FCPPN/L 2026: Measurement of inclusive production of charmonium states in b-hadron decays via their decay into $\phi\phi$ with LHCb

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IJCLab (CNRS and Paris-Saclay University)

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Introduction

Studying inclusive production from b-hadrons decays: $b \to c\bar{c}X$ with LHCb Run 2 ($5.9\,\text{fb}^{-1}$)

$c\bar{c}$ reconstructed with $c\bar{c} \to \phi\phi$ and $\phi \to K^+K^-$

States allowed: $\eta_c(1S)$, χ_{c0}, χ_{c1}, χ_{c2}, $\eta_c(2S)$

LHCb Run 1 ($3.0\,\text{fb}^{-1}$) by A. Usachov, M. Teklishyn, J. He, S. Barsuk *[Eur. Phys. J.* **C77** *(2017) 609]*

Other usual final state is $p\bar{p}$: recent LHCb Run 2 analysis *[Eur. Phys. J.* **C84** *(2024) 1274]*, thesis by V. Zhovkovska *[Université Paris-Saclay (2023)]*

Using low ϕ background and narrow ϕ peak

Prompt production not studied: higher background, less efficient triggers

Considering potential interference effects with $\phi\phi$ background

Preprint available here

Introduction

Measuring:

$$\frac{\mathcal{B}(b \to \chi_{cJ} X)}{\mathcal{B}(b \to \chi_{c0} X)},$$

$$\mathcal{B}(b \to \chi_{cJ} X),$$

$$\mathcal{B}(b \to \eta_c(2S) X) \times \mathcal{B}(\eta_c(2S) \to \phi\phi) \text{ (as } \mathcal{B}(\eta_c(2S) \to \phi\phi) \text{ unknown)}$$

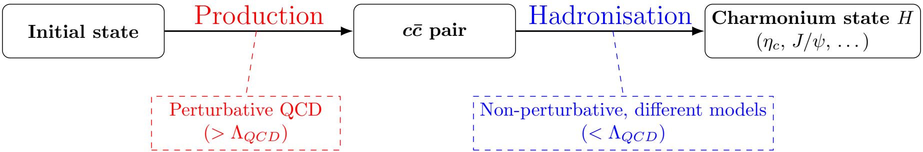
Using $\frac{\mathcal{B}(b \to AX) \times \mathcal{B}(A \to \phi\phi)}{\mathcal{B}(b \to BX) \times \mathcal{B}(B \to \phi\phi)} = \frac{\sum_{\text{year}} N_A^{\text{year}} / \varepsilon_A^{\text{year}}}{\sum_{\text{year}} N_B^{\text{year}} / \varepsilon_B^{\text{year}}}$

$\eta_c(1S)$ as normalization for absolute branching ratios

Also extracting masses and widths except $\Gamma_{\chi_{c1}}$ and $\Gamma_{\chi_{c2}}$

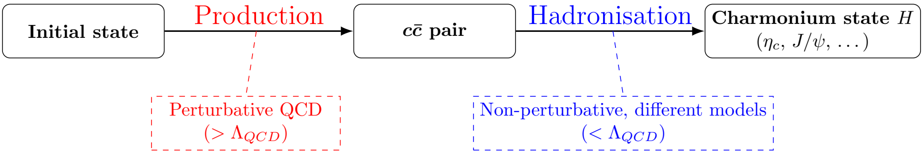
Charmonium production models

$$d\sigma_{A+B\to H+X} = \sum_n d\sigma_{A+B\to c\bar{c}(n)+X} \times \langle \vartheta^H(n) \rangle \qquad \textbf{(Factorisation)}$$



Charmonium production models

$$d\sigma_{A+B\to H+X} = \sum_n d\sigma_{A+B\to c\bar{c}(n)+X} \times \langle \vartheta^H(n) \rangle$$ **(Factorisation)**



Color Singlet Model: $c\bar{c}$ colorless, same quantum numbers J^{PC} as H

Charmonium production models

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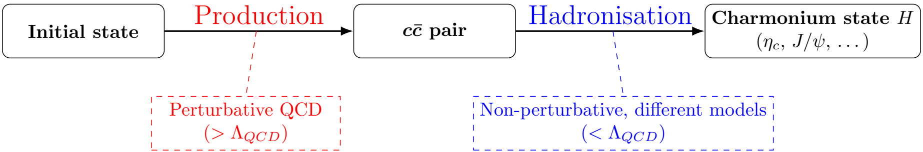


Color Singlet Model: $c\bar{c}$ colorless, same quantum numbers J^{PC} as H

NRQCD: all colors and J^{PC} allowed, giving H with probability given by Long Distance Matrix Element (LDME) $\langle \vartheta^H(n) \rangle$, need experimental constraints

Universality: LDMEs independant of context

The LHCb detector in Run 2 (2015-2018)

Single arm spectrometer at the Large Hadron Collider (LHC)

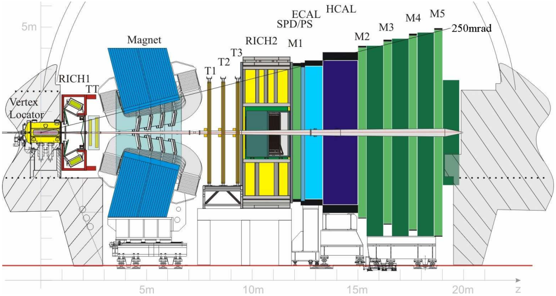
[JINST **3** *(2008) S08005, Int. J. Mod. Phys.* **A30** *(2015) 1530022]*

Rapidity range of $2 < \eta < 4.5$

Forward peaked correlated heavy flavour production

$\Rightarrow$ 4% of solid angle, 25% of heavy quark pairs

Flexible trigger for different studies



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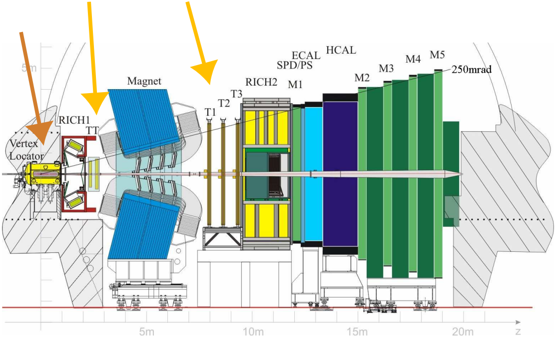
Tracking subdetectors:

Very good vertexing thanks to the **VErtex LOcator**

Momentum also from **tracking system**



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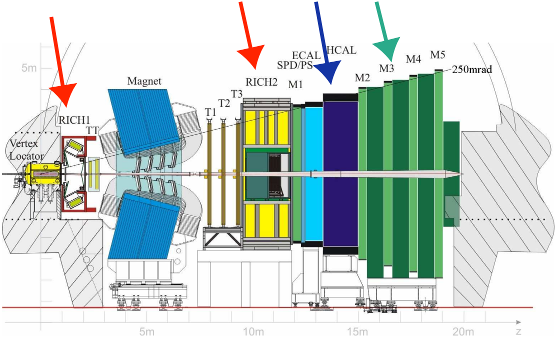
Identification subdetectors:

Charged particle identification with **Ring Imaging CHerenkov** detectors

Trigger and neutral particle information with **calorimeters**

Muon identification and information with **muon stations**



Selection steps

Analysis steps

Triggers, stripping and additional requirements

- Quality of the reconstruction
- Rapidity requirements
- Particle identification
- Momentum requirements
- Mass range
- Exploiting *b*-hadron flight
- Remove cloned tracks

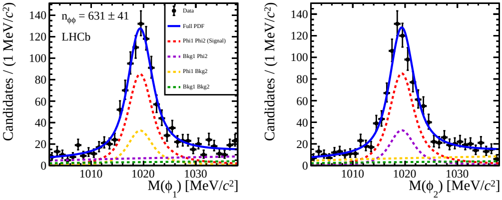
	Requirement
K	Track $\chi^2/\text{ndf} < 2$
	$\chi^2_{IPPV} > 3$
	$p \in [5000; 200000]$
	$p_{\text{T}} > 650$
	$\eta \in [2; 4.5]$
	$PIDk > 0$
	$PNNk > 0.1$
All KK	$\|\Delta\phi\| > 2\,\text{mrad}$
ϕ	VTX $\chi^2 < 9$
	$p_{\text{T}} > 800$
	$M_{KK} \in [1002; 1038]$
$\phi\phi$	VTX $\chi^2 < 3$
	$M_{\phi\phi} \in [2800; 4000]$
	$y \in [2; 4.5]$
	$\chi^2_{FD} > 9$
	$t_z > 0.3\,\text{ps}$

Analysis steps

Triggers, stripping and additional requirements

↓

2D fits to extract $\phi\phi$ yield bin by bin



Splitting the data into bins of $M_{\phi\phi}$

Extracting $\phi\phi$ component in each bin to construct full $M(\phi\phi)$ histogram

Analysis steps

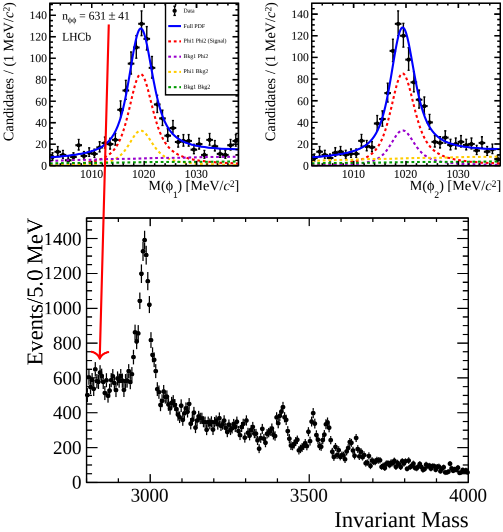
Triggers, stripping and additional requirements

↓

2D fits to extract $\phi\phi$ yield bin by bin

↓

Fit to $\phi\phi$ mass distribution



Now only true $\phi\phi$ in distribution

Fitting it to extract the yields, masses and widths

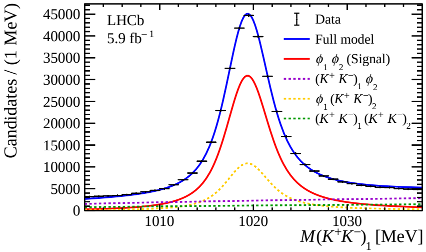
2D $(K^+K^-)(K^+K^-)$ fit

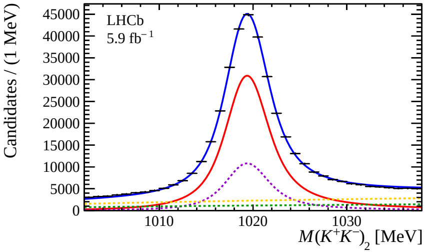
2D unbinned maximum likelihood fit

3 components: $\phi\phi$, $\phi(KK)$, $(KK)(KK)$

Signal with Relativistic Breit-Wigner (RBW) (floating M_ϕ and Γ_ϕ) convoluted with a Gaussian, times $\sqrt{M_{KK} - 2M_K}$ for phase space

Background: $\sqrt{M_{KK} - 2M_K}$





Nominal model of the fit

Interference model

Adding interference term with non-resonant $\phi\phi$ background

Plane wave $(\frac{K(x)}{K(M)})^L Ce^{i\varphi}$ allowed to interfere with RBW, thanks to Feng-Kun Guo

Creates interfering term with the sum of a $\sin\varphi$ and a $\cos\varphi$ contribution

$$RBW_I(x) = \frac{x \times \Gamma_f}{(\Delta M^2)^2 + M^2\Gamma_f^2} + 2C(\frac{K(x)}{K(M)})^L \frac{\sqrt{x \times \Gamma_f} \times (\Delta M^2 \cos\varphi - M\Gamma_f \sin\varphi)}{(\Delta M^2)^2 + M^2\Gamma_f^2}$$

But degeneracy between $\sin\varphi$ part and RBW, so introducing $D = C\cos\varphi$ and the simpler model:

$$RBW_I^{simple} = \frac{x \times \Gamma_f}{(x^2 - M^2)^2 + M^2\Gamma_f^2} + 2D(\frac{K(x)}{K(M)})^L \frac{\sqrt{x \times \Gamma_f} \times (x^2 - M^2)}{(x^2 - M^2)^2 + M^2\Gamma_f^2}$$

Impact on mass and width modelled perfectly

But $\sin\varphi$ part absorbed in the RBW shape

$\Rightarrow$ Extra systematic on ratios by comparing without interference

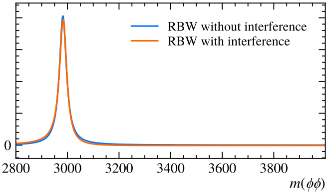
Interference for each state

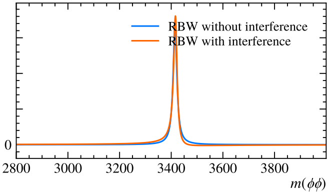
Strong hints of interference for $\eta_c(1S)$ ($\sim 3\sigma$)

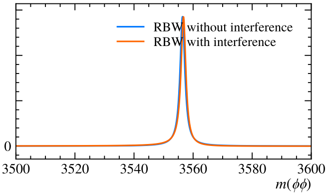
Expansion in $\phi\phi$ system L, interference for χ_{c0}, χ_{c2} ($L = 0$), $\eta_c(1S)$, $\eta_c(2S)$ ($L = 1$)

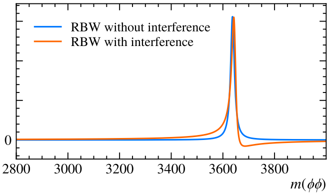
Significance of 3.8σ of adding interference for all four peaks together

Small shape changes, largest relatively for $\eta_c(2S)$









$\phi\phi$ invariant mass fit

Binned χ^2 simultaneous fit to $\phi\phi$ mass distribution for all four years

Signals: RBW $\circledast$ 2G, lowest available L used

Background: 2^{nd} order Chebychev polynomial

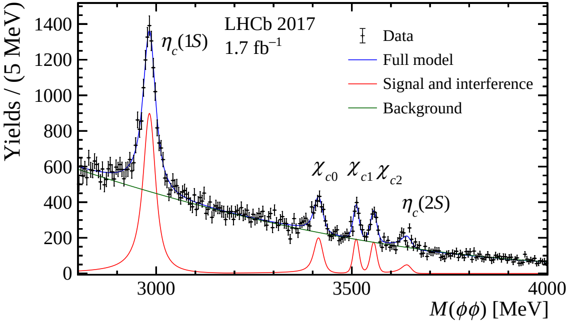
$\eta_c(1S)$, χ_{c0}, χ_{c1}, χ_{c2}, $\eta_c(2S)$

All masses floating

$\Gamma_{\eta_c(1S)}$, $\Gamma_{\chi_{c0}}$, $\Gamma_{\eta_c(2S)}$ floating

Ratios between Gaussian yields and widths constrained from MC:

$\Rightarrow$ Only $\sigma_1(\eta_c(1S))$ floating



Simplified interference model added for $\eta_c(1S)$, χ_{c0}, χ_{c2}, $\eta_c(2S)$

Systematic uncertainties

Systematic uncertainty sources

Considered sources:

- Single Gaussian resolution
- Recomputed $\sigma_{c\bar{c}}/\sigma_{\eta_c(1S)}$ ratios from a fit
- Adding $\chi_{c1}(3872)$ and $\chi_{c0}(3915)$ states
- Changing L for χ_{c0} and χ_{c2} from 0 to 2, only case where another L is possible
- 3rd order Chebychev polynomial background
- Masses of χ_{c1}, χ_{c2} at world average values *(not on χ_{c1}, χ_{c2} masses)*
- Varying the RBW radial parameter
- Changing the bin width to 10 MeV/c^2
- Removing interference *(only on ratios)*
- Residual prompt contribution of maximum 2% *(only on ratios)*
- Momentum scaling, comparing MC with $\delta_{scale} = \pm 3 \times 10^{-4}$ *[JINST **19** (2024) P02008]* *(not on ratios)*

500 toys averaged per systematic, based on nominal model

Also used to validate nominal fit procedure, small corrections applied

Systematic uncertainties on ratios

Uncertainties in percent:

Source	$\frac{\chi_{c0}}{\eta_c(1S)}$	$\frac{\chi_{c1}}{\eta_c(1S)}$	$\frac{\chi_{c1}}{\chi_{c0}}$	$\frac{\chi_{c2}}{\eta_c(1S)}$	$\frac{\chi_{c2}}{\chi_{c0}}$	$\frac{\eta_c(2S)}{\eta_c(1S)}$
Single Gaussian resolution	0.9	2.5	1.6	2.8	3.6	0.4
$\sigma_{c\bar{c}}/\sigma_{\eta_c(1S)}$ from fit	< 0.1	0.7	0.6	0.5	0.5	0.3
With $\chi_{c1}(3872)$ and $\chi_{c0}(3915)$	< 0.1	0.2	0.2	0.5	0.5	1.0
New $\phi\phi$ mass fit background	< 0.1	< 0.1	0.2	0.5	0.6	0.1
Alternative efficiencies	0.7	1.3	0.7	2.2	1.6	1.4
$M_{\chi_{c1}}$, $M_{\chi_{c2}}$ at known values	0.2	1.7	1.4	**4.6**	**4.9**	0.4
Changing L for χ_{c0} and χ_{c2}	**3.8**	0.2	**3.7**	1.6	2.1	0.7
New RBW radial parameter	0.2	0.3	0.2	0.3	0.1	1.7
10 MeV bin width	0.2	0.1	0.3	0.3	0.2	0.3
No interference	1.6	**4.4**	**5.0**	**5.1**	**5.1**	**15.8**
Residual prompt contribution	2.0	2.0	2.0	2.0	2.0	2.0
Total systematic uncertainties	4.7	5.9	7.0	8.2	8.6	16.2
Statistical uncertainties	9.7	7.8	11.3	13.8	16.8	15.4

Main sources are removing interference, fixing $M_{\chi_{c1}}$ and $M_{\chi_{c2}}$, changing L for χ_{c0} and χ_{c2}

Systematics for masses and widths

Uncertainties in percent:

Source	$M_{\eta_c(1S)}$	$M_{\chi_{c0}}$	$M_{\chi_{c1}}$	$M_{\chi_{c2}}$	$M_{\eta_c(2S)}$	$\Gamma_{\eta_c(1S)}$	$\Gamma_{\chi_{c0}}$	$\Gamma_{\eta_c(2S)}$
Single Gaussian resolution	<0.001	<0.001	<0.001	0.009	0.005	0.9	1.7	4.5
$\sigma_{c\bar{c}}/\sigma_{\eta_c(1S)}$ from fit	<0.001	<0.001	<0.001	<0.001	<0.001	1.5	0.4	3.6
With $\chi_{c1}(3872)$, $\chi_{c0}(3915)$	<0.001	<0.001	<0.001	<0.001	0.005	<0.1	<0.1	0.7
New $\phi\phi$ mass fit bkg	<0.001	<0.001	<0.001	<0.001	0.002	<0.1	0.3	<0.1
$M_{\chi_{c1}}$, $M_{\chi_{c2}}$ known values	<0.001	<0.001	/	/	0.002	0.5	1.7	1.9
Changing L for χ_{c0}, χ_{c2}	<0.001	0.003	<0.001	<0.001	0.002	<0.1	0.9	0.5
New RBW radial parameter	0.004	<0.001	<0.001	<0.001	<0.001	<0.1	0.1	<0.1
10 MeV bin width	<0.001	0.001	<0.001	<0.001	0.001	0.8	0.5	0.9
Changing momentum scaling	**0.015**	**0.018**	**0.018**	**0.019**	**0.019**	<0.1	0.3	0.3
Total systematics	0.015	0.018	0.018	0.021	0.021	2.0	2.7	6.2
Statistical uncertainties	0.016	0.044	0.017	0.039	0.096	3.2	17.8	27.4

No simplification of peak shape from simplified interference model

So not adding the fit without interference as systematic

Momentum scaling dominates for the masses

Results

Branching fractions

$\mathcal{B}(\eta_c(1S), \chi_c \to \phi\phi)$ and $\mathcal{B}(b \to \eta_c(1S)X)$ from world average

	Run 1 $c\bar{c} \to \phi\phi$	Run 2 $c\bar{c} \to \phi\phi$	2018 $c\bar{c} \to p\bar{p}$
$\mathcal{B}_{b\to\chi_{c0}X}[\times 10^{-3}]$	$1.75 \pm 0.27 \pm 0.13 \pm 0.48$	$1.34 \pm 0.13 \pm 0.06 \pm 0.37$	$3.05 \pm 0.54 \pm 0.08 \pm 0.29$
$\mathcal{B}_{b\to\chi_{c1}X}[\times 10^{-3}]$	$1.73 \pm 0.38 \pm 0.14 \pm 0.48$	$1.58 \pm 0.12 \pm 0.09 \pm 0.44$	$5.11 \pm 1.20 \pm 0.14 \pm 0.50$
$\mathcal{B}_{b\to\chi_{c2}X}[\times 10^{-3}]$	$0.66 \pm 0.11 \pm 0.04 \pm 0.19$	$0.55 \pm 0.08 \pm 0.05 \pm 0.15$	$1.54 \pm 1.13 \pm 0.04 \pm 0.15$
$\mathcal{B}_{b\to\chi_{c1}X} / \mathcal{B}_{b\to\chi_{c0}X}$	$1.00 \pm 0.22 \pm 0.02 \pm 0.06$	$1.18 \pm 0.13 \pm 0.08 \pm 0.07$	$1.68 \pm 0.66 \pm 0.05 \pm 0.10$
$\mathcal{B}_{b\to\chi_{c2}X} / \mathcal{B}_{b\to\chi_{c0}X}$	$0.39 \pm 0.07 \pm 0.01 \pm 0.03$	$0.41 \pm 0.07 \pm 0.04 \pm 0.03$	$0.50 \pm 0.36 \pm 0.04 \pm 0.03$
$\mathcal{B}(b \to \eta_c(2S)X) \times \mathcal{B}(\eta_c(2S) \to \phi\phi)[\times 10^{-7}]$	$4.0 \pm 1.1 \pm 0.4 \pm 1.1$	$4.0 \pm 0.6 \pm 0.6 \pm 1.1$	/

Comparison with Run 1 analysis *[Eur. Phys. J.* **C77** *(2017) 609]* and other LHCb analysis with $p\bar{p}$ final state *[Eur. Phys. J.* **C84** *(2024) 1274]*

Compatible with Run 1

Difference with $p\bar{p}$ probably due to $\mathcal{B}(\eta_c(1S) \to \phi\phi)$ normalisation

Small tension between world average of $\mathcal{B}(\eta_c(1S) \to \phi\phi) = (1.4 \pm 0.4) \times 10^{-3}$ vs average from Belle *[Phys. Rev. Lett.* **91** *(2003) 241802]* and BaBar *[Phys. Rev. D* **70** *(2004) 011101]* of $(2.7 \pm 0.7) \times 10^{-3}$, which would give closer results

Comparison with NRQCD

Using computations from Beneke, Maltoni and Rothstein for χ_c production in b-decays *[Phys. Rev.* **D59** *(1999) 054003]*:

$$\mathcal{B}(b \to \chi_{c0}X) = \frac{-0.0148}{m_c^2}\langle O_1^{\chi_{c0}}(^3P_0)\rangle + 0.195\langle O_8^{\chi_{c0}}(^3S_1)\rangle$$

$$\mathcal{B}(b \to \chi_{c1}X) = \frac{-0.00783}{m_c^2}\langle O_1^{\chi_{c1}}(^3P_1)\rangle + 0.195\langle O_8^{\chi_{c1}}(^3S_1)\rangle$$

$$\mathcal{B}(b \to \chi_{c2}X) = \frac{-0.0120}{m_c^2}\langle O_1^{\chi_{c2}}(^3P_2)\rangle + 0.195\langle O_8^{\chi_{c2}}(^3S_1)\rangle$$

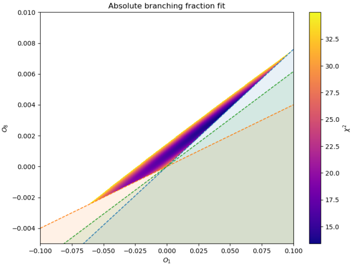
Approximate results, with $\sim 20\%$ uncertainty from c, b quark mass and $\sim 40\%$ (10%) uncertainty from scale for $^3P_J^{(1)}$ ($^3S_1^{(8)}$), on top of χ_{c1} CS uncertainty

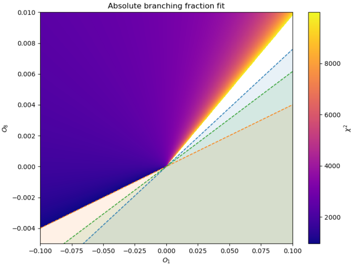
Using $\langle O_1^{\chi_{cJ}}(^3P_0)\rangle = (2J+1)\langle O_1^{\chi_{c0}}(^3P_0)\rangle$, $\langle O_8^{\chi_{cJ}}(^3S_1)\rangle = (2J+1)\langle O_8^{\chi_{c0}}(^3S_1)\rangle$ at first order in v^2

Scanning possible values of $\langle O_1^{\chi_{c0}}(^3P_0)\rangle$, $\langle O_8^{\chi_{c0}}(^3S_1)\rangle$, like A. Usachov thesis method *[Université Paris Saclay (2019)]*

Ignoring correlations between measurements

No compatible values, minimum χ^2 of 13 for absolute $\mathcal{B}$ and 950 for ratios





Masses and width

	Run 1	Run 2	World average
$M_{\eta_c(1S)}$ [MeV]	$2982.8 \pm 1.0 \pm 0.5$	$2984.1 \pm 0.5 \pm 0.5$	2984.1 ± 0.4
$M_{\chi_{c0}}$ [MeV]	$3413.0 \pm 1.9 \pm 0.6$	$3416.7 \pm 1.5 \pm 0.6$	3414.71 ± 0.30
$M_{\chi_{c1}}$ [MeV]	$3508.4 \pm 1.9 \pm 0.7$	$3511.8 \pm 0.6 \pm 0.7$	3510.67 ± 0.05
$M_{\chi_{c2}}$ [MeV]	$3557.3 \pm 1.7 \pm 0.7$	$3556.7 \pm 1.4 \pm 0.7$	3556.17 ± 0.07
$M_{\eta_c(2S)}$ [MeV]	$3636.4 \pm 4.1 \pm 0.7$	$3644.3 \pm 3.5 \pm 0.8$	3637.8 ± 0.6
$\Gamma_{\eta_c(1S)}$ [MeV]	$31.4 \pm 3.5 \pm 2.0$	$34.0 \pm 1.1 \pm 0.7$	30.5 ± 0.5
$\Gamma_{\chi_{c0}}$ [MeV]	—	$16.3 \pm 2.9 \pm 0.4$	10.9 ± 0.6
$\Gamma_{\eta_c(2S)}$ [MeV]	—	$18.6 \pm 5.1 \pm 1.2$	11.6 ± 1.4

Masses within 2σ of world average, widths within 2.5σ

Best single measurement of $\eta_c(1S)$ mass

Conclusion

Study of inclusive charmonium production in b-decays

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Adding interference model
$\Rightarrow$ Hints of interference for $\eta_c(1S)$ in particular

Measured χ_{cJ} production incompatible with current models

Best single measurement of $\eta_c(1S)$ mass

Can combine with $p\overline{p}$ results to get best precision for all 3 χ_c states

Thank you for listening

APPENDIX

Efficiencies

State	$\varepsilon_{\text{tot}}^{2015}$ [%]	$\varepsilon_{\text{tot}}^{2016}$ [%]	$\varepsilon_{\text{tot}}^{2017}$ [%]	$\varepsilon_{\text{tot}}^{2018}$ [%]
$\eta_c(1S)$	0.161 ± 0.004	0.388 ± 0.007	0.441 ± 0.008	0.275 ± 0.006
χ_{c0}	0.249 ± 0.005	0.456 ± 0.009	0.505 ± 0.009	0.343 ± 0.007
χ_{c1}	0.279 ± 0.006	0.465 ± 0.009	0.534 ± 0.010	0.339 ± 0.007
χ_{c2}	0.272 ± 0.006	0.475 ± 0.009	0.525 ± 0.010	0.336 ± 0.007
$\eta_c(2S)$	0.317 ± 0.006	0.516 ± 0.009	0.556 ± 0.010	0.384 ± 0.007

Corrections to tracking and PID efficiencies from calibration samples *[JINST* **10** *(2015) P02007, LHCb-PUB-2016-021 (2016)]*

Mostly goes up with charmonium mass

In 2015, fewer triggers available

Lower trigger efficiencies in 2018, stricter requirements

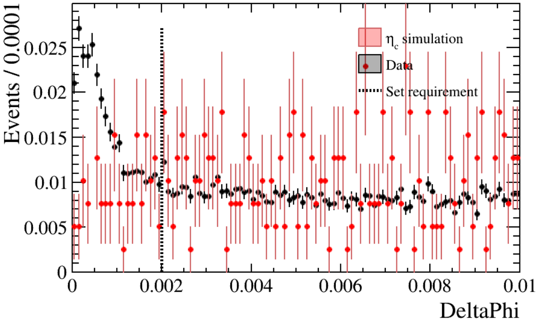
Cloned tracks

Cut $|\Delta\phi| > 2\,\text{mrad}$ on all kaon combinations

Selection to removed cloned tracks

Around 4% of events eliminated

Similar distribution for other combinations and years



Events with multiple candidates

Around 1.5% of events with multiple candidates in the data before the 2D $(KK)(KK)$ fit

Can't know remaining ones after the 2D $(KK)(KK)$ fit

Mostly absorbed in the backgrounds and remainder should compensate between states in the ratios

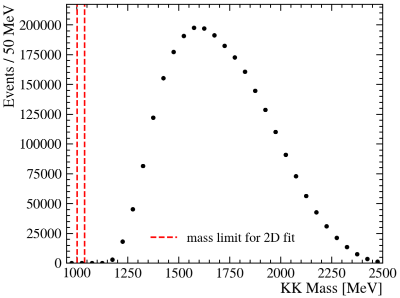
$\Rightarrow$ Negligible compared to current uncertainties

Different kaon combinations

Check that other kaon combinations do not peak under the ϕ mass (range in red)

No contribution seen

Similar distribution for other years and combinations (except $K_1^+ + K_1^-$ and $K_2^+ + K_2^-$)



Invariant mass of the $K_1^+ + K_2^-$ combinations for 2016

Remaining prompt contribution

Using prompt η_c MC, $\frac{\varepsilon_{\text{prompt}}}{\varepsilon_{\text{nonprompt}}} \approx 0.1\%$ for all years

Then the proportion of prompt remaining in the data is $\frac{\varepsilon_{\text{prompt}}}{\varepsilon_{\text{nonprompt}}} \frac{\sigma_{\text{prompt}}}{\sigma_{\text{nonprompt}}}$

$\frac{\sigma_{\text{prompt}}}{\sigma_{\text{nonprompt}}}$ not available for all states

But all studies on charmonia (J/ψ, $\psi(2S)$, $\eta_c(1S)$, χ_{c1} and χ_{c2}) show $\frac{\sigma_{\text{prompt}}}{\sigma_{\text{nonprompt}}} \lesssim 10$

2% uncertainty taken on ratios conservatively, as this effect should also in part cancel out

Studies on charmonium prompt vs nonprompt

Reference	State	$\sqrt{s}$	$y \in$	$p_{\mathrm{T}} \in$	$\frac{\sigma_{\text{prompt}}}{\sigma_{\text{nonprompt}}}$
[Eur. Phys. J. **C80** *(2020) 191]*	$\eta_c(1S)$	13	$2.0 - 4.5$	$6.5 - 14.0$	13.0 ± 2.8
[Eur. Phys. J. **C84** *(2024) 1274]*	$\eta_c(1S)$	13	$2.0 - 4.0$	$5.0 - 14.0$	10.0 ± 2.1
[Eur. Phys. J. **C71** *(2011) 1645]*	J/ψ	7	$2.0 - 4.5$	$0.0 - 14.0$	$9.2^{+2.3}_{-2.6}$
[JHEP **10** *(2015) 172]*	J/ψ	13	$2.0 - 4.5$	$0.0 - 14.0$	6.7 ± 0.6
[Eur. Phys. J. **C80** *(2020) 185]*	$\psi(2S)$	7	$2.0 - 4.5$	$3.5 - 14.0$	3.7 ± 0.3
[Eur. Phys. J. **C80** *(2020) 185]*	$\psi(2S)$	13	$2.0 - 4.5$	$2.0 - 20.0$	3.4 ± 0.3
[JHEP **07** *(2014) 154]*	χ_{c1}	7	$-0.75 - 0.75$	$12.0 - 14.0$	4.3 ± 1.42
[JHEP **07** *(2014) 154]*	χ_{c2}	7	$-0.75 - 0.75$	$12.0 - 14.0$	$9.1^{+4.7}_{-6.7}$

All studies of charmonium production show $\frac{\sigma_{\text{prompt}}}{\sigma_{\text{nonprompt}}} \lesssim 10$

Allowed orbital angular momenta

J^{PC} for ϕ: 1^{--}

From parity conservation: $P = (-1)^L$

System of two identical bosons ($\phi\phi$): $(-1)^L(-1)^S = 1 \Rightarrow L + S$ even *[Phys. Rev.* **77** *(1950) 242–245]*

Using conservation of total angular momentum

State	J^{PC}	Possible (L, S)	$\rightarrow$ with $P = (-1)^L$	$\rightarrow$ with $L + S$ even
$\eta_c(1S)$	0^{-+}	(0,0),(1,1),(2,2)	(1,1)	(1,1)
χ_{c0}	0^{++}	(0,0),(1,1),(2,2)	(0,0),(2,2)	(0,0),(2,2)
χ_{c1}	1^{++}	(0,1),(1,0),(1,1) (1,2),(2,1) (2,2),(3,2)	(0,1),(2,1),(2,2)	(2,2)
χ_{c2}	2^{++}	(0,2),(2,0),(1,1) (1,2),(2,1),(2,2) (3,1),(3,2),(4,2)	(0,2),(2,0),(2,1),(2,2),(4,2)	(0,2),(2,0),(2,2),(4,2)
$\eta_c(2S)$	0^{-+}	(0,0),(1,1),(2,2)	(1,1)	(1,1)

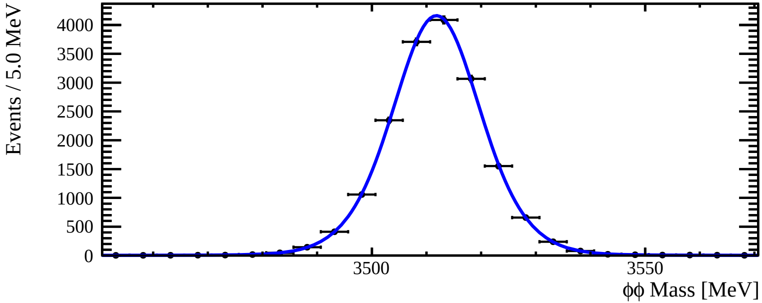
MC constraints

Extract the variables necessary to constrain the fit to $\phi\phi$ mass:

- Ratio between the Gaussian σs
- Ratio between the Gaussian yields
- Ratio between the σs of all states to the ones of χ_{c1} (see plot)

From 2016, 2017, 2018 simulation, no 2D fit before $\phi\phi$ fit

$\sigma_1^{\eta_c} = 4.55 \pm 0.45$ with MC and $\sigma_1^{\eta_c} = 4.18 \pm 0.24$ on data



Interference details for each state

When interference added to states individually:

	$D(\times 10^{-4})$	M_{noint}	M_{int}	Γ_{noint}	Γ_{int}
$\eta_c(1S)$	-1.1 ± 0.3	2982.44 ± 0.34	2983.93 ± 0.52	33.06 ± 1.09	33.50 ± 1.10
χ_{c0}	-2.7 ± 1.5	3414.49 ± 0.87	3416.39 ± 1.37	17.26 ± 2.96	17.43 ± 3.00
χ_{c1}	-1.6 ± 7.8	3511.98 ± 0.53	3511.80 ± 0.85	/	/
χ_{c2}	-0.7 ± 6.2	3556.40 ± 0.61	3556.29 ± 1.18	/	/
$\eta_c(2S)$	-5.8 ± 3.5	3637.17 ± 1.73	3642.26 ± 3.41	17.34 ± 4.74	19.05 ± 4.96

Expansion in L for interference, 3σ away from 0 for $\eta_c(1S)$,

$\Rightarrow$ Interference effects added for χ_{c0}, χ_{c2} ($L = 0$), η_c, $\eta_c(2S)$ ($L = 1$)

When all, $D_{\eta_c(1S)} = -0.00013 \pm 0.00003$, $D_{\chi_{c0}} = -0.00033 \pm 0.00019$, $D_{\chi_{c2}} = -0.00025 \pm 0.00079$ and $D_{\eta_c(2S)} = -0.00090 \pm 0.00041$

NRQCD predictions

Using computations from Beneke, Maltoni and Rothstein *[Phys. Rev.* **D59** *(1999) 054003]*

Both ${}^3P_J^{(1)}$ and ${}^3S_1^{(8)}$ contribute to χ_{cJ} production at leading order in v

$$\mathcal{B}(b \to \chi_{c0} X) = \Gamma[{}^3P_0^{(1)}]\langle O_1^{\chi_{c0}}({}^3P_0)\rangle + \Gamma[{}^3S_1^{(8)}]\langle O_8^{\chi_{c0}}({}^3S_1)\rangle$$
$$\mathcal{B}(b \to \chi_{c1} X) = \Gamma[{}^3P_1^{(1)}]\langle O_1^{\chi_{c1}}({}^3P_1)\rangle + \Gamma[{}^3S_1^{(8)}]\langle O_8^{\chi_{c1}}({}^3S_1)\rangle$$
$$\mathcal{B}(b \to \chi_{c2} X) = \Gamma[{}^3P_2^{(1)}]\langle O_1^{\chi_{c2}}({}^3P_2)\rangle + \Gamma[{}^3S_1^{(8)}]\langle O_8^{\chi_{c2}}({}^3S_1)\rangle$$

Large coefficient $\mathcal{C}_{[8]}^2$ in front of ${}^3S_1^{(8)}$ production, higher velocity orders not contributing significantly

Small, scaled dependent coefficient $\mathcal{C}_{[1]}^2(\mu)$ in front of ${}^3P_1^{(1)}$) production, so NLO contribution important

But with NLO, negative decay rate $\Rightarrow$ need NNLO *[Phys. Lett. B* **328** *(1994) 153–161]*

Expansion in α_s and $\mathcal{C}_{[1]}/\mathcal{C}_{[8]}$ (instead of just α_s), so should remove $\alpha_s\mathcal{C}_{[1]}^2$ and add $\alpha_s^2\mathcal{C}_{[8]}^2$

Not all terms computed for $\alpha_s^2\mathcal{C}_{[8]}^2$, using approximation

Because of that, large uncertainty on CS contribution to χ_{c1}, maybe up to 100%

Cross-check for decays in flight

In MC, $\sim 1.4\%$ of events with decay in flight of a kaon after nominal selection (S)

Applying harsher (H) selection to see potential effects on the results, lowering proportion to $\sim 0.4\%$

No significant effect on measured interference or yield ratios

	$\eta_c(1S)$	χ_{c0}	χ_{c2}	$\eta_c(2S)$
$D_{\mathrm{S}}(\times 10^{-4})$	-1.3 ± 0.3	-3.3 ± 1.9	-2.5 ± 7.9	-9.0 ± 4.1
$D_{\mathrm{H}}(\times 10^{-4})$	-1.5 ± 0.3	-3.0 ± 2.0	-4.2 ± 12.6	-8.7 ± 4.5

	$\chi_{c0}/\eta_c(1S)$	$\chi_{c1}/\eta_c(1S)$	χ_{c1}/χ_{c0}	$\chi_{c2}/\eta_c(1S)$	χ_{c2}/χ_{c0}	$\eta_c(2S)/\eta_c(1S)$
R_{S}	0.113 ± 0.011	0.067 ± 0.005	0.591 ± 0.067	0.067 ± 0.009	0.592 ± 0.100	0.039 ± 0.006
R_{H}	0.109 ± 0.011	0.064 ± 0.006	0.587 ± 0.074	0.068 ± 0.015	0.626 ± 0.152	0.038 ± 0.007

Cross-check for PID requirement and background in 2D fit

PID requirement of *ProbNNk* > 0.1 chosen

Comparing with results for *ProbNNk* > 0.2

	$\chi_{c0}/\eta_c(1S)$	$\chi_{c1}/\eta_c(1S)$	χ_{c1}/χ_{c0}	$\chi_{c2}/\eta_c(1S)$	χ_{c2}/χ_{c0}	$\eta_c(2S)/\eta_c(1S)$
R_1	0.113 ± 0.011	0.067 ± 0.005	0.591 ± 0.067	0.067 ± 0.009	0.592 ± 0.100	0.039 ± 0.006
R_2	0.115 ± 0.012	0.065 ± 0.005	0.568 ± 0.066	0.066 ± 0.010	0.577 ± 0.103	0.038 ± 0.007

Tried adding exponential part to the background in the 2D $(K^+K^-)(K^+K^-)$ fit

	$\chi_{c0}/\eta_c(1S)$	$\chi_{c1}/\eta_c(1S)$	χ_{c1}/χ_{c0}	$\chi_{c2}/\eta_c(1S)$	χ_{c2}/χ_{c0}	$\eta_c(2S)/\eta_c(1S)$
R_{const}	0.113 ± 0.011	0.067 ± 0.005	0.591 ± 0.067	0.067 ± 0.009	0.592 ± 0.100	0.039 ± 0.006
R_{exp}	0.113 ± 0.011	0.067 ± 0.005	0.592 ± 0.067	0.066 ± 0.009	0.588 ± 0.095	0.039 ± 0.006

Again, no significant difference for either change

Checks of dataset consistency with luminosity

Yields corrected for efficiencies are divided by the luminosity per year

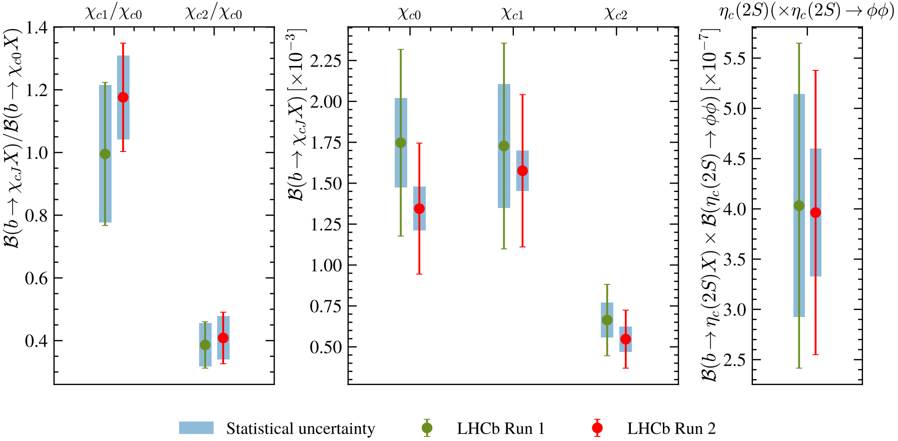
This should be constant apart from uncertainties

Deviation from weighted average never higher than 2σ

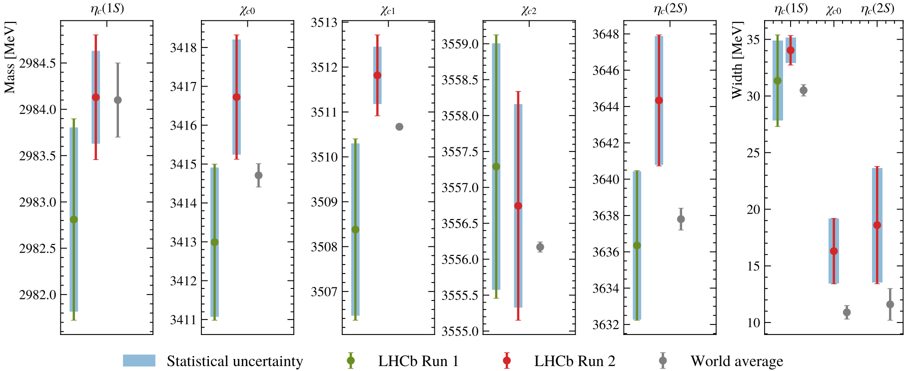
$\Rightarrow$ No unreasonable deviation

State	2015	2016	2017	2018
$\eta_c(1S)$	$16.65 \pm 1.22 \pm 0.96$	$14.82 \pm 0.52 \pm 0.65$	$15.02 \pm 0.51 \pm 0.68$	$15.88 \pm 0.56 \pm 0.83$
χ_{c0}	$1.34 \pm 0.42 \pm 0.11$	$1.52 \pm 0.20 \pm 0.10$	$2.12 \pm 0.25 \pm 0.12$	$1.69 \pm 0.21 \pm 0.10$
χ_{c1}	$0.52 \pm 0.24 \pm 0.08$	$0.96 \pm 0.11 \pm 0.08$	$0.94 \pm 0.11 \pm 0.08$	$1.21 \pm 0.11 \pm 0.09$
χ_{c2}	$0.98 \pm 0.26 \pm 0.10$	$1.01 \pm 0.16 \pm 0.09$	$1.06 \pm 0.17 \pm 0.09$	$1.02 \pm 0.17 \pm 0.09$
$\eta_c(2S)$	$1.02 \pm 0.27 \pm 0.15$	$0.56 \pm 0.12 \pm 0.11$	$0.62 \pm 0.13 \pm 0.12$	$0.57 \pm 0.13 \pm 0.11$

Branching fraction plot



Masses and widths plot



Dependence on p_T, η, event multiplicity, $|cos(\theta_\phi)|$

Checking the dependence on p_T, pseudorapidity, event multiplicity and $|cos(\theta_\phi)|$ of yield ratios with η_c

No significant dependence found
Fluctuations in η_c can mimic tendencies as those are ratios (last bin for $|cos(\theta_\phi)|$)