Generalized CDF Matrix Method for a Measurement of the Diphoton Cross-Section

The vector **W** holds the number of events classified according to the truth information. γ denotes prompt photons, while j denotes all other photon candidates (non-prompt, fakes).

$$\mathbf{W} = \begin{pmatrix} W_{\gamma\gamma} \\ W_{\gamma j} \\ W_{jj} \end{pmatrix}$$

The vector **N** holds the number of events according to the two cuts. P means that a photon candidate passes the 1$^{\text{st}}$ cut (from now on called *quality criterion*), F means that is does not. Accordingly I means that the candidate passes the 2$^{\text{nd}}$ cut (*isolation criterion*), N means that is does not. The indices for **N** follow the notation for conditional probabilies: $N_{\mathrm{PP|IN}}$ means that both photon candidates pass the quality criterion. One of the photons passes the isolation criterion, while the other one does not. **W** and **N** are connected by the matrix E:

$$\mathbf{N} = \begin{pmatrix} N_{\mathrm{II|PP}} \\ N_{\mathrm{IN|PP}} \\ N_{\mathrm{NN|PP}} \\ N_{\mathrm{II|PF}} \\ N_{\mathrm{IN|PF}} \\ N_{\mathrm{NI|PF}} \\ N_{\mathrm{NN|PF}} \\ N_{\mathrm{II|FF}} \\ N_{\mathrm{IN|FF}} \\ N_{\mathrm{NN|FF}} \end{pmatrix} = E \cdot \mathbf{W} = E \cdot \begin{pmatrix} W_{\gamma\gamma} \\ W_{\gamma j} \\ W_{jj} \end{pmatrix}$$

E depends on the following single photon efficiencies and fake rates:

- $\epsilon \equiv p(\mathrm{P}|\gamma)$: the probability that a prompt photon passes the quality criterion. This efficiency is supposed to be the same for all type of prompt photons (hard scattering, bremsstrahlung) – independant of the nature of the hard process.
- $f_{\gamma j} \equiv p_{\gamma j}(\mathrm{P}|j)$: the fake rate for the quality criterion for non-prompt and fake photons in γ+jet events.
- $f_{jj} \equiv p_{jj}(\mathrm{P}|j)$: the fake rate for the quality criterion for non-prompt and fake photons in dijet events. It is supposed to differ from $f_{\gamma j}$ due to the differences in the quark-/gluon-jet decomposition of γ+jet and dijet events.
- $\hat{\epsilon}_P \equiv p(\mathrm{I}|\mathrm{P}|\gamma)$: the probability that a prompt photon which has already passed the quality criterion also passes the isolation criterion. This efficiency is again supposed to be independant of the type of the prompt photon and the nature of the hard process.
- $\hat{\epsilon}_F \equiv p(\mathrm{I}|\mathrm{F}|\gamma)$: the probability that a prompt photon which did not pass the quality criterion *does* pass the isolation criterion. This efficiency is again supposed to be independant of the type of the prompt photon and the nature of the hard process.

- $\hat{f}_{\gamma j,P} \equiv p_{\gamma j}(\mathrm{I}|\mathrm{P}|j)$: the probability that a non-prompt or fake photon which has already passed the quality criterion also passes the isolation criterion in γ+jet events.
- $\hat{f}_{\gamma j,F} \equiv p_{\gamma j}(\mathrm{I}|\mathrm{F}|j)$: the probability that a non-prompt or fake photon which did not pass the quality criterion *does* pass the isolation criterion in γ+jet events.
- $\hat{f}_{jj,P} \equiv p_{jj}(\mathrm{I}|\mathrm{P}|j)$: the probability that a non-prompt or fake photon which has already pass the quality criterion also passes the isolation criterion in dijet events.
- $\hat{f}_{jj,F} \equiv p_{jj}(\mathrm{I}|\mathrm{F}|j)$: the probability that a non-prompt or fake photon which did not pass the quality criterion *does* pass the isolation criterion in dijet events.

E can be decomposed into two matrices $E_{\mathrm{isolation}}$ and E_{quality}:

$$\mathbf{N} = E \cdot \mathbf{W} = E_{\mathrm{isolation}} \cdot E_{\mathrm{quality}} \cdot \mathbf{W} = E_{\mathrm{isolation}} \cdot \mathbf{N}' = E_{\mathrm{isolation}} \cdot \begin{pmatrix} N'_{\mathrm{PP}|\gamma\gamma} \\ N'_{\mathrm{PF}|\gamma\gamma} \\ N'_{\mathrm{FF}|\gamma\gamma} \\ N'_{\mathrm{PP}|\gamma j} \\ N'_{\mathrm{PF}|\gamma j} \\ N'_{\mathrm{FP}|\gamma j} \\ N'_{\mathrm{FF}|\gamma j} \\ N'_{\mathrm{PP}|jj} \\ N'_{\mathrm{PF}|jj} \\ N'_{\mathrm{FF}|jj} \end{pmatrix}$$

$$
\begin{aligned}
E_{\text{quality}} &= \begin{pmatrix}
\epsilon\cdot\epsilon & 0 & 0 \\
2\cdot\epsilon\cdot(1-\epsilon) & 0 & 0 \\
(1-\epsilon)\cdot(1-\epsilon) & 0 & 0 \\
0 & \epsilon\cdot f_{\gamma j} & 0 \\
0 & \epsilon\cdot(1-f_{\gamma j}) & 0 \\
0 & (1-\epsilon)\cdot f_{\gamma j} & 0 \\
0 & (1-\epsilon)\cdot(1-f_{\gamma j}) & 0 \\
0 & 0 & f_{jj}\cdot f_{jj} \\
0 & 0 & 2\cdot f_{jj}\cdot(1-f_{jj}) \\
0 & 0 & (1-f_{jj})\cdot(1-f_{jj})
\end{pmatrix} \\
&= \begin{pmatrix}
\epsilon^2 & 0 & 0 \\
2\cdot\epsilon\cdot(1-\epsilon) & 0 & 0 \\
(1-\epsilon)^2 & 0 & 0 \\
0 & \epsilon\cdot f_{\gamma j} & 0 \\
0 & \epsilon\cdot(1-f_{\gamma j}) & 0 \\
0 & (1-\epsilon)\cdot f_{\gamma j} & 0 \\
0 & (1-\epsilon)\cdot(1-f_{\gamma j}) & 0 \\
0 & 0 & f_{jj}^2 \\
0 & 0 & 2\cdot f_{jj}\cdot(1-f_{jj}) \\
0 & 0 & (1-f_{jj})^2
\end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
E_{\text{isolation}} \quad = \quad & \begin{pmatrix}
\hat{\epsilon}_P \cdot \hat{\epsilon}_P & 0 & 0 & \hat{\epsilon}_P \cdot \hat{f}_{\gamma j,P} & 0 & 0 & 0 & \hat{f}_{jj,P} \cdot \hat{f}_{jj,P} & 0 & 0 \\
2 \cdot \hat{\epsilon}_P \cdot (1-\hat{\epsilon}_P) & 0 & 0 & \hat{\epsilon}_P \cdot (1-\hat{f}_{\gamma j,P}) + (1-\hat{\epsilon}_P) \cdot \hat{f}_{\gamma j,P} & 0 & 0 & 0 & 2 \cdot \hat{f}_{jj,P} \cdot (1-\hat{f}_{jj,P}) & 0 & 0 \\
(1-\hat{\epsilon}_P) \cdot (1-\hat{\epsilon}_P) & 0 & 0 & (1-\hat{\epsilon}_P) \cdot (1-\hat{f}_{\gamma j,P}) & 0 & 0 & 0 & (1-\hat{f}_{jj,P}) \cdot (1-\hat{f}_{jj,P}) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \\
+ \quad & \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \hat{\epsilon}_P \cdot \hat{\epsilon}_F & 0 & 0 & \hat{\epsilon}_P \cdot \hat{f}_{\gamma j,F} & \hat{\epsilon}_F \cdot \hat{f}_{\gamma j,P} & 0 & 0 & \hat{f}_{jj,P} \cdot \hat{f}_{jj,F} & 0 \\
0 & \hat{\epsilon}_P \cdot (1-\hat{\epsilon}_F) & 0 & 0 & \hat{\epsilon}_P \cdot (1-\hat{f}_{\gamma j,F}) & (1-\hat{\epsilon}_F) \cdot \hat{f}_{\gamma j,P} & 0 & 0 & \hat{f}_{jj,P} \cdot (1-\hat{f}_{jj,F}) & 0 \\
0 & (1-\hat{\epsilon}_P) \cdot \hat{\epsilon}_F & 0 & 0 & (1-\hat{\epsilon}_P) \cdot \hat{f}_{\gamma j,F} & \hat{\epsilon}_F \cdot (1-\hat{f}_{\gamma j,P}) & 0 & 0 & (1-\hat{f}_{jj,P}) \cdot \hat{f}_{jj,F} & 0 \\
0 & (1-\hat{\epsilon}_P) \cdot (1-\hat{\epsilon}_F) & 0 & 0 & (1-\hat{\epsilon}_P) \cdot (1-\hat{f}_{\gamma j,F}) & (1-\hat{\epsilon}_F) \cdot (1-\hat{f}_{\gamma j,P}) & 0 & 0 & (1-\hat{f}_{jj,P}) \cdot (1-\hat{f}_{jj,F}) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \\
+ \quad & \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \hat{\epsilon}_F \cdot \hat{\epsilon}_F & 0 & 0 & 0 & \hat{\epsilon}_F \cdot \hat{f}_{\gamma j,F} & 0 & 0 & \hat{f}_{jj,F} \cdot \hat{f}_{jj,F} \\
0 & 0 & 2 \cdot \hat{\epsilon}_F \cdot (1-\hat{\epsilon}_F) & 0 & 0 & 0 & \hat{\epsilon}_F \cdot (1-\hat{f}_{\gamma j,F}) + (1-\hat{\epsilon}_F) \cdot \hat{f}_{\gamma j,F} & 0 & 0 & 2 \cdot \hat{f}_{jj,F} \cdot (1-\hat{f}_{jj,F}) \\
0 & 0 & (1-\hat{\epsilon}_F) \cdot (1-\hat{\epsilon}_F) & 0 & 0 & 0 & (1-\hat{\epsilon}_F) \cdot (1-\hat{f}_{\gamma j,F}) & 0 & 0 & (1-\hat{f}_{jj,F}) \cdot (1-\hat{f}_{jj,F})
\end{pmatrix}
\end{aligned}
$$

This is a system of 10 equations with 9 free parameters if we can take the efficiencies for prompt photons from a different source:

$$\begin{aligned}\epsilon &= \tilde{\epsilon} \pm \sigma(\tilde{\epsilon})\\ \hat{\epsilon}_P &= \tilde{\hat{\epsilon}}_P \pm \sigma(\tilde{\hat{\epsilon}}_P)\\ \hat{\epsilon}_F &= \tilde{\hat{\epsilon}}_F \pm \sigma(\tilde{\hat{\epsilon}}_F)\end{aligned}$$

A solution of this overconstraint system can be found by minimising a χ^2 function, poissonian probabilities being approximated by Gaussians:

$$\chi^2 = \sum_{i=1}^{10} \frac{\Big(N_i - N_i(f_{\gamma j}, f_{jj}, \hat{f}_{\gamma j,P}, \hat{f}_{\gamma j,F}, \hat{f}_{jj,P}, \hat{f}_{jj,F}, W_{\gamma\gamma}, W_{\gamma j}, W_{jj})\Big)^2}{N_i(f_{\gamma j}, f_{jj}, \hat{f}_{\gamma j,P}, \hat{f}_{\gamma j,F}, \hat{f}_{jj,P}, \hat{f}_{jj,F}, W_{\gamma\gamma}, W_{\gamma j}, W_{jj})}$$