Plug-and-Play Mass Mapping with Uncertainty Quantification

Hubert Leterme

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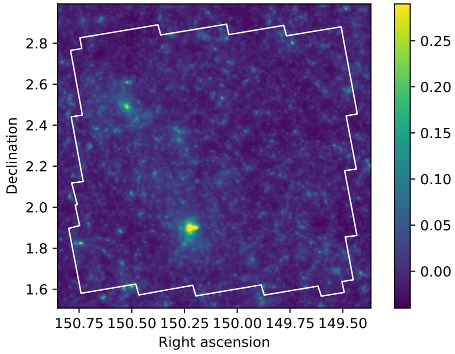
Joint work with Andreas Tersenov, Jalal Fadili and Jean-Luc Starck

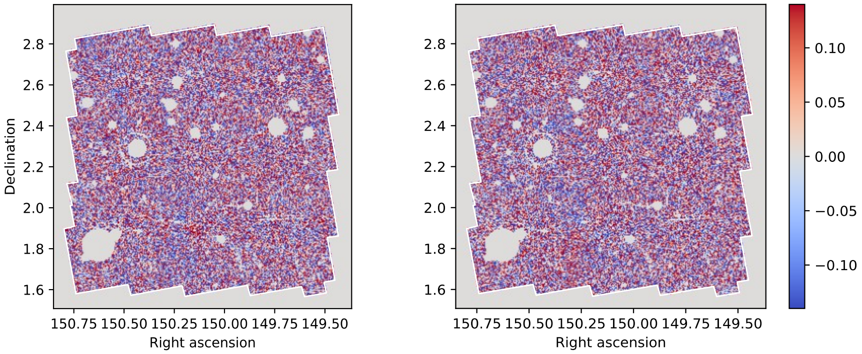
CosmoStat Days, 12th February 2026, CEA Paris-Saclay

Context and objectives

Convergence map $\boldsymbol{\kappa}$



Noisy shear map $\boldsymbol{\gamma}$ (real and imaginary parts)



Sources: κTNG simulated dataset[1] and COSMOS shape catalog[2]

- Relation between the noisy shear $\boldsymbol{\gamma}$ (observable) and the convergence $\boldsymbol{\kappa}$ (qty of interest):

$$\boldsymbol{\gamma} = \mathbf{A}\boldsymbol{\kappa} + \boldsymbol{n}, \qquad \text{with} \qquad \boldsymbol{n} \sim \mathcal{N}(\mathbf{0},\, \boldsymbol{\Sigma}).$$

- Noise level (standard deviation per pixel): $\boldsymbol{\Sigma}[k,\, k] = \sigma \,/\, N_k$.
- **Objective:** get a point estimate with error bars, with coverage guarantees.

[1] K. Osato, J. Liu, and Z. Haiman, “κTNG: effect of baryonic processes on weak lensing with IllustrisTNG simulations,” MNRAS, 2021.
[2] T. Schrabback et al., “Evidence of the accelerated expansion of the Universe from weak lensing tomography with COSMOS,” A&A, 2010.

Related work and proposed approach

	Accurate	Flexible	Fast rec.	Fast UQ
End-to-end method → DeepMass[1]	✓	✗*	✓	✓
Sampling with denoising score matching → DeepPosterior[2]	✓	✓	✗	✗
PnPMass (ours)	✓	✓	✓	✓

Notes. *Requires specific retraining for each new observation.

Proposed approach: Method based on plug-and-play (PnP) forward-backward splitting (FBS):

- Iterative method; at each iteration $i \in \{1, \ldots, N_{\text{niter}}\}$:
  - Forward step: $\boldsymbol{\kappa}_0^{(i)} = \boldsymbol{\kappa}^{(i-1)} + \tau \mathbf{A}^\top \boldsymbol{\Sigma}^{-1/2} \left(\boldsymbol{\gamma} - \mathbf{A}\boldsymbol{\kappa}^{(i-1)} \right);$
  - Backward step: $\boldsymbol{\kappa}^{(i)} = \mathcal{D}_{\hat{\boldsymbol{\theta}}}(\boldsymbol{\kappa}_0^{(i)}, \tau).$

- Training phase independent of the noise covariance matrix Σ → **flexibility**.

$$\boldsymbol{\gamma} = \mathbf{A}\boldsymbol{\kappa} + \boldsymbol{n}, \qquad \text{with} \qquad \boldsymbol{n} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}).$$

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Gradient descent step

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  Step size, depends on $\|\Sigma\|$

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  "Noise-aware" deep denoiser, trained on a range of white noise levels

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  - Backward step: $\boldsymbol{\kappa}^{(i)} = \mathcal{D}_{\hat{\boldsymbol{\theta}}}(\boldsymbol{\kappa}_0^{(i)}, \boxed{\tau})$.

Step size, also equal to the white noise standard deviation

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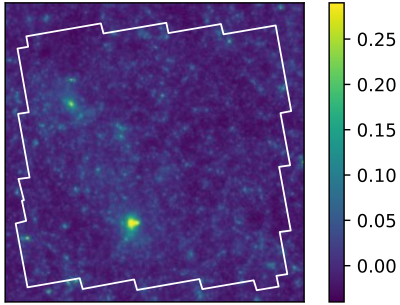
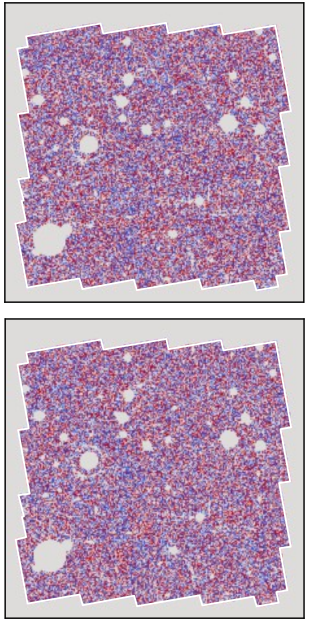
PnPMass step by step



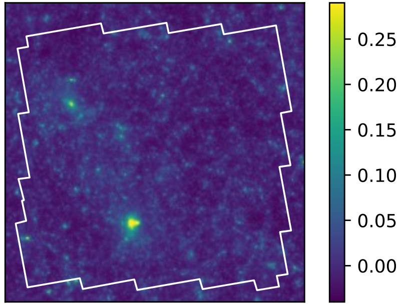
Ground truth κ



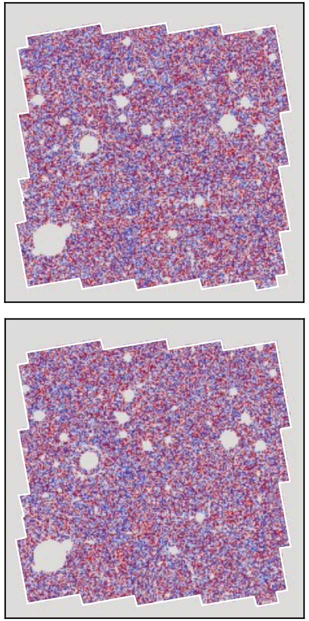
Input γ

PnPMass step by step

Iteration $i=1$



Ground truth $\boldsymbol{\kappa}$

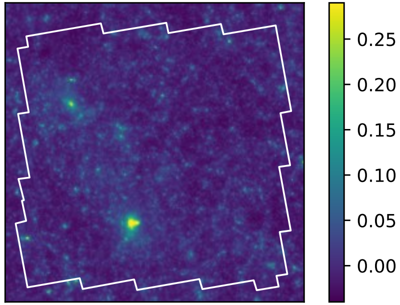


Input γ

Initialization

$\boldsymbol{\kappa}^{(0)} = \mathbf{0}$

PnPMass step by step



Ground truth $\boldsymbol{\kappa}$

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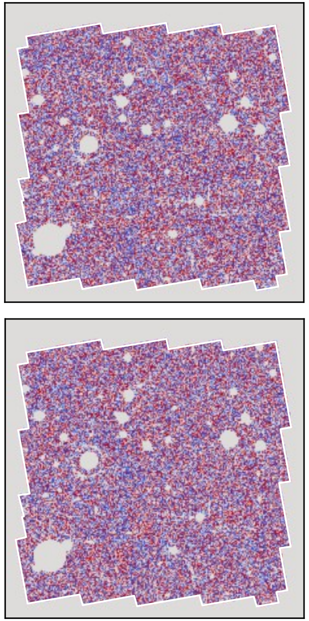
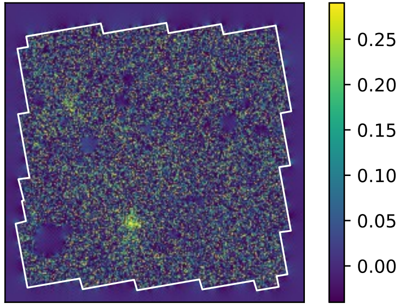


Input γ

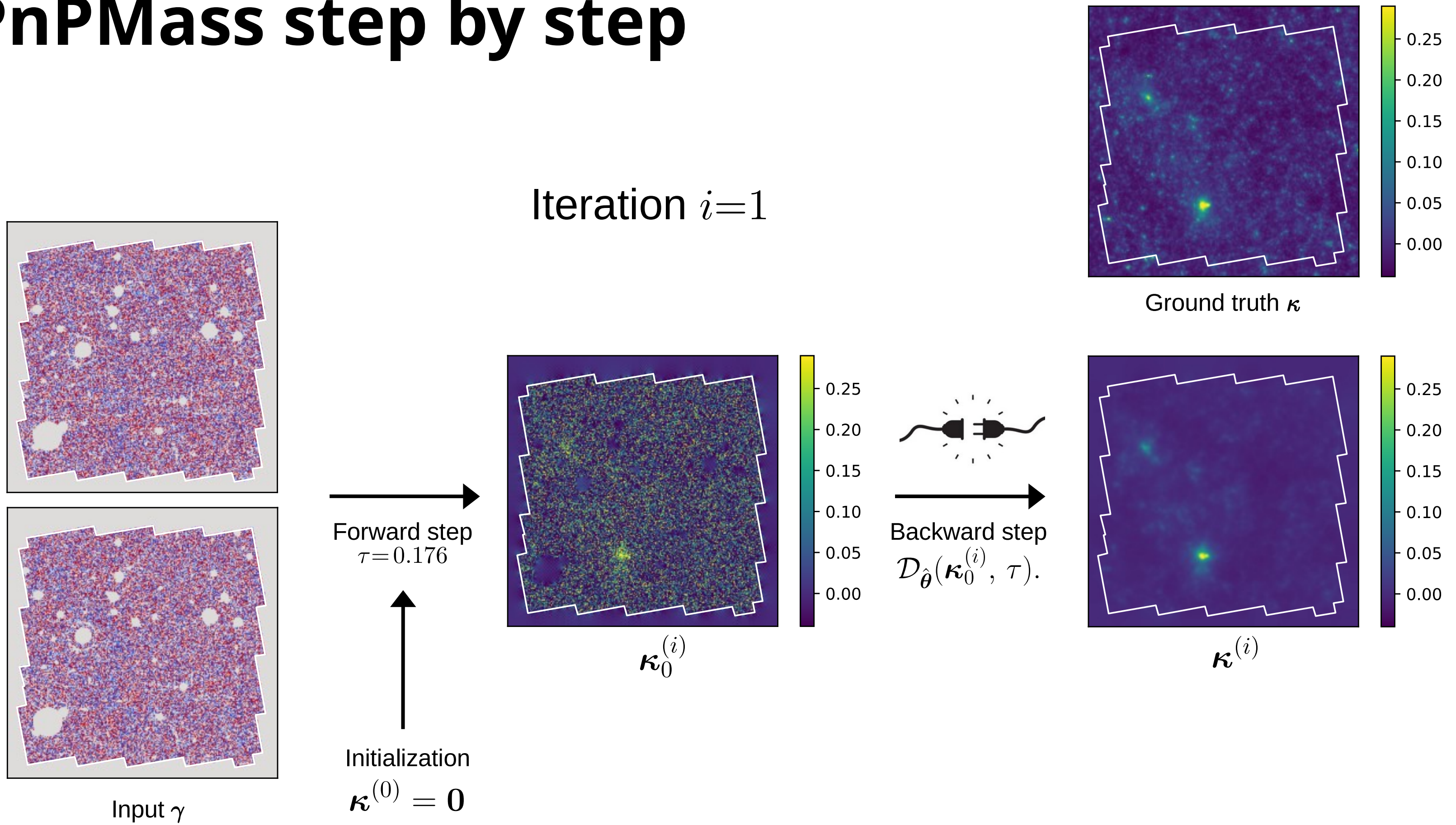
Forward step
$\tau=0.176$

Initialization
$\boldsymbol{\kappa}^{(0)} = \mathbf{0}$



$\boldsymbol{\kappa}_0^{(i)}$

PnPMass step by step



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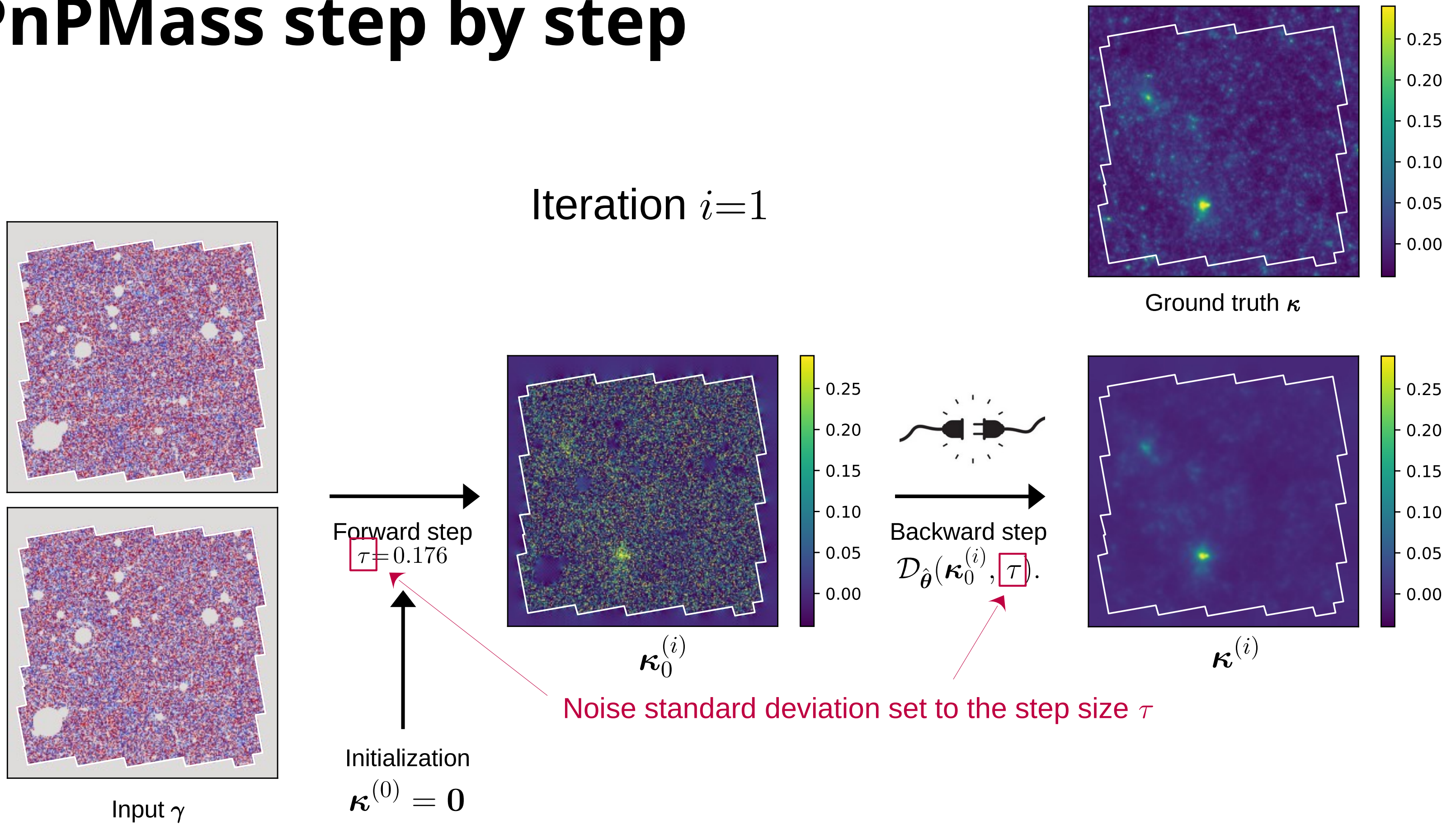

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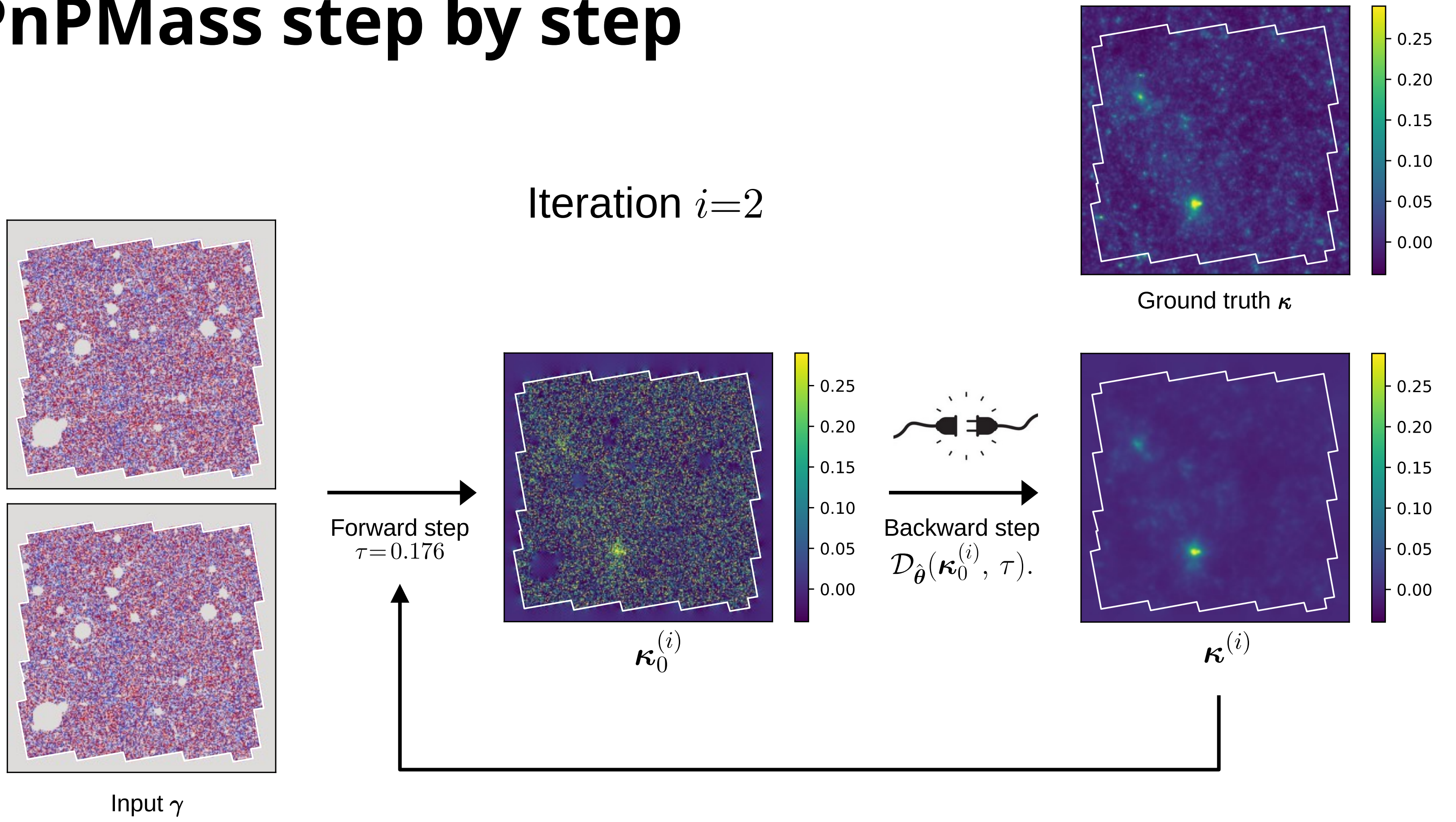

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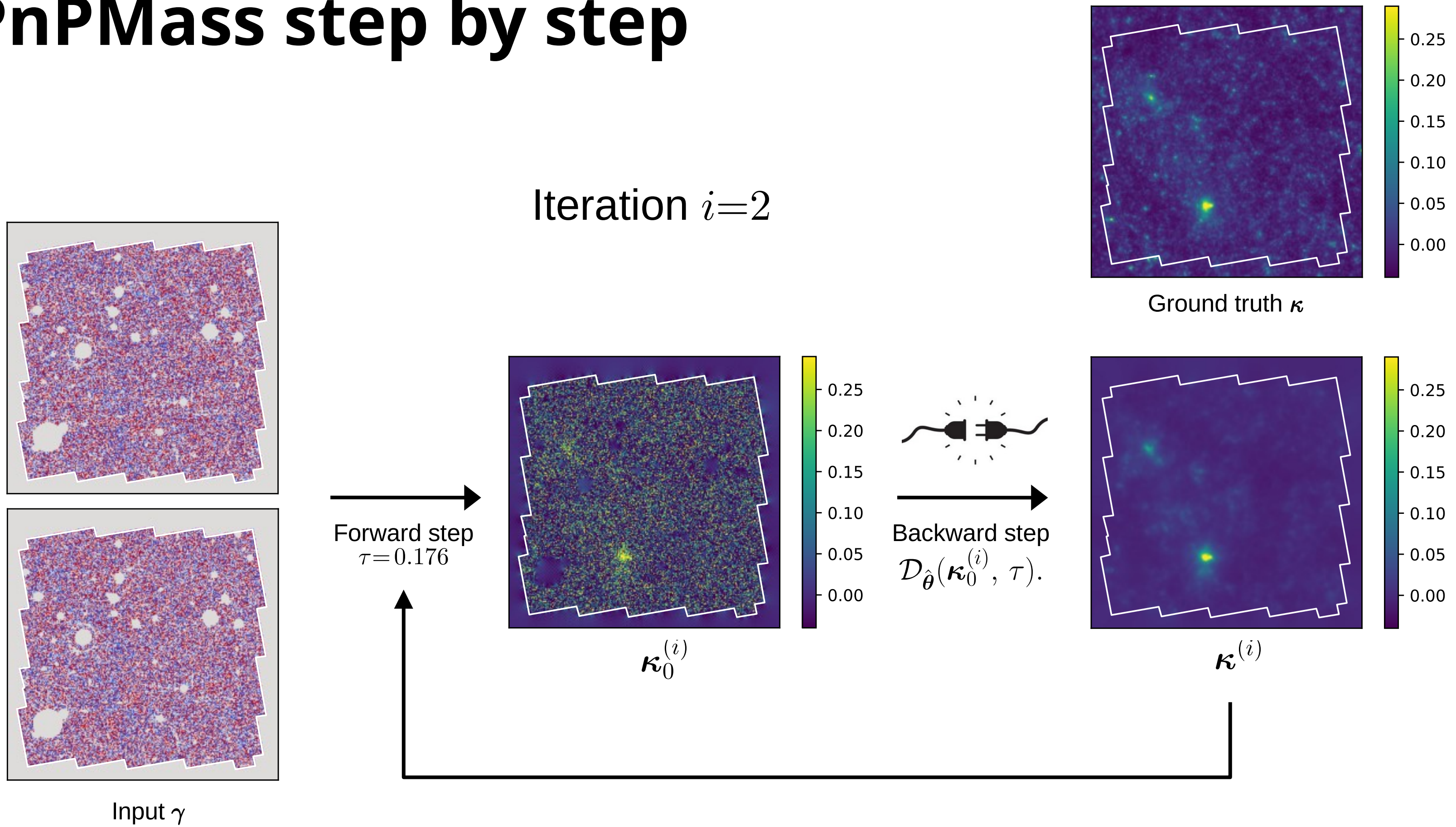

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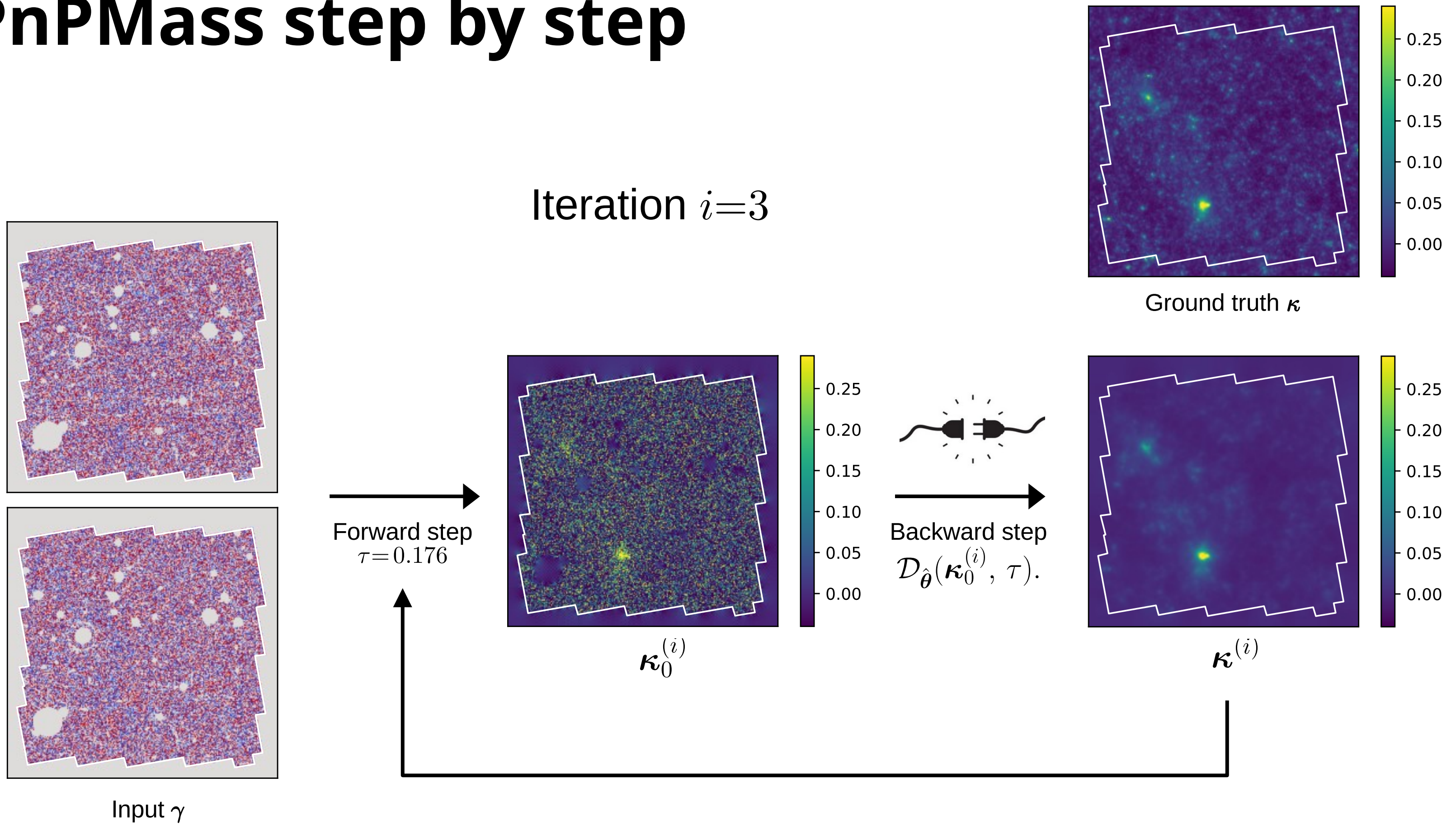

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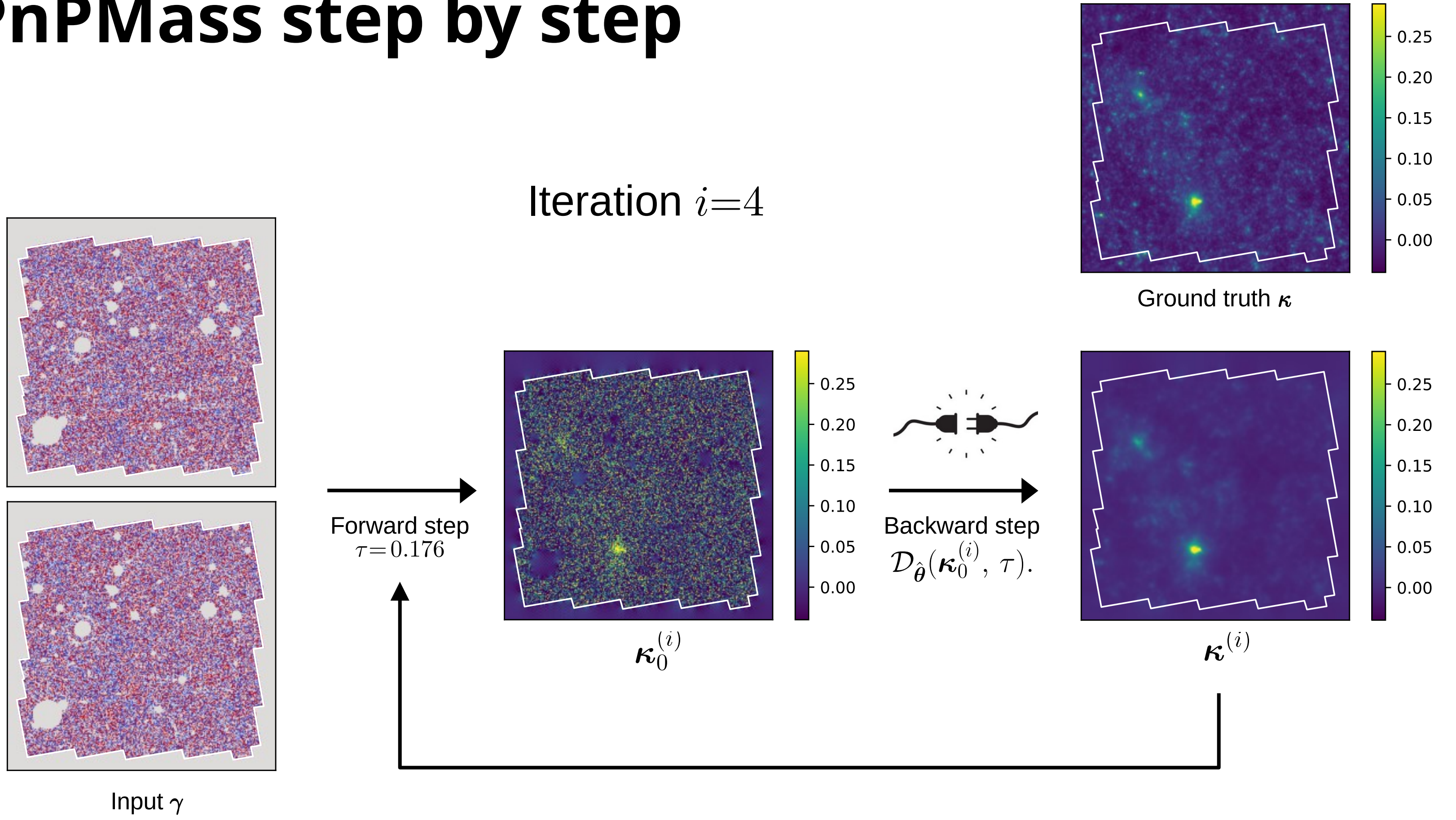
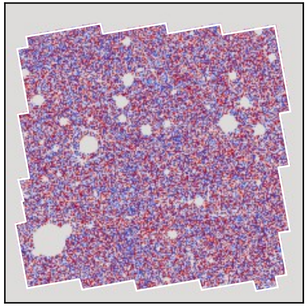
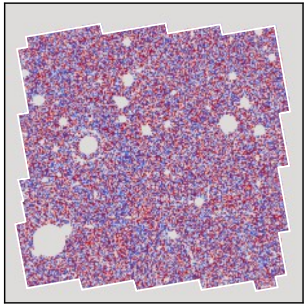


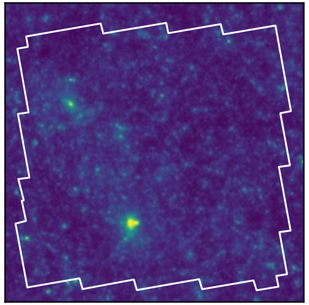
PnPMass on residuals

- **Main idea:** include knowledge about underlying physics.
- Decompose κ into Gaussian / non-Gaussian components.





Input γ



Ground truth κ

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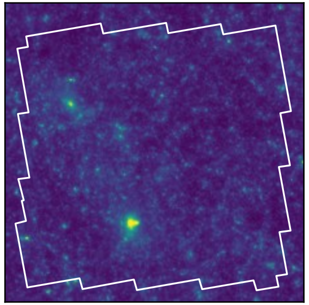

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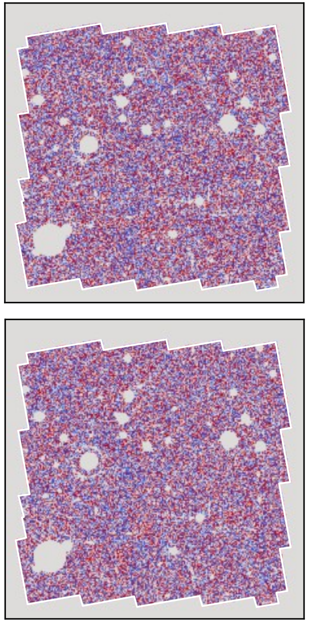
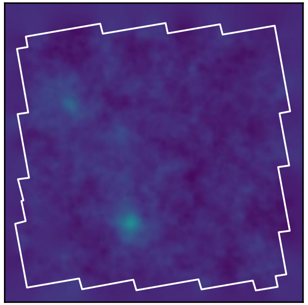


Input γ

Wiener filter →



Gaussian component $\boldsymbol{\kappa}_{\mathrm{g}}$

PnPMass on residuals

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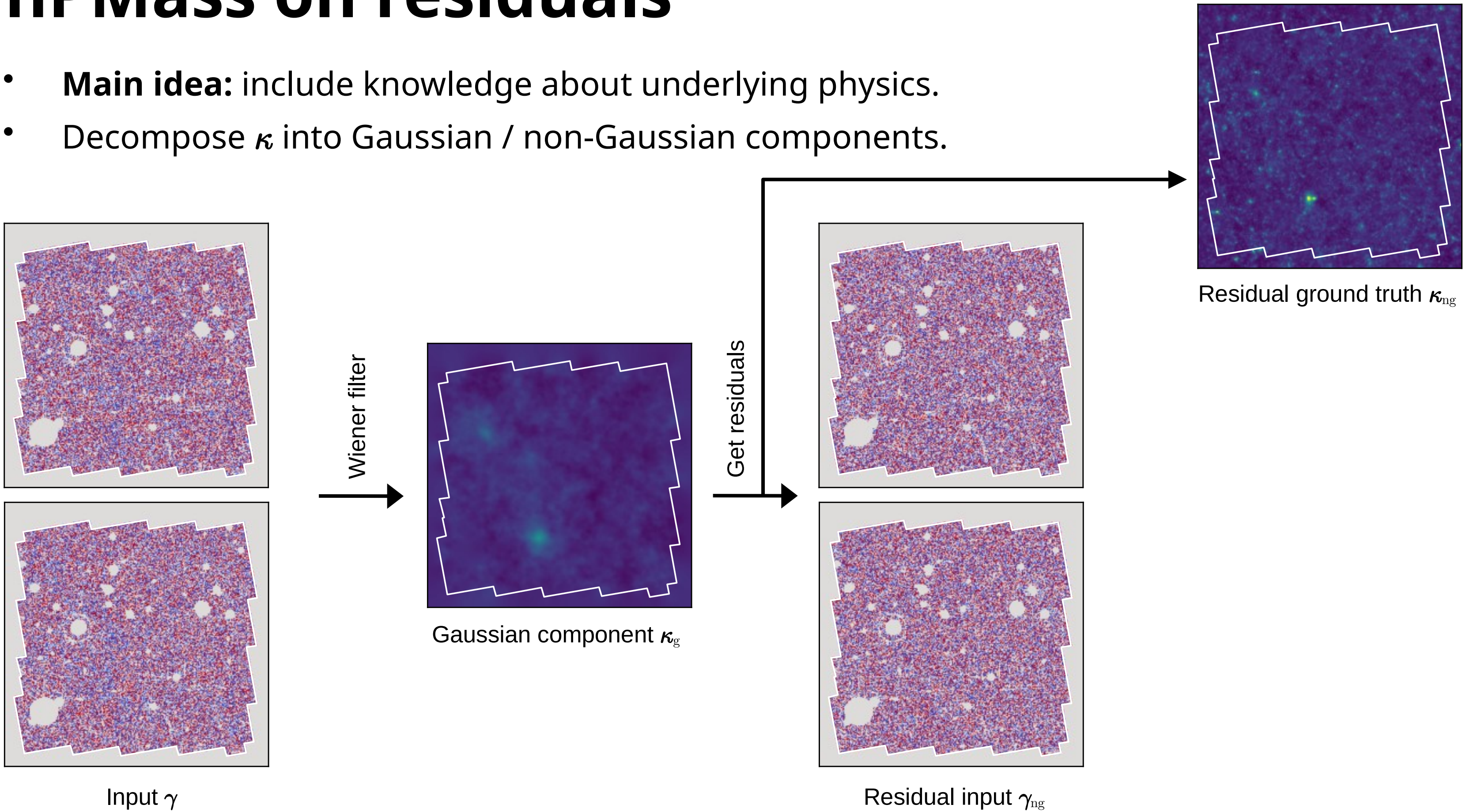

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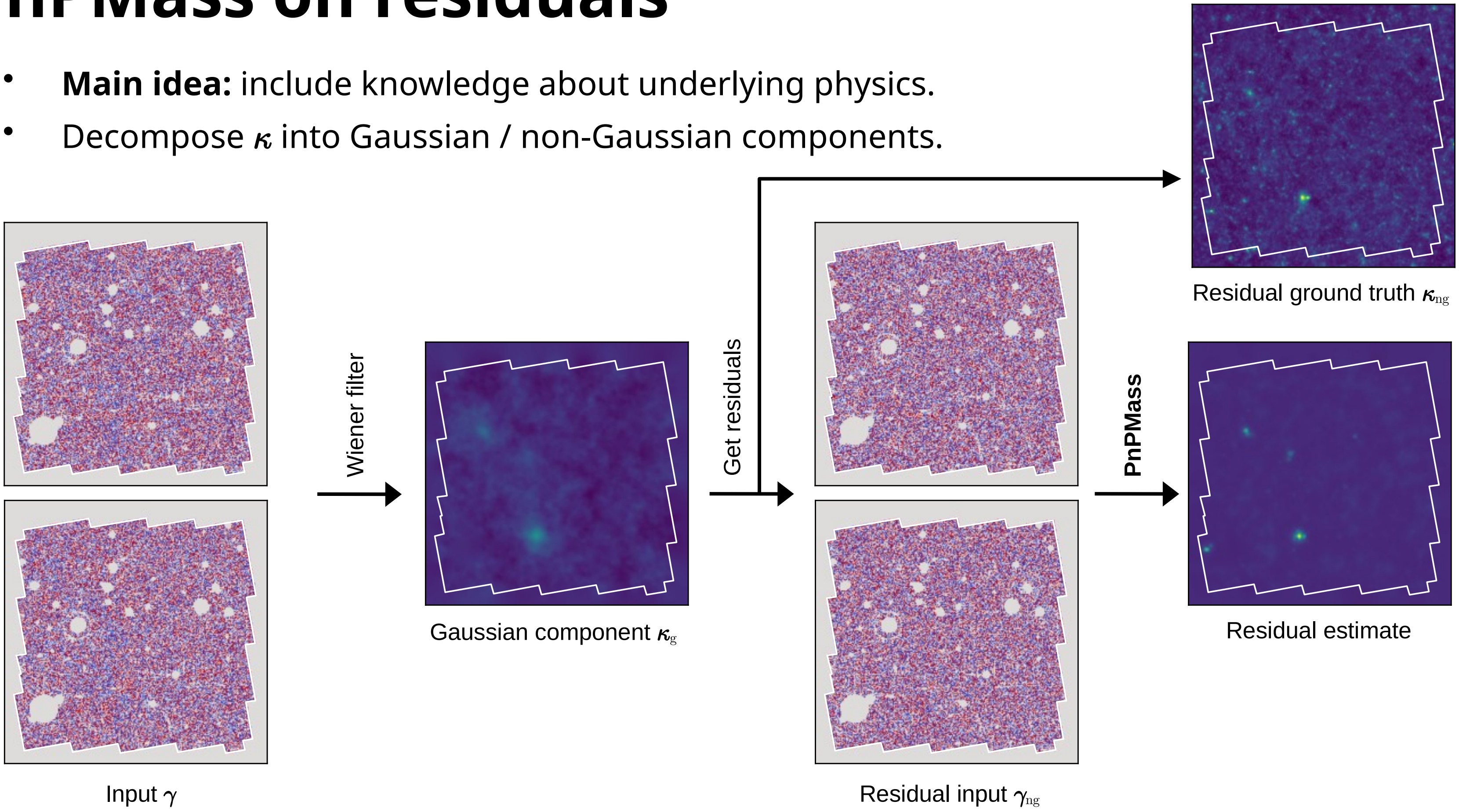

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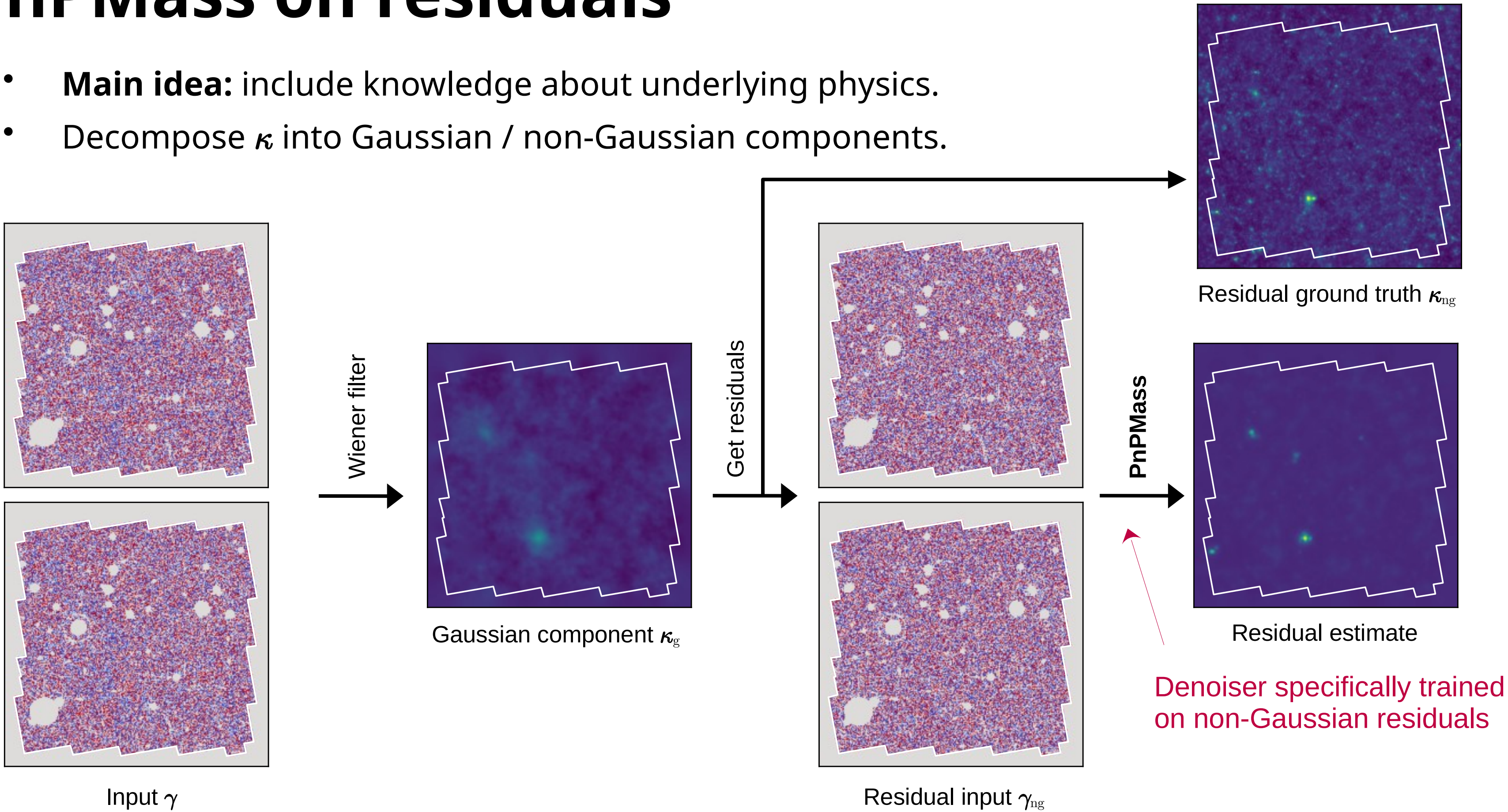

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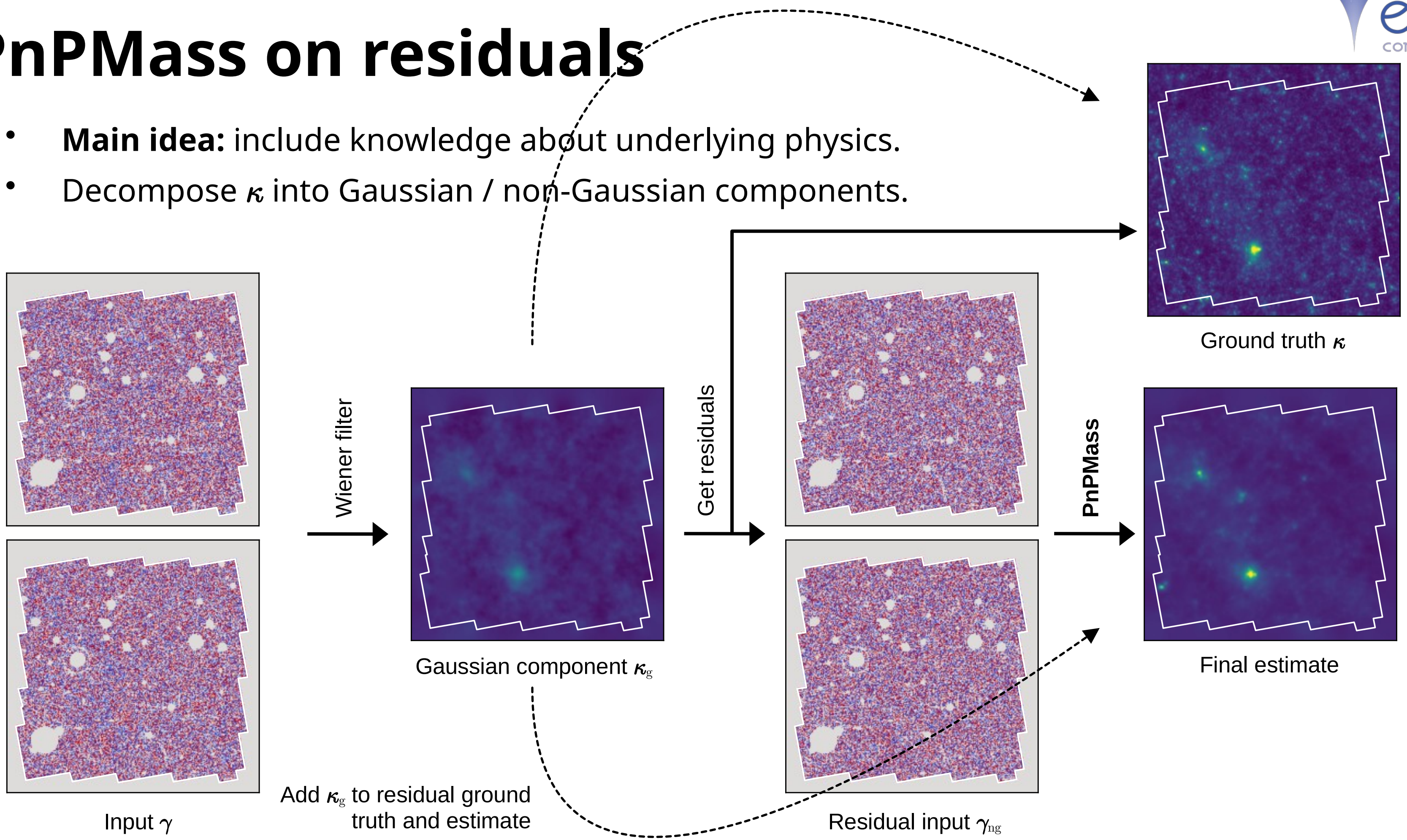
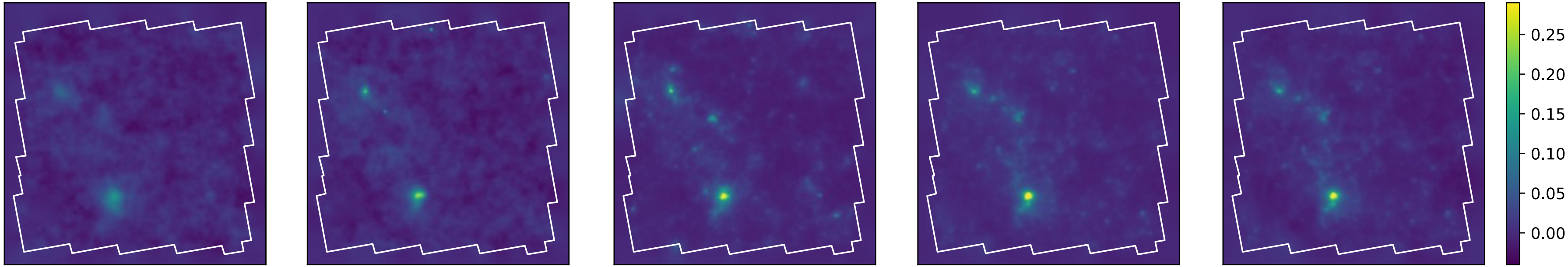


Visual comparison with other methods



Iterative Wiener[1]
NRMSE $= 0.886$

MCALens[2]
NRMSE $= 0.874$

DeepMass[3]
NRMSE $= 0.853$

PnPMass
NRMSE $= 0.864$

PnPMass (residual)
NRMSE $= 0.858$

[1] J. Bobin, J.-L. Starck, F. Sureau, and J. Fadili, "CMB Map Restoration," Advances in Astronomy, 2012.
[2] J.-L. Starck, K. E. Themelis, N. Jeffrey, A. Peel, and F. Lanusse, "Weak-lensing mass reconstruction using sparsity and a Gaussian random field," A&A, 2021.
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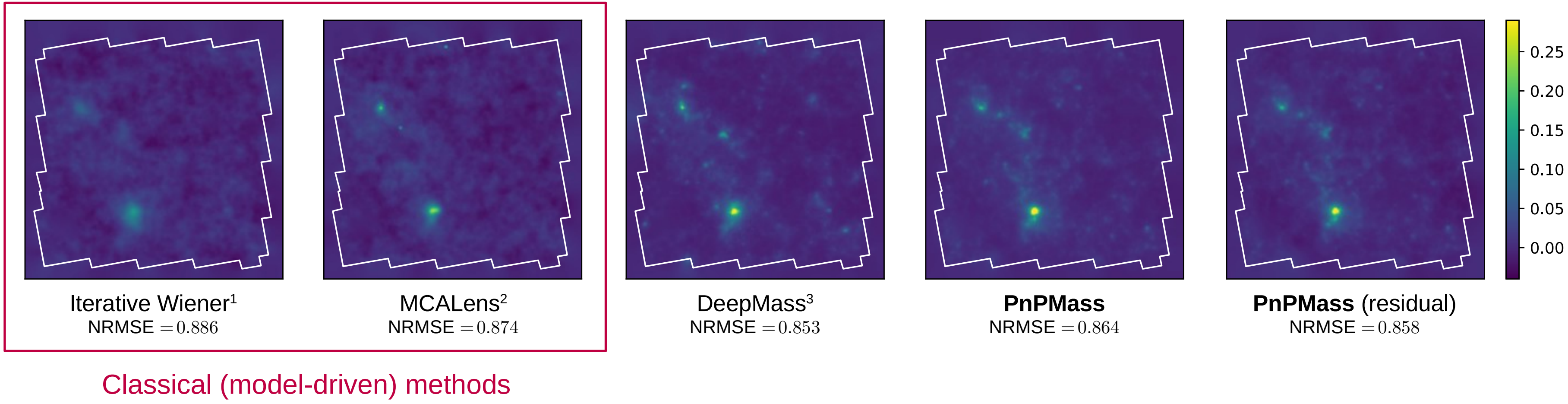

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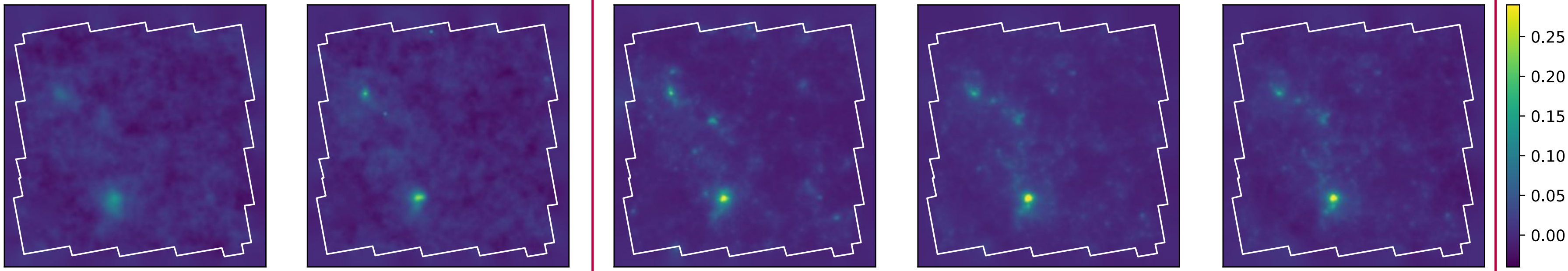

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Deep-learning-based (data driven) methods

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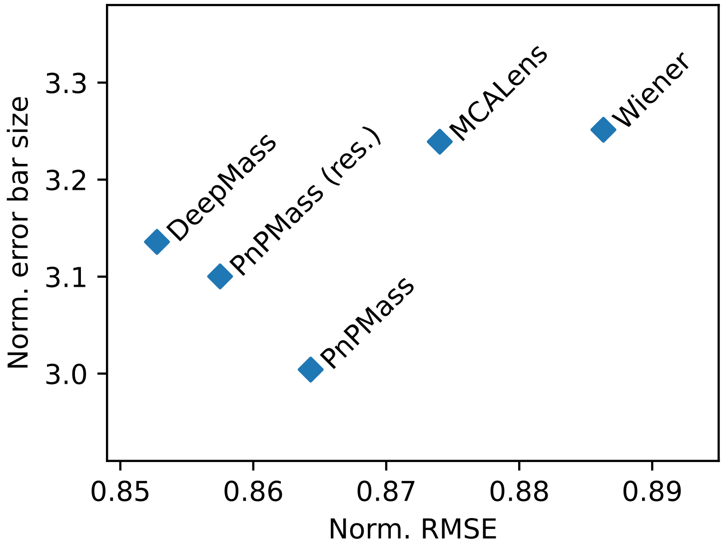
Results – Accuracy vs Error bar size

- Test set: 512 images from κTNG simulations;
- Uncertainty quantification with calibration: coverage guarantees for all methods;
- Target miscoverage rate set to 4.6% (2σ-confidence).



Comments:

- PnPMass (residual version) slightly less accurate than DeepMass, but much more flexible;
- Smaller error bars for PnPMass than DeepMass (in 100% of the test examples);
- **Possible explanation:** DeepMass recovers more peaks, but also hallucinate more → bias / variance trade-off? Check with CHEM.[1]

[1]J. Li, I. Rosellon-Inclan, G. Kutyniok, and J.-L. Starck, “CHEM: Estimating and Understanding Hallucinations in Deep Learning for Image Processing,” arXiv, 2025

Toward tomographic mass mapping

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- So far, source galaxies from all redshifts:

$$\boldsymbol{\kappa} := \int_0^{z_{\max}} \boldsymbol{\kappa}_{\mathrm{s}}(z_{\mathrm{s}}) n(z_{\mathrm{s}}) \, \mathrm{d}z_{\mathrm{s}}.$$

Toward tomographic mass mapping

- So far, source galaxies from all redshifts:

Convergence at a given source redshift

$$\boldsymbol{\kappa} := \int_0^{z_{\max}} \boldsymbol{\kappa}_{\mathrm{s}}(z_{\mathrm{s}}) n(z_{\mathrm{s}})\,\mathrm{d}z_{\mathrm{s}}.$$

Toward tomographic mass mapping

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Source distribution

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Toward tomographic mass mapping

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- New objective: perform mass mapping per redshift bin → lower SNR!

$$\boldsymbol{\kappa}_i := \int_{z_{i-1}}^{z_i} \boldsymbol{\kappa}_{\mathrm{s}}(z_{\mathrm{s}}) n(z_{\mathrm{s}}) \, \mathrm{d}z_{\mathrm{s}}.$$

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Integrate over the i-th redshift bin

Toward tomographic mass mapping

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- Each binned convergence map is a line-of-sight integration:

$$\boldsymbol{\kappa}_i \propto \int_0^{z_i} \frac{w_i(z)}{a(z)} \boldsymbol{\delta}(z) \, \mathrm{d}z$$

Plug-and-Play Mass Mapping with Fast Uncertainty Quantification

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Matter (over)density field

Toward tomographic mass mapping

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$$\boldsymbol{\kappa}_i \propto \int_0^{z_i} \frac{w_i(z)}{a(z)} \boldsymbol{\delta}(z) \, \mathrm{d}z$$

Lensing kernel for the i-th redshift bin

Toward tomographic mass mapping

- So far, source galaxies from all redshifts:

$$\boldsymbol{\kappa} := \int_0^{z_{\max}} \boldsymbol{\kappa}_{\mathrm{s}}(z_{\mathrm{s}}) n(z_{\mathrm{s}}) \, \mathrm{d}z_{\mathrm{s}}.$$

- New objective: perform mass mapping per redshift bin → lower SNR!

$$\boldsymbol{\kappa}_i := \int_{z_{i-1}}^{z_i} \boldsymbol{\kappa}_{\mathrm{s}}(z_{\mathrm{s}}) n(z_{\mathrm{s}}) \, \mathrm{d}z_{\mathrm{s}}.$$

- Each binned convergence map is a line-of-sight integration:

$$\boldsymbol{\kappa}_i \propto \int_0^{z_i} \frac{w_i(z)}{a(z)} \boldsymbol{\delta}(z) \, \mathrm{d}z$$

Foreground mass taken into account
→ correlations between bins

Toward tomographic mass mapping

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- Linear combination of bins to get projected mass in each bin:[1]

$$\boldsymbol{\kappa}_{\mathrm{b}\,i} := \sum_{j=(i-2)}^{i} b_{ij}\, \boldsymbol{\kappa}_j .$$

[1]F. Bernardeau, T. Nishimichi, and A. Taruya, “Cosmic shear full nulling: sorting out dynamics, geometry and systematics,” MNRAS, Dec. 2014.

Toward tomographic mass mapping

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BNT weights, depend on the source distribution AND on the cosmology

[1] F. Bernardeau, T. Nishimichi, and A. Taruya, “Cosmic shear full nulling: sorting out dynamics, geometry and systematics,” MNRAS, Dec. 2014.

Toward tomographic mass mapping

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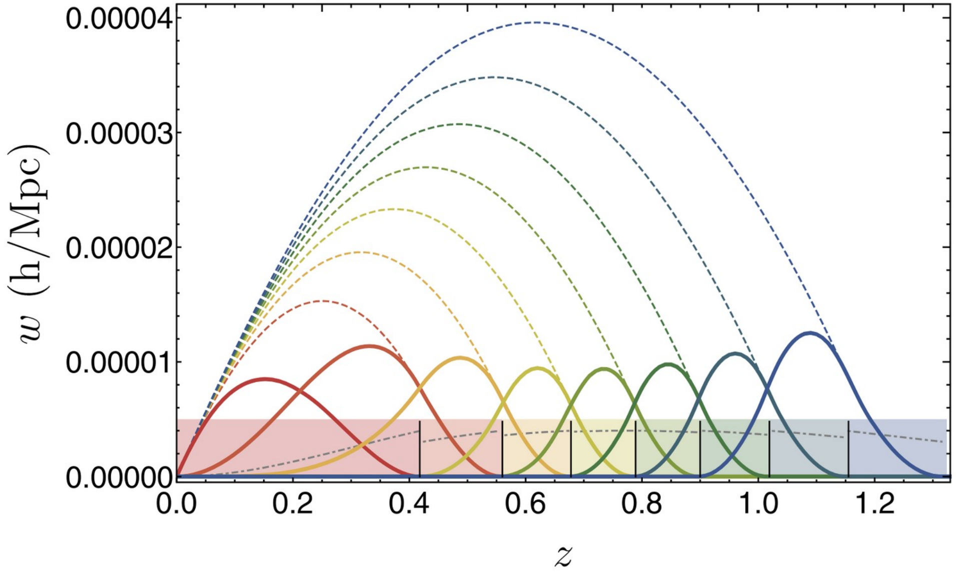
- Linear combination of bins to get projected mass in each bin:[1]

$$\boldsymbol{\kappa}_{\mathrm{b}\,i} := \sum_{j=(i-2)}^{i} b_{ij}\, \boldsymbol{\kappa}_j.$$

Decorrelated maps; mass distribution averaged over each redshift bin

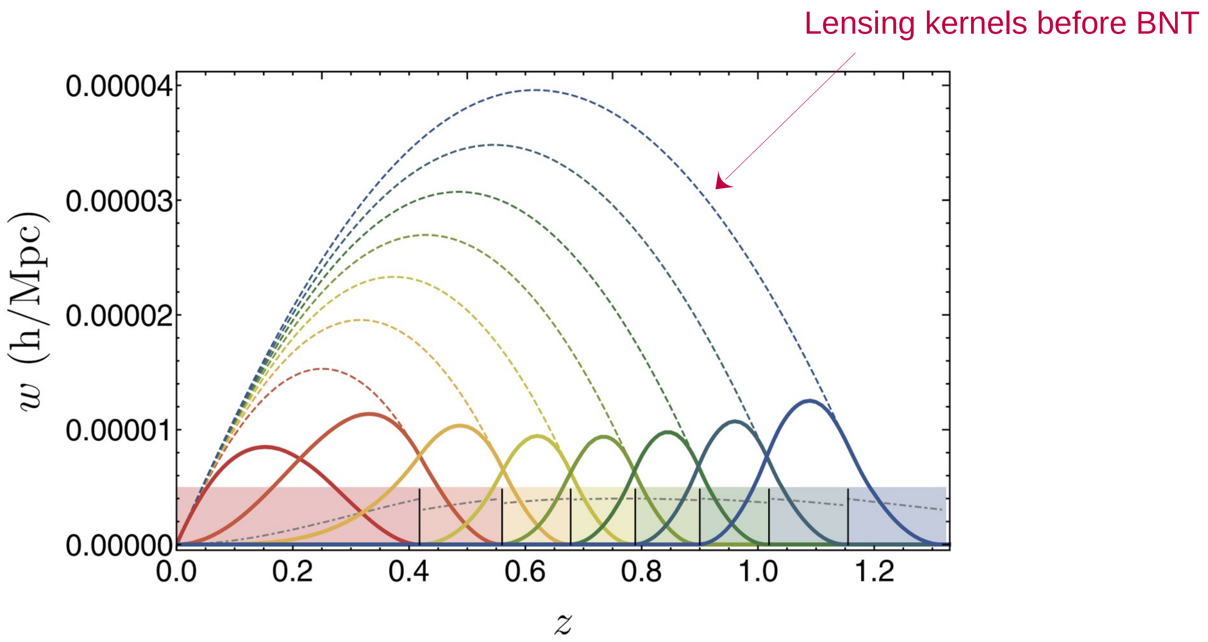
[1]F. Bernardeau, T. Nishimichi, and A. Taruya, "Cosmic shear full nulling: sorting out dynamics, geometry and systematics," MNRAS, Dec. 2014.

Toward tomographic mass mapping



Plot from: A. Barthelemy et al., “Numerical complexity of the joint nulled weak-lensing probability distribution function,” Phys. Rev. D, Feb. 2022

Toward tomographic mass mapping



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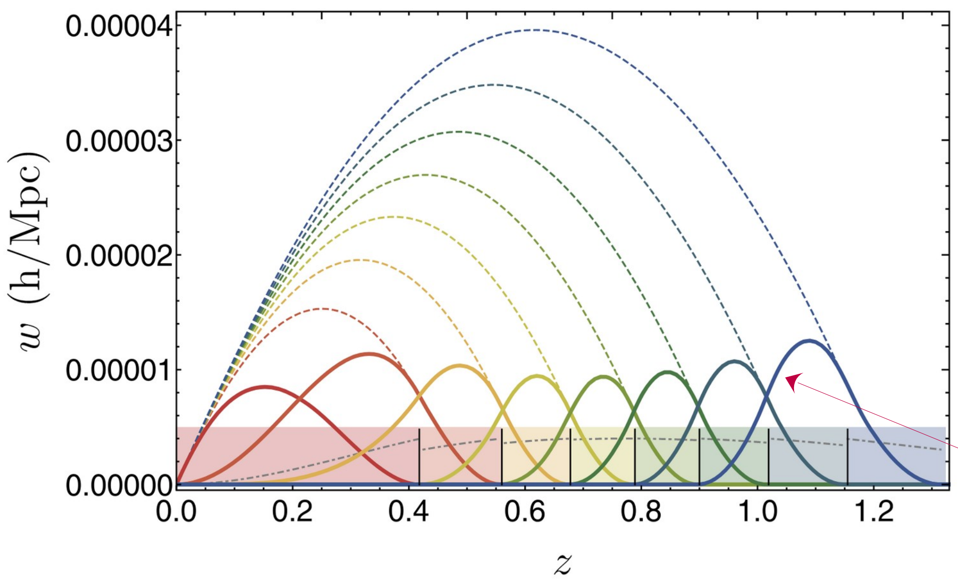
Toward tomographic mass mapping



Lensing kernels after BNT

Plot from: A. Barthelemy et al., "Numerical complexity of the joint nulled weak-lensing probability distribution function," Phys. Rev. D, Feb. 2022

Toward tomographic mass mapping

Proposed solutions

- **Option 1:** Joint reconstruction of $\boldsymbol{\kappa}_i$, then BNT. Corresponding inverse problem:

$$\bar{\boldsymbol{\gamma}} = \bar{\mathbf{A}}\bar{\boldsymbol{\kappa}} + \bar{\boldsymbol{n}},$$

with:

$$\bar{\boldsymbol{\gamma}} := \begin{bmatrix} \boldsymbol{\gamma}_1 \\ \boldsymbol{\gamma}_2 \\ \vdots \\ \boldsymbol{\gamma}_J \end{bmatrix}; \quad \bar{\boldsymbol{\kappa}} := \begin{bmatrix} \boldsymbol{\kappa}_1 \\ \boldsymbol{\kappa}_2 \\ \vdots \\ \boldsymbol{\kappa}_J \end{bmatrix}; \quad \bar{\mathbf{A}} := \begin{bmatrix} \mathbf{A} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A} \end{bmatrix}; \quad \bar{\boldsymbol{n}} \sim \mathcal{N}(\mathbf{0}, \bar{\boldsymbol{\Sigma}}), \quad \text{with} \quad \bar{\boldsymbol{\Sigma}} := \begin{bmatrix} \boldsymbol{\Sigma}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{\Sigma}_J \end{bmatrix}.$$

  - Model trained for joint denoising across redshift bins;
  - PnPMass applied to this new problem;
  - Then, apply BNT to the joint estimate.

Toward tomographic mass mapping

Proposed solutions

- **Option 2:** BNT directly embedded in the inverse problem:

$$\bar{\gamma} = \bar{\mathbf{A}}\bar{\mathbf{B}}^{-1}\bar{\boldsymbol{\kappa}}_{\mathrm{b}} + \bar{\boldsymbol{n}},$$

  - New forward operator: $\bar{\mathbf{A}}_{\mathrm{b}} := \bar{\mathbf{A}}\bar{\mathbf{B}}^{-1}$.
  - Apply PnPMass on this new problem.
  - New model trained on BNT convergence maps:
    - Clean maps are (almost) uncorrelated along zbins;
    - However the noise is correlated → joint denoising.

Toward tomographic mass mapping

Proposed solutions

- **Option 2:** BNT directly embedded in the inverse problem:

$$\bar{\gamma} = \bar{\mathbf{A}}\bar{\mathbf{B}}^{-1}\bar{\boldsymbol{\kappa}}_{\mathrm{b}} + \bar{\boldsymbol{n}},$$

Decorrelated maps to estimate

  - New forward operator: $\bar{\mathbf{A}}_{\mathrm{b}} := \bar{\mathbf{A}}\bar{\mathbf{B}}^{-1}$.
  - Apply PnPMass on this new problem.
  - New model trained on BNT convergence maps:
    - Clean maps are (almost) uncorrelated along zbins;
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Toward tomographic mass mapping

Proposed solutions

- **Option 2:** BNT directly embedded in the inverse problem:

$$\bar{\gamma} = \bar{\mathbf{A}}\bar{\mathbf{B}}^{-1}\bar{\boldsymbol{\kappa}}_{\mathrm{b}} + \bar{\boldsymbol{n}},$$

Inverse BNT transform

  - New forward operator: $\bar{\mathbf{A}}_{\mathrm{b}} := \bar{\mathbf{A}}\bar{\mathbf{B}}^{-1}$.
  - Apply PnPMass on this new problem.
  - New model trained on BNT convergence maps:
    - Clean maps are (almost) uncorrelated along zbins;
    - However the noise is correlated → joint denoising.

Toward tomographic mass mapping

Proposed solutions

- **Option 2:** BNT directly embedded in the inverse problem:

$$\bar{\gamma} = \bar{\mathbf{A}}\boxed{\bar{\mathbf{B}}^{-1}\bar{\boldsymbol{\kappa}}_{\mathrm{b}}} + \bar{\boldsymbol{n}},$$

$$\bar{\boldsymbol{\kappa}}$$

  - New forward operator: $\bar{\mathbf{A}}_{\mathrm{b}} := \bar{\mathbf{A}}\bar{\mathbf{B}}^{-1}$.
  - Apply PnPMass on this new problem.
  - New model trained on BNT convergence maps:
    - Clean maps are (almost) uncorrelated along zbins;
    - However the noise is correlated $\rightarrow$ joint denoising.

Toward tomographic mass mapping

Proposed solutions

- **Option 2:** BNT directly embedded in the inverse problem:

$$\bar{\gamma} = \bar{\mathbf{A}}\bar{\mathbf{B}}^{-1}\bar{\boldsymbol{\kappa}}_{\mathrm{b}} + \bar{\boldsymbol{n}},$$

Correlates the noise across redshift bins!

  - New forward operator: $\bar{\mathbf{A}}_{\mathrm{b}} := \bar{\mathbf{A}}\bar{\mathbf{B}}^{-1}$.
  - Apply PnPMass on this new problem.
  - New model trained on BNT convergence maps:
    - Clean maps are (almost) uncorrelated along zbins;
    - However the noise is correlated $\rightarrow$ joint denoising.

Conclusion and future work

- **Take-home messages:**
  - PnPMass: iterative method based on deep-learning denoising, fast and flexible;
  - Near state-of-the-art accuracy, with smaller error bars than existing methods;
  - Tomographic mass mapping with BNT transform (implementation in progress): take advantage of the correlations across redshift bins (either in the signal, or in the noise).

- **Next steps:**
  - Use PnPMass for cosmological parameter inference: size of contours? Bias? Benchmark against Kaiser-Squires and MCALens (paper Andreas[1]).
  - Extend the method to spherical data;
  - Integrate PnPMass into Euclid's Science Ground Segment.

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[1]A. Tersenov, L. Baumont, J.-L. Starck, and M. Kilbinger, A&A, vol. 698, p. A25, Jun. 2025.