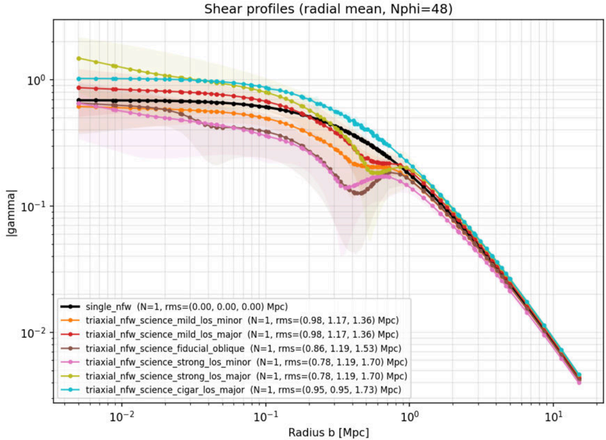


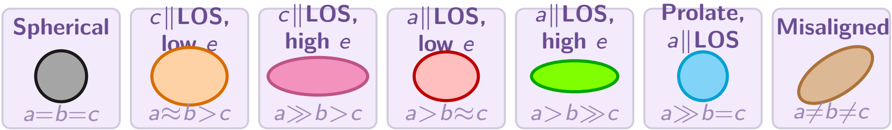
Figure 8: Ratio of numerical to analytical shear profile for an NFW halo.

Validation of lensing quantities

Compare simulated κ and γ profiles of an NFW halo to the analytical NFW predictions.

Current work: model errors on mass and shape





Sky-plane projections ($\perp$ LOS)

Conclusion and outlook

1 Relativistic tensor chain

Beyond Born, thin-lens & single-plane

$g_{\mu\nu}$ → $\Gamma^{\mu}{}_{\alpha\beta}$ → $R^{\mu}{}_{\nu\alpha\beta}$ → (*Sachs eq.*) → $\mathcal{D}_{ab}$

2 EXCALIBUR pipeline

Modular, validated against analytical NFW

Metric → Integrator → $\kappa,\ \gamma,\ \omega$

3 Halo shape systematics

7 NFW geometries, triaxiality effects

Perspectives

Lens geometry effects
NFW potentials in FLRW background

N-body raytracing
Full coupling with cosmological simulations

LSST-ready systematics
Long term: quantify biases for stage-IV surveys

Take home message

excalibur: a modular relativistic raytracing code to quantify lensing effects and systematics. Lens geometry can bias shear profiles by $\approx 20\%$