Standard Model and Beyond
(with a focus on LHC Physics)

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PLAN TODAY

- SR + QM ---→ QFT
- Fields & properties : scalar (Klein-Gordon eq), spin $\frac{1}{2}$ (Dirac eq.)
  spin 1 (E.M., Maxwell eq.)
- Scattering & interactions in QFT : Feynman diagrams
  Cross sections
- Interactions from symmetries (GAUGE INVARIANCE)
  - QED
  - YANG-MILLS THEORIES
- EWSB : boson (& fermion) masses, Higgs mechanism
- Higgs pheno @ LHC

PLAN TOMORROW

- SM pheno @ LHC
- SM shortcomings
- BSM : some examples (↳ DIRECT SEARCHES)
- New Physics through precision measurements (↳ INDIRECT SEARCHES)
- (SM) Effective Field Theory

"Elementary Particle Physics is the quadrant of Nature whose laws can be written in a few lines with absolute precision and the greatest empirical adequacy"

R. Barbieri

⇒ Find elementary constituents of Nature and understand how they interact

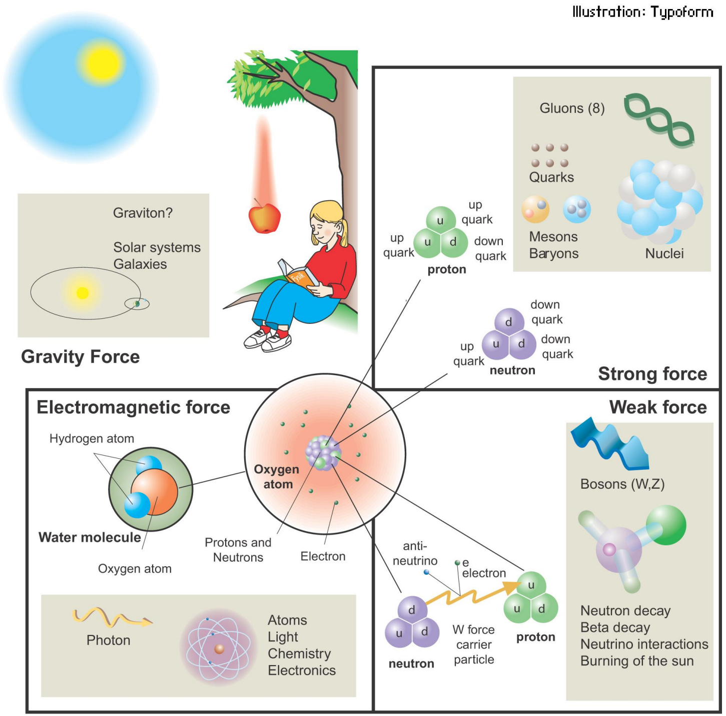
How do things interact?

- Forces we experience daily: gravity & electromagnetism
- electricity and magnetism are one and the same thing
- Maxwell [~1865]: unification = simplicity
  ⇒ 4 equations, many phenomena

How do things interact?

Intensity: 1
Range: ∞

Intensity: 10^{38}
Range: 10^{-15} m

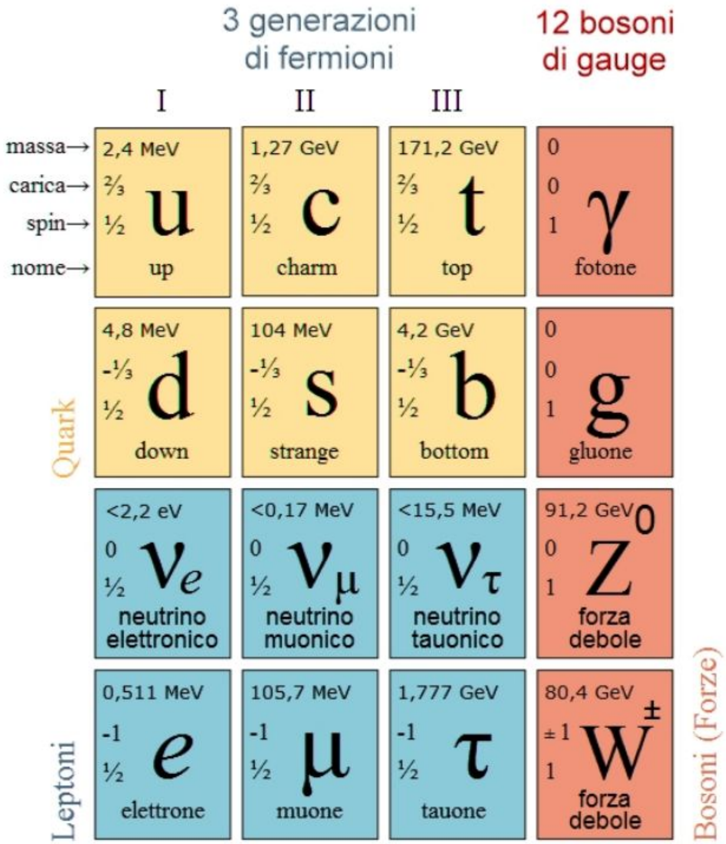


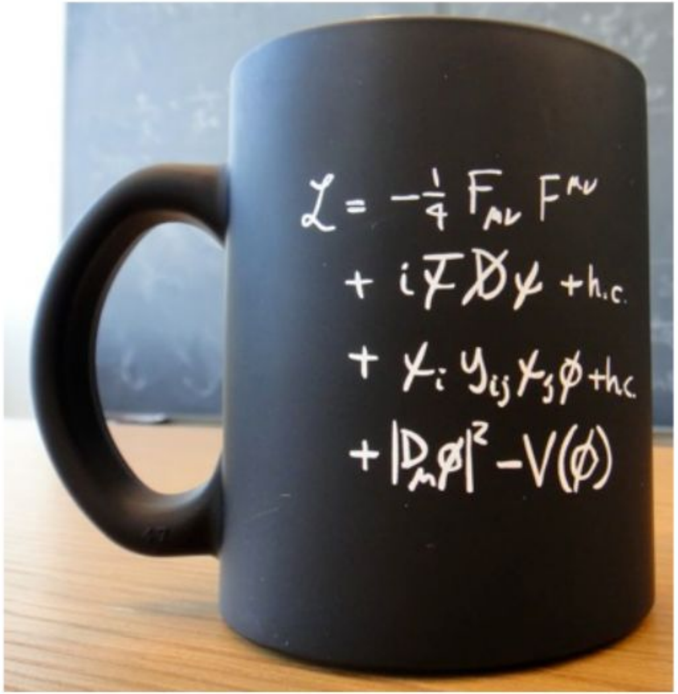
Intensity: 10^{36}
Range: ∞

Intensity: 10^{31}
Range: 10^{-18} m

⇒ Weak force carriers need be massive

The SM of Particle Physics, in a nutshell





The SM of Particle Physics: theory

- All observed elementary particles and interactions [except gravity]
- QFT: unified language of QM and special relativity
- guiding principle: simplicity & (gauge) symmetry
  → Interactions are mediated by gauge bosons, EW unification

- The SM is a non-abelian gauge theory:

$$\boxed{SU(3)_c \times SU(2)_L \times U(1)_Y\text{, where } SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}}$$

- SM is renormalizable [finite number of inputs → plethora of predictions]
- Extremely successful, *e.g.* particles predicted before their discovery
- Masses of gauge bosons → Electroweak Symmetry Breaking (EWSB)

[Feynman,Schwinger,Tomonaga,Glashow,Salam,Weinberg,'t Hooft,Veltman,Gross,Politzer,Wilczek,...]

SPECIAL RELATIVITY I (BASICS)

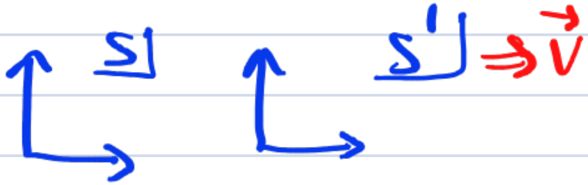
2 POSTULATES
1) laws of Physics are the same in all inertial frames
2) speed of light (in vacuum) is the same for all inertial observers

Maxwell equations: fundamental + invariant under Lorentz transformation (L.T.)

Invariance under L.T. $\Rightarrow$ Newton mechanics no longer correct

SPECIAL RELATIVITY II (spacetime, mechanics)

- spacetime : $(t, \bar{x}) \longrightarrow x^\mu = (ct, \vec{x})$
- L.T. frame S $\rightarrow$ frame S'



$$x'^\mu = \Lambda^\mu{}_\nu \, x^\nu \qquad \Lambda^\mu{}_\nu = \begin{bmatrix} \gamma & -\gamma v & 0 & 0 \\ -\gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Invariant distance & Minkowski space : $(ds)^2 = (c\,dt)^2 - (d\bar{x})^2$
- Lorentz group : $SO(1,3)$ $\qquad \Lambda^T g \Lambda = g$
- Mechanics changed :
  - $p^\mu = m U^\mu = (\gamma m c, \gamma m \vec{u})$
  - $p^2 = m^2 \Rightarrow E^2 = (mc^2)^2 + (\vec{p}c)^2$
  - $\dfrac{d\vec{p}}{dt} = \vec{F} \rightsquigarrow \dfrac{dp^\mu}{d\tau} = f^\mu$

SPECIAL RELATIVITY II (E.M.)

- E.M. is <u>not</u> changed. Maxwell's equation can be written using tensor notation:

$$\partial_\mu F^{\mu\nu} = J^\nu$$

$\partial_\mu = (\partial_t, \vec{\nabla})$
$\partial^\mu = (\partial_t, -\vec{\nabla})$

4-current $J^\nu = (\rho, \vec{J})$

Faraday's tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

$A^\mu = (\varphi, \vec{A})$ potentials

- Tensor notation $\leftrightarrow$ eq. is invariant under L.T.:

$$S \to S' \qquad \partial_\mu F^{\mu\nu} = J^\nu \longrightarrow \partial'_\mu F'^{\mu\nu} = J'^\nu$$

SPECIAL RELATIVITY (LAGRANGIAN OF E.M. FIELD)

- E.M. can be formulated with Hamilton principle:

  $S = \int d^4x \, \mathcal{L} \qquad \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - J_\mu A^\mu$

  → least action principle → Euler-Lagrange equations

  → with above S: Euler-Lagrange equations are $\partial_\mu F^{\mu\nu} = J^\nu$

- Notice: S must be a Lorentz scalar (invariant)

  ⇒ $\mathcal{L}$ must be a scalar

QUANTUM MECHANICS

- Nature is intrinsically probabilistic
- State of system → wavefunction $|\psi(t)\rangle$, $\psi(t,\vec{x})$
- Observables → hermitian operators. $\hat{x}, \hat{p}, \hat{H}, \vec{L}, \dots$ $[\hat{x},\hat{p}] = i\hbar$
- Evolution in time : Schroedinger equation $i\hbar\partial_t |\psi\rangle = \hat{H}|\psi\rangle$
- Probability : $\rho(t,\vec{x}) = |\psi(t,\vec{x})|^2$, $\int d^3x\, \rho = 1$

  continuity equation: $\partial_t \rho + \vec{\nabla}\cdot\vec{J} = 0$

- $\mathrm{Prob}(A\to B) = \left|\langle B|\exp\left(-i\frac{Ht}{\hbar}\right)|A\rangle\right|^2$
- Main assumptions: FIXED NUMBER OF PARTICLES → HILBERT SPACE

  PARTICLE POSITION ← WAVEFUNCTION

NOTATION, UNITS

A^μ, B^μ 4-momenta $\qquad A\cdot B = AB = A^\mu B_\mu = A^0 B^0 - \vec{A}\cdot\vec{B}$

$kx = k^0 x^0 - \vec{k}\cdot\vec{x}$

Natural units: $\hbar = c = 1$, $\varepsilon_0 = 1$ $\rightarrow$

$\rightarrow [\hat{x}, \hat{p}] = i$

$\rightarrow \left(-\frac{1}{2m}\nabla^2 + V\right)\psi = i\partial_t \psi$

$\rightarrow$ Maxwell: $\vec{\nabla}\cdot\vec{E} = \rho$ .....

$\rightarrow x^\mu = (t, \vec{x}) \qquad E^2 = m^2 + \vec{p}^{\,2}$

$\rightarrow$ Wave eq. $(\partial_t^2 - \nabla^2)\varphi = 0$

$\frac{d^4k}{(2\pi)^3}\,\delta(k^2 - m^2)\,\Theta(k^0)$ is Lorentz invariant

It's equal to: $[dk] = \frac{d^3k}{(2\pi)^3}\frac{1}{2E_k} \qquad E_k = \sqrt{m^2 + \vec{k}^2}$

QM + SR : problems I

- Problems : 1) Schroedinger eq. is NOT Lorentz invariant

$$i\hbar\partial_t\Psi = \hat{H}\Psi \qquad \hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + V$$

1st order 2nd order

2) QM : fixed number of particles

SR : $E = mc^2 \Rightarrow$ particles can be created $pp \to pp\,p\bar{p}$

$pp \to$ Higgs $+ \ldots$

decays : $n \to p\, e^-\, \bar{\nu}_e$, $\mu \to e^-\, \bar{\nu}_e\, \nu_\mu$

3) QM : $\Psi(t,\vec{x})$: evolves istanteneously over all space

SR : no signals faster than light $\Rightarrow [O(x), O(y)] = 0$

if $(x-y)^2 < 0$

QM + SR : problems II

- Schroedinger eq. $\leftarrow$ quantizing dispersion relation

$$E = \frac{\vec{p}^{\,2}}{2m} \longrightarrow \begin{cases} E \to i\hbar\partial_t \\ \vec{p} \to -i\hbar\vec{\nabla} \end{cases} \to i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi$$

- Let's try with $E^2 = \vec{p}^{\,2} + m^2$ [from here $c = \hbar = 1$]

Klein-Gordon equation: $(\partial_t^2 - \nabla^2 + m^2)\varphi = 0 \qquad (\Box + m^2)\varphi = 0$

K.G. Lagrangian: $\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi)(\partial^\mu\varphi) - \frac{m^2}{2}\varphi^2$

Relativistic wave equation, free particle, no spin

- Solution to K.G. equation has several problems,

QM + SR : problems III

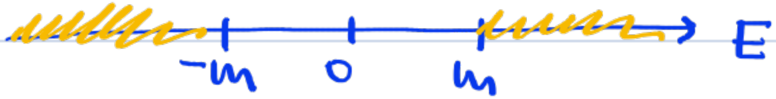
① Negative energy solutions

$$\varphi(t,\vec{x}) = \int [dk] \left(a_-(\vec{k})\, e^{-ikx} + a_+(\vec{k})\, e^{+ikx} \right)$$

$\uparrow \varphi_-$ $\uparrow \varphi_+$

$$E \mapsto i\partial_t \quad \Rightarrow \quad i\partial_t \varphi_\pm = \pm E_k \varphi_\pm \qquad E_k = \sqrt{m^2 + \vec{k}^2}$$

- Not surprising! We started from $E^2 = \vec{k}^2 + m^2$
- Energy spectrum (free particle)



not bounded from below.

★ Trying to ignore φ_- (or starting from $E = \sqrt{m^2 + \vec{k}^2}$)

DOES NOT FIX the problems [see e.g. Alvarez Gaumé]

QM + SR : problems IV

② Negative probability

QM: Schroedinger eq. admits a continuity equation

$$\rho = |\psi(t,\vec{x})|^2 \qquad \vec{J} = \frac{-i}{2m}\left(\psi^* \vec{\nabla}\psi - \psi\, \vec{\nabla}\psi^*\right)$$

$$\partial_t \rho + \vec{\nabla}\cdot\vec{J} = 0$$

↳ $\rho > 0$: can be interpreted as a probability density

$\frac{d}{dt}\int d^3x\, \rho$ conserved $\Rightarrow$ conservation of probability

KG : $\rho = i\left(\varphi^* \partial_t \varphi - \varphi\, \partial_t \varphi^*\right) \qquad \vec{J} = -i\left(\varphi^* \vec{\nabla}\varphi - \varphi\, \vec{\nabla}\varphi^*\right)$

↳ ρ is NOT POSITIVE DEFINITE (due to states with $E<0$)

QM + SR: from particles to fields (QFT)

Solution: quantize fields

particles ~~are~~ fundamental → • fields are fundamental

• particles: elementary excitations of fields

Real scalar field: $\varphi(t,\vec{x}) = \int [dk] \left(a(\vec{k})\, e^{-ikx} + a^*(\vec{k})\, e^{ikx} \right)$

$\varphi_{\vec{k}} = a\, e^{-ikx} + a^* e^{ikx}$: $\ddot{\varphi}_k + \omega^2 \varphi_k = 0$ $\quad \omega = \sqrt{m^2 + \vec{k}^2}$

→ $\varphi(x)$: collection of infinite independent harmonic oscillators

QUANTIZATION: $a, a^\dagger$: $[a(\vec{k}), a^\dagger(\vec{k}')] = (2\pi)^3 2E_k\, \delta(\vec{k} - \vec{k}')$

$[a, a] = 0 \quad [a^\dagger, a^\dagger] = 0$

$\hat{\varphi}$ has become an operator-valued function, that satisfies the KG equation

QFT: states & field

STATES : $|0\rangle$ such that $a(\vec{k})|0\rangle = 0$

$$|\vec{k}\rangle = a^\dagger(\vec{k})|0\rangle$$

$$|\vec{k}_1, \vec{k}_2\rangle = a^\dagger(\vec{k}_1)\, a^\dagger(\vec{k}_2)|0\rangle$$

$\vdots$

FOCK SPACE : $\mathcal{F} = \bigoplus_{N=0}^{\infty} \mathcal{H}_N$

Hilbert space for N particles

$$\hat{\varphi}(x) = \int [dk] \left\{ a(\vec{k}) e^{-ikx} + a^\dagger(\vec{k}) e^{ikx} \right\}$$

QFT: how problems are solved

$\hat{\varphi}$ is a operator-valued function, solution of KG equation

→ RELATIVISTIC EQ ✓

$\hat{\varphi}$ creates and annihilates

→ NUMBER OF PARTICLES NOT FIXED ✓

$\hat{\varphi}$ is an operator that depends on t (and $\vec{x}$), states $|\rangle$ do not depend on t → Heisenberg picture

It holds that $[\varphi(x), \varphi(y)] = 0$ if $(x-y)^2 < 0$

→ (MICRO) CAUSALITY ✓

$\hat{\varphi}(t=0, \vec{x})\,|0\rangle = \int [dk]\, e^{-i\vec{k}\cdot\vec{x}}\, |\vec{k}\rangle$ linear comb. of momentum eigenstates

AN INTERLUDE

$$\mathcal{H} = \pi\dot{\varphi} - \mathcal{L} \quad , \pi = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}}$$

$$H = \int d^3x\, \mathcal{H} \overset{KG}{=} \int d^3x\, \frac{1}{2}\left[\pi^2 + |\nabla\varphi|^2 + m^2\right]$$

- POSITIVE DEFINITE ✓

After quantization: $\hat{H} = \int [dk]\, E_{\vec{k}} \left[N(\vec{k}) + \frac{1}{2}\delta(\vec{0}) \right]$

number operator

- indeed: sum of ∞ harmonic oscillators
- $\langle 0 | \hat{H} | 0 \rangle = \infty$

- Vacuum energy is ∞
- only energy differences are measured
- Formally: $:\hat{H}: = \hat{H} - \langle 0|\hat{H}|0\rangle$

QFT: CANONICAL QUANTIZATION I

1) Fundamental object: action and $\mathcal{L}$

2) Classical fields: $\varphi(x)$, $\pi(x) = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}}$ → they become $\hat{\varphi}, \hat{\pi}$

3) Quantization: coefficient of Fourier modes get FORMALLY PROMOTED to creation/annihilation operators

Requirement: $[\hat{\varphi}(x), \hat{\pi}(y)] = i\,\delta(\bar{x}-\bar{y})$ if $x^0 = y^0$ (equal time)

... this is also important because it ensures that

$$[\hat{O}(x), \hat{O}(y)] = 0 \quad \text{if } (x-y)^2 < 0$$

(important for causality)

4) If field $\hat{\varphi}$ has other quantum numbers (~ index), then $a, a^\dagger$ will carry such quantum number. ⇒ STATES (→ particles) WILL HAVE IT

QFT: CANONICAL QUANTIZATION II

5) QFT: naturally formulated in Heisenberg picture.

$| \rangle \rightarrow$ no time dependence

$\mathcal{O}(t,\bar{x}) = \mathcal{O}(x) \rightarrow$ time dependence

As in QM: $i \frac{d}{dt} \hat{\varphi} = [\hat{\varphi}, \hat{H}]$

$$\Rightarrow \mathcal{O}(t,\bar{x}) = e^{iHt}\, \mathcal{O}(t=0,\bar{x})\, e^{-iHt}$$

Notice that, so far, $\mathcal{L}$ was bilinear in the fields

↳ free fields

DIRAC EQUATION I

- Attempt to find another generalization of the Schroedinger equation
- Instead of eq quadratic in ∂_t^2 and $\vec{\nabla}^2$, try with linear
- We will find an equation for a fermionic field
  - ↳ extra SPINORIAL index attached to ψ is unavoidable

- $i\partial_t \psi = \hat{H} \psi \qquad (*)$

$\rightsquigarrow$ try $\hat{H} = -i\alpha_i \partial^i + \beta m = +i\,\vec{\alpha}\cdot\vec{\nabla} + \beta m$

α_i, β : $n\times n$ matrices, n unknown $\qquad \psi = \psi_\sigma \quad \sigma = 1 \dots n$

$\rightsquigarrow$ S.R. : $E^2 = \vec{p}^{\,2} + m^2 \Rightarrow (*)$ "squared" $\equiv$ K.G.

DIRAC EQUATION II

$$\partial_t \psi = \left(\vec{\alpha}\cdot\vec{\nabla} - i\beta m \right)\psi$$

$$\partial_t^2 = \frac{1}{2}\{\alpha_i, \alpha_j\}\partial^i\partial^j - \beta^2 m^2 - \frac{i}{2}\{\alpha_i, \beta\}\partial_t\partial^i$$

KG: $\partial_t^2 = \partial^i\partial^i - m^2$

$$\Rightarrow \{\alpha_i, \alpha_j\} = 2\delta_{ij}\mathbb{1}_n, \quad \{\alpha_i, \beta\} = 0, \quad \beta^2 = \mathbb{1}_n$$

Define $\beta = \gamma^0$ $\quad \beta\alpha_j = \gamma_j$ $\quad \Rightarrow \boxed{\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\mathbb{1}_n}$ $[\gamma]$

Eq. (*) gives: $\quad (i\gamma^\mu\partial_\mu - m)\psi = 0 \quad$ Dirac eq.

In order to realise $[\gamma]$: $n = 4$ $\quad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$ $\quad \sigma^\mu = (1, \vec{\sigma})$, $\bar{\sigma}^\mu = (1, -\vec{\sigma})$

DIRAC FIELD: PROPERTIES I

- $\psi, \psi^\dagger$ ~ more useful to use $\bar{\psi} = \psi^\dagger \gamma^0$
- $\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$
- As anticipated, ψ_α has 4 components → not a spin 0 field
- $\gamma_5 =: i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\psi = \begin{vmatrix} \psi_L \\ \psi_R \end{vmatrix} = \begin{vmatrix} \psi_- \\ \psi_+ \end{vmatrix} \qquad \psi_{L/R} = P_{L/R}\,\psi \qquad P_{L/R} = \frac{1 \mp \gamma_5}{2}$$

- If $m=0$, then ψ_L and ψ_R are completely decoupled:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu)\psi = i\psi_L^\dagger \bar{\sigma}^\mu \partial_\mu \psi_L + i\psi_R^\dagger \sigma^\mu \partial_\mu \psi_R$$

→ ψ_L and ψ_R (Weyl spinors, 2 components) are more fundamental than ψ

→ A priori, ψ_L and ψ_R can have different interactions (AS IN THE SM)

DIRAC FIELD : PROPERTIES II

• Properties: ① under 3-dim rotations
$$\psi_L \to e^{-i\vec{\theta}\cdot\frac{\vec{\sigma}}{2}}\psi_L$$
$$\psi_R \to e^{-i\vec{\theta}\cdot\frac{\vec{\sigma}}{2}}\psi_R$$

$\Rightarrow \vec{J} = \frac{\vec{\sigma}}{2}$ for ψ_L and ψ_R $\Rightarrow$ SPIN $\frac{1}{2}$!!!

② under parity $\psi_L \leftrightarrow \psi_R$

③ Weyl equation : $(\partial_0 \pm \vec{\sigma}\cdot\vec{\nabla})\psi_\pm = 0$

$\Rightarrow \partial_\mu\partial^\mu\psi_\pm = 0$ $\Rightarrow$ MASSLESS (expected)

solution: $\psi_\pm(x) \sim u_\pm(\vec{k})\, e^{-ikx}$, with $k^0 = |\vec{k}|$

Inserting in Weyl equation : $\left(\frac{1}{2}\vec{\sigma}\cdot\frac{\vec{k}}{|\vec{k}|}\right)u_\pm = \pm\frac{1}{2}u_\pm$

$\Rightarrow u_\pm$ are eigenvectors of helicity operator $\vec{S}\cdot\frac{\vec{k}}{|\vec{k}|}$

$u_+ \to$ right handed $\quad u_- \to$ left handed

DIRAC FIELD: PROPERTIES III

- Adding back m: $\mathcal{L} = i\psi_L^\dagger \bar{\sigma}^\mu \partial_\mu \psi_L + i\psi_R^\dagger \sigma^\mu \partial_\mu \psi_R +$

$$- m\left(\psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L\right)$$

$\to$ mass term mixes L and R fields

VECTOR FIELDS

- In full generality, let's assume it can also be massive.
- $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu$ [Proca lagrangian]
- Under a Lorentz transformation: $A^\mu \to A'^\mu = \Lambda^\mu{}_\nu A^\nu$

$\rightsquigarrow$ under rotations in 3d: $A^0 \to A^0$

$\vec{A} \to R\vec{A}$

$\Rightarrow$ SPIN = 1

- Notice that, if I require gauge symmetry, i.e.

Under $A^\mu \to A^\mu + \partial^\mu \alpha$, $\mathcal{L}$ invariant $\Rightarrow$ NO MASS TERM ALLOWED

☆ General properties of spin $0, \frac{1}{2}, 1$ fields ARE CONSEQUENCES OF Lorentz / Poincaré invariance

QUANTIZATION

• Dirac field:

$$\psi(x) = \sum_s \int [dk] \left\{ b_s(\vec{k})\, u_s(\vec{k})\, e^{-ikx} + d^+(\vec{k})\, v_s(\vec{k})\, e^{ikx} \right\}$$

$$\rightarrow (i\gamma^\mu \partial_\mu - m)\psi = 0 \qquad \Rightarrow \begin{cases} (\not{k} - m)\, u_s = 0 \\ (\not{k} + m)\, v_s = 0 \end{cases}$$

$$\rightarrow \text{spin } \tfrac{1}{2} \Rightarrow \cancel{[\psi, \pi] = i\delta \ldots} \qquad \{\psi_\alpha(x), \pi_\beta(y)\} = i\,\delta(\bar{x}-\bar{y}) \text{ if } x^0 = y^0$$

realized if $\{d_s(\vec{k}), d^+_{s'}(\vec{k}')\} = (2\pi)^3\, 2E_k\, \delta(\vec{k}-\vec{k}')\, \delta_{ss'}$

• Vector field

$$A^\mu(x) = \sum_\lambda \int [dk] \left\{ a(\vec{k},\lambda)\, \epsilon^\mu(\vec{k},\lambda)\, e^{-ikx} + a^+(\vec{k},\lambda)\, \epsilon^{*\mu}(\vec{k},\lambda)\, e^{ikx} \right\}$$

$$\rightarrow \text{if } k^\mu = (k^0, 0, 0, |\vec{k}|) \qquad \Rightarrow \begin{cases} \epsilon_{T1} = (0100) \\ \epsilon_{T2} = (0010) \\ \epsilon_L = \left(\frac{|\vec{k}|}{m}, 0, 0, \frac{k^0}{m}\right) \end{cases}$$

$m = 0 \Rightarrow$ ONLY TRANSVERSE POLARIZATIONS

SCATTERING, INTERACTIONS, S-MATRIX, PERT. THEORY & FEYNMAN DIAGRAMS

- So far, no interaction, but field will interact --- $\mathcal{L}_{int}$
- In general, solving EoM (or Heisenberg eq.) for full $H = H_0 + H_{int}$ NOT POSSIBLE
- We'd still like to use $a, a^\dagger$: contact with particles, etc...
- Use INTERACTION PICTURE

$$H = H_0 + H_{int} \quad \rightarrow \quad |\psi(t)\rangle_I = \exp(i H_0 t)\,|\psi(t)\rangle_S \qquad [\rightarrow \text{no int} \Rightarrow H = I]$$

$$i\,\partial_t O_I(t) = [O_I, H_0]$$

$$i\,\partial_t |\psi(t)\rangle_I = H_{int}\,|\psi(t)\rangle_I$$

O_I : EASY, evolve with H_0

Complication is shifted to $|\psi(t)\rangle_I$

SCATTERING, INTERACTIONS, S-MATRIX, PERT. THEORY & FEYNMAN DIAGRAMS

Scattering :

$|i\rangle \longrightarrow |f\rangle$

Assumption: $|i\rangle$ = free state $= |\Psi(t=-\infty)\rangle_I$

$|f\rangle$ = free state $= |\Psi(t=+\infty)\rangle_I$

$$|f\rangle = S|i\rangle$$

↑ unitary operator. $S \rightsquigarrow$ scattering

Finding x-section ← compute $\langle f|S|i\rangle$

From $i\partial_t|\Psi\rangle_I = H_{int}|\Psi\rangle_I$

$$|\Psi(t)\rangle_I = |i\rangle - i\int_{-\infty}^{t} dt_1\, H_{int}(t_1)\, |\Psi(t_1)\rangle_I$$

Iterate $$|\Psi(t)\rangle_I = |i\rangle - i\int_{-\infty}^{t} dt_1\, H_{int}(t_1)\,|i\rangle + (-i)^2\int_{-\infty}^{t} dt_1 \int_{-\infty}^{t_1} dt_2\, H(t_1)\, H(t_2)\,|i\rangle + \dots$$

SCATTERING, INTERACTIONS, S-MATRIX, PERT. THEORY & FEYNMAN DIAGRAMS

- $$S = \sum_{n=0}^{\infty} (-i)^n \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 \cdots \int_{-\infty}^{t_{n-1}} dt_n \; H(t_1) \cdots H(t_n)$$

Equivalent to: $$S = T \exp\left(-i \int_{-\infty}^{\infty} dt \; V_I(t)\right) = T \exp\left(+i \int d^4x \, \mathcal{L}_{int}\right)$$

- Use perturbation theory, assuming that $V_I \sim g$, $g \ll 1$

coupling constant

SCATTERING, INTERACTIONS, S-MATRIX, PERT. THEORY & FEYNMAN DIAGRAMS

- Example: in QED, $\mathcal{L}_{int} = -q\bar{\Psi}\gamma^\mu\Psi A_\mu$ ← 3 fields ≡ interaction
- $\langle f|S|i\rangle \sim iq\int d^4x\, \langle f|\bar{\Psi}\gamma^\mu\Psi A_\mu|i\rangle$

A_μ: $a\,\varepsilon^\mu + a^\dagger\,\varepsilon^{*\mu}$

Ψ: $b\,u + d^\dagger v$

$\bar{\Psi}$: $d\,\bar{v} + b^\dagger \bar{u}$

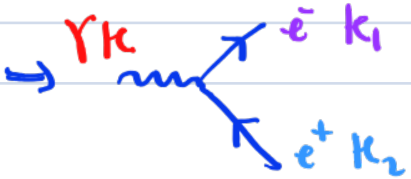
- Interaction DOES NOT PRESERVE N_{tot} ✓
- $|i\rangle = a^\dagger(k)|0\rangle \qquad |f\rangle = b^\dagger(k_1)\, d^\dagger(k_2)|0\rangle$
- $\langle f|S|i\rangle \simeq -iq\int d^4x\, \langle 0|b(k_1)\, d(k_2) \begin{smallmatrix} d \\ b^\dagger \end{smallmatrix}\, \gamma^\mu \begin{smallmatrix} b \\ d^\dagger \end{smallmatrix}\, \begin{smallmatrix} \boxed{a} \\ a^\dagger \end{smallmatrix}\, \boxed{a^\dagger(k)}|0\rangle$

$aa^\dagger = a^\dagger a + [a,a^\dagger]$

$bb^\dagger = -b^\dagger b + \{b,b^\dagger\}$

$dd^\dagger = -d^\dagger d + \{d,d^\dagger\}$

$$\simeq -iq\,\bar{u}(k_1)\gamma^\mu v(k_2)\,\varepsilon_\mu(k)$$

→ 

$= -iq\,\bar{u}\gamma^\mu v\,\varepsilon_\mu \quad \Rightarrow \text{vertex} = -iq\gamma^\mu$

SCATTERING, INTERACTIONS, S-MATRIX, PERT. THEORY & FEYNMAN DIAGRAMS

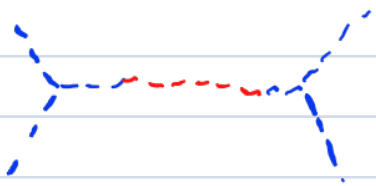
- Example in φ^3 : $\mathcal{L}_{int} = g^3 \varphi^3$
- $\varphi(k_1) + \varphi(k_2) \rightarrow \varphi(k_3) + \varphi(k_4)$

$$\langle f | S | i \rangle = (ig^3)^2 \int d^4x_1\, d^4x_2 \, \langle 0 | a(3) a(4) \; \varphi^3(x_1) \, \varphi^3(x_2) \; a^\dagger(1) a^\dagger(2) | 0 \rangle$$

$$a \; a \; a \qquad a \; a \; a$$
$$a^\dagger a^\dagger a^\dagger \qquad a^\dagger a^\dagger a^\dagger$$

- Highlighted 1 possible combination

$$\Rightarrow (ig^3)^2 \langle 0 | T(\varphi(x_1)\varphi(x_2)) | 0 \rangle$$



$$\text{Propagator} = \int \frac{d^4k}{(2\pi)^4} \frac{i e^{-ik\cdot(x-y)}}{k^2 - m^2 + i\varepsilon}$$

FEYNMAN RULES & CROSS SECTIONS

1) Draw all diagrams [higher order $\to$ higher complexity]

2) Diagrams $\to$ Amplitude

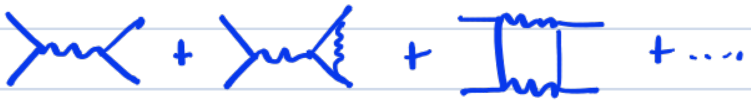
a) EXT LINES $\to$ WAVEFUNCTIONS $\quad 1 \quad u, v \quad \epsilon^{\mu}$

b) VERTEXES $\leftarrow$ from $\mathcal{L}_{int}$

c) INT LINES $\to$ PROPAGATORS $\quad \dfrac{i}{p^2-m^2} \quad \dfrac{i(\not{p}-m)}{p^2-m^2} \quad \dfrac{i}{p^2}(-g^{\mu\nu})$

⋮ Loops, symmetry factors, etc..

$e^+e^- \to \mu^+\mu^-$



3) $\langle f | S | i \rangle \sim \mathcal{M}_{i\to f}$

4) $d\sigma_{2\to n} \sim [dk_1]\dots[dk_n]\,(2\pi)^4\,\delta^4(p_1+p_2-k_1\dots-k_n)\,|\mathcal{M}_{2\to n}|^2\,\dfrac{1}{2s}$

$d\Gamma_{M\to n} \sim [dk_1]\dots[dk_n]\,(2\pi)^4\,\delta^4(p-k_1\dots-k_n)\,|\mathcal{M}_{1\to n}|^2\,\dfrac{1}{2M}$

LAGRANGIANS & INTERACTIONS FROM SYMMETRIES

- SM is a theory with "matter content" ($\rightarrow$ fermions) that interact according to the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$
- $\mathcal{L}_{SM}$ is symmetric under such group

- SYMMETRIES: fundamental to describe Nature

external (on spacetime) $\quad \vec{x}' = R\vec{x}$ continuous
$\quad \vec{x}' = -\vec{x}$ discrete

internal (on wavefunctions/fields) $\quad \psi(x) \rightarrow \psi'(x) = e^{i\alpha}\psi(x) \quad U(1)$

$\psi(x) \rightarrow \psi'(x) = \exp\left(i\vec{\alpha}\cdot\frac{\vec{\tau}}{2}\right)\psi(x) \quad SU(2)$

SYMMETRIES

- Noether's theorem: continuous symmetry → conserved current & conserved charge
- System is symmetric ⟷ $\mathcal{L}$ invariant
- Examples:

symmetry	cons. quantity	
time transl $\left(\frac{dL}{dt}=0\right)$	Energy	E
space transl.	momentum	$\vec{p}$
rotations	ang. momentum	$\vec{L}$
phase of ψ	electric charge	

- Fundamental interactions (E.M., weak, strong force) are based on <u>local</u> internal symmetries

$\alpha = \alpha(x)$

LOCAL (GAUGE) SYMMETRIES

Paradigm : - free Lagrangian with global symmetry
- require local symmetry "gauge symmetry"

⇒ STRUCTURE OF THE INTERACTIONS EMERGES

⤳ gauge vector fields mediate interactions

⤳ theory is predictive and unitary
(more details later)

GAUGE INVARIANCE AND E.M. IN Q.M.

- The above paradigm has its roots in QM
- Charge in e.m. field $\quad H = \frac{(\vec{p} - q\vec{A})^2}{2m} + qV \qquad A^\mu = (V, \vec{A})$

  where $\vec{B} = \vec{\nabla} \wedge \vec{A} \qquad \vec{E} = -\vec{\nabla} V - \partial_t \vec{A}$

- EM is invariant if $\begin{array}{l} \vec{A} \to \vec{A} + \vec{\nabla} \alpha(x) \\ V \to V - \partial_t \alpha(x) \end{array}$ (gauge transf.)

- Schroeding. equation must be invariant under gauge transf.
- Properties of ψ such that

$$i\partial_t \psi = \hat{H}(V, A)\psi \quad \Leftrightarrow \quad i\partial_t \psi' = \hat{H}'(V', \vec{A}')\psi'$$

$$\Rightarrow \psi(t, \bar{x}) \longrightarrow \psi'(t, \bar{x}) = e^{iq\alpha(x)} \psi(t, \bar{x}) \qquad U(1) \text{ local}$$

FROM DIRAC FREE LAGRANGIAN TO QED I

- Free Dirac field: $(i\gamma^\mu \partial_\mu - m)\psi = 0$ $\qquad \mathcal{L}_0 = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$
- $\mathcal{L}$ is invariant under global $U(1)$: $\psi(x) \to e^{i\alpha}\psi(x)$
- Gauge paradigm : require that $\mathcal{L}$ invariant under $U(1)$ local $\cdot\, q\alpha(x)$

$$\mathcal{L} \xrightarrow[U(1)]{} \mathcal{L}' = \bar{\psi}(i\gamma^\mu \partial_\mu - m - q\gamma^\mu \partial_\mu \alpha)\psi$$

- $\partial_\mu \to D_\mu =: \partial_\mu + iqA_\mu$ $\qquad D_\mu$: covariant derivative

  $\to$ property $(D_\mu\psi) \to (D_\mu\psi)' = e^{i\alpha(x)}(D_\mu\psi)$ (transform as ψ)

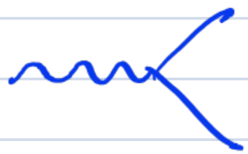
  $\to$ it works if $A^\mu \xrightarrow[U(1)]{} A'^\mu = A^\mu - \partial^\mu\alpha$

  proof: $D'_\mu\psi' = (\partial_\mu + iqA - iq\partial^\mu\alpha)e^{iq\alpha}\psi = e^{iq\alpha}(\partial_\mu + iqA_\mu)\psi$

- $U(1)$ local $\Rightarrow \mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi = \mathcal{L}_0 - qA_\mu\bar{\psi}\gamma^\mu\psi$

From Dirac Free Lagrangian to QED II

$$\mathcal{L}_{int} = -q A_\mu \bar{\Psi} \gamma^\mu \Psi$$

 $= -iq\gamma^\mu$

- Notice formal similarity with $-A_\mu J^\mu$

  $\leftarrow J^\mu = q \bar{\Psi} \gamma^\mu \Psi$

- Vector field A^μ is dynamic $\rightarrow -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

$$\mathcal{L}_{QED} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Invariant U(1) local : $\Psi \rightarrow \Psi' = e^{iq\alpha(x)} \Psi \quad A^\mu \rightarrow A'^\mu = A^\mu - \partial^\mu \alpha$

QED

• loop diagrams UV divergent

RENORMALIZATION

- FINITE set of EXP inputs
- absorb ALL divergences
- running coupling (observed in data ✓)

• GAUGE INVARIANCE CRUCIAL FOR RENORMALIZATION

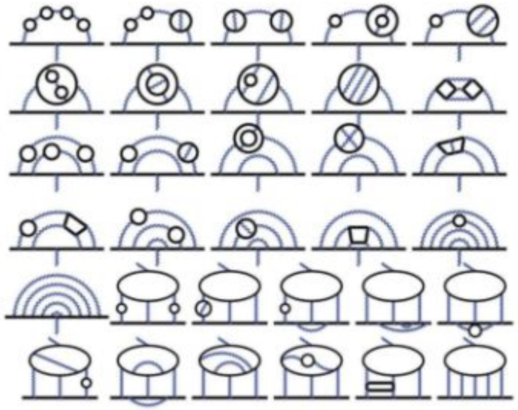
  • predictive at all energies
  • works spectacularly well

• Notice: mass term is NOT gauge invariant

QED AND DATA

SM: it works impressively well:
- e.g. electron g-2 @ 5 loops

$$a_e = \frac{(g-2)_e}{2} \quad \begin{array}{llll} 1\,159\,652\,181\,64.3 & \pm & 76.4\ (\times 10^{-12}) & \text{[TH]} \\ 1\,159\,652\,180\,59 & \pm & 13\ \ (\times 10^{-12}) & \text{[EX]} \end{array}$$



Credit: T. Aoyama et al., Phys. Rev. Lett., 2012

NON ABELIAN GAUGE THEORIES (Yang-Mills theories)

- Fundamental interactions require also NON ABELIAN transformation

ex: Ψ doublet of $SU(2)$ $\quad \Psi = \begin{vmatrix} \psi_1 \\ \psi_2 \end{vmatrix}$

$$\psi(x) \xrightarrow[SU(2)]{} \psi'(x) = U(\theta(x))\psi(x) \qquad U(\theta(x)) = \exp\left(i g \theta^a(x) \frac{\tau^a}{2}\right)$$

Pauli matrices

$a = 1, 2, 3$

- Non abelian because $U(\theta_1)U(\theta_2) \neq U(\theta_2)U(\theta_1)$

- Possible to have $SU(N)$ local invariance:

$\dim SU(N) = N^2 - 1 \dashrightarrow a = 1 \dots N^2 - 1$

$$\boxed{\mathcal{L}_{YM} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi - \frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}}$$

YANG MILLS THEORIES I

$$D_\mu = \partial_\mu - ig W_\mu^a t^a$$

t^a GENERATORS (hermitian traceless)

introduce N^2-1 vector fields ⇐

$$F_{\mu\nu}^a = \partial_\mu W_\nu - \partial_\nu W_\mu - g f^{abc} W_\mu^b W_\nu^c$$

extra term, new interactions ⇐

$\mathcal{L}_{YM}$ invariant under:

$$\psi(x) \to \psi'(x) = U(x)\psi(x) \qquad U(x) = \exp\left(ig\,\theta^a(x)\, t^a\right)$$

$$W_\mu^a t^a \longrightarrow W_\mu'^a t^a = U W_\mu^a t^a U^\dagger - ig (\partial_\mu U) U^\dagger$$

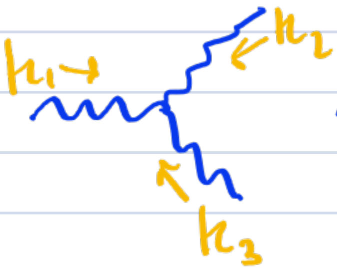
- Notice: mass term IS NOT gauge invariant

$$m^2 W^{a,\mu} W_\mu^a \sim m^2 \, \mathrm{Tr}\left[W_\mu t \, W^\mu t\right]$$

YANG MILLS THEORIES II

- Possible to build interactions using non abelian gauge groups
- Theory is RENORMALIZABLE
- Force mediators : $SU(N) \sim N^2-1$ vectors
- Mass terms for vectors NOT ALLOWED
- Interactions :

$\sim g$ similar to QED



$\sim g$ [DERIVATIVE VERTEX]

$\sim k^\mu$

$\sim g^2$

MASSIVE VECTOR BOSONS

- Weak interactions : short-range $\longleftrightarrow$ EM interactions : long-range

  MASSIVE mediator — massless mediator

- Weak interactions through gauge symmetry $\leftarrow$ Need mechanism for MASSIVE VECTOR BOSONS $\oplus$ GAUGE SYMMETRY

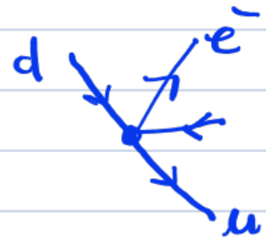
- Next slides : β-decay $\rightarrow$ Effective Theory $\rightarrow$ need of massive vectors
  Higgs mechanism

WEAK FORCE & FERMI THEORY

- β decay $\quad n \to p + e^- + \bar{\nu}_e \qquad [n = |udd\rangle \quad p = |uud\rangle]$

$$\Rightarrow d \to u\, e^-\, \bar{\nu}_e$$

- Fermi theory (1933) : 4-fermion interaction



$$\mathcal{L}_F = -\frac{G_F}{\sqrt{2}} \underbrace{\left(\bar{u}_L \gamma^\mu d_L\right)}_{\text{current}} \cdot \underbrace{\left(\bar{e}_L \gamma_\mu \nu_L\right)}_{\text{current}} + h.c.$$

same point in spacetime $\leftarrow$ force with NO RANGE

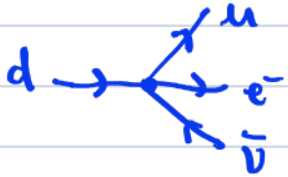
$$G_F \simeq 1.1 \times 10^{-5}\, (\text{GeV})^{-2}$$ $\leftarrow$ "small" : WEAK FORCE

- Low energy : Fermi theory VERY SUCCESSFUL

- High energy : $\mathcal{L}_F \to$ unitarity violation, non renormalizable $\Rightarrow$ Approximation to more general theory

WEAK FORCE & FERMI THEORY

- $\mathcal{L}_F$: low energy limit of force mediated by MASSIVE BOSONS



Fermi SM

- Weak force : range $\sim 10^{-18}$ m $(\neq 0)$

$\Rightarrow$ Force carrier must have a mass

$\hookrightarrow W^{\pm}, Z$ bosons

- Realized through NON ABELIAN GAUGE THEORY $(SU(2))$

$\hookrightarrow g$ COUPLING

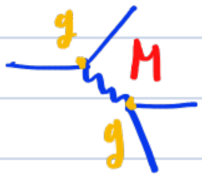
- Observation : ONLY LEFT-HANDED FIELDS $\rightarrow SU(2)_L$

- More precisely : high energy : $SU(2)_L \times U(1)_Y$

low energy : $U(1)_{EM}$

EWSB

INTERLUDE

★ 

$$\mathcal{M} \sim \frac{g^2}{q^2 - M^2} \xrightarrow{q^2 \ll M^2} -\frac{g^2}{M^2}$$

$$\mathcal{M} \sim \frac{G_F}{\sqrt{2}}$$

$$\Rightarrow \frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$

★ $$-\frac{G_F}{\sqrt{2}} \qquad J^{had}_{pn} \cdot J^{lept}$$

$$-\frac{G_F}{\sqrt{2}} V^{ud}_{CKM} \quad J^{quark}_{ud} \cdot J^{lept}$$

THE STANDARD MODEL (without Higgs)

- Matter content (fermions) + Gauge group $\Rightarrow \mathcal{L}$ FIXED
- STRONG FORCE $SU(3)_c$ EW FORCE $SU(2)_L \otimes U(1)_Y$

quarks/gluons ← quarks/leptons/$W^\pm$ Z A

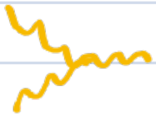
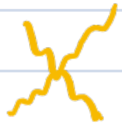
- Let's focus on a single leptonic family ← NO $SU(3)_c$

- couplings / mediators $SU(2)_L : g / W^a_\mu$ $U(1)_Y : g' / B_\mu$
- Matter fields : e_L e_R ν_L (ν_R)

	$SU(2)_L$	$U(1)_Y$	$SU(2)_L$	$U(1)_Y$
$L(x) = \begin{vmatrix} \nu_L \\ e_L \end{vmatrix}$	doublet	$Y = -1$	$L \to \underbrace{U(x)}_{g} L$	$\exp\left(ig'\alpha(x)\frac{Y}{2}\right) L$
$e_R(x)$	singlet (no transf)	$Y = -2$	$e_R \to e_R$	$\exp\left(ig'\alpha(x)\frac{Y}{2}\right) e_R$

THE STANDARD MODEL (without Higgs)

- $\mathcal{L}: \quad \bar{\Psi}\, i\gamma^\mu D_\mu \Psi \quad - \frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}$

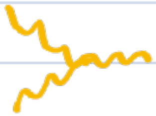
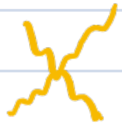




- $SU(2)_L \times U(1)_Y$: 4 gauge bosons $\{ W_1^\mu \;\; W_2^\mu \;\; W_3^\mu \;\; B^\mu \}$
- With proper rotation $(W_3^\mu, B^\mu) \rightarrow (Z^\mu, A^\mu)$
- Above $\mathcal{L}$ (+ quark sector) (+ 3 families) looks "perfect"
- What's missing?

⋮

THE STANDARD MODEL (without Higgs)

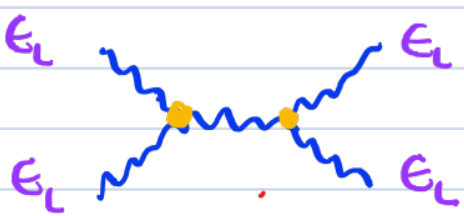
- $\mathcal{L}: \quad \bar{\Psi}\, i\gamma^\mu D_\mu \Psi \quad - \frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}$

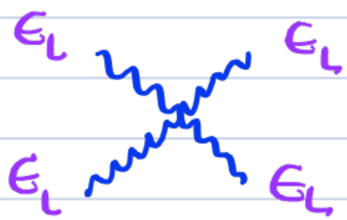
- $SU(2)_L \times U(1)_Y$: 4 gauge bosons $\{ W_1^\mu \;\; W_2^\mu \;\; W_3^\mu \;\; B^\mu \}$
- With proper rotation $(W_3^\mu, B^\mu) \to (Z^\mu, A^\mu)$
- Above $\mathcal{L}$ (+ quark sector) (+ 3 families) looks "perfect"
- What's missing?
    - masses for gauge bosons $\sim M^2 V_\mu V^\mu$ GAUGE INVARIANCE !
    - masses for fermions $\sim m(\bar{e}_L e_R + \bar{e}_R e_L)$ GAUGE INVARIANCE !

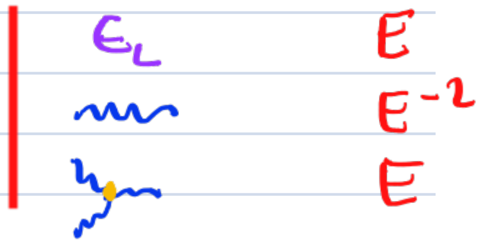
MASSIVE GAUGE BOSONS (WITHOUT HIGGS)

- Let's forget all the TH conceptual issues : mass by hand ?
- Massive vector boson : 2 TRANSVERSE ⊕ 1 LONG Pol's
- $p^\mu = (E, 0, 0, |\vec{p}|)$ $\qquad \epsilon_L^\mu = \frac{1}{M}(|\vec{p}|, 0, 0, E)$
- $W_L Z_L \to W_L Z_L$ "Longitudinal Vector Boson Scattering"



+ u-channel +





- $\mathcal{M} \sim E^4 + E^2$ $\qquad$ • E^4 cancel (gauge sym)
- E^2 remains $\Rightarrow$ Unitary violation @ $E \simeq 800$ GeV

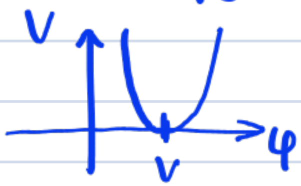
HIGGS MECHANISM : SSB

- Main goal: GAUGE BOSONS MASSIVE ($\leftarrow$ allows also fermion masses)
- VEV (Vacuum Expectation Value)

$$\varphi(x) = \int [dk] \left(a(k)\, e^{-ikx} + a^\dagger(k)\, e^{ikx} \right) \qquad \text{VEV} = \langle 0|\varphi|0\rangle = 0$$

- Interested in quantum fluctuations around VEV : φ : fluctuations about VEV
- VEV $\leftarrow$ minimum of classical potential

a) $$\mathcal{L}_{KG} = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{m^2}{2}\varphi^2 \quad \leftarrow V = \frac{m^2}{2}\varphi^2 \quad \leftarrow \text{minimum } \varphi = 0$$

b) 

$\leftarrow$ expand around v: $\varphi = v + \tilde{\varphi}$

$\nwarrow$ physical DoF

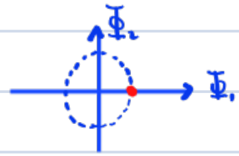
- Vacuum is unique.
- SPONTANEOUS SYMMETRY BREAKING $\equiv$ $\mathcal{L}$ has a symmetry BUT VACUUM IS NOT SYMMETRIC

HIGGS MECHANISM: ABELIAN GAUGE THEORY I

- SSB of gauge symmetry $\to$ Higgs mechanism $\to$ massive vectors
- Abelian $U(1)$ gauge theory $\quad D_\mu = \partial_\mu + ie A_\mu$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \Phi)(D^\mu \Phi)^* - V(\Phi) \quad \Big| \quad V(\Phi) = \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2 \quad \langle 0 | \Phi | 0 \rangle = \frac{v}{\sqrt{2}}$$

$\Phi = \frac{1}{\sqrt{2}} (\Phi_1 + i \Phi_2)$ complex scalar field



- $\mathcal{L}$ invariant if $\Phi \to e^{ie\alpha(x)} \Phi$. VEV not invariant under $U(1)$ local
- $\Phi = \frac{v}{\sqrt{2}} + \varphi(x) \qquad \varphi(x) = \frac{1}{\sqrt{2}} (\varphi_1 + i\varphi_2)$
- $|D_\mu \Phi|^2 = \frac{1}{2} \sum_i (\partial_\mu \varphi_i)(\partial^\mu \varphi_i) + ev A_\mu \partial^\mu \varphi_2 + \frac{e^2 v^2}{2} A_\mu A^\mu + \cdots$

  massive vector boson ☺

  $M = ev$
- $V(\Phi) = \lambda v^2 \varphi_1^2 + \cdots$

  massive scalar (HIGGS) $\quad m^2 = 2\lambda v^2$

HIGGS MECHANISM: ABELIAN GAUGE THEORY II

- Change of variables: $B^\mu = A^\mu + \frac{1}{ev}\partial^\mu \varphi_2$
- Rewrite $\mathcal{L}$ using B^μ: $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ is invariant: $F^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 + \underline{\frac{1}{2}(\partial_\mu \varphi_1)^2} + \underline{\frac{e^2 v^2}{2} B_\mu B^\mu} + \underline{\lambda v^2 \varphi_1^2}$$

  - B^μ: massive, $\partial_\mu \varphi_2$ absorbed in B ← LONGIT. POLARIZATION
  - φ_2: disappeared from $\mathcal{L}$ (Goldstone boson eaten)
  - φ_1: massive (Higgs field)

- DoF: A^μ massless + (φ_1, φ_2) 2+2 DoF [before SSB]

  A^μ massive + φ_1 3+1 DoF [after SSB]

- VEV $\neq 0$ + SSB ⇒ massive gauge bosons + Higgs field

HIGGS MECHANISM IN THE SM I

- Add to $\mathcal{L}$ a scalar field, doublet $SU(2)_L$ and charged $U(1)_Y$
- $\phi = \begin{vmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{vmatrix}$ 4 DoF $\dashrightarrow$ 3 eaten by $\{W^{\pm}, Z\}$ + 1 Higgs boson
- $V(\phi)$ such that $\langle 0|\phi|0\rangle$ $\rightarrow$ Mexican Hat potential

$$\Rightarrow \quad V(\phi) = -\mu^2 |\phi|^2 + \lambda |\phi|^4$$

$$|\phi_{min}| = \sqrt{\frac{\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}}$$

- vacuum $\phi_{min} = \frac{1}{\sqrt{2}} \begin{vmatrix} 0 \\ v \end{vmatrix}$ $\qquad \phi = e^{i\left(\tau^a \frac{\theta^a(x)}{v}\right)} \begin{vmatrix} 0 \\ \frac{v + H(x)}{\sqrt{2}} \end{vmatrix}$

- After SSB $\Rightarrow$ $\left.\begin{matrix} 3 \text{ massive bosons } W^{\pm}, Z \\ 1 \text{ massless boson } \gamma \end{matrix}\right\}$ ✓ $\quad SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$

1 massive scalar field $H(x)$ HIGGS BOSON

The Higgs Mechanism

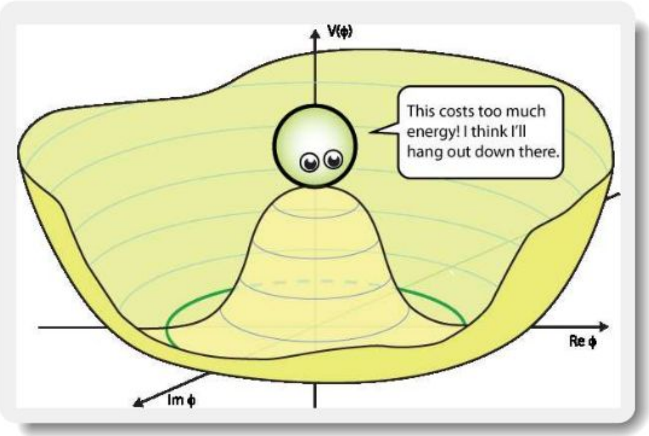
[Brout,Englert,Higgs 1964]

- One complex (doublet) scalar field Φ [4 d.o.f.]

potential: $V(\Phi) = \mu^2|\Phi|^2 + \lambda|\Phi|^4$

- spontaneously broken symmetry

$\mu^2 < 0 : \quad \langle\Phi\rangle \neq 0$



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[Brout,Englert,Higgs 1964]

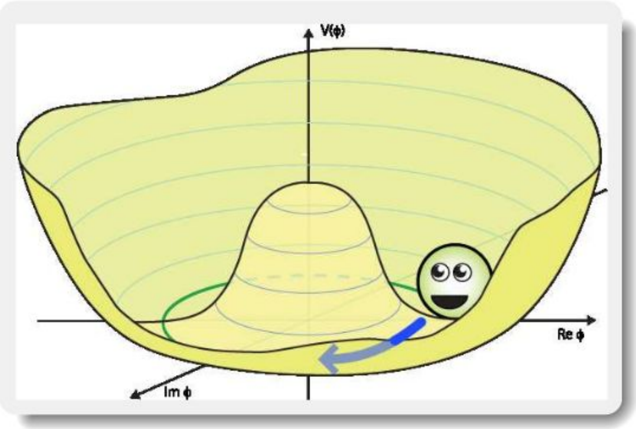
- One complex (doublet) scalar field Φ [4 d.o.f.]

potential: $V(\Phi) = \mu^2|\Phi|^2 + \lambda|\Phi|^4$

- spontaneously broken symmetry

$$\mu^2 < 0 : \quad \langle\Phi\rangle \neq 0$$

- 3 “angular” modes
  $\Rightarrow$ longitudinal W and Z



The Higgs Mechanism

[Brout,Englert,Higgs 1964]

- One complex (doublet) scalar field Φ [4 d.o.f.]

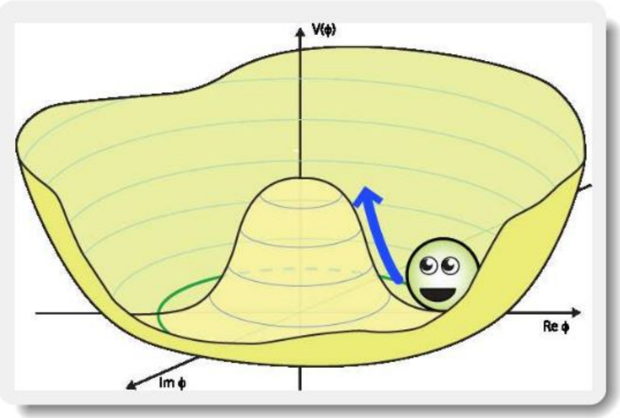
potential: $V(\Phi) = \mu^2|\Phi|^2 + \lambda|\Phi|^4$

- spontaneously broken symmetry

$$\mu^2 < 0 : \quad \langle\Phi\rangle \neq 0$$

- 3 "angular" modes
  $\Rightarrow$ longitudinal W and Z

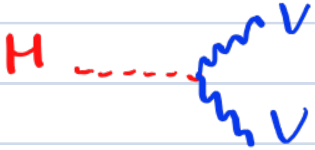
- radial mode: massive
  $\Rightarrow$ the "Higgs boson"



HIGGS MECHANISM IN SM II

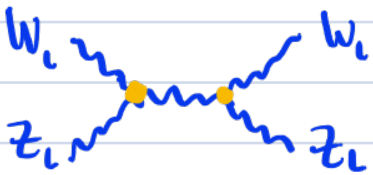
- Higgs boson mass : $m_H^2 = 2\lambda v^2$

- Higgs couplings to gauge bosons

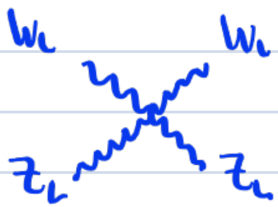


$\sim i\, g\, M_V$ "H-SM-SM" coupling $\sim M_{SM}$

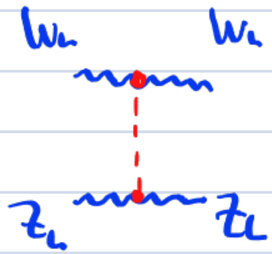
- Higgs unitarizes $V_L V_L$ x-section



+ u-channel +

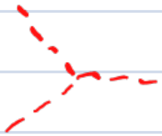


+



$\sim E^2$

- Higgs potential ⇢ Higgs self couplings



$\sim \lambda v$

$\sim \lambda$

YUKAWA LAGRANGIAN & FERMION MASSES

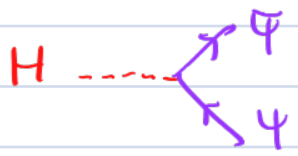
- We need to include a Dirac mass, keeping gauge invariance
- Focus on leptons for simplicity, and 1 family only
- $\mathcal{L}_{Yuk}$: only possible extra term to $\mathcal{L}_{SM}$ that is gauge invariant & renormalizable

$$\mathcal{L}_{Yuk} = -h_e \bar{L}(x)\,\phi(x)\,e_R(x) + h.c. \xrightarrow[EWSB]{} -h_e \bar{L} \begin{vmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{vmatrix} e_R + h.c. = -h_e \frac{v+H}{\sqrt{2}} \left(\bar{e}_L e_R + \bar{e}_R e_L \right)$$

$$\mathcal{L}_{Yuk} = -h_e \underbrace{\frac{v}{\sqrt{2}}}_{\text{mass term}} \Big(1 + \underbrace{\frac{H}{v}}_{\text{coupling}}\Big) \bar{\psi}_e \psi_e$$

☺

$$h_e \frac{v}{\sqrt{2}} = m_e$$

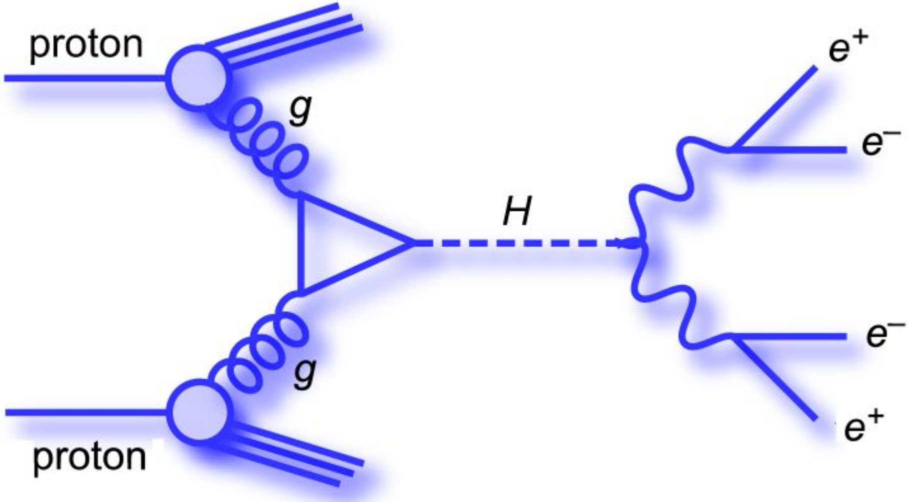


$$\sim -i\,\frac{m_\psi}{v}$$

YUKAWA COUPLING (proportional to fermion mass)

- Yukawa Lagrangian ⊕ 3 families (of quarks) $\dashrightarrow$ $\begin{cases} \text{CKM matrix} \\ \cancel{CP} \end{cases}$

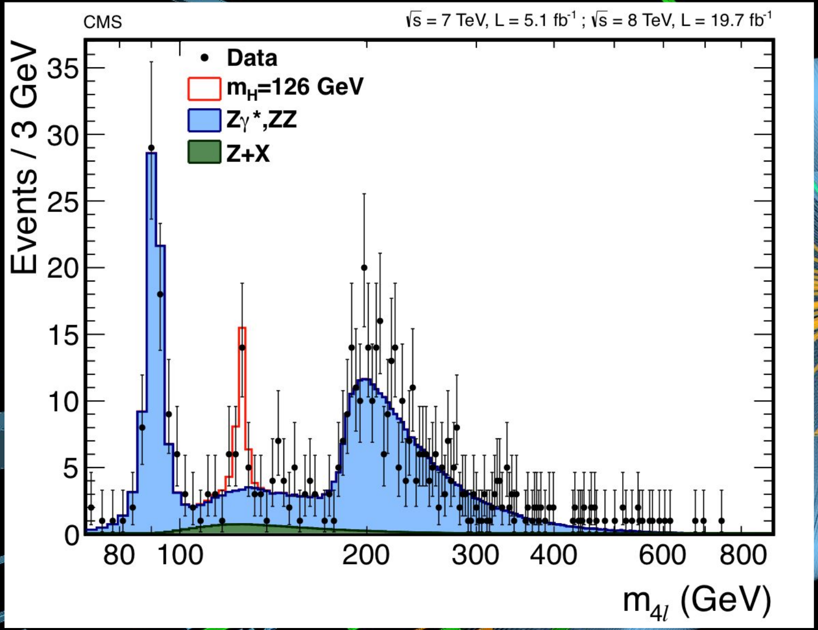
The Higgs boson discovery

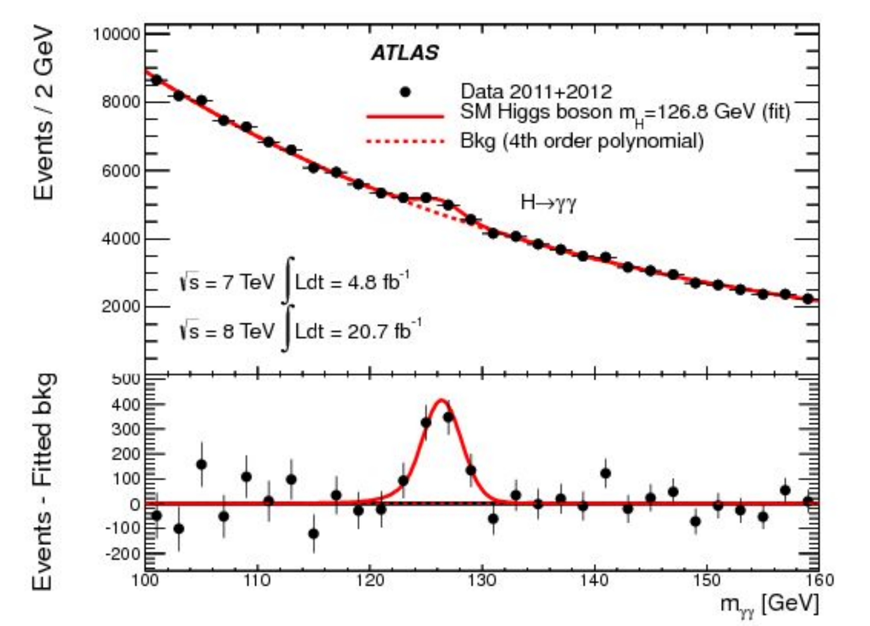


ultimately the Higgs boson discovery was an experimental challenge, but with a lot of cross-talk between theorists and experimentalists

CMS Experiment at the LHC, CERN
Data recorded: 2012-May-27 23:35:47.271030 GMT
Run/Event: 195099 / 137440354

The Higgs boson discovery



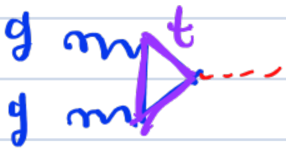


m_H = 125 GeV

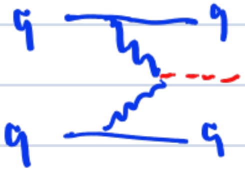
HIGGS AT THE LHC I

(main) production channels

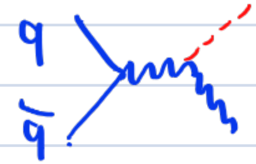
Decay channels



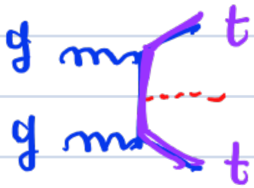
Gluon Fusion



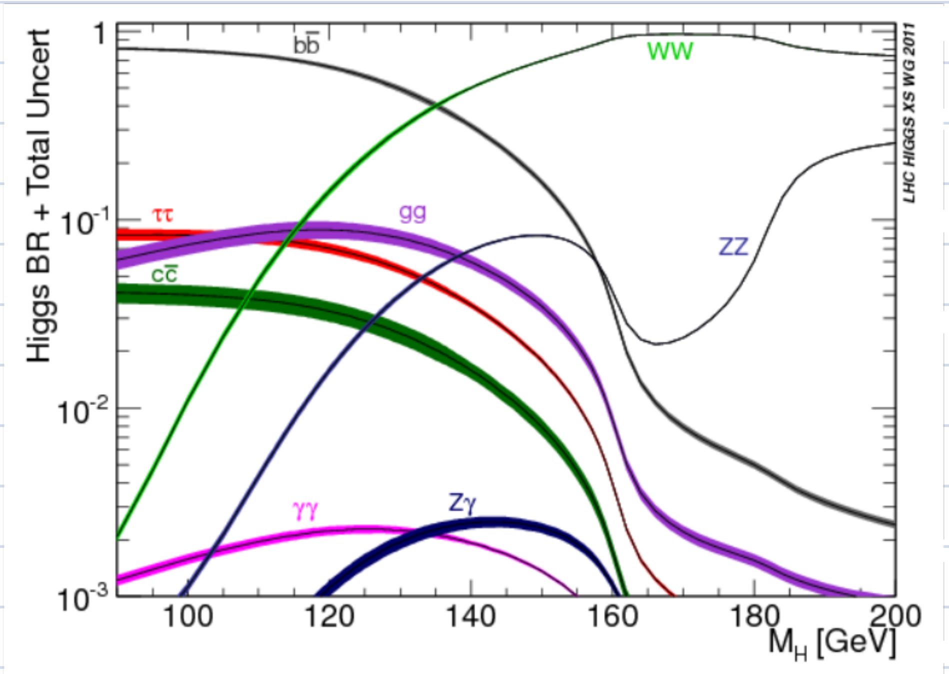
Vector Boson Fusion



VH



$t\bar{t} + H$

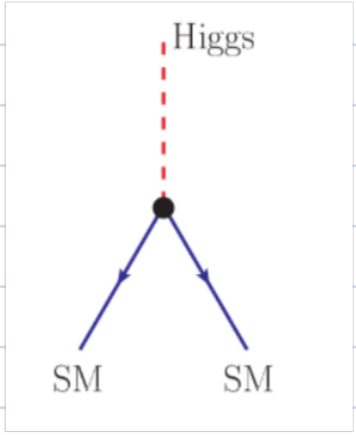


⇒ study its properties : · exactly as in the SM ?

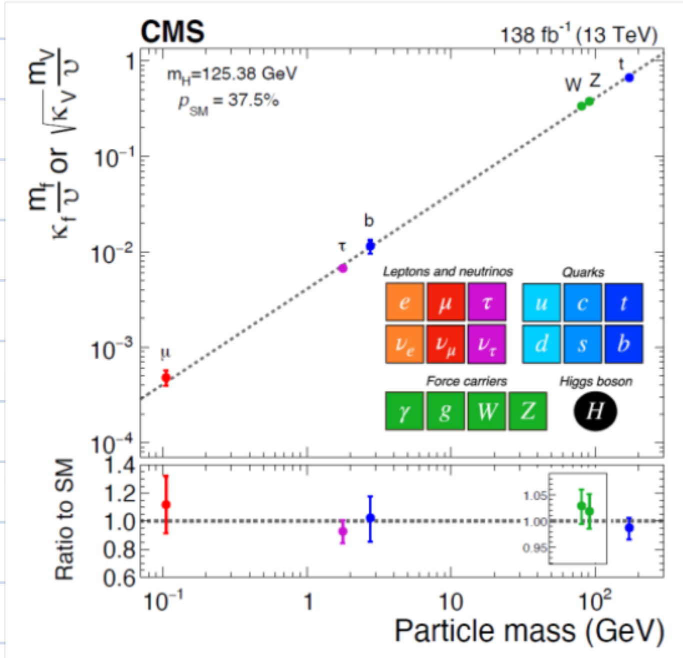
· deviations ?

HIGGS AT THE LHC II [STATUS]

- Higgs couplings with SM particles



H-SM-SM $\sim M_{SM}$ $\Rightarrow$



HIGGS : · SM-like within ~ 10-15 % accuracy

· Some tensions in some channel, no smoking gun!

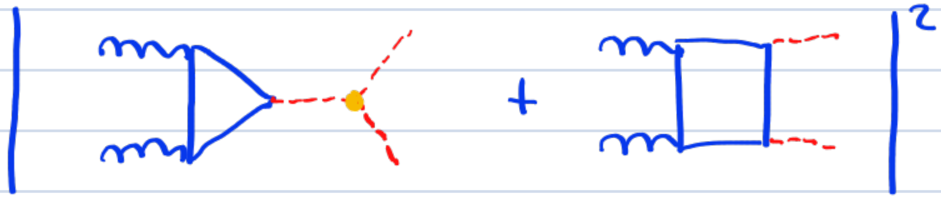
HIGGS TRILINEAR COUPLING

• $V(\phi) = -\mu^2|\phi|^2 + \lambda|\phi|^4 = \lambda\left(|\phi|^2 - \frac{v^2}{2}\right)^2$ $\quad [\mu^2 = \lambda v^2]$

$m_H^2 = 2\lambda v^2$ $\quad$ v known from G_F $\quad \rightarrow \lambda$ extracted (assuming SM)

 $= i\lambda v$ $\quad \rightarrow$ Measure λ directly

• Process : $pp \rightarrow HH$



challenging : $\begin{cases} \text{small x-section} \\ \text{distructive interference} \\ \text{large backgrounds} \end{cases}$ $\Rightarrow$ HL-LHC !