Skew-Normal Likelihood for non-Gaussian astronomical data

Zeynab Ashuri

Dr. Elena Sellentin, Dr. Daniela Grandon

Leiden Observatory

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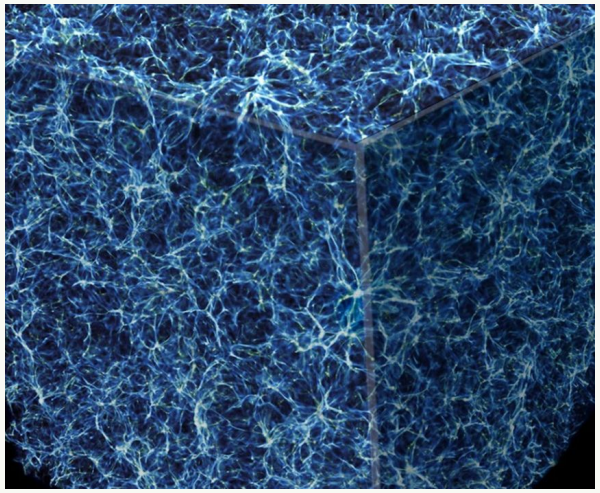
Which non-Gaussianity are we talking about?

The Underlying Field

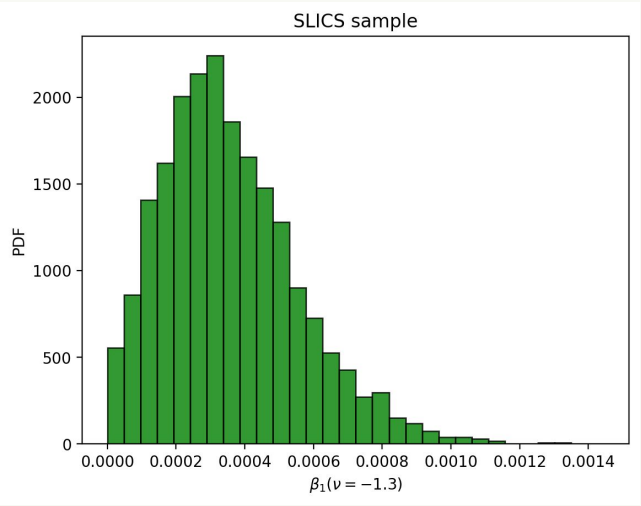
Domain: Physical 3D Space.
Cause: Non-linear gravity, etc.
Result: Information leaks into higher-order statistics.

The Likelihood

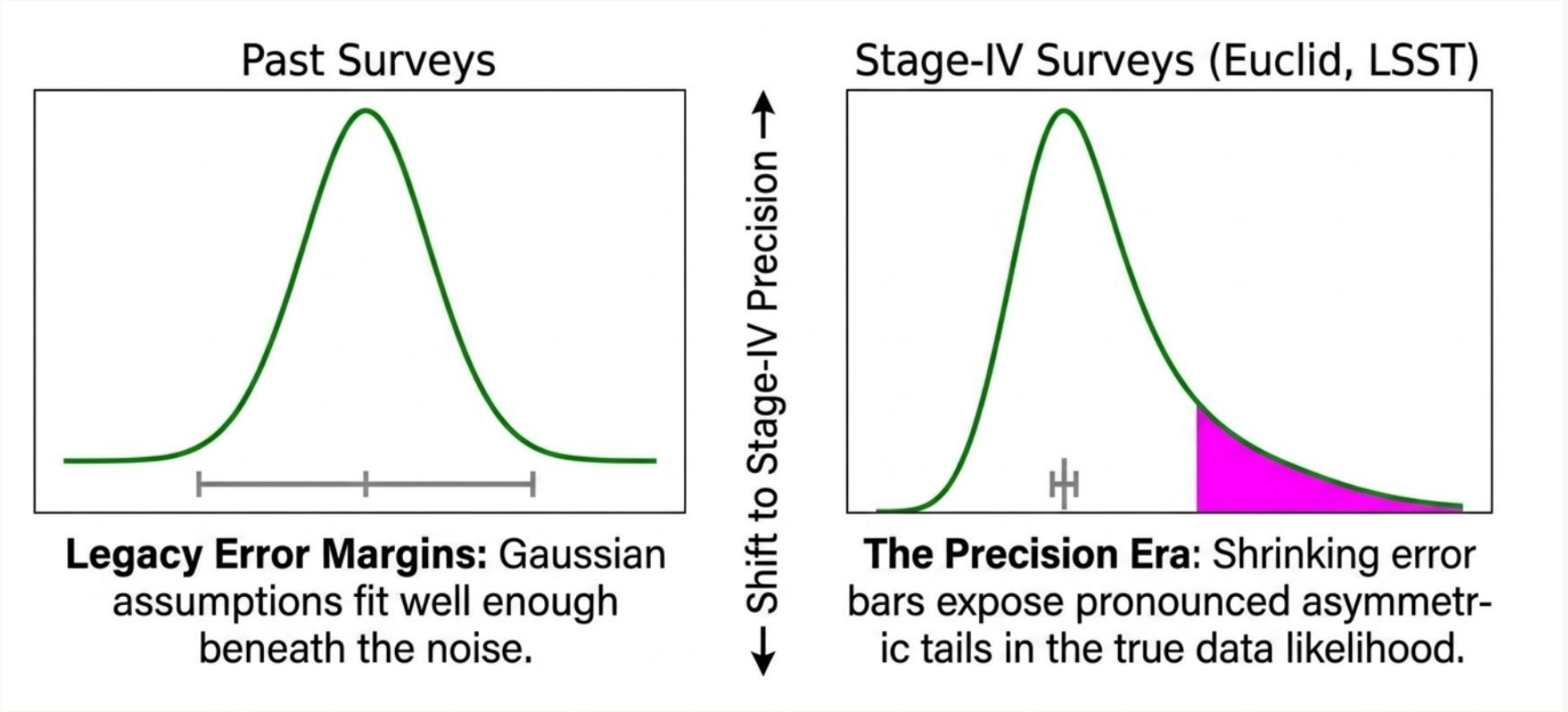
Domain: Statistical Data Vector Space.
Cause: Cosmic variance, etc.
Result: The PDF of summary statistics is asymmetric (skewed). Requires non-Gaussian modeling.



Credit: NASA, ESA, and E. Hallman (University of Colorado, Boulder)

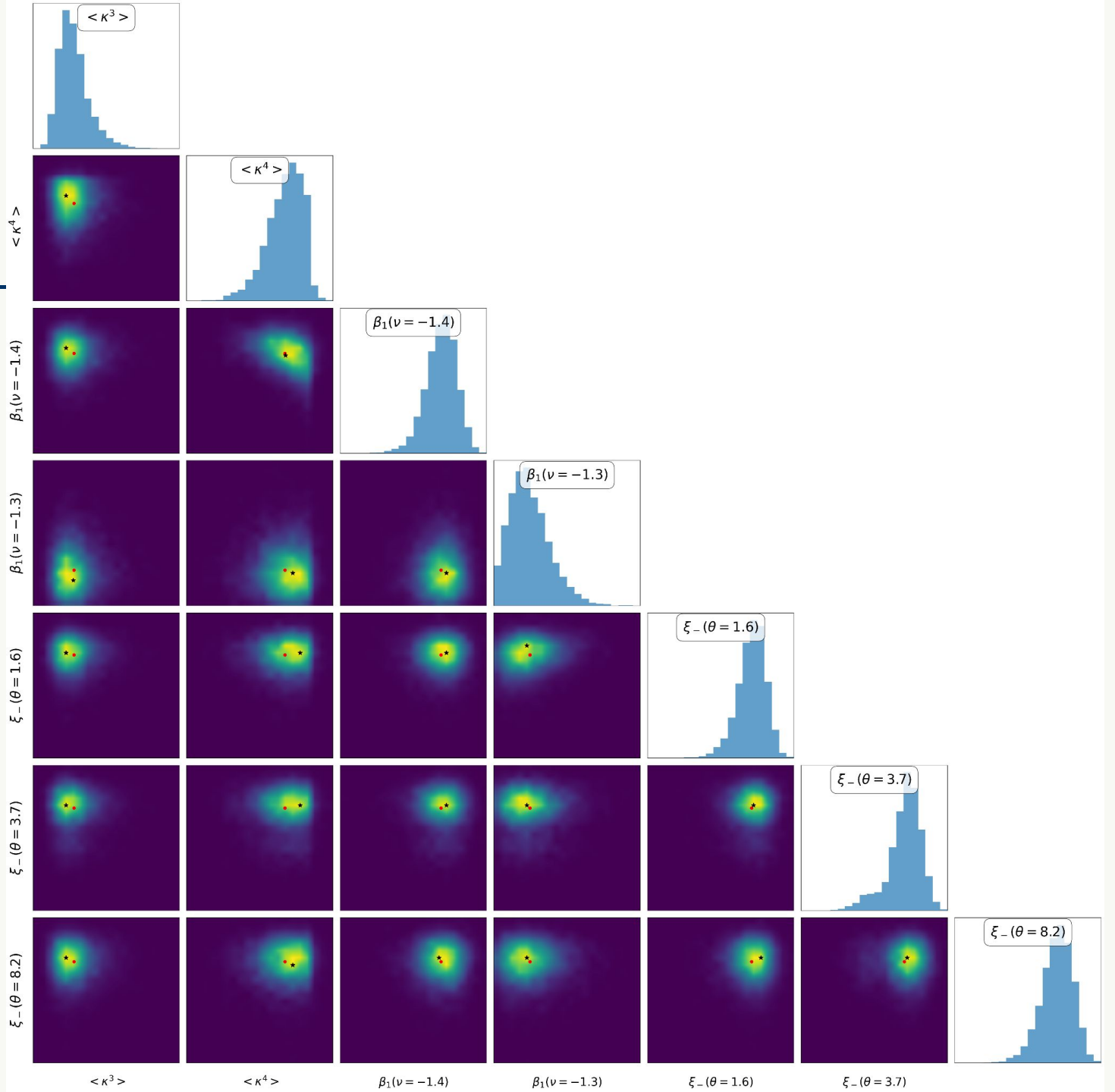


Why is it important for us?

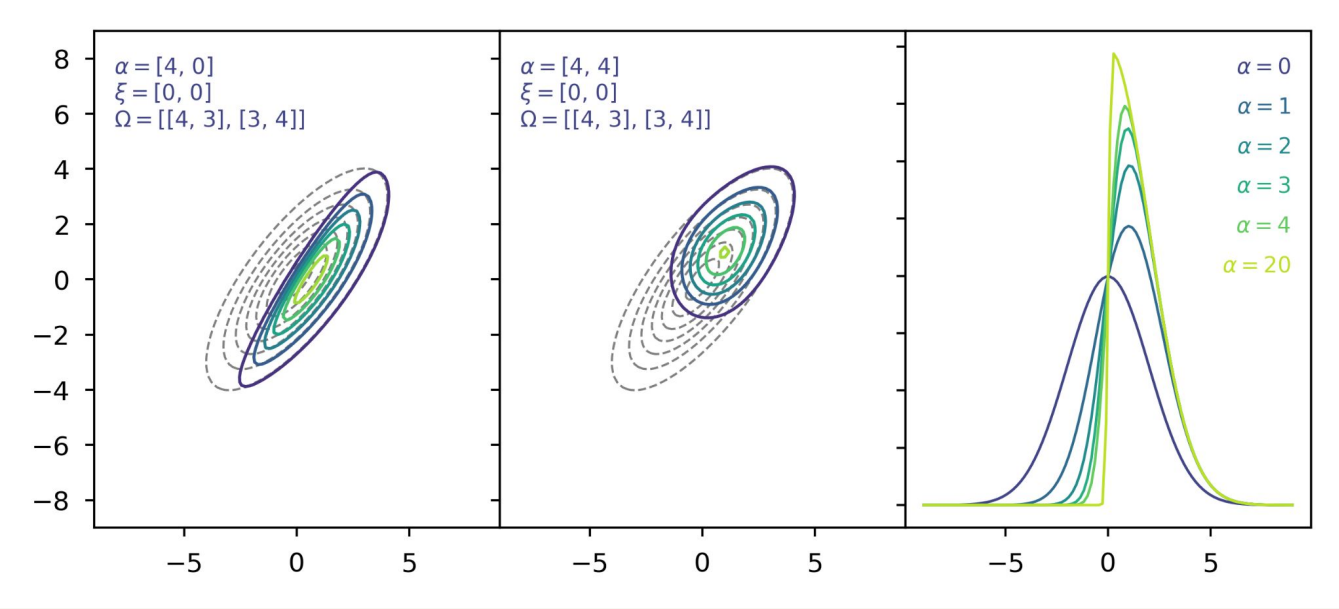


How pronounced is the non gaussianity of the likelihood in WL data?

- Some selected data points from SLICS simulation with high non-gaussianity
- **SLICS**: N-body sim with 7640 realizations - 25 deg2 light cones and 4.8 arcsec resolution



Anatomy of the Skew-Normal Distribution



$$\mathrm{SN}(x;\alpha,\xi,\Omega) = 2 \times N(x;\xi,\Omega) \times \Phi(\alpha\omega^{-1}(x-\xi))$$

The Base Normal

The Perturbation Factor

The Slant Parameter

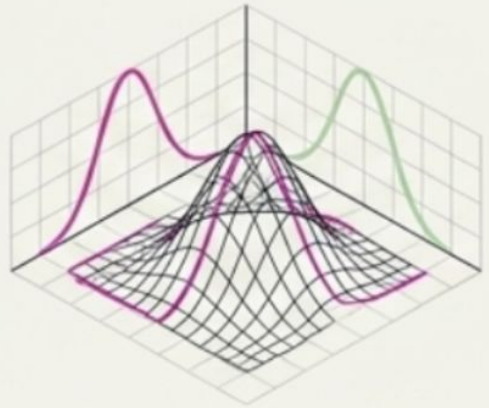
Following the notation of Azzalini (1985)

What make SN a good option for using as likelihood?

- converges to Gaussian likelihood at α = 0
- Adding Gaussian noise is theoretically straightforward
- Theoretical form for the marginals

Exact Gaussian Convergence

Closure Under Normal Noise

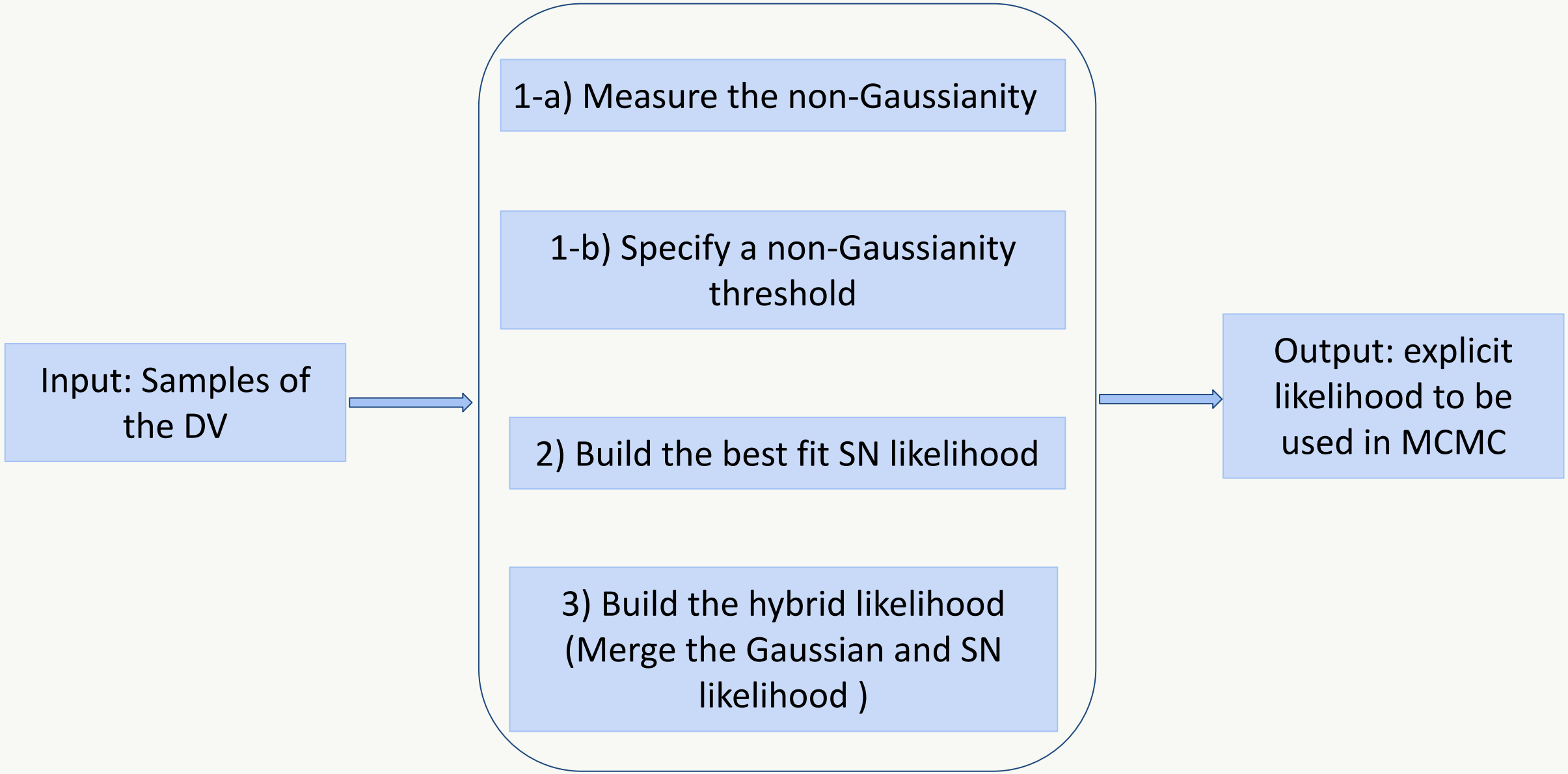


Closed-Form Marginals

Building the SN likelihood

	Method of Moments	Bayesian	Profile likelihood	The marginal method
Speed	Fast	Slow	Scalable	Fast
Scalability	Fails in extreme cases. Skeweness estimator becomes ill-conditioned while approching the symmetric case	Expensive	The most accurate/efficient method when enough samples are provided	Not as accurate as the profile likelihood but works more reliably with small sample size

Structure of the likelihood pipeline



Non-Gaussianity metric

SMAPE

$$y_i = (\mathbf{D}_i - \langle \mathbf{D} \rangle)^{\mathrm{T}} \, \mathrm{COV}^{-1} \, (\mathbf{D}_i - \langle \mathbf{D} \rangle)$$

$$\mathcal{S}_i = \frac{\left| P_{\mathrm{obs}}(y_i) - P_{\chi^2}(y_i) \right|}{|P_{\mathrm{obs}}(y_i)| + \left| P_{\chi^2}(y_i) \right|}$$

$$\mathcal{S}_{\mathrm{lim}} = \langle \mathcal{S}_{\mathrm{obs}}^{\mathrm{gauss}} \rangle + 2\,\sigma \left(\mathcal{S}_{\mathrm{obs}}^{\mathrm{gauss}} \right)$$

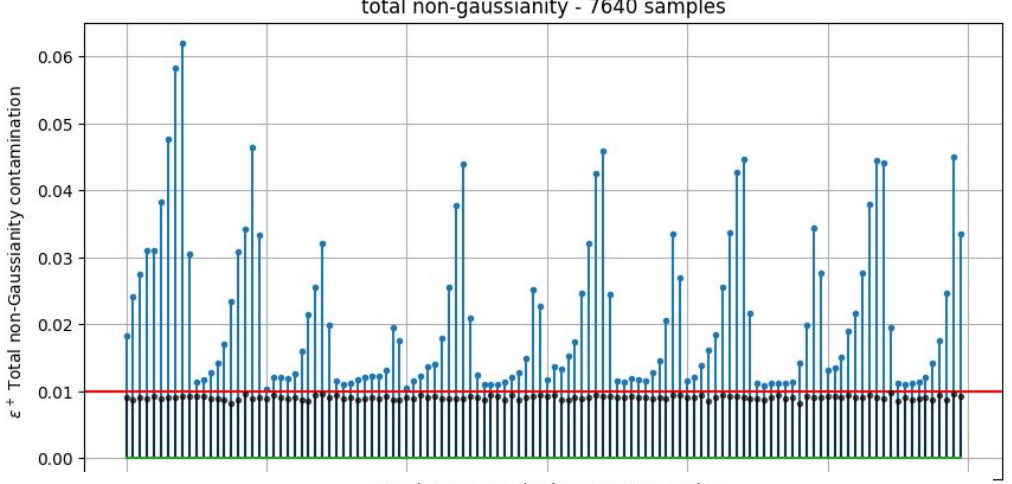
Vinciguerra et al. (2025)

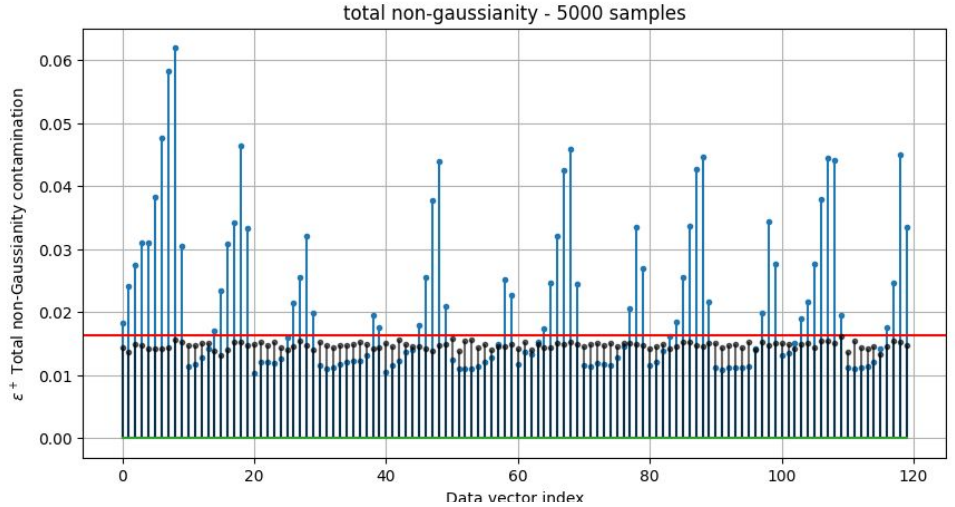
MISE (Summation)

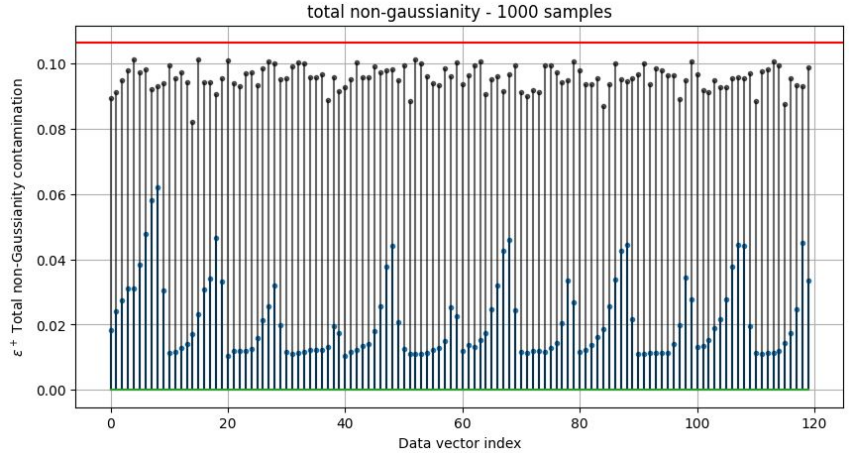
$$\frac{1}{B} \sum_{b=1}^{B} [\mathcal{H}_b(s_i^{e,f}) - \mathcal{G}(0,2)]^2 =: \mathsf{S}_{e,f}^{+}$$

$$\epsilon_e^{\mathrm{tot},+} = \sum_{f \neq e} \mathsf{S}_{e,f}^{+}$$

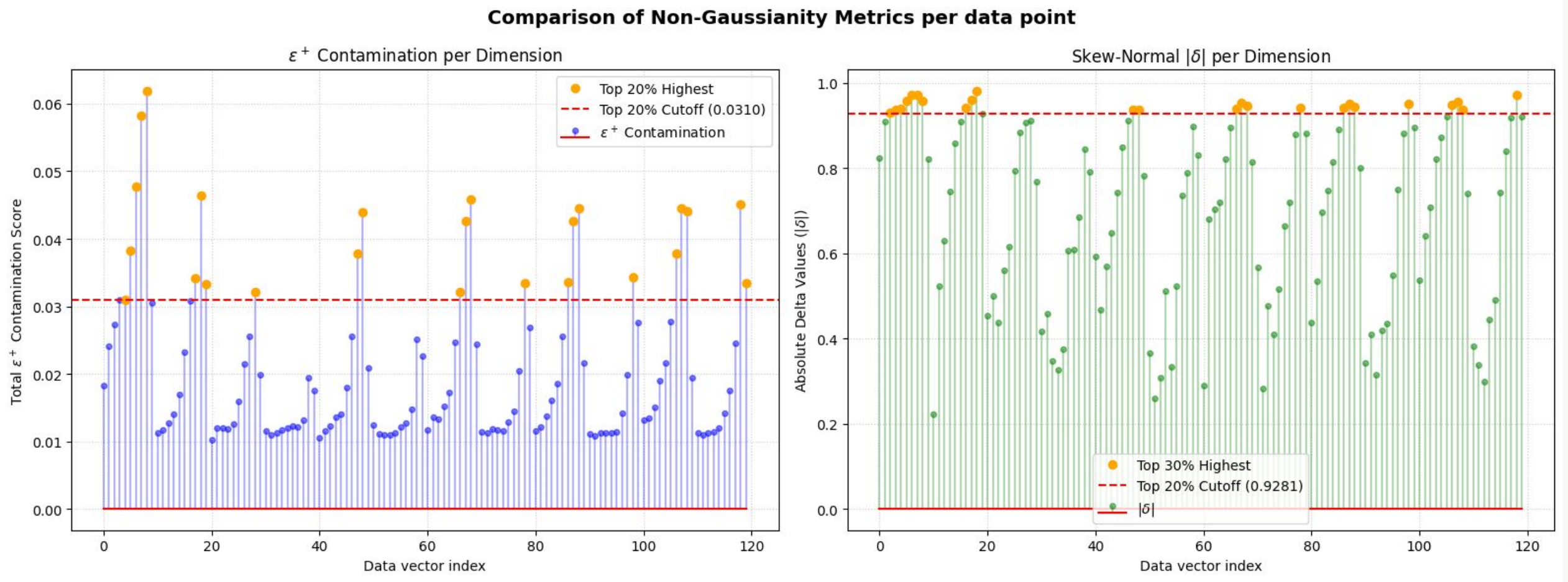
Sellentin et al. (2017)



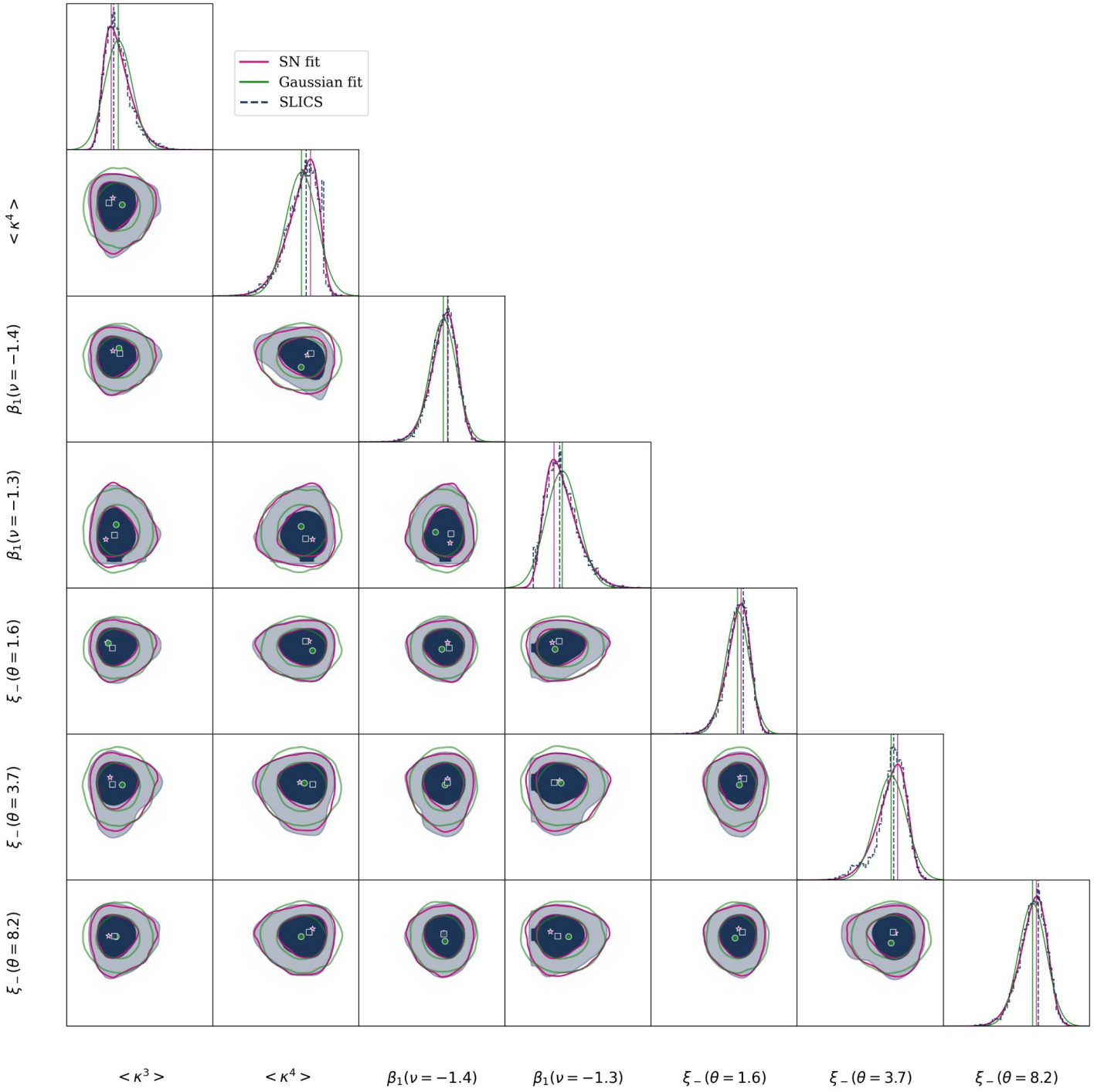




Using a different threshold for non-Gaussianity

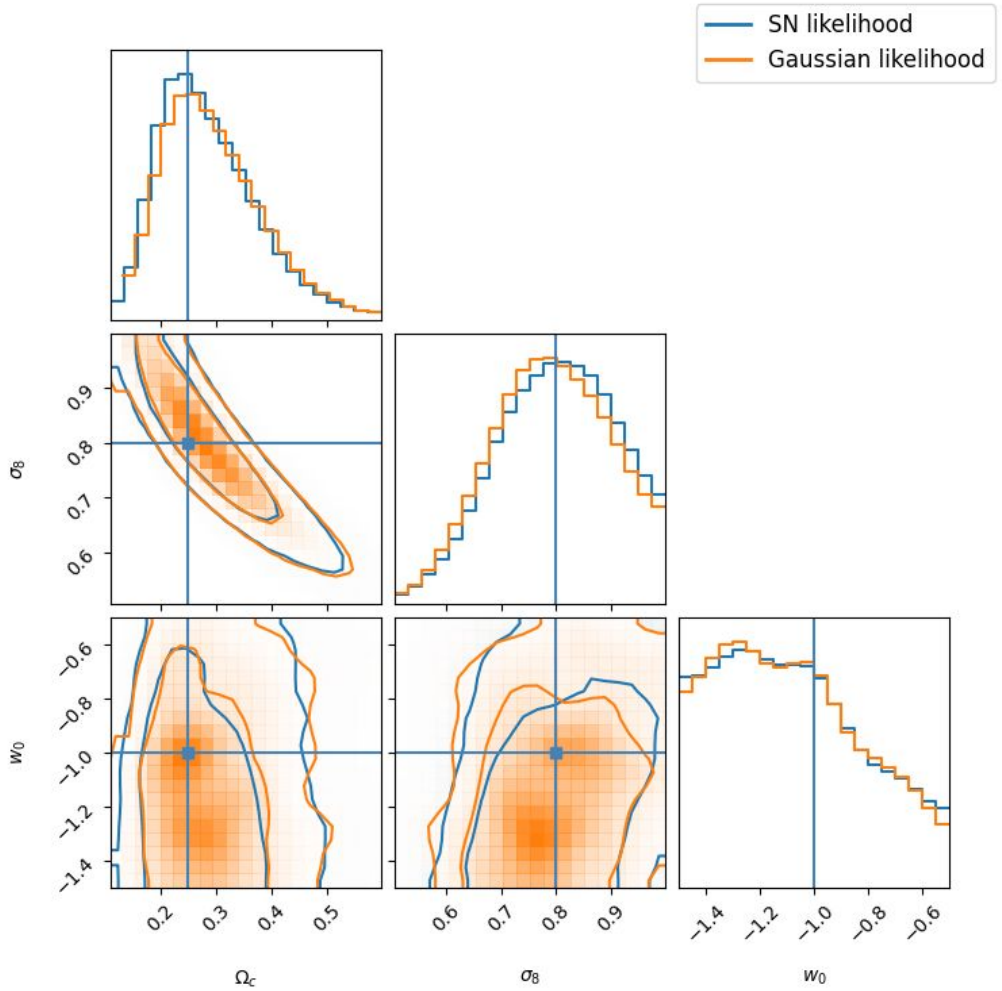


Case study: SLICS shear-2PCF

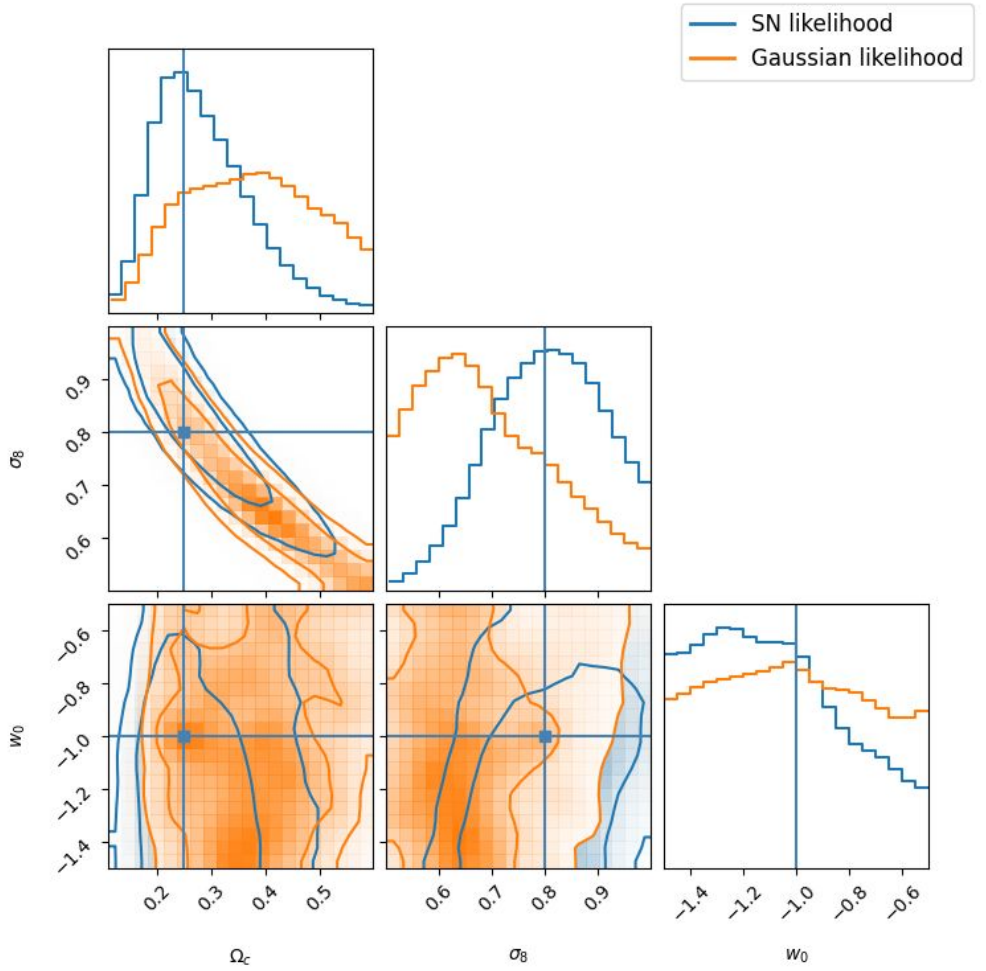


MCMC result: SLICS shear-2PCF

- All the data points included in both likelihoods: 0.1σ shift in S8.

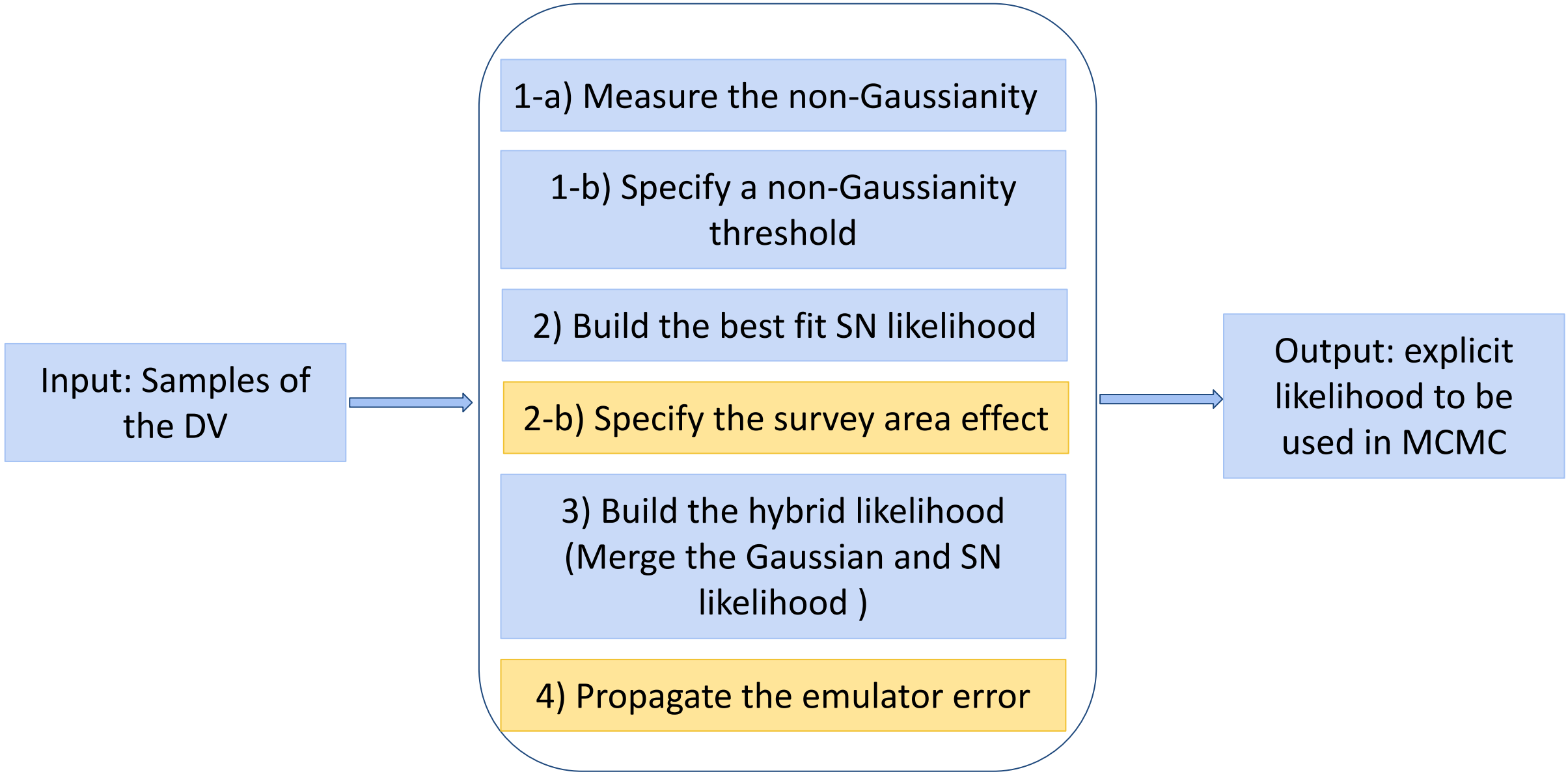


- Data points with non-Gaussian contamination have been removed in the Gaussian likelihood.



Data vector includes notomography and auto z-bins (length: 120)

Things to be done...



Conclusion

- The non-Gaussian metrics have significant details that change the valid range of the fisher forecast.
- **At the fitting level:** The skew normal likelihood fits the distribution of the data vectors better than the gaussian likelihood depending on the contamination of the data
- **At the inference level**: The skew normal likelihood can shift the S8 parameter using shear2pcf data up to 0.1σ in comparison to the Gaussian likelihood.
- If we assume that the Gaussian likelihood is not valid above a certain threshold this shift can go significantly higher and also due to the fact that we are excluding some of the non-Gaussian data points, the uncertainty of the parameters inference will expand.

Thank You!