University of Montpellier

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Foundations and Interpretations of Quantum Mechanics

Master’s Degree in CCP

Supervised by Gilbert Moultaka

Quantum mechanics

1. Quantisation Principle
2. Superposition principle
3. Schrödinger's equation
4. Born's rule
5. Collapse of the wave function

$$|\psi\rangle \in \mathcal{H}$$

$$\hat{X}, \hat{P}, \hat{H}$$

$$|\psi\rangle = \alpha\,|\psi_1\rangle + \beta\,|\psi_2\rangle \quad , \quad \alpha, \beta \in \mathbb{C}$$

$$i\hbar \frac{\partial\,|\psi, t\rangle}{\partial t} = \hat{H}\,|\psi, t\rangle$$

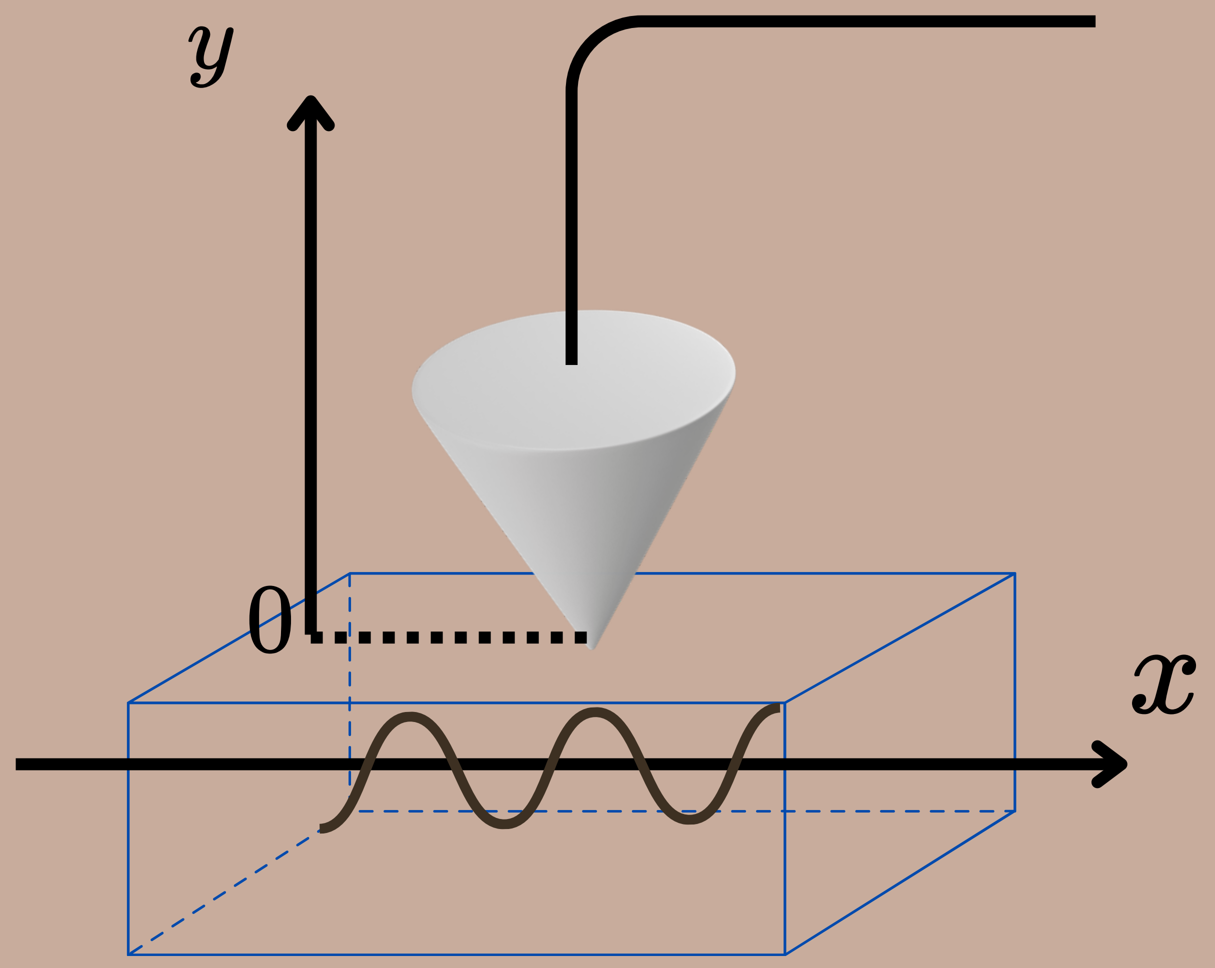
$$\mathcal{P} = |\,\langle \vec{x}|\psi\rangle\,|^2 d^3\vec{x}$$

Measurement Problem

Two postulates for dynamics

Decoherence

Interpretations



Bohmian theory

1. Non-Duality Principle $\psi(\vec{x},t) = R(\vec{x},t)e^{iS(\vec{x},t)}$
2. Wave function dynamics $i\hbar\frac{\partial\psi(\vec{x},t)}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi(\vec{x},t) + V(\vec{x},t)\psi(\vec{x},t)$
3. Particle motion $\dot{\vec{X}}(t) = \frac{1}{m}\vec{\nabla}S(\vec{x},t)|_{\vec{x}=\vec{X}(t)}$
4. Born's rule $R^2(\vec{x},t)d^3\vec{x}$

Conclusion

Demonstration of Born's rule

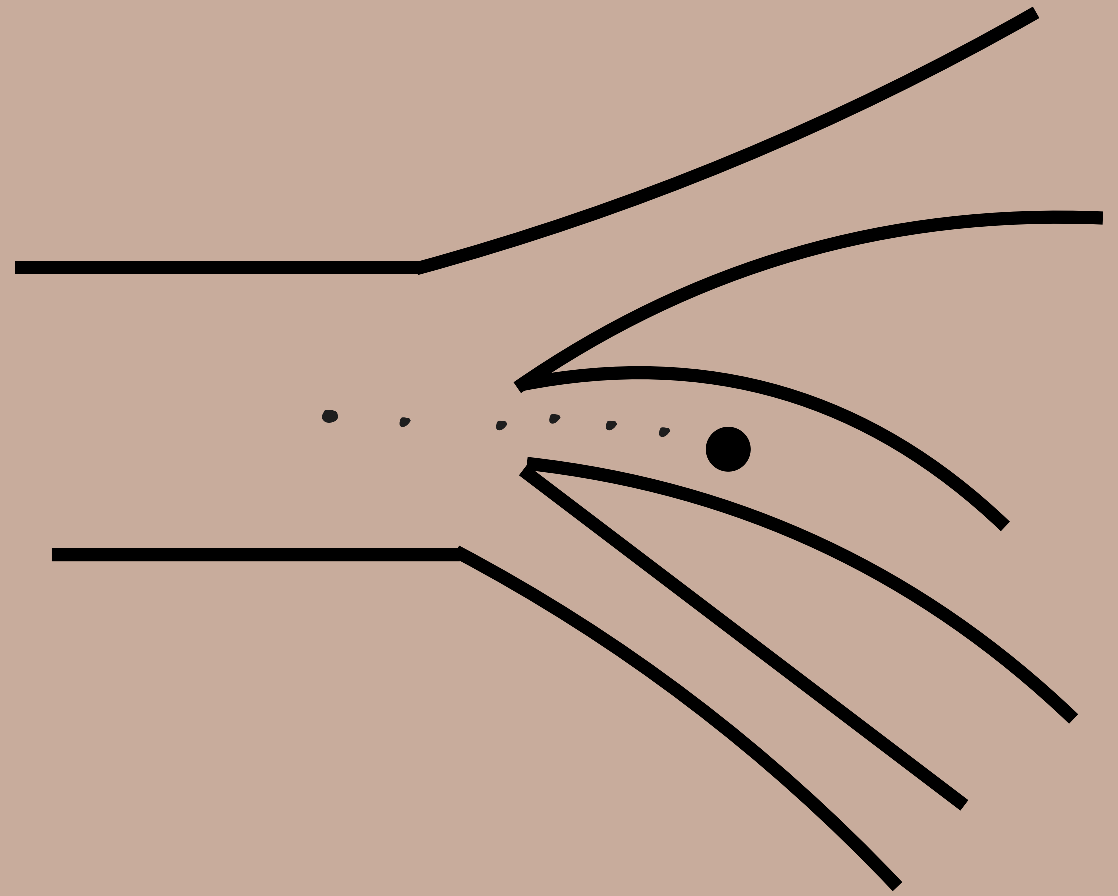
Determinism

Aknowledgements

I would like to thank my supervisor, with whom I have always had passionate discussions and who has always been able to express patience and provide constructive criticism regarding the review of this report.

Measurement and non-locality

$$\Psi(x,y,t) = \sum_{n=0}^{N} \psi_n(x)\phi(y-\lambda E_n t)e^{-iE_n t/\hbar}$$



$$\dot{\vec{X}}(t) = \frac{1}{m}\vec{\nabla}S(\vec{x},t)|_{\vec{x}=\vec{X}(t)}$$

Tension with relativity

Fourth postulate and Quantum potential

$$\begin{cases} \partial_t S + \frac{(\vec{\nabla} S)^2}{2m} - \frac{\hbar^2}{2m}\frac{\Delta R}{R} + V = 0 \\ \partial_t R^2 + \vec{\nabla}.(\frac{R^2 \vec{\nabla} S}{m}) = 0 \end{cases}$$

$$Q = -\frac{\hbar^2}{2m}\frac{\Delta R}{R}$$

$$E = \frac{p^2}{2m} + V + Q$$

$$P_0(x) = R_0^2(x) \longrightarrow P(x,t) = R^2(x,t)$$

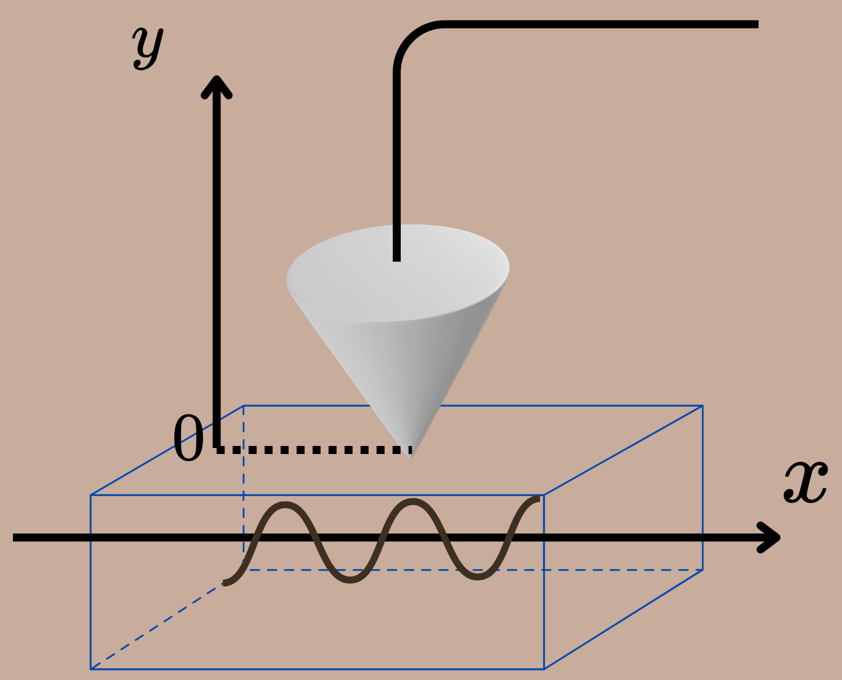
Quantum equilibrium

Postulates

1. Non-Duality Principle $$\psi(\vec{x},t) = R(\vec{x},t)e^{iS(\vec{x},t)}$$
2. Wave function dynamics $$i\hbar\frac{\partial\psi(\vec{x},t)}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi(\vec{x},t) + V(\vec{x},t)\psi(\vec{x},t)$$
3. Particle motion $$\dot{\vec{X}}(t) = \frac{1}{m}\vec{\nabla}S(\vec{x},t)|_{\vec{x}=\vec{X}(t)}$$
4. Born's rule $$R^2(\vec{x},t)d^3\vec{x}$$

Measurement Problem

$$\hat{H}_{int} = \lambda \hat{H}_S \hat{p}_y = -i\hbar\lambda \hat{H}_S \frac{\partial}{\partial y}$$



$$\Psi(x, y, 0) = \chi(x)\phi(y) = \left(\sum_{n=0}^{N} c_n \psi_n(x)\right)\phi(y)$$

$$\longrightarrow \quad \Psi(x, y, t) = \sum_{n=0}^{N} \psi_n(x)\phi(y - \lambda E_n t)e^{-iE_n t/\hbar}$$

Pair	*particle 1*	*particle 2*	*Occurrence*
1	{+1;+1;+1}	{-1;-1;-1}	C_1
2	{+1;+1;-1}	{-1;-1;+1}	C_2
3	{+1;-1;+1}	{-1;+1;-1}	C_3
4	{+1;-1;-1}	{-1;+1;+1}	C_4
5	{-1;+1;+1}	{+1;-1;-1}	C_5
6	{-1;-1;+1}	{+1;+1;-1}	C_6
7	{-1;+1;-1}	{+1;-1;+1}	C_7
8	{-1;-1;-1}	{+1;+1;+1}	C_8

$$\begin{cases} P_{az}(++) = C_2 + C_4 \\ P_{ab}(++) = C_3 + C_4 \\ P_{bz}(++) = C_2 + C_7 \end{cases}$$

$$C_2 + C_4 \leqslant C_2 + C_4 + C_3 + C_7$$

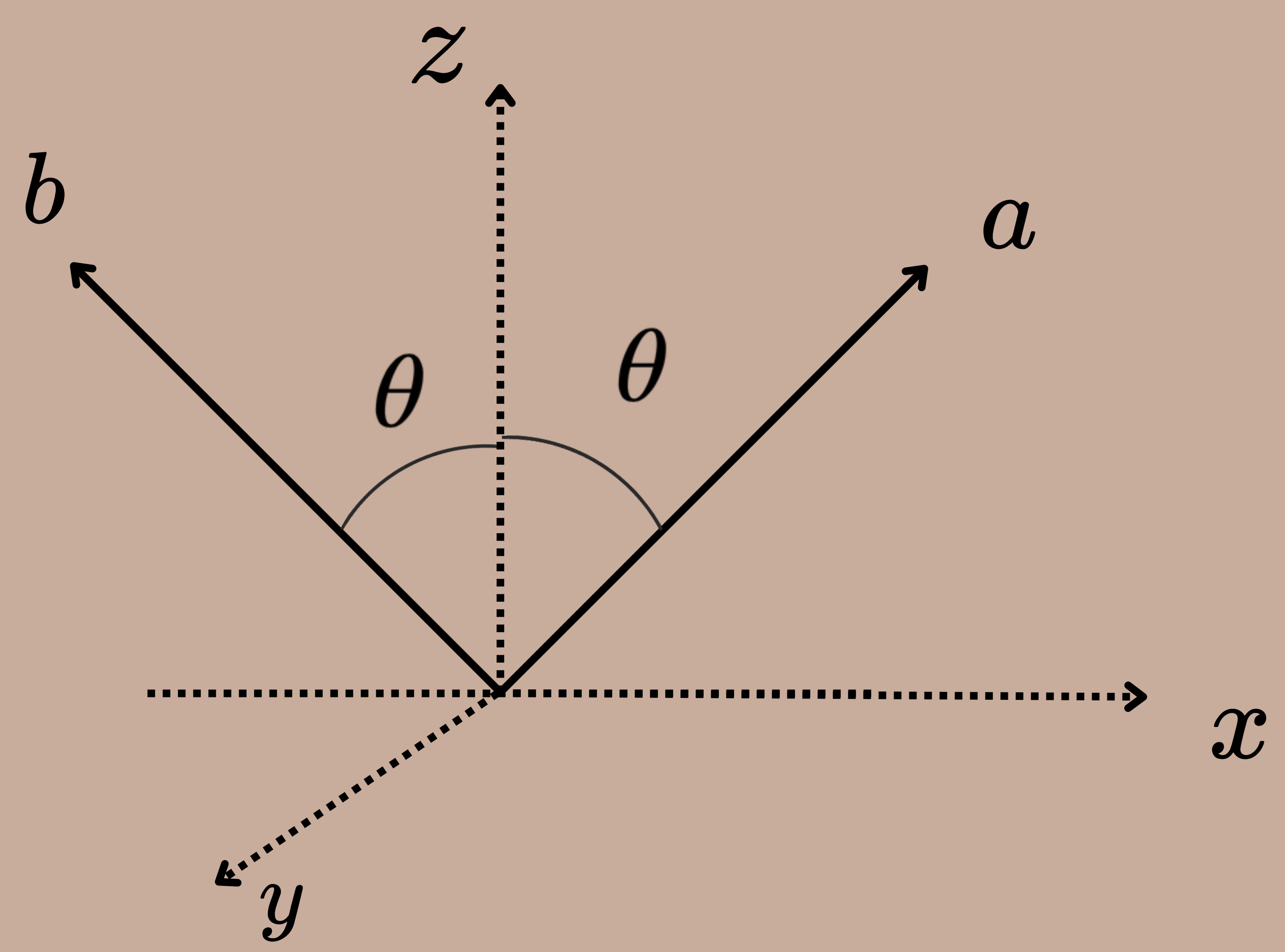
$$P_{az}(++) \leqslant P_{bz}(++) + P_{ab}(++)$$

→ Violation of Bell's inequalities

Bell theorem

$$|\psi\rangle = \frac{1}{\sqrt{2}}\,|\psi_1\rangle_{a+}\,|\psi_2\rangle_{a-} - \frac{1}{\sqrt{2}}\,|\psi_1\rangle_{a-}\,|\psi_2\rangle_{a+}$$

Pair	*particle 1*	*particle 2*	*Occurrence*
1	{+1;+1;+1}	{-1;-1;-1}	C_1
2	{+1;+1;-1}	{-1;-1;+1}	C_2
3	{+1;-1;+1}	{-1;+1;-1}	C_3
4	{+1;-1;-1}	{-1;+1;+1}	C_4
5	{-1;+1;+1}	{+1;-1;-1}	C_5
6	{-1;-1;+1}	{+1;+1;-1}	C_6
7	{-1;+1;-1}	{+1;-1;+1}	C_7
8	{-1;-1;-1}	{+1;+1;+1}	C_8



Bell theorem

Hidden local variables

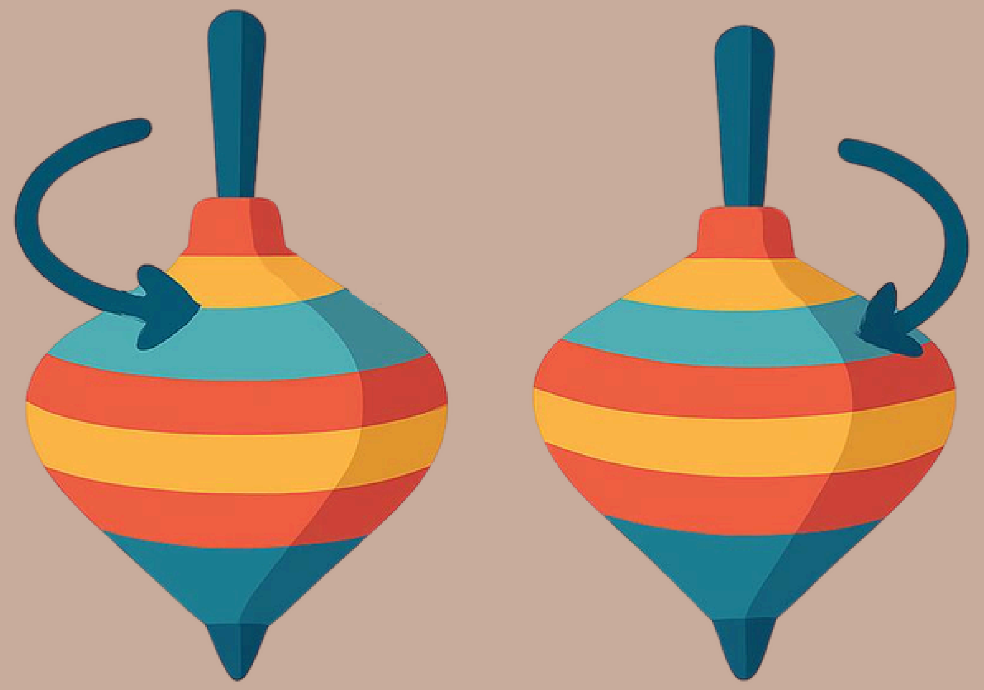
Pair	*Particle 1*	*Particle 2*	*Occurrence*
1	{+1;+1}	{-1;-1}	25%
2	{+1;-1}	{-1;+1}	25%
3	{-1;+1}	{+1;-1}	25%
4	{-1;-1}	{+1;+1}	25%

EPR Argument

Entanglement

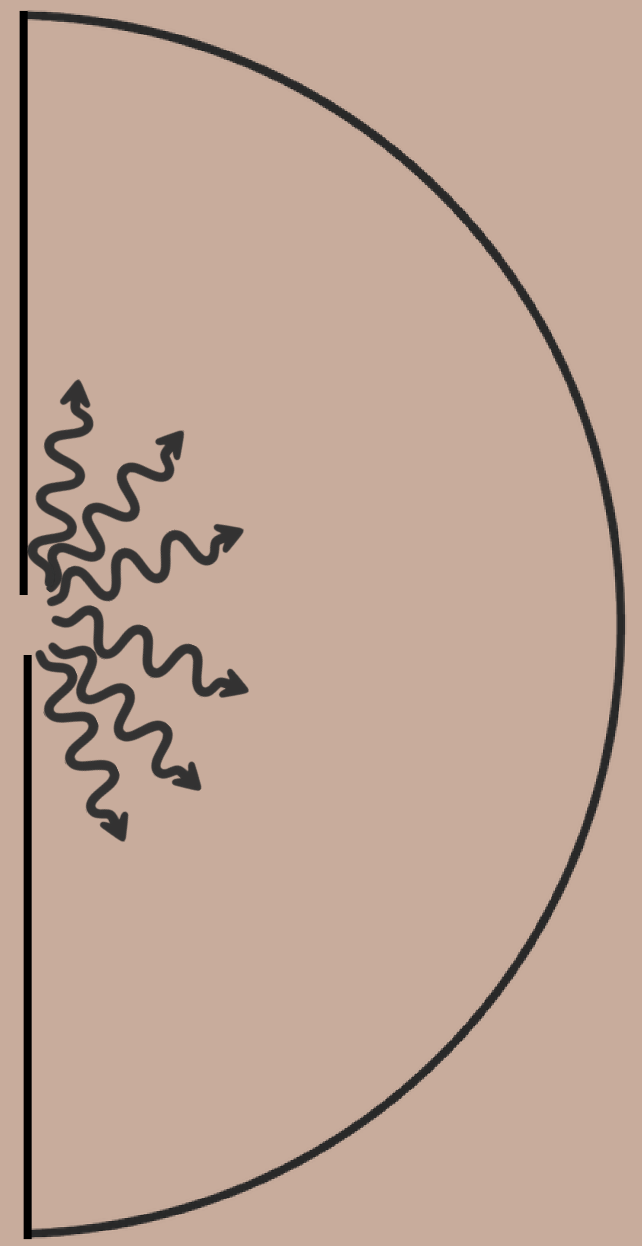
Two entangled spins

Reality principle



$$\longrightarrow \quad |\Psi\rangle = \frac{1}{\sqrt{2}} |\psi_1\rangle_+ |\psi_2\rangle_- - \frac{1}{\sqrt{2}} |\psi_1\rangle_- |\psi_2\rangle_+$$

Einstein Boxes

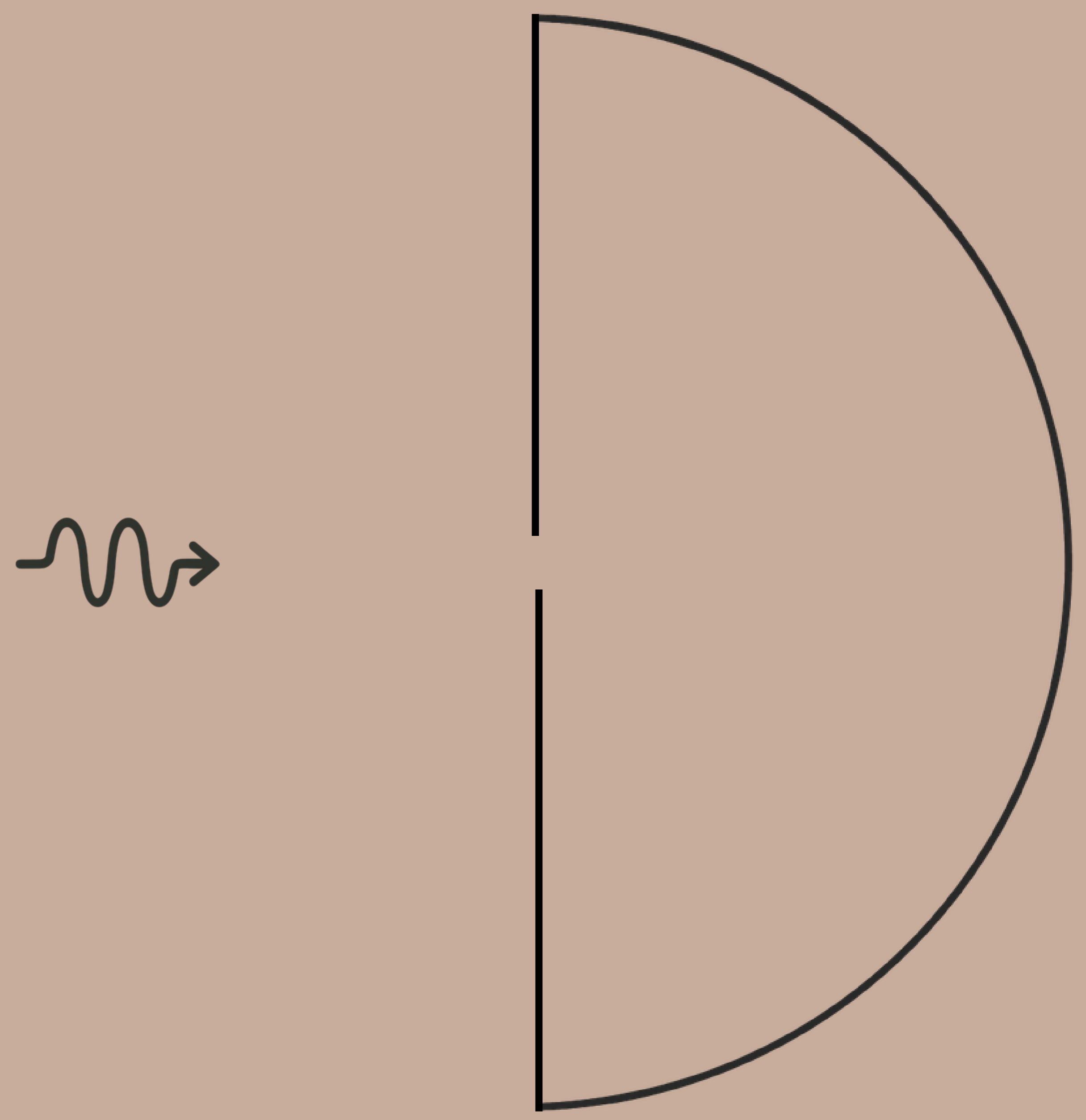


Non-locality

Tension with relativity ?

Non -signaling theorem

Einstein Boxes



Quantum Potential

$$\partial_t S + \frac{p^2}{2m} - \frac{\hbar^2}{2m}\frac{\Delta R}{R} + V = 0$$

$$E(X(t), t) = -\partial_t S(x, t)|_{x=X(t)}$$

$$Q = -\frac{\hbar^2}{2m}\frac{\Delta R}{R}$$

$$\frac{d}{dt}(m\dot{\vec{X}}(t)) = -\vec{\nabla} V - \vec{\nabla} Q$$

Quantum Potential

$$\begin{cases} \partial_t S + \dfrac{(\vec{\nabla} S)^2}{2m} - \dfrac{\hbar^2}{2m}\dfrac{\Delta R}{R} + V = 0 \\ \partial_t R^2 + \vec{\nabla}.(\dfrac{R^2 \vec{\nabla} S}{m}) = 0 \end{cases}$$

$$\partial_t S + \frac{p^2}{2m} - \frac{\hbar^2}{2m}\frac{\Delta R}{R} + V = 0$$

Quantum Potential

$$i\hbar\partial_t\psi(\vec{x},t) = -\frac{\hbar^2}{2m}\Delta\psi(\vec{x},t) + V(\vec{x},t)\psi(\vec{x},t)$$

$$\psi(\vec{x},t) = R(\vec{x},t)e^{iS(\vec{x},t)}$$

Weaker fourth postulate

$$P_0(x) \; = \; R_0^2(x)$$

$$\partial_t R^2(\vec{X}(t), t) + \vec{\nabla}.\left(\vec{v}(t) R^2(\vec{X}(t), t)\right) = 0$$

$$f(\vec{x}, t) = \frac{P(\vec{X}, t)}{R^2(\vec{X}, t)}$$

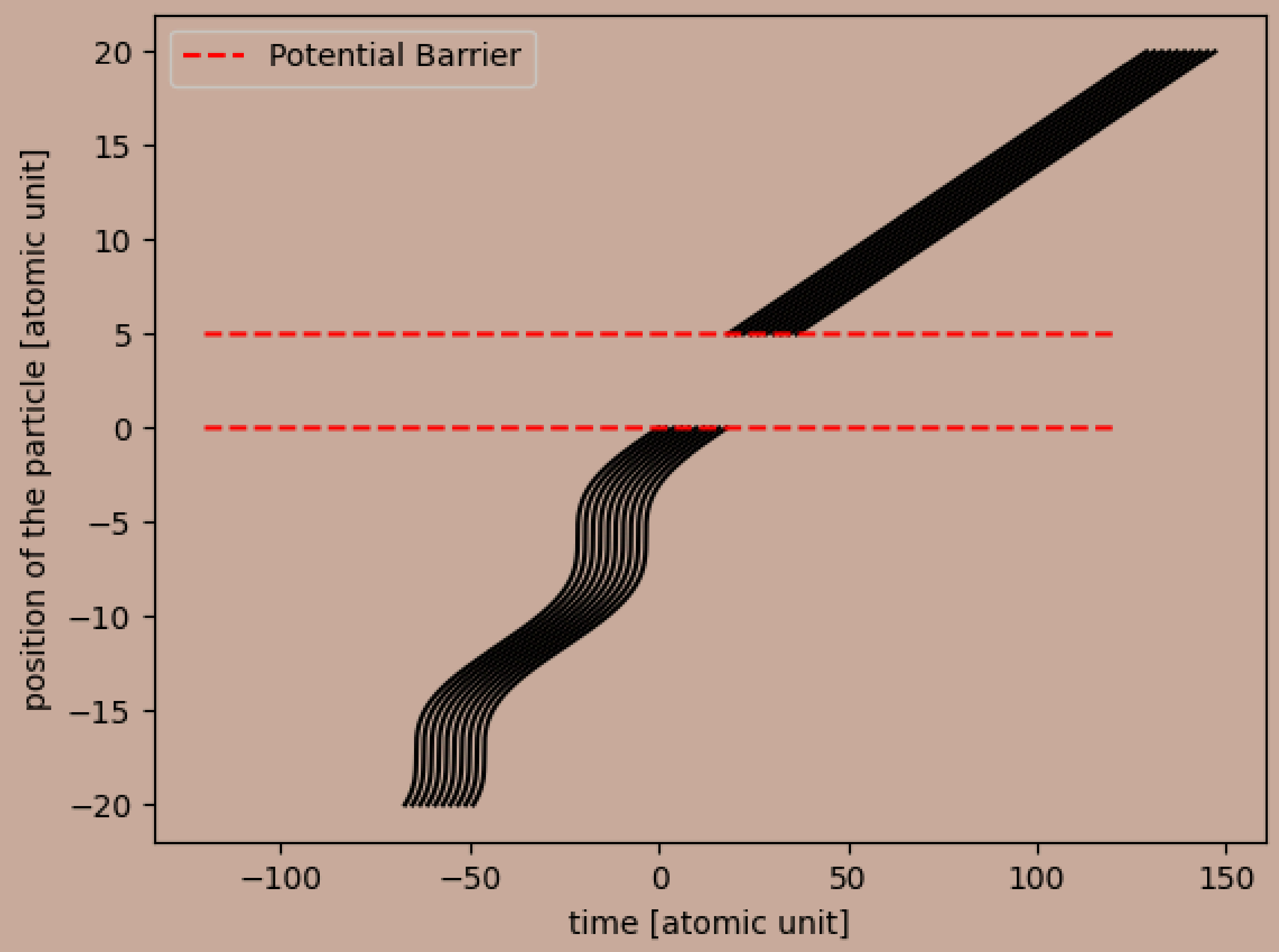
$$\partial_t P(\vec{X}, t) + \vec{\nabla}.\left(\vec{v} P(\vec{X}, t)\right) = 0$$

$$\longrightarrow \frac{d}{dt} f(\vec{X}, t) = 0$$

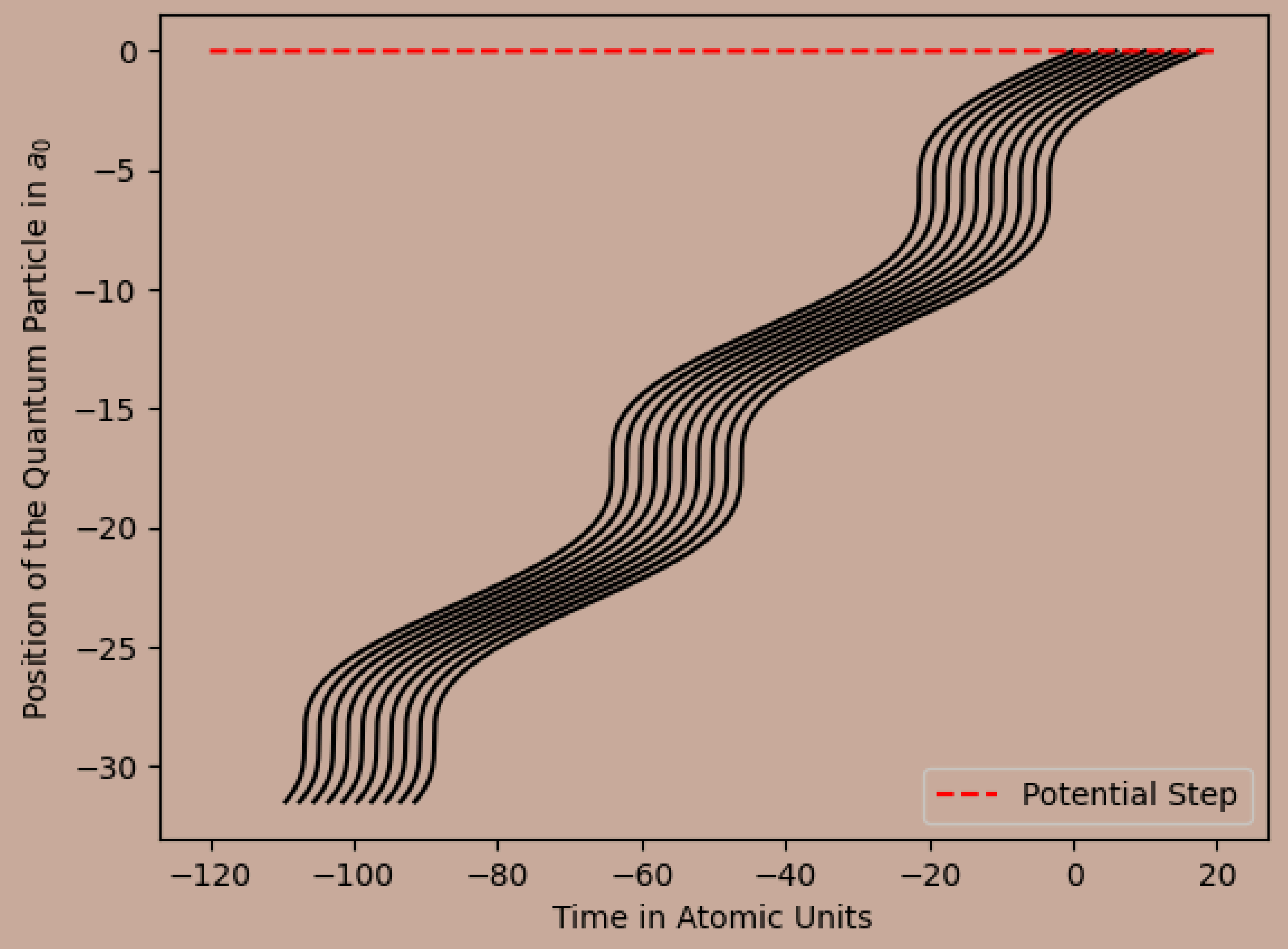
Tunnelling

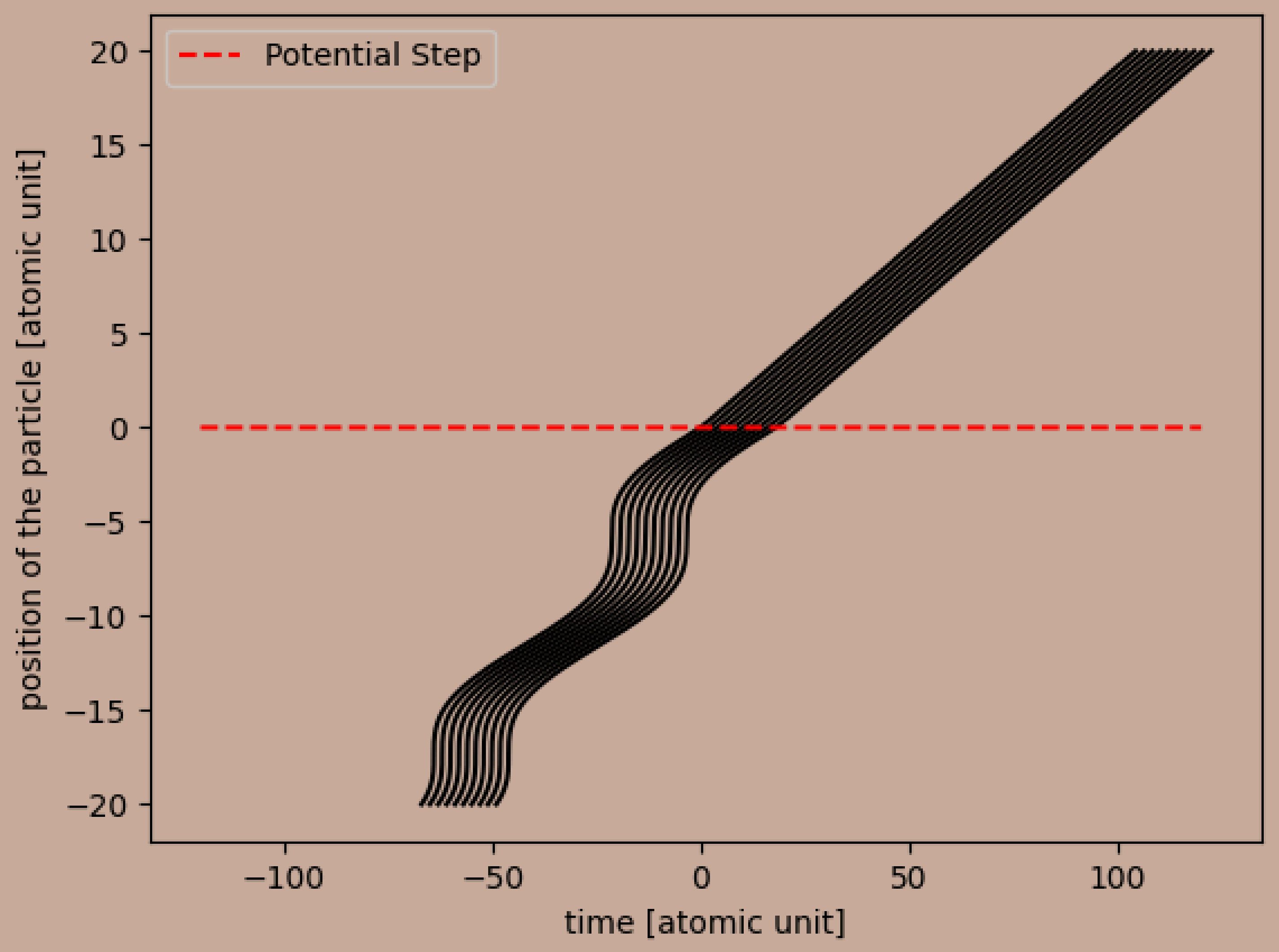
$$\Psi(x,t) = \begin{cases} e^{ikx} + re^{-ikx} & \text{if } x < 0 \ , \\ e^{\kappa x} + e^{-\kappa x} & \text{if } 0 < x < a \ , \\ te^{ikx} & \text{else,} \end{cases}$$

$$\frac{dX}{dt} = \begin{cases} \frac{\hbar k}{m} \frac{1}{1+a\cos(2kX(t))} & \text{if } x < 0 \ , \\ 0 & \text{if } 0 < x < a \ , \\ \frac{\hbar}{m} tk & \text{else.} \end{cases}$$



Step of potential



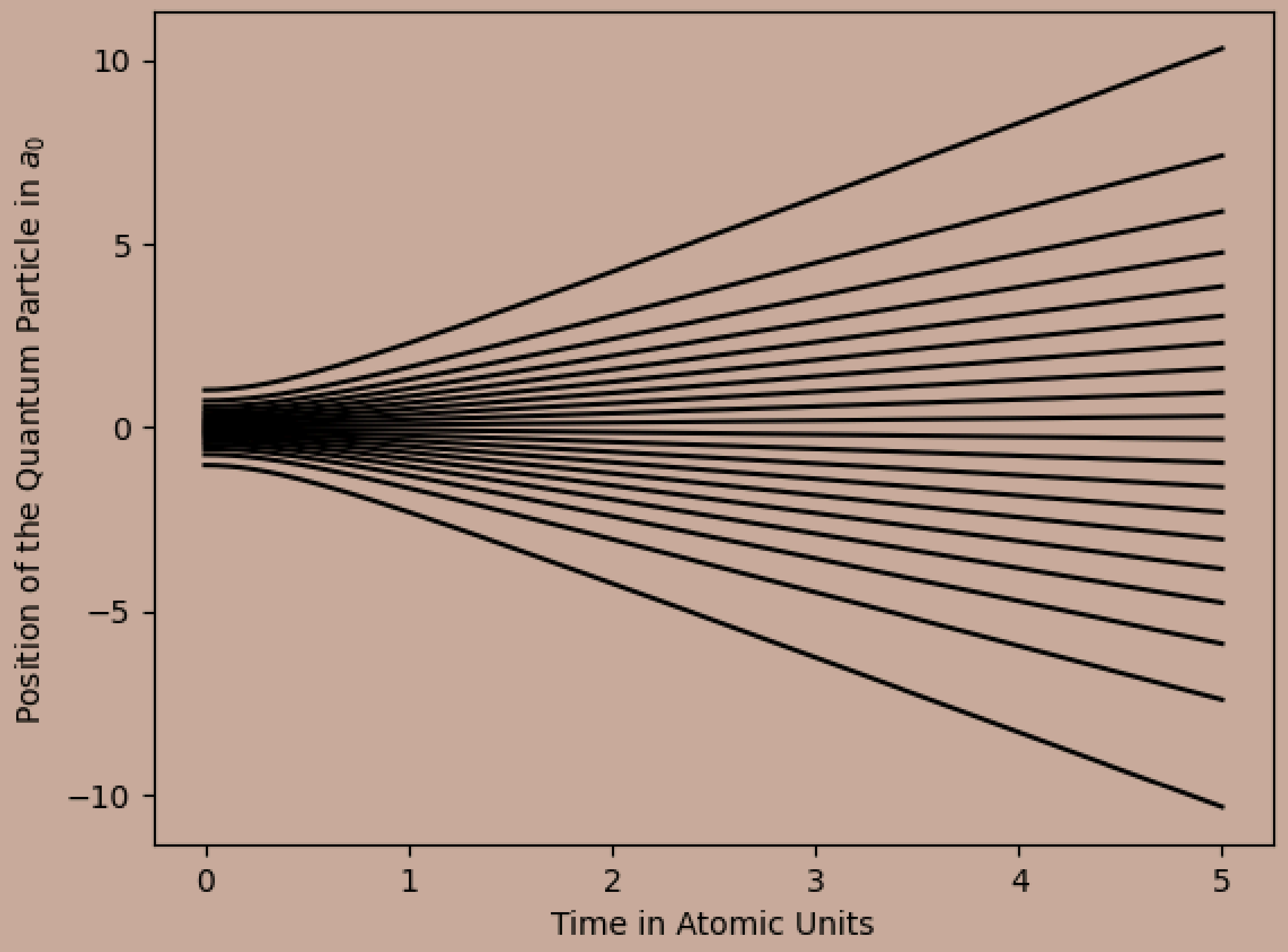


Spreading of a wave packet

$$\psi(x,0) = Ae^{-x^2/4\sigma^2}$$

$$\psi(x,t) = N(t)e^{-x^2/(4(\sigma^2+i\hbar t/4m^2))}$$

$$X(t) = X_0\left(1+\frac{t^2}{4m^2\sigma^4/\hbar^2}\right)^{1/2}$$



Particle Motion

$$\vec{p} = \hbar \vec{k}$$

$$\psi(x,t) \sim e^{i\vec{k}.\vec{x}}$$

$$\vec{v} = \frac{\hbar}{m}\vec{k}$$

$$\psi(\vec{x},t) = R(\vec{x},t)e^{iS(\vec{x},t)}$$

$$\longrightarrow \quad \vec{v} = \frac{\hbar}{m}\vec{\nabla}S(x,t)$$