

# Astrophysics & EoS

## *dense-matter EoS and compact-objects properties*

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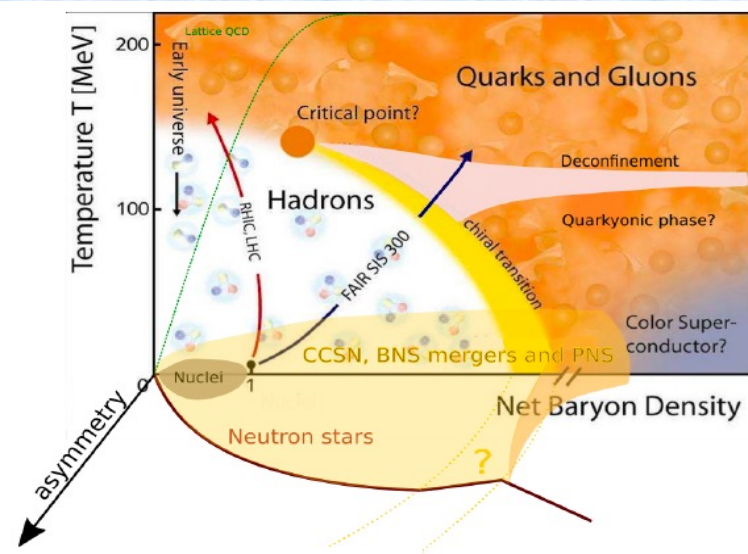
# Outline

- ❖ Motivations:
  - Intro on astrophysical sites: (proto-)neutron stars ((P)NS), core-collapse supernovae (CCSN), and binary neutron star (BNS) merger
  - Connection with the nuclear equation of state (EoS)
- ❖ Astrophysical sites at  $T = 0$  (NS):
  - EoS: generalities
  - Application to NS (crust & core)
- ❖ Astrophysical sites at  $T > 0$  (CCSN/PNS):
  - EoS: generalities
  - Applications to CCSN and PNS
  - Rates (CCSN)





# Compact objects: sites for (hot) dense matter

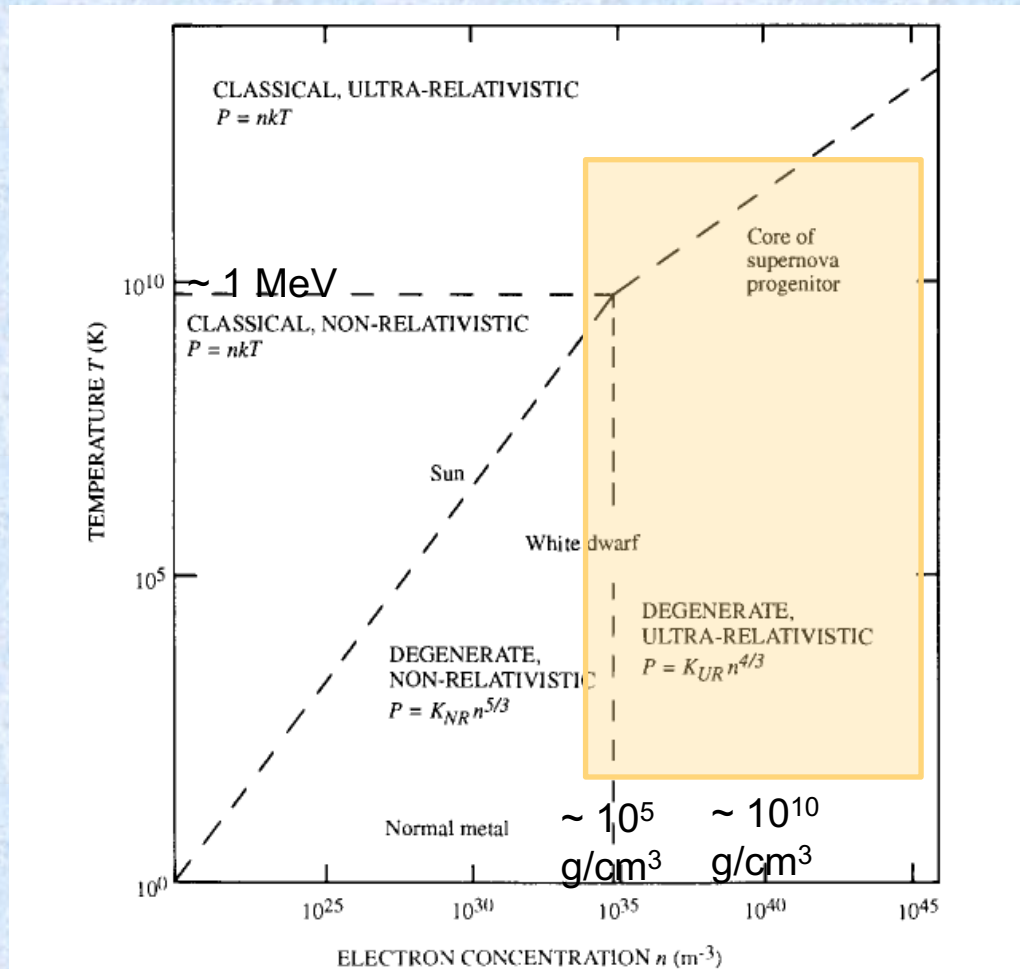


Abac et al., arXiv:2503.12263 (ET BlueBook)

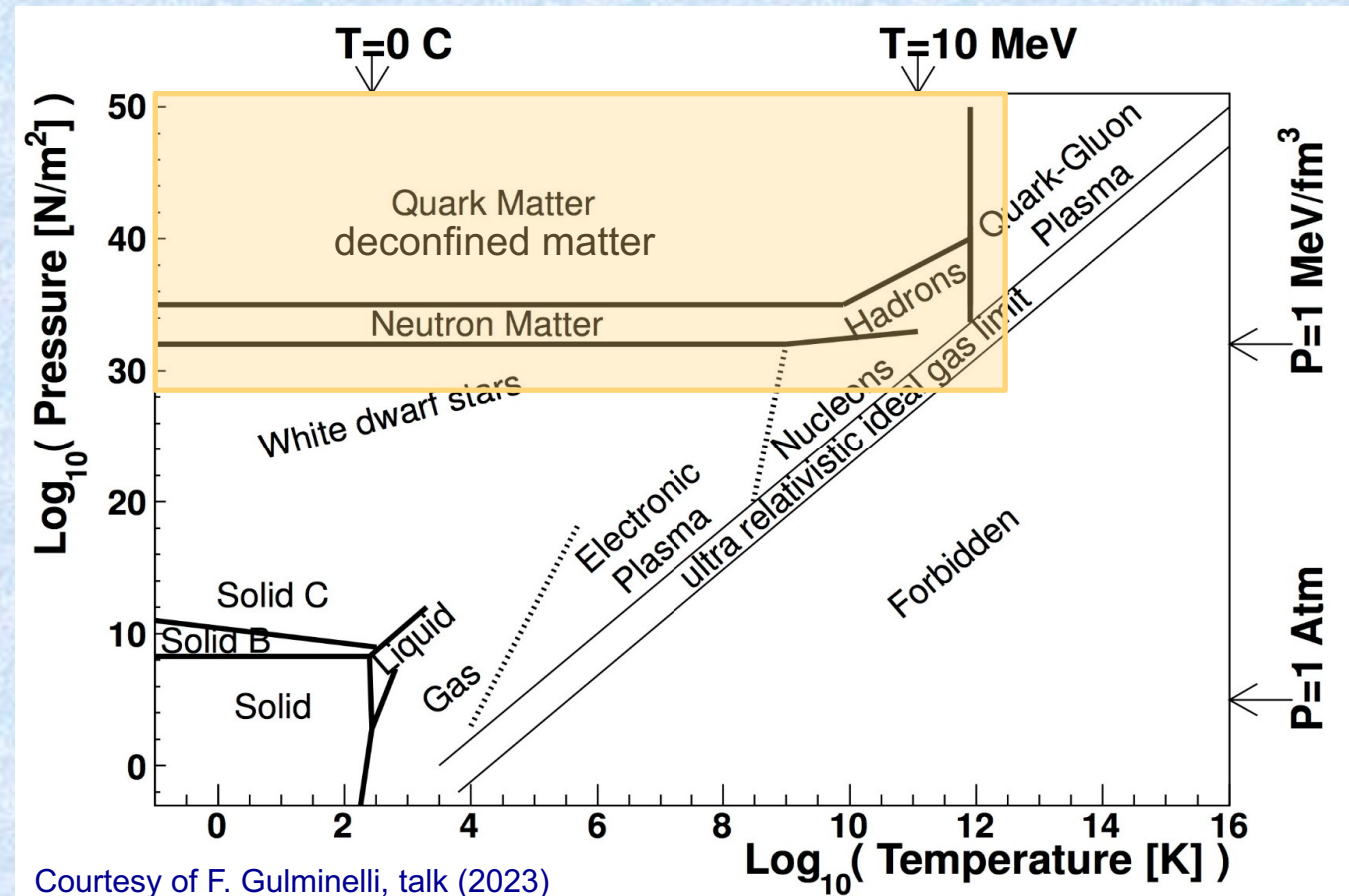
| Object                                  | Thermo                                                                                           | Equilibrium                               | Micro inputs                                                   |
|-----------------------------------------|--------------------------------------------------------------------------------------------------|-------------------------------------------|----------------------------------------------------------------|
| <b>CCSN<br/>BNS (post-<br/>)mergers</b> | $n_B \in (0, \sim 10n_{\text{sat}})$<br>$T \in (0, \sim 100) \text{ MeV}$<br>$Y_e \in (0, 0.6)$  | strong                                    | $P(n_B, T, Y_e)$<br>(weak), $\nu$ rates<br>$\rightarrow$ compo |
| <b>PNS</b>                              | $n_B \in (0, \sim 10n_{\text{sat}})$<br>$T \in (0, \sim 20) \text{ MeV}$<br>$(Y_e \in (0, 0.5))$ | strong<br>(+weak)                         | $P(n_B, T, (Y_e))$<br>(+ rates<br>$\rightarrow$ composition)   |
| <b>NS (old)<br/>binary inspiral</b>     | $n_B \in (0, \sim 10n_{\text{sat}})$<br>$T = 0$                                                  | strong+weak<br>$\rightarrow \beta$ equil. | $P(n_B)$                                                       |



# What matter can we expect?



Phillips, "The Physics of Stars"



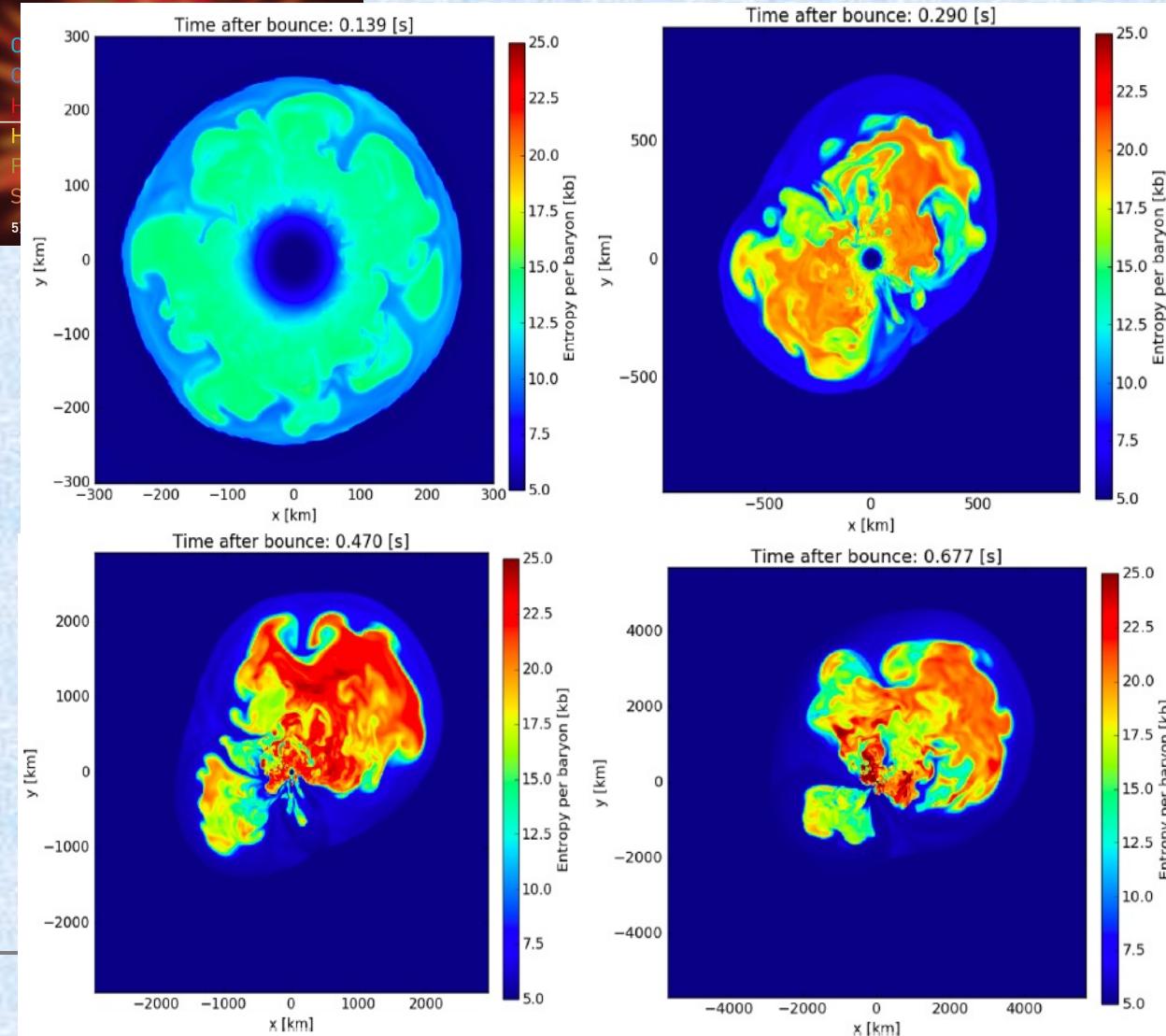
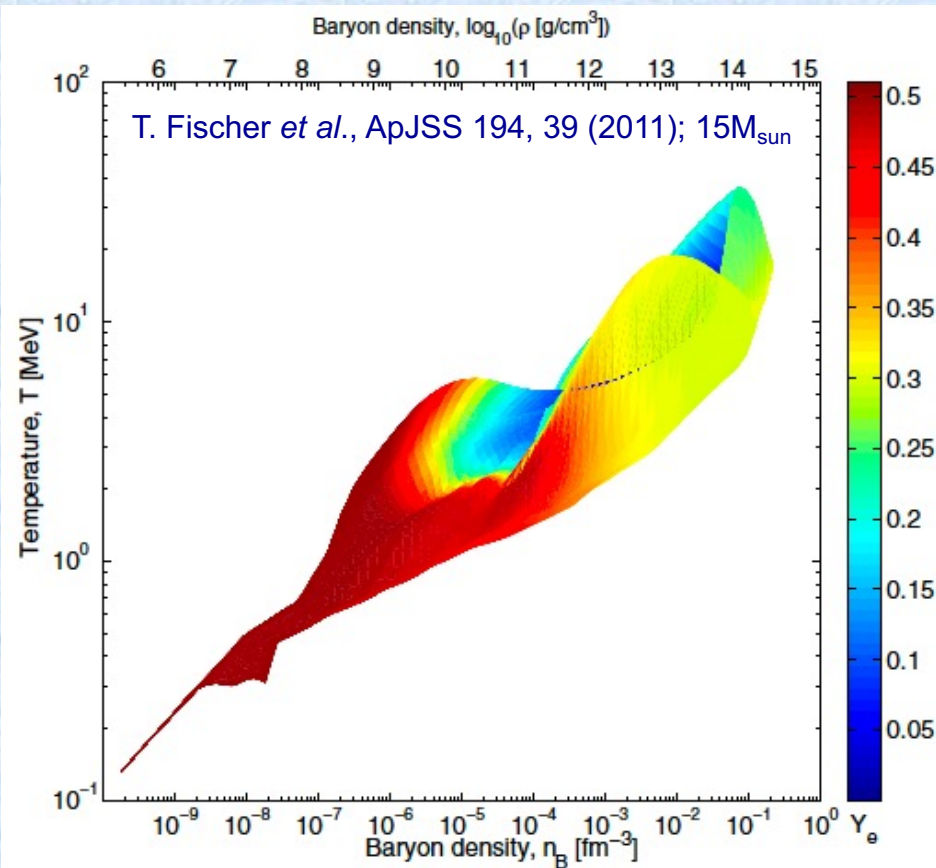
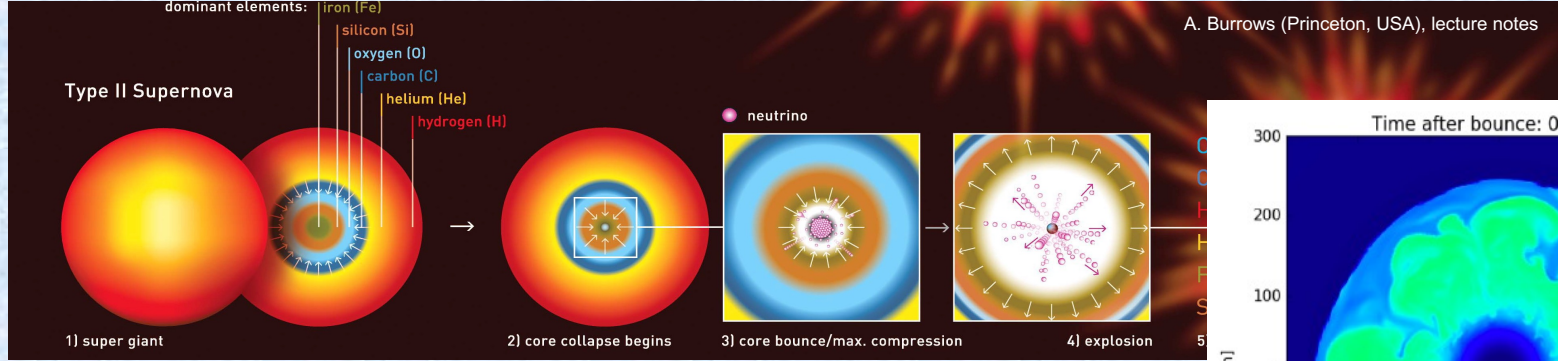
Courtesy of F. Gulminelli, talk (2023)

- Matter (almost always) degenerate and relativistic e-
- Possible additional d.o.f. → "exotic" matter





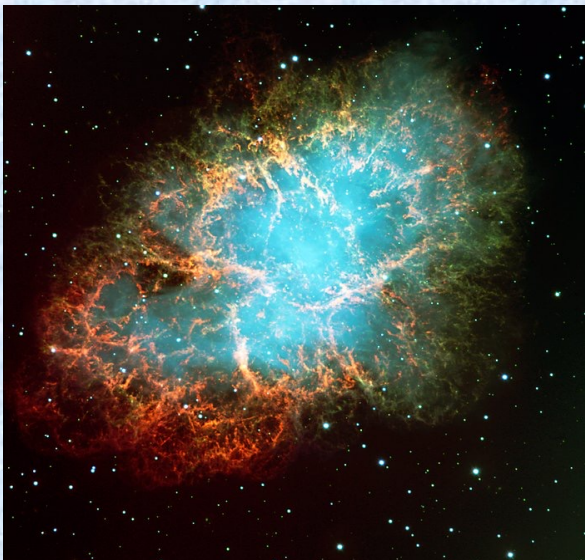
# Core-collapse supernovae (CCSN)







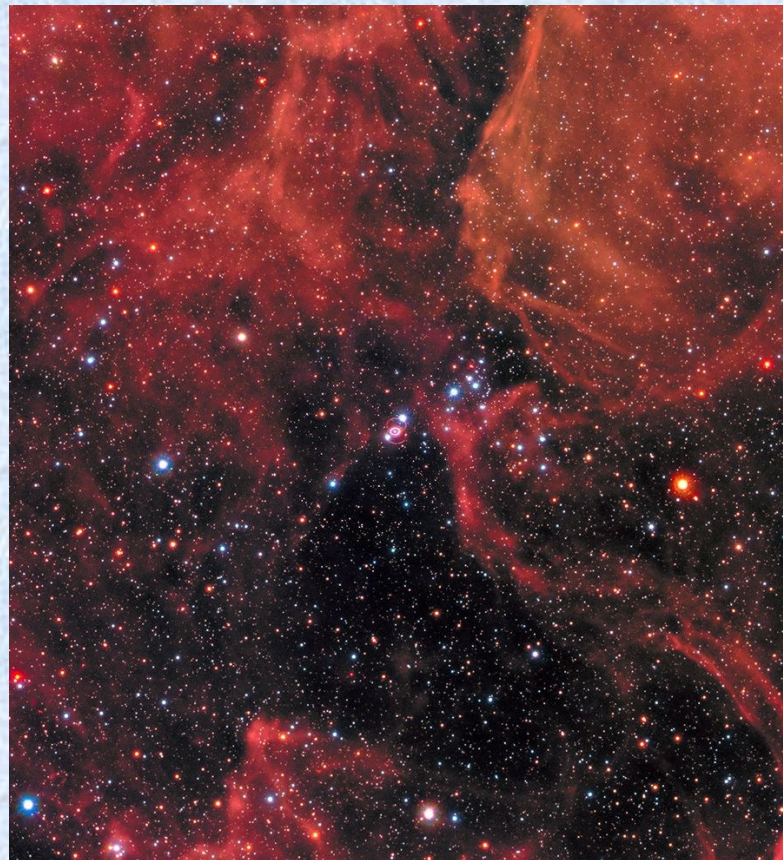
# Core-collapse supernovae (CCSN): observations



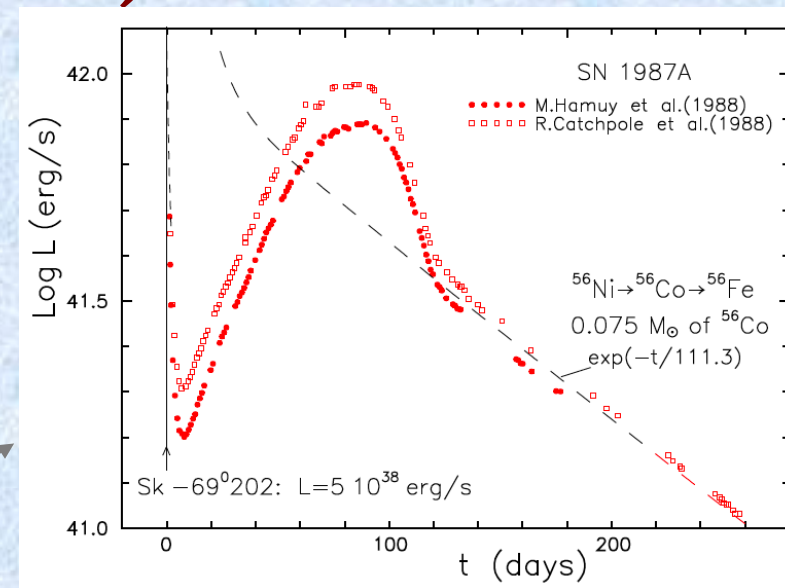
Crab Nebula (1054). Credit: ESO



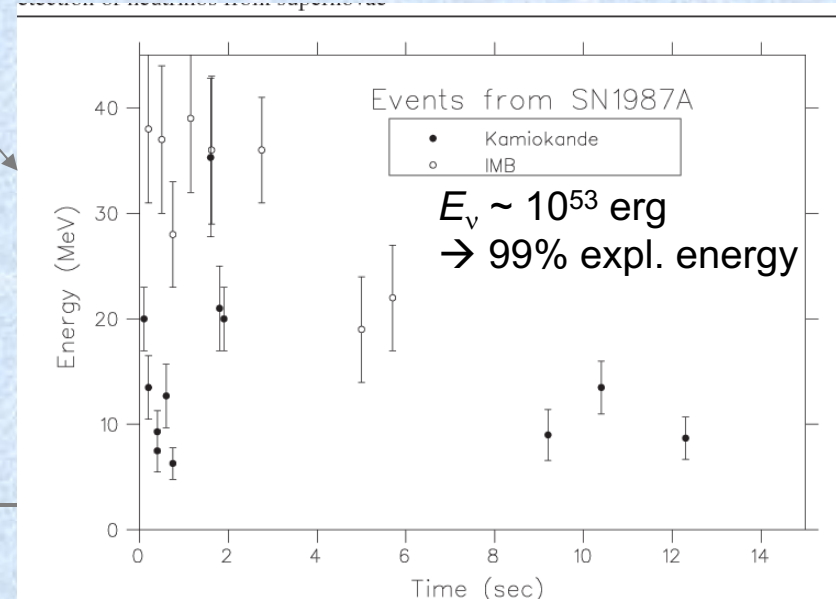
Cassiopea A. Credit: NASA, ESA, CSA, STScI  
A. F. Fantina



SN1987A. Credit: NASA, ESA

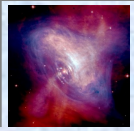


Light curve; Nadyozhin&Imshennik, arXiv:0501.002

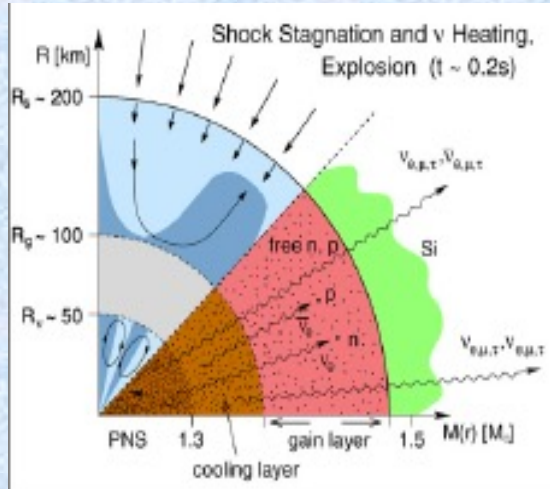


Boyd et al., J.Phys.G 29, 11 (2003)

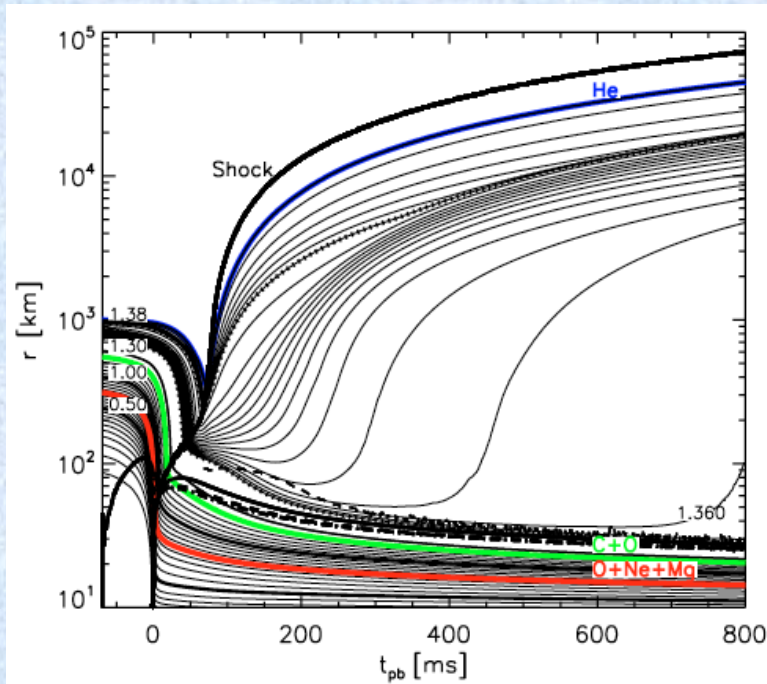




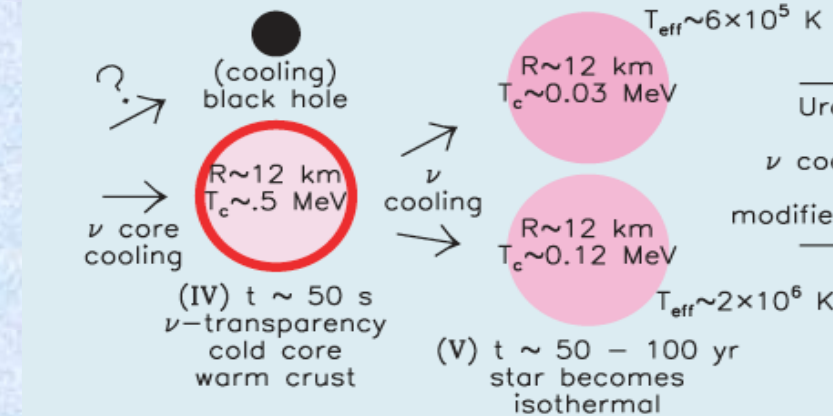
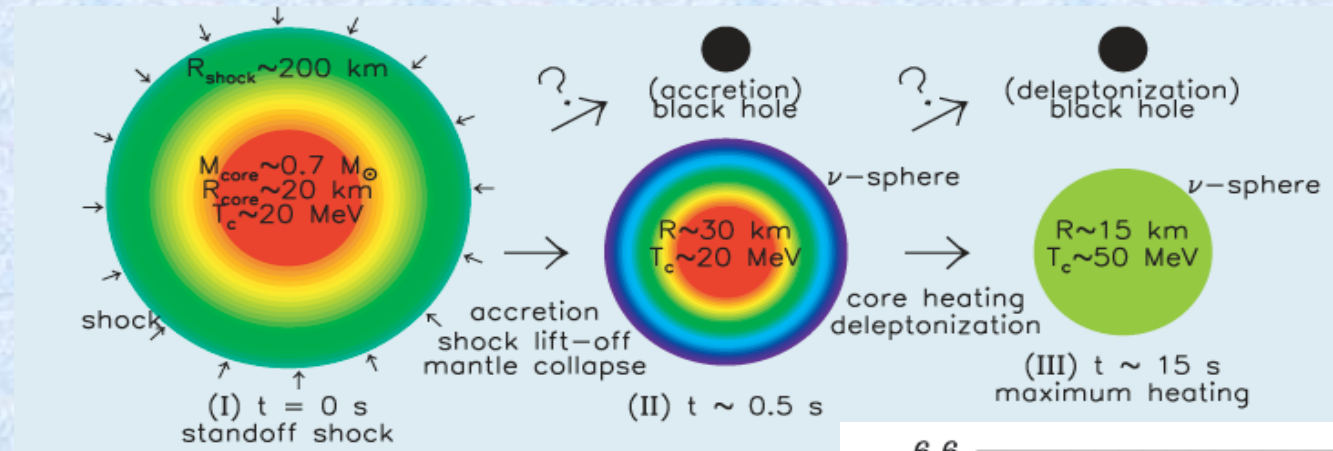
# Proto-neutron stars (PNS)



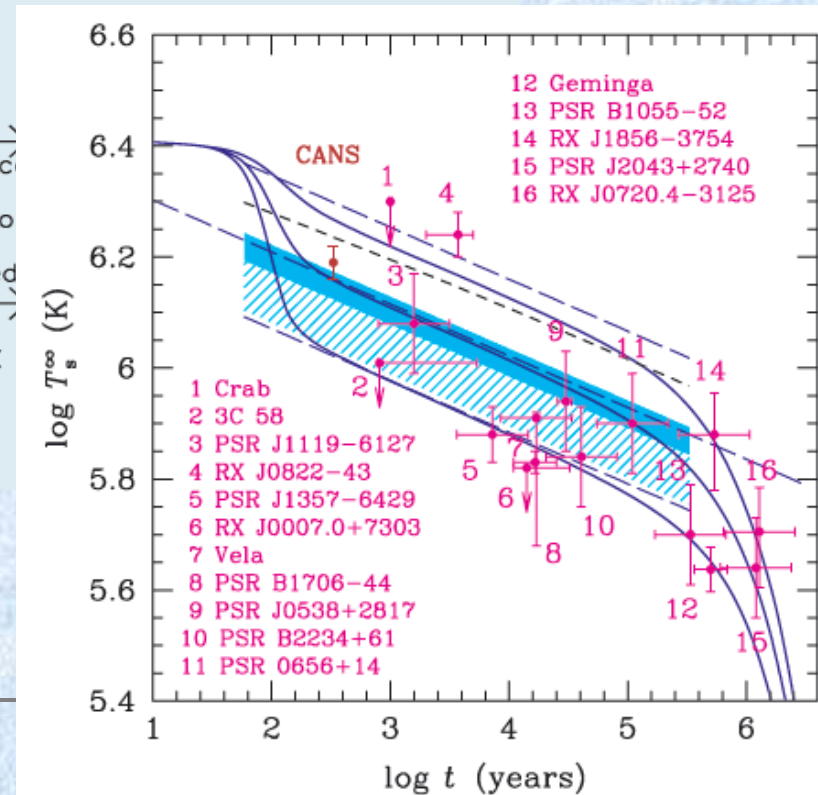
Janka et al., Phys. Rep. 2007



Kitaura et al., A&A 450, 345 (2006). O-Ne-Mg SN



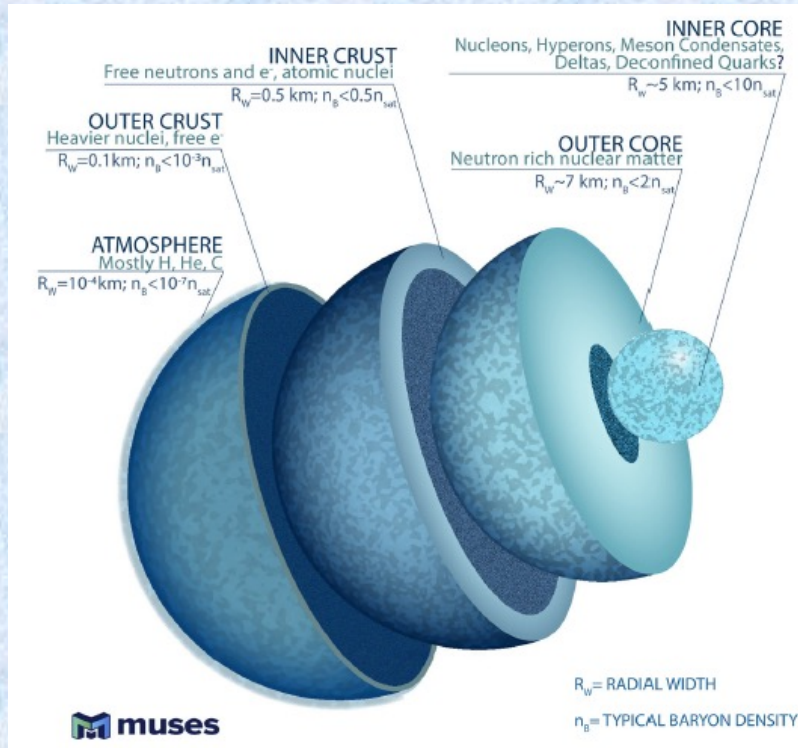
Lattimer & Prakash, Science 304, 536 (2004)



Yakovlev et al., MNRAS 411, 1977 (2011)



# Neutron-stars (NS)



Kumar et al., Liv. Rev. Rel. 27, 3 (2024)

$$M \approx 1 - 2M_{\odot}$$

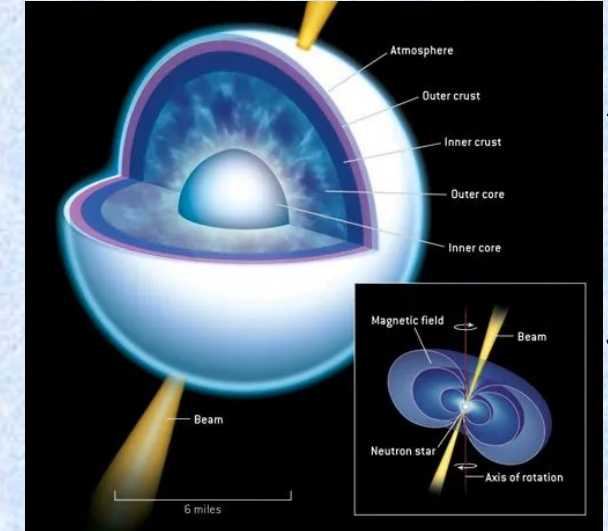
$$R \approx 10 \text{ km}$$

$$\bar{\rho} \approx 10^{14} - 10^{15} \text{ g cm}^{-3}$$

$$T < 10^8 \text{ K} \quad \text{NB: } T = 0 \Leftrightarrow T \ll T_F$$

rotation up to 716 Hz ( $v_{\text{equat}} \sim \frac{1}{4} c$ )

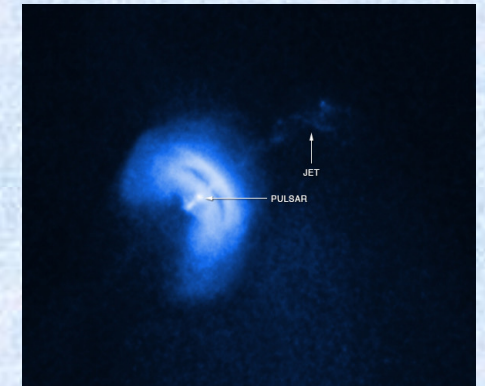
high magnetic fields,  $B > 10^{12} \text{ G}$



Credits: <https://www.astronomy.com/science/>

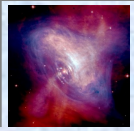


Crab, X-ray: NASA/CXC/SAO; Optical: NASA/STScI; IR: NASA-JPL-Caltech



Vela, Credit: NASA/CXC/Univ of Toronto/ M.Durant et al





# Neutron-stars (NS): fortuitous discovery



Jocelyn Bell in 1966

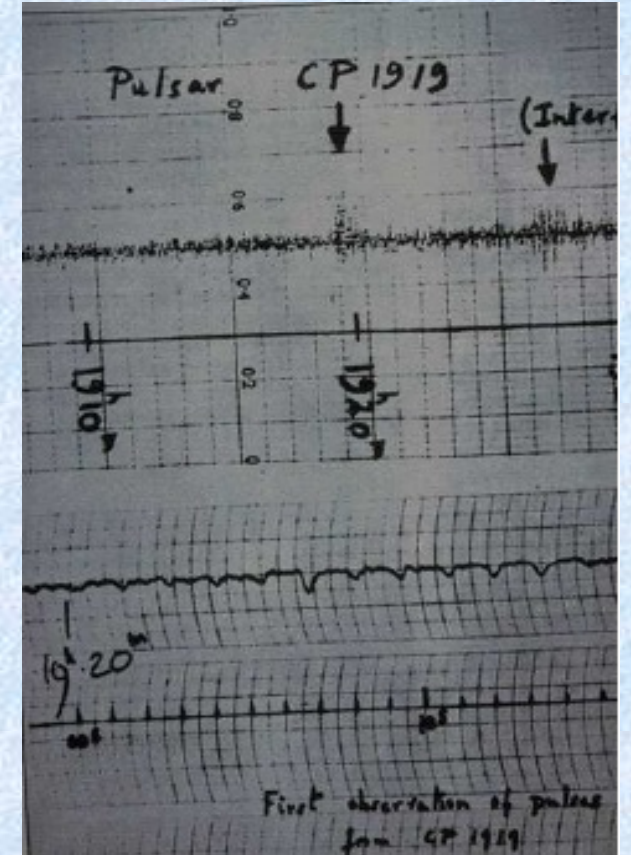
1967: J. Bell, PhD thesis at Cavendish Laboratory, Cambridge on radio sources. With a 3.7m diameter telescope, she discovered a very regular source, with a period of 1.3373012 s.

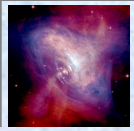
This source was called LGM ("Little Green Man"); a journalist from Daily Telegraph called this source "*Pulsar*".

→ It is now known as PSR B1919+21.

In 1974, A. Hewish (Bell's PhD supervisor) received the Nobel prize...

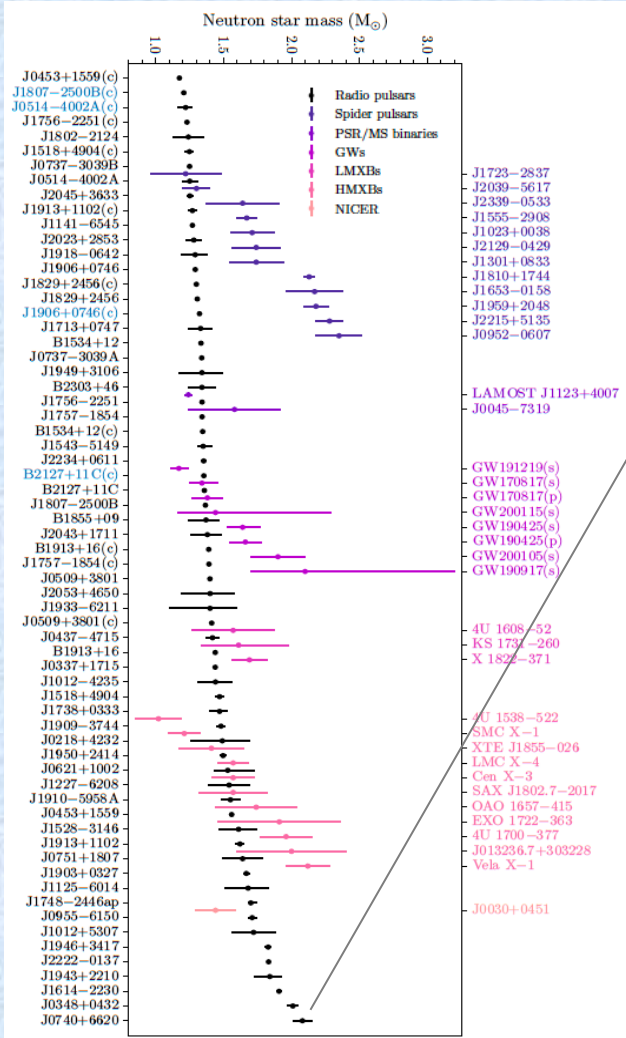
Now, over 3000 pulsars known!



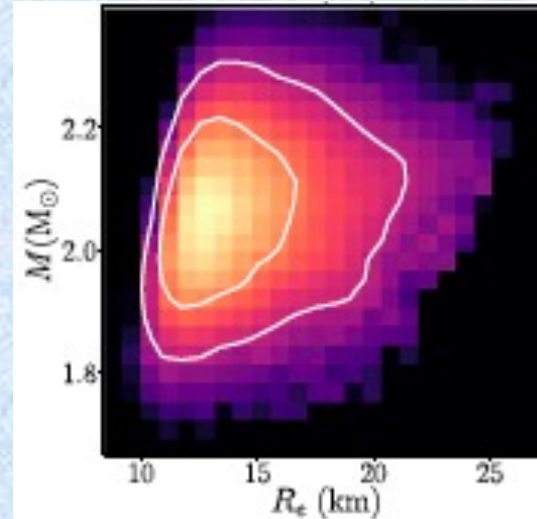


# Neutron-stars (NS): observations

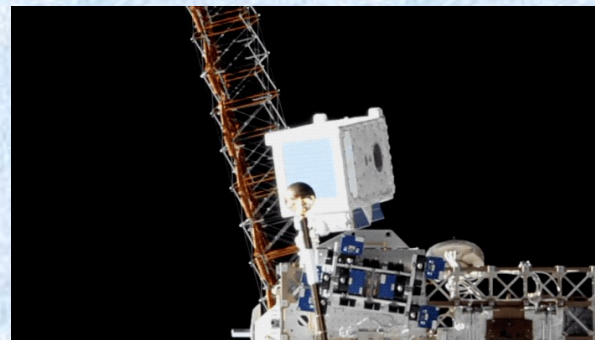
## MASSSES



## RADII



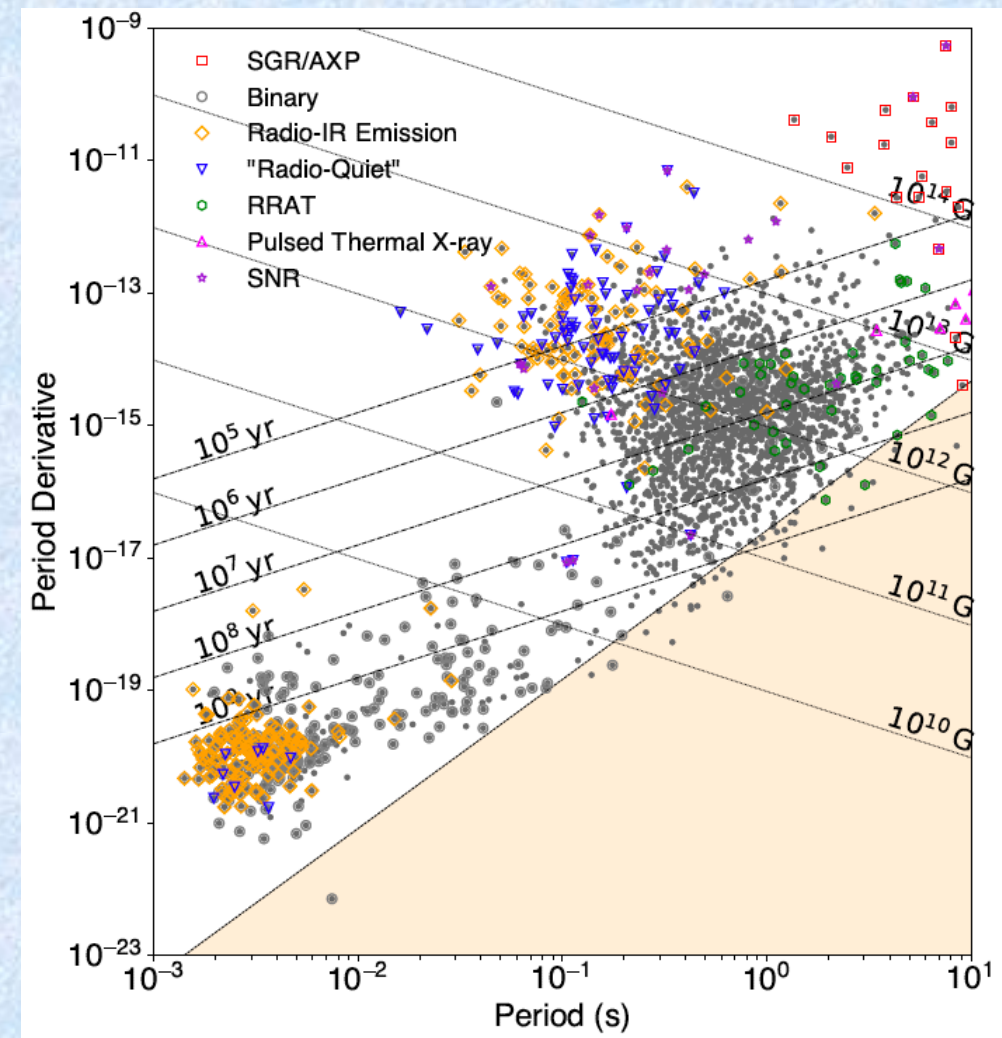
NICER observations (Miller et al., ApJ 2019, 2021; Riley et al., ApJL 2019, 2021; Rutherford et al., ApJL 2024; Mauviard et al., arXiv 2025)



NICER on the ISS

Credits: <https://heasarc.gsfc.nasa.gov/docs/nicer/>

## SPIN PERIOD



Credits: ATNF Pulsar Catalogue, <https://www.atnf.csiro.au/research/pulsar/psrcat/>; Manchester et al. AJ 129, 1993 (2005); plot generated with Python psrqpy package

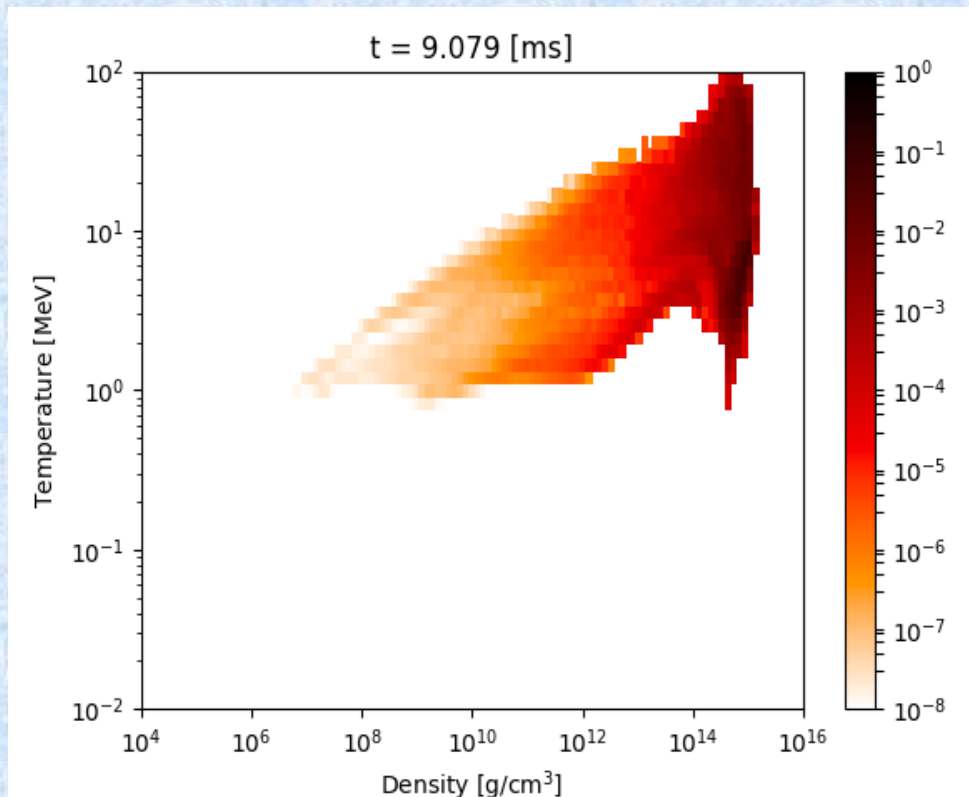
You et al., Nat. Astr. 9, 552 (2025); [https://www3.mpifr-bonn.mpg.de/staff/pfreire/NS\\_masses.html](https://www3.mpifr-bonn.mpg.de/staff/pfreire/NS_masses.html)

A. F. Fantina

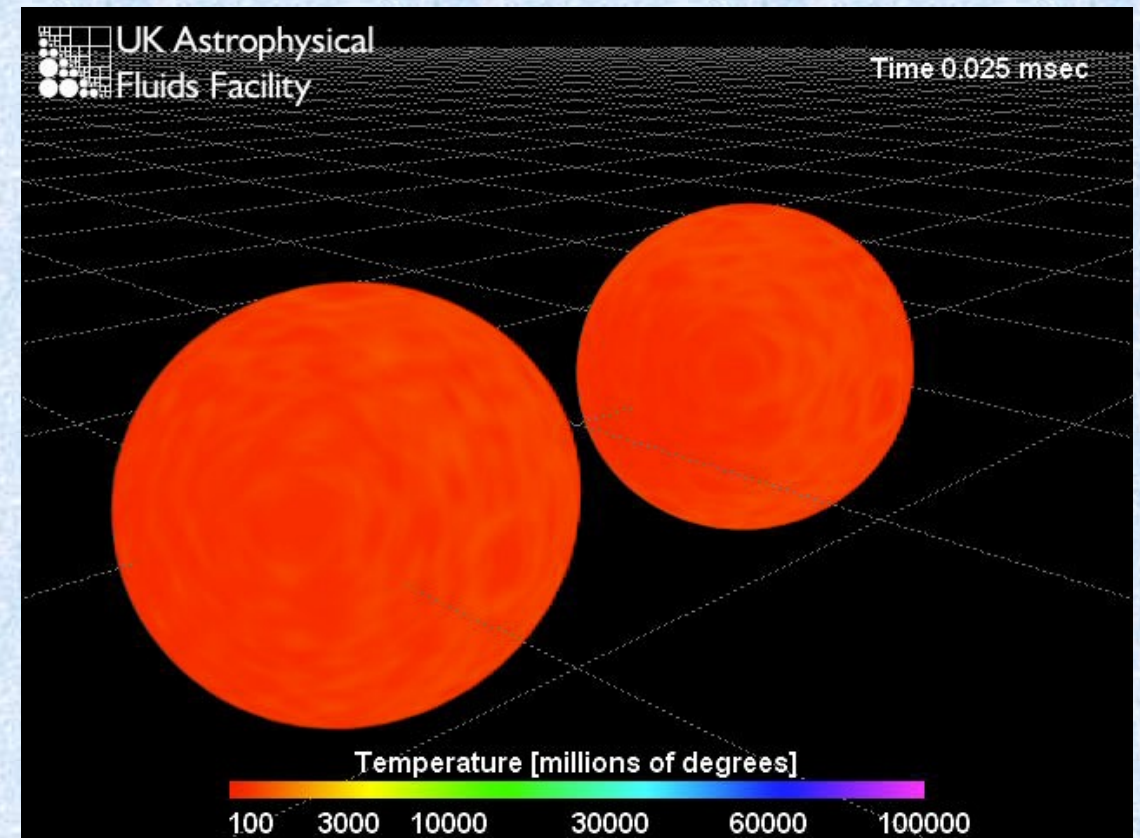




# Neutron-star (NS) mergers (1)



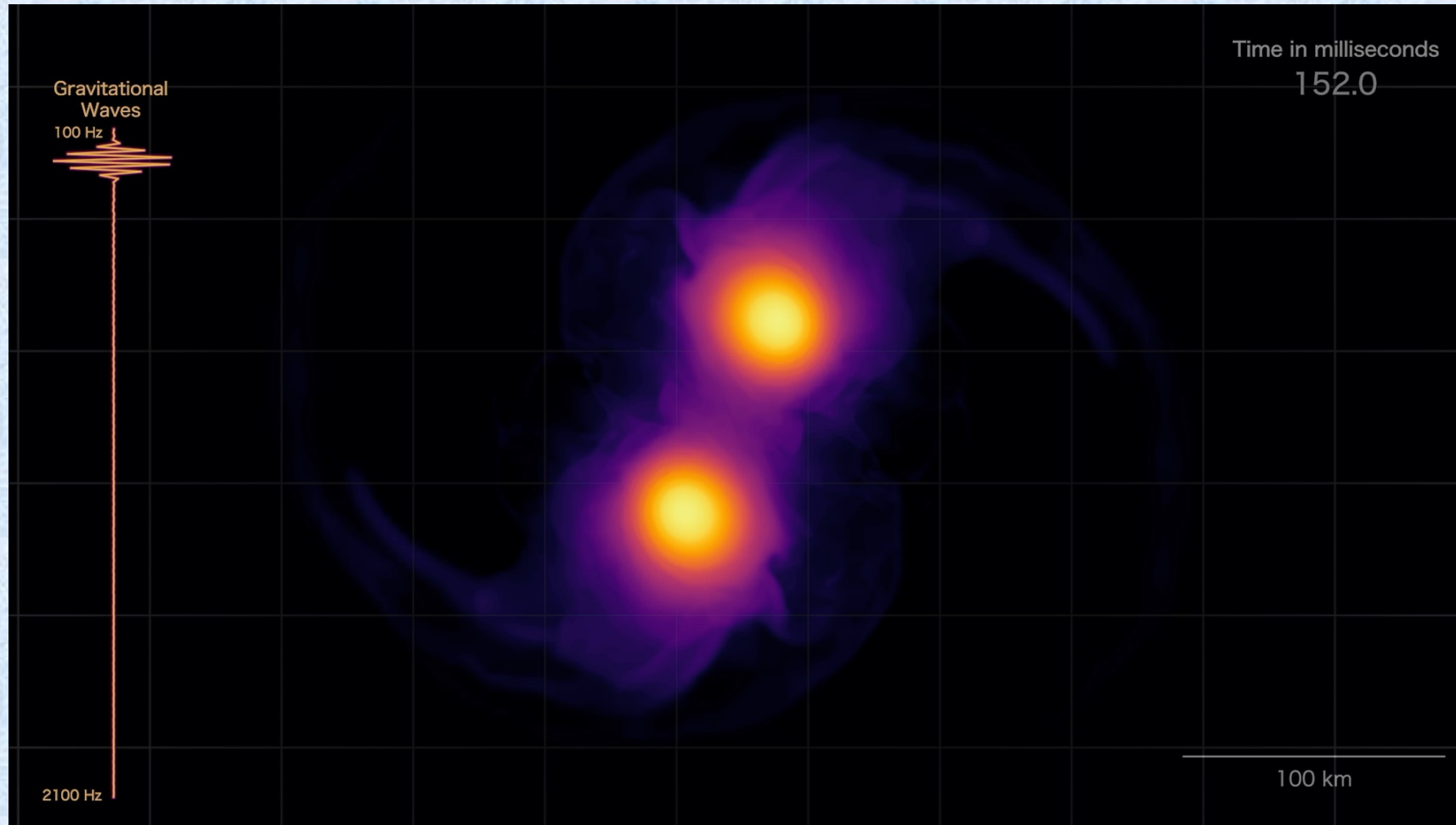
Perego et al., EPJA 55, 124 (2019)



Simulation by S. Rosswog, visualisation R. West  
<http://www.ukaff.ac.uk/movies/nsmerger/>



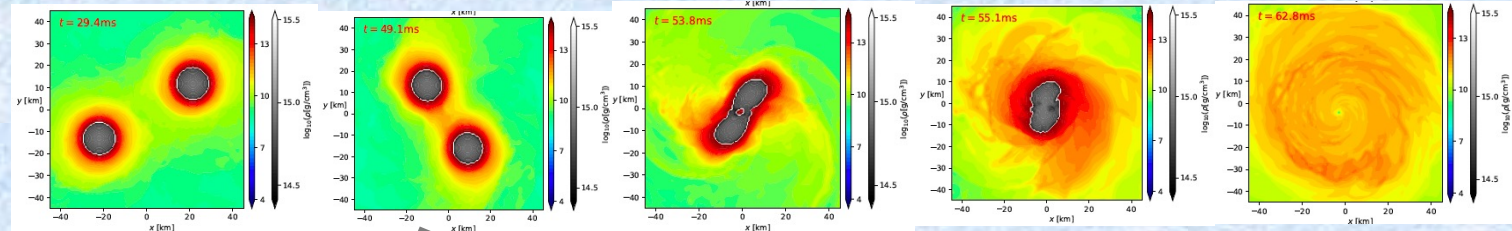
# Neutron-star (NS) mergers (2)



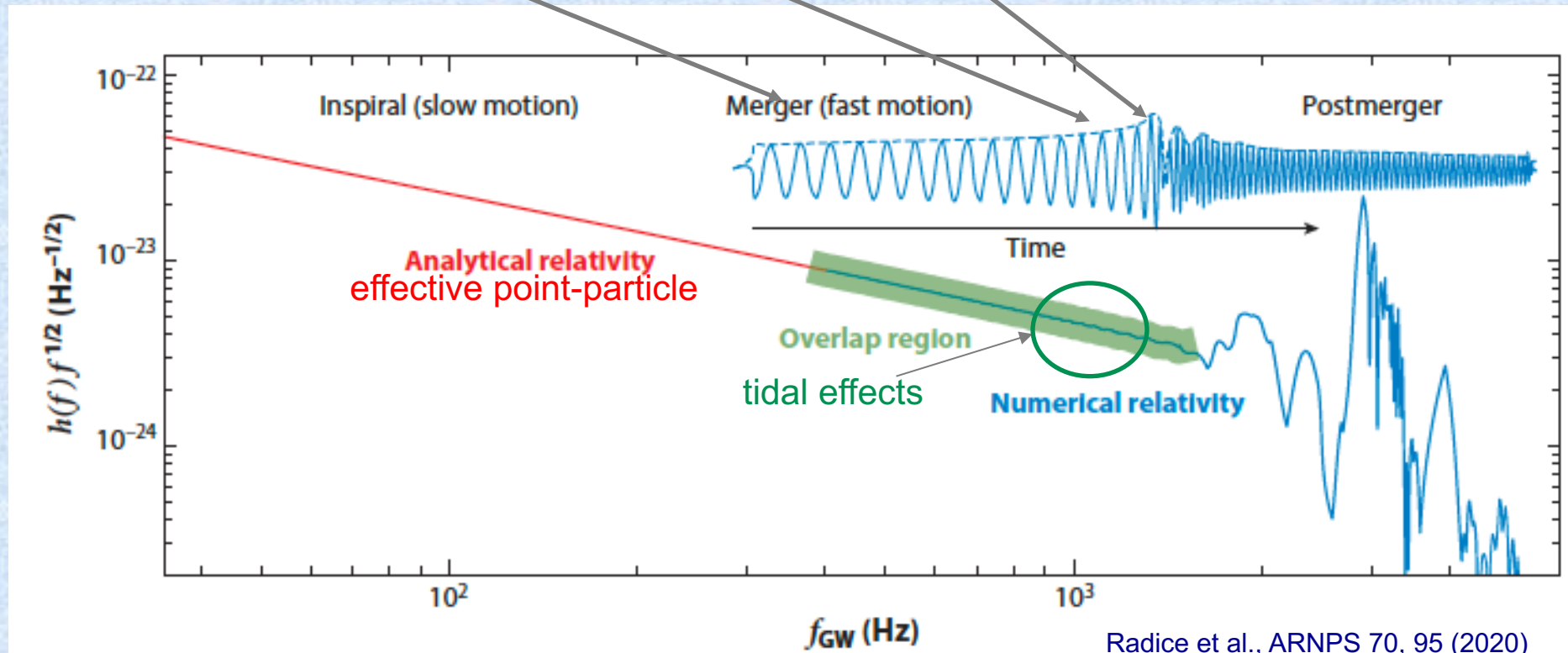




# Neutron-star (NS) mergers (3)



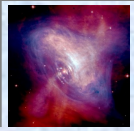
Giegl et al., Particle 2, 23 (2019)



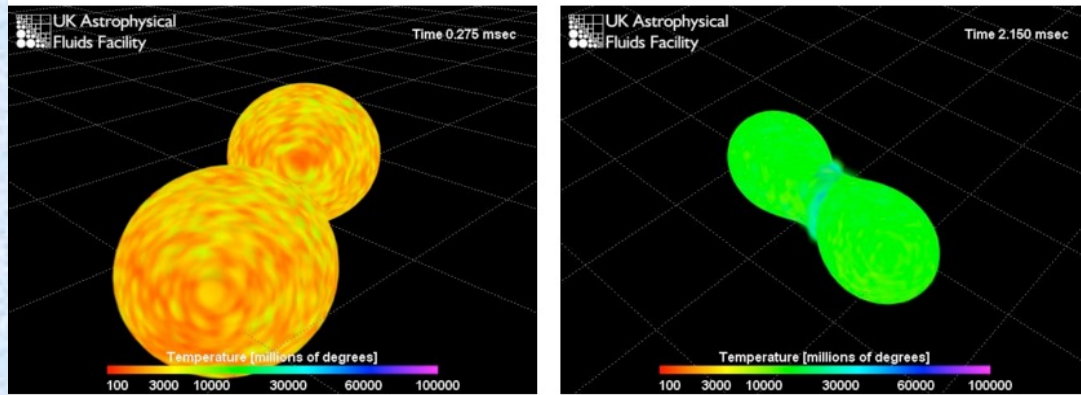
Radice et al., ARNPS 70, 95 (2020)

- (Late) inspiral (before merging): modification wrt point particle but still “(hydro)static”
- Temperatures not so high → like “ $T = 0$ ”





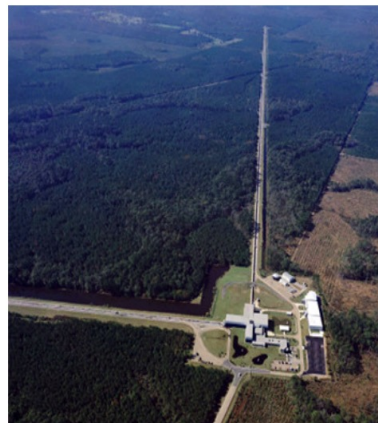
# NS mergers: observations



Simulation by S. Rosswog, visualisation R. West <http://www.ukaff.ac.uk/movies/nsmerger/>



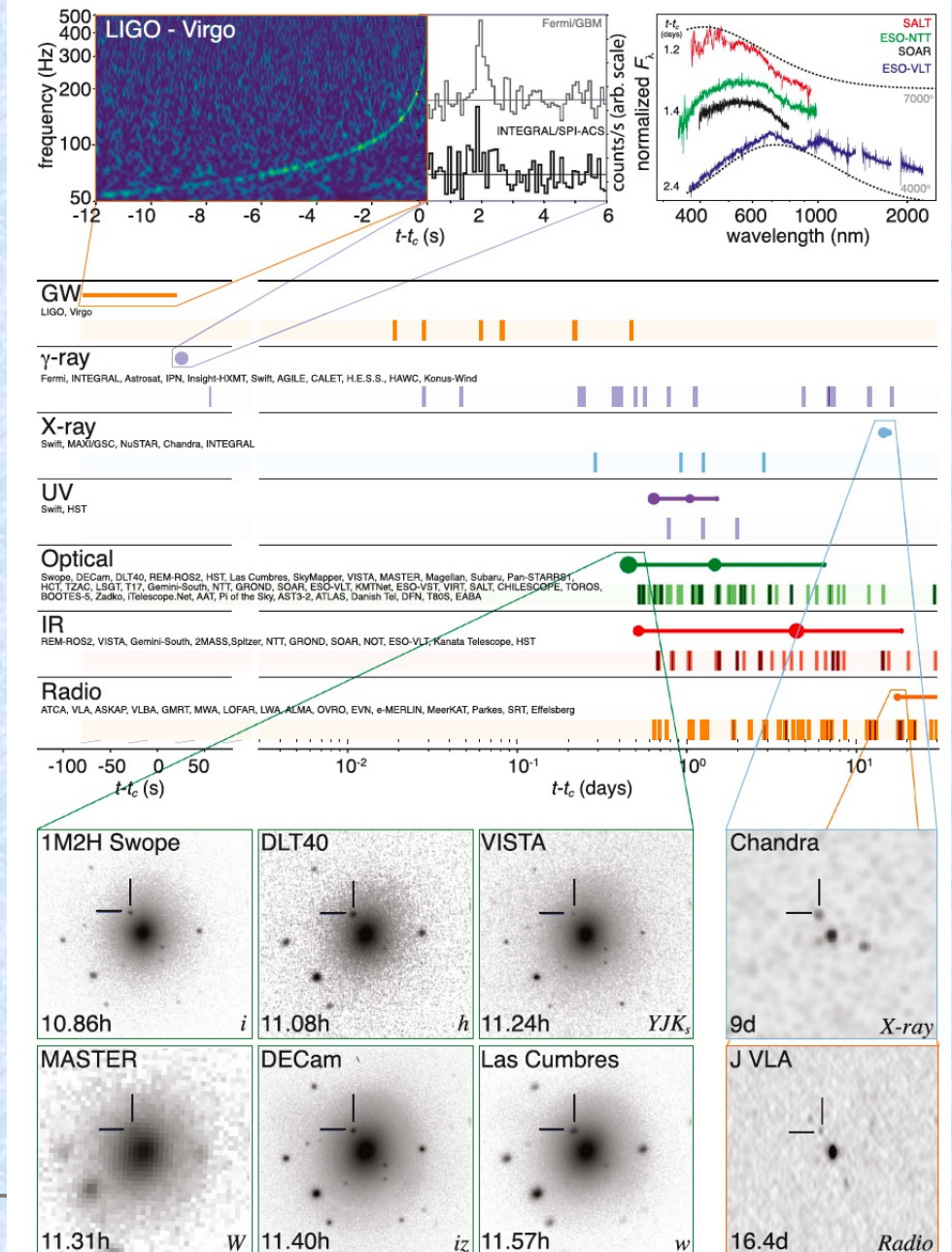
LIGO Hanford Observatory, Washington  
(Credits: C. Gray)



LIGO Livingston Observatory, Louisiana  
(Credits: J. Giaime)



Virgo detector, Italy  
(Credits: Virgo Collaboration)

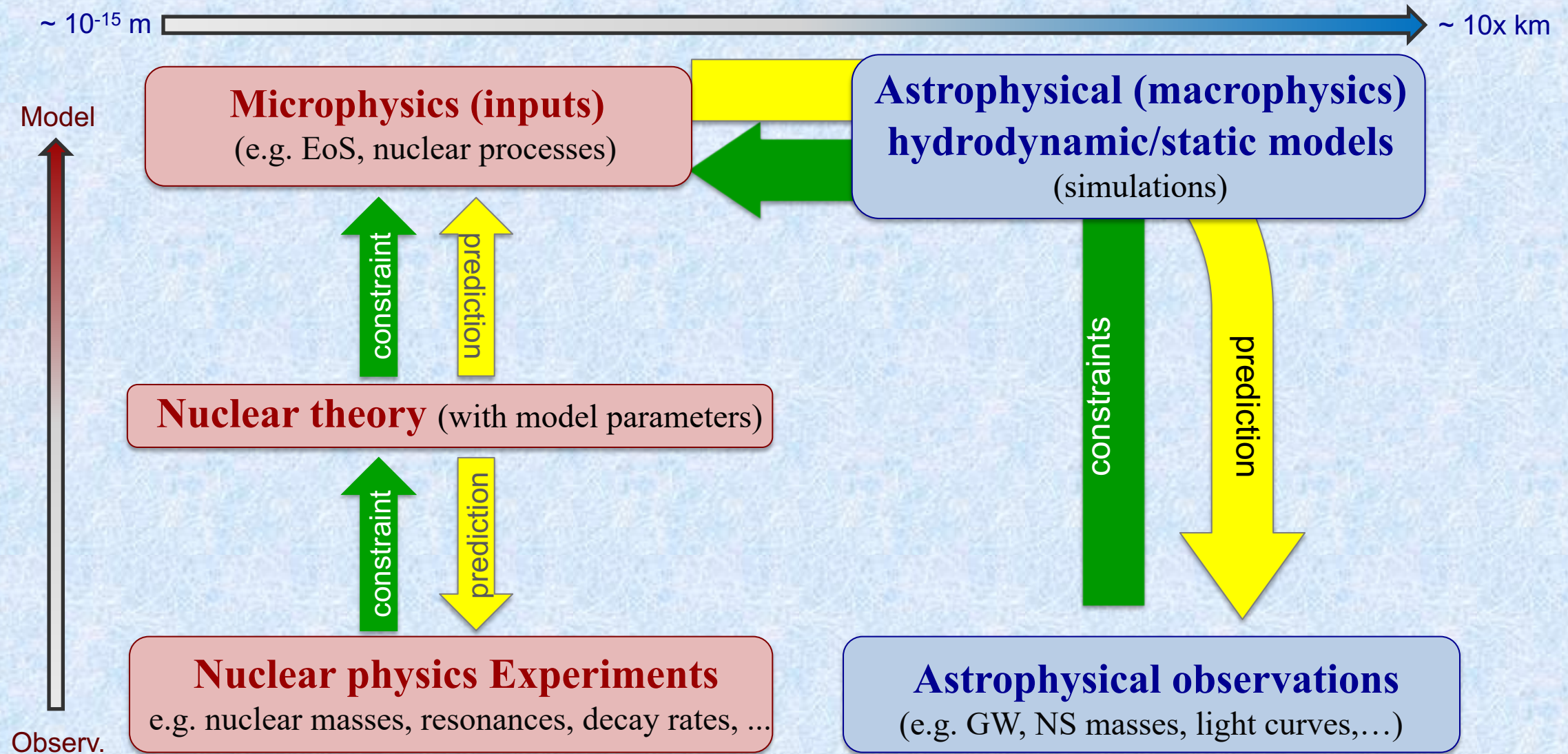


Abbott et al., ApJL 848, L12 (2017)





# Micro to macro through modelling and scales





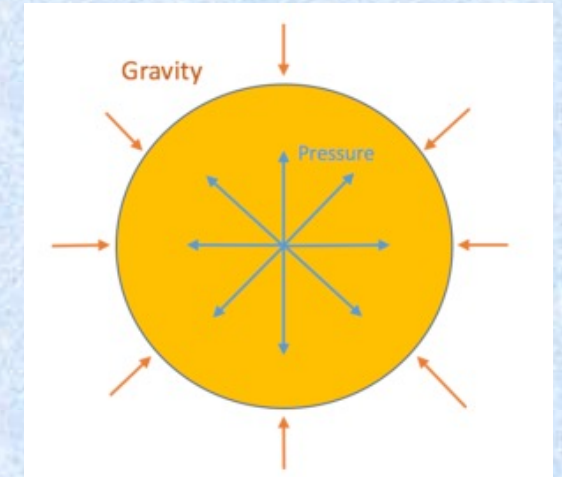
# (cold) NS $\rightarrow$ static $\rightarrow$ EoS $\leftrightarrow$ NS observables (1)

- **TOV  $\rightarrow M(R)$**  (Tolmann 1939; Oppenheimer&Volkoff 1939; see also Haensel et al. Springer 2007)

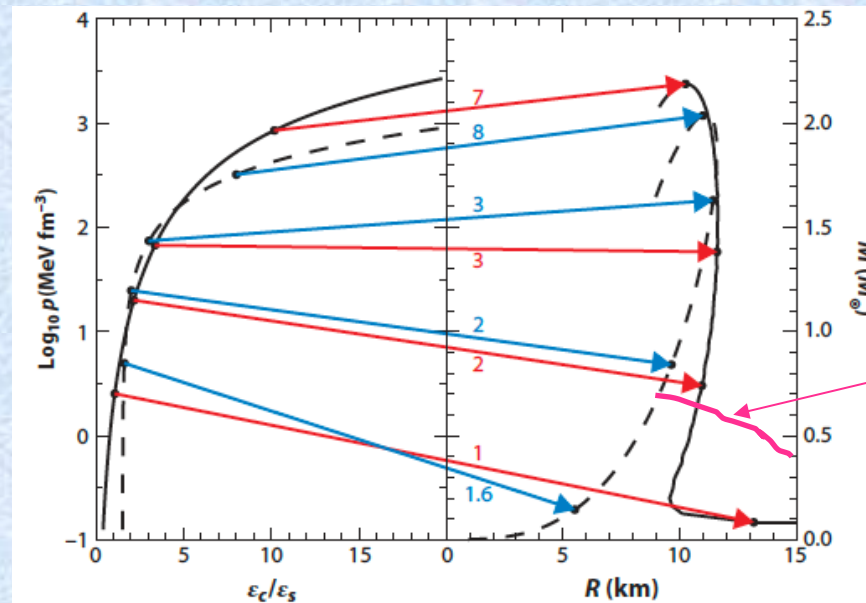
$$\frac{dP(r)}{dr} = -\frac{G\rho(r)\mathcal{M}(r)}{r^2} \left[ 1 + \frac{P(r)}{c^2\rho(r)} \right] \left[ 1 + \frac{4\pi P(r)r^3}{c^2\mathcal{M}(r)} \right] \left[ 1 - \frac{2G\mathcal{M}(r)}{c^2r} \right]^{-1}$$

$$\mathcal{M}(r) = 4\pi \int_0^r \rho(r') r'^2 dr' \quad \Rightarrow \quad P(\mathcal{E}) \quad \text{EoS only needed!}$$

$$\forall P_c \text{ (or } \mathcal{E}_c) \rightarrow R = r(P = 0), M = \mathcal{M}(r = R) \rightarrow \text{Mass \& Radius}$$



Credits: J. Stayner, Stanford Univ.



Lattimer, Annu. Rev. Part. Nucl. Sci. 62, 485 (2012)

using EoS of free n gas  
 $\rightarrow$  nuclear interaction !!!

(Oppenheimer&Volkoff 1939; for a review on Mmax:  
Chamel et al., IJMPE 22, 1330018 (2013))



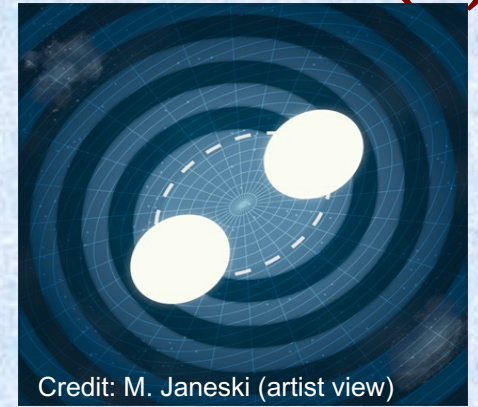
# **(cold) NS $\rightarrow$ static $\rightarrow$ EoS $\leftrightarrow$ NS observables (2)**

- **NS in external, inhomogeneous gravitational field  $\rightarrow$  tidal deformation**

(Thorne & Campolattaro 1967; Hinderer, ApJ 2008; Hinderer et al., PRD 2010; Blanchet, Liv. Rev. Relat. 2014)

$$Q_{ij} = -\lambda \mathcal{E}_{ij} \quad \rightarrow \text{quadrupolar } (l=2) \text{ momentum in response to tidal field} \\ (\text{hp linear order, source of tidal field far away})$$

$$k_2 = \frac{3}{2} \frac{G}{R^5} \lambda \quad \rightarrow \text{tidal Love number (dimensionless)}$$

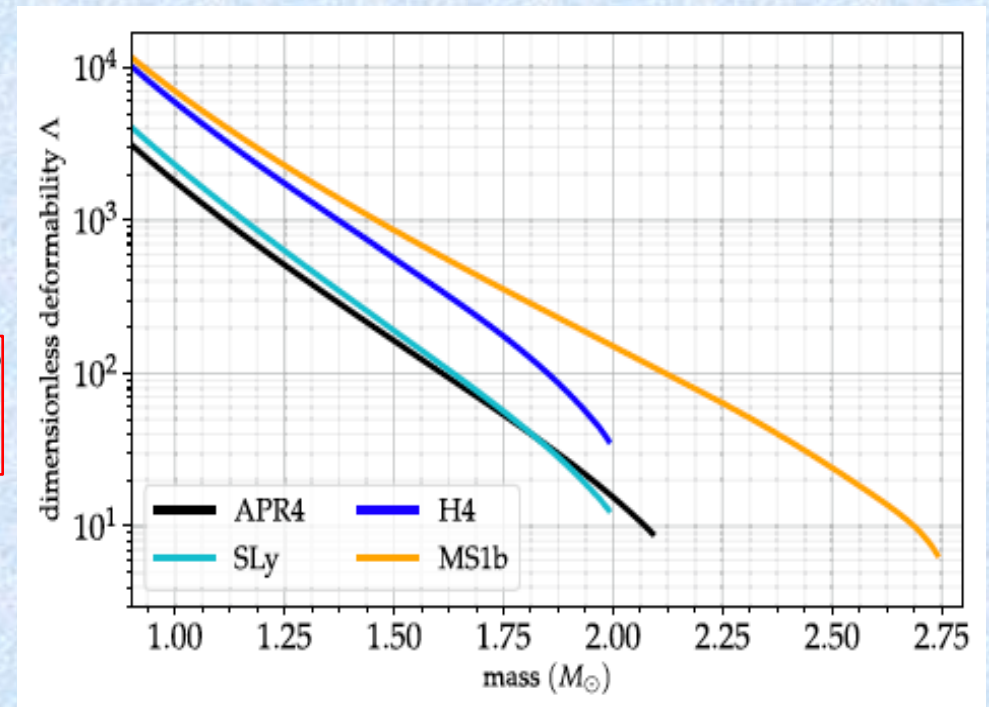


- **Perturbation to the equilibrium metric  $\rightarrow$  eq. for  $y(r)$**

$$r \frac{dy}{dr} + y(r)^2 + F(r)y(r) + Q(r) = 0$$

$\swarrow$   $P(r), \mathcal{E}(r)$

$$\rightarrow k_2 = k_2(y(r=R), M/R) \rightarrow \Lambda = \frac{2}{3} k_2 \left( \frac{Rc^2}{GM} \right)^5$$



Dietrich et al., Gen. Rel. Gravit. 53, 27 (2021)



# Binary parameters and GW signal

- **Binary parameters extracted from GW signal**  
(compare  $h(t)$  with templates)

$$h(t) \propto A(t) \exp^{i\Phi(t)} \rightarrow \tilde{h}(f) \propto \mathcal{A} f^{-7/6} \exp^{i\phi(f)}$$

$$\mathcal{A} \propto \mathcal{M}^{5/6} \quad \rightarrow \text{chirp mass} \quad \mathcal{M} = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}}$$

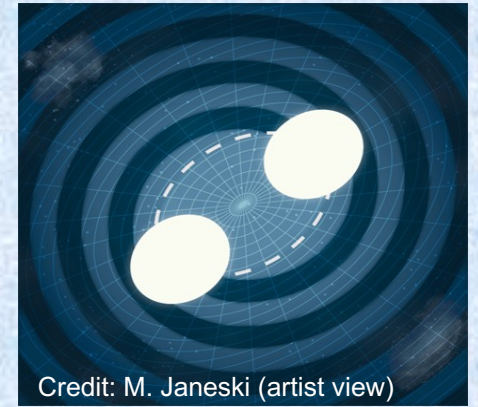
$$\phi(f) = \phi_{\text{p.p.}} + \phi_{\text{tidal}}$$

In post-newtonian ( $\rightarrow$  expansion in  $(v/c)^2$ ) and for non-spinning objects (Henri et al., arXiv:2005.13367; Wade et al., PRD 2014):

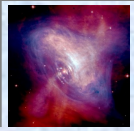
$$\phi(f) = 2\pi f t_c - 2\phi_0 - \frac{\pi}{4} + \frac{3}{128} \frac{(\pi G(M_1 + M_2) f)^{-5/3} (M_1 + M_2)^2}{M_1 M_2} \left[ 1 - \frac{39}{2} \tilde{\Lambda} \left( \frac{\pi G(M_1 + M_2) f}{c^3} \right)^{10/3} \right]$$

➤ Leading mass dependence:  $\frac{(M_1 + M_2)^{1/3}}{M_1 + M_2} \rightarrow \mathcal{M}^{-5/3}$

➤ Tidal effects at 5PN  $\rightarrow \tilde{\Lambda} = \frac{16}{13} \frac{(M_1 + 12M_2)M_1^4 \Lambda_1 + (M_2 + 12M_1)M_2^4 \Lambda_2}{(M_1 + M_2)^5}$







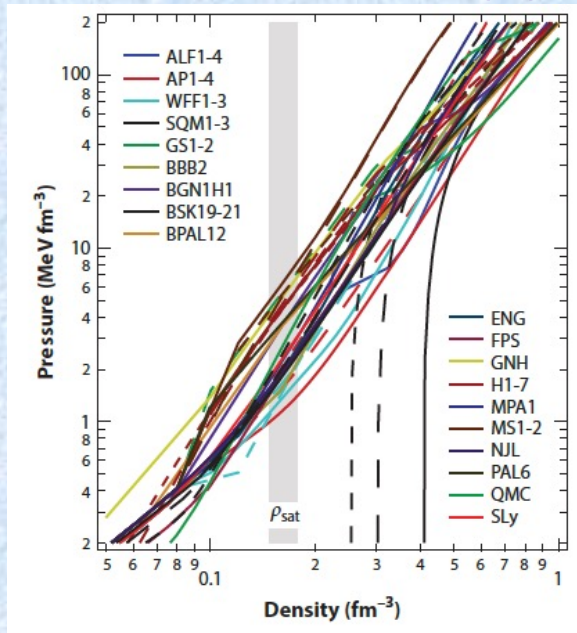
# EoS $\leftrightarrow$ NS observables $\leftrightarrow$ $M(R)$ , $\Lambda(R)$ ?

→ GR → direct correspondence  
EoS  $\leftrightarrow$  NS static properties

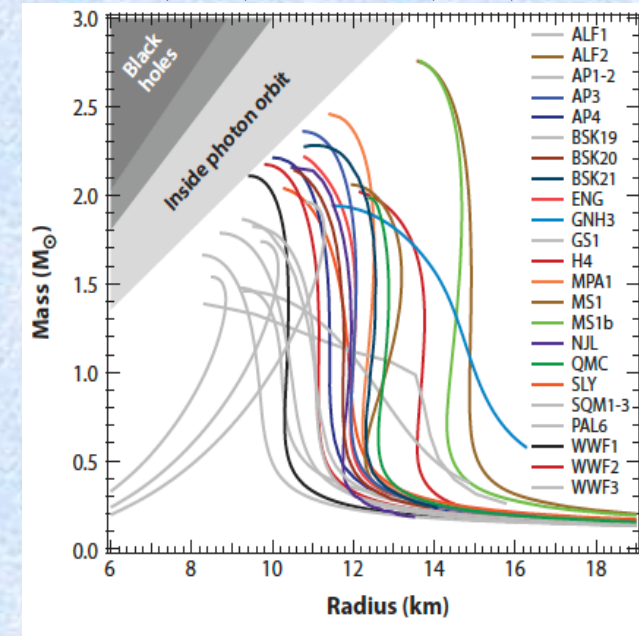
?  
→ trace back to EoS  
(and composition?)

but:

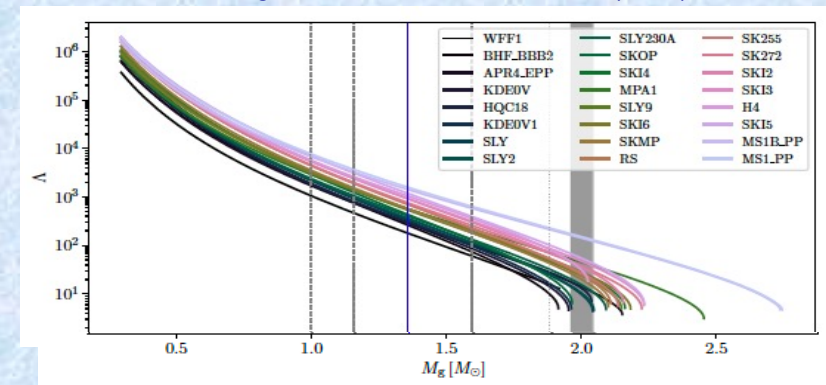
- ✗ EoS model dependent ! (higher uncertainties at higher  $\rho$ )
- ✗ no ab-initio dense-matter calculations  
in all regimes → phenomenological models
- ✗ composition  $\leftrightarrow$  EoS →  $M(R)$  ?



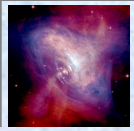
Ozel & Freire, ARAA 54, 401 (2016)



Ozel & Freire, ARAA 54, 401 (2016)  
see also Burgio & Fantina, ASSL 457, 255 (2018)



Abbott et al., Class. Quantum Grav. 37, 045006 (2020)



# EoS at $T = 0$ : generalities (1)

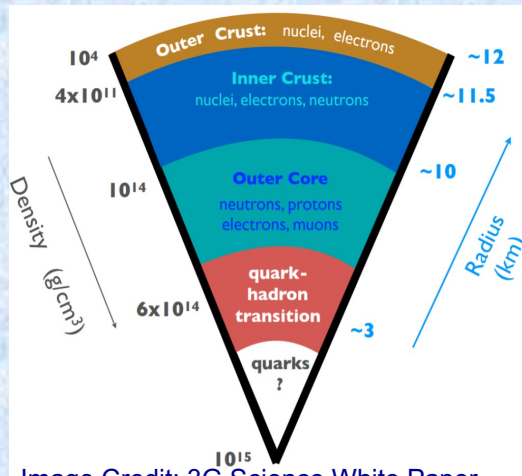


Image Credit: 3G Science White Paper

- **NS hot** ( $\sim 10^{12}$  K  $\rightarrow$  finite  $T$  important)  $\rightarrow$  cool in days to  $\sim 10^9$  K  $\rightarrow T = 0 \rightarrow$  **full equilibrium (strong+weak)**  $\rightarrow T \ll T_F$
- EoS needed to compute NS properties ( $M$ ,  $R$ , etc)
- Composition input for different observables (e.g. cooling, elastic properties, oscillations, magneto-rotational evolution, ...)
- Different phases of matter (solid crust, “liquid” core, superfluid, QGP?, ...)

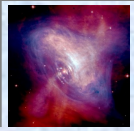
- **NB:** matter decouples from gravity!  $\rightarrow$  when calculating EoS, gravity can be neglected (this also at  $T > 0$ )

$$ds^2 = - \left( 1 - \frac{2Gm(r)}{rc^2} \right) c^2 dt^2 + \left( 1 - \frac{2Gm(r)}{rc^2} \right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

Variation of the metric from NS centre to surface:  $\frac{g_{00}(r = R, m = M)}{g_{00}(r = 0, m = 0)} = 1 - \frac{2GM}{Rc^2} \approx 0.4$  if  $M = 2M_\odot, R = 10\text{km}$

Variation of the metric on a nuclear scale:  $\frac{g_{00}(r + \delta r)}{g_{00}(r)} \approx 1 + \frac{\delta r}{r} \approx 1 + 10^{-19}$





# EoS at $T = 0$ : generalities (2)

- $T = 0 \rightarrow$  full equilibrium (cold catalysed matter)
- EoS only density dependent  $P(n_B)$  (or  $P(\varepsilon)$ )  $\rightarrow$  one-dimensional
- EoS at each  $n_B \rightarrow$  minimise the total energy density
- However, different matter constituents  $\rightarrow E(\{n_i\})$ , with  $n_i(r) \rightarrow P(\{n_i\})$   
but: conserved densities:  $n_B, n_L, n_Q, (n_S)$ : baryon, lepton, charge, (strange)  
in the core: infinite matter  $\rightarrow$  thermo limit:  $n_i(r) = n_i$
- Baryons & leptons decoupled:

$$\mathcal{E} = \mathcal{E}_B + \mathcal{E}_L$$

$$\text{If only baryons + e: } \mathcal{E} = \mathcal{E}_B(n_n, n_p) + \mathcal{E}_e(n_e = n_p)$$

$$\text{If } \beta \text{ equilibrium: 1 variable } n_B \text{ as } n_B = n_n + n_p, \quad n_e = n_p = Y_p n_B$$

$$\mu_n - \mu_p = \mu_e$$

$$P = - \left. \frac{dE}{dV} \right|_{\{N_j\}, T} = n_B^2 \frac{de}{dn_B}$$

$$\text{If only baryons + electrons: } P = P_B + P_e$$

NB:  $n_B$  = baryon number density,  $\rho c^2 = \varepsilon$  mass-energy density !

NB: Leptons are treated as free Fermi gas, but baryons interact!

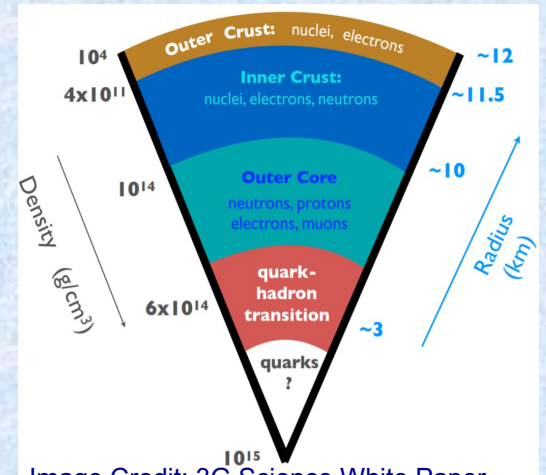


Image Credit: 3G Science White Paper



# Leptons (electrons) at $T = 0$ (1)

- Fermi gas, degenerate matter

$$f(k) = \frac{1}{1 + e^{(e(k) - \mu)/(k_B T)}}$$

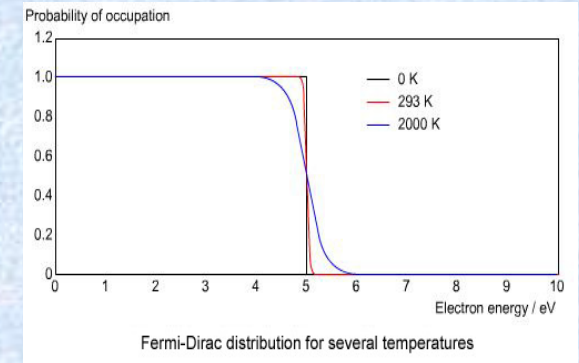
$$f(k) = \Theta(k)$$

- density: 
$$n_e = \frac{2}{(2\pi)^3} \int d^3k f(k) = \frac{1}{\pi^2} \int_0^\infty dk k^2 \Theta(k) = \frac{1}{\pi^2} \int_0^{k_F} dk k^2 = \frac{k_{F,e}^3}{3\pi^2}$$

- electron energy: 
$$e_e(k) = \sqrt{(m_e c^2)^2 + (\hbar c k)^2}$$

- NR limit:  $p_e c \ll m_e c^2 : \mathcal{E}_e = \frac{2}{(2\pi^2)^3} \int_0^{k_F} dk 4\pi k^2 \left( m_e c^2 + \frac{\hbar^2 c^2 k^2}{2m_e c^2} \right) = n_e m_e c^2 + \frac{\hbar^2 c^2 k_{e,F}^5}{10\pi^2 m_e c^2}$

$$P_e = n_e^2 \frac{d(\mathcal{E}_e/n_e)}{dn_e} = \frac{\hbar^2 c^2 k_{e,F}^5}{15\pi^2 m_e c^2} \propto n_e^{5/3} \rightarrow P = \mathcal{K} n^\gamma \rightarrow \text{Polytropic EoS}$$



Credit: University of Cambridge



NB: same for muons, with  $\mu_\mu = \mu_e$ . They appear when  $\mu_\mu = m_\mu$





# Leptons (electrons) at $T = 0$ (2)



## ▪ Fermi gas, degenerate matter

- UR limit:  $p_e c \gg m_e c^2$  :  $\mathcal{E}_e = \frac{2}{(2\pi^2)^3} \int_0^{k_F} dk 4\pi k^2 \hbar c k = \frac{\hbar c k_{e,F}^4}{4\pi^2}$

$$P_e = n_e^2 \frac{d\mathcal{E}_e}{dn_e} = \frac{\hbar c k_{e,F} n_e}{4} \propto n_e^{4/3} \rightarrow P = \mathcal{K} n^\gamma \rightarrow \text{Polytropic EoS}$$

e<sup>-</sup> relativistic when  $p_e \sim m_e c \rightarrow \rho_B = \frac{A}{Z} n_e m_p \approx \frac{A}{Z} n_e m_p \frac{1}{3\pi^2 \lambda_{e,C}^3} \approx 10^6 \text{ g cm}^{-3}$

$$\lambda_{e,C} = \frac{m_e c^2}{\hbar c}$$

- more general expressions:  $\mathcal{E}_e = \frac{m_e c^2}{8\pi^2 \lambda_{e,C}^3} \left[ 2x^3 \sqrt{1+x^2} + x \sqrt{1+x^2} - \ln(x + \sqrt{1+x^2}) \right]$   $x = \frac{\hbar c (3\pi^2 n_e)^{1/3}}{m_e c^2}$

$$P_e = \frac{m_e c^2}{8\pi^2 \lambda_{e,C}^3} \left[ x \left( \frac{2}{3} x^2 - 1 \right) \sqrt{1+x^2} + \ln(x + \sqrt{1+x^2}) \right]$$

N.B.: there are corrections to this formula, e.g.:

- electron screening
- exchange energy

NB: same for muons, with  $\mu_\mu = \mu_e$ . They appear when  $\mu_\mu = m_\mu$



# Leptons (electrons) at $T > 0$

- Fermi distribution function at  $T > 0$  not step function  $f(k) = \frac{1}{1 + e^{(e(k) - \mu)/(k_B T)}}$

$$e_e = \sqrt{(m_e c^2)^2 + (\hbar c k)^2} \quad \text{relativistic particles}$$

$$n_e = \frac{2}{(2\pi)^3} \int d^3 k f(k) = \frac{4}{\sqrt{\pi} \lambda_e^3} \left[ I_{1/2}(\chi, k_B T / m_e c^2) + \frac{k_B T}{m_e c^2} I_{3/2}(\chi, k_B T / m_e c^2) \right]$$

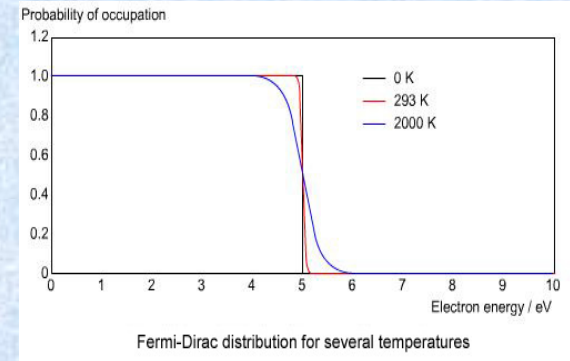
$$\mathcal{E}_e = \frac{4}{\sqrt{\pi}} \frac{k_B T}{\lambda_e^3} \left[ I_{3/2}(\chi, k_B T / m_e c^2) + \frac{k_B T}{m_e c^2} I_{5/2}(\chi, k_B T / m_e c^2) \right] \quad \lambda_e = \left( \frac{2\pi (\hbar c)^2}{m_e c^2 k_B T} \right)^{1/2}$$

$$P_e = \frac{8}{3\sqrt{\pi}} \frac{k_B T}{\lambda_e^3} \left[ I_{3/2}(\chi, k_B T / m_e c^2) + \frac{k_B T}{2m_e c^2} I_{5/2}(\chi, k_B T / m_e c^2) \right] \quad \chi = \frac{\mu_e - m_e c^2}{k_B T}$$

→ Fermi integral  $I_k \rightarrow$  numerical resolution

$$I_k(\chi, t) \equiv \int_0^\infty \frac{x^k (1 + tx/2)^{1/2}}{1 + \exp(x - \chi)} dx$$

NB: for relatively small  $T$  ( $T/T_F \ll 1$ ) → Sommerfeld expansion in powers of  $T^2$



NB: same for muons





# Baryons (n,p) at $T = 0$

NN interaction not known; nuclear physics = non-perturbative regime of QCD

→ *Two different approaches*

➤ **Ab-initio (“microscopic”) approaches**

based on quantum many-body theory from realistic NN interactions;

different techniques: **variational methods**, (D)BHF, **chiral EFT**, Monte-Carlo, Green’s functions, ...

→ usually restricted to homogeneous matter (NS core)

see L. Jokiniemi’s and P. Arhuis’s lecture

➤ **Phenomenological approaches**

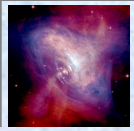
based on effective interactions (start from Hamiltonian/Lagrangian) with parameters

adjusted to reproduced nuclear properties: **EDF**  
e.g. **Skyrme / Gogny**, **RMF**, **meta-models**, ...

→ also for inhomogeneous matter (crust+core)

see M. Bender’s lecture

see also S. Lenzi’s lecture



# Baryons (n,p) at $T = 0$

NN interaction not known; nuclear physics = non-perturbative regime of QCD

→ Two different approaches

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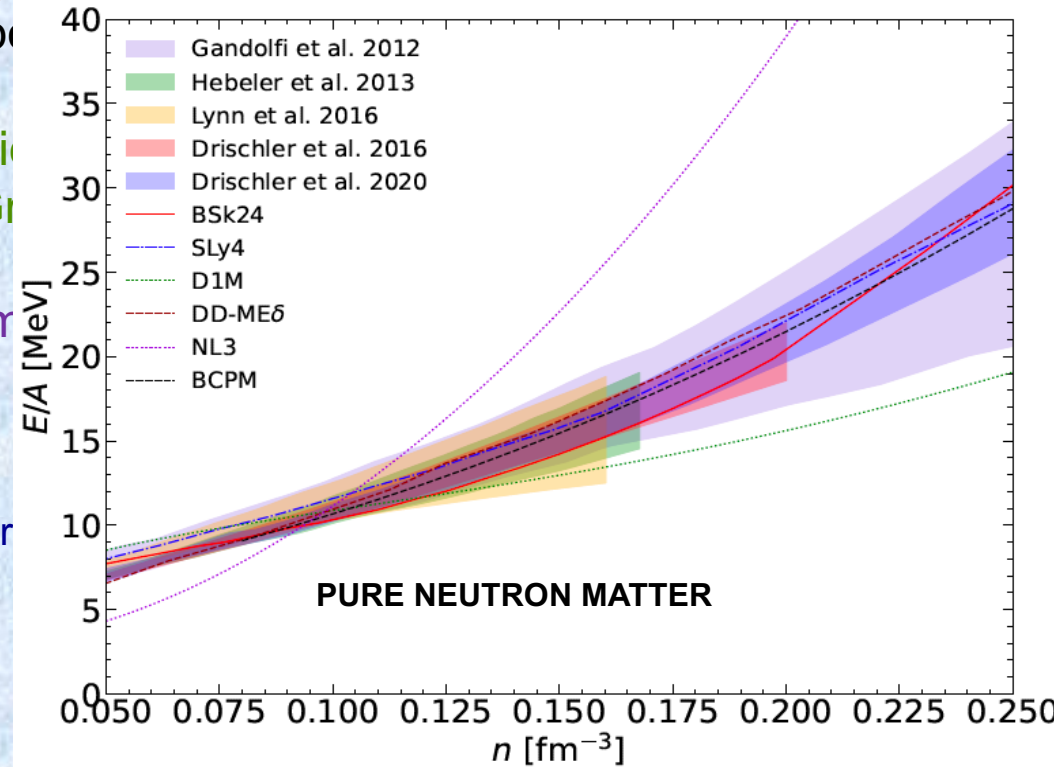
see L. Jokiniemi’s and P. Ar

## ➤ Phenomenological approaches

effective interactions (start from  
agrangian) with parameters  
produced nuclear properties: **EDF**  
**Gogny**, **RMF**, **meta-models**, ...

homogeneous matter (crust+core)

. Bender’s lecture



Fantina & Gulminelli, J.Phys. Conf. Ser. 2586, 012112 (2023)





# Baryons (n,p) at $T = 0$ : mean-field approx (1)

- Fermions (see electron part), but interactions!  
→ similar to electron gas but with “effective” masses and “effective” chemical potentials
- Independent (quasi-)particles (non-relativistic,  $q = n, p$ ):  $e_q(k) = \frac{\hbar^2 k^2}{2m_q} + U_q(k)$   
→  $U_q(k)$  from effective interactions (DFT)
- In infinite nuclear matter,  $T = 0$   $\sum_q \sum_k \rightarrow \sum_q g_{s,q} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \Theta_q(k) = \sum_q g_{s,q} \int_0^{k_{F,q}} \frac{dk}{2\pi^2} k^2$   
(transl. invariant):
- Matter energy density:  $\mathcal{E}_q = g_{s,q} \int_{k \leq k_F} \frac{d^3\mathbf{k}}{(2\pi)^3} e_q(k) = \mathcal{E}_{F,q} + g_{s,q} \underbrace{\int_{k \leq k_F} \frac{d^3\mathbf{k}}{(2\pi)^3} U_q(k)}_{\text{EDF} \rightarrow \text{energy functional}}$   
NB: rest-mass to be added
- Examples: *Skyrme* (0-range), *Gogny* (finite-range), *Fayans* (more flexibility than Skyrme), *BCPM* (inspired to BHF calculations), *KIDS* (based on power expansion of nuclear matter energy in terms of  $k_F$ ), *meta-model* (based on Taylor expansion in density around saturation density)
- NB: here effects beyond mean-field not discussed see M. Bender’s lecture



# Baryons (n,p) at $T = 0$ : mean-field approx (2)

❖ Example of **Skyrme** EDF (Skyrme, Nucl. Phys. 9, 615 (1959)):

Matter energy (infinite matter): central term + density-dep. term + eff. mass term ~~+ s-o, grad, tensor~~

$$\mathcal{E}_{\text{Sky}} = \sum_q m_q c^2 n_q + \sum_q \mathcal{E}_{F,q} + \underline{C_{1,\text{is}} n_B^2} + \sum_q \underline{C_{1,\text{is}} n_q^2} \quad n_B = n_n + n_p$$

$$+ \underline{C_{2,\text{is}} n_B^{\alpha+2}} + \sum_q \underline{C_{2,\text{iv}} n_q^\alpha n_B^2} + \underline{C_{m,\text{is}} n_B^{5/3}} + \sum_q \underline{C_{m,\text{iv}} n_q^{5/3}} \quad n^\alpha \text{ terms} \rightarrow \text{mimic many-body effect}$$

NB:  $\mathcal{E}_{F,q} \propto n_q^{5/3} \Rightarrow$

$$\mathcal{E}_{\text{Sky}} = \sum_q m_q n_q + \sum_q \mathcal{E}_{F,q}^* + \underline{C_{1,\text{is}} n_B^2} + \sum_q \underline{C_{1,\text{is}} n_q^2}$$

$$+ \underline{C_{2,\text{is}} n_B^{\alpha+2}} + \sum_q \underline{C_{2,\text{iv}} n_q^\alpha n_B^2}$$

$$\frac{\hbar^2}{2m_q} \rightarrow \frac{\hbar^2}{2m_q^*}$$



$\Rightarrow$  Landau (k) effective mass (momentum dependence)

$$\Rightarrow e_q = \frac{\hbar^2 k^2}{2m_q^*} + U_q \quad \frac{\hbar^2}{2m_q^*} = \frac{\delta E_{\text{EDF}}}{\delta \tau_q}, U_q = \frac{\delta E_{\text{EDF}}}{\delta n_q}$$

$\tau_q$  kin.energy dens.; no gradient in infinite matter!

Model parameters  $\rightarrow$  fitted to exp.

NB: Extensions (e.g. added density-dep terms)





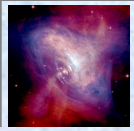
# Baryons (n,p) at $T = 0$ : mean-field approx (3)

- **RMF** : - nucleons interact via field, e.g.  $\omega$  (repulsion, short range) and  $\sigma$  (attraction, medium range)  
but others can be added, e.g.  $\rho$  needed to account for isospin!  
 - meson field are replaced by average values
- Relativistic particle:  $e_q(k) = \sqrt{(m_q^*c^2)^2 + (\hbar ck)^2} + V_q$
- Scalar field  $\rightarrow$  effective mass:  $m^* = m - \underline{g_\sigma \sigma_0}$   

$$m_\sigma^2 \sigma_0 = g_\sigma \sum_q \frac{g_{s,q}}{2\pi^2} \int_0^{k_{F,q}} k^2 dk \frac{m_q^*}{\sqrt{\hbar^2 k^2 + m_q^{*2}}} \rightarrow n_s \text{ (scalar density)}$$
- Vector field  $\rightarrow$  shifts energy  $\rightarrow V_q$ :  $g_\omega \omega_0 (\pm g_\rho \rho)$
- Matter energy density:  $\mathcal{E}_{\text{RFM}, \sigma\omega(\rho)} = \sum_q \frac{g_{s,q}}{2\pi^2} \int_0^{k_{F,q}} k^2 dk \sqrt{k^2 + m_q^{*2}} + \frac{1}{2} m_\sigma^2 \sigma_0^2 - \frac{1}{2} m_\omega^2 \omega_0^2 + \underline{g_\omega \omega_0 n_B}$   

$$\left( -\frac{1}{2} m_\rho^2 \rho_0^2 + \frac{1}{2} \underline{g_\rho \rho_0^3} (n_p - n_n) \right) \quad \text{Model parameters} \rightarrow \text{fitted to exp.}$$

NB: Extensions: density-dependent (DD) couplings or non-linear (NL) self-interaction



# Effective mass

- Note: Landau (NR) & Dirac (R) effective masses are different (different definition) !
- Nucleons interact in medium  $\rightarrow$  mass not the same as in vacuum

❖ Landau  $k$ -mass (at  $k_F$ ) (momentum dep.)  $m_L^* = k \left( \frac{dE}{dk} \right)^{-1}$

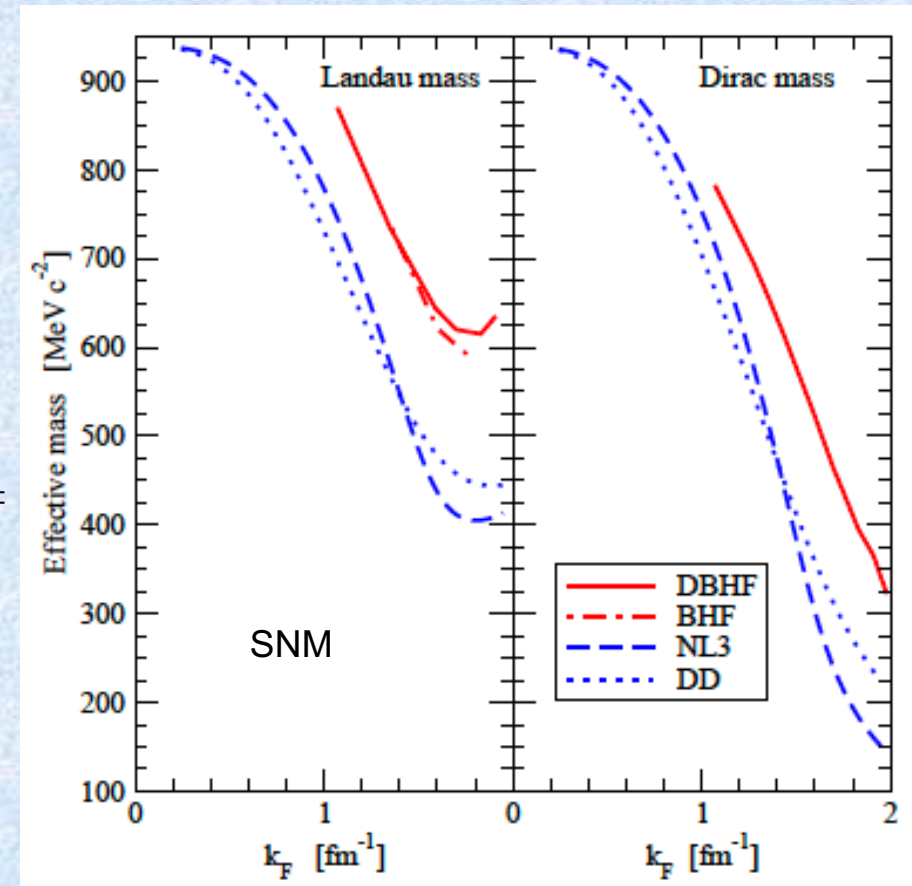
❖ Dirac mass

$$m_D^* = m - g_\sigma \sigma_0 = m - \frac{g_\sigma^2}{m_\sigma^2} \sum_q \frac{g_{s,q}}{2\pi^2} \int_0^{k_F} k^2 dk \frac{m_q^*}{\sqrt{\hbar^2 k^2 + m_q^{*2}}}$$

$\rightarrow$  different functional dependencies and values!

$\rightarrow$  mapping not straightforward

$$m_L^* \stackrel{?}{\approx} \sqrt{m_D^{*2} + (\hbar k_F)^2}$$



Van Dalen & M  ther, IJMPA 19, 2077 (2010); arXiv:1004.0144





# Baryons (n,p) at $T > 0$

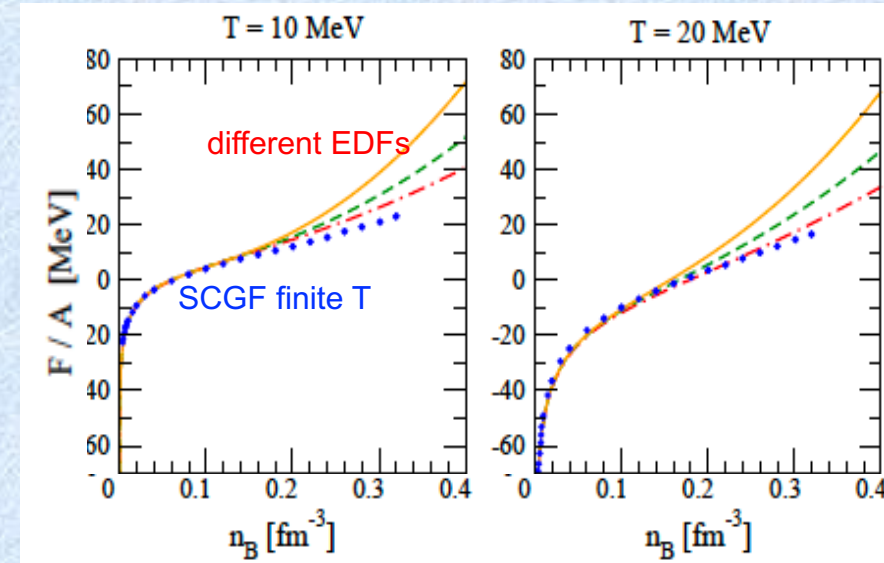
- At  $T > 0 \rightarrow$  Fermi function 
$$f(k) = \frac{1}{1 + e^{(e(k) - \mu)/(k_B T)}}$$
- In principle, same approach as before but  $f(k) \rightarrow T$ -dep in occupation
- With good approx., only kinetic part depend on  $T \rightarrow m^*$  term!  
(beyond mean-field effect can add  $T$ -dep but small for  $T < 10$ -20 MeV)

❖ Non-relativistic particles

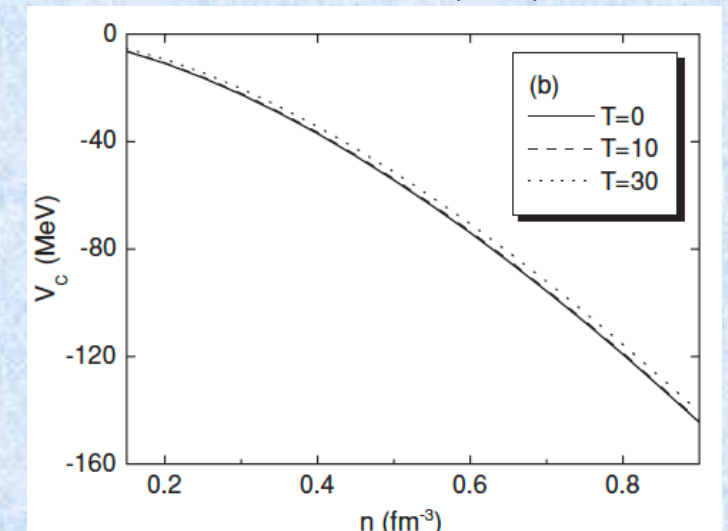
$$e_q = \frac{\hbar^2 c^2}{2m_q^*} + U_q, \quad \tilde{\mu}_q = \mu_q - U_q$$

$$\rightarrow \mathcal{E}_{\text{kin},q} = \frac{4}{\sqrt{\pi}} \frac{k_B T}{\lambda_q^3} I_{3/2} \left( \frac{\tilde{\mu}_q}{k_B T} \right)$$

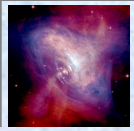
caveat: what about extrapolation at much higher  $T$   
( $\sim 100$  MeV, e.g. SN, BNS mergers?)



Fantina et al., JPCS 342, 012003 (2012)

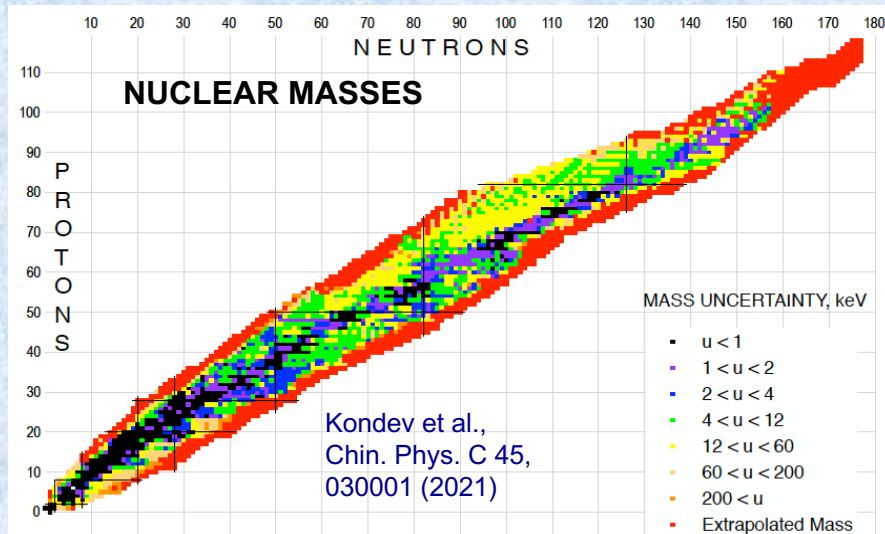


Moustakidis & Panos, PRC 79, 045806 (2009);  
(see also e.g. Lu et al., PRC 2019, Shang et al., PRC 2020)

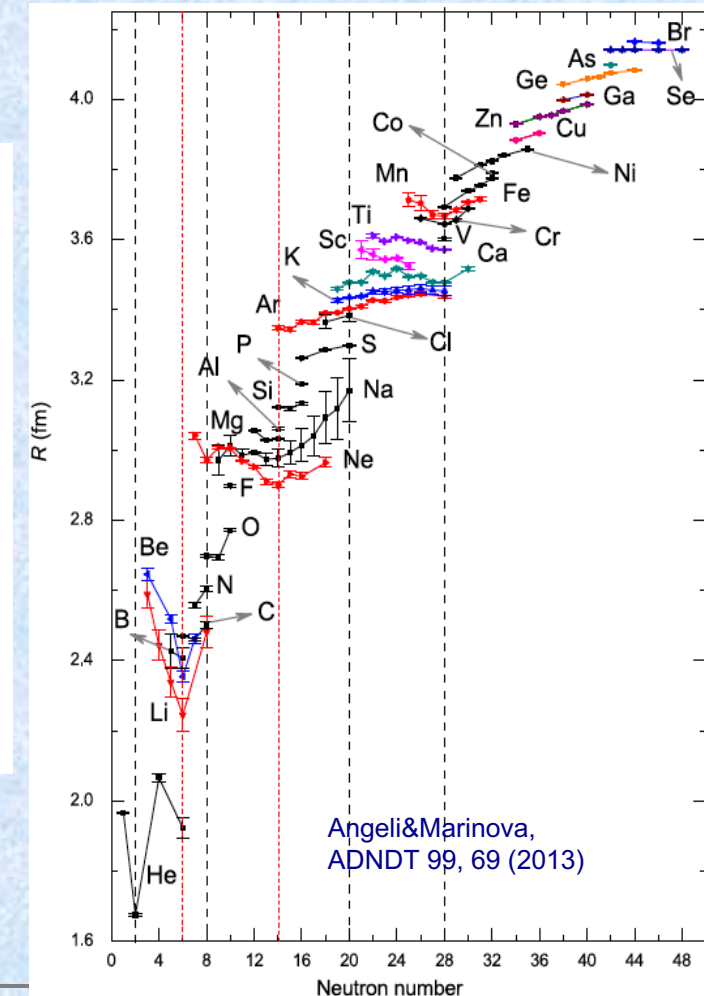
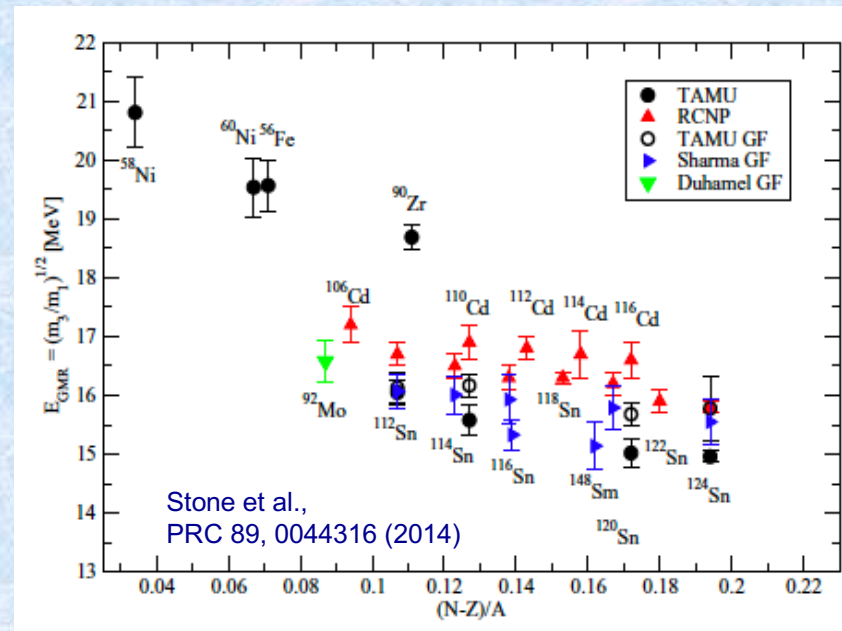


# Connection with nuclear physics experiments (1)

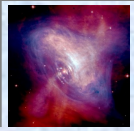
- EDF (with parameters) can model nuclei ( $T = 0$ )  
 → Parameters fitted on nuclear physics experiments



see M. Bender's lecture







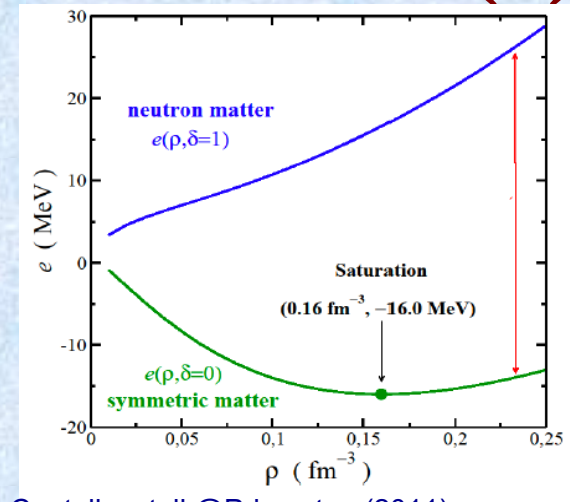
# Connection with nuclear physics experiments (2)

- EoS of infinite matter can be Taylor expanded around saturation

$$e(n_B, \delta) \approx e_{\text{is}}(n_B) + e_{\text{iv}}(n_B)\delta^2 \quad \text{but attention: kinetic part!} \quad \delta = (n_n - n_p)/n$$

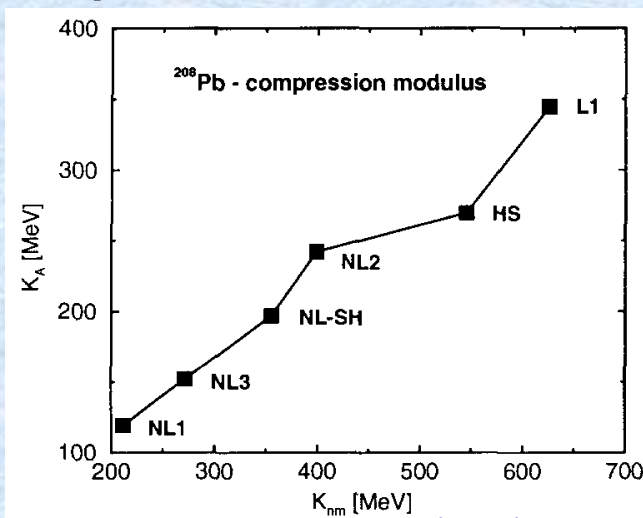
$$e_{\text{is}} = E_{\text{sat}} + \frac{1}{2}K_{\text{sat}}x^2 + \frac{1}{6}Q_{\text{sat}}x^3 + \dots \rightarrow e_{\text{sat}}(n_B, \delta=0)$$

$$e_{\text{iv}} = E_{\text{sym}} + L_{\text{sym}}x + \frac{1}{2}K_{\text{sym}}x^2 + \frac{1}{6}Q_{\text{sym}}x^3 + \dots \rightarrow e_{\text{sym}}(n_B) = e(n_B, \delta=1) - e(n_B, \delta=0)$$

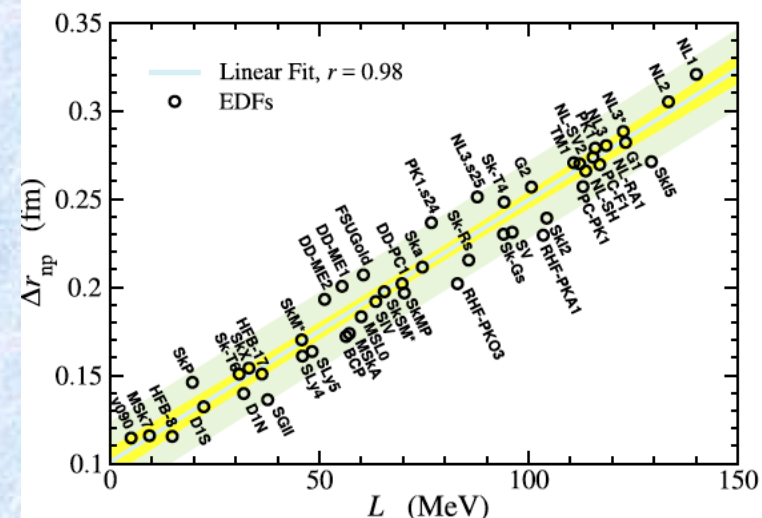


Centelles, talk@Primosten (2011)

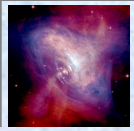
**nuclear empirical parameters** = successive derivative of energy functional at  $n_{\text{sat}} \rightarrow$  from experiments



Vretenar et al., NPA 621, 853 (1997)

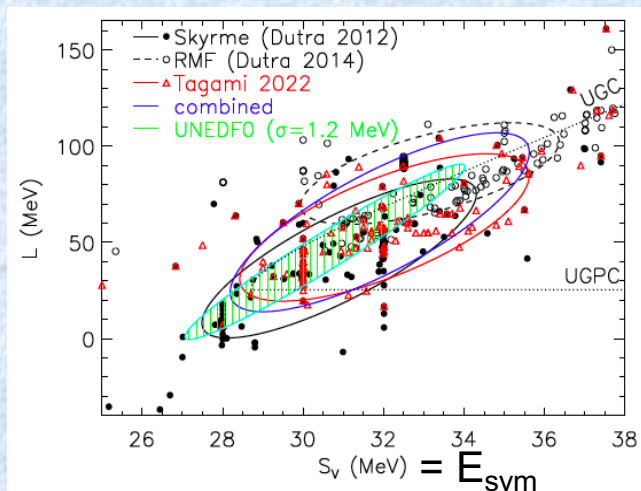


Roca Maza & Paar, PNP 101, 96 (2018)

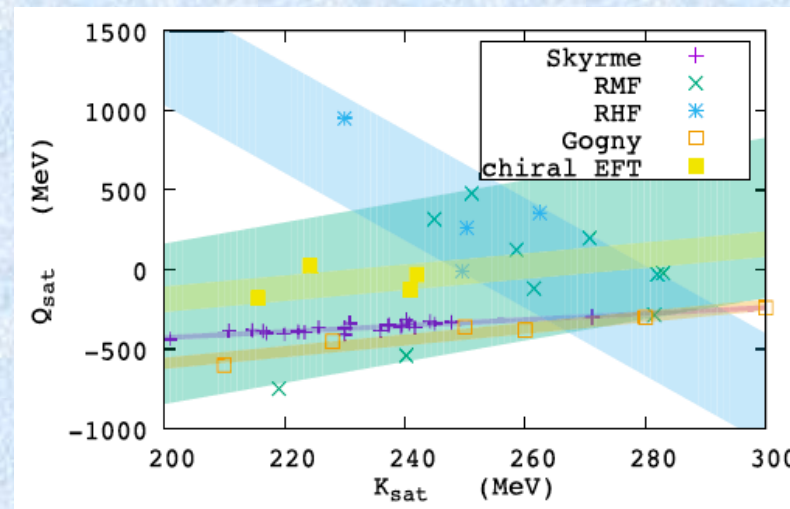


# ... but choice not unique!

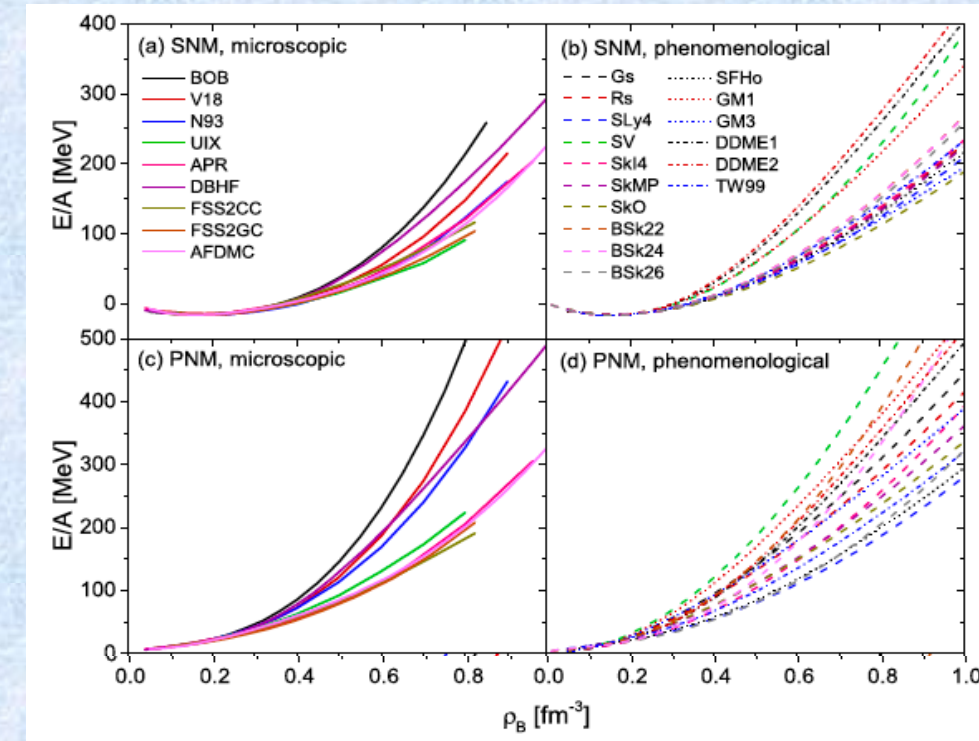
- but: choice of the functional, data set & fitting protocol not unique  
 → variety of models (even if same potential!)  
 → spread at high density (extrapolation far from experimental info)
- Even for ab-initio, different many-body techniques, w/ or w/o 3BF  
 → variety of models



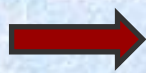
Lattimer, Particles 6, 30 (2023)



Margueron et al., PRC 97, 025805 (2018)

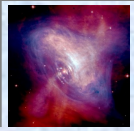


Burgio et al., PPNP 120, 103879 (2021)



How this affects astro?





# From nuclear models to astro prediction

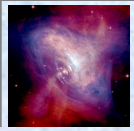
*Two ways of investigating impact of nuclear physics in astro*

## 1. Choose your “favourite” model(s)

- More “microscopic” model (accurately calibrated EDFs)  
→ study specific model dependence
- **Benchmark calculations (if simulations long)**
- Calculate several nuclear inputs consistently for nuclear physics and astrophysics
- X No parameter-space exploration

## 2. Statistical model exploration

- More flexible, faster computation model (EDF, meta-model-like)  
→ Parameter-space exploration
- **Bayes analysis**
- Run millions of models and evaluate uncertainties
- X Less microscopic



# NS atmosphere

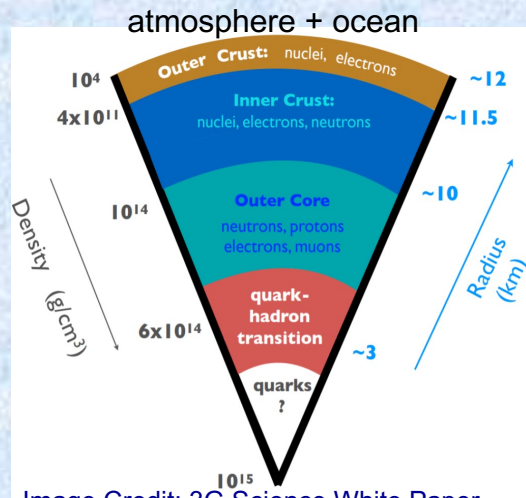
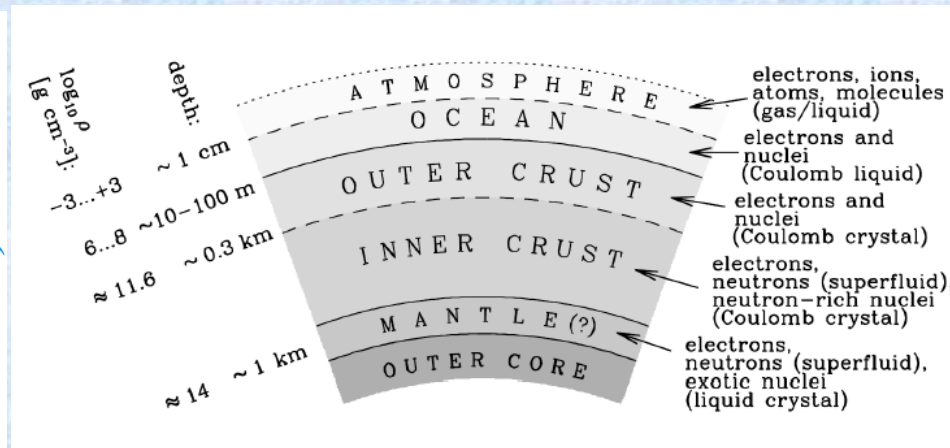
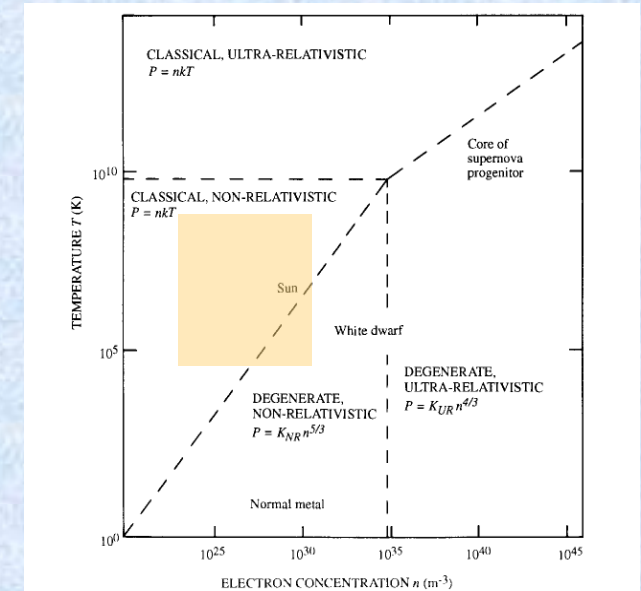


Image Credit: 3G Science White Paper



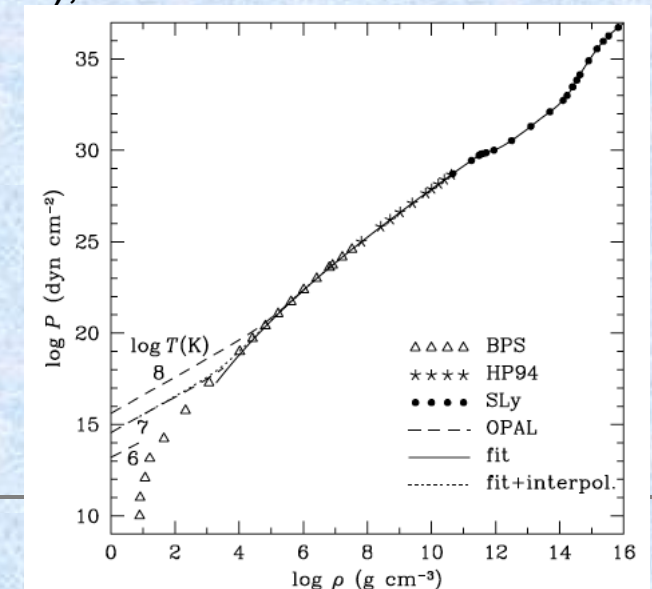
Haensel et al., Neutron Star 1 (Springer 2007)



Phillips, "The Physics of Stars"

- **Atmosphere:** (few cm) of light to intermediate-mass elements (e.g. H, He, C),  $e^-$  + ions + atoms + molecules; gas/liquid
  - Emission of thermal radiation  $\rightarrow T_{\text{surf}}$ , surface gravity, etc.
  - Very low density ( $\rho < \sim 1 - 100 \text{ g/cm}^3$ )  $\rightarrow$  EoS ideal gas  $P = nRT$

NB: below  $\sim 10^6 \text{ g/cm}^3$  EoS is  $T$ -dependent ( $\rightarrow T=0$  approx not valid)



Haensel et al., Neutron Star Structure 1 (Springer 2007); Nattila et al., ApJ 971, 37 (2024)





# NS ocean

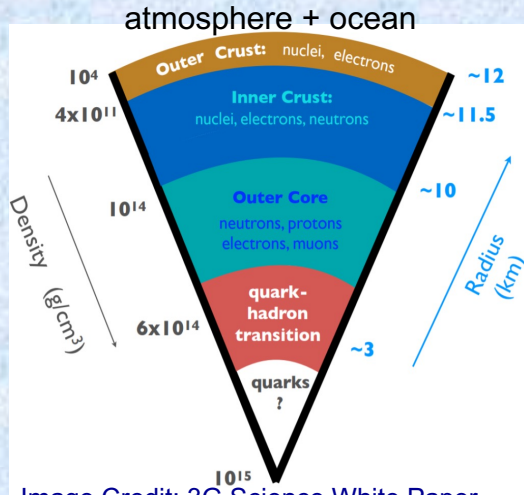
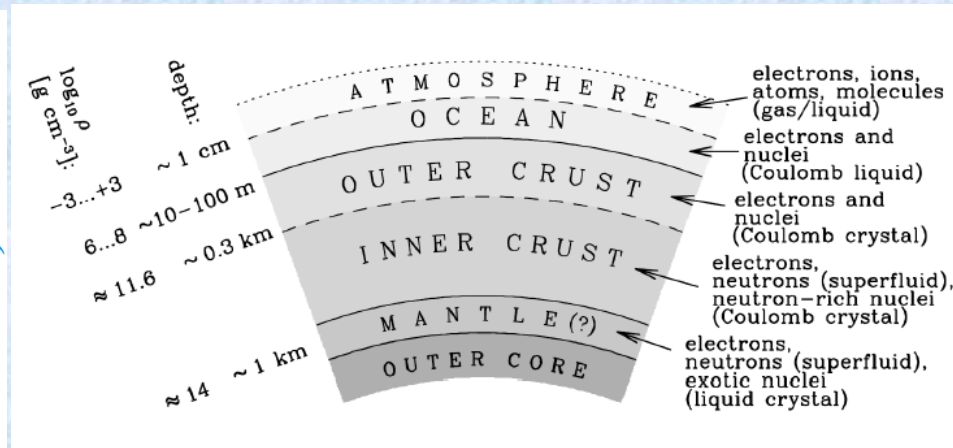


Image Credit: 3G Science White Paper



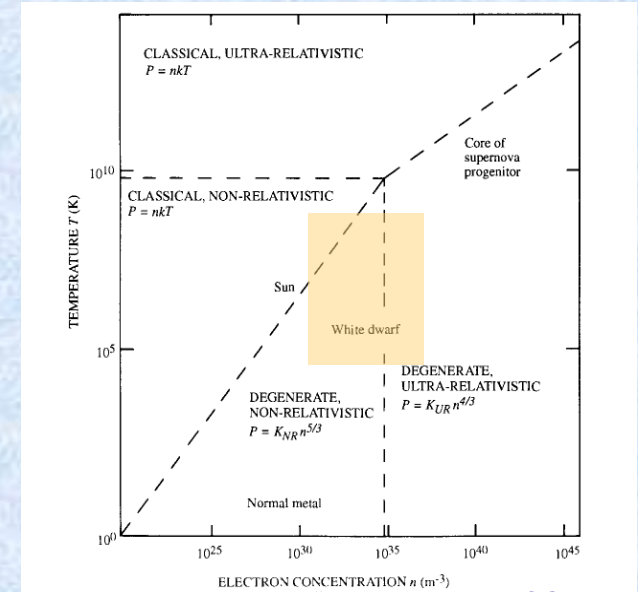
Haensel et al., Neutron Star 1 (Springer 2007)

## ■ Ocean: a few 10 m, ions + electrons

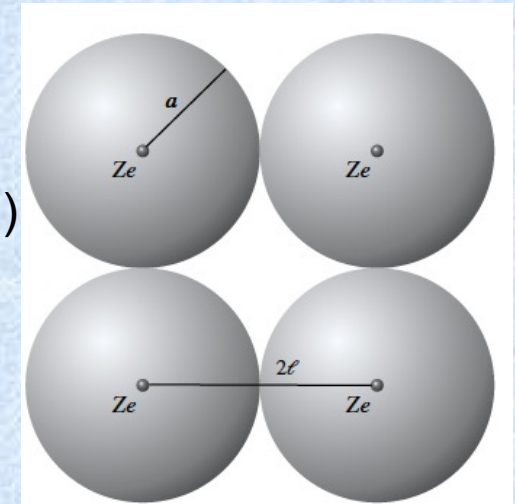
- Expected to be made by Fe nuclei (at least at the bottom) + electrons
- Low density ( $\rho < \sim 10^8 \text{ g/cm}^3$ )  $\rightarrow$  EoS dominated by degenerate  $e^-$  gas
- Atoms completely ionised! When this occurs?  
 $\rightarrow$  inter-atom spacing  $\sim$  atom radius (found from  $E_{\text{kin}} + E_{\text{coul}} = 0$  when atom bound)

$$l = \left( \frac{4\pi n_N}{3} \right)^{-1/3} \approx a = \left( \frac{3\pi^2}{16} \right)^{1/3} \frac{a_0}{Z^{1/3}}$$

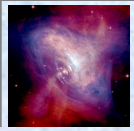
$$\rightarrow \rho_B \gg \rho_i = m_N n_N \approx A m_p n_N \approx 1.4 A Z \approx 10^3 \text{ g cm}^{-3}$$



Phillips, "The Physics of Stars"



N. Chamel (2026)



# NS crust: general picture

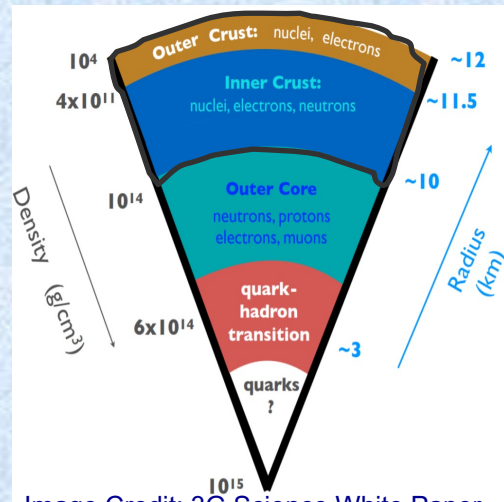
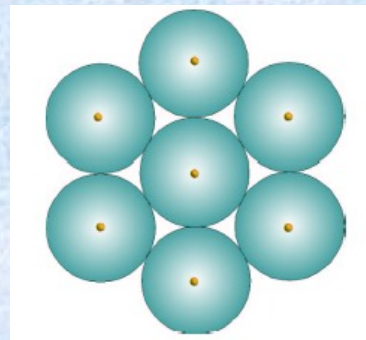
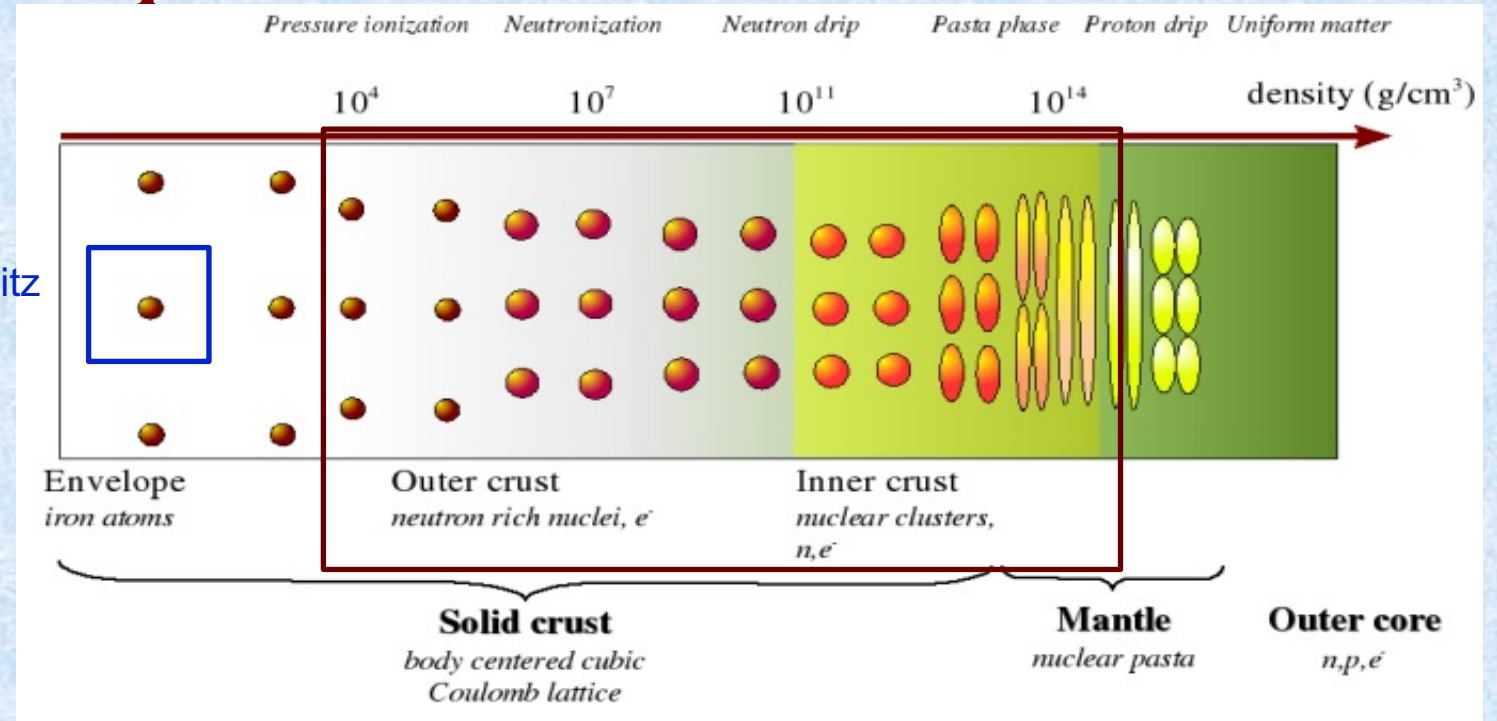


Image Credit: 3G Science White Paper



Wigner-Seitz  
(WS) cell



Chamel & Haensel, Liv. Rev. Relativ. 11, 10 (2008); see also Blaschke & Chamel, ASSL 457, 337 (Springer, 2018)

Haensel et al., Neutron Star Structure 1 (Springer 2007)

- To obtain the EoS → **minimisation**  $\varepsilon(n_B) = E_{WS}/V_{WS} = \min$
- One nuclear species at each density point → **one component or single nucleus approx.**
- At  $T = 0$  → bcc lattice





# NS crust: matter neutronisation

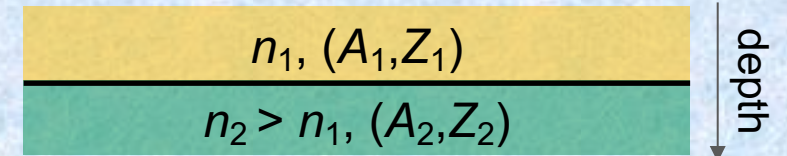
- With increasing density, matter becomes more neutron rich

$$\mu_n - \mu_p = \mu_e \quad \rightarrow \text{electron Fermi energy changes with } n_B$$

- charge neutrality:  $n_e = n_p$

- Le Chatelier-Braun principle:  $n_2 > n_1 \rightarrow \frac{Z_2}{A_2} > \frac{Z_1}{A_1}$

$\rightarrow \beta$  eq: n/p ratio adjust  $\rightarrow$  matter gets more n rich



- When does this occur? Ignoring (Coulomb) interaction and  $\nu$  mass:



| $Z$ | $N$ | $A$ | $\bar{n}_{\min}$      | $\bar{n}_{\max}$      |
|-----|-----|-----|-----------------------|-----------------------|
| 26  | 30  | 56  | —                     | $4.93 \times 10^{-9}$ |
| 28  | 34  | 62  | $5.08 \times 10^{-9}$ | $1.63 \times 10^{-7}$ |

Pearson et al., MNRAS 481, 2994 (2018)

$$M(A, Z)c^2 + \mu_e > M(A, Z - 1)c^2$$

$$\text{if } M(A, Z) \approx (A - Z)m_n + Zm_p \rightarrow \mu_e > (m_n - m_p)c^2$$

$$\text{if UR } e^- \rightarrow \mu_e \approx \hbar c (3\pi^2 n_e)^{1/3}$$

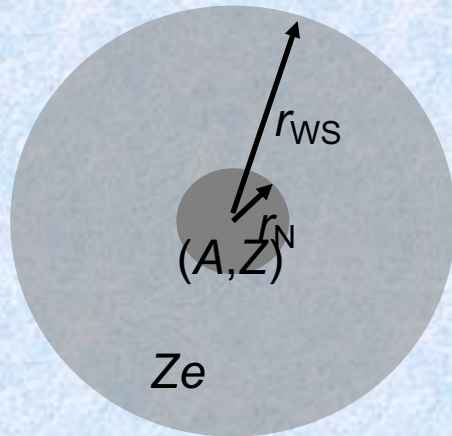
$$\rightarrow \rho_\beta = \frac{1}{3\pi^2} \frac{A}{Z} m_p \left( \frac{(m_n - m_p)c^2}{\hbar c} \right)^3 \sim 10^7 \frac{A}{Z} \text{ g cm}^{-3} \longrightarrow \text{first transition in NS crust}$$



# NS crust: Coulomb interaction



## ▪ Gauss theorem in WS cell

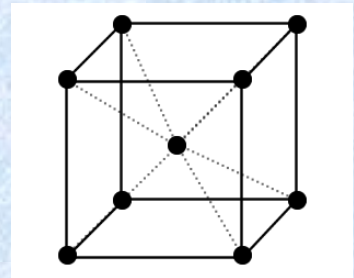


$$E_{\text{Coul}} = \frac{3}{5} \frac{1}{4\pi\epsilon_0} \frac{(Ze)^2}{r_N} \left[ 1 + \frac{1}{2} \left( \frac{r_N}{r_{\text{WS}}} \right)^3 - \frac{3}{2} \frac{r_N}{r_{\text{WS}}} \right]$$

nucleus (sphere)  
(Gaussian units  $1/(4\pi\epsilon_0)=1$ )

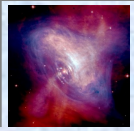
Lattice energy (e-e, e-N):  $E_L = -\frac{9}{10} \frac{(Ze)^2}{r_{\text{WS}}}$

for a bcc:  $\sim 0.896 \sim 9/10$



Lattice pressure:  $P_L = \frac{1}{3} E_L < 0 \rightarrow \text{soften the EoS}$





# NS outer crust

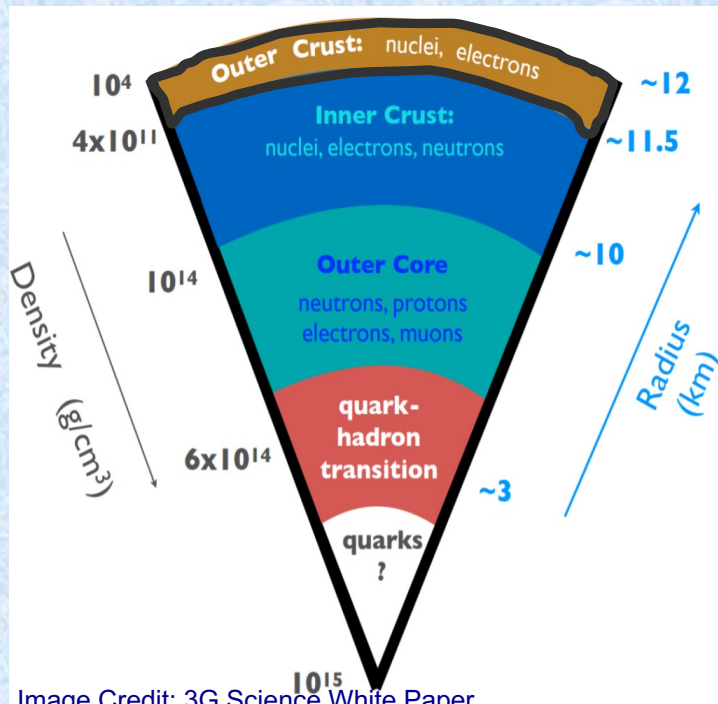
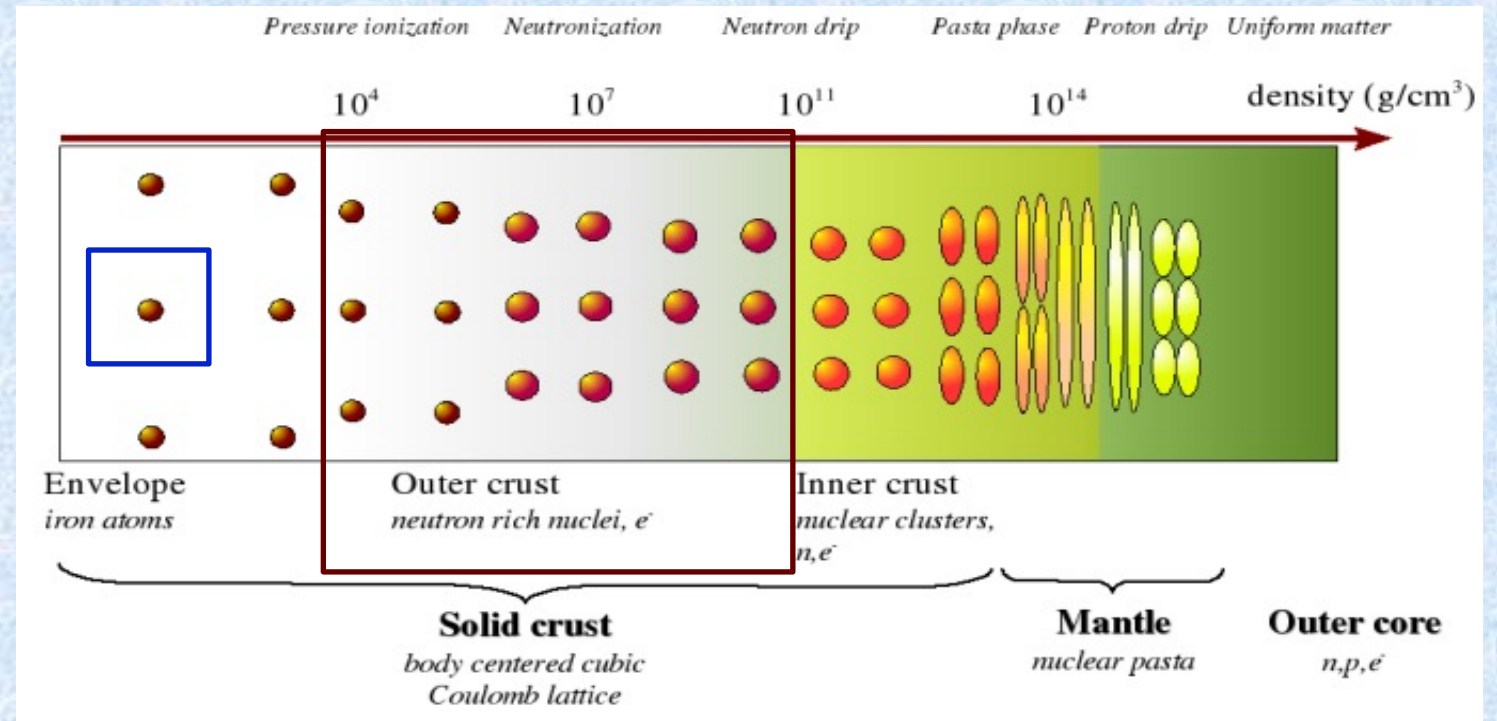
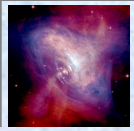


Image Credit: 3G Science White Paper



Chamel & Haensel, Liv. Rev. Relativ. 11, 10 (2008); see also Blaschke & Chamel, ASSL 457, 337 (Springer, 2018)



# NS outer crust (1)

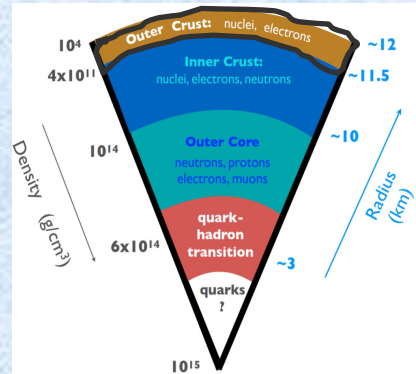
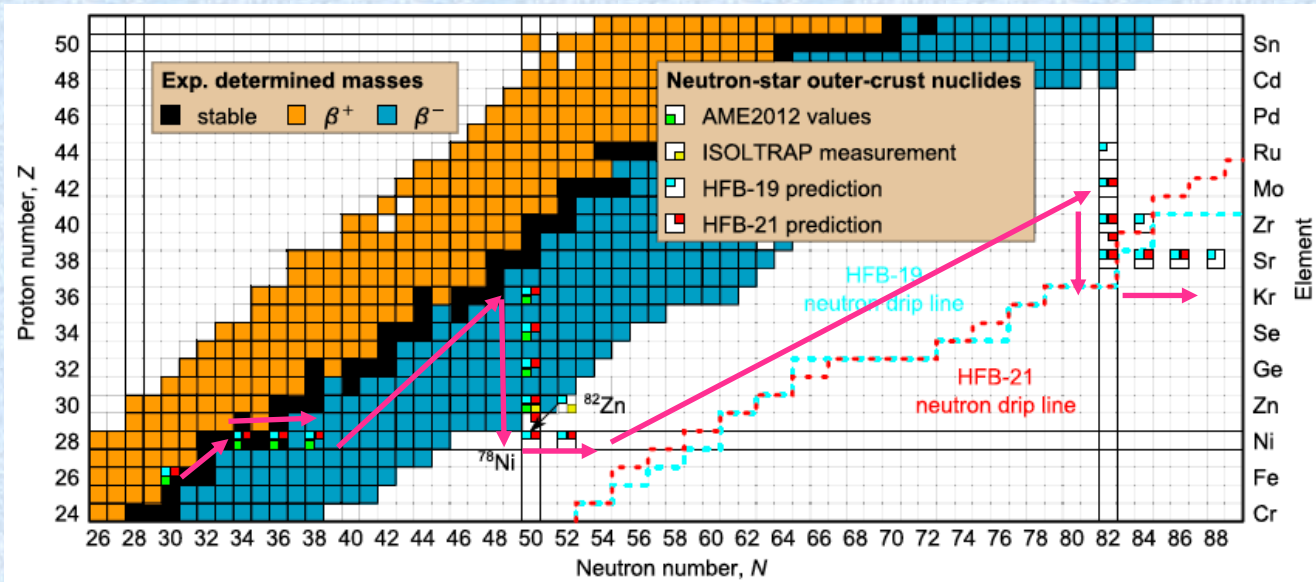


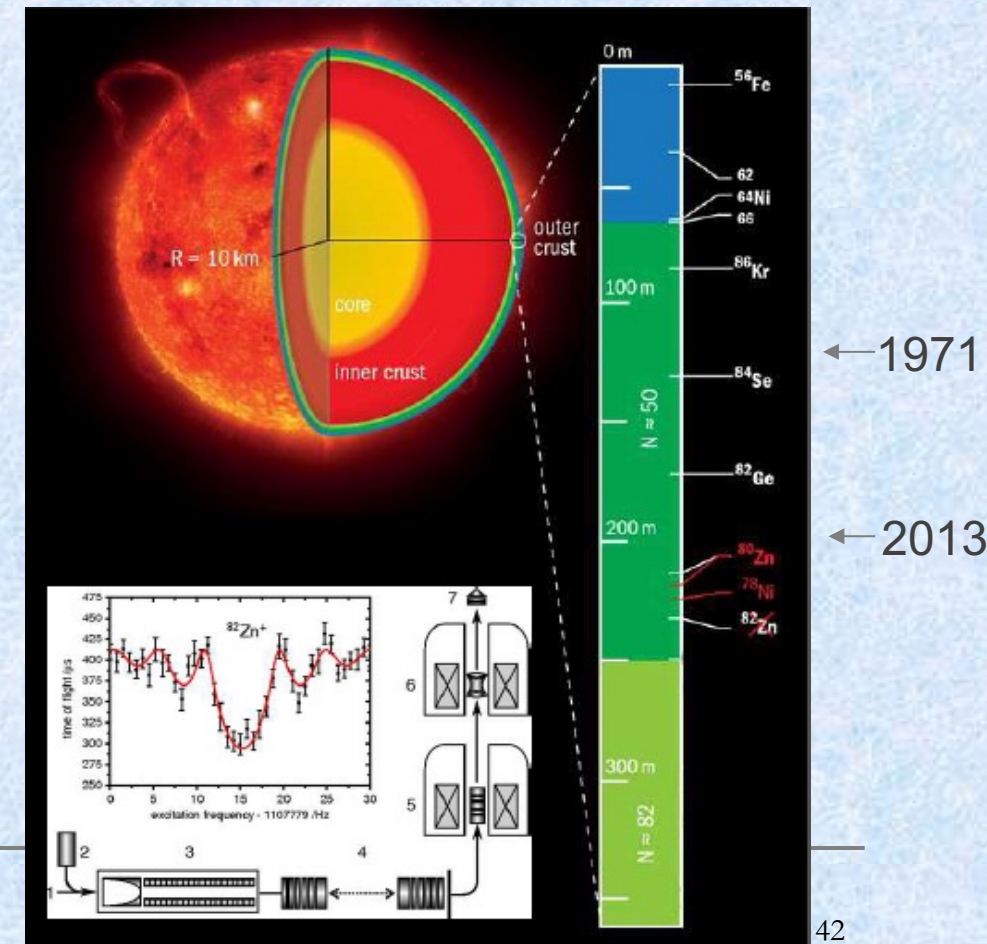
Image Credit: 3G Science White Paper

- Nuclei in bcc lattice + electrons:  $\mathcal{E}_{\text{WS}}(n_B) = \frac{M(A,Z)c^2}{V_{\text{WS}}} + \mathcal{E}_e + \mathcal{E}_{\text{Coul}}$   
e-e and e-i int.

Only microscopic inputs are nuclear masses → Experimental or mass models



Wolf *et al.*, PRL 110, 041101 (2013) ; ISOLTRAP collaboration at CERN ( $^{82}\text{Zn}$ )

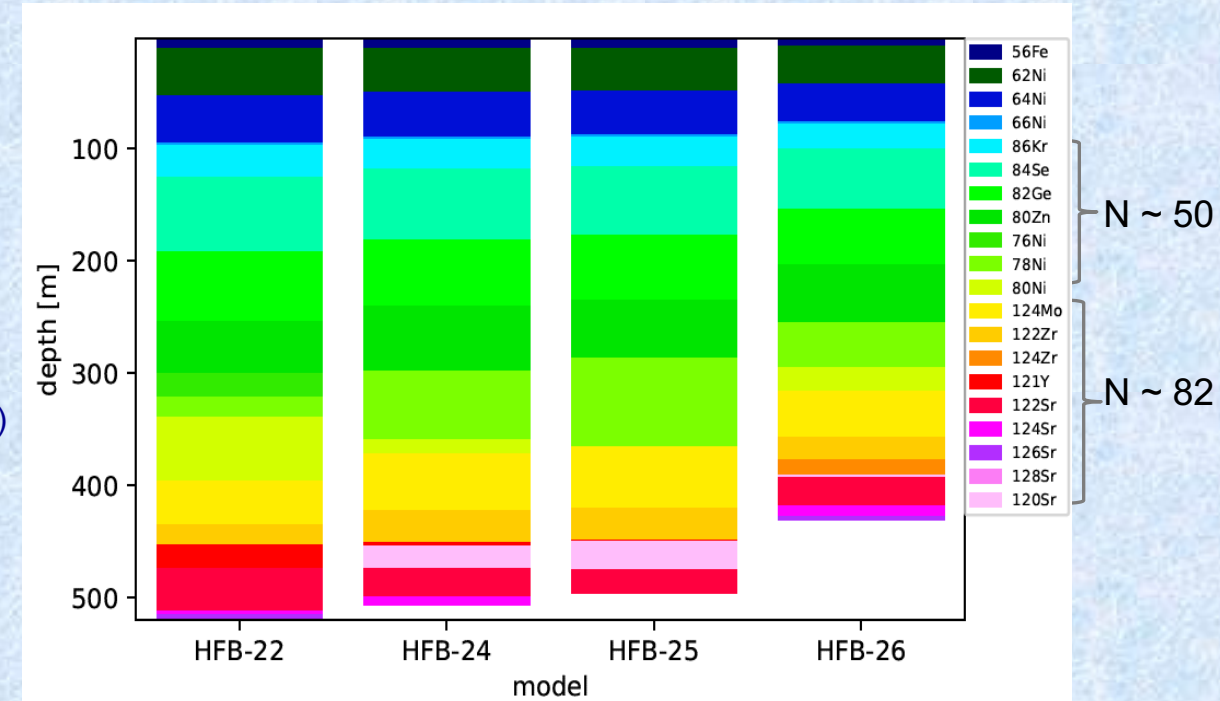
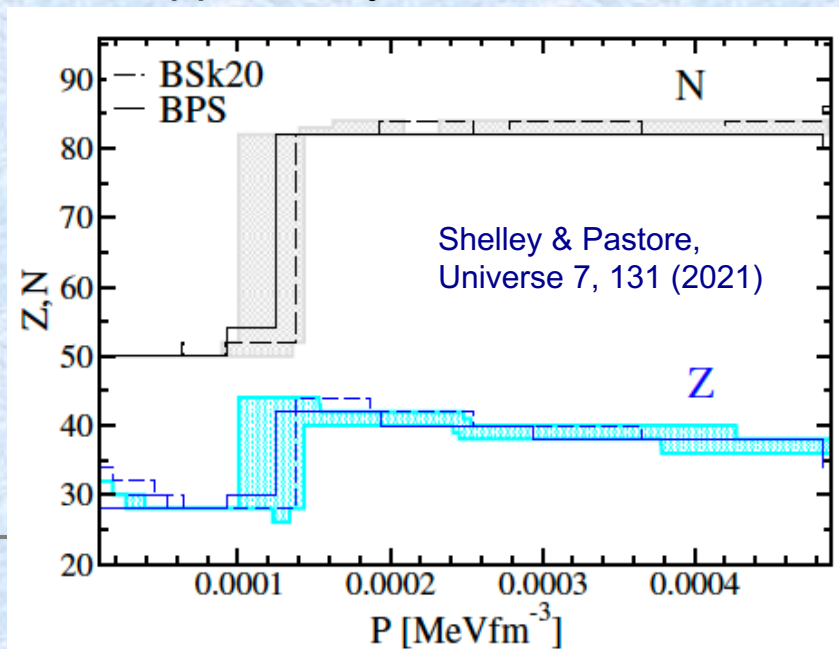


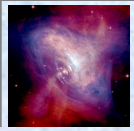




# NS outer crust (2)

- Experimental info on masses → model independent results
- Beyond → nuclear mass models → model dependence
- Imprint of magic nuclei ( $N \sim 50, 82$ )  
(less on  $Z$  because of weak equilibrium and charge neutrality, few layers with  $Z=28$ )
- Machine learning used to fit nuclear mass data  
(e.g. Utama et al., PRC 93, 014311 (2016); Shelley & Pastore, Universe 7, 131 (2021))  
but: extrapolations, applicability for inner crust and core ?





# Neutron drip

- At some point, nuclei become so neutron rich  $\rightarrow$  n drip out of nuclei  $\rightarrow$  **neutron drip**
- In nuclear physics, neutron dripline  $\rightarrow$  lightest isotope such that
 
$$M'(A, Z) - M'(A - 1, Z) > m_n$$
- But: nuclei (with structure effects) in a medium (background gas, lattice effects)  $\rightarrow$  **dripline shifted !**
- Drip  $\rightarrow$  neutrons occupy delocalised continuum states

NB: - two representations can be mapped  
 - neutron superfluid fraction (i.e. those not co-moving with lattice) important for e.g. glitches, elastic properties, ...  
 $\rightarrow$  “free” neutrons in e-cluster representation (Almirante & Urban, PRL 2025)

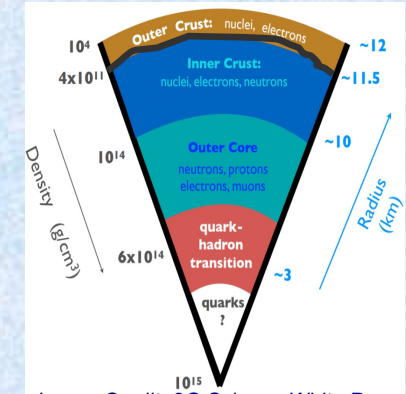
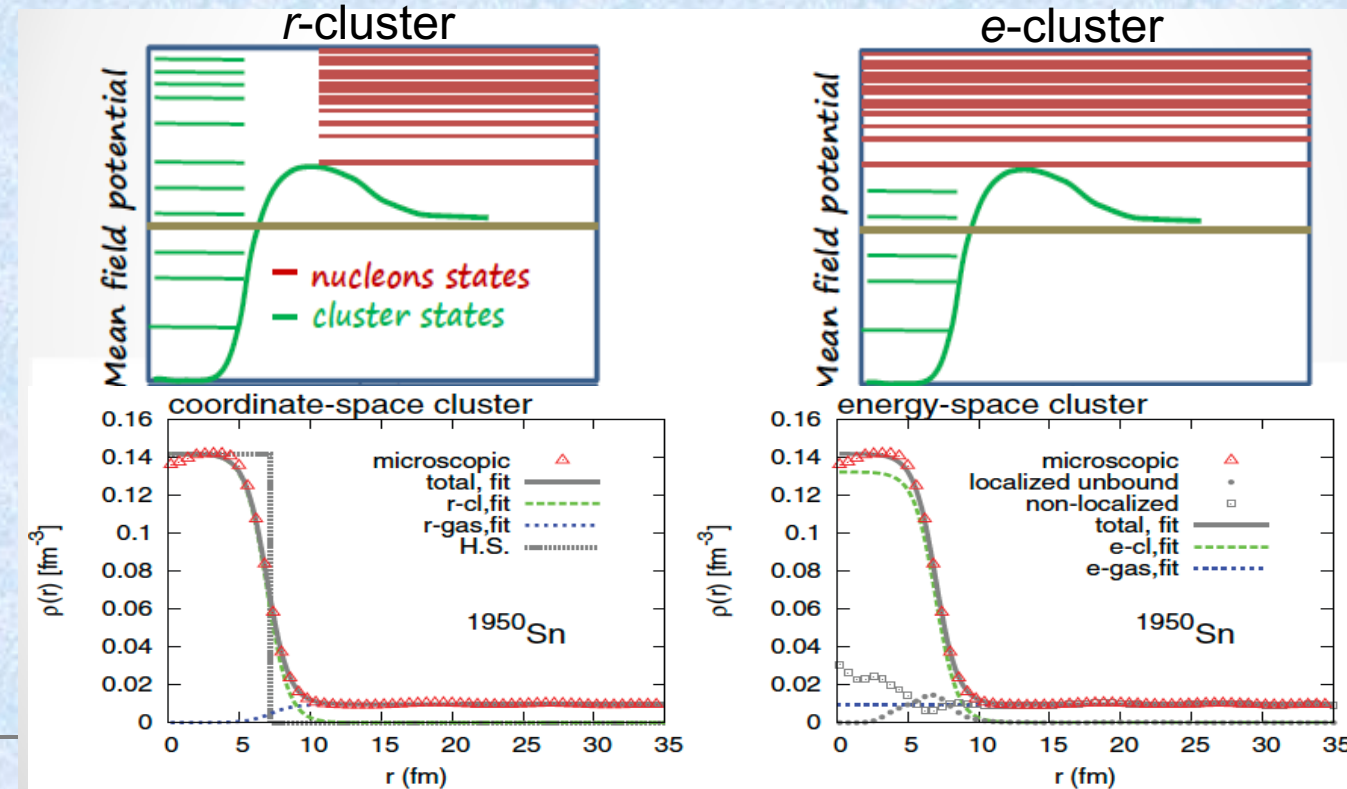
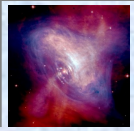


Image Credit: 3G Science White Paper







# NS inner crust

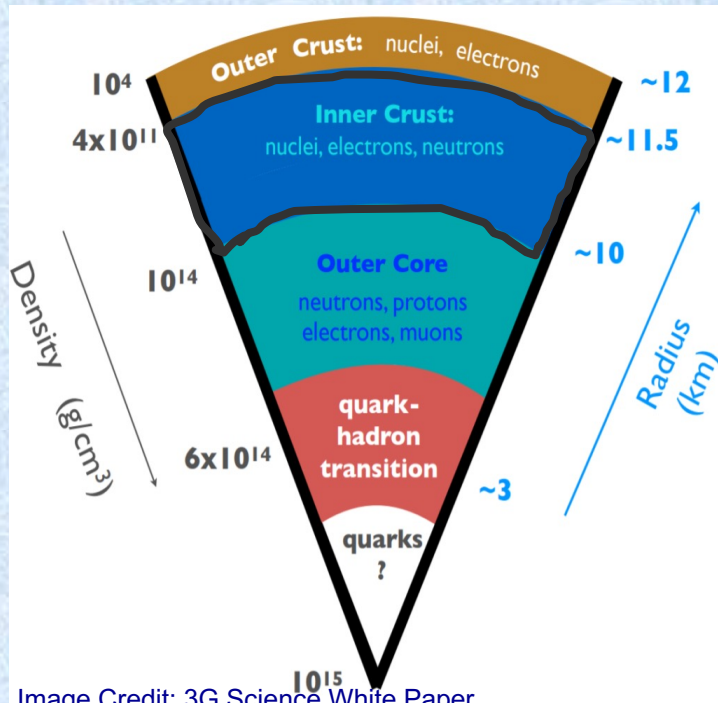
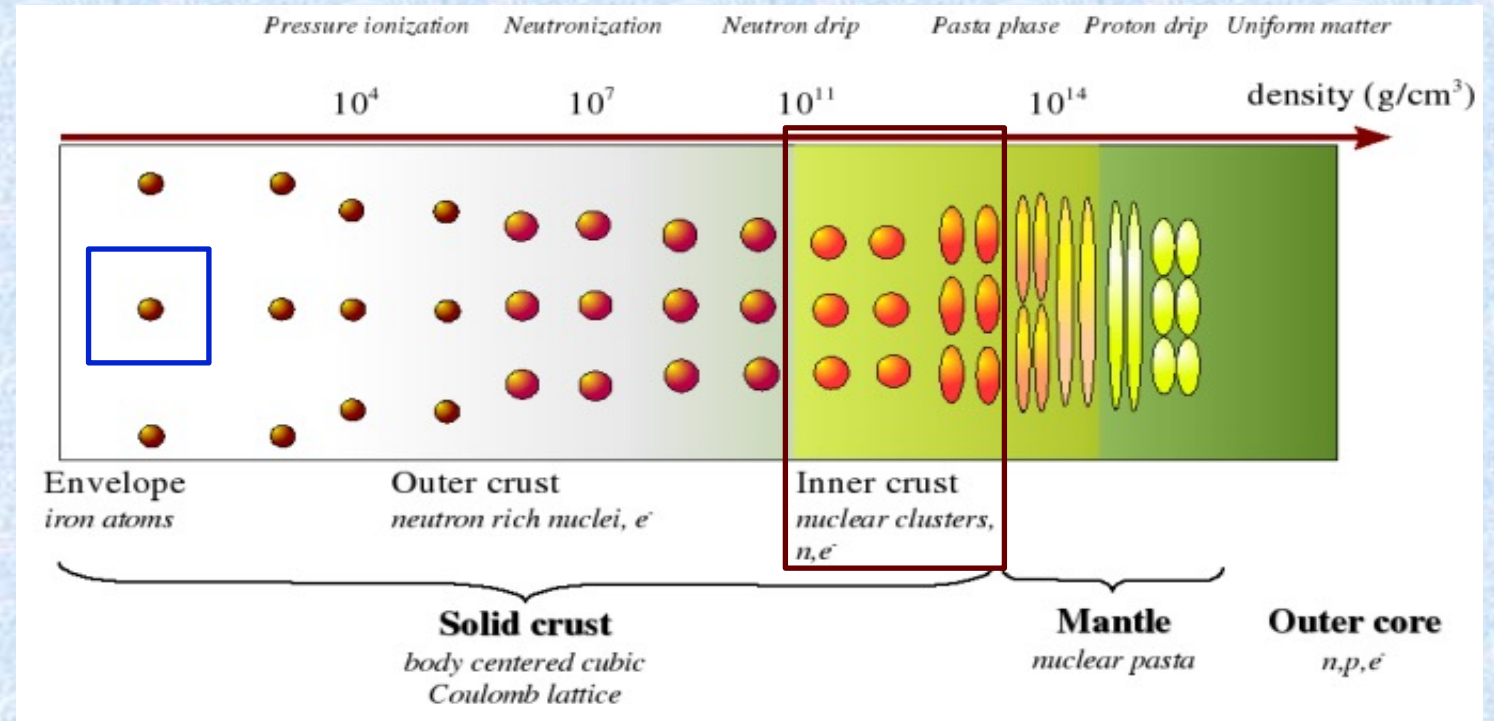
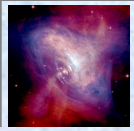


Image Credit: 3G Science White Paper



Chamel & Haensel, Liv. Rev. Relativ. 11, 10 (2008); see also Blaschke & Chamel, ASSL 457, 337 (Springer, 2018)

NB not to scale !



# NS inner crust

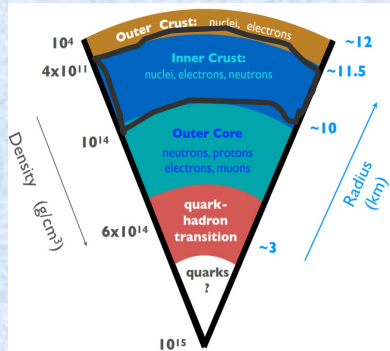


Image Credit: 3G Science White Paper

- Nuclei + electrons + free neutrons

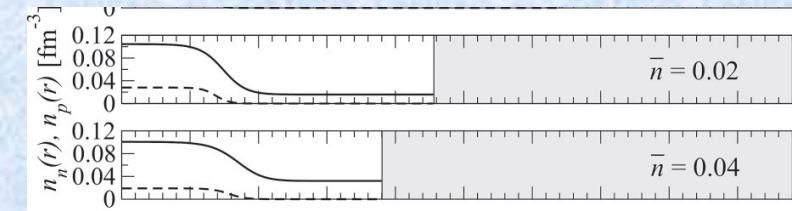
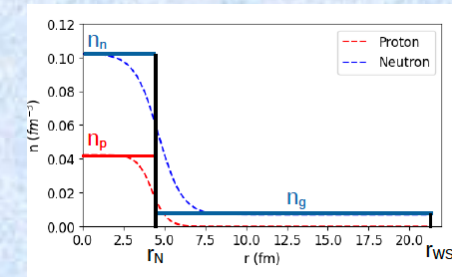
$$\mathcal{E}_{\text{WS}}(n_B) = \frac{M(A, Z)c^2}{V_{\text{WS}}} + \mathcal{E}_e(n_e) + \mathcal{E}_{\text{Coul}} + \mathcal{E}_n(n_{\text{gn}}) + \mathcal{E}_{\text{int}}$$

exp./theo masses      Coulomb interaction      homog. matter      nucleus-gas interaction

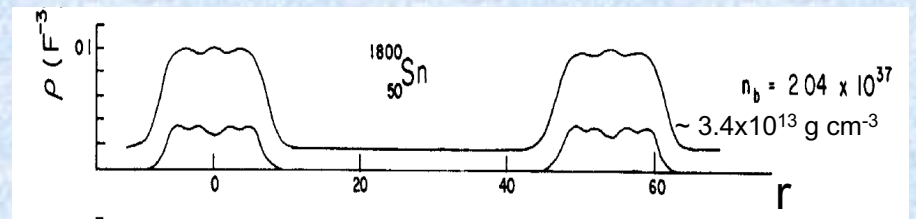
→ different methods:

1. **Compressible liquid-drop model (CLDM)**  
→ classical approach
2. **(Extended) Thomas-Fermi ((E)TF)**  
→ (semi) classical approach
3. **Hartree-Fock / Hartree-Fock Bogoliubov**  
→ quantum approach

→ variational problem (in beta equilibrium)

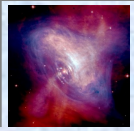


Pearson et al., PRC 85, 065803 (2012)



Negele & Vauterin, NPA 207, 298 (1973)





# Classical (“macroscopic”) approach (1)

## 1. Compressible liquid-drop model (CLDM)

- separation nucleons “inside”/”outside” nuclei → definition of “cluster”
- nuclear energy given by sum of contribution (bulk, surface, Coulomb)

$$\mathcal{E} = \frac{E_i}{V_{WS}} + \mathcal{E}_e + \mathcal{E}_{\text{gas}} \left( 1 - \frac{V_N}{V_{WS}} \right)$$

$e^-$  and gas energy include rest mass

NB: at  $T = 0$ , only n gas! At  $T > 0$ , p gas also possible

$$E_i = M_i c^2 + E_{\text{bulk}} + E_{\text{Coul}} + E_{\text{surf}}$$

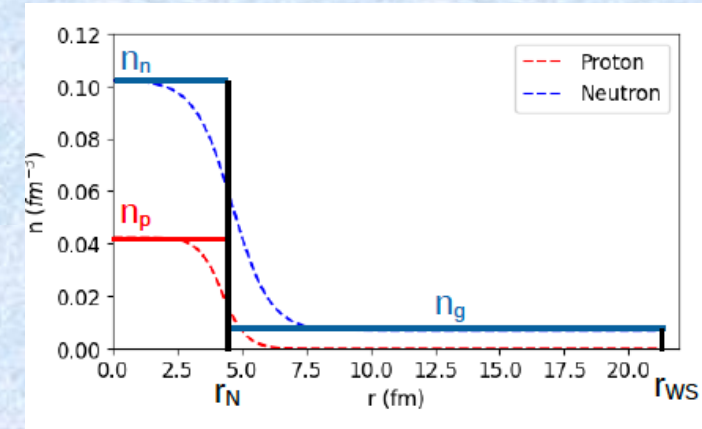
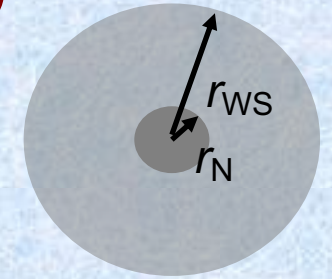
infinite matter

$$E_{\text{surf}} = 4\pi^2 r_N^2 \sigma_s$$

parametrizations often used for  $\sigma_s$

→ parameters to be fitted (exp or theo calculations)

(e.g. Lattimer&Swesty, NPA 535, 331 (1991))



Courtesy of T. Diverriès



# Classical (“macroscopic”) approach (1)

→ variational problem (in beta equilibrium) → Lagrange multipliers

$$d \left[ \mathcal{E}(n_B) - \lambda_1 \left( \frac{N_{WS}}{V_{WS}} - n_n \right) - \lambda_2 \left( \frac{Z_{WS}}{V_{WS}} - n_p \right) \right] = 0$$

Lagrange multipliers →  $\mu_n, \mu_p$

$Z_{WS} = Z$  if no free p ( $T=0$ );  
charge neutrality:  $n_p = n_e$

→ at each  $n_B$ , solve a system of coupled eqs.

(baryon number conservation + charge neutrality)

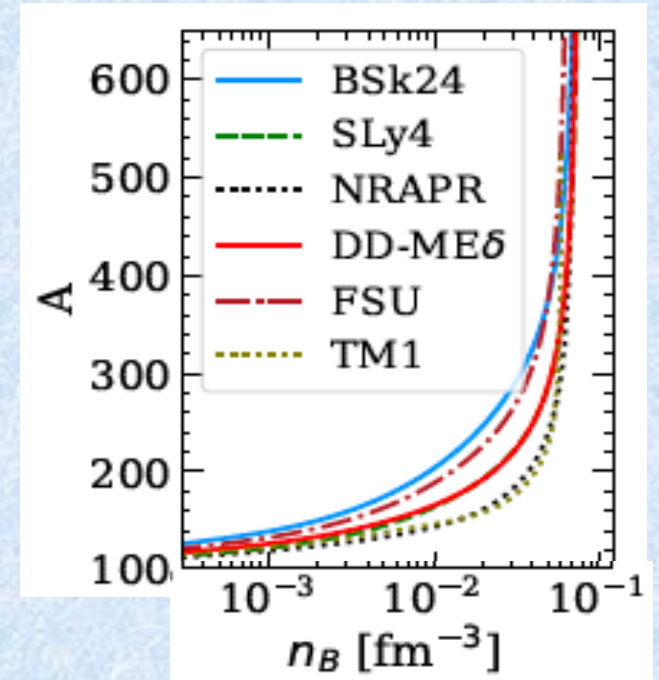
→ obtain variational variables ( $A, Z, n_i, V_{WS}, n_{gn}$ ) (or alternative variables)  
+ shape of WS cell if non-spherical cells (*pasta*, see later)

NB:  $T=0 \leftrightarrow$  One nuclear species at each density point

→ **one component or single nucleus approx (WS approx)**

✓ Fast computation → good for Bayes analyses

✗ No structure (shell) effects



Dinh Thi, PhD thesis (2023)





# (Semi) classical approach (1)

## 2. (Extended) Thomas-Fermi ((E)TF)

- energy => functional of particle densities and their derivatives
- **extended** → expansion up to order  $\hbar^N$  (e.g. Onsi et al., PRC 50, 3139 (1997))  
→ energy function of particle densities and Nth derivatives

$$E = \int_{V_{\text{WS}}} d^3\mathbf{r} \mathcal{E}[n_q(\mathbf{r}), n_e(\mathbf{r})]$$

- consistent treatment nucleons “inside”/“outside” → smooth profiles

$$\mathcal{E} = n_n m_n c^2 + m_p n_p c^2 + \mathcal{E}_{\text{nuc}} + \mathcal{E}_e + \mathcal{E}_{\text{Coul}}$$

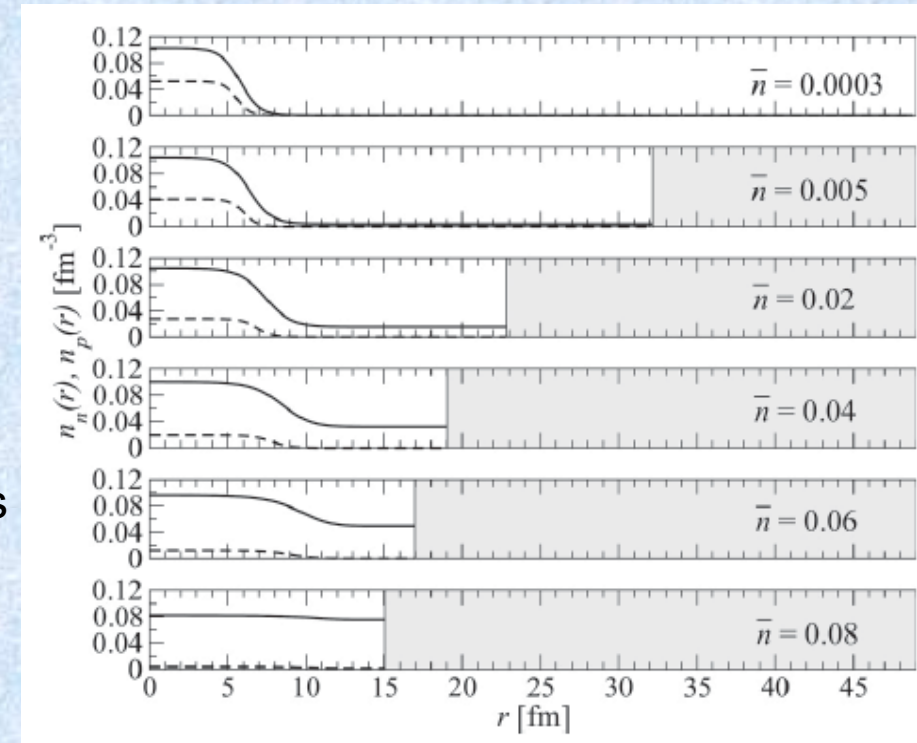
e<sup>-</sup> energy include  $m_e$

- minimization (Lagrange multipliers) requires functional derivatives

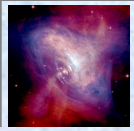
$$\sum_q \int_{V_{\text{WS}}} d^3\mathbf{r} \left[ \frac{\delta E}{\delta n_q(\mathbf{r})} \delta n_q(\mathbf{r}) - \mu_q \delta n_q(\mathbf{r}) \right] + \int_{V_{\text{WS}}} d^3\mathbf{r} \left[ \frac{\delta E}{\delta n_e(\mathbf{r})} \delta n_e(\mathbf{r}) - \mu_e \delta n_e(\mathbf{r}) \right]$$

- (non-linear) differential equations

- density profiles often parametrized (e.g. Fermi-like profiles) and electrons uniformly distributed
- reduced variational problem to minimization of  $E$  wrt reduced number of parameters



Pearson et al., PRC 85, 065803 (2012)



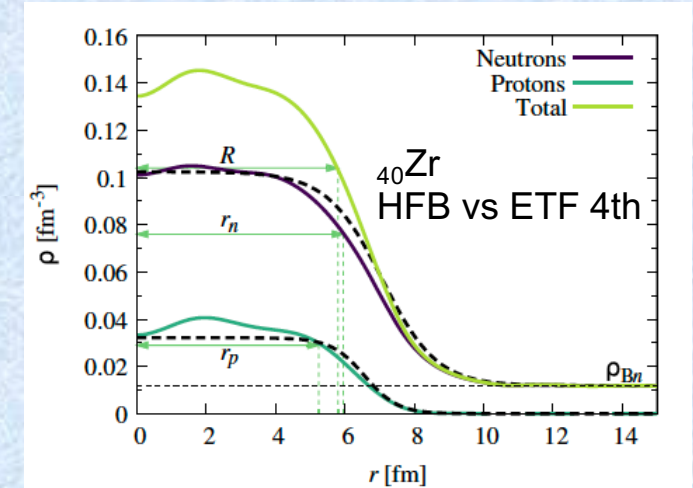
# (Semi) classical approach (2)

but: semi-classical approach → no shell effects !

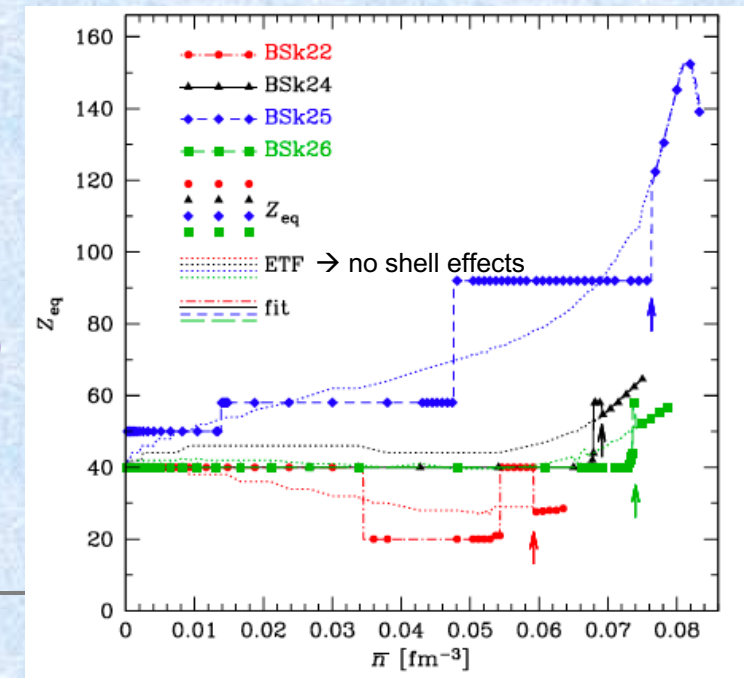
- can be added via Strutinski integral (e.g. Onsi et al., PRC 77, 065805 (2008))  
usually only for protons
- added perturbatively (iterating once HF equation starting from smooth ETF profiles)

$$E_{\text{nuc+sh}} = E_{\text{nuc}} + E_{\text{sh}}^p \quad \text{(E)TF}$$

- ✓ Faster computation than full HF(B)
- ✗ Less fast than CLDM (→ Bayes?)
- ✗ No structure (shell) effects → but can be included (perturbatively)



Pecak et al., PRX 14, 041054 (2024); see also Shelley&Pastore, Universe 6, 206 (2020)

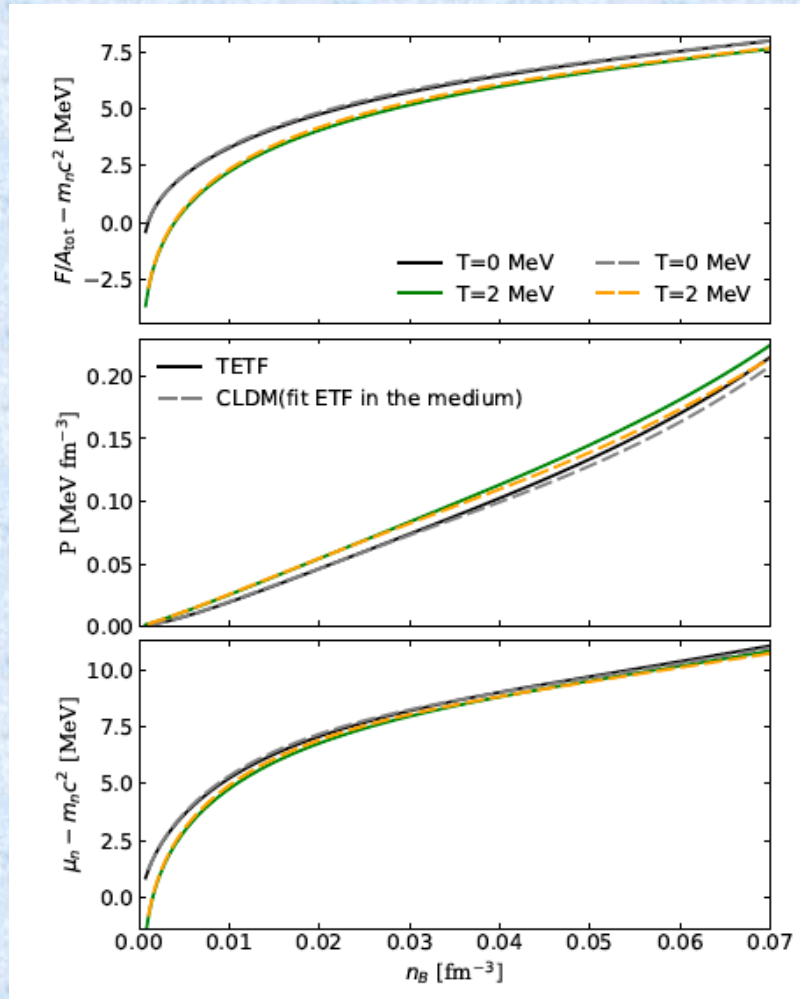


Pearson et al., MNRAS 481, 2994 (2018)

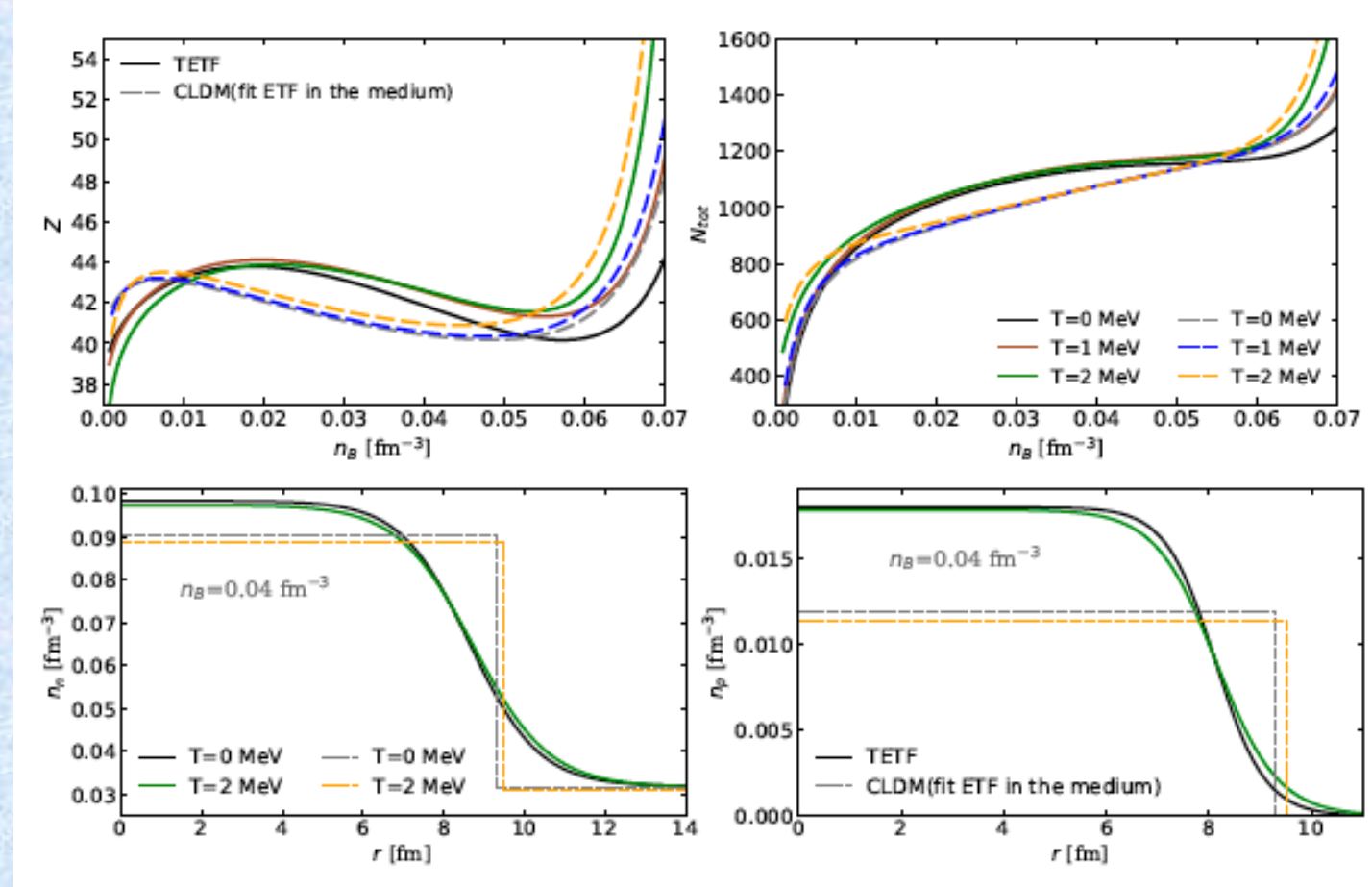




# CLDM vs ETF



Grams, Shchechilin, Diverrès, et al., Universe 11, 172 (2025)



- Thermo quantities → good agreement
- Composition / profiles → more micro approach



# Quantum approach

## 3. Hartree-Fock / Hartree-Fock Bogoliubov

- HF  $\rightarrow$  g.s. wave function = Slater determinant  
 $\rightarrow$  minimization of expectation value

$$\langle \Phi | \hat{H} | \Phi \rangle = 0 \rightarrow \langle \Phi | \hat{H}_{\text{eff}} | \Phi \rangle = 0$$

wrt single particle wave function

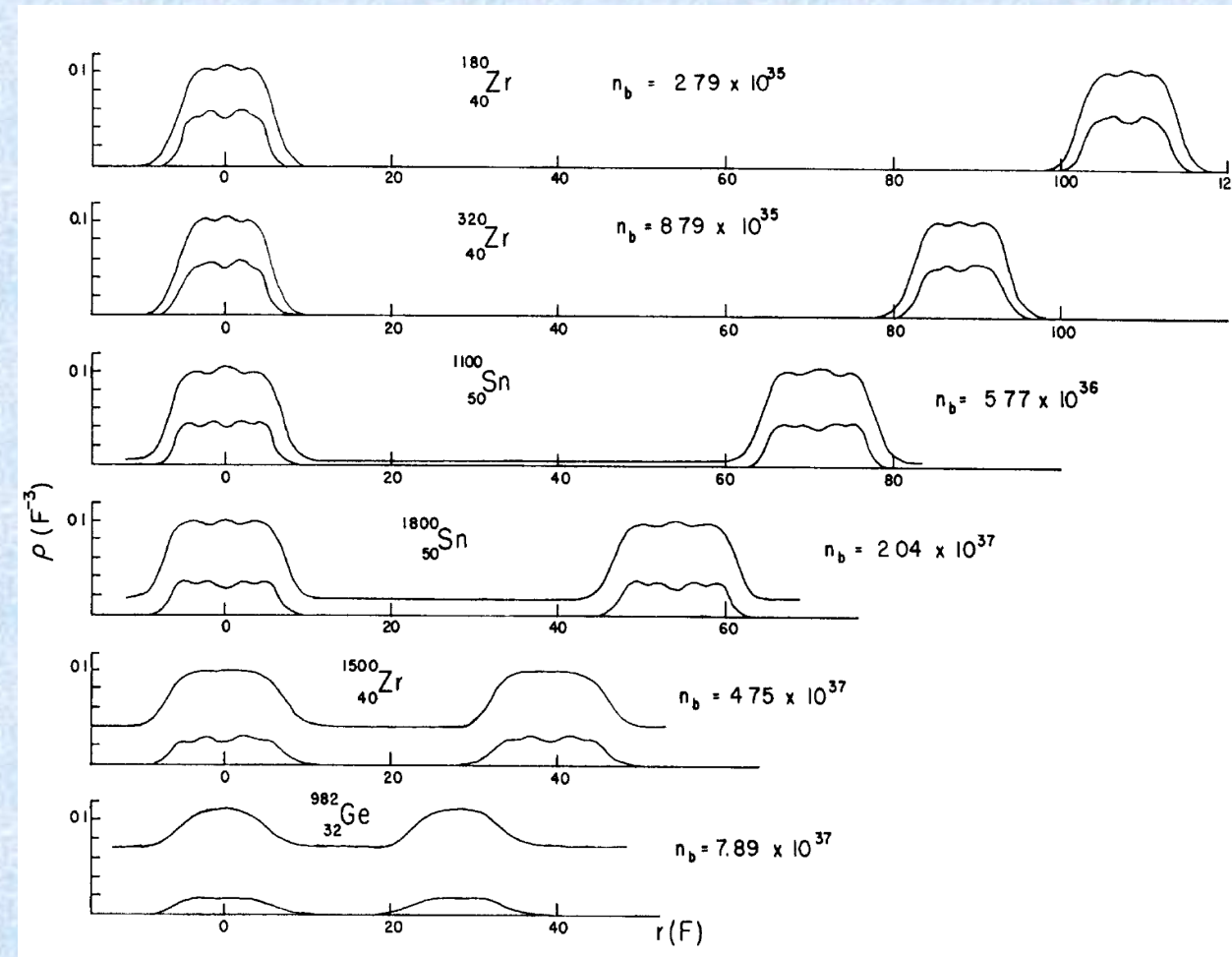
$\rightarrow$  set of sp Schrodinger eq. (solved in box)

$$\left[ -\nabla \frac{\hbar^2}{2m_q^*} \nabla + U_q(\mathbf{r}) \right] \phi_{q,i}(\mathbf{r}) = \epsilon_{q,i} \phi_{q,i}(\mathbf{r})$$

(here s-o omitted  $\rightarrow$  reduced computational cost)

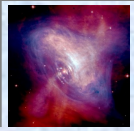
NB: self-consistent solution of HF eqs.

- ✓ Quantum  $\rightarrow$  shell effects (pairing can be added)
- ✗ Numerical challenge
- ✗ Spurious shell effects  
 (nucleons confined with periodic b.c.)



Negele & Vautherin, NPA 207, 298 (1973)





# NS “pasta” mantle

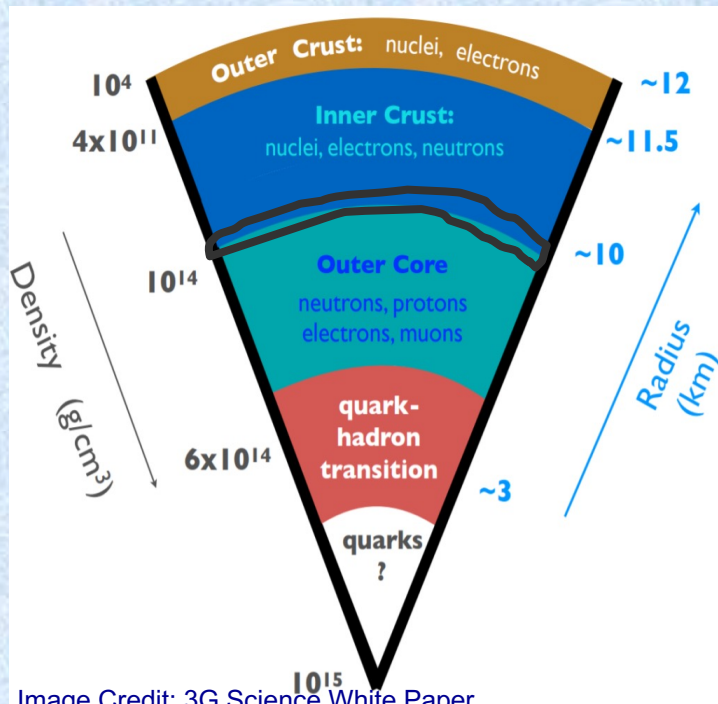
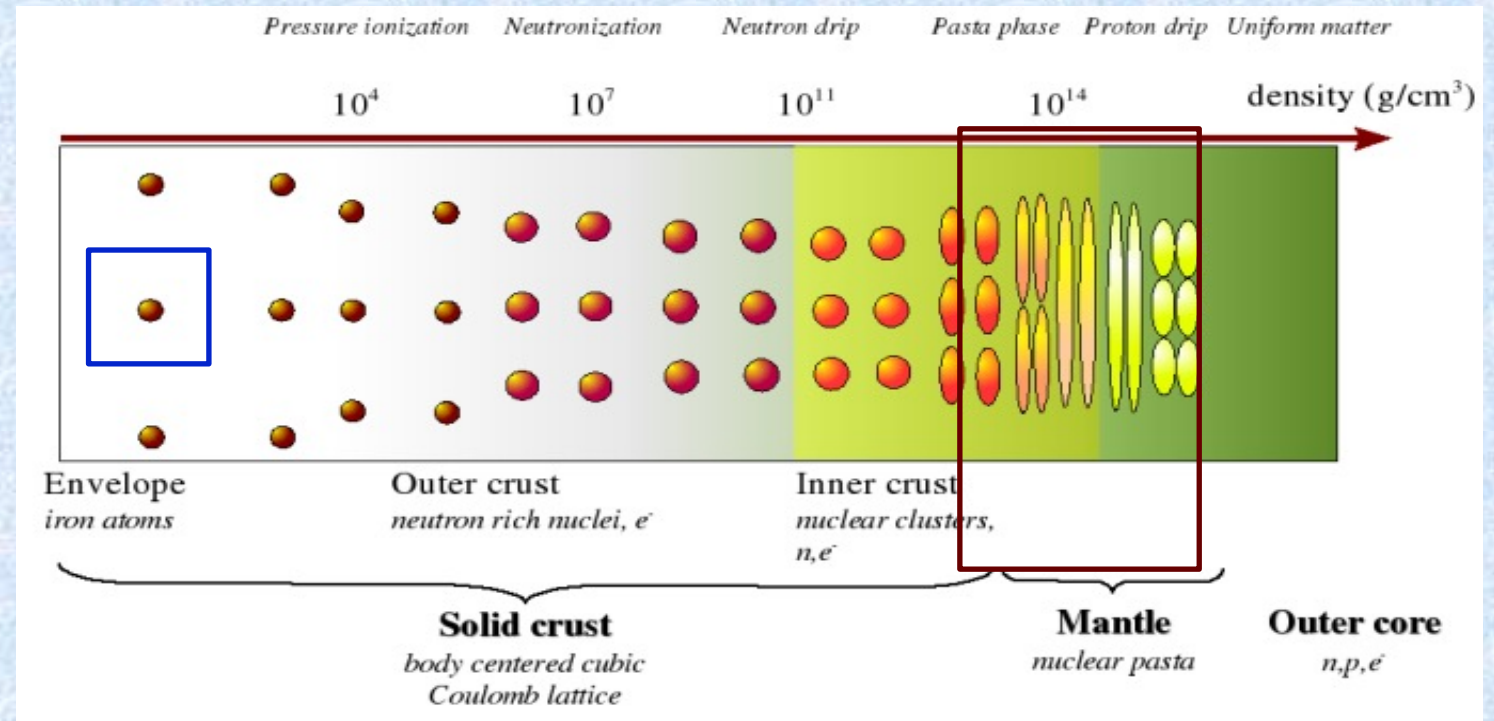
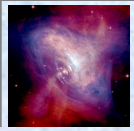


Image Credit: 3G Science White Paper



Chamel & Haensel, Liv. Rev. Relativ. 11, 10 (2008); see also Blaschke & Chamel, ASSL 457, 337 (Springer, 2018)





# Pasta phases (1)

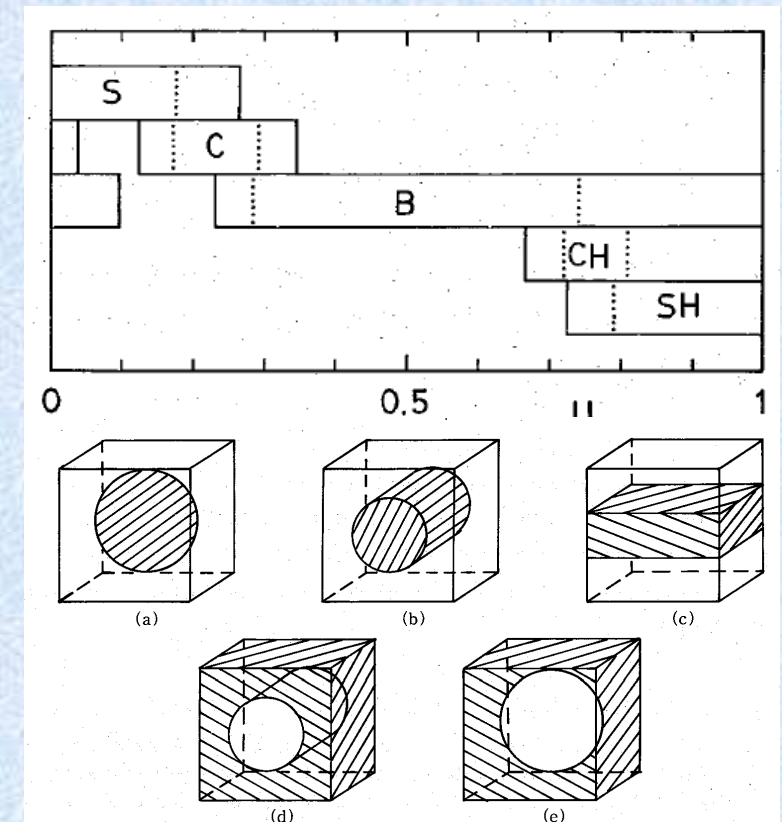
- Predicted already in 1971  
(Baym et al., NPA 175, 225 (1971))

The fraction of space,  $u$ , occupied by nuclei is seen in table 2 to increase monotonically until the nuclei begin to touch. For a bcc lattice the value  $u = \sqrt{3}\pi/8 = 0.68$  corresponds to the nuclei just touching. The picture of nuclei as spherical drops is certainly not valid beyond this point. The nuclear parameters  $A$ ,  $Z$  and  $n_N$ , given for  $\rho = 2.26 \times 10^{14}$  and  $2.39 \times 10^{14}$  g/cm<sup>3</sup> should be regarded as highly tentative since they are particularly sensitive to the precise way in which the surface energy tends to zero as  $n_n/n \rightarrow 1$ . Furthermore, our model has neglected deformations of the nuclei, which become important here. In fact, it might be more favorable, beyond  $u = 0.5$ , for the nuclei to “turn inside out”, that is, for the neutron gas to exist as a lattice of droplets in a sea of nuclear matter.

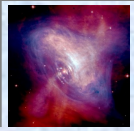
- Competition between surface ( $\rightarrow$  short range, favours compact, spherical shapes) and Coulomb energy ( $\rightarrow$  long range, try to keep protons apart)  $\rightarrow$  “frustrated” state
- Importance of surface tension at large isospin values (not constrained by experiments)

- Nuclear pasta shown to appear when  $V_N/V_{WS} > \sim 0.2$

(Hashimoto et al., PTP 71, 320 (1984),  
Oyamatsu et al., PTP 72, 373 (1984);  
Ravenhall et al., PRL 50, 2066 (1983))

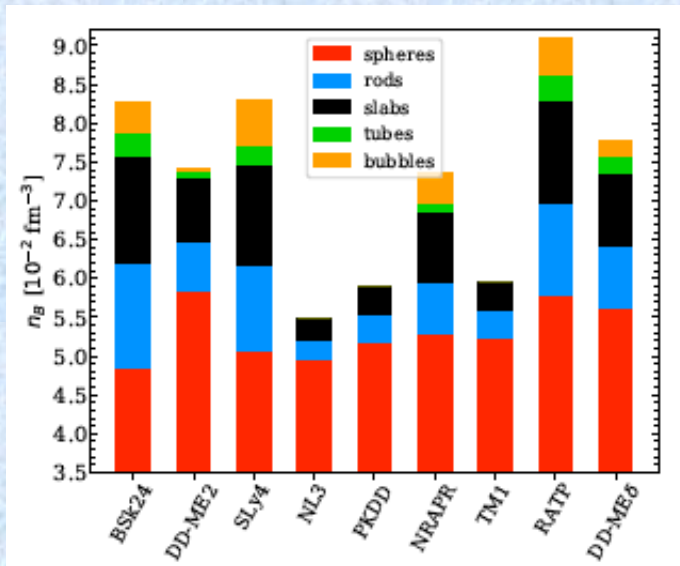




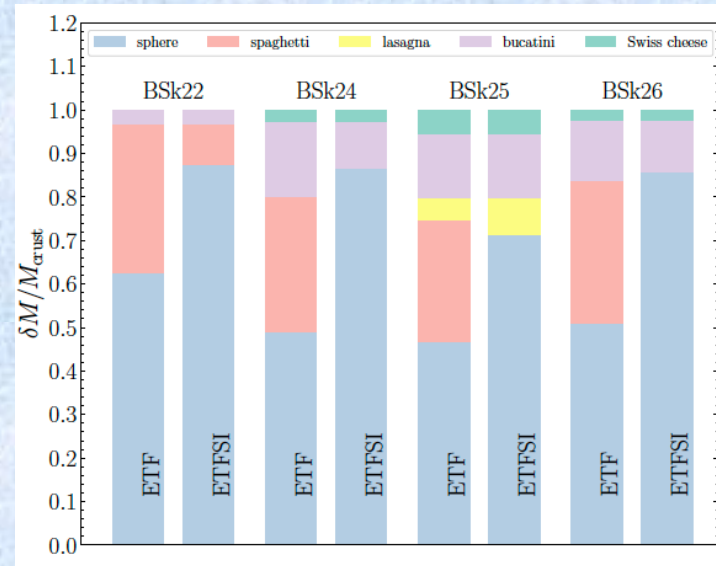


# Pasta phases (2)

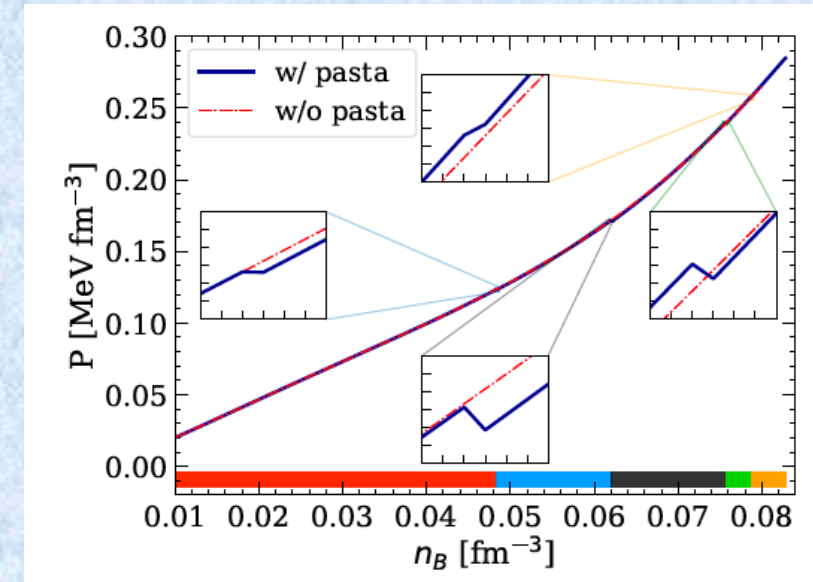
- More recent calculations with CLDM / ETF(SI) → similar sequence



Dinh Thi, PhD thesis 2023; Dinh Thi et al., A&A 654, A114 (2021), EPJA 57, 296 (2021), CLDM



Shchechilin et al., PRC 108, 025805 (2023);  
ETF(SI) see also Pearson & Chamel, PRC 105, 015803 (2021); Shchechilin et al., PRC 109, 055802 (2024)



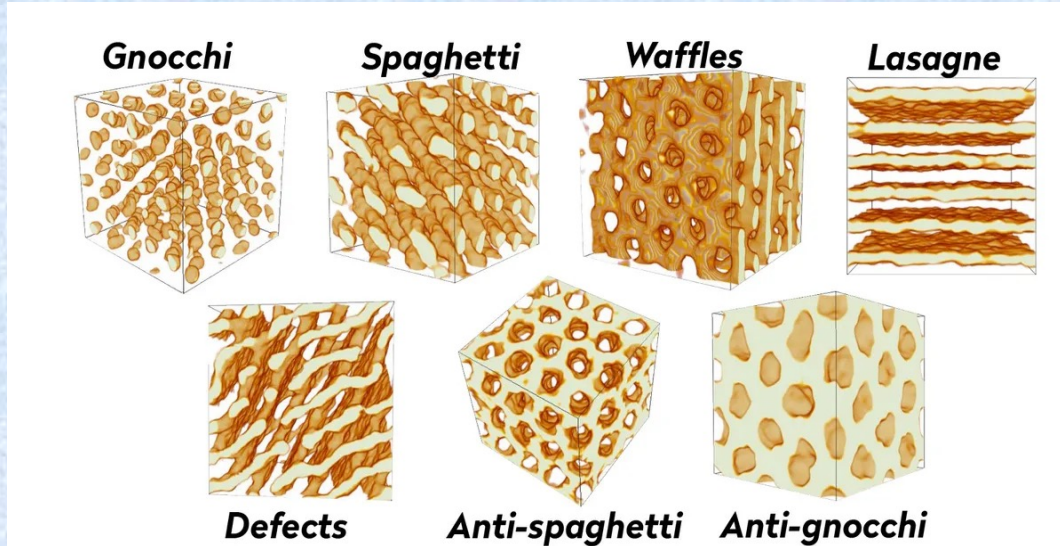
Dinh Thi, PhD thesis 2023

- details of sequence (e.g. transition density) depend on the functional + many-body method
- EoS negligibly affected by pasta, but elastic/transport properties are

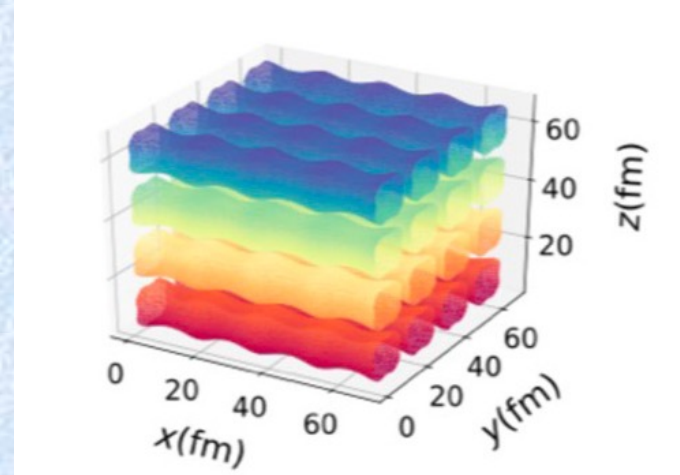


# Pasta phases (3)

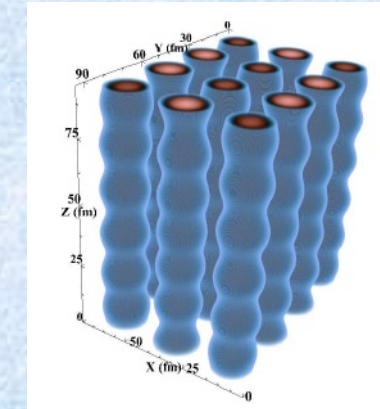
- Calculations done in different frameworks (CLDM, ETF, HFB, CMD, QMD ...)



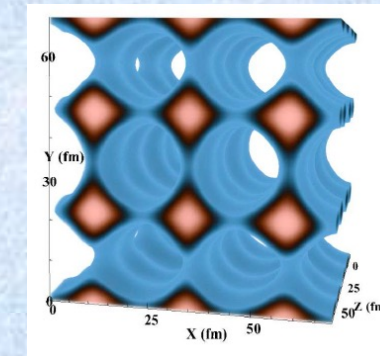
Credit: Caplan, Horowitz, CMD (BBC Sky at Night Magazine)  
(usually at fixed proton fraction)



Newton et al., PRC 105, 025806 (2022),  
3D HF+BCS



$n_B \sim 0.07 \text{ fm}^{-3}$

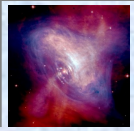


$n_B \sim 0.075 \text{ fm}^{-3}$

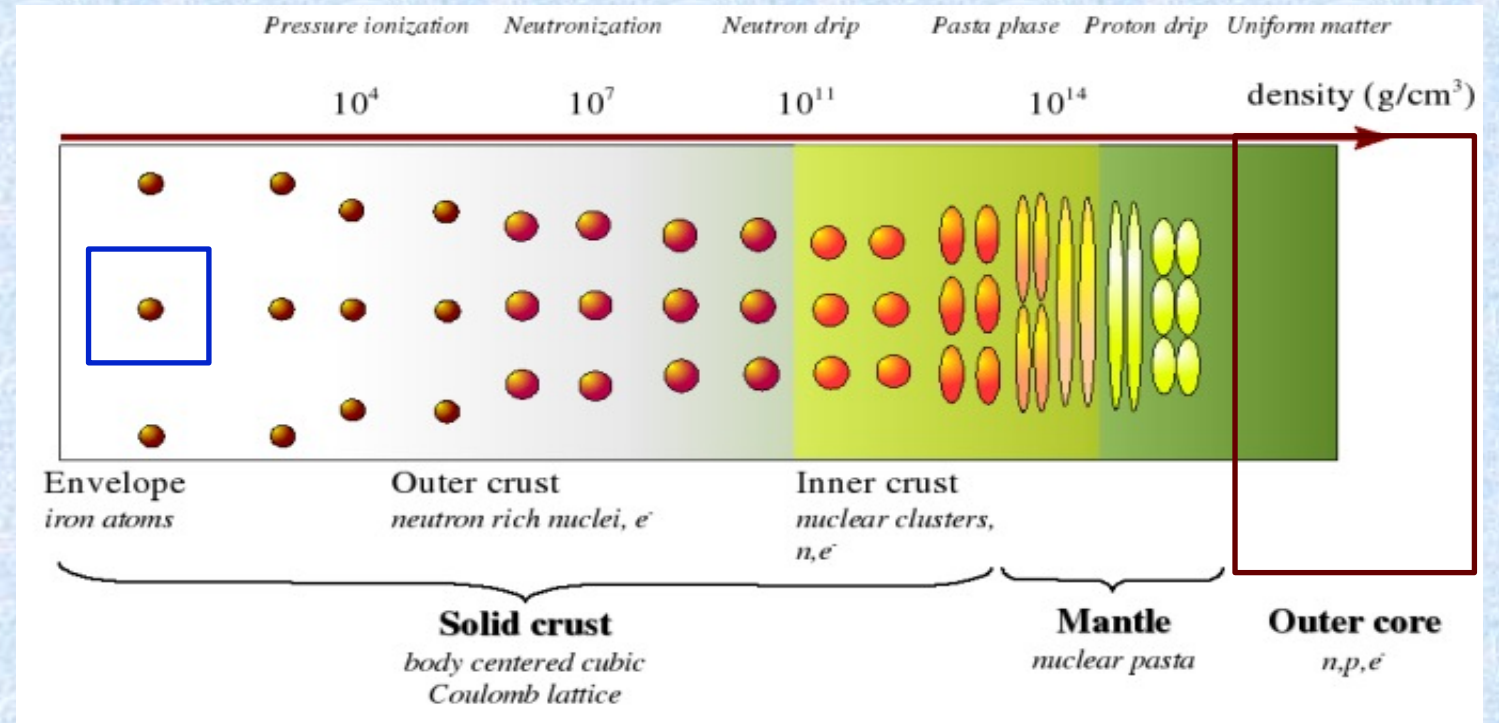
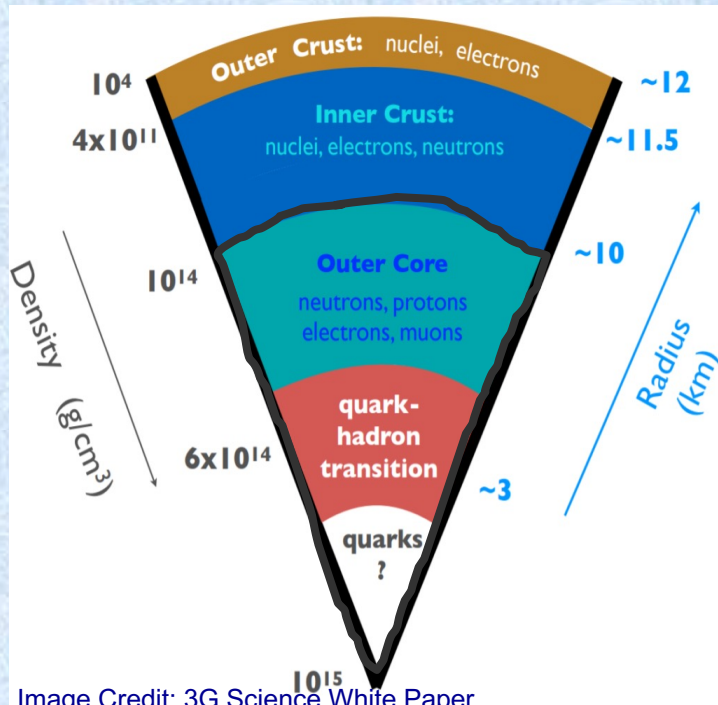
Shchechilin, PhD thesis (2026),  
3D HF+BCS

- up to 50% crust mass (e.g. Dinh Thi et al. 2021, Baillet et al. 2021)
- potential implications for magneto-rotational evolution, transport properties, “mountains”, ...  
(e.g. Gearheart et al., MNRAS 2011, Pons et al., Nat. Astron. 2013, Horowitz et al., PRL 2015)

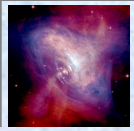




# NS core



Chamel & Haensel, Liv. Rev. Relativ. 11, 10 (2008); see also Blaschke & Chamel, ASSL 457, 337 (Springer, 2018)



# NS core: homogeneous matter

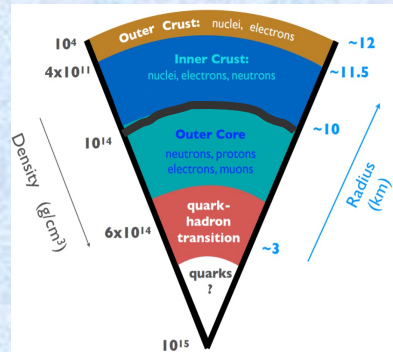
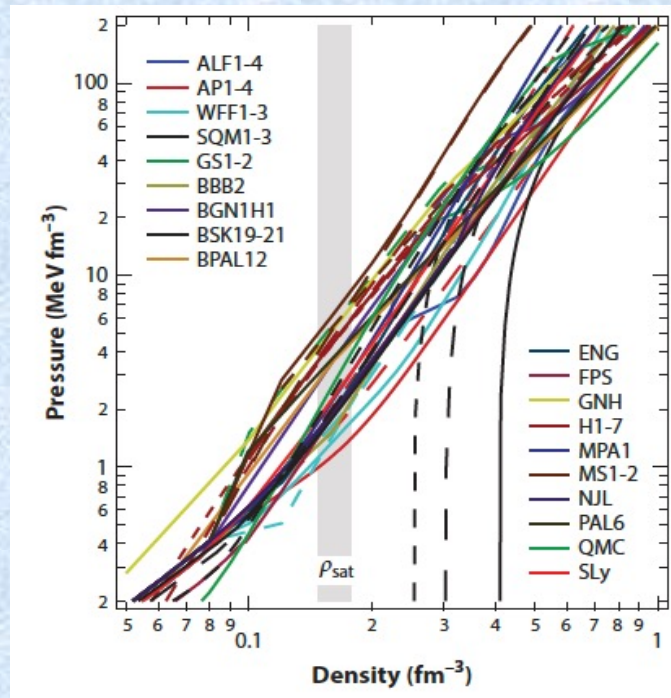


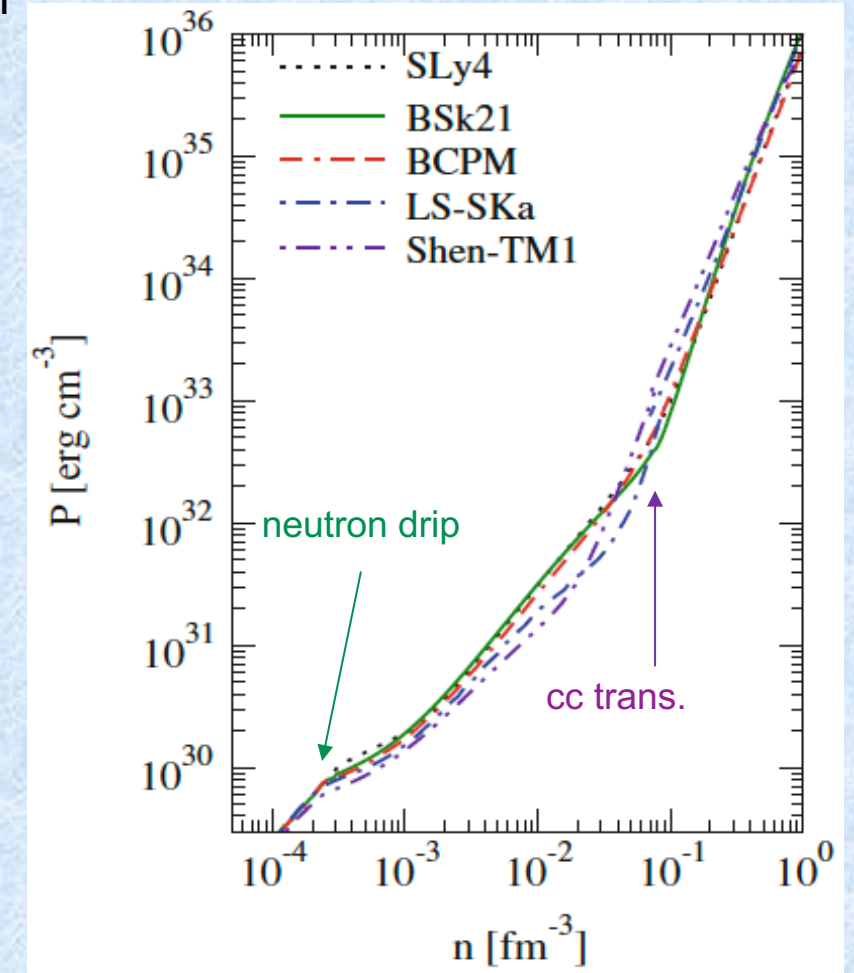
Image Credit: 3G Science White Paper

$$\mathcal{E}_{\text{homog-matter}} < \mathcal{E}_{\text{inhomog-matter}} \rightarrow \text{cc transition}$$

- Core EoS  $\rightarrow$  homogeneous matter +  $\beta$  equilibrium



Ozel & Freire, ARAA 54, 401 (2016)



Burgio & Fantina, ASSL 457 (2018)

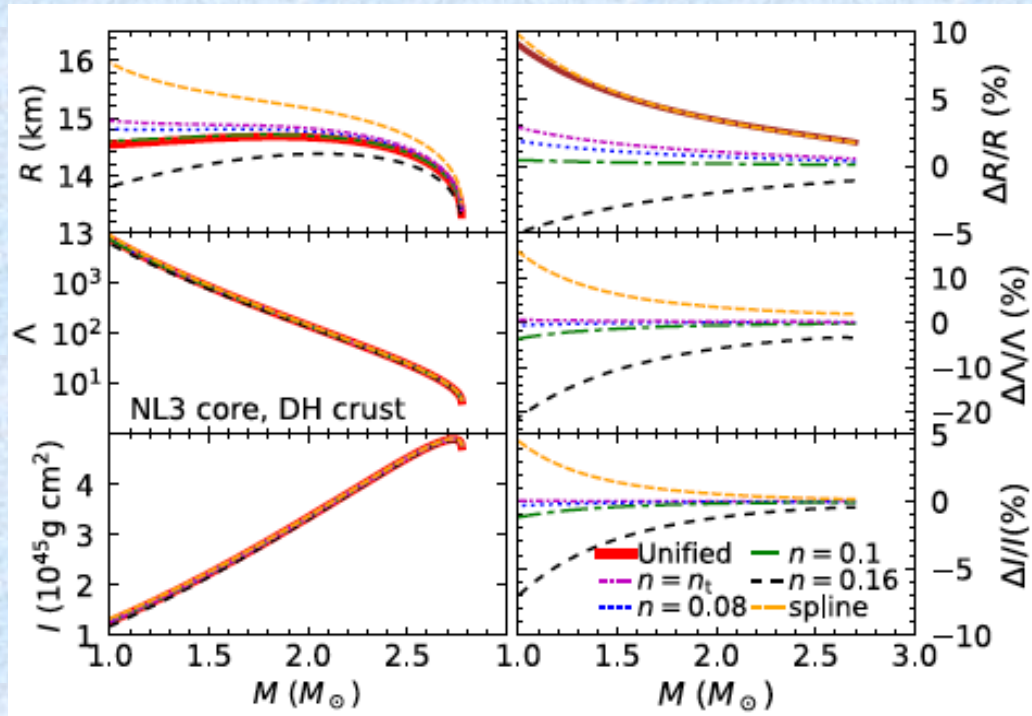
➤ EoS model dependence increases with density !



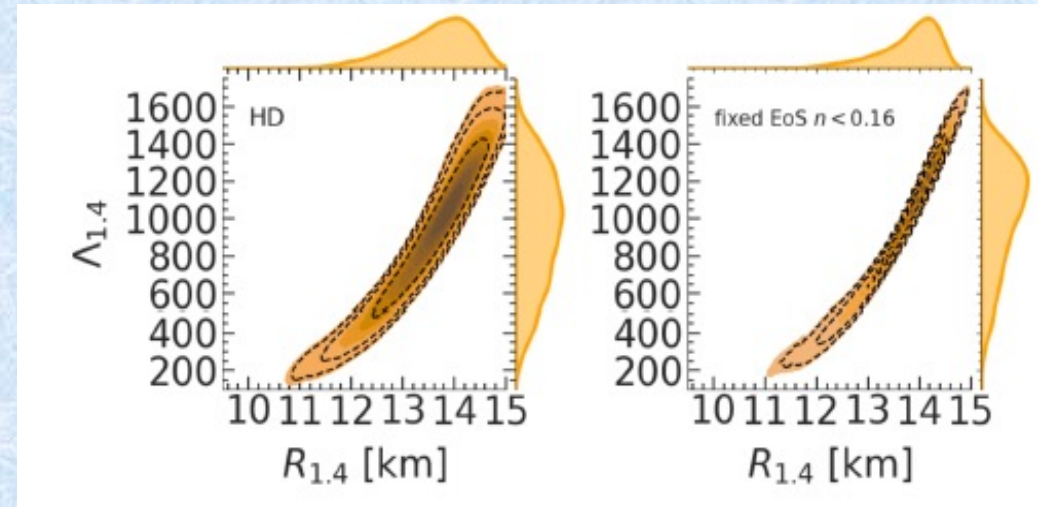


# Matching crust & core or *unified* approach?

- If crust & core EoS matched inconsistently → spurious non-physical effects in numerical modelling



Suleiman et al., PRC 104, 015801 (2021)



Davis et al., A&A 687, A44 (2024); Fantina & Gulminelli, PoS QNP2024, 153 (2025)  
CUTER code

see also Fortin et al., PRC 94, 035804 (2016),  
Ferreira & Providência, Universe 6, 220 (2020)

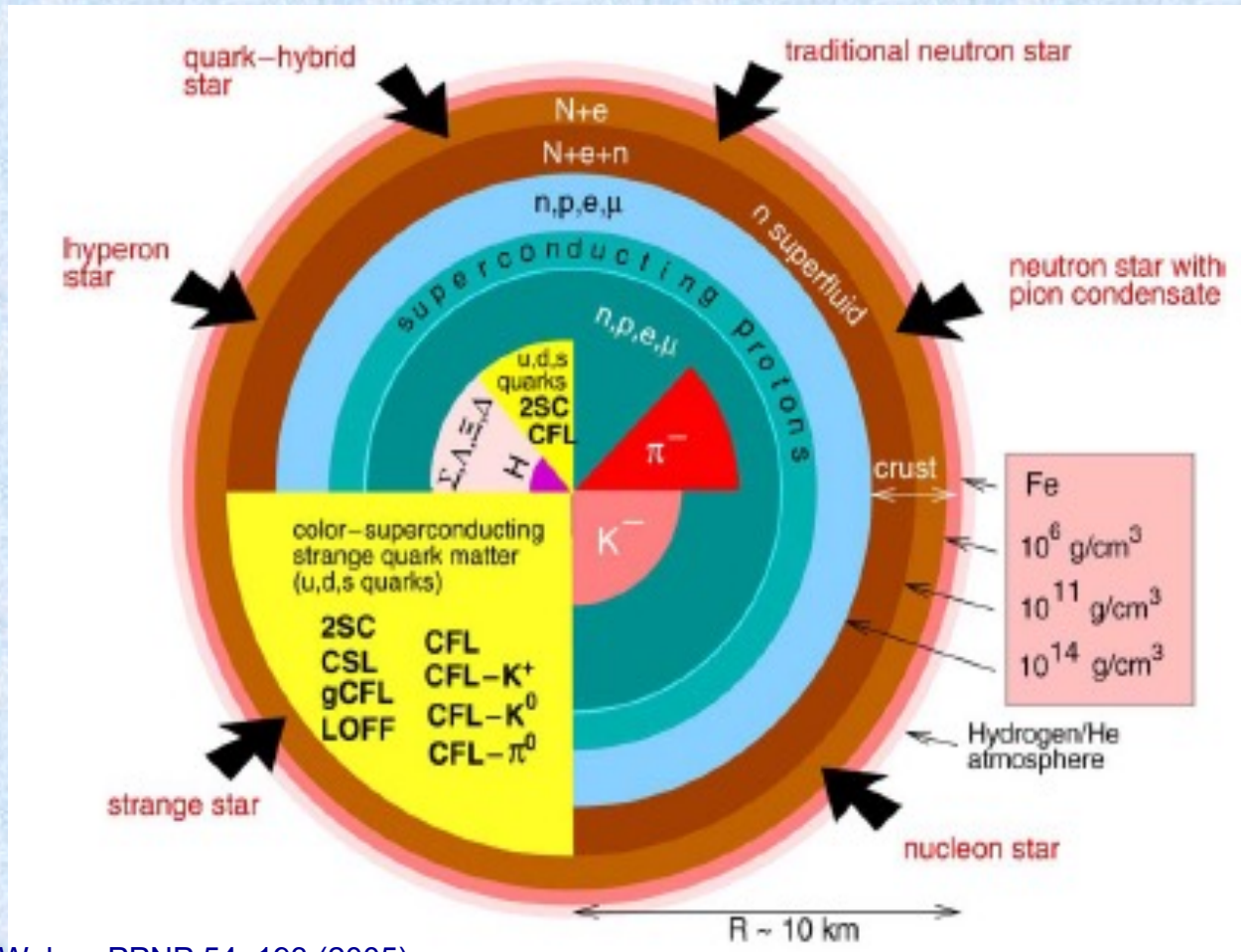
➤ Error due to inconsistency can be > error in many-body treatment of the crust!

➡ **Thermodynamically consistent and unified EoSs needed for astro modelling & inference analyses**

(but not many available, e.g. Douchin&Haensel, A&A 2001; Fantina et al., A&A 2013; Raduta&Gulminelli, NPA 2019; Viñas et al., Symmetry 2021; Pearson et al., MNRAS 2018; Grams et al., EPJA 2022; Xia et al., PRC 2022; Scurto et al., PRD 2024; Davis et al., A&A 2024, see CompOSE database)



# NS core $\rightarrow$ nucleons, $e^-$ , ... what else?



Weber, PPNP 54, 193 (2005)

$$\mu_i = B_i \mu_B + Q_i \mu_Q + S_i \mu_S + L_i^e \mu_{le} + L_i^\mu \mu_{l\mu}$$

- New particle appears if:

$$\mu_i > m_i c^2 + U_i$$

- **Muons:** ( $m_\mu \sim 106 \text{ MeV}/c^2$ )

“Free” particle  $\rightarrow U = 0 \rightarrow \mu_\mu > m_\mu c^2$

two independent densities:  $n_B, n_Q$

charge neutrality (& beta equilibrium):  $\mu_\mu = \mu_e$

$\rightarrow$  even in “standard” NS they appear



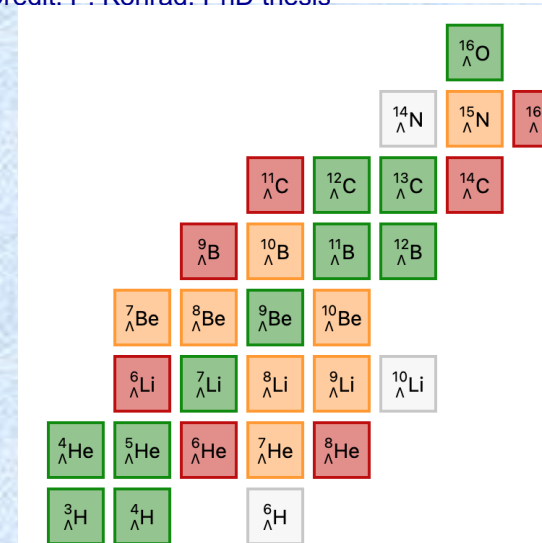
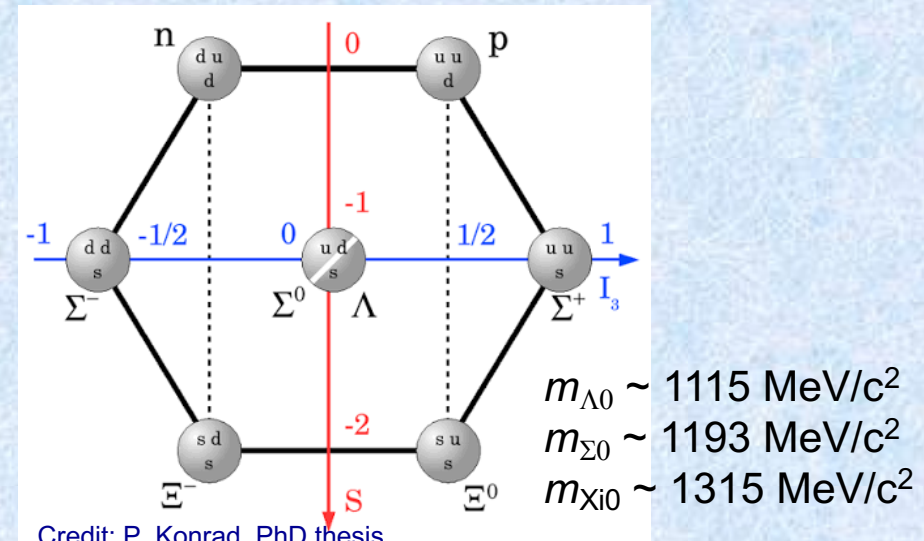


# NS core: hyperons (1)

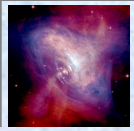
- Interacting particle  $\rightarrow \mu_i > m_i c^2 + U_i$   
three independent densities:  $n_B, n_Q, n_S \rightarrow$  corresponding  $\mu_B, \mu_Q, \mu_S$   

$$\mu_{\Lambda^0} = \mu_{\Sigma^0} = \mu_B$$

$$\mu_{\Sigma^\pm} = \mu_{\Xi^\pm} = \mu_B \pm \mu_Q$$
- If no potential  $\rightarrow \Lambda$  appears first, but: depends on interaction!
- Info on YN potential based on (few) hypernuclear data  
Calculations based on ab-initio and EDF (recently also Gamow-shell model for hypernuclei, Dassie et al., PRC 114, 024331 (2026)).  
 $\rightarrow U_{\Lambda N}(n_{\text{sat}}) \sim -30 \text{ MeV}; U_{\Sigma N}(n_{\text{sat}}) \sim ?$   
 $\rightarrow U_{YY} ?$  (from B.E. difference between single- and double-Y nuclei but large error bars!)

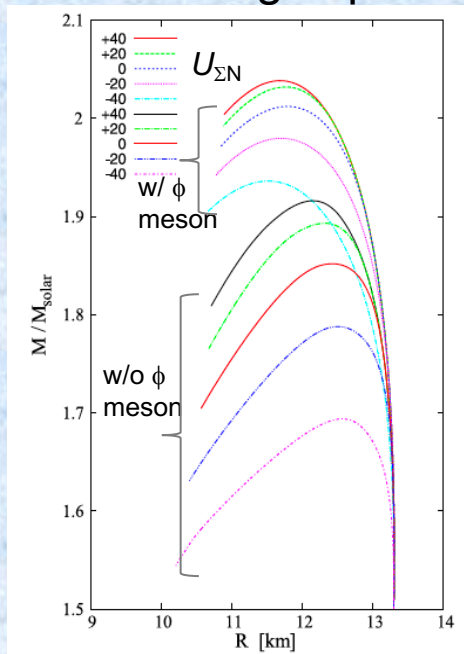


Hypernuclear data: <https://hypernuclei.kph.uni-mainz.de>

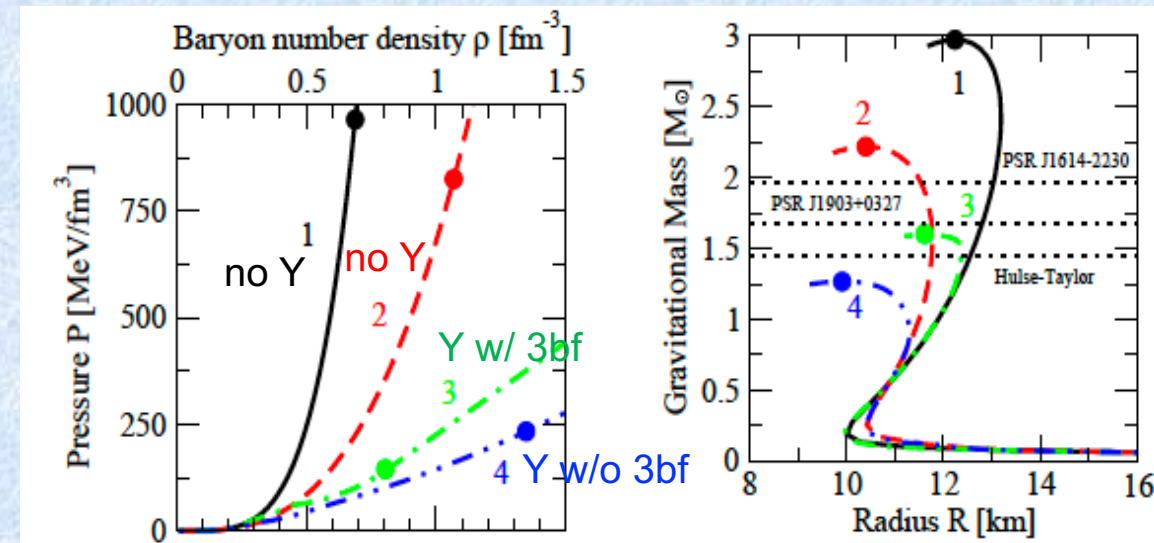


# NS core: hyperons (2)

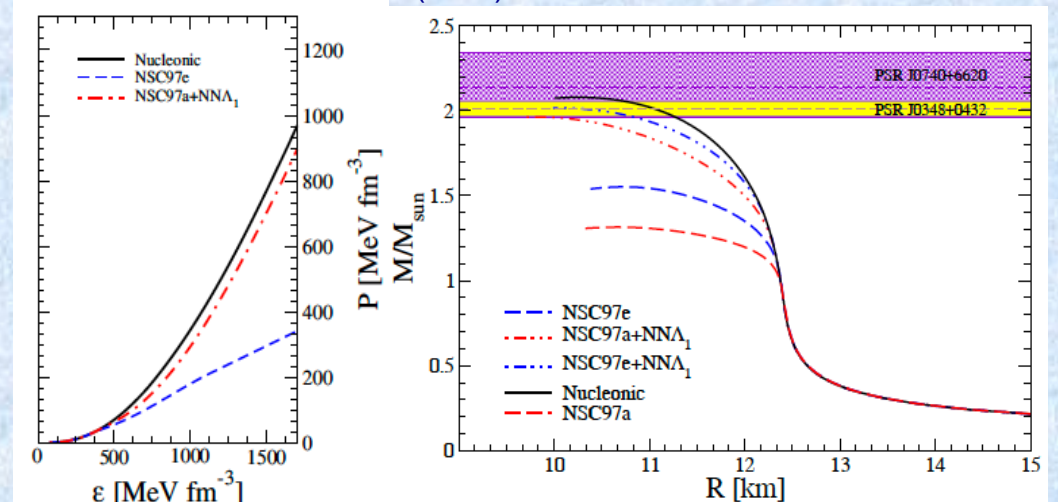
- Appearance of Y depends on interaction!
- New d.o.f. → **softening of EoS** → “hyperon puzzle”  
but: Y potential very poorly known!  
→ model dependent results
- $2M_{\text{sun}}$  reached if repulsion (e.g. 3bf from N2LO chiral inter., or parameter tuning in pheno models)



Weissenborn et al., NPA 881, 62 (2012)



Vidana et al., EPL 94, 11002 (2011)



Logoteta et al., EPJA 55, 207 (2019)



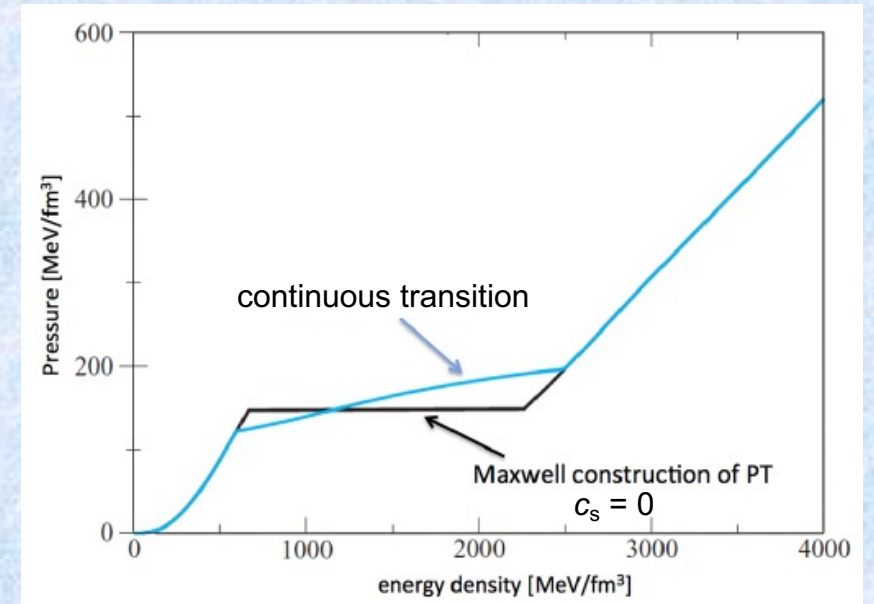


# NS core: quarks (1)

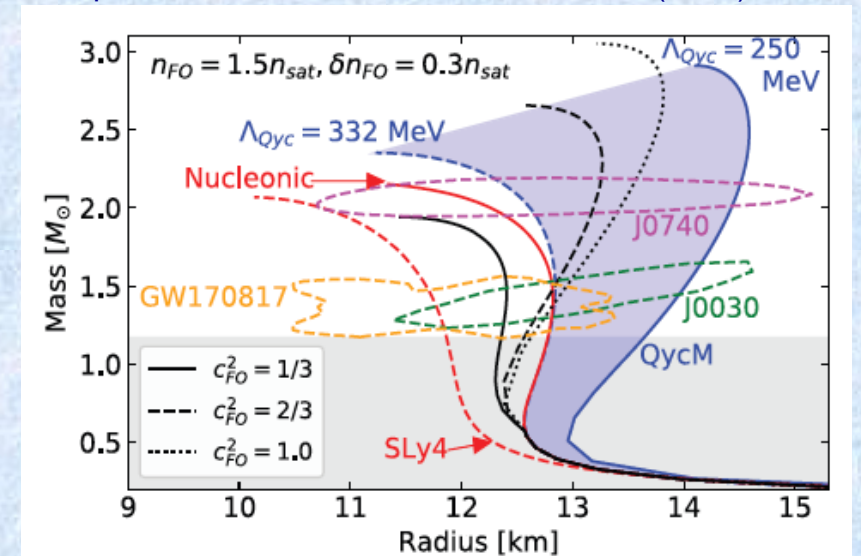
- Quarks  $\rightarrow$  deconfinement  $\rightarrow$  free quarks (u,d,s)  
three independent densities:  $n_B, n_Q, n_S \rightarrow \mu_B, \mu_Q, \mu_S$
- but: no unified model to treat transition  
 $\rightarrow$  effective models for quarks (e.g. NJL, MIT Bag model, etc.)  
 $\rightarrow$  sharp transition modelled via Maxwell construction  
 $P_H = P_Q$  &  $\mu_H = \mu_Q$

caveat: if inhomogeneous phases  $\rightarrow$  continuous transition

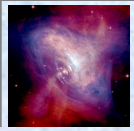
- Model dependencies  
 $\rightarrow$  effects of quarks less clear (soften / stiffen)



adapted from Schramm et al., EPJA 52, 14 (2016)



Somasundaram & Margueron, EPL 138, 14002 (2022)



# NS core: quarks (2)

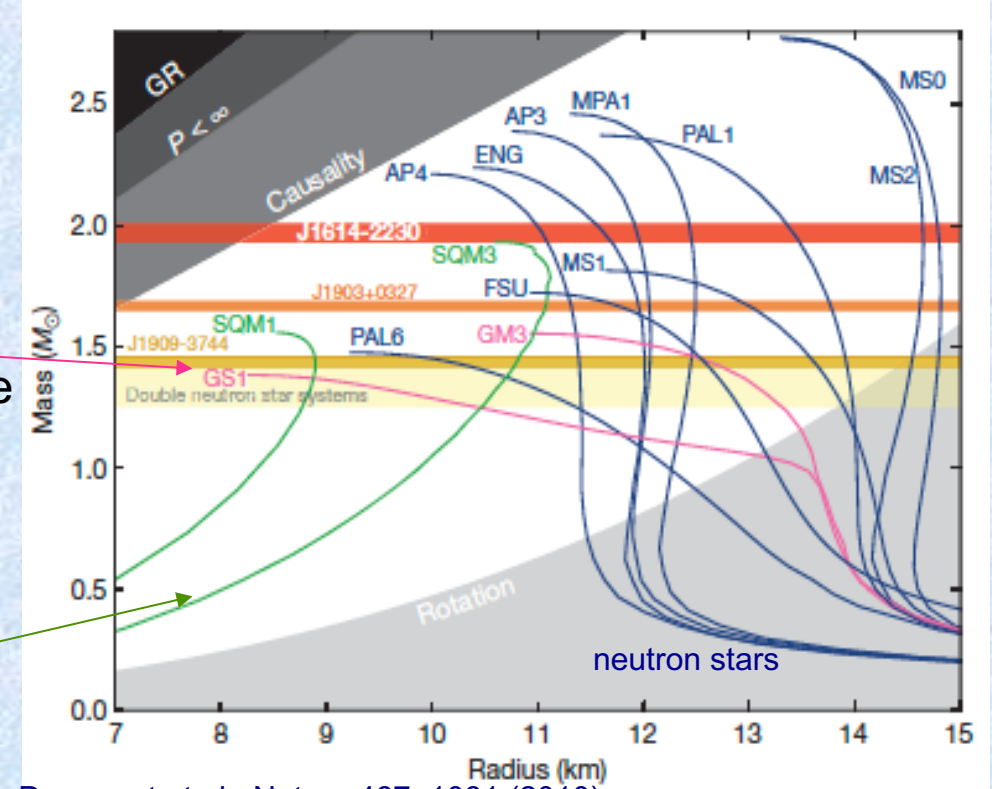
Different “families” : hybrid vs quark NS

$e_{\text{quark}} < e_{\text{hadron}}$  in some density range (NS core)  
 → **hybrid stars**  
 typically with crust + nucleonic outer core

$$e_{\text{quark}} < \frac{E_{\text{mass}, 56\text{Fe}}}{56} \sim 930 \text{ MeV}$$

Bodmer-Witten hypothesis

→ **quark stars**  
 stable (strong forces) (e.g. arXiv:1612.00755)  
 typically more compact

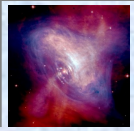


Demorest et al., Nature 467, 1081 (2010)

Can we observationally distinguish?

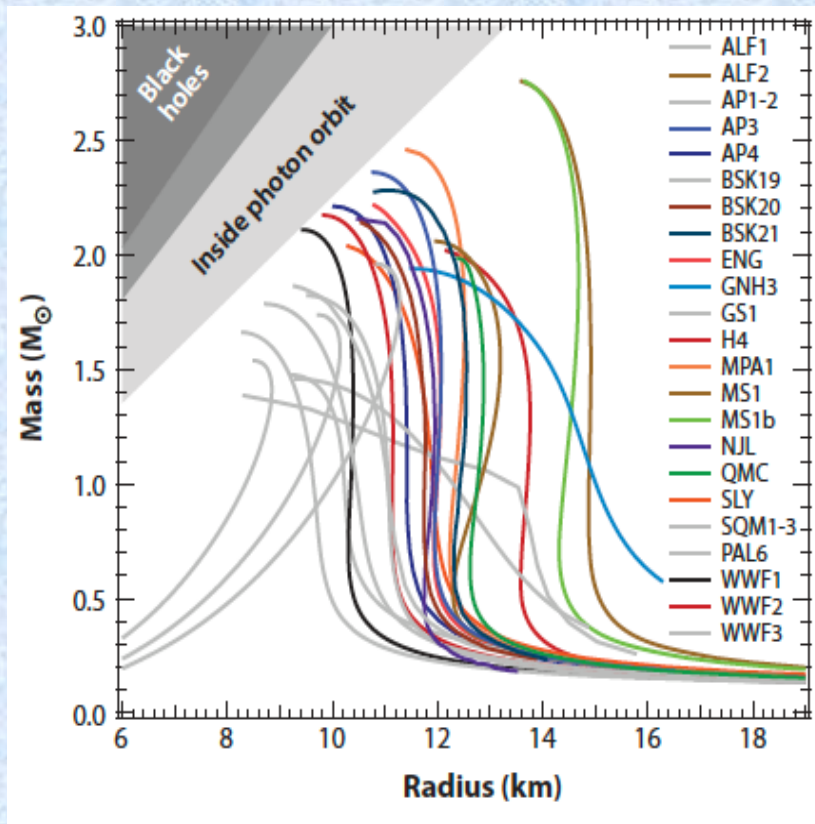
(see e.g. Bombaci, arXiv:astro-ph/0002524, Andersson et al., MNRAS 337, 1224 (2002), Xia et al., Int. J. Mod. Phys A (2025), arXiv:2511.01146)



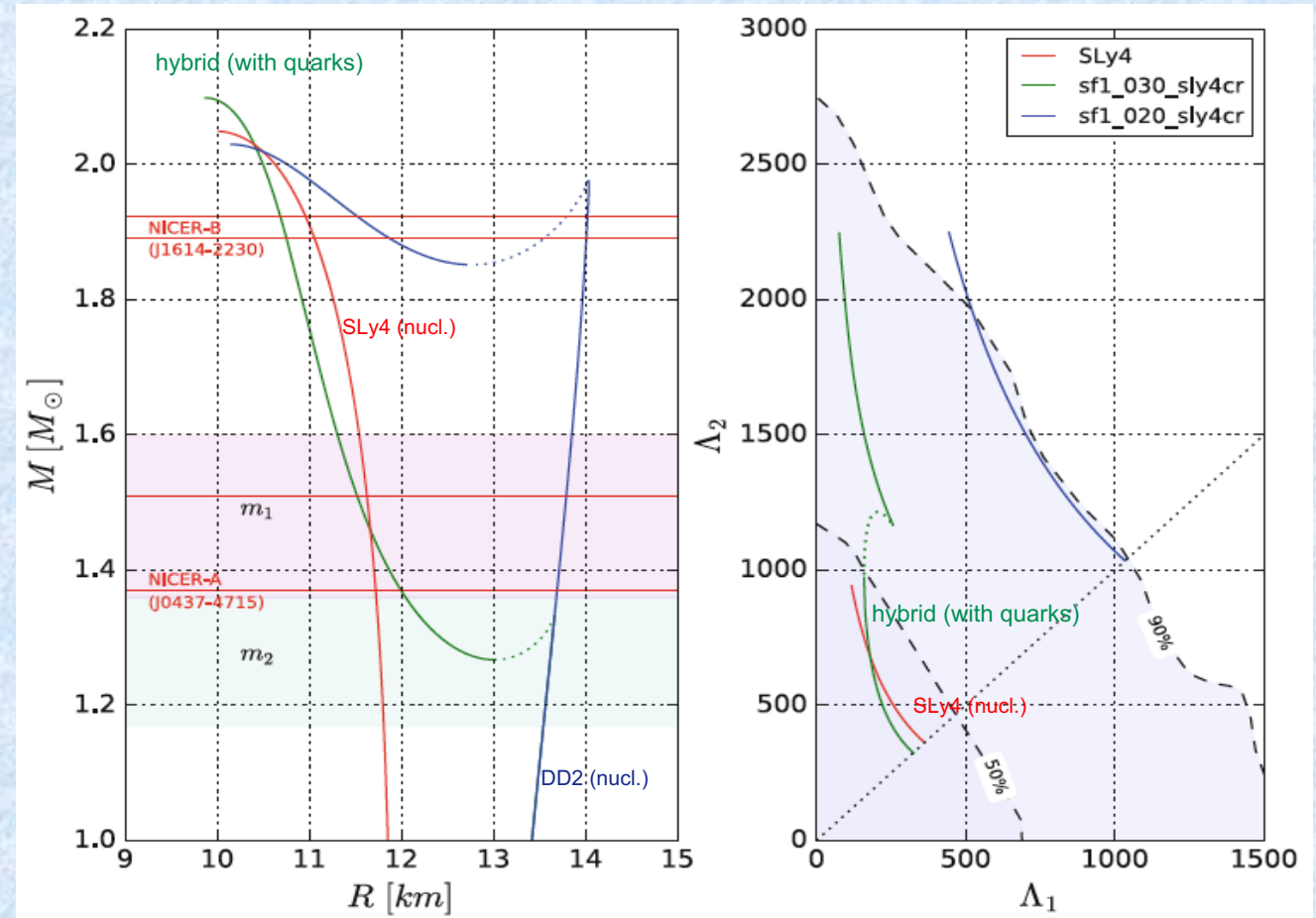


# Masquerade !

Up to no not yet clear observational evidence of hybrid / (strange) quark stars!



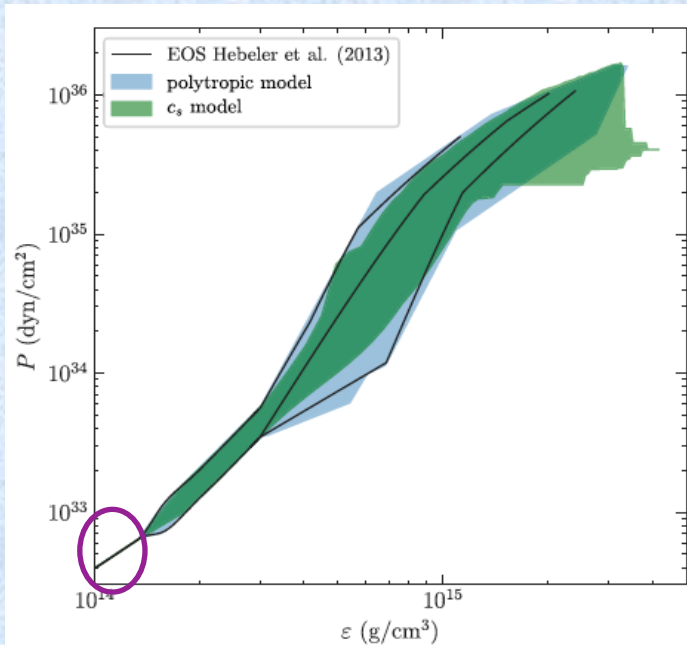
Ozel & Freire, ARAA 54, 401 (2016)  
see also Burgio & Fantina, ASSL 457, 255 (2018)



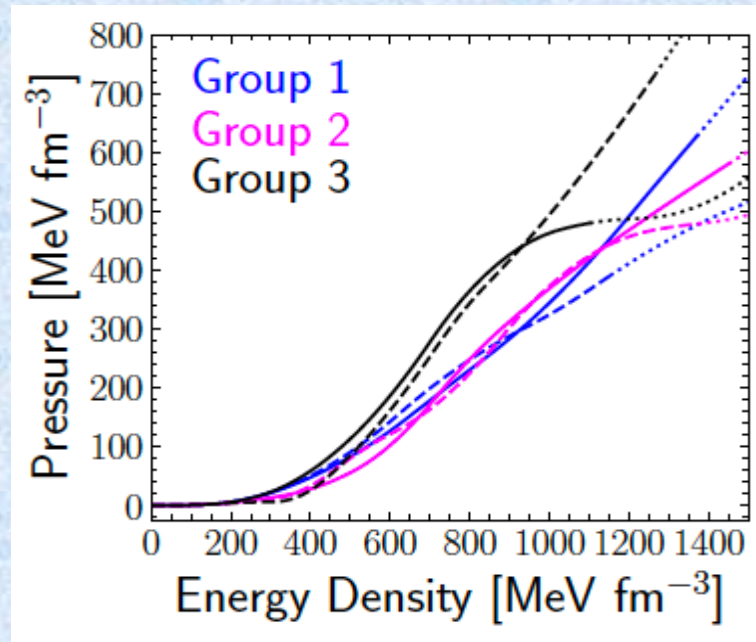
Blaschke & Chamel, ASSL 457, 337 (2018);

# “Non-nuclear-physics” approaches to the NS core

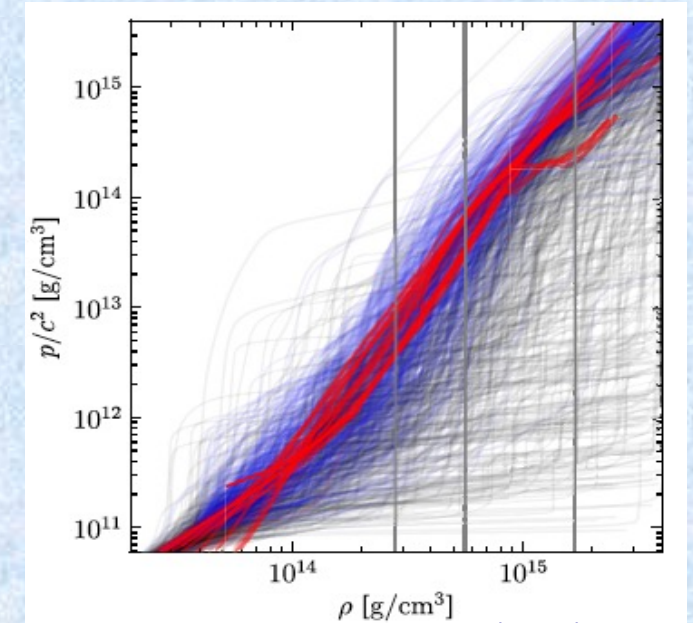
- High model dependence & large uncertainties in the NS-core EoS, mainly due to appearance of new d.o.f.
- **Agnostic** (“non-nuclear”) approaches for NS core (conditioned by astro)
  - \* Parametric (e.g. piecewise polytropes,  $c_s$  models, ...)
  - \* Non-parametric approaches (e.g. Gaussian Process)



Hebeler, Phys. Rep. 980, 1 (2021)



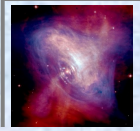
Somasundaram et al., PRC 107, 025801 (2023)



Essick et al., PRD 101, 063007 (2020)

- ✓ powerful → no underlying hypotheses
- ✓ used in bayesian inferences
- ✗ no info on composition
- ✗ often unique (non-consistent) low-density EoS





# Model exploration: Bayesian inference

$$p_{\text{post}}(\vec{X}|\vec{c}) \propto p(\vec{X}) \prod_i p(c_i|\vec{X})$$

$$p(\vec{X})$$

prior  $\rightarrow$  large parameter space  
(non-informative / nuc.-phys. informed)

$$p(c_{\text{nuc}}|\vec{X})$$

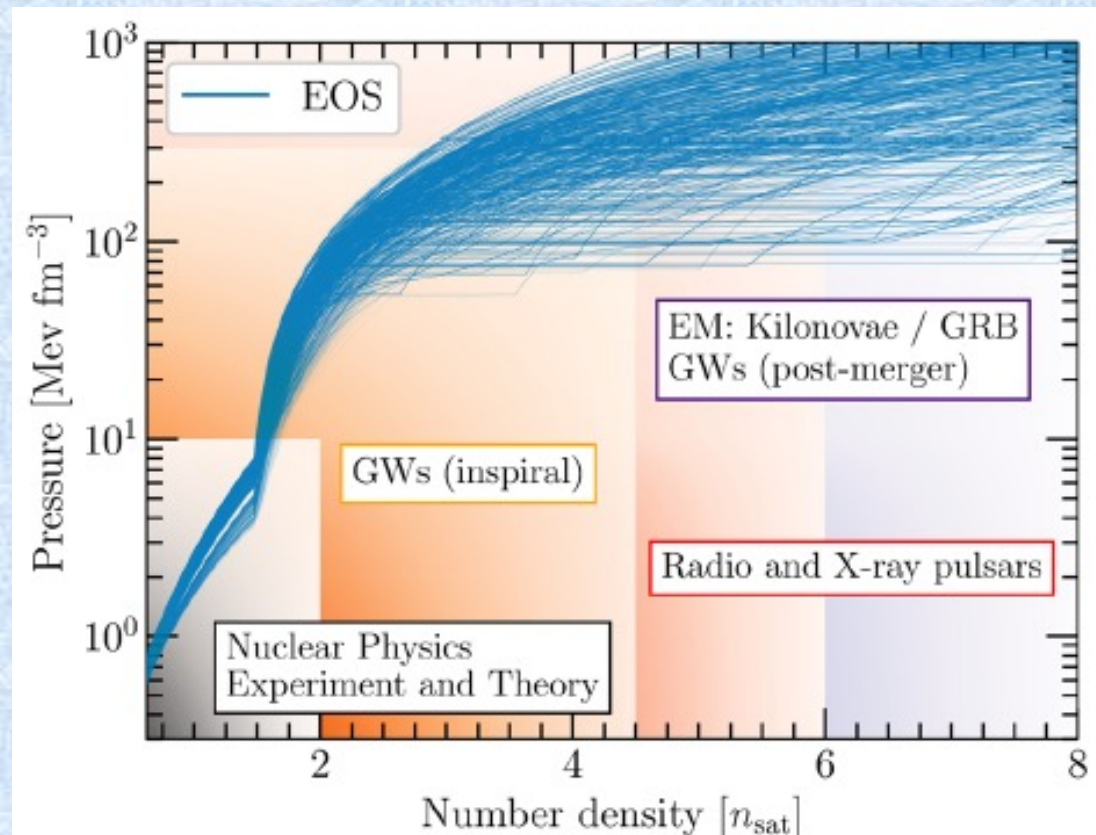
Nuclear data (e.g. nuclear experiments)  
Ab-initio calculations

$\rightarrow$  Low-density constraints

$$p(c_{\text{astro}}|\vec{X})$$

Causality + thermo stability  
Astro data:  $M_{\text{max}}$ , GW, NICER

$\rightarrow$  High-density constraints

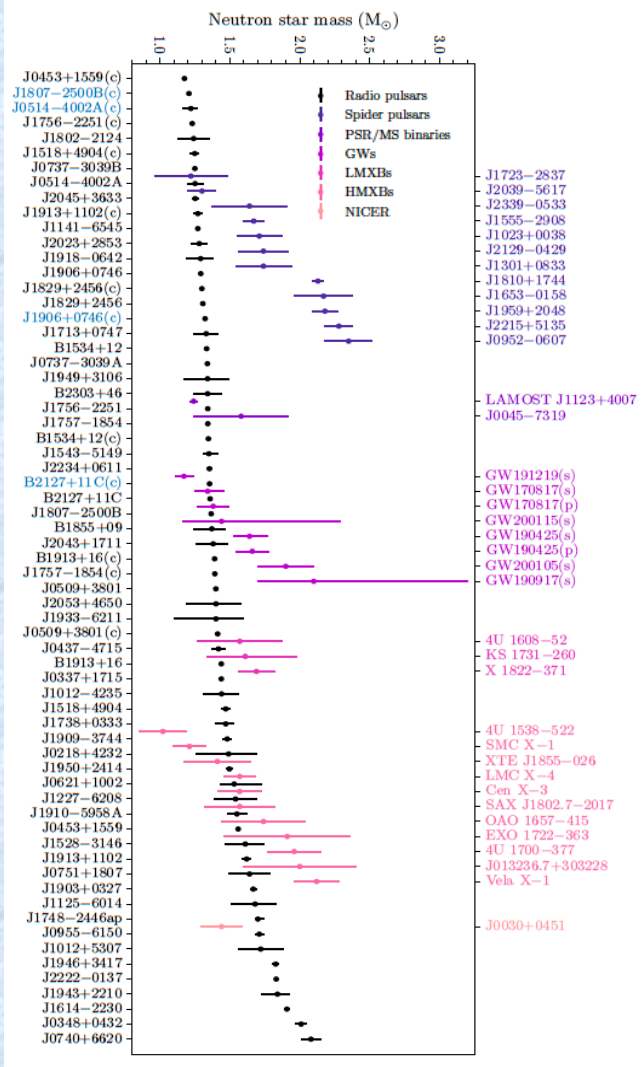


Pang et al., Nat. Comm. 14, 8352 (2023)

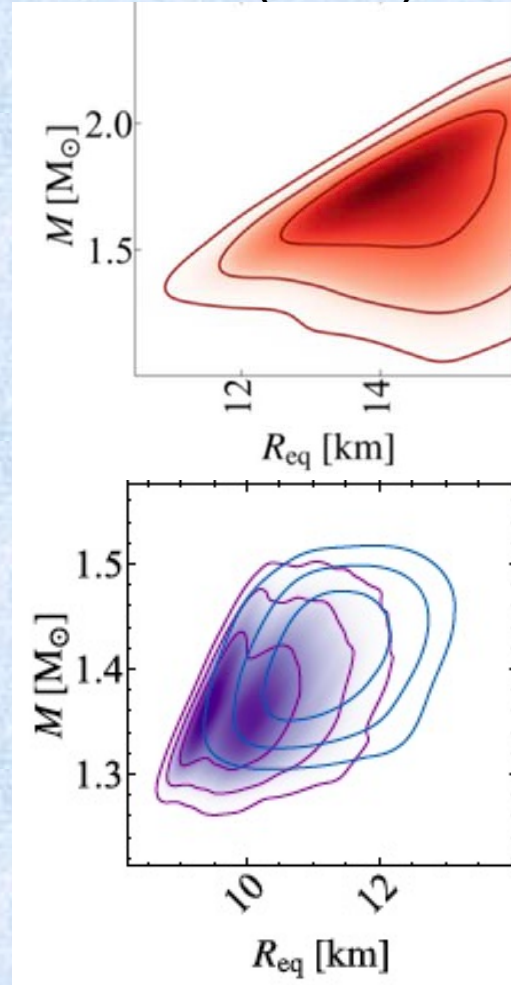


# Constraints from astrophysics

## MASSES

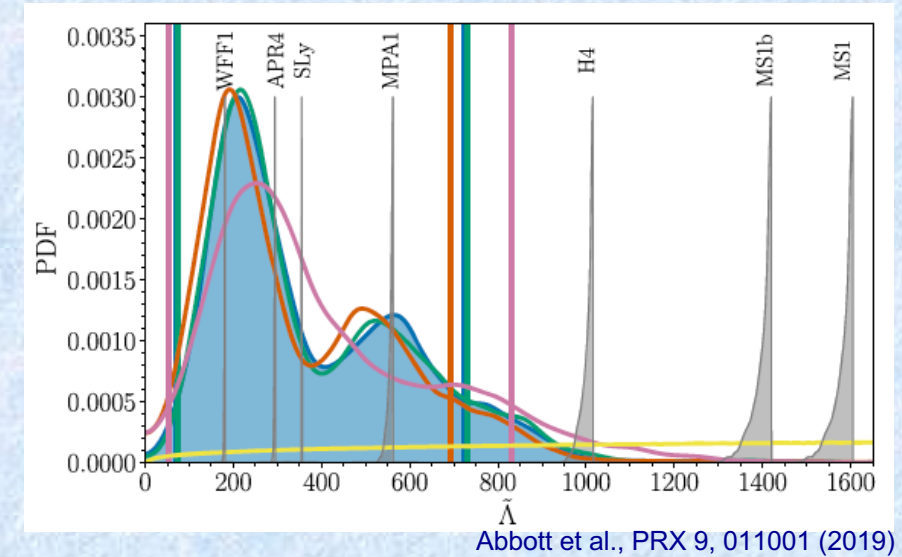


## RADII (NICER)



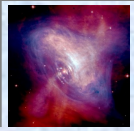
Choudhury et al., ApJL 971, L20 (2024);  
Vinciguerra et al., ApJ 961, 62 (2024)  
see also Miller et al., Riley et al., ApJL 2019, 2021

## TIDAL DEFORMABILITY



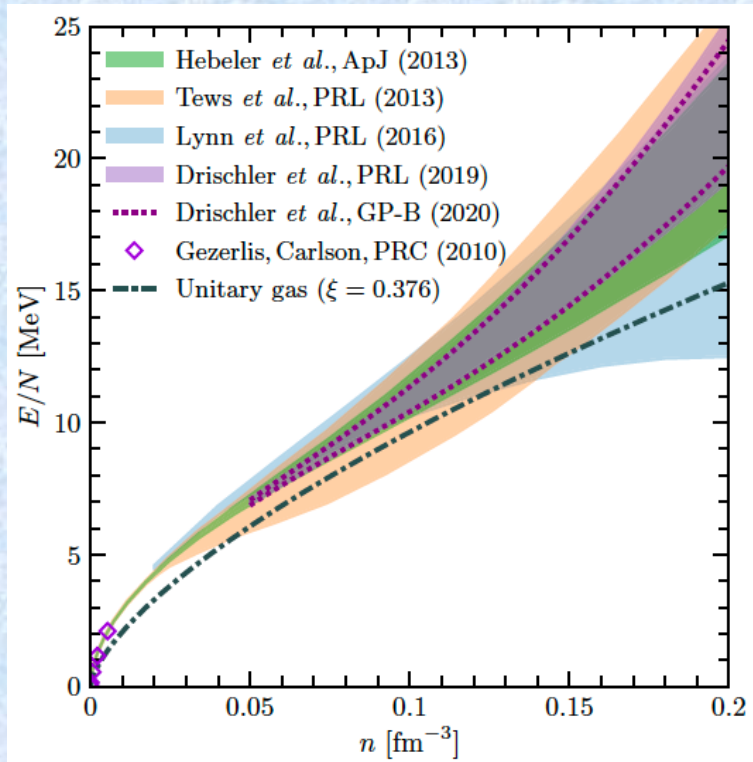
You et al., Nat. Astr. 9, 552 (2025);  
[https://www3.mpi-fr-bonn.mpg.de/staff/pfreire/NS\\_masses.html](https://www3.mpi-fr-bonn.mpg.de/staff/pfreire/NS_masses.html)





# Constraints from nuclear physics: theory

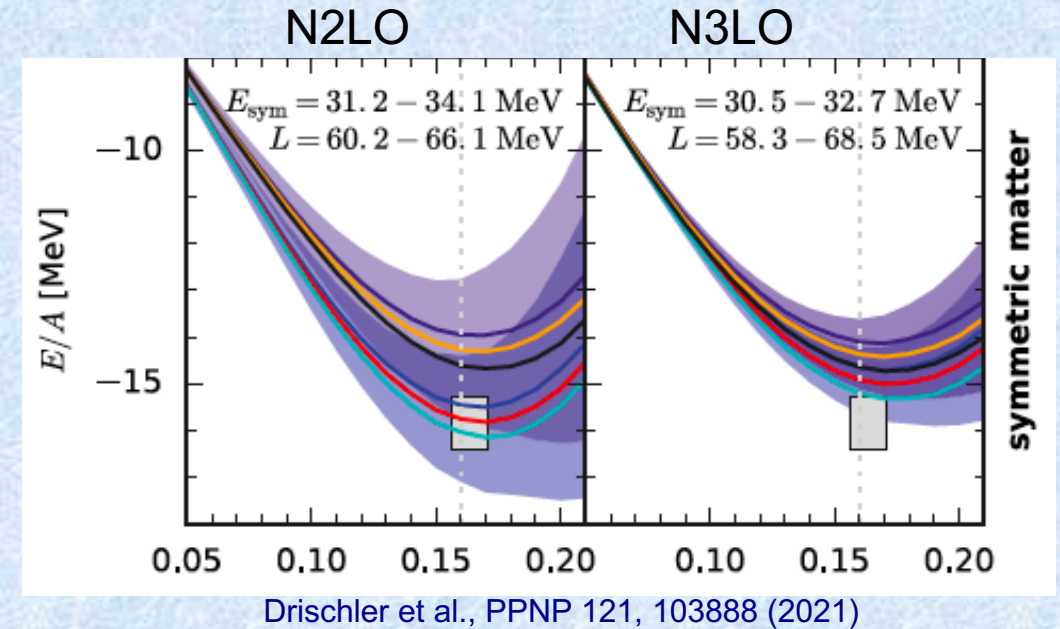
## PURE NEUTRON MATTER



Huth et al., PRC 103, 025803 (2021)

see L. Jokiniemi's lecture

## SYMMETRIC MATTER



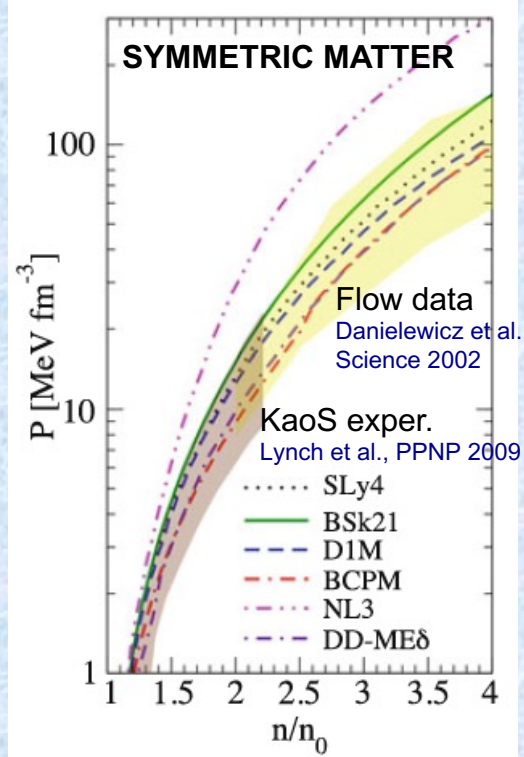
Drischler et al., PPNP 121, 103888 (2021)

- Different many-body methods → “uncertainty band” but reasonable agreement up to  $\sim n_{\text{sat}}$  ( $\sim$  low density)
- PNM calculations more reliable

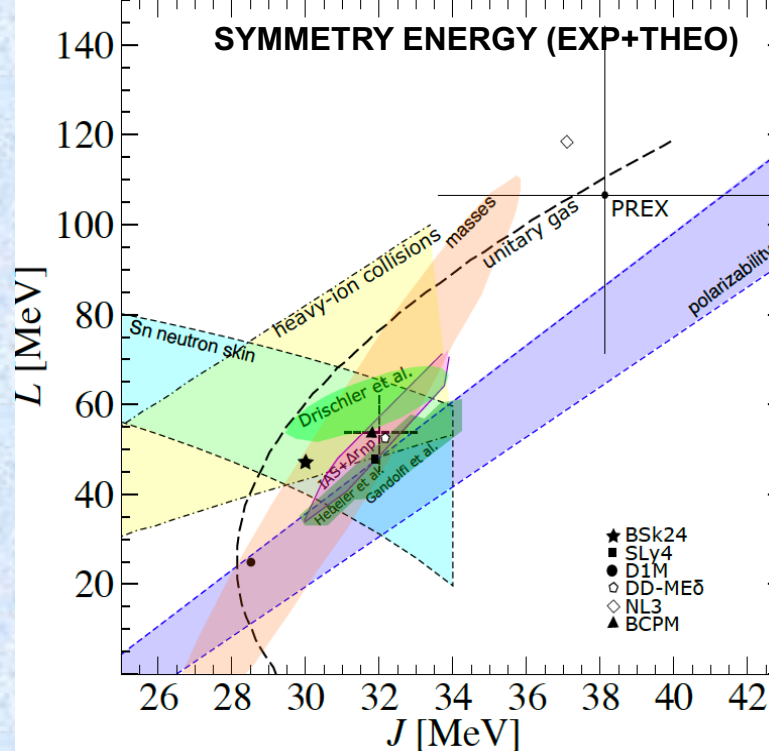
see also Oertel et al., Rev. Mod. Phys. 89, 015007 (2017), Burgio&Fantina, ASSL Springer 2018, Burgio et al., PPNP 120, 103879 (2021)



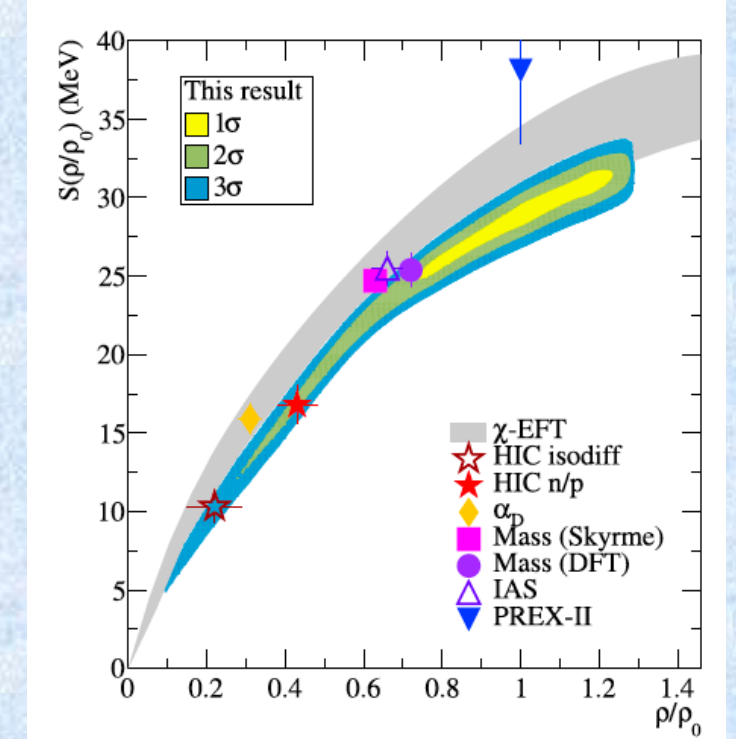
# Constraints from nuclear physics: experiments (1)



Burgio&Fantina, ASSL 457, 255 (2018)



Gulminelli&Fantina, Nucl. Phys. News 31, 9 (2021);  
Fantina&Gulminelli, J.Phys.Conf.Ser. 2586, 012112 (2023)



Ciampi et al., PLB 868, 139815 (2025)



“compilation” of results (“bands”) → filter for Bayes analysis

→ Constraints at “low” densities → low-order parameters  
N.B.: model dependence of constraints





# Constraints from nuclear physics: experiments (2)

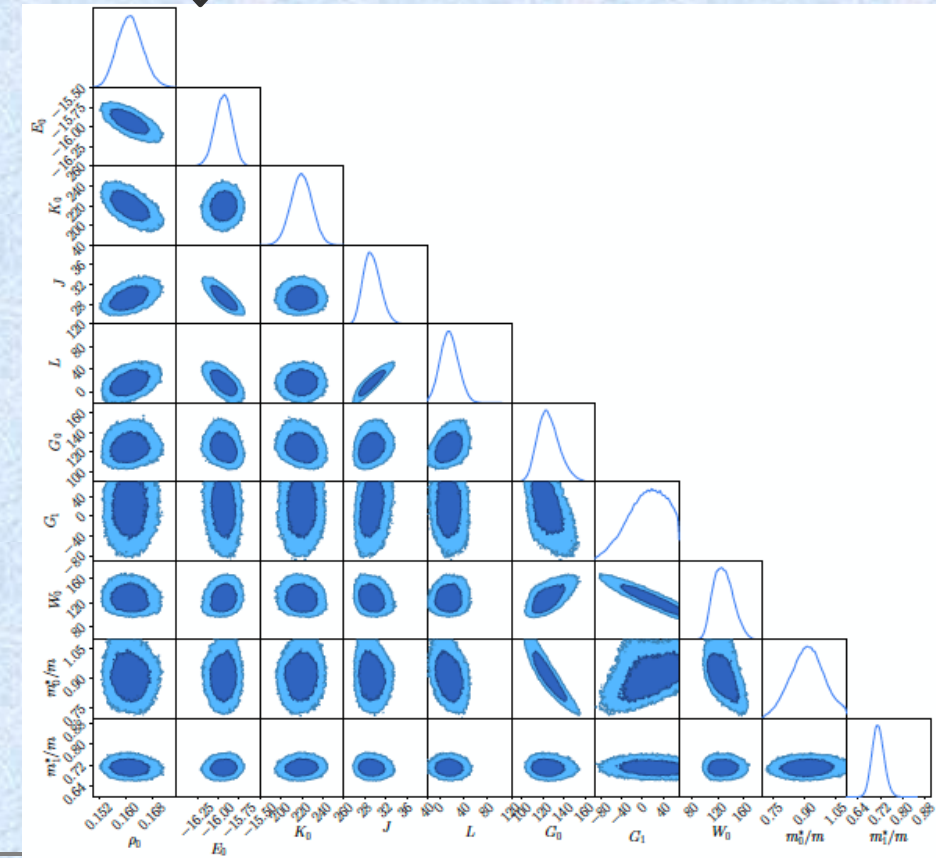
| Ground-state properties |                                        |                                    |                              |
|-------------------------|----------------------------------------|------------------------------------|------------------------------|
|                         | $B.E.$ [MeV]                           | $R_{\text{ch}}$ [fm]               | $\Delta E_{\text{SO}}$ [MeV] |
| $^{208}\text{Pb}$       | $1636.4 \pm 2.0^*$                     | $5.50 \pm 0.05^*$                  | $2.02 \pm 0.50^*$            |
| $^{48}\text{Ca}$        | $416.0 \pm 2.0^*$                      | $3.48 \pm 0.05^*$                  | $1.72 \pm 0.50^*$            |
| $^{40}\text{Ca}$        | $342.1 \pm 2.0^*$                      | $3.48 \pm 0.05^*$                  | -                            |
| $^{56}\text{Ni}$        | $484.0 \pm 2.0^*$                      | -                                  | -                            |
| $^{68}\text{Ni}$        | $590.4 \pm 2.0^*$                      | -                                  | -                            |
| $^{100}\text{Sn}$       | $825.2 \pm 2.0^*$                      | -                                  | -                            |
| $^{132}\text{Sn}$       | $1102.8 \pm 2.0^*$                     | $4.71 \pm 0.05^*$                  | -                            |
| $^{90}\text{Zr}$        | $783.9 \pm 2.0^*$                      | $4.27 \pm 0.05^*$                  | -                            |
| Isoscalar resonances    |                                        |                                    |                              |
|                         | $E_{\text{GMR}}^{\text{IS}}$ [MeV]     | $E_{\text{GQR}}^{\text{IS}}$ [MeV] |                              |
| $^{208}\text{Pb}$       | $13.5 \pm 0.5^*$                       | $10.9 \pm 0.5^*$                   |                              |
| $^{90}\text{Zr}$        | $17.7 \pm 0.5^*$                       | -                                  |                              |
| Isovector properties    |                                        |                                    |                              |
|                         | $\alpha_{\text{D}}$ [fm <sup>3</sup> ] | $m(1)$ [MeV fm <sup>2</sup> ]      | $A_{\text{PV}}$ (ppb)        |
| $^{208}\text{Pb}$       | $19.60 \pm 0.60$                       | $961 \pm 22$                       | $550 \pm 18$                 |
| $^{48}\text{Ca}$        | $2.07 \pm 0.22$                        | -                                  | $2668 \pm 113$               |

Klausner et al., PRC 111, 014311 (2025)



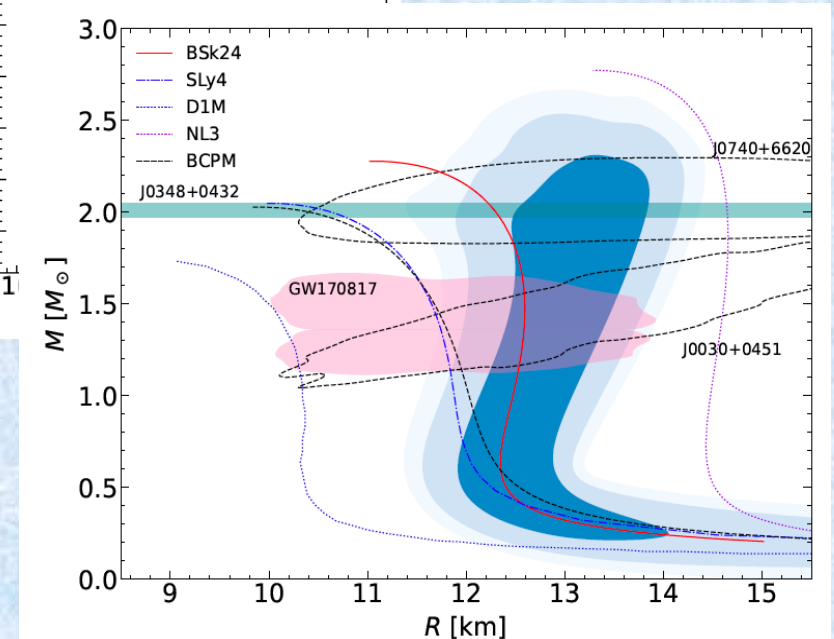
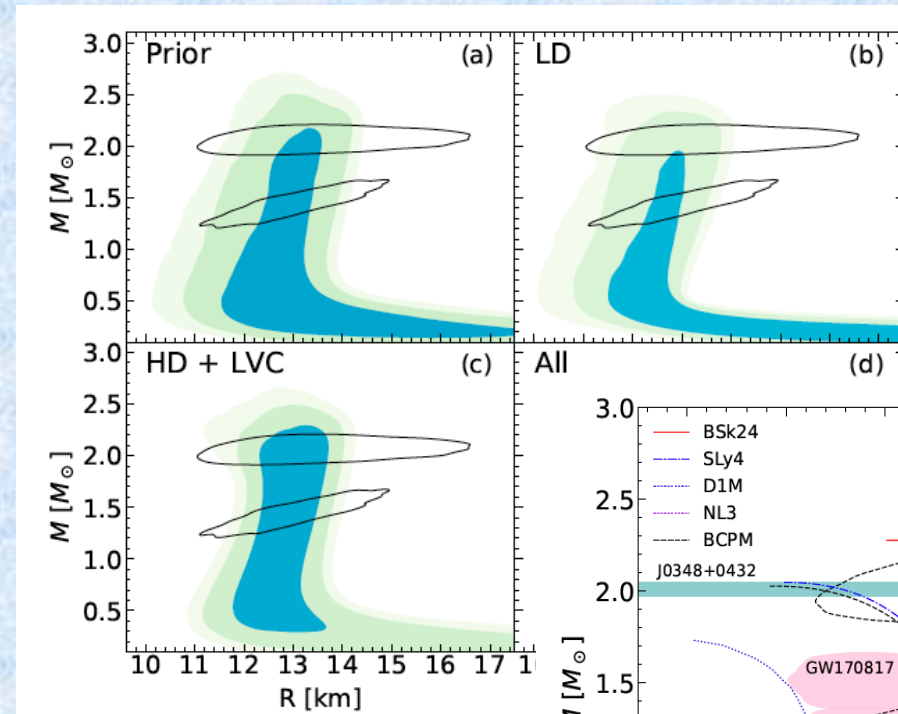
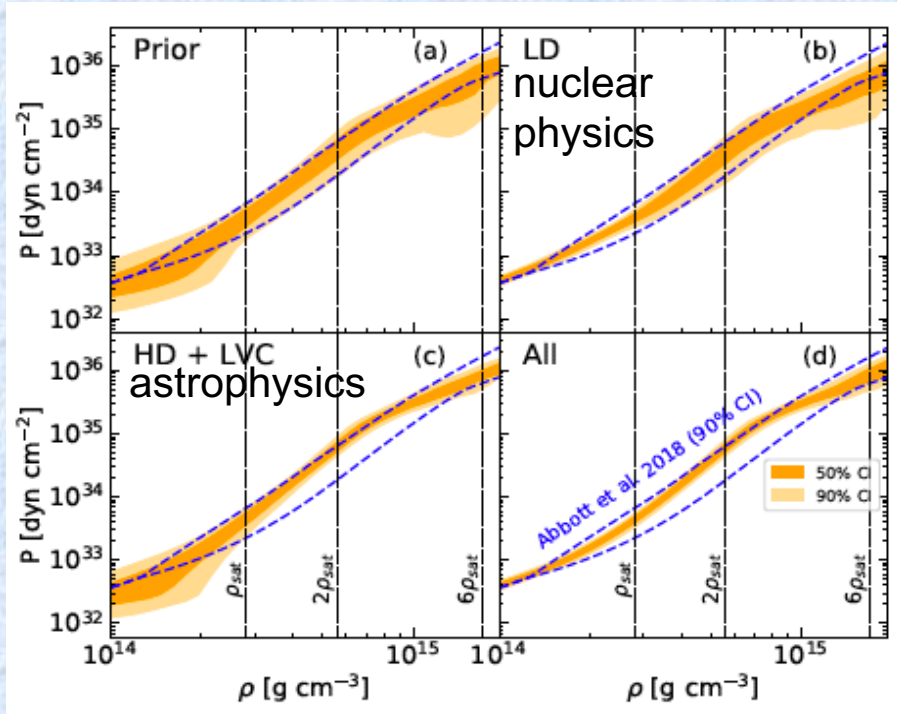
observables directly included in model fit  
 $\rightarrow$  nuc.-phy. informed prior

parameter fit  
 through emulator (e.g. MADAI package)





# Predictions informed by nuclear physics (1)



Dinh Thi et al., Universe 7, 373 (2021); Dinh Thi et al., A&A 654, A114 (2021)

- Nuclear constraints act up to 1-2  $n_{\text{sat}}$
- Some EoSs can be discarded
- Nucleonic hypothesis (still) compatible with observations  
→ observations not yet constraining enough!

see also Lim&Holt, EPJA 2019, Malik et al., ApJ 2022

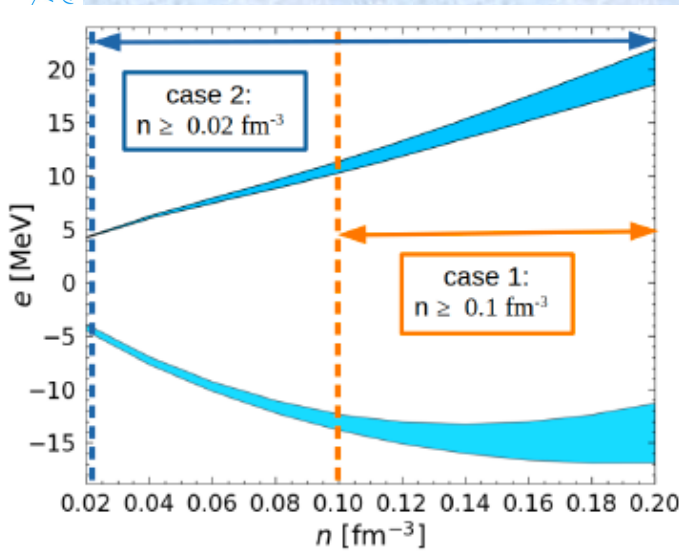
Gulminelli & Fantina, NPN 31, 9 (2021);  
Fantina & Gulminelli, J.Phys. Conf.Ser. 2596, 012112 (2023)





# Predictions informed by nuclear physics (2)

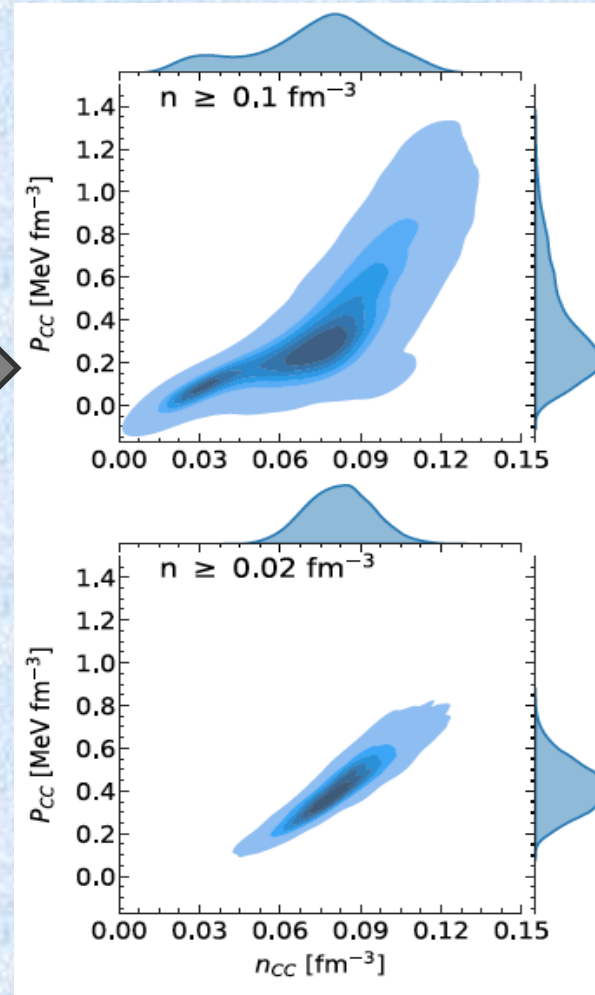
## AB INITIO CONSTRAINTS



Dinh Thi, talk (2021); Drischler et al. PNM calculations

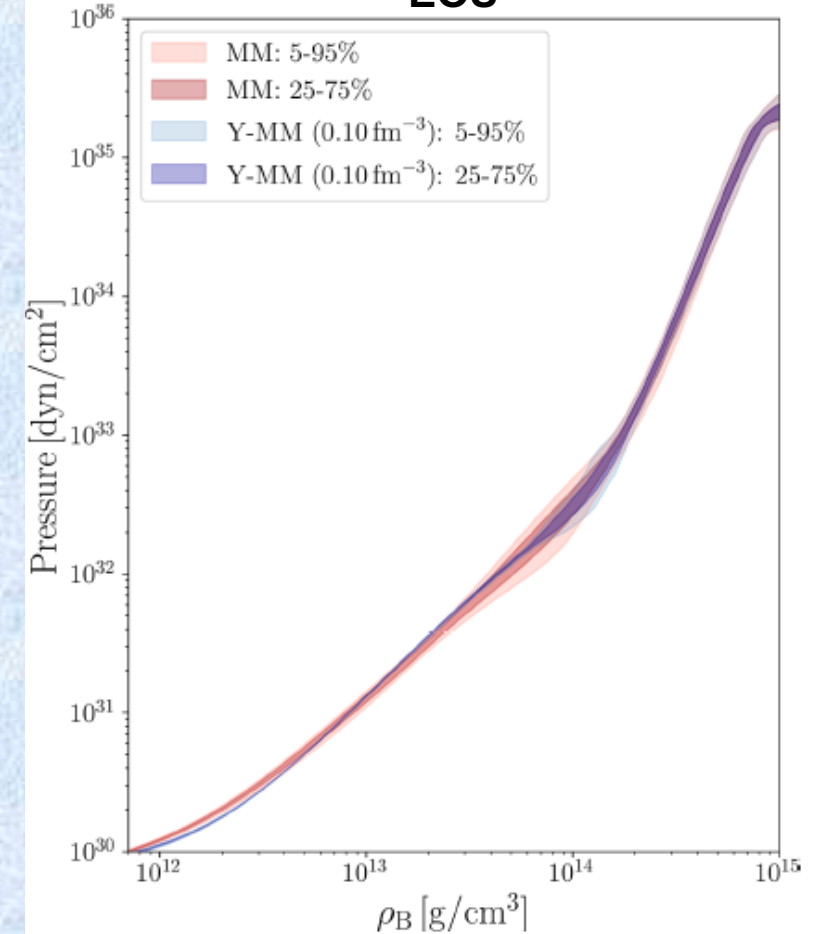
➤ Importance of very low-density EoS

## CC TRANSITION



Dinh Thi et al., A&A 654, A114 (2021);  
EPJA 57, 296 (2021)

## EOS

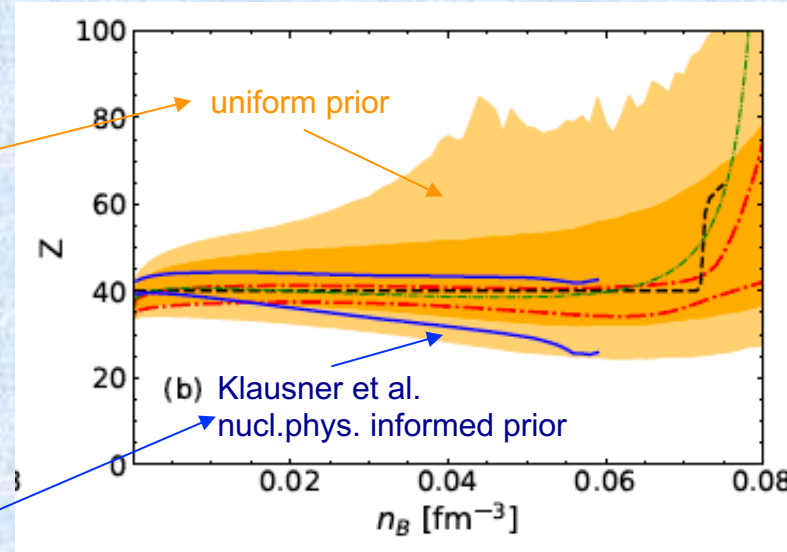
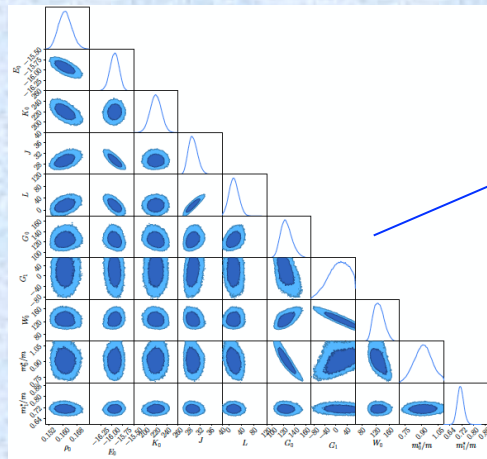
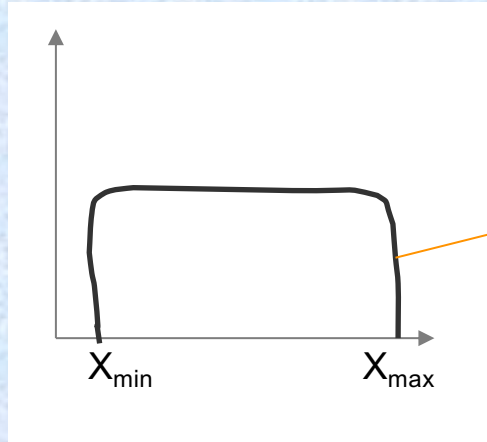


Burrello et al., PRC 112, 035802 (2025)  
inclusion of ab-initio inspired functional in EDF

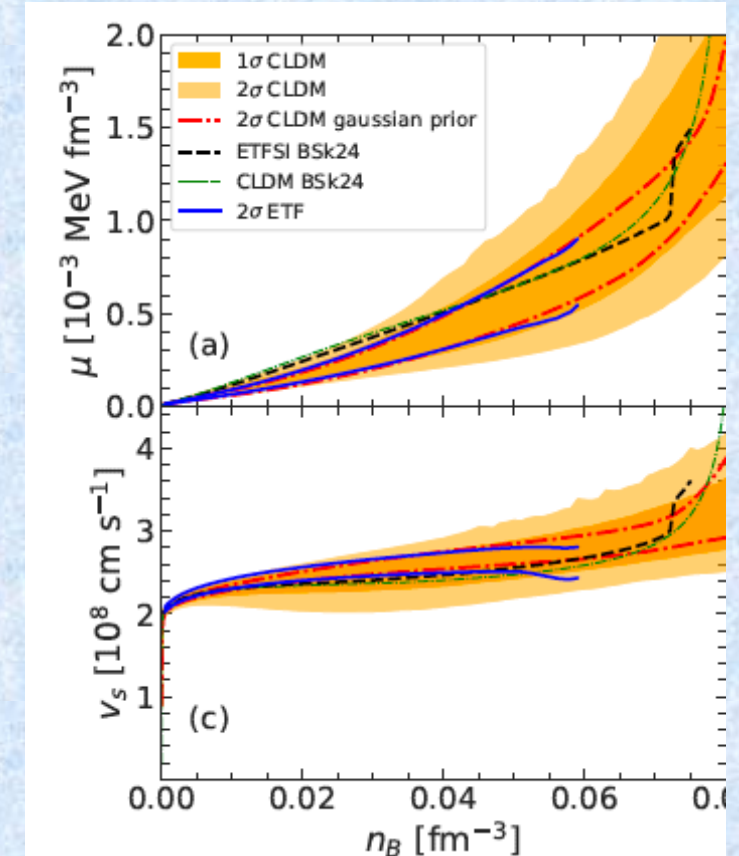


# Predictions informed by nuclear physics (3)

## NUCL. PHYS. EXP. & COMPOSITION / ELASTIC PROPERTIES OF CRUST



Diverrès et al., A&A 712, A160 (2026)



- inclusion of nuclear phys. exp. observables constrains NS-crust properties  
→ constraints on elastic ( / transport) properties





# Which nuclear parameters matter ?

|                   |           |           |           |           |           |           |           |           |           |           |            |       |               |         |
|-------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|------------|-------|---------------|---------|
| $n_{CC}$          | -0.09     | -0.05     | 0.15      | -0.11     | 0.00      | -0.34     | -0.69     | -0.22     | 0.55      | -0.16     | 0.10       | 0.43  | -0.17         | -0.10   |
| $P_{CC}$          | -0.05     | -0.02     | 0.13      | -0.03     | -0.03     | -0.07     | -0.62     | -0.38     | 0.41      | -0.04     | 0.05       | 0.12  | -0.02         | -0.05   |
| $R_{crust}^{1.4}$ | 0.09      | -0.18     | 0.19      | 0.15      | 0.09      | -0.01     | -0.14     | 0.20      | 0.64      | 0.01      | -0.09      | -0.08 | 0.04          | 0.08    |
| $R_{1.4}$         | 0.15      | -0.22     | 0.13      | 0.20      | 0.14      | -0.03     | 0.41      | 0.65      | 0.42      | 0.06      | -0.14      | -0.12 | 0.02          | 0.13    |
| $\Lambda_{1.4}$   | 0.16      | -0.24     | 0.16      | 0.23      | 0.14      | -0.17     | 0.26      | 0.60      | 0.50      | 0.10      | -0.15      | 0.00  | -0.01         | 0.14    |
| $\chi_p^{1.4}$    | -0.24     | 0.32      | -0.41     | -0.50     | -0.09     | 0.39      | 0.60      | 0.75      | 0.58      | 0.04      | 0.22       | -0.15 | -0.06         | -0.20   |
| $R_{crust}^{2.0}$ | 0.15      | -0.23     | 0.27      | 0.31      | 0.19      | -0.16     | -0.26     | 0.08      | 0.53      | 0.12      | -0.14      | 0.00  | 0.05          | 0.14    |
| $R_{2.0}$         | 0.21      | -0.27     | 0.24      | 0.38      | 0.25      | -0.19     | 0.10      | 0.36      | 0.37      | 0.18      | -0.19      | -0.01 | 0.04          | 0.18    |
| $\Lambda_{2.0}$   | 0.22      | -0.29     | 0.27      | 0.42      | 0.27      | -0.28     | -0.03     | 0.27      | 0.36      | 0.21      | -0.20      | 0.05  | 0.03          | 0.19    |
| $\chi_p^{2.0}$    | -0.20     | 0.19      | -0.39     | -0.57     | -0.29     | 0.20      | 0.31      | 0.51      | 0.62      | 0.25      | 0.19       | -0.00 | -0.12         | -0.18   |
|                   | $E_{sat}$ | $n_{sat}$ | $K_{sat}$ | $Q_{sat}$ | $Z_{sat}$ | $E_{sym}$ | $L_{sym}$ | $K_{sym}$ | $Q_{sym}$ | $Z_{sym}$ | $\sigma_0$ | $b_s$ | $\sigma_{0c}$ | $\beta$ |

Dinh Thi et al., A&A 654, A114 (2021); EPJA 57, 296 (2021); Universe 7, 00373 (2021)

- importance of higher order parameters (isovector sector) & surface properties
- typically NS observations have little effect on low-order NEP → nucl. phys. constraints

N.B.: Many works within Bayesian analysis trying to constrain NEP

see also Beznogov & Raduta, PRC 107, 045803 (2023); Ghosh et al., EPJA 58, 37 (2022); Char et al., PRD 108, 103405 (2023); Imam et al., PRD 109, 103025 (2024); Zhu et al., ApJ 943, 163 (2023); Huang et al., arXiv:2303.17518, ...



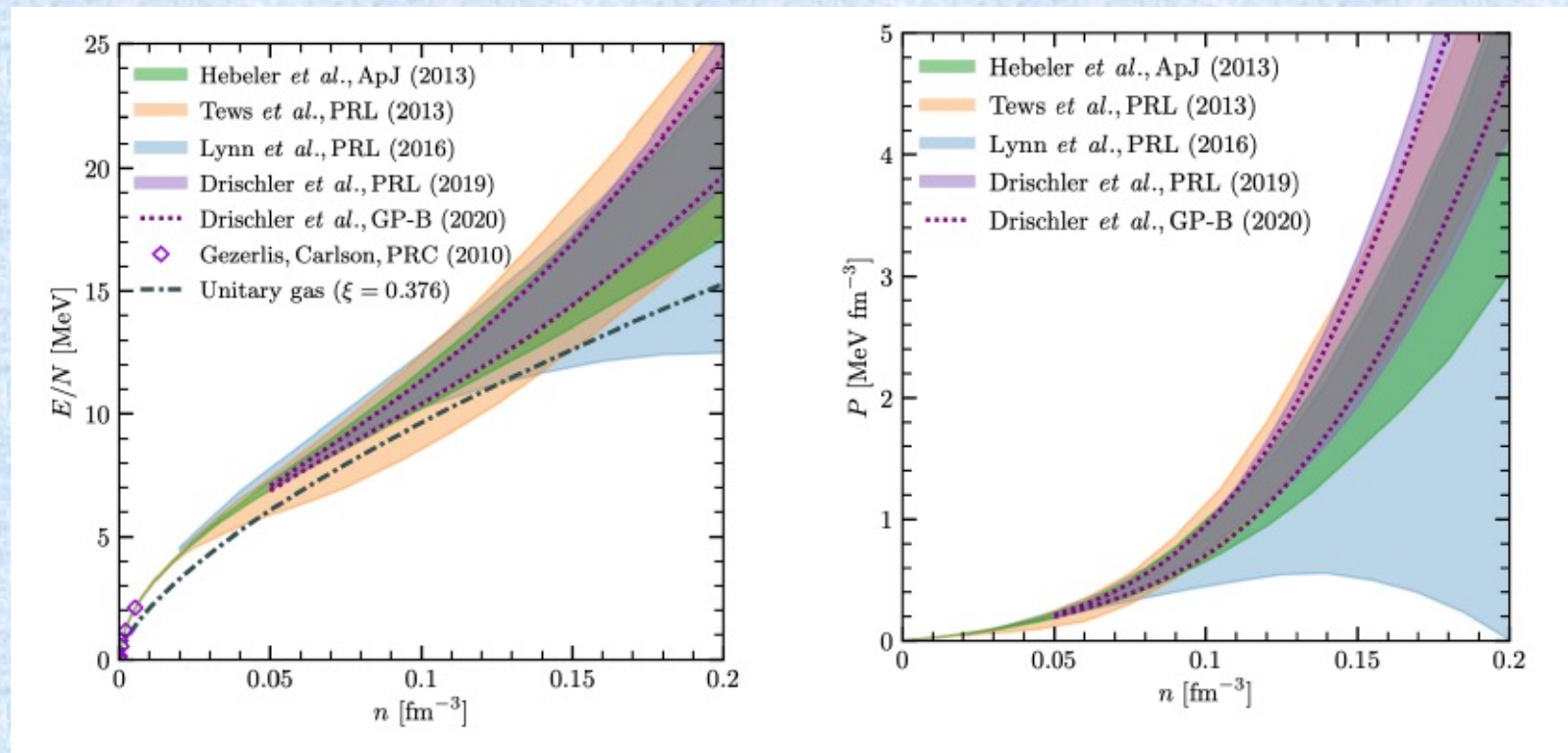
# How to discriminate models? (theory)

❖ **Nuclear physics (theory + experiments)** → information up to  $\sim 1.5 - 2 n_{\text{sat}}$

➤ *Ab-initio prediction of derived quantities (not only  $E/A$ , but also  $P$ , etc.)*

→ propagation of uncertainties

→ constraints on more phenomenological functionals



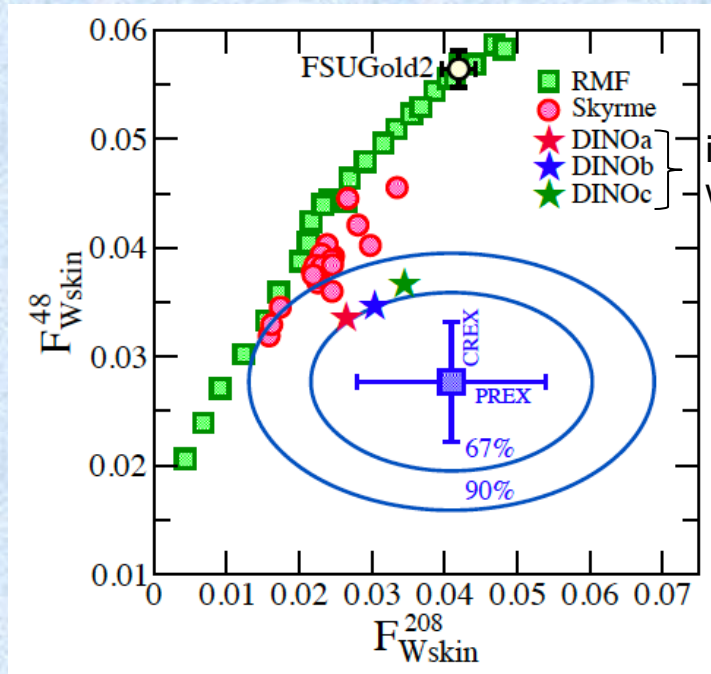
Hebel et al., talk @MICRA2023; Huth et al., Nature 606, 276 (2022)



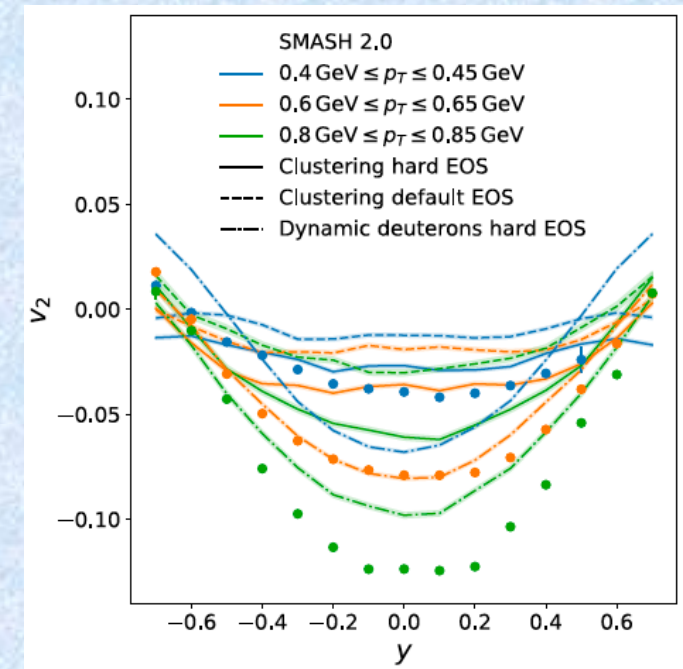


# How to discriminate models? (nuc. phys. exp)

- ❖ **Nuclear physics (theory + experiments)** → information up to  $\sim 1.5 - 2 n_{\text{sat}}$ 
  - “PREX/CREX dilemma” vs reduced error bar in (neutron skin) measurements vs other observables?
    - constraints on low-order parameters in isospin sector
  - constraints at high density (e.g. HADES, FOPI, Asy-EoS → transport model vs data, systematics?)
    - constraints on isoscalar / isovector sector (at  $\sim 2n_{\text{sat}}$ )



Reed et al., PRC 109, 035803 (2024)  
see also Kunjipurayil et al., arXiv:2503.07405



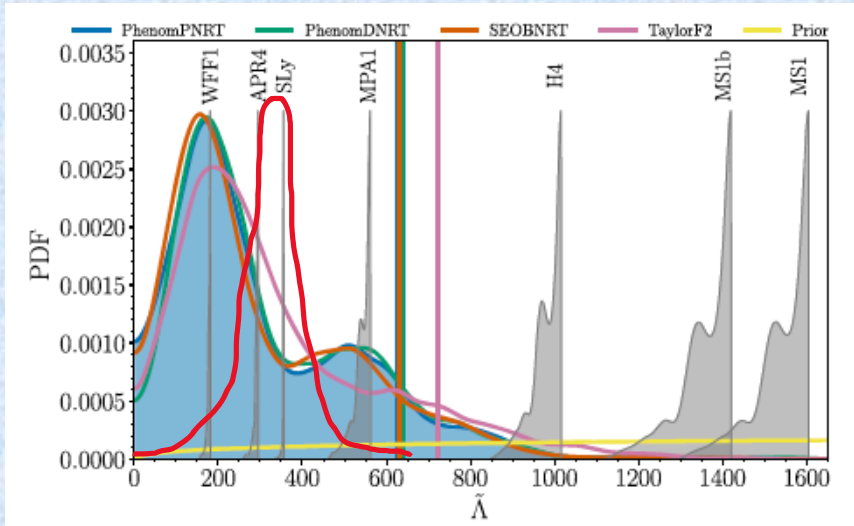
Mohs et al., PRC 105, 034906 (2022)



# How to discriminate models? (astro1)

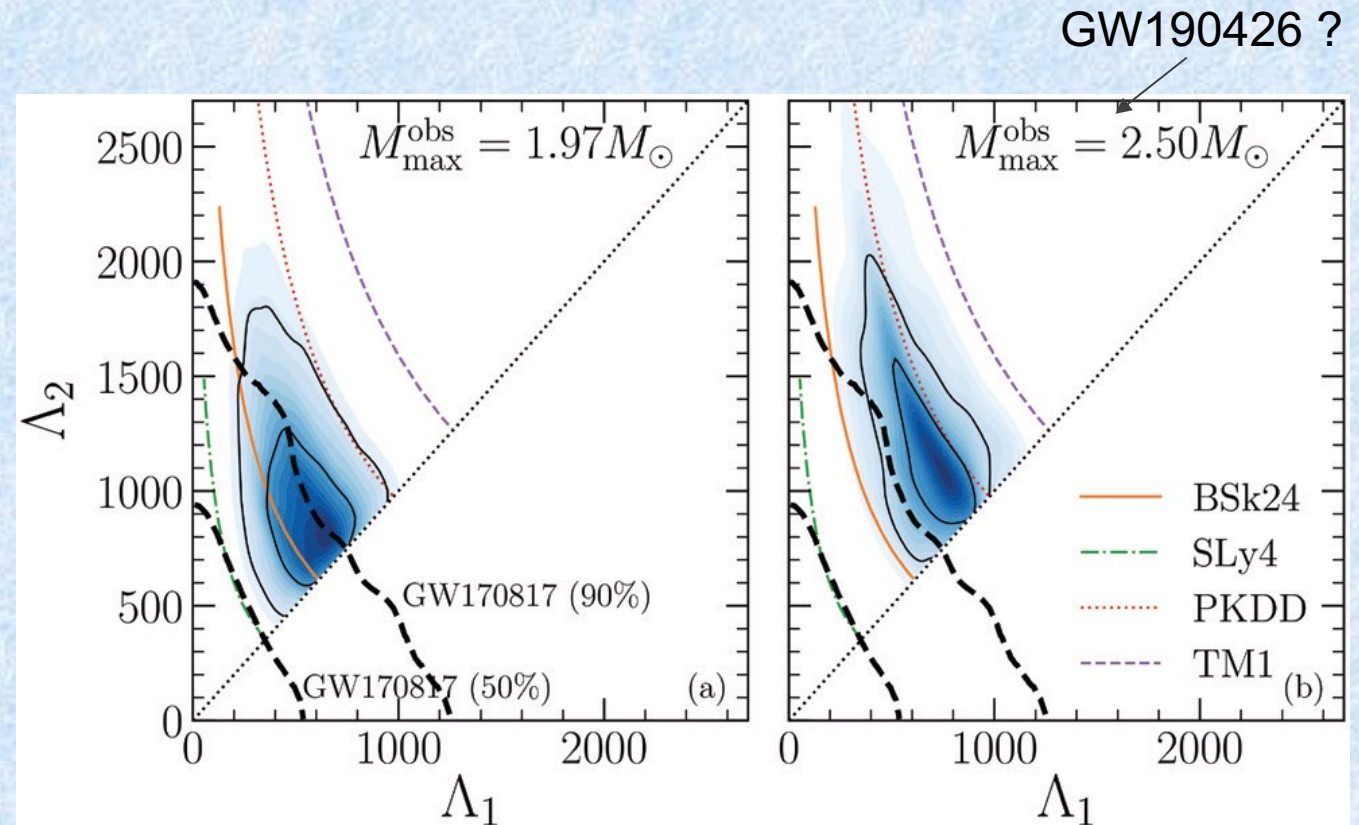
- more precise data (e.g.  $M$ ,  $R$ ,  $\Lambda$ , ... )
- “smoking gun” observation!

hypothetical observation



Abbott et al., Phys. Rev. X 9, 011001 (2019)

current constraint:  $\tilde{\Lambda} \sim [0-800]$



Gulminelli & Fantina, Nucl. Phys. News 31, 2 (2021); T. Carreau, PhD Thesis (2020)

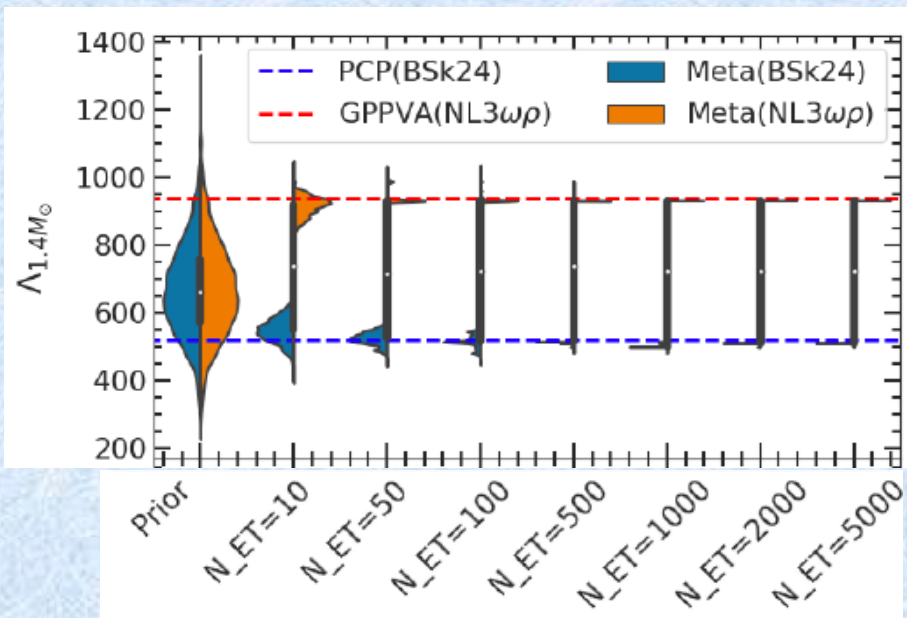
- posterior with nucleonic matter compatible with observations
- but: if  $M_{\max} \sim 2.5 M_{\text{sun}} \rightarrow$  challenge for nucleonic hypothesis !  $\rightarrow$  “exotica” ?
- $\rightarrow$  Nucleonic hp can be used as null hp!



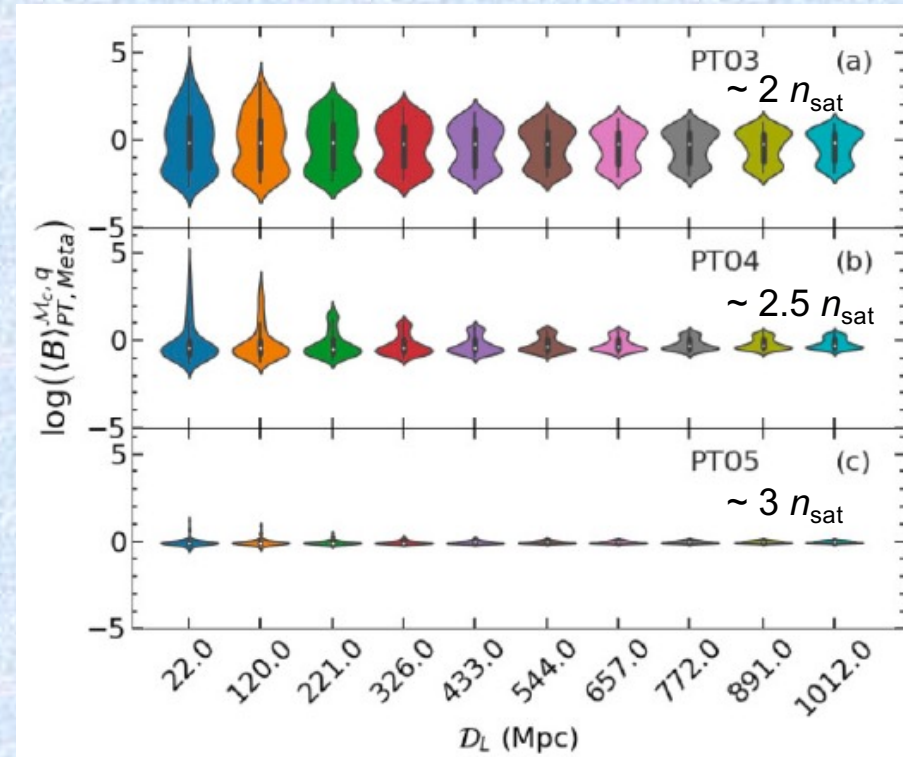


# How to discriminate models? (astro2)

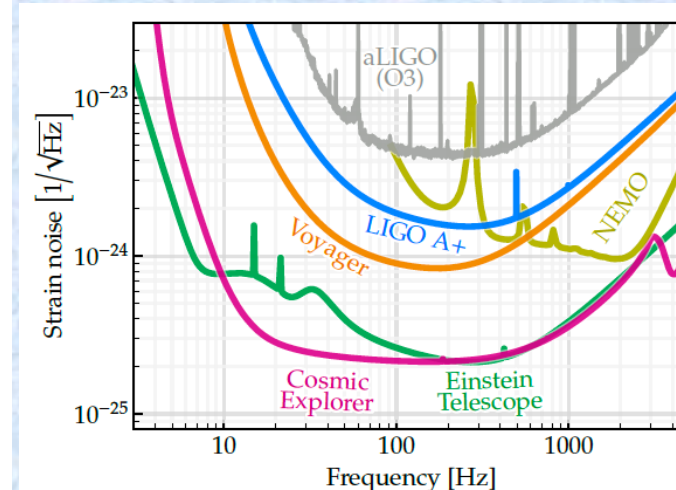
- more numerous (and precise) data (e.g.  $M$ ,  $R$ ,  $\Lambda$ , ... )
- more sensitive detectors → new generation (ET, CE)  
→ possible identification of PT if close detection and early PT



Iacovelli et al., PRD 2023



Mondal et al., MNRAS 524, 3464 (2023)



Hall, Galaxies 10, 90 (2022)

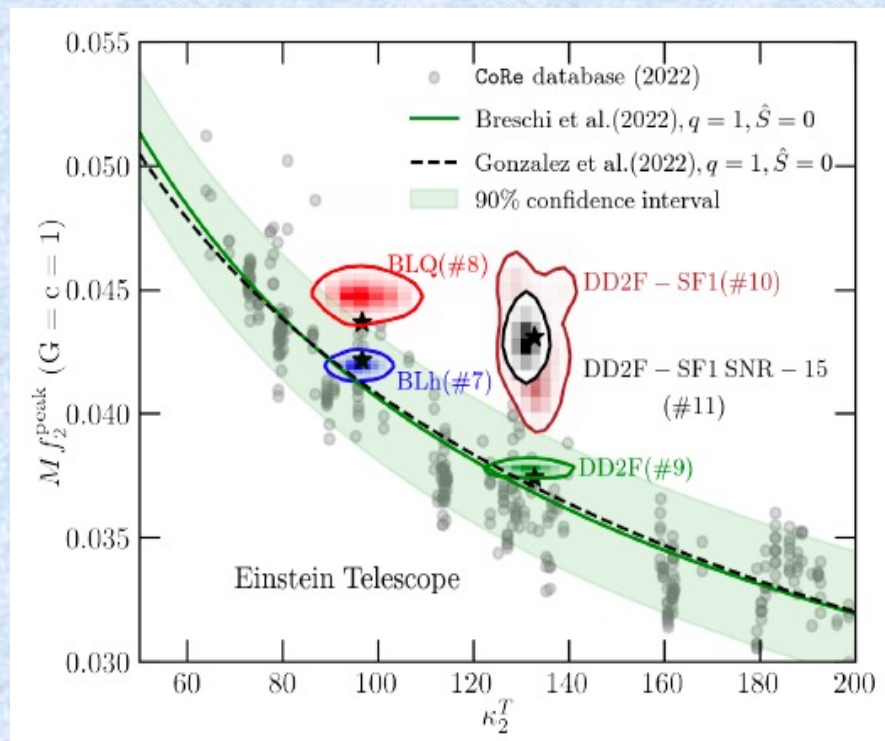
but: if phase transition “late” → hard to disentangle from other uncertainties



# How to discriminate models? (astro 3)

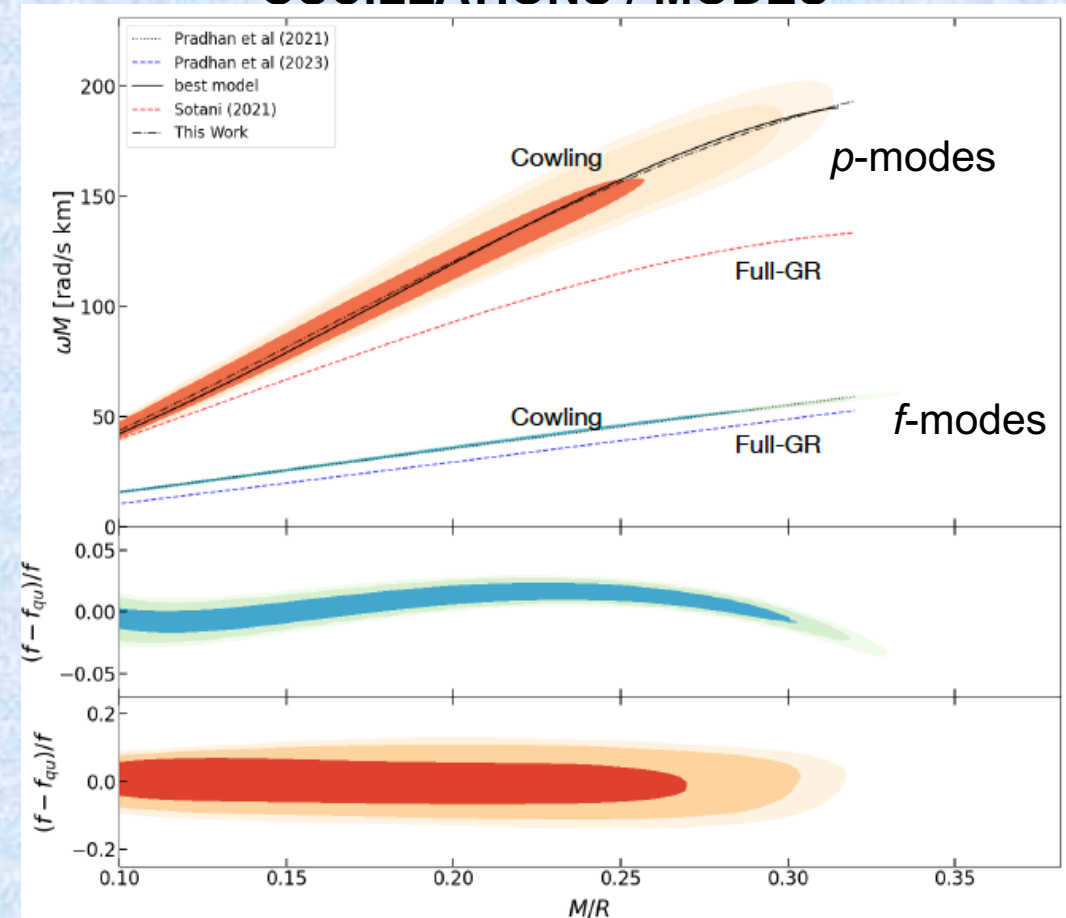
- more sensitive detectors → new generation (ET, CE)
  - possible identification of properties from (quasi-)universal relations (or discrepancy from them)

## PHASE TRANSITION → POST-MERGER



Prakash et al., PRD 109, 103008 (2024); see also e.g. Bauswein et al. 2019, Kochanovksi et al. 2025

## OSCILLATIONS / MODES



Montefusco et al., A&A 694, A150 (2025)





# Compact objects: sites for (hot) dense matter

| <i>Object</i>                       | <i>Thermo</i>                                                                                      | <i>Equilibrium</i>              | <i>Micro inputs</i>                                |
|-------------------------------------|----------------------------------------------------------------------------------------------------|---------------------------------|----------------------------------------------------|
| <b>CCSN<br/>BNS (post-)mergers</b>  | $n_B \in (0, \sim 10n_{\text{sat}})$<br>$T \in (0, \sim 100) \text{ MeV}$<br>$Y_e \in (0, 0.6)$    | strong                          | $P(n_B, T, Y_e)$<br>(weak), $\nu$ rates<br>→ compo |
| <b>PNS</b>                          | $n_B \in (0, \sim 10n_{\text{sat}})$<br>$T \in (0, \sim 20) \text{ MeV}$<br>( $Y_e \in (0, 0.5)$ ) | strong<br>(+weak)               | $P(n_B, T, (Y_e))$<br>(+ rates<br>→ composition)   |
| <b>NS (old)<br/>binary inspiral</b> | $n_B \in (0, \sim 10n_{\text{sat}})$<br>$T = 0$                                                    | strong+weak<br>→ $\beta$ equil. | $P(n_B)$                                           |

Memo:  $1 \text{ MeV} \leftrightarrow \sim 10^{10} \text{ K}$   
 $10^{-4} \text{ fm}^{-3} \leftrightarrow \sim 10^{11} \text{ g/cm}^3$ ,  $n_{\text{sat}} \sim 0.16 \text{ fm}^{-3} \rightarrow \sim 2.7 \cdot 10^{14} \text{ g/cm}^3$



# CCSN / NS mergers → hydro + rates

## ■ Hydro eqs (in Newtonian and spherical symmetry)

(Mihalas & Mihalas, "Foundation of radiation hydrodynamics")

$$\begin{aligned} \frac{D\rho}{Dt} + \rho \frac{1}{r^2} \frac{\partial(r^2 v)}{\partial r} &= 0 && \text{Mass conserv.} \\ \frac{Dv}{Dt} &= -4\pi r^2 \frac{DP}{Dm} - \frac{Gm}{r^2} + \frac{1}{\rho} Q_M^0 && \text{Momentum conserv.} \\ \frac{D\epsilon}{Dt} + P \frac{D(1/\rho)}{Dt} &= Q_{th} + \frac{1}{\rho} Q_E^0 && \text{Energy cons.} \\ \frac{DY_e}{Dt} &= R_{Y_e}^0 \end{aligned}$$

reactions (pink arrow pointing to  $Q_{th}$ )

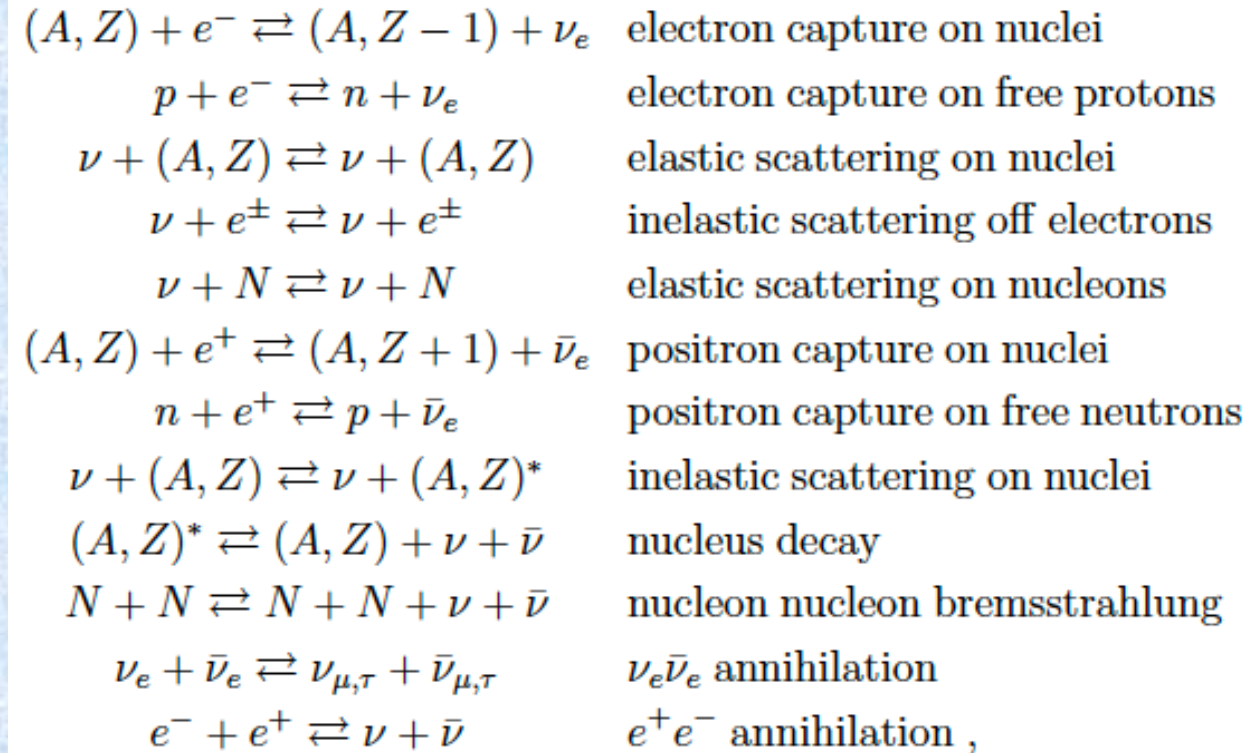
EC rates (green arrow pointing to  $R_{Y_e}^0$ )

$\nu$  energy & momentum (purple arrow pointing to  $Q_M^0$  and  $Q_E^0$ )

**NB:** SN / merger needs 3D after bounce !  
GR included in simulations!

## ■ Weak reactions

(e.g. Langanke & Martinez-Pinedo, Rev.Mod.Phys. 2003)



- Need of **EoS + matter composition, rates**
- Neutrino transport (numerically challenging)





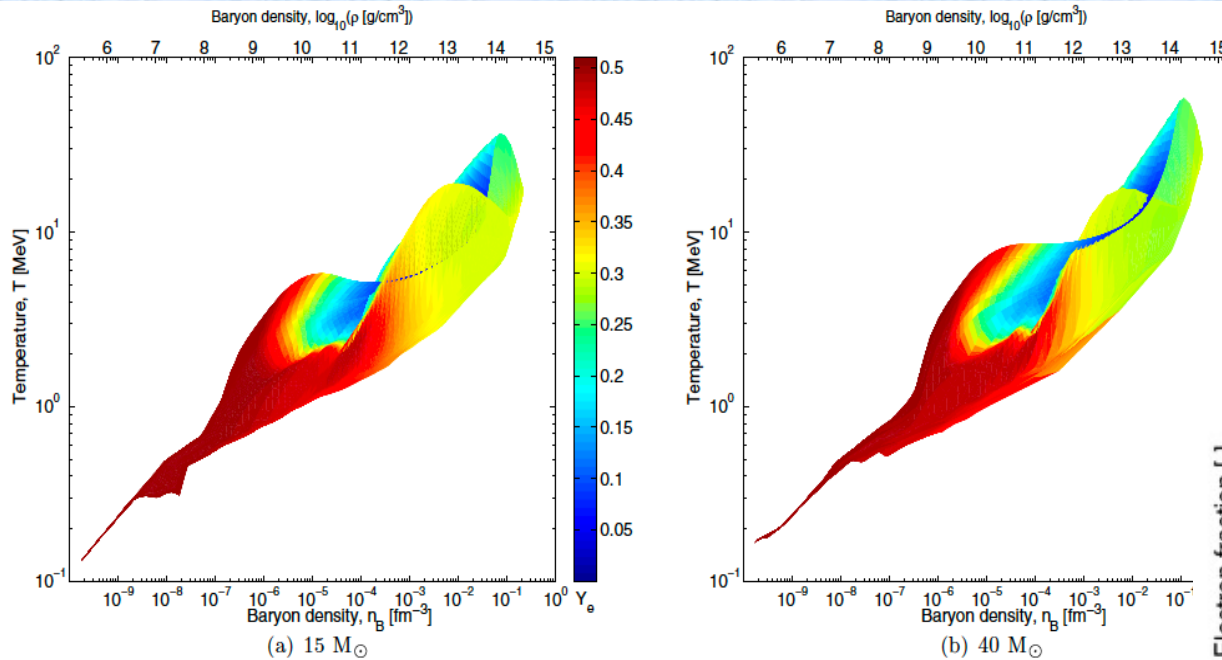
# EoS at finite temperature: generalities (1)

- Most general EoS:  $P(\{n_{\text{cons}}\}, T)$ , with conserved densities (charges)  $n_B, n_L, n_Q, n_s$
- For CCSN / BNS merger often  $P(n_B, Y_e = Y_Q, T) \rightarrow$  **general purpose EoS**

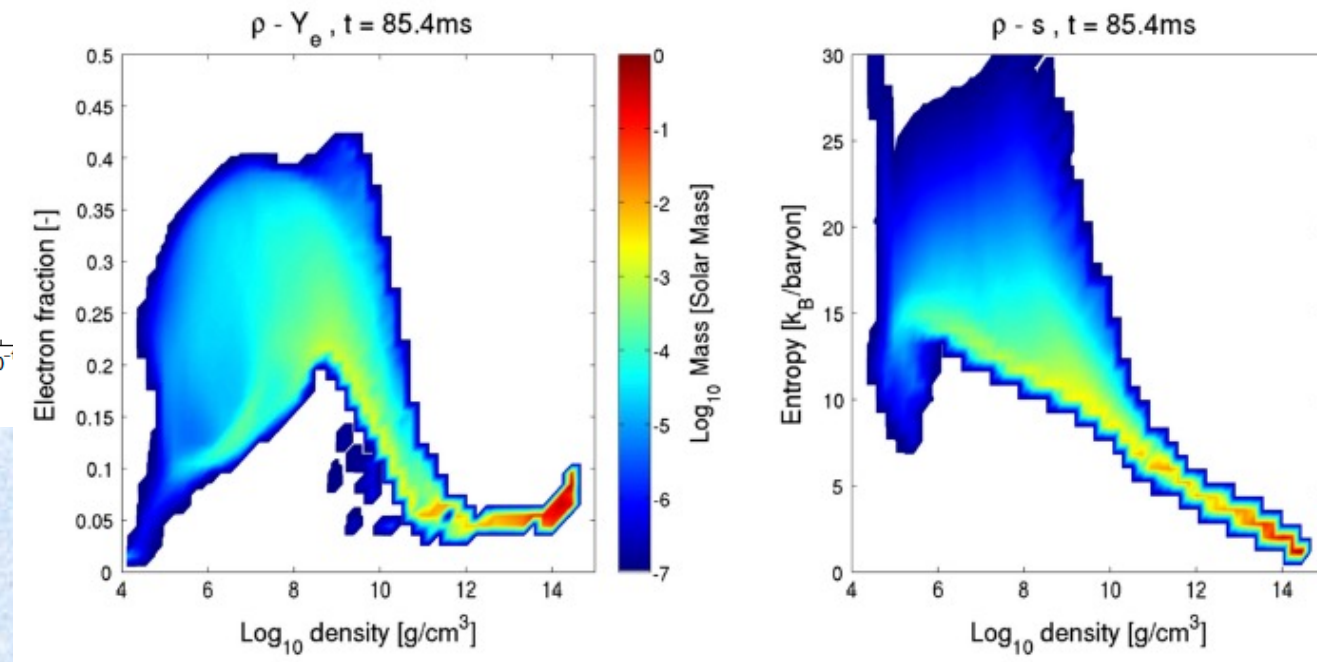
$$n_B \in (0, \sim 10n_{\text{sat}})$$

$$T \in (0, \sim 100) \text{ MeV}$$

$$Y_e \in (0, 0.6)$$



Fischer et al., ApJS 194, 39 (2011)



Perego et al., MNRAS 443, 3134 (2014)

for a review, e.g. Oertel et al., RMP 89, 015007 (2017)



# EoS at finite temperature: generalities (2)

- Temperature enters in FD distribution:  $f(e) = \frac{1}{1 + \exp^{(e - \mu)/(k_B T)}}$  → effects depend on  $(e - \mu)/T$  ratio
- Photons → free Bose gas:  $\mathcal{E}_{\text{phot}} = \frac{\pi^2}{15\hbar^3 c^3} (k_B T)^4$        $P_{\text{phot}} = \frac{1}{3} \mathcal{E}_{\text{phot}} \propto (k_B T)^4$
- Leptons → free Fermi gas at finite  $T$  (see slides 21-23)
- Hadrons (bound/unbound) → different regimes:
  - ❖ Low density &  $T$  ( $< \sim 0.5$  MeV) → nuclear reaction network needed (e.g. stellar evolution, nucleosynthesis)
  - ❖ Low density &  $T$  ( $> \sim 0.5$  MeV) → nuclear statistical equilibrium (NSE)
  - ❖ Intermediate density & low-intermediate  $T$  → interactions in medium → “modified” NSE
  - ❖ High density &  $T$  → cluster dissolution → nucleons + new d.o.f. ?
- Minimisation of  $E \rightarrow$  minimization of  $F$        $\mathcal{E} \rightarrow \mathcal{F} = \mathcal{E} - k_B T S$   
NB: - also p gas in the medium!  
- photons & leptons decouple from hadrons







# NSE

- Hp: mixture of nucleons and clusters  $X (A_j, Z_j)$  in chemical equilibrium:  $\mu_X = N\mu_n + Z\mu_p$
- If non-interacting particles (simplest model):
  - partition function factorises
  - apply Boltzmann statistics (ideal-gas like):  $f_{\text{MB}} = e^{-\frac{e_i - \mu_i}{k_B T}}$
  - non-relativistic particles:  $e_i = m_i c^2 + \frac{\hbar k_i^2}{2m_i}$
- number density (Saha-like):  $n_i = g_{s,i} e^{-(m_i c^2 - \mu_i)/k_B T} \left( \frac{m_i k_B T}{2\pi \hbar^2} \right)^{3/2} = g_{s,i} \frac{z_i}{\lambda_i^3}$ 

fugacity  
↙
- Energy density:  $\mathcal{E} = \sum_i m_i c^2 n_i + \frac{3}{2} k_B T \sum_i n_i$
- Pressure:  $P = k_B T \sum_i n_i$
- ❖ Masses and  $g_{s,i} = (2J_i + 1)$  (g.s.) from exp/theo data tables
- ❖ Possible inclusion of excited states (but no exp data → theo level density)
- ❖ and Coulomb interaction (screening – often fitting formulae)





# (Extended) NSE

- but : interactions → in medium effects → different ways, e.g.

- ❖ **Virial expansion** (e.g. Horowitz&Schwenk, NPA 2006)

Expansion in fugacity  $z_i$  of grand canonical potential → valid when  $z_i \ll 1$

Coefficients of expansion → many-body correlations of system at low density

- ❖ **Light clusters with energy shifts** (e.g. Typel et al., PRC 2010, Pais et al., PRC 2018)

Light clusters (2H, 3H, 3He, alpha) added as d.o.f. (quasi-particles)

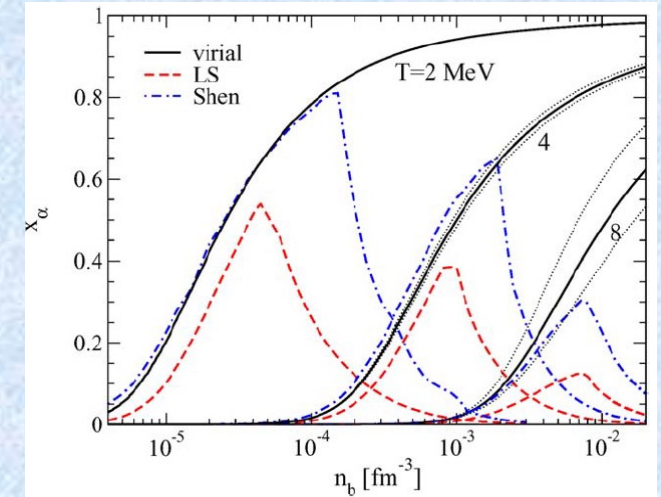
$$m_{A,Z}^* = m_{A,Z} - g_{\sigma,AZ} \langle \sigma \rangle$$

- ❖ **Excluded volume approach**

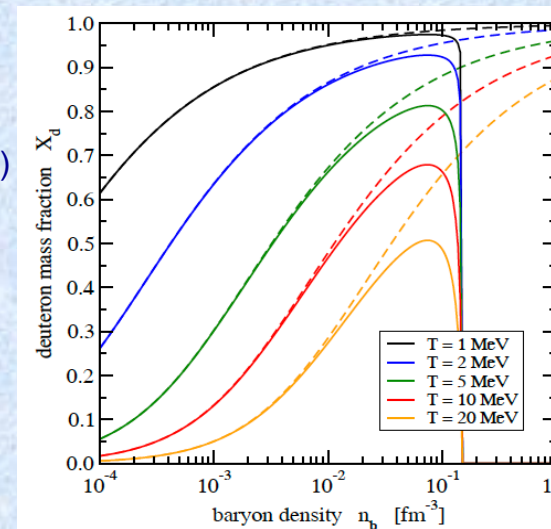
(e.g. Lattimer&Swesty, NPA 1991, Gulminelli&Raduta, PRC 2015, ...)

Volume occupied by cluster → excluded to others

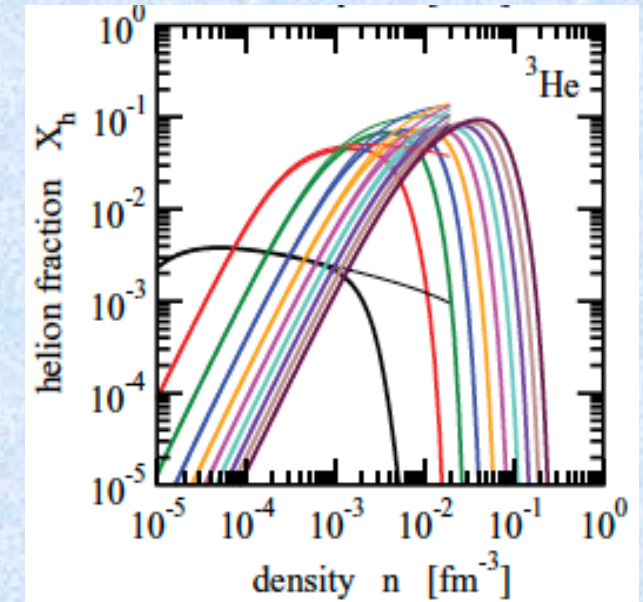
→ dissolution of clusters at high density  
but: model dependence !



Horowitz&Schwenk, NPA 776, 55 (2006)



Typel, EPJA 52, 16 (2016)



Typel et al., PRC 81, 015803 (2010)

for a review, e.g. Oertel et al., RMP 89, 015007 (2017), Burgio & Fantina, ASSL 457 (2018) and refs therein





# Conclusions & open questions

- ❖ Nuclear physics + astrophysics → constraints on EoS but still hard to discriminate  
→ nucleonic hp compatible with observations, no (dis)proof of exotic matter to now

➡  
✓ need of (microscopic) reliable theoretical model + experimental data to calibrate models  
✓ need of (more precise / numerous) astrophysical observations + post-merger?  
→ phase transition, modes (?)

- ❖ Importance of treatment at finite  $T$  → ensemble of nuclei
- 

- Extrapolation from raw data → **model dependence of the constraints**
- Lab. exper. mostly “low” density ( $\sim$  saturation density), low  $T$  probed  
→ **extrapolation to astro conditions (high  $T$  and density, asymmetry, charge neutral) ?**
- Uncertainties in high-density EoS → **blurring of different effects ?**
- Astro simulations vs microphysics inputs → **effects of microphysics inputs in astro modelling ?**
  - consistent implementation
  - systematic studies / bayesian analysis → uncertainties, implementation of consistent probab. distributions
  - static vs dynamical properties