Gravitational lensing of neutrinos

Using gravitational lenses as astrophysical probes to neutrino mass constraints

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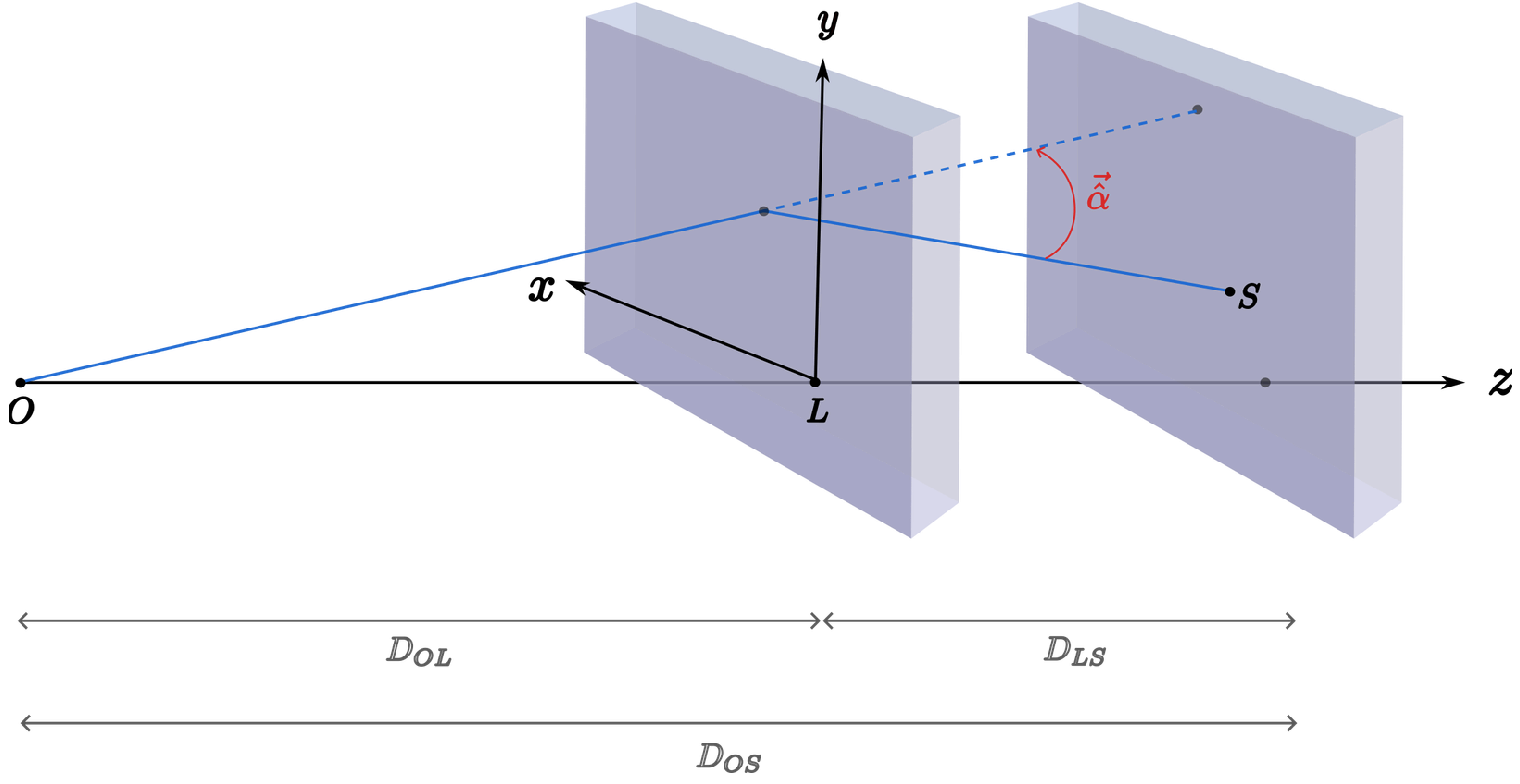
Lensing problem



Notation :

$$\vec{\hat{\alpha}} = \begin{array}{c} \begin{bmatrix} \hat{\alpha}^1 \\ \hat{\alpha}^2 \end{bmatrix} \end{array} \begin{array}{l} \rightarrow (xLy) \\ \rightarrow (yLz) \end{array}$$

Deflection angle

Geodesic equation

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\lambda^2} = -\Gamma^\mu{}_{\nu\rho} \frac{\mathrm{d}x^\nu}{\mathrm{d}\lambda} \frac{\mathrm{d}x^\rho}{\mathrm{d}\lambda}$$

Deflection angle

$$\vec{\hat{\alpha}} = (2 + \bar{\gamma}^{-2}) \vec{\nabla} \hat{\psi}$$

neutrino term correction

Lorentz factor : $\gamma = \frac{1}{\sqrt{1 - v^2}} = \frac{E}{m}$

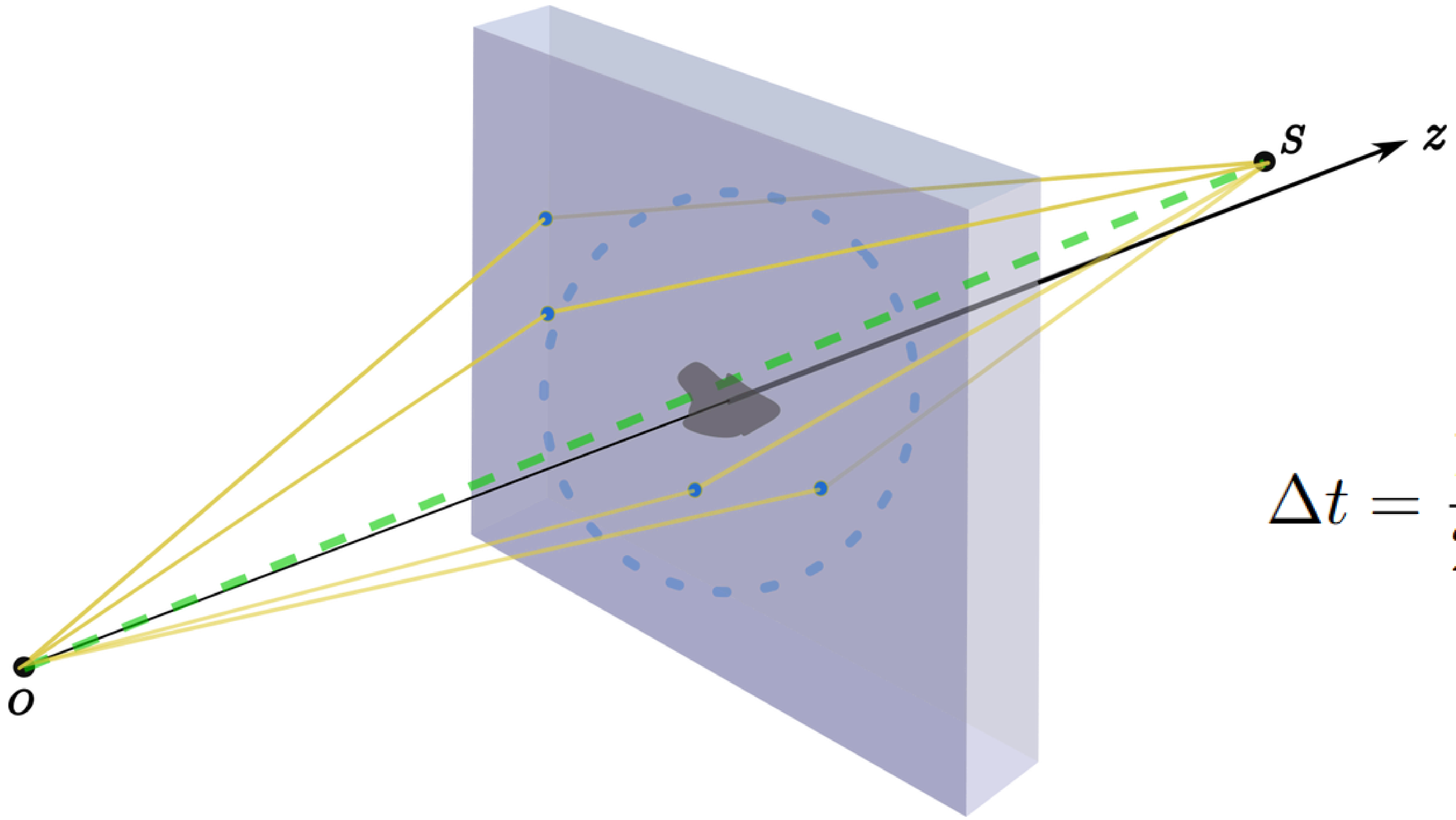
Integrated potential along dz : $\hat{\psi} = \int_{z_{in}}^{z_f} \phi(x, y, z) dz$

Time delay

between deviated and non deviated paths

$$g\left(\frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}, \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}\right) = -1$$

$$\frac{\mathrm{d}t}{\mathrm{d}l}$$



Geometric

$$\Delta t = \frac{1}{2}(2+\bar{\gamma}^{-2})\frac{D_{OL}D_{OS}}{D_{LS}}\left(\frac{\|\vec{\theta}-\vec{\beta}\|^2}{2} - \psi(\vec{\theta}, z)\right)$$

Shapiro

Actual observables : time delay between images

Connecting with neutrino flavours

- Detected neutrino flavour depend on their travel time
- Gravitational lensing creates multiple images with time delays

Time delay appears as the time in the oscillation probability

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\beta\alpha} - 2\sum_{i>k} \mathrm{Re}\left(U_{\alpha' i}U^*_{\alpha' k}U^*_{\alpha i}U_{\alpha k}\right)\left(1 - \cos\frac{\Delta m^2_{ki}}{2E}\Delta t\right)$$
$$+ 2\sum_{i>k} \mathrm{Im}\left(U_{\alpha' i}U^*_{\alpha' k}U^*_{\alpha i}U_{\alpha k}\right)\sin\frac{\Delta m^2_{ki}}{2E}\Delta t$$

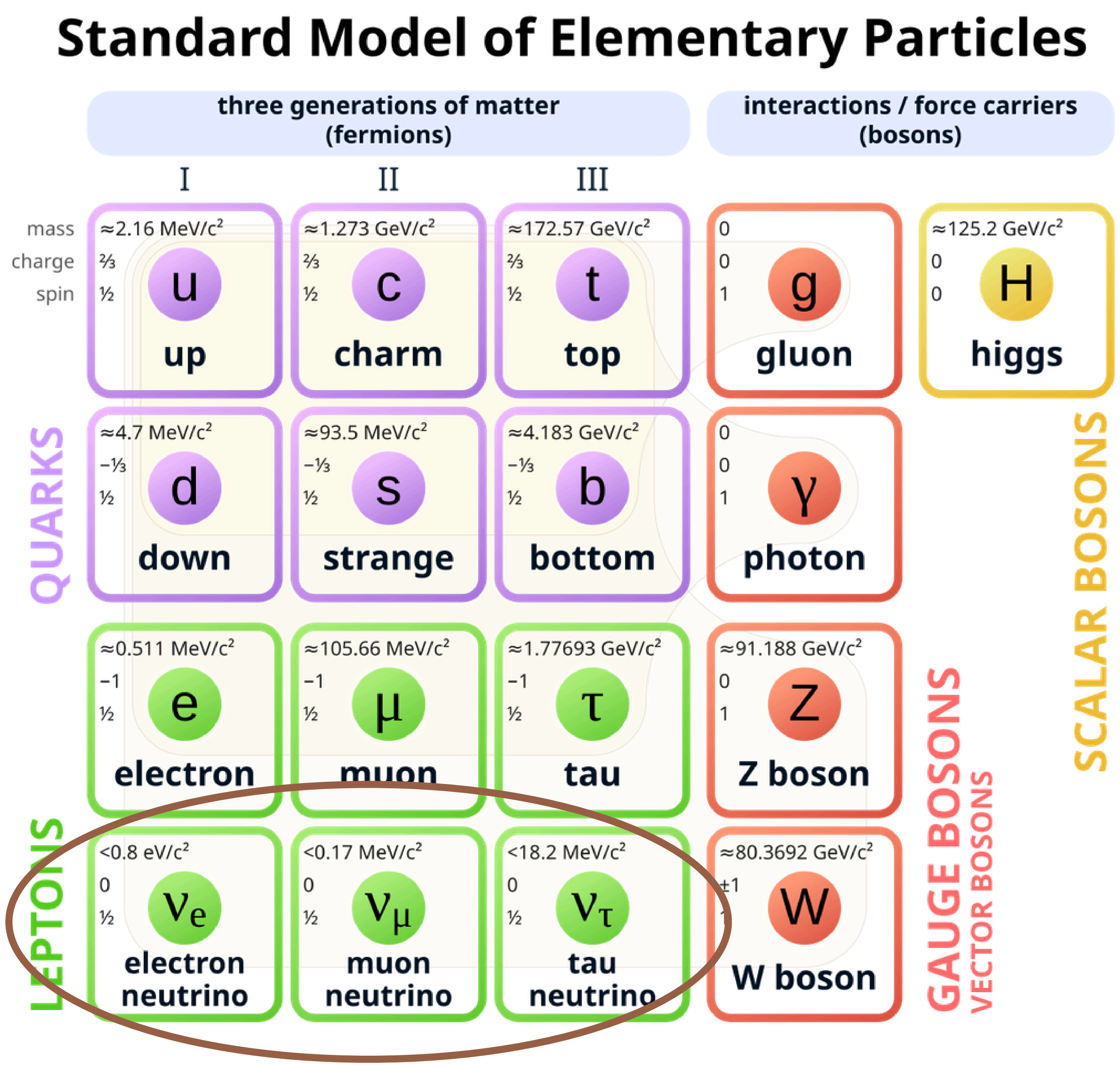
Conclusion and outlook

Time delays induced by gravitational lensing can help constrain neutrino masses, provided that the elements of the PMNS matrix are known with sufficient precision

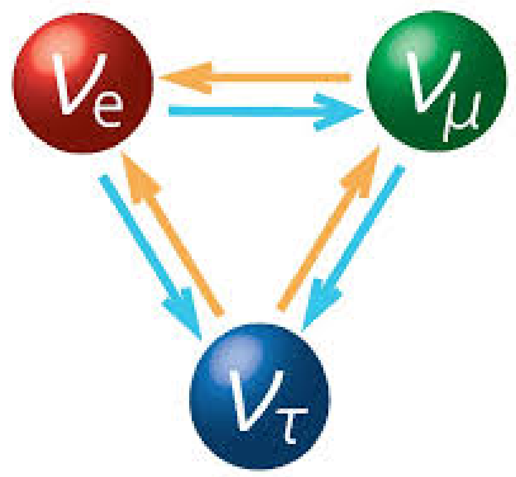
Some ideas to pursue :

- Quantum properties : interference effects
- Multi messenger astrophysics

Neutrinos



- Sensitive only to weak interaction
- Interact very little with matter
- Flavour oscillations



2024 (KATRIN) : $m_{\nu_e} < 0.45 \ \mathrm{eV/c^2}$

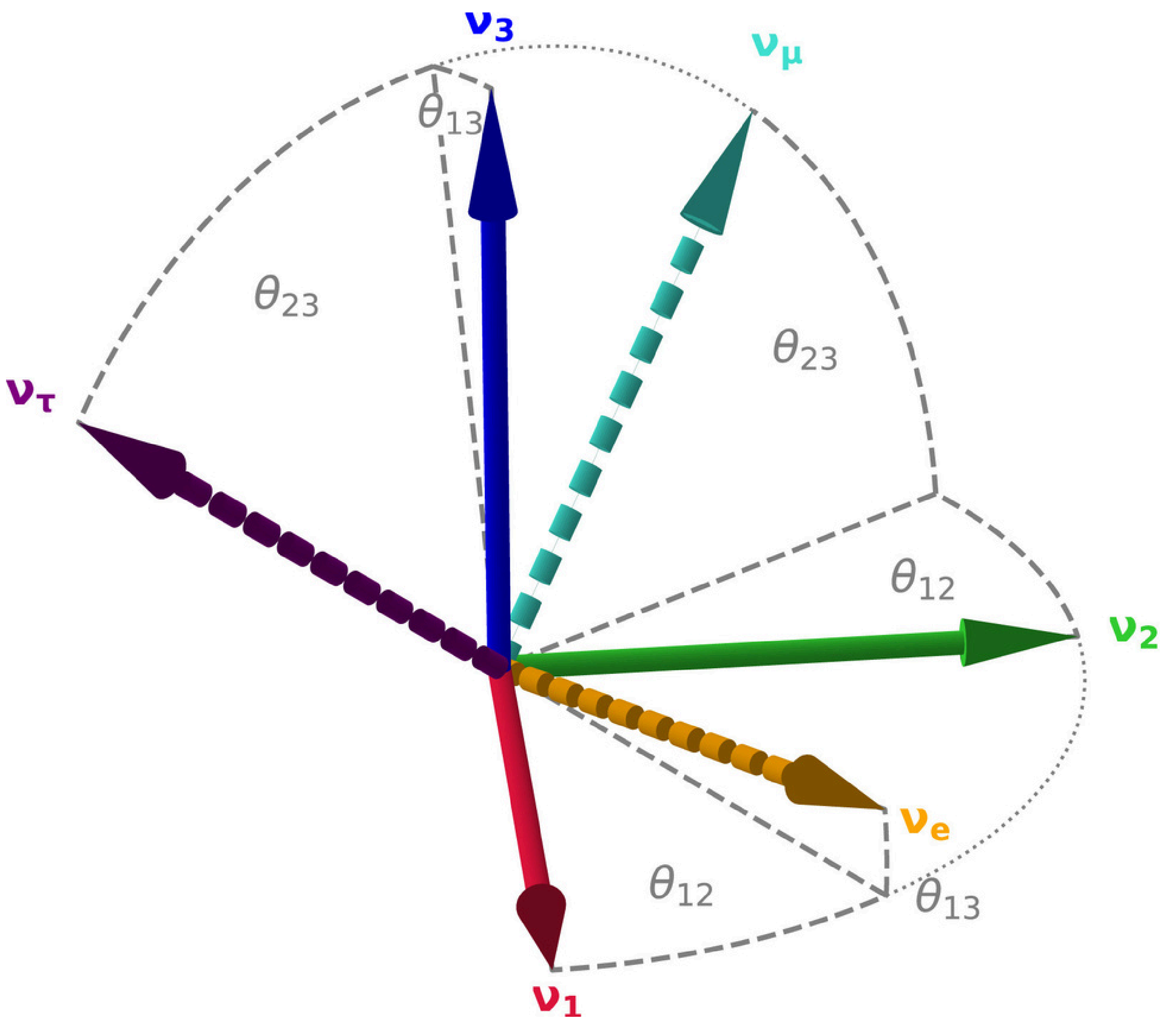
Neutrino oscillations

Flavour

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U^*_{\alpha k} |\nu_k\rangle$$

Mass

Mixing matrix “PMNS”



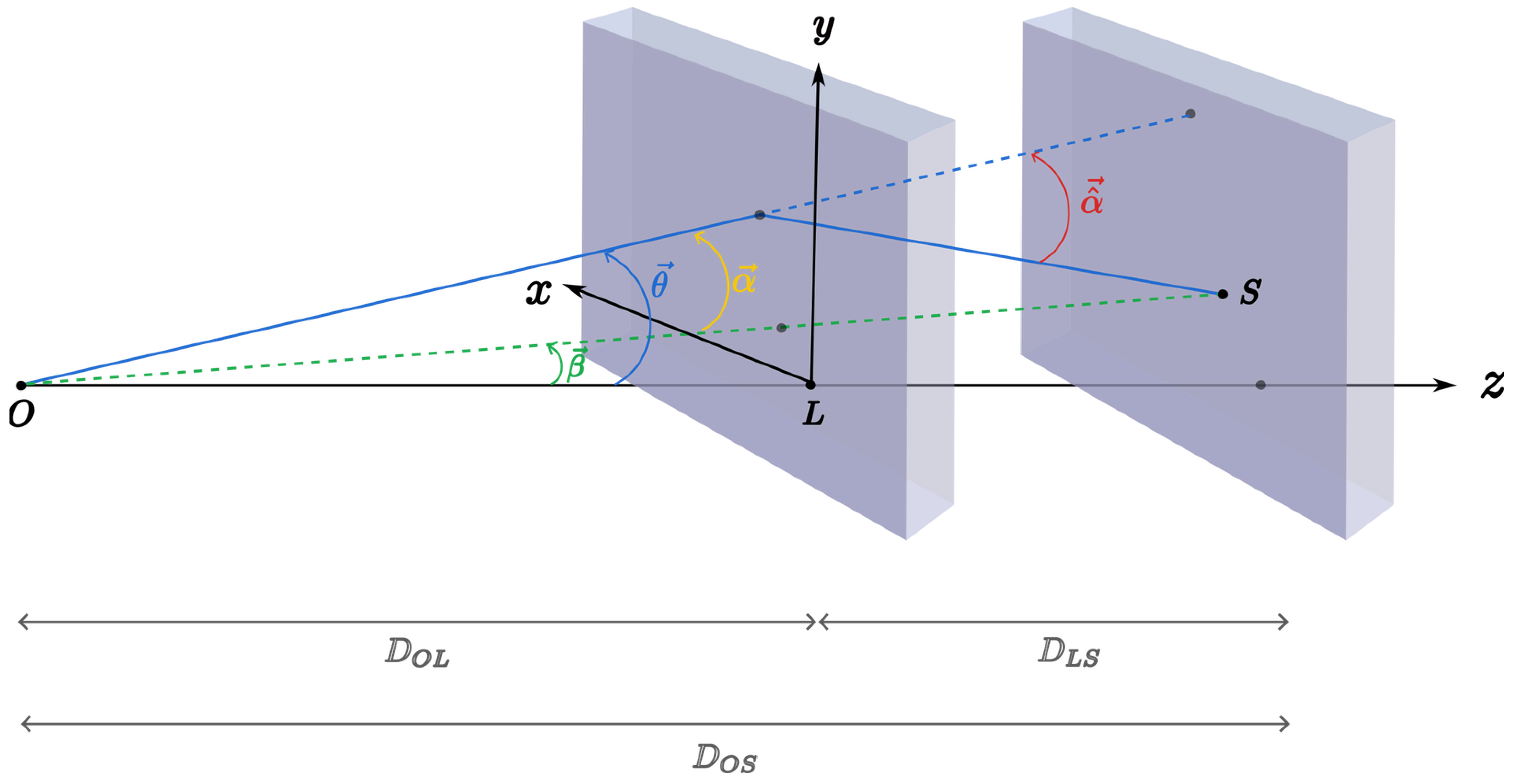
Transition probability (in vacuum)

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\beta\alpha} - 2\sum_{i>k} \mathrm{Re}\left(U_{\alpha' i} U^*_{\alpha' k} U^*_{\alpha i} U_{\alpha k}\right)\left(1 - \cos\frac{\Delta m^2_{ki}}{2E} t\right)$$
$$+ 2\sum_{i>k} \mathrm{Im}\left(U_{\alpha' i} U^*_{\alpha' k} U^*_{\alpha i} U_{\alpha k}\right) \sin\frac{\Delta m^2_{ki}}{2E} t$$

Observables :
- Energie E
- *Travel time t*

Mass constraint : $\Delta m^2_{ik} = m_i^2 - m_k^2$

Lens equation



Lens equation :

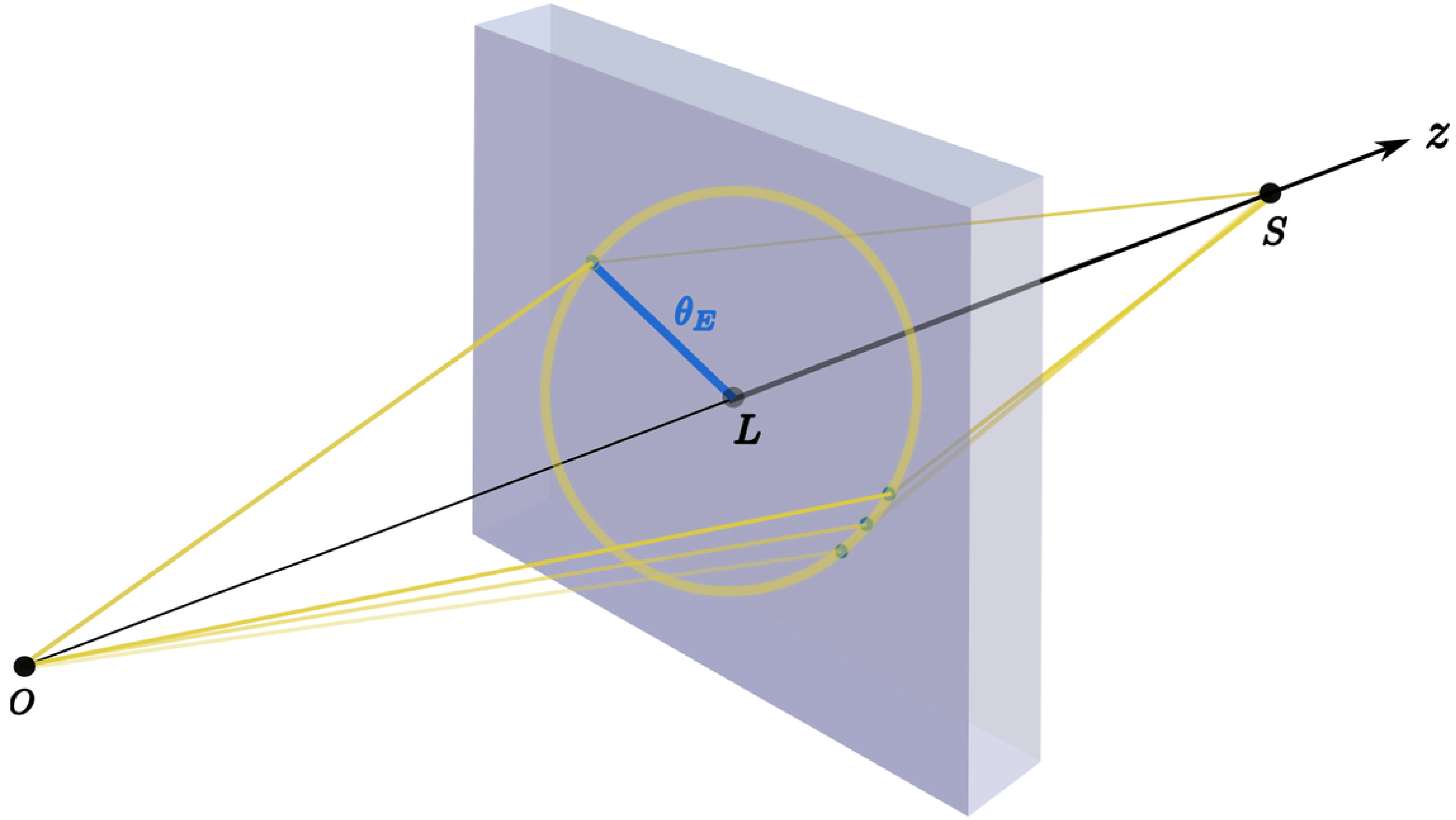
$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

$$\vec{\alpha} = \vec{\hat{\alpha}}(\vec{\theta}, D_{\mathrm{OL}})\frac{D_{LS}}{D_{OS}}$$

Point lens

$$\vec{\beta} = \vec{\theta}(1 - \frac{\theta_E^2}{\theta^2})$$

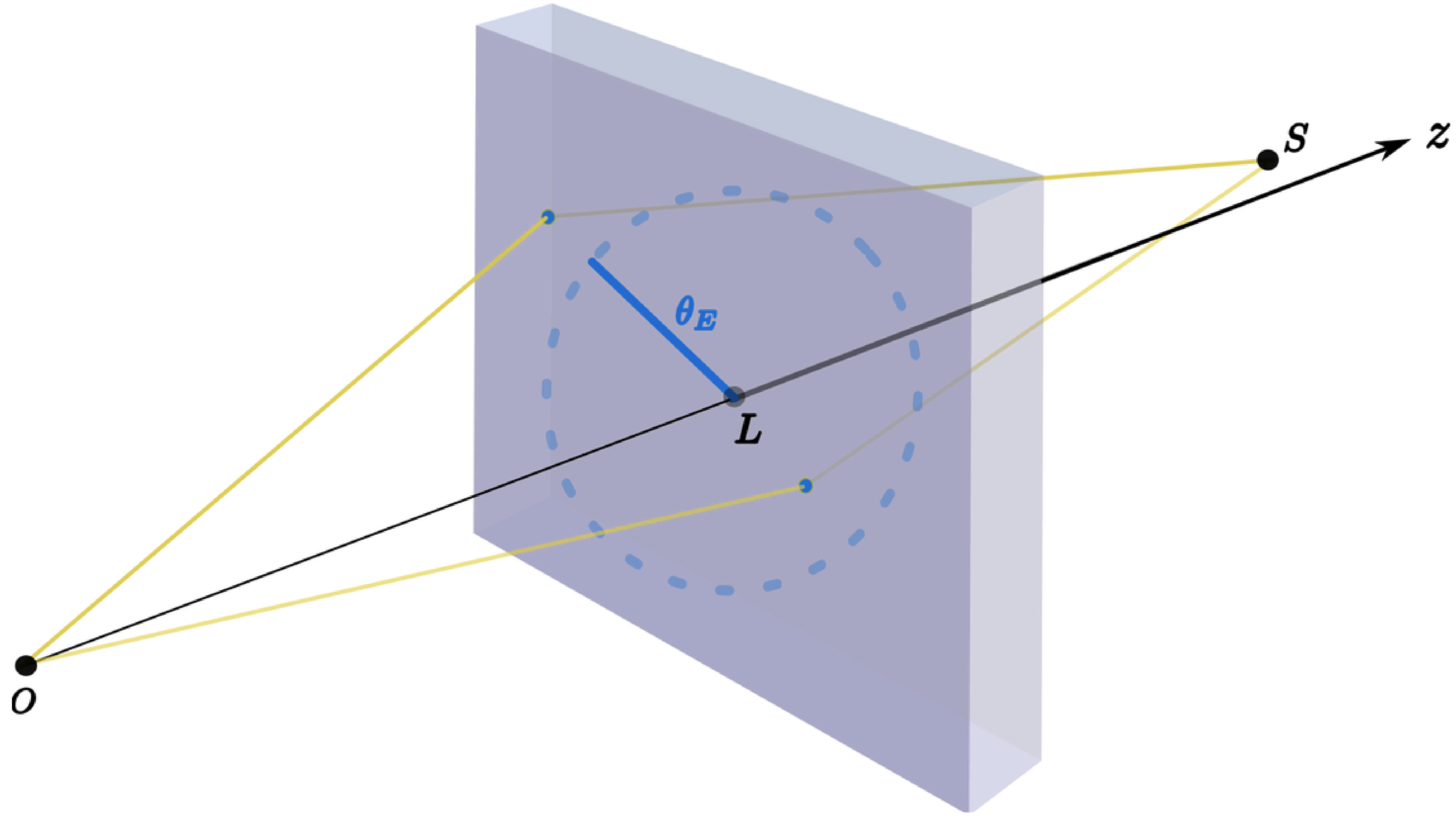
O, L and S aligned : Einstein ring

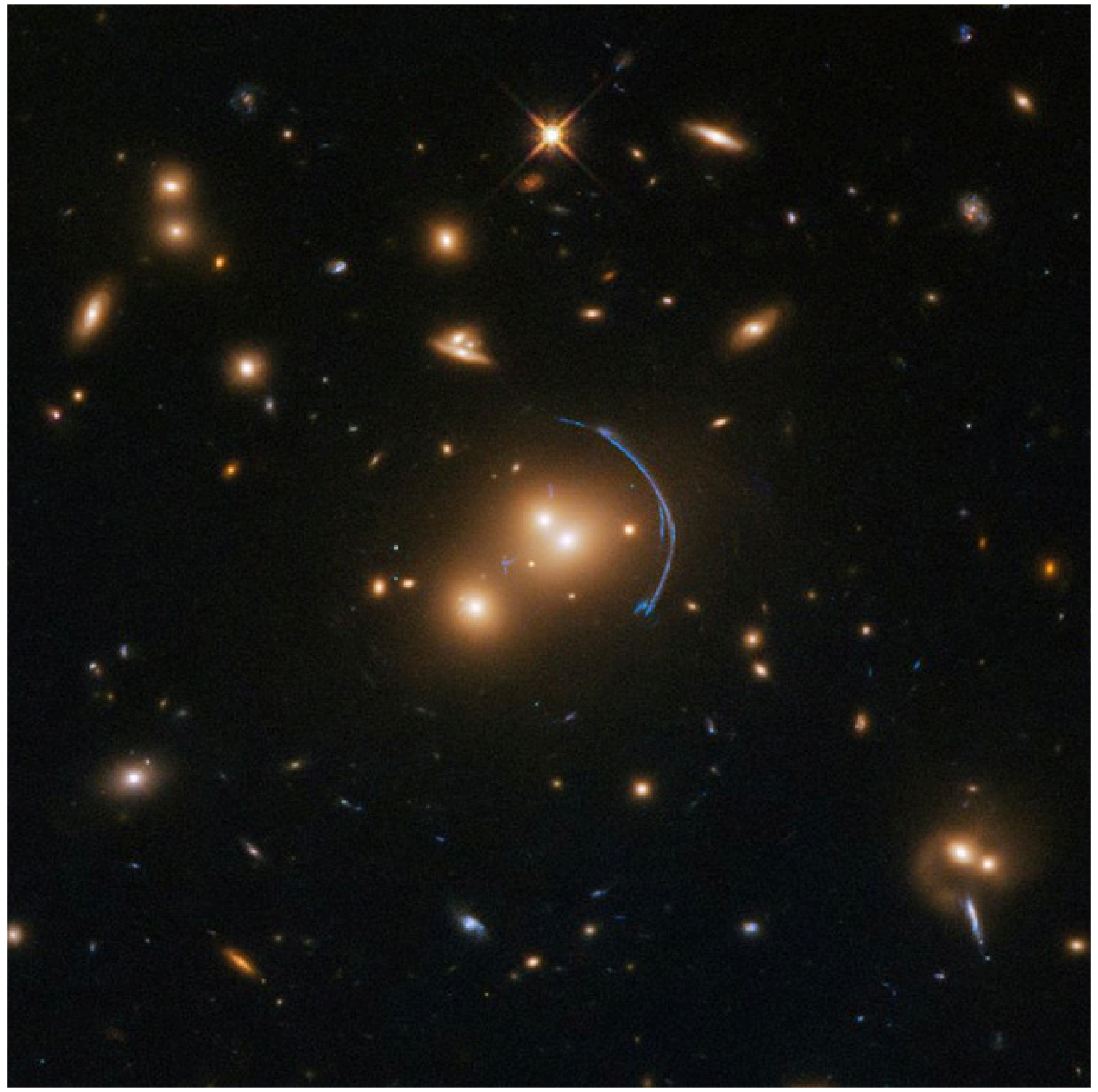


Point lens

$$\vec{\beta} = \vec{\theta}(1 - \frac{\theta_E^2}{\theta^2})$$

O, L and S no aligned : 2 images





Observable	Valeur actuelle (95 % C.L.)	Perspective proche
θ_{23}	$45^o \pm 10^o$	$P(\nu_\mu \rightarrow \nu_\mu)$ - MINOS et CNGS
θ_{12}	$33^o \pm 3^o$	SNO (NC) et KamLAND
θ_{13}	$< 13^o$ si $\Delta m^2_{13} = 2 \times 10^{-3}$	Réacteurs (P($\overline{\nu_e} \rightarrow \overline{\nu_e}$)) et Long baseline ($P(\nu_\mu \rightarrow \nu_e)$)
Δm^2_{13}	$(2.0 - 2.5 \pm 1.0) \times 10^{-3}$ eV^2	$P(\nu_\mu \rightarrow \nu_\mu)$ - MINOS et CNGS
Signe de Δm^2_{13}	?	$P(\nu_\mu \rightarrow \nu_e)$ et ($P(\overline{\nu_\mu} \rightarrow \overline{\nu_e})$)- Long baseline
Δm^2_{12}	$(7.3 \pm 1.0) \times 10^{-5}$ eV^2	KamLAND (P($\overline{\nu_e} \rightarrow \overline{\nu_e}$))
Signe de Δm^2_{12}	Connu via l'effet MSW : +	
δ (CP)	?	$P(\nu_\mu \rightarrow \nu_e)$ et ($P(\overline{\nu_\mu} \rightarrow \overline{\nu_e})$)- Long baseline
Majorana ?	?	Expériences $\beta\beta0\nu$
Φ_2	?	Expériences $\beta\beta0\nu$ (peut-être)
Φ_3	?	Sans espoir !
m_ν	$\Sigma m_\nu < 1$ eV	Cosmologie, Désintégration $\beta\beta0\nu$, Désintégration β

Source : D. Vignaud. Comment la masse vint aux neutrinos. 2004. ⟨in2p3-00022196⟩

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\mathrm{CP}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & c_{23}c_{13} \end{pmatrix}$$

$$U = \begin{pmatrix} 0.79 - 0.86 & 0.50 - 0.61 & 0. - 0.16 \\ 0.24 - 0.52 & 0.44 - 0.69 & 0.63 - 0.79 \\ 0.26 - 0.52 & 0.47 - 0.71 & 0.60 - 0.77 \end{pmatrix}$$

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