Amplitude Analysis of $B_s^0 \to K_S^0\pi^+\pi^-$ with Run 1+2 data

Fit Fractions Results and discussion: GLASS and DP signal PDF

S. Ordonez-Soto

For Clermont group and the KShh μ-group

LHCb@LPCA meeting

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ÉCOLE DOCTORALE
DES SCIENCES FONDAMENTALES
Université Clermont Auvergne

News and prospects for next weeks

Timescale

- The conveners contacted me to ask for updates. I've said that we plan to enter WG review in September $\Rightarrow$ There is no defined date.

Prospects

- Documentation:
  - To keep track of the internal review of the analysis I've created this CodiMD.
  - Reminder: Twiki **to be created** $\Rightarrow$ Stéphane
- Tasks:
  - ANA note v0 $\Rightarrow$ **~90% completed**
  - Fit fractions with the corresponding uncertainties from toys $\Rightarrow$ **Done**
  - Systematic uncertainties $\Rightarrow$ **~70% completed**
- To discuss:
  - Alternative models for the $K_S^0\pi$ and $\pi\pi$ S-waves
  - Dalitz signal PDF

Fit fractions calculation

- The FF are evaluated from the fitted amplitude coefficients using no-efficiency phase-space integrals:
  - The coefficients are obtained from a fit that includes the real category-dependent efficiencies.
  - The final physics fit fractions are evaluated from those fitted coefficients using phase-space integrals with unit efficiency.
- Separate B_s^0 and $\bar{B}_s^0$ fractions are combined with their relative total intensities:

$$\boxed{\mathrm{FF}_i^{\text{combined}} = \frac{D_i + \overline{D}_i}{N + \overline{N}} = \frac{(x_i^2 + y_i^2)S_i + (\bar{x}_i^2 + \bar{y}_i^2)\overline{S}_i}{N + \overline{N}} = w\,\mathrm{FF}_i^B + (1-w)\,\mathrm{FF}_i^{\bar{B}}}$$

$$\boxed{\mathrm{IFF}_{ij}^{\text{combined}} = \frac{C_{ij} + \overline{C}_{ij}}{N + \overline{N}} = \frac{(x_i x_j + y_i y_j)R_{ij} + (y_i x_j - x_i y_j)J_{ij}}{N + \overline{N}} = w\,\mathrm{IFF}_{ij}^B + (1-w)\,\mathrm{IFF}_{ij}^{\bar{B}}}$$

By construction:

$$\boxed{\sum_i \mathrm{FF}_i^{\text{combined}} + \sum_{i<j} \mathrm{IFF}_{ij}^{\text{combined}} = 1}$$

Checked ✓

Fit fractions results

- Combined fit fractions. The statistical uncertainties are obtained from toy fits (next slide).

Resonance	Fit fraction (%)	Raw CP asymm.
$K^*(892)$	$21.87^{+1.91}_{-1.56}$	0.158 ± 0.075
$(K\pi)^*_0$	$50.46^{+4.68}_{-4.45}$	-0.143 ± 0.065
$K^*_2(1430)$	$6.95^{+1.94}_{-1.64}$	-0.247 ± 0.232
$K^*(1680)$	$7.39^{+1.78}_{-1.74}$	-0.002 ± 0.276
$\rho^0(770)$	$9.47^{+1.72}_{-1.90}$	–
$f_0(500)$	$1.84^{+2.10}_{-0.99}$	–
$f_0(1370)$	$4.97^{+2.46}_{-2.00}$	–
NR	$20.16^{+6.92}_{-6.42}$	–

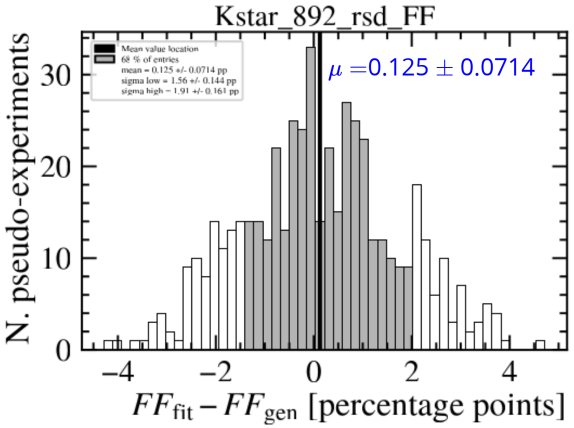
No corrections due to bias are included for the moment. To be checked.

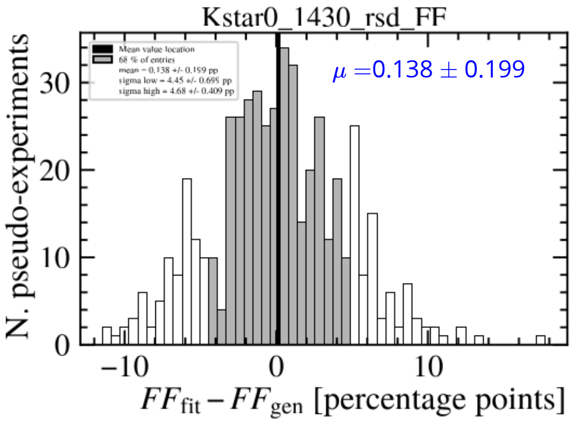
Interference fit fractions

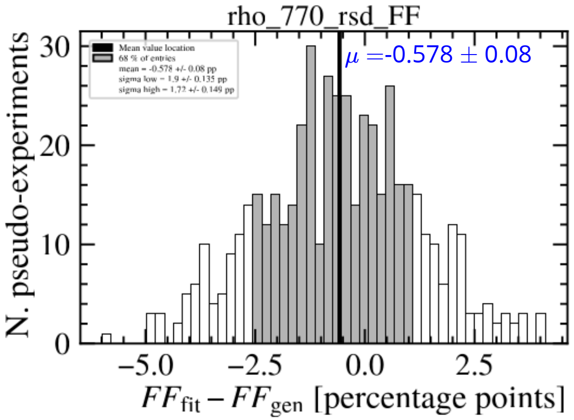
Resonance	$K^*(892)$	$(K\pi)^*_0$	K^*_2	$K^*(1680)$	ρ^0	$f_0(500)$	$f_0(1370)$	NR
$K^*(892)$	21.87	0.00	0.70	1.82	0.12	−0.02	0.56	0.00
$(K\pi)^*_0$		50.46	2.15	0.00	1.28	0.14	1.49	−21.15
K^*_2			6.95	0.29	−0.42	−0.25	0.63	−4.45
$K^*(1680)$				7.39	−0.87	−0.67	0.53	0.00
ρ^0					9.47	0.00	0.00	0.00
$f_0(500)$						1.84	0.16	2.34
$f_0(1370)$							4.97	−7.48
NR								20.16

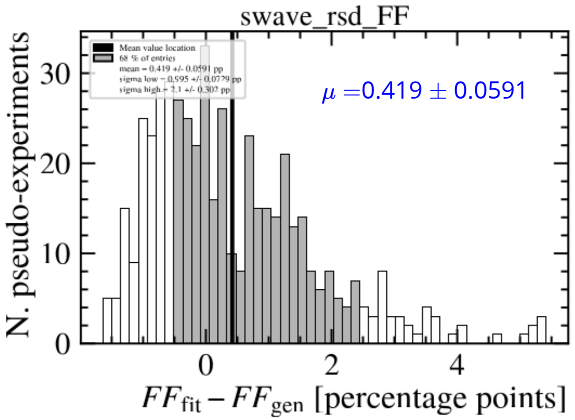
Fit fraction statistical uncertainties

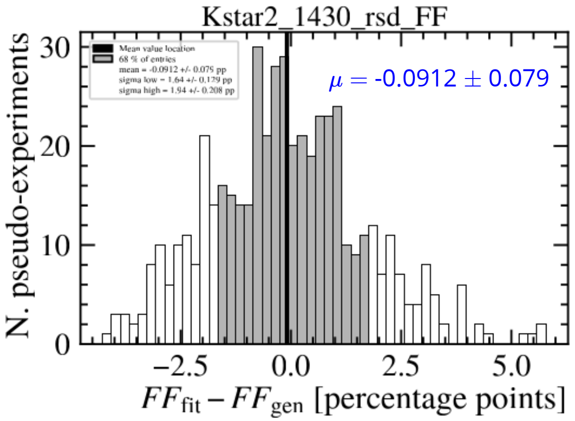
- The stats. uncertainties are obtained from fits to toys
  - The distribution of residuals (absolute difference between nominal and toy after expressing them as percenatges) is built for each observable.
  - The mean can be used as a correction to the nominal fitted value, if necessary.
  - The interval containing 68% of the dist. around the central value is taken as stat. uncertainty.

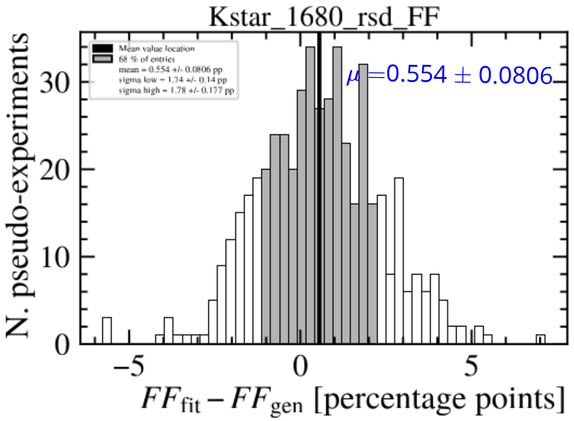


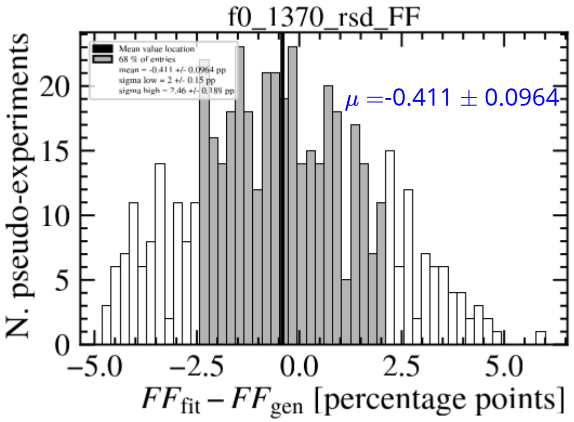


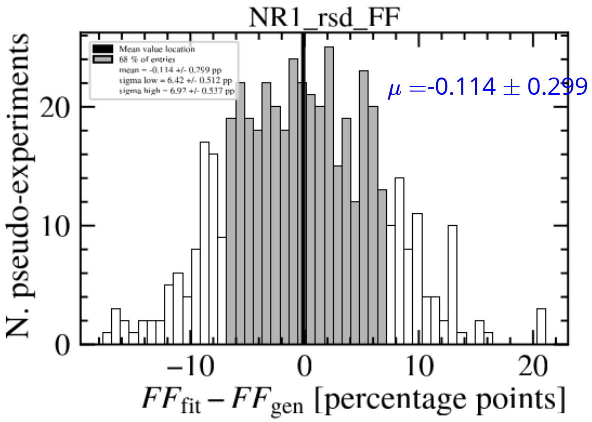








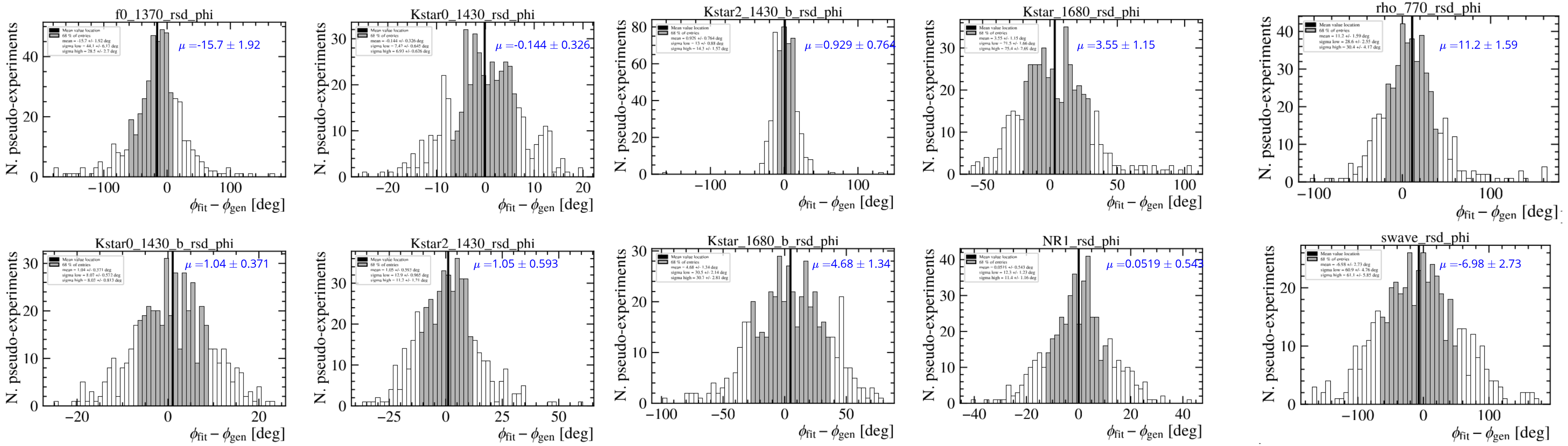




Phase statistical uncertainties

Resonance	Real part	Imaginary part	Magnitude	Phase (°)
NR	-1.76 ± 0.27	0.28 ± 0.27	1.78 ± 0.28	$171.1^{+11.4}_{-12.3}$
$(K\pi)^{*+}_0$	-2.81 ± 0.22	-1.10 ± 0.30	3.02 ± 0.21	$-158.7^{+6.9}_{-7.4}$
$(K\pi)^{*-}_0$	-2.61 ± 0.19	-0.07 ± 0.33	2.61 ± 0.19	$-178.4^{+8.0}_{-8.1}$
$f_0(500)$	0.14 ± 0.48	0.52 ± 0.15	0.54 ± 0.18	$75.4^{+61.1}_{-60.9}$
$f_0(1370)$	-0.31 ± 0.38	-0.83 ± 0.21	0.89 ± 0.15	$-110.2^{+28.5}_{-44.1}$
$K^{*+}(892)$	1.71 ± 0.13	0.00	1.71 ± 0.13	0.0 ± 0.0
$K^{*-}(892)$	-2.00	0.00	2.00	180.0
$\rho^0(770)$	-0.47 ± 0.55	1.13 ± 0.29	1.22 ± 0.13	$112.7^{+30.4}_{-28.6}$
$K^{*+}(1680)$	0.17 ± 0.34	1.07 ± 0.20	1.08 ± 0.20	$81.1^{+25.4}_{-21.5}$
$K^{*-}(1680)$	0.15 ± 0.48	-1.07 ± 0.21	1.08 ± 0.21	$-82.2^{+30.7}_{-30.5}$
$K^{*+}_2(1430)$	1.16 ± 0.18	0.14 ± 0.22	1.17 ± 0.17	$6.8^{+11.7}_{-12.9}$
$K^{*-}_2(1430)$	0.56 ± 0.17	-0.72 ± 0.22	0.91 ± 0.20	$-52.1^{+14.3}_{-13.0}$

No corrections due to bias are included for the moment



Systematic uncertainties study

Categories distinction

The sources of systematic uncertainty will be divided in two categories:

Experimental systematic uncertainties

- **biases** related to the Dalitz fit to the data,
- the **signal fraction** extracted from the mass fit,
- uncertainties on the **selection, trigger and tracking efficiency** across the sqDP,
- the **binning choice** for the signal efficiency histograms,
- uncertainty on the **combinatorial background** across the sqDP,
- uncertainty on the $\boldsymbol{B^0 \to K_S^0 \pi^+ \pi^-}$ **component** across the sqDP.
- symmetrization of:
  - efficiency maps
  - combinatorial maps

Model systematic uncertainties

- **fixed parameters** of the resonance line-shapes models,
- addition or removal of **marginal components** to the nominal model,
- **alternative models** for the $K_S^0\pi$ and $\pi\pi$ S-waves,

GLASS

- Do we want to include GLASS as a new "dynamical function" in CRAFT or we aim to build an analogous lookup table to that of the EFKLLM model?
  - GLASS contains more parameters than the EFKLLM
  - We somehow fix in a reasonable way the GLASS shape and let CRAFT fit the usual complex coefficients $\Rightarrow$ Similar to what was done for EFKLLM
- What is GLASS?

Thus, modifications to the LASS lineshape are necessary. This is achieved with the GLASS lineshape, which introduces independent magnitude parameters (B and R) and phase offsets (ϕ_{SVP} and $\phi_{K_0^*}$) to the two terms. Writing

$$\delta_{\mathrm{SVP}} = \delta'_{\mathrm{SVP}} + \phi_{\mathrm{SVP}} \quad \text{and} \quad \delta_{K_0^*} = \delta'_{K_0^*} + \phi_{K_0^*}\,,$$

the GLASS lineshape is

$$T_{\mathrm{GLASS}}(s) = B \sin\delta_{\mathrm{SVP}} e^{i\delta_{\mathrm{SVP}}} + R \sin\delta_{K_0^*} e^{i\delta_{K_0^*}} e^{2i\delta_{\mathrm{SVP}}}\,. \tag{39}$$

- How GLASS is coupled to the decay amplitude?

Decoupling of the GLASS lineshape into resonance-like term
The total GLASS amplitude can be written as

$$A_{\mathrm{GLASS}}(s) \equiv a_{K_0^*}(\sin\delta_{K_0^*} e^{i\delta_{K_0^*}} e^{2i\delta_{\mathrm{SVP}}} + a_{\mathrm{SVP}} \sin\delta_{\mathrm{SVP}} e^{i\delta_{\mathrm{SVP}}})\,, \tag{40}$$

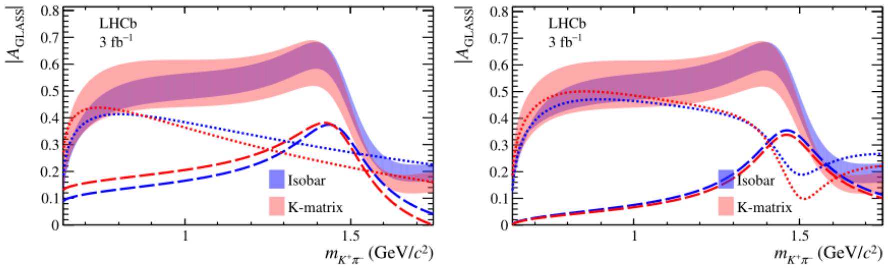


Figure 39: Magnitude of GLASS line shape, where long-dashed line and short-dashed line are the resonance and slow varying part, respectively. Left is the nominal while right is for modified decoupling of GLASS. The total GLASS amplitude (filled area) is unchanged between the two.

GLASS

- GLASS parameter values from B2Kpipi analysis $\Rightarrow$ Not universal GLASS constants

Table 21: Lineshape parameters of the Isobar S-wave approach, where the uncertainties are statistical, systematic and model-induced, respectively.

Parameter	Value
$m_0[K_0^*(1430)^0]$ (GeV/c^2)	$1.461 \pm 0.002 \pm 0.001 \pm 0.002$
$\Gamma_0[K_0^*(1430)^0]$ (GeV)	$0.216 \pm 0.004 \pm 0.004 \pm 0.002$
$a[K_0^*(1430)^0]$ (c/ GeV)	$6.900 \pm 0.244 \pm 0.771 \pm 0.697$
$r[K_0^*(1430)^0]$ (c/ GeV)	$-2.199 \pm 0.051 \pm 0.172 \pm 0.080$
$\phi_R[K_0^*(1430)^0]$	$3.373 \pm 0.011 \pm 0.015 \pm 0.022$
$\phi_F[K_0^*(1430)^0]$	$0.386 \pm 0.014 \pm 0.040 \pm 0.020$
$m_0[f_0(980)]$ (GeV/c^2)	$0.961 \pm 0.002 \pm 0.002 \pm 0.001$
$g_{\pi\pi}[f_0(980)]$ (GeV^2/c^4)	$0.117 \pm 0.003 \pm 0.005 \pm 0.002$
$g_{K\bar{K}/\pi\pi}[f_0(980)]$	$2.571 \pm 0.122 \pm 0.182 \pm 0.022$

- CRAFT's existing machinery is naturally designed for **fixed dynamical functions with floating external coefficients**.

Revisiting the signal PDF

The CRAFT machinery currently uses a signal PDF in which the $\boldsymbol{\Delta\Gamma_s}$ dependent mixing-decay interference term is **neglected**:

$$\boxed{\mathcal{P}_{\text{untag}}(s_+, s_-) \propto \varepsilon_{\text{DP}}(s_+, s_-)\left(|\mathcal{A}|^2 + |\bar{\mathcal{A}}|^2\right)}$$

After reviewing the literature, this is just an approximation in the case of a B_s^0 decay. In the very end of the PDF derivation, there are terms that **don't cancel upon normalization**:

$$\mathcal{P}_{\text{untag}}(s_+, s_-) \propto \left[|\mathcal{A}(s_+, s_-)|^2 + |\bar{\mathcal{A}}(s_+, s_-)|^2\right]\left[1 - yK(s_+, s_-)\right].$$

Including the experimental time acceptance (following the Bs2KSKpi ANA note):

$$\boxed{\mathcal{P}_{\text{untag}}(s_+, s_-) \propto |\mathcal{A}|^2 + |\bar{\mathcal{A}}|^2 - 2D\mathcal{R}\left[\frac{q}{p}\mathcal{A}^*\bar{\mathcal{A}}\right].}$$

with:

$$D = \frac{\int_0^\infty \epsilon(t)e^{-\Gamma t}\sinh(\Gamma y t)\,dt}{\int_0^\infty \epsilon(t)e^{-\Gamma t}\cosh(\Gamma y t)\,dt}.$$

For uniform time acceptance: $D = y_s = \frac{\Delta\Gamma_s}{2\Gamma_s}$

Revisiting the signal PDF

How did they proceed in the Bs2KSKpi analysis?

- First, they could not resolve the two amplitudes, so they introduced a single effective amplitude.
- They state that they ignore the contribution of the last term in the PDF.
  - D is numerically suppressed by the relatively small width difference. In the uniform-acceptance limit they quote: $D = y_s \approx 6.5\%$
    - Not guarantee that its effect is negligible, but makes treating it as a correction plausible.
- They also assumed that D did not vary across the Dalitz plot.
- **Toy study**:
  - They generated 500 psudoexperiments using the complete model and fitted those samples using their simplified model. They tested several scenarios and found reasonable agreement between generated and recovered fit fractions within uncertainties.
    - They quote this as validation of the approximation.

How do we proceed?

- Keep the Bs2KSKpi AmAn strategy (which might be the best LHCb reference we have).
  - **Keep the current fit as the nominal baseline for now.**
  - Determine the decay-time acceptance, construct a more complete PDF, toys, etc.

Backup

Selection

Selection

SELECTION - TRIGGER LINES

→ L0

2011-2012	2015-2016	2017-2018
B_L0HadronDecision_TOS B_L0DiMuonDecision_TIS B_L0MuonDecision_TIS B_L0ElectronDecision_TIS B_L0PhotonDecision_TIS B_L0HadronDecision_TIS	B_L0HadronDecision_TOS B_L0DiMuonDecision_TIS B_L0MuonDecision_TIS B_L0ElectronDecision_TIS B_L0PhotonDecision_TIS B_L0HadronDecision_TIS B_L0MuonEWDecision_TIS B_L0JetE1Decision_TIS B_L0JetPhDecision_TIS	B_L0HadronDecision_TOS B_L0DiMuonDecision_TIS B_L0MuonDecision_TIS B_L0ElectronDecision_TIS B_L0PhotonDecision_TIS B_L0HadronDecision_TIS B_L0MuonEWDecision_TIS

→ HLT1

Run1	Run2
B_Hlt1TrackAllL0Decision_TOS	B_Hlt1TrackMVADecision_TOS B_Hlt1TwoTrackMVADecision_TOS

→ HLT2

2011	2012	Run2
B_Hlt2Topo2BodyBBDTDecision_TOS B_Hlt2Topo3BodyBBDTDecision_TOS B_Hlt2Topo4BodyBBDTDecision_TOS B_Hlt2Topo2BodySimpleDecision_TOS B_Hlt2Topo3BodySimpleDecision_TOS B_Hlt2Topo3BodySimpleDecision_TOS	B_Hlt2Topo2BodyBBDTDecision_TOS B_Hlt2Topo3BodyBBDTDecision_TOS B_Hlt2Topo4BodyBBDTDecision_TOS	B_Hlt2Topo2BodyDecision_TOS B_Hlt2Topo3BodyDecision_TOS B_Hlt2Topo4BodyDecision_TOS

Hadron - TOS
Physical - TIS

Standard $B^0_{(d,s)} \to K^0_S h^+ h^{-\prime}$ selection criteria

Selection

SELECTION – PRESELECTION AND VETOES

→ Preselection : applied as preparation to the actual MVA (*Multi Variate Analysis*)

Preselection cut	Description
$\chi^2_{\mathrm{IsoVtx}}(B) > 4$	B vertex isolation variable
$Z_{vtx}(K^0_S) - Z_{vtx}(B) > 30$ mm	K^0_S vertex separation w.r.t. the B vertex
$h(1,2)\_isMuon == 0$	Reject $h^\pm$ candidates compatible with the muon hypothesis
$3000 \leq p(h^\pm) \leq 100000\,\mathrm{MeV}$	Fiducial cut (for RICHs)
$\min\chi^2_{\mathrm{IP}}(h^\pm) > 4$	Minimum IP χ^2 of the charged daughters w.r.t. the related PV
$p_T(h^\pm) > 250\,\mathrm{MeV}$	Minimum transverse momentum of the charged daughters

→ Vetoes :

$$D^\pm \to \pi^\pm K^0_S,\ D^\pm \to K^\pm K^0_S$$

$$D^\pm_s \to \pi^\pm K^0_S,\ D^\pm_s \to K^\pm K^0_S$$

$$D^0 \to \pi^+\pi^-,\ D^0 \to K^+K^-,\ D^0 \to K^\pm\pi^\mp$$

$$\Lambda^+_c \to pK^0_S$$

$$J/\psi \to \pi^+\pi^-,\ J/\psi \to K^+K^-$$

$$\chi_{c,0} \to \pi^+\pi^-,\ \chi_{c,0} \to K^+K^-$$

Selection

MVA ON TOPOLOGICAL VARIABLES

→ Used to reduce the amount of combinatorial background

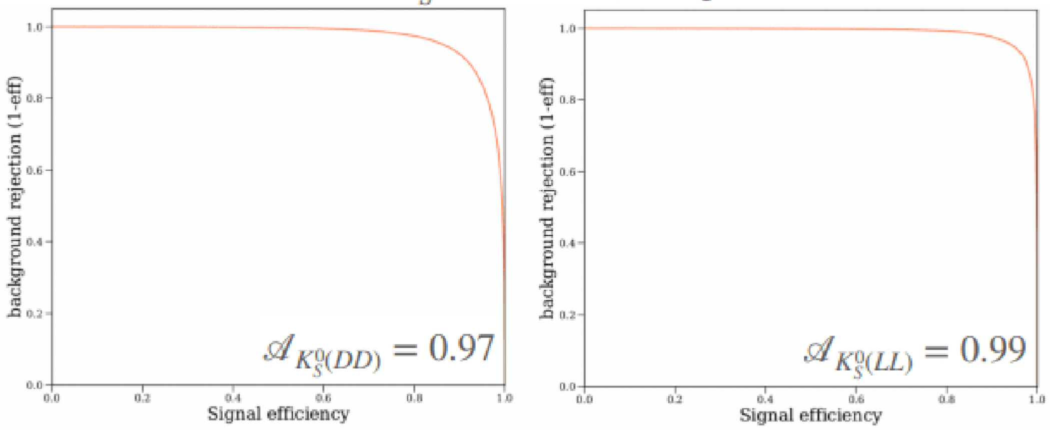
→ Discriminating variables that are uncorrelated to the B-meson mass and to the Dalitz-plot kinematics

→ Trained on $B^0 \to K^0_S \pi^+ \pi^-$ samples - Signal from MC and Background in upper sideband of the data samples

Good background-signal separation

ROC curves for the topological MVA

$B^0 \to K^0_S \pi^+ \pi^-$ (2018) sample

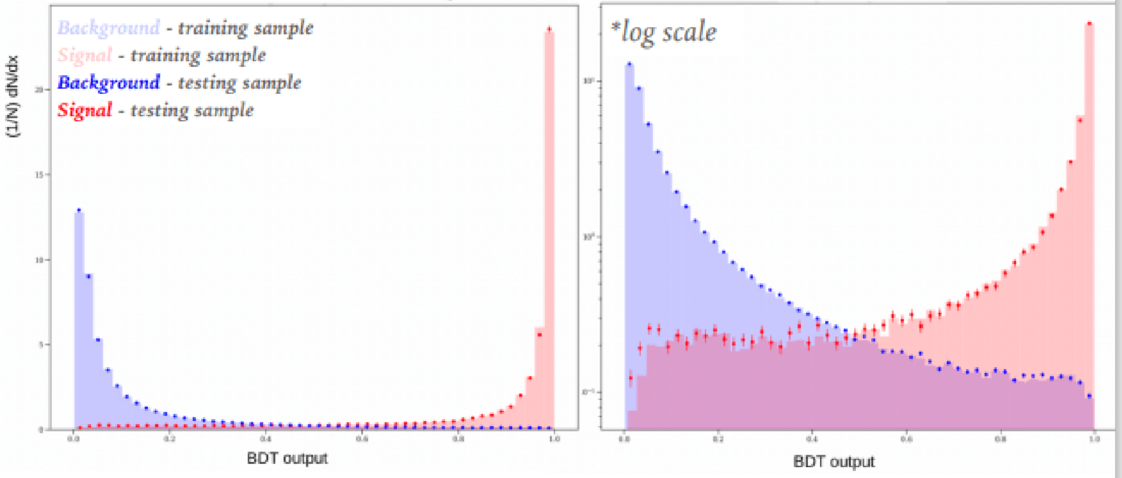


*Interpretation: indication of the classifying power
ideal machine learning tool → area under a ROC curve close to 1
poorly performing tool → area under a ROC curve ~0.5

No over-training

Distribution of the topological MVA output variable

$B^0 \to K^0_S(\mathrm{DD}) \pi^+ \pi^-$ (2018) sample



*Interpretation: similar performances on the training sample and the testing sample → no over-fitting

Selection

MVA ON PID VARIABLES

→ Used to reduce the amount of Crossfeeds (events from the miss-ID of h or h')

→ PID variables used : ProbNN for kaons, pions and protons

→ Trained on MC samples for Signal and Crossfeeds

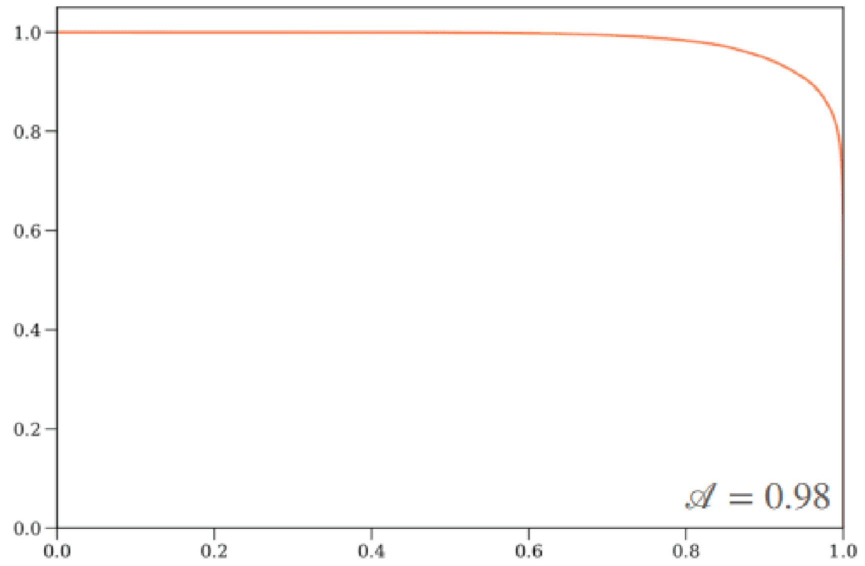
*Crossfeed example:

Events from $B^0 \to K_S^0\pi\pi$ found in the $K_S^0K\pi$ spectrum when one of the π is miss-ID as a K

Good background-signal separation

ROC curves for the PID MVA

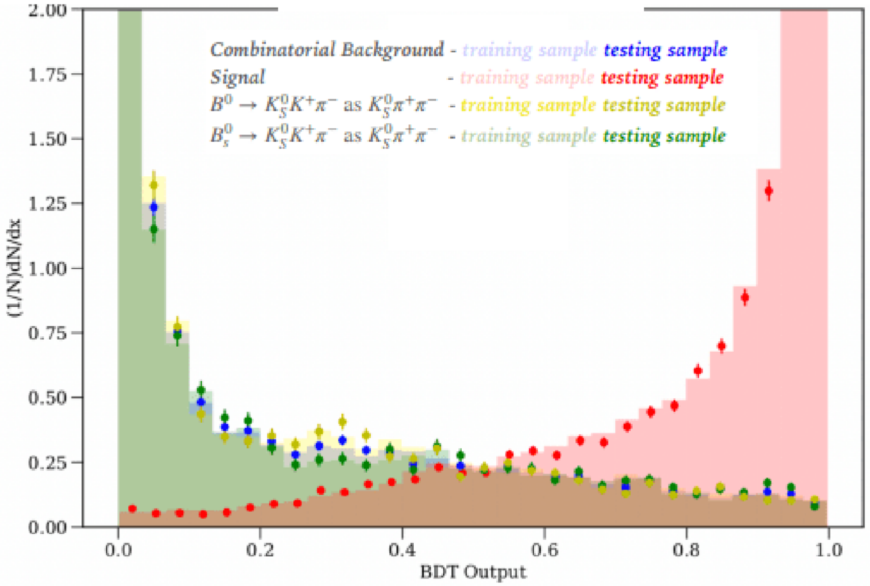
2018 sample



No over-training

Distribution of the PID MVA output variable

$B^0 \to K_S^0(\mathrm{DD})\pi^+\pi^-$ (2018) sample



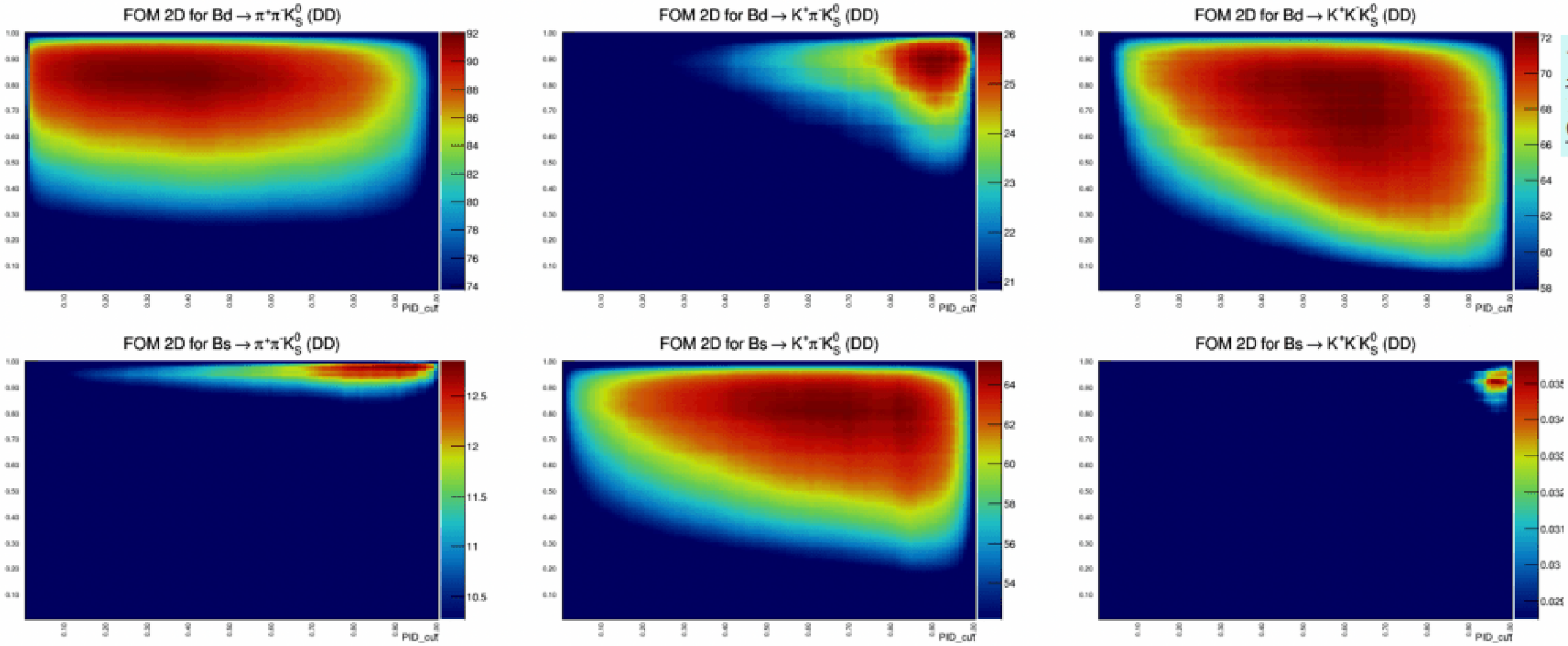
Selection

2D OPTIMISATION

→ Two well-performing correlated BDTs ⇒ 2D optimisation

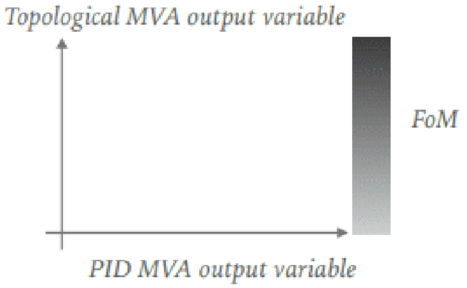
→ Optimisation by finding the maximum of a figure of merit, *FoM* (Observed mode: standard, Unobserved mode: Punzi)

→ Separate optimisation for B^0 and B_s^0 with each channel and each K_S^0 reconstruction



Evaluation of the FoM on the 2D-MVA plane

2018 sample



Backup

Inputs for the $B_S^0 \to K_S\pi^+\pi^-$ DP fit

Inputs for the $B_s^0 \to K_S^0 \pi^+ \pi^-$ DP fit

Fraction of signal and bkg. in the B_s^0 mass window

Strategy

- Signal and part. reco. shapes fixed from fits to MC.
- Data: Simultaneous fit among different categories
  - The slope of the right-hand sideband is shared among all sub-samples and is floated.
  - The $\mu_{B_d^0}$ and $\sigma_{B_d^0}$ are shared among sub-samples of the same (KS reco, Run) category
  - Gaussian constraint to the mass-difference between B_d^0 and B_s^0.

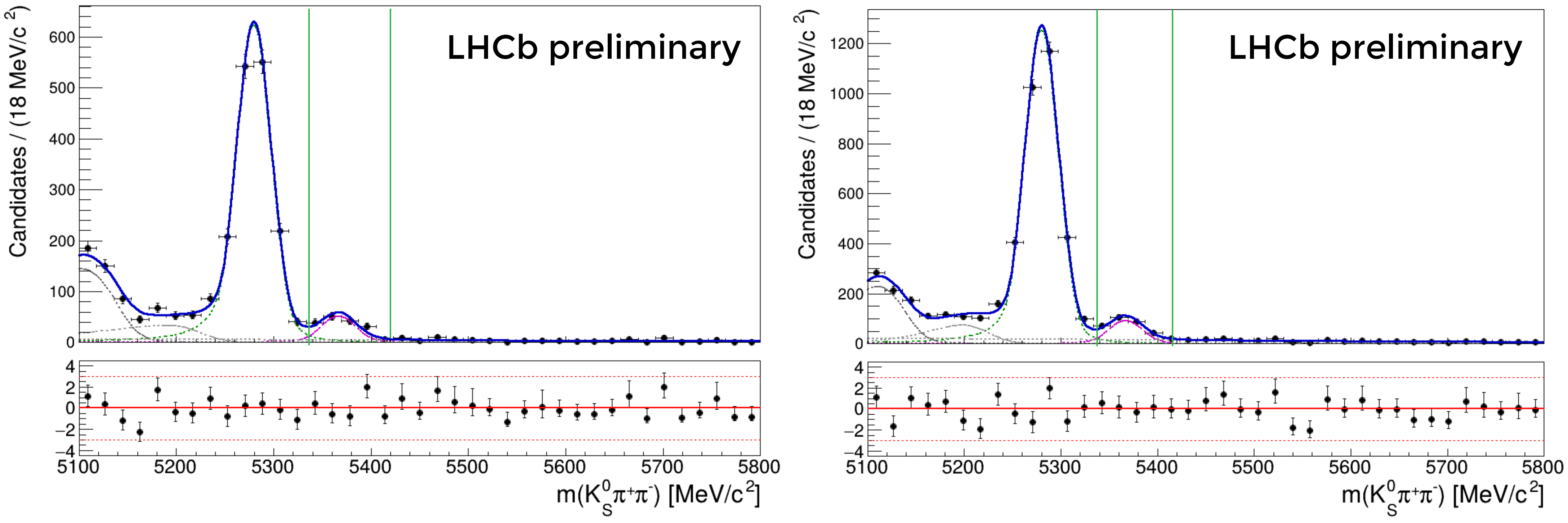
Parameter	2011	2012a	2012b	2015	2016	2017	2018
K_S^0 (LL)							
$\mu(B^0)$	$5283.82{\pm0.63}$	$5283.82{\pm0.63}$	$5283.82{\pm0.63}$	$5279.68{\pm0.29}$	$5279.68{\pm0.29}$	$5279.68{\pm0.29}$	$5279.68{\pm0.29}$
$\mu(B_s^0)$	$5371.03{\pm0.65}$	$5371.03{\pm0.65}$	$5371.03{\pm0.65}$	$5366.88{\pm0.32}$	$5366.88{\pm0.32}$	$5366.88{\pm0.32}$	$5366.88{\pm0.32}$
$\sigma(B_{(s)}^0)$	$17.66{\pm0.54}$	$17.66{\pm0.54}$	$17.66{\pm0.54}$	$17.85{\pm0.25}$	$17.85{\pm0.25}$	$17.85{\pm0.25}$	$17.85{\pm0.25}$
$a_{\rm comb}$	$-0.544{\pm0.060}$	$-0.544{\pm0.060}$	$-0.544{\pm0.060}$	$-0.544{\pm0.060}$	$-0.544{\pm0.060}$	$-0.544{\pm0.060}$	$-0.544{\pm0.060}$
K_S^0 (DD)							
$\mu(B^0)$	$5284.22{\pm0.55}$	$5284.22{\pm0.55}$	$5284.22{\pm0.55}$	$5280.16{\pm0.23}$	$5280.16{\pm0.23}$	$5280.16{\pm0.23}$	$5280.16{\pm0.23}$
$\mu(B_s^0)$	$5371.43{\pm0.56}$	$5371.43{\pm0.56}$	$5371.43{\pm0.56}$	$5367.36{\pm0.26}$	$5367.36{\pm0.26}$	$5367.36{\pm0.26}$	$5367.36{\pm0.26}$
$\sigma(B_{(s)}^0)$	$18.42{\pm0.46}$	$18.42{\pm0.46}$	$18.42{\pm0.46}$	$17.55{\pm0.19}$	$17.55{\pm0.19}$	$17.55{\pm0.19}$	$17.55{\pm0.19}$
$a_{\rm comb}$	$-0.544{\pm0.060}$	$-0.544{\pm0.060}$	$-0.544{\pm0.060}$	$-0.544{\pm0.060}$	$-0.544{\pm0.060}$	$-0.544{\pm0.060}$	$-0.544{\pm0.060}$

Inputs for the $B_s^0 \to K_S^0\pi^+\pi^-$ DP fit

Fraction of signal and bkg. in the B_s^0 mass window

Invariant-mass fit plots

Selection of simultaneous $K_S^0\pi^+\pi^-$ invariant-mass fit results. This is for 2018 data, both K_S^0 reconstruction categories: LL (left) and DD (right).



The vertical green lines indicate the selected signal window around the B_s^0 peak.

Inputs for the $B_s^0 \to K_S^0\pi^+\pi^-$ DP fit

Fraction of signal and bkg. in the B_s^0 mass window

Fitted yields in data

Fit component	2011	2012a	2012b	2015	2016	2017	2018	Sum
K_S^0 (LL)								
$B_d^0 \to K_S^0\pi^+\pi^-$	293 ± 18	87 ± 10	467 ± 23	204 ± 15	1487 ± 41	1689 ± 43	1676 ± 43	5904 ± 81
$B_s^0 \to K_S^0\pi^+\pi^-$	16 ± 5	10 ± 3	44 ± 8	17 ± 5	116 ± 13	93 ± 12	139 ± 15	435 ± 26
Combinatorial bkg.	32 ± 9	3 ± 3	67 ± 13	12 ± 6	150 ± 20	130 ± 19	173 ± 22	567 ± 40
B^0 PR bkg.	45 ± 8	15 ± 4	104 ± 15	34 ± 6	317 ± 22	331 ± 21	343 ± 24	1189 ± 43
B_s^0 PR bkg.	29 ± 9	13 ± 4	37 ± 15	33 ± 6	185 ± 23	134 ± 20	209 ± 26	639 ± 44
K_S^0 (DD)								
$B_d^0 \to K_S^0\pi^+\pi^-$	353 ± 20	331 ± 19	567 ± 25	307 ± 19	2150 ± 49	2855 ± 57	3272 ± 61	9836 ± 105
$B_s^0 \to K_S^0\pi^+\pi^-$	36 ± 7	25 ± 7	55 ± 8	21 ± 6	169 ± 17	204 ± 18	243 ± 20	753 ± 35
Combinatorial bkg.	29 ± 9	62 ± 12	31 ± 9	29 ± 9	394 ± 31	444 ± 34	548 ± 37	1536 ± 62
B^0 PR bkg.	74 ± 9	55 ± 9	101 ± 14	53 ± 8	432 ± 26	481 ± 30	553 ± 29	1750 ± 53
B_s^0 PR bkg.	18 ± 7	13 ± 8	42 ± 15	46 ± 8	244 ± 27	381 ± 34	352 ± 31	1097 ± 57

- Total: ~1.2K signal candidates
- Run 1:
  - Total: ~186
  - DD: ~116
  - LL: ~70
- Run 2:
  - Total: ~1K
  - DD: ~637
  - LL: ~365

Fractions in the B_s^0 signal window $[\mu_{B_s^0} - 1.5\sigma_{B_s^0}, \mu_{B_s^0} + 3.0\sigma_{B_s^0}]$

Component	2011	2012a	2012b	2015	2016	2017	2018
K_S^0 (LL)							
B_s^0	$(71.74\pm7.71)\%$	$(93.27\pm4.06)\%$	$(76.86\pm3.57)\%$	$(80.58\pm6.20)\%$	$(75.80\pm2.65)\%$	$(71.59\pm3.64)\%$	$(77.01\pm2.36)\%$
B_d^0	$(8.34\pm1.69)\%$	$(3.22\pm1.27)\%$	$(5.98\pm1.10)\%$	$(10.54\pm2.50)\%$	$(9.68\pm0.97)\%$	$(13.63\pm1.56)\%$	$(8.83\pm0.66)\%$
Combinatorial	$(19.92\pm6.97)\%$	$(3.51\pm4.36)\%$	$(17.16\pm3.52)\%$	$(8.87\pm3.84)\%$	$(14.52\pm2.17)\%$	$(14.79\pm2.58)\%$	$(14.16\pm2.02)\%$
K_S^0 (DD)							
B_s^0	$(86.02\pm3.54)\%$	$(67.59\pm6.91)\%$	$(85.49\pm2.59)\%$	$(76.50\pm6.86)\%$	$(69.84\pm2.61)\%$	$(69.83\pm2.39)\%$	$(70.06\pm2.34)\%$
B_d^0	$(3.70\pm0.73)\%$	$(7.55\pm1.76)\%$	$(7.38\pm1.29)\%$	$(8.87\pm2.21)\%$	$(6.82\pm0.51)\%$	$(8.22\pm0.58)\%$	$(7.25\pm0.46)\%$
Combinatorial	$(10.27\pm3.22)\%$	$(24.87\pm5.90)\%$	$(7.13\pm2.17)\%$	$(14.63\pm5.41)\%$	$(23.34\pm2.43)\%$	$(21.95\pm2.24)\%$	$(22.69\pm1.94)\%$

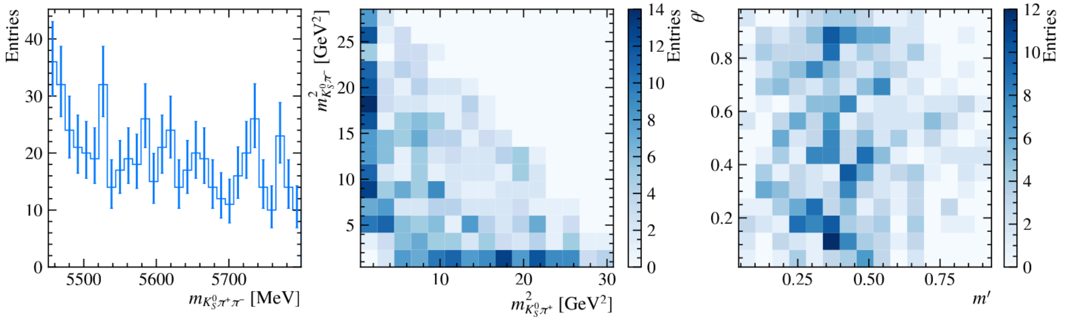
The approximate number of $B_s^0 \to K_S^0\pi^+\pi^-$ signal candidates in the signal window and hence entering the DP fit is 1.02K

Inputs for the $B_s^0 \to K_S^0\pi^+\pi^-$ DP fit

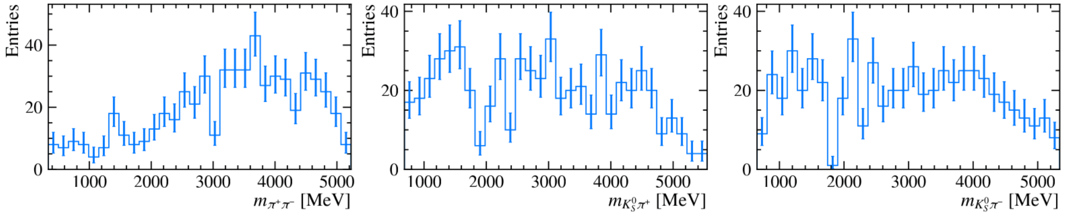
Combinatorial background model

- The combinatorial background in the signal region is estimated with candidates from the upper mass side-band $5450\ \mathrm{MeV} \leq m_{K_S^0\pi^+\pi^-} \leq 5800\ \mathrm{MeV}$
  - Veto on $B^+ \to K_S^0\pi^+$ applied: $|m_{K_S^0\pi^\pm} - m_{B^+}| \geq 20\ \mathrm{MeV}$
- A combined map of all data-taking periods is obtained for each K_S^0 category
- The SqDP maps are smoothed and are inputs for the DP fit.

Combined dataset (all years-DD)

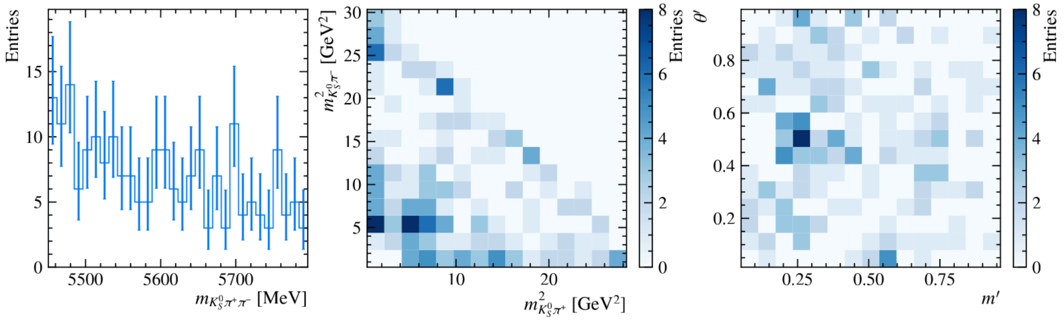


(a) $K_S^0\pi^+\pi^-$ invariant-mass distribution (left), the Dalitz plot (middle), and the square Dalitz plot (right)



(b) Two-body invariant-mass distributions: $m_{\pi^+\pi^-}$ (left), $m_{K_S^0\pi^+}$ (middle), and $m_{K_S^0\pi^-}$ (right).

Combined dataset (all years-LL)



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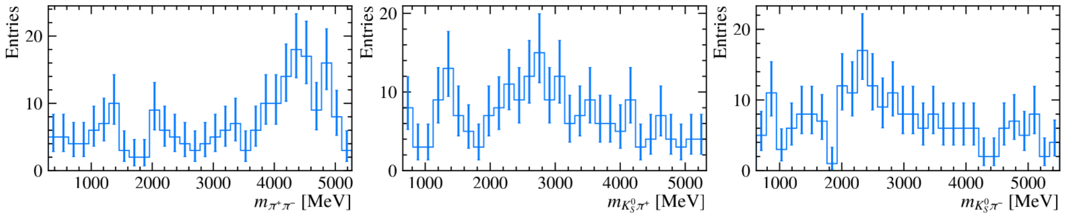

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Inputs for the $B_s^0 \to K_S^0\pi^+\pi^-$ DP fit

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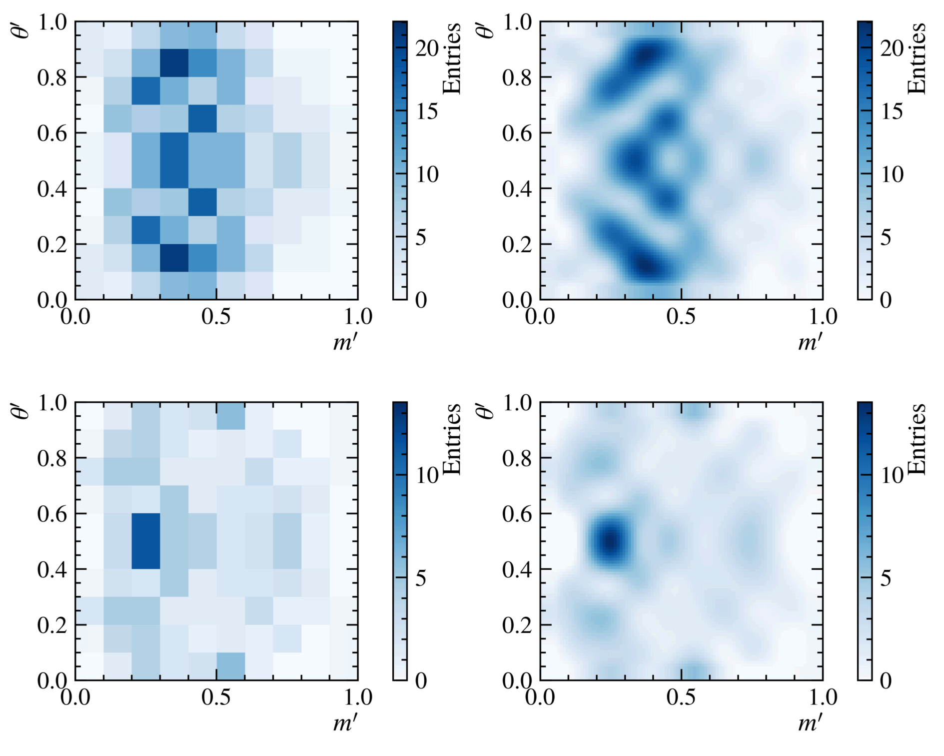
- A combined map of all data-taking periods is obtained for each K_S^0 category
- The SqDP maps are symmetrized and smoothed, and are inputs for the DP fit.



⇒DD

⇒LL

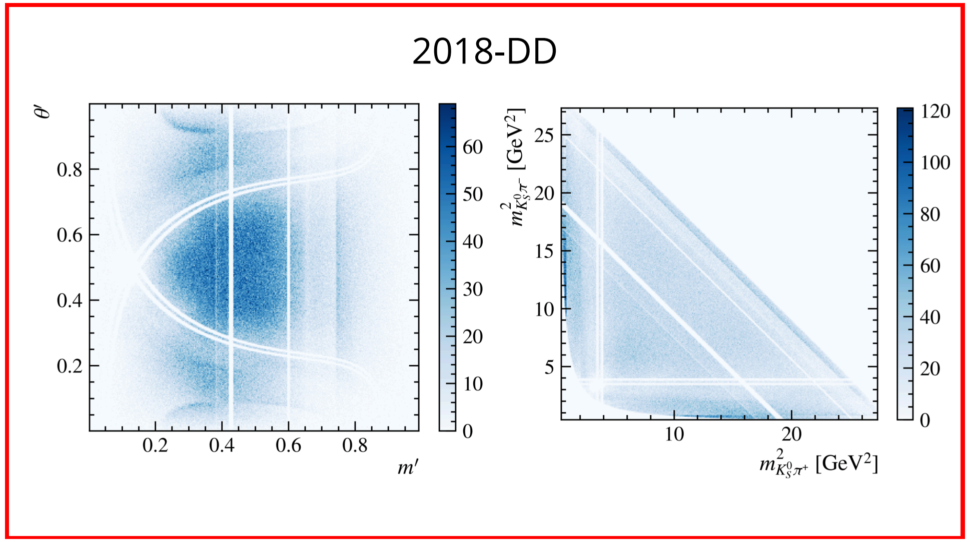
Inputs for the $B_s^0 \to K_S^0\pi^+\pi^-$ DP fit

$B_d^0 \to K_S^0\pi^+\pi^-$ background model

Estimated using the model for this decay from Run I:

- Paper: AmAn of the decay $B_d^0 \to K_S^0\pi^+\pi^-$ and first observation of the CP asymmetry in $B_d^0 \to K^*(892)\pi^+$.
- Large toy MC generated using CRAFT, weighted by the corresponding efficiency map.

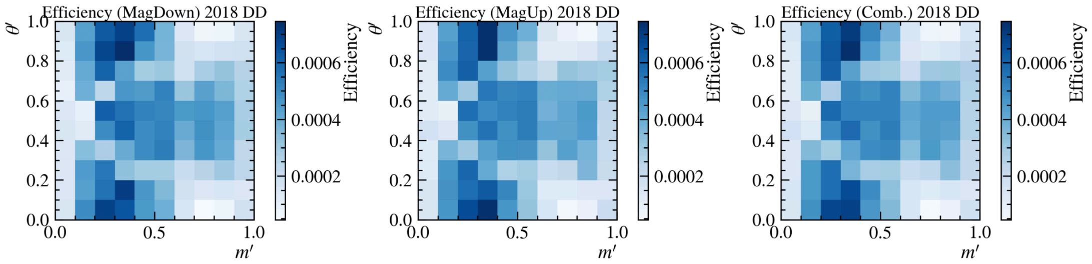
Resonance	Parameters	Lineshape
$K^{*\pm}(892)$	$m_0 = 891.66 \pm 0.26$ $\Gamma_0 = 50.8 \pm 0.9$	RBW
$(K\pi)_0^{*\pm}$	$\mathrm{Re}(c_0) = 0.204 \pm 0.103$ $\Im(c_0) = 0$ $\mathrm{Re}(c_1) = 1$ $\Im(c_1) = 0$	EFKLLM
$K_2^{*\pm}(1430)$	$m_0 = 1425.6 \pm 1.5$ $\Gamma_0 = 98.5 \pm 2.7$	RBW
$K^{*\pm}(1680)$	$m_0 = 1717 \pm 27$ $\Gamma_0 = 332 \pm 110$	Flatté
$f_0(500)$	$\mathrm{m}_0 = 513 \pm 32$ $\Gamma_0 = 335 \pm 67$ $\kappa = 0$	Red. K-Matrix = RBW
$\rho^0(770)$	$m_0 = 775.26 \pm 0.25$ $\Gamma_0 = 149.8 \pm 0.8$	GS
$f_0(980)$	$m_0 = 965 \pm 10$ $g_\pi = 165 \pm 18$ $g_K = 695 \pm 93$	Flatté
$f_0(1500)$	$m_0 = 1505 \pm 6$ $\Gamma_0 = 109 \pm 7$	RBW
χ_{c0}	$m_0 = 3414.75 \pm 0.31$ $\Gamma_0 = 10.5 \pm 0.6$	RBW
NR		DP flat



Inputs for the $B_s^0 \to K_S^0 \pi^+ \pi^-$ DP fit

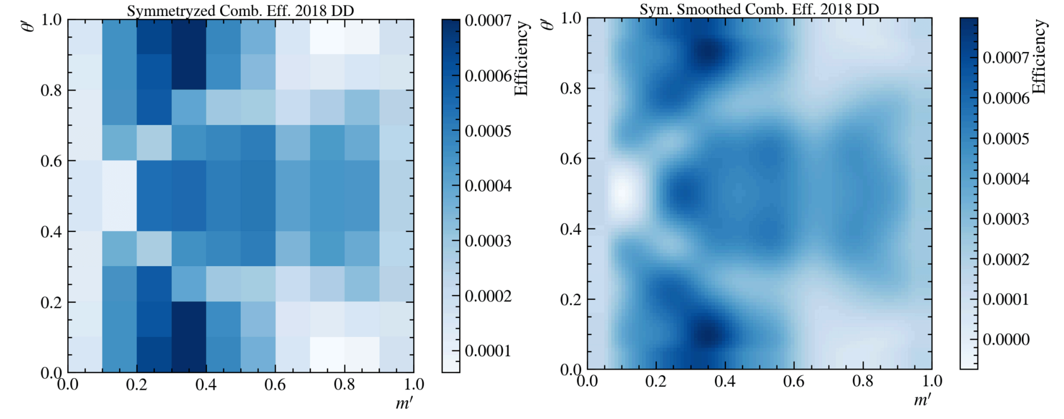
Efficiency across DP

- The reconstruction efficiency depends on the position of the candidate in the phase space.
- The efficiency is computed as function of the square Dalitz plot (SqDP) variables using MC events generated flat in SqDP and with corrections included (PID, Trigger, Tracking).
  - They are combined according to the MU-MD fractions available in the RunDB.
- The efficiency maps are smoothed out using the spline technique. This is the baseline for the efficiency modelling.



$$\epsilon_{\text{tot}}^{\text{corrected}} = \left(\epsilon_{\text{tot}}^{\text{TOS}} \times C_{\text{L0corr}}^{\text{TOS}} \times \frac{f_{\text{TOS}}^{\text{data}}}{f_{\text{TOS}}^{\text{MC}}} + \epsilon_{\text{tot}}^{\text{TIS\&!TOS}} \times C_{\text{L0corr}}^{\text{TIS\&!TOS}} \times \frac{f_{\text{TIS\&!TOS}}^{\text{data}}}{f_{\text{TIS\&!TOS}}^{\text{MC}}} \right) \times C_{\text{tracking}}$$

Computed in bins of the SqDP



Backup

Dissimilarity test

Point-to-point dissimilarity test review

Introduction

When doing an Amplitude Analyses, a crucial step is the unbinned maximum likelihood fit of a PDF to the data. This raises the question:

- Which statistical figure of merit can be used to assess the fit's performance?

Which can be re-stated as:

- What is the level of agreement between the fit PDF and the data?

Remarks:

- The GoF is one of the metrics used to determine contributions to the fit and define the baseline model.
- Why not simply use the χ^2 approach?
  - This requires binning $\Rightarrow$ sparse or overly coarse bins and loss of information, so unbinned goodness-of-fit methods are preferable.
- The point-to-point dissimilarity test (PPDT) shall be presented: http://arxiv.org/abs/1006.3019

Point-to-point dissimilarity test review

Foundations of the PPDT

Notation and terminology:

- $\vec{s} = (s_-, s_+) = (s_{K_S\pi^-}, s_{K_S\pi^+})$
- $f(\vec{s})$ and $f_0(\vec{s})$: parent/true PDF of the data and test PDF, respectively.
- T: test statistic (TS) $\Rightarrow$ Larger values correspond to a worse level of agreement.
- H_0: Null hypothesis $\Rightarrow$ "The two samples are drawn from the same PDF".
- p: p-value

Recipe:

- Ideally, if the parent PDF is known, one can define the following statistic:

$$T = \frac{1}{2}\int \left(f(\vec{s}) - f_0(\vec{s})\right)^2 \, d\vec{s}$$

- Then, one needs to get the distribution PDF of T, $g_{f_0}(T)$, for the case $f = f_0$
- This allows to finally compute the p-value:

$$p = \int_T^{\infty} g_{f_0}(T')\, dT'$$

Point-to-point dissimilarity test review

T-statistic

One can compute T by introducing a weighting function (WF) $\psi(|\vec{s}-\vec{s}'|)$ that correlates the differences between the PDFs at different points:

The average kernel value between pairs of points if both are drawn from distribution f

$$T = \frac{1}{2}\iint \left(f(\vec{s}) - f_0(\vec{s})\right)\left(f(\vec{s}') - f_0(\vec{s}')\right)\psi(|\vec{s}-\vec{s}'|)\, d\vec{s}\, d\vec{s}'.$$

$$T = \frac{1}{2}\iint \left[f(\vec{s})f(\vec{s}') + f_0(\vec{s})f_0(\vec{s}') - 2f(\vec{s})f_0(\vec{s}')\right]\psi(|\vec{s}-\vec{s}'|)\, d\vec{s}\, d\vec{s}'.$$

This can be approximated by:

$$\boxed{T = \frac{1}{n_d(n_d-1)}\sum_{i,j>i}^{n_d}\psi(|\vec{s}_i^{\,d}-\vec{s}_j^{\,d}|) + \frac{1}{n_{\rm mc}(n_{\rm mc}-1)}\sum_{i,j>i}^{n_{\rm mc}}\psi(|\vec{s}_i^{\,\rm mc}-\vec{s}_j^{\,\rm mc}|) - \frac{1}{n_d n_{\rm mc}}\sum_{i,j}^{n_d, n_{\rm mc}}\psi(|\vec{s}_i^{\,d}-\vec{s}_j^{\,\rm mc}|).}$$

$$\psi_{\rm exp}(|\vec{s}_i-\vec{s}_j|) = e^{-|\vec{s}_i-\vec{s}_j|^2/2\sigma(\vec{s}_i)\sigma(\vec{s}_j)}$$
$$\psi_{\rm Log}(|\vec{s}_i-\vec{s}_j|) = \ln(|\vec{s}_i-\vec{s}_j|+\epsilon)$$

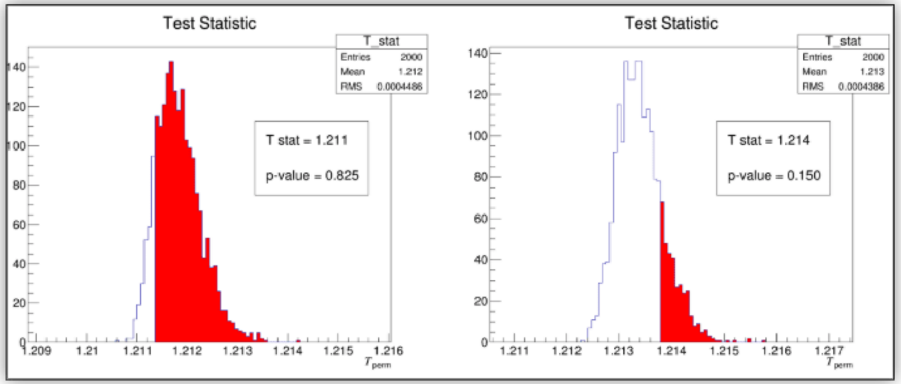
1. **Calibration under the null:** p–value to be uniformly distributed on [0,1] when comparing data vs. MC drawn from exactly the same PDF.
2. **Rejection power under alternatives:** give a high fraction of p-values below a threshold when comparing data vs. a mis-specified PDF.

Point-to-point dissimilarity test review

Permutation test

The distribution of T for the case $f = f_0$ is not known. How do we estimate a p-value?

- Combine all data points from both samples into a single pool $n_d + n_{mc}$
- Randomly reassign points into two new "data" and "MC" samples with sizes n_d and n_{mc}
- Compute the test statistic T_{perm} for each permuted pair.
- Repeat this process n_{perm} times to obtain $\{T^1_{perm}, \ldots, T^{n_{perm}}_{perm}\} \Rightarrow \; g_{f_0}(T)$



- If p-value is small:
  - The observed dissimilarity is unlikely under H_0.
- If p-value is large:
  - The observed dissimilarity could happen by chance.

Backup

Preliminary systematic detailed

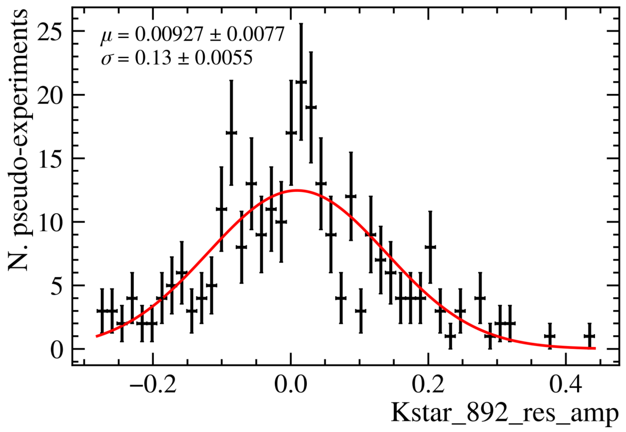
Preliminary systematics

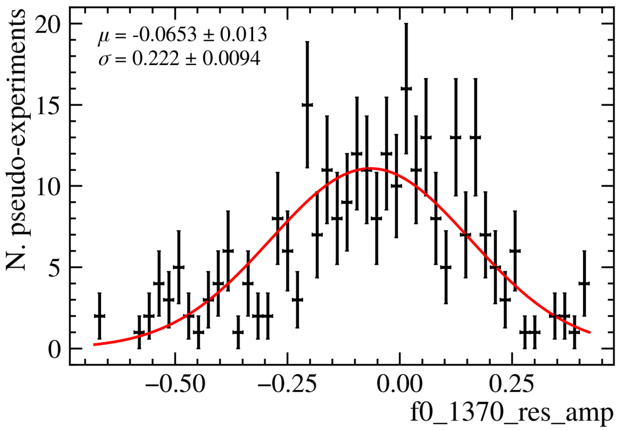
Experimental: DP fit biases

- Large set of pseudo-experiments were generated from the nominal model and fitted back.
- The difference between the values of the parameters from the pseudo-experiment fits and the ones from the nominal fit are used to evaluate the syst. uncertainty.
- The mean value of the residual distribution (μ_{residual}) is used to correct the mean value measured in the data fit and its uncertainty is assigned as the syst. uncertainty.

Resonance	Magnitude	
	Correction	Systematic
NR	0.008	0.019
$(K\pi)_0^{*+}(1430)$	0.009	0.014
$(K\pi)_0^{*-}(1430)$	0.042	0.012
$f_0(500)$	0.034	0.010
$f_0(1370)$	-0.065	0.013
$K^{*+}(892)$	0.009	0.008
$K^{*-}(892)$	–	–
$\rho^0(770)$	-0.034	0.009
$K^{*+}(1680)$	0.015	0.012
$K^{*-}(1680)$	0.042	0.013
$K_2^{*+}(1430)$	-0.009	0.011
$K_2^{*-}(1430)$	-0.041	0.014

$$a_{\text{corr}} = a_{\text{nominal}} - \mu_{\text{residual}}$$



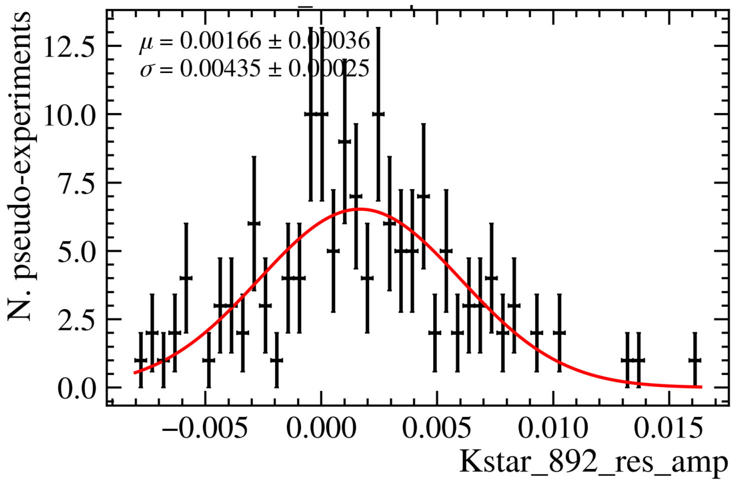


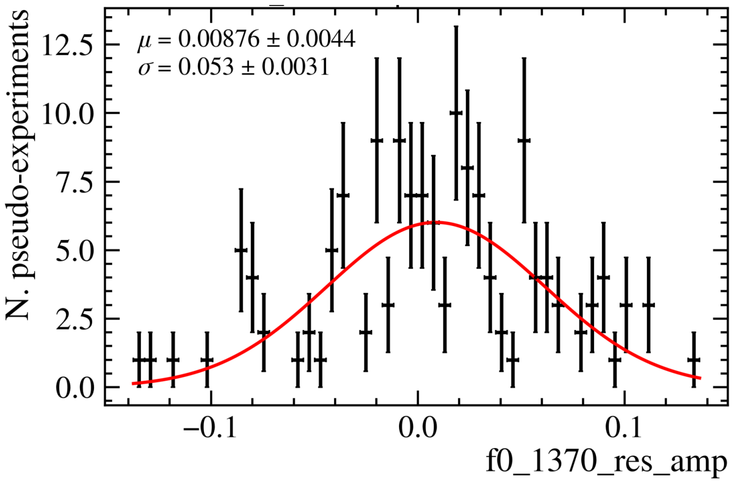
Preliminary systematics

Experimental: combinatorial background

- The histograms describing the combinatorial background distributions are fluctuated within their statistical uncertainties to generate a large set of them.
- Fits to data with these fluctuated maps are performed. The difference between the values of the parameters from these fits and the ones from the nominal are used to estimate the syst. uncertainty.
- The standard deviation (σ) of the distribution of residuals is used as syst. uncertainty.

Resonance	Magnitude
NR	0.055
$(K\pi)_0^{*+}(1430)$	0.023
$(K\pi)_0^{*-}(1430)$	0.024
$f_0(500)$	0.035
$f_0(1370)$	0.053
$K^{*+}(892)$	0.004
$K^{*-}(892)$	–
$\rho^0(770)$	0.009
$K^{*+}(1680)$	0.037
$K^{*-}(1680)$	0.038
$K_2^{*+}(1430)$	0.017
$K_2^{*-}(1430)$	0.024





Preliminary systematics

Experimental: binning choice for efficiency maps

- The fit on data is repeated with alternative efficiency-map binning (7,8,9,11,12,13).
- For the systematic uncertainty, we use the **full spread** of the observables across the considered binnings, rather than only the shift wrt. the baseline (10x10).
- The efficiency-binning systematic is computed as: $\boxed{\sigma_{\text{eff-binning}} = \dfrac{|x_{\max} - x_{\min}|}{\sqrt{12}}}$

Resonance	Magnitude
NR	0.018
$(K\pi)_0^{*+}(1430)$	0.022
$(K\pi)_0^{*-}(1430)$	0.022
$f_0(500)$	0.023
$f_0(1370)$	0.024
$K^{*+}(892)$	0.003
$K^{*-}(892)$	–
$\rho^0(770)$	0.018
$K^{*+}(1680)$	0.062
$K^{*-}(1680)$	0.068
$K_2^{*+}(1430)$	0.035
$K_2^{*-}(1430)$	0.035

Backup

Last BnoC talk main contents

Introduction

Physics motivation

The charmless decay $B_s^0 \to K_S^0\pi^+\pi^-$ will provide access to the first establishment of:

- The direct CP asymmetries of the quasi-two-body decays with an untagged time-integrated (TI) Dalitz analysis:
  - Flavour specific: $\bar{B}_s^0 \to K_0^{*+}(1430)\pi^-$, $\bar{B}_s^0 \to K^{*+}(892)\pi^-$
  - CP-eigenstate: $B_s^0 \to \rho^0 K_S^0$
- Fit Fractions of the different amplitudes contributing to the decay.
- These amplitudes can serve as measurements of ϕ_s or used in CKM γ angle determination w/ charmless decays.

Data samples

- This analysis inherits samples and selections from the B2KShh' Branching Fraction Analysis.
  - 14 samples: Run 1 (2011, 2012a, 2012b), Run 2 (2015-2018). Two K_S^0 reco categories: DD and LL.
  - The selection is optimized with a tight optimization targeting $B_s^0 \to K_s^0\pi^+\pi^-$ signal candidates.
    In contrast to the BF analysis, the topo MVA is optimized using as FoM: significance $\times$ purity.
  - Charm and charmonia vetoes applied:
    - Charmonia vetoes (48 MeV): $J/\psi \to \pi^+\pi^-$
    - Charm meson vetoes (30 MeV): $D_{(s)}^+ \to K_S^0\pi^+$, $D^0 \to \pi^+\pi^-$

Analysis strategy

Precedents

- This analysis follows a similar strategy to that of the DP analysis of the decay $B_d^0 \to K_S^0\pi^+\pi^-$ with Run I data.
  - The results were obtained with CRAFT (Clermont Roofit-based Amplitude Fitter Tool).
  - Analysis note: ANAnote Run I B02KSpipi
  - WG database: Analysis Run I B02KSpipi TI Dalitz Analysis
- Previous WG talk (12/02/2026): Inputs for the $B_s^0 \to K_S^0\pi^+\pi^-$ DP fit and baseline model

Stages

1. Selection of $B_s^0 \to K_S^0\pi^+\pi^-$ signal candidates
2. Fit the invariant-mass spectrum $K_S^0\pi^+\pi^-$, define a signal window around B_s^0 signal peak, and determine the fraction of signal and bkg. (both combinatorial and $B_d^0 \to K_S^0\pi^+\pi^-$)
3. Obtain the spline histogram of the efficiency variation across the DP, evaluated from signal MC.
4. Determine a model for the different background components .
5. Fit simultaneously the different samples and educate the final model by adding relevant new contributions to the baseline model and decide the nominal DP model of the analysis.

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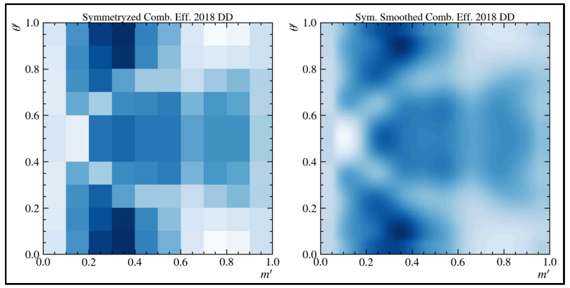
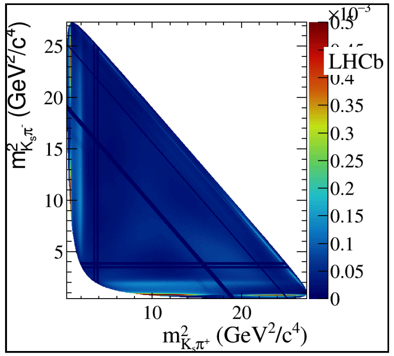
Today

Analysis Strategy

Schematic of the analysis workflow





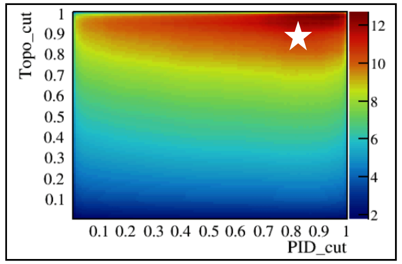
Selection of the signal candidates

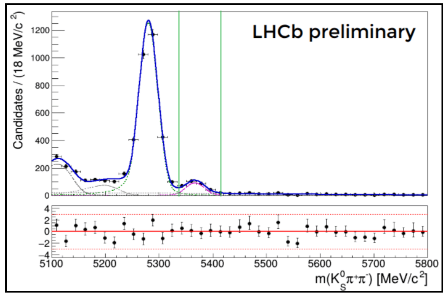
Signal and bkg. fractions from invariant-mass fit

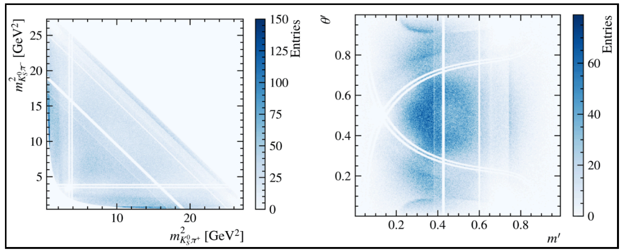
Efficiency evaluation over the sqDP from signal MC

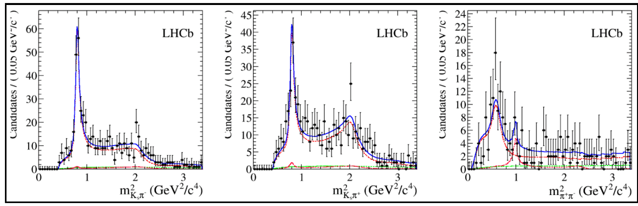
Background models: combinatorial & Bd2KSpipi

Simultaneous DP Fit: nominal model and observables determination









Towards nominal model

- A preliminary Dalitz plot fit on data is conducted using a **Baseline model**.
- The model parameterizes the signal amplitude as the **isobar** sum of four resonances (and conjugates) plus a non-resonant (NR) contribution.

$$\mathcal{S}(s_+, s_-) \propto |\mathcal{A}(s_+, s_-)|^2 + |\bar{\mathcal{A}}(s_+, s_-)|^2 \quad \Rightarrow \quad |\mathcal{A}(s_+, s_-)|^2 = \left|\sum_j a_j e^{i\phi_j} F_j(s_+, s_-)\right|^2$$

Untagged time-integrated Dalitz signal PDF.

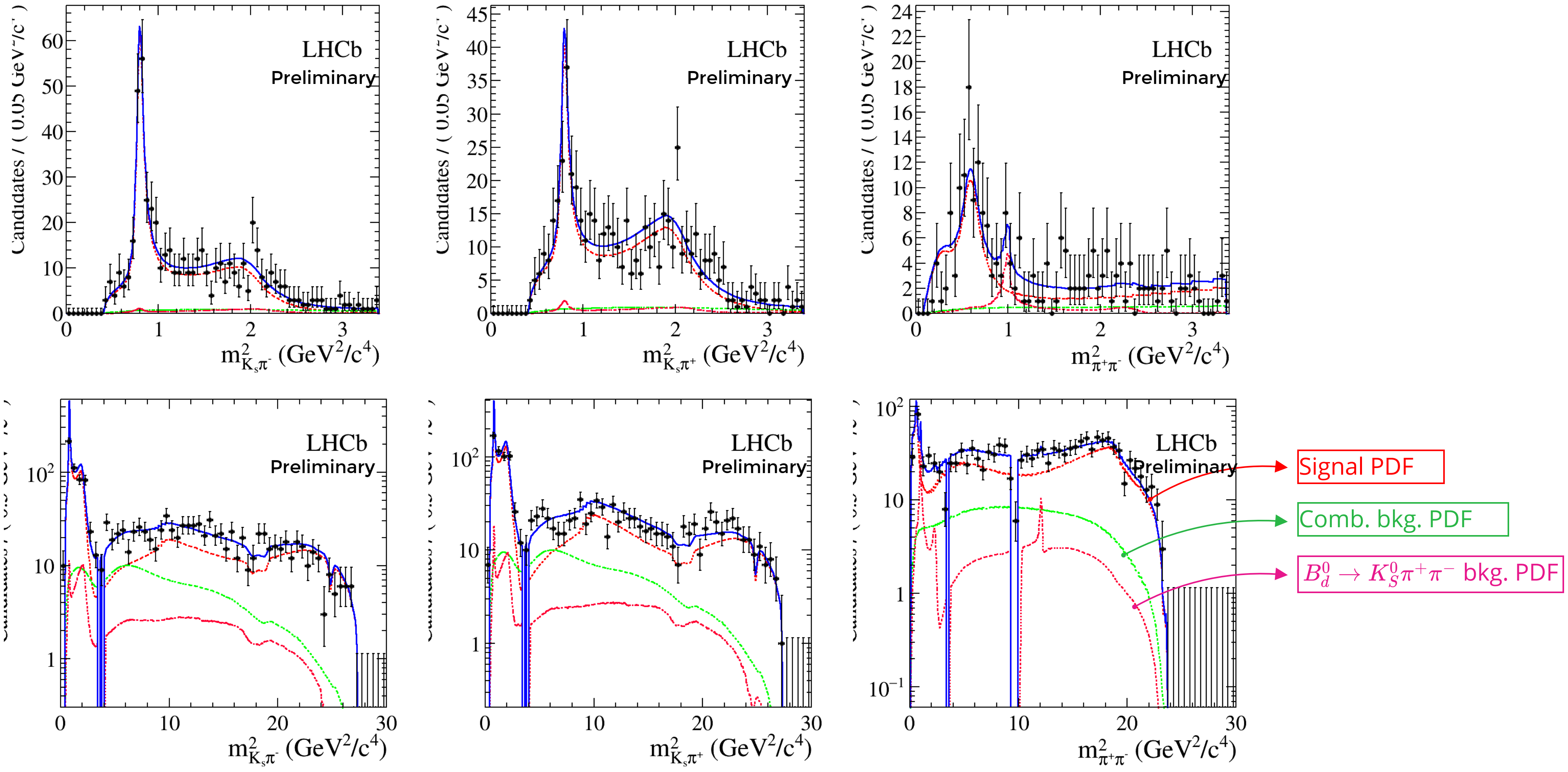
a_j and ϕ_j (x_j, y_j in cartesian) describe the relative magnitude and phase. **Floating parameters in the fit.**

Resonance	Parameters	Lineshape	CP asymmetry
$K^{*\pm}(892)$	$m_0 = 891.66 \pm 0.26$ $\Gamma_0 = 50.8 \pm 0.9$	RBW	allowed
$(K\pi)_0^{*\pm}$	$\mathrm{Re}(c_0) = 0.204 \pm 0.103$ $\Im(c_0) = 0$ $\mathrm{Re}(c_1) = 1$ $\Im(c_1) = 0$	EFKLLM	allowed
$f_0(500)$	$m_0 = 513 \pm 32$ $\Gamma_0 = 335 \pm 67$ $\kappa = 0$	Red. K-Matrix	not allowed
$\rho^0(770)$	$m_0 = 775.26 \pm 0.25$ $\Gamma_0 = 149.8 \pm 0.8$	GS	not allowed
NR		Flat	not allowed

- The $K^*(892)$ resonance is used as **reference** $\Rightarrow x = 2$ and $y = \bar{y} = 0$.
- This already gives a reasonable description of the data, where all the components are well identified.

Towards nominal model

Dalitz plot fit with Baseline model



- The $K^*(892)$ resonance is used as **reference** $\Rightarrow x = -2$ and $y = \bar{y} = 0$.
- This already gives a reasonable description of the data, where all the components are well identified.

Towards nominal model

Relevant additional contributions have been tested, one by one, and accepted according to the requirements (**at least one of the following criteria must be satisfied**):

- The **dissimilarity test** (backup) of the model including the additional contribution is increasing.
- The additional component is considered to be significant only when it causes a change greater than 11.83 (16.25) units of $\mathbf{-2\Delta nll}$ **(3σ deviation)**, in case of 2 (4) degrees-of-freedom.
- The magnitude of the additional contribution is measured far from zero by $\boldsymbol{3\sigma_a}$ (**magnitude precision better than 33.3%**), where σ_a is the statistical error on the amplitude.

Additional contribution	p-value	-2Δnll	Mag. precision (%)
baseline	0.24	–	–
(baseline) + $K_2^*(1430)$	0.29	40.56	(16.4, 18.4)
(baseline) + $K^*(1680)$	0.13	52.04	(16.6, 17.3)
(baseline) + $f_0(980)$	–	0.82	95.8
(baseline) + $f_2(1270)$	–	0.52	67.9
(baseline) + $f_0(1370)$	0.11	8.6	17.5
(baseline) + $f_0(1500)$	0.15	2.98	30.5
(baseline) + χ_{c0}	–	0.0	69.0
(baseline + $K_2^*(1430)$) + $K^*(1680)$	0.47	67.14	(16.6, 18.2)
(baseline + $K_2^*(1430)$) + $f_0(1370)$	0.17	12.16	17.5
(baseline + $K_2^*(1430)$) + $f_0(1500)$	0.14	7.82	30.53
Nominal model $\Leftarrow$ (baseline + $K_2^*(1430)$ + $K^*(1680)$) + $f_0(1370)$	0.33	4.72	15.3
(baseline + $K_2^*(1430)$ + $K^*(1680)$) + $f_0(1500)$	0.39	3.66	57.9

Towards nominal model

Nominal model

Resonance	Parameters	Lineshape
NR		Flat
S-wave		
$(K\pi)_0^{*\pm}$	$\mathrm{Re}(c_0) = 0.204 \pm 0.103$ $\Im(c_0) = 0$ $\mathrm{Re}(c_1) = 1$ $\Im(c_1) = 0$	EFKLLM
$f_0(500)$	$m_0 = 513 \pm 32$ $\Gamma_0 = 335 \pm 67$ $\kappa = 0$	Red. K-Matrix
$f_0(1370)$	$m_0 = 1380 \pm 70$ $\Gamma_0 = 220 \pm 80$	T-Matrix pole
P-wave		
$K^{*\pm}(892)$	$m_0 = 891.66 \pm 0.26$ $\Gamma_0 = 50.8 \pm 0.9$	RBW
$\rho^0(770)$	$m_0 = 775.26 \pm 0.25$ $\Gamma_0 = 149.8 \pm 0.8$	GS $\boxed{*}$
$K^{*\pm}(1680)$	$m_0 = 1717 \pm 27$ $\Gamma_0 = 332 \pm 110$	Flatté
D-wave		
$K_2^{*\pm}(1430)$	$m_0 = 1425.6 \pm 1.5$ $\Gamma_0 = 98.5 \pm 2.7$	RBW

*In this analysis the $\rho^0(770)K_S^0$ amplitude is CP-averaged.

- EFKLLM parameterization of $K_S^0\pi$ S-wave:

$$R(m) = F(m)\left(\frac{c_0}{m^2} + c_1\right)$$

the c_0 and c_1 coefficients are fixed.

- Reduced K-matrix for the $\pi\pi$ S-wave:

$$K(m) = K_{\mathrm{res}}(m) + K_{\mathrm{non-res}} = \frac{m_0\Gamma(m)}{(m_0^2 - m^2)\,\rho(m)} + \kappa$$

- T-matrix pole for $f_0(1370)$:

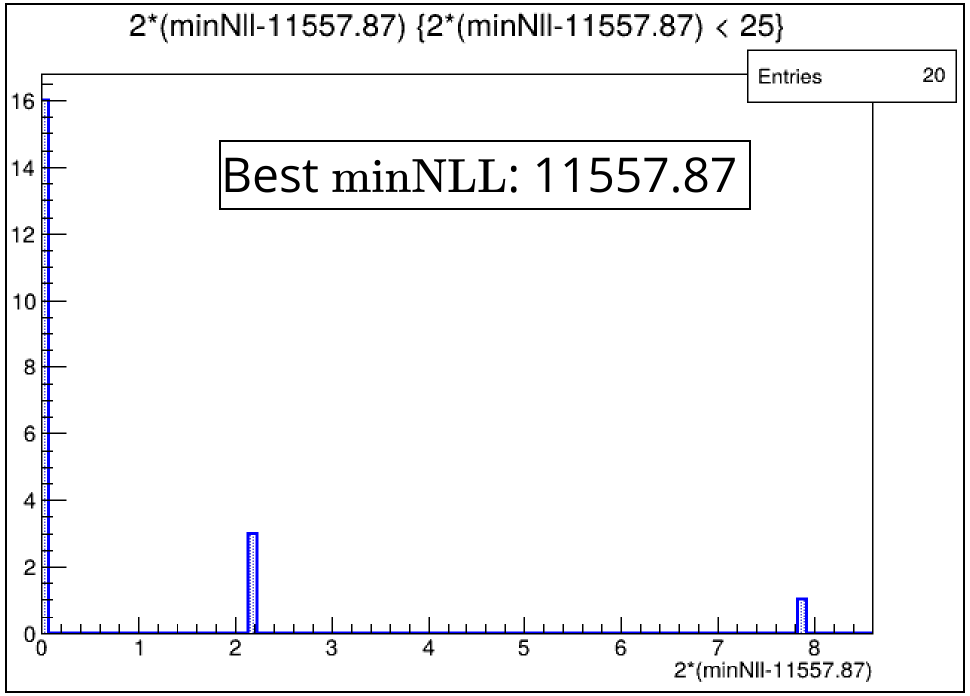
$$T(s) = \frac{1}{s_0 - s}$$

where $\sqrt{s_0} = m_0 - i\Gamma_0$ is the pole position.

Fit results with the nominal model

Finding the best solution

- Simultaneous fits to the data (all sub-samples) have been conducted using the nominal model but varying randomly the initial values of the isobar parameters for each fit.
- Six solutions are found, from which three are in the range of 25 units of $-2\Delta\mathrm{nll}$ from the best minNLL:

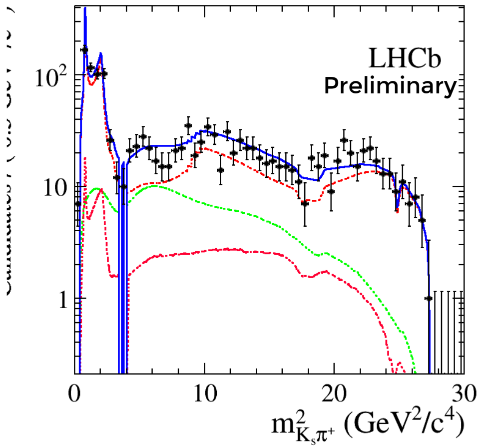


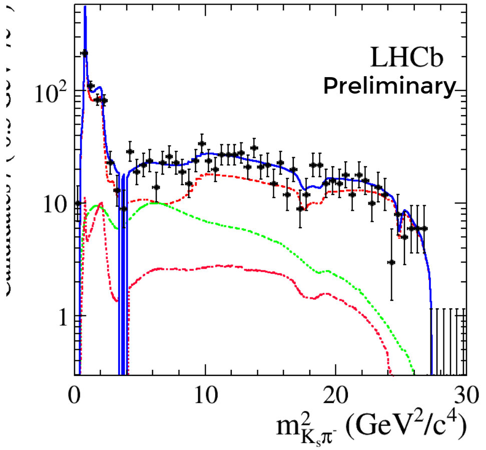
- The global minimum (best solution) clearly emerges.

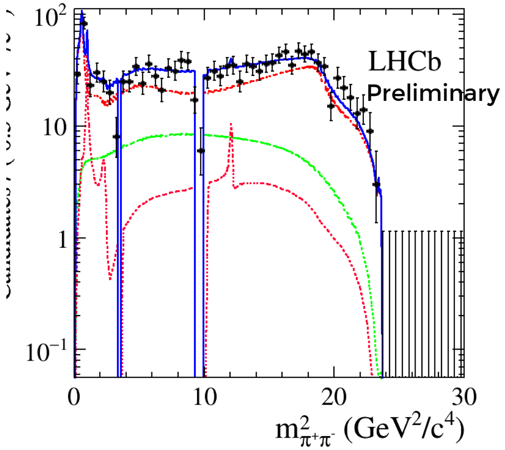
Fit results with the nominal model

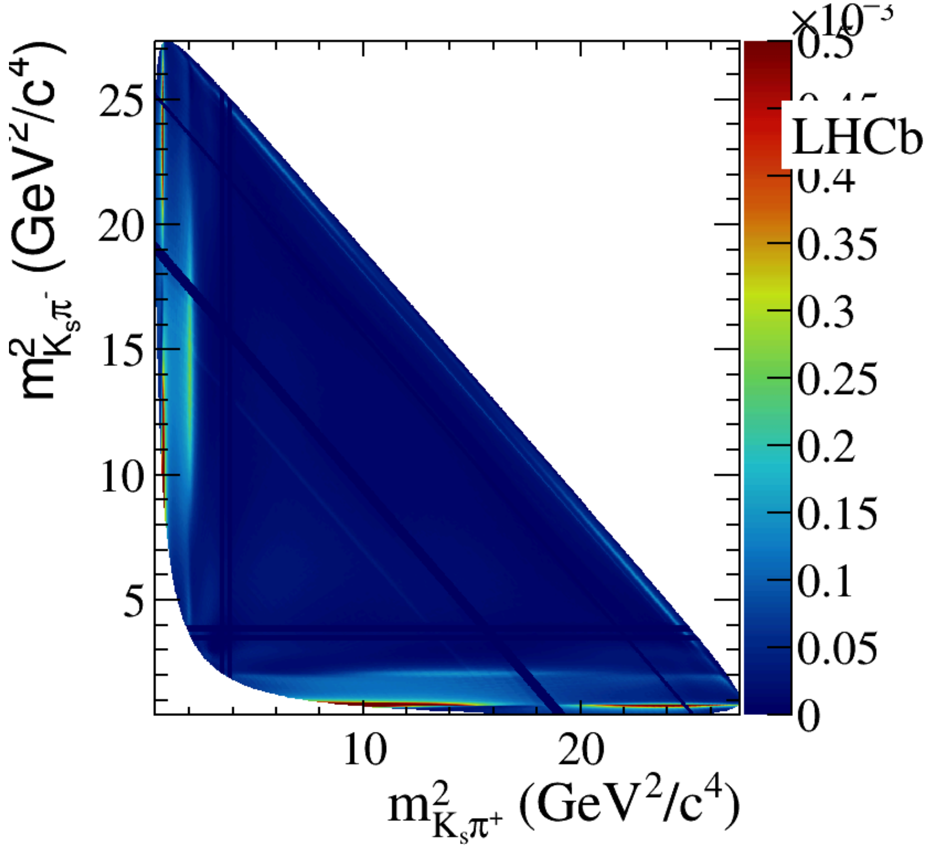
Plots of the projections in the whole spectrum

- All 14 sub-samples (7 data-taking periods x 2 K_S^0 reconstruction categories) are combined in the plots below.





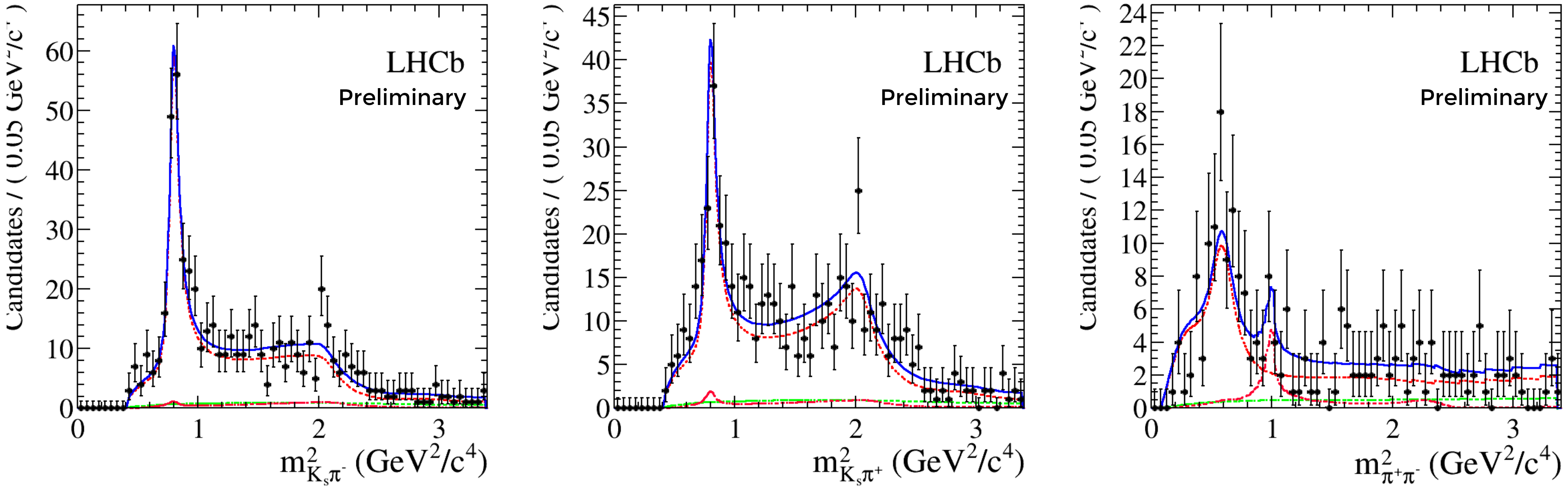




Fit results with the nominal model

Zoom plots of the projections

- All 14 sub-samples (7 data-taking periods x 2 K_S^0 reconstruction categories) are combined in the plots below.



- All projections of the DP fit are found to be satisfactorily described by the model ($p\text{-value} = 0.33$).

Towards nominal model

Fit results with the nominal model: Isobar parameters

- The magnitude and phase are built up from the cartesian parameters, which are the floating parameters in the fit.

Resonance	Real part (x)	Imaginary part (y)	Magnitude (a)	Phase (ϕ)
NR	-1.76 ± 0.24	0.28 ± 0.26	1.78 ± 0.25	171.1 ± 7.8
$(K\pi)_0^{*+}$	-2.81 ± 0.21	-1.10 ± 0.30	3.02 ± 0.21	-158.7 ± 5.8
$(K\pi)_0^{*-}$	-2.61 ± 0.19	-0.07 ± 0.32	2.61 ± 0.19	-178.4 ± 7.0
$f_0(500)$	0.14 ± 0.32	0.52 ± 0.13	0.54 ± 0.15	75.4 ± 32.7
$f_0(1370)$	-0.31 ± 0.31	-0.83 ± 0.20	0.89 ± 0.14	-110.2 ± 22.0
$K^{*+}(892)$	1.71 ± 0.13	0.00	1.71 ± 0.13	0.0
$K^{*-}(892)$	-2.00	0.00	2.00	180.0
$\rho^0(770)$	-0.47 ± 0.58	1.13 ± 0.31	1.22 ± 0.13	112.7 ± 30.1
$K^{*+}(1680)$	0.17 ± 0.34	1.07 ± 0.20	1.08 ± 0.19	81.1 ± 18.1
$K^{*-}(1680)$	0.15 ± 0.41	-1.07 ± 0.20	1.08 ± 0.21	-82.2 ± 21.4
$K_2^{*+}(1430)$	1.16 ± 0.18	0.14 ± 0.22	1.17 ± 0.17	6.8 ± 11.1
$K_2^{*-}(1430)$	0.56 ± 0.17	-0.72 ± 0.22	0.91 ± 0.20	-52.1 ± 12.0

Disclaimer: The phase uncertainties will be determined from pseudo-experiments.

Systematic uncertainties study

Categories distinction

The sources of systematic uncertainty will be divided in two categories:

Experimental systematic uncertainties

- **biases** related to the Dalitz fit to the data,
- the **signal fraction** extracted from the mass fit,
- uncertainties on the **selection, trigger and tracking efficiency** across the sqDP,
- the **binning choice** for the signal efficiency histograms,
- uncertainty on the **combinatorial background** across the sqDP,
- uncertainty on the $\boldsymbol{B^0 \to K^0_S \pi^+ \pi^-}$ **component** across the sqDP.
- symmetrization of: efficiency maps and combinatorial maps?

Model systematic uncertainties

- **fixed parameters** of the resonance line-shapes models,
- addition or removal of **marginal components** to the nominal model,
- **alternative models** for the $K^0_S\pi$ and $\pi\pi$ S-waves,

Preliminary systematic uncertainty determination

WORK IN PROGRESS

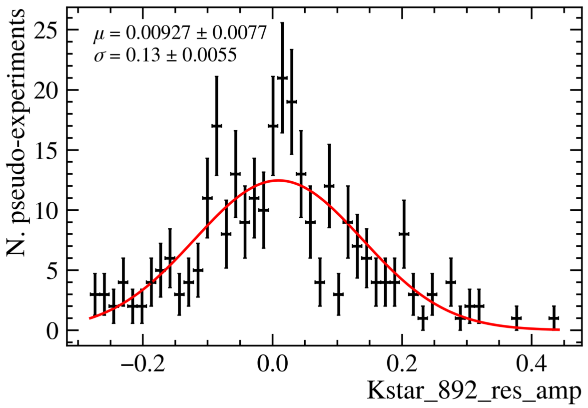
Experimental: DP fit biases, eff. binning choice, comb. bkg. model

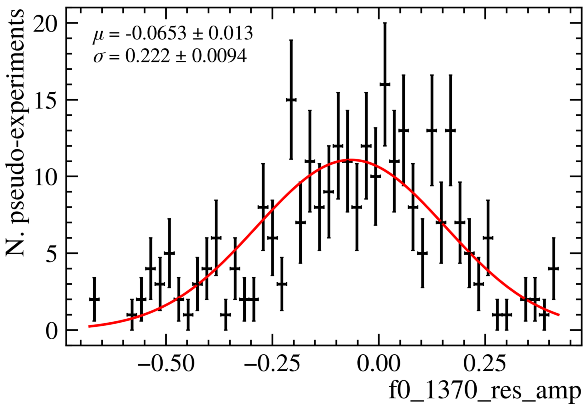
- **DP fit biases:** Computed using toys fitted with the nominal model. The mean value of the residual distribution μ_{residual} is quoted as bias correction and its uncertainty is assigned as syst.
- **Comb. background model:** fits to data are performed with comb. bkg. histograms fluctuated within their statistical uncertainties. The σ of the distribution of residuals is used as syst.
- **Eff.-binning choice:** fits to data are repeated with alternative efficiency-map binning $(7 \times 7, \ldots, 13 \times 13)$. The associated syst. is computed as: $\sigma_{\text{eff-binning}} = \frac{|x_{\max} - x_{\min}|}{\sqrt{12}}$

Systematic uncertainties on the fitted amplitude magnitudes

Resonance	Correction	Fit-bias syst.	Comb. syst.	Eff.-binning syst.
NR	0.008	0.019	0.055	0.018
$(K\pi)_0^{*+}$	0.009	0.014	0.023	0.022
$(K\pi)_0^{*-}$	0.042	0.012	0.024	0.022
$f_0(500)$	0.034	0.010	0.035	0.023
$f_0(1370)$	-0.065	0.013	0.053	0.024
$K^{*+}(892)$	0.009	0.008	0.004	0.003
$K^{*-}(892)$	–	–	–	–
$\rho^0(770)$	-0.034	0.009	0.009	0.018
$K^{*+}(1680)$	0.015	0.012	0.037	0.062
$K^{*-}(1680)$	0.042	0.013	0.038	0.068
$K_2^{*+}(1430)$	-0.009	0.011	0.017	0.035
$K_2^{*-}(1430)$	-0.041	0.014	0.024	0.035

$$\boxed{a_{\text{corr}} = a_{\text{nominal}} - \mu_{\text{residual}}}$$





The corrections are ~5% of the stats. unc.

Summary and outlook

Summary

- The nominal model has been defined according to an "objective" pre-defined algorithm, including information from: dissimilarity test, negative likelihood and magnitude precision of the added component.
- Results from the nominal model have been presented, including momentarily the isobar parameters (cartesian and polar) and the raw CP asymmetry.
- The systematic uncertainties have been introduced and results are presented for some of them (experimental category).

Outlook

- Complete the fit validation study with Likelihood scans.
- Continue with the computation of systematic uncertainties for the Fit Fractions.
- Correct the raw CP asymmetry by accounting for the production asymmetry and the asymmetry due to the interactions of the final state pions with the detector.
- Update the ANA with the recent results.
- Completion of the analysis expected before Summer.

Thank you!

Fit fractions calculation: quick review

For each flavour, the amplitude is written as:

$$\mathcal{A}(\Phi) = \sum_i c_i F_i(\Phi), \qquad \overline{\mathcal{A}}(\Phi) = \sum_i \bar{c}_i \bar{F}_i(\Phi)$$

Fit fraction component is:

$$\mathrm{FF}_i = \frac{(x_i^2 + y_i^2) S_i}{N}$$

with:

$$N = \int |\mathcal{A}(\Phi)|^2 \, d\Phi = \sum_i (x_i^2 + y_i^2) S_i + \sum_{i<j} \left[(x_i x_j + y_i y_j) R_{ij} + (y_i x_j - x_i y_j) J_{ij} \right], \qquad S_i = \int |F_i(\Phi)|^2 \, d\Phi$$

Similarly for the interference fit fractions:

$$\mathrm{IFF}_{ij} = \frac{(x_i x_j + y_i y_j) R_{ij} + (y_i x_j - x_i y_j) J_{ij}}{N}.$$

with:

$$I_{ij} = \int F_i(\Phi) F_j^*(\Phi) \, d\Phi \qquad \Rightarrow \qquad R_{ij} = 2\,\mathrm{Re}(I_{ij}), \quad J_{ij} = -2\,\mathrm{Im}(I_{ij})$$

Fit fractions calculation: quick review

Sketch of the combination. The FF can be rewritten for each amplitude as:

$$\mathrm{FF}_i^B = \frac{D_i}{N}, \qquad \mathrm{FF}_i^{\bar{B}} = \frac{\overline{D}_i}{\overline{N}} \qquad \overline{N} = \int |\overline{\mathcal{A}}(\Phi)|^2 \, d\Phi$$

with:

$$D_i = (x_i^2 + y_i^2) S_i \qquad \overline{D}_i = (\bar{x}_i^2 + \bar{y}_i^2) \overline{S}_i$$

The combined fit fraction can be written as:

$$\boxed{\mathrm{FF}_i^{\mathrm{combined}} = \frac{D_i + \overline{D}_i}{N + \overline{N}} = \frac{(x_i^2 + y_i^2) S_i + (\bar{x}_i^2 + \bar{y}_i^2) \overline{S}_i}{N + \overline{N}} = w\, \mathrm{FF}_i^B + (1-w)\, \mathrm{FF}_i^{\bar{B}}}$$

with:

$$1 - w = \frac{\overline{N}}{N + \overline{N}}$$

Similarly:

$$\boxed{\mathrm{IFF}_{ij}^{\mathrm{combined}} = \frac{C_{ij} + \overline{C}_{ij}}{N + \overline{N}} = \frac{(x_i x_j + y_i y_j) R_{ij} + (y_i x_j - x_i y_j) J_{ij}}{N + \overline{N}} = w\, \mathrm{IFF}_{ij}^B + (1-w)\, \mathrm{IFF}_{ij}^{\bar{B}}}$$