Workshop: The Non-Gaussian Universe

Heraklion, Crete • June 2026

Going Below and Beyond: A Dive Into One-Point Statistics

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UNIVERSITÄT BIELEFELD

erc

Euclid consortium

image OpenAI ChatGPT (DALL·E)

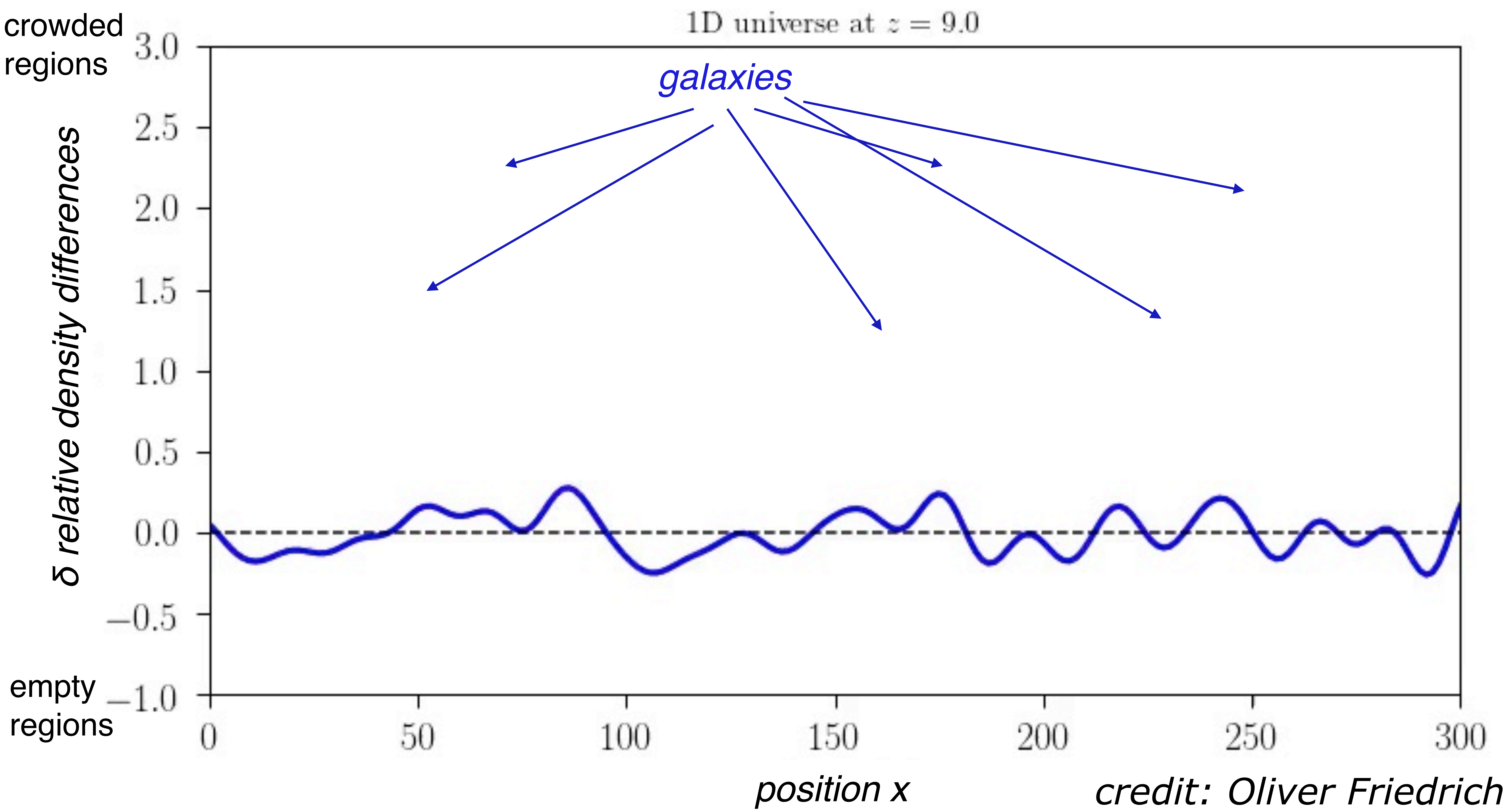
Cosmic Structure Formation

early: one snapshot
linear, Gaussian

late: motion picture
nonlinear, non-Gaussian

Non-Gaussian Universe

nonlinear dynamics → non-Gaussian statistics

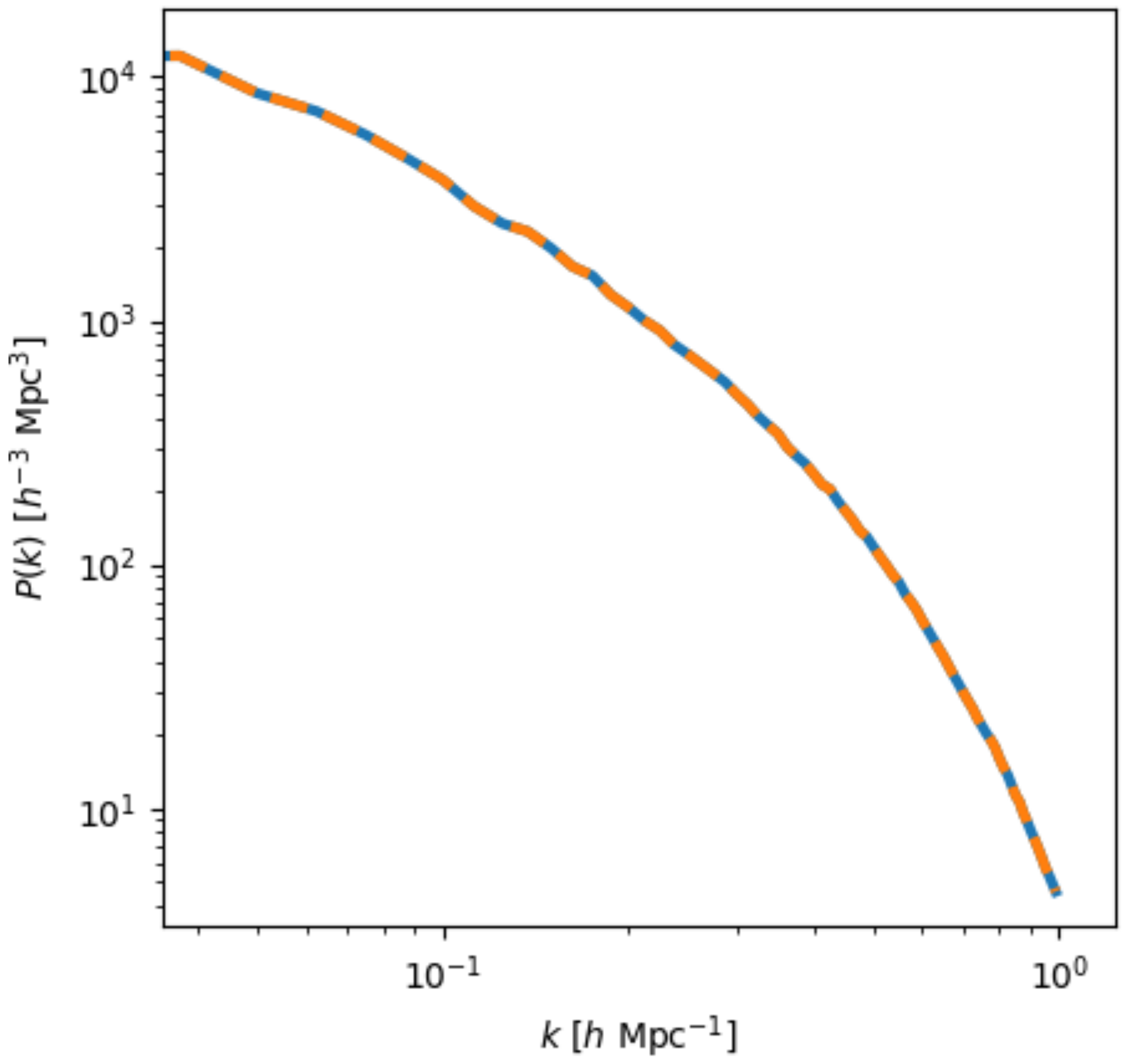


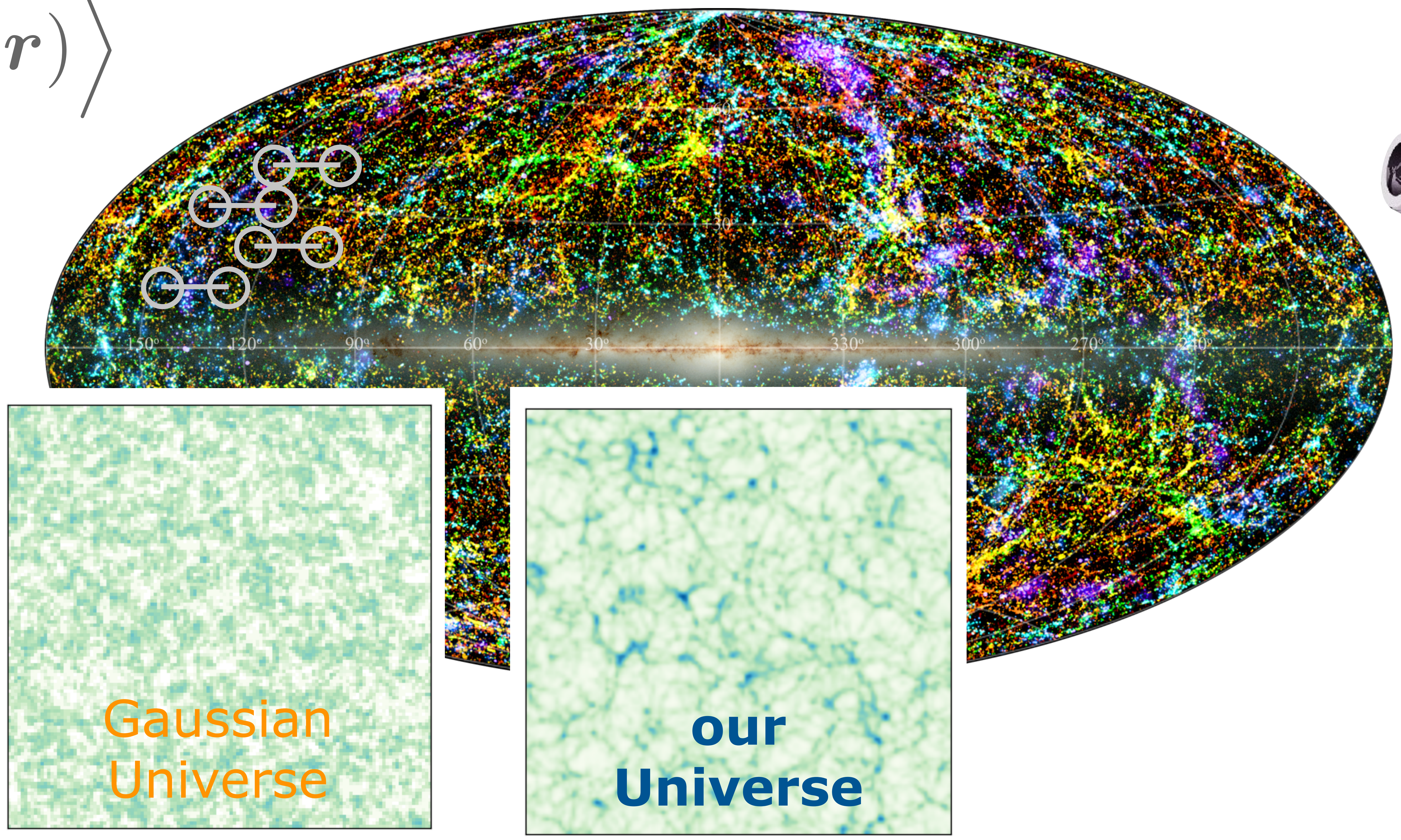
credit: Oliver Friedrich

Non-Gaussian Universe

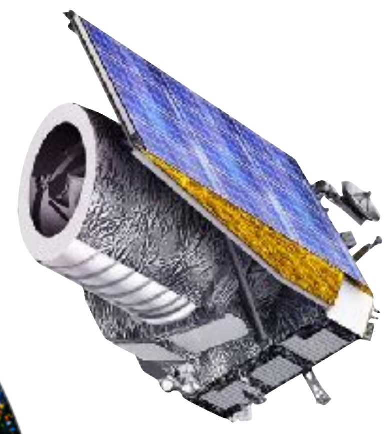
current: 2-point correlation averages over all densities

$$\xi(r) = \left\langle \delta(\boldsymbol{x})\delta(\boldsymbol{x}+\boldsymbol{r}) \right\rangle$$





Euclid

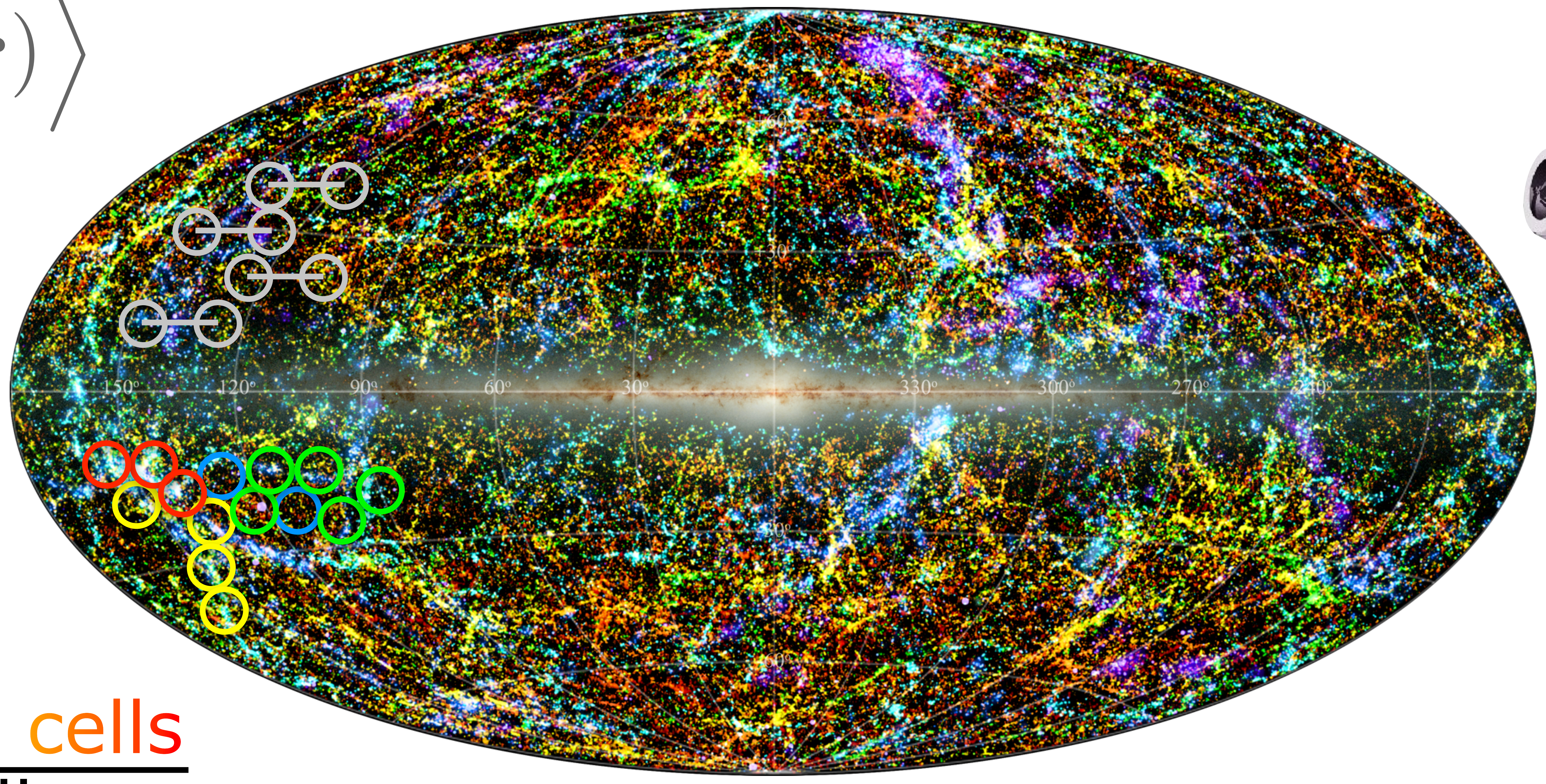


credit: Natalia Porqueres

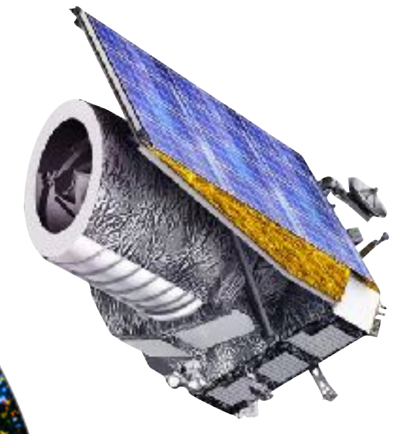
Non-Gaussian Universe

current: 2-point correlation averages over all densities

$$\xi(r) = \Big\langle \delta(\boldsymbol{x})\delta(\boldsymbol{x}+\boldsymbol{r}) \Big\rangle$$



Euclid



beyond average:

1-point statistics split density environments

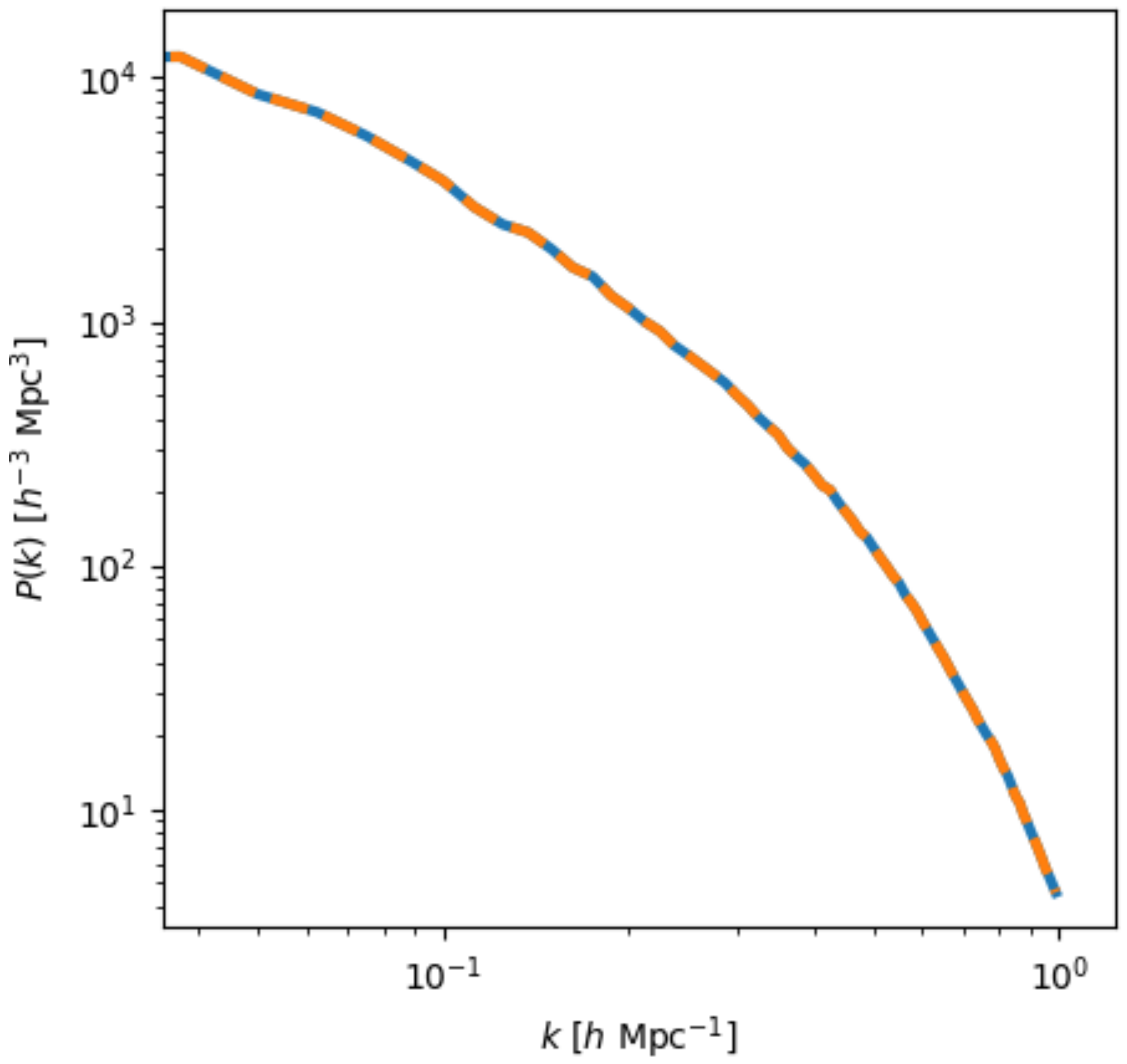
$$\mathcal{P}_R(\delta) \propto \frac{\text{\#coloured cells}}{\text{\#all cells}}$$

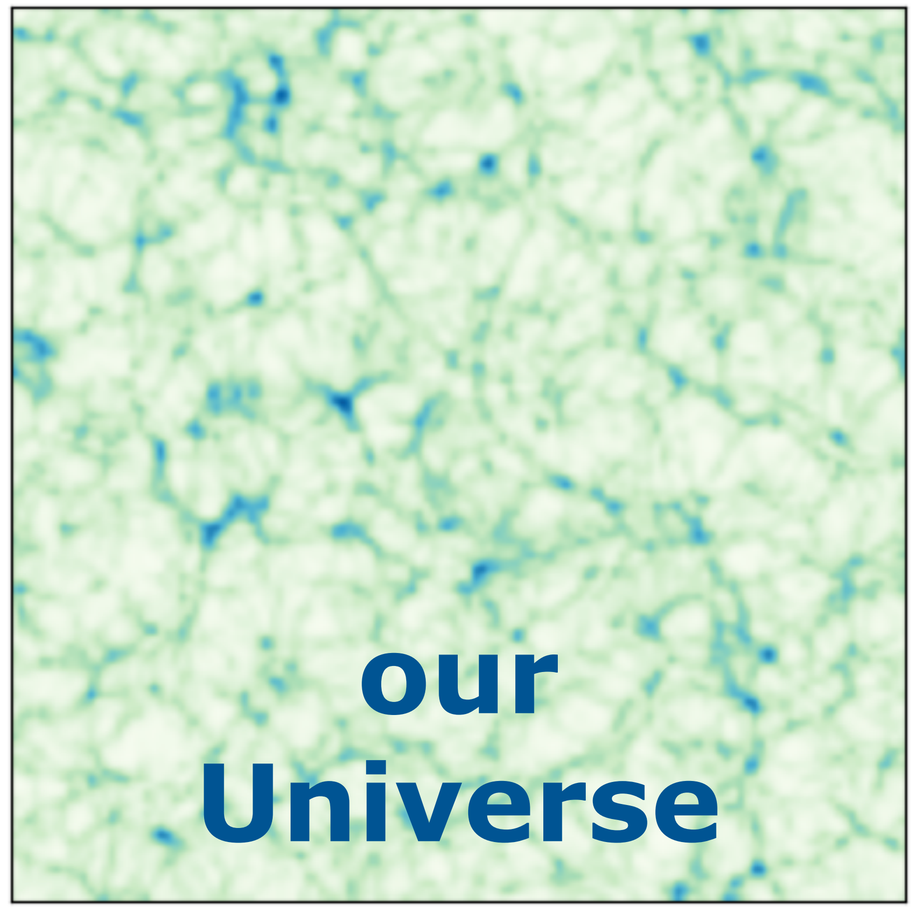
predictable on mildly nonlinear scales

Non-Gaussian Universe

CU++ '19

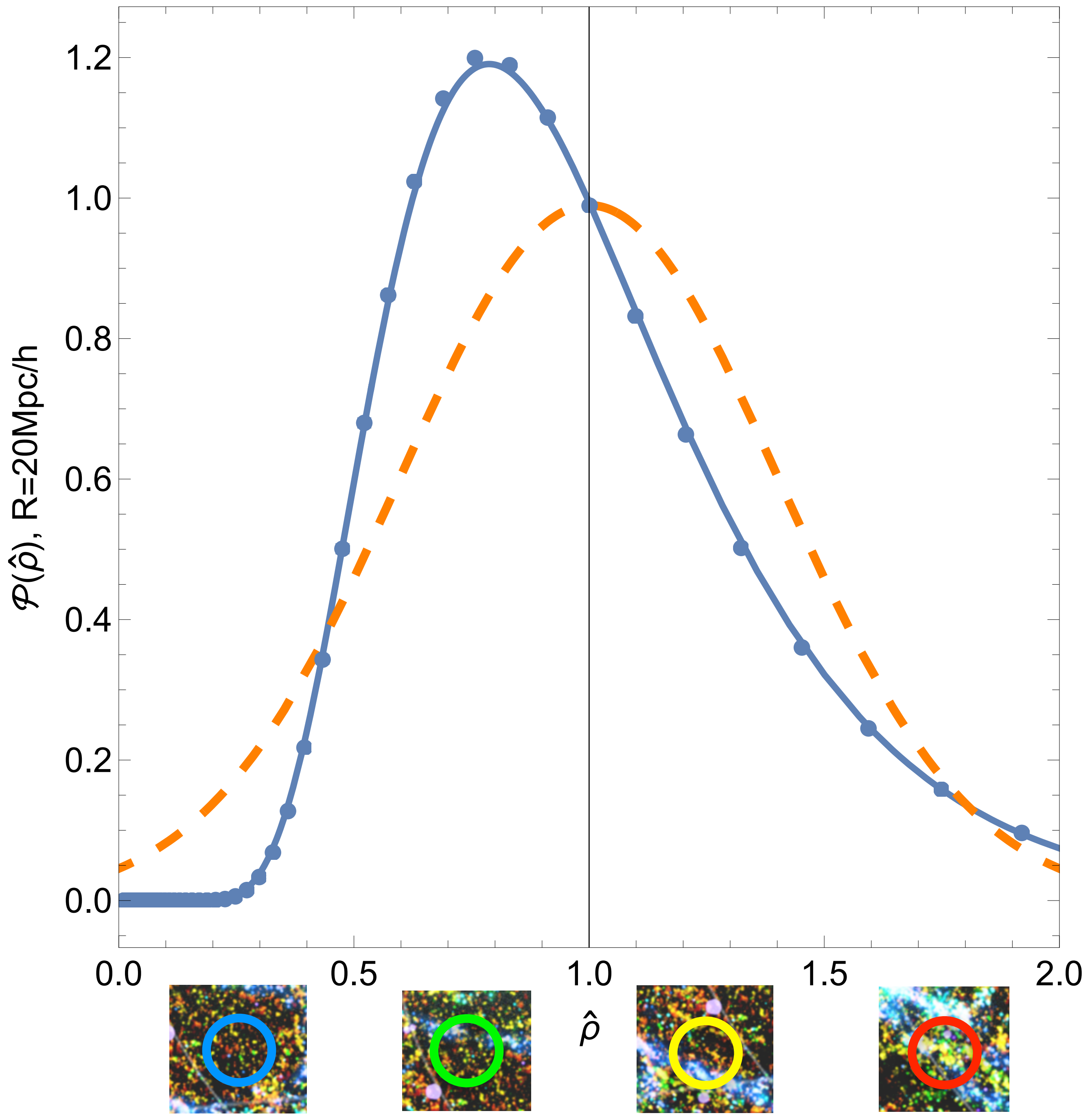
standard
2-point statistics





**goal: (1+2)-point
survey analysis**

predictable
non-Gaussian
1-point statistics



including higher-order
cumulants: skewness,...

Large Deviations Theory

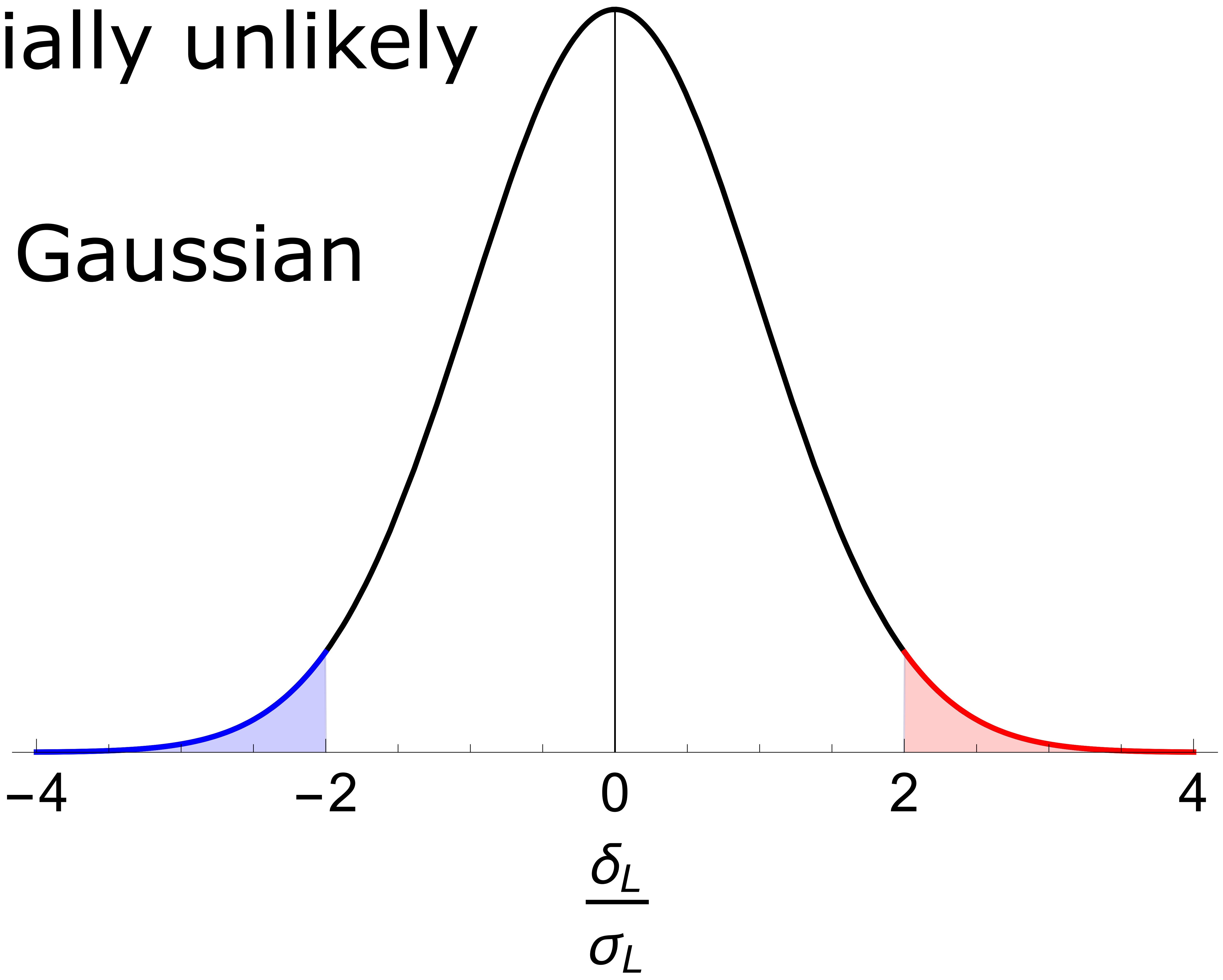
densities in spherical cells

large deviations exponentially unlikely

$$\mathcal{P}_r^{\mathrm{ini}}(\delta_L) \sim \exp\left[-\frac{\delta_L^2}{2\sigma_L^2(r)}\right]$$ Gaussian

want to predict PDF for

$$\rho = 1 + \delta_{\mathrm{NL}}$$



Large Deviations Theory

guiding principle: steepest descent

linear statistics known $P(\delta_L) \propto \exp(-\Psi(\delta_L))$

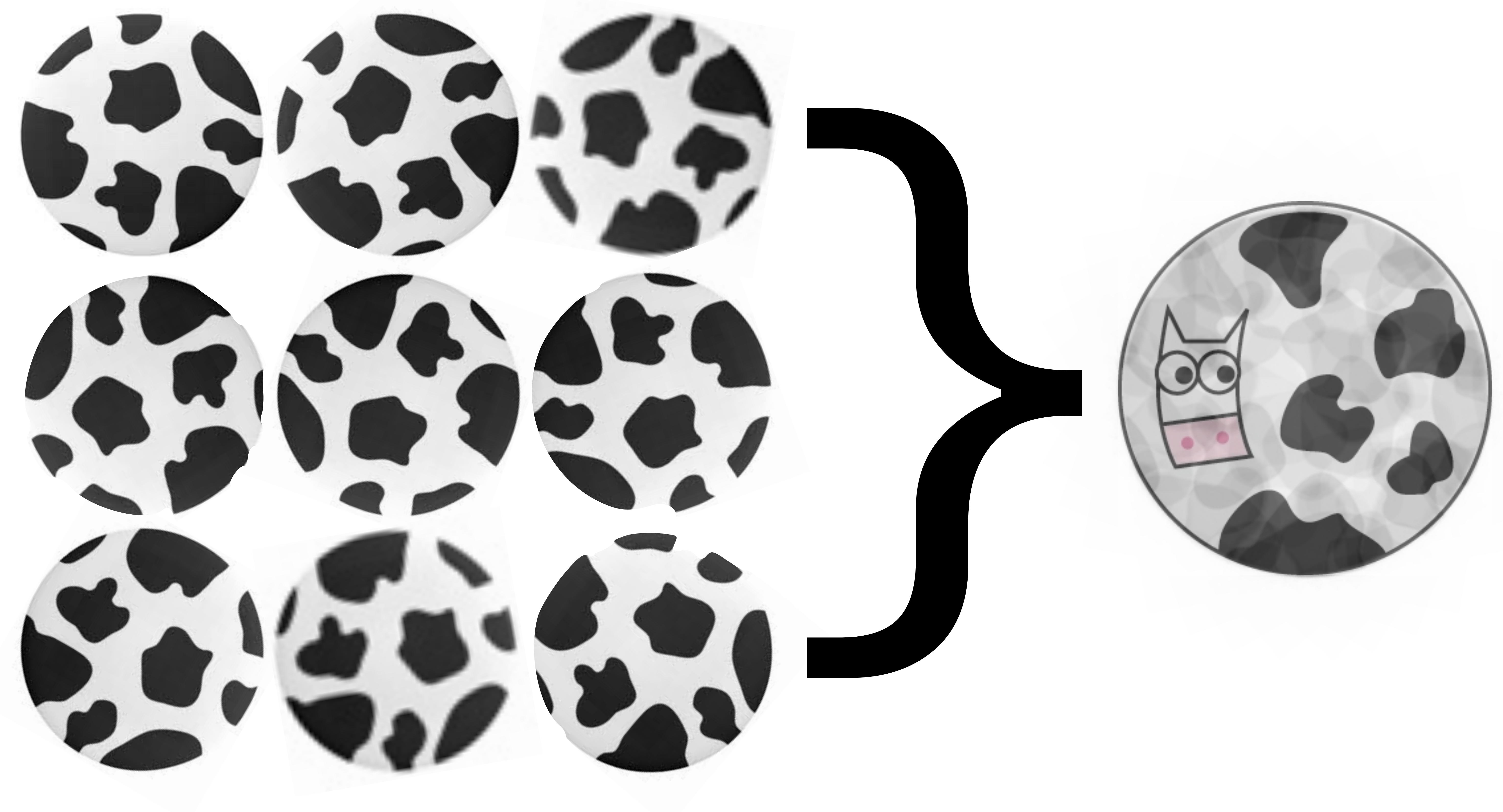
nonlinear **cumulant** generating function as path integral

$$\exp[\varphi(\lambda)] = \int d\rho\, P(\rho) \exp(\lambda\rho)$$
$$= \int \mathcal{D}[\delta_L(\boldsymbol{x})] P[\delta_L(\boldsymbol{x})] \exp\left(\lambda\rho[\delta_L(\boldsymbol{x})]\right)$$

dominated by least suppressed contribution
the one respecting symmetry

Clustering Statistics

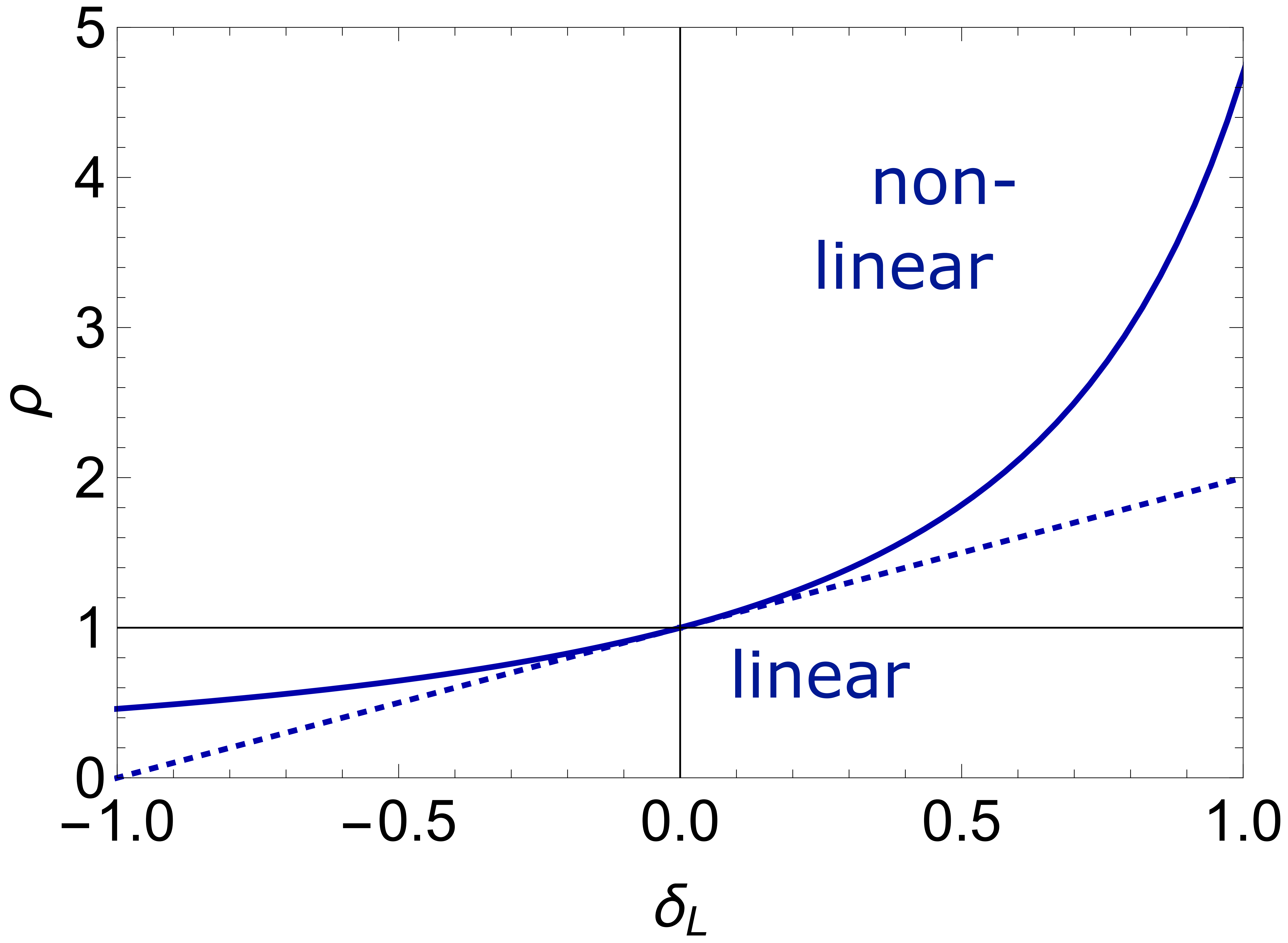
symmetry statistics ⟷ (large deviations theory) dynamics



relies on explicit smoothing (spherical top-hat)

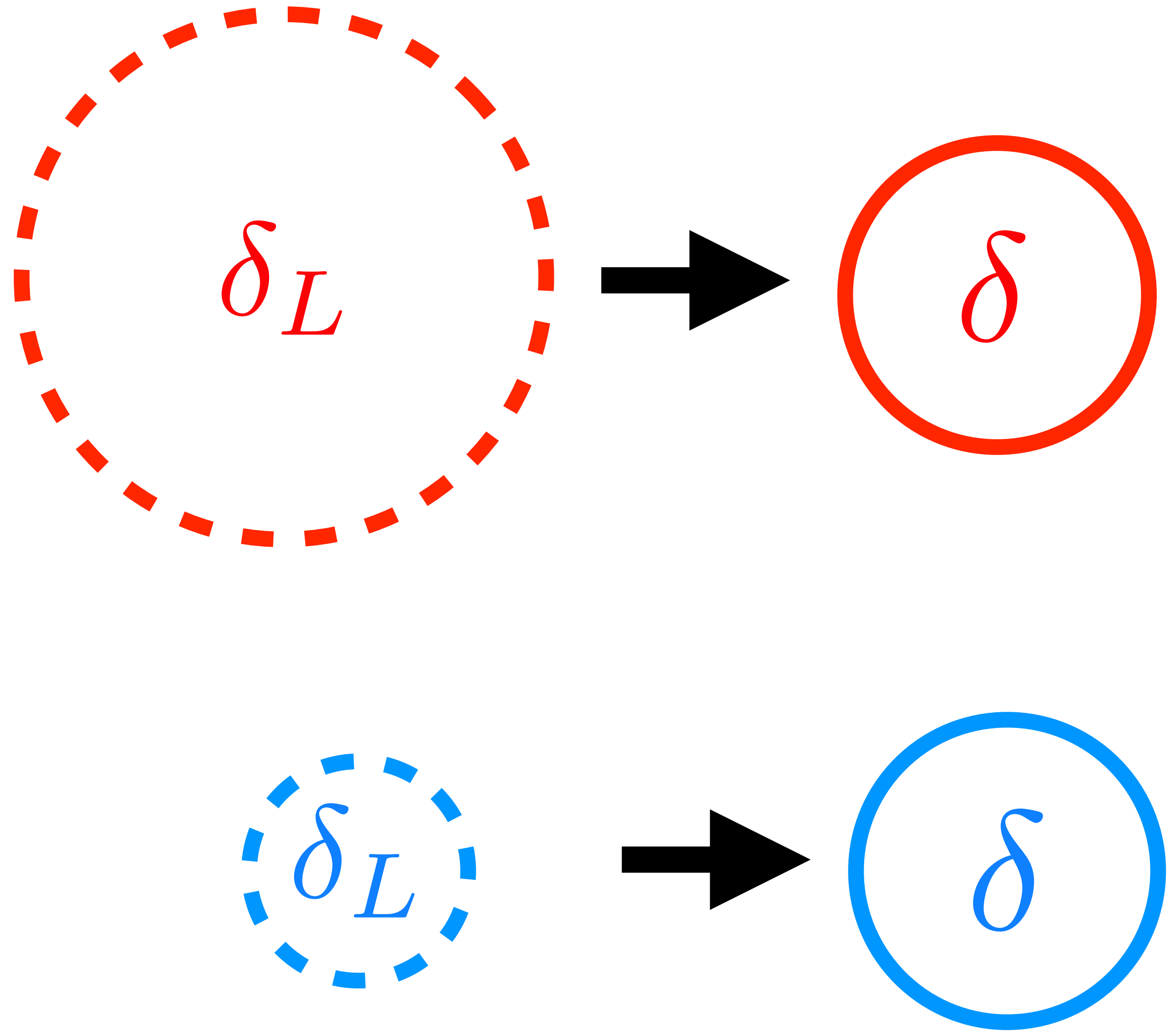
Spherical Dynamics

spherical collapse $\delta_L(\rho) \simeq \frac{21}{13}\left(1 - \rho^{-\frac{13}{21}}\right)$



mass conservation

$r = \rho^{1/3} R$



Matter density PDF from LDT

most likely path dominates $\rho = 1 + \delta_{\rm NL}$

spherical collapse

$$\mathcal{P}_{R,z}(\rho) \sim \exp\left[-\frac{\delta_L(\rho)^2}{2\sigma_L^2(z, r(R,\rho))}\frac{\sigma_L^2}{\sigma_{\rm NL}^2}\right]$$

Bernardeau 94

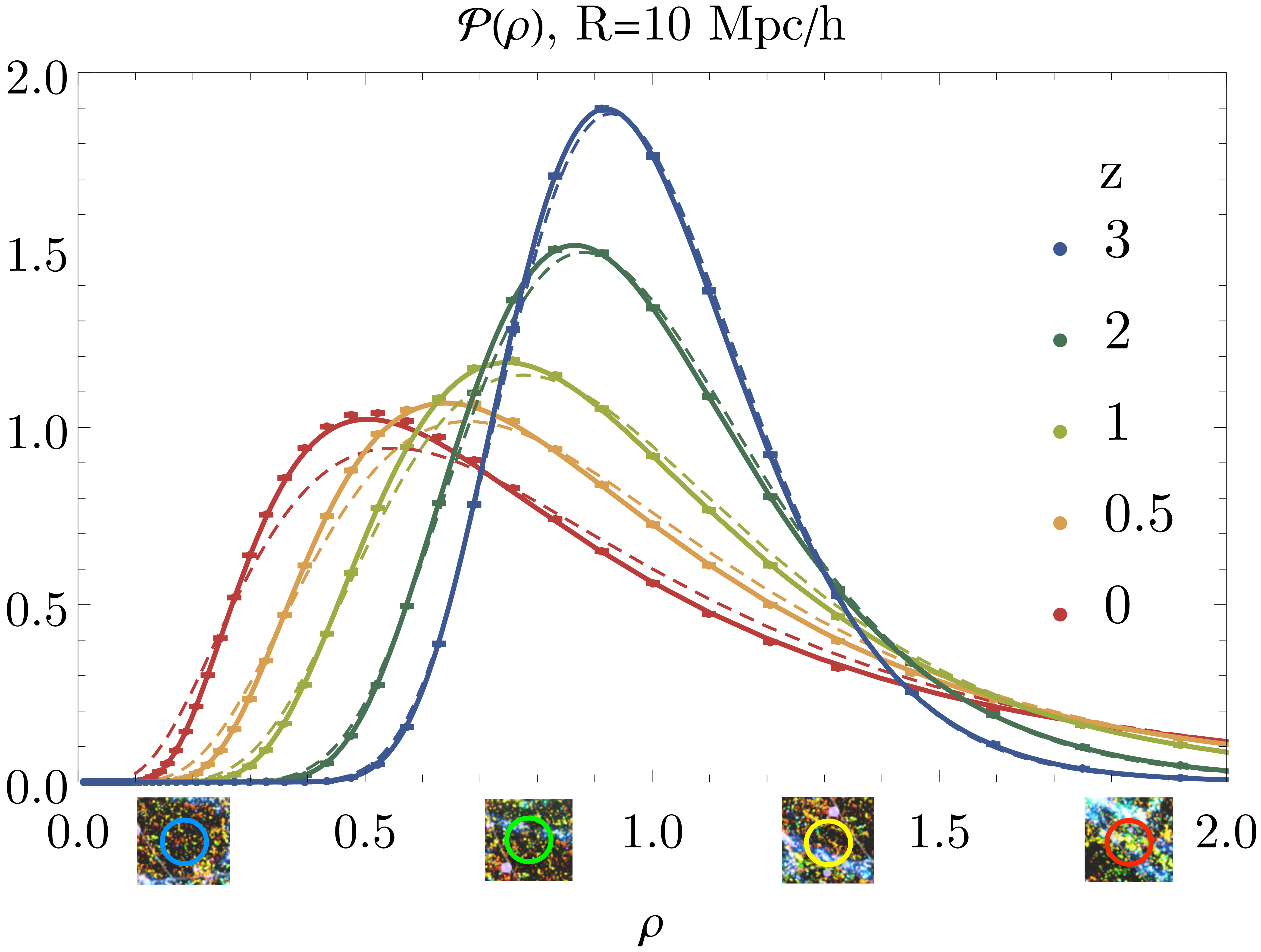
CU++ 16

initial conditions

clever saddle-point for inverse Laplace

Dark Matter 1-Point Statistics

dark matter: %-level accurate predictions in bulk, R$\gtrsim$ 10Mpc/h



work in progress:
*unlock tails with **SBI***

David Gebauer

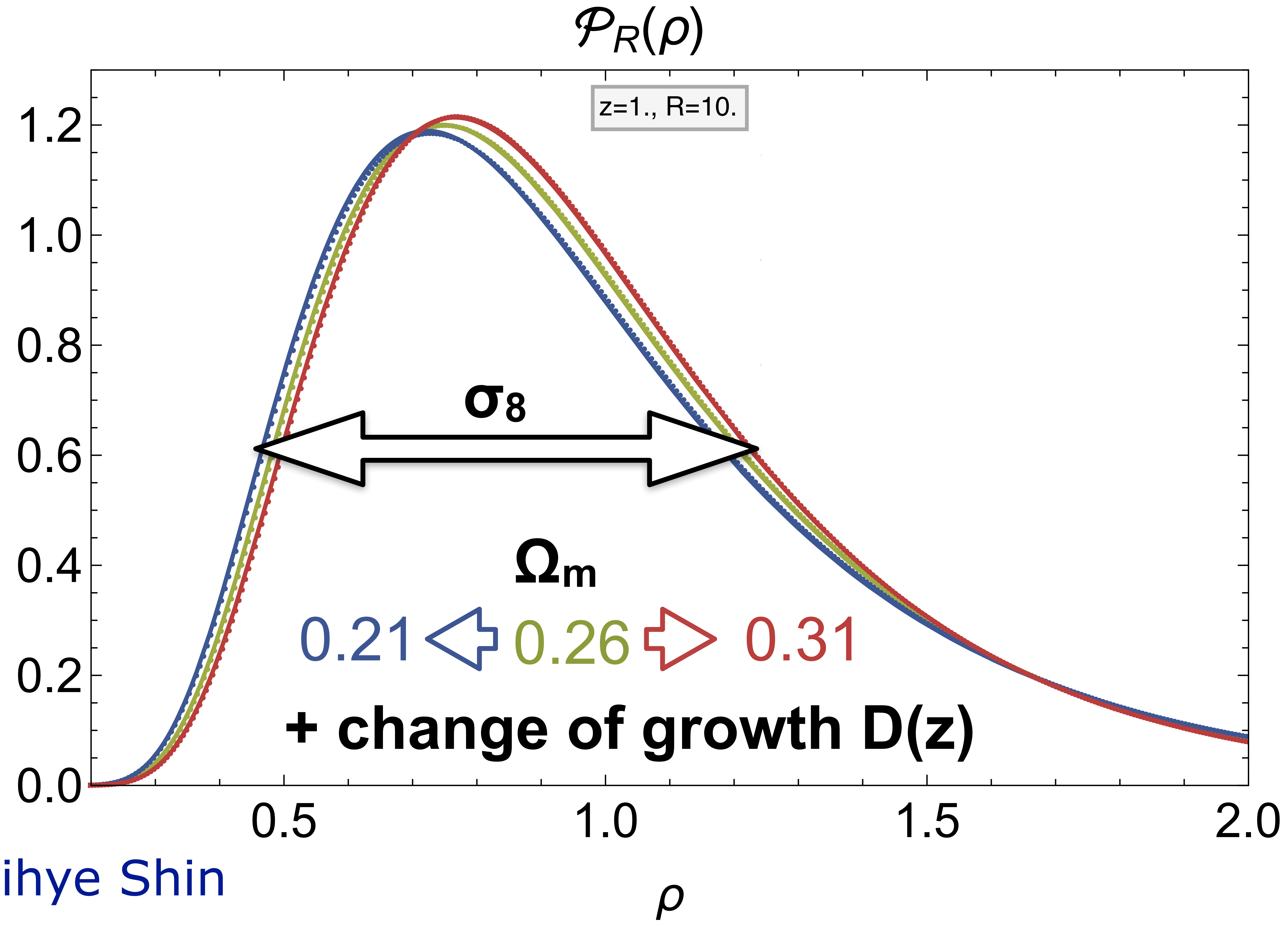
Jed Homer @ LMU

simulations:
Quijote

CU++ 20

Dark Matter 1-Point Statistics

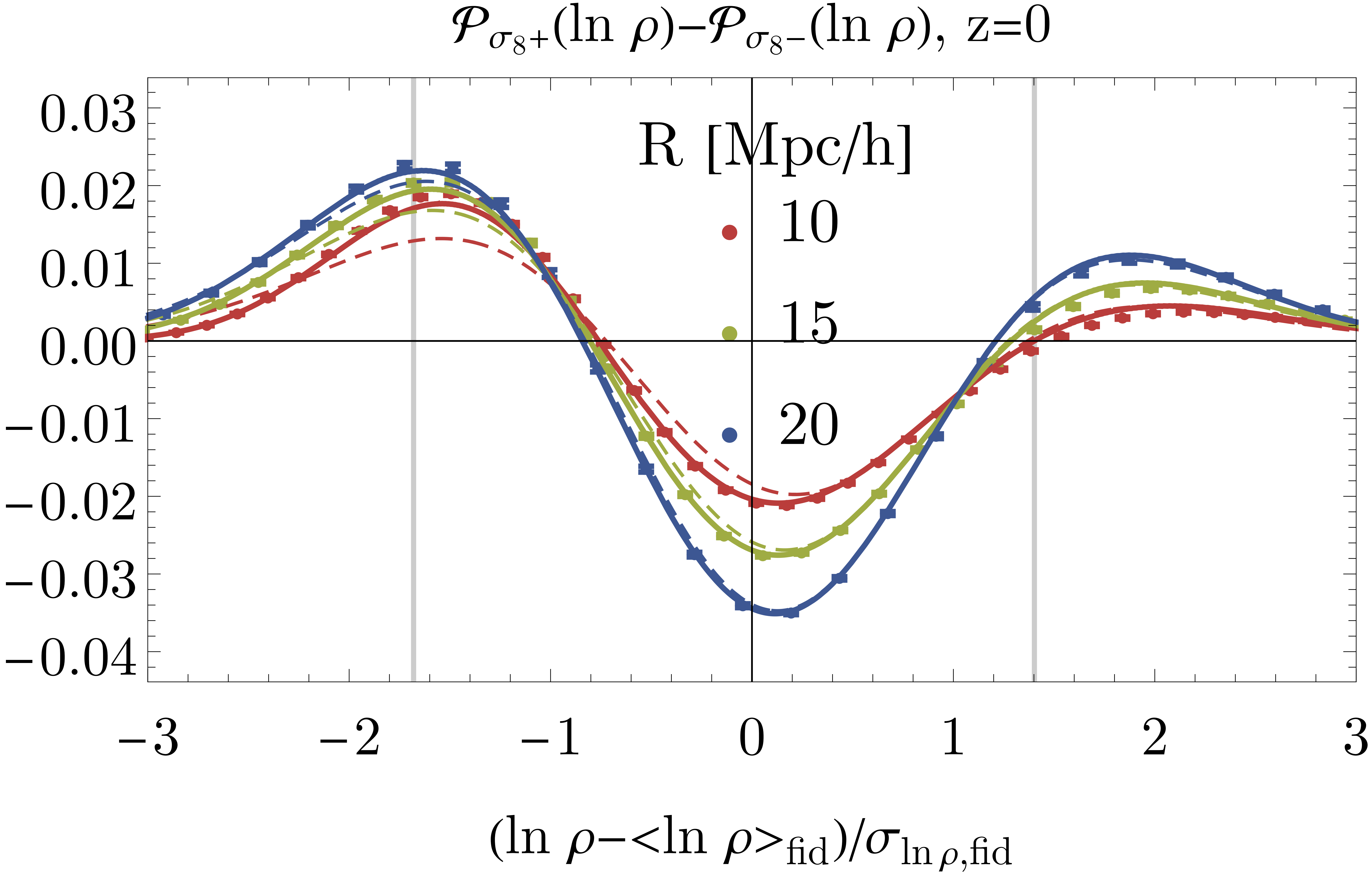
cosmology dependence - matter content σ_8 & Ω_m: width & tilt



sims: Jihye Shin

Dark Matter 1-Point Statistics

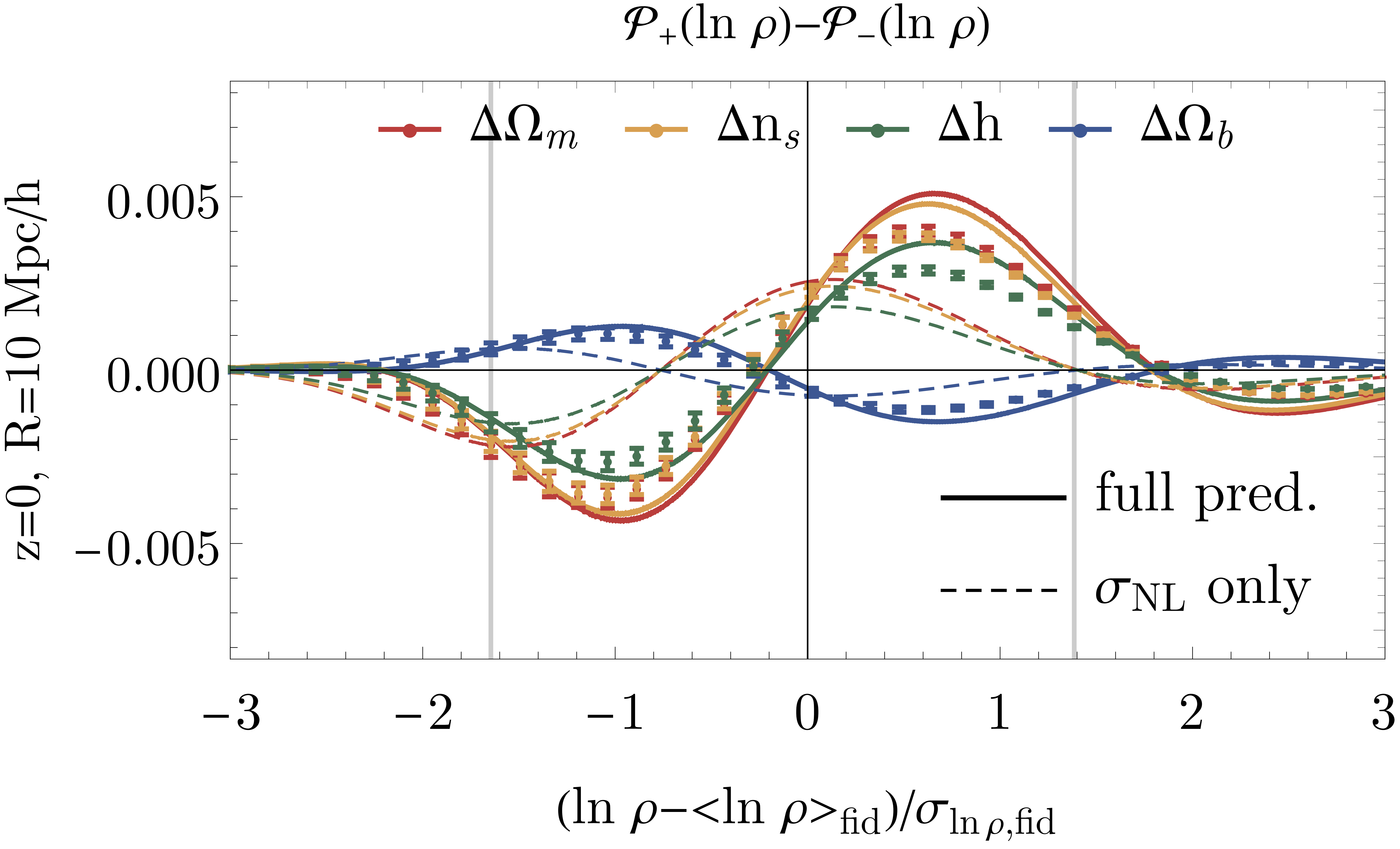
cosmology dependence - clustering amplitude σ_8: width



CU++ 20

Dark Matter 1-Point Statistics

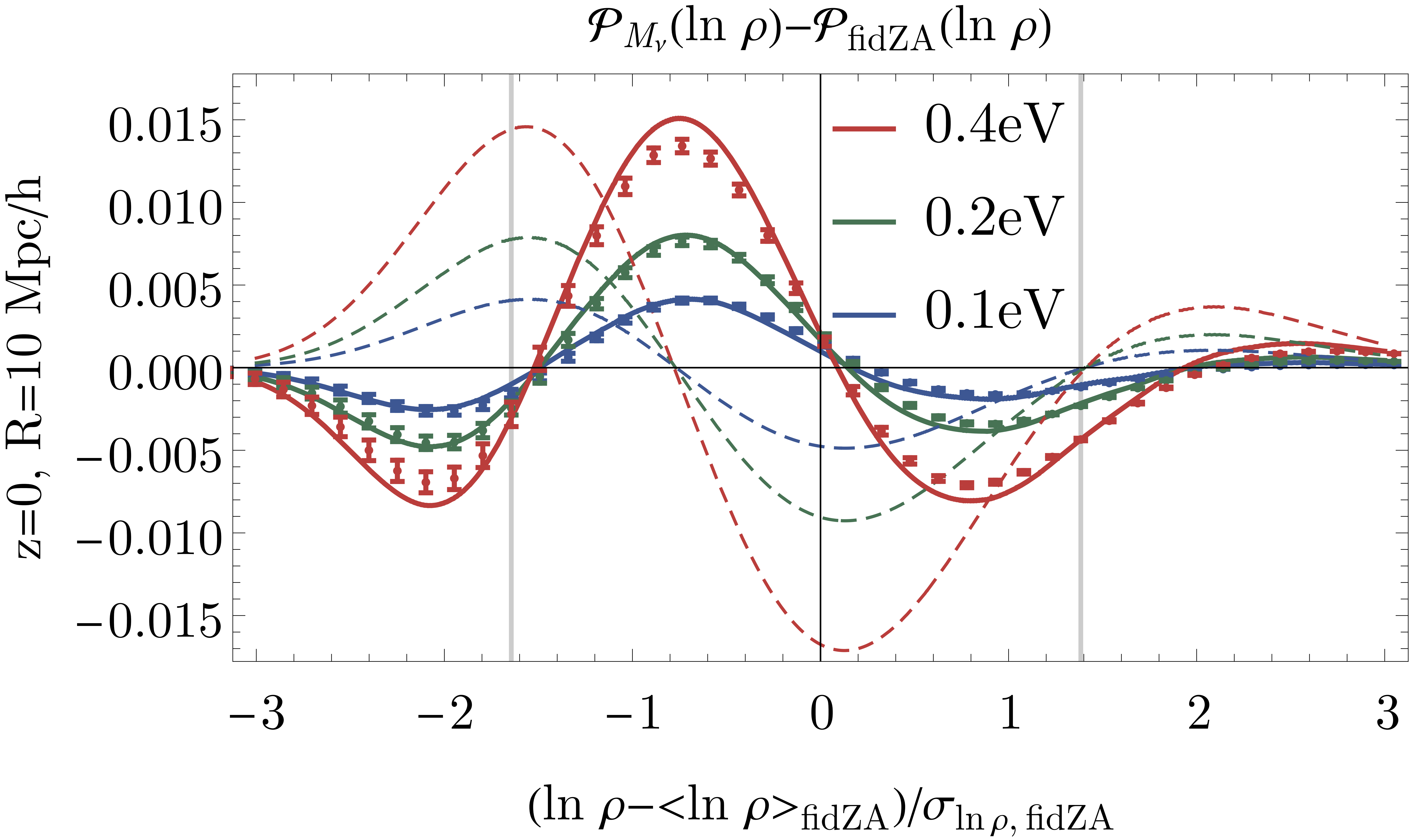
cosmology dependence - matter density Ω_m & initial n_s: tilt



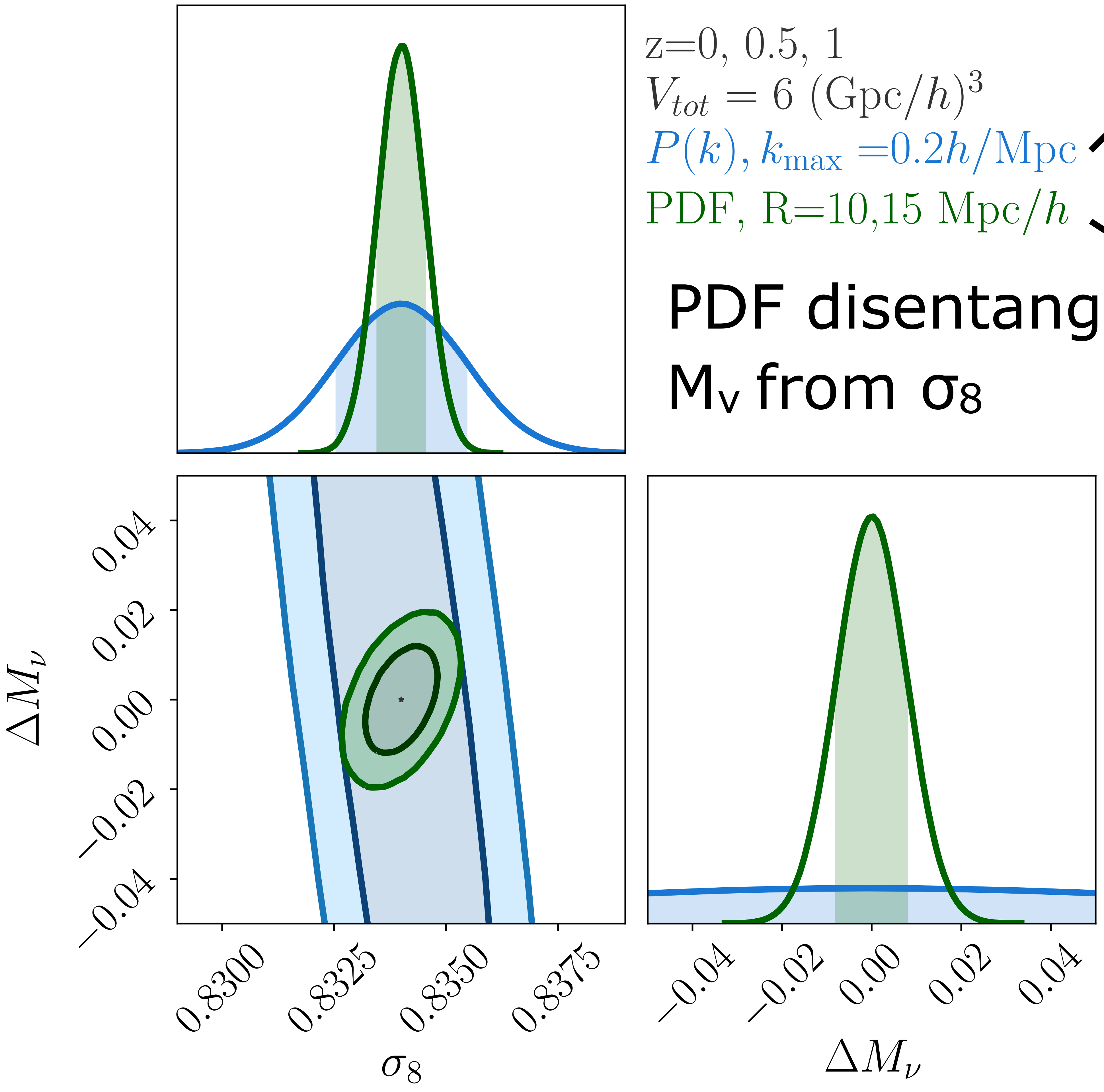
CU++ 20

Dark Matter 1-Point Statistics

cosmology dependence - neutrino mass M_ν: density dependence



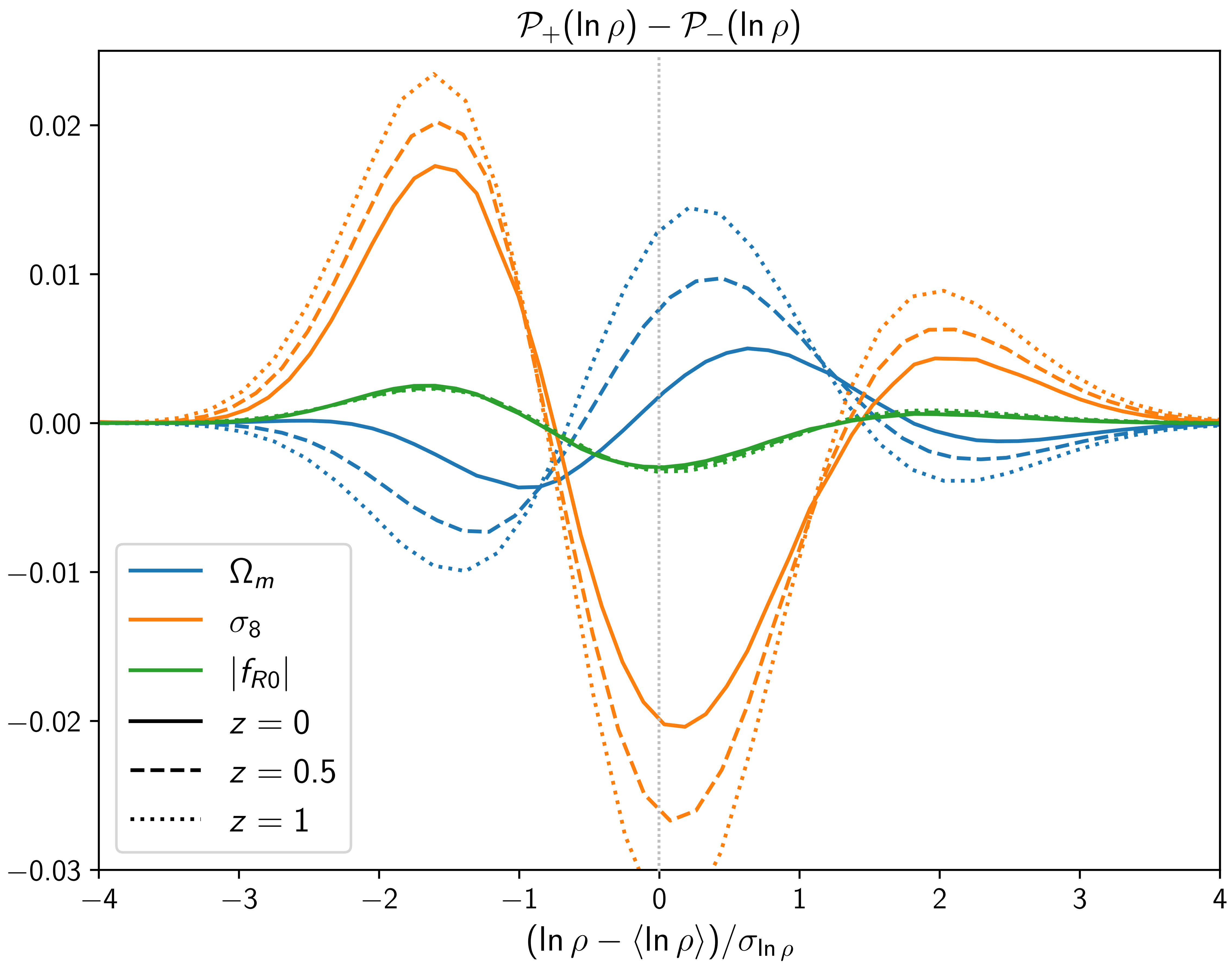
Dark Matter 1-Point Statistics

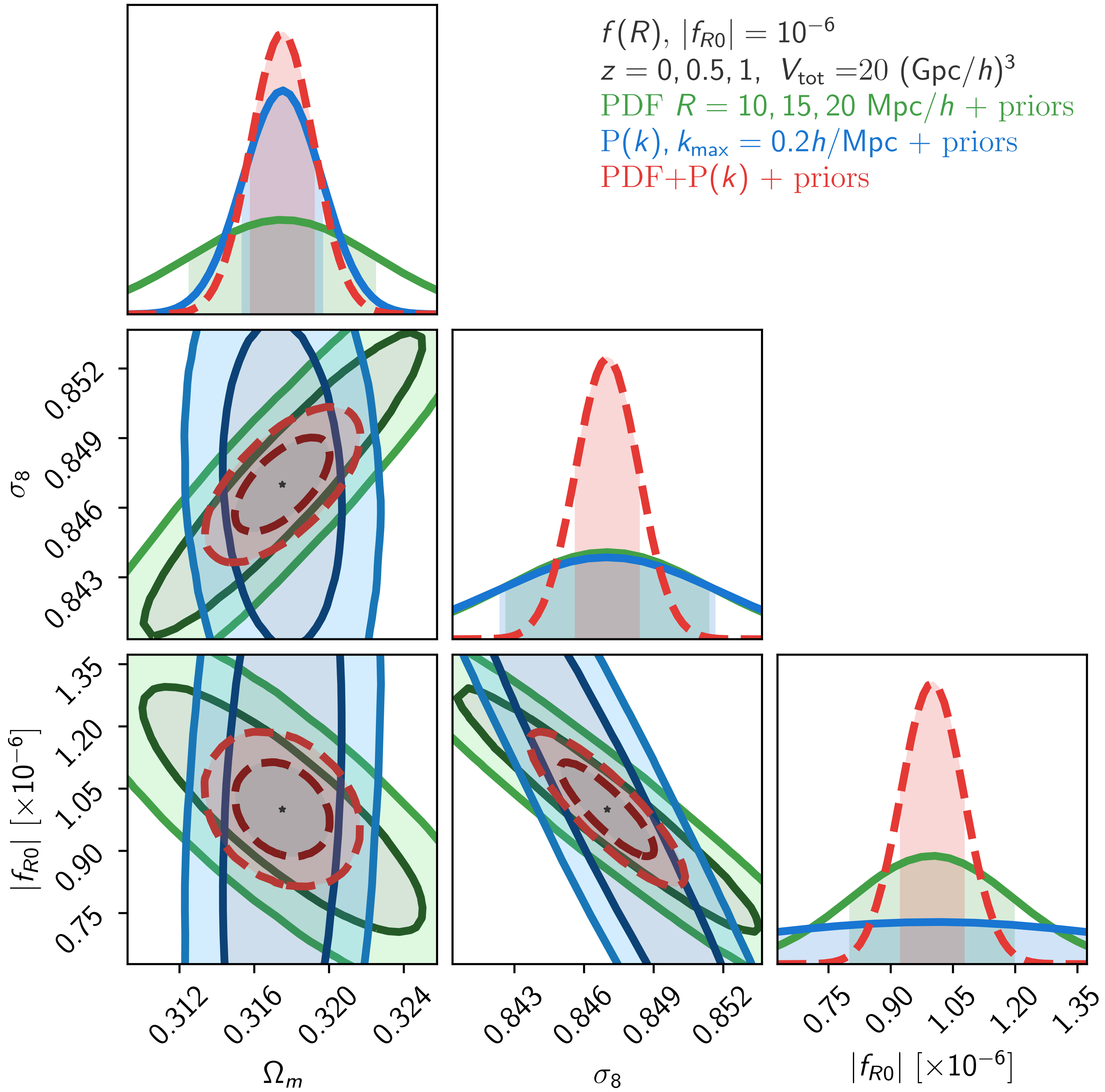


PDF disentangles M_ν from σ_8

Dark Matter 1-Point Statistics

modified gravity: density dependence





Cataneo, **CU** ++ 21

forecast with theory derivatives

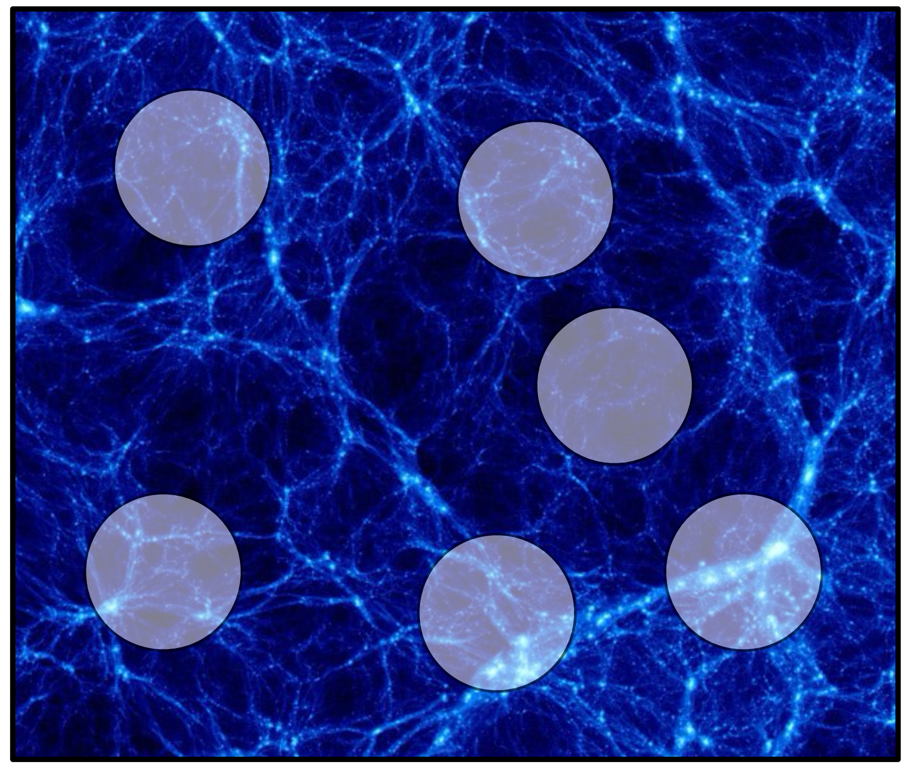
by Alex Gough

Dark Matter 1-Point Statistics

Powerful statistics

non-Gaussian, beyond perturbation theory

robust & accurate predictions



Cosmology & fundamental physics

$\boldsymbol{\Omega_m, \sigma_8, M_\nu}$ **CU**, Friedrich++ 19

$\boldsymbol{f_{NL}}$ Friedrich, **CU**++ 19
Coulton++ 24 (kNN)

$\boldsymbol{w_{0,a}, \Omega_{rc}, f_{R0}}$ Cataneo, **CU**++ 21

Reality: no 3D matter field

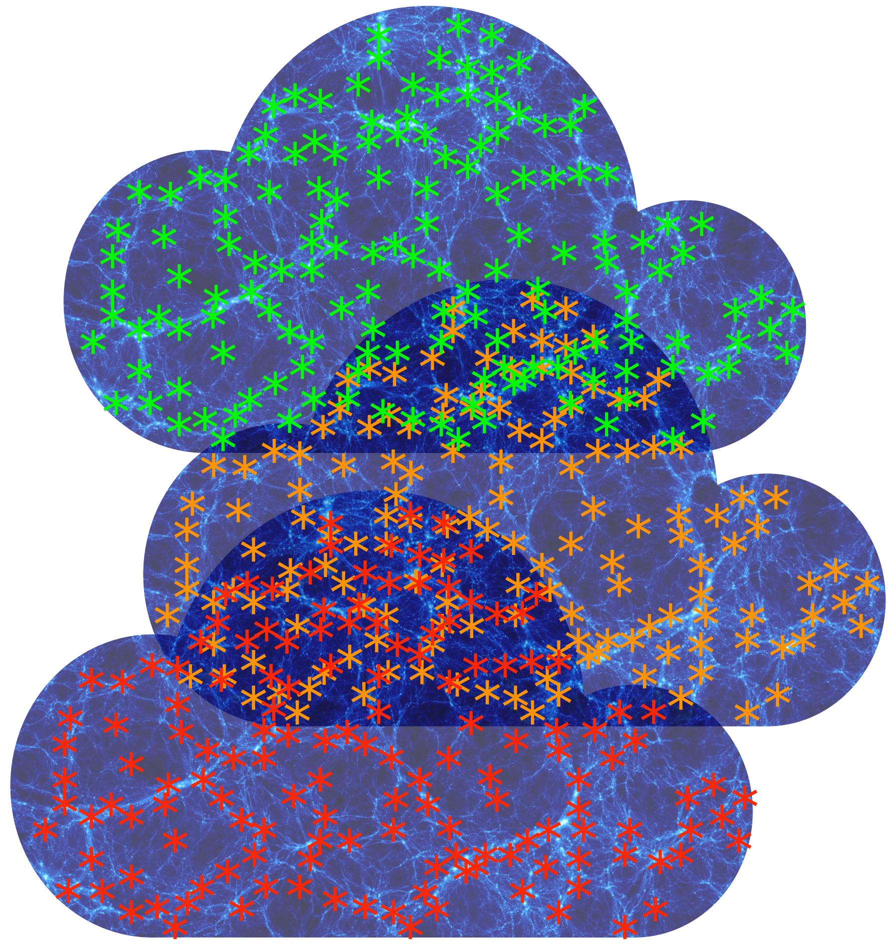
weak lensing: projected matter

galaxy clustering: bias & stochasticity

Clustering Statistics

1-point clustering statistics: observables

3D



(dark) matter

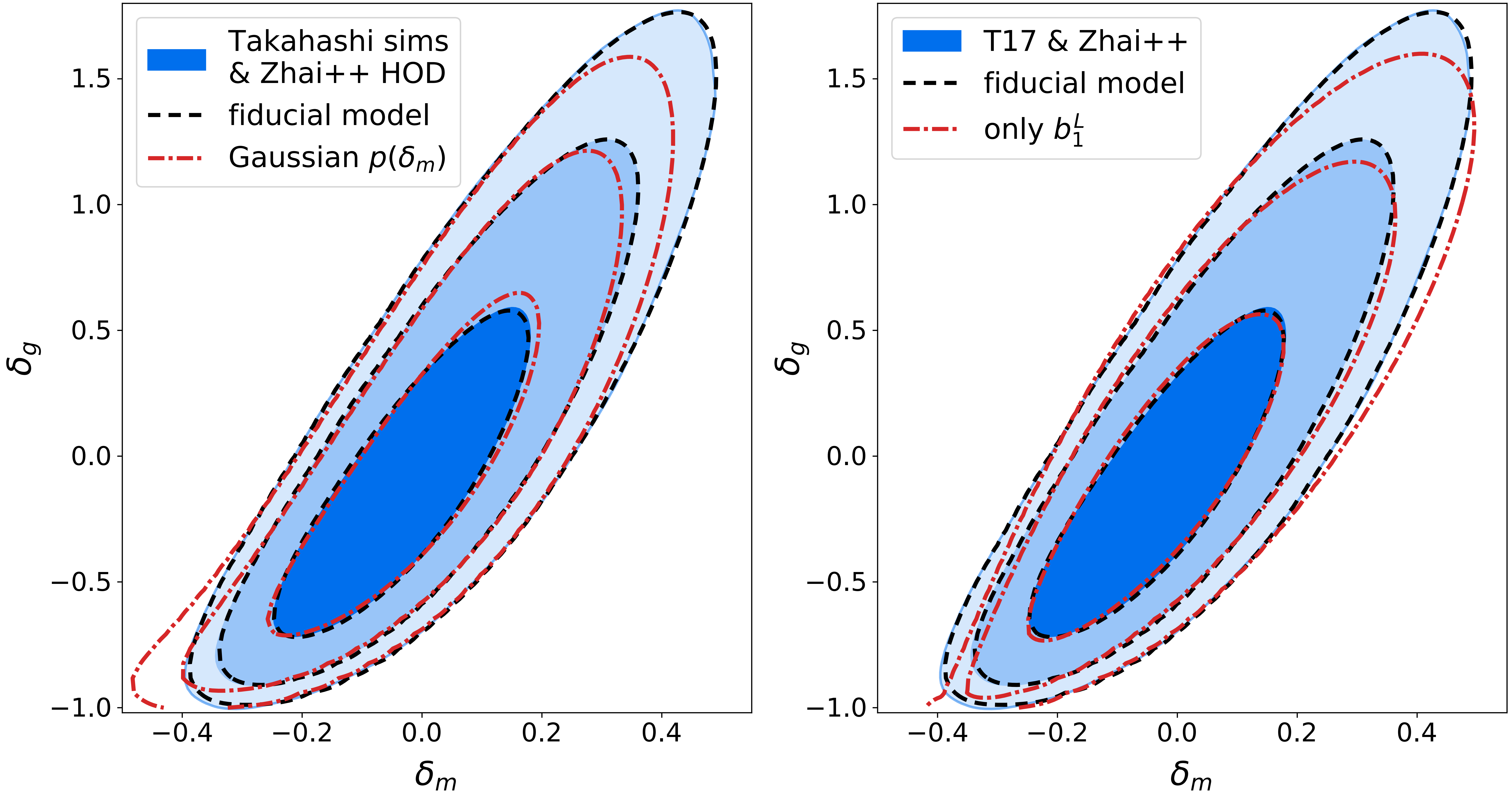
spec-z tracer counts (N_{spec})

PDF $\mathcal{P}(N_{spec})$

+ 2-point analysis

Tracer 1-Point Statistics

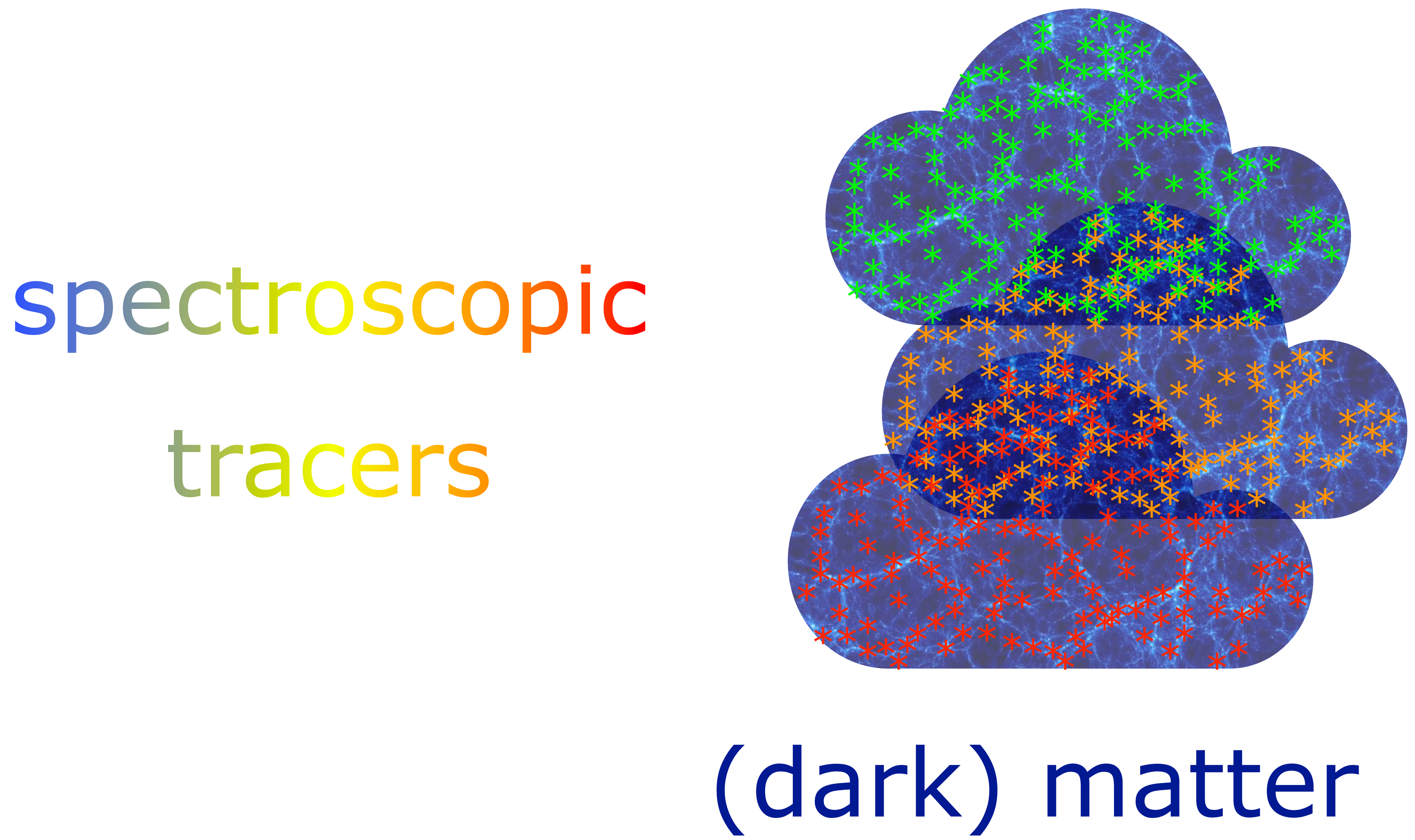
challenge: non-Gaussian matter & nonlinear bias



computed with CosMomentum code Friedrich+(**CU**)+ 21

Tracer 1-Point Statistics

dark matter -> galaxy clustering

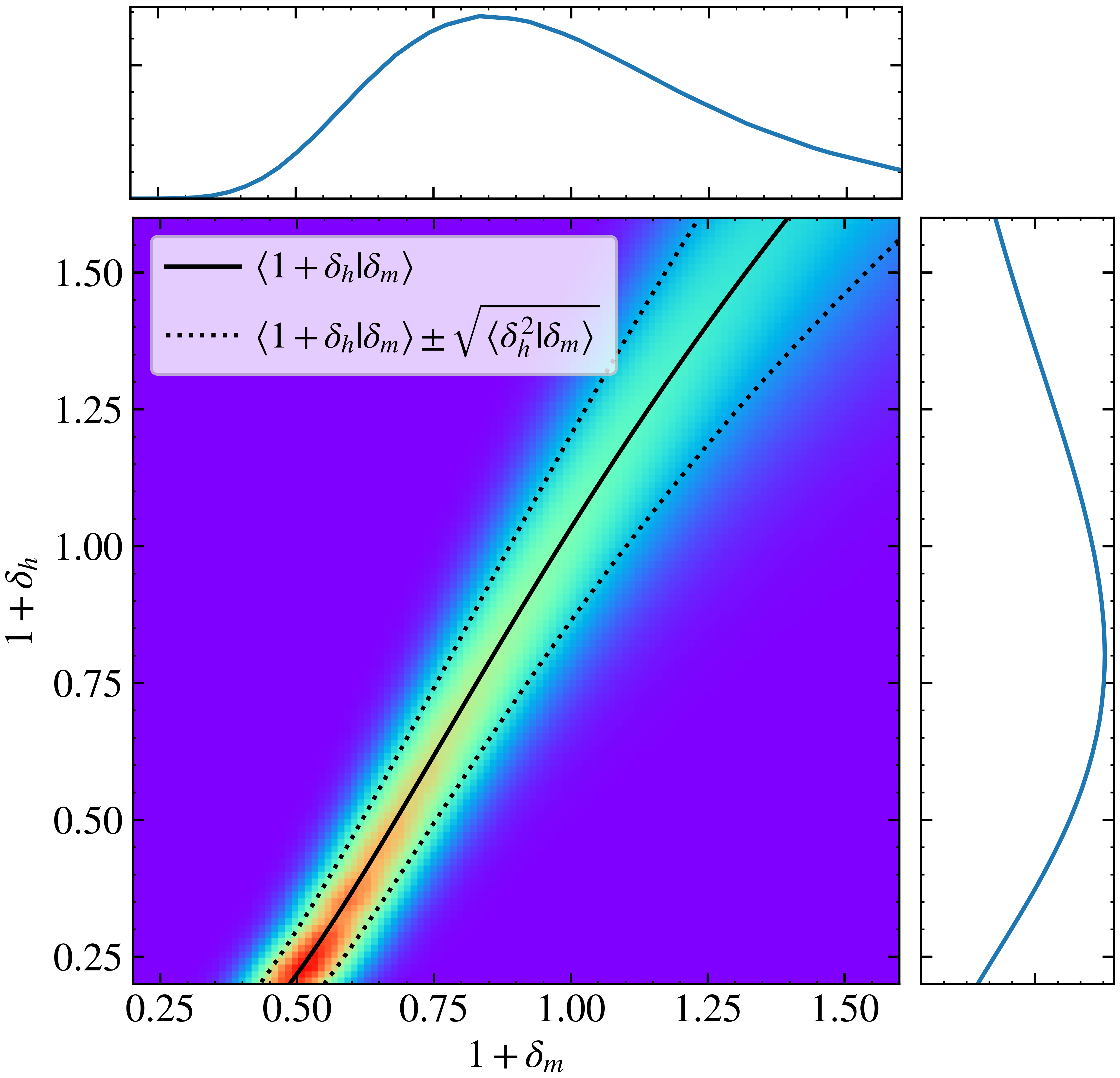


$$\mathcal{P}_t(N_t) = \int \mathcal{P}(N_t|\delta_m)\mathcal{P}_m(\delta_m)d\delta_m$$

parametrise:

nonlinear bias $<\delta_h|\delta_m>$

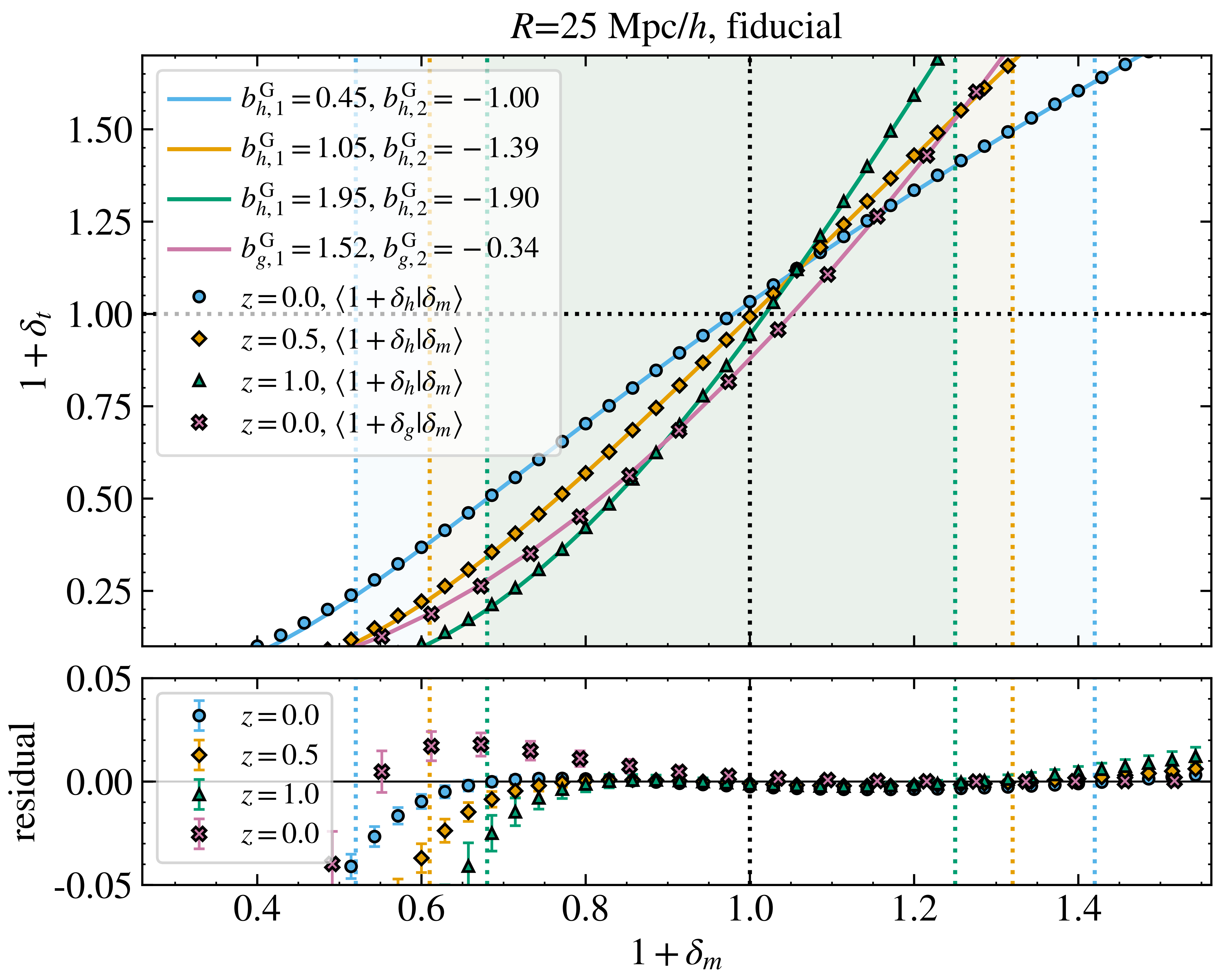
non-Poissonian shot noise $<\delta_h^2|\delta_m>$



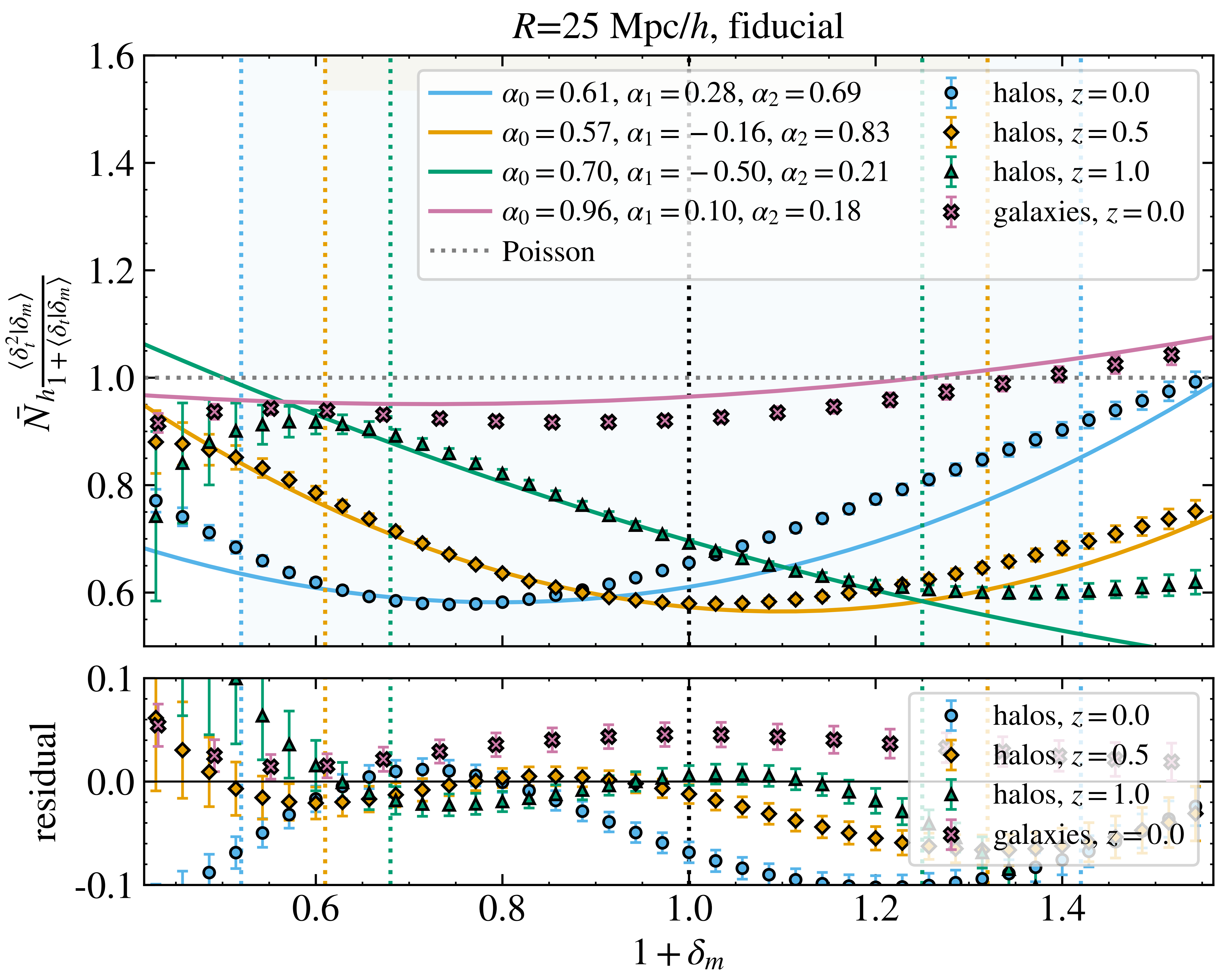
Beth Gould et al. 2024

Tracer 1-Point Statistics

challenge: nonlinear bias $\langle\delta_h|\delta_m\rangle$, non-Poisson shot noise $\langle\delta_h^2|\delta_m\rangle$



Gaussian Lagrangian bias



quadratic stochasticity

Beth Gould et al. 2024

Tracer 1-Point Statistics

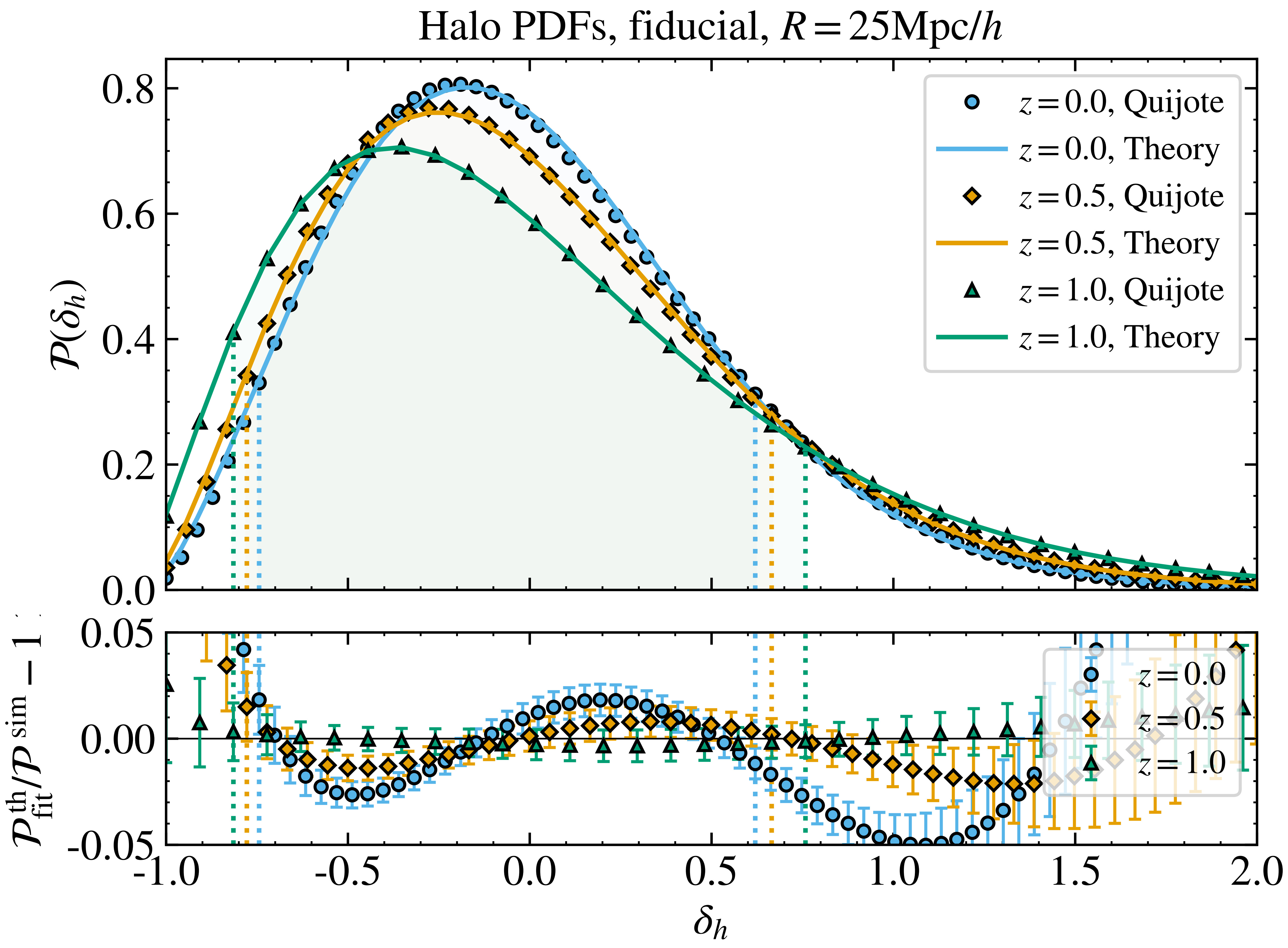
dark matter -> galaxy clustering

real space:

Gaussian local Lagrangian bias

quadratic non-Poisson shot noise

simplified theory SDSS analysis
Repp & Szapudi 2020



Beth Gould et al. 2024

Tracer 1-Point Statistics

dark matter -> intensity mapping

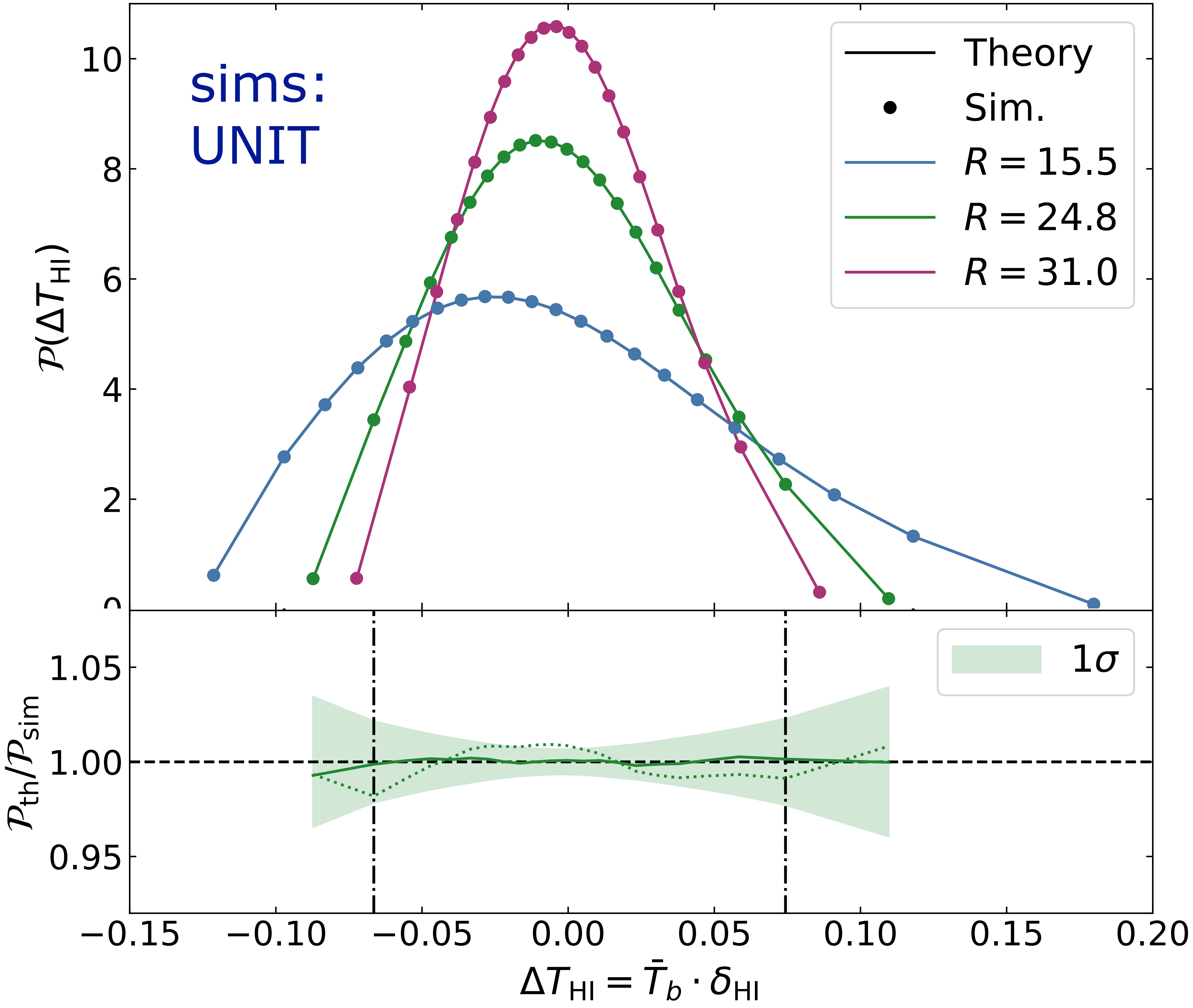
spectroscopic

HI intensity map

(dark) matter

+ systematics

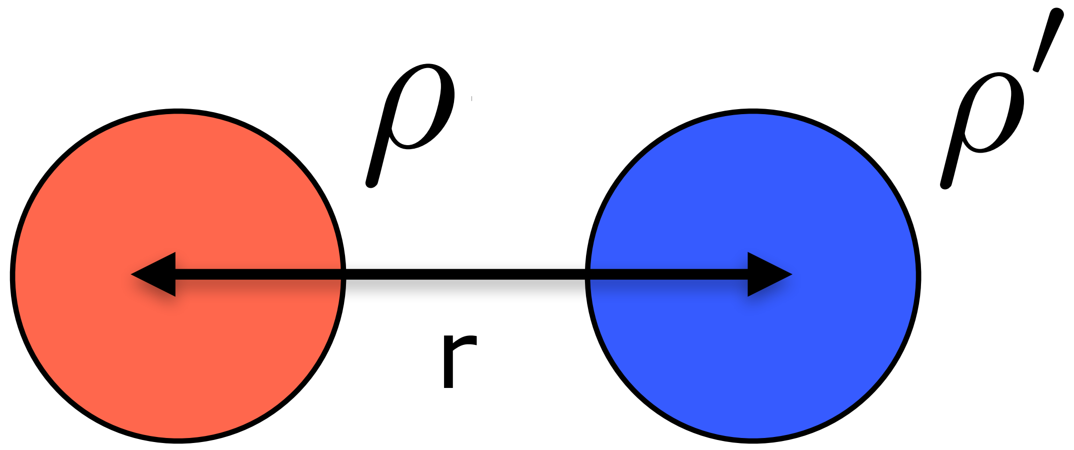
-> Bernhard Vos Gines talk



Density-Split Clustering

local densities modulate correlation

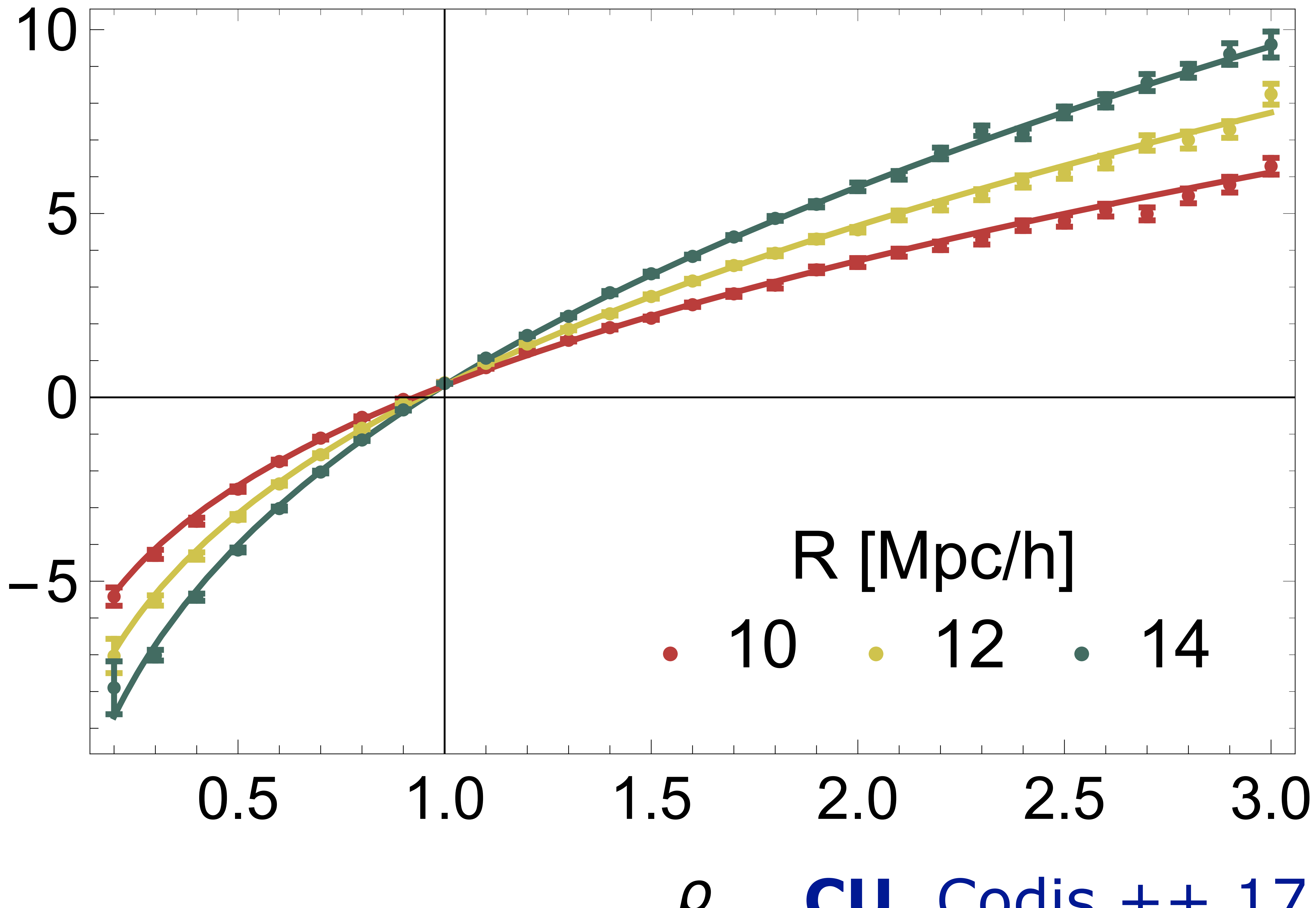
$$\frac{P(\rho(x),\rho'(x+r))}{P(\rho)P(\rho')} = 1 + \xi(r)b(\rho)b(\rho')$$



average density at separation r

$$b_R(\rho) = \frac{\langle \rho'_R(r)|\rho_R\rangle - 1}{\xi_R(r)}$$

large-deviations theory,
Gaussian ICs &
spherical collapse



CU, Codis ++ 17

Density-Split Clustering

dark matter -> galaxy clustering

map with tracer bias function

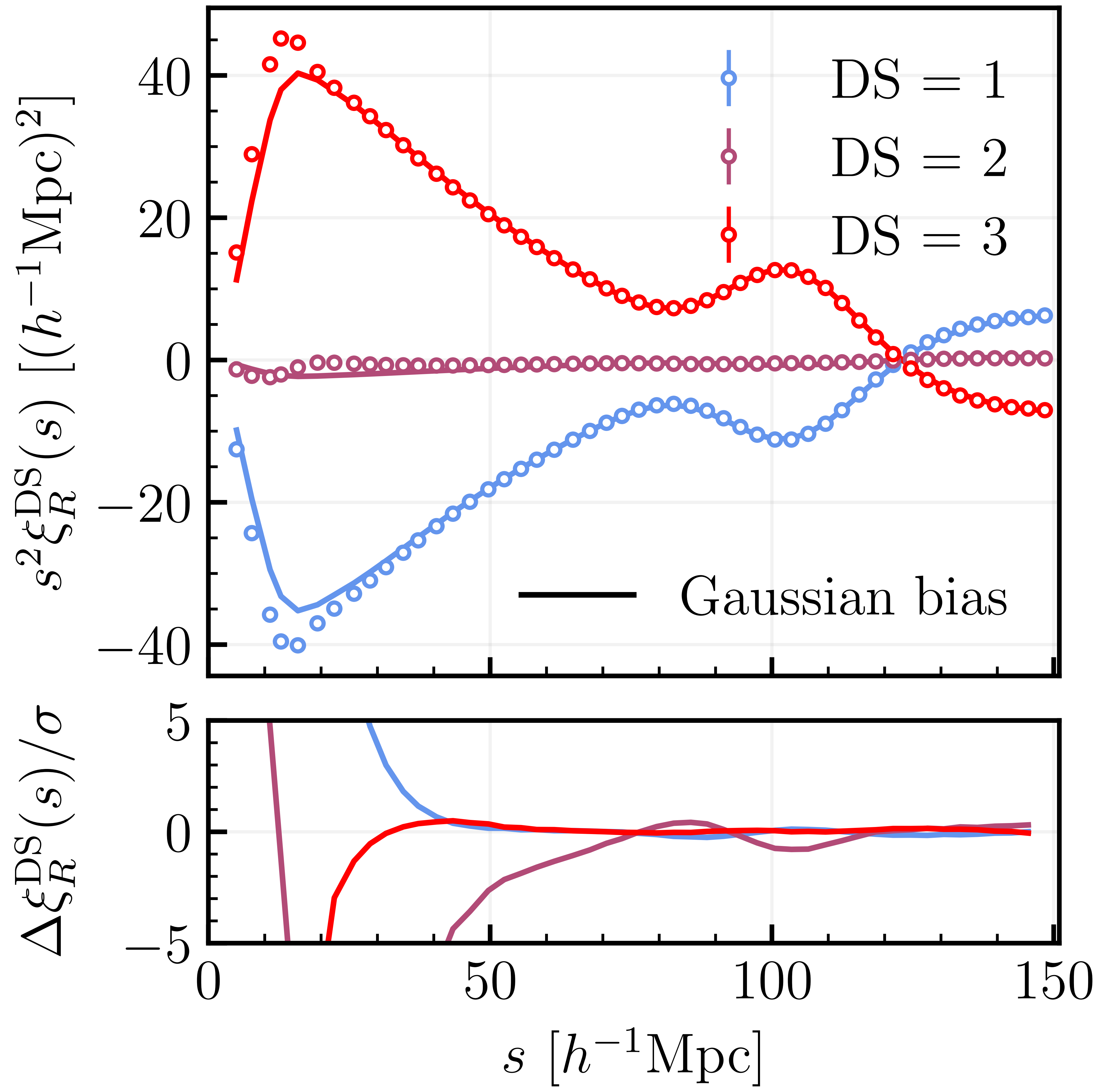
$$\frac{P(\rho(x), \rho'(x+r))}{P(\rho)P(\rho')} = 1 + \xi(r)b(\rho)b(\rho')$$

SBI-based analysis of BOSS CMASS

Paillas et al. 2024 (to 1 Mpc/h)

Mathilde Pinon et al. 2025

ELG-like HOD



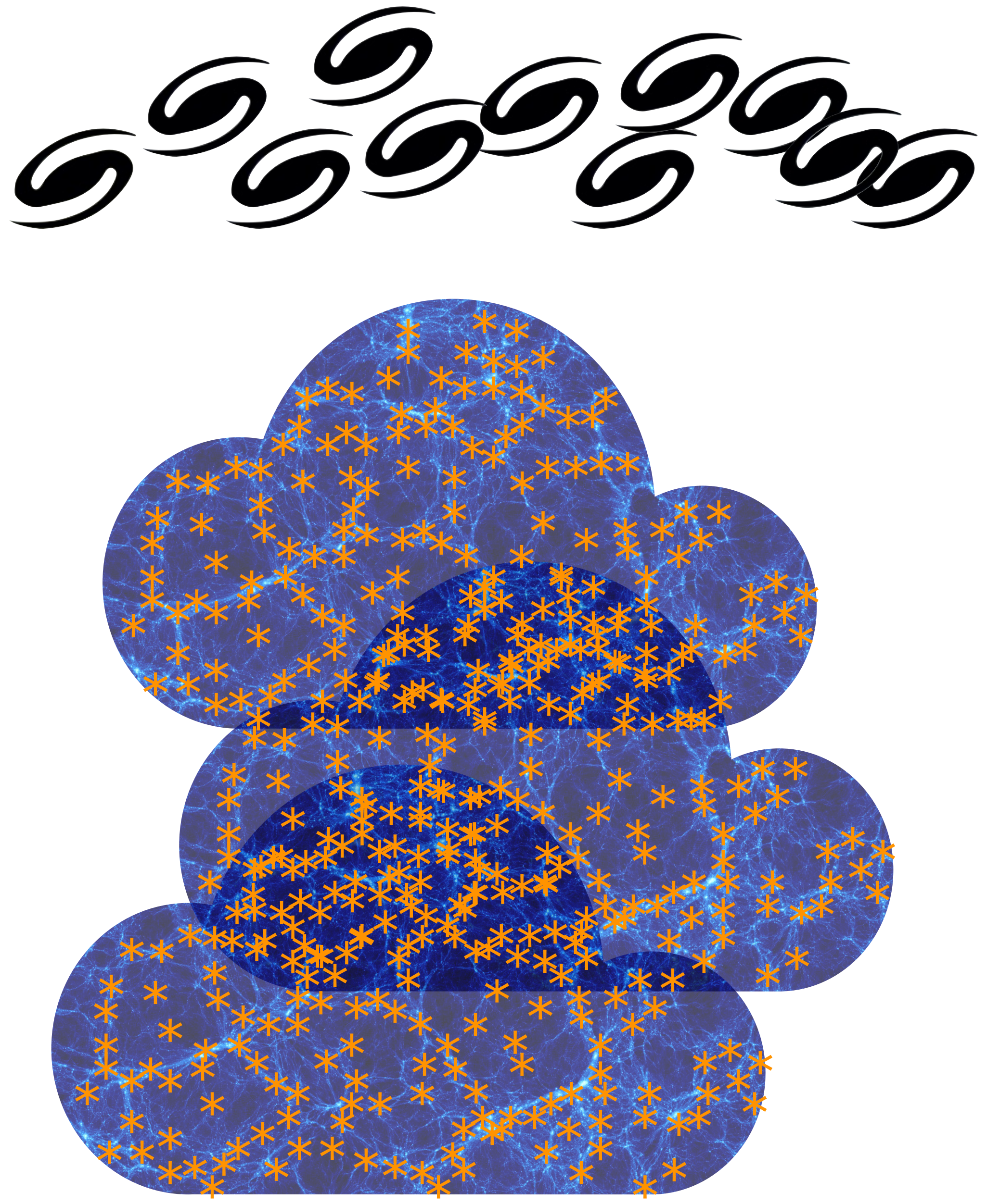
Clustering Statistics

1-point clustering statistics: observables

2D

galaxy shapes
(κ)

photo-z galaxy
counts (N_{ph})

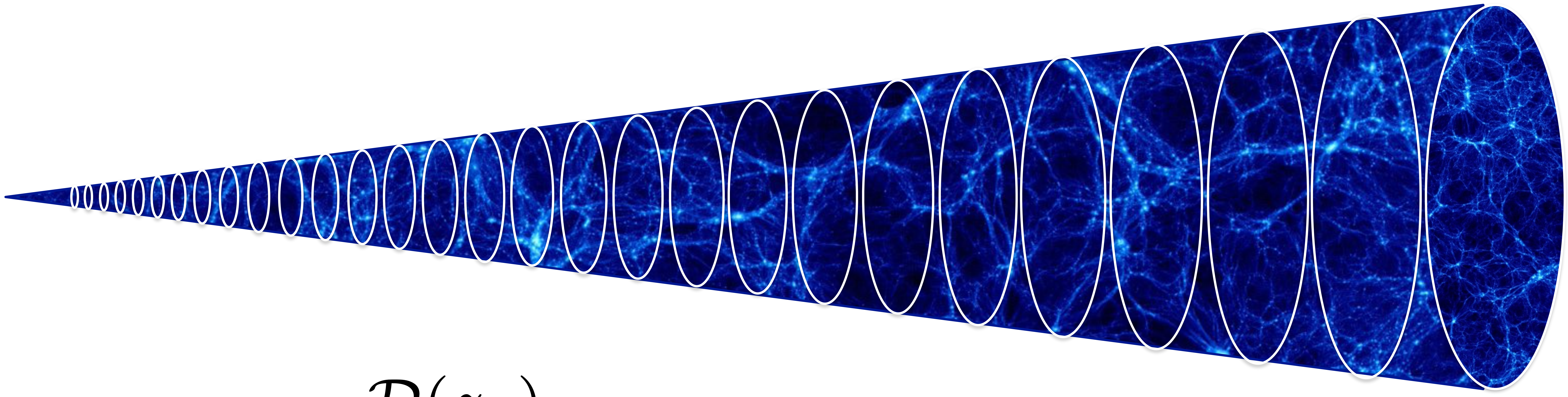


joint PDF $\mathcal{P}(\kappa, N_{ph})$
+ 3x2-point analysis

(dark)
matter

Projected 1-Point Statistics

weight 2D density slices



$$\left.\begin{array}{c}\kappa_{<\theta}\\ \delta_{g,<\theta}\end{array}\right\}=\int_0^{\mathcal{D}(z_s)} d\mathcal{D}(z)\,\delta^{\text{disk}}_{<\theta\mathcal{D}(z)}\,w(z,z_s)$$

cylindrical collapse

weights

→ construct PDF

can also build aperture mass

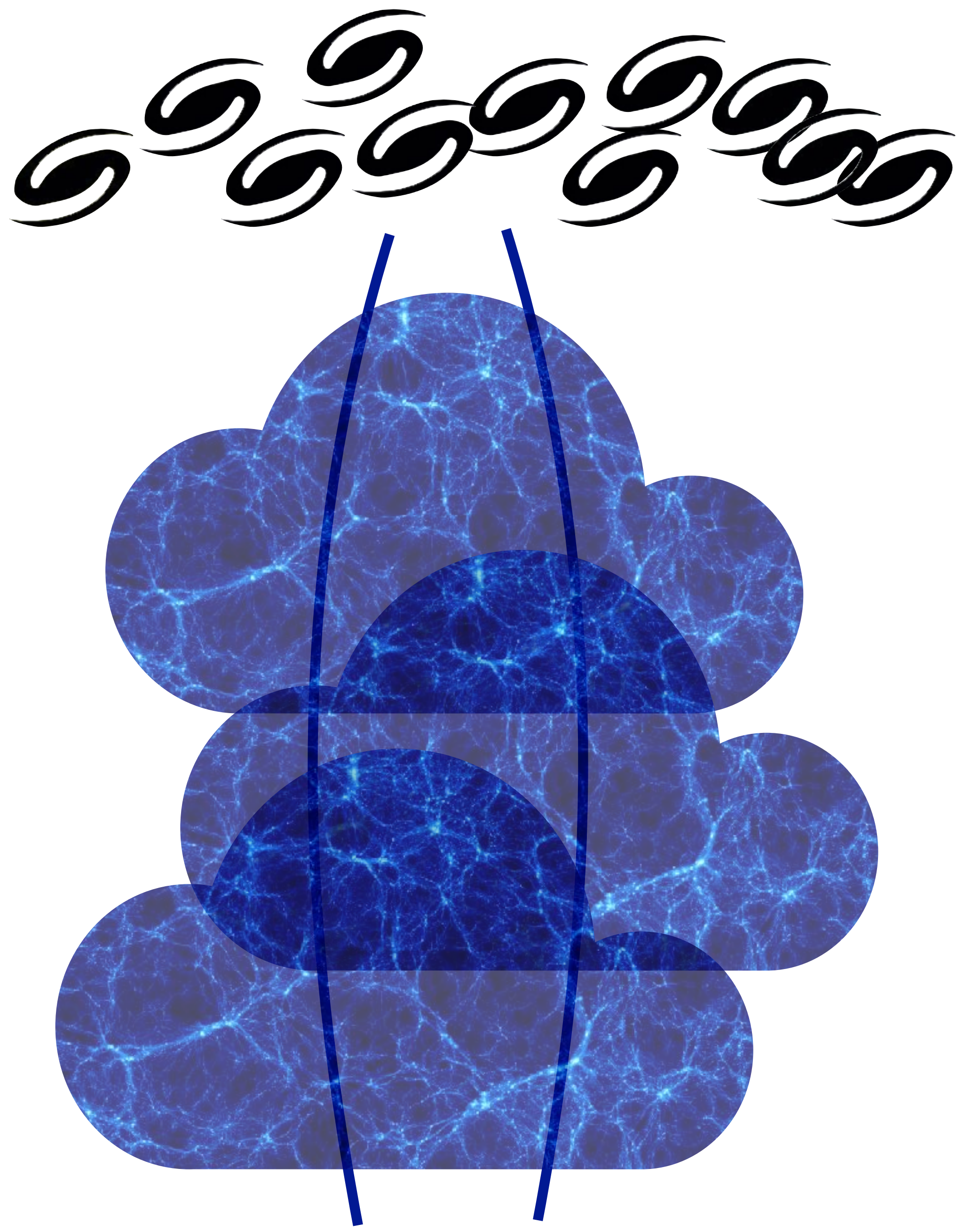
Bernardeau & Valageas `00
Barthelemy et al. '19

Weak Lensing 1-Point Statistics

dark matter -> weak lensing

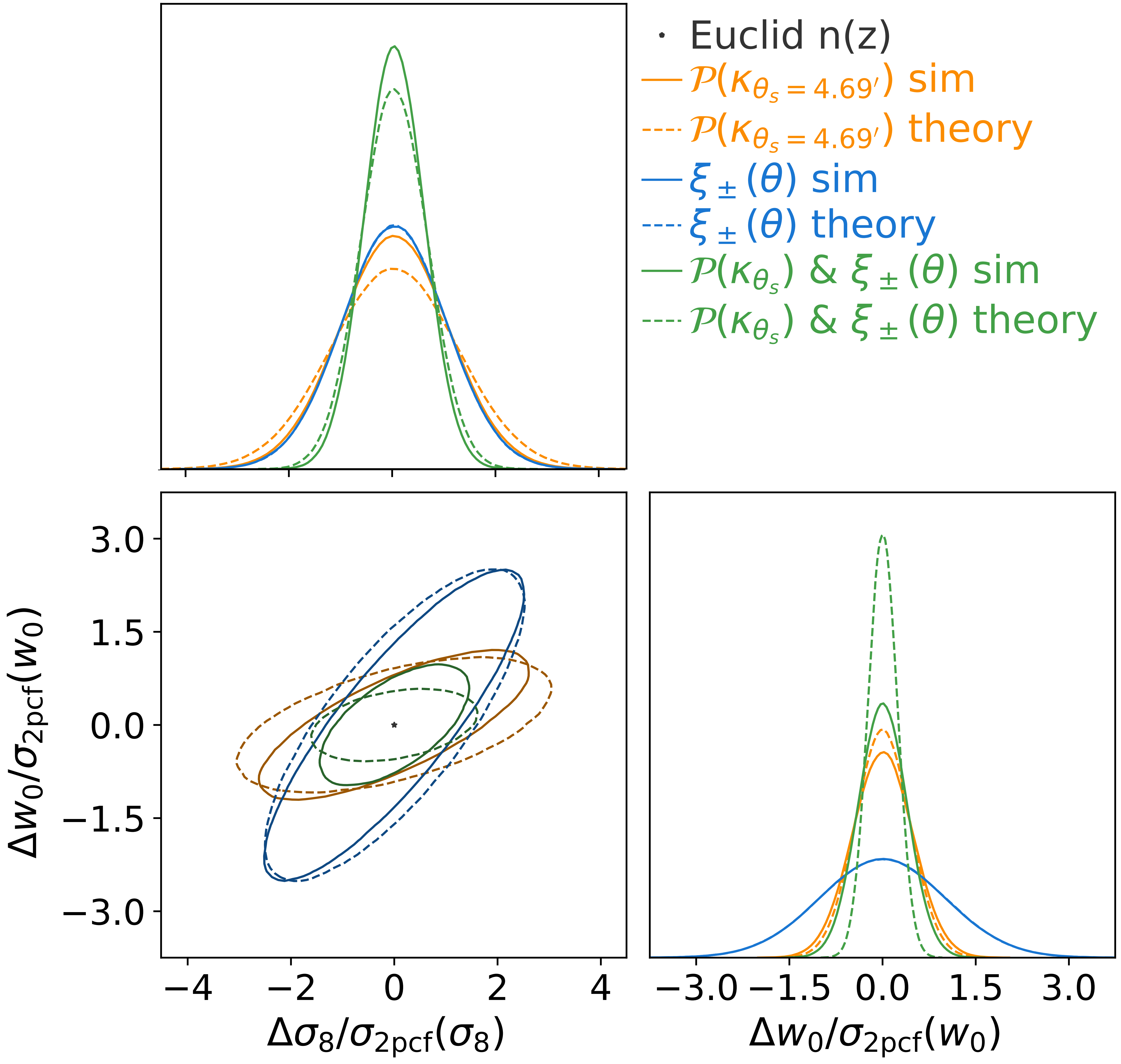
galaxy shear

-> convergence



(dark) matter

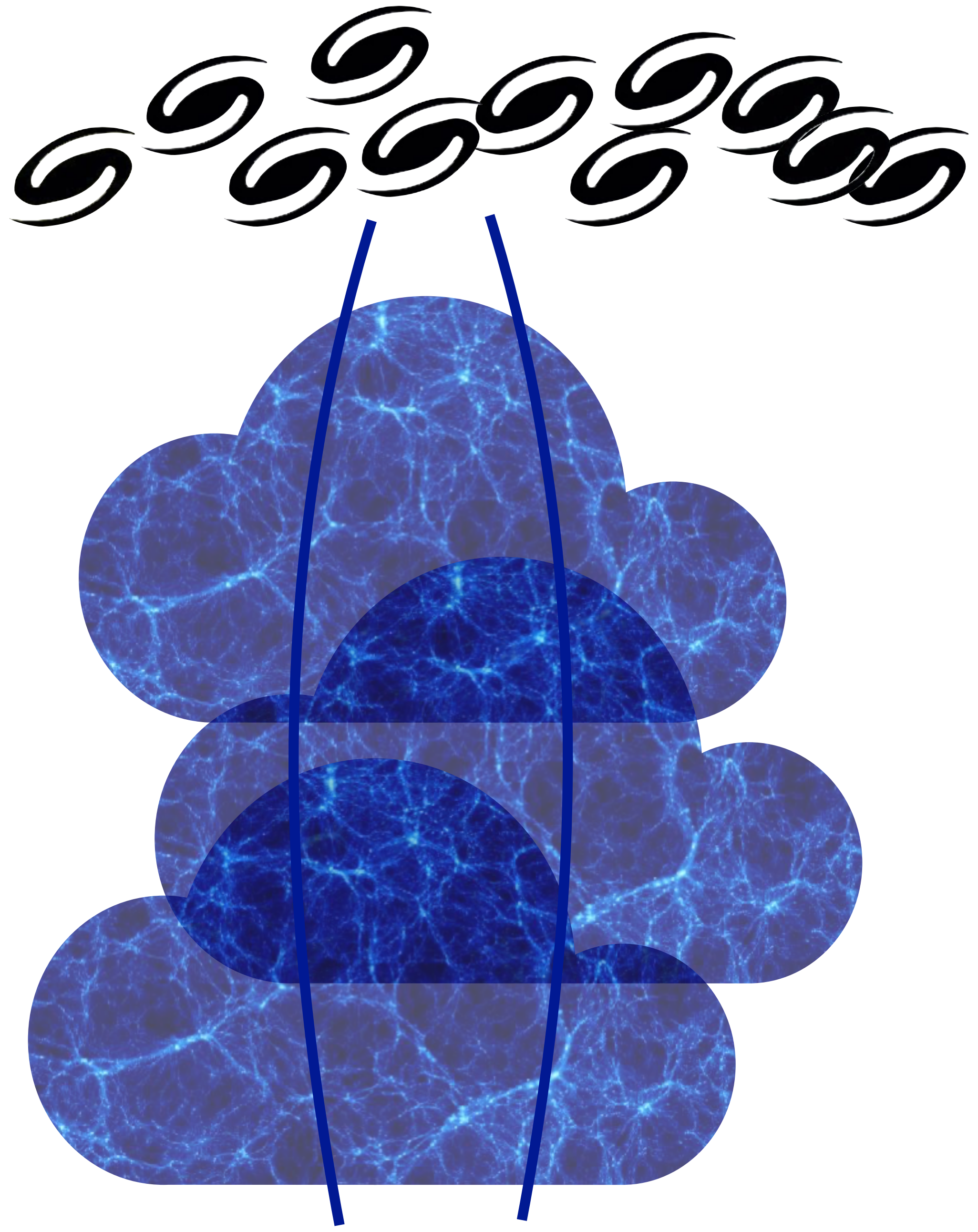
Euclid Overview 2025



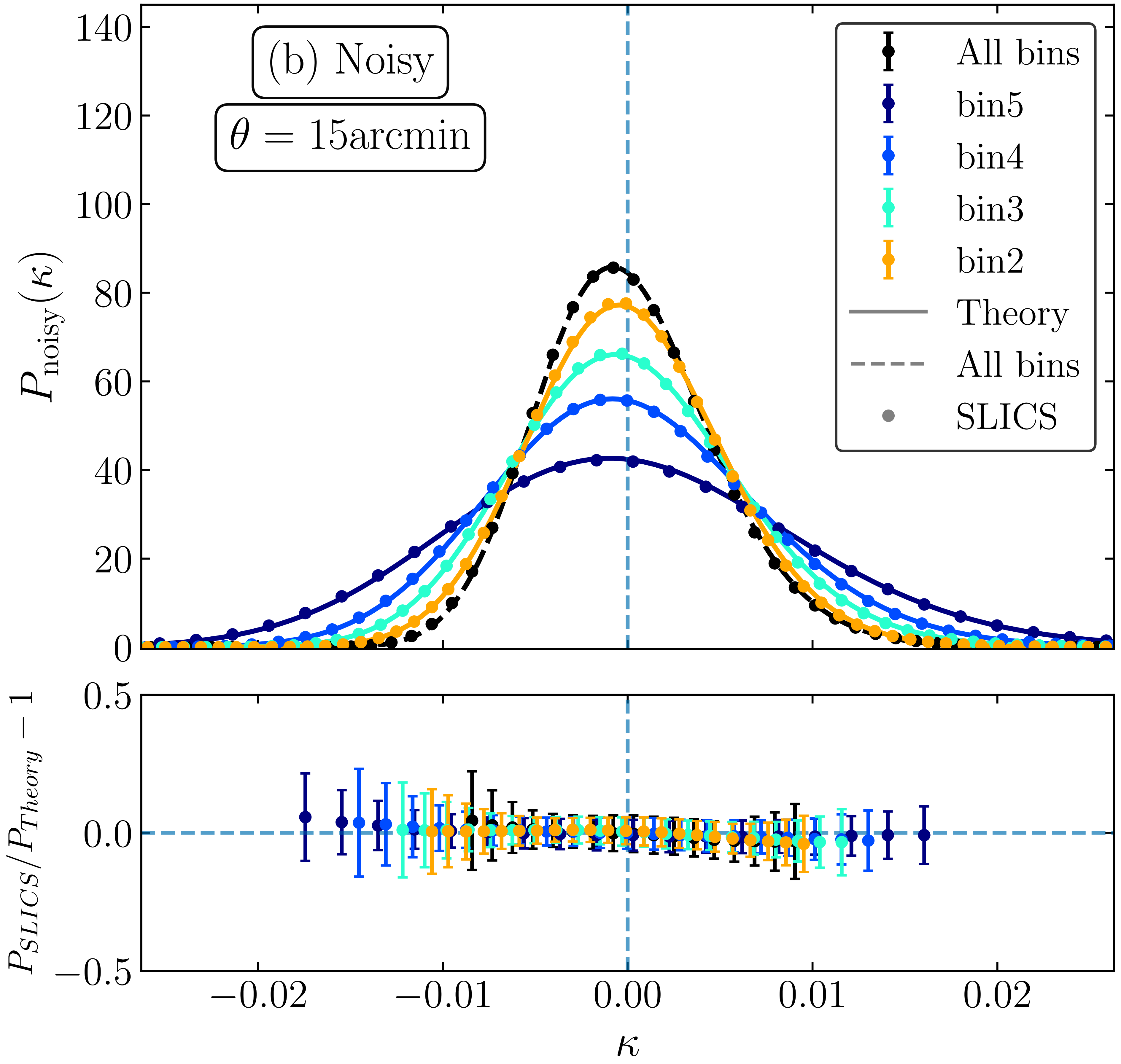
Weak Lensing 1-Point Statistics

Alexandre Barthelemy et al. 2023

challenges
mass mapping: Kaiser-Squires
IA: NLA
baryons: κ/σ

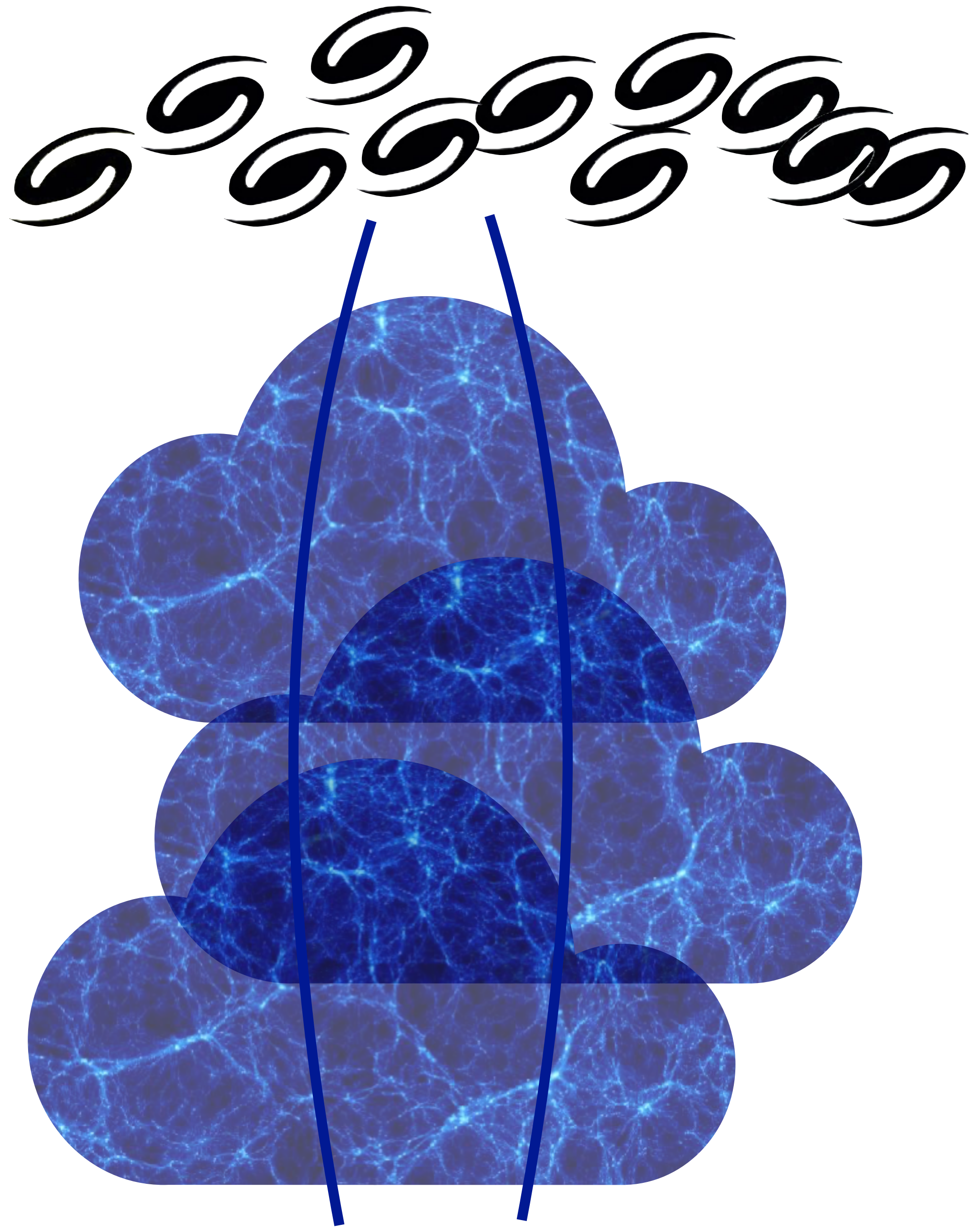


sources Euclid-like n(z) 5 bins



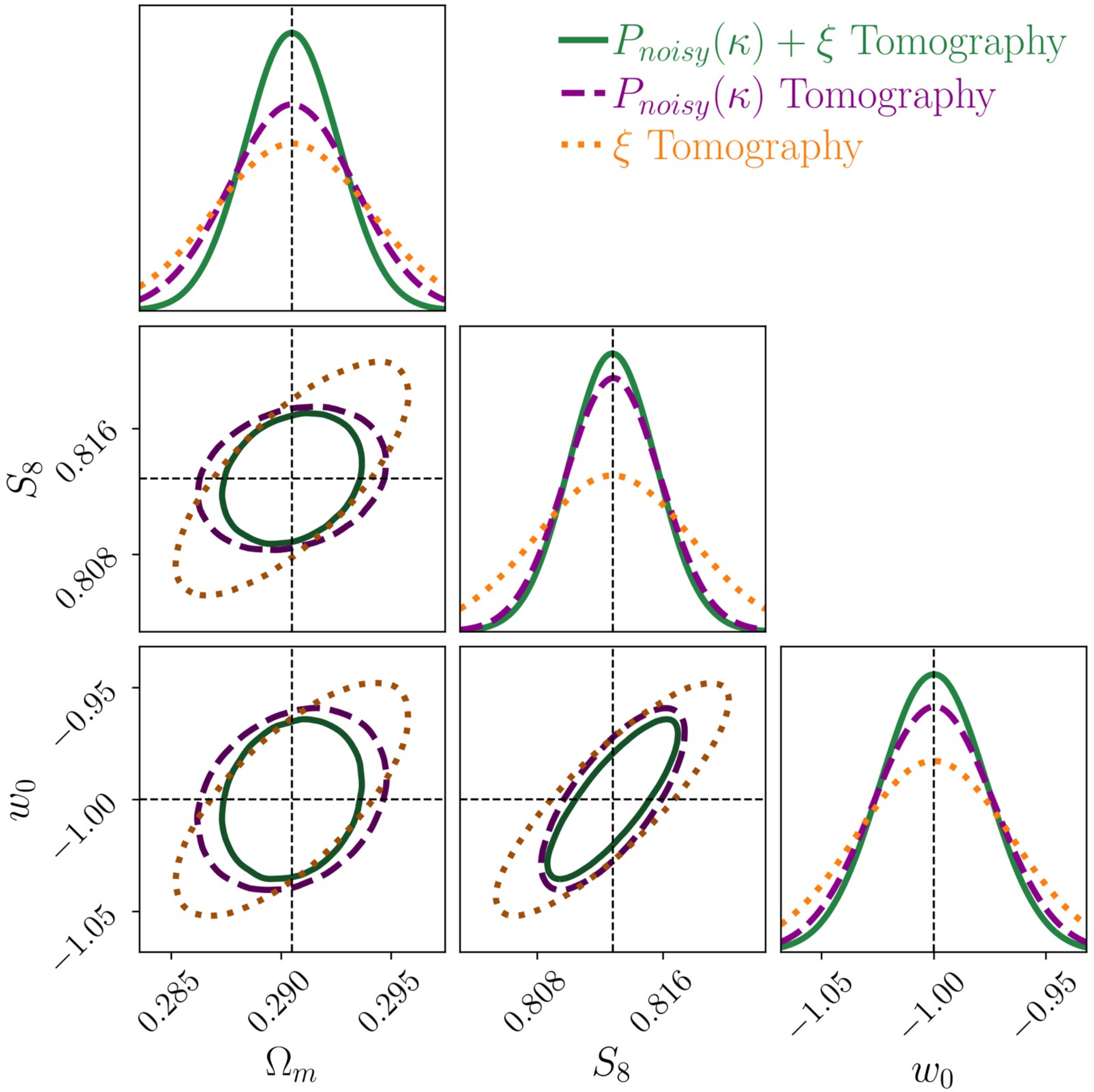
Lina Castiblanco et al. 2024

Weak Lensing 1-Point Statistics



(dark) matter

Lina Castiblanco et al. 2024



dark matter

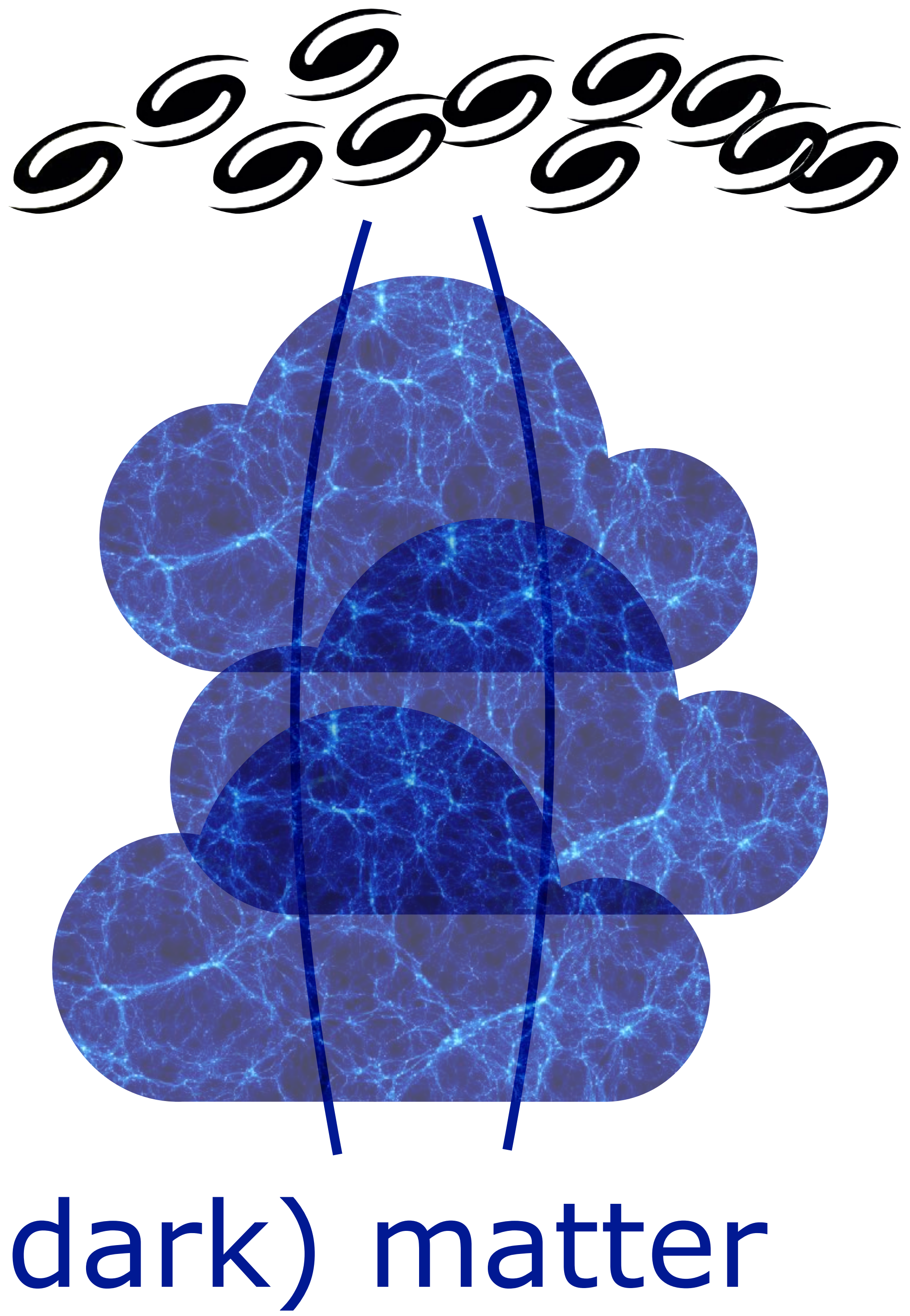
dark energy

Higher-order Lensing Statistics

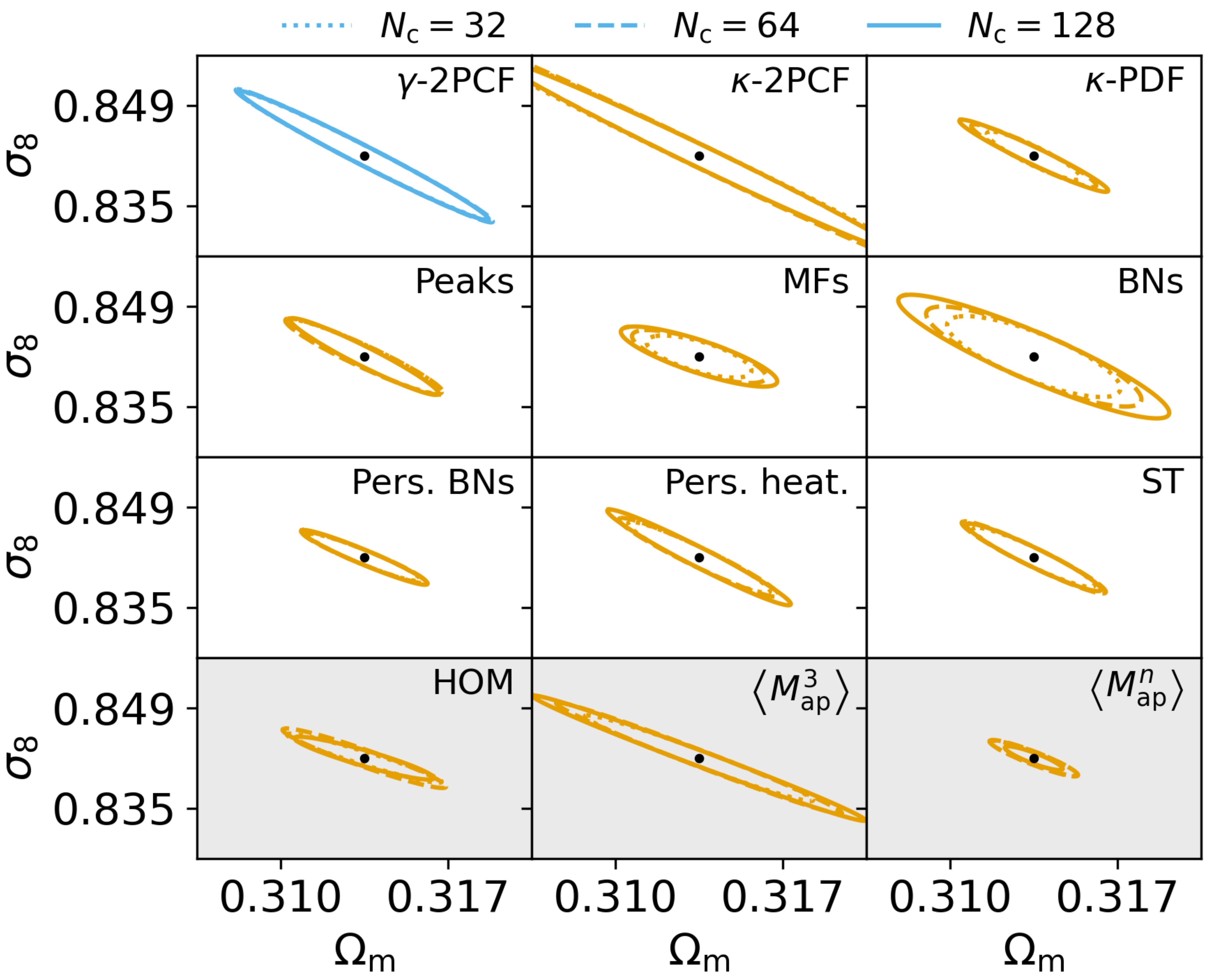
dark matter -> weak lensing

galaxy shear

-> convergence



Euclid Preparation XXIX: Forecasts for 10 different WL HOS



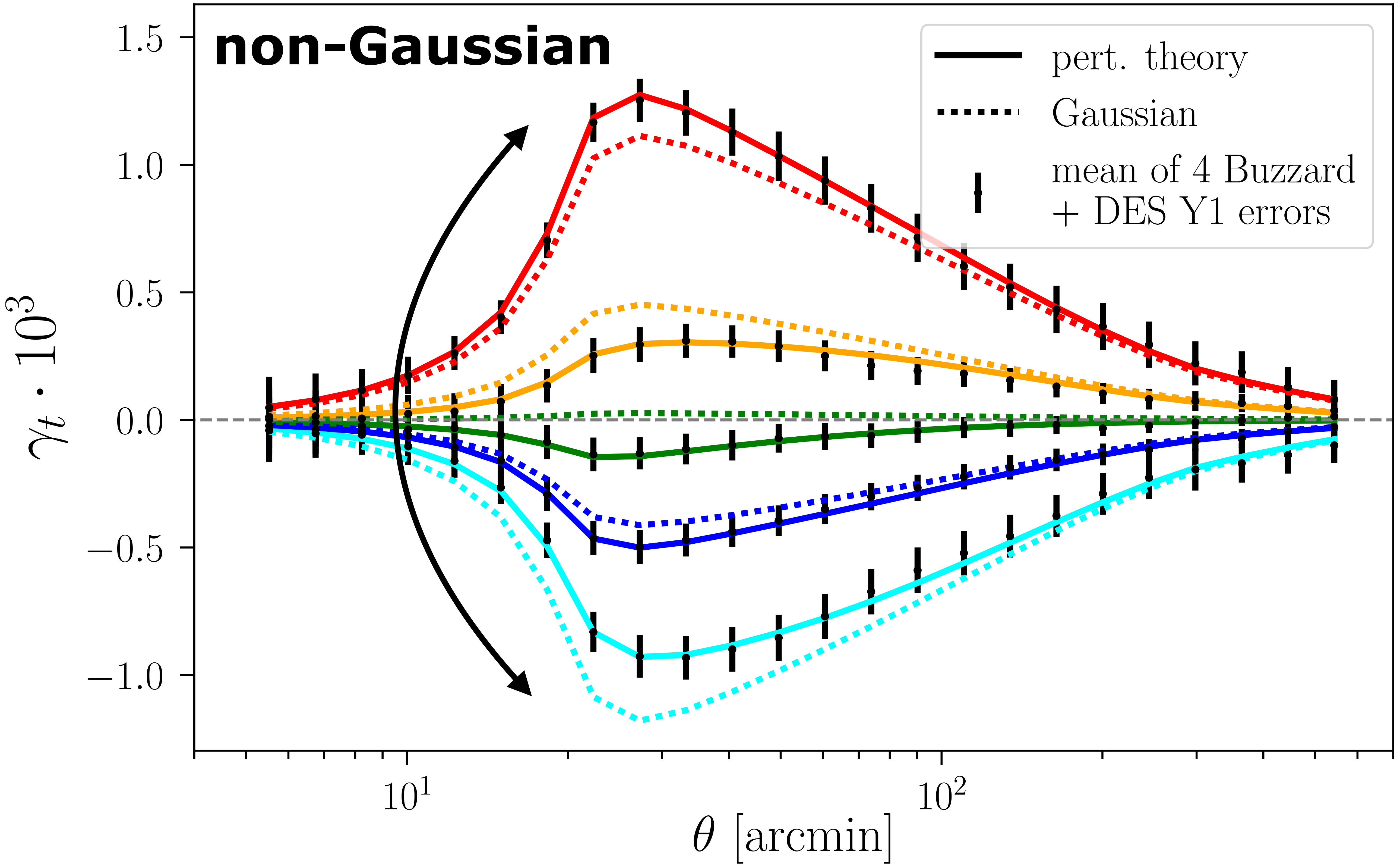
-> Nicolas Martinet's talk

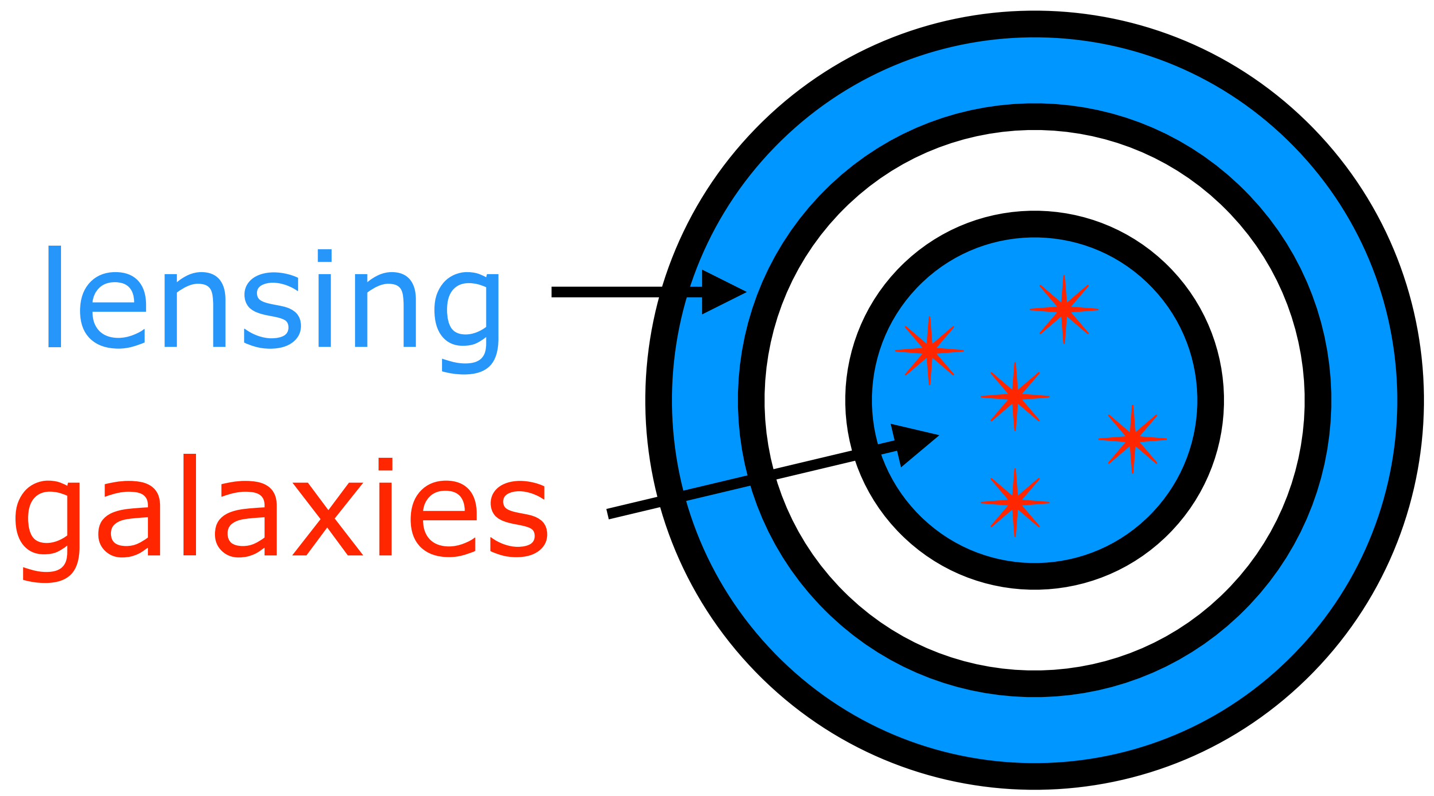
Beyond 3x2pt: Density-Split Statistics

density-split statistics on real data

DESY1: 10% Euclid area

competitive with 3x2pt





DES: Friedrich++ 18,
Gruen++ 18

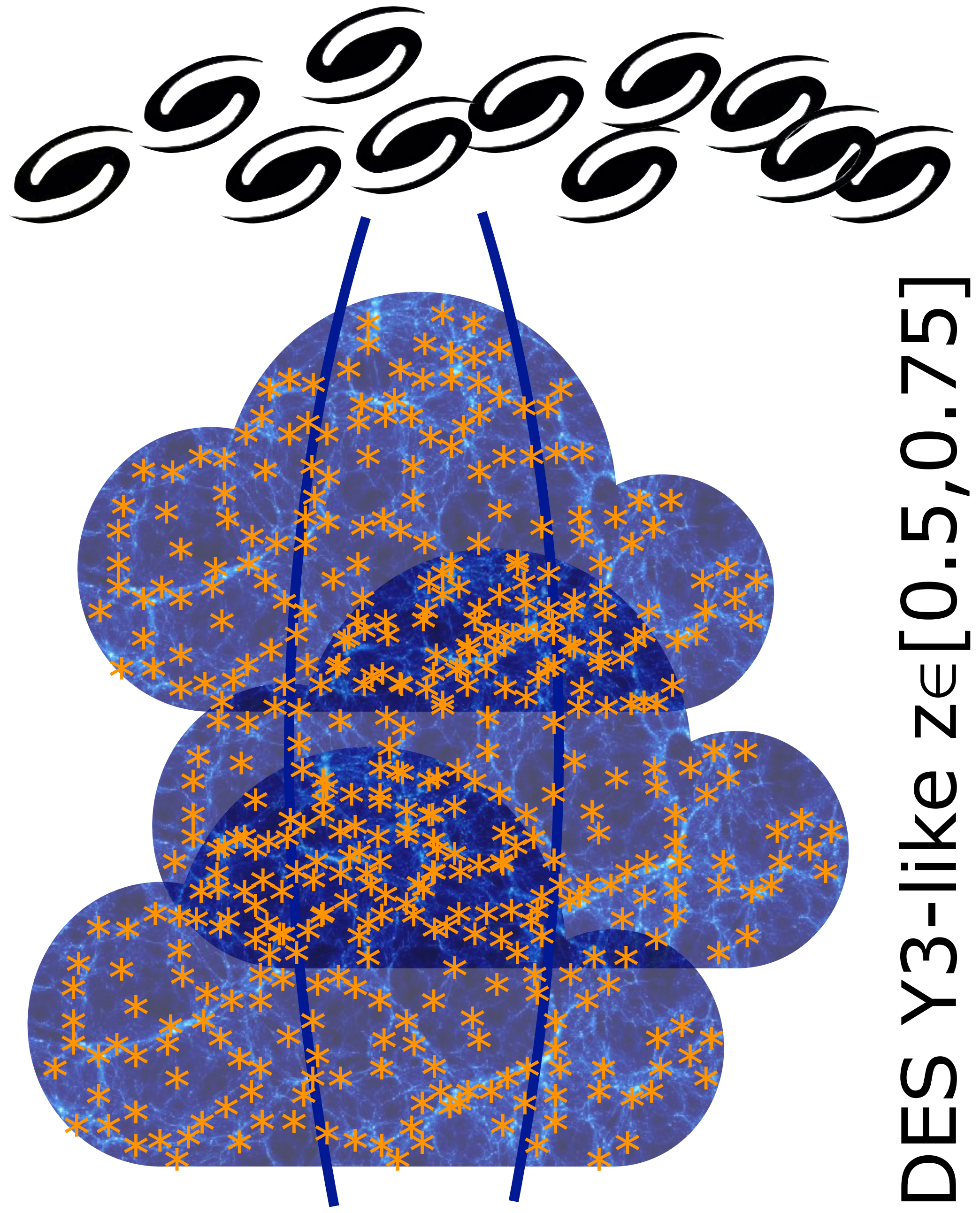
Beyond 3x2pt: Joint 1Pt Statistics

dark matter -> lensing & clustering

sources Euclid-like z∈[0.75,1.5]

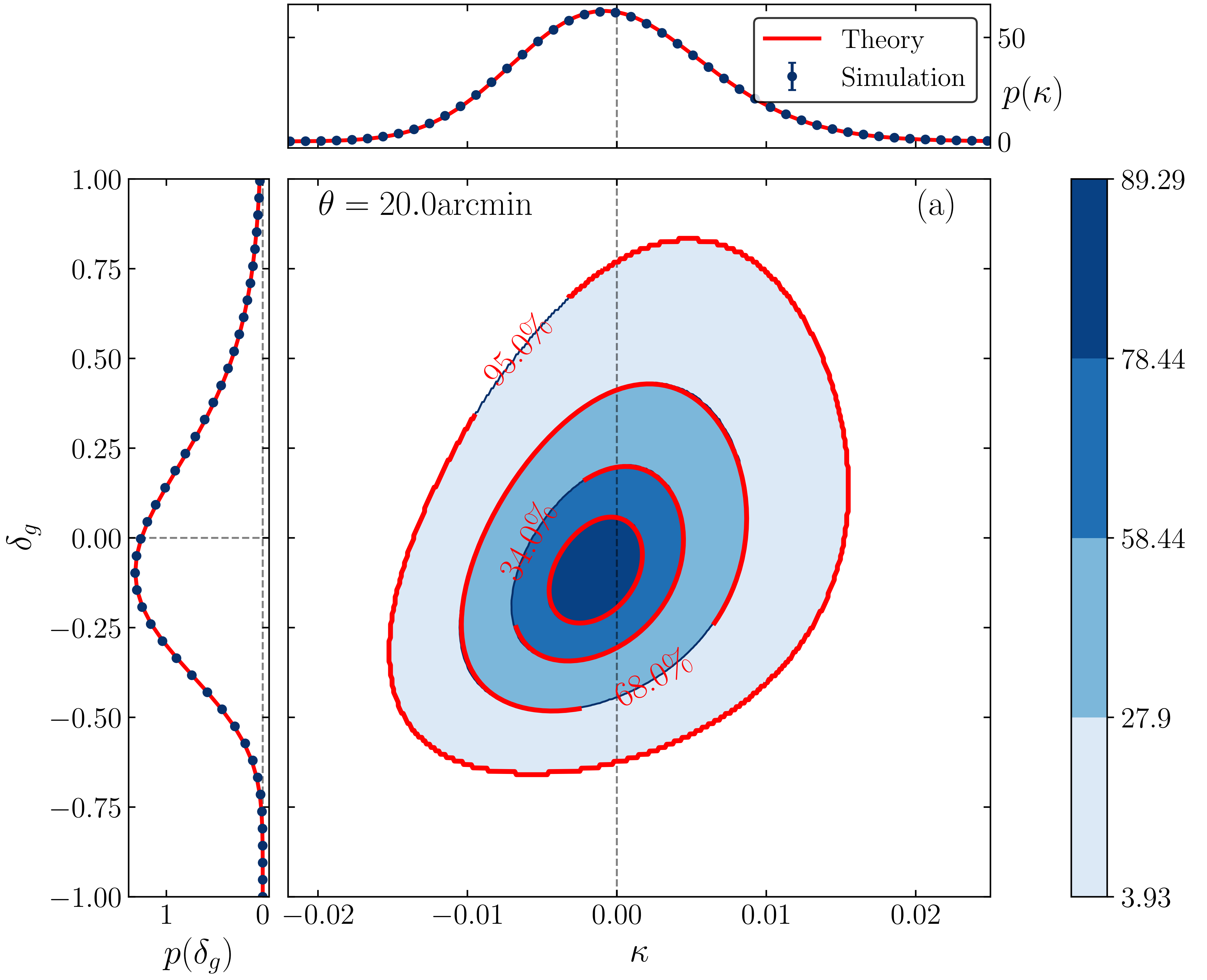
galaxy shapes

photometric galaxy counts



(dark) matter

lenses DES Y3-like z∈[0.5,0.75]



Friedrich et al. 2025

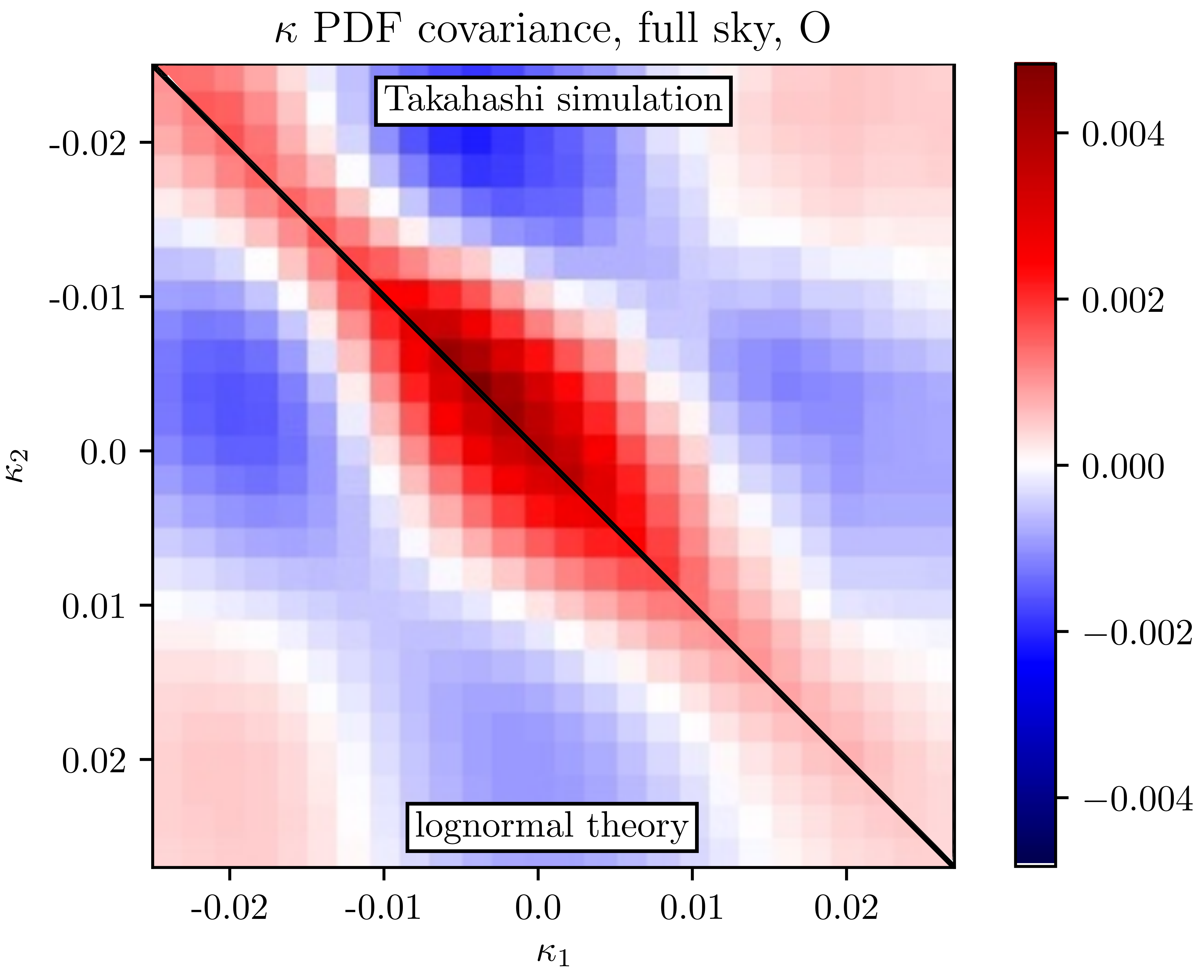
computed with CosMomentum code

including systematics
work in progress

1-Point Covariances

1pt PDF covariance from $\langle \mathcal{P}(\rho_1)\mathcal{P}(\rho_2)\rangle = \int dr\, P_d(r) \mathcal{P}(\rho_1, \rho_2; r)$ **2-pt PDF**

distance distribution



large-sep. expansion

super-sample covariance

finite sampling terms

shifted lognormal models

full covariance

CU ++ 23

Going Below & Beyond

non-Gaussian info in 1-point

probing different mildly nonlinear densities

rich physics, including beyond SM

features

accessible, interpretable, predictable

robust: smoothing & bulk region

versatile: spec-z, photo-z + weak lensing

challenges

nonlinear galaxy bias, non-Poisson stochasticity,

RSD, systematics, scale mixing in projection