How does the graviton appear in String Theory ?

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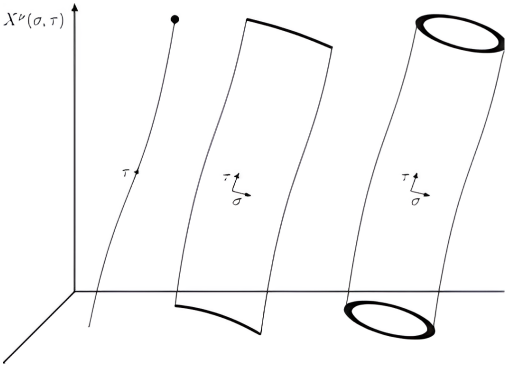
Point particle and string parametrization



- Minkowskian metric $\eta^{\mu\nu} = diag(-1, +1, +1, ..., +1)$
- Coordinates of the point particle : $x^\mu(\tau)$
- Coordinates of the string : $X^\mu(\tau, \sigma)$

Equation of motion

- We define $\dot{X}^{\mu} = \frac{\partial X^{\mu}}{\partial \tau}$ and $X^{\mu\,\prime} = \frac{\partial X^{\mu}}{\partial \sigma}$

Nambu-Goto action S

$$S = -\frac{1}{2\pi\alpha'}\int d\tau \int_0^{\pi} d\sigma \sqrt{(\dot{X}\cdot X')^2 - (\dot{X})^2(X')^2}$$

- The equation of motion is a wave equation with c = 1 :

Equation of motion

$$\ddot{X}^{\mu} - X^{\mu\prime\prime} = 0$$

Solving the wave equation

- Wave equation implies

$$X^\mu(\tau,\sigma) = \frac{1}{2}(f(\tau+\sigma) + g(\tau-\sigma))$$

- Neumann boundary conditions at $\sigma = 0 : X^\mu(\tau,\sigma) = \frac{1}{2}(f(\tau+\sigma) + f(\tau-\sigma))$
- Neumann boundary conditions at $\sigma = \pi : f'(\tau+2\pi) = f'(\tau)$

General solution for X^μ

$$X^\mu(\tau,\sigma) = {x_0}^\mu + \sqrt{2\alpha'}\alpha_0^\mu\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\alpha_n^\mu e^{in\tau}\frac{cos(n\sigma)}{n}$$

Creation and annihilation operators

- By construction, $\alpha_0^\mu = \sqrt{2\alpha'} p^\mu$
- Generalization of $[x^\mu, p^\nu] = i\hbar\eta^{\mu\nu}$, etc. :

Commutation relations of the α's

$$[\alpha_m^\mu, \alpha_n^\nu] = m\delta_{m+n,0}\eta^{\mu\nu} \quad , \quad [\alpha_m^\mu, \alpha_n^{\nu\dagger}] = \delta_{m,n}\eta^{\mu\nu}$$

- For the closed string :

$$X^\mu(\tau,\sigma) = x_0^\mu + \sqrt{2\alpha'}\alpha_0^\mu\tau + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}(\alpha_n^\mu e^{in\sigma} + \bar{\alpha}_n^\mu e^{-in\sigma})$$

Graviton

- Perturbation of the metric : $g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$ with $h^{\mu\nu}$ a symmetric and traceless tensor, so the graviton states can be written as :

$$R_{IJ} a_p^{IJ\dagger} |\Omega\rangle \quad , R: \text{ a symmetric traceless tensor}$$

- Moreover, $p^2 h^{IJ} = 0 \implies$ mass of the graviton $= 0$
- Since $\alpha_n^\dagger$'s are creation operators for n>0, the first general oscillating closed string state you can write is :

$$|\psi\rangle = \xi_{IJ} \alpha_1^{I\dagger} \bar{\alpha}_1{}^{J\dagger} |p\rangle$$

- For every matrix you can write

$$\xi_{IJ} = S_{IJ} + A_{IJ} = \hat{S}_{IJ} + A_{IJ} + S' \delta_{IJ}$$

Conclusion

- Mass of the string state :

$$\frac{1}{2}\alpha' M^2 = N + \bar{N} - 2 \implies M \hat{S}_{IJ}\, \alpha_1^{I\dagger}\, \bar{\alpha}_1{}^{J\dagger}\, |p\rangle = 0$$

- Finally we can identify the two states

$$\hat{S}_{IJ}\, \alpha_1^{I\dagger}\, \bar{\alpha}_1{}^{J\dagger}\, |p\rangle \longleftrightarrow R_{IJ}\, a_p^{IJ\dagger}\, |\Omega\rangle$$

We have a graviton in String Theory !

Thank you for your attention !