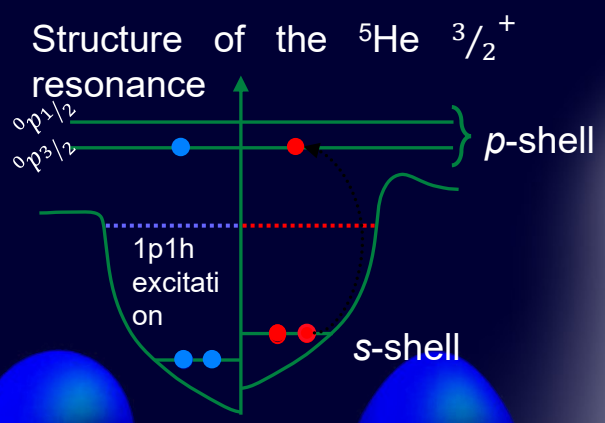
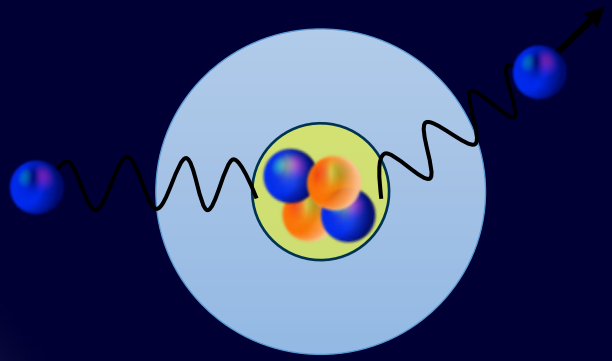
Astrophysics and reactions





Nuclear structure from nuclear reactions

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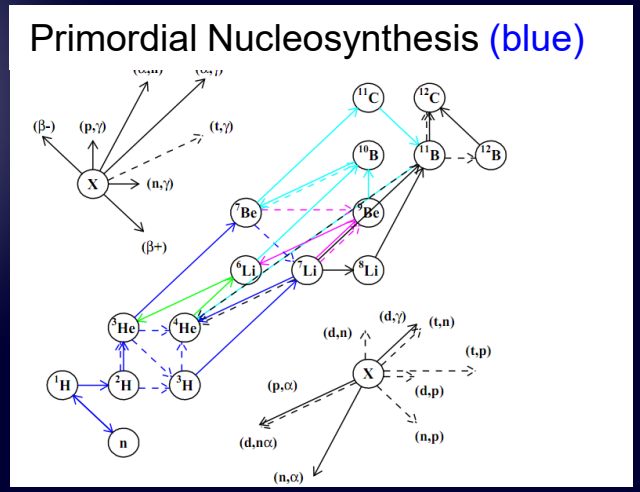

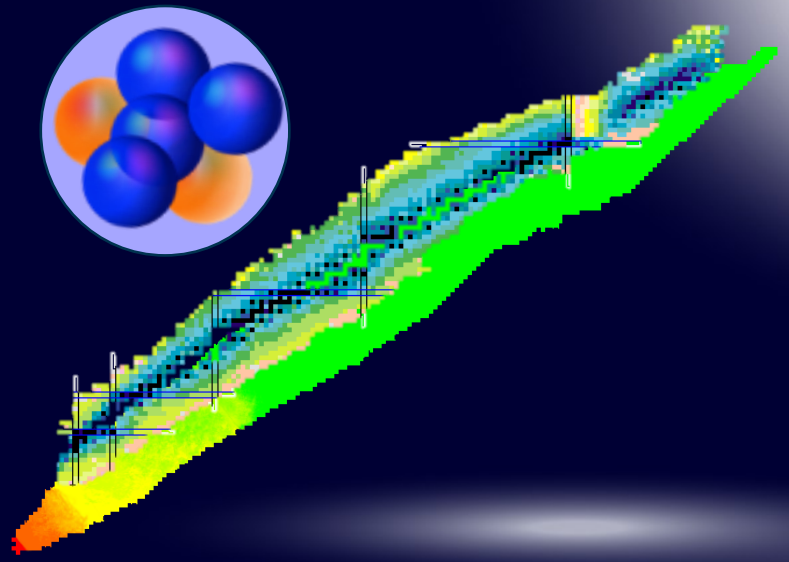

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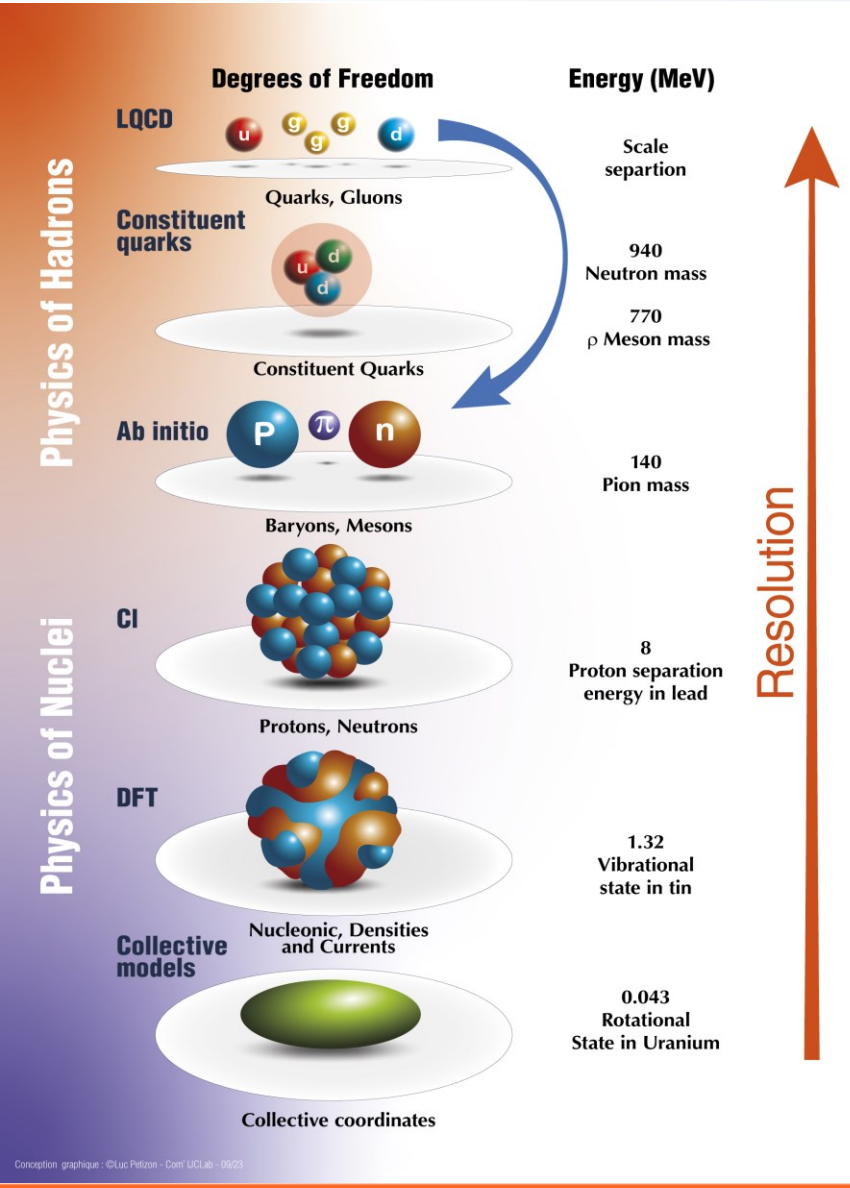
An incomplete list of References (textbooks)

- Quantum Collision Theory C. J. Joachain, ISBN 978-0-444-86773-5, North-Holland, 1975.
- Scattering Theory, J. R. Taylor, 978-0-486-45013-1, Dover publications, 1983.
- Introduction to Nuclear Reactions, C. A. Bertulani and P. Danielewicz,978-0-750-30932-5, Taylor & Francis, 2003.
- Nuclear Reactions for Astrophysics, I. J. Thompson and F. M. Nunes, 978-0-524-85635-5, Cambridge University press, 2009.
- Nuclear Reactions, H.P. gen. Schieck, 978-3-642-53985-5, Springer, 2014.
- Introduction to Nuclear Reactions, G.R. Satchler, 0-333-25907-6, Macmillan Press, 1980.

This presentation brings together material from lectures by several authors to provide a comprehensive overview of the different approaches required to understand and model nuclear reactions in astrophysics.

- Carl Brune
- James deBoer
- Chloë Hebborn
- Stephane Hilaire
- Denis Lacroix

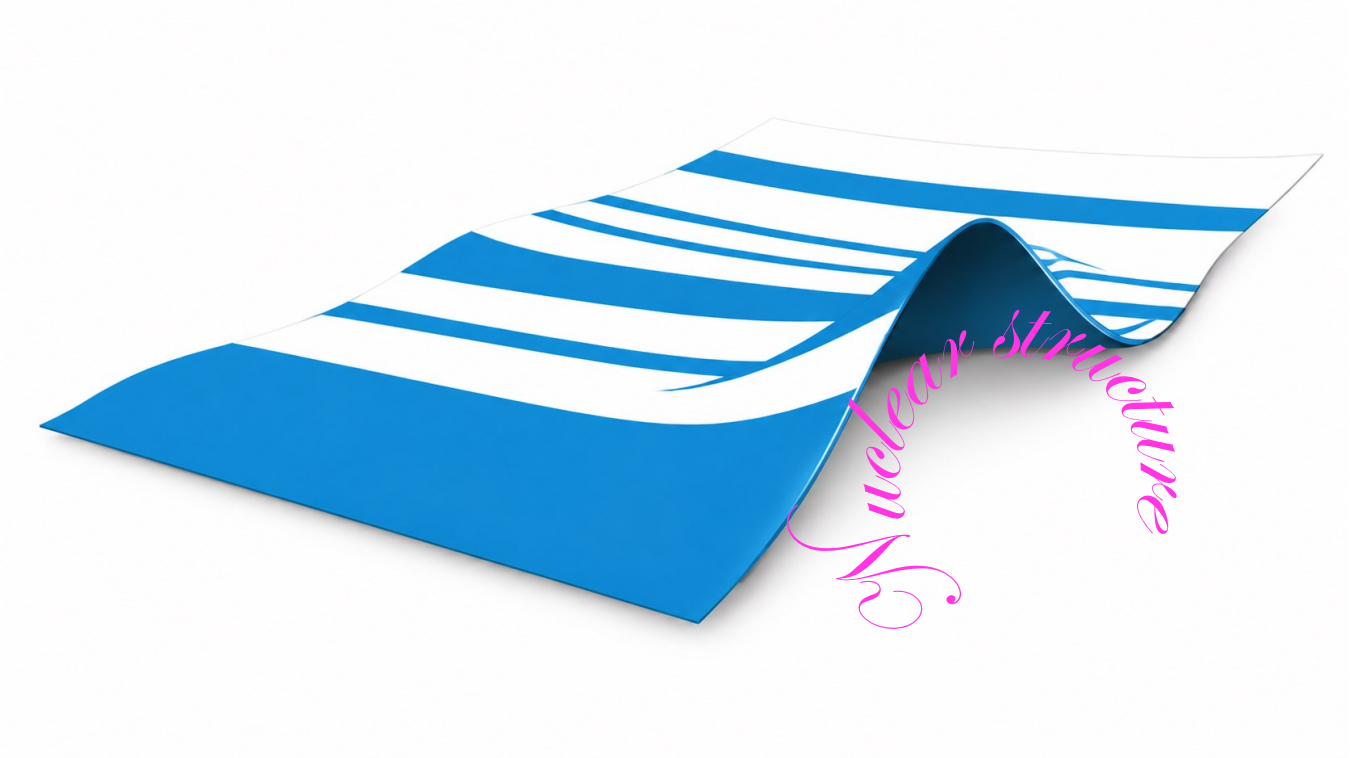
ab initio in nuclear physics



- **First principles for Nuclear Physics: QCD**
  - ➢ Non-perturbative at low energies;
  - ➢ Still outstanding to perform $a \to 0$ LQCD.
- Assuming degrees of freedom to be **nucleons (including antinucleons and hyperons…):**
  - ➢ Nuclei made of nucleons;
  - ➢ Interacting by nucleon-nucleon and three-nucleon potentials.

ab initio means
- All nucleons are active;
- Exact Pauli principle;
- Realistic inter-nucleon interactions;
  - ➢ Accurate description of NN and 3N data
- Controllable approximations.

Reaction theory



Reaction theory

The nuclear many-body problem

1. Assume *A* active nucleons with a spatial, spin, and isospin components

$$\vec{r_i} = \{\vec{r_i}, \vec{\sigma_i}, \vec{\tau_i}\} \quad i = 1,2 \dots A$$

2. For non-relativistic energies, the microscopic *A*-nucleon Hamiltonian

$$H^{(A)} = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{1 \le i < j}^{A} V^{NN}(r_{ij}) + \sum_{1 \le i < j < j}^{A} V_{ijk}^{3N} + \cdots$$

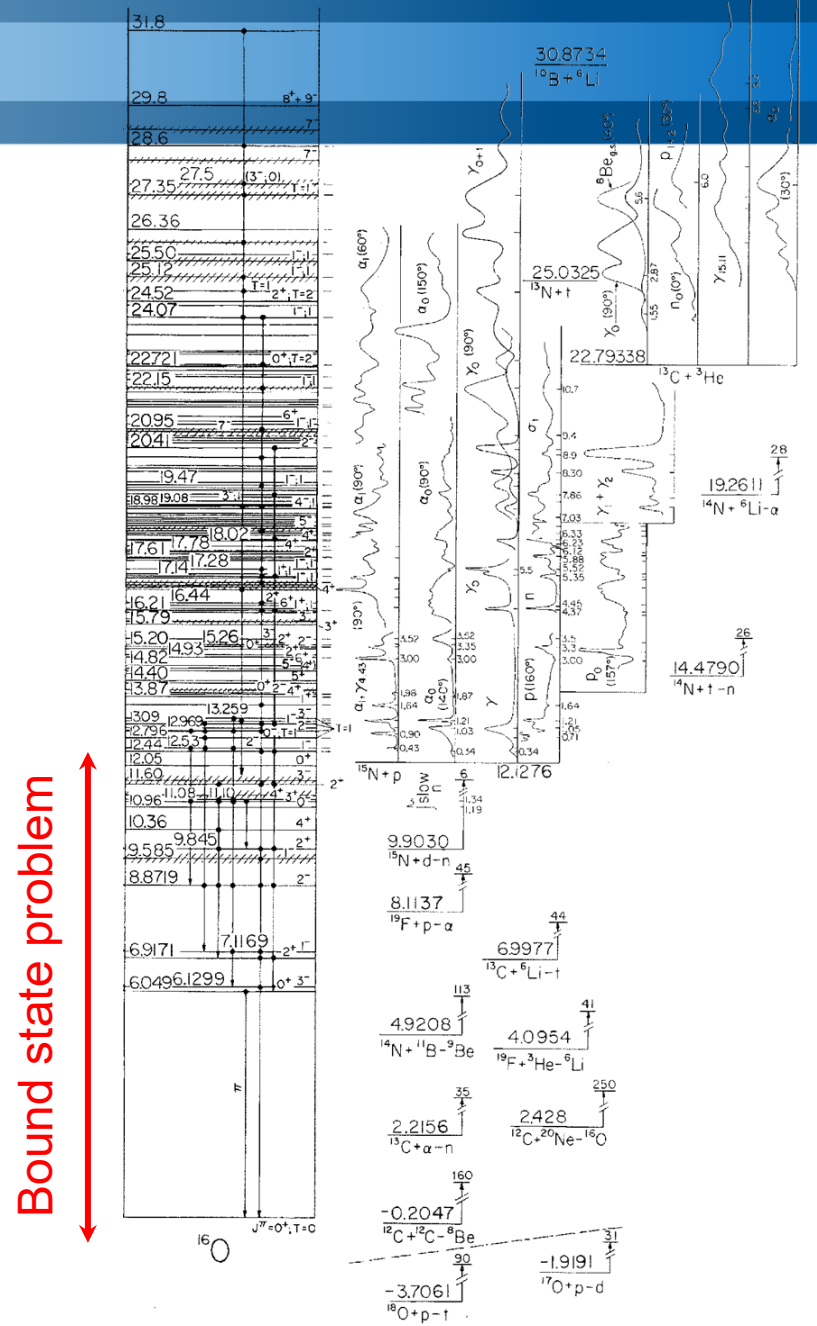
Nucleons are composite systems and nuclei is highly correlated

3. Find the state vector that minimize the energy by solving the many-body Schrödinger equation

$$H^{(A)} \Psi_\epsilon^{(A)}(\vec{r_1} \dots \vec{r_A}) = E_\epsilon \Psi_\epsilon^{(A)}(\vec{r_1} \dots \vec{r_A})$$

I. $E - E_{\text{threshold}} < 0$ are bound-states and subject to vanishing boundary conditions
Find eigenfunctions and eigenenergies

II. $E - E_{\text{threshold}} > 0$ are continuum states



Nuclear forces

Symmetries and associated conservation of strong force between n and p

- Any 3D rotation ↔ total angular momentum J conserved
- Point reflection ↔ total parity (π) conserved
- Translation and Galilean invariance ↔ no dependence on $\boldsymbol{R}_{\mathrm{CM}}$ and $\boldsymbol{P}_{\mathrm{CM}}$
- Particle exchange ($P_r P_\sigma P_\tau$) ↔ total wave-function is antisymmetric
- Time reversal ↔ Hamiltonian is real symmetric
- Isospin rotations (in first approximation) ↔ partially conserved T

The nucleon-nucleon interaction can be cast either in a local form

$$V^{NN} = V_C(r_{12}) + V_S(r_{12})\vec{\sigma}_1 \cdot \vec{\sigma}_1 + V_{LS}(r_{12})\vec{L} \cdot \vec{S} + V_T(r_{12})\vec{S}_{12}(\vec{r}) + \cdots$$

or a general non-local interaction (generally softer)

$$V^{NN} = V_C(p'_{12}, p_{12}) + V_S(p'_{12}, p_{12})\vec{\sigma}_1 \cdot \vec{\sigma}_1 + V_{LS}(p'_{12}, p_{12})\left[-i\vec{S} \cdot (\vec{q} \times \vec{K})\right] + \cdots$$

$$\begin{cases} \vec{L} = \vec{r}_{12} \times \vec{p}_{12} \\ \vec{S} = \dfrac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2) \end{cases}$$

$$\begin{cases} \vec{q} = p'_{12} - p_{12} \\ \vec{K} = \dfrac{p'_{12} + p_{12}}{2} \end{cases}$$

The nuclear many-body wave function

Nucleons are fermions → only the antisymmetric wave function is relevant

$$\Psi_{\epsilon}^{(A)}\left(\vec{r_1}\ldots,\vec{r_i},\ldots,\vec{r_j},\ldots,\vec{r_A}\right) = -\Psi_{\epsilon}^{(A)}\left(\vec{r_1}\ldots,\vec{r_j},\ldots,\vec{r_i},\ldots,\vec{r_A}\right)$$

Conserved total angular momentum J and parity π

- And approximately conserved total isospin T

$$\Psi_{\epsilon}^{(A)} = \Psi_{\epsilon}^{J^{\pi}T\ (A)}$$

Due to translational invariance all nucleonic operators are defined in the intrinsic frame (C.M. frame is irrelevant)

- The wave function must be translational invariant

$$\Psi_{\epsilon}^{(A)} \coloneqq \Psi_{\epsilon}^{(A)}\left(\vec{\xi_0}\ldots,\vec{\xi_j},\ldots,\overrightarrow{\xi_{A-1}}\right)$$

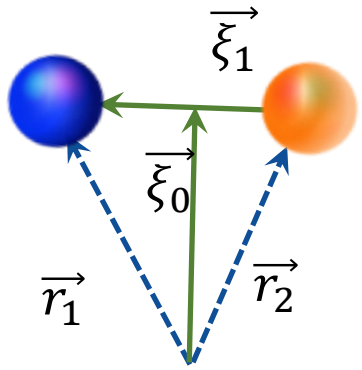
$$T^t\Psi_{\epsilon}^{(A)}\left(\vec{\xi_0}\ldots,\vec{\xi_j},\ldots,\overrightarrow{\xi_{A-1}}\right) = \Psi_{\epsilon}^{(A)}\left(\vec{\xi_0}\ldots,\vec{\xi_j},\ldots,\overrightarrow{\xi_{A-1}}\right)$$

- Traditionally in nuclear physic with can either break symmetry and restore it or keep it enforced:
  - Work with $A-1$ translational invariant coordinates
  - Work with A single-particle coordinates and achieved restoration

$$\Psi_{\epsilon}^{(A)}\left(\vec{r_1}\ldots,\vec{r_i},\ldots,\vec{r_j},\ldots,\vec{r_A}\right) = \Psi_{\epsilon}^{(A)}\left(\vec{\xi_0}\ldots,\vec{\xi_j},\ldots,\overrightarrow{\xi_{A-1}}\right)\Psi_{\epsilon}\left(\vec{R}_{\text{C.M.}}\right)$$

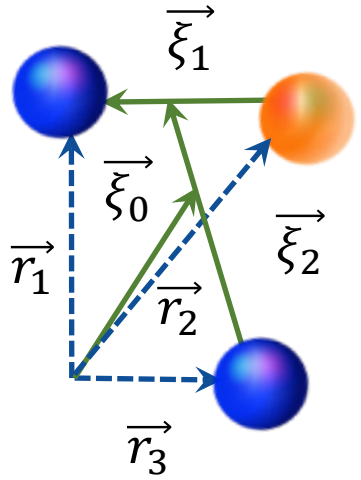
Coordinate systems…

- Two-nucleon system



$$\begin{cases} \vec{\xi}_0 = \frac{1}{\sqrt{2}}(\vec{r}_1 + \vec{r}_2) \propto \vec{R}_{\mathrm{CM}} \\ \vec{\xi}_1 = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) \propto \vec{r}_{12} \end{cases} \qquad \begin{cases} \vec{r}_1 = \frac{1}{\sqrt{2}}(\vec{\xi}_0 + \vec{\xi}_1) \\ \vec{r}_2 = \frac{1}{\sqrt{2}}(\vec{\xi}_0 - \vec{\xi}_1) \end{cases}$$

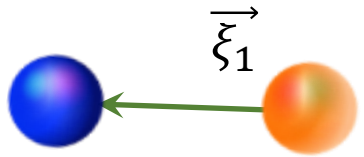
- Three-nucleon system



$$\begin{cases} \vec{\xi}_0 = \frac{1}{\sqrt{3}}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3) \\ \vec{\xi}_1 = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) \\ \vec{\xi}_2 = \frac{\sqrt{2}}{\sqrt{3}}\left(\frac{1}{2}(\vec{r}_1 + \vec{r}_2) - \vec{r}_3\right) \end{cases} \qquad \begin{cases} \vec{r}_1 = \frac{1}{\sqrt{3}}\vec{\xi}_0 + \frac{1}{\sqrt{2}}\vec{\xi}_1 + \frac{\sqrt{2}}{\sqrt{6}}\vec{\xi}_2 \\ \vec{r}_2 = \frac{1}{\sqrt{3}}\vec{\xi}_0 - \frac{1}{\sqrt{2}}\vec{\xi}_1 + \frac{\sqrt{2}}{\sqrt{6}}\vec{\xi}_2 \\ \vec{r}_3 = \frac{1}{\sqrt{3}}\vec{\xi}_0 - \frac{\sqrt{2}}{\sqrt{3}}\vec{\xi}_2 \end{cases}$$

Two-nucleon system

The NN interaction is a rank 2 tensor we need an antisymmetrized two-nucleon state basis



$$= \vec{r}_1 - \vec{r}_2$$

1. Start with two-body basis states (LS coupled)

$$\langle \boldsymbol{\xi}_1 \boldsymbol{\sigma}_1 \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \boldsymbol{\tau}_2 | \epsilon l_{12} s_{12} j_{12} t_{12} \rangle$$

$$= R_{\epsilon l_{12}}(\xi_1) \left[Y_{l_{12}}(\boldsymbol{\xi}_1) \left[\chi_{\frac{1}{2}}(\boldsymbol{\sigma}_1) \chi_{\frac{1}{2}}(\boldsymbol{\sigma}_2) \right]^{s_{12}} \right]^{j_{12}} \left[\chi_{\frac{1}{2}}(\boldsymbol{\tau}_1) \chi_{\frac{1}{2}}(\boldsymbol{\tau}_2) \right]^{t_{12}}$$

2. Keep only antisymmetric part of the basis (- sign if two nucleons are permuted)

$$\hat{P}_{12} | \epsilon l_{12} s_{12} j_{12} t_{12} \rangle = (-1)^{(l_{12}+s_{12}+t_{12})} | \epsilon l_{12} s_{12} j_{12} t_{12} \rangle$$

$$\Longrightarrow (-1)^{l_{12}+s_{12}+t_{12}} = -1$$

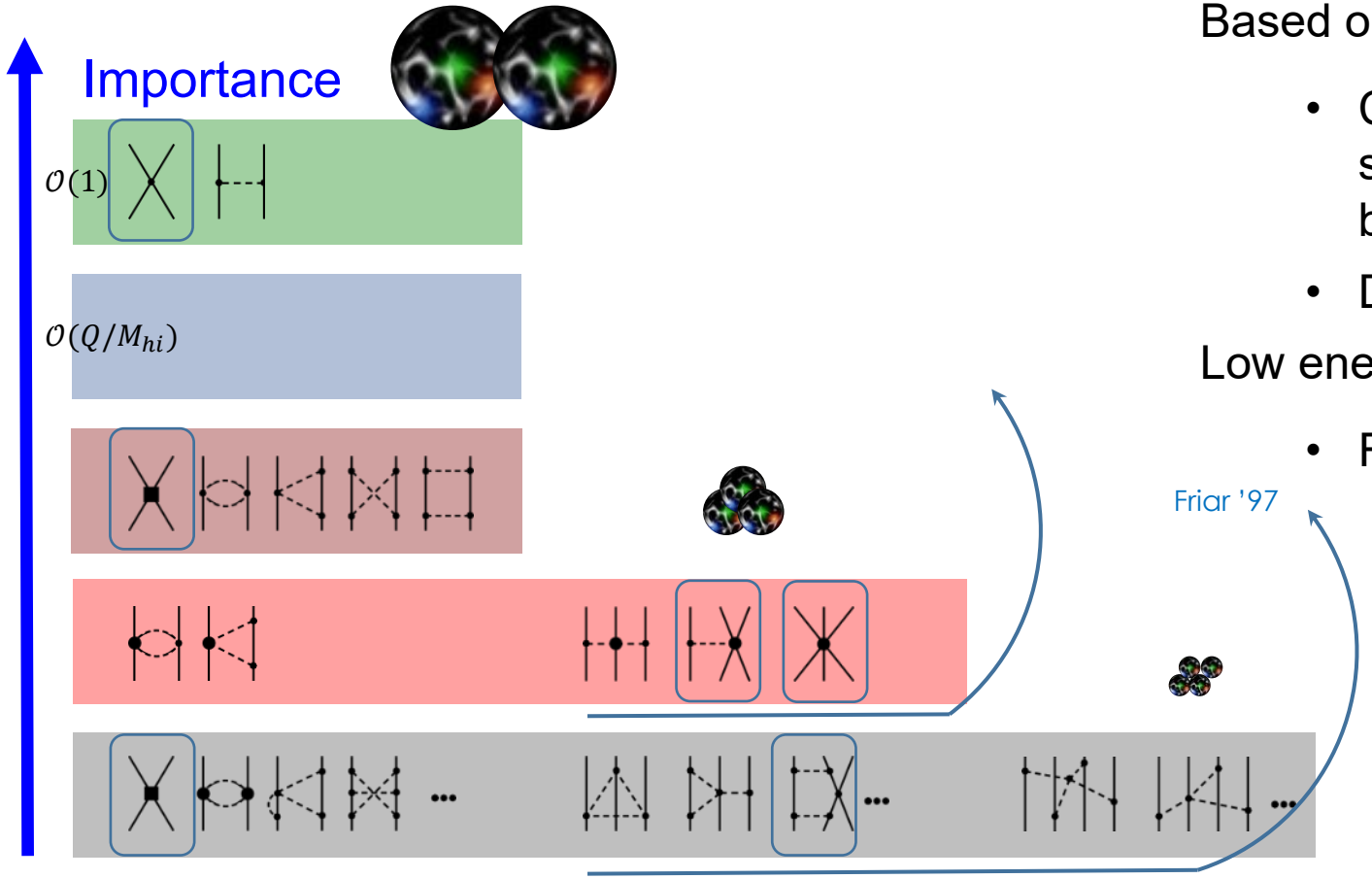
$$\begin{cases} s_{12} \in \{0,1\} \\ t_{12} \in \{0,1\} \\ \pi = (-1)^{l_{12}} \end{cases}$$

3. Nucleon-nucleon channels: ${}^{2s_{12}+1}l_{j_{12}}$

S	T	n/p state	$J=0$	$J=1$	$J=2$
0	1	nn, pp, (np + pn)	${}^1\mathrm{S}_0$	⛔	${}^1\mathrm{D}_2$
1	0	(np − pn)	⛔	${}^3\mathrm{S}_1, {}^3\mathrm{D}_1$	⛔
0	0	(np − pn)	⛔	${}^1\mathrm{P}_1$	⛔
1	1	nn, pp, (np + pn)	${}^3\mathrm{P}_0$	⛔	${}^3\mathrm{P}_2, {}^3\mathrm{F}_2$

Chiral Effective Field Theory

Weinberg '90'91'92 Ordóñez and vK '92 etc…



Inter-nucleon interactions can be derived from chiral effective field theory:

Based on the symmetries of QCD

- Chiral symmetry of QCD $(m_u \approx m_d \approx 0)$, spontaneously broken with π as the Goldstone boson
- Degrees of freedom: nucleons and pions

Low energy constants (LEC)

- Fitted to scattering data, deuteron, ^{3}He and triton.

- Systematic low-momentum expansion to a given order (Q/Λ);
- Hierarchy;
- Consistency.

The NN interaction from chiral EFT

Nowadays chiral NN potential up to N^{5}LO

Set of six potentials constructed

- Sequence of LO, NLO,…,N^{5}LO
- Uncertainty quantification

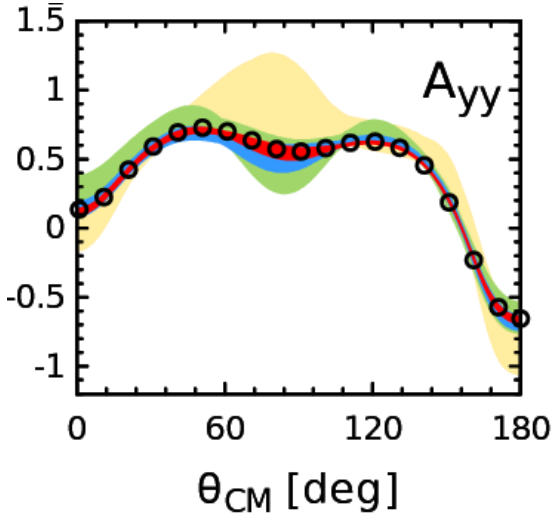
At N^{3}LO and N^{4}LO:

- 24 NN LECs fitted to the *np* and *pp* scattering data and the deuteron properties

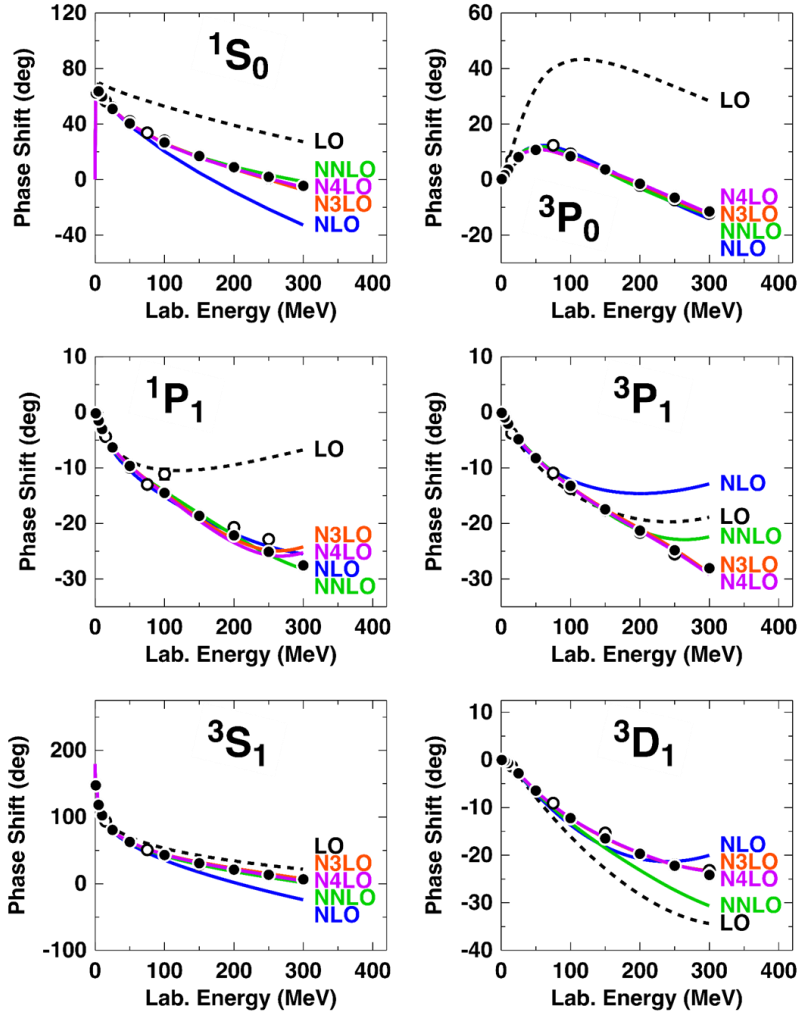
  Including c_i LECs $(i = 1 - 4)$ from pion-nucleon scattering or Rey-Steiner equation/pionic atoms

- 2 3N LECs (at N^{2}LO)

N^{4}LO NN fitted to data up to pion production threshold with χ^2/datum~1.15



E. Epelbaum, *et al.* PRL115 (2015)



D.R. Entem *et al.* PRC96 (2017)

Solver in CI with Jacobi coordinate system

The Harmonic-Oscillator (HO) basis expansion:

$$R_{\epsilon l_{12}}(\xi_1) = \sum_{n_{12}\geq 0} x_n R_{n_{12} l_{12}}(\xi_1)$$

The A-body basis can be build adding one particle at a time thank to the gaussian generator function of the HO wave function.

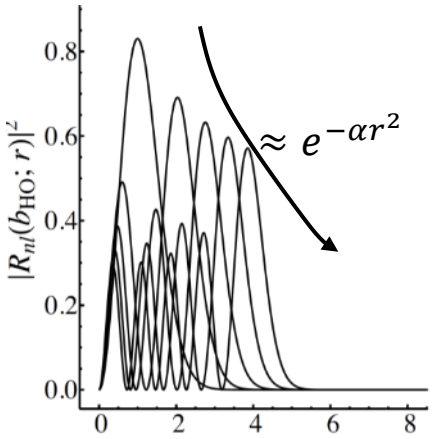
Antisymmetrized two-nucleon states

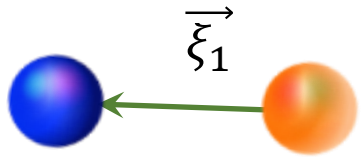
- Start with two-body basis states (LS coupled)

$$\langle \boldsymbol{\xi}_1 \boldsymbol{\sigma}_1 \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \boldsymbol{\tau}_2 | n_{12} l_{12} s_{12} j_{12} t_{12} \rangle$$
$$= R_{n_{12} l_{12}}(\xi_1) \left[Y_{l_{12}}(\boldsymbol{\xi}_1) \left[\chi_{\frac{1}{2}}(\boldsymbol{\sigma}_1) \chi_{\frac{1}{2}}(\boldsymbol{\sigma}_2) \right]^{s_{12}} \right]^{j_{12}} \left[\chi_{\frac{1}{2}}(\boldsymbol{\tau}_1) \chi_{\frac{1}{2}}(\boldsymbol{\tau}_2) \right]^{t_{12}}$$

- This wave function is solution of the traditional HO Hamiltonian $H = \frac{p^2}{2m} + \frac{m}{2}\Omega^2 r^2$; very simple to work either in momentum or position basis
- Total energy

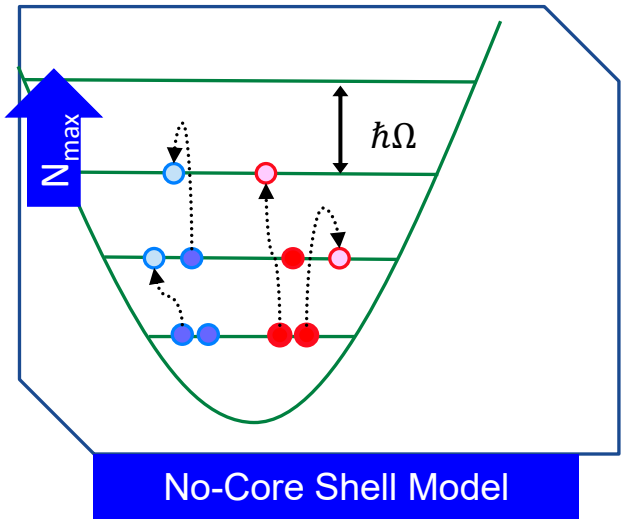
$$\epsilon_N = \left(N + \frac{3}{2} \right) \hbar\Omega; \quad N = 2n_{12} + l_{12}; \quad N \leq N_{\max}$$





$= \vec{r}_1 - \vec{r}_2$

$R_{n_{12}}(\xi_1)$ depends on HO length $b = \sqrt{\hbar/m\Omega}$



Solver in CI with Jacobi coordinate system

Add one more nucleon

$$\left\langle \boldsymbol{\xi}_2 \boldsymbol{\sigma}_3 \boldsymbol{\tau}_3 \middle| n_3 l_3 j_3 \frac{1}{2} \right\rangle = R_{n_3 l_3}(\xi_2) \left[Y_{l_3}(\boldsymbol{\xi}_3) \chi_{\frac{1}{2}}(\boldsymbol{\sigma}_3) \right]^{j_3} \chi_{\frac{1}{2}}(\boldsymbol{\tau}_3)$$

Three-nucleon basis (JJ coupled)

$$\langle \boldsymbol{\xi}_1 \boldsymbol{\xi}_2 \boldsymbol{\sigma}_1 \boldsymbol{\sigma}_2 \boldsymbol{\sigma}_3 \boldsymbol{\tau}_1 \boldsymbol{\tau}_2 \boldsymbol{\tau}_3 | n_{12} l_{12} s_{12} j_{12} t_{12}, n_3 l_3 j_3 t_3, JT \rangle$$

$$|n_{12} l_{12} s_{12} j_{12} t_{12}, n_3 l_3 j_3 t_3, JT\rangle = \sum_{m_{12} m3} C^{JM}_{j_{12} m_{12}, j_3 m_3}$$

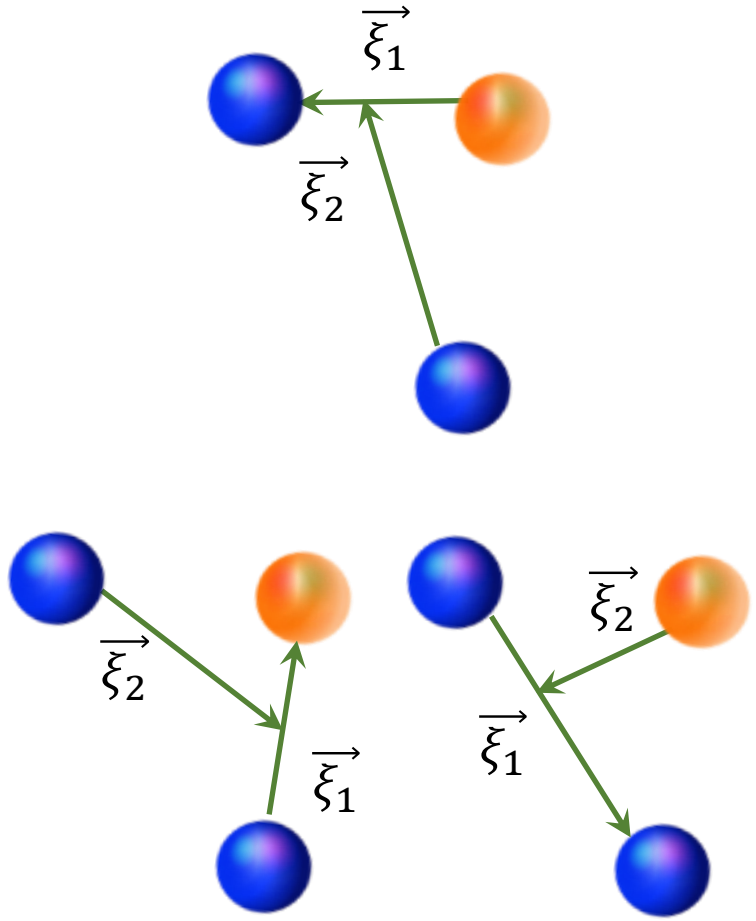
$$\sum_{m_{12}^t m_3^t} C^{TM^t}_{t_{12} m_{12}^t, t_3 m_3^t} \left| n_{12} l_{12} s_{12} j_{12} t_{12} \right\rangle \left| n_3 l_3 j_3 \frac{1}{2} \right\rangle$$

Total energy:

$$\epsilon_N = (N+3)\hbar\Omega; \quad N = 2n_{12} + l_{12} + 2n_3 + l_3$$

To find totally antisymmetric states, diagonalize:

$$\hat{A} = \frac{1}{3}(1 - \hat{P}_{13} - \hat{P}_{23})$$

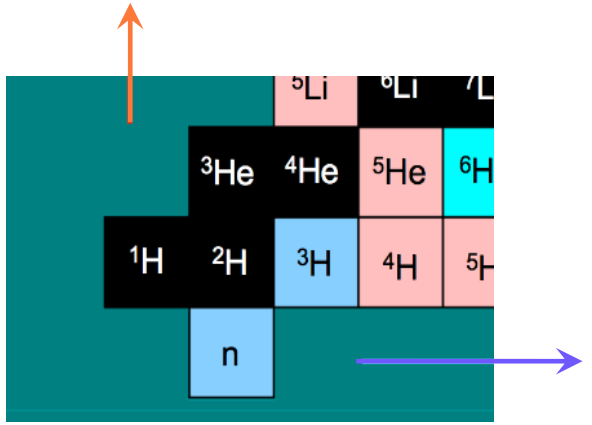


How to solve the many-body Schrödinger equation?

The nuclear wave function must factorize the center of mass motion.

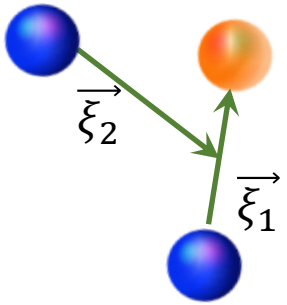
$$\Psi_{\epsilon}^{(A)}(\vec{r_1}\dots,\vec{r_i},\dots,\vec{r_j},\dots,\vec{r_A}) = \Psi_{\epsilon}^{(A)}(\vec{\xi_1}\dots,\vec{\xi_j},\dots,\overrightarrow{\xi_{A-1}})\Psi_{\epsilon}(\vec{R}_{\text{C.M.}})$$

$$\Psi_{\epsilon}(\vec{R}_{\text{C.M.}}) = \exp\left(-\frac{i\boldsymbol{P}_{CM}\boldsymbol{R}_{CM}}{\hbar}\right) \text{ and } E = E_{CM} + \frac{P_{CM}^2}{2Am}$$



The few-body option (in first quantization) is to solve eigenvalue problem for the intrinsic Hamiltonian ($\widehat{H}_{int}$)

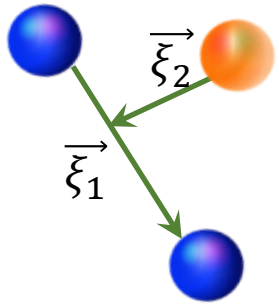
- The C.M. motion is not present.
- We work with one-less spatial coordinates, i.e. 3(*A*-1) spatial degrees of freedom
- PROBLEM: It is very complicated to restrict the problem to the antisymmetric part of the Hamiltonian since Jacobi coordinates do not treat the nucleons in a symmetric way, i.e. $\mathrm{P_{ij}}\Psi_{\epsilon}^{(A)}(\vec{\xi_1}\dots,\vec{\xi_j},\dots,\overrightarrow{\xi_{A-1}}) = \sum_{\epsilon'} R_{\epsilon\epsilon'}\Psi_{\epsilon'}^{(A)}(\vec{\xi_1}\dots,\vec{\xi_j},\dots,\overrightarrow{\xi_{A-1}})$



$$\begin{cases} \vec{\xi_1} = \frac{1}{\sqrt{2}}(\vec{r_1} - \vec{r_2}) \\ \vec{\xi_2} = \frac{\sqrt{2}}{\sqrt{3}}\left(\frac{1}{2}(\vec{r_1} + \vec{r_2}) - \vec{r_3}\right) \end{cases}$$

$2 \leftrightarrow 3$

$$\begin{cases} \vec{\xi_1'} = \frac{1}{\sqrt{2}}(\vec{r_1} - \vec{r_3}) \\ \vec{\xi_2'} = \frac{\sqrt{2}}{\sqrt{3}}\left(\frac{1}{2}(\vec{r_1} + \vec{r_3}) - \vec{r_2}\right) \end{cases}$$



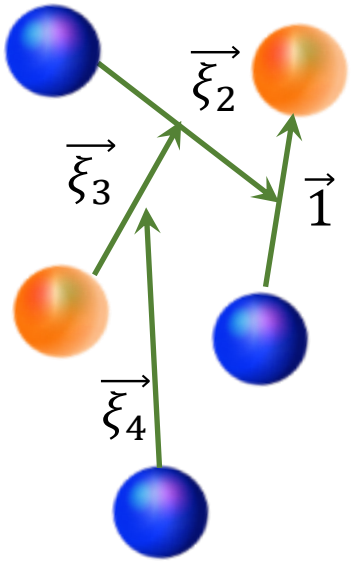
Jacobi relative coordinates for A nucleons

The vector proportional to the C.M. coordinate

$$\begin{cases} \vec{\xi}_0 = \sqrt{\frac{1}{A}} \sum_{i=1}^{A} \vec{r}_i \\ \vec{R}_{C.M.} = \sqrt{\frac{1}{A}} \vec{\xi}_0 \end{cases}$$

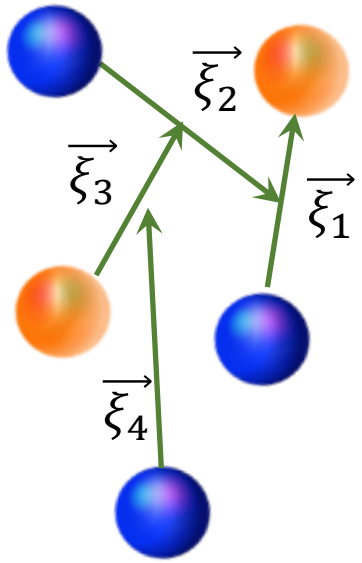
The relative coordinate between the C.M. of the first k nucleons and the $(k+1)$th nucleon

$$\begin{cases} \vec{\xi}_1 = \frac{1}{\sqrt{2}} (\vec{r}_1 - \vec{r}_2) \\ \vdots \\ \vec{\xi}_j = \sqrt{\frac{j}{j+1}} \left[\frac{1}{j} \sum_{i=1}^{j} \vec{r}_i - \vec{r}_{j+1} \right] \\ \vdots \\ \vec{\xi}_{A-1} = \sqrt{\frac{A-1}{A}} \left[\frac{1}{A-1} \sum_{i=1}^{A-1} \vec{r}_i - \vec{r}_A \right] \end{cases}$$



Jacobi relative coordinates for A nucleons

$$\begin{cases} \vec{\xi}_1 = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) \\ \vdots \\ \vec{\xi}_j = \sqrt{\frac{j}{j+1}}\left[\frac{1}{j}\sum_{i=1}^{j}\vec{r}_i - \vec{r}_{j+1}\right] \\ \vdots \\ \vec{\xi}_{A-1} = \sqrt{\frac{A-1}{A}}\left[\frac{1}{A-1}\sum_{i=1}^{A-1}\vec{r}_i - \vec{r}_A\right] \end{cases}$$



$$\begin{cases} \vec{r}_1 = \frac{1}{\sqrt{A}}\vec{\xi}_0 + \sum_{i=1}^{A-1}\frac{1}{\sqrt{i(i-1)}}\vec{\xi}_i \\ \vdots \\ \vec{r}_j = \frac{1}{\sqrt{A}}\vec{\xi}_0 - \sqrt{\frac{j-1}{j}}\vec{\xi}_{j-1} + \sum_{i=j}^{A-1}\frac{1}{\sqrt{i(i-1)}}\vec{\xi}_{i-1} \\ \vdots \\ \vec{r}_A = \frac{1}{\sqrt{A}}\vec{\xi}_0 - \sqrt{\frac{A-1}{A}}\vec{\xi}_{A-1} \end{cases}$$

Transformation is orthogonal and has Jacobian $J = 1$

$$\int \Psi_\epsilon^{(A)^*}(\vec{r}_1, \dots, \vec{r}_A)\hat{O}\Psi_\epsilon^{(A)}(\vec{r}_1, \dots, \vec{r}_A)d\vec{r}_1, \dots, d\vec{r}_A = \int \Psi_\epsilon^{(A)^*}(\vec{\xi}_0, \dots, \vec{\xi}_{A-1})\hat{O}\Psi_\epsilon^{(A)}(\vec{\xi}_0, \dots, \vec{\xi}_{A-1})d\vec{\xi}_0, \dots, d\vec{\xi}_{A-1}$$

For Harmonic Oscillator wave function the transformation is analytic iteratively:

$$\varphi_{n_1 l_1}(\vec{r}_1)\varphi_{n_2 l_2}(\vec{r}_2) = \sum_{nlNL}\{NL, nl, \Lambda; n_1 l_1, n_2 l_2 \Lambda\}\,\varphi_{NL}(\vec{\xi}_0)\varphi_{nl}(\vec{\xi}_1)$$

How to solve the many-body Schrödinger equation

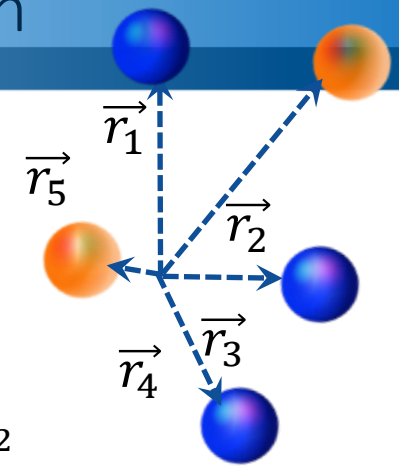
The many-body option (second quantization) is to solve eigenvalue problem for the intrinsic Hamiltonian plus center of mass Hamiltonian ($\hat{H}_{int}$+ $\hat{H}_{C.M.}$)

$$\hat{H}^A = \sum_i^A \frac{p_i^2}{2m} + V^{MF}(r_i) + \sum_{i>j=1}^A V^{NN}(r_{ij}) - \sum_i^A V^{MF}(r_i)$$

The Mean-Field Hamiltonian is a the auxiliary one-body fields (φ) to approximate the energy of the system it is obtained as solution of $\left(\frac{p^2}{2m} + V_k^{MF}(\boldsymbol{r})\right)\varphi_k(\boldsymbol{r}) = \epsilon_k \varphi_k(\boldsymbol{r})$

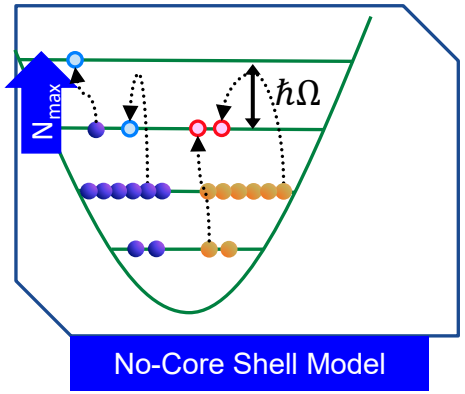
The total wave function is then an antisymmetrized product state $\Psi_\epsilon^{(A)} \approx \mathcal{N}\hat{\mathcal{A}}\varphi_1(\boldsymbol{r}_1)\ldots\varphi_A(\boldsymbol{r}_A)$. This basis is complete and orthonormalized. It can be cast as

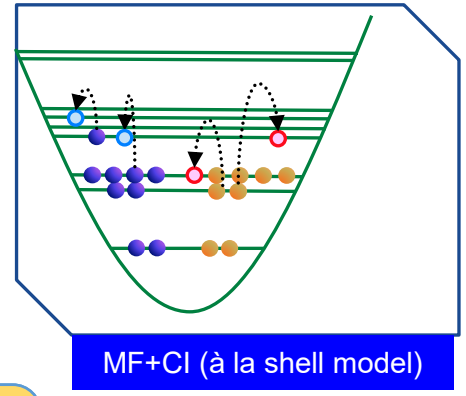
$$\Psi_\epsilon^{(A)} \approx \frac{1}{\sqrt{A!}}\begin{bmatrix} \varphi_1(\boldsymbol{r_1}) & \cdots & \varphi_A(\boldsymbol{r_1}) \\ \vdots & \ddots & \vdots \\ \varphi_1(\boldsymbol{r_A}) & \cdots & \varphi_A(\boldsymbol{r_A}) \end{bmatrix}$$



$$H_i^{MF} = \frac{p_i^2}{2m} + \frac{m}{2}\Omega^2 r_i^2$$

$$H_i^{MF} = \frac{p^2}{2m} + V^{MF}(\boldsymbol{r}_i)$$





The mean field determines at first order the shell structure.

How to solve the many-body Schrödinger equation

The mean-field basis is the best naïve CI basis to be used for constructing the many-body Hamiltonian. It permits to minimize the particle-hole to account for in the CI space.
However, the mean-field potential V^{MF} renders the fields non-invariance with respect to translations. The symmetry breaking can find lower-energy solutions however one need to apply projector techniques:

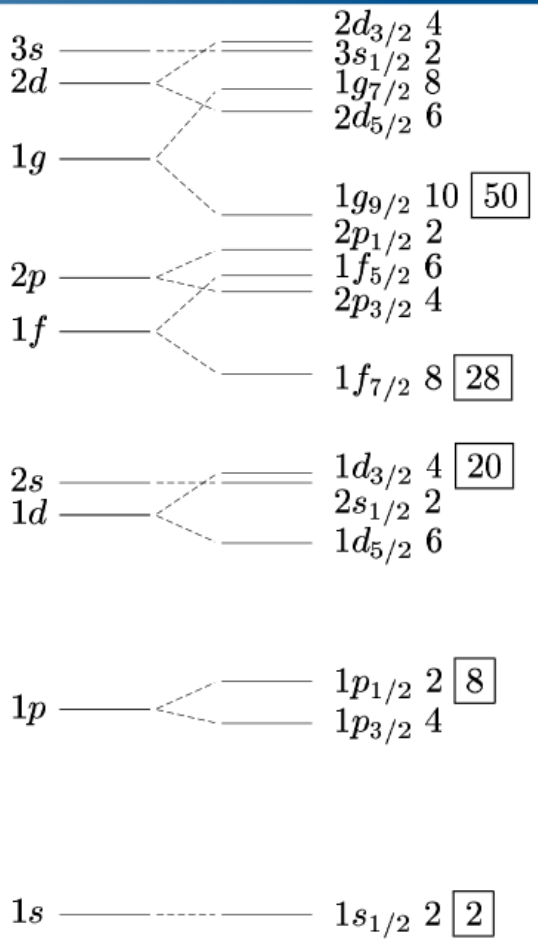
$$P^I = \int d\Omega D^I(\Omega)\hat{R}(\Omega) \text{ with } \begin{cases} D^I(\boldsymbol{a}) = \dfrac{1}{(2\pi\hbar)^3}\exp\left(-\dfrac{i}{\hbar}\boldsymbol{a}\cdot\boldsymbol{p}\right) \\ \hat{R}(\boldsymbol{a}) = \exp\left(-\dfrac{i}{\hbar}\boldsymbol{a}\cdot\hat{\boldsymbol{p}}\right) \end{cases}$$

If not, the C.M. motion is no longer separable:

$$\tilde{\Psi}^{(A)}_{\tilde{\epsilon}} = \sum_{\epsilon\epsilon'} \Psi^{(A)}_{\epsilon}\,\varphi_{\epsilon'}(\boldsymbol{R}_{C.M.})$$

The factorization for H_{int} only when **exact solution is obtained.**
The Harmonic Oscillator (HO) potential is exactly separable:

$$\sum_{i=1}^{A}\frac{m}{2}\Omega^2 r_i^2 = \sum_{i=1}^{A-1}\frac{m}{2}\Omega^2\xi_i^2 + A\frac{m}{2}\Omega^2 R^2_{C.M.}$$



Major shells (HO eigenbasis)

+spin orbit splitting

No-Core Shell Model (NCSM)

NCSM is an *ab initio* approach to solve the many-body Schrödinger equation for bound states starting from

- High-precision NN+3N interactions either in coordinate or momentum space; local or not.

Uses large and finite expansions in HO many-body basis states.

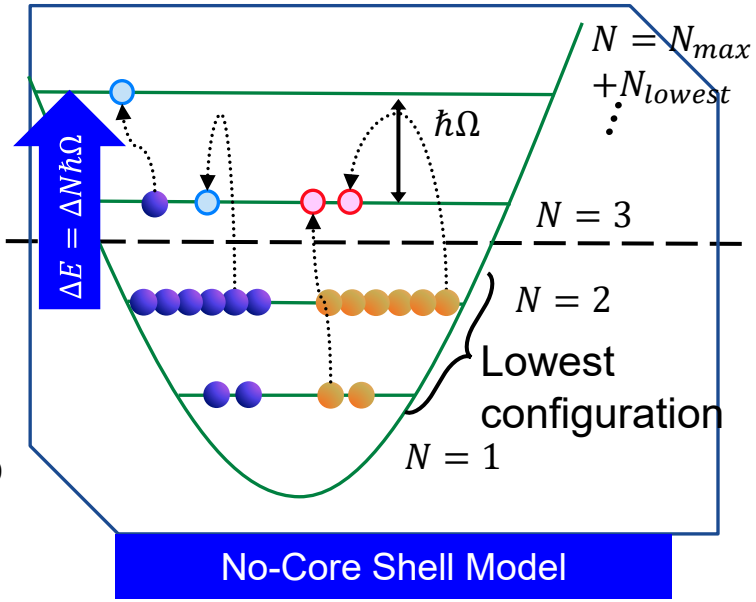
It works either in Jacobi (intrinsic frame) or single-particle coordinates (lab. frame). The former is more tedious for many-nucleon due to antisymetrization.

- Translational invariance of the internal wave function is preserved also when single-particle Slater Determinant (SD) basis is used with $N_{\max}$ truncation.

$$\Psi^{(A)}_{Jacobi} = |A\lambda J^{\pi} T\rangle = \sum_{N=0}^{N_{\max}} \sum_{i} c_{Ni} \; |Aij_z{}^{\pi} t_z\rangle = \sum_{N=0}^{N_{\max}} \sum_{i} c_{Ni} \Psi^{(A)}_{i} \left(\vec{\xi}_0, \ldots, \vec{\xi}_{A-1}\right)$$

$$\Psi^{(A)}_{\mathrm{SD}} = |A\lambda J^{\pi} T\rangle = \sum_{N=0}^{N_{\max}} \sum_{i} c_{Ni} \Psi^{(A)}_{\mathrm{SD},\epsilon}(\vec{r_1}, \ldots, \vec{r_A}) = \Psi^{(A)}_{Jacobi} \varphi_{nl}(\boldsymbol{R}_{C.M.})$$

The method is variational and so converge by above to exact result.



- $N_{\max}$ is the maximal allowed HO excitation above the lowest possible A-nucleon configuration
- Full $N_{\max}$ space: All basis states with $N \leq N_{\max}$ kept

Multi-particle states in the Slater Determinant basis

Many-body HO Slater determinants

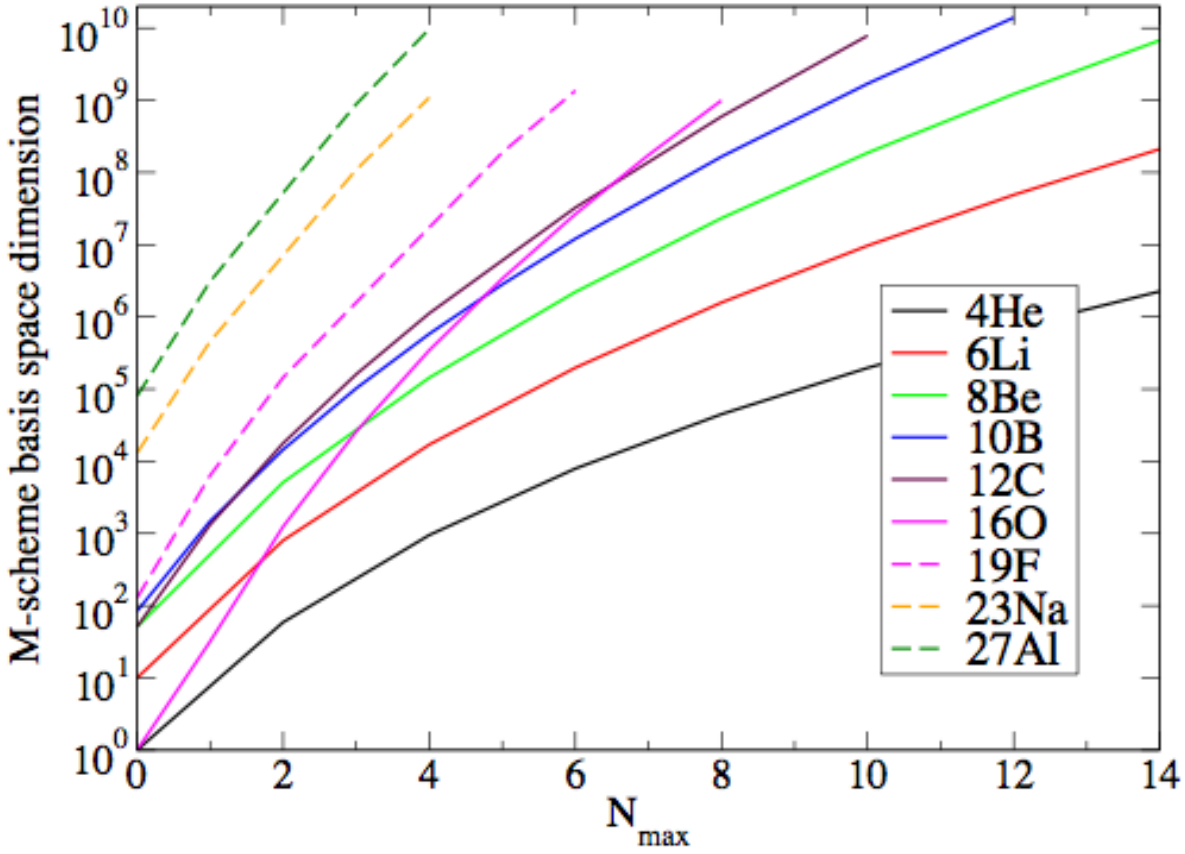
$$\Psi_{\epsilon}^{(\mathrm{A})} = \begin{cases} \mathcal{N}\hat{\mathcal{A}}\varphi_1(\mathbf{r_1}\boldsymbol{\sigma}_1\boldsymbol{\tau}_1)\ldots\varphi_{\mathrm{A}}(\mathbf{r_A}\boldsymbol{\sigma}_{\mathrm{A}}\boldsymbol{\tau}_{\mathrm{A}}) \\ \dfrac{1}{\sqrt{\mathrm{A}!}}\begin{bmatrix} \varphi_1(\mathbf{r_1}\boldsymbol{\sigma}_1\boldsymbol{\tau}_1) & \cdots & \varphi_{\mathrm{A}}(\mathbf{r_1}\boldsymbol{\sigma}_1\boldsymbol{\tau}_1) \\ \vdots & \ddots & \vdots \\ \varphi_1(\mathbf{r_A}\boldsymbol{\sigma}_{\mathrm{A}}\boldsymbol{\tau}_{\mathrm{A}}) & \cdots & \varphi_{\mathrm{A}}(\mathbf{r_A}\boldsymbol{\sigma}_{\mathrm{A}}\boldsymbol{\tau}_{\mathrm{A}}) \end{bmatrix} \end{cases}$$

The one-body state reads (are fixed!)

$$\varphi_{\mathrm{n}_1 l_1 j_1 m_j \frac{1}{2} m_t}(\mathbf{r_1}\boldsymbol{\sigma}_1\boldsymbol{\tau}_1) = R_{n_3 l_3}(r_1)\left[Y_{l_3}(\hat{r}_1)\chi_{\frac{1}{2}}(\boldsymbol{\sigma}_3)\right]^{j_3 m_j}\chi_{\frac{1}{2} m_t}(\boldsymbol{\tau}_3)$$

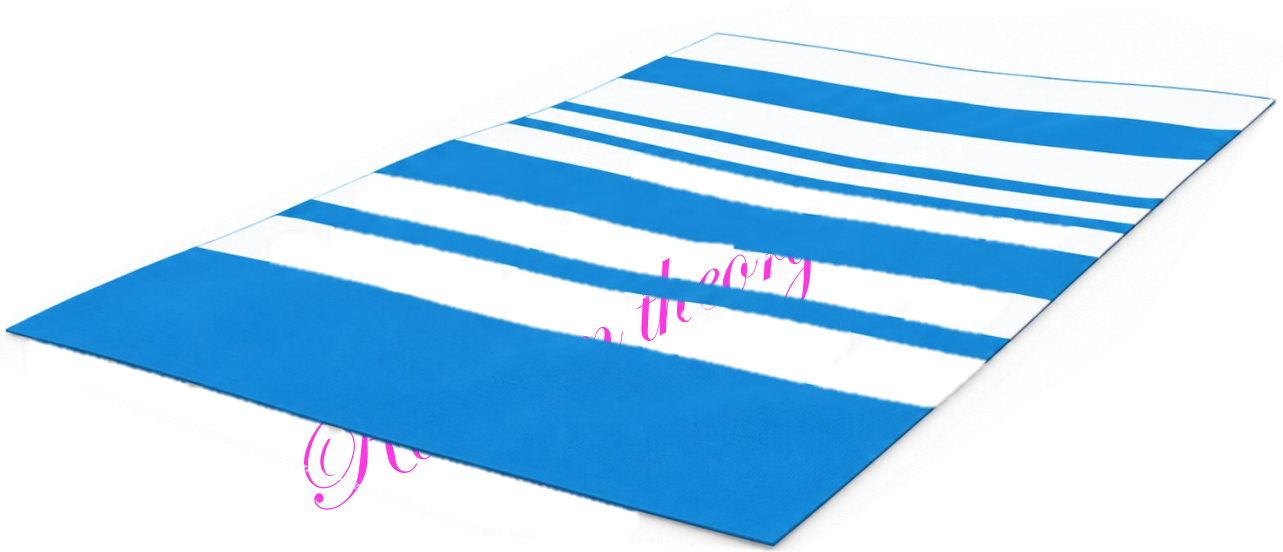
The total wave function is anti symmetric and has good M_J, M_T and parity quantum numbers, but not J and T. They are restored when the p-h truncation is consistent.

Very large number of basis state to reach accurate results (but sparse matrices).



Courtesy of J.P. Vary and P. Maris

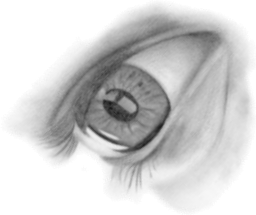
Reaction theory



Nuclear structure

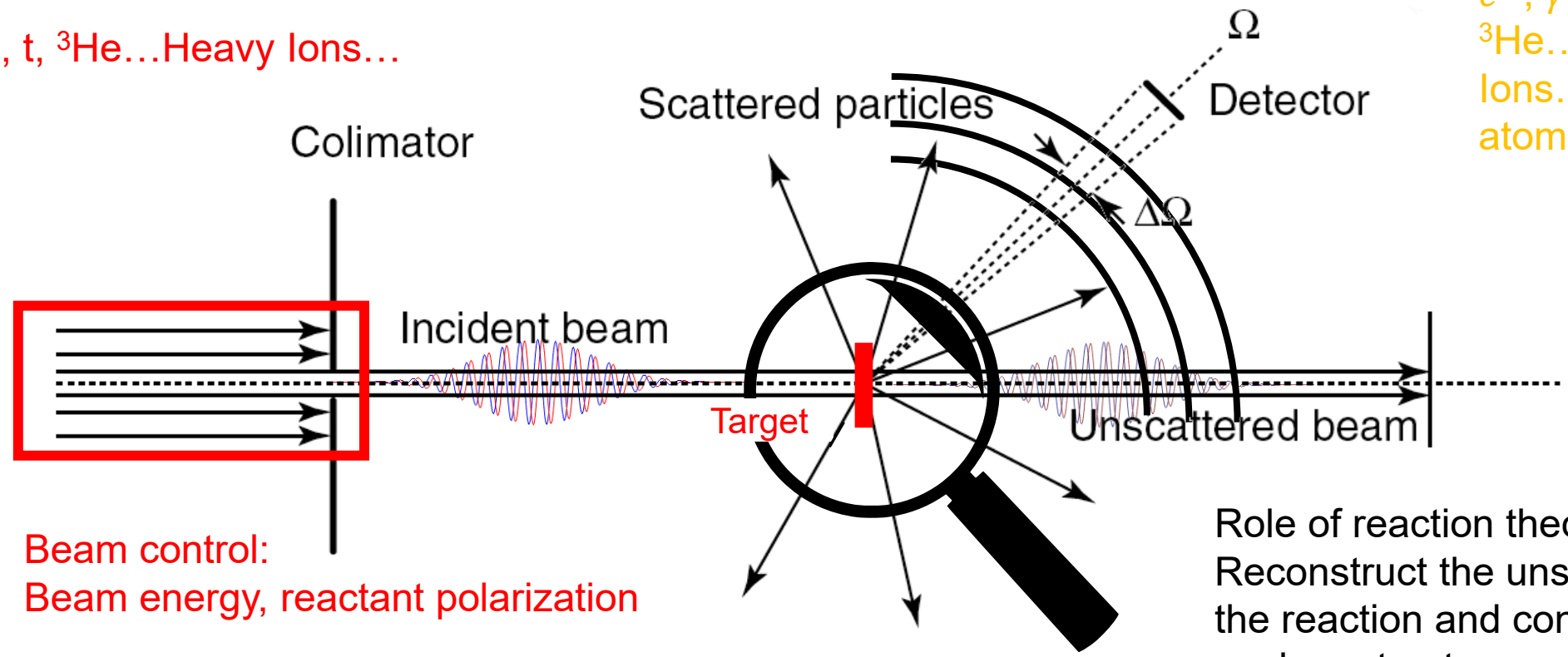
A nuclear reaction in the laboratory

Typical experiment:



e^-, γ, p, n, t, ^{3}He…Heavy Ions…

e^-, γ, p, n, t, ^{3}He…Heavy Ions…polarimeter, atomic transitions…



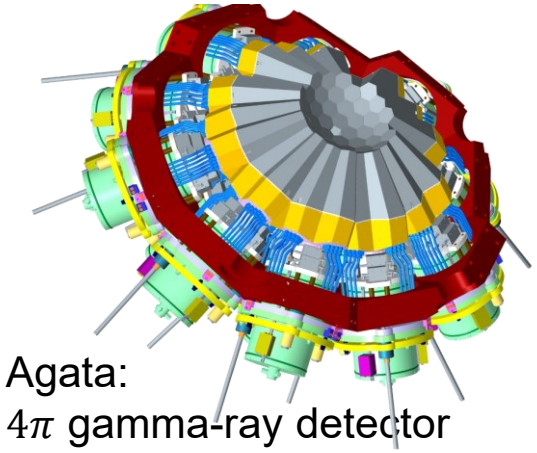
Beam control:
Beam energy, reactant polarization

Role of reaction theory:
Reconstruct the unseen history of the reaction and connect to nuclear structure properties

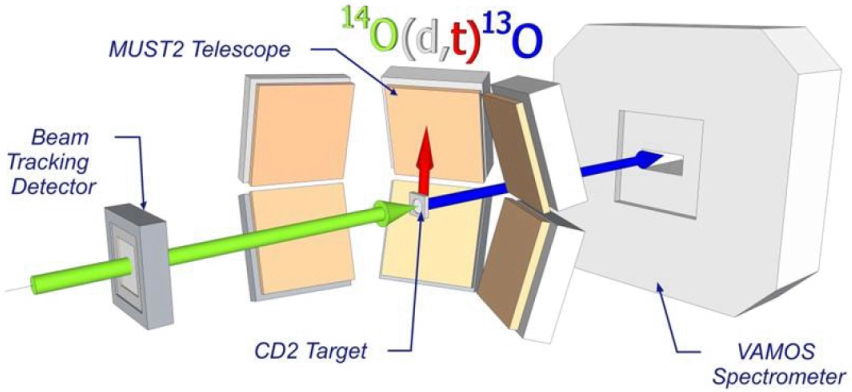
Today's nuclear reactions in the laboratory

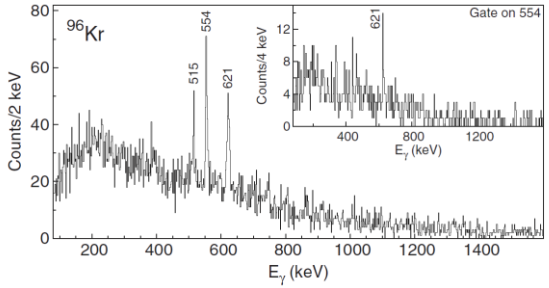
An example of experiment:

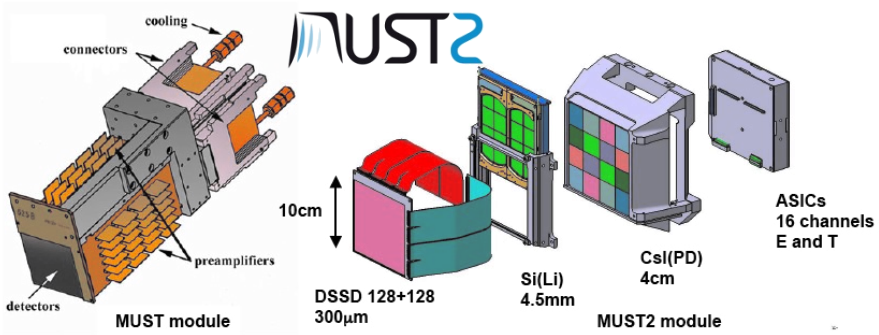
- Determination of the nature of the products (PID)
- Energy and angular resolution
- Momentum distributions
- γ measurements

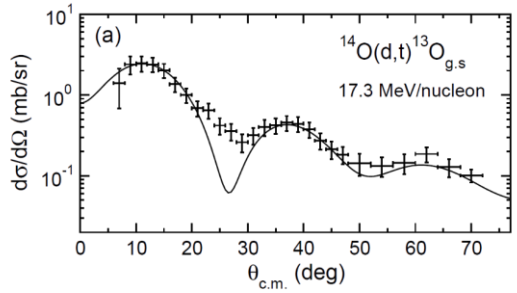


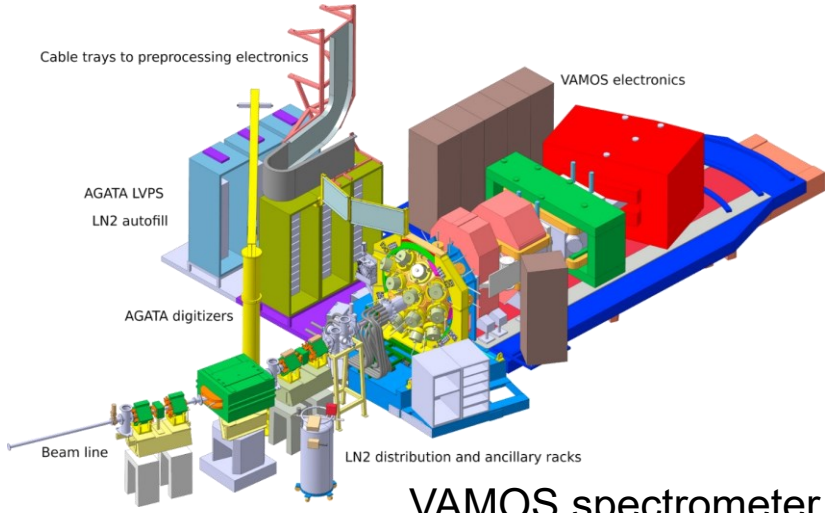
Agata:
4π gamma-ray detector
FWHM 6keV @ 1 MeV
ε~43%







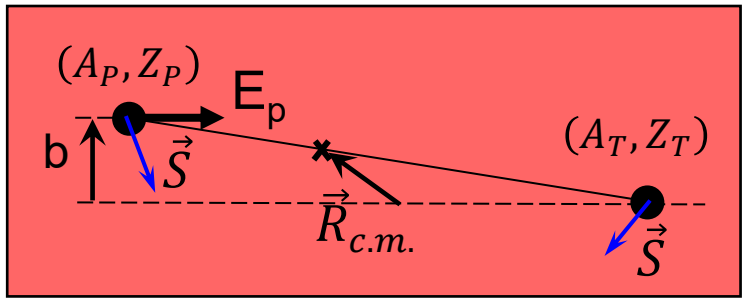




VAMOS spectrometer

Classification of reactions

Entrance channel



Exit channel

Inclusive/exclusive
Strongly detectors dependent

No energy transfer

$A + a \rightarrow A + a$ — Elastic scattering $A(a,a)A$

$A + \vec{a} \rightarrow \vec{A} + a$ — Spin transfer $A(\vec{a},a)\vec{A}$

Energy transfer

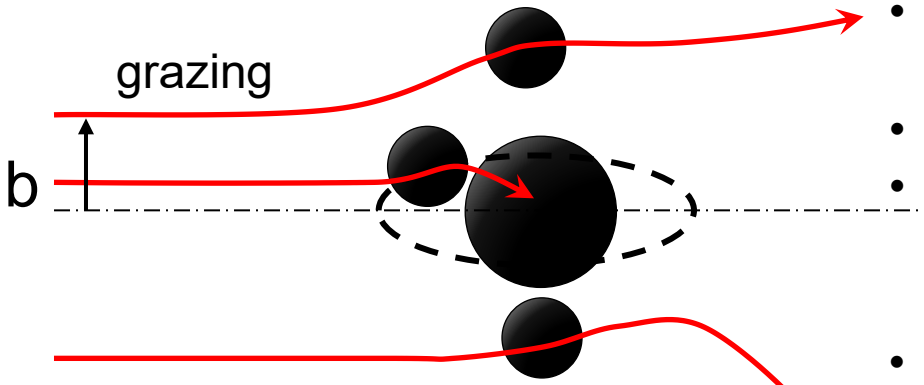
$A + a \rightarrow B + b$ — Transfer reaction $A(a,b)B$

$A + a \rightarrow A + a + \gamma$ — Nuclear Bremsstrahlung

$A + a \rightarrow B^* + b^* + \cdots$ — Inelastic scattering
Deeply inelastic…

$A + a \rightarrow C^* \rightarrow \text{decay}$ — Compound nucleus

Nuclear reactions by classifications

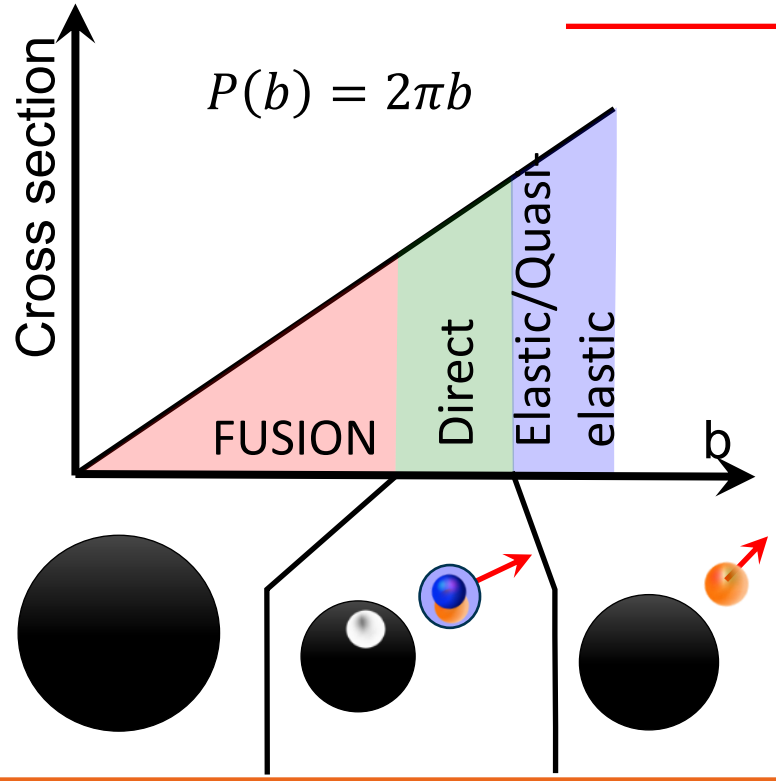


- Direct reaction
- Elastic and quasi-elastic

- Fusion
- Large amplitude Collective motion

- Multi-nucleon transfer
- Deep inelastic

Adapted from : W. Nörenberg and H.A. Weidenmüller, “Introduction to Heavy-Ion theory”, Springer-Verlag 1981.

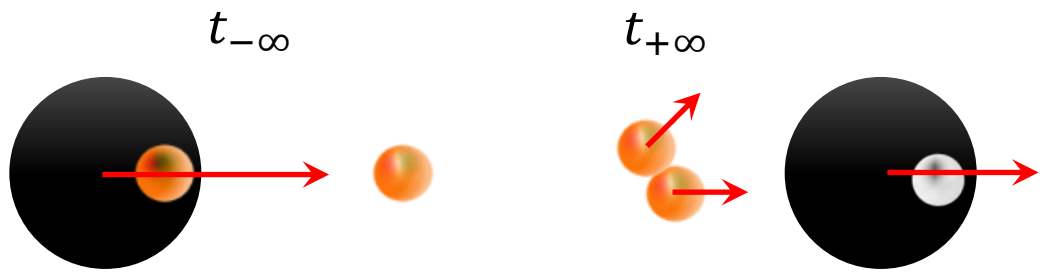


The more peripheral is the collision the highest is the associated cross-section

From reaction observables to nuclei

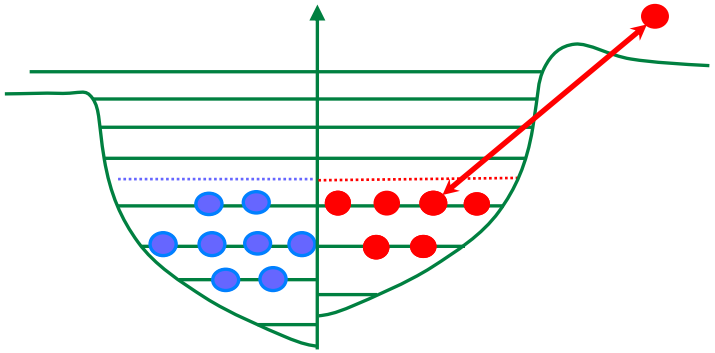
Nuclear structure

Knockout reactions (high energy)



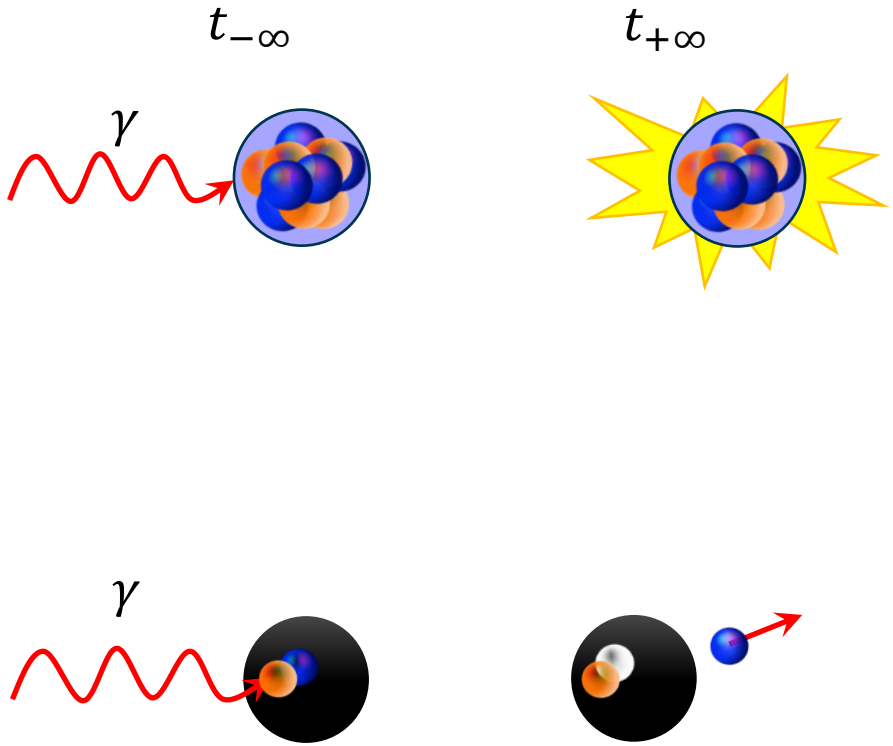
Transfer reactions (low energy)

Spectroscopic information

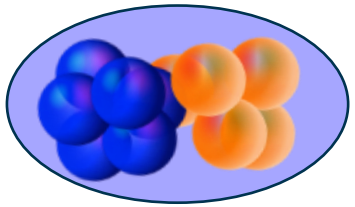


Nuclear spectroscopy

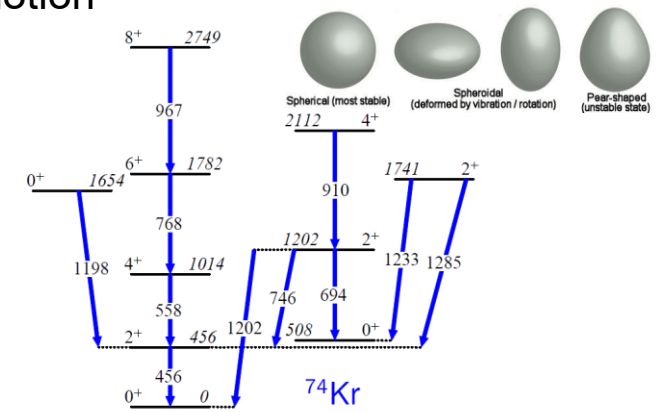
Photonuclear reactions



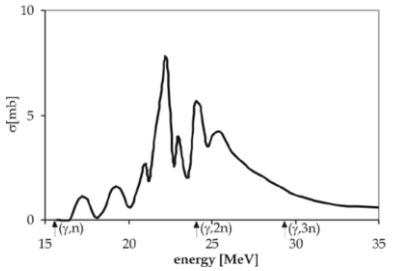
GDR excitation



Collective motion



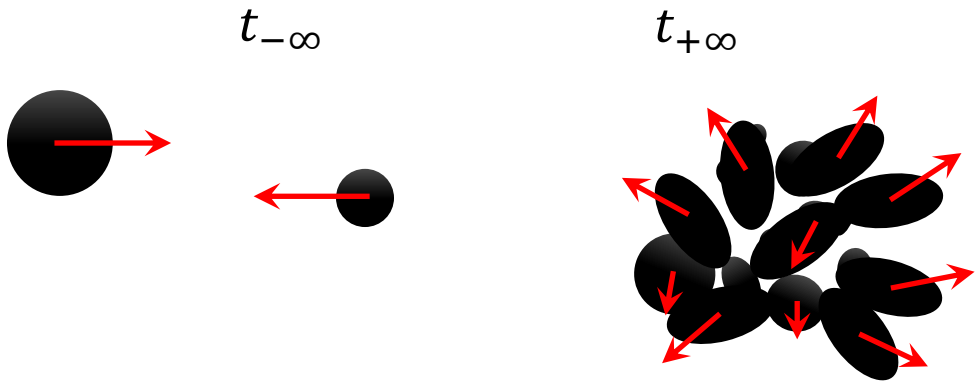
Bremsstrahlung, radiative capture.
Nuclear force imprints low-energy resonances.
Sensitive to off-shellness.

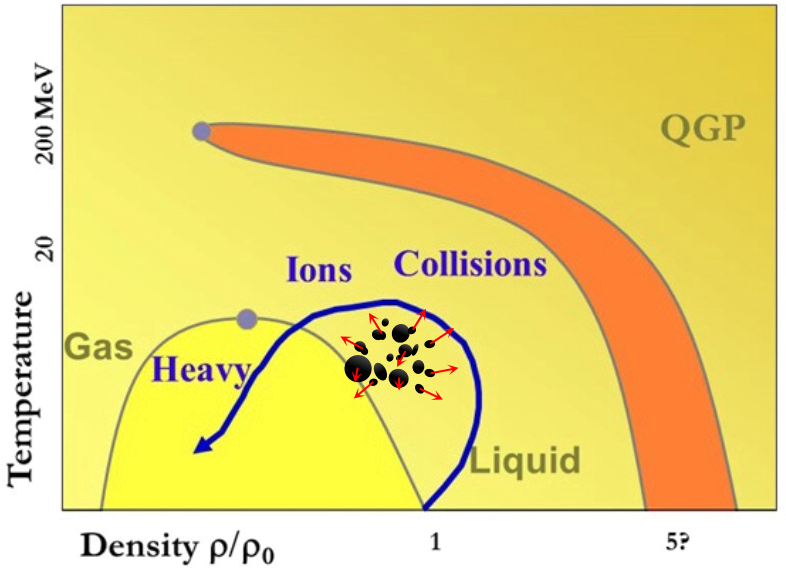


From reaction observables to nuclei

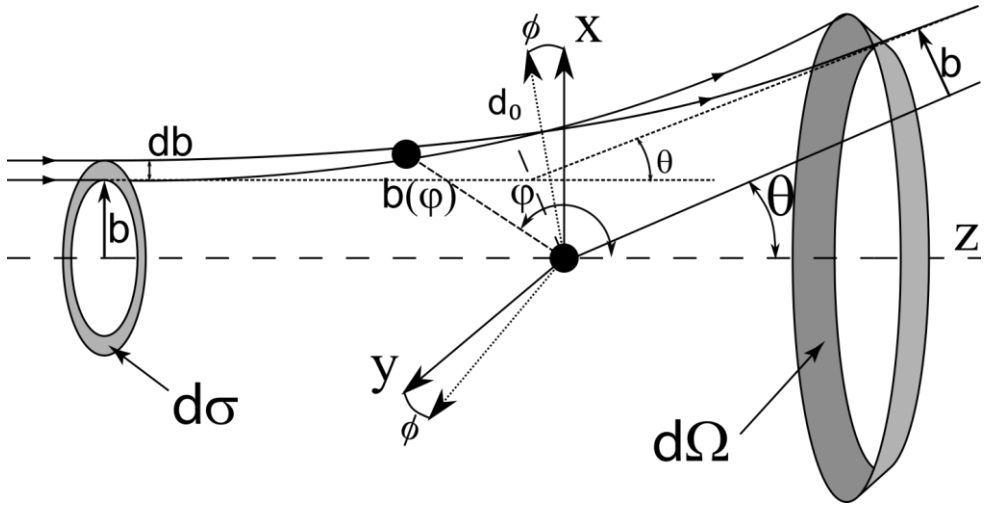
Out of equilibrium properties (away from nuclear density)

Reactions at Fermi energy





Definitions of cross-section and other quantities



Differential cross-section

$$\frac{d\sigma_{\text{channel}}(\theta,\varphi)}{d\Omega} = \frac{(\#\text{ scattered particle}.s^{-1})(\theta,\varphi)}{(\text{Inc. flux})(\#\text{ scatt. centers})(\text{unit of }\Omega)}$$

$$\xrightarrow{n=1} \frac{j_f(\theta,\varphi)}{j_i}$$

We define the outgoing solution

$$|\psi^+\rangle = |\phi_i\rangle + |\phi_f\rangle$$

$$\psi_k^+(\vec{r}) \xrightarrow{r\to\infty} A\left(e^{i\vec{k}.\vec{r}} + f(\Theta,\varphi)\frac{e^{ikr}}{r}\right)$$

The flux is given by

$$\vec{J} = \frac{\hbar}{2\mu i}\left((\psi_k^+)^*\nabla\psi_k^+ - \psi_k^+\nabla(\psi_k^+)^*\right)$$

We find the cross section to be:

$$\frac{d\sigma(\theta,\varphi)}{d\Omega} = |f(\Theta,\varphi)|^2$$

Wave effects

$$|\psi^+\rangle = |\phi_i\rangle + |\phi_f\rangle$$

$$\psi_k^+(\vec{r}) \xrightarrow{r\to\infty} A\left(e^{i\vec{k}.\vec{r}} + f(\Theta,\varphi)\frac{e^{ikr}}{r}\right)$$

Quantum current is defined as

$$\vec{J} = \frac{\hbar}{2\mu i}\left((\psi_k^+)^*\nabla\psi_k^+ - \psi_k^+\nabla(\psi_k^+)^*\right)$$

$$\vec{J} = \vec{j}_i + \vec{j}_f + \vec{j}_{\text{int}}$$

We obtain:

$$\begin{cases} j_f(\theta,\varphi) = \vec{J}\cdot\hat{r}\,r^2 \\ j_i = \vec{J}\cdot\hat{z} \end{cases}$$

$$j_f(\theta,\varphi) = v|A|^2|f(\Theta,\varphi)|^2$$

$$j_i = v|A|^2$$

$$j_{\text{int}} = 0 \text{ for } \theta \neq 0$$

$^1/_r$ leading contributions

For $\theta = 0$

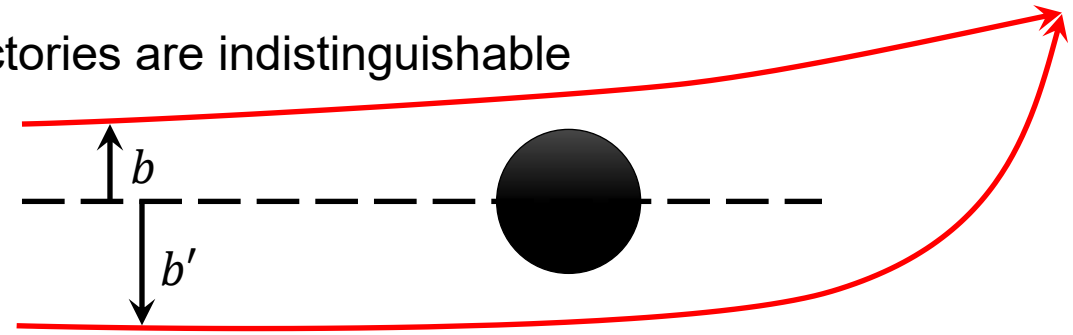
$\Phi_{\text{int}} = r^2\int d\Omega\, j_{\text{int}} = -\frac{4\pi\hbar}{m}\Im(f(0))$ Since the flux ($\Phi_{\text{int}} + \Phi_{sc} = 0$) is conserved we obtain that $\sigma_{tot} = \frac{4\pi}{k}\Im(f(0))$

$\vec{j}_{\text{int}}$ represents the flux removed from the incident beam by destructive interference in the forward direction. That lost forward flux exactly accounts for the flux scattered out of the beam.

Quantum effects

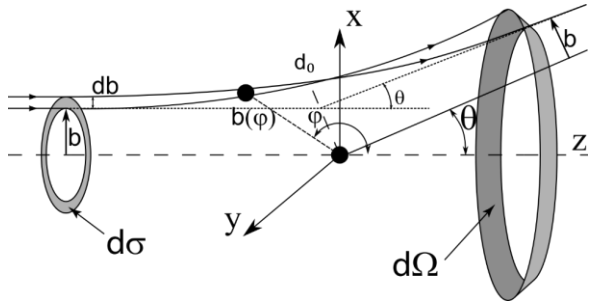
Few implications of quantum mechanics

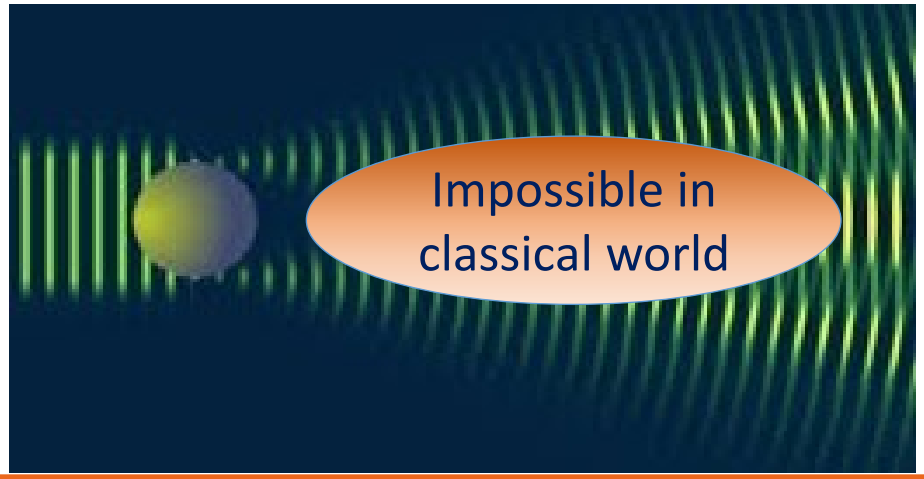
Trajectories are indistinguishable



- Impact parameter is not defined in the quantum world
- Quantum interference phenomenon
- Interfering trajectories can be constructive or destructive.

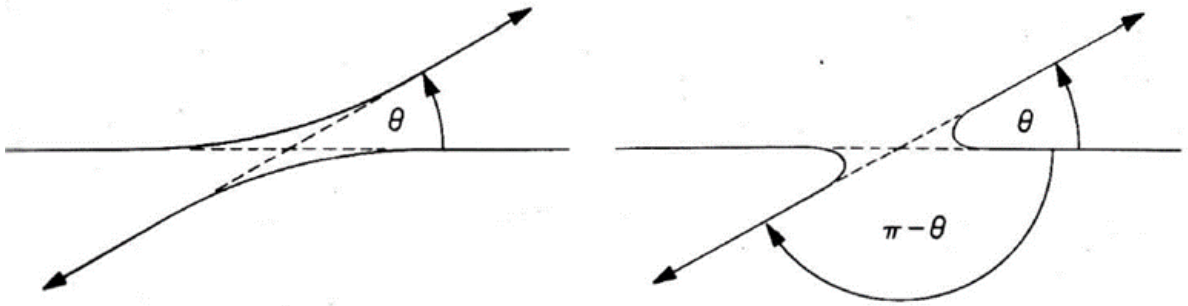
Particle-Target motion must be described by wave functions





Quantum effect

If target/beam are identical particles or nuclei then



This leads to

One cannot distinguish the two cases: w.f. is symmetric or antisymmetric depending on the spin

$$\frac{d\sigma(\theta,\varphi)}{d\Omega} = |f(\Theta,\varphi) + f(\pi-\Theta,\varphi)|^2$$

causes interferences !

scattering of ^{12}C + ^{12}C @ 5 MeV

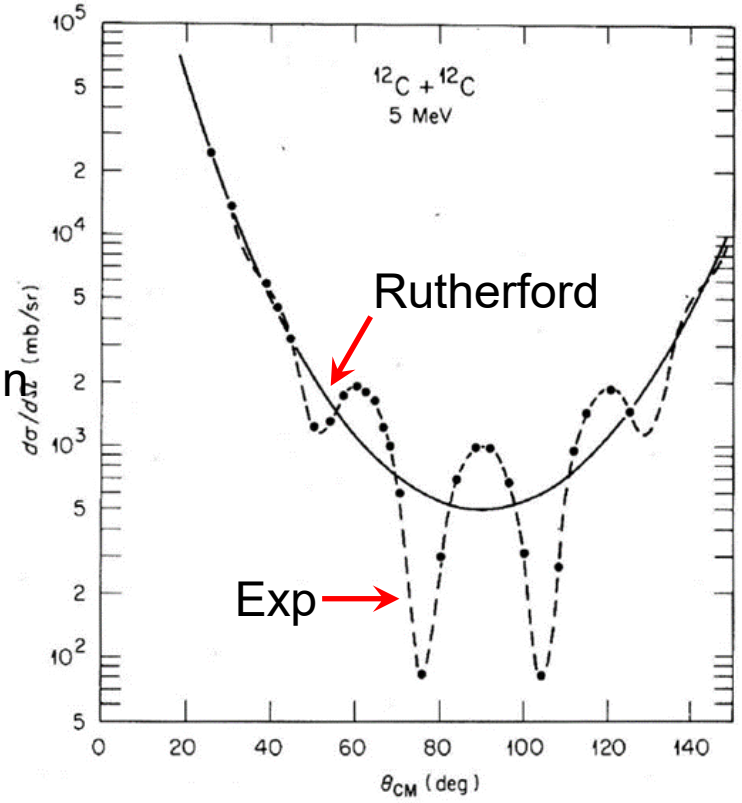
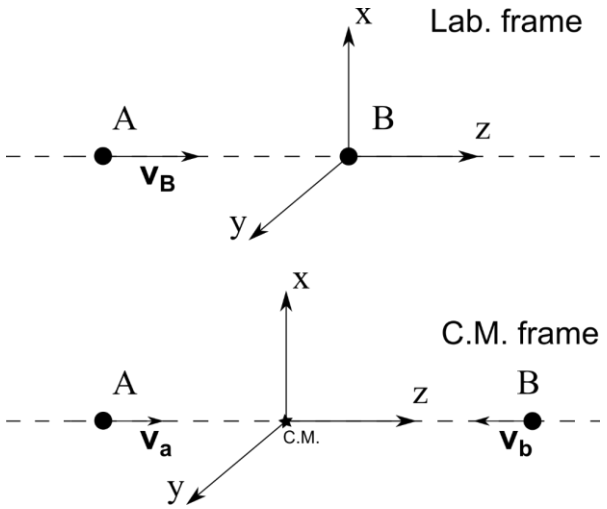
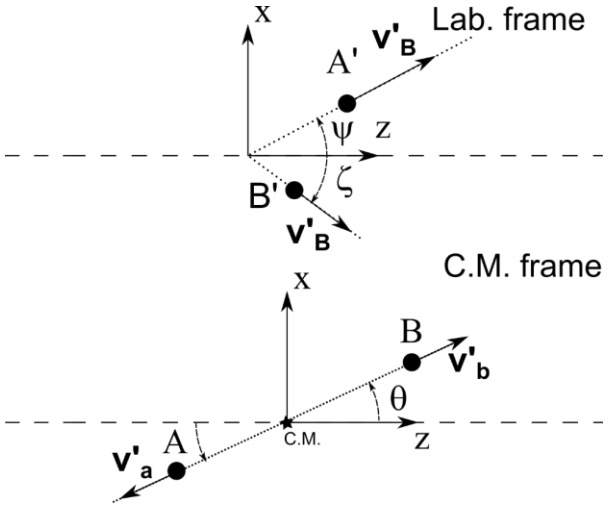


Illustration from G. R. Satchler
"Introduction to direct reactions"

Note that the same happens with Coulomb

Translational invariance: C.M. frame





The Schrödinger equation

$$\left[-\frac{\hbar^2}{2m_A}\nabla_A^2 - \frac{\hbar^2}{2m_B}\nabla_B^2 + V(|\vec{r}_A - \vec{r}_B|)\right]\psi_k^+(\vec{r}_A, \vec{r}_B) = E\psi_k^+(\vec{r}_A, \vec{r}_B)$$

reduces in the c.m. frame

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V(|\vec{r}|)\right]\psi_k^+(\vec{r}\,) = E_{\text{C.M.}}\psi_k^+(\vec{r}\,)$$

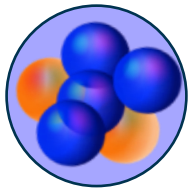
phase velocity

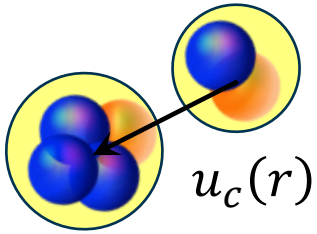
$$w = \frac{E}{\hbar} = \frac{\hbar k^2}{2\mu} = kv$$

Bound state E<0

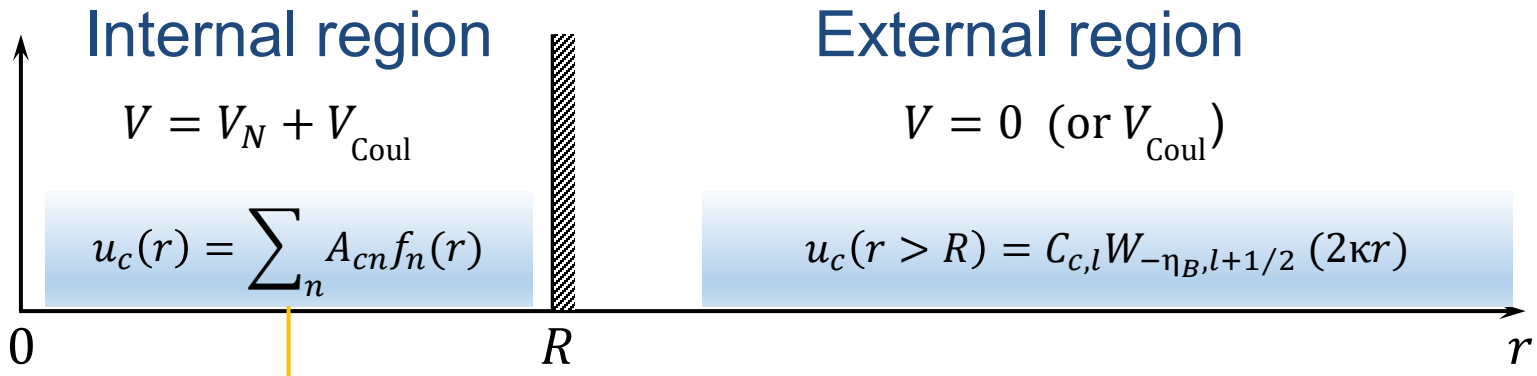
$$\Psi^{J^\pi}_{A+a} = S^A_{a,A}\left[\Psi_a \Psi_A \chi^f_{k_\beta}(\vec{r})\right]^{J^\pi}$$

- $S^A_{a,A}$ spectroscopic amplitude
- $\left|S^A_{a,2}\right|^2$~ probability to form the configuration $(a + A)$ in the nucleus A





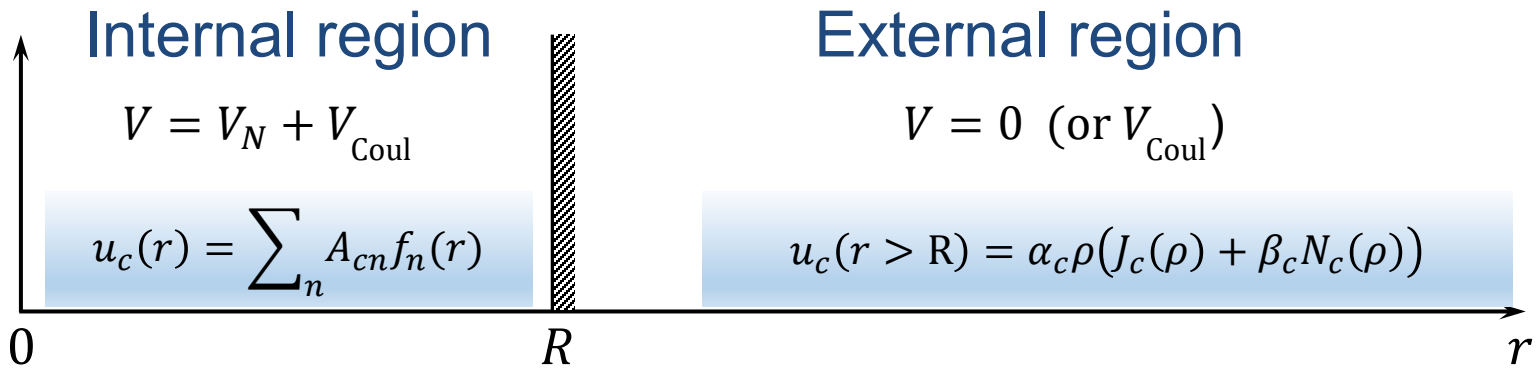
The Asymptotic Normalization Coefficient (ANC) $C_c = \frac{u_c(r)}{W_{-\eta_B, l+1/2}\,(2\kappa r)}$ pour $r > R$.



The tail of the wave function is determined by E_B and κ

$$u_c(r \to \infty) \sim x^{-\eta_B} e^{-\frac{2\kappa r}{2}},\ \kappa = \sqrt{-\frac{2\mu E_B}{\hbar^2}}$$

Phase shifts, resonances E>0



Matching conditions on $u_{k,l\equiv c}(\rho)$ and $^{du_{k,l\equiv c}(\rho)}/_{dr}$ at $r = R$ between region I and II determines β_c.

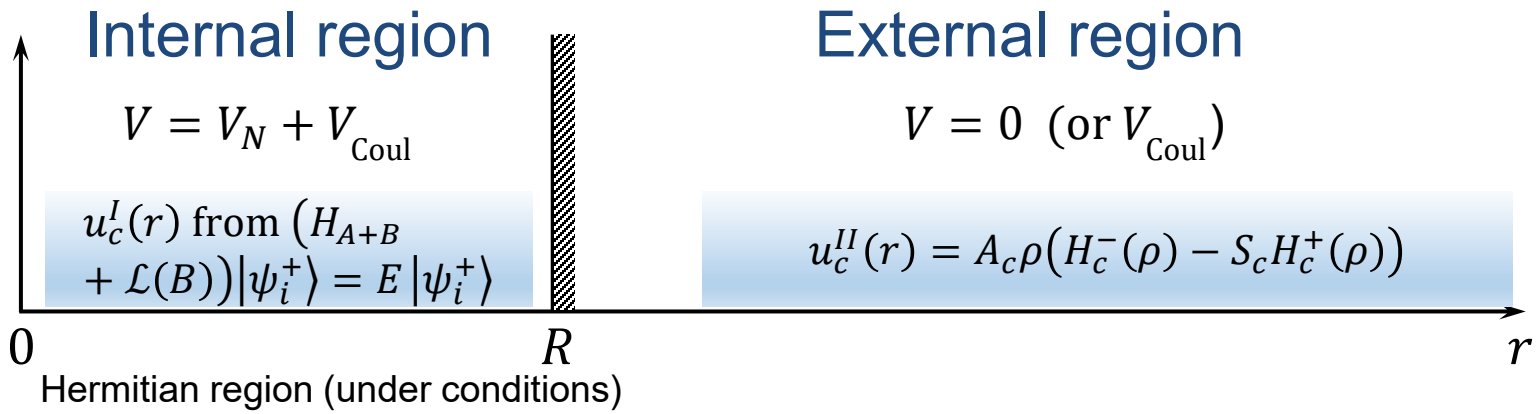
If we write $\beta_c = \tan\delta_c$, δ_c called the phase shift since:

$$u_c(r \to \infty) \sim \sin(\rho - l\,\pi/2 + \delta_c)$$

While the plane wave gives

$$\sin(\rho - l\,\pi/2)$$

Solution of the scattering problem with R-matrix method



We know expand the interieur wave function on a square integrable basis, i.e.

$$u_c^I(r) = \sum c_j \varphi_j(r)$$

B correspond to a choice of logarithmic boundary condition

$$\langle c|\frac{\partial}{\partial r_\alpha} r_\alpha|\psi_i^+\rangle = b_c\langle c|\psi_i^+\rangle$$

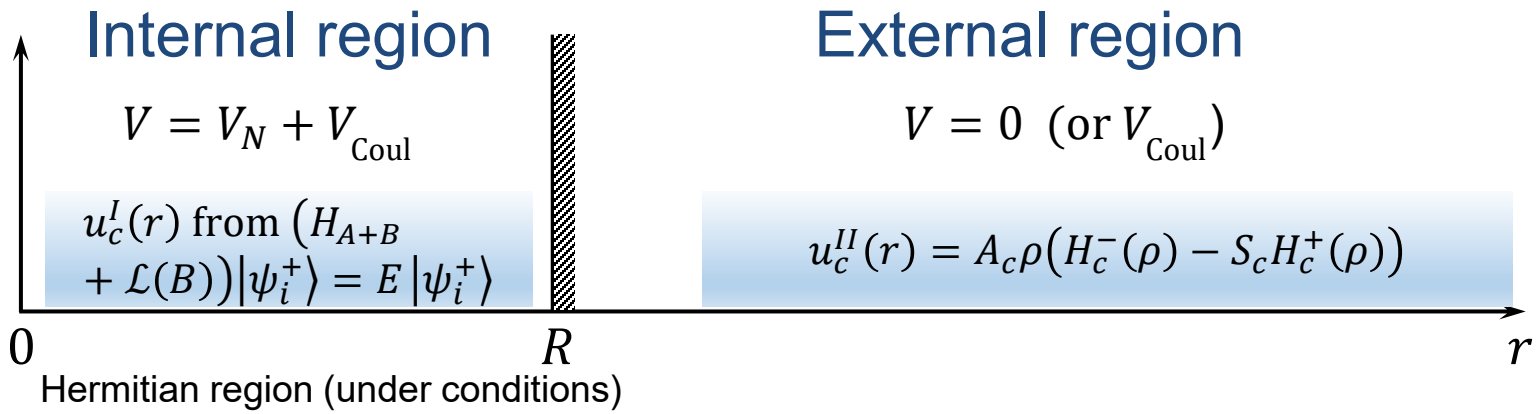
The R-matrix is defined by the differential equation

$$(3) \quad \rightarrow \; u_c(R) = \mathcal{R}_c(E)[Ru'_c(R) - Bu_c(R)], \text{B is arbitrary}$$

To have an Hermitian problem in I, we must use the Bloch-Schrodinger equation

$$(H_l + \mathcal{L}(B) - E)u_c^I(r) = \mathcal{L}(B)u_c^{II}(r) \text{ where } \mathcal{L}(B) = \frac{\hbar^2}{2\mu}\delta(r-R)\left(\frac{d}{dr} - \frac{B}{r}\right)$$

Solution of the scattering problem with R-matrix method



Application of $\mathcal{L}(B)$ to $u_c^{II}(r)$ gives the right-hand side of (3) times $\left(\mathcal{R}_c(E)\right)^{-1}$

We only must solve [formally] the linear equation for the c_j introduce them in the expansion of $u_c^I(r)$ and use the differential equation that defines $\mathcal{R}_c(E)$ to find that

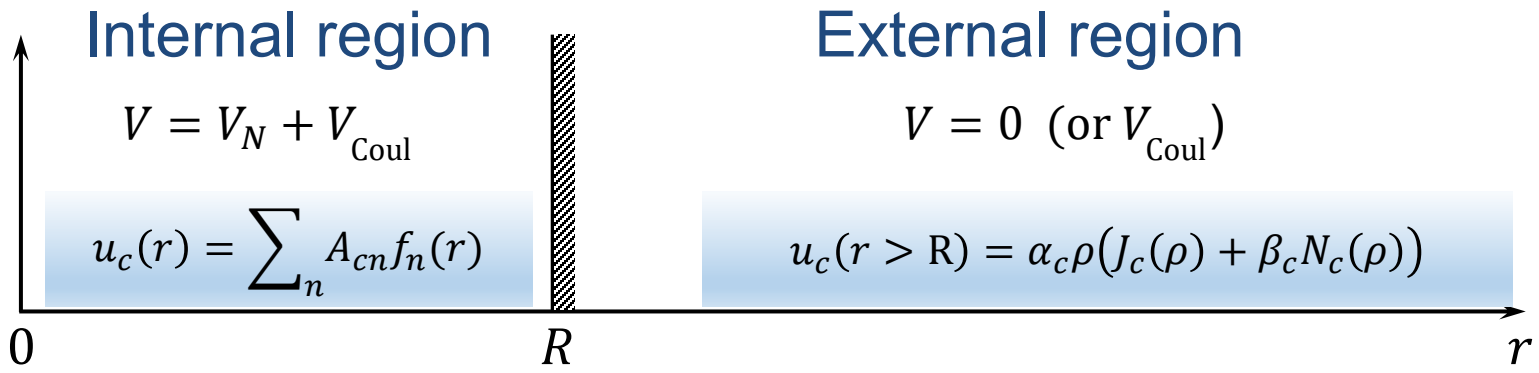
$$\mathcal{R}_c(E) = \frac{\hbar^2}{2\mu}\sum \varphi_i^*(R)[C^{-1}]_{i,j}\,\varphi_j(R)$$

Where the collision matrix is defined as $[C]_{i,j} = \langle\varphi_i|H_{A+B} + \mathcal{L}(B) - E|\varphi_j\rangle$. In the eigen-basis of C, we have

$$\mathcal{R}_c(E) = \sum \frac{\gamma_i^2}{E_i - E}$$

NB: $S_c(E) = (f \circ \mathcal{R}_c)(E)$

Solution of the scattering problem with R-matrix method



Decomposition on a Lagrange mesh.

NCSMC can be cast as Bloch-Schrödinger equation:

$$(C - EI)\vec{X} = Q(B)$$

And solved using R-matrix, which in the eigen basis of $C - EI$ reads:

$$R_{cc'} = \sum_{\lambda} \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E}$$

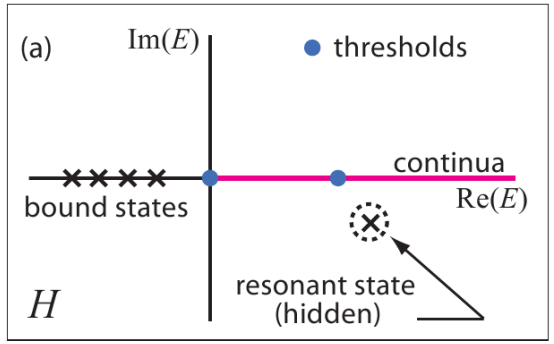
In general, this is used as a parametrization and fitted to data: phenomenological R-matrix

$$b_c = S_c(E) = \Re\left(\frac{a_c}{O_c}\frac{\partial O_c}{\partial r_c}\right)_{a_c}$$

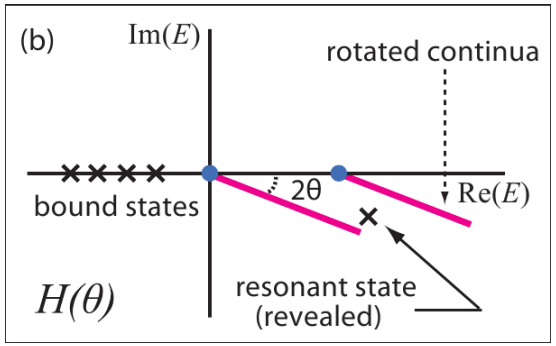
$$a_c = 1.4(A_1^{\frac{1}{3}} + A_2^{\frac{1}{3}})\ \text{fm}$$

Simple for binary reacting system, more involved for neutral ternary system and extremely challenging for charged breakup.

Asymptotically vanishing equivalent problem



Complex scaling



Kruppa *et al.* PRC**89** (2014)

The complex scaling and the resonance states

$$\hat{H}(r) = \hat{T} + \hat{V}(r)$$

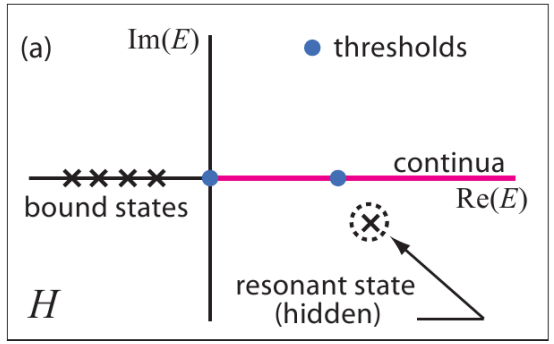
$$\hat{H}(\theta) = e^{-2i\theta}\hat{T} + \hat{V}\left(re^{i\theta}\right)$$
$$\hat{H}(r) = \hat{U}(\theta)\hat{H}(r)\hat{U}^{-1}(\theta)$$

Aguilar-Balslev-Combes theorem: the resonant states of the original Hamiltonian are invariant and the non-resonant scattering states are rotated and distributed on a 2θ ray that cuts the complex energy plane with a corresponding threshold being the rotation point. Math. Phys. **22**, 269 (1971)

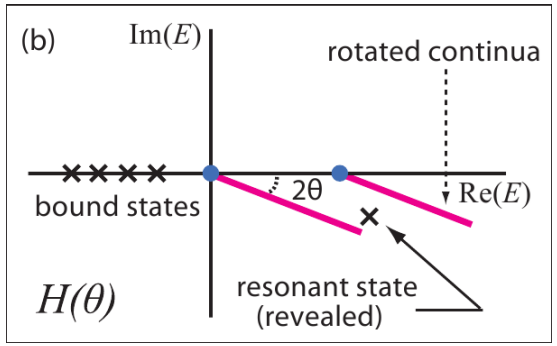
$$\hat{H}(r,\theta)\psi(r,\theta) = \left(E - \frac{i\Gamma}{2}\right)\psi(r,\theta)$$

Energy ↑ ↑ Half-life

Asymptotically vanishing equivalent problem



Complex scaling



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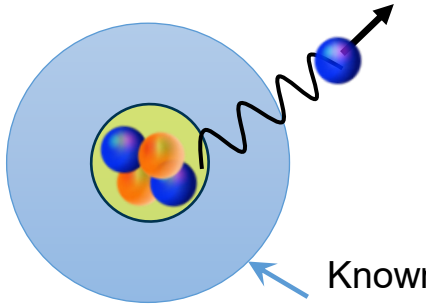
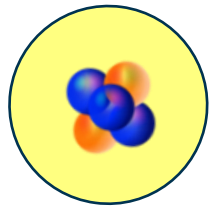
$$\hat{H}(r) = \hat{U}(\theta)\hat{H}(r)\hat{U}^{-1}(\theta)$$



Known asymptotic

$$U(\theta)H(r)U(\theta)^{-1}$$

$$\psi(r,\theta) \underset{\infty}{\sim} e^{-\boldsymbol{kr}\,sin\,\theta}$$



Spatially extended but exponential fall off

Boundary limit problem

Bound state problem

From phase shifts to cross-sections

$$H_{A+B}\left|\psi_i^+\right\rangle = E_i\left|\psi_i^+\right\rangle$$

Solve

$$\left[-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2}V(r)\right]u_{k,l}(r\,) = k^2 u_{k,l}(r\,)$$

$$f_l(k) = \frac{e^{2i\delta_l(k)} - 1}{2ik}$$

Match

$$\psi_k^+(\vec{r}) \longrightarrow \left(e^{i\vec{k}.\vec{r}} + f(\Theta,\varphi)\frac{e^{ikr}}{r}\right)$$

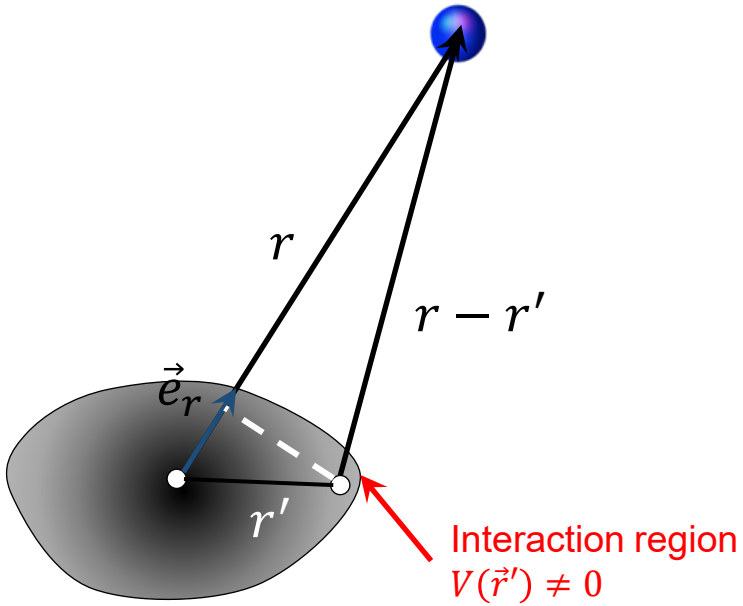
$$\frac{d\sigma\,(\theta,\varphi)}{d\Omega},\ \sigma_{\text{tot}}$$

Calculate

Differential cross-sections for central potential

$$\frac{d\sigma\,(\theta,\varphi)}{d\Omega} = \frac{1}{4k^2}\left|\sum(2l+1)\left(1 - e^{2i\delta_l(k)}\right)P_l(\cos\Theta)\right|^2$$

Reaction theory as a series



The Lippmann-Schwinger equation gives

$$\psi_{\boldsymbol{k}}^{\pm}(\vec{r}) = \frac{e^{i\vec{k}\cdot\vec{r}}}{(2\pi)^{3/2}} - \frac{2\mu}{\hbar^2}\int d^3r' \frac{e^{\pm ik|r-r'|}}{4\pi|r-r'|} V(\vec{r}')\psi_{\boldsymbol{k}}^{\pm}(\vec{r}')$$

with $\vec{k}' = k\vec{e}_r$

$k' = k$

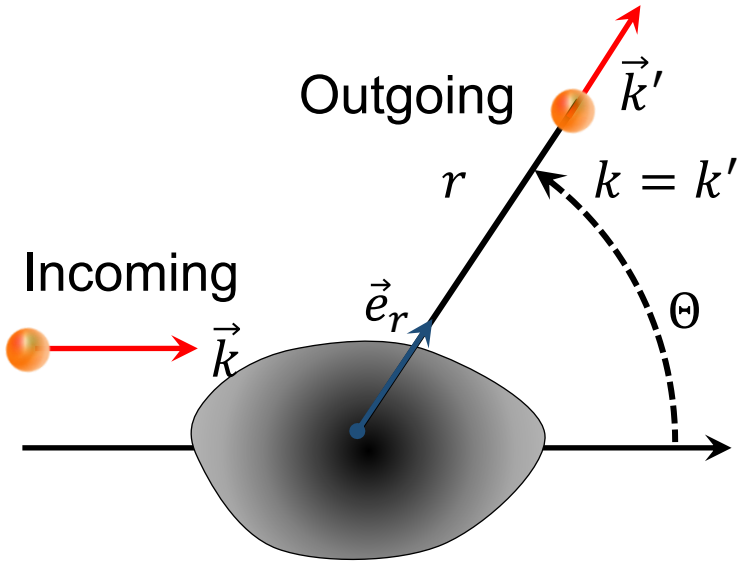
Looking for the solution following

$$\psi_k^{+}(\vec{r}) \longrightarrow \frac{1}{(2\pi)^{3/2}}\left(e^{i\vec{k}.\vec{r}} + f(\Theta,\varphi)\frac{e^{ikr}}{r}\right)$$

We obtain the scattering amplitude:

$$f(\theta,\varphi) = -2\pi^2 \frac{2\mu}{\hbar^2}\int d^3r' \frac{e^{-i\vec{k}'\cdot\vec{r}'}}{(2\pi)^{3/2}} V(\vec{r}')\psi_{\boldsymbol{k}}^{+}(\vec{r}')$$

Reaction theory as a series



T-matrix is defined as

$$f(\theta,\varphi) = -2\pi^2 \frac{2\mu}{\hbar^2} \langle \varphi_{0,\boldsymbol{k}'} | V | \psi_{\boldsymbol{k}}^{+} \rangle = -2\pi^2 \frac{2\mu}{\hbar^2} T_{\boldsymbol{k}',\boldsymbol{k}}$$

We deduce the differential cross-section as:

$$\frac{d\sigma(\theta,\varphi)}{d\Omega} = \left(2\pi^2 \frac{2\mu}{\hbar^2}\right)^2 \left|T_{\boldsymbol{k}',\boldsymbol{k}}\right|^2$$

$T_{\boldsymbol{k}',\boldsymbol{k}}$ is the on-shell $[k = k']$ T-matrix element and relates to the S-matrix by

$$S_{\boldsymbol{k}',\boldsymbol{k}} = \delta(\vec{k} - \vec{k}') - 2\pi i \delta(E_k - E_{k'}) T_{\boldsymbol{k}',\boldsymbol{k}}$$

So $T_l(E) = -{}^1/_{\pi}\, e^{i\delta_l(E)} \sin\delta_l(E)$. Similarly, $T_l(E) = \frac{2\mu}{\pi\hbar^2} \int dr\, r J_l(kr)\, V(r) u_l(r)$

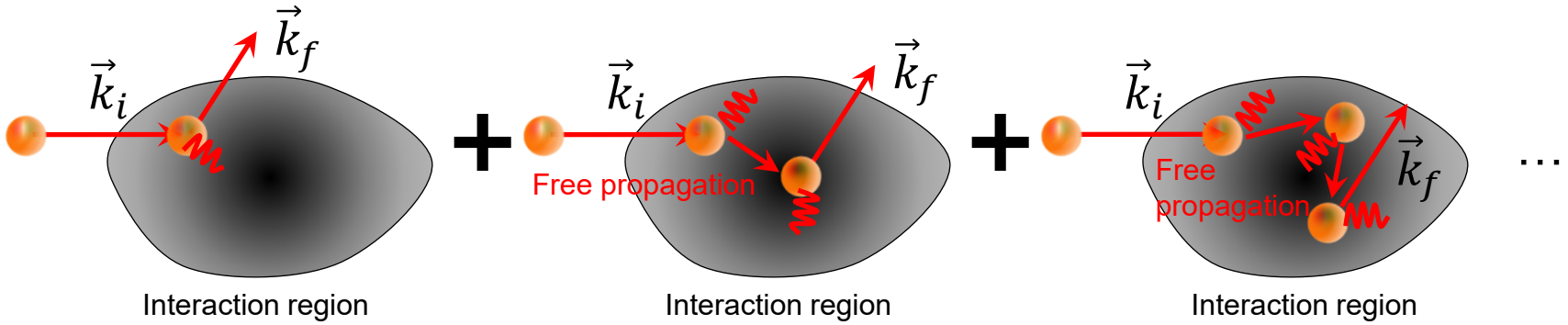
Reaction observables carry imprints of the nuclear force

Representation of the born series

From the Lippmann-Schwinger equation we define the Born series expansion

$$f(\Theta,\varphi) = -(2\pi)^2 \frac{\mu}{\hbar^2} \left\langle \varphi_{0,\boldsymbol{k}'} \middle| V \sum \left(\frac{2\mu}{\hbar^2} G_0 V\right)^n \middle| \varphi_{0,\boldsymbol{k}} \right\rangle$$

We can understand that the unperturbed plane wave undergoes a sequences of multiples scattering events from inside the potential region:

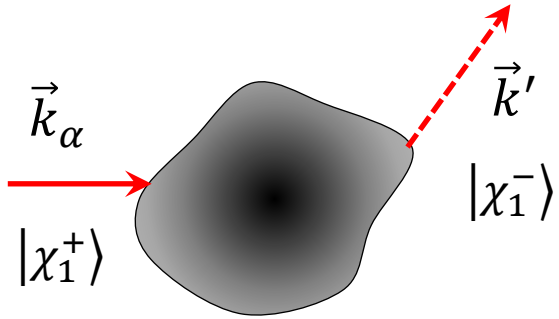


But the series may not converge until all terms are including if the potential is strong enough.

The Distorted Wave-Born Approximation (DWBA)

Born Approximation

$$\chi_k^+(\vec{r}) = \frac{e^{i\vec{k}_\alpha\cdot\vec{r}}}{(2\pi)^{3/2}} - \frac{2\mu}{\hbar^2}\int d^3r' \frac{e^{ik_\beta|r-r'|}}{4\pi|r-r'|} V(\vec{r}')\chi_k^+(\vec{r}')$$



Systematic constructive treatment

$$f_{\text{Born}} = -(2\pi)^2 \frac{\mu}{\hbar^2}\langle \boldsymbol{k}'|V|\mathbf{k}\rangle$$

In some cases, the free-wave approximation is rather poor starting point.

Suppose $V = V_{\text{MF}} + V_{\text{res}}$ and the solutions of $(\nabla^2 + k^2 - V_{\text{MF}})\chi_1(\boldsymbol{k},\boldsymbol{r}) = 0$ are known/computable

One can show $f = f_1 - (2\pi)^2 \frac{\mu}{\hbar^2}\int d^3r' \chi_1^-(\boldsymbol{k},\vec{r})V_{\text{res}}(\vec{r}')\chi_k^+(\vec{r}')$

The DWBA approximation consists in:

$\chi_k^+ \longrightarrow \chi_1^+(\boldsymbol{k},\boldsymbol{r})$ then $f = f_1 - (2\pi)^2 \frac{\mu}{\hbar^2}\langle\chi_1^-|V_{\text{res}}|\chi_1^+\rangle$

Multichannel reactions

$$f(\Theta,\varphi) = \sum_{\beta} f_{\beta}(\Theta,\varphi)$$

All energetically allowed opened channels β

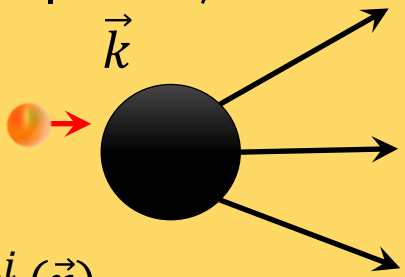
Note that, in a finite basis, closed channels also contribute but to reaction observables

$t = t_0$

Incoming wave

$|a + A\rangle$

$\vec{k}$



$\Psi_a \Psi_A \chi_k^i(\vec{r})$

$\chi_k^i(\vec{r}) \propto e^{i\vec{k}.\vec{r}}$

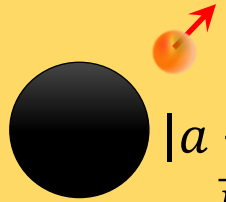
$H_a\Psi_a = E_0\Psi_a$

$H_A\Psi_A = E_0\Psi_A$

Ψ_a and Ψ_A quantum numbers defined conservation of total relative angular momentum

$t \gg t_0$

Elastic channel, always opened



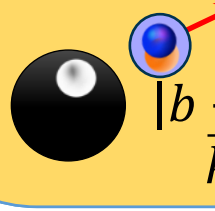
$|a + A\rangle$ $\quad \Psi_a \Psi_A \chi_{k_\beta}^f(\vec{r})$

$\vec{k}_\beta$ $\quad H_a\Psi_a = E_0\Psi_a$

$H_A\Psi_A = E_0\Psi_A$

$|C^*\rangle$ $\quad \Psi_{C^*}$

$\vec{k}_\beta$ $\quad H_C\Psi_{C^*} = E_n\Psi_{C^*}$



$|b + C\rangle$ $\quad \Psi_b \Psi_C \chi_{k_\beta}^f(\vec{r})$

$\vec{k}_\beta$ $\quad H_b\Psi_b = E_0\Psi_b$

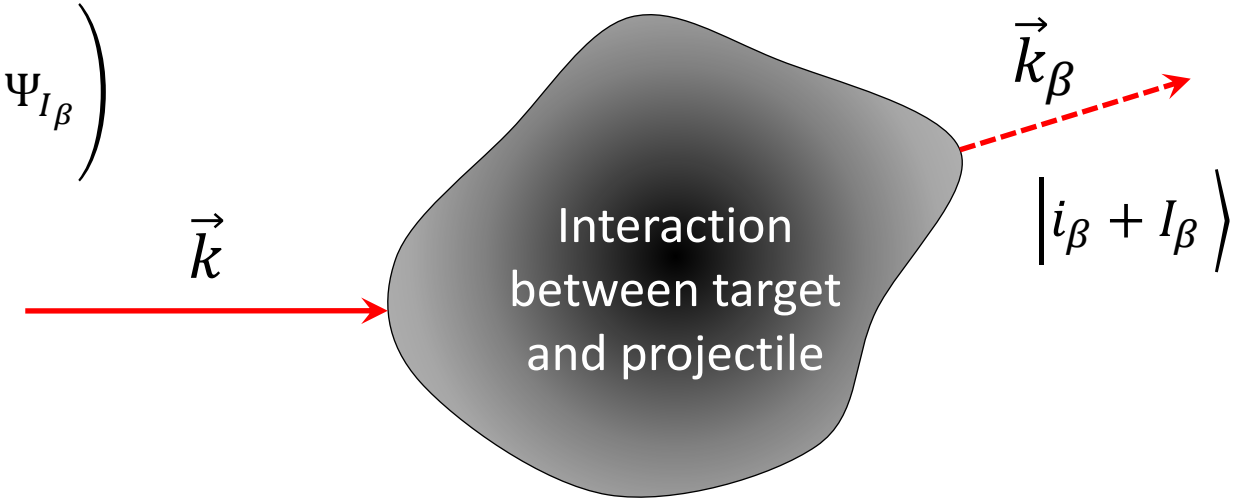
$H_C\Psi_C = E_0\Psi_C$

channel Q-value

Multichannel reactions

Scattering wave-function

$$\psi_k^+(\vec{r}) \longrightarrow \left(e^{i\vec{k}.\vec{r}}\Psi_a\Psi_A + \sum_\beta f_\beta(\Theta,\varphi)\frac{e^{ik_\beta r}}{r}\Psi_{i_\beta}\Psi_{I_\beta} \right)$$



Partial cross-section

$$\frac{d\sigma_\beta}{d\Omega} = \frac{\vec{j}_f \cdot d\vec{S}/r^2}{\vec{j}_i \cdot \hat{k}}$$

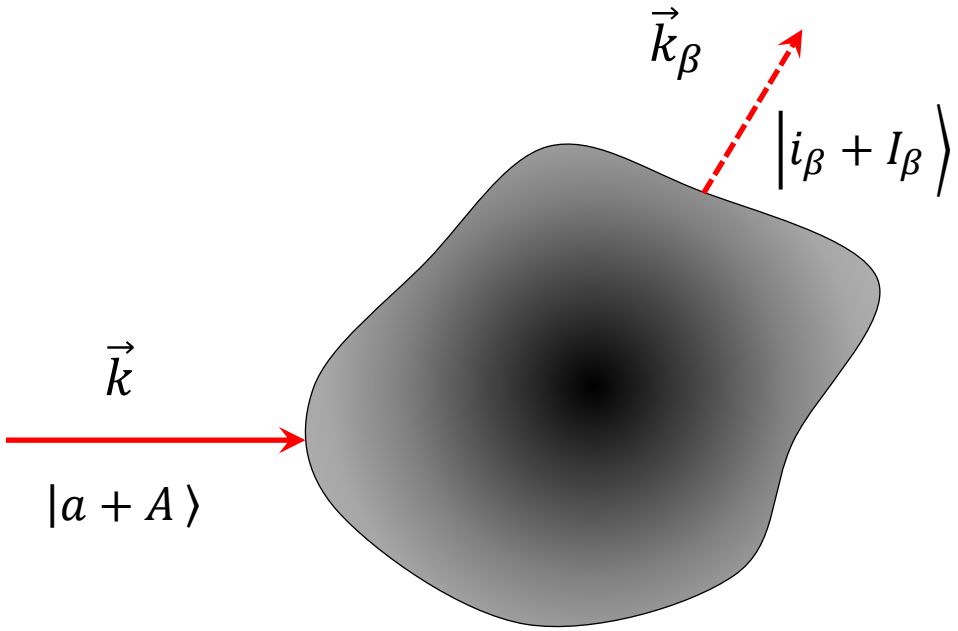
Since $\vec{j} = \rho\vec{v}$ with $\vec{v}$ the wave vector, we have that :

$$\frac{d\sigma_\beta}{d\Omega} = \frac{v_\beta}{v}\left|f_\beta(\Theta,\varphi)\right|^2$$

- Elastic scattering $\quad v_\beta/v = 1$
- Inelastic scattering $\quad v_\beta/v \neq 1$

Multichannel reactions

$$\frac{d\sigma_\beta}{d\Omega} = \frac{v_\beta}{v}\left|\breve{f}_\beta(\Theta,\varphi)\right|^2$$



The scattering amplitude contributing to the *same* final quantum state will interfere, and *channel coupling affects all amplitudes dynamically*.

$|a + A\rangle$

Elastic channel with $\frac{v_\beta}{v} = 1$, always opens

$|a^* + A^*\rangle$

Inelastic scattering $\frac{v_\beta}{v} \neq 1$,
Energetically opens if $E_{\text{c.m.}}$ is greater than the reaction threshold

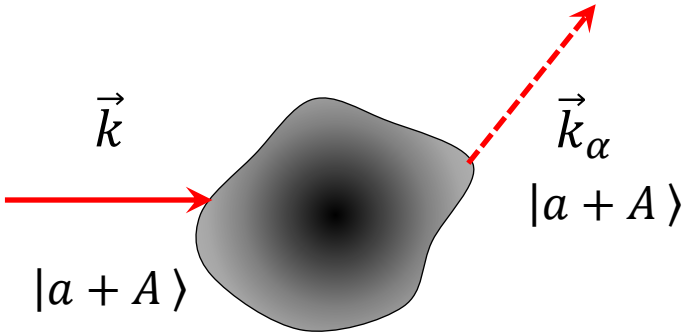
$|b + C\rangle$

$|b + d + C\rangle$

All other reaction channels energetically allowed ($\frac{v_\beta}{v} \neq 1$)

Inelastic channels

Influence of the non-elastic channels on cross-section



With only elastic channel

$$\psi_k^+(\vec{r}) \longrightarrow \left(e^{i\vec{k}.\vec{r}} + f(\Theta,\varphi)\frac{e^{ik_\alpha r}}{r}\right)$$

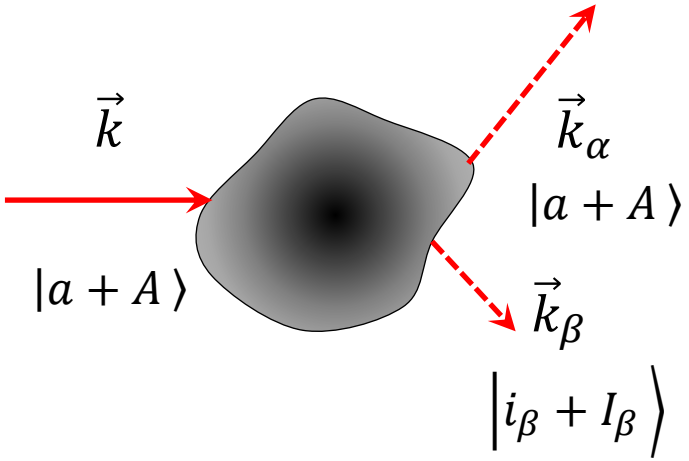
$$u_{\alpha,l}(r > \mathrm{R}) = A_{\alpha,l}\rho\left(H_l^-(\rho) - S_{\alpha,l}H_l^+(\rho)\right)$$

Elastic channel has the same wave number so $k = k_\alpha$, the conservation of the flux [which implies the unitarity of the S-matrix i.e. $S_{\alpha,l}{S_{\alpha,l}}^* = 1$] means $S_{\alpha,l} = e^{2i\delta_l}, \delta \in \mathbb{R}$

Adding non-elastic channels

$$u_{\alpha\alpha,l}(r > \mathrm{R}) = A_{\alpha\alpha,l}\rho\left(H_l^-(\rho) - S_{\alpha\alpha,l}H_l^+(\rho)\right)$$

$$u_{\beta,l}(r > \mathrm{R}) = -A_{\beta,l}\rho S_{\beta\alpha,l}H_l^+(\rho)$$

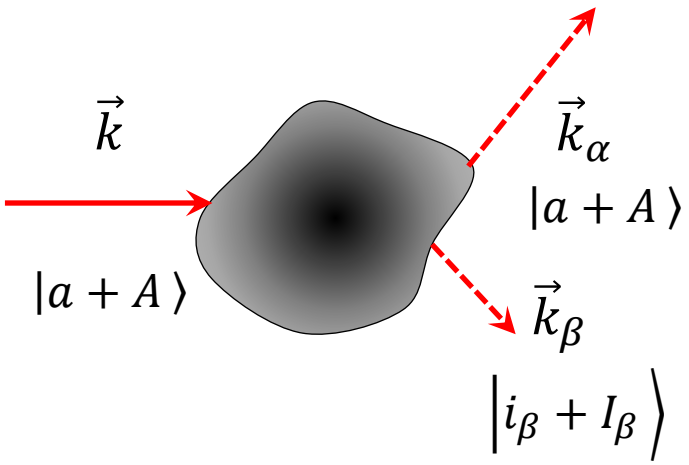


Where $S_{\beta\alpha,l} = \sqrt{v/v_\beta}\,\tilde{S}_{\beta\alpha,l}$ and $S_{\alpha\alpha,l} = e^{2i\delta_l}, \delta \in \mathbb{C}$. Total energy is conserved but $k_\beta \neq k$ due to energy consumed by the Q value. The flux is distributed among channels:

$$\left|S_{\alpha\alpha,l}(E)\right|^2 + \sum\left|S_{\beta\alpha,l}(E)\right|^2 = 1 = \left|S_{\alpha\alpha,l}(E)\right|^2 + T_{\alpha,l}$$

If PW are coupled (tensor force) then $S_{\alpha\alpha,l} \rightarrow S_{\alpha\alpha,ll'}$ (more parameters).

Elastic, Reaction and Total cross section



Elastic channel:

$$\sigma_{\text{el}} = \frac{\pi}{k^2}\sum(2l+1)\left|1-\tilde{S}_{\alpha\alpha,l}\right|^2$$

Inelastic channels:

$$\sigma_{\text{in}} = \frac{\pi}{k^2}\sum(2l+1)\left|\tilde{S}_{\beta\alpha,l}\right|^2$$

Sum of all inelastic channels (absorption cross-sec.):

$$\sigma_{\text{abs}} = \frac{\pi}{k^2}\sum(2l+1)\left(1-\left|\tilde{S}_{\alpha\alpha,l}\right|^2\right) = \frac{\pi}{k^2}\sum(2l+1)T_{\alpha,l}$$

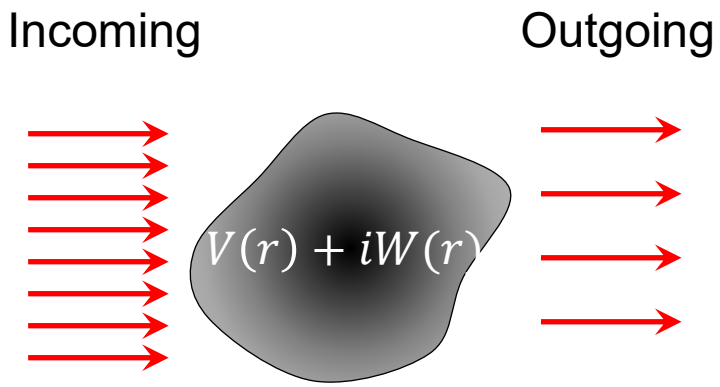
from $\left|S_{\alpha\alpha,l}\right|^2 + \sum\left|S_{\beta\alpha,l}\right|^2 = 1$, and total cross-section

The transmission coefficient use in HF

$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{abs}}$$

$$= \frac{2\pi}{k^2}\sum(2l+1)(1-\text{Re}(\tilde{S}_{\alpha\alpha,l}))$$

Optical potential



Scattering equation with an imaginary potential

$$\left(\Delta + k^2 - \frac{2\mu}{\hbar^2}(V(\vec{r}) + iW(\vec{r})\right)\varphi(\vec{r}) = 0$$

The standard conservation of matter

$$\frac{d\rho}{dt} = -\vec{\nabla}\cdot\vec{J}$$

Is modified with a sink term $\frac{2}{\hbar}W(r)\rho(\mathrm{r})$.

Thus we have $\hbar\,\vec{\nabla}\cdot\vec{J} = \frac{2}{\hbar}W(r)\rho(\mathrm{r})$ we immediately obtained the lost outgoing flux as $-\frac{2}{\hbar}\int d^3rW(r)\rho(\mathrm{r})$, the absorption cross-section reads

$$\sigma_{\mathrm{abs}} = -\frac{2}{\hbar v}\int d^3rW(r)\rho(\mathrm{r})$$

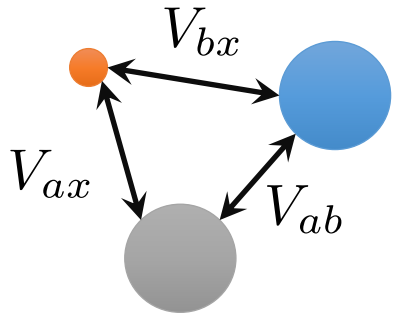
In practice, $W(r) < 0$ is parametrized by various Woods-Saxon functions:

$$W(r,E) = -W^{\mathrm{vol}}(E,r) + 4a^{\mathrm{sur}}W^{\mathrm{sur}}(E)\frac{d}{dr}f(r) + W^{so}(r,E)$$

At high energies, we assume internal dof are frozen (spectator nucleons) only the transferred particle (n, p, d, …) ic considered explicitly:

$$A + b \rightarrow a + B \implies (a+x) + b \longrightarrow a + (b+x)$$

The interaction potentials should be between all constituents [3-body problem]:



Entrance

$$\Psi^{J^\pi}_{A=(a+x)} = S^A_{a,x}\left[\Psi_a \Psi_x \chi^f_{k_\beta}(\vec{r})\right]^{J^\pi} + \cdots$$

Exit

$$\Psi^{J^\pi}_{B=(b+x)} = S^B_{b,x}\left[\Psi_b \Psi_x \chi^f_{k_{\beta'}}(\vec{r})\right]^{J^\pi} + \cdots$$

χ_1^- χ_1^+

$$\langle \chi_1^- | V_{\text{res}} | \chi_1^+ \rangle \propto \left\langle \Psi_b \Psi_x \chi^f_{k_{\beta'}}(\vec{r}) \middle| V_{\text{res}} \middle| \Psi_a \Psi_x \chi^f_{k_\beta}(\vec{r}) \right\rangle$$

Scattering observables at very low energy

$$\sigma_{\text{tot}} = \frac{4\pi}{k^2}\sum(2l+1)(\sin\delta_l(k))^2$$

Requires to estimate the phase-shift for all l, i.e. solve

$$\left[-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2}V(r)\right]u_{k,l}(r) = k^2 u_{k,l}(r)$$

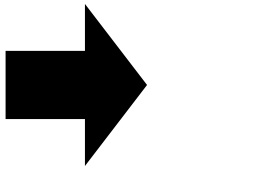
Need two conditions:

1. Request regular solution at $r \longrightarrow 0$, i.e. $u_{k,l}(0) = 0$
2. At long relative distance should match free particle solution

- At high energy many l contributes
- At very low-energy $k \to 0$ the cross section is dominated by $l = 0$ (s-wave scattering)

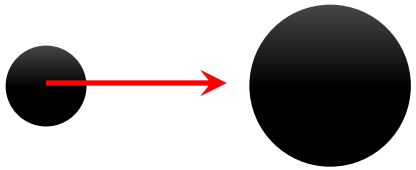
$$\sigma_{\text{tot}} = \frac{4\pi}{k^2}(sin\,\delta_0(k \to 0))^2$$

If $(sin\,\delta_0)^2 \sim (ka)^2$



$$\sigma = 4\pi a^2$$

Equivalent to hard-sphere scattering



Scattering length: a

It characterize the scattering potential at very low energy in an effective way.

A standard definition is

$$\lim_{k\to 0} k\cot\delta_0 \cong -\frac{1}{a}$$

This expression is sometimes generalized to (called finite range expansion)

$$k\cot\delta_0 = -\frac{1}{a} + \frac{1}{2}k^2 \blacksquare + O(k^4)$$

effective range

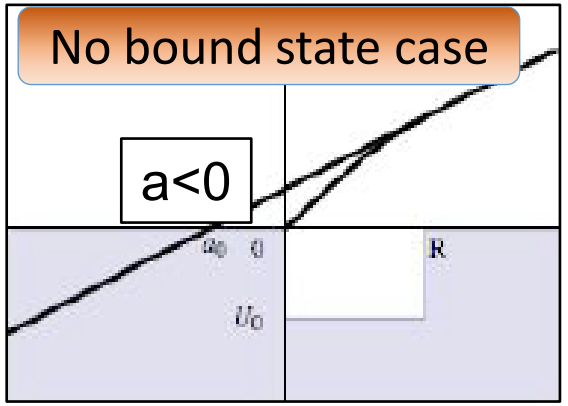
This can be interpreted as the solution of

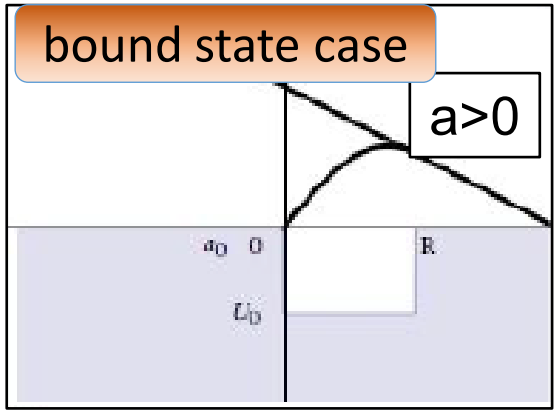
$$\left[-\frac{d^2}{dr^2} + \frac{2\mu}{\hbar^2}(V(r) - E)\right] u_{k,0}(r\,) = 0$$

when $V(r)$ and E goes to zero

$$\Rightarrow \quad \frac{d^2}{dr^2}u_{k,0}(r\,) = 0 \quad \Rightarrow \quad u_{k,0}(r\,) \sim \sin(kr + \delta_0) \cong kr + \delta_0 \cong k(r - a)$$

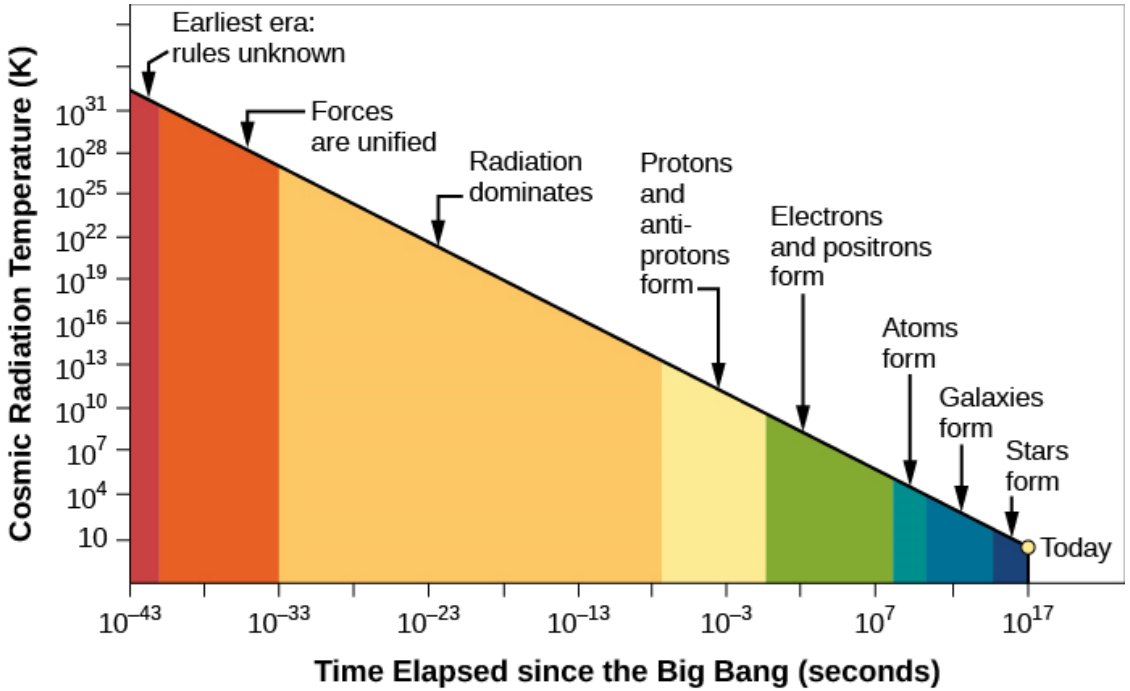
Near threshold rule

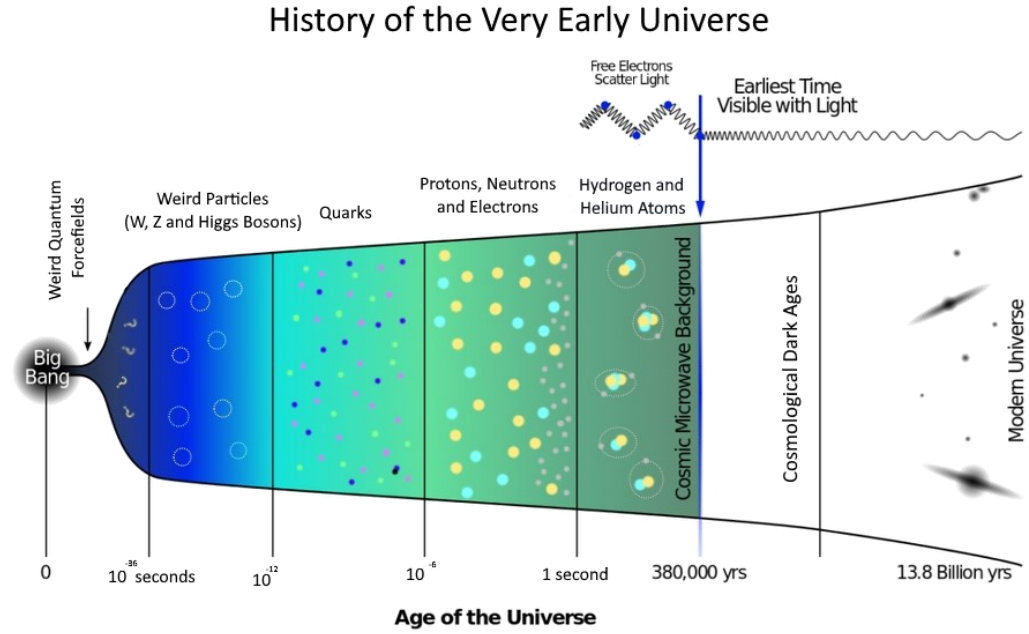




Nucleosynthesis since $t = t_0$

Some schematic view of the Big Bang

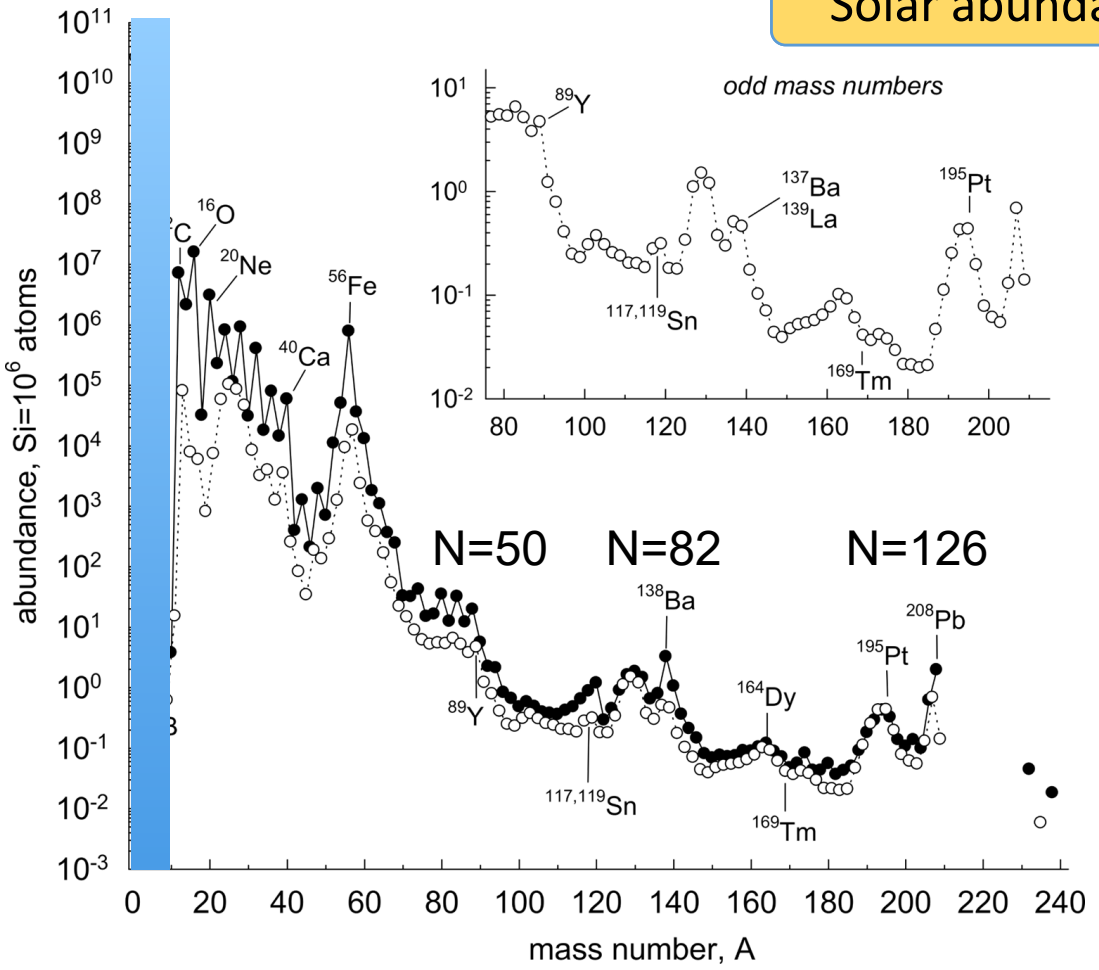




To understand this one should be able to trace back the creation of light nuclei from nucleons and heavier nuclei from light nuclei [fusion, capture with EM emission].

Observation of today's solar abundancy

Solar abundancy



Light nuclei formation

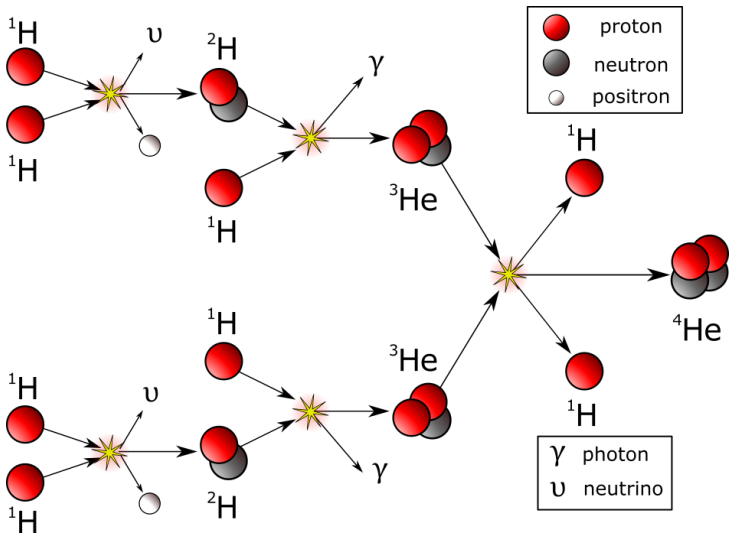
$p + p \rightarrow d + e^+ + \nu_e$ $\quad Q = 0.42$ MeV

$p + e^- + p \rightarrow d + \nu_e$ $\quad Q = 1.42$ MeV

$d + p \rightarrow {}^3He + \gamma$ $\quad Q = 5.49$ MeV

${}^3He + {}^3He \rightarrow {}^4He + 2p$ $\quad Q = 12.96$ MeV

PPI chain is 91 % sun energy



${}^3He + {}^4He \rightarrow {}^7Be + \nu_e$

$e^- + {}^7Be \rightarrow {}^7Li + \nu_e$ $\quad Q = 0.86$ MeV

$p + {}^7Li \rightarrow 2\ {}^4He$

${}^7Be + p \rightarrow {}^8B + \gamma$

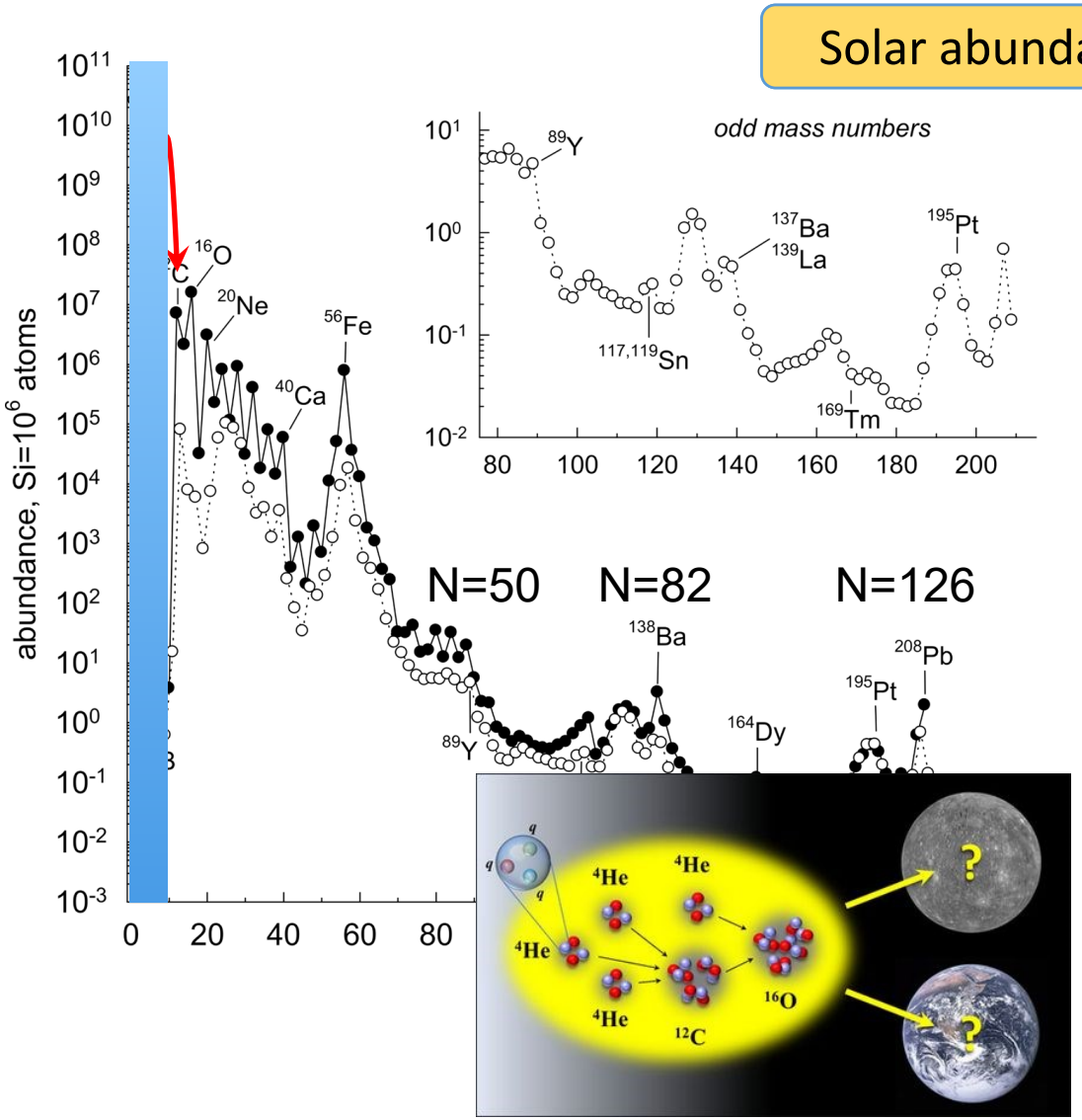
${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$

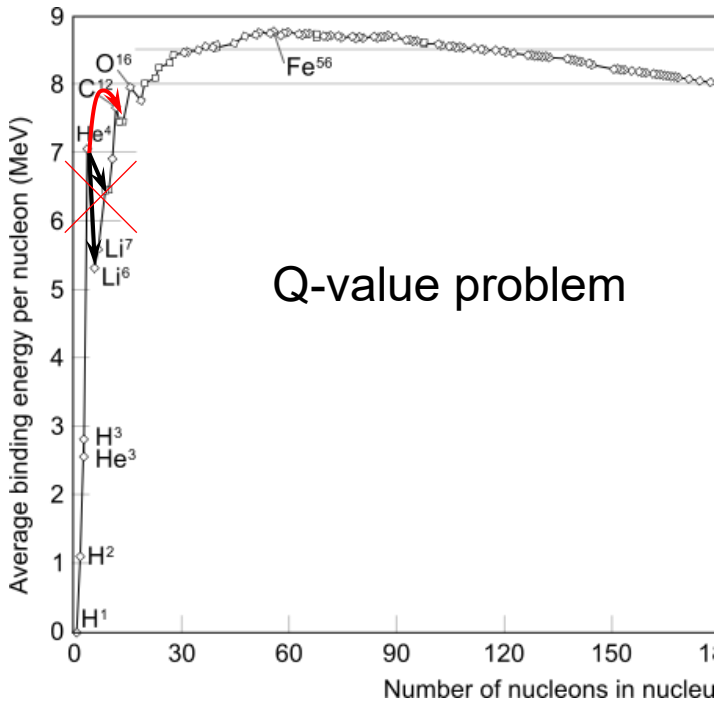
${}^8Be^* \rightarrow 2\ {}^4He$ ...

How to have heavier systems?

Observation of today's solar abundancy

Solar abundancy





Q-value problem

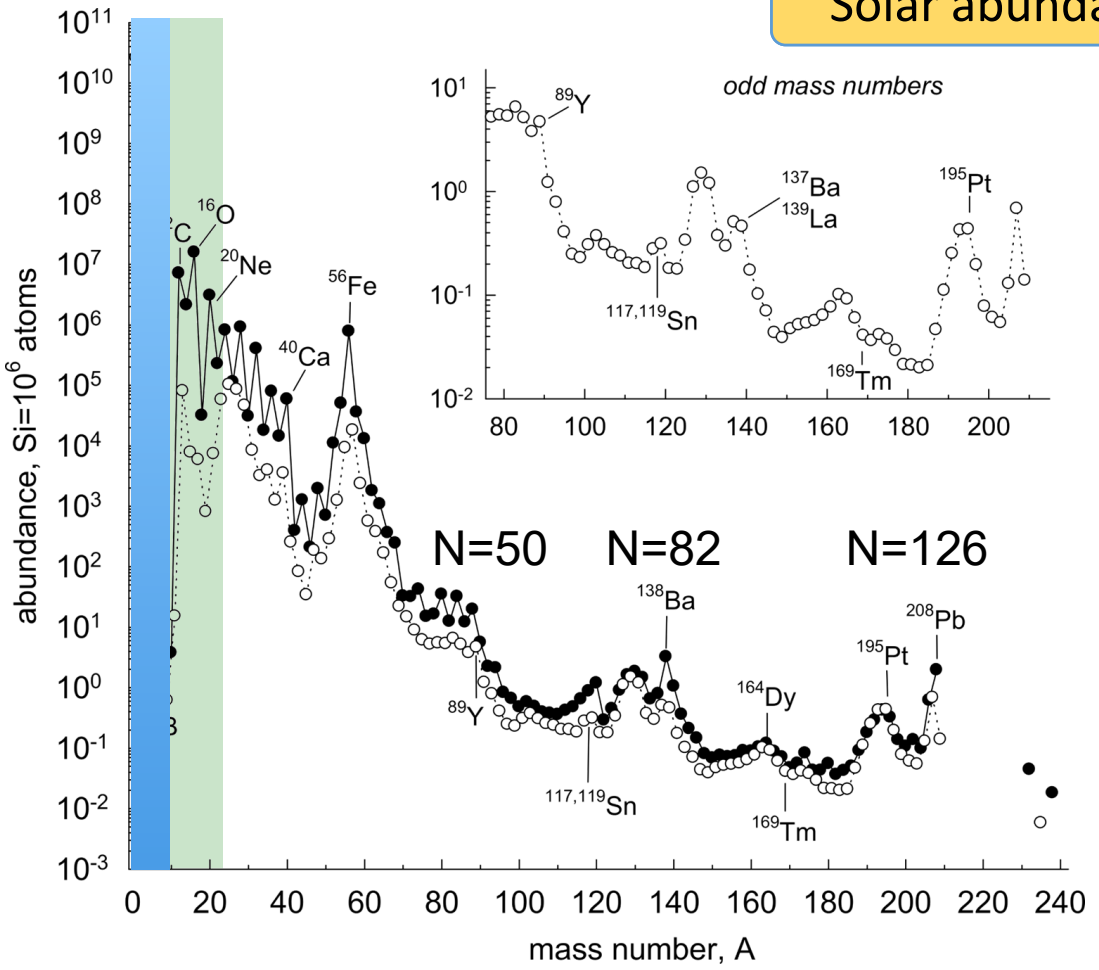
The guess of F. Hoyle

$$\alpha + \alpha + \alpha \longrightarrow {}^{12}C^{*} \longrightarrow {}^{12}C$$

Very low reaction rate but synthesis of C possible thanks to a resonance located around the threshold

Observation of today's solar abundancy

Solar abundancy



CNO cycle

$$^{12}C + p \rightarrow {}^{13}N + \gamma$$

$$^{13}N \rightarrow {}^{13}C + e^{+} + \nu_e$$

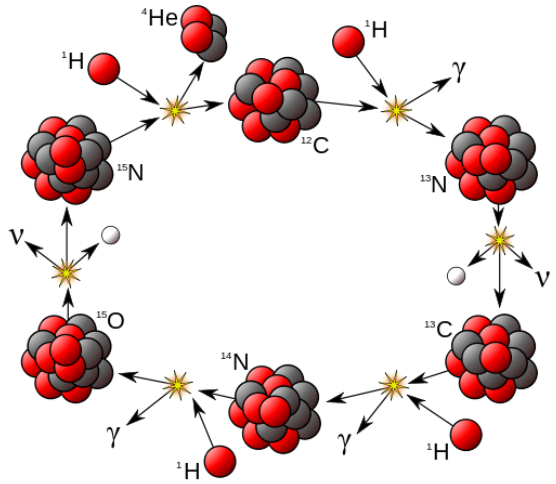
$$^{13}C + p \rightarrow {}^{14}N + \gamma$$

$$^{14}N + p \rightarrow {}^{15}O + \gamma$$

$$^{15}O \rightarrow {}^{15}N + e^{+} + \nu_e$$

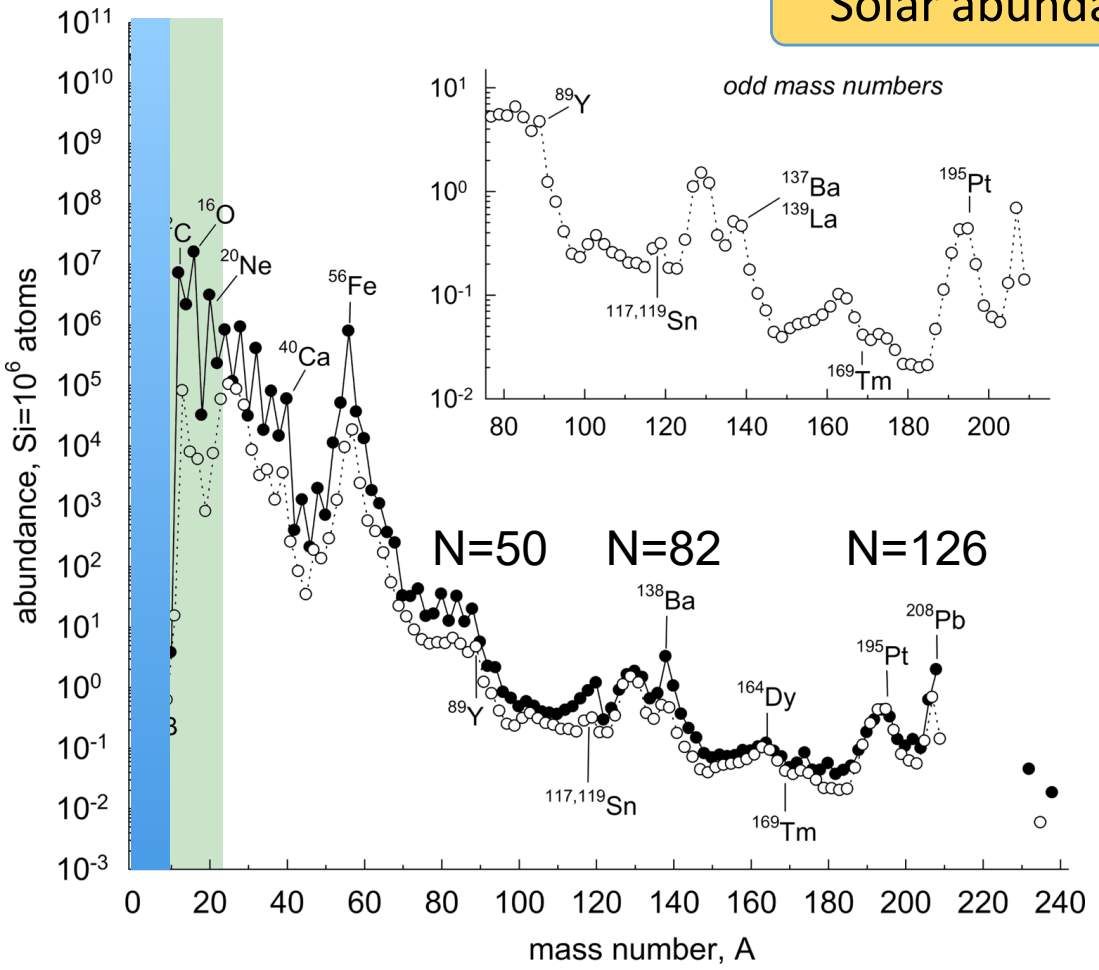
$$^{15}N + p \rightarrow {}^{12}C + {}^{4}He$$

2 % sun energy

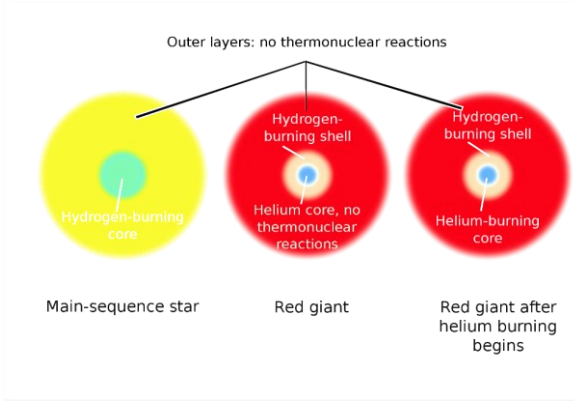


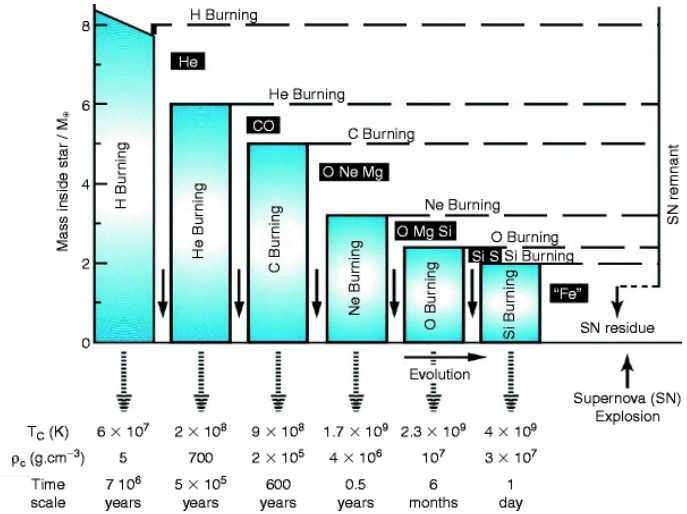
Observation of today's solar abundancy

Solar abundancy



Alpha burning $\alpha + \alpha + \alpha \longrightarrow {}^{12}C$





$^{4}He + {}^{12}C \rightarrow {}^{16}O + \gamma \quad Q = 7.16\ MeV$

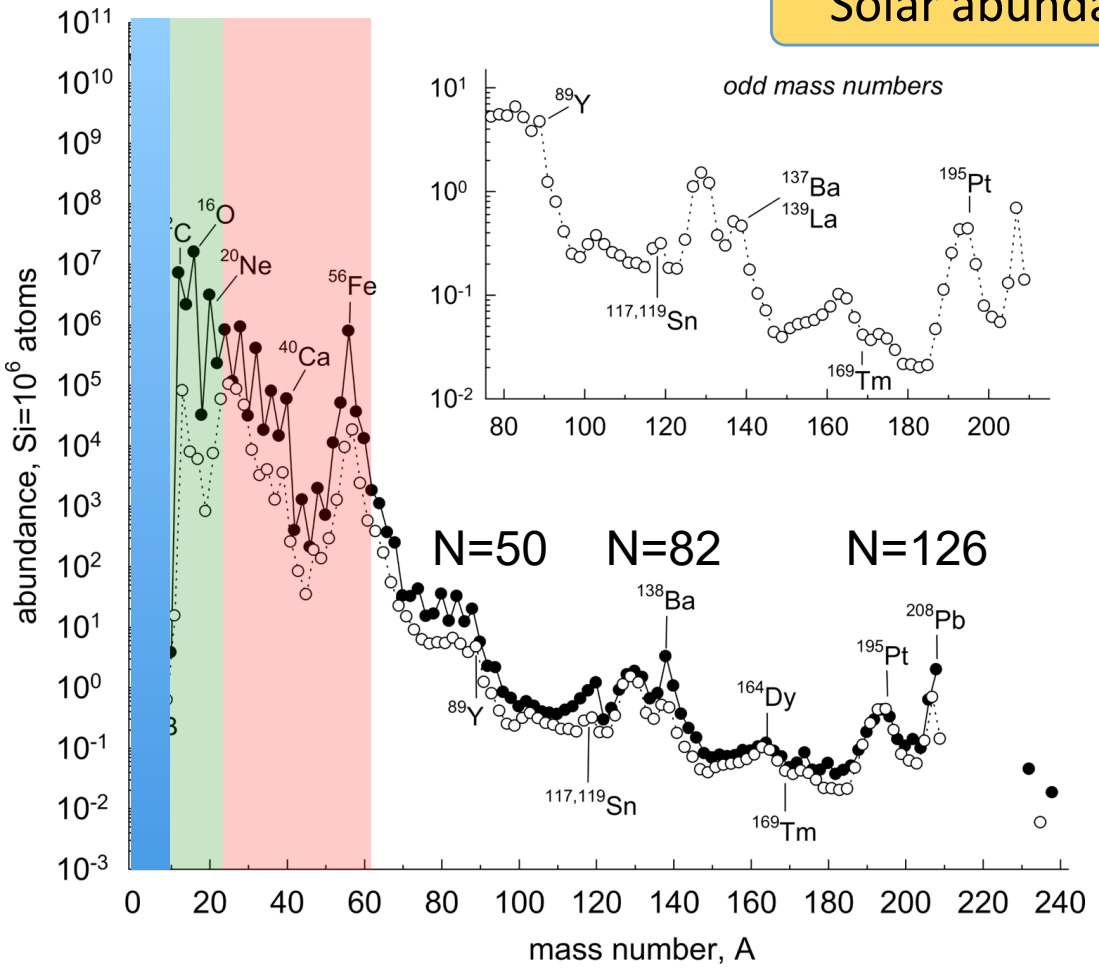
$^{4}He + {}^{16}O \rightarrow {}^{20}Ne + \gamma \quad Q = 4.73\ MeV$

$^{20}Ne + \gamma \rightarrow {}^{4}He + {}^{16}O$

$^{4}He + {}^{20}Ne \rightarrow {}^{24}Mg + \gamma$

Observation of today's solar abundancy

Solar abundancy



Synthesis of nuclei with $A < 60$

$^{12}C + {}^{12}C \rightarrow {}^{20}Ne + {}^{4}He$

$^{12}C + {}^{12}C \rightarrow {}^{23}Na + p$

$^{12}C + {}^{12}C \rightarrow {}^{23}Mg + n$

$^{12}C + {}^{12}C \rightarrow {}^{24}Mg + \gamma$

$^{16}O + {}^{16}O \rightarrow {}^{24}Mg + 2\,{}^{4}He$

$^{16}O + {}^{16}O \rightarrow {}^{28}Si + {}^{4}He$

$^{16}O + {}^{16}O \rightarrow {}^{31}P + p$

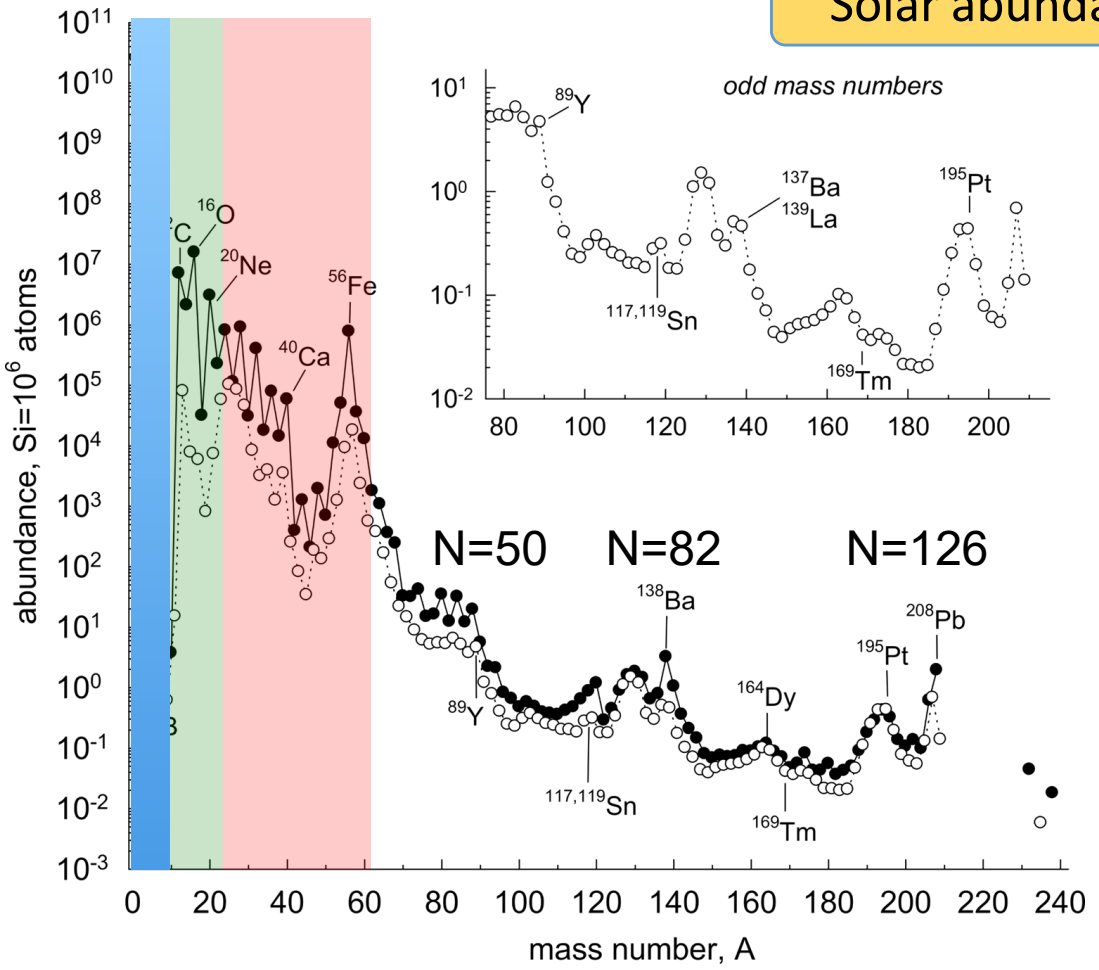
$^{16}O + {}^{16}O \rightarrow {}^{31}S + n$

$^{16}O + {}^{16}O \rightarrow {}^{32}S + \gamma$

Light-ion fusion or incomplete fusion

Observation of today's solar abundancy

Solar abundancy



Synthesis of nuclei with $A > 60$

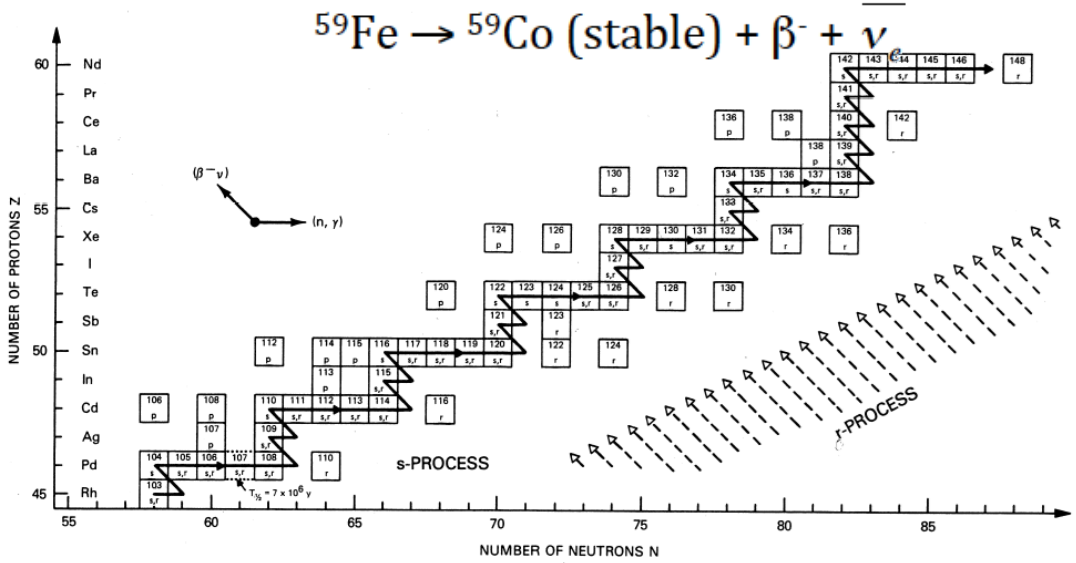
s-process: competition between neutron radiative capture and beta decay

Example: $^{56}\mathrm{Fe} + \mathrm{n} \rightarrow {}^{57}\mathrm{Fe}\ (\mathrm{stable}) + \gamma$

$^{57}\mathrm{Fe} + \mathrm{n} \rightarrow {}^{58}\mathrm{Fe}\ (\mathrm{stable}) + \gamma$

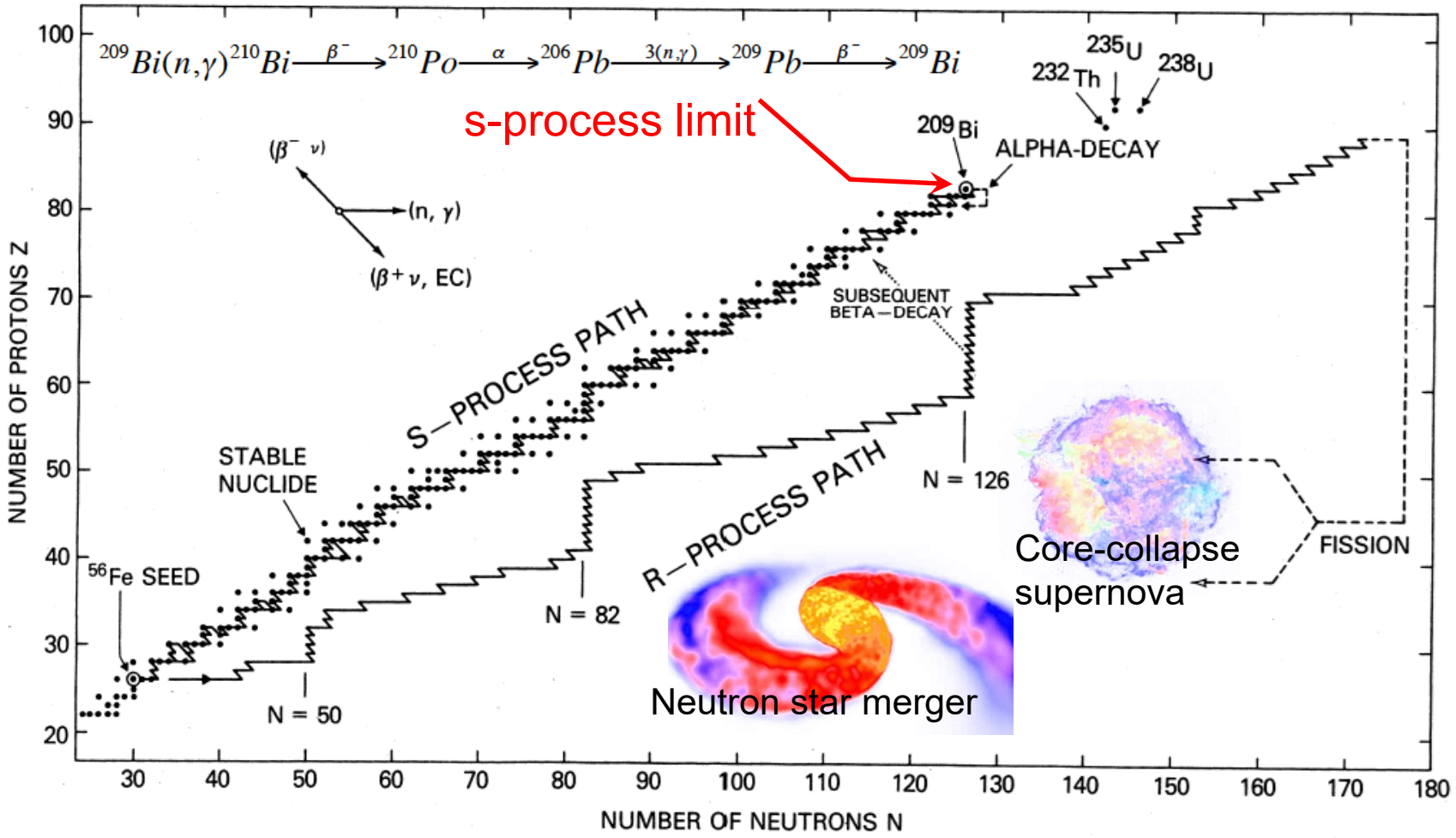
$^{58}\mathrm{Fe} + \mathrm{n} \rightarrow {}^{59}\mathrm{Fe}\ (t_{1/2} = 44.5\ \mathrm{d}) + \gamma$

$^{59}\mathrm{Fe} \rightarrow {}^{59}\mathrm{Co}\ (\mathrm{stable}) + \beta^- + \bar{\nu}$

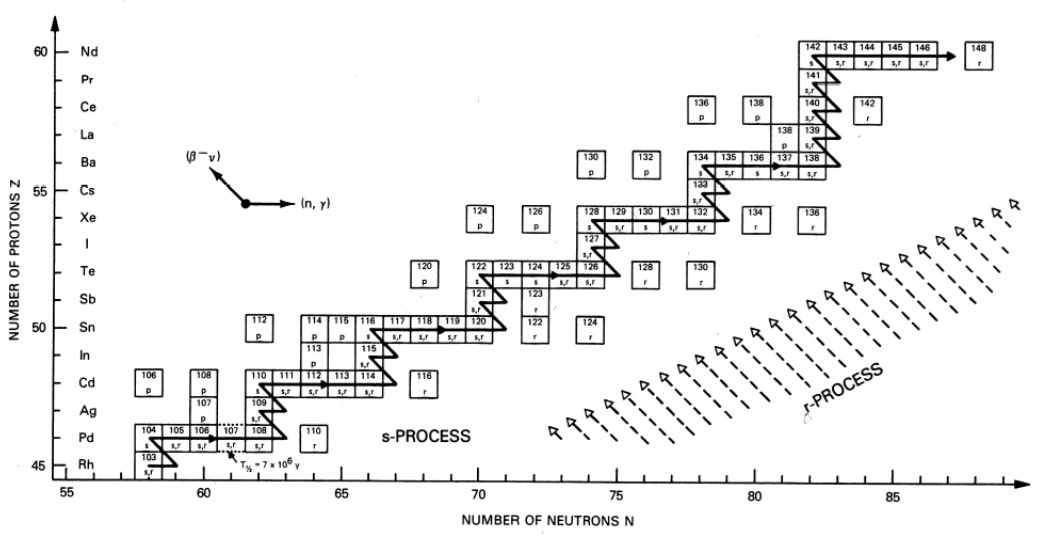


Observation of today's solar abundancy

The s-process compete with the r-process (rapid neutron capture), when neutron density is high [very astrophysical site dependent]



Nuclear inputs for astrophysics



Synthesis of nuclei in a stellar environment is modeled by a differential equation:

$$\left(\frac{dn_i}{dt}\right)_{\rho=\text{cst}} = \sum N_j^i r_j + N_{j,k}^i r_{j,k} \cdots$$

Corresponding to interaction with non-nuclear fields (weak decays, etc.) two-body nuclear reaction and so on.

The coefficient N_j^i specifies how many particles of species i are created in the reaction while r_j is the nuclear reaction rate

A cross section between a target j and a projectile k is $\sigma = \frac{r/n_k}{n_i v}$ so, we define the average number of reactions per cm^3 and s as

$$r_{j,k} = \int \sigma |v_k - v_j| d^3 n_j d^3 n_k$$

Nuclear inputs for astrophysics

In general, for a plasma the species obeys a Maxwell-Boltzmann distribution

$$d^3 n_j = n_j \left(\frac{m_j}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{m_j^2 v_j}{2kT}} d^3 v_j$$

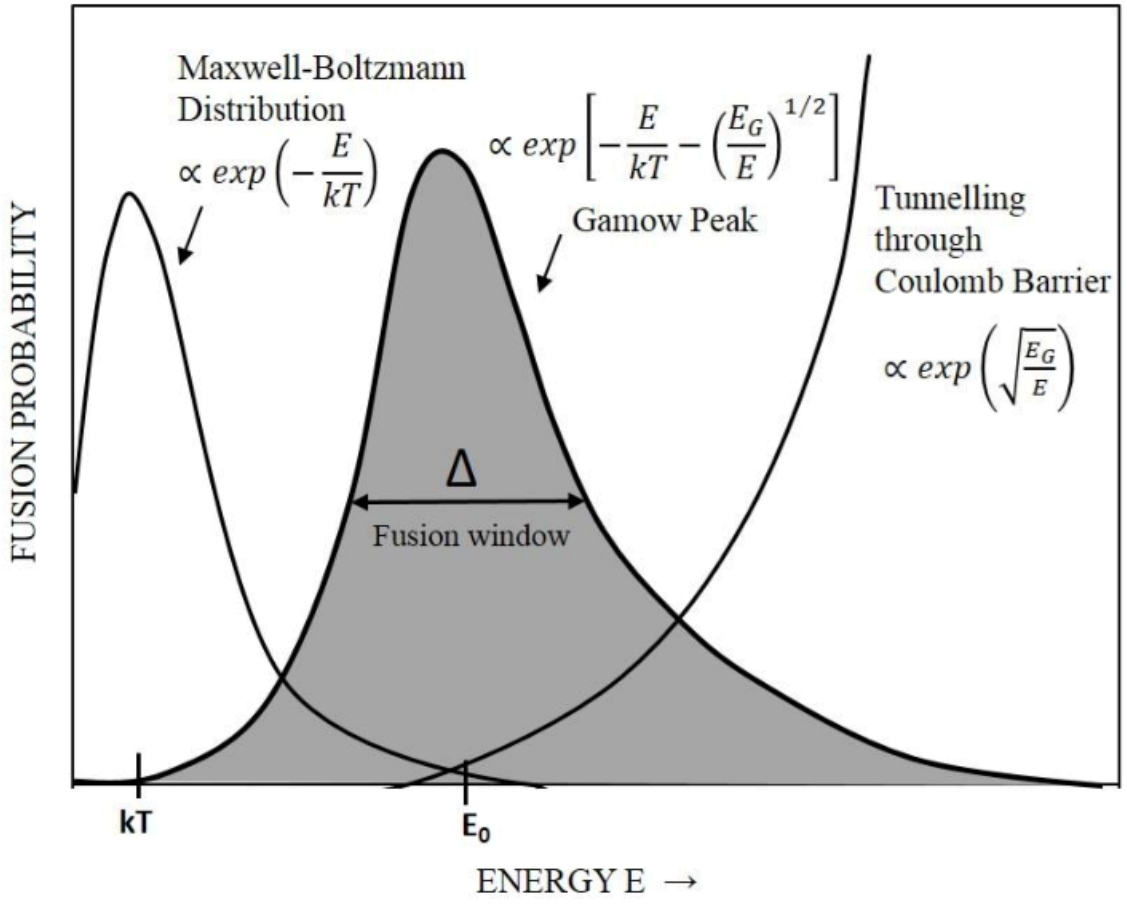
The reaction rate is often written as a function of the astrophysical S-factor that attempts to deconvolute the effect of the Coulomb barrier (we work at energy under the Coulomb barrier)

$$\mathrm{S-factor} = E\sigma(E)e^{2\pi\eta}$$

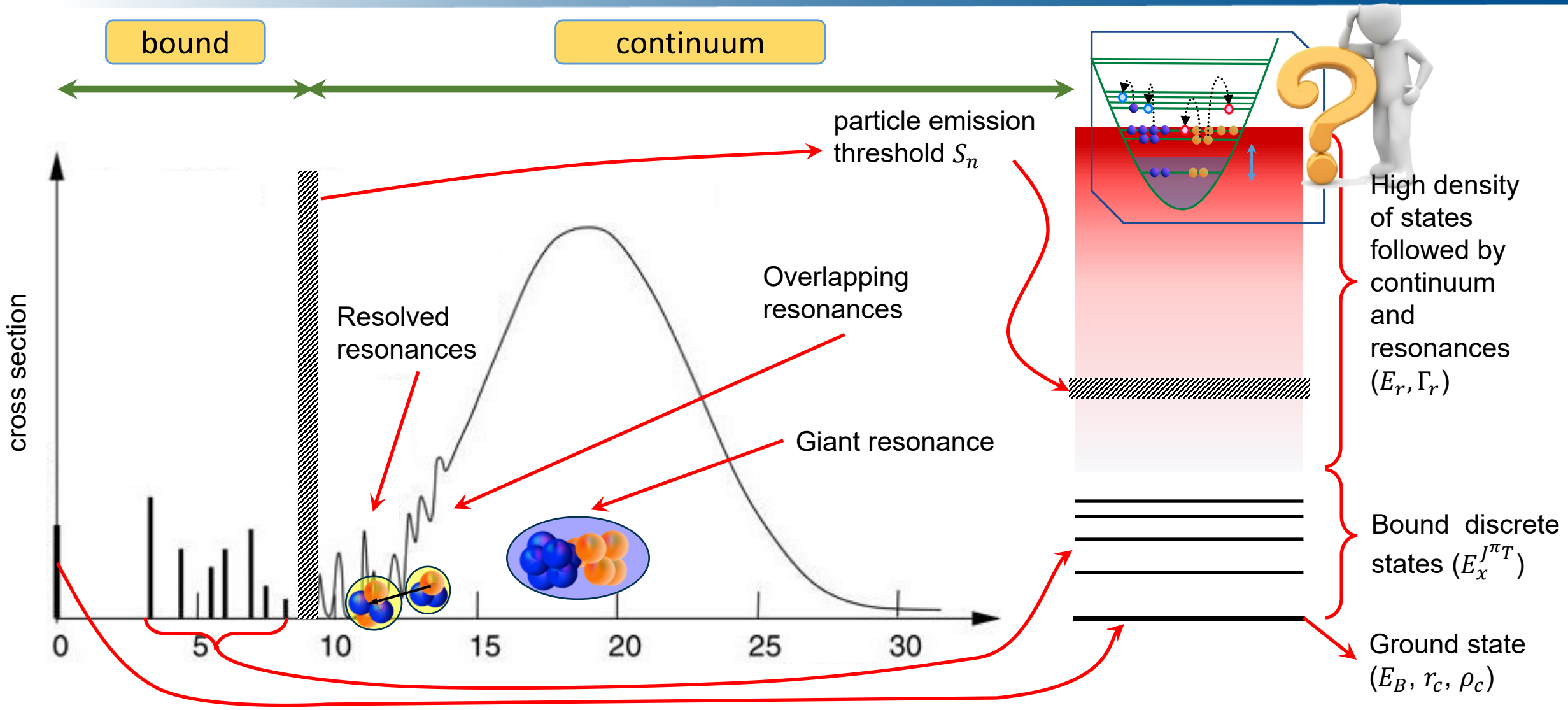
The reaction rate then reads

$$r_{j,k} = \frac{n_j n_k}{1+\delta_{i,k}} \langle \sigma v \rangle$$

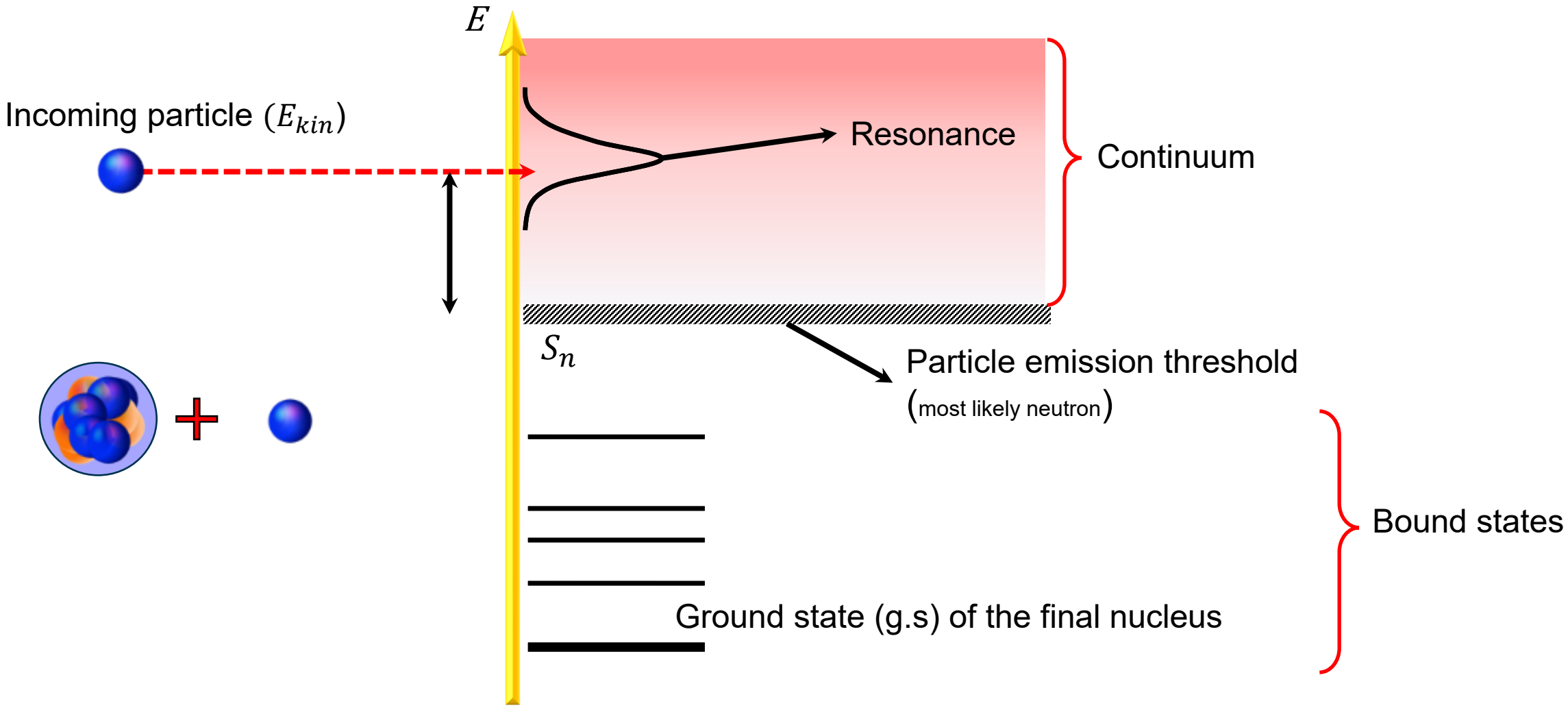
$$\langle \sigma v \rangle = \left(\frac{8}{\pi\mu}\right)^{\frac{1}{2}} \left(\frac{1}{kT}\right)^{\frac{3}{2}} \int S(E) e^{-\frac{E}{kT} - \left(\frac{E_g}{E}\right)^{\frac{1}{2}}} dE$$



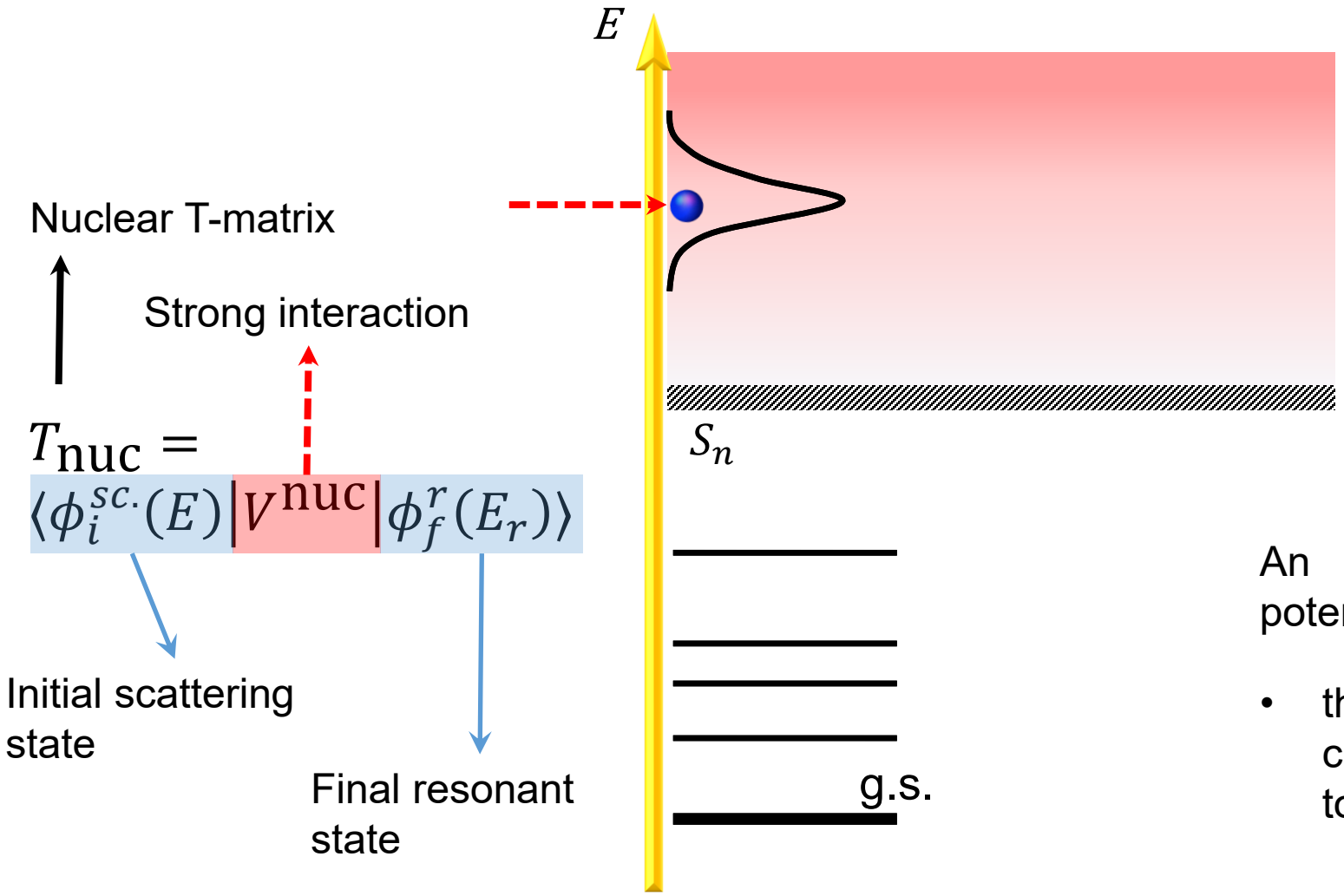
Nuclear structure viewed from nuclear reactions



Capture processes: resonant capture



Capture processes: resonant capture



Draw a pot

An incoming particle gets trapped in the potential pocket of an isolated resonance:

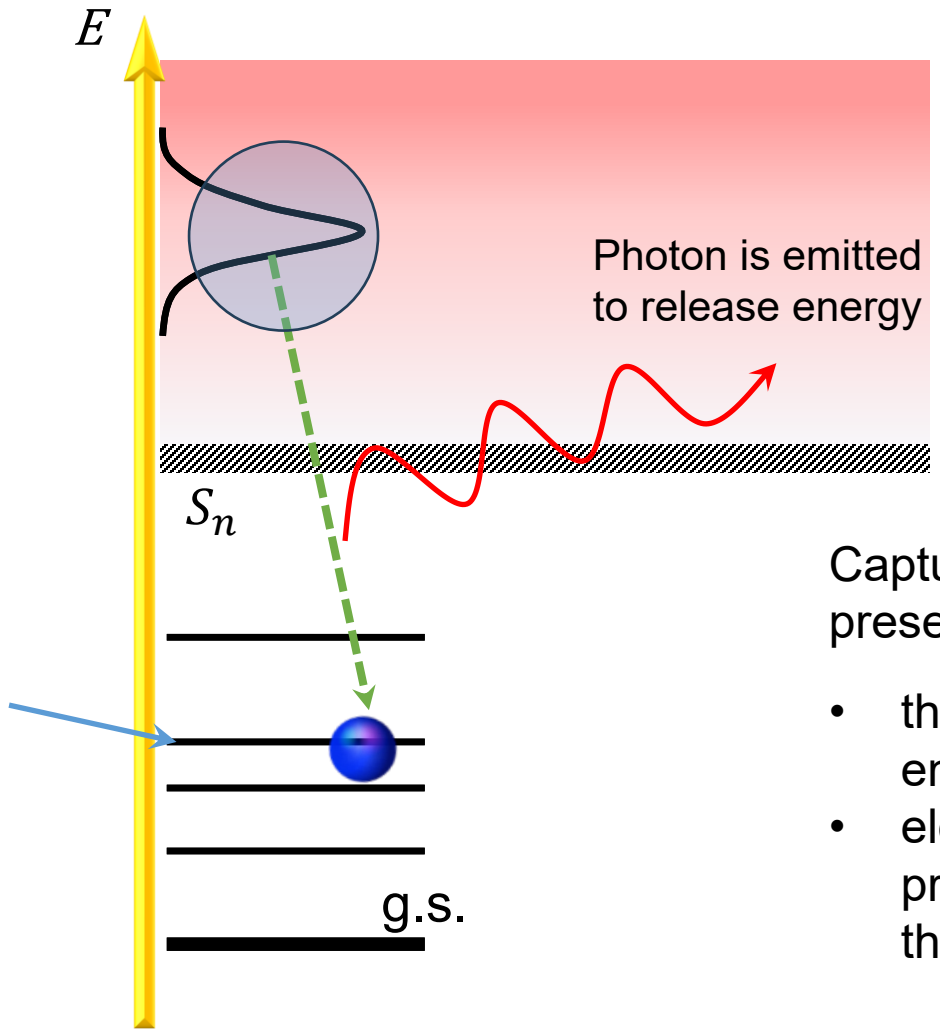
- the T-matrix is the amplitude for the capture process from the scattering state to the resonance.

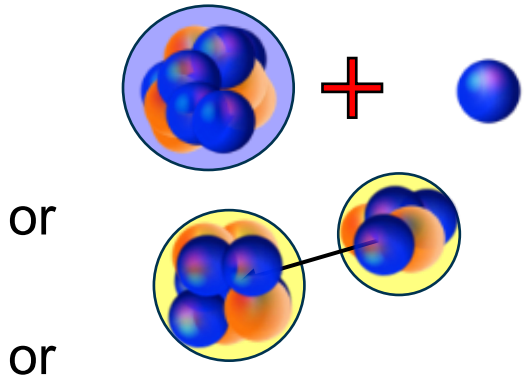
Capture processes: resonant capture

Electromagnetic interaction

$$T_\gamma = \langle \phi_i^r(E_r)|V|\phi_f\rangle$$

Final bound state (g.s. or ex. state)

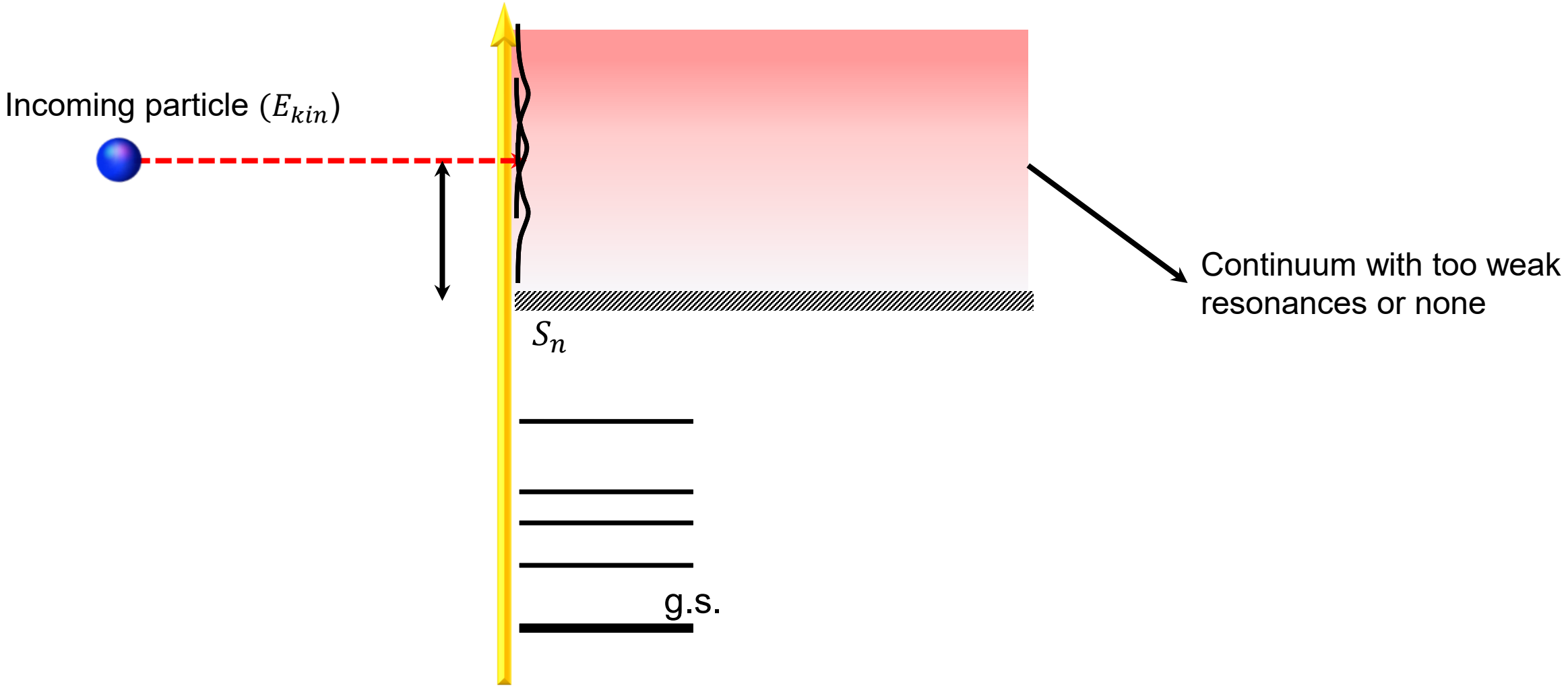




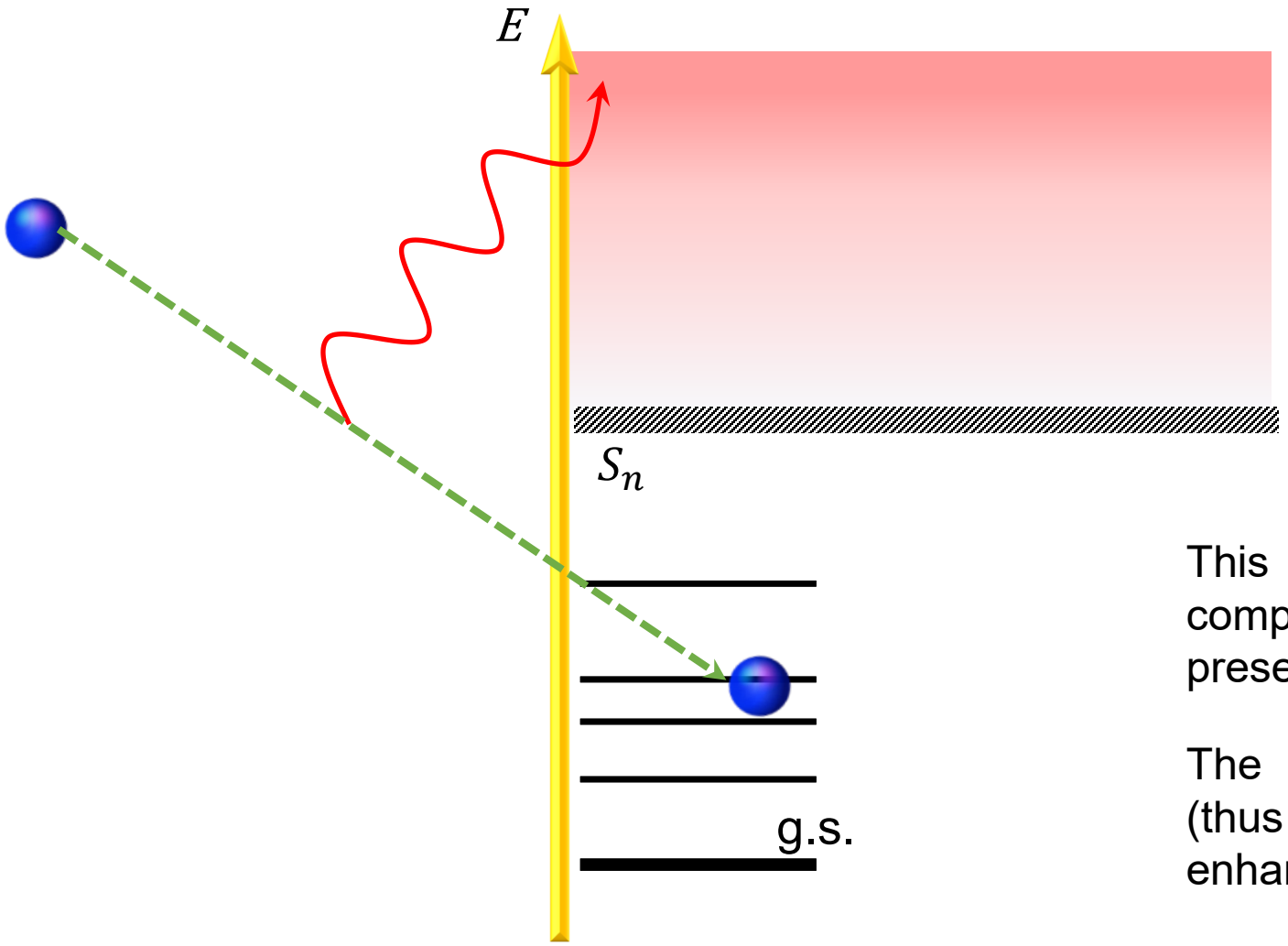
Capture is favored if nuclear resonances are present:

- the photon emission competes with particle emission;
- electromagnetic T-matrix accounts for the probability to decay from the resonance to the final bound state.

Capture processes: direct capture



Capture processes: direct capture

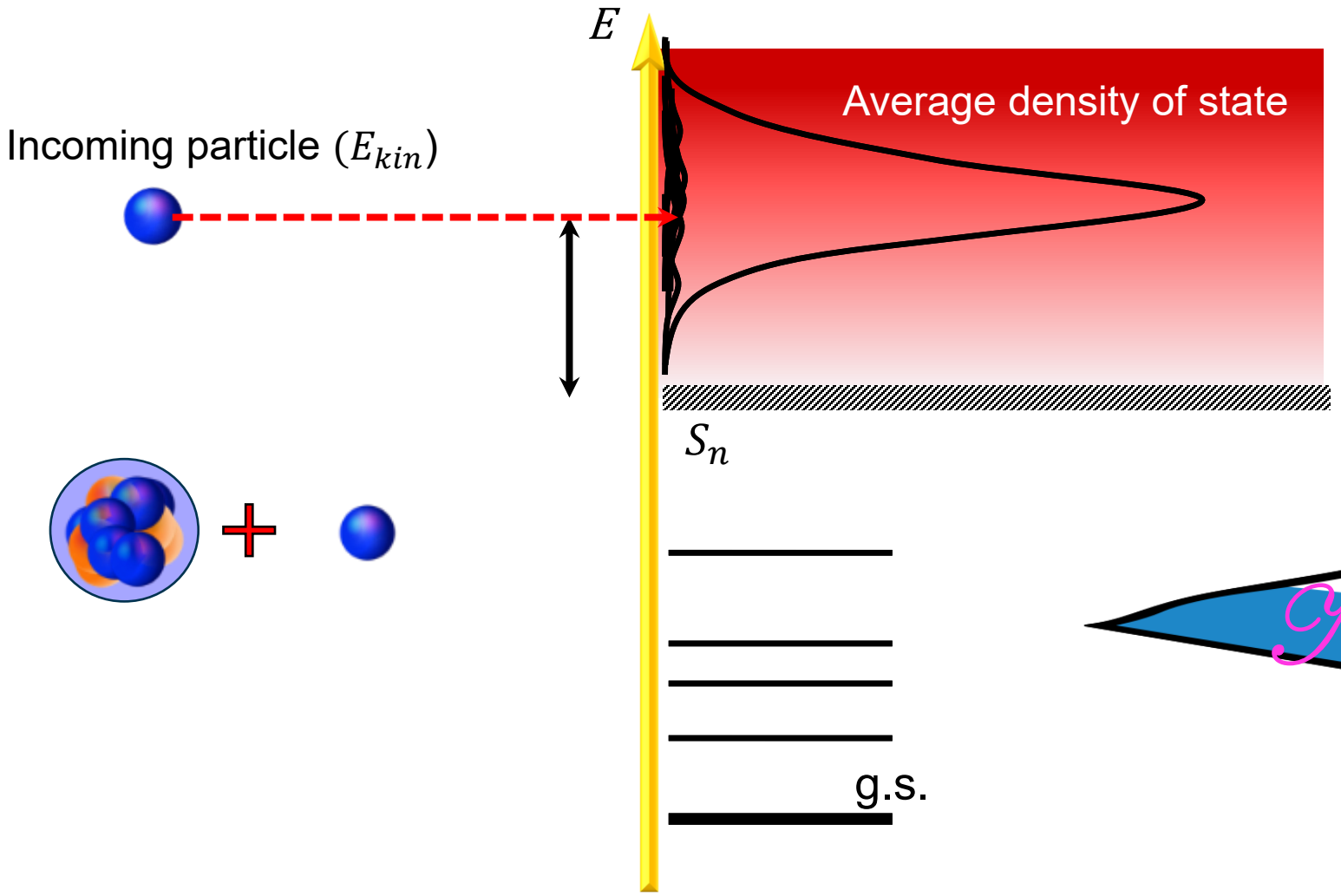


$$T_{f\leftarrow i} = \langle \phi_i^{sc.}(E_{kin}) | V | \phi_f \rangle$$

This process is favored (by lack of competitors) if strong resonances are not present.

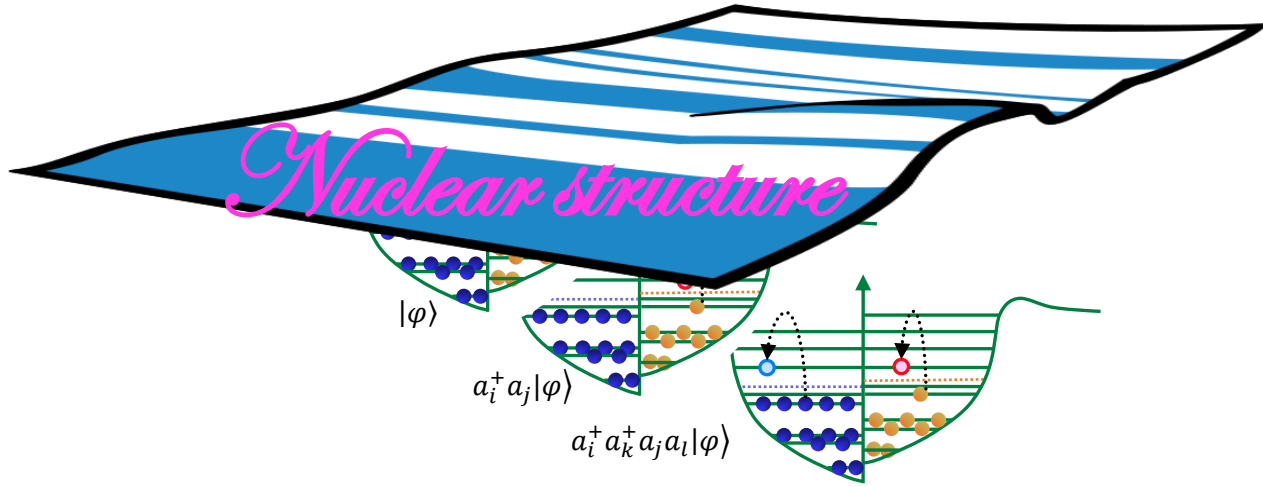
The probability to capture is uniquely E.M. (thus much weaker than nuclear resonantly enhanced).

Capture processes: compound capture

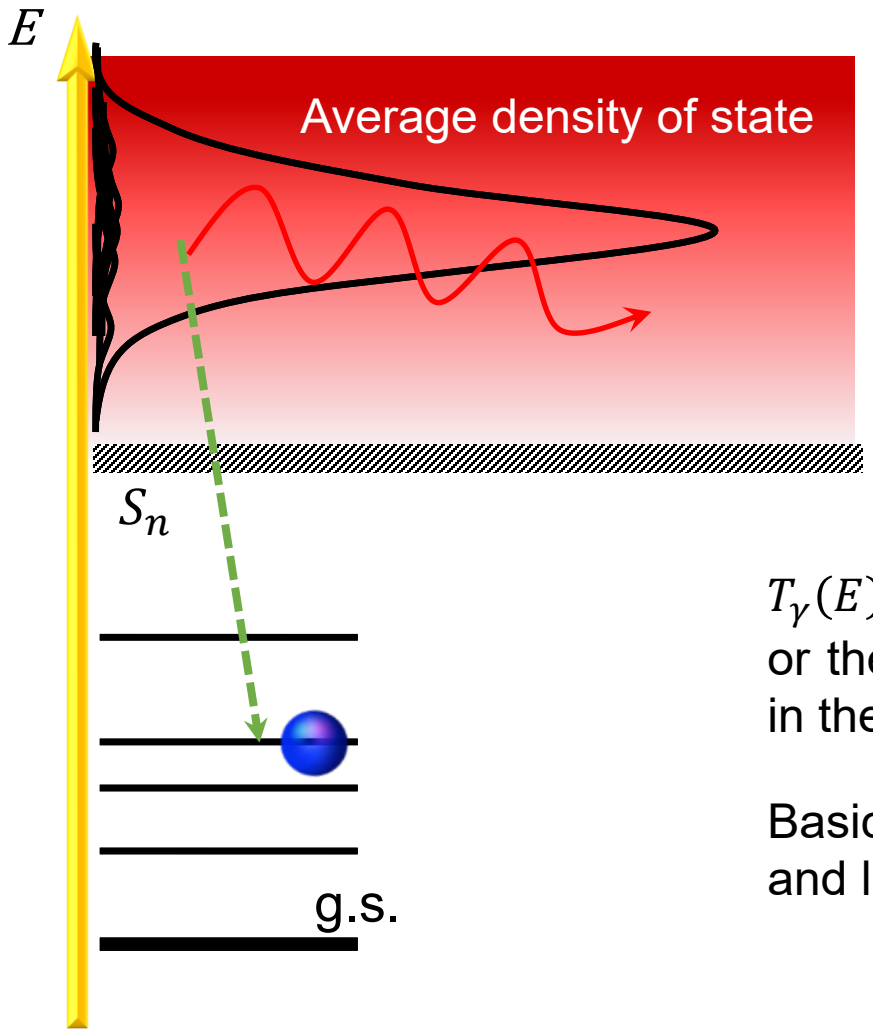


Dense continuum:

- High-level density
- Many overlapping resonances



Capture processes: compound capture

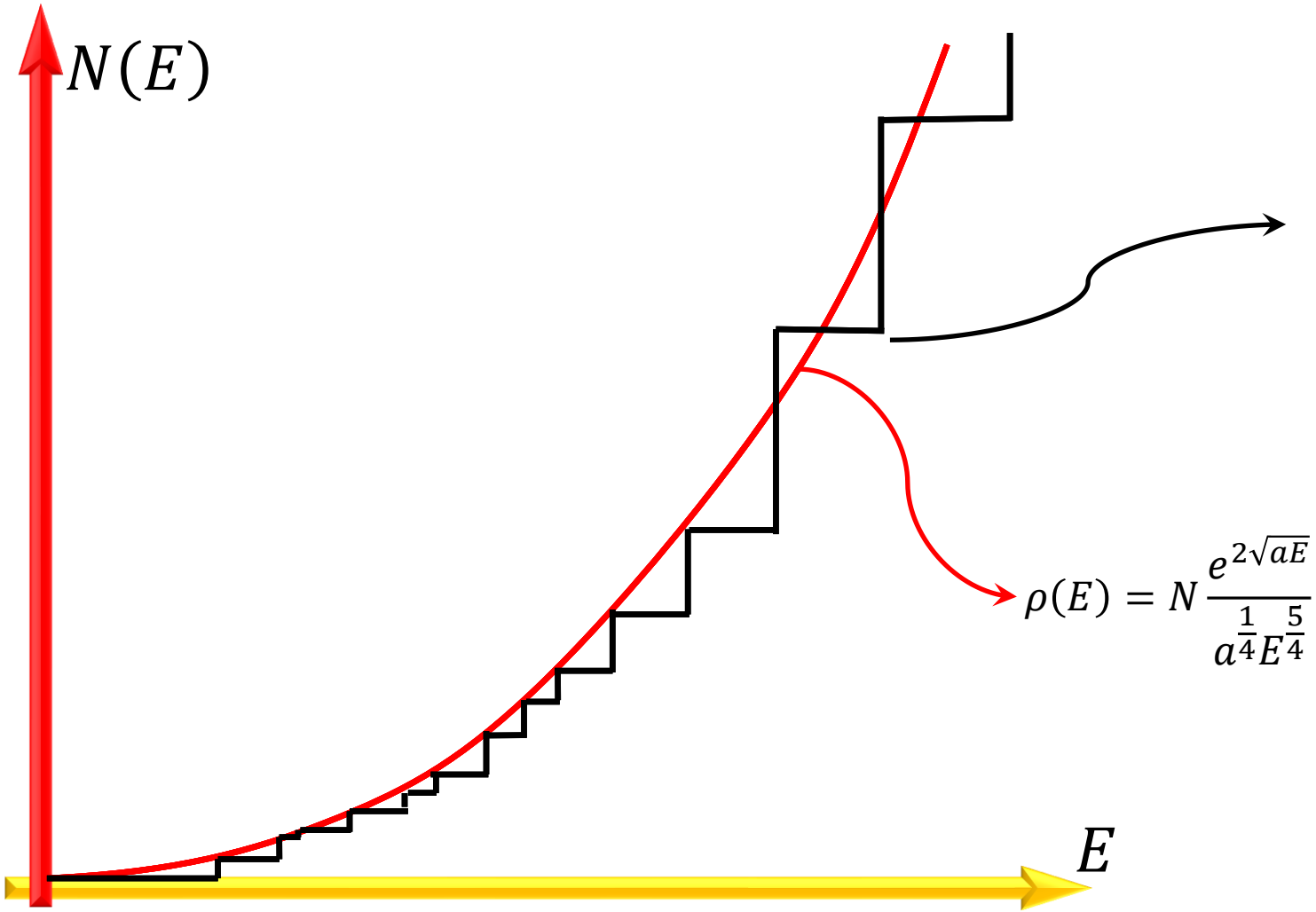


$$T_\gamma^c \longrightarrow T_\gamma(E)$$

$T_\gamma(E)$ is the γ channel transmission coefficient or the probability to decay per E.M. transition in the compound nucleus.

Basically calculated from γ-strength functions and level densities.

Hauser-Feshbach: $\lim_{N\to\infty} \rho(E)$



$$N(E) = \sum_n \Theta(E - E_n)$$

$$\rho(E) = \frac{dN(E)}{dE} = \sum_n \delta(E - E_n)$$

The level density $\rho(E)$ is very high

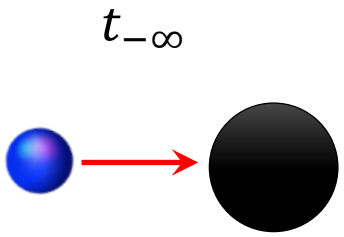
and

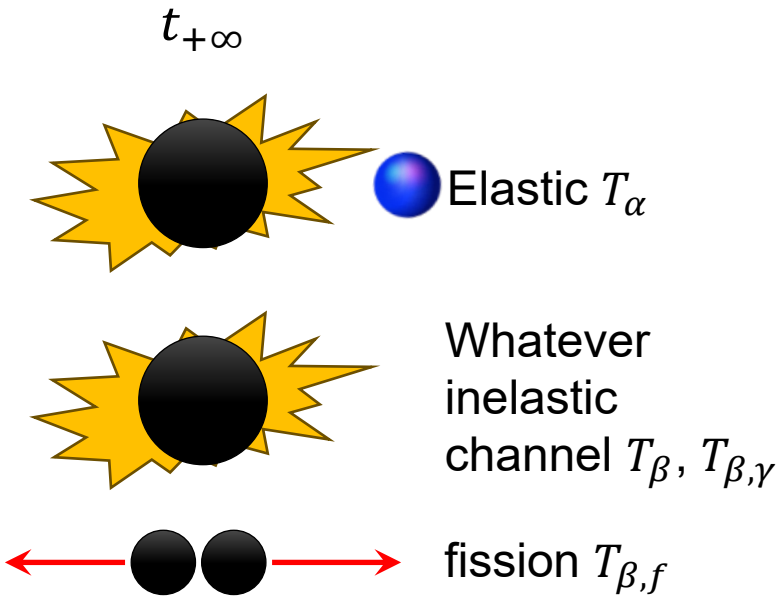
The experimental conditions impose that resonances are not resolved:

- step function replaced by continuous function;
- $\rho(E)$ is purely a statistical inference of data.

Hauser-Feshbach: « Memoryless »

Compound nucleus formation described by transmission coefficient $T_{\alpha,n}(E)$





Decay channels are described by transmission coefficients:

$T_\alpha, T_\beta(T_{\beta,\gamma}), T_{\beta,f}$ etc..

Hypothesis of independence of probability (no memory of formation):

$$T_{\alpha,i}P = \frac{T_{\alpha,i}T_{\beta,f}}{\sum T_\beta}$$

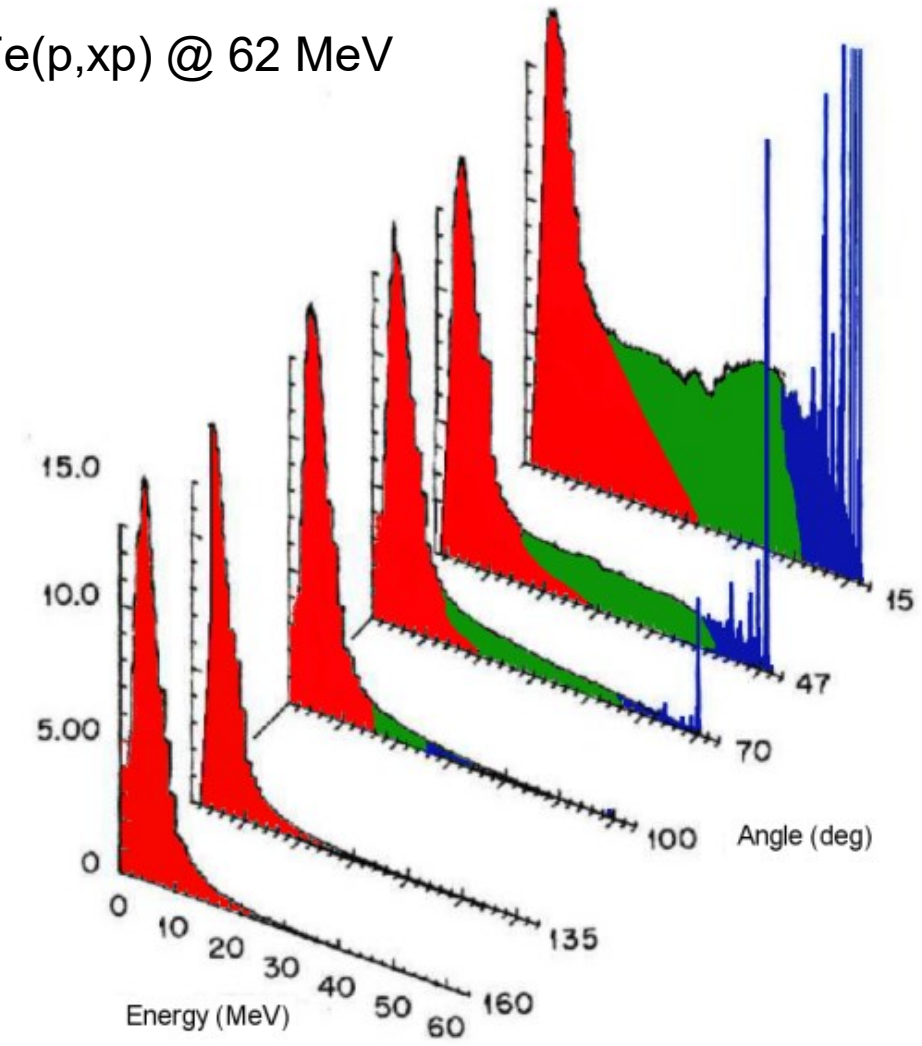
After averaging over unresolved compound-nucleus resonances, channel couplings are characterized by transmission coefficients $T_c(E)$, rather than by individual resonance amplitudes.

On needs various models to compute the transmission coefficients:

- particle emission and absorption: optical model;
- γ emission and absorption: γ-strength functions from electromagnetic response (E1, M1, E2…);
- fission: fission barriers.

Typical emission spectrum from compound reaction

^{56}Fe(p,xp) @ 62 MeV



In red: evaporation

Slow reaction time 10^{-14}s.
Isotropic angular distributions.

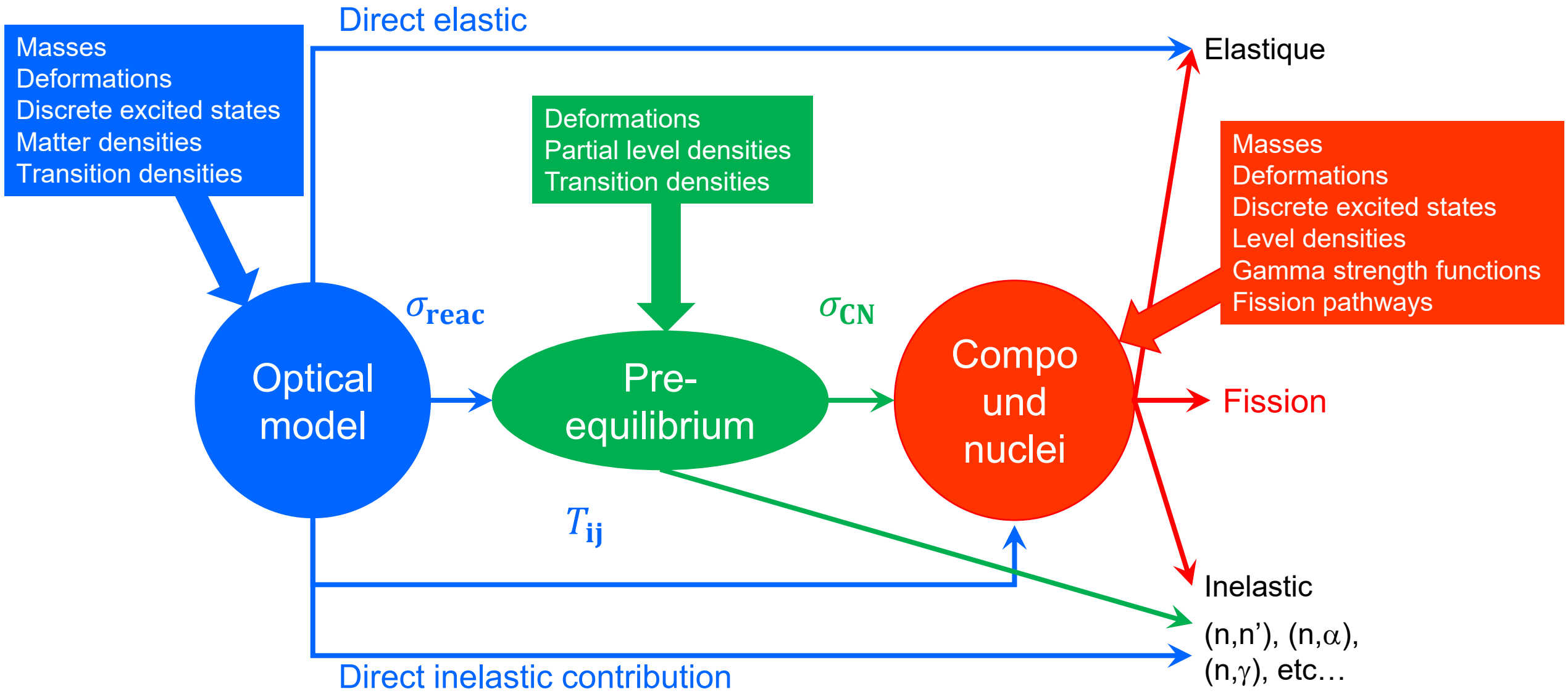
In green: “flat” region

Intermediate reaction time.
Anisotropic angular distributions that evolve slowly toward the forward direction as the emission energy increases.

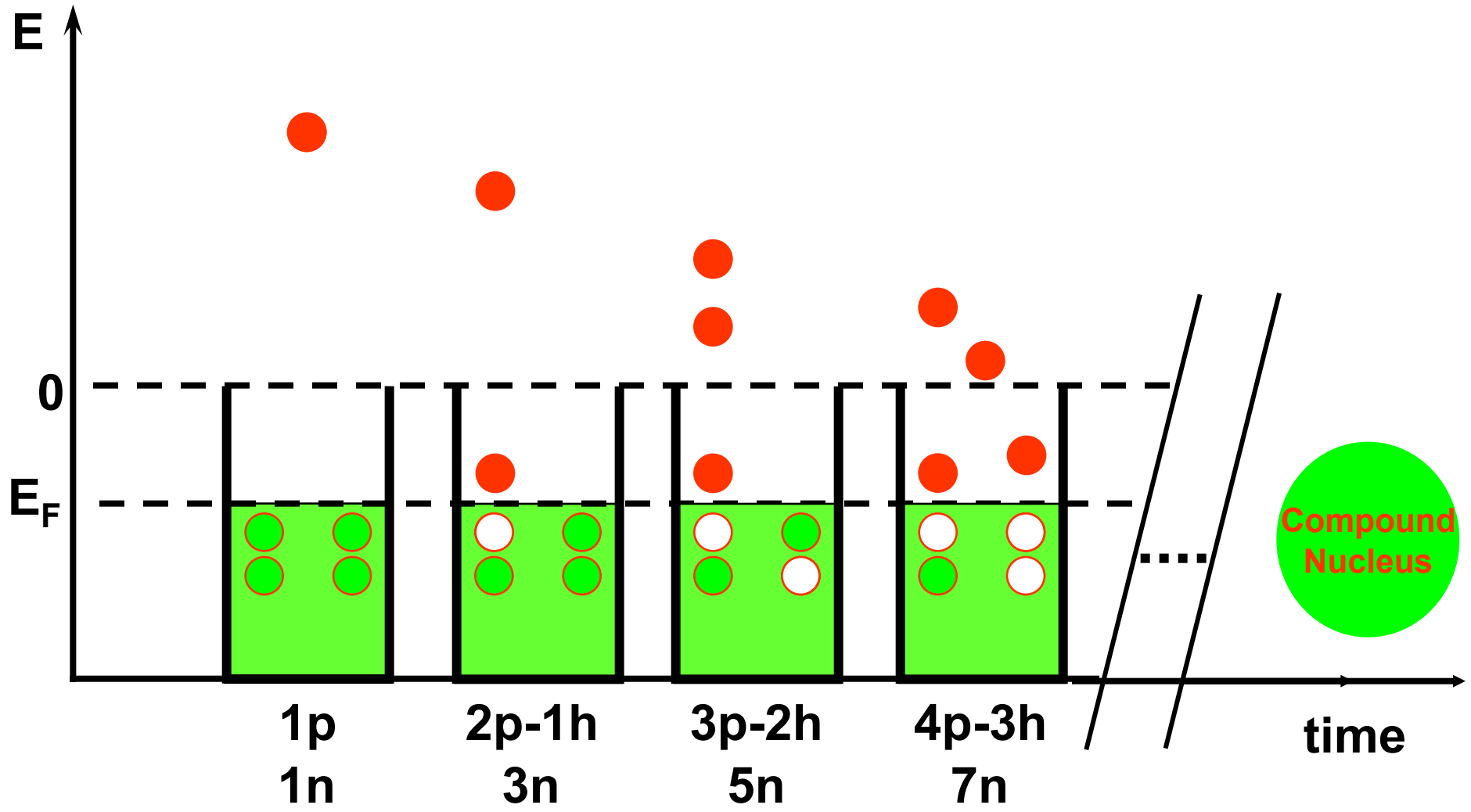
In blue: discrete peaks

Fast reaction time 10^{-21}s.
Angular distributions peaked in the forward direction and showing oscillations.

TALYS: flow chart



TALYS pre-equilibrium: exciton model principle



TALYS pre-equilibrium: master equation exciton model

We want to track in time the evolution of different species (excited composite system to equilibrate. We define:

- $P(n,E,t)$ is the probability to find for at time t the composite system with an energy E and an exciton number n.
- $\lambda_{i\rightarrow f}(E)$ is the transition rate from an initial state i a to a state f for a given energy E.

The time evolution is a reaction network-like and thus follows (for each n):

- $\frac{dP(n,E,t)}{dt} = P(n-2,E,t)\lambda_{n-2\rightarrow n}(E) + P(n+2,E,t)\lambda_{n+2\rightarrow n}(E) - P(n,E,t)\left(\lambda_{n\rightarrow n+2}(E) + \lambda_{n\rightarrow n-2}(E) + \int \sum_c \lambda_{n\rightarrow c}(E,\varepsilon_c)d\varepsilon_c\right)$

Then compute the quantities of desire.. the emission cross-section in channel “c”:

$$\frac{d\sigma_c}{d\varepsilon_c} = \sigma_{\text{reac}} \int_0^\infty \sum_{n,\Delta n=2} P(n,E,t)\,\lambda_{n\rightarrow c}(E,\varepsilon_c)\,dt$$

$$\sigma_{\text{reac}} = \frac{\pi}{k^2}\sum_{l,j}(2J+1)T_{lj}^{\text{entrance}}$$

TALYS pre-equilibrium: initialization & transition rates

Initialisation:

$P(n, E, 0) = \delta_{n,n_0}$ with for instance $n\_0 = 3$ for nucleon induced reactions

Transition rates:

Fermi's golden rule: $\frac{2\pi}{\hbar}$ |Matrix elements|^2 x density of final states

$$\lambda_{n\to n-2}(E) = \frac{2\pi}{\hbar}\langle M^2\rangle\, w(p,h,E) \text{ with } p+h = n-2$$

$$\lambda_{n\to n+2}(E) = \frac{2\pi}{\hbar}\langle M^2\rangle\, w(p,h,E) \text{ with } p+h = n+2$$

Particle emission:

$$\lambda_{n\to c}(E,\varepsilon_c) = \frac{2S_c+1}{\pi^2\hbar^3}\mu_c\varepsilon_c\sigma_{c,\mathrm{inv}}(\varepsilon_c)\frac{w(\text{"residual conf"}, E-\varepsilon_c-\mathrm{BE}_c)}{w(p,h,E)}\mathrm{Q_c(n)F_c}$$

Original formulation

From detailed balanced

corrections for proton-neutron distinguishability & complex particle emission

State densities:

$w(p,h,E)$ = number of ways of distributing p particles and h holes among accessible single particle levels with the available excitation energy E,

$$w(p,h,E) \approx \frac{\rho_{s.p.}^{p+h}E^{p+h-1}}{p!\,h!\,(p+h-1)!}$$

TALYS compound nucleus model

Compound nucleus hypothesis

- Continuum of excited levels
- Independence between incoming channel i and outgoing channel f

$$\sigma_{i\to f} = \sigma_i^{\mathrm{CN}} P_f$$

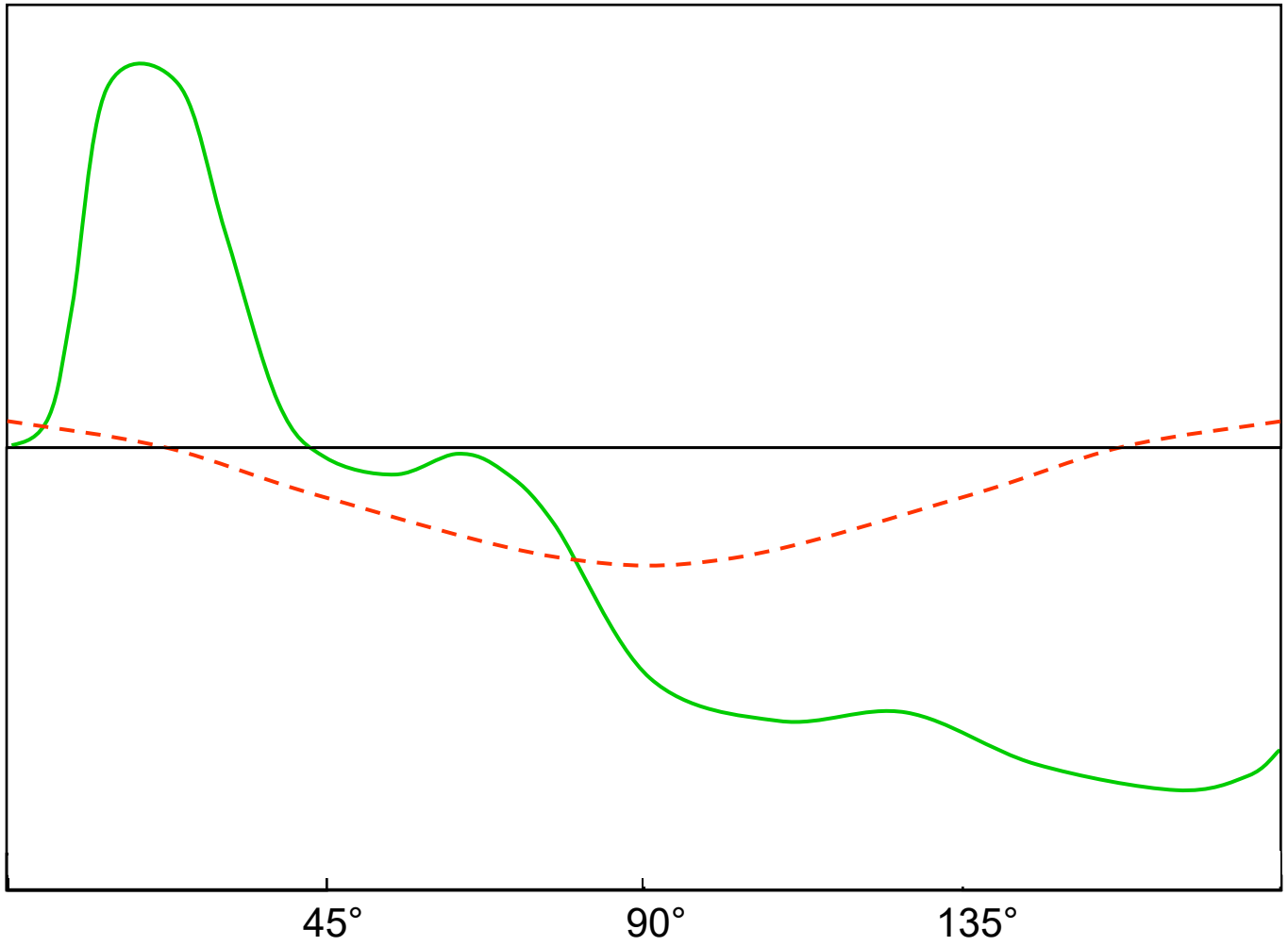
$$\sigma_i^{\mathrm{CN}} = \frac{\pi}{k_i^2} T_i \qquad P_f = \frac{T_f}{\sum_d T_d}$$

We recover the first step of the Hauser-Feshbach formula

$$\sigma_{i\to f} = \frac{\pi}{k_i^2} \frac{T_i T_f}{\sum_d T_d}$$

TALYS compound nucleus model: observable signature

Compound angular distribution & direct angular distributions



TALYS compound nucleus model : width fluctuation

Cross-section from a Breit-Wigner resonance (assumption: sharp resonance):

$$\sigma_{i\to f}^{r}(E) = \frac{\pi}{k_i^2}\frac{\Gamma_i^{\mathrm{r}}\Gamma_f^{\mathrm{r}}}{(E-E_r)^2 + \frac{\Gamma^{r\,2}_{\mathrm{tot}}}{4}}.$$

integrated and averaged ($D = \frac{\Delta E}{\#\mathrm{res}}$ is the mean CN resonance spacing) over an energy width corresponding to the incident beam dispersion to give:

$$\langle\sigma_{i\to f}\rangle = \frac{\pi}{k_i^2}\frac{2\pi}{D}\left\langle\frac{\Gamma_{\mathrm{i}}\Gamma_f}{\Gamma_{tot}}\right\rangle$$

The weak-coupling/narrow-width relation gives $T_i \approx \frac{2\pi\langle\Gamma_i\rangle}{D}$

$$\Rightarrow \begin{cases} \langle\sigma_{i\to f}\rangle = \frac{\pi}{k_i^2}\frac{T_i T_f}{\sum_d T_d} W_{i\to f} \\ W_{i\to f} = \frac{\left\langle\frac{\Gamma_{\mathrm{i}}\Gamma_f}{\Gamma_{tot}}\right\rangle}{\frac{\langle\Gamma_i\rangle\langle\Gamma_f\rangle}{\langle\Gamma_{tot}\rangle}} \end{cases}$$

TALYS compound nucleus model: cross-sections

$$\sigma_{\mathrm{CN}} = \sum_{f} \sigma_{i \to f}$$

The sum over the final state replace by an average given the statistical number of final channels:

$$\sum_{f} \to \int \rho(E_f) dE_f$$

The final state (mass partition) can be a γ, n , p, d, t,…, fission $\otimes$ residual nucleus

$$\sigma_{i \to f} = \frac{\pi}{k_i^2} \sum_{J,\pi} \sum_{\alpha,\beta} \frac{2J+1}{(2S+1)(2I+1)} T_{l_\alpha, j_\alpha}^{J^\pi}(\alpha) \frac{\left\langle T_f^{J^\pi}(\beta) \right\rangle}{\sum_\delta \left\langle T_d^{J^\pi}(\delta) \right\rangle} W_{i \to f}$$

With the quantum numbers $J = l_\alpha + s_\alpha + I_i = j_\alpha + I_i$ and $\pi = (-1)^{l_\alpha} \pi_i$ composed of channel quantum numbers plus intrinsic (particle plus residual nucleus f).

The $\left\langle T_f^{J^\pi}(\beta) \right\rangle$ is the transmission coefficient to the outgoing channel β with the outgoing particle plus residual nucleus f.

TALYS compound nucleus model: decay channels

Possible decays:

1. Emission to a discrete level with energy E_d

$$\langle T_f(\beta) \rangle = T^{J_f^\pi}_{l_\beta j_\beta}(\beta)$$

The transmission coefficient is calculated from the optical potential

2. Emission in the level continuum

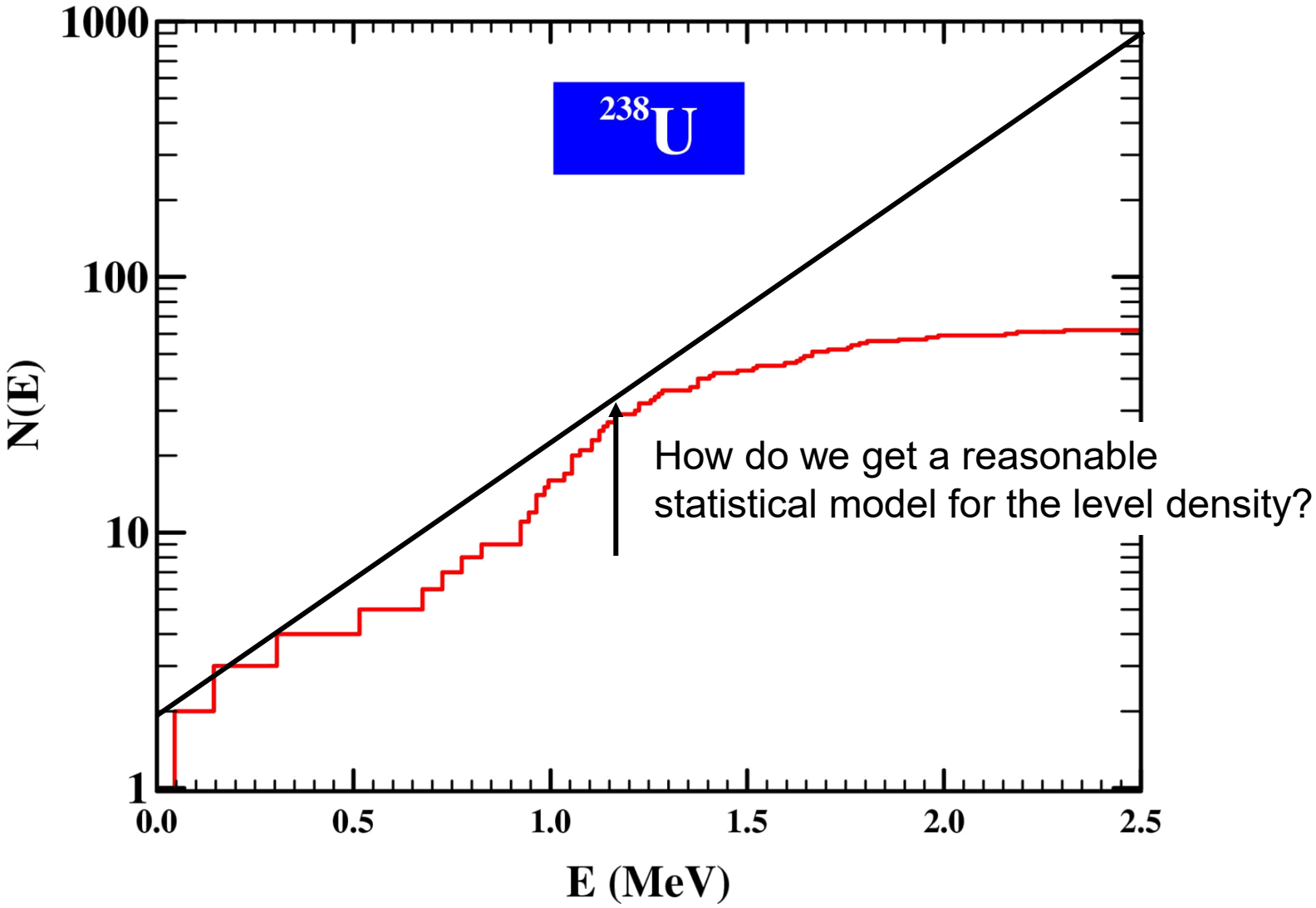
$$\langle T_f(\beta) \rangle = \int_E^{E+\Delta E} T^{J_f^\pi}_{l_\beta j_\beta}(\beta)\, \rho\left(E, J_f, \pi\right) dE$$

$\rho\left(E, J_f, \pi\right)$ is the level density of the residual nucleus with quantum number J_f^π and excitation E

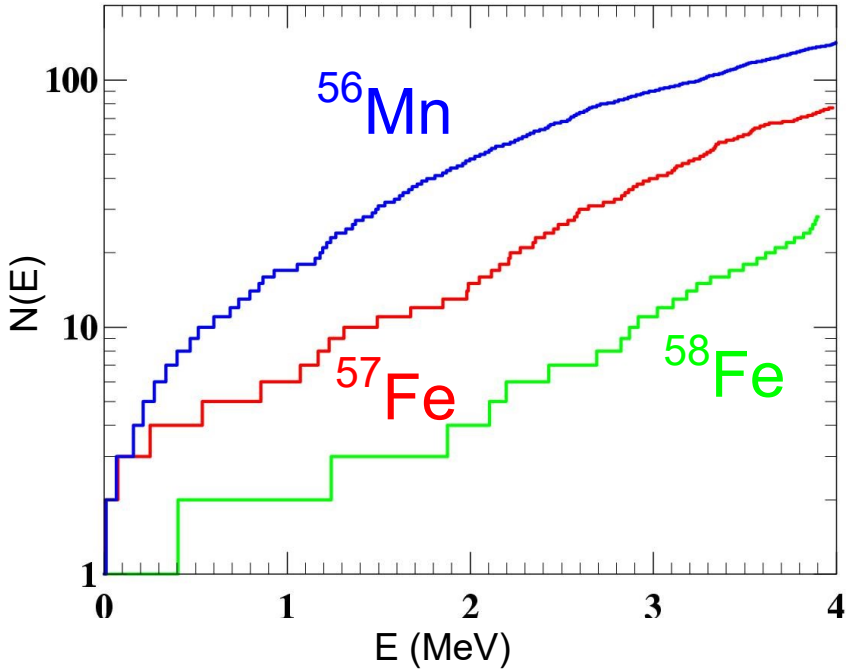
3. Emission of photons, fission

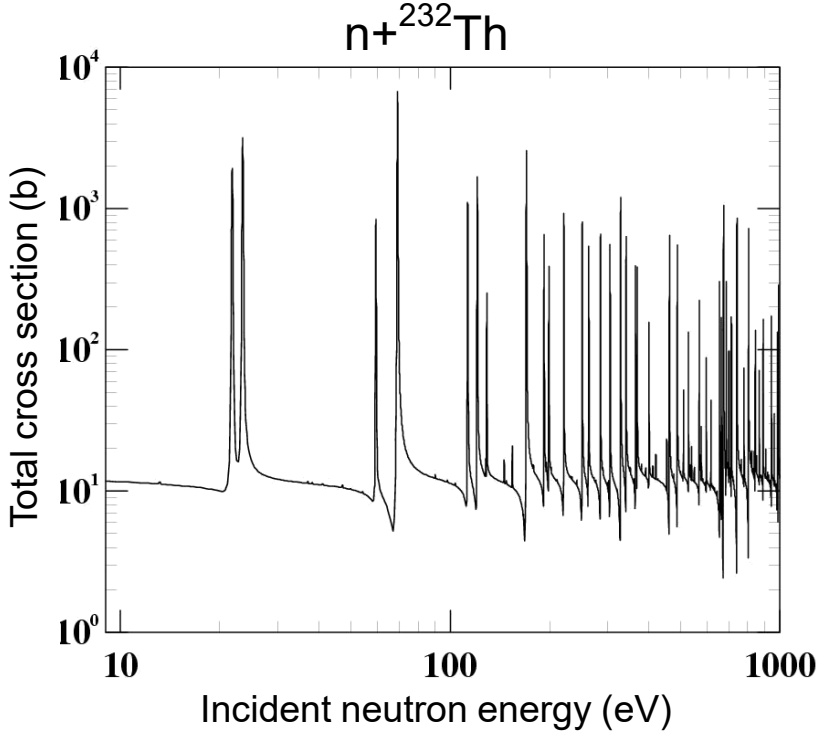
Specific treatment (not discussed here)

TALYS: model for level densities



TALYS: level densities microphysics





- Exponential increase of the cumulated number of discrete levels $N(E)$ with energy

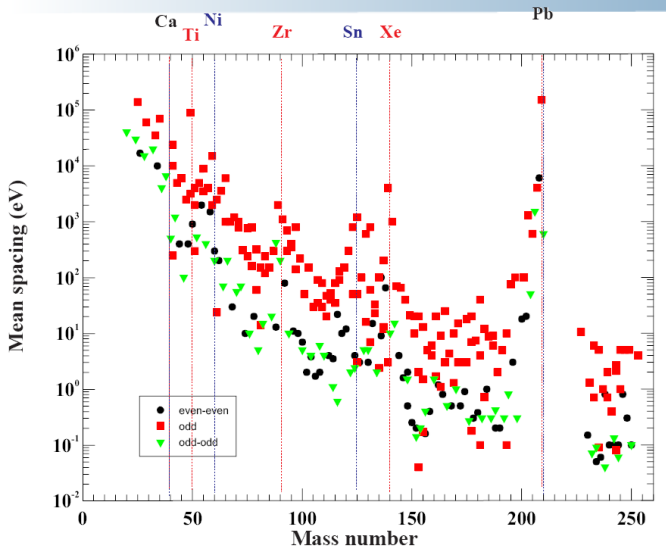
  $\rho(E) = \frac{dN(E)}{dE}$ increases exponentially

  odd-even effects are important

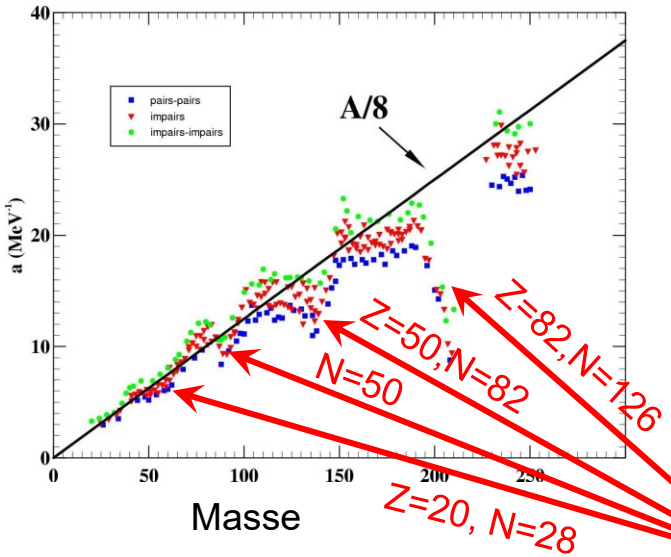
- Mean spacings of s-wave neutron resonances at E_n of the order of few eV

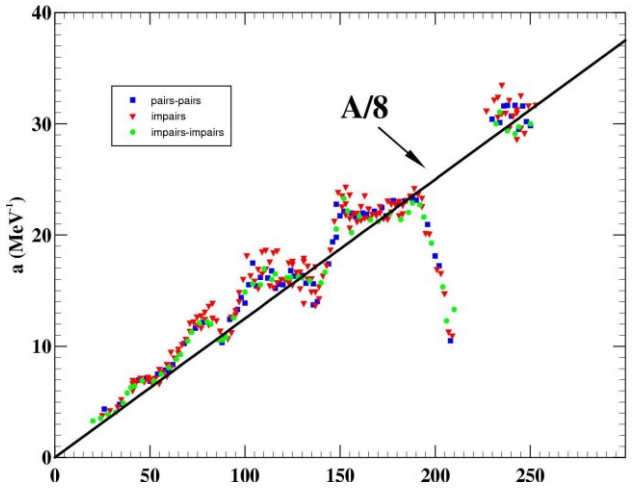
  $\rho(E_n)$ is of the order of 10^4– 10^6 levels per MeV (hence the statistical approximation)

TALYS: level densities microphysics



$$\rho(E,J,\pi) = \frac{1}{2}\frac{\sqrt{p}}{12}\frac{e^{2\sqrt{aE}}}{a^{\frac{1}{4}}E^{\frac{5}{4}}}\frac{2J+1}{2\sqrt{2\pi\sigma^3}}e^{-\frac{\left(J+\frac{1}{2}\right)^2}{2\sigma^2}}$$





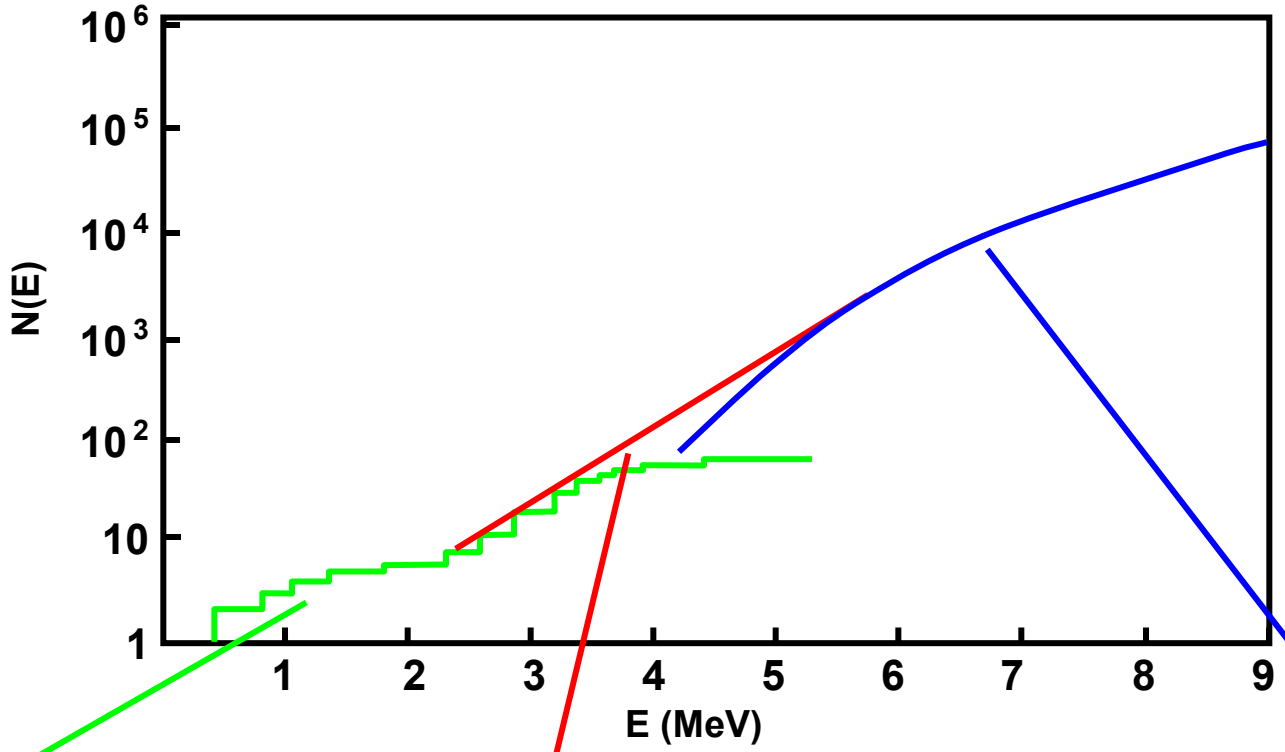
Shell effects

With $\sigma^2 = I_{\text{rig}}\sqrt{\frac{E}{a}}$ odd even effects are accounted via

$$E \longrightarrow E^* = E - \Delta$$

$$\Delta = \begin{cases} 0 & \text{odd}-\text{odd} \\ \frac{12}{\sqrt{A}} & \text{odd}-\text{even} \\ \frac{24}{\sqrt{A}} & \text{even}-\text{even} \end{cases}$$

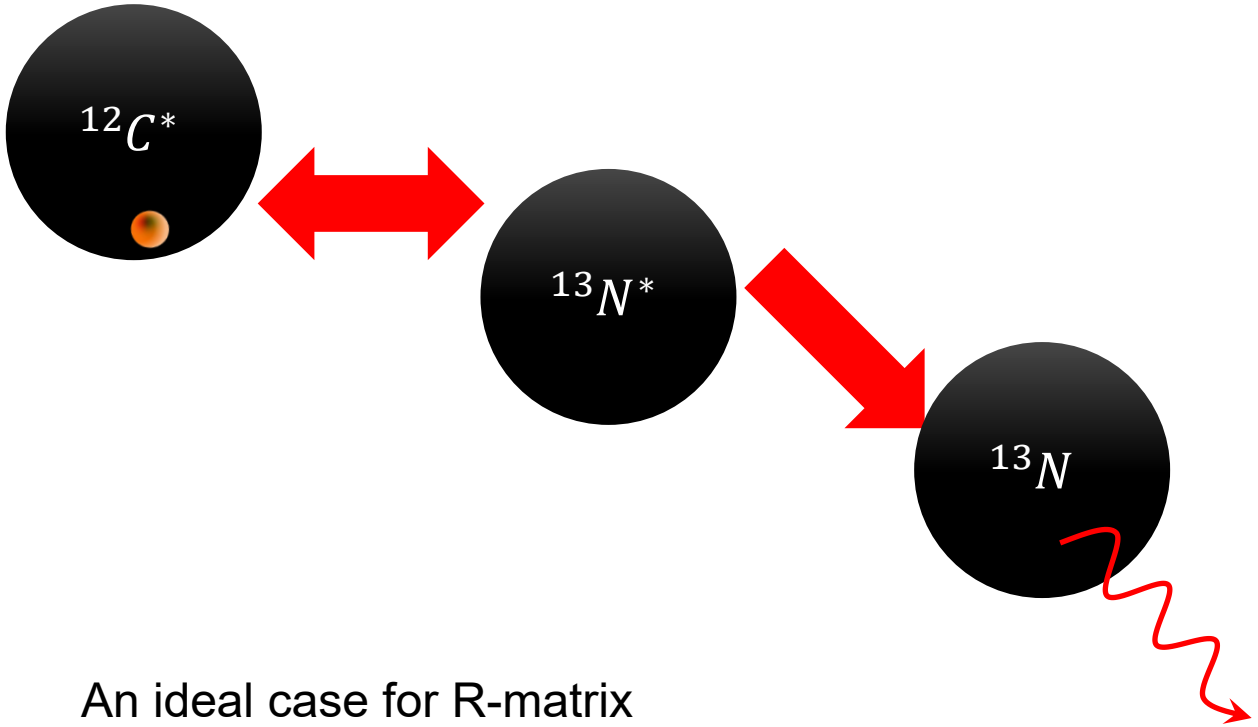
TALYS: level density regimes in literature



$$N(E) = e^{(\frac{E-E_0}{T})}$$

$$\rho(E) \propto \frac{e^{(2\sqrt{aU^*})}}{a^{\frac{1}{4}}U^{*\frac{5}{4}}}$$

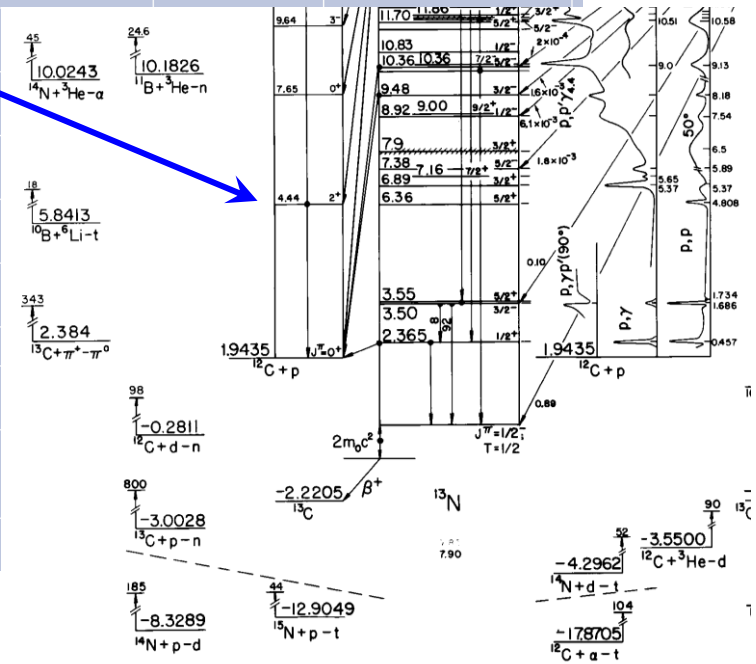
AZURE2 example case: level Structure of ^{13}N review



$S_p = 1.9435$ MeV

E_{level} (keV)	XREF	Jπ	$T_{1/2}$
0.0	A DE GHIJKLM	1/2-	9.965 m 4 % ε = 100
2364.9 6	DEFGHI K	1/2+	31.7 keV 8 % IT = 0.00158 13 % p = 100
3502 2	A DEFGHIJK	3/2-	62 keV 4 % IT = 0.0011 % p = 100
3547 4	D FGHI K	5/2+	47 keV 7 % p = 100 % IT < 4.3E-6
6364 9	CD FGH KL	5/2+	11 keV % p = 100
6886 8			
7155 5			
7376 9	A		
7900			
8918 11	A		
9000			
9476 8	A		
1025E+1 15			
10360	A		

Almost 3 MeV gap

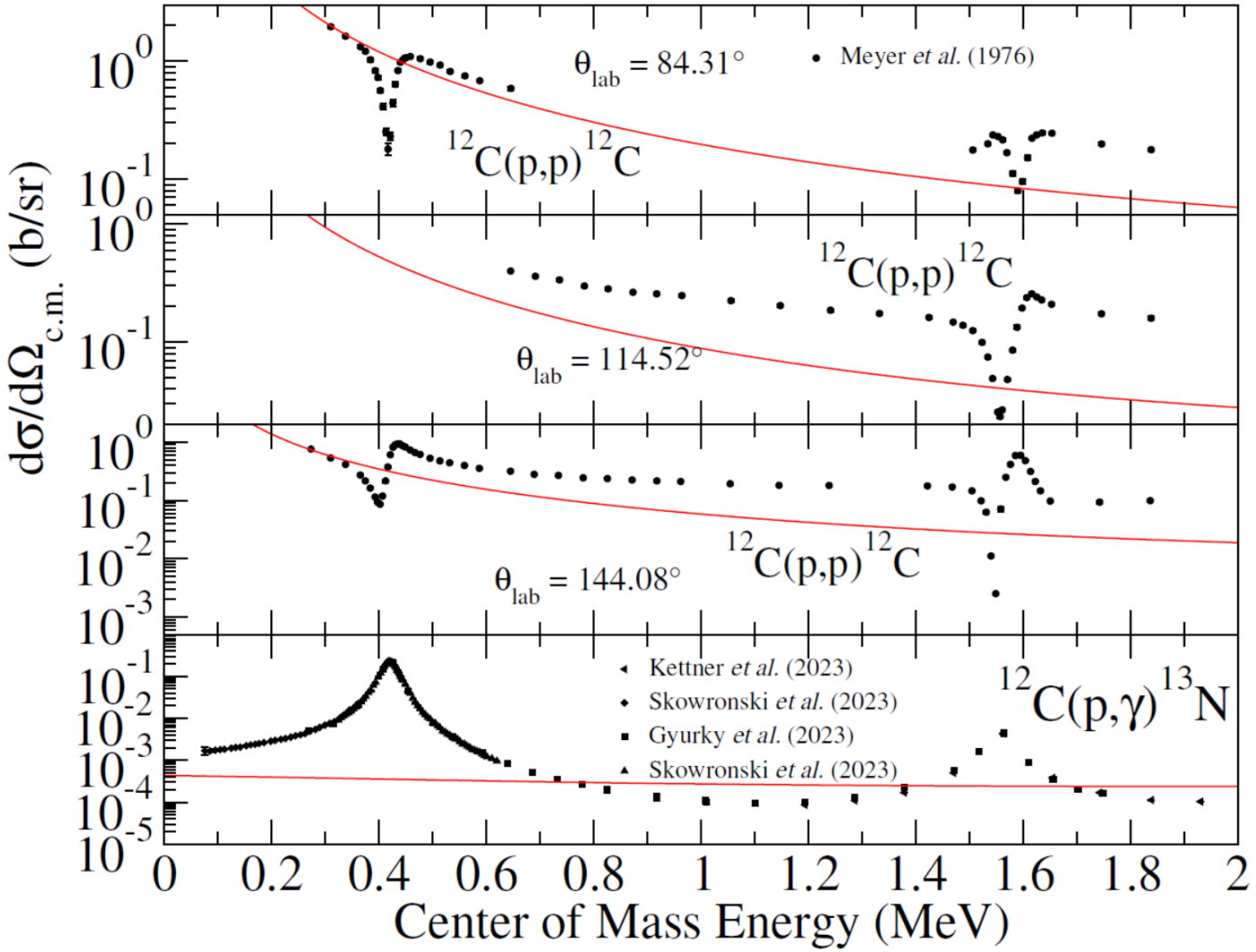


An ideal case for R-matrix

- Low level density;
- Isolated set of levels near threshold;
- Resonances are broad enough to easily measure but not too broad so that their shape are not overlapping strongly.

https://nndc.bnl.gov
https://nucldata.tunl.duke.edu

AZURE2: Individual contributions of Coulomb and radiative direct capture

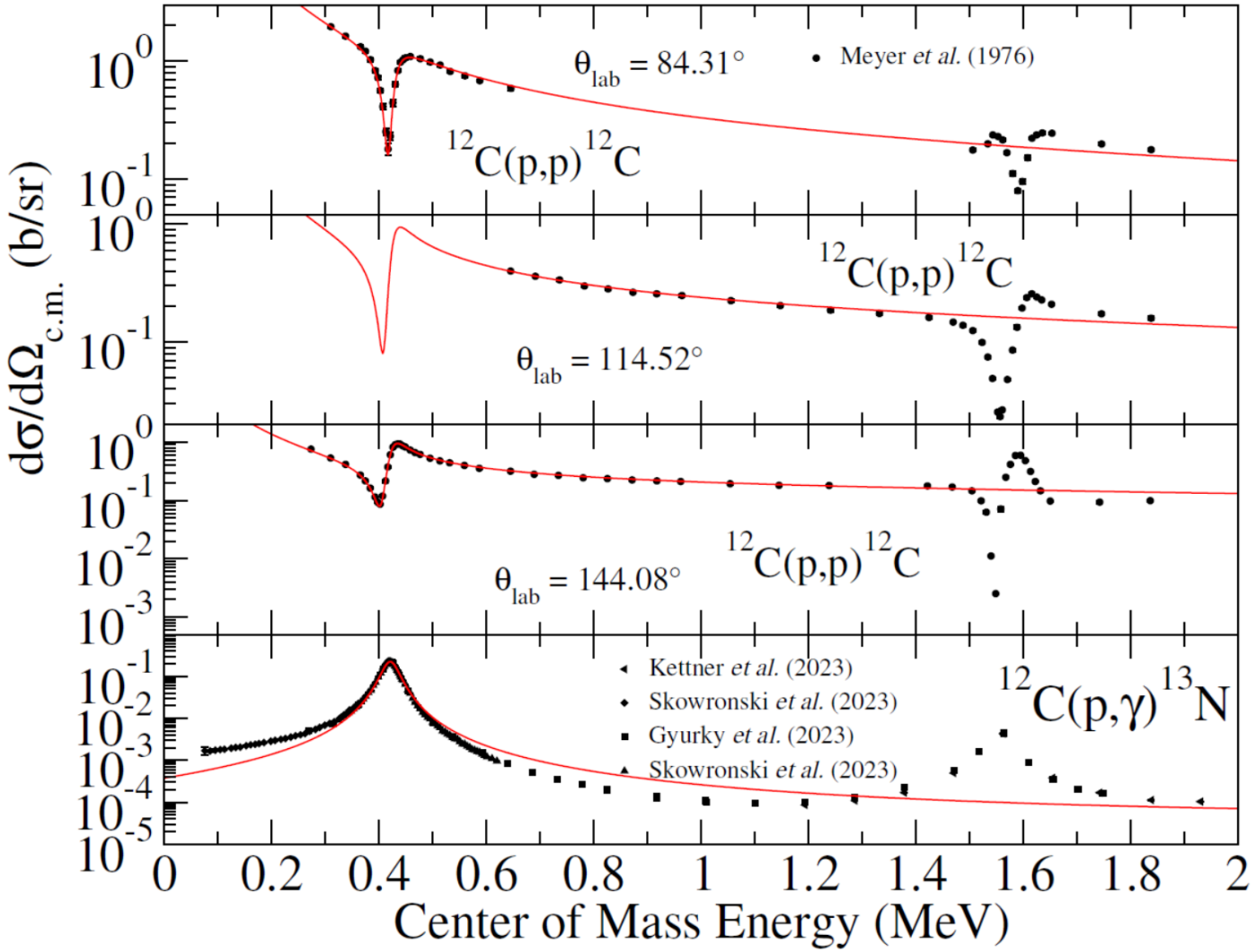


E_{level} (keV)	XREF	Jπ	$T_{1/2}$
0.0	A DE GHIJKLM	1/2-	9.965 m *4* % ε = 100
2364.9 *6*	DEFGHI K	1/2+	31.7 keV *8* % IT = 0.00158 *13* % p = 100
3502 *2*	A DEFGHIJK	3/2-	62 keV *4* % IT = 0.0011 % p = 100
3547 *4*	D FGHI K	5/2+	47 keV *7* % p = 100 % IT < 4.3E-6
6364 *9*	CD FGH KL	5/2+	11 keV % p = 100
6886 *8*	CD FGH K	3/2+	115 keV *5* % p = 100

Level parameters:

- Ground state proton ANC $= 1.65\ \text{fm}^{-\frac{1}{2}}$
- Where does it come from?
  - Fits to this data
  - Proton transfer measurements (direct reaction studies)

AZURE2: Individual contributions of lowest energy proton-unbound level

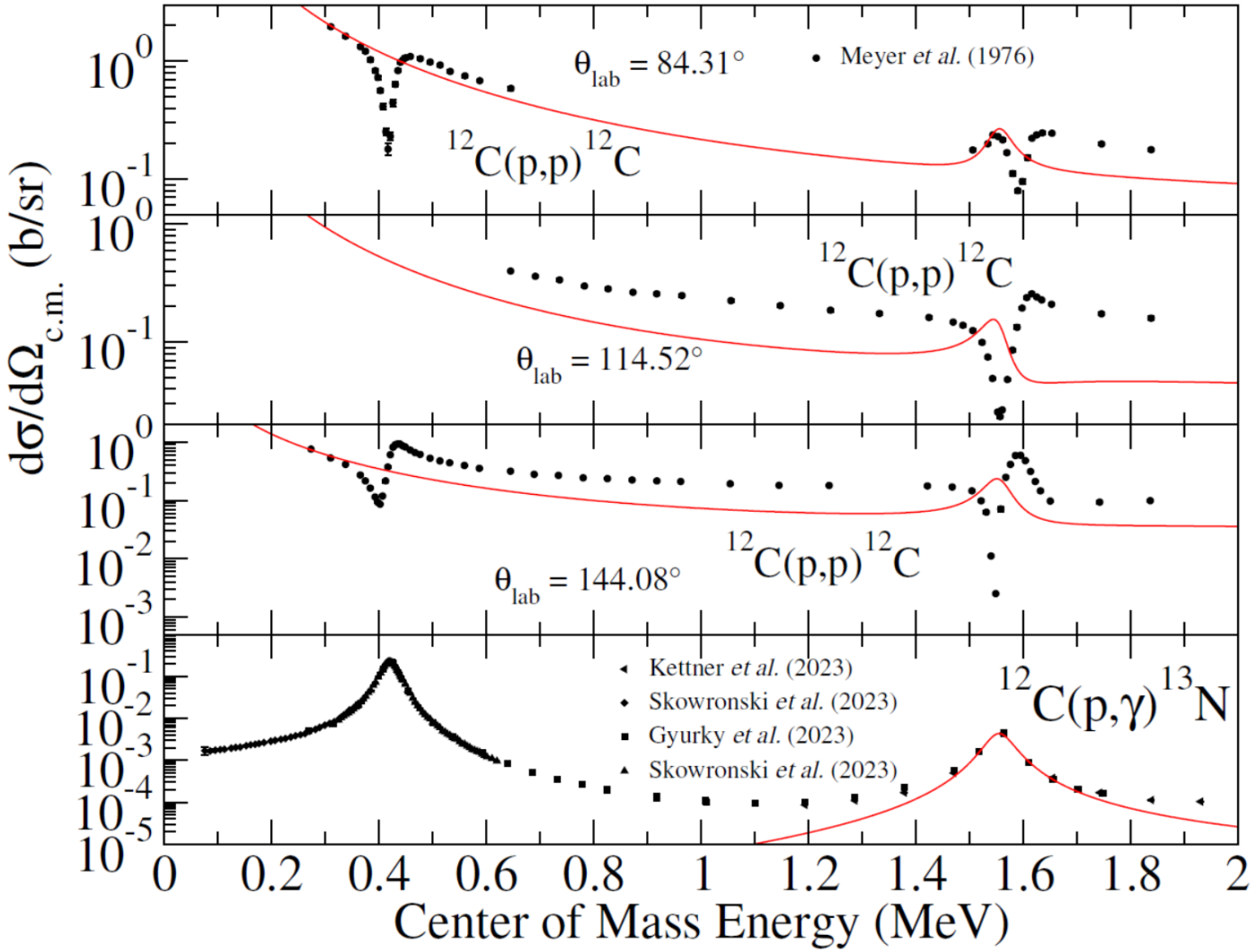


E_{level} (keV)	XREF	Jπ	$T_{1/2}$
0.0	A DE GHIJKLM	1/2-	9.965 m *4* % ε = 100
2364.9 *6*	DEFGHI K	1/2+	31.7 keV *8* % IT = 0.00158 *13* % p = 100
3502 *2*	A DEFGHIJK	3/2-	62 keV *4* % IT = 0.0011 % p = 100
3547 *4*	D FGHI K	5/2+	47 keV *7* % p = 100 % IT < 4.3E-6
6364 *9*	CD FGH KL	5/2+	11 keV % p = 100
6886 *8*	CD FGH K	3/2+	115 keV *5* % p = 100

Level parameters:

- First excited state's J^{π}, E, Γ_p and Γ_{γ};
- Where do they come from?
  - Fits to this data;
  - Many other measurements;

AZURE2: Individual contributions of second excited state



E_{level} (keV)	XREF	Jπ	$T_{1/2}$
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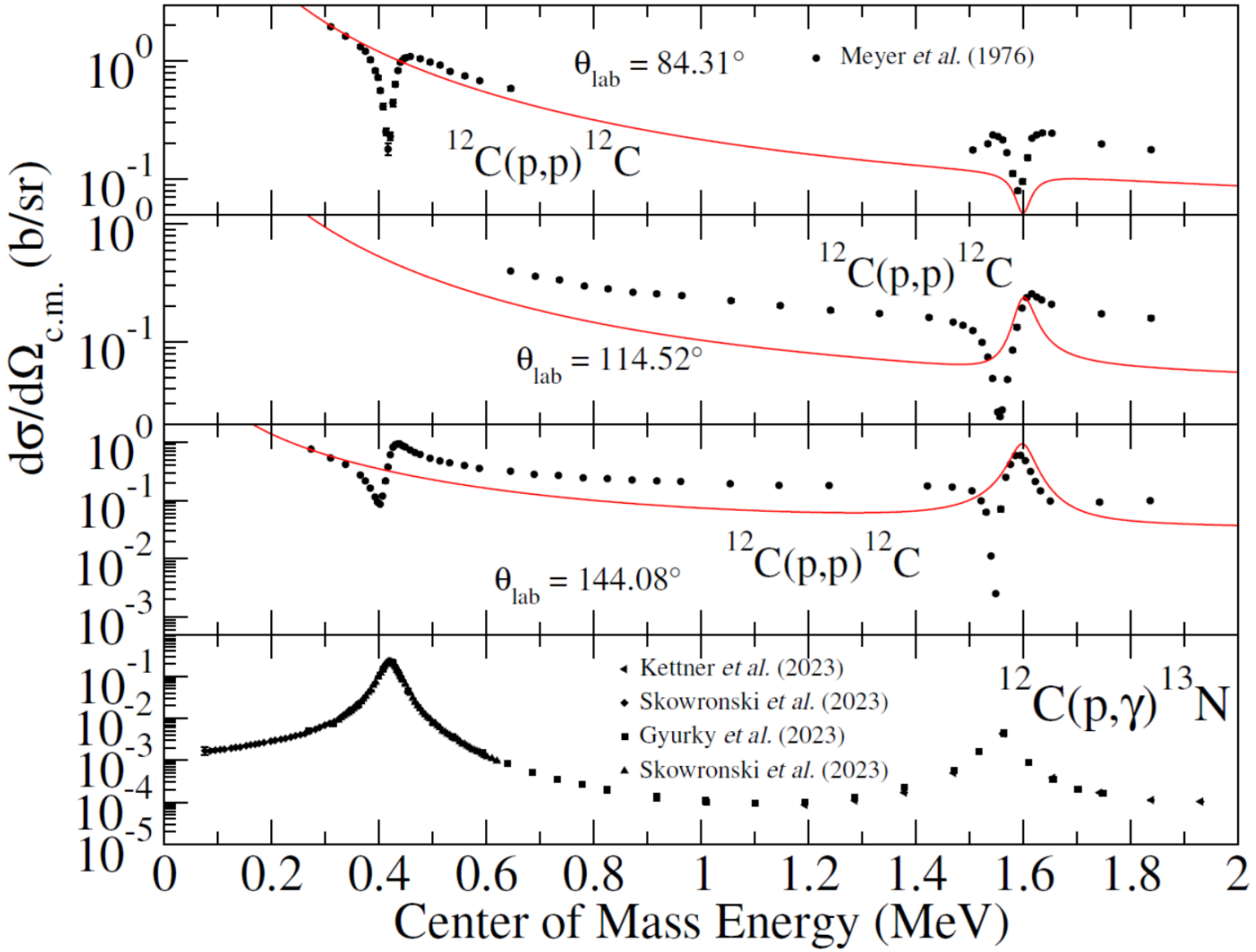
2364.9 *6*	DEFGHI K	1/2+	31.7 keV *8* % IT = 0.00158 *13* % p = 100
3502 *2*	A DEFGHIJK	3/2-	62 keV *4* % IT = 0.0011 % p = 100
3547 *4*	D FGHI K	5/2+	47 keV *7* % p = 100 % IT < 4.3E-6
6364 *9*	CD FGH KL	5/2+	11 keV % p = 100
6886 *8*	CD FGH K	3/2+	115 keV *5* % p = 100

Level parameters:

- Second excited state's J^{π}, E, Γ_p and Γ_{γ};
- Where do they come from?
  - Fits to this data;
  - Many other measurements;

AZURE2: Individual contributions of the third excited state

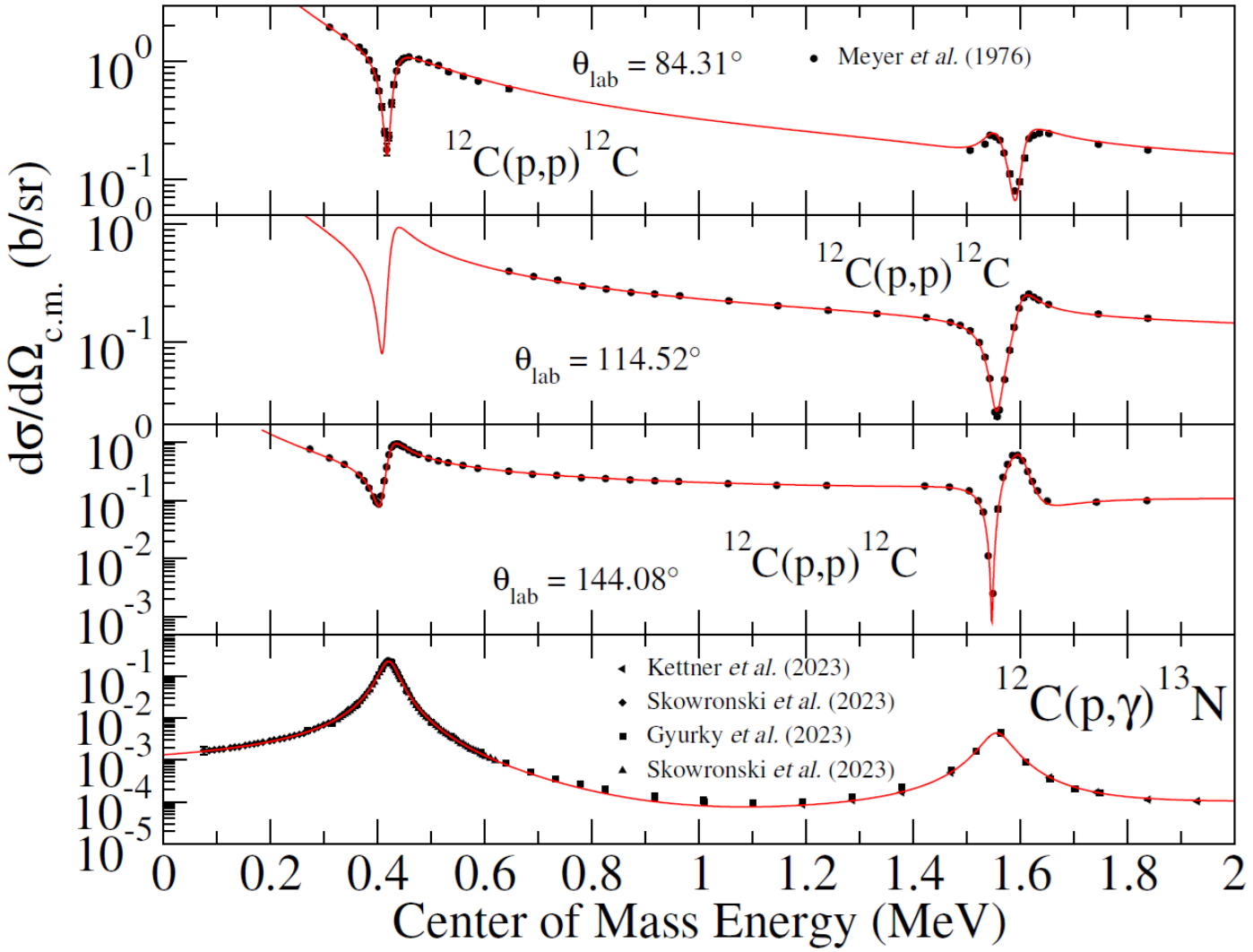


E_{level} (keV)	XREF	Jπ	$T_{1/2}$
0.0	A DE GHIJKLM	1/2-	9.965 m *4* % ε = 100
2364.9 *6*	DEFGHI K	1/2+	31.7 keV *8* % IT = 0.00158 *13* % p = 100
3502 *2*	A DEFGHIJK	3/2-	62 keV *4* % IT = 0.0011 % p = 100
3547 *4*	D FGHI K	5/2+	47 keV *7* % p = 100 % IT < 4.3E-6
6364 *9*	CD FGH KL	5/2+	11 keV % p = 100
6886 *8*	CD FGH K	3/2+	115 keV *5* % p = 100

Level parameters:

- Third excited state's J^{π}, E, Γ_p and Γ_{γ};
- Where do they come from?
  - Fits to this data;
  - Many other measurements;

AZURE2: all contributions



E_{level} (keV)	XREF	Jπ	$T_{1/2}$
0.0	A DE GHIJKLM	1/2-	9.965 m *4* % ε = 100
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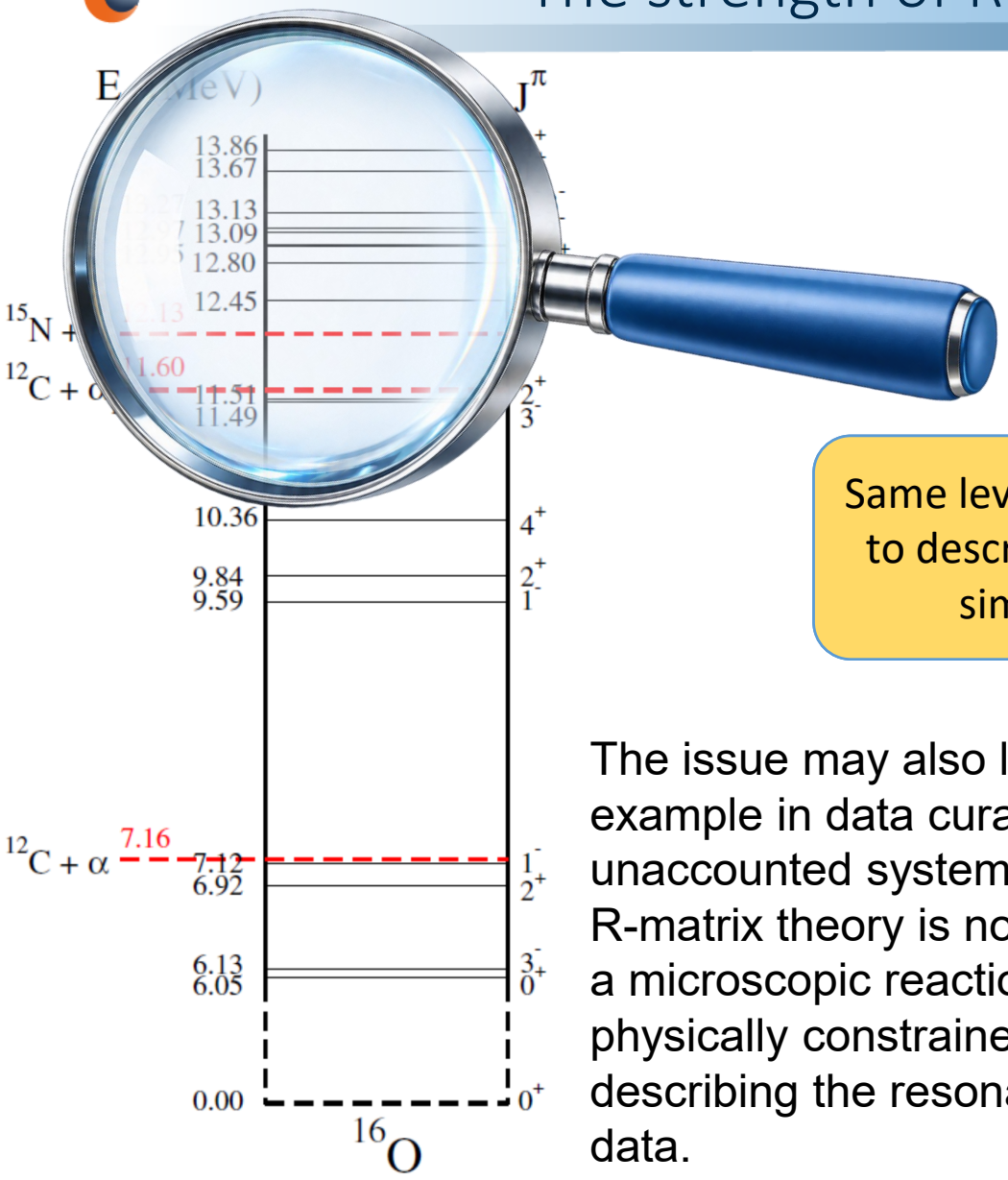
3547 *4*	D FGHI K	5/2+	47 keV *7* % p = 100 % IT < 4.3E-6
6364 *9*	CD FGH KL	5/2+	11 keV % p = 100
6886 *8*	CD FGH K	3/2+	115 keV *5* % p = 100

Level parameters:

- All excited state's J^{π}, E, Γ_p and Γ_{γ};

Note that to extract accurate values for each of the level parameters from this data, this fit is about the minimum amount of complexity

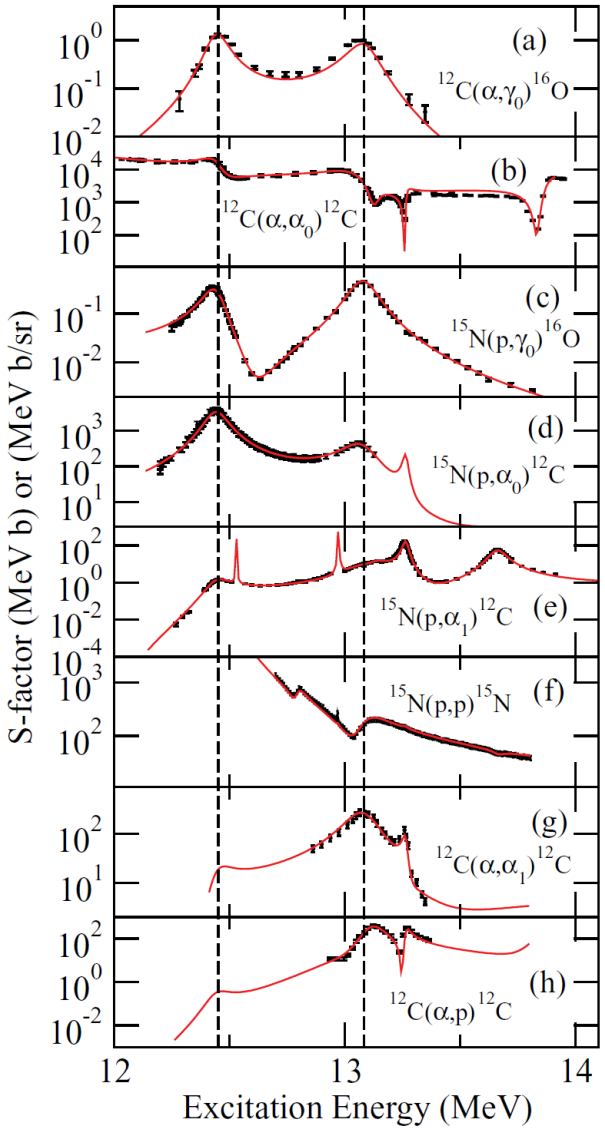
The strength of R-matrix evaluation



Data to level scheme in the continuum

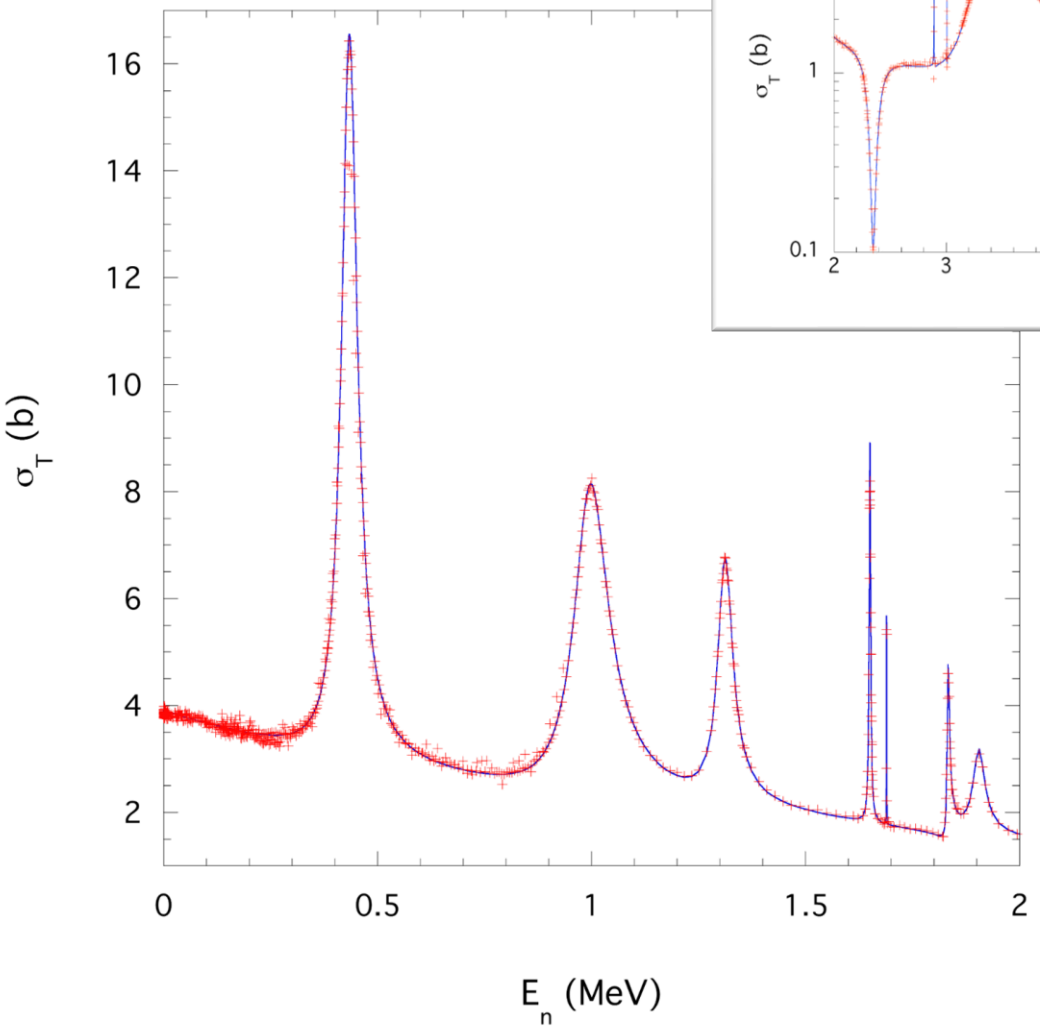
Same level parameters used to describe all of this data simultaneously!

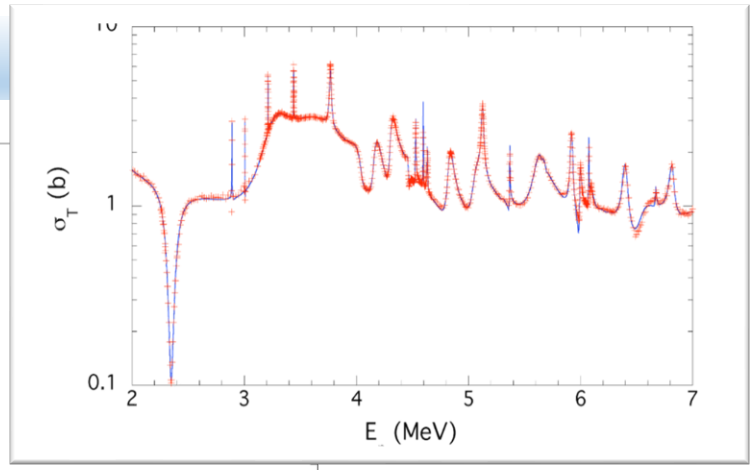
The issue may also lie on the experimental side, for example in data curation, normalization, or unaccounted systematic uncertainties.
R-matrix theory is not predictive in the same sense as a microscopic reaction model; rather, it provides a physically constrained framework for extracting and describing the resonance information contained in the data.



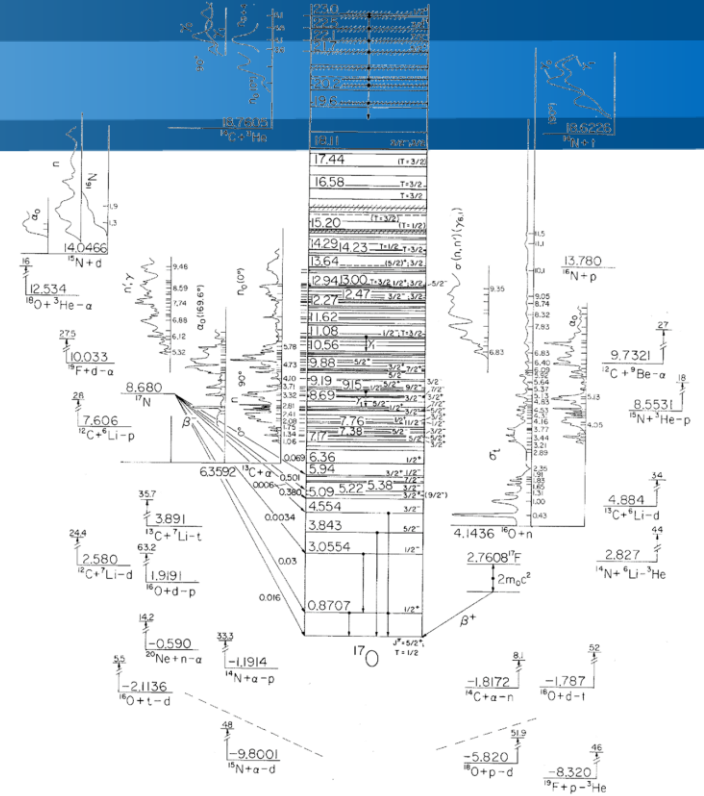
Extracting nuclear data for astrophysics

n-^{17}O R-matrix analyses of data





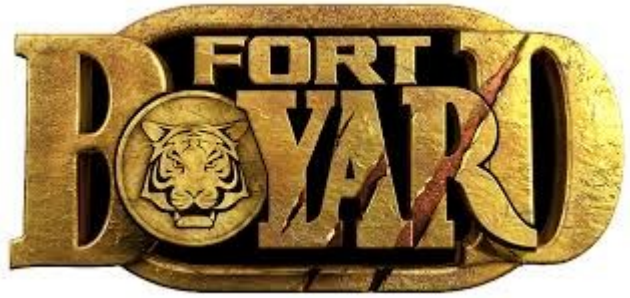
M.W. Paris and G.M. Hale R-matrix workshop 2016



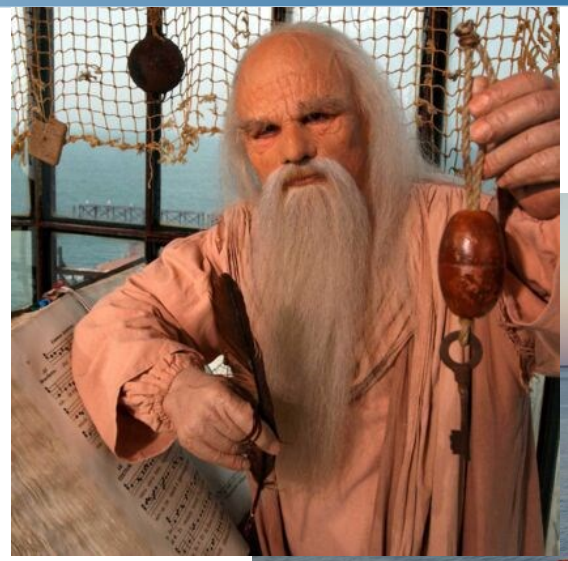
Phenomenological R-matrix is

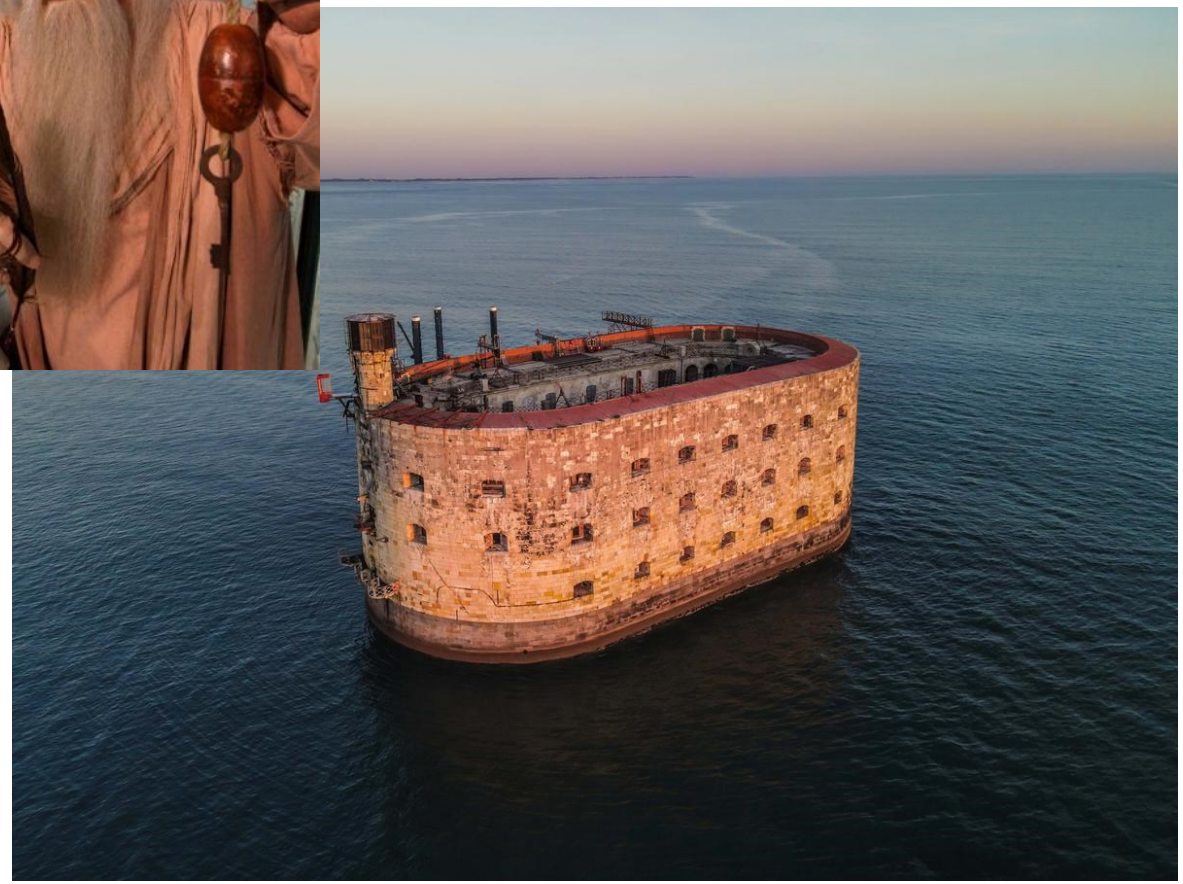
- Very efficient at extracting resonance properties and to evaluate data
- Well, situated for nuclear astrophysics
- Not systematically improvable

Conclusion



You have 5 minutes:





Mon premier ne porte aucun vêtement.
Mon deuxième peut ouvrir bien des portes.
Mon troisième tombe du ciel, coule dans les rivières et remplit les océans.
Mon quatrième rassemble en peu de mots l'essentiel d'un long discours.

Qui suis-je ?