Amplitude Analysis of $B_s^0 \to K_S^0\pi^+\pi^-$ with Run 1+2 data

Final studies with GLASS + Fit validation

S. Ordonez-Soto

LHCb@LPCA meeting

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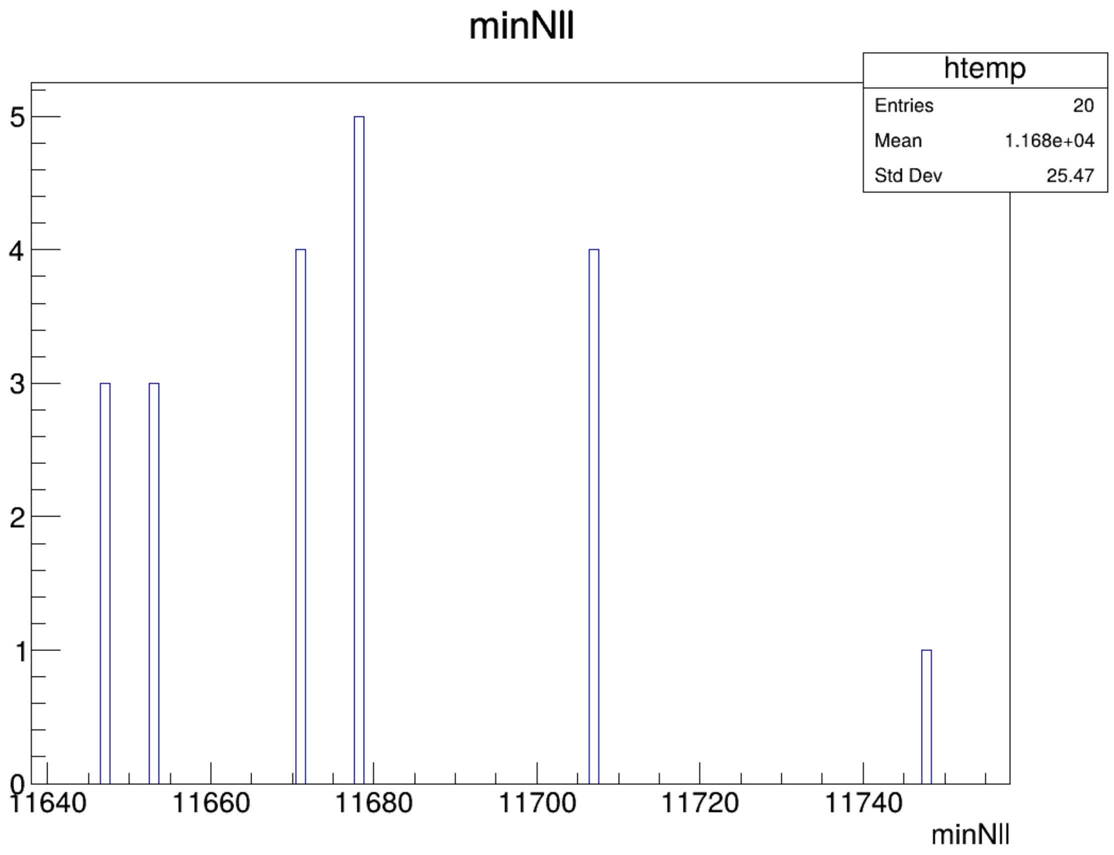
LPC Particules pLasmas Univers applicationS
Laboratoire de Physique de Clermont

ÉCOLE DOCTORALE
DES SCIENCES FONDAMENTALES
Université Clermont Auvergne

Model building with GLASS

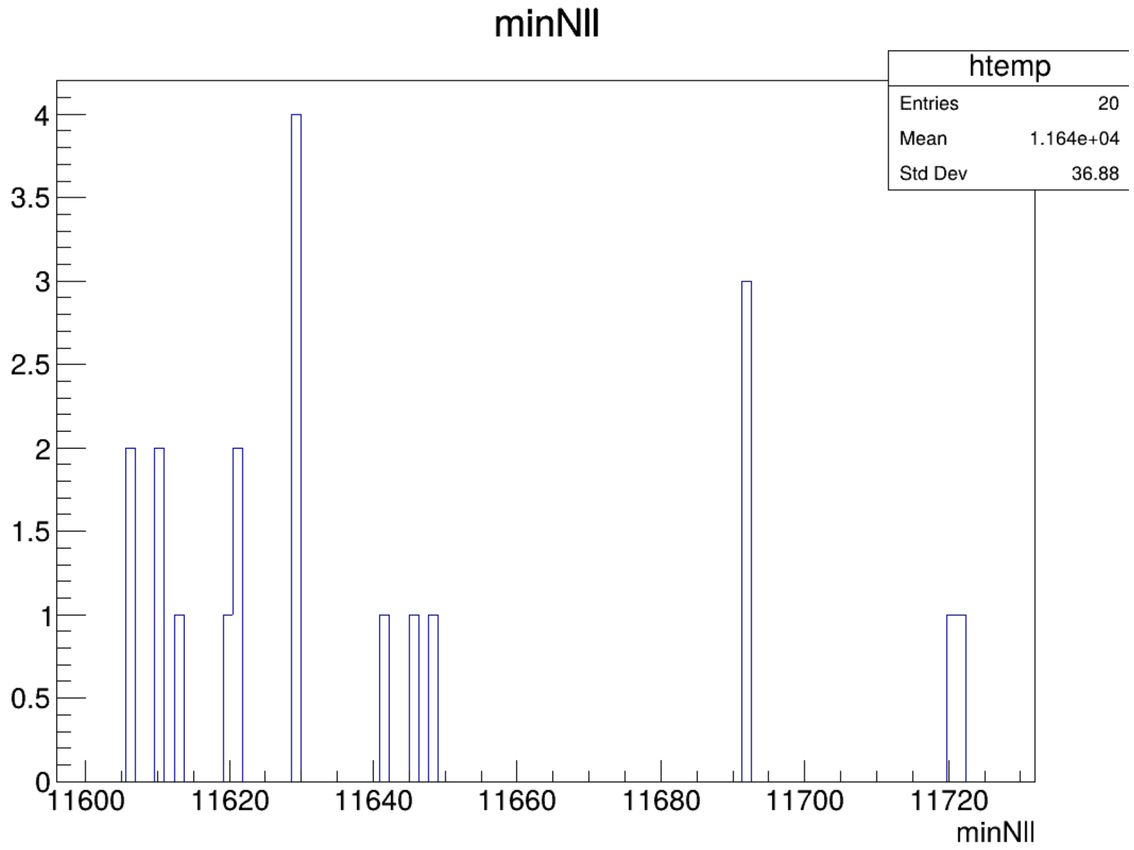
Baseline



best_minNll_in_cluster
11647.5409579884
11653.4519869079
11671.1214408644
11678.4859884714
11706.4593275998
11748.3590711584

Model building with GLASS

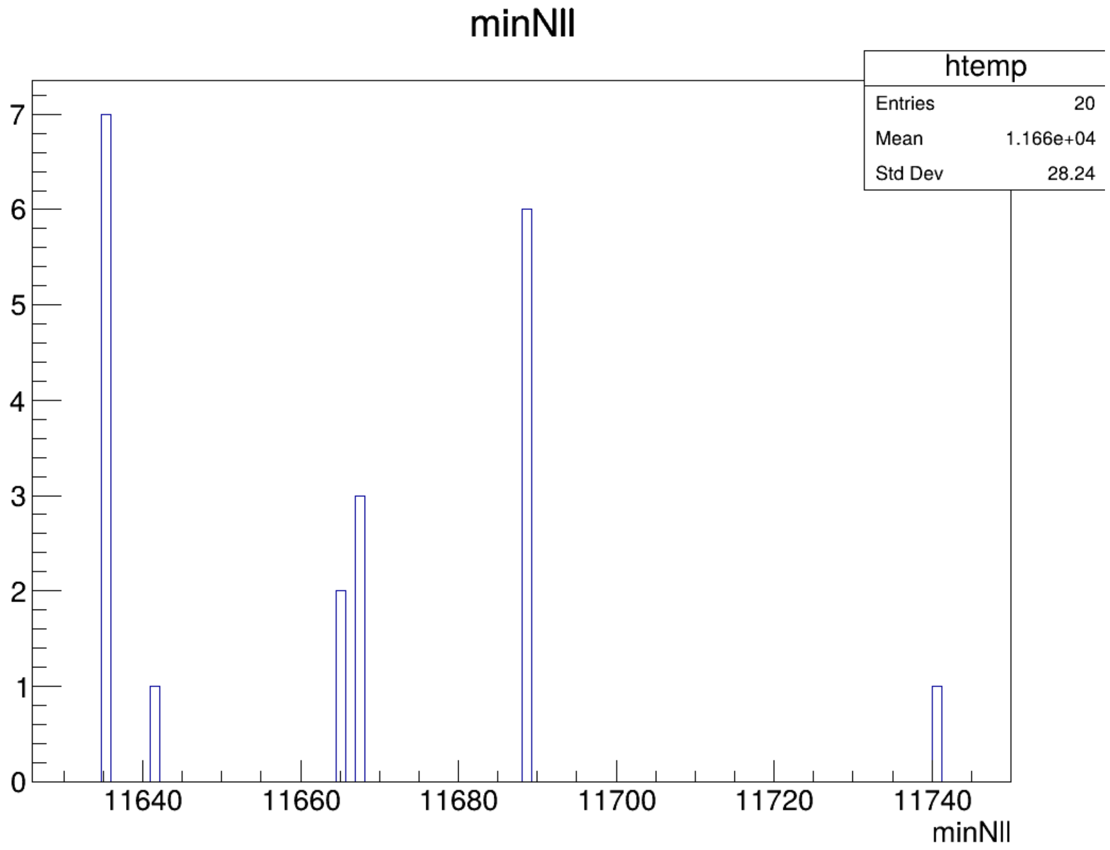
(Baseline) + kstar2_1430



best_minNll_in_cluster
11606.5247458524
11609.8807097827
11613.2175075841
11619.584108686
11621.4553602747
11621.5666184397
11629.9346345282
11641.9262246295
11645.0320828556
11648.741178015
11692.0048900385
11692.2536361516
11719.9045879006
11721.2882379818

Model building with GLASS

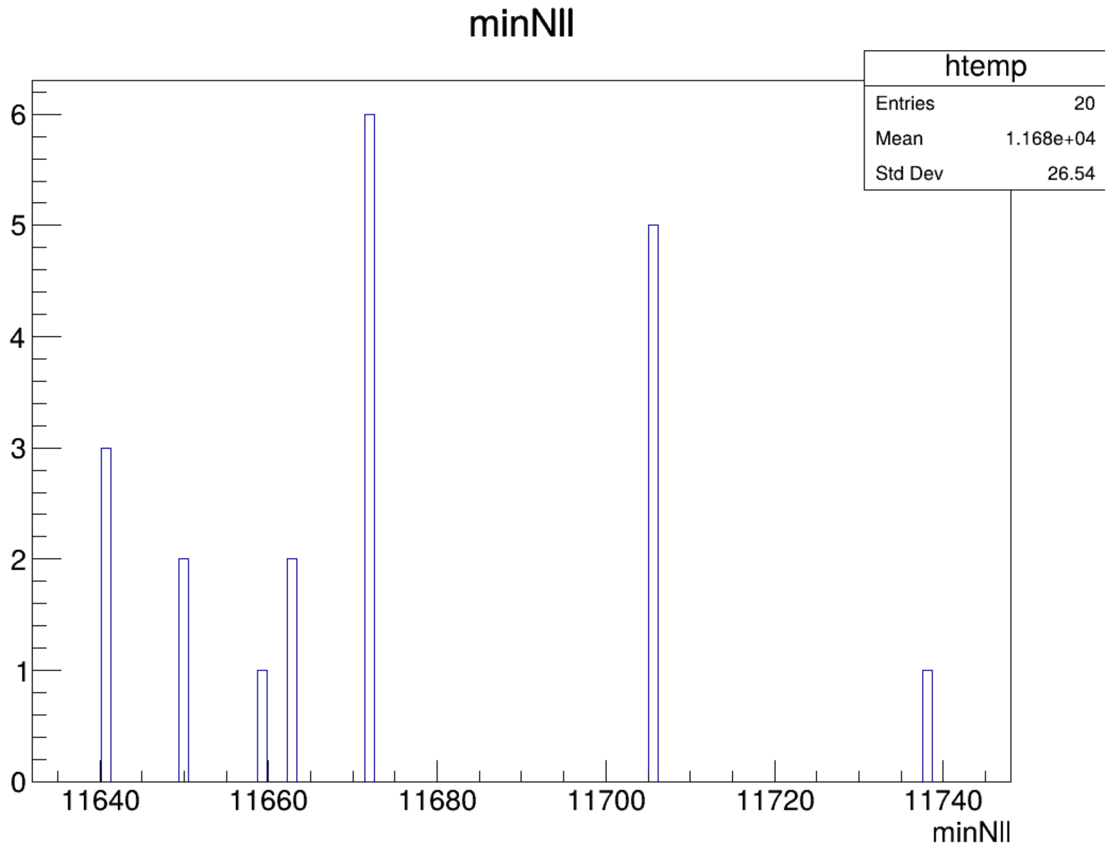
(Baseline) + kstar_1680



best_minNll_in_cluster
11635.3538778002
11641.1564443882
11665.1518843589
11667.0906704781
11688.79906905
11741.3124463052

Model building with GLASS

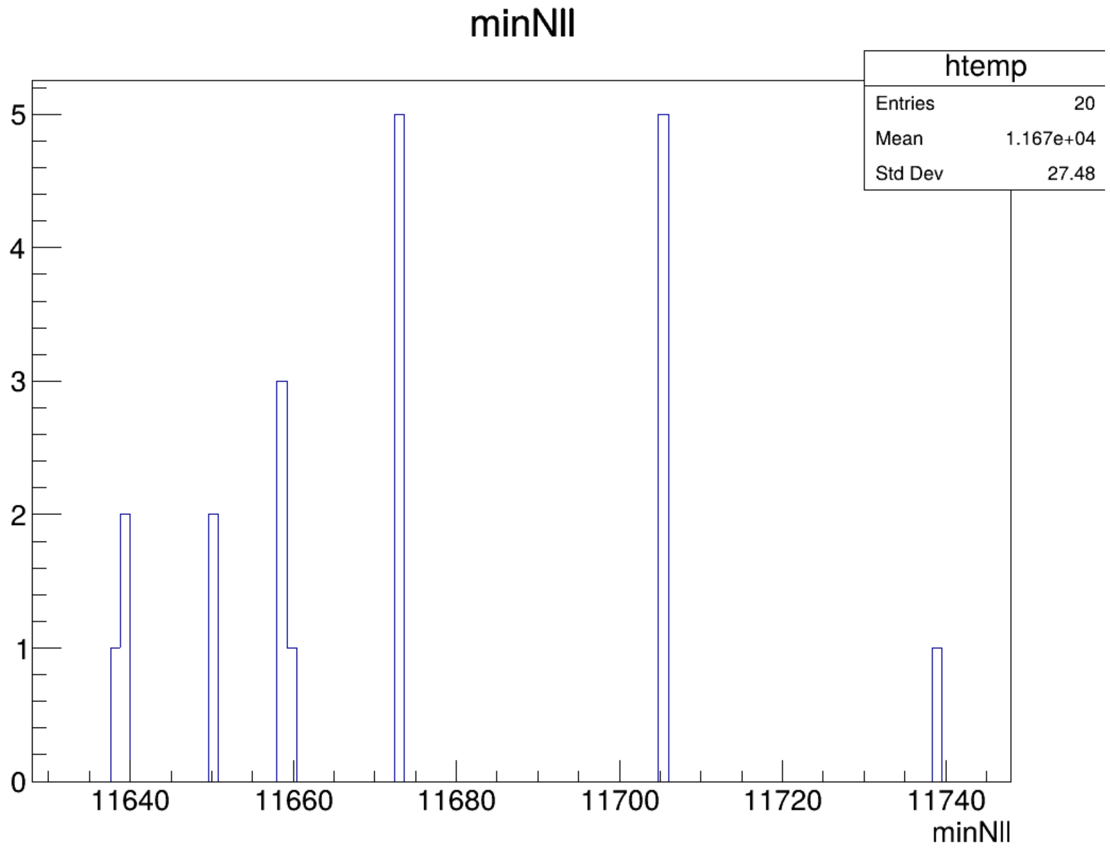
(Baseline) + f2_1270



best_minNll_in_cluster
11640.5790245312
11649.4681855916
11659.761331896
11663.0417776558
11672.5958873379
11705.419408793
11738.6465313737

Model building with GLASS

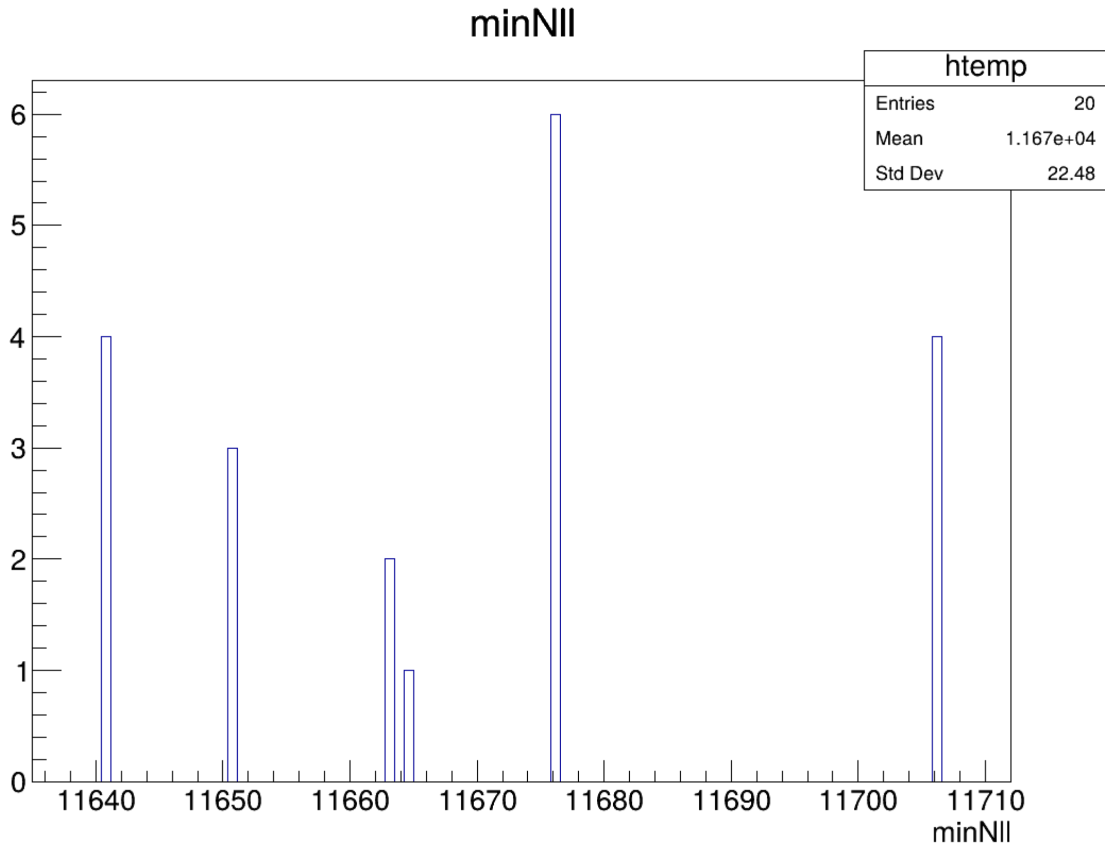
(Baseline) + f0_1370



best_minNll_in_cluster
11637.6521053849
11639.3354186122
11650.4430476042
11658.3503924989
11659.7380064566
11672.5707195581
11673.0599805671
11705.9813712159
11738.9407568589

Model building with GLASS

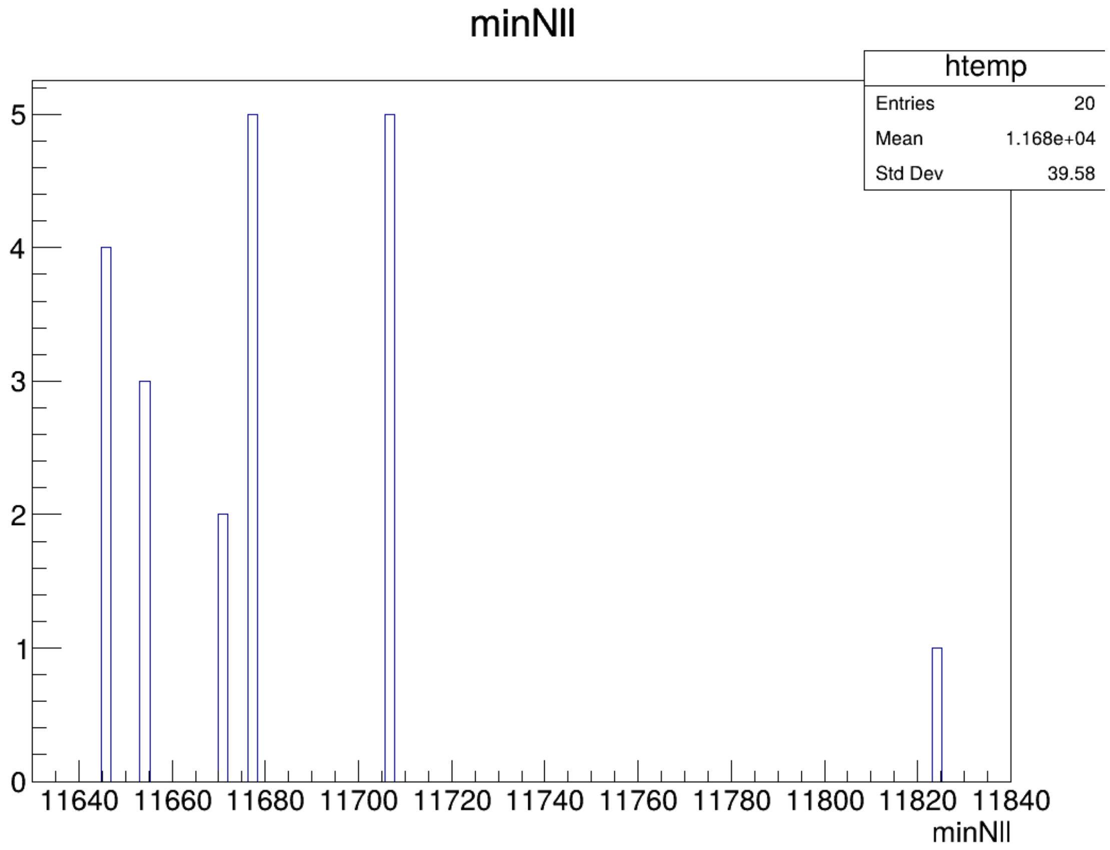
(Baseline) + f0_1500



best_minNll_in_cluster
11640.9385405322
11650.8082821656
11663.4266286384
11664.8192747105
11676.2873592452
11706.3098570615

Model building with GLASS

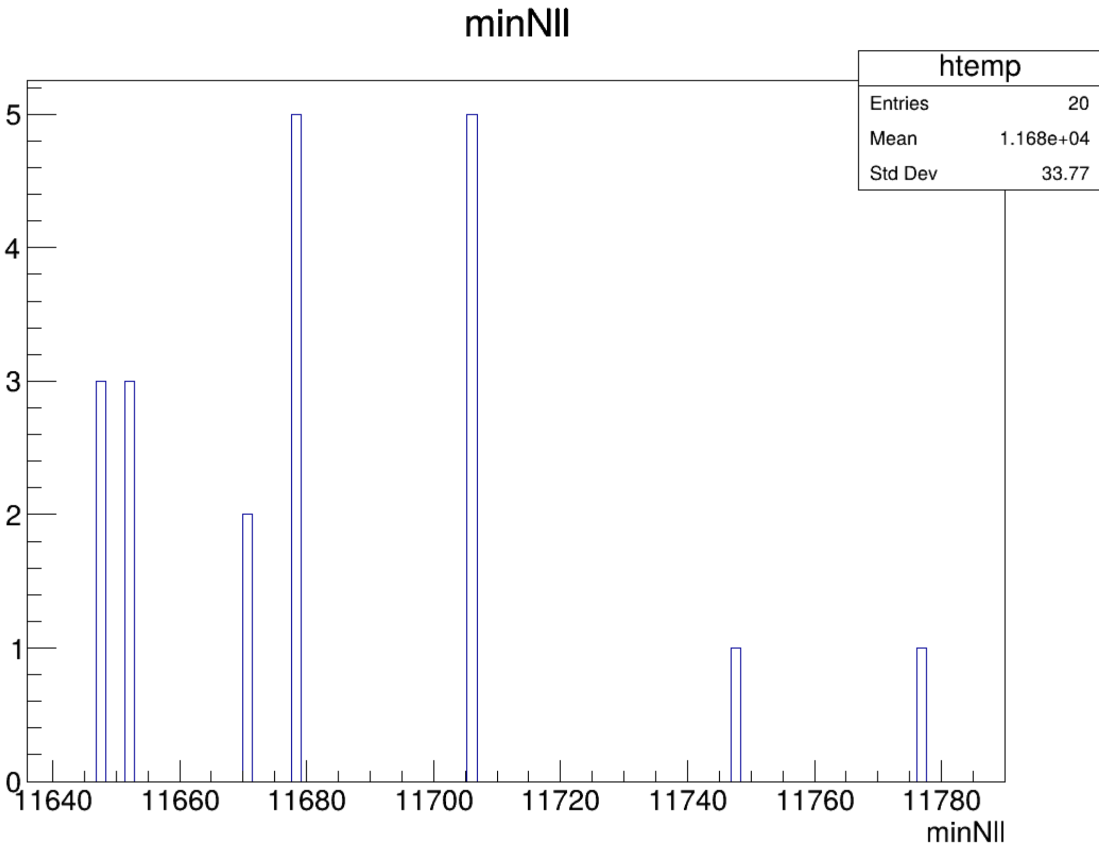
(Baseline) + f0_980



best_minNll_in_cluster
11646.1906015344
11653.4259871355
11670.0367934904
11670.8828343157
11677.3848573399
11706.3826766156
11824.8554176959

Model building with GLASS

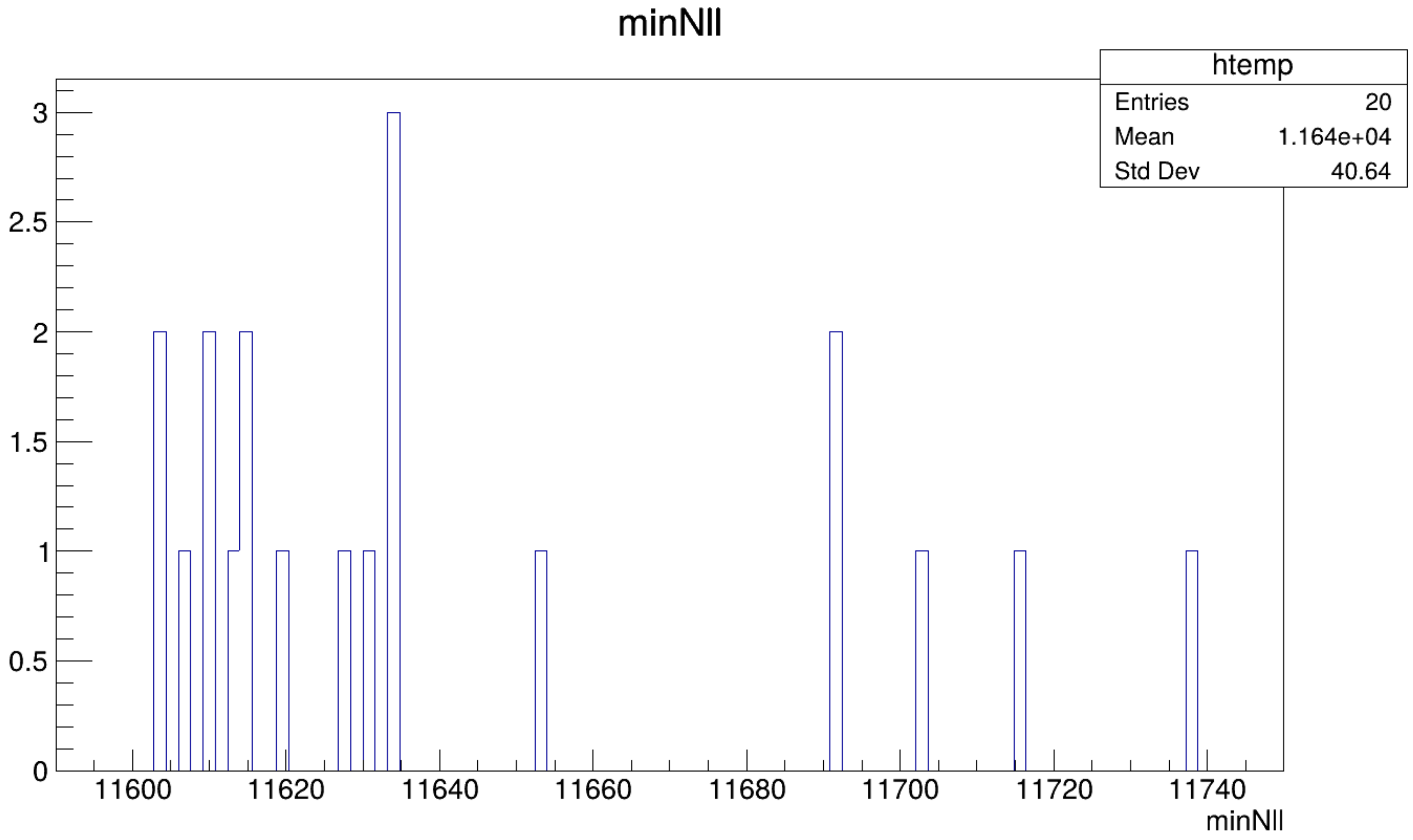
(Baseline) + chic0



best_minNll_in_cluster
11647.1807842243
11652.3315936196
11670.7811596216
11677.5985689289
11705.7985690826
11748.2058284765
11777.180265386

Model building with GLASS

(Baseline + kstar2_1430) + f2(1270)



best_minNll_in_cluster
11602.9428118262
11607.486055569
11609.4812884023
11613.7468017968
11614.4843370528
11619.0813693943
11627.355484143
11630.3261556062
11633.243923132
11653.4751558728
11691.4336778861
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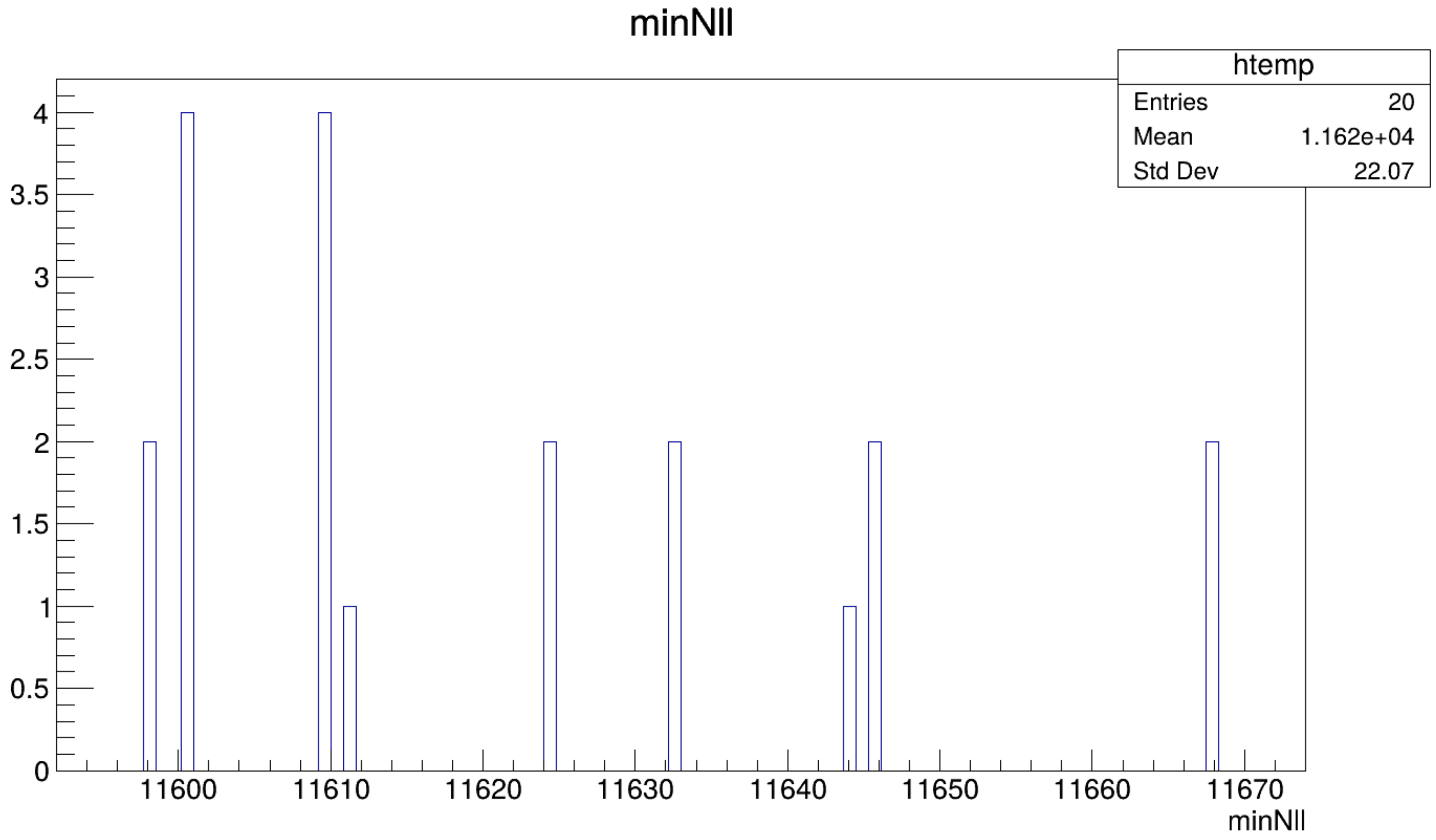
11714.9096735239
11738.4215242444

Model building with GLASS

(Baseline + kstar2_1430) + kstar_1680



best_minNll_in_cluster
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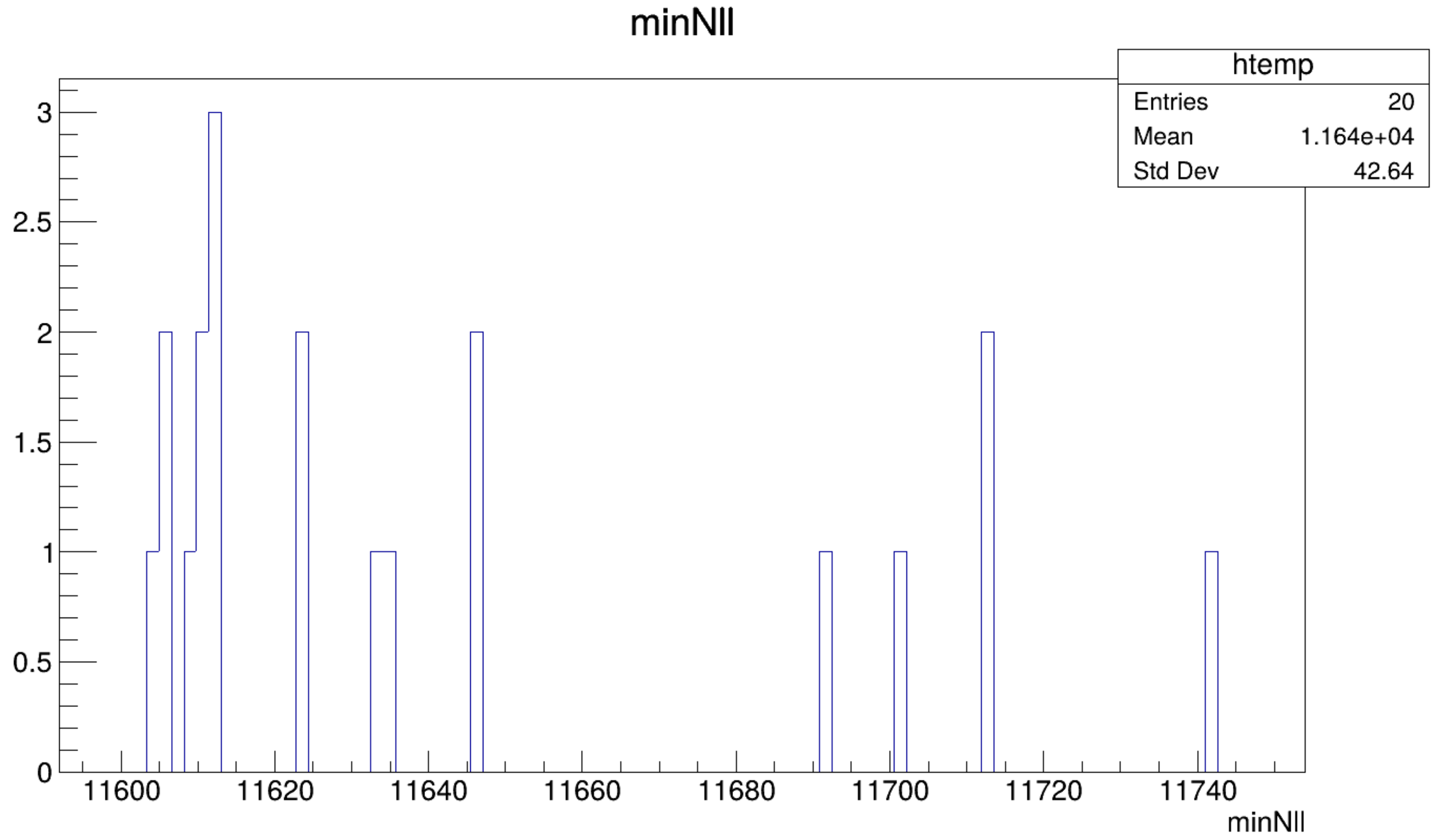
11600.350365619
11609.768100245
11611.0638557088
11624.6826940978
11632.8796726113
11643.8382372175
11645.7506107117
11667.6450789579

Model building with GLASS

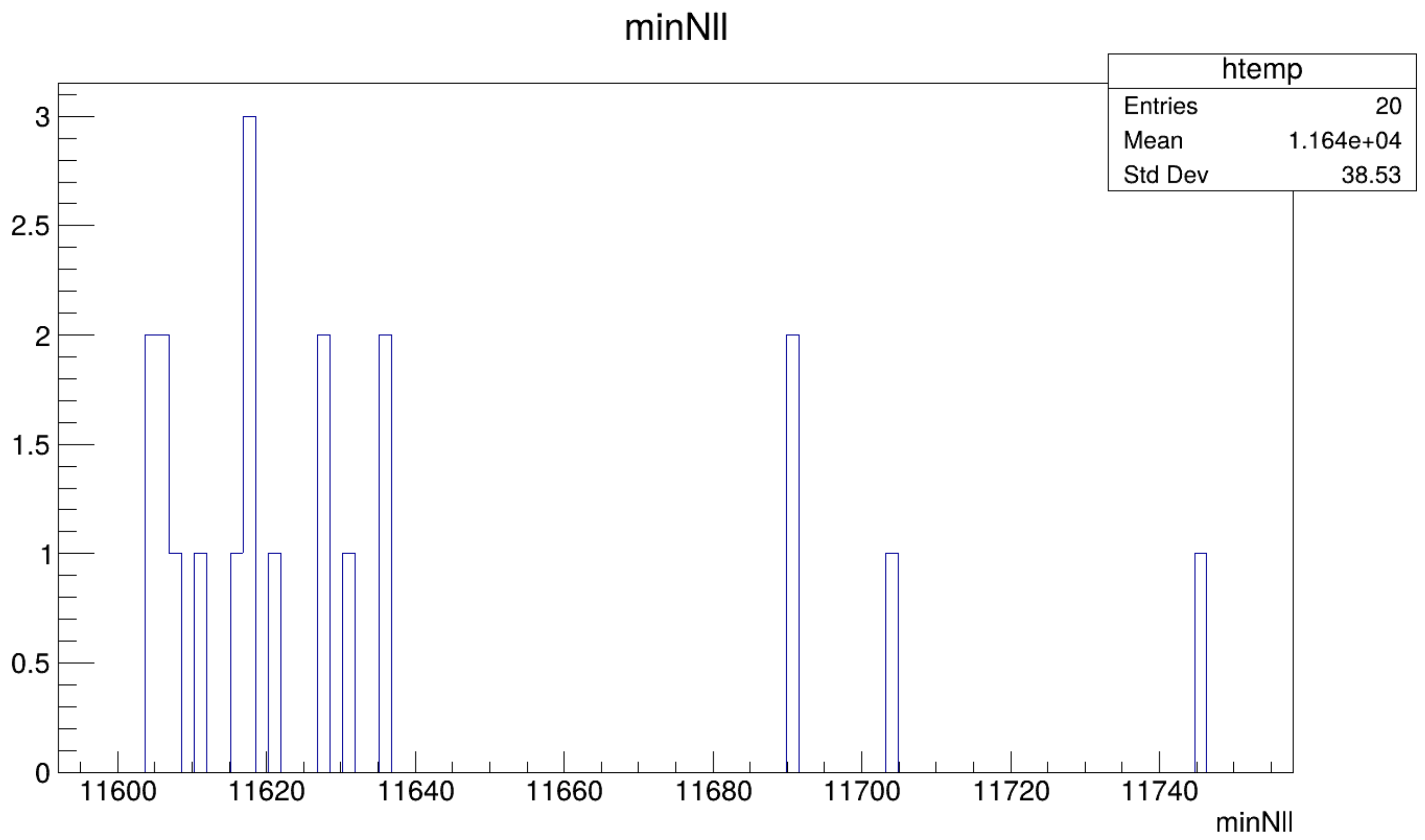
(Baseline + kstar2_1430) + f0_1370



best_minNll_in_cluster
11604.7760096481
11606.1963201344
11608.3234383979
11609.8250296513
11612.9000524562
11623.7696598963
11624.0187374796
11633.2118294878
11634.2190867554
11645.8958856134
11691.7804428729
11700.6930872749
11713.0643374297
11713.0654855188
11741.1611601342

Model building with GLASS

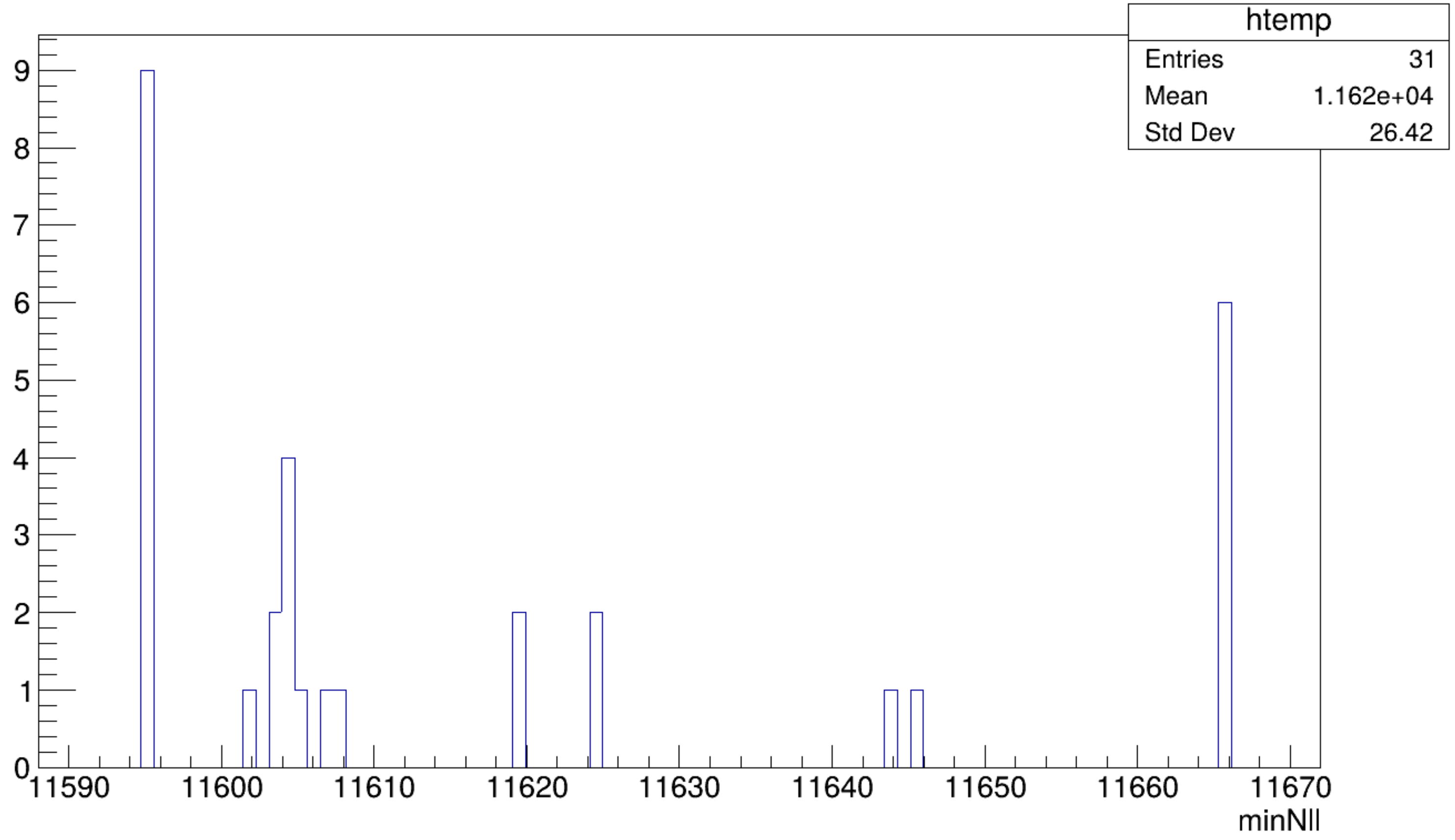
(Baseline + kstar2_1430) + f0_1500



best_minNll_in_cluster
11604.8888473784
11606.77602754
11608.5688912527
11610.6724394609
11615.5408200848
11618.456489952
11618.506378189
11618.5499449242
11627.0291665872
11628.2080542891
11636.0474879733
11636.3229476782
11691.5119611307
11704.6432555899
11745.7806354651

Model building with GLASS

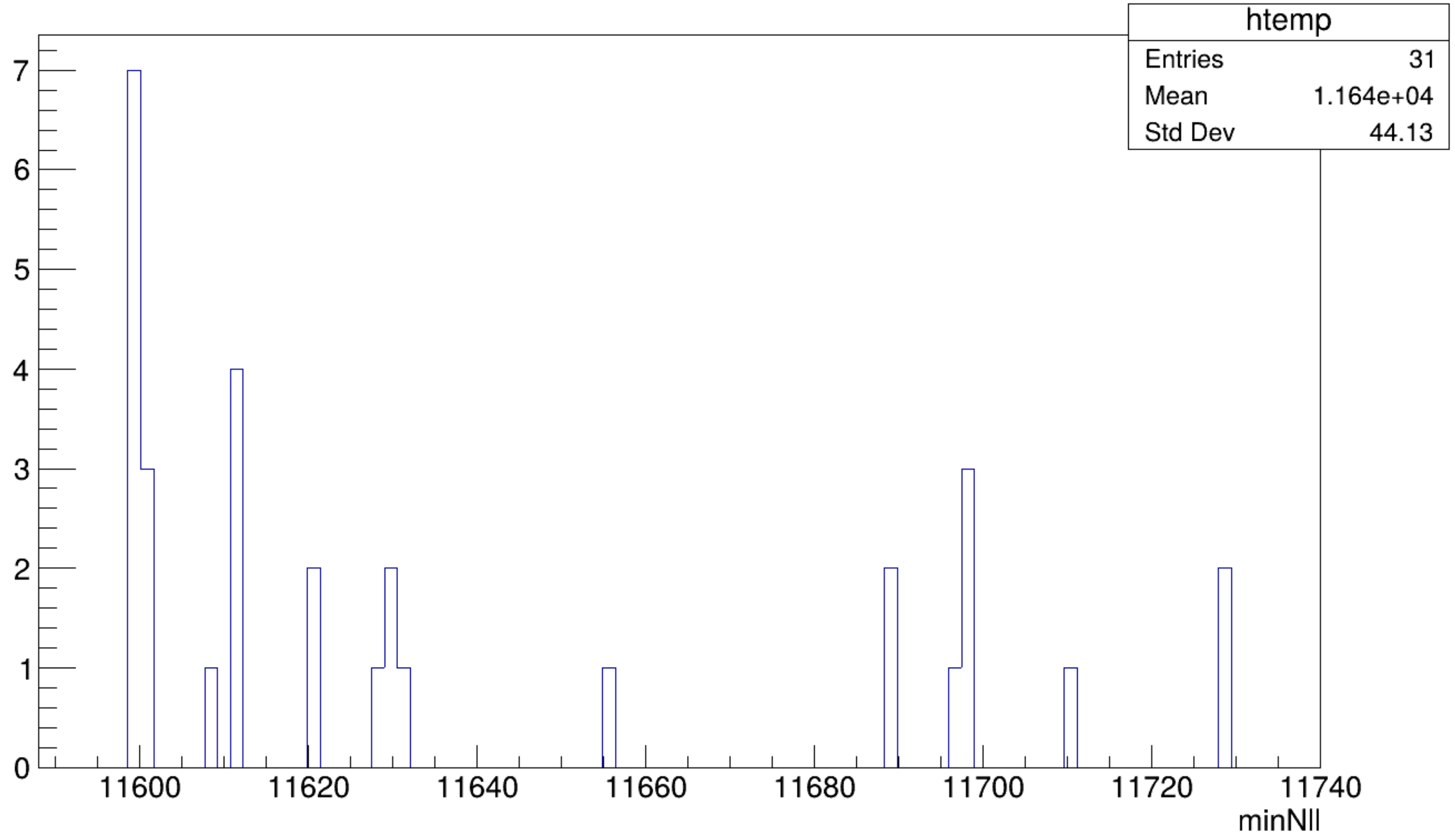
(Baseline + kstar2_1430 + f2(1270)) + kstar_1680



best_minNll_in_cluster
11595.0053430455
11595.4940822868
11602.2041761813
11603.869311048
11604.062423139
11605.2113305512
11607.6614864297
11619.8878650188
11624.2529549856
11643.8831132553
11645.9506504301
11665.4943131895
11665.6447312723

Model building with GLASS

(Baseline + kstar2_1430 + f2(1270)) + f0_1370



best_minNll_in_cluster
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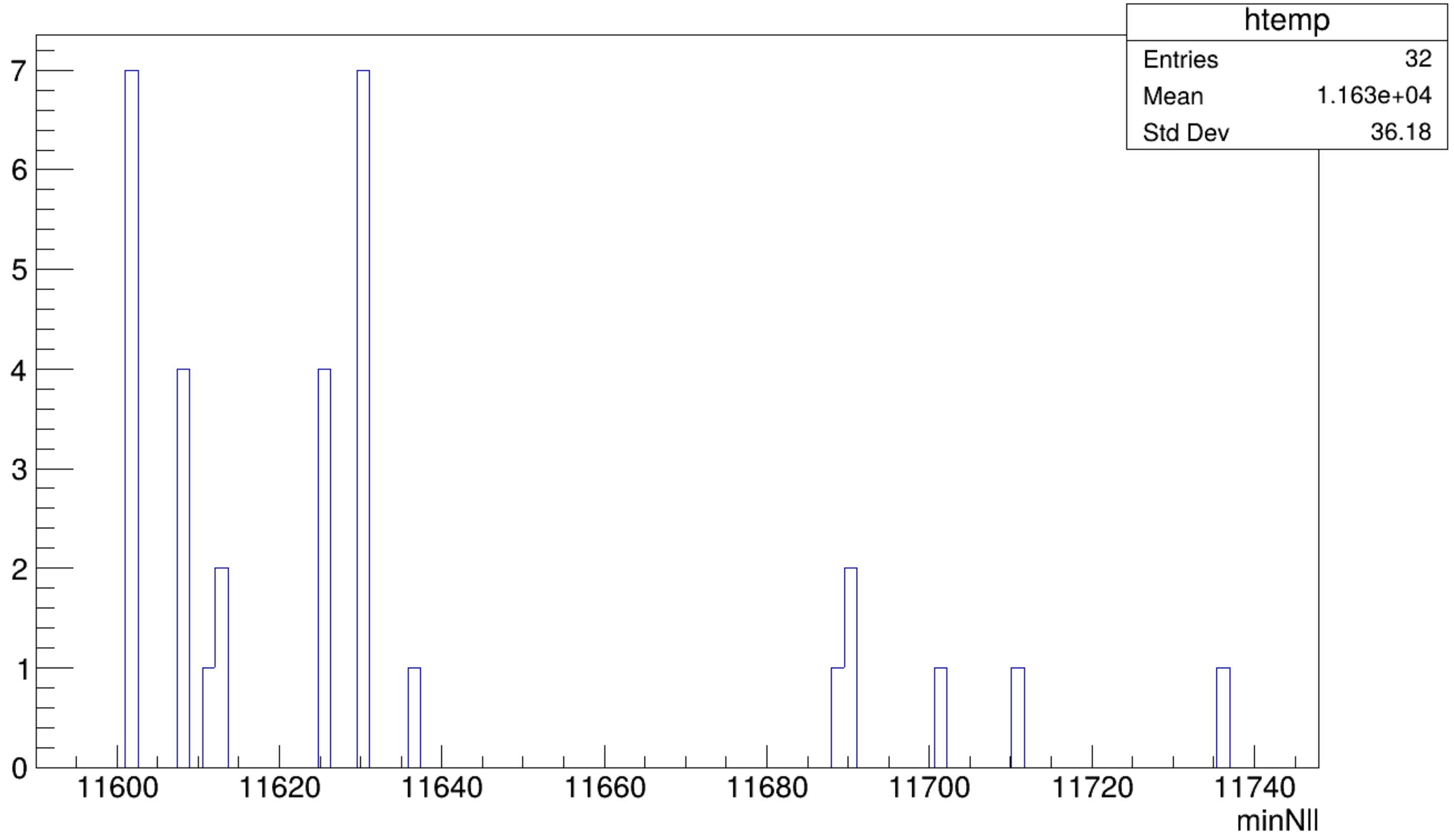
11600.2499426312
11600.4196315114
11608.5559704411
11611.7791889401
11620.1286141815
11620.5537773334
11628.0833515359
11629.1873935773
11630.3246530022
11630.7389201891
11655.8465006436
11689.1014921946
11696.6640266411
11697.9270537252
11709.862998637
11728.2074697203

Model building with GLASS

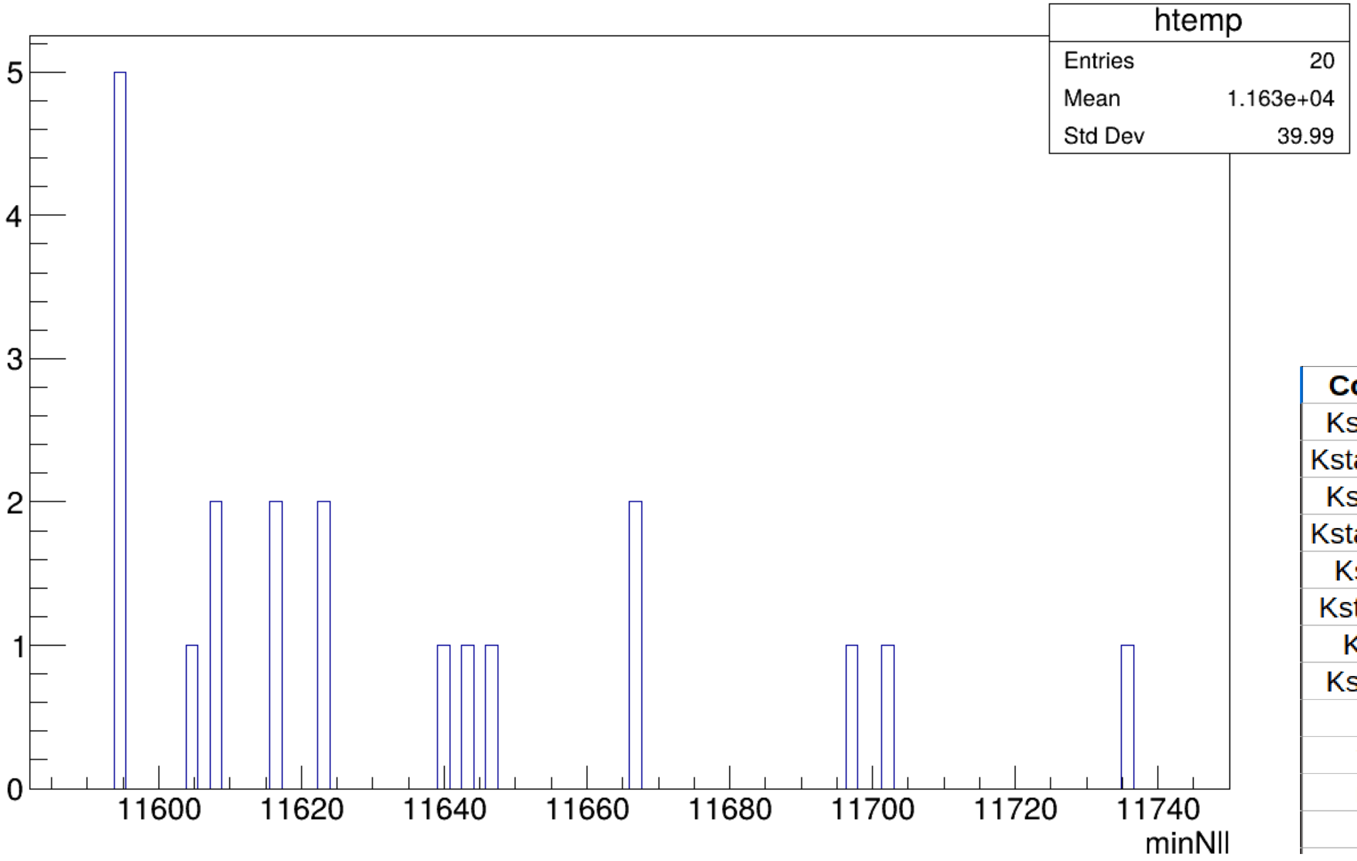
(Baseline + kstar2_1430 + f2(1270)) + f0_1500



best_minNll_in_cluster
11602.1114231242
11602.3697180799
11602.5449435139
11607.3904734272
11607.3944991484
11612.233550062
11612.7308255872
11625.1934849093
11625.226719599
11630.6498202925
11637.0937437597
11689.3197109468
11689.9097943343
11701.0904041217
11710.7855521415
11736.5270807858

Model building with GLASS

(Baseline + kstar2_1430 + kstar_1680) + f0(1370)

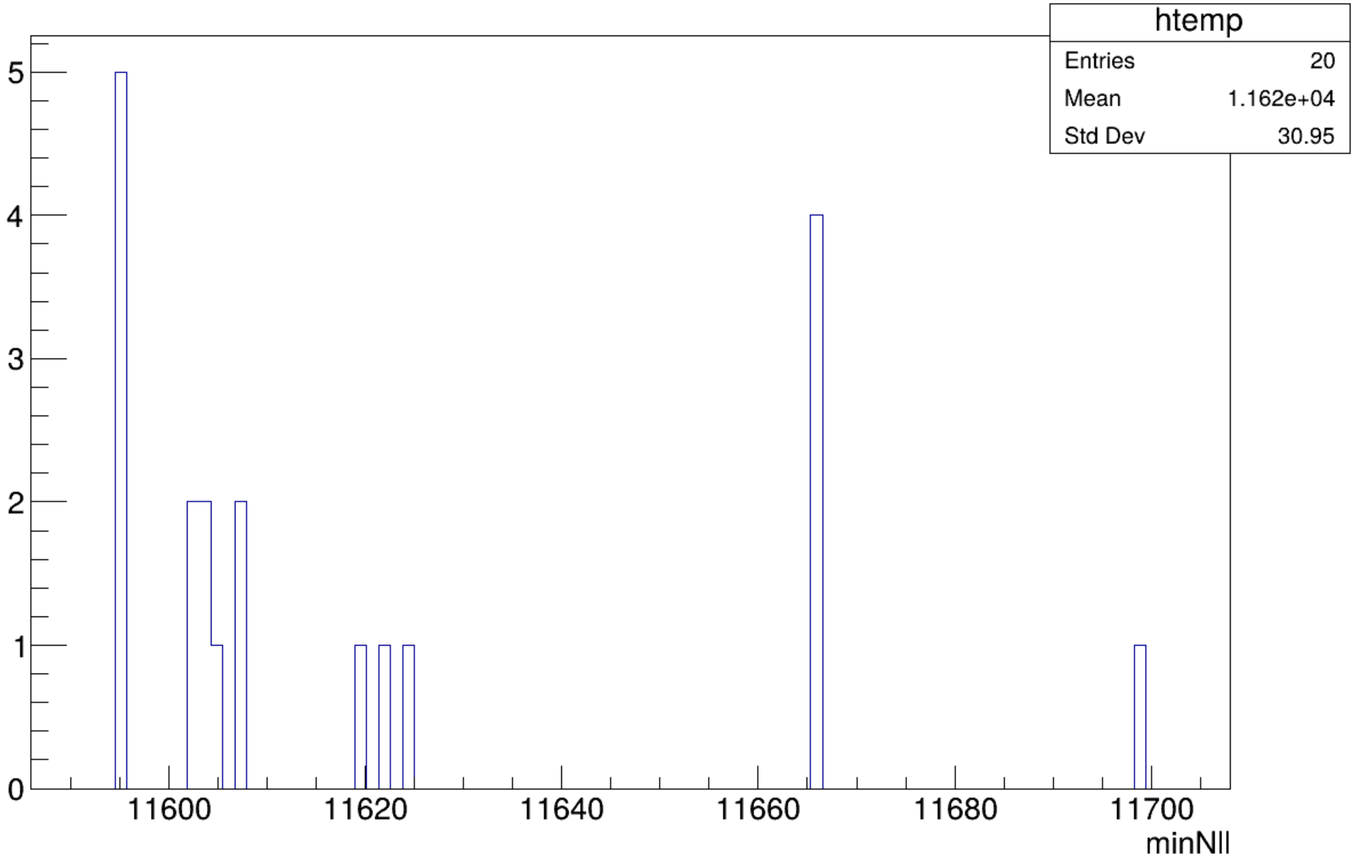


best_minNll_in_cluster
11595.0329477688
11595.4259606333
11604.0846184603
11607.3196234889
11617.2675811041
11622.7090020857
11623.8522184805
11640.736837619
11642.9827091865
11646.0371304036
11666.3498815395
11696.2784940471
11702.119557967
11736.2912102279

Component	Type	Value	Uncertainty	Precision_pct
Kstar0_1430	Amplitude	2.84535052808651	0.237282402530829	8.3393030204402
Kstar0_1430_b	Amplitude	1.90611282470163	0.245238606190628	12.8659019032106
Kstar2_1430	Amplitude	1.24841024125383	0.174147767233651	13.9495625299218
Kstar2_1430_b	Amplitude	1.29915345650738	0.194116552700001	14.9417724078463
Kstar_1680	Amplitude	0.746234634545246	0.261185710537873	35.0004808738259
Kstar_1680_b	Amplitude	1.00904203304597	0.226365602840481	22.4337139016059
Kstar_892	Amplitude	1.6496530649327	0.128087714153036	7.76452436429483
Kstar_892_b	Amplitude	2	0	0
NR1	Amplitude	1.98612236568395	0.1837330495253	9.25084238009825
f0_1370	Amplitude	0.584137182015958	0.158127779293373	27.0703157000976
rho_770	Amplitude	0.966910105964061	0.117552876560538	12.1575807136003
swave	Amplitude	0.872670405712415	0.116373524015036	13.3353352254489

Model building with GLASS

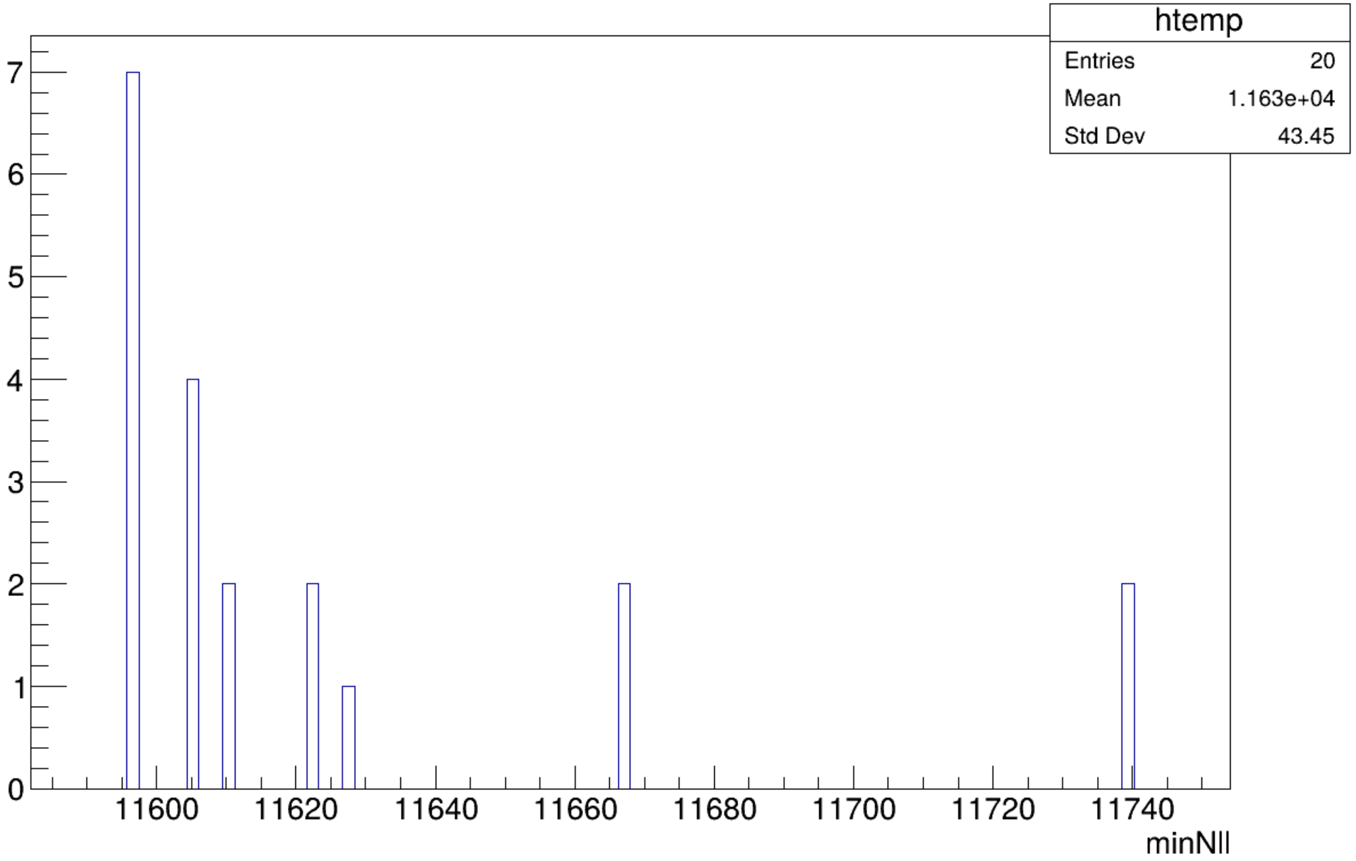
(Baseline + kstar2_1430 + kstar_1680) + f2(1270)



best_minNll_in_cluster
11595.0053367916
11595.4941281229
11602.2042071505
11604.0624270776
11605.2113157399
11607.6613868875
11619.8879168887
11624.2530512228
11665.4943171233
11665.6447428131
11698.3162767094

Model building with GLASS

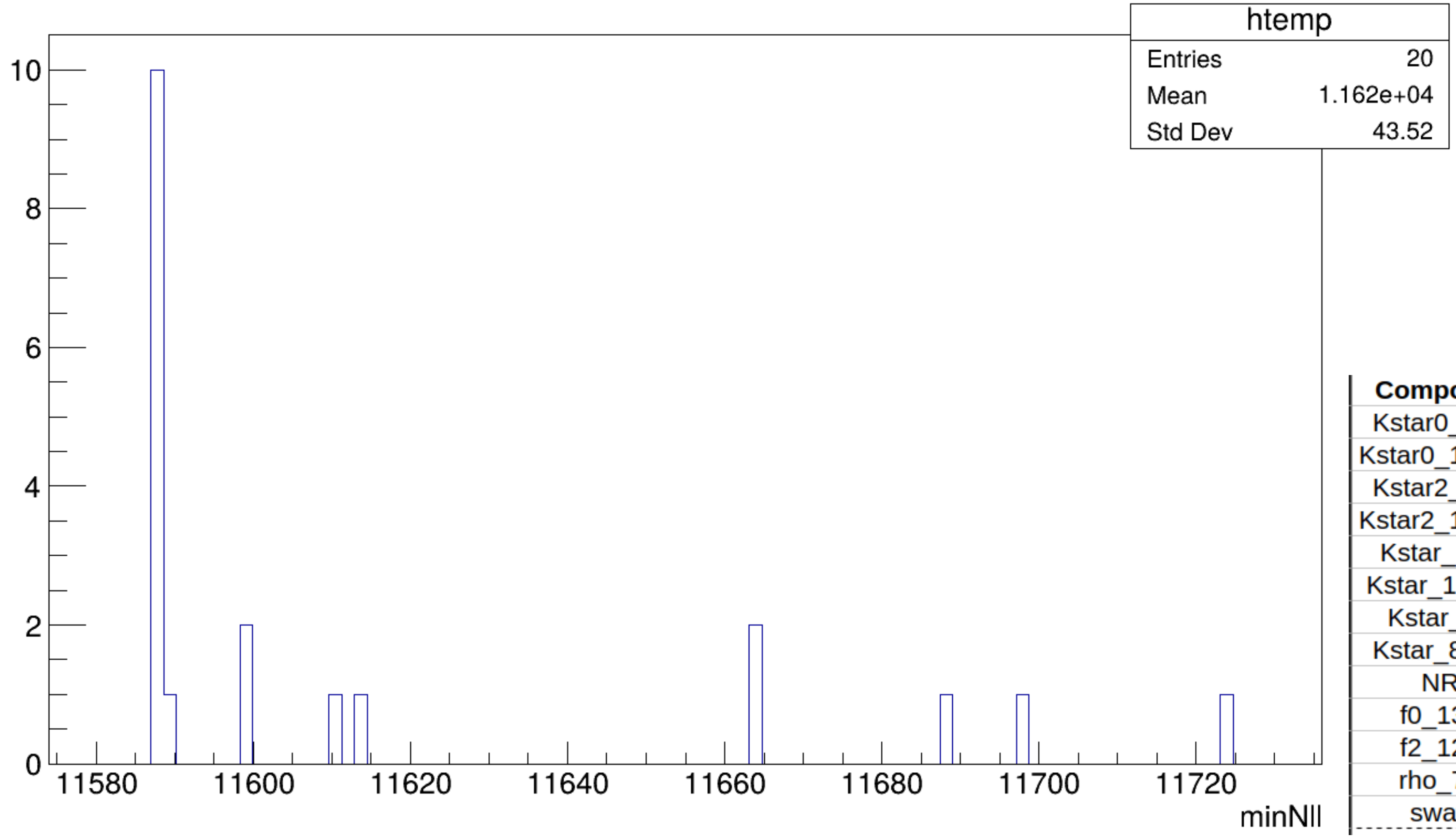
(Baseline + kstar2_1430 + kstar_1680) + f0(1500)



best_minNll_in_cluster
11596.1889643855
11596.375452665
11605.9075870469
11609.7296679958
11609.9806395744
11622.0114199446
11622.0942597631
11627.7987792194
11667.2830282553
11740.0819605903

Model building with GLASS

(Baseline + kstar2_1430 + kstar_1680 + f0(1370)) + f2(1270)

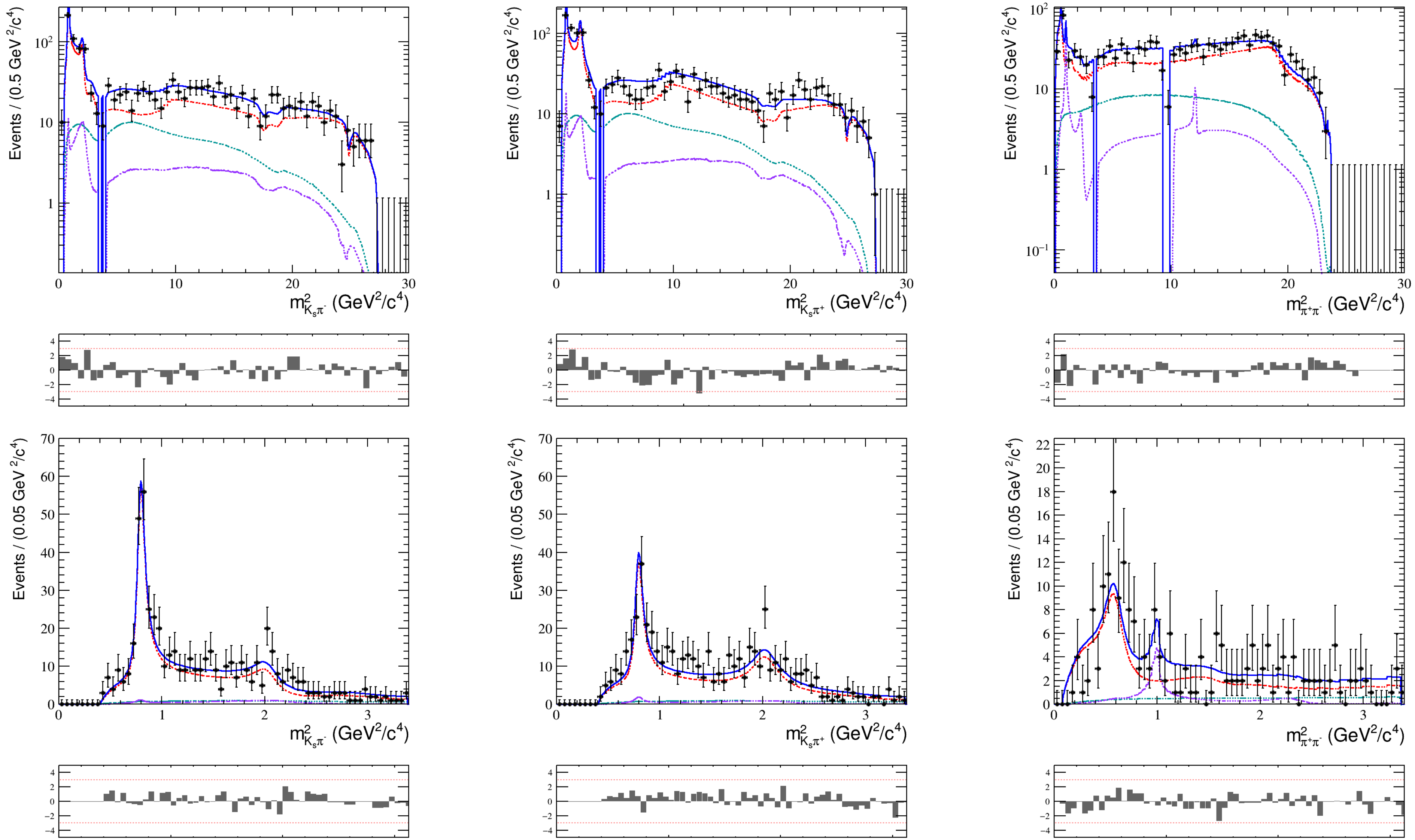


best_minNll_in_cluster
11587.2930460137
11588.8690327981
11598.3866113969
11599.1821566291
11610.915008829
11612.9858663179
11663.9326948597
11688.0775323463
11698.5929217873
11724.0188782023

Component	Type	Value	Uncertainty	Precision_pct
Kstar0_1430	Amplitude	2.71458198049994	0.217999347037708	8.03067833661665
Kstar0_1430_b	Amplitude	2.0156802046892	0.234145478642099	11.6162017217509
Kstar2_1430	Amplitude	1.30538473759456	0.156507644625758	11.9893882713962
Kstar2_1430_b	Amplitude	1.18681699416254	0.197305177469241	16.6247347686882
Kstar_1680	Amplitude	0.843949878814444	0.220585065280549	26.1372234083877
Kstar_1680_b	Amplitude	0.959911956635667	0.222764623236264	23.206776589909
Kstar_892	Amplitude	1.6613700827725	0.12783172237766	7.69435562269995
Kstar_892_b	Amplitude	2	0	0
NR1	Amplitude	2.02701208040878	0.179948793641091	8.87753927962785
f0_1370	Amplitude	0.91410016315298	0.164139696343585	17.9564234818013
f2_1270	Amplitude	0.690781663850424	0.175317557558512	25.3795905035015
rho_770	Amplitude	0.885148570525072	0.133428856635318	15.0741763674959
swave	Amplitude	0.879254279316144	0.122237281884056	13.9023812291398

Model building with GLASS

(Baseline + kstar2_1430 + kstar_1680 + f0(1370)) + f2(1270)



Model building with GLASS

Additional contribution	p-value	nll	-2Δnll	Mag. precision (%)
baseline	–	11647.54 *	–	–
(baseline) $+K_2^*(1430)$	–	11606.52	82.04	$(14.23, 9.71)$
(baseline) $+K^*(1680)$	–	11635.35	24.38	$(39.13, 19.14)$
(baseline) $+f_0(980)$	–	11646.19	2.70	59.82
(baseline) $+f_2(1270)$	–	11640.58	13.92	23.02
(baseline) $+f_0(1370)$	–	11637.65	19.78	24.56
(baseline) $+f_0(1500)$	–	11640.93	13.22	28.26
(baseline) $+\chi_{c0}$	–	11647.18	0.72	116.30
(baseline + $K_2^*(1430)$) $+f_2(1270)$	–	11602.94	7.16	30.43
(baseline + $K_2^*(1430)$) $+K^*(1680)$	–	11598.26	16.52	$(38.89, 27.29)$
(baseline + $K_2^*(1430)$) $+f_0(1370)$	–	11604.77	3.50	48.12
(baseline + $K_2^*(1430)$) $+f_0(1500)$	–	11604.88	3.28	49.12
(baseline + $K_2^*(1430)$ + $f_2(1270)$) $+K^*(1680)$?	–	11595.01	15.86	$(38.69, 29.34)$
(baseline + $K_2^*(1430)$ + $f_2(1270)$) $+f_0(1370)$?	–	11599.44	7.00	19.23
(baseline + $K_2^*(1430)$ + $f_2(1270)$) $+f_0(1500)$	–	11602.11	1.66	47.64
(baseline + $K_2^*(1430)$ + $K^*(1680)$) $+f_2(1270)$?	–	11595.00	6.52	30.26
(baseline + $K_2^*(1430)$ + $K^*(1680)$) $+f_0(1370)$?	–	11595.03	6.46	27.07
(baseline + $K_2^*(1430)$ + $K^*(1680)$) $+f_0(1500)$	–	11596.19	4.14	43.54
(baseline + $K_2^*(1430)$ + $K^*(1680)$ + $f_0(1370)$) $+f_2(1270)$	–	11587.29	15.48	25.37

*As reference, for the baseline model with EFKLLM the NLL was found to be: 11615.03

and the one for the nominal model is: 11557.87

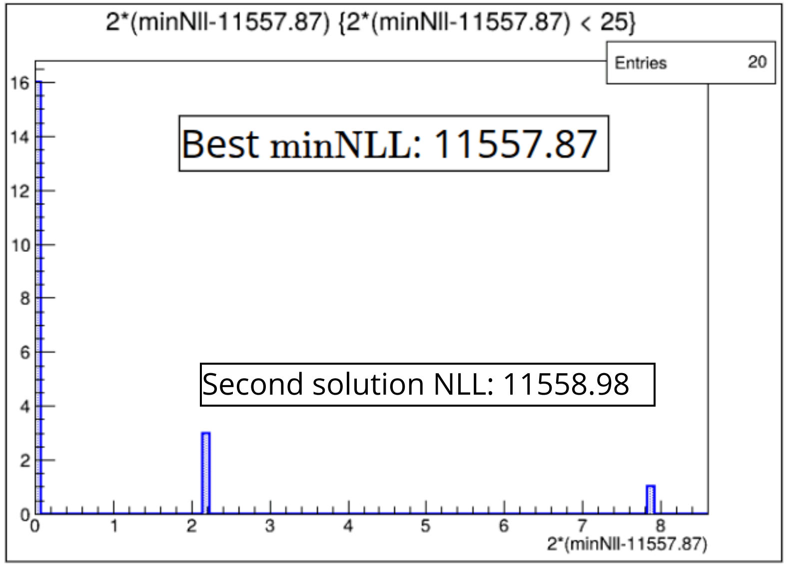
Comparison with the nominal model (EFKLLM)

Component	nominal_magnitude	glass_magnitude	shift_glass_minus_nominal	systematic
swave	0.538809103621658	0.87276466163382	0.333955558012162	0.333955558012162
f0_1370	0.885701572042709	0.584127037260657	-0.301574534782052	0.301574534782052
Kstar_892	1.70581005618553	1.6496661952043	-0.0561438609812264	0.0561438609812264
Kstar_892_b	2	2	0	0
rho_770	1.22324086914523	0.966788294822179	-0.256452574323053	0.256452574323053
Kstar_1680	1.08151181798359	0.746074460125627	-0.335437357857963	0.335437357857963
Kstar_1680_b	1.07909825105783	1.00923557634507	-0.0698626747127615	0.0698626747127615
Kstar2_1430	1.16965693372191	1.24863389904852	0.0789769653266099	0.0789769653266099
Kstar2_1430_b	0.909263080121219	1.29892158258424	0.389658502463025	0.389658502463025

For reference of the stats. uncertainties, below the results for the nominal fit (EFKLLM):

Resonance	Real part	Imaginary part	Magnitude	Phase (°)
NR	-1.76 ± 0.27	0.28 ± 0.27	1.78 ± 0.28	$171.1^{+11.4}_{-12.3}$
$(K\pi)_0^{*+}$	-2.81 ± 0.22	-1.10 ± 0.30	3.02 ± 0.21	$-158.7^{+6.9}_{-7.4}$
$(K\pi)_0^{*-}$	-2.61 ± 0.19	-0.07 ± 0.33	2.61 ± 0.19	$-178.4^{+8.0}_{-8.1}$
$f_0(500)$	0.14 ± 0.48	0.52 ± 0.15	0.54 ± 0.18	$75.4^{+61.1}_{-60.9}$
$f_0(1370)$	-0.31 ± 0.38	-0.83 ± 0.21	0.89 ± 0.15	$-110.2^{+28.5}_{-44.1}$
$K^{*+}(892)$	1.71 ± 0.13	0.00	1.71 ± 0.13	0.0 ± 0.0
$K^{*-}(892)$	-2.00	0.00	2.00	180.0
$\rho^0(770)$	-0.47 ± 0.55	1.13 ± 0.29	1.22 ± 0.13	$112.7^{+30.4}_{-28.6}$
$K^{*+}(1680)$	0.17 ± 0.34	1.07 ± 0.20	1.08 ± 0.20	$81.1^{+25.4}_{-21.5}$
$K^{*-}(1680)$	0.15 ± 0.48	-1.07 ± 0.21	1.08 ± 0.21	$-82.2^{+30.7}_{-30.5}$
$K_2^{*+}(1430)$	1.16 ± 0.18	0.14 ± 0.22	1.17 ± 0.17	$6.8^{+11.7}_{-12.9}$
$K_2^{*-}(1430)$	0.56 ± 0.17	-0.72 ± 0.22	0.91 ± 0.20	$-52.1^{+14.3}_{-13.0}$

Nominal model: Second solution results



$$2\Delta\text{nll} = 2(11558.98 - 11557.87) = 2.22$$

Fitted Cartesian isobar coefficients

Resonance	Second solution		Nominal solution	
	Real part	Imaginary part	Real part	Imaginary part
NR	-1.80 ± 0.25	0.51 ± 0.27	-1.76 ± 0.27	0.28 ± 0.27
$(K\pi)_0^{*+}$	-2.85 ± 0.23	-1.07 ± 0.32	-2.81 ± 0.22	-1.10 ± 0.30
$(K\pi)_0^{*-}$	-2.60 ± 0.20	-0.10 ± 0.33	-2.61 ± 0.19	-0.07 ± 0.33
$f_0(500)$	0.20 ± 0.30	0.36 ± 0.16	0.14 ± 0.48	0.52 ± 0.15
$f_0(1370)$	-0.30 ± 0.28	0.18 ± 0.23	-0.31 ± 0.38	-0.83 ± 0.21
$K^{*+}(892)$	1.70 ± 0.13	0.00	1.71 ± 0.13	0.00
$K^{*-}(892)$	-2.00	0.00	-2.00	0.00
$\rho^0(770)$	-0.81 ± 0.43	0.93 ± 0.44	-0.47 ± 0.55	1.13 ± 0.29
$K^{*+}(1680)$	0.07 ± 0.38	1.16 ± 0.20	0.17 ± 0.34	1.07 ± 0.20
$K^{*-}(1680)$	0.55 ± 0.48	-1.08 ± 0.25	0.15 ± 0.48	-1.07 ± 0.21
$K_2^{*+}(1430)$	1.20 ± 0.18	0.09 ± 0.23	1.16 ± 0.18	0.14 ± 0.22
$K_2^{*-}(1430)$	0.61 ± 0.17	-0.77 ± 0.23	0.56 ± 0.17	-0.72 ± 0.22

Magnitudes

Resonance	Second solution	Nominal solution
NR	1.87 ± 0.25	1.78 ± 0.28
$(K\pi)_0^{*+}$	3.04 ± 0.22	3.02 ± 0.21
$(K\pi)_0^{*-}$	2.60 ± 0.20	2.61 ± 0.19
$f_0(500)$	0.41 ± 0.12	0.54 ± 0.18
$f_0(1370)$	0.35 ± 0.21	0.89 ± 0.15
$K^{*+}(892)$	1.70 ± 0.13	1.71 ± 0.13
$K^{*-}(892)$	2.00	2.00
$\rho^0(770)$	1.23 ± 0.13	1.22 ± 0.13
$K^{*+}(1680)$	1.16 ± 0.20	1.08 ± 0.20
$K^{*-}(1680)$	1.21 ± 0.23	1.08 ± 0.21
$K_2^{*+}(1430)$	1.20 ± 0.17	1.17 ± 0.17
$K_2^{*-}(1430)$	0.98 ± 0.19	0.91 ± 0.20

Fit fractions and raw CP asymmetries

Resonance	Second solution		Nominal solution	
	Fit fraction (%)	Raw CP asymm.	Fit fraction (%)	Raw CP asymm.
$K^*(892)$	21.51	0.163 ± 0.077	$21.87^{+1.91}_{-1.56}$	0.158 ± 0.075
$(K\pi)_0^*$	50.12	-0.156 ± 0.066	$50.46^{+4.68}_{-4.55}$	-0.143 ± 0.065
$K_2^*(1430)$	7.56	-0.198 ± 0.222	$6.95^{+1.94}_{-1.64}$	-0.247 ± 0.232
$K^*(1680)$	8.79	0.045 ± 0.254	$7.39^{+1.78}_{-1.74}$	-0.002 ± 0.276
$\rho^0(770)$	9.52	–	$9.47^{+1.72}_{-1.90}$	–
$f_0(500)$	1.06	–	$1.84^{+2.10}_{-0.99}$	–
$f_0(1370)$	0.78	–	$4.97^{+2.46}_{-2.00}$	–
NR	21.90	–	$20.16^{+6.92}_{-6.42}$	–

Bias corrections to FF and phases

- To avoid correcting tiny or poorly established biases, I'm considering to introduce the following criteria:

$$\boxed{\frac{b}{\sigma_b} \geq 2} \quad \text{AND} \quad \boxed{\frac{b}{\sigma_{\text{stat}}^{\text{dir}}} \geq 0.10}$$

where, σ_b is the unc. on the mean residual and because the stat. unc. are asymmetric, the criterion uses the unc. in the direction of the correction.

- So, we apply the bias correction only if the mean residual is determined with **at least 2σ significance** and the correction is **at least 10% of the relevant stat. unc**.

Phases

Contribution	Mean residual (°)	$\lvert b\rvert/\sigma_b$	$\lvert b\rvert/\sigma_{\text{stat}}^{\text{dir}}$	Selected
NR	0.1 ± 0.5	0.10	0.004	no
$(K\pi)_0^{*+}$	-0.1 ± 0.3	0.44	0.021	no
$(K\pi)_0^{*-}$	1.0 ± 0.4	2.84	0.129	yes
$f_0(500)$	-7.0 ± 2.8	2.52	0.114	yes
$f_0(1370)$	-15.7 ± 2.0	7.98	0.551	yes
$\rho^0(770)$	11.2 ± 1.6	6.92	0.390	yes
$K^{*+}(1680)$	3.6 ± 1.2	3.00	0.166	yes
$K^{*-}(1680)$	4.7 ± 1.4	3.43	0.153	yes
$K_2^{*+}(1430)$	1.1 ± 0.6	1.76	0.081	no
$K_2^{*-}(1430)$	0.9 ± 0.8	1.20	0.071	no

Fit fractions

Contribution	Mean residual (pp)	$\lvert b\rvert/\sigma_b$	$\lvert b\rvert/\sigma_{\text{stat}}^{\text{dir}}$	Selected
$K^*(892)$	0.13 ± 0.07	1.77	0.080	no
$(K\pi)_0^*$	0.14 ± 0.20	0.68	0.031	no
$K_2^*(1430)$	-0.09 ± 0.08	1.16	0.047	no
$K^*(1680)$	0.55 ± 0.08	6.66	0.317	yes
$\rho^0(770)$	-0.58 ± 0.08	7.43	0.337	yes
$f_0(500)$	0.42 ± 0.06	7.07	0.422	yes
$f_0(1370)$	-0.41 ± 0.10	4.22	0.167	yes
NR	-0.11 ± 0.30	0.39	0.017	no

Bias corrections to FF and phases

Before corrections

Resonance	Real part	Imaginary part	Magnitude	Phase (°)
NR	-1.76 ± 0.27	0.28 ± 0.27	1.78 ± 0.28	$171.1^{+11.4}_{-12.3}$
$(K\pi)_0^{*+}$	-2.81 ± 0.22	-1.10 ± 0.30	3.02 ± 0.21	$-158.7^{+6.9}_{-7.4}$
$(K\pi)_0^{*-}$	-2.61 ± 0.19	-0.07 ± 0.33	2.61 ± 0.19	$-178.4^{+8.0}_{-8.1}$
$f_0(500)$	0.14 ± 0.48	0.52 ± 0.15	0.54 ± 0.18	$75.4^{+61.1}_{-60.9}$
$f_0(1370)$	-0.31 ± 0.38	-0.83 ± 0.21	0.89 ± 0.15	$-110.2^{+28.5}_{-44.1}$
$K^{*+}(892)$	1.71 ± 0.13	0.00	1.71 ± 0.13	0.0 ± 0.0
$K^{*-}(892)$	-2.00	0.00	2.00	180.0
$\rho^0(770)$	-0.47 ± 0.55	1.13 ± 0.29	1.22 ± 0.13	$112.7^{+30.4}_{-28.6}$
$K^{*+}(1680)$	0.17 ± 0.34	1.07 ± 0.20	1.08 ± 0.20	$81.1^{+25.4}_{-21.5}$
$K^{*-}(1680)$	0.15 ± 0.48	-1.07 ± 0.21	1.08 ± 0.21	$-82.2^{+30.7}_{-30.5}$
$K_2^{*+}(1430)$	1.16 ± 0.18	0.14 ± 0.22	1.17 ± 0.17	$6.8^{+11.7}_{-12.9}$
$K_2^{*-}(1430)$	0.56 ± 0.17	-0.72 ± 0.22	0.91 ± 0.20	$-52.1^{+14.3}_{-13.0}$

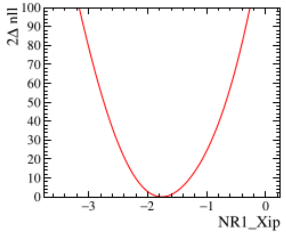
Resonance	Fit fraction (%)	Raw CP asymm.
$K^*(892)$	$21.87^{+1.91}_{-1.56}$	0.158 ± 0.075
$(K\pi)_0^*$	$50.46^{+4.68}_{-4.45}$	-0.143 ± 0.065
$K_2^*(1430)$	$6.95^{+1.94}_{-1.64}$	-0.247 ± 0.232
$K^*(1680)$	$7.39^{+1.78}_{-1.74}$	-0.002 ± 0.276
$\rho^0(770)$	$9.47^{+1.72}_{-1.90}$	–
$f_0(500)$	$1.84^{+2.10}_{-0.99}$	–
$f_0(1370)$	$4.97^{+2.46}_{-2.00}$	–
NR	$20.16^{+6.92}_{-6.42}$	–

After corrections

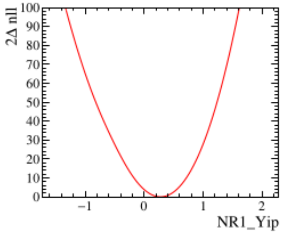
Resonance	Real part	Imaginary part	Magnitude	Phase (°)
NR	-1.76 ± 0.27	0.28 ± 0.27	1.78 ± 0.28	$171.1^{+11.4}_{-12.3}$
$(K\pi)_0^{*+}$	-2.81 ± 0.22	-1.10 ± 0.30	3.02 ± 0.21	$-158.7^{+6.9}_{-7.4}$
$(K\pi)_0^{*-}$	-2.61 ± 0.19	-0.07 ± 0.33	2.61 ± 0.19	$-179.5^{+8.0}_{-8.1}$
$f_0(500)$	0.14 ± 0.48	0.52 ± 0.15	0.54 ± 0.18	$82.4^{+61.1}_{-60.9}$
$f_0(1370)$	-0.31 ± 0.38	-0.83 ± 0.21	0.89 ± 0.15	$-94.5^{+28.5}_{-44.1}$
$K^{*+}(892)$	1.71 ± 0.13	0.00	1.71 ± 0.13	0.0 ± 0.0
$K^{*-}(892)$	-2.00	0.00	2.00	180.0
$\rho^0(770)$	-0.47 ± 0.55	1.13 ± 0.29	1.22 ± 0.13	$101.5^{+30.4}_{-28.6}$
$K^{*+}(1680)$	0.17 ± 0.34	1.07 ± 0.20	1.08 ± 0.20	$77.6^{+25.4}_{-21.5}$
$K^{*-}(1680)$	0.15 ± 0.48	-1.07 ± 0.21	1.08 ± 0.21	$-86.9^{+30.7}_{-30.5}$
$K_2^{*+}(1430)$	1.16 ± 0.18	0.14 ± 0.22	1.17 ± 0.17	$6.8^{+11.7}_{-12.9}$
$K_2^{*-}(1430)$	0.56 ± 0.17	-0.72 ± 0.22	0.91 ± 0.20	$-52.1^{+14.3}_{-13.0}$

Resonance	Fit fraction (%)	Raw CP asymm.
$K^*(892)$	$21.87^{+1.91}_{-1.56}$	0.158 ± 0.075
$(K\pi)_0^*$	$50.46^{+4.68}_{-4.45}$	-0.143 ± 0.065
$K_2^*(1430)$	$6.95^{+1.94}_{-1.64}$	-0.247 ± 0.232
$K^*(1680)$	$6.83^{+1.78}_{-1.74}$	-0.002 ± 0.276
$\rho^0(770)$	$10.05^{+1.72}_{-1.90}$	–
$f_0(500)$	$1.42^{+2.10}_{-0.99}$	–
$f_0(1370)$	$5.38^{+2.46}_{-2.00}$	–
NR	$20.16^{+6.92}_{-6.42}$	–

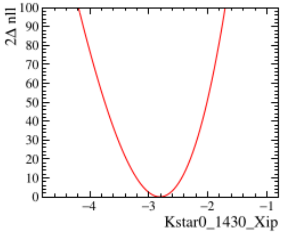
Fit validation review: likelihood scans



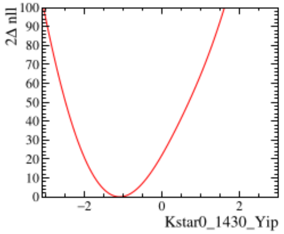
(a) x_{NR}



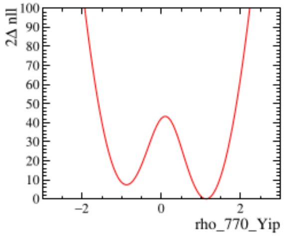
(b) y_{NR}



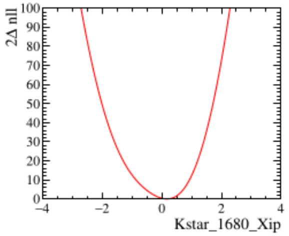
(c) $x_{(K\pi)_0^{*+}}$



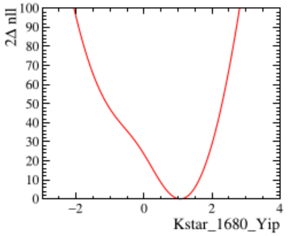
(d) $y_{(K\pi)_0^{*+}}$



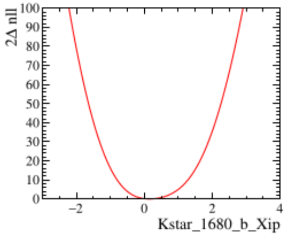
(m) $y_{\rho^0(770)}$



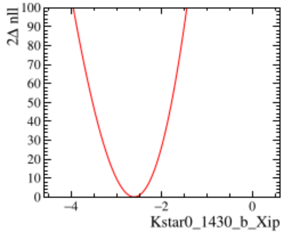
(n) $x_{K^{*+}(1680)}$



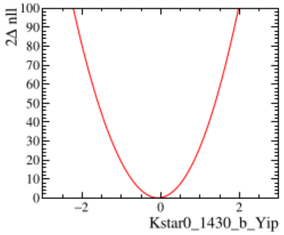
(o) $y_{K^{*+}(1680)}$



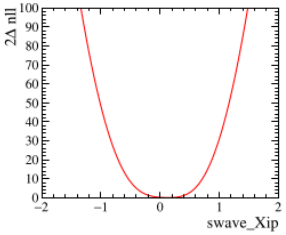
(p) $x_{K^{*-}(1680)}$



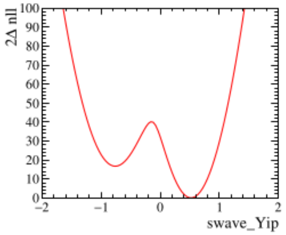
(e) $x_{(K\pi)_0^{*-}}$



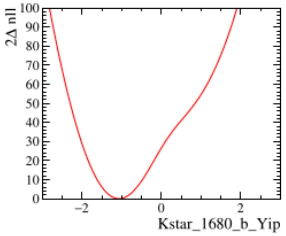
(f) $y_{(K\pi)_0^{*-}}$



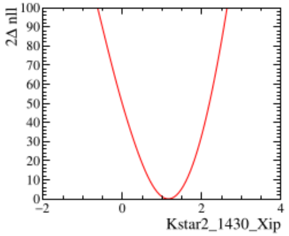
(g) $x_{f_0(500)}$



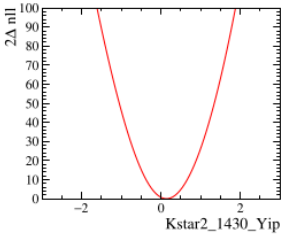
(h) $y_{f_0(500)}$



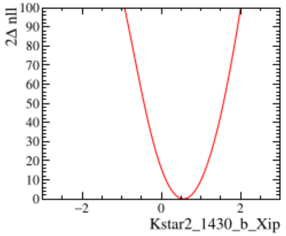
(q) $y_{K^{*-}(1680)}$



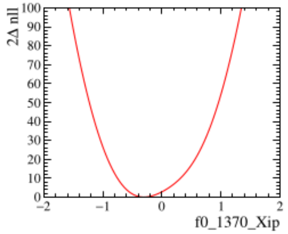
(r) $x_{K_2^{*+}(1430)}$



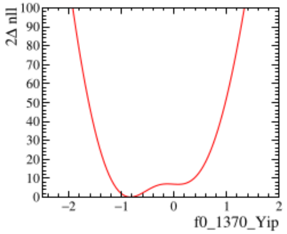
(s) $y_{K_2^{*+}(1430)}$



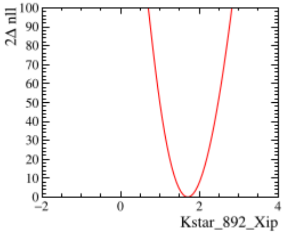
(t) $x_{K_2^{*-}(1430)}$



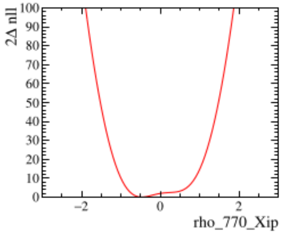
(i) $x_{f_0(1370)}$



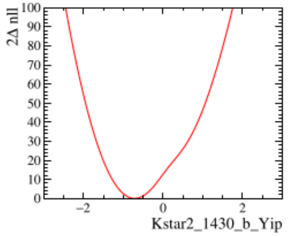
(j) $y_{f_0(1370)}$



(k) $x_{K^{*+}(892)}$

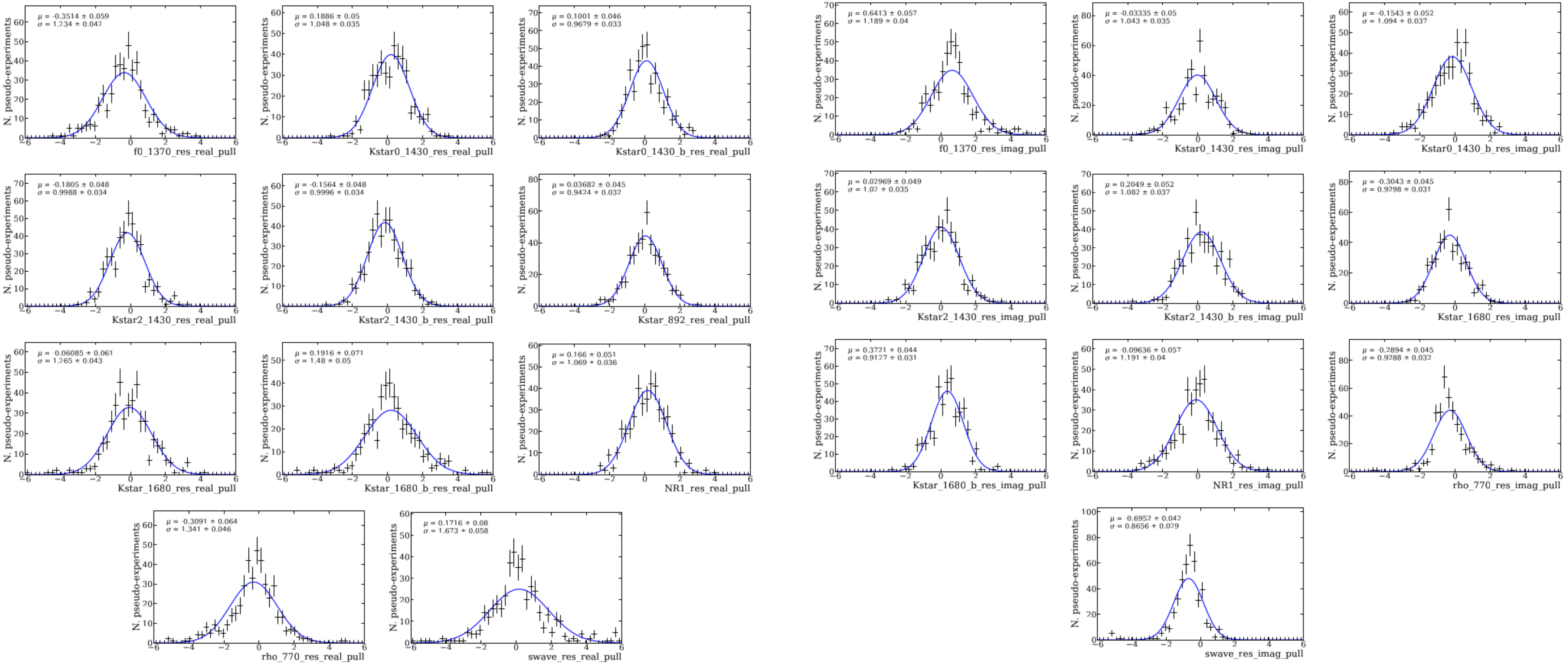


(l) $x_{\rho^0(770)}$

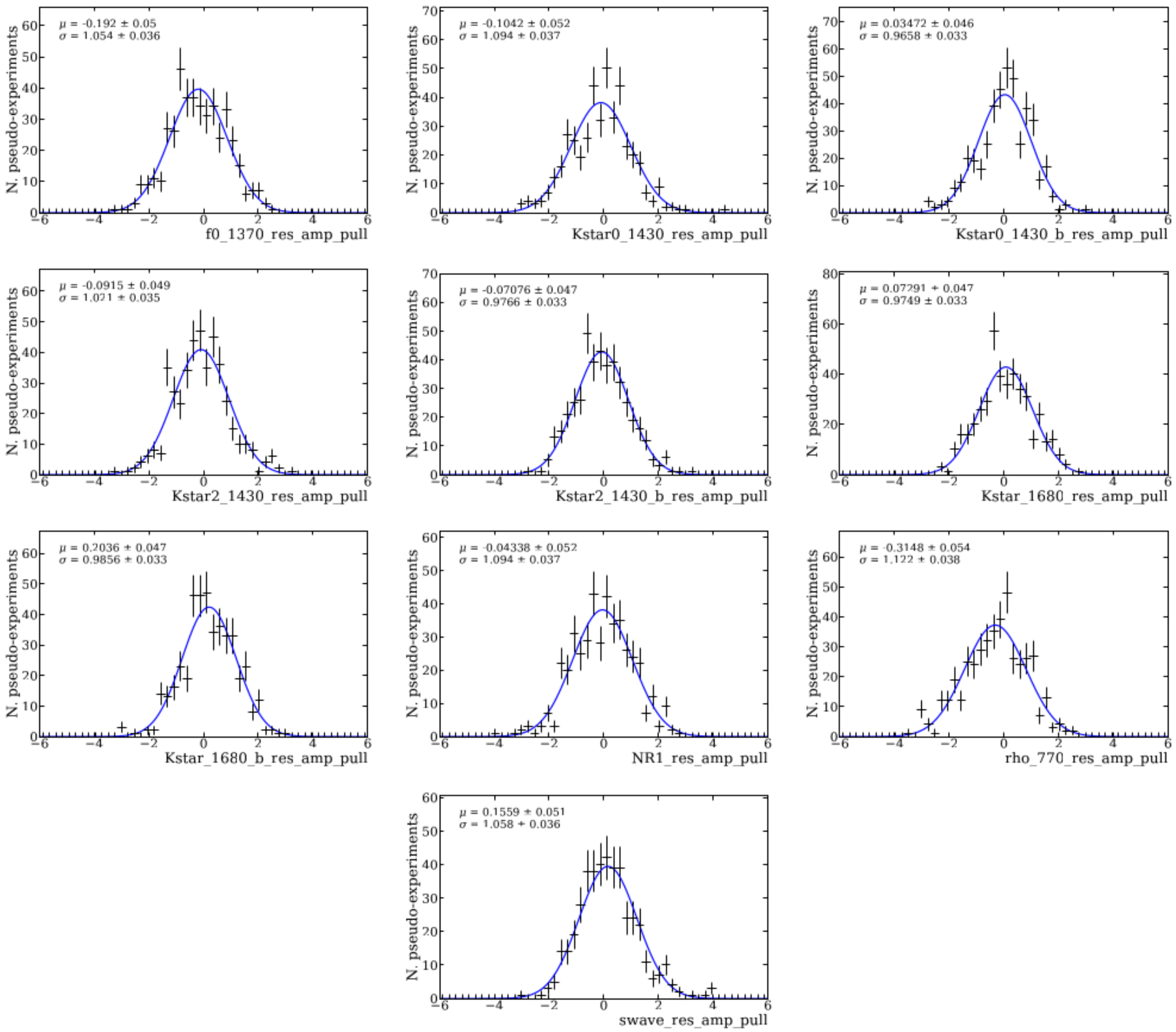


(u) $y_{K_2^{*-}(1430)}$

Fit validation review: pulls for cartesian parameters



Fit validation review: pulls for magnitudes



Fit validation review: pulls summary

Cartesian parameters

	Real part		Imaginary part	
Resonance	Pull mean	Pull width	Pull mean	Pull width
NR	0.166 ± 0.051	1.069 ± 0.036	-0.096 ± 0.057	1.191 ± 0.040
$(K\pi)_0^{*+}$	0.189 ± 0.050	1.048 ± 0.035	-0.033 ± 0.050	1.043 ± 0.035
$(K\pi)_0^{*-}$	0.100 ± 0.046	0.968 ± 0.033	-0.154 ± 0.052	1.094 ± 0.037
$f_0(500)$	0.172 ± 0.080	1.673 ± 0.058	-0.695 ± 0.042	0.866 ± 0.029
$f_0(1370)$	-0.351 ± 0.059	1.234 ± 0.042	0.641 ± 0.057	1.189 ± 0.040
$K^{*+}(892)$	0.037 ± 0.045	0.942 ± 0.032	–	–
$\rho^0(770)$	-0.309 ± 0.064	1.341 ± 0.046	-0.289 ± 0.045	0.929 ± 0.032
$K^{*+}(1680)$	-0.061 ± 0.061	1.265 ± 0.043	-0.304 ± 0.045	0.930 ± 0.031
$K^{*-}(1680)$	0.192 ± 0.071	1.480 ± 0.050	0.372 ± 0.044	0.913 ± 0.031
$K_2^{*+}(1430)$	-0.180 ± 0.048	0.999 ± 0.034	0.030 ± 0.049	1.020 ± 0.035
$K_2^{*-}(1430)$	-0.156 ± 0.048	1.000 ± 0.034	0.205 ± 0.052	1.082 ± 0.037

Magnitudes

Resonance	Pull mean	Pull width
NR	-0.043 ± 0.052	1.094 ± 0.037
$(K\pi)_0^{*+}$	-0.104 ± 0.052	1.094 ± 0.037
$(K\pi)_0^{*-}$	0.035 ± 0.046	0.966 ± 0.033
$f_0(500)$	0.156 ± 0.051	1.058 ± 0.036
$f_0(1370)$	-0.192 ± 0.050	1.054 ± 0.036
$\rho^0(770)$	-0.315 ± 0.054	1.122 ± 0.038
$K^{*+}(1680)$	0.073 ± 0.047	0.975 ± 0.033
$K^{*-}(1680)$	0.204 ± 0.047	0.986 ± 0.033
$K_2^{*+}(1430)$	-0.091 ± 0.049	1.021 ± 0.035
$K_2^{*-}(1430)$	-0.071 ± 0.047	0.977 ± 0.033

Backup

Selection

Selection

SELECTION - TRIGGER LINES

→ **L0**

2011-2012	2015-2016	2017-2018
B_L0HadronDecision_TOS	B_L0HadronDecision_TOS	B_L0HadronDecision_TOS
B_L0DiMuonDecision_TIS	B_L0DiMuonDecision_TIS	B_L0DiMuonDecision_TIS
B_L0MuonDecision_TIS	B_L0MuonDecision_TIS	B_L0MuonDecision_TIS
B_L0ElectronDecision_TIS	B_L0ElectronDecision_TIS	B_L0ElectronDecision_TIS
B_L0PhotonDecision_TIS	B_L0PhotonDecision_TIS	B_L0PhotonDecision_TIS
B_L0HadronDecision_TIS	B_L0HadronDecision_TIS	B_L0HadronDecision_TIS
	B_L0MuonEWDecision_TIS	B_L0MuonEWDecision_TIS
	B_L0JetE1Decision_TIS	
	B_L0JetPhDecision_TIS	

Hadron - TOS
Physical - TIS

→ **HLT1**

Run1	Run2
B_Hlt1TrackAllL0Decision_TOS	B_Hlt1TrackMVADecision_TOS
	B_Hlt1TwoTrackMVADecision_TOS

→ **HLT2**

2011	2012	Run2
B_Hlt2Topo2BodyBBDTDecision_TOS	B_Hlt2Topo2BodyBBDTDecision_TOS	B_Hlt2Topo2BodyDecision_TOS
B_Hlt2Topo3BodyBBDTDecision_TOS	B_Hlt2Topo3BodyBBDTDecision_TOS	B_Hlt2Topo3BodyDecision_TOS
B_Hlt2Topo4BodyBBDTDecision_TOS	B_Hlt2Topo4BodyBBDTDecision_TOS	B_Hlt2Topo4BodyDecision_TOS
B_Hlt2Topo2BodySimpleDecision_TOS		
B_Hlt2Topo3BodySimpleDecision_TOS		
B_Hlt2Topo3BodySimpleDecision_TOS		

Standard $B^0_{(d,s)} \to K^0_S h^+ h^{-\prime}$ selection criteria

Selection

SELECTION – PRESELECTION AND VETOES

→ Preselection : applied as preparation to the actual MVA (*Multi Variate Analysis*)

Preselection cut	Description
$\chi^2_{\text{IsoVtx}}(B) > 4$	B vertex isolation variable
$Z_{vtx}(K^0_S) - Z_{vtx}(B) > 30$ mm	K^0_S vertex separation w.r.t. the B vertex
$h(1,2)\_isMuon == 0$	Reject $h^\pm$ candidates compatible with the muon hypothesis
$3000 \leq p(h^\pm) \leq 100000\,\text{MeV}$	Fiducial cut (for RICHs)
$\min\chi^2_{\text{IP}}(h^\pm) > 4$	Minimum IP χ^2 of the charged daughters w.r.t. the related PV
$p_T(h^\pm) > 250\,\text{MeV}$	Minimum transverse momentum of the charged daughters

→ Vetoes :

$$D^\pm \to \pi^\pm K^0_S,\ D^\pm \to K^\pm K^0_S$$

$$D^\pm_s \to \pi^\pm K^0_S,\ D^\pm_s \to K^\pm K^0_S$$

$$D^0 \to \pi^+\pi^-,\ D^0 \to K^+K^-,\ D^0 \to K^\pm\pi^\mp$$

$$\Lambda^+_c \to pK^0_S$$

$$J/\psi \to \pi^+\pi^-,\ J/\psi \to K^+K^-$$

$$\chi_{c,0} \to \pi^+\pi^-,\ \chi_{c,0} \to K^+K^-$$

Selection

MVA ON TOPOLOGICAL VARIABLES

→ Used to reduce the amount of combinatorial background

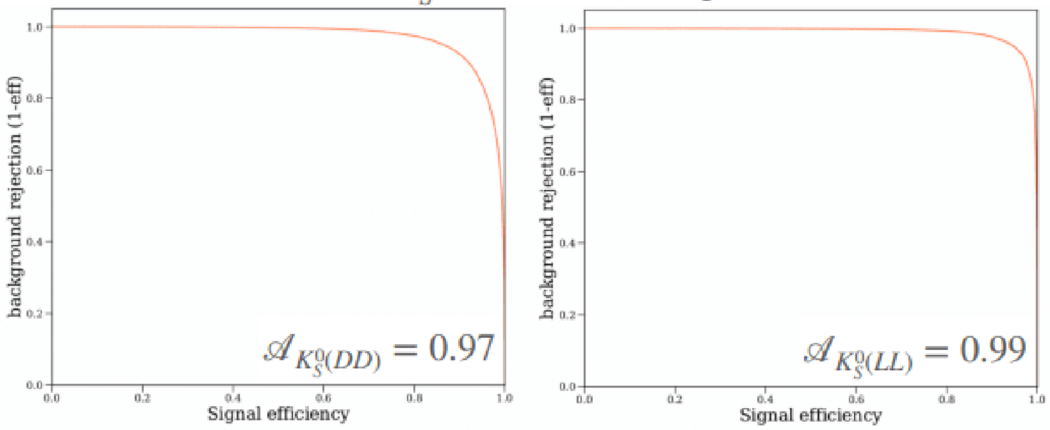
→ Discriminating variables that are uncorrelated to the B-meson mass and to the Dalitz-plot kinematics

→ Trained on $B^0 \to K_S^0 \pi^+ \pi^-$ samples - Signal from MC and Background in upper sideband of the data samples

Good background-signal separation

ROC curves for the topological MVA

$B^0 \to K_S^0 \pi^+ \pi^-$ (2018) sample

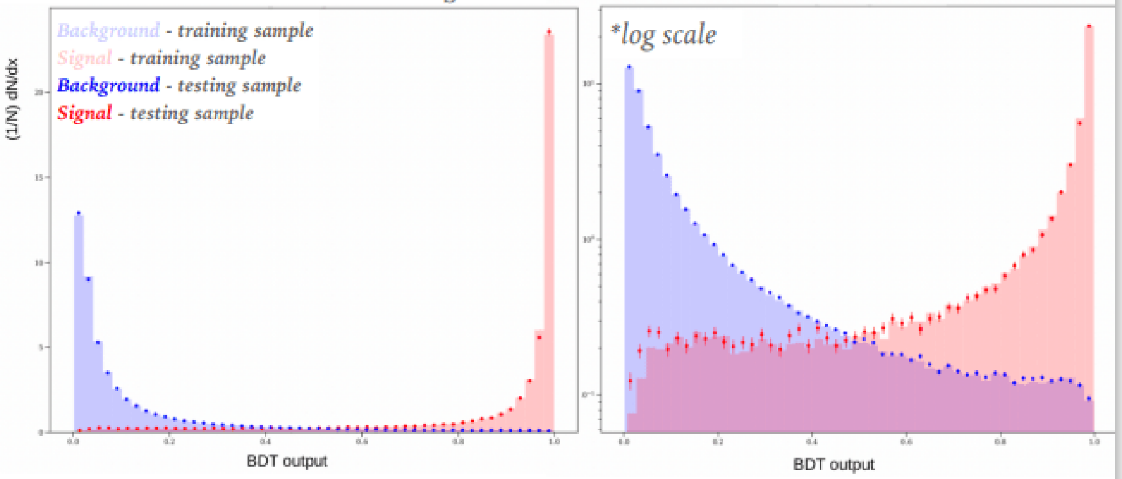


*Interpretation: indication of the classifying power
ideal machine learning tool → area under a ROC curve close to 1
poorly performing tool → area under a ROC curve ~0.5*

No over-training

Distribution of the topological MVA output variable

$B^0 \to K_S^0(DD) \pi^+ \pi^-$ (2018) sample



Interpretation: similar performances on the training sample and the testing sample → no over-fitting

Selection

MVA ON PID VARIABLES

- Used to reduce the amount of Crossfeeds (events from the miss-ID of h or h')
- PID variables used : ProbNN for kaons, pions and protons
- Trained on MC samples for Signal and Crossfeeds

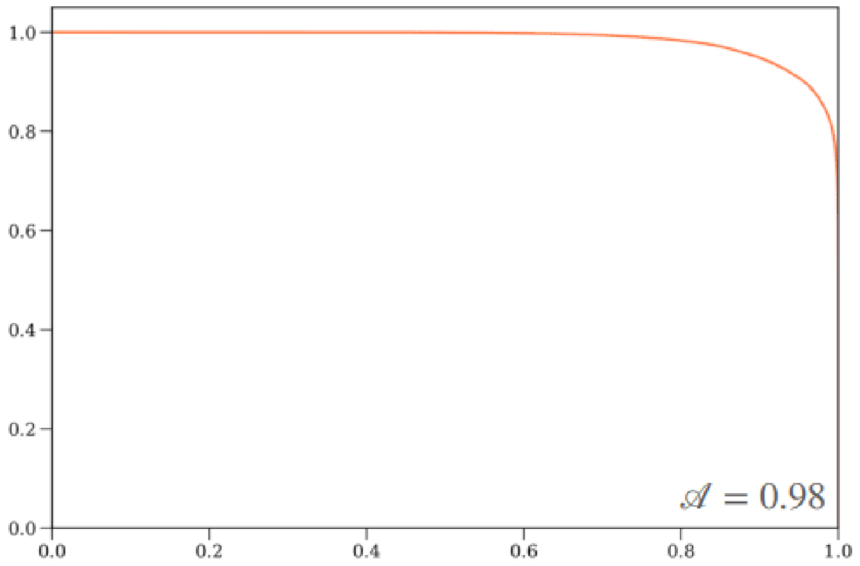
**Crossfeed example:*

Events from $B^0 \to K_S^0\pi\pi$ found in the $K_S^0K\pi$ spectrum when one of the π is miss-ID as a K

Good background-signal separation

ROC curves for the PID MVA

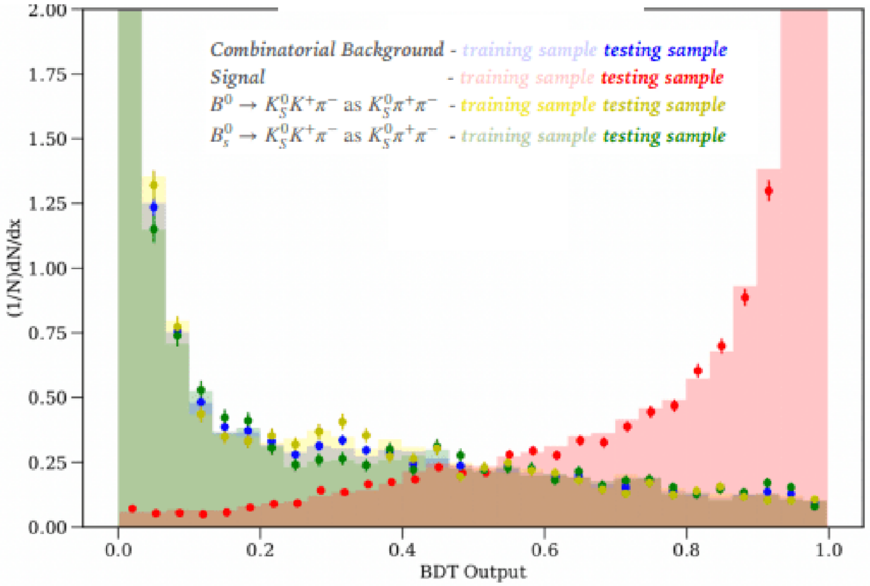
2018 sample



No over-training

Distribution of the PID MVA output variable

$B^0 \to K_S^0(\mathrm{DD})\pi^+\pi^-$ (2018) sample



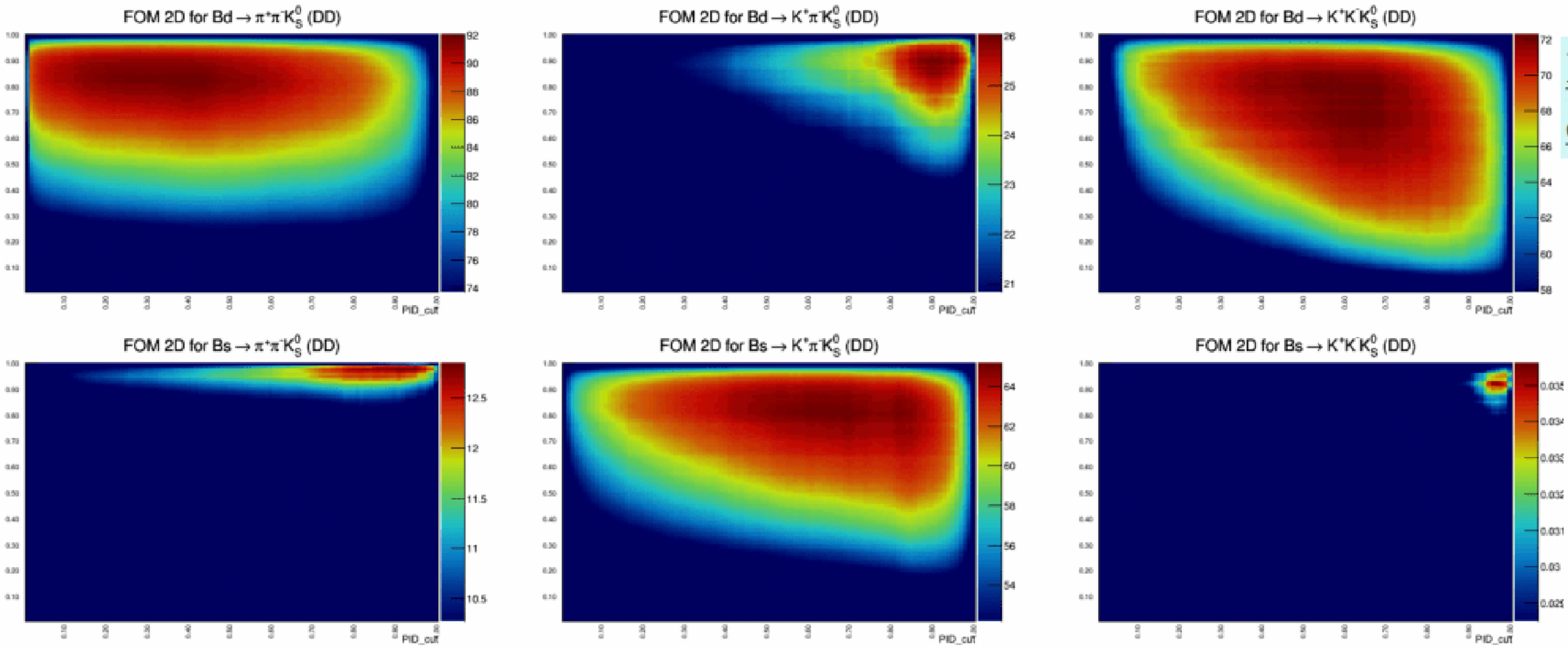
Selection

2D OPTIMISATION

→ Two well-performing correlated BDTs ⇒ 2D optimisation

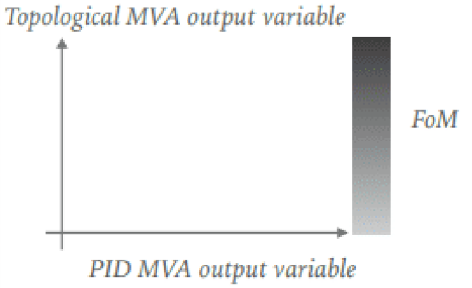
→ Optimisation by finding the maximum of a figure of merit, *FoM* (Observed mode: standard, Unobserved mode: Punzi)

→ Separate optimisation for B^0 and B_s^0 with each channel and each K_S^0 reconstruction



Evaluation of the FoM on the 2D-MVA plane

2018 sample



Backup

Inputs for the $B_S^0 \to K_S \pi^+ \pi^-$ DP fit

Inputs for the $B_s^0 \to K_S^0\pi^+\pi^-$ DP fit

Fraction of signal and bkg. in the B_s^0 mass window

Strategy

- Signal and part. reco. shapes fixed from fits to MC.
- Data: Simultaneous fit among different categories
  - The slope of the right-hand sideband is shared among all sub-samples and is floated.
  - The $\mu_{B_d^0}$ and $\sigma_{B_d^0}$ are shared among sub-samples of the same (KS reco, Run) category
  - Gaussian constraint to the mass-difference between B_d^0 and B_s^0.

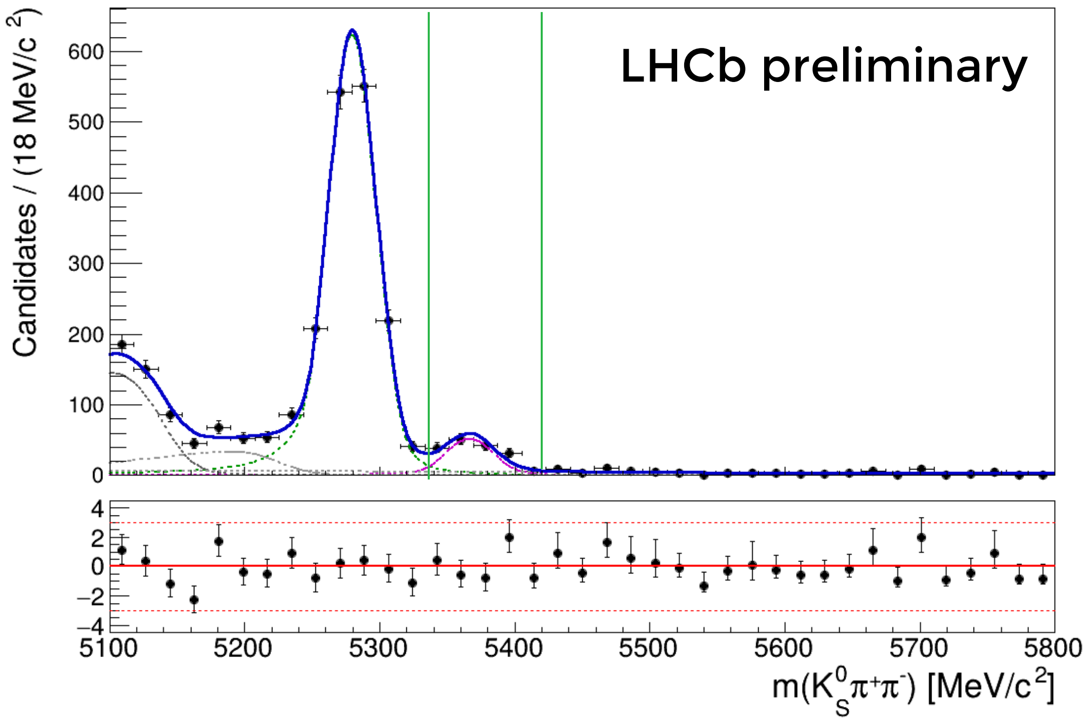
Parameter	2011	2012a	2012b	2015	2016	2017	2018
K_S^0 (LL)							
$\mu(B^0)$	5283.82 ± 0.63	5283.82 ± 0.63	5283.82 ± 0.63	5279.68 ± 0.29	5279.68 ± 0.29	5279.68 ± 0.29	5279.68 ± 0.29
$\mu(B_s^0)$	5371.03 ± 0.65	5371.03 ± 0.65	5371.03 ± 0.65	5366.88 ± 0.32	5366.88 ± 0.32	5366.88 ± 0.32	5366.88 ± 0.32
$\sigma(B_{(s)}^0)$	17.66 ± 0.54	17.66 ± 0.54	17.66 ± 0.54	17.85 ± 0.25	17.85 ± 0.25	17.85 ± 0.25	17.85 ± 0.25
a_{comb}	-0.544 ± 0.060	-0.544 ± 0.060	-0.544 ± 0.060	-0.544 ± 0.060	-0.544 ± 0.060	-0.544 ± 0.060	-0.544 ± 0.060
K_S^0 (DD)							
$\mu(B^0)$	5284.22 ± 0.55	5284.22 ± 0.55	5284.22 ± 0.55	5280.16 ± 0.23	5280.16 ± 0.23	5280.16 ± 0.23	5280.16 ± 0.23
$\mu(B_s^0)$	5371.43 ± 0.56	5371.43 ± 0.56	5371.43 ± 0.56	5367.36 ± 0.26	5367.36 ± 0.26	5367.36 ± 0.26	5367.36 ± 0.26
$\sigma(B_{(s)}^0)$	18.42 ± 0.46	18.42 ± 0.46	18.42 ± 0.46	17.55 ± 0.19	17.55 ± 0.19	17.55 ± 0.19	17.55 ± 0.19
a_{comb}	-0.544 ± 0.060	-0.544 ± 0.060	-0.544 ± 0.060	-0.544 ± 0.060	-0.544 ± 0.060	-0.544 ± 0.060	-0.544 ± 0.060

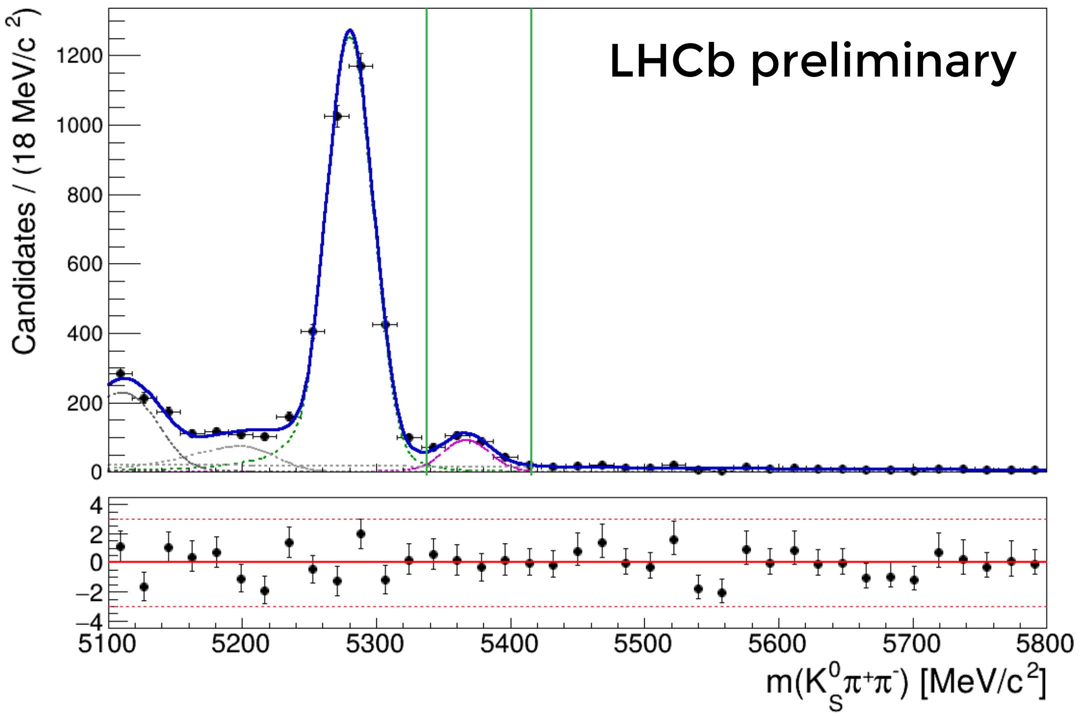
Inputs for the $B_s^0 \to K_S^0\pi^+\pi^-$ DP fit

Fraction of signal and bkg. in the B_s^0 mass window

Invariant-mass fit plots

Selection of simultaneous $K_S^0\pi^+\pi^-$ invariant-mass fit results. This is for 2018 data, both K_S^0 reconstruction categories: LL (left) and DD (right).





The vertical green lines indicate the selected signal window around the B_s^0 peak.

Inputs for the $B_s^0 \to K_S^0\pi^+\pi^-$ DP fit

Fraction of signal and bkg. in the B_s^0 mass window

Fitted yields in data

Fit component	2011	2012a	2012b	2015	2016	2017	2018	Sum
K_S^0 (LL)								
$B_d^0 \to K_S^0\pi^+\pi^-$	293 ± 18	87 ± 10	467 ± 23	204 ± 15	1487 ± 41	1689 ± 43	1676 ± 43	5904 ± 81
$B_s^0 \to K_S^0\pi^+\pi^-$	16 ± 5	10 ± 3	44 ± 8	17 ± 5	116 ± 13	93 ± 12	139 ± 15	435 ± 26
Combinatorial bkg.	32 ± 9	3 ± 3	67 ± 13	12 ± 6	150 ± 20	130 ± 19	173 ± 22	567 ± 40
B^0 PR bkg.	45 ± 8	15 ± 4	104 ± 15	34 ± 6	317 ± 22	331 ± 21	343 ± 24	1189 ± 43
B_s^0 PR bkg.	29 ± 9	13 ± 4	37 ± 15	33 ± 6	185 ± 23	134 ± 20	209 ± 26	639 ± 44
K_S^0 (DD)								
$B_d^0 \to K_S^0\pi^+\pi^-$	353 ± 20	331 ± 19	567 ± 25	307 ± 19	2150 ± 49	2855 ± 57	3272 ± 61	9836 ± 105
$B_s^0 \to K_S^0\pi^+\pi^-$	36 ± 7	25 ± 7	55 ± 8	21 ± 6	169 ± 17	204 ± 18	243 ± 20	753 ± 35
Combinatorial bkg.	29 ± 9	62 ± 12	31 ± 9	29 ± 9	394 ± 31	444 ± 34	548 ± 37	1536 ± 62
B^0 PR bkg.	74 ± 9	55 ± 9	101 ± 14	53 ± 8	432 ± 26	481 ± 30	553 ± 29	1750 ± 53
B_s^0 PR bkg.	18 ± 7	13 ± 8	42 ± 15	46 ± 8	244 ± 27	381 ± 34	352 ± 31	1097 ± 57

- Total: ~1.2K signal candidates
- Run 1:
  - Total: ~186
  - DD: ~116
  - LL: ~70
- Run 2:
  - Total: ~1K
  - DD: ~637
  - LL: ~365

Fractions in the B_s^0 signal window $[\mu_{B_s^0} - 1.5\sigma_{B_s^0}, \mu_{B_s^0} + 3.0\sigma_{B_s^0}]$

Component	2011	2012a	2012b	2015	2016	2017	2018
K_S^0 (LL)							
B_s^0	$(71.74\pm7.71)\%$	$(93.27\pm4.06)\%$	$(76.86\pm3.57)\%$	$(80.58\pm6.20)\%$	$(75.80\pm2.65)\%$	$(71.59\pm3.64)\%$	$(77.01\pm2.36)\%$
B_d^0	$(8.34\pm1.69)\%$	$(3.22\pm1.27)\%$	$(5.98\pm1.10)\%$	$(10.54\pm2.50)\%$	$(9.68\pm0.97)\%$	$(13.63\pm1.56)\%$	$(8.83\pm0.66)\%$
Combinatorial	$(19.92\pm6.97)\%$	$(3.51\pm4.36)\%$	$(17.16\pm3.52)\%$	$(8.87\pm3.84)\%$	$(14.52\pm2.17)\%$	$(14.79\pm2.58)\%$	$(14.16\pm2.02)\%$
K_S^0 (DD)							
B_s^0	$(86.02\pm3.54)\%$	$(67.59\pm6.91)\%$	$(85.49\pm2.59)\%$	$(76.50\pm6.86)\%$	$(69.84\pm2.61)\%$	$(69.83\pm2.39)\%$	$(70.06\pm2.34)\%$
B_d^0	$(3.70\pm0.73)\%$	$(7.55\pm1.76)\%$	$(7.38\pm1.29)\%$	$(8.87\pm2.21)\%$	$(6.82\pm0.51)\%$	$(8.22\pm0.58)\%$	$(7.25\pm0.46)\%$
Combinatorial	$(10.27\pm3.22)\%$	$(24.87\pm5.90)\%$	$(7.13\pm2.17)\%$	$(14.63\pm5.41)\%$	$(23.34\pm2.43)\%$	$(21.95\pm2.24)\%$	$(22.69\pm1.94)\%$

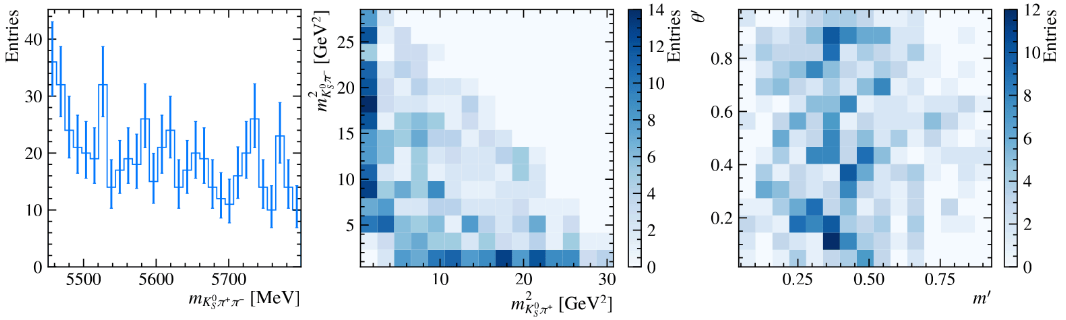
The approximate number of $B_s^0 \to K_S^0\pi^+\pi^-$ signal candidates in the signal window and hence entering the DP fit is 1.02K

Inputs for the $B_s^0 \to K_S^0\pi^+\pi^-$ DP fit

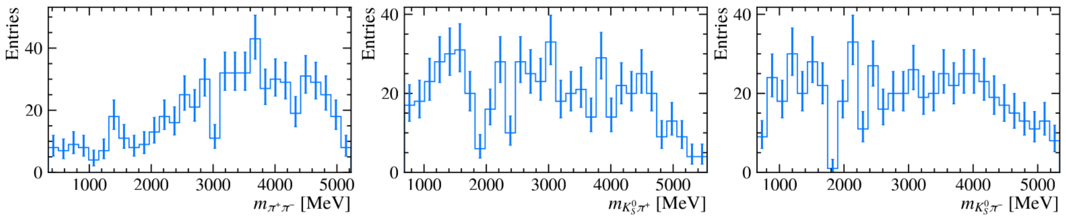
Combinatorial background model

- The combinatorial background in the signal region is estimated with candidates from the upper mass side-band $5450\ \mathrm{MeV} \leq m_{K_S^0\pi^+\pi^-} \leq 5800\ \mathrm{MeV}$
  - Veto on $B^+ \to K_S^0\pi^+$ applied: $|m_{K_S^0\pi^\pm} - m_{B^+}| \geq 20\ \mathrm{MeV}$
- A combined map of all data-taking periods is obtained for each K_S^0 category
- The SqDP maps are smoothed and are inputs for the DP fit.

Combined dataset (all years-DD)

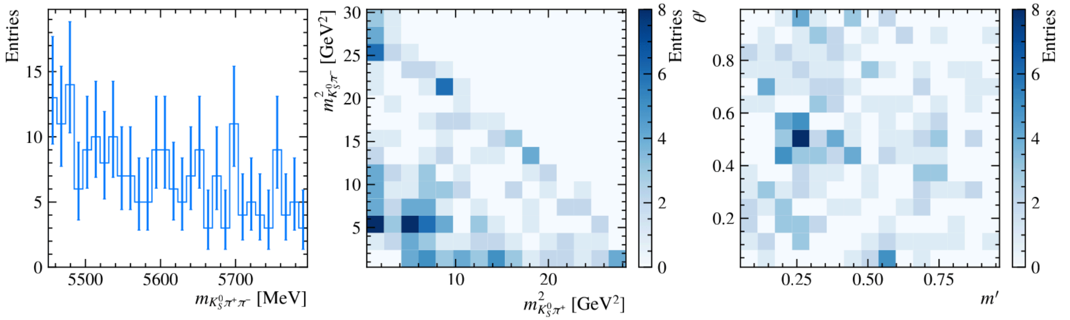


(a) $K_S^0\pi^+\pi^-$ invariant-mass distribution (left), the Dalitz plot (middle), and the square Dalitz plot (right)

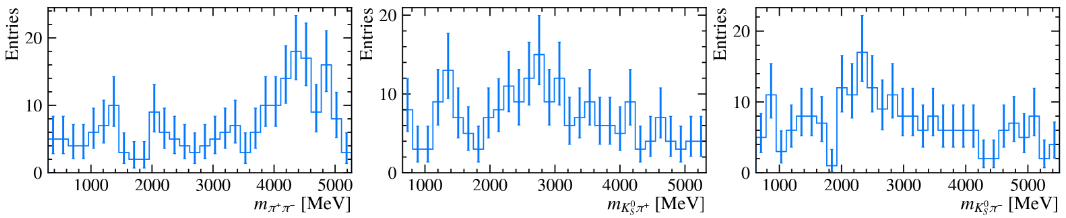


(b) Two-body invariant-mass distributions: $m_{\pi^+\pi^-}$ (left), $m_{K_S^0\pi^+}$ (middle), and $m_{K_S^0\pi^-}$ (right).

Combined dataset (all years-LL)



(a) $K_S^0\pi^+\pi^-$ invariant-mass distribution (left), the Dalitz plot (middle), and the square Dalitz plot (right)

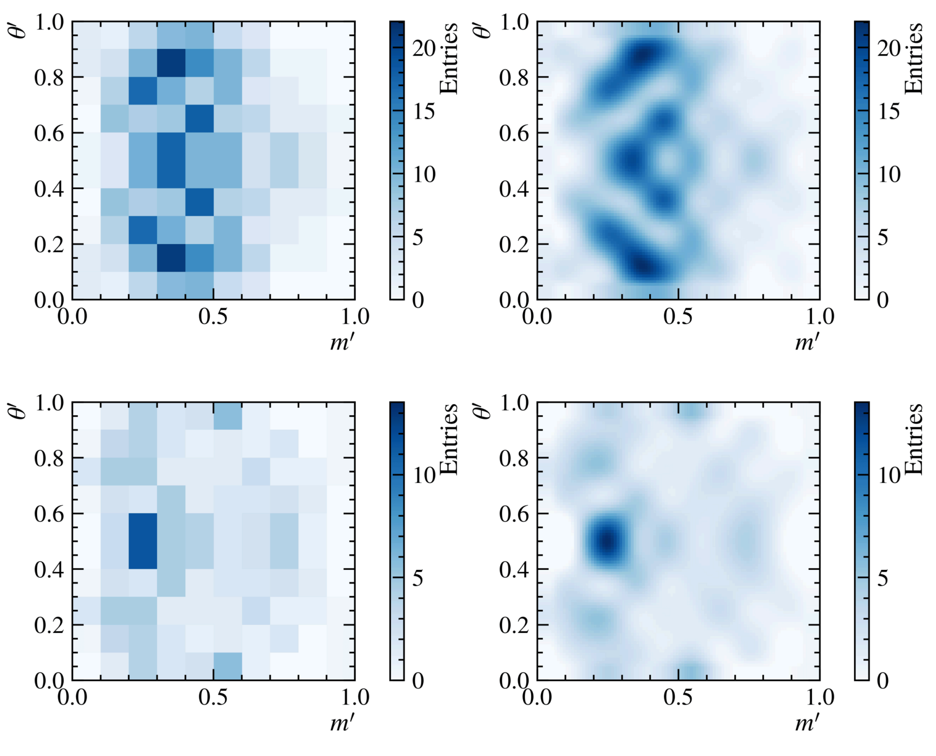


(b) Two-body invariant-mass distributions: $m_{\pi^+\pi^-}$ (left), $m_{K_S^0\pi^+}$ (middle), and $m_{K_S^0\pi^-}$ (right).

Inputs for the $B_s^0 \to K_S^0\pi^+\pi^-$ DP fit

Combinatorial background model

- The combinatorial background in the signal region is estimated with candidates from the upper mass side-band $5450\ \mathrm{MeV} \leq m_{K_S^0\pi^+\pi^-} \leq 5800\ \mathrm{MeV}$
  - Veto on $B^+ \to K_S^0\pi^+$ applied: $|m_{K_S^0\pi^\pm} - m_{B^+}| \geq 20\ \mathrm{MeV}$
- A combined map of all data-taking periods is obtained for each K_S^0 category
- The SqDP maps are symmetrized and smoothed, and are inputs for the DP fit.



$\Rightarrow$DD

$\Rightarrow$LL

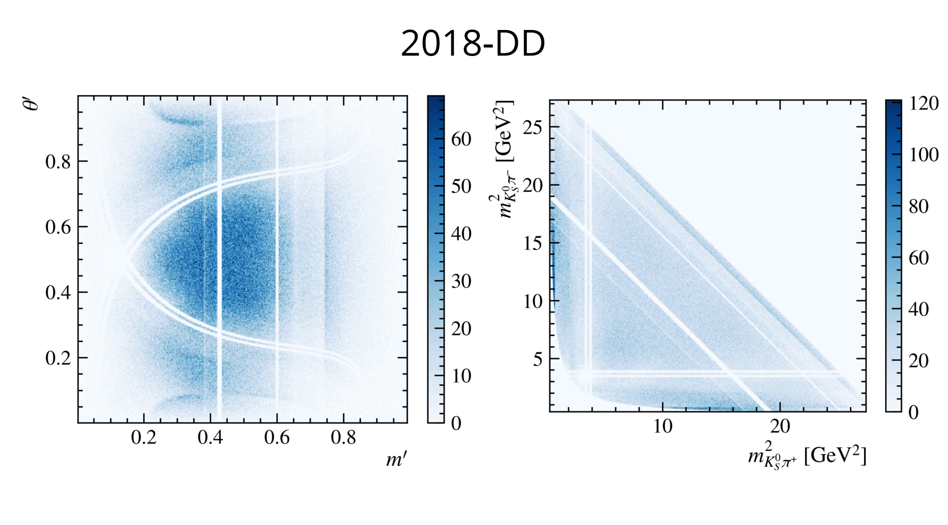
Inputs for the $B_s^0 \to K_S^0\pi^+\pi^-$ DP fit

$B_d^0 \to K_S^0\pi^+\pi^-$ background model

Estimated using the model for this decay from Run I:

- Paper: AmAn of the decay $B_d^0 \to K_S^0\pi^+\pi^-$ and first observation of the CP asymmetry in $B_d^0 \to K^*(892)\pi^+$.
- Large toy MC generated using CRAFT, weighted by the corresponding efficiency map.

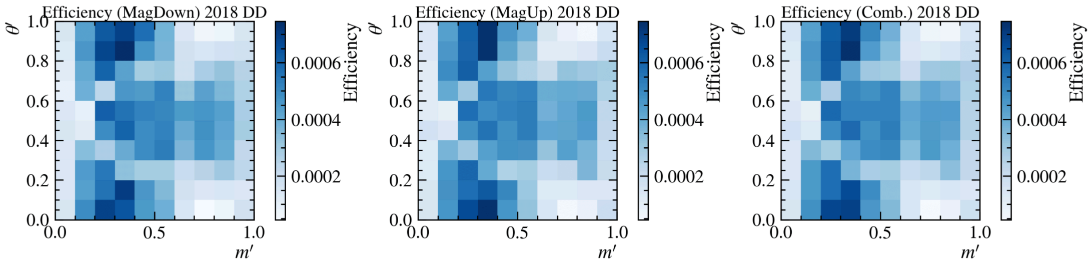
Resonance	Parameters	Lineshape
$K^{*\pm}(892)$	$m_0 = 891.66 \pm 0.26$ $\Gamma_0 = 50.8 \pm 0.9$	RBW
$(K\pi)_0^{*\pm}$	$\mathrm{Re}(c_0) = 0.204 \pm 0.103$ $\Im(c_0) = 0$ $\mathrm{Re}(c_1) = 1$ $\Im(c_1) = 0$	EFKLLM
$K_2^{*\pm}(1430)$	$m_0 = 1425.6 \pm 1.5$ $\Gamma_0 = 98.5 \pm 2.7$	RBW
$K^{*\pm}(1680)$	$m_0 = 1717 \pm 27$ $\Gamma_0 = 332 \pm 110$	Flatté
$f_0(500)$	$\mathrm{m}_0 = 513 \pm 32$ $\Gamma_0 = 335 \pm 67$ $\kappa = 0$	Red. K-Matrix = RBW
$\rho^0(770)$	$m_0 = 775.26 \pm 0.25$ $\Gamma_0 = 149.8 \pm 0.8$	GS
$f_0(980)$	$m_0 = 965 \pm 10$ $g_\pi = 165 \pm 18$ $g_K = 695 \pm 93$	Flatté
$f_0(1500)$	$m_0 = 1505 \pm 6$ $\Gamma_0 = 109 \pm 7$	RBW
χ_{c0}	$m_0 = 3414.75 \pm 0.31$ $\Gamma_0 = 10.5 \pm 0.6$	RBW
NR		DP flat



Inputs for the $B_s^0 \to K_S^0\pi^+\pi^-$ DP fit

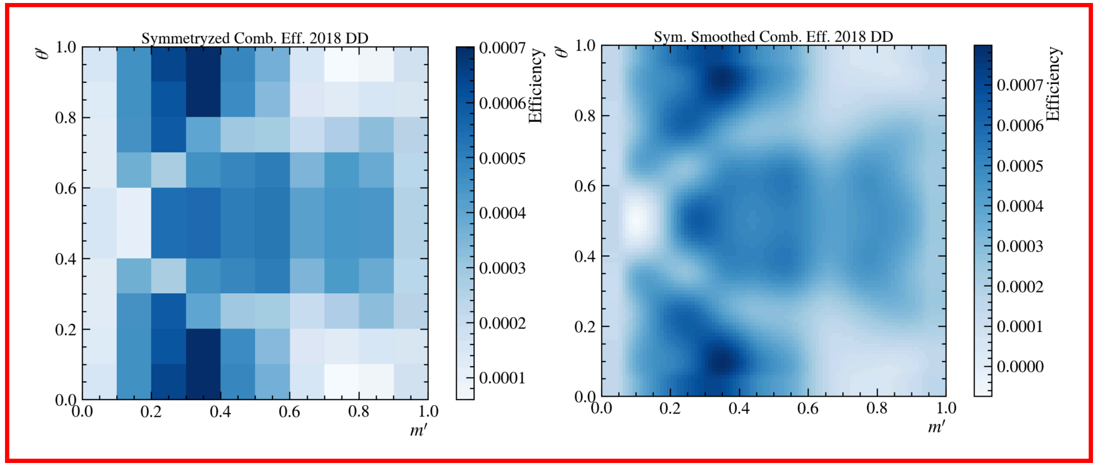
Efficiency across DP

- The reconstruction efficiency depends on the position of the candidate in the phase space.
- The efficiency is computed as function of the square Dalitz plot (SqDP) variables using MC events generated flat in SqDP and with corrections included (PID, Trigger, Tracking).
  - They are combined according to the MU-MD fractions available in the RunDB.
- The efficiency maps are smoothed out using the spline technique. This is the baseline for the efficiency modelling.



$$\epsilon^{\mathrm{corrected}}_{\mathrm{tot}} = \left(\epsilon^{\mathrm{TOS}}_{\mathrm{tot}} \times C^{\mathrm{TOS}}_{\mathrm{L0corr}} \times \frac{f^{\mathrm{data}}_{\mathrm{TOS}}}{f^{\mathrm{MC}}_{\mathrm{TOS}}} + \epsilon^{\mathrm{TIS\&!TOS}}_{\mathrm{tot}} \times C^{\mathrm{TIS\&!TOS}}_{\mathrm{L0corr}} \times \frac{f^{\mathrm{data}}_{\mathrm{TIS\&!TOS}}}{f^{\mathrm{MC}}_{\mathrm{TIS\&!TOS}}}\right) \times C_{\mathrm{tracking}}$$

Computed in bins of the SqDP



Backup

Dissimilarity test

Point-to-point dissimilarity test review

Introduction

When doing an Amplitude Analyses, a crucial step is the unbinned maximum likelihood fit of a PDF to the data. This raises the question:

- Which statistical figure of merit can be used to assess the fit's performance?

Which can be re-stated as:

- What is the level of agreement between the fit PDF and the data?

Remarks:

- The GoF is one of the metrics used to determine contributions to the fit and define the baseline model.
- Why not simply use the χ^2 approach?
    - This requires binning $\Rightarrow$ sparse or overly coarse bins and loss of information, so unbinned goodness-of-fit methods are preferable.
- The point-to-point dissimilarity test (PPDT) shall be presented: http://arxiv.org/abs/1006.3019

Point-to-point dissimilarity test review

Foundations of the PPDT

Notation and terminology:

- $\vec{s} = (s_-, s_+) = (s_{K_S\pi^-}, s_{K_S\pi^+})$
- $f(\vec{s})$ and $f_0(\vec{s})$: parent/true PDF of the data and test PDF, respectively.
- T: test statistic (TS) $\Rightarrow$ Larger values correspond to a worse level of agreement.
- H_0: Null hypothesis $\Rightarrow$ "The two samples are drawn from the same PDF".
- p: p-value

Recipe:

- Ideally, if the parent PDF is known, one can define the following statistic:

$$T = \frac{1}{2}\int \left(f(\vec{s}) - f_0(\vec{s})\right)^2 \, d\vec{s}$$

- Then, one needs to get the distribution PDF of T, $g_{f_0}(T)$, for the case $f = f_0$
- This allows to finally compute the p-value:

$$p = \int_T^{\infty} g_{f_0}(T')\, dT'$$

Point-to-point dissimilarity test review

T-statistic

One can compute T by introducing a weighting function (WF) $\psi(|\vec{s} - \vec{s}'|)$ that correlates the differences between the PDFs at different points:

The average kernel value between pairs of points if both are drawn from distribution f

$$T = \frac{1}{2}\iint \left(f(\vec{s}) - f_0(\vec{s})\right)\left(f(\vec{s}') - f_0(\vec{s}')\right)\psi(|\vec{s} - \vec{s}'|)\, d\vec{s}\, d\vec{s}'.$$

$$T = \frac{1}{2}\iint \left[f(\vec{s})f(\vec{s}') + f_0(\vec{s})f_0(\vec{s}') - 2f(\vec{s})f_0(\vec{s}')\right]\psi(|\vec{s} - \vec{s}'|)\, d\vec{s}\, d\vec{s}'.$$

This can be approximated by:

$$\boxed{T = \frac{1}{n_d(n_d-1)}\sum_{i,j>i}^{n_d}\psi(|\vec{s}_i^{\,d} - \vec{s}_j^{\,d}|) + \frac{1}{n_{\mathrm{mc}}(n_{\mathrm{mc}}-1)}\sum_{i,j>i}^{n_{\mathrm{mc}}}\psi(|\vec{s}_i^{\,\mathrm{mc}} - \vec{s}_j^{\,\mathrm{mc}}|) - \frac{1}{n_d n_{\mathrm{mc}}}\sum_{i,j}^{n_d, n_{\mathrm{mc}}}\psi(|\vec{s}_i^{\,d} - \vec{s}_j^{\,\mathrm{mc}}|).}$$

1. **Calibration under the null:** p–value to be uniformly distributed on [0,1] when comparing data vs. MC drawn from exactly the same PDF.
2. **Rejection power under alternatives:** give a high fraction of p-values below a threshold when comparing data vs. a mis-specified PDF.

$$\psi_{\exp}(|\vec{s}_i - \vec{s}_j|) = e^{-|\vec{s}_i - \vec{s}_j|^2/2\sigma(\vec{s}_i)\sigma(\vec{s}_j)}$$

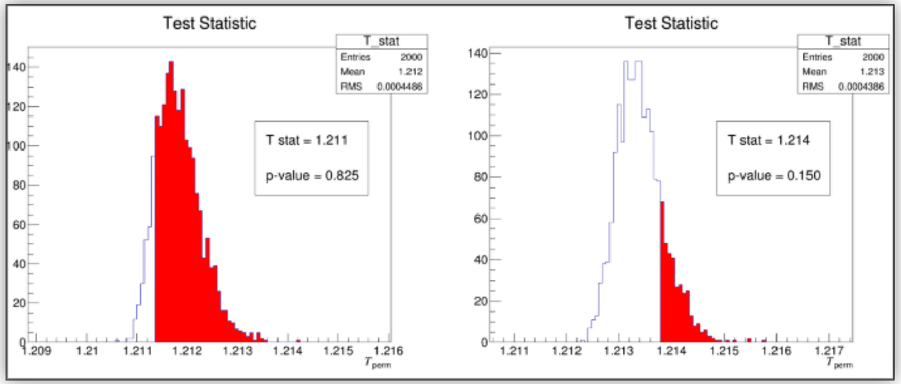
$$\psi_{\mathrm{Log}}(|\vec{s}_i - \vec{s}_j|) = \ln(|\vec{s}_i - \vec{s}_j| + \epsilon)$$

Point-to-point dissimilarity test review

Permutation test

The distribution of T for the case $f = f_0$ is not known. How do we estimate a p-value?

- Combine all data points from both samples into a single pool $n_d + n_{mc}$
- Randomly reassign points into two new "data" and "MC" samples with sizes n_d and n_{mc}
- Compute the test statistic T_{perm} for each permuted pair.
- Repeat this process n_{perm} times to obtain $\{T^1_{perm}, ..., T^{n_{perm}}_{perm}\} \Rightarrow \; g_{f_0}(T)$



- If p-value is small:
  - The observed dissimilarity is unlikely under H_0.
- If p-value is large:
  - The observed dissimilarity could happen by chance.

Backup

Preliminary systematic detailed

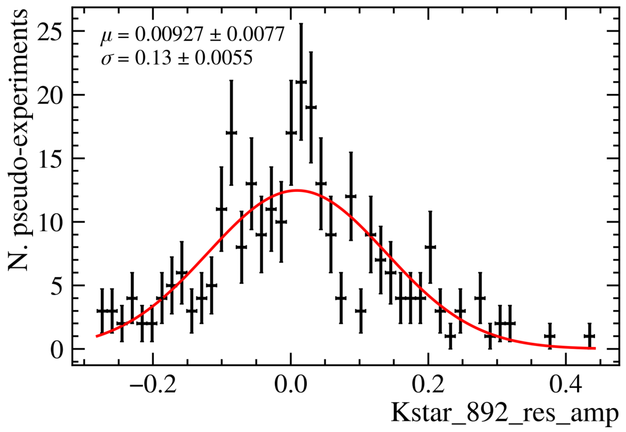
Preliminary systematics

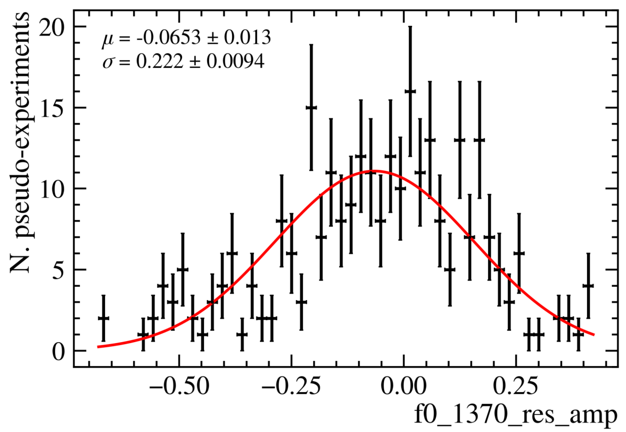
Experimental: DP fit biases

- Large set of pseudo-experiments were generated from the nominal model and fitted back.
- The difference between the values of the parameters from the pseudo-experiment fits and the ones from the nominal fit are used to evaluate the syst. uncertainty.
- The mean value of the residual distribution (μ_{residual}) is used to correct the mean value measured in the data fit and its uncertainty is assigned as the syst. uncertainty.

Resonance	Magnitude	
	Correction	Systematic
NR	0.008	0.019
$(K\pi)_0^{*+}(1430)$	0.009	0.014
$(K\pi)_0^{*-}(1430)$	0.042	0.012
$f_0(500)$	0.034	0.010
$f_0(1370)$	-0.065	0.013
$K^{*+}(892)$	0.009	0.008
$K^{*-}(892)$	–	–
$\rho^0(770)$	-0.034	0.009
$K^{*+}(1680)$	0.015	0.012
$K^{*-}(1680)$	0.042	0.013
$K_2^{*+}(1430)$	-0.009	0.011
$K_2^{*-}(1430)$	-0.041	0.014

$$\boxed{a_{\mathrm{corr}} = a_{\mathrm{nominal}} - \mu_{\mathrm{residual}}}$$



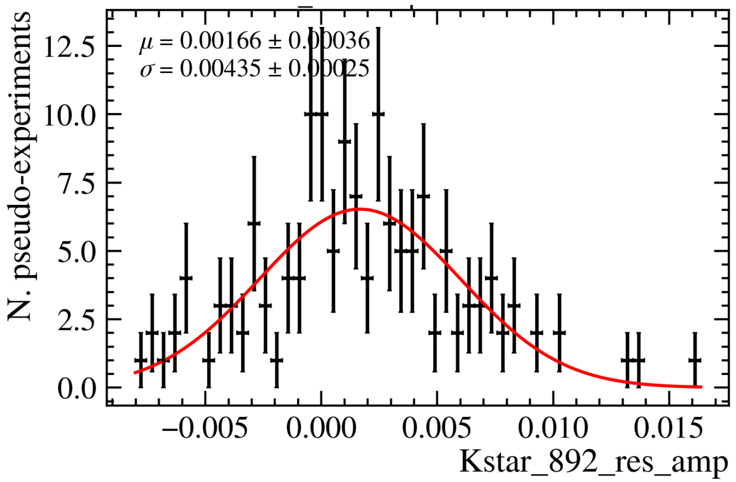


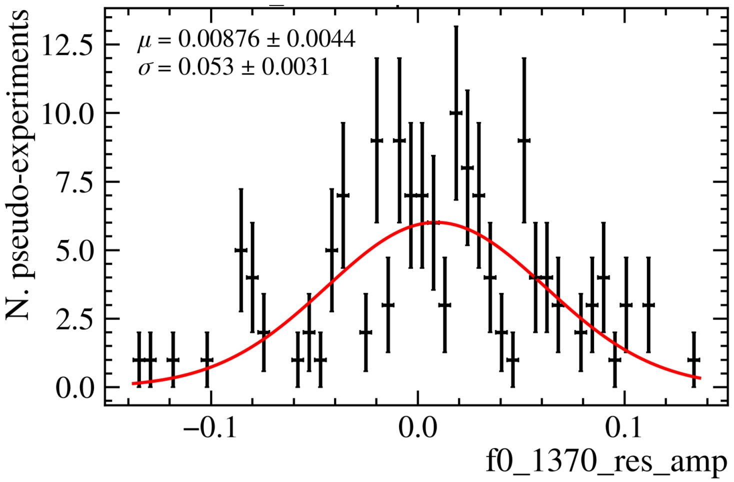
Preliminary systematics

Experimental: combinatorial background

- The histograms describing the combinatorial background distributions are fluctuated within their statistical uncertainties to generate a large set of them.
- Fits to data with these fluctuated maps are performed. The difference between the values of the parameters from these fits and the ones from the nominal are used to estimate the syst. uncertainty.
- The standard deviation (σ) of the distribution of residuals is used as syst. uncertainty.

Resonance	Magnitude
NR	0.055
$(K\pi)_0^{*+}(1430)$	0.023
$(K\pi)_0^{*-}(1430)$	0.024
$f_0(500)$	0.035
$f_0(1370)$	0.053
$K^{*+}(892)$	0.004
$K^{*-}(892)$	–
$\rho^0(770)$	0.009
$K^{*+}(1680)$	0.037
$K^{*-}(1680)$	0.038
$K_2^{*+}(1430)$	0.017
$K_2^{*-}(1430)$	0.024





Preliminary systematics

Experimental: binning choice for efficiency maps

- The fit on data is repeated with alternative efficiency-map binning (7,8,9,11,12,13).
- For the systematic uncertainty, we use the **full spread** of the observables across the considered binnings, rather than only the shift wrt. the baseline (10x10).
- The efficiency-binning systematic is computed as: $\boxed{\sigma_{\text{eff-binning}} = \frac{|x_{\max} - x_{\min}|}{\sqrt{12}}}$

Resonance	Magnitude
NR	0.018
$(K\pi)_0^{*+}(1430)$	0.022
$(K\pi)_0^{*-}(1430)$	0.022
$f_0(500)$	0.023
$f_0(1370)$	0.024
$K^{*+}(892)$	0.003
$K^{*-}(892)$	–
$\rho^0(770)$	0.018
$K^{*+}(1680)$	0.062
$K^{*-}(1680)$	0.068
$K_2^{*+}(1430)$	0.035
$K_2^{*-}(1430)$	0.035

Backup

Last BnoC talk main contents

Introduction

Physics motivation

The charmless decay $B_s^0 \to K_S^0\pi^+\pi^-$ will provide access to the first establishment of:

- The direct CP asymmetries of the quasi-two-body decays with an untagged time-integrated (TI) Dalitz analysis:
  - Flavour specific: $\bar{B}_s^0 \to K_0^{*+}(1430)\pi^-$, $\bar{B}_s^0 \to K^{*+}(892)\pi^-$
  - CP-eigenstate: $B_s^0 \to \rho^0 K_S^0$
- Fit Fractions of the different amplitudes contributing to the decay.
- These amplitudes can serve as measurements of ϕ_s or used in CKM γ angle determination w/ charmless decays.

Data samples

- This analysis inherits samples and selections from the B2KShh' Branching Fraction Analysis.
  - 14 samples: Run 1 (2011, 2012a, 2012b), Run 2 (2015-2018). Two K_S^0 reco categories: DD and LL.
  - The selection is optimized with a tight optimization targeting $B_s^0 \to K_s^0\pi^+\pi^-$ signal candidates.
    In contrast to the BF analysis, the topo MVA is optimized using as FoM: significance $\times$ purity.
  - Charm and charmonia vetoes applied:
    - Charmonia vetoes (48 MeV): $J/\psi \to \pi^+\pi^-$
    - Charm meson vetoes (30 MeV): $D_{(s)}^+ \to K_S^0\pi^+$, $D^0 \to \pi^+\pi^-$

Analysis strategy

Precedents

- This analysis follows a similar strategy to that of the DP analysis of the decay $B_d^0 \to K_S^0\pi^+\pi^-$ with Run I data.
  - The results were obtained with CRAFT (Clermont Roofit-based Amplitude Fitter Tool).
  - Analysis note: ANAnote Run I B02KSpipi
  - WG database: Analysis Run I B02KSpipi TI Dalitz Analysis
- Previous WG talk (12/02/2026): Inputs for the $B_s^0 \to K_S^0\pi^+\pi^-$ DP fit and baseline model

Stages

1. Selection of $B_s^0 \to K_S^0\pi^+\pi^-$ signal candidates
2. Fit the invariant-mass spectrum $K_S^0\pi^+\pi^-$, define a signal window around B_s^0 signal peak, and determine the fraction of signal and bkg. (both combinatorial and $B_d^0 \to K_S^0\pi^+\pi^-$)
3. Obtain the spline histogram of the efficiency variation across the DP, evaluated from signal MC.
4. Determine a model for the different background components .
5. Fit simultaneously the different samples and educate the final model by adding relevant new contributions to the baseline model and decide the nominal DP model of the analysis.

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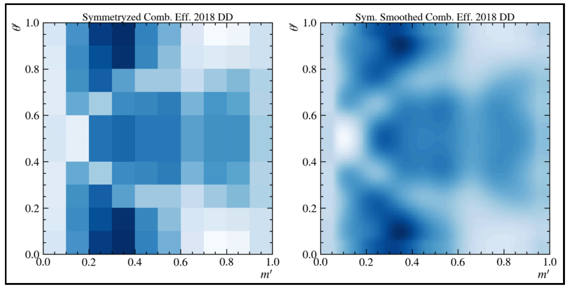
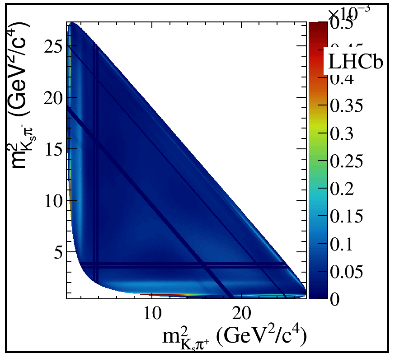
Today

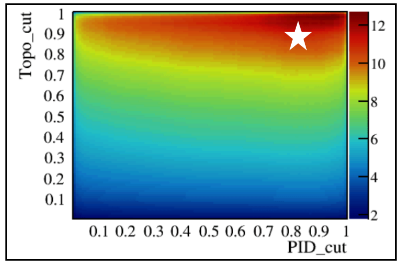
Analysis Strategy

Schematic of the analysis workflow

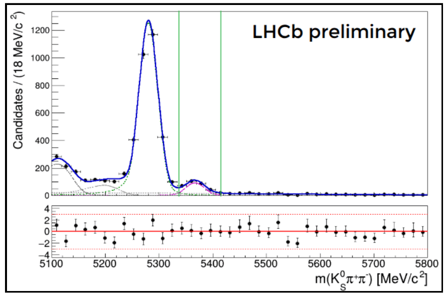




Selection of the signal candidates

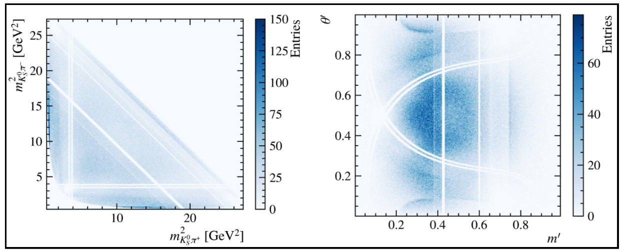


Signal and bkg. fractions from invariant-mass fit

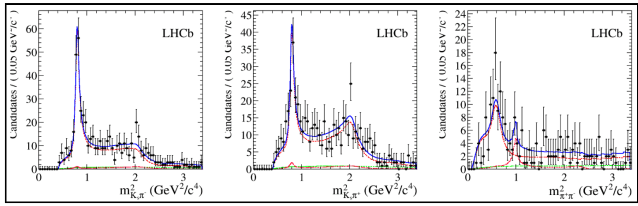


Efficiency evaluation over the sqDP from signal MC

Background models: combinatorial & Bd2KSpipi



Simultaneous DP Fit: nominal model and observables determination



Towards nominal model

- A preliminary Dalitz plot fit on data is conducted using a **Baseline model**.
- The model parameterizes the signal amplitude as the **isobar** sum of four resonances (and conjugates) plus a non-resonant (NR) contribution.

$$\mathcal{S}(s_+, s_-) \propto |\mathcal{A}(s_+, s_-)|^2 + |\bar{\mathcal{A}}(s_+, s_-)|^2 \quad \Longrightarrow \quad |\mathcal{A}(s_+, s_-)|^2 = \left| \sum_j a_j e^{i\phi_j} F_j(s_+, s_-) \right|^2$$

Untagged time-integrated Dalitz signal PDF.

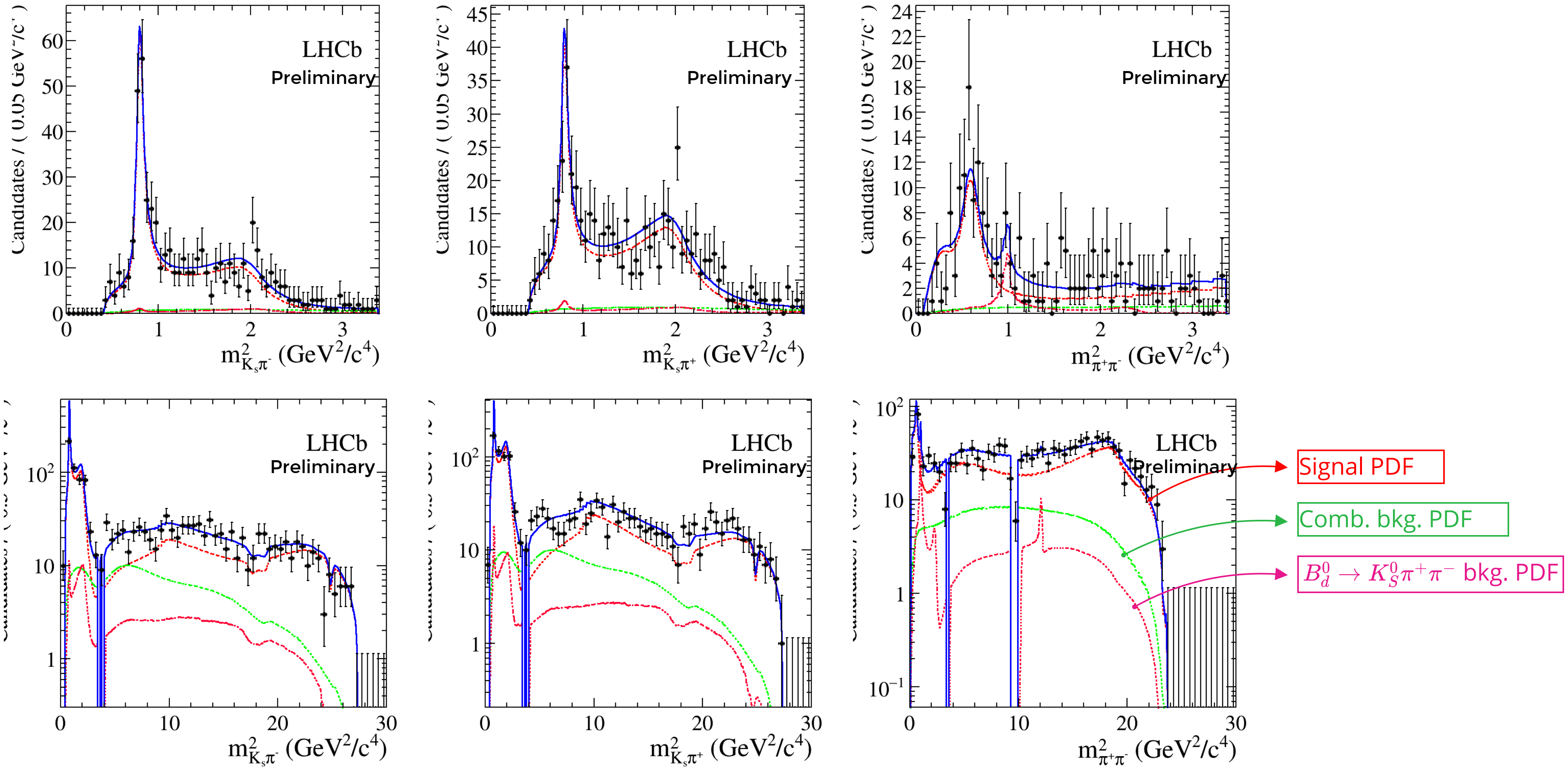
a_j and ϕ_j (x_j, y_j in cartesian) describe the relative magnitude and phase. **Floating parameters in the fit.**

Resonance	Parameters	Lineshape	CP asymmetry
$K^{*\pm}(892)$	$m_0 = 891.66 \pm 0.26$ $\Gamma_0 = 50.8 \pm 0.9$	RBW	allowed
$(K\pi)_0^{*\pm}$	$\text{Re}(c_0) = 0.204 \pm 0.103$ $\Im(c_0) = 0$ $\text{Re}(c_1) = 1$ $\Im(c_1) = 0$	EFKLLM	allowed
$f_0(500)$	$m_0 = 513 \pm 32$ $\Gamma_0 = 335 \pm 67$ $\kappa = 0$	Red. K-Matrix	not allowed
$\rho^0(770)$	$m_0 = 775.26 \pm 0.25$ $\Gamma_0 = 149.8 \pm 0.8$	GS	not allowed
NR		Flat	not allowed

- The $K^*(892)$ resonance is used as **reference** $\Rightarrow x = 2$ and $y = \bar{y} = 0$.
- This already gives a reasonable description of the data, where all the components are well identified.

Towards nominal model

Dalitz plot fit with Baseline model



- The $K^*(892)$ resonance is used as **reference** $\Rightarrow x = -2$ and $y = \bar{y} = 0$.
- This already gives a reasonable description of the data, where all the components are well identified.

Towards nominal model

Relevant additional contributions have been tested, one by one, and accepted according to the requirements (**at least one of the following criteria must be satisfied**):

- The **dissimilarity test** (backup) of the model including the additional contribution is increasing.
- The additional component is considered to be significant only when it causes a change greater than 11.83 (16.25) units of $\mathbf{-2\Delta nll}$ **(3σ deviation)**, in case of 2 (4) degrees-of-freedom.
- The magnitude of the additional contribution is measured far from zero by $\boldsymbol{3\sigma_a}$ (**magnitude precision better than 33.3%**), where σ_a is the statistical error on the amplitude.

Additional contribution	p-value	-2Δnll	Mag. precision (%)
baseline	0.24	–	–
(baseline) + $K_2^*(1430)$	0.29	40.56	(16.4, 18.4)
(baseline) + $K^*(1680)$	0.13	52.04	(16.6, 17.3)
(baseline) + $f_0(980)$	–	0.82	95.8
(baseline) + $f_2(1270)$	–	0.52	67.9
(baseline) + $f_0(1370)$	0.11	8.6	17.5
(baseline) + $f_0(1500)$	0.15	2.98	30.5
(baseline) + χ_{c0}	–	0.0	69.0
(baseline + $K_2^*(1430)$) + $K^*(1680)$	0.47	67.14	(16.6, 18.2)
(baseline + $K_2^*(1430)$) + $f_0(1370)$	0.17	12.16	17.5
(baseline + $K_2^*(1430)$) + $f_0(1500)$	0.14	7.82	30.53
Nominal model $\Leftarrow$ (baseline + $K_2^*(1430)$ + $K^*(1680)$) + $f_0(1370)$	0.33	4.72	15.3
(baseline + $K_2^*(1430)$ + $K^*(1680)$) + $f_0(1500)$	0.39	3.66	57.9

Towards nominal model

Nominal model

Resonance	Parameters	Lineshape
NR		Flat
S-wave		
$(K\pi)_0^{*\pm}$	$\mathrm{Re}(c_0) = 0.204 \pm 0.103$ $\Im(c_0) = 0$ $\mathrm{Re}(c_1) = 1$ $\Im(c_1) = 0$	EFKLLM
$f_0(500)$	$m_0 = 513 \pm 32$ $\Gamma_0 = 335 \pm 67$ $\kappa = 0$	Red. K-Matrix
$f_0(1370)$	$m_0 = 1380 \pm 70$ $\Gamma_0 = 220 \pm 80$	T-Matrix pole
P-wave		
$K^{*\pm}(892)$	$m_0 = 891.66 \pm 0.26$ $\Gamma_0 = 50.8 \pm 0.9$	RBW
$\rho^0(770)$	$m_0 = 775.26 \pm 0.25$ $\Gamma_0 = 149.8 \pm 0.8$	GS $\boxed{*}$
$K^{*\pm}(1680)$	$m_0 = 1717 \pm 27$ $\Gamma_0 = 332 \pm 110$	Flatté
D-wave		
$K_2^{*\pm}(1430)$	$m_0 = 1425.6 \pm 1.5$ $\Gamma_0 = 98.5 \pm 2.7$	RBW

*In this analysis the $\rho^0(770)K_S^0$ amplitude is CP-averaged.

- EFKLLM parameterization of $K_S^0\pi$ S-wave:

$$R(m) = F(m)\left(\frac{c_0}{m^2} + c_1\right)$$

the c_0 and c_1 coefficients are fixed.

- Reduced K-matrix for the $\pi\pi$ S-wave:

$$K(m) = K_{\mathrm{res}}(m) + K_{\mathrm{non-res}} = \frac{m_0\Gamma(m)}{(m_0^2 - m^2)\,\rho(m)} + \kappa$$

- T-matrix pole for $f_0(1370)$:

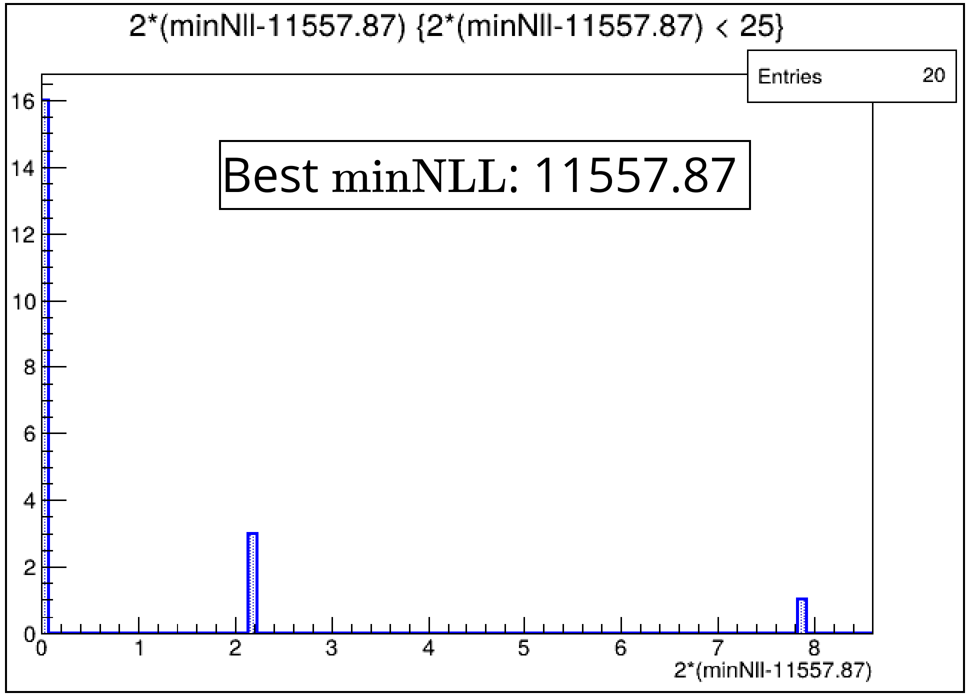
$$T(s) = \frac{1}{s_0 - s}$$

where $\sqrt{s_0} = m_0 - i\Gamma_0$ is the pole position.

Fit results with the nominal model

Finding the best solution

- Simultaneous fits to the data (all sub-samples) have been conducted using the nominal model but varying randomly the initial values of the isobar parameters for each fit.
- Six solutions are found, from which three are in the range of 25 units of $-2\Delta\mathrm{nll}$ from the best minNLL:

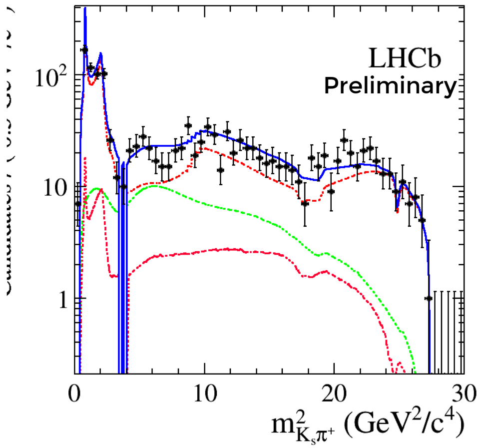


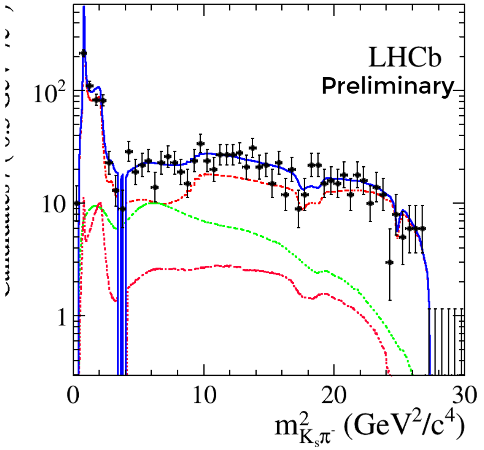
- The global minimum (best solution) clearly emerges.

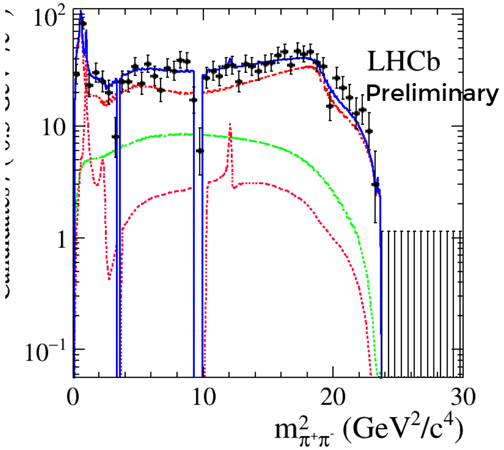
Fit results with the nominal model

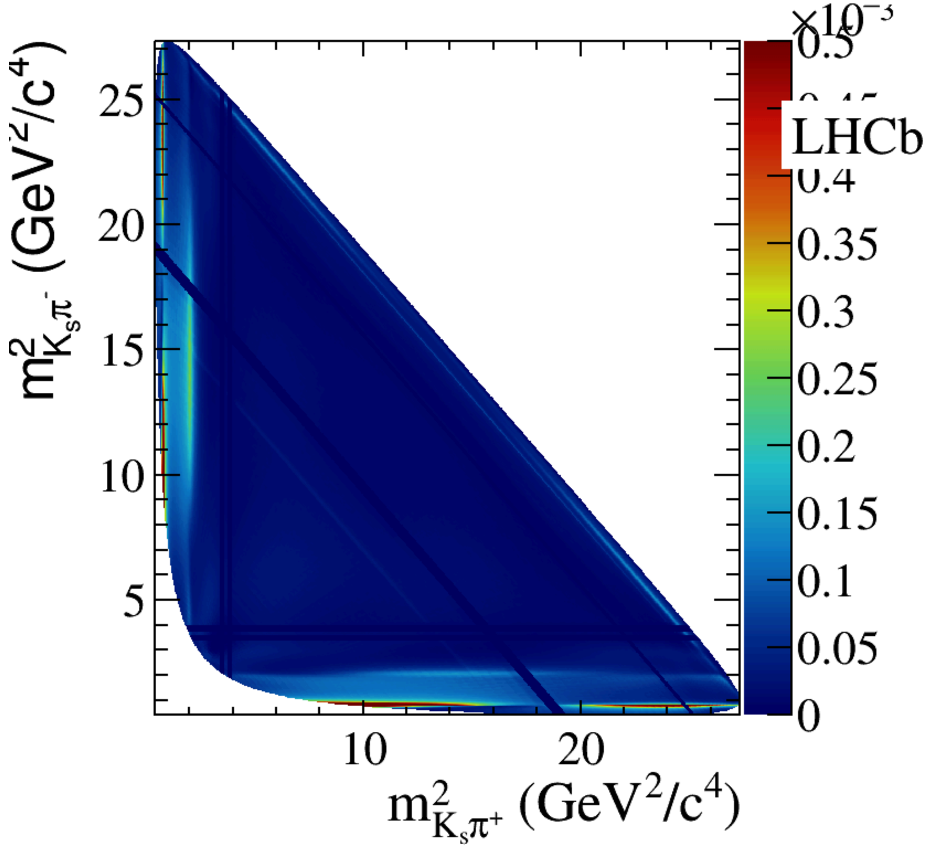
Plots of the projections in the whole spectrum

- All 14 sub-samples (7 data-taking periods x 2 K_S^0 reconstruction categories) are combined in the plots below.





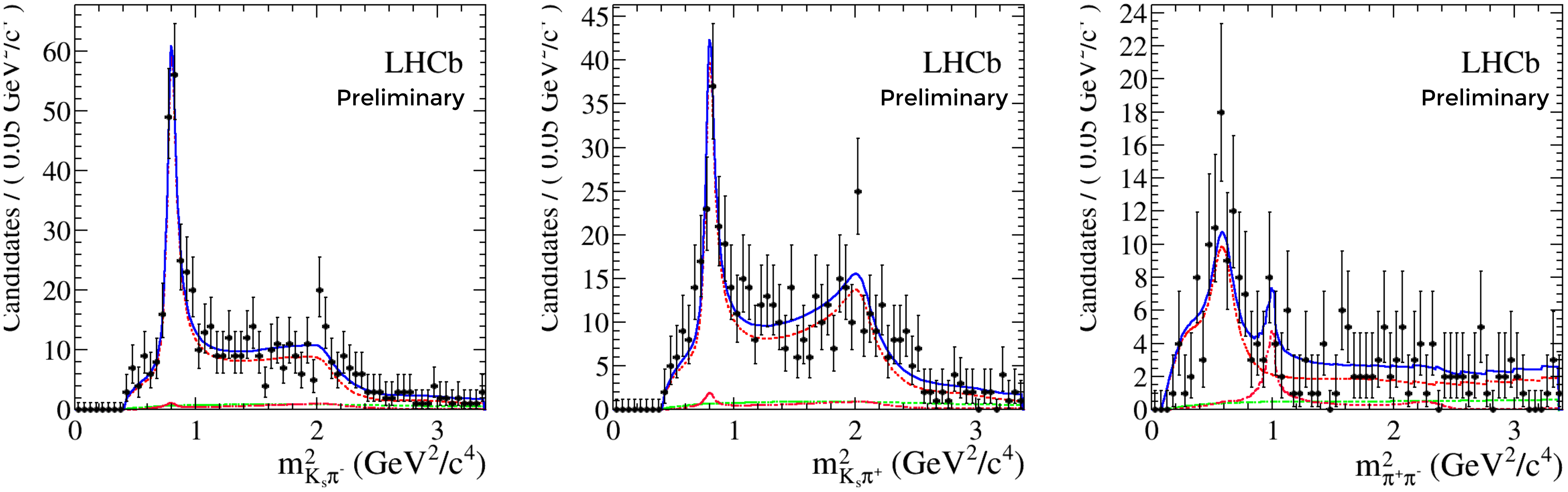




Fit results with the nominal model

Zoom plots of the projections

- All 14 sub-samples (7 data-taking periods x 2 K_S^0 reconstruction categories) are combined in the plots below.



- All projections of the DP fit are found to be satisfactorily described by the model ($p\text{-value} = 0.33$).

Towards nominal model

Fit results with the nominal model: Isobar parameters

- The magnitude and phase are built up from the cartesian parameters, which are the floating parameters in the fit.

Resonance	Real part (x)	Imaginary part (y)	Magnitude (a)	Phase (ϕ)
NR	-1.76 ± 0.24	0.28 ± 0.26	1.78 ± 0.25	171.1 ± 7.8
$(K\pi)_0^{*+}$	-2.81 ± 0.21	-1.10 ± 0.30	3.02 ± 0.21	-158.7 ± 5.8
$(K\pi)_0^{*-}$	-2.61 ± 0.19	-0.07 ± 0.32	2.61 ± 0.19	-178.4 ± 7.0
$f_0(500)$	0.14 ± 0.32	0.52 ± 0.13	0.54 ± 0.15	75.4 ± 32.7
$f_0(1370)$	-0.31 ± 0.31	-0.83 ± 0.20	0.89 ± 0.14	-110.2 ± 22.0
$K^{*+}(892)$	1.71 ± 0.13	0.00	1.71 ± 0.13	0.0
$K^{*-}(892)$	-2.00	0.00	2.00	180.0
$\rho^0(770)$	-0.47 ± 0.58	1.13 ± 0.31	1.22 ± 0.13	112.7 ± 30.1
$K^{*+}(1680)$	0.17 ± 0.34	1.07 ± 0.20	1.08 ± 0.19	81.1 ± 18.1
$K^{*-}(1680)$	0.15 ± 0.41	-1.07 ± 0.20	1.08 ± 0.21	-82.2 ± 21.4
$K_2^{*+}(1430)$	1.16 ± 0.18	0.14 ± 0.22	1.17 ± 0.17	6.8 ± 11.1
$K_2^{*-}(1430)$	0.56 ± 0.17	-0.72 ± 0.22	0.91 ± 0.20	-52.1 ± 12.0

Disclaimer: The phase uncertainties will be determined from pseudo-experiments.

Systematic uncertainties study

Categories distinction

The sources of systematic uncertainty will be divided in two categories:

Experimental systematic uncertainties

- **biases** related to the Dalitz fit to the data,
- the **signal fraction** extracted from the mass fit,
- uncertainties on the **selection, trigger and tracking efficiency** across the sqDP,
- the **binning choice** for the signal efficiency histograms,
- uncertainty on the **combinatorial background** across the sqDP,
- uncertainty on the $\boldsymbol{B^0 \to K_S^0 \pi^+ \pi^-}$ **component** across the sqDP.
- symmetrization of: efficiency maps and combinatorial maps?

Model systematic uncertainties

- **fixed parameters** of the resonance line-shapes models,
- addition or removal of **marginal components** to the nominal model,
- **alternative models** for the $K_S^0\pi$ and $\pi\pi$ S-waves,

Preliminary systematic uncertainty determination

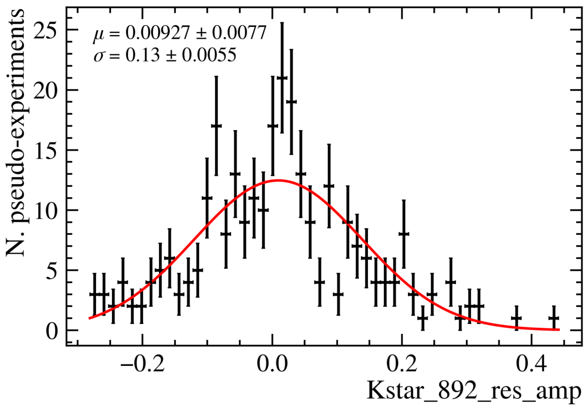
Experimental: DP fit biases, eff. binning choice, comb. bkg. model

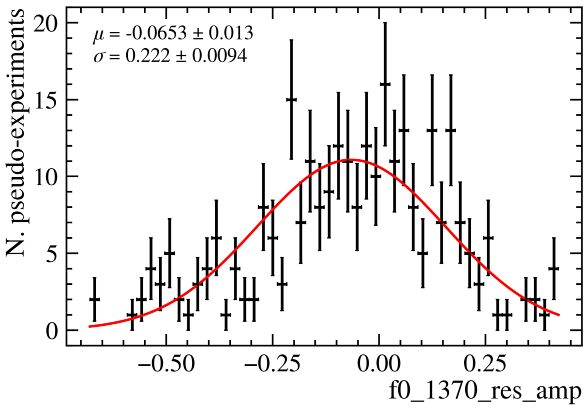
- **DP fit biases:** Computed using toys fitted with the nominal model. The mean value of the residual distribution μ_{residual} is quoted as bias correction and its uncertainty is assigned as syst.
- **Comb. background model:** fits to data are performed with comb. bkg. histograms fluctuated within their statistical uncertainties. The σ of the distribution of residuals is used as syst.
- **Eff.-binning choice:** fits to data are repeated with alternative efficiency-map binning ($7 \times 7, \ldots, 13 \times 13$). The associated syst. is computed as: $\sigma_{\text{eff-binning}} = \frac{|x_{\max} - x_{\min}|}{\sqrt{12}}$

Systematic uncertainties on the fitted amplitude magnitudes

Resonance	Correction	Fit-bias syst.	Comb. syst.	Eff.-binning syst.
NR	0.008	0.019	0.055	0.018
$(K\pi)_0^{*+}$	0.009	0.014	0.023	0.022
$(K\pi)_0^{*-}$	0.042	0.012	0.024	0.022
$f_0(500)$	0.034	0.010	0.035	0.023
$f_0(1370)$	-0.065	0.013	0.053	0.024
$K^{*+}(892)$	0.009	0.008	0.004	0.003
$K^{*-}(892)$	–	–	–	–
$\rho^0(770)$	-0.034	0.009	0.009	0.018
$K^{*+}(1680)$	0.015	0.012	0.037	0.062
$K^{*-}(1680)$	0.042	0.013	0.038	0.068
$K_2^{*+}(1430)$	-0.009	0.011	0.017	0.035
$K_2^{*-}(1430)$	-0.041	0.014	0.024	0.035

$$\boxed{a_{\text{corr}} = a_{\text{nominal}} - \mu_{\text{residual}}}$$





The corrections are ~5% of the stats. unc.

Summary and outlook

Summary

- The nominal model has been defined according to an "objective" pre-defined algorithm, including information from: dissimilarity test, negative likelihood and magnitude precision of the added component.
- Results from the nominal model have been presented, including momentarily the isobar parameters (cartesian and polar) and the raw CP asymmetry.
- The systematic uncertainties have been introduced and results are presented for some of them (experimental category).

Outlook

- Complete the fit validation study with Likelihood scans.
- Continue with the computation of systematic uncertainties for the Fit Fractions.
- Correct the raw CP asymmetry by accounting for the production asymmetry and the asymmetry due to the interactions of the final state pions with the detector.
- Update the ANA with the recent results.
- Completion of the analysis expected before Summer.

Thank you!

Fit fractions calculation: quick review

For each flavour, the amplitude is written as:

$$\mathcal{A}(\Phi) = \sum_i c_i F_i(\Phi), \qquad \overline{\mathcal{A}}(\Phi) = \sum_i \bar{c}_i \bar{F}_i(\Phi)$$

Fit fraction component is:

$$\mathrm{FF}_i = \frac{(x_i^2 + y_i^2) S_i}{N}$$

with:

$$N = \int |\mathcal{A}(\Phi)|^2 \, d\Phi = \sum_i (x_i^2 + y_i^2) S_i + \sum_{i<j} \left[(x_i x_j + y_i y_j) R_{ij} + (y_i x_j - x_i y_j) J_{ij} \right], \qquad S_i = \int |F_i(\Phi)|^2 \, d\Phi$$

Similarly for the interference fit fractions:

$$\mathrm{IFF}_{ij} = \frac{(x_i x_j + y_i y_j) R_{ij} + (y_i x_j - x_i y_j) J_{ij}}{N}.$$

with:

$$I_{ij} = \int F_i(\Phi) F_j^*(\Phi) \, d\Phi \qquad \Rightarrow \qquad R_{ij} = 2\,\mathrm{Re}(I_{ij}), \quad J_{ij} = -2\,\mathrm{Im}(I_{ij})$$

Fit fractions calculation: quick review

Sketch of the combination. The FF can be rewritten for each amplitude as:

$$\mathrm{FF}_i^B = \frac{D_i}{N}, \qquad \mathrm{FF}_i^{\bar{B}} = \frac{\overline{D}_i}{\overline{N}} \qquad \overline{N} = \int |\overline{\mathcal{A}}(\Phi)|^2 \, d\Phi$$

with:

$$D_i = (x_i^2 + y_i^2) S_i \qquad \overline{D}_i = (\bar{x}_i^2 + \bar{y}_i^2) \overline{S}_i$$

The combined fit fraction can be written as:

$$\boxed{\mathrm{FF}_i^{\mathrm{combined}} = \frac{D_i + \overline{D}_i}{N + \overline{N}} = \frac{(x_i^2 + y_i^2) S_i + (\bar{x}_i^2 + \bar{y}_i^2) \overline{S}_i}{N + \overline{N}} = w \, \mathrm{FF}_i^B + (1 - w) \, \mathrm{FF}_i^{\bar{B}}}$$

with:

$$1 - w = \frac{\overline{N}}{N + \overline{N}}$$

Similarly:

$$\boxed{\mathrm{IFF}_{ij}^{\mathrm{combined}} = \frac{C_{ij} + \overline{C}_{ij}}{N + \overline{N}} = \frac{(x_i x_j + y_i y_j) R_{ij} + (y_i x_j - x_i y_j) J_{ij}}{N + \overline{N}} = w \, \mathrm{IFF}_{ij}^B + (1 - w) \, \mathrm{IFF}_{ij}^{\bar{B}}}$$