

Sorbonne Université – Bachelor Project

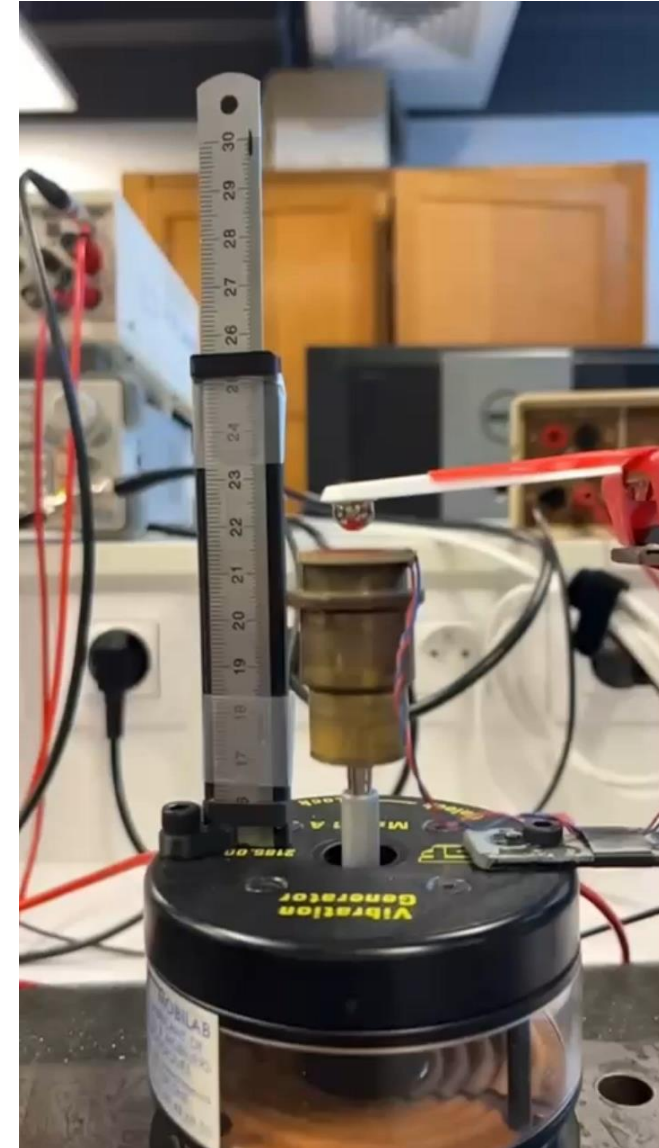
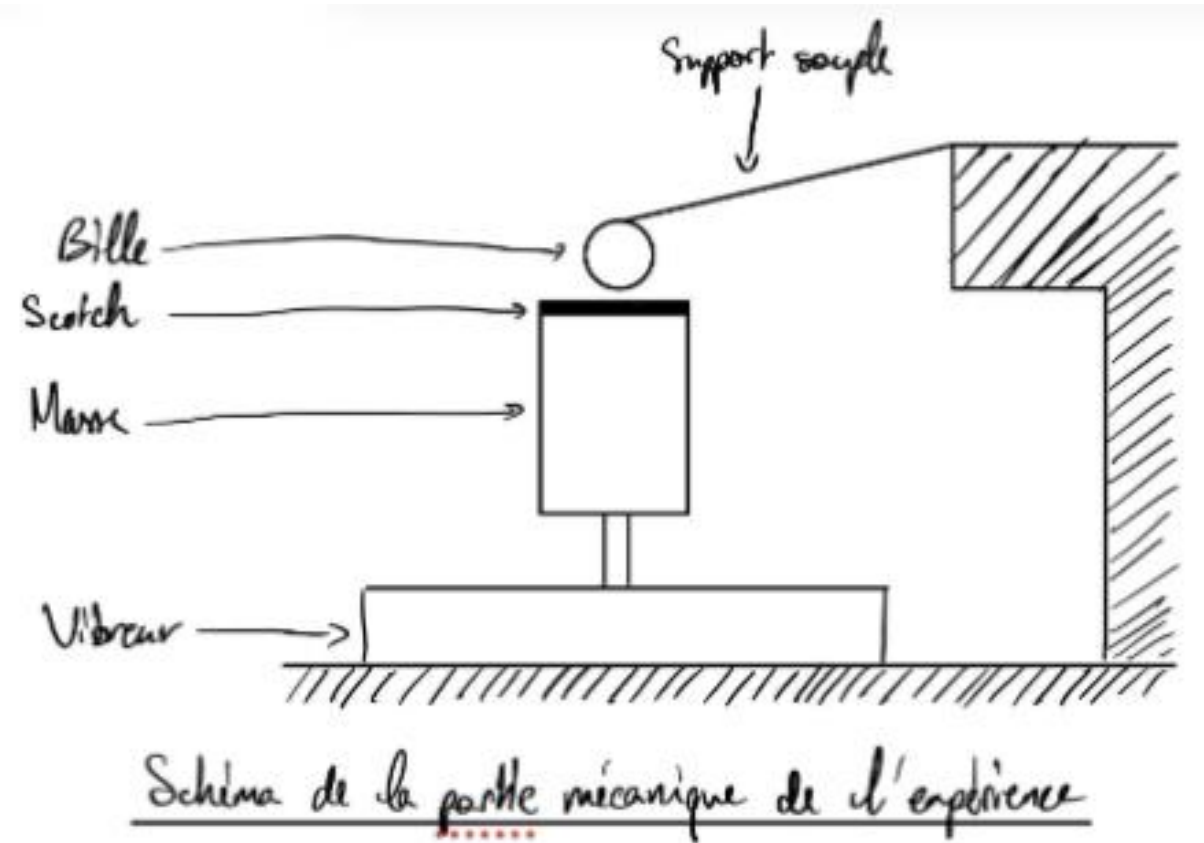
Chaotic Bouncing Ball

- 1) Experimental Study
- 2) Numerical Study

Supervising professors – Lionel FORET, Benoît FORGET

Experimental Study

a) Set Up



Experimental Study

b) Bifurcation Diagram

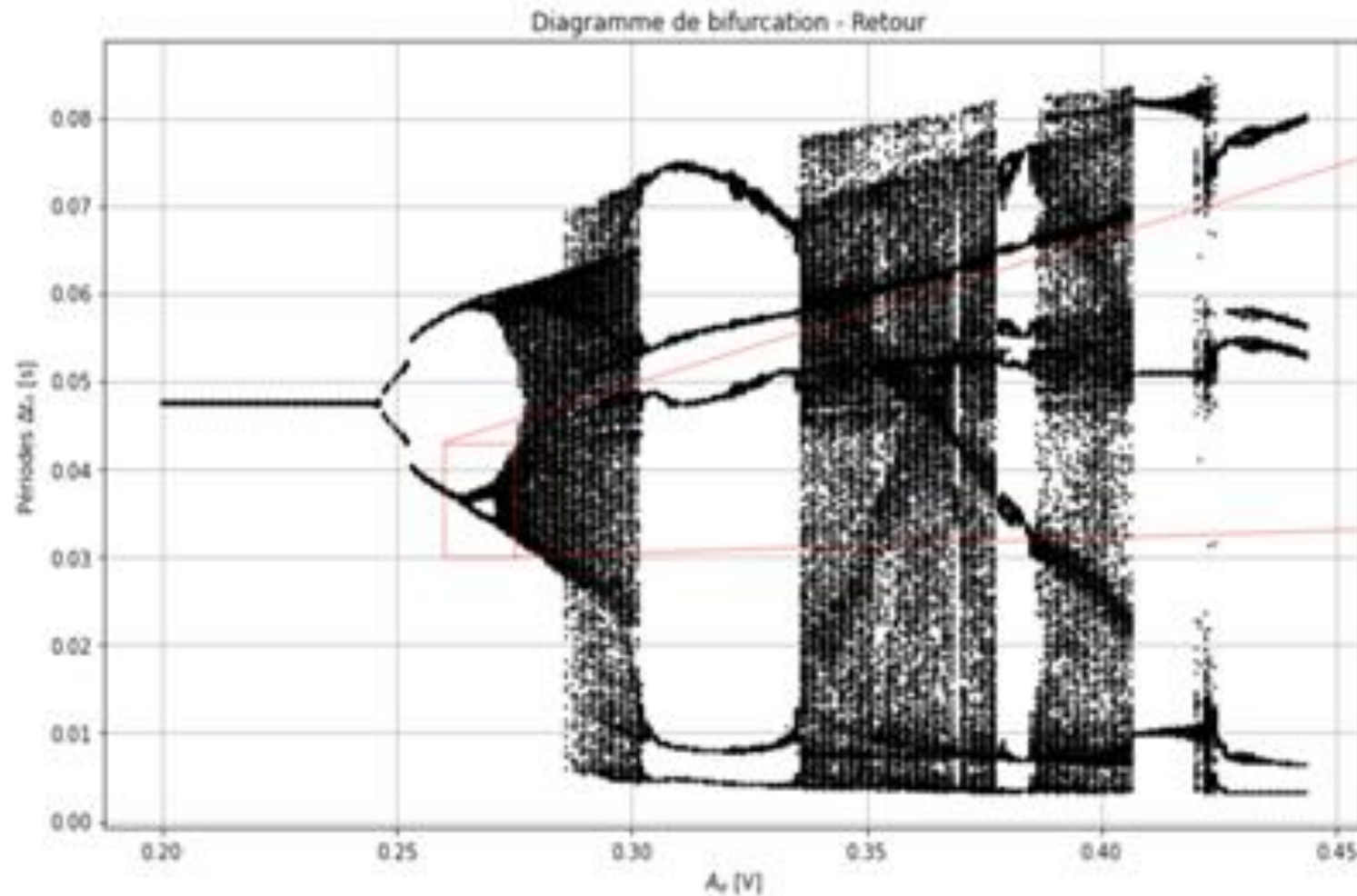
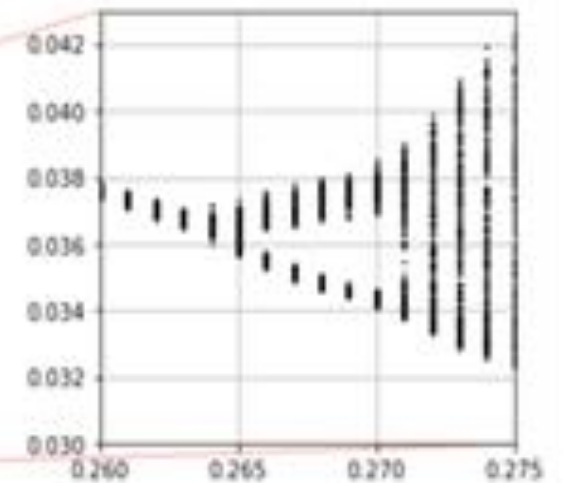


Figure 2



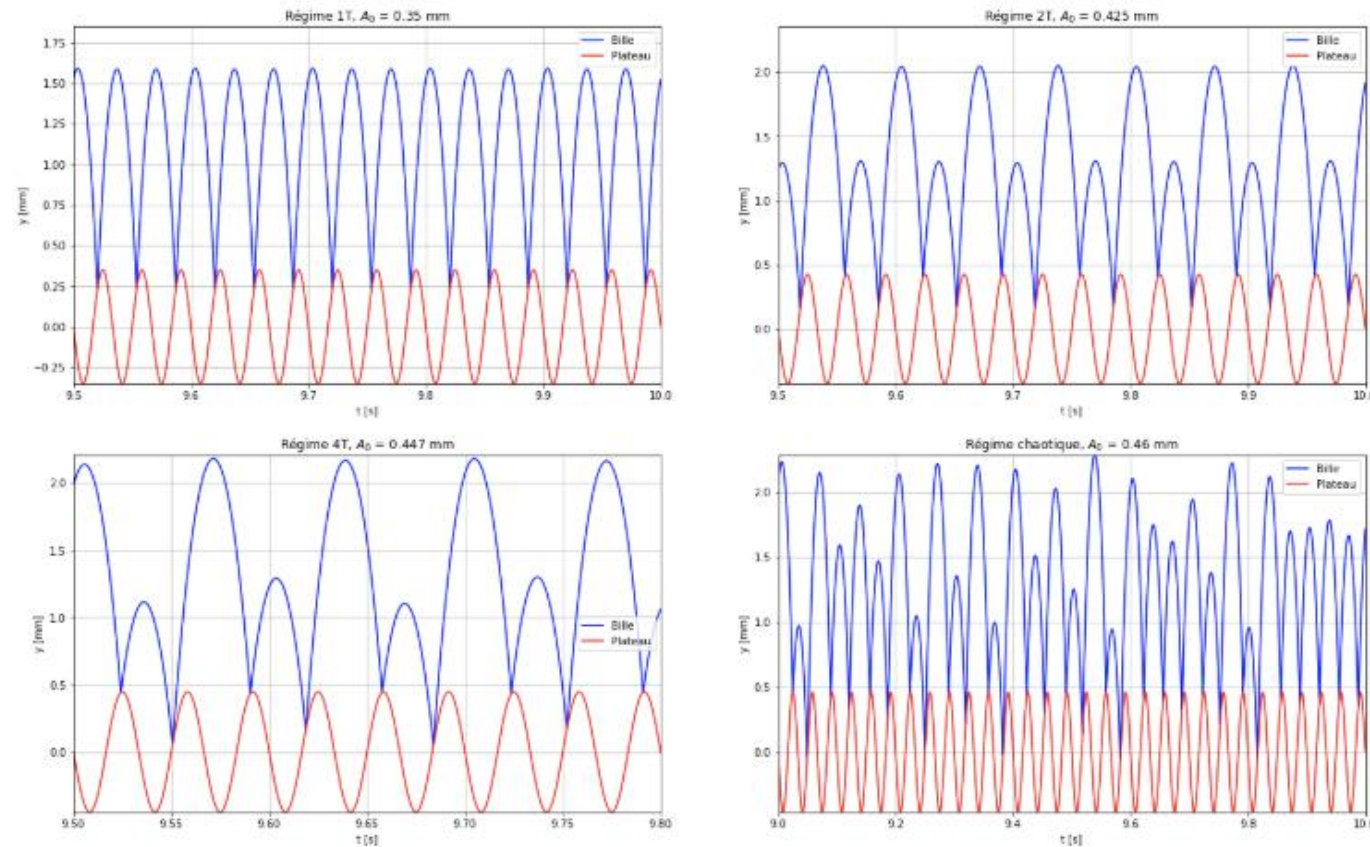
Numerical Study

a) Python Model

Collision equation :

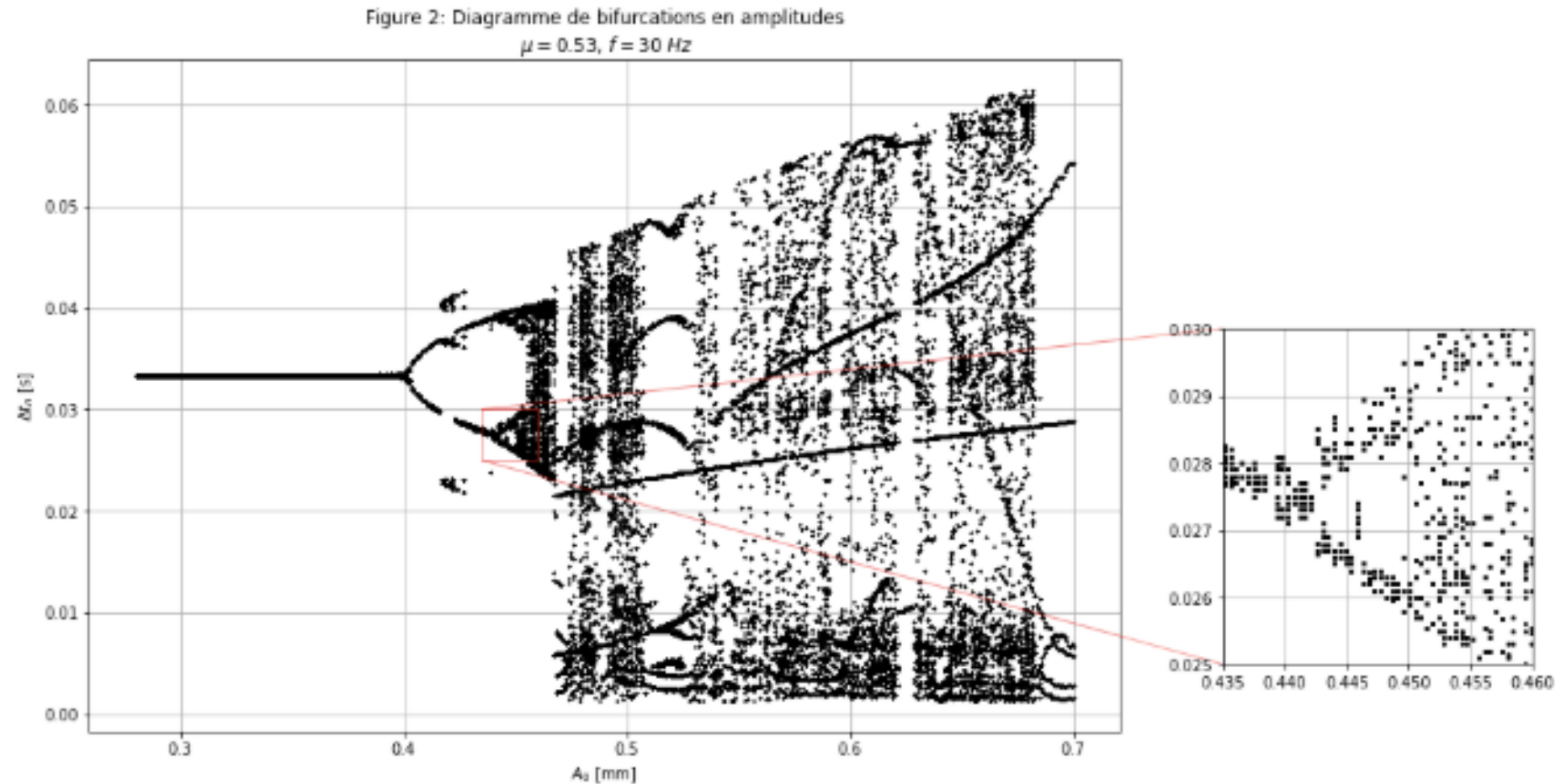
$$v_n - v_P(t_n) = -\mu \left(v_{B,n-1}(t_n) - v_P(t_n) \right)$$

Figure 1: Exemples de dynamiques de la bille en régime permanent



Numerical Study

b) Bifurcation Diagram



Numerical Study

c) Strange Attractor

Figure 6: Section de Poincaré (coordonnées polaires)
 $A_0 = 0.6 \text{ mm}$, $f = 30 \text{ Hz}$, $\mu = 0.87$

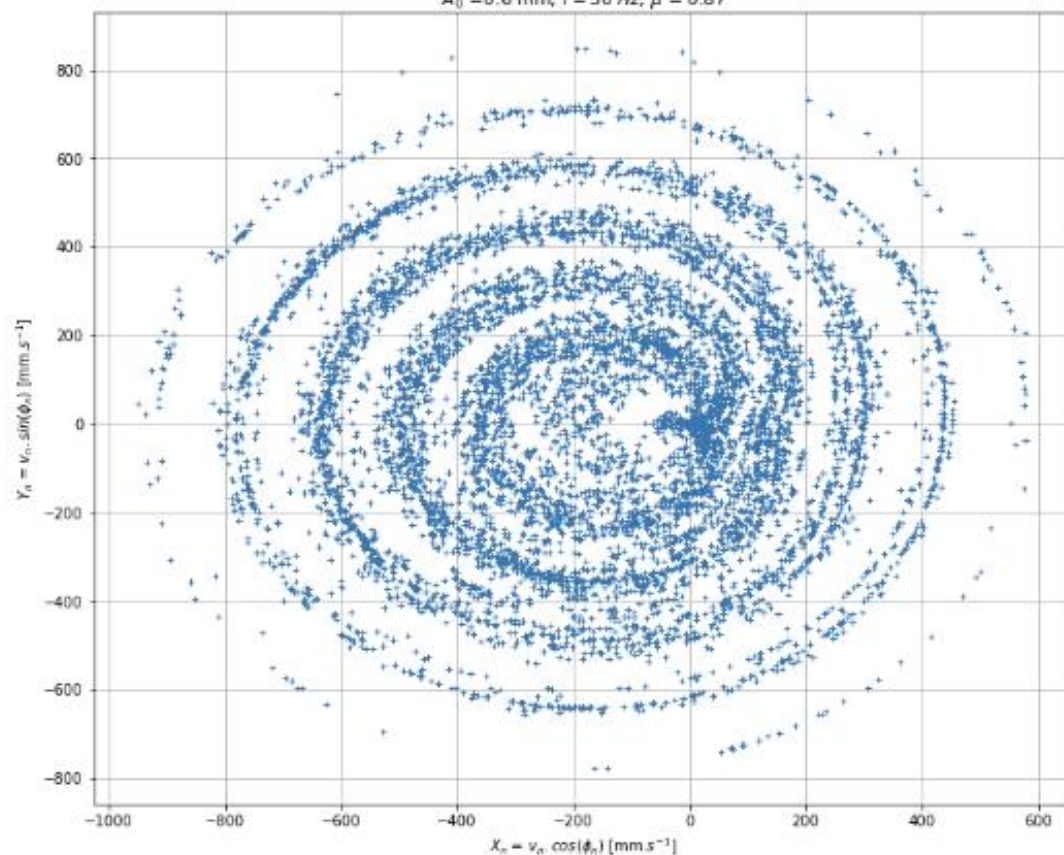
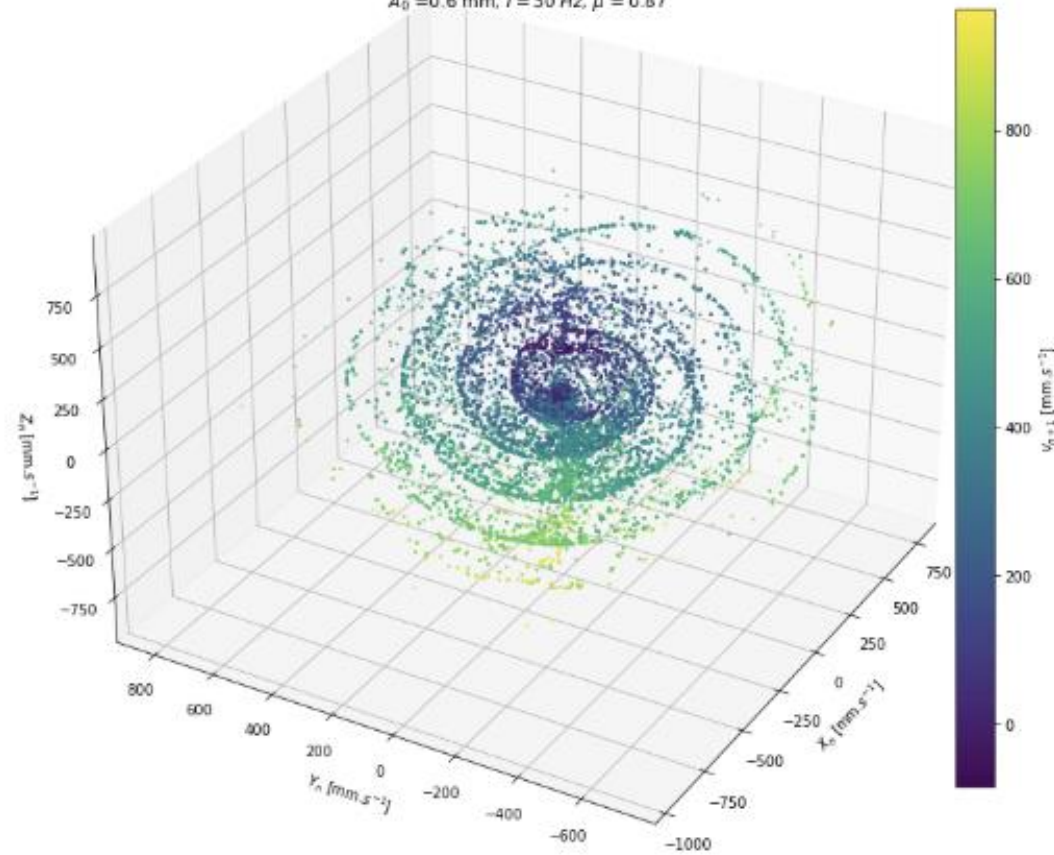


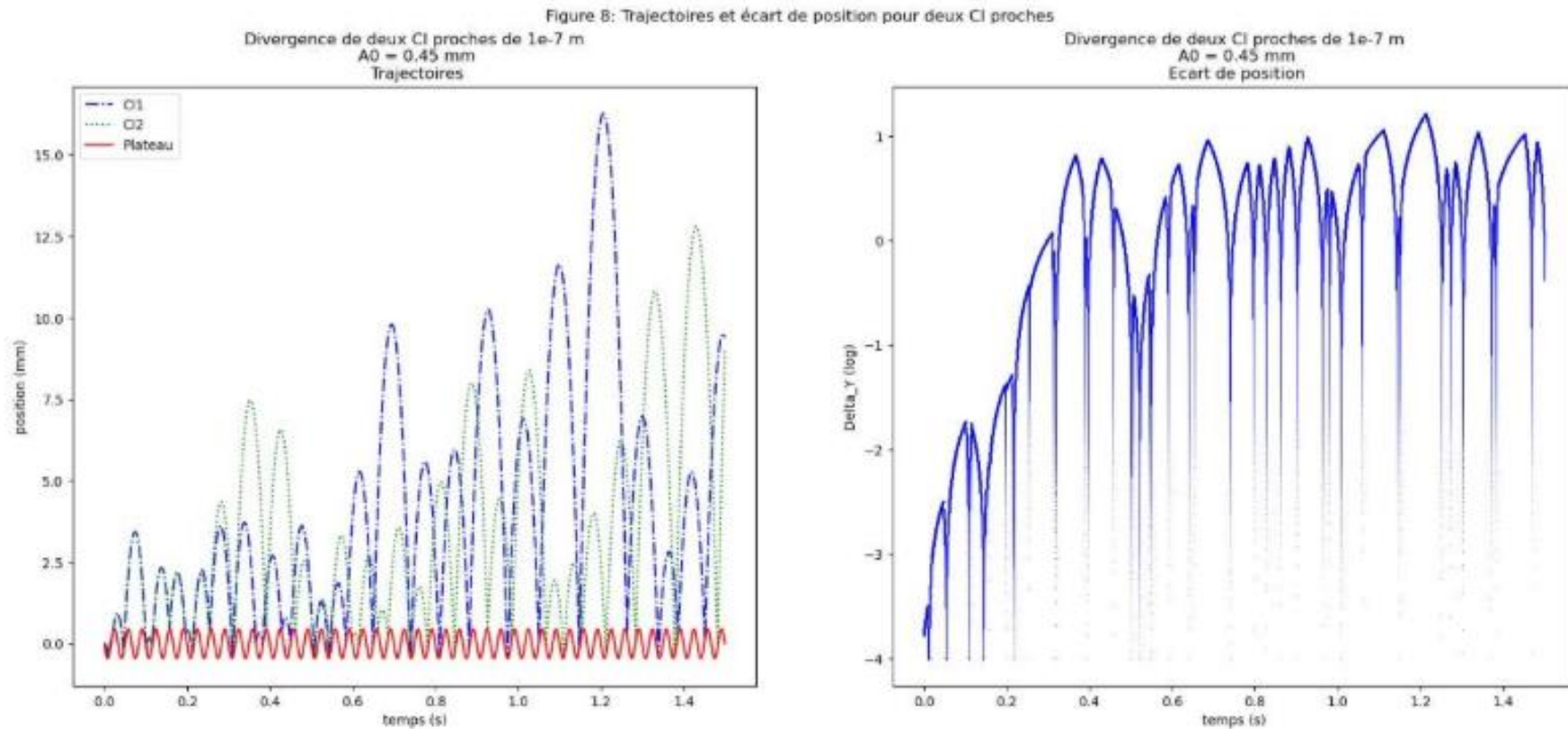
Figure 7: Représentation 4D de l'attracteur étrange (coordonnées sphériques)
 $A_0 = 0.6 \text{ mm}$, $f = 30 \text{ Hz}$, $\mu = 0.87$



Numerical Study

d) Lyapunov Exponent

$$|y_1(t) - y_2(t)| = d_0 \exp(\lambda t)$$



Thank you for listening

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References

- [1] *La bille qui rebondit : une expérience simple pour aborder la physique du chaos*, Pierre Boissel, 1992
- [2] *The dynamics of repeated impacts with a sinusoidally vibrating table*, P. J. Holmes, 1981
- [3] *Fractal dimension of the strange attractor of the bouncing ball circuit*, B. K. Clark and al., 1993