# Beyond Fick & Fourier: The Kinetic Exclusion Process

From Microscopic Rules to Time Crystals

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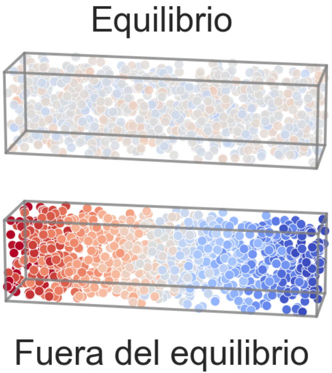
# The problem: transport out of equilibrium

**Fick and Fourier** describe diffusive transport well... close to equilibrium.

But real systems feature:

- Sustained net currents (out of equilibrium).



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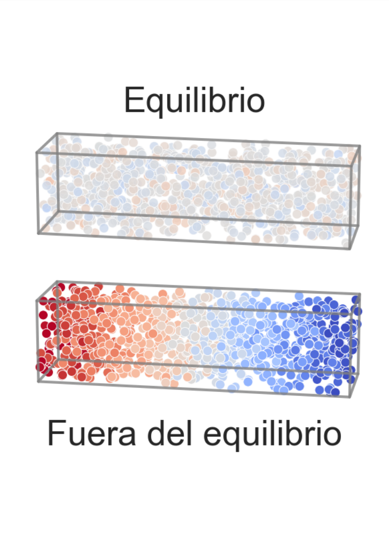

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But real systems feature:

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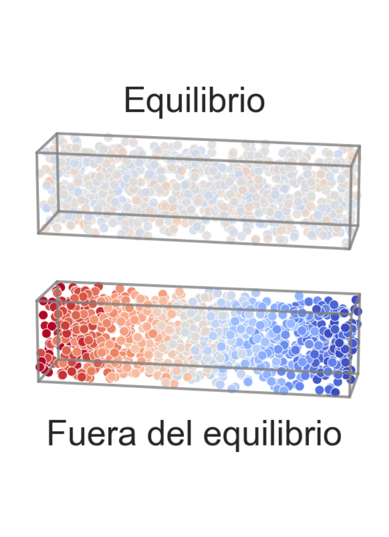
- Coupled conserved fields (mass, energy).
- No universal macroscopic framework for their fluctuations.



# The model: Kinetic Exclusion Process (KEP)

A 1D lattice gas where particles carry mass and energy stochastically and in a coupled way.

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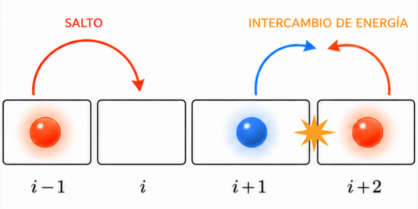

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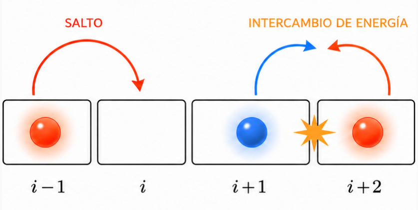

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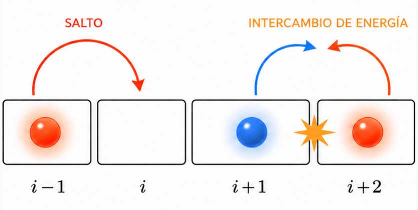
- **Macroscopic Fluctuation Theory (MFT):** rare-event statistics, beyond the Gaussian limit.



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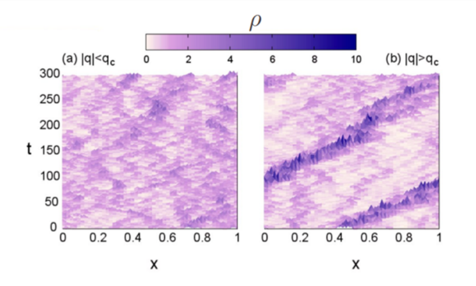
- For $|q| < q_c$: a profile that is stationary on average.

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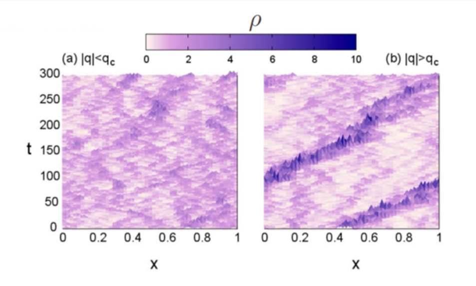


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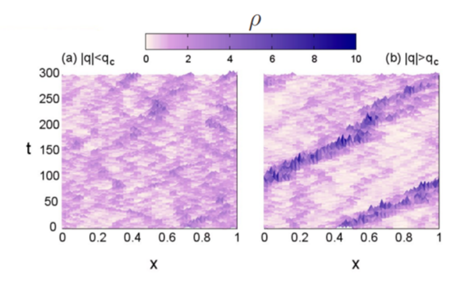
- This is the signature of a **time crystal** as the optimal mechanism to sustain the fluctuation.

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- First realized experimentally in isolated quantum systems (trapped ions, NV centers) – a very active frontier in condensed matter and quantum computing (stable periodic drives, robust qubits, frequency references).
- Our result suggests a classical, dissipative route to time-crystal-like behaviour: it emerges as the optimal strategy to sustain a rare, extreme fluctuation – not as a special property of isolated quantum systems.