Simple Views of a Complex System

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Outline

- The context; SM-CI
- Elliott's SU3
- Nuclear Shapes
- Coexistence at the Islands of Inversion

Basic elements of the SM-CI approach

- A valence space, computationally tractable, encompassing the targeted physics.
- An effective interaction for the valence space, that usually is expressed as a set of single particle energies and two body matrix elements (TBME) -and ideally three body contributions as well.
- Shell Model codes to build and diagonalize the (huge dimensional) matrices involved, or other, approximated, mean field based methods (MCSM, DNO-SM, TAURUS, etc) to solve the secular problem.
- The present generation of SM codes includes, Antoine, Nushell, and K-shell, among others.

The spherical mean field and the magic numbers

- The starting point is Independent Particle Model (IPM) a la Meyer-Jensen, with the magic numbers and the doubly magic nuclei as its cornerstones
- However, the collective manifestations of the nuclear dynamics are well beyond the reach of the IPM.
- Item more, the vulnerability of the magic closures, as exemplified by the coexistence in doubly magic nuclei of IPM-like ground states with low lying well deformed states of many particle-hole nature, is a real challenge.
- All in all, the principal asset of the IPM turns out to be to provide a basis in Fock space in which the nuclear many-body problem can be approximately solved

The Nuclear A-body Problem

- The goal is to find $\Psi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots \vec{r}_A)$ such that
- $H\Psi = E\Psi$, with
- $H = \sum_{i}^{A} T_i + \sum_{i,j}^{A} V_{2b}(\vec{r}_i, \vec{r}_j) + \sum_{i,j,k}^{A} V_{3b}(\vec{r}_i, \vec{r}_j, \vec{r}_k)$

The Interacting Shell Model (ISM, SM-CI, LSSM)

Is an approximation to the exact solution of the nuclear A-body problem using effective interactions in restricted valence spaces

The single particle states (i,j, k, ...), which are the solutions of the Independent Particle Model (IPM, aka Spherical Shell Model), provide as well a basis in the space of the occupation numbers (Fock space). The many body wave functions are Slater determinants:

$$\Phi = a^\dagger_{i_1}, a^\dagger_{i_2}, a^\dagger_{i_3}, \ldots a^\dagger_{i_A} |0\rangle$$

We can distribute the A particles in all the possible ways in the available single particle states. This provides a complete basis in the Fock space.

A formal solution to the A-body problem

Therefore, the "exact" solution can be expressed as a linear combination of the basis states:

$$\Psi = \sum_{\alpha} \Phi_{\alpha}$$

and the solution of the many body Schödinger equation

$$H\Psi = E\Psi$$

is transformed in the diagonalization of the matrix:

$$\langle \Phi_{\alpha} | H | \Phi_{\beta} \rangle$$

whose eigenvalues and eigenvectors provide the "physical" energies and wave functions

A formal solution to the A-body problem

This would be a No Core Shell Model calculation.

Which is not doable except for very light nuclei

Therefore, we are bound to define an inert core and a finite set of active orbits, i. e. a valence space

Effective Interactions: ”Ab initio” PHASE-I

- The interactions were renormalized G-matrices (Brueckner) obtained from bare NN interaction that reproduced the deuteron data and the NN phase shifts. The most popular ones were those of Kuo and Brown in the late 60’s. Had they had a powerful enough computer and shell model code, they could have calculated the spectrum of ^{48}Cr, getting the following results:

	ΔE in MeV				B(E2)(J$\rightarrow$J-2) in e^2fm^4		
	0^+	2^+	4^+	6^+	2^+	4^+	6^+
KB	0.0	0.65	1.66	3.14	268	376	389
EXP	0.0	0.75	1.86	3.45	320(40)	330(50)	300(80)

- An excellent Spherical Shell Model description of the yrast band of a well deformed nucleus indeed!! Somehow, the ”ab initio” description of nuclear collectivity dates back to 50 years ago

”Ab initio”, PHASE-I, fifty years ago

- And why not to try ^{56}Ni? In this case the results would have puzzled them quite a lot !

	ΔE in MeV				B(E2)(J$\rightarrow$J-2) in e^2fm^4		
	0^+	2^+	4^+	6^+	2^+	4^+	6^+
KB	0.0	0.52	1.65		545	770	
EXP	0.0	2.70	3.92		98(25)		

- Because the KB interaction makes ^{56}Ni a perfect oblate rotor, with β= 0.43, and E(4^+)/E(2^+)= 3.2. The doubly magic N=Z=28 configuration is completely absent in the yrast states' wave functions, which are, on average, of 8p-8h nature instead. A 4p-4h prolate band of similar deformation shows up about 2 MeV higher.
- The reason is that KB does not produce the right N=28, Z=28 spherical gaps. These are early fingerprints of the MONOPOLE anomalies of the realistic G-matrices.

A game changer; the monopole multipole decomposition. (A. P. Zuker)

The effective hamiltonian can be decomposed in two parts

- $H = \mathcal{H}_m + \mathcal{H}_M$. **Monopole and Multipole.**
- $\mathcal{H}_m$ **determines the spherical mean field and its evolution (aka shell evolution). Realistic two body interactions do not describe correctly this evolution. Hence it is necessary to modify empirically the two body centroids or the explicit inclusion of 3B forces to comply with experiment. A single monopole shift turns the KB interaction into the well behaved KB3 family.**
- $\mathcal{H}_M$ **contains the terms responsible for the correlations** *i.e.* **pairing, quadrupole, etc. This part is correctly given by the realistic two body effective interactions.**

The monopole hamiltonian

$$\mathcal{H}_m = H_{sp} + \sum \left[\frac{1}{(1+\delta_{ij})} a_{ij}\, n_i (n_j - \delta_{ij}) + \sum \Omega_{i,j,k} n_i n_j n_k \right.$$

$$\left. + \frac{1}{2} b_{ij} \left(T_i \cdot T_j - \frac{3n_i}{4} \delta_{ij} \right) \right].$$

the coefficients a and b are defined in terms of the centroïds as:

$$a_{ij} = \frac{1}{4}(3V_{ij}^1 + V_{ij}^0), b_{ij} = V_{ij}^1 - V_{ij}^0, \quad V_{ij}^T = \frac{\sum_J V_{ijij}^{JT}[J]}{\sum_J [J]}$$

the sums run over Pauli allowed values.

Multipole Universality and the NNN interactions. ”Ab initio” PHASE-II

Dufour-Zuker analysis of the multipole part of an *sd*-shell effective interactions obtained from Chiral Perturbation Theory: Coherent particle-particle and particle-hole content. The results are similar for other valence spaces.

	pp(JT)			ph($\lambda\tau$)			
	10	01	21	20	40	10	11
NN (*)	-6.06	-4.38	-2.92	-3.35	-1.31	+1.03	+2.49
NN+NNN (*)	-6.40	-4.36	-2.91	-3.28	-1.23	+1.10	+2.43

(*) S. K. Bogner, *et al.* IM-SRG matrix elements, PRL 113, 142501 (2014)

"Ab initio" PHASE-II

Hence, in view of these results, and considering the success of the monopole corrected two body "ab initio" effective interactions, it seems sensible to submit that the three body interactions should contribute mainly to the monopole hamiltonian, *i. e.* providing the right evolution of the spherical mean field in the valence space. Recently, the Napoli group has provided evidence supporting this view (L. Coraggio, *et al.* PPNP 134, 104079 (2024).

This antsatz will simplify enormously the VS-IMSRG and Coupled Cluster calculations.

”Ab initio” PHASE-II, pros and cons

- The ”Ab initio” PHASE-II approach using ChPT interactions and VS-SRG, solves some of the problems related to the short range repulsion of the bare NN-interaction
- Via the 3B terms, sensible saturation properties are obtained, and by the same token, the monopole evolution of the spherical mean field is very much improved.
- A recurrent problem is the over-abundance of ChPT interactions, and even more, their truncation levels and cut-offs

”Ab initio” PHASE-II, pros and cons

- The zero range operators like $(\vec{\sigma} \cdot \vec{\tau})$ - GT, M1 - are correctly renormalized as well
- On the contrary, long range operators like the E2, are not. Somehow, the coupling to the GQR is not yet correct
- The approach has still problems to deal with intruder states

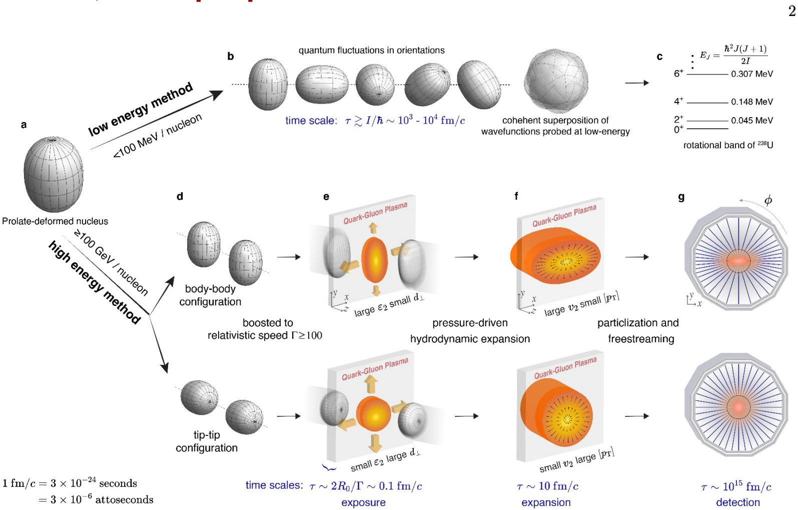
Nuclear Shape

- To assign a shape to the nucleus it is necessary to define an intrinsic reference frame, hence the rotational (and reflection) invariances must be broken. In addition, we usually rely on semiclassical models, liquid-drop like, to describe properties akin to the concept of shape.
- A surface in 3D can be expressed in the basis of the spherical harmonics $Y_{\lambda,\mu}(\theta, \phi)$. The coefficients of the development, $\alpha_{\lambda,\mu}$, are the shape parameters.

The meaning of the nuclear collective modes

- When dealing with the "collective modes (or excitations)" of the nucleus, quite often the textbooks resort to semiclassical -liquid drop like- analogies
- But analogies are dangerous, because we are lazy. Hence, sometimes we can overlook, trying to be more "physical", the quantal nature of the nucleus
- For instance, vibrational excitations are explained as governed by the competition of the coulomb repulsion and the surface tension (as in a charged drop). But the vibrational nuclei do not vibrate (classically, they have phonon-like excitations of purely nuclear origin
- In a similar -provocative- mood: Deformed nuclei do not rotate (classically), they just have J(J+1) excitation spectra
- Obviously, there are no time scales associated to these "modes"

Indeed, some people think there are !!



from the STAR collaboration

Quadrupole Collectivity: Elliott's SU(3)

- The mechanism that produces permanent deformation and rotational spectra in nuclei is much better understood in the framework of the SU(3) symmetry of the isotropic harmonic oscillator and its implementation in Elliott's model.
- The basic simplifications of the model are:
  i) the valence space is limited to one major HO shell,
  ii) the two-body interaction is quadrupole-quadrupole.
  This implies (mainly) that the spin orbit splitting and the pairing interaction are put to zero.

The SU3 symmetry of the harmonic oscillator

Let's then start with the spherical HO which in units m=1 ω=1 can be written as:

$$H_0 = \frac{1}{2}(p^2 + r^2) = \frac{1}{2}(\vec{p} + i\vec{r})(\vec{p} - i\vec{r}) + \frac{3}{2}\hbar = \hbar(\vec{A}^\dagger \vec{A} + \frac{3}{2})$$

$$\vec{A}^\dagger = \frac{1}{\sqrt{2\hbar}}(\vec{p} + i\vec{r}) \;\; \vec{A} = \frac{1}{\sqrt{2\hbar}}(\vec{p} - i\vec{r})$$

which have bosonic commutation relations.

The SU3 symmetry of the harmonic oscillator

H_0 is invariant under all the transformations which leave invariant the scalar product $\vec{A}^\dagger \vec{A}$. As the vectors are three dimensional and complex, the symmetry group is U(3). We can built the generators of U(3) as bi-linear operators in the A's. The anti-symmetric combinations produce the three components of the orbital angular momentum L_x, L_y and L_z, which are on turn the generators of the rotation group O(3). From the six symmetric bi-linears we can retire the trace that is a constant; the mean field energy. Taking it out we move into the group SU(3). The five remaining generators are the five components of the quadrupole operator:

$$q^{(2)}_\mu = \frac{\sqrt{6}}{2\hbar}(\vec{r} \wedge \vec{r})^{(2)}_\mu + \frac{\sqrt{6}}{2\hbar}(\vec{p} \wedge \vec{p})^{(2)}_\mu$$

The SU3 symmetry of the harmonic oscillator

The generators of SU(3) transform single nucleon wavefunctions of a given p (principal quantum number) into themselves. In a single nucleon state there are p oscillator quanta which behave as l=1 bosons. When we have several particles we need to construct the *irreps* of SU(3) which are characterized by the Young's tableaux (n_1, n_2, n_3) with $n_1 \geq n_2 \geq n_3$ and $n_1+n_2+n_3$=Np (N being the number of particles in the open shell). The states of one particle in the p shell correspond to the representation (p,0,0). Given the constancy of Np the *irreps* can be labeled with only two numbers. Elliott's choice was $\lambda=n_1-n_3$ and $\mu=n_2-n_3$. In the cartesian basis we have; $n_x=a+\mu$, $n_y=a$, and $n_z=a+\lambda+\mu$, with $3a+\lambda+2\mu=Np$.

The SU3 symmetry of the harmonic oscillator

The quadratic Casimir operator of SU(3) is built from the generators

$$\vec{L} = \sum_{i=1}^{N} \vec{l}(i) \qquad Q_{\alpha}^{(2)} = \sum_{i=1}^{N} q_{\alpha}^{(2)}(i)$$

as:

$$C_{SU(3)} = \frac{3}{4}(\vec{L} \cdot \vec{L}) + \frac{1}{4}(Q^{(2)} \cdot Q^{(2)})$$

and commutes with them. With the usual group theoretical techniques, it can be shown that the eigenvalues of the Casimir operator in a given representation (λ, μ) are:

$$C(\lambda, \mu) = \lambda^2 + \lambda\mu + \mu^2 + 3(\lambda + \mu)$$

Once these tools ready we come back to the physics problem as posed by Elliott's hamiltonian

$$H = H_0 + \chi(Q^{(2)} \cdot Q^{(2)})$$

that can be rewritten as:

$$H = H_0 + 4\chi C_{SU(3)} - 3\chi(\vec{L} \cdot \vec{L})$$

The eigenvectors of this problem are thus characterized by the quantum numbers λ, μ, and L. We can choose to label our states with these quantum numbers because O(3) is a subgroup of SU(3) and therefore the problem has an analytical solution:

Elliott's Model

$$E(\lambda,\mu,L) = \hbar\omega(p+\frac{3}{2})+4\chi(\lambda^2+\lambda\mu+\mu^2+3(\lambda+\mu))-3\chi L(L+1)$$

This final result can be interpreted as follows: For an attractive quadrupole quadrupole interaction ($\chi < 0$) the ground state of the problem pertains to the representation which maximizes the value of the Casimir operator, and this corresponds to maximizing λ or μ (the choice is arbitrary). If we look at that in the cartesian basis, this state is the one which has the maximum number of oscillator quanta in the Z-direction, thus breaking the symmetry at the intrinsic level. We can then speak of a deformed solution even if its wave function conserves the good quantum numbers of the rotation group, i.e. L and L_z.

Elliott's Model

$$E(\lambda,\mu,L) = \hbar\omega(p+\frac{3}{2})+4\chi(\lambda^2+\lambda\mu+\mu^2+3(\lambda+\mu))-3\chi L(L+1)$$

For this one (and for every) (λ,μ) representation, there are different values of L which are permitted, for instance for the representation $(\lambda,0)$ L=0,2,4$\ldots\lambda$. And their energies satisfy the L(L+1) law (or J(J+1)), thus giving the spectrum of a quantum rigid rotor. The problem of the description of deformed nuclear rotors is thus formally solved.

Variants of Elliott's SU(3)

As it was realized later, there are variants of SU(3) which provide similar solutions. Pseudo-SU(3) applies when the relevant valence space contains the orbits of a major HO shell except the orbit with larger angular momentum (the so called intruder orbit) and they are quasi-degenerated. Quasi-SU(3), if the space comprises the intruder orbit and its quadrupole partners with $\Delta j = \Delta l = 2, 4, \ldots$.

Given that the quadrupole-quadrupole term is dominant in the effective NN interaction, Elliott's model and its variants, Pseudo-SU3 and Quasi-SU3, provide the heuristic toolkit to delineate the minimal valence spaces in which the quadrupole collectivity (deformation) can develop.

Kumar-Cline invariants: β, γ and their variances

- The only rigorous method to relate the intrinsic shape parameters to the laboratory-frame observables is provided by the so-called quadrupole invariants Q^n of the second-rank quadrupole operator Q_2 introduced by Kumar.
- The calculation of β and γ requires the knowledge of the expectation values of the second- and third-order invariants defined, respectively, by $\hat{Q}^2 = \hat{Q} \cdot \hat{Q}$ and $\hat{Q}^3 = (\hat{Q} \times \hat{Q}) \cdot \hat{Q}$ (where $\hat{Q} \times \hat{Q}$ is the coupling of $\hat{Q}$ with itself to a second-rank operator).

Fluctuations

Indeed, it is not very meaningful to assign effective values to β and γ without also studying their fluctuations. Our aim is to go beyond the extraction of effective values of these intrinsic parameters and obtain their variances.

With this goal, we calculate:

$$\sigma(\hat{Q}^2) = (\langle \hat{Q}^4 \rangle - \langle \hat{Q}^2 \rangle^2)^{1/2} \tag{1}$$

and

$$\sigma(\hat{Q}^3) = (\langle \hat{Q}^6 \rangle - \langle \hat{Q}^3 \rangle^2)^{1/2} \,. \tag{2}$$

see A. Poves, F. Nowacki and Y. Alhassid, PRC 101, 054307 (2020), for the details on how to compute them exactly in a shell model context.

The intrinsic quadrupole moment Q_0 and the average values of the Bohr-Mottelson shape parameters β and γ can be calculated from the expectation values of the second- and third-order invariants using

$$Q_0 = \sqrt{\frac{16\pi}{5}} \langle \hat{Q}^2 \rangle^{1/2} , \tag{3}$$

$$\beta = \frac{4\pi}{3r_0^2} \frac{\langle \hat{Q}^2 \rangle^{1/2}}{A^{5/3}} , \tag{4}$$

with r_0=1.2 fm, and

$$\cos 3\gamma = -\sqrt{\frac{7}{2}} \frac{\langle \hat{Q}^3 \rangle}{\langle \hat{Q}^2 \rangle^{3/2}} \tag{5}$$

Fluctuations in β and γ

$$\frac{\Delta\beta}{\beta} = \frac{1}{2}\frac{\sigma\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle}\,. \tag{6}$$

$$\frac{\sigma^2(\cos 3\gamma)}{(\overline{\cos 3\gamma})^2} = \frac{\sigma^2\langle\hat{Q}^3\rangle}{\langle\hat{Q}^3\rangle^2} + \frac{9}{4}\frac{\sigma^2\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle^2} - 3\frac{\langle\hat{Q}^5\rangle - \langle\hat{Q}^3\rangle\langle\hat{Q}^2\rangle}{\langle\hat{Q}^3\rangle\langle\hat{Q}^2\rangle}\,. \tag{7}$$

Notice that the covariance term in (7) requires the knowledge of $\langle\hat{Q}^5\rangle$. The range of γ values at 1σ is given by:

$$\cos^{-1}(\cos 3\gamma \pm \sigma(\cos 3\gamma)) \tag{8}$$

Mind your words !!

It is important to fix the language

- Does axial means either γ=0° (prolate) or γ=60° (oblate) ?
- By the way, does spherical means β=0 ?
- How large are the fluctuations in γ that we are ready to accept before dropping out the concept of axial (or triaxial) shape?

A menagerie of shapes. Try to name them.

- ^{34}Si: 0_1^+, β = 0.18$\pm$ 0.10 γ= 40° span 0° – 60°
- ^{48}Ca: β = 0.15$\pm$ 0.05 γ= 33° span 0° – 60°
- ^{56}Ni: β = 0.21$\pm$ 0.07 γ= 40.5° span 13° – 60°
- ^{68}Ni: 0_1^+, β = 0.11$\pm$ 0.06 γ= 36° span 0° – 60°
- ^{34}Si: 0_2^+, β = 0.42$\pm$ 0.08 γ= 40° span 30° – 60°
- ^{48}Cr: β = 0.31$\pm$ 0.06 γ= 13° span 0° – 20°
- ^{68}Ni: 0_2^+, β = 0.19$\pm$ 0.05 γ= 38° span 23° – 60°
- ^{68}Ni: 0_3^+, β = 0.29$\pm$ 0.05 γ= 16° span 0° – 24°
- ^{64}Cr: β = 0.29$\pm$ 0.06 γ= 16° span 0° – 24°

**If you can't, put the blame on Nature,
don't put the blame on me. (Rita Hayworth, Gilda, 1946)**

**Only in the strict SU(3) limit
the variances of β and γ are zero**

Miscellaneous results

	β	$\Delta\beta$	$\frac{\sigma\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle}$	γ	γ range
^{20}Ne	0.62	0.07	0.24	3°	0° — 9°
^{22}Ne	0.59	0.08	0.27	13°	2° — 18°
^{24}Mg	0.60	0.07	0.25	18°	12° — 22°
^{28}Si	0.46	0.09	0.41	50°	38° — 60°
^{48}Cr	0.31	0.06	0.41	13°	0° — 20°
^{34}Si	0.18	0.10	1.07	40°	0° — 60°
0^+_1	0.42	0.08	0.37	40°	30° — 60°
^{44}S	0.30	0.07	0.43	27°	0° — 45°
0^+_1	0.31	0.07	0.43	21°	0° — 34°

Do the intrinsic shape parameters β and γ survive in the laboratory frame?

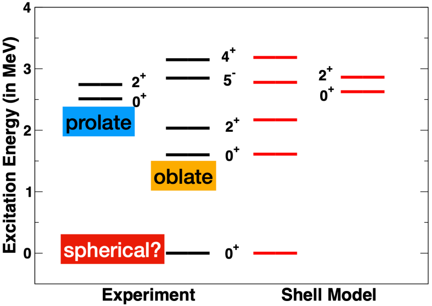
- β: yes although nuclei are most often β-soft
- γ: rather not. The fluctuations in γ amount to 20°- 30°. In some cases the oblate or prolate character survives. In others, both sectors of the β-γ sextant are equally probable
- β and γ only have small fluctuations when the nucleus approaches the SU3 limit. And surely that's what happens in the well deformed heavy nuclei.

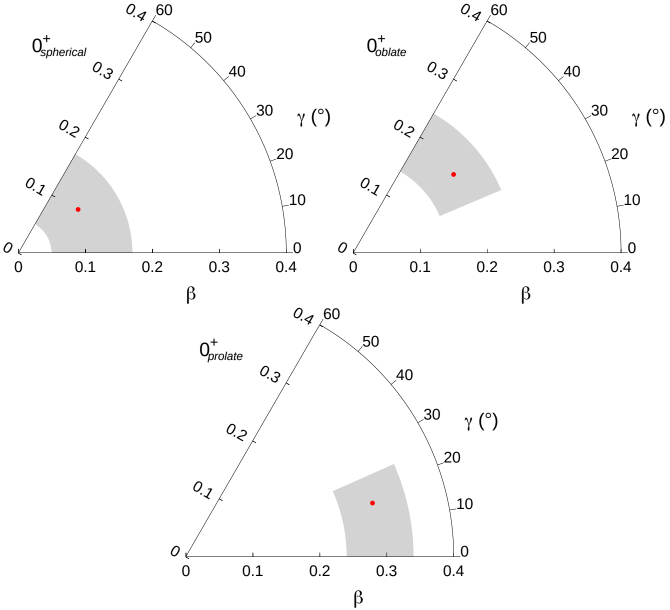
A bit of semantics: Are doubly magic nuclei spherical?

- Doubly magic nuclei are NOT spherical, they have NO shape
- Hence, there are NO spherical nuclei at all as seen below
- ^{56}Ni, β = 0.21$\pm$ 0.07 γ= 40.5° span 13° – 60°
- ^{48}Ca, β = 0.15$\pm$ 0.05 γ= 33° span 0° – 60°
- Discomforting isn't it?

The K-plots are a representation in the (β, γ) sextant of the locus of their variances at 1σ.

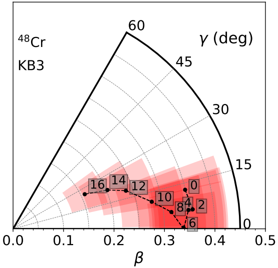
Let's look at ^{68}Ni with this Kumar lens

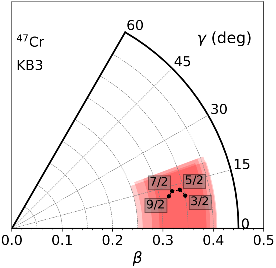


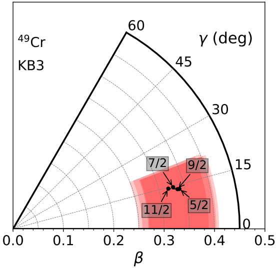


$0^+_{spherical}$
0^+_{oblate}
$0^+_{prolate}$
γ (°)
β
0
0.1
0.2
0.3
0.4
10
20
30
40
50
60

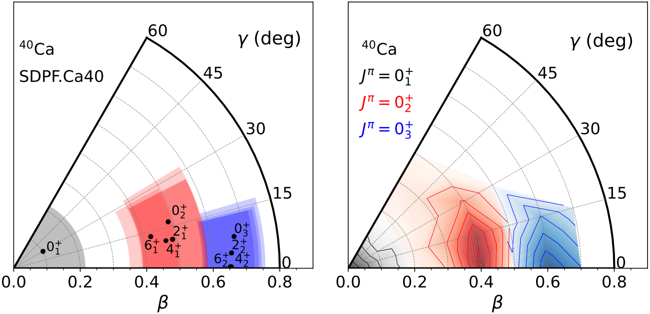
The Kumar invariants for J > 0
with D. Frycz and J. Menéndez







The Kumar invariants for J $>$ 0



The plot to the right is a PHFB calculation with the same interaction

The Islands of Inversion (IoI) and SU(3)

- It is now common knowledge that the IoI's occur when the neutron magic gaps in the very neutron rich isotopes are quenched, provided the intruder configurations maximize their correlation energy, mostly of quadrupole type.
- Indeed, for that to happen, the orbits close to the Fermi level must pertain to some of the SU(3) variants mentioned above.

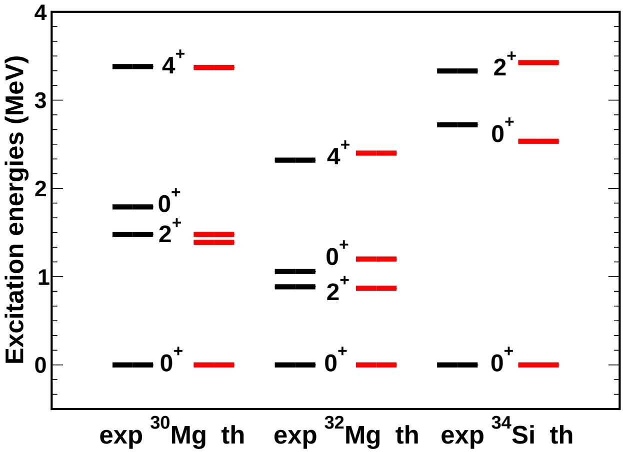
Where all that started; ^{32}Mg at N=20

The valence space for the IoI N=20

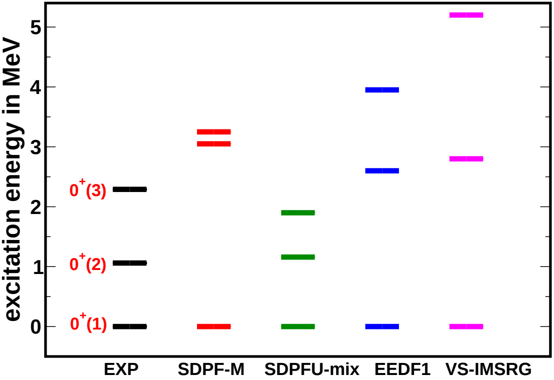
Neutrons	Protons
$1p_{1/2}$	$1p_{1/2}$
$0f_{5/2}$	$0f_{5/2}$
$1p_{3/2}$	$1p_{3/2}$
$0f_{7/2}$	$0f_{7/2}$
$0d_{3/2}$	$0d_{3/2}$
$1s_{1/2}$	$1s_{1/2}$
$0d_{5/2}$	$0d_{5/2}$

Quasi-SU3 doublets. The valence space includes the red and green orbits, blue orbits are blocked.
SDPF-U-MIX effective interaction.

Shape Coexistence in ^{30}Mg and ^{34}Si

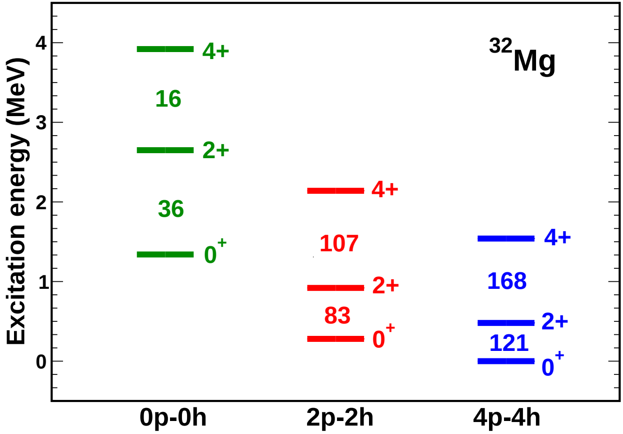


A landmark in the Iol's, ^{32}Mg



And a headache for the theorists !

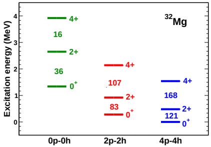
^{32}Mg; Semi-Magic, Deformed and Super-deformed configurations. B(E2)'s in e^2fm^4



The remarkable structure of the three physical 0^+'s of ^{32}Mg

- They are rather weird; the ground state is 9% 0p-0h, 54% 2p-2h, and 35% 4p-4h, thus, it is a mixture of deformed and superdeformed prolate shapes and it makes sense to speak of shape mixing.
- However, the first excited 0^+ (K. Wimmer's state) has 33% 0p-0h, 12% 2p-2h, and 54% 4p-4h, a surprising hybrid of semi-magic and superdeformed, whose direct mixing matrix element is strictly zero. Clearly, it is not a case of shape mixing, could it be an example of *shape entanglement*?
- Finally the second excited 0^+ turns out to be an even mixture of semi-magic, deformed and super-deformed. Quite exotic as well !

K-plots for ^{32}Mg; the np-nh configulations

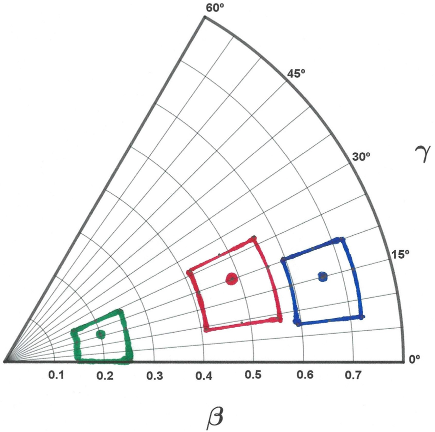


$\mathbf{0}^{+}_{0p-0h}$

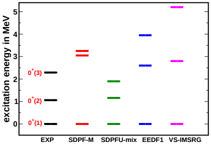
$\mathbf{0}^{+}_{2p-2h}$

$\mathbf{0}^{+}_{4p-4h}$

A neat case of shape coexistence !



BUT, look at the three lowest (physical) 0^+ states

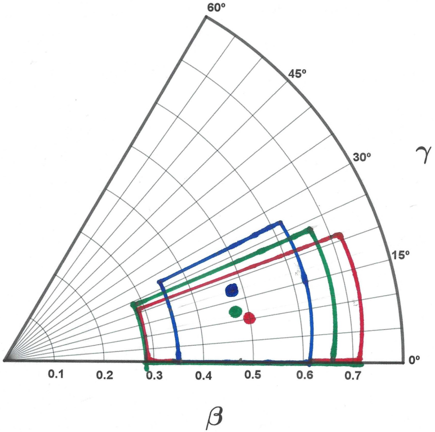


$\mathbf{0_1^+}$

$\mathbf{0_2^+}$

$\mathbf{0_3^+}$

this is rather a case of shape mixing or entanglement



^{32}Mg under the Kumar lens; the physical states

- The ground state 0_1^+ has $\frac{\sigma\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle}$=0.62 and β=0.48$\pm$0.13, $\frac{\sigma\langle\hat{Q}^3\rangle}{\langle\hat{Q}^3\rangle}$=1.16, γ=17° with a spread 0° – 27° at 1σ
- The excited 0_2^+ has $\frac{\sigma\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle}$=0.88 and β=0.50$\pm$0.21, $\frac{\sigma\langle\hat{Q}^3\rangle}{\langle\hat{Q}^3\rangle}$=1.03, γ=10° with a spread 0° – 21° at 1σ
- The excited 0_3^+ has $\frac{\sigma\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle}$=0.77 and β=0.47$\pm$0.18, $\frac{\sigma\langle\hat{Q}^3\rangle}{\langle\hat{Q}^3\rangle}$=1.15, γ=12° with a spread 0° – 23° at 1σ

Finale

Thanks for your attention !