Astrophysics & EoS

dense-matter EoS and compact-objects properties

Anthea F. Fantina

(anthea.fantina[AT]ganil.fr)

Ecole Joliot Curie,

Saint-Pierre-d'Oléron (France), 13 – 18 September 2026

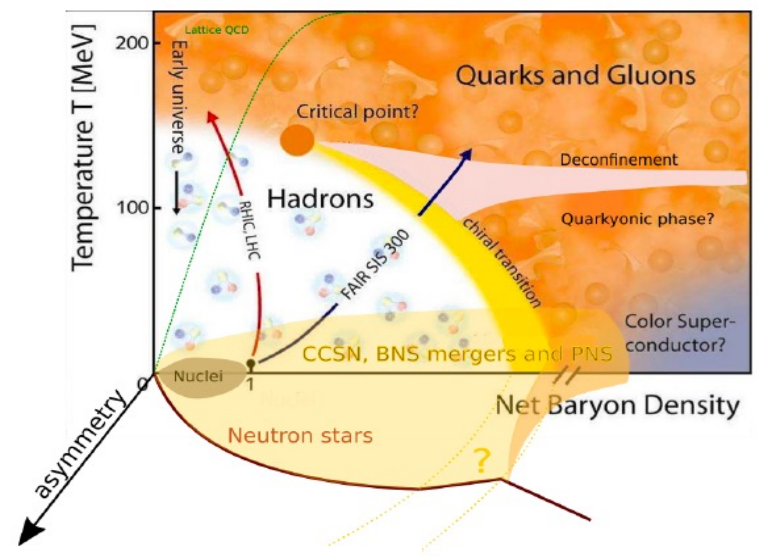
Outline

- Motivations:
  - Intro on astrophysical sites: (proto-)neutron stars ((P)NS), core-collapse supernovae (CCSN), and binary neutron star (BNS) merger
  - Connection with the nuclear equation of state (EoS)

- Astrophysical sites at $T = 0$ (NS):
  - EoS: generalities
  - Application to NS (crust & core)

- Astrophysical sites at $T > 0$ (CCSN/PNS):
  - EoS: generalities
  - Applications to CCSN and PNS
  - Rates (CCSN)

Compact objects: sites for (hot) dense matter

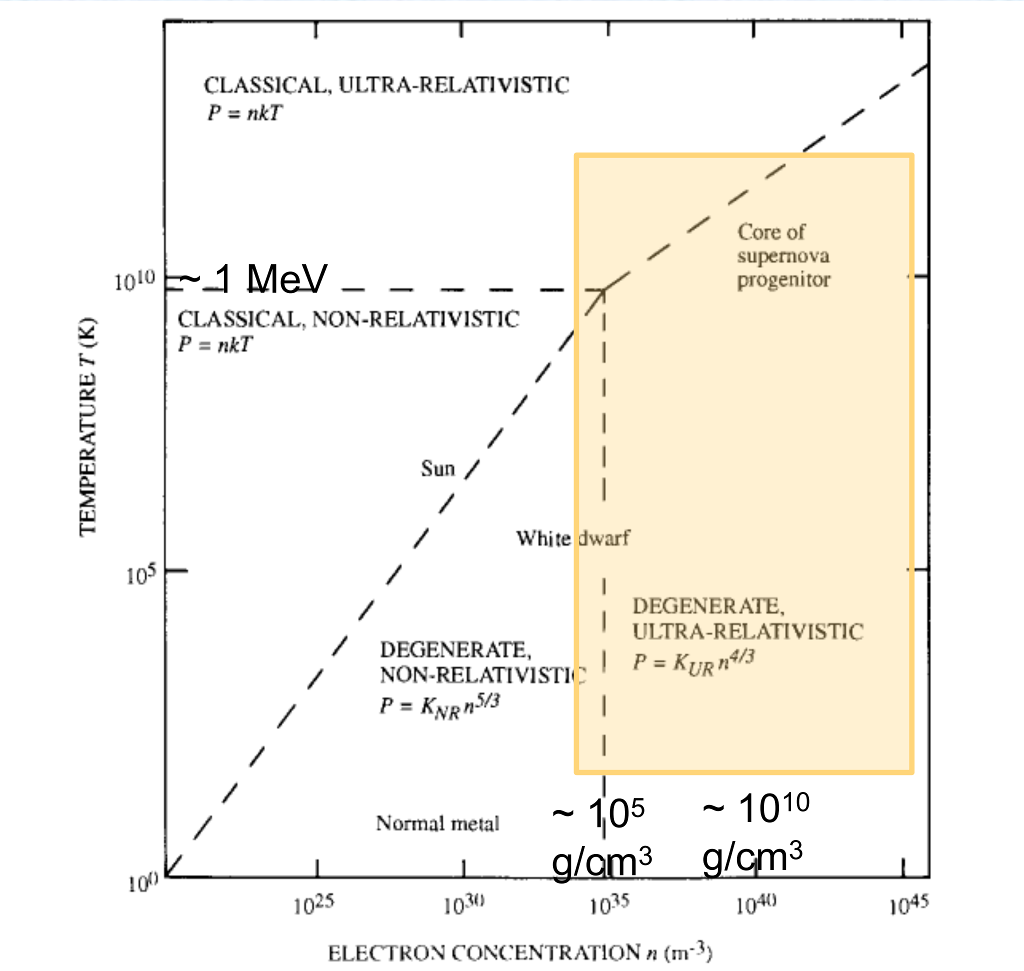


Abac et al., arXiv:2503.12263 (ET BlueBook)

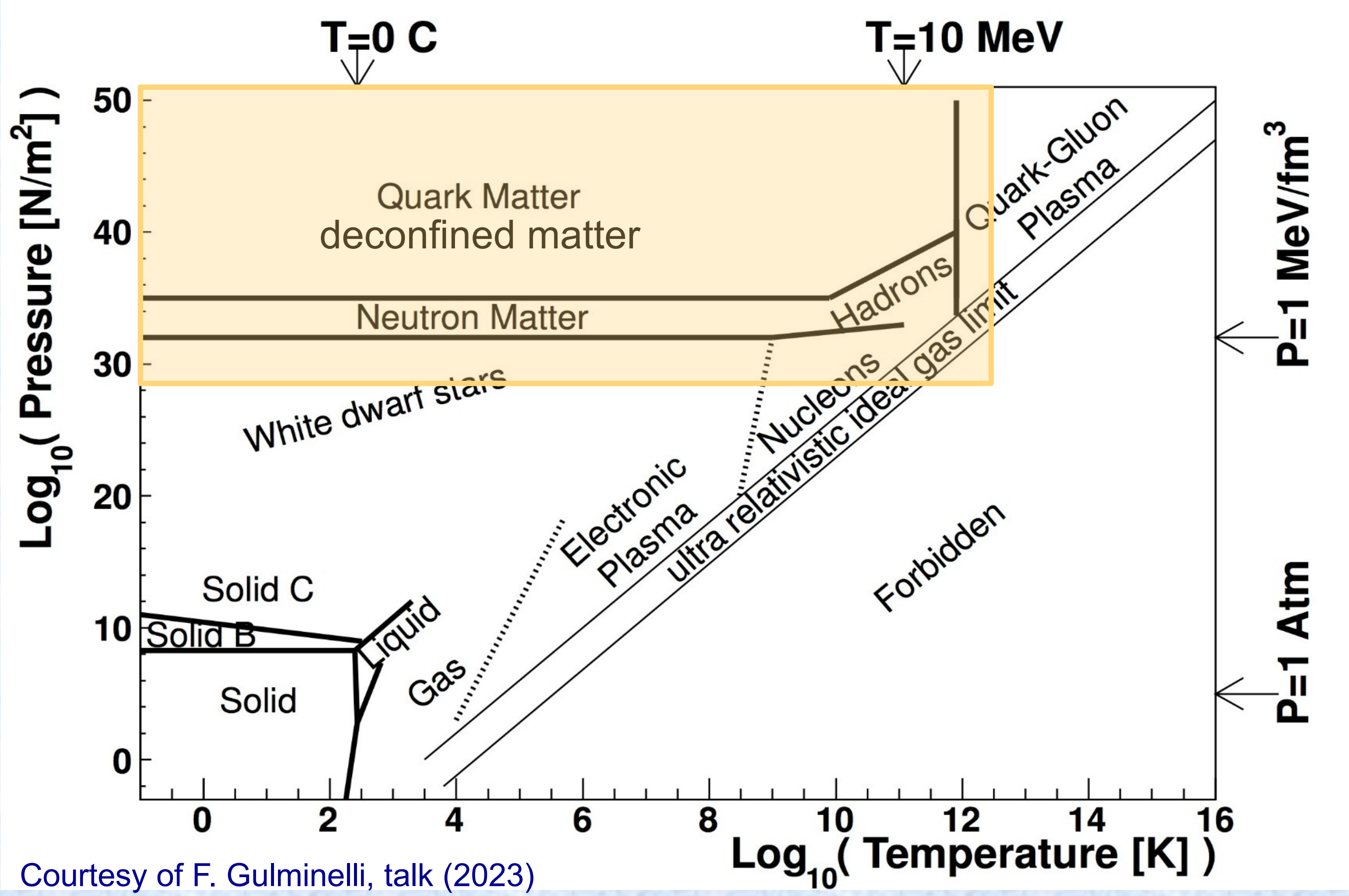
Object	***Thermo***	***Equilibrium***	***Micro inputs***
CCSN BNS (post-)mergers	$n_B \in (0, \sim 10 n_{\rm sat})$ $T \in (0, \sim 100)$ MeV $Y_e \in (0, 0.6)$	strong	$P(n_B, T, Y_e)$ (weak), ν rates → compo
PNS	$n_B \in (0, \sim 10 n_{\rm sat})$ $T \in (0, \sim 20)$ MeV $(Y_e \in (0, 0.5))$	strong (+weak)	$P(n_B, T, (Y_e))$ (+ rates → composition)
NS (old) binary inspiral	$n_B \in (0, \sim 10 n_{\rm sat})$ $T = 0$	strong+weak → β equil.	$P(n_B)$

<u>Memo</u>: 1 MeV ←→ ~ 10^{10} K
10^{-4} fm^{-3} ←→ ~ 10^{11} g/cm^3 , $n_{\rm sat}$ ~ 0.16 fm^{-3} → ~ 2.7 10^{14} g/cm^3

What matter can we expect?



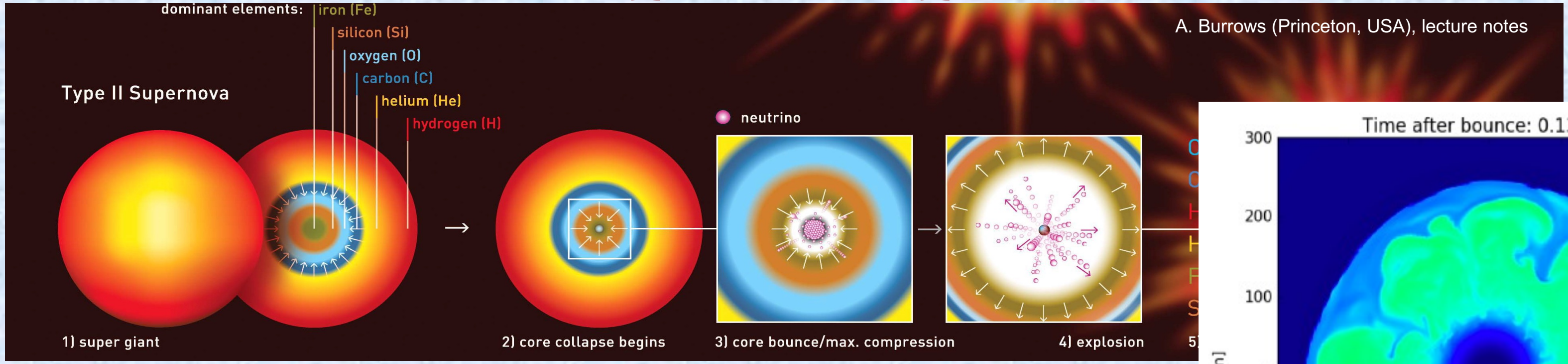
Phillips, "The Physics of Stars"

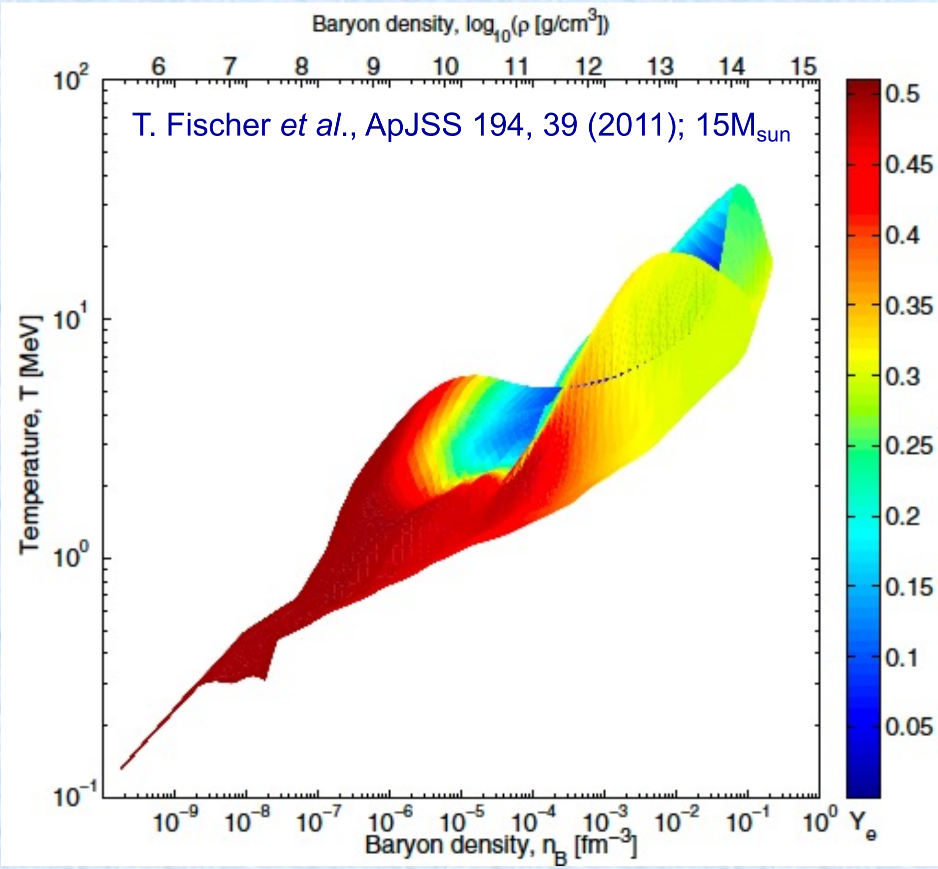


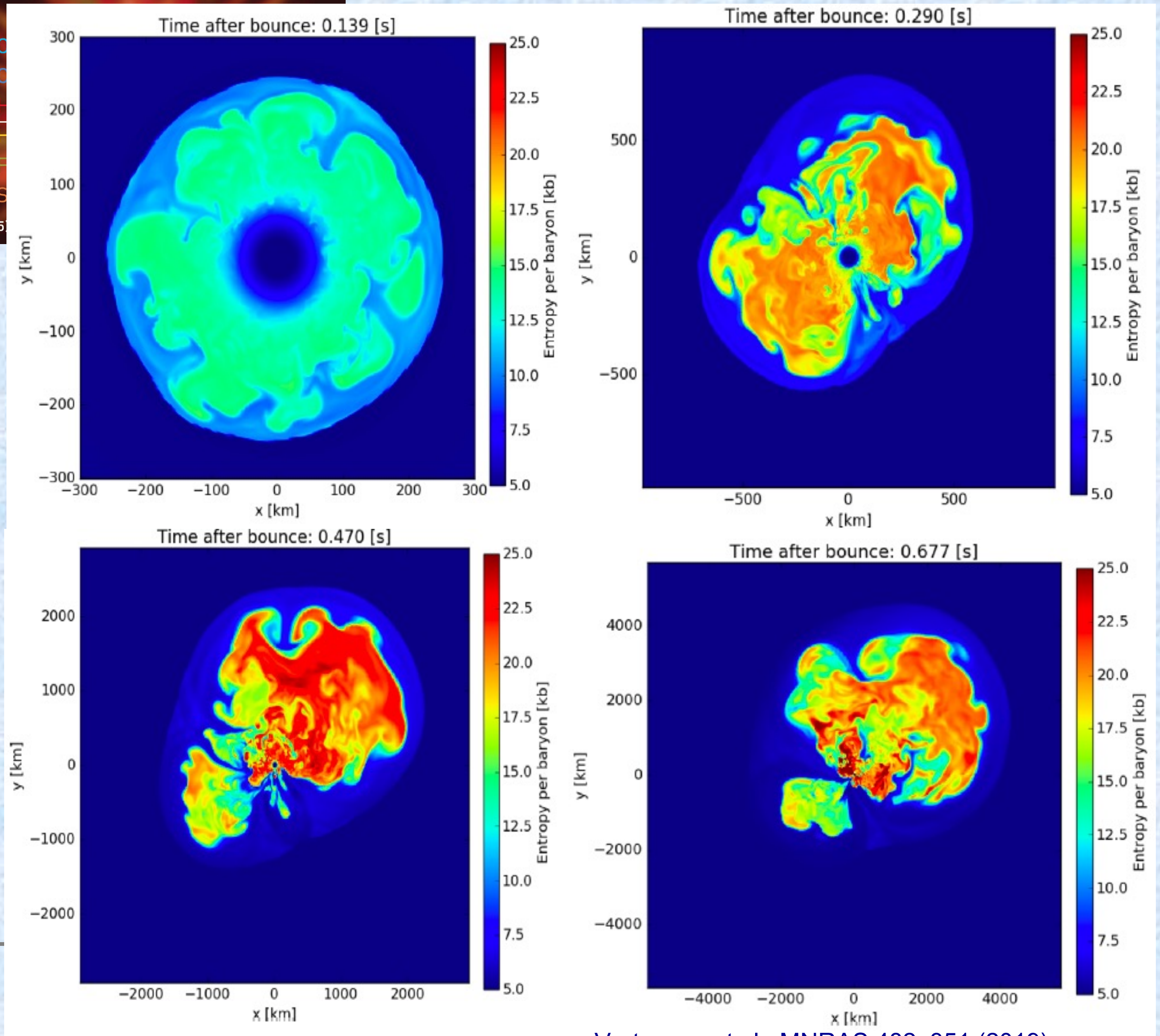
Courtesy of F. Gulminelli, talk (2023)

- Matter (almost always) degenerate and relativistic e^-
- Possible additional d.o.f. → "exotic" matter

Core-collapse supernovae (CCSN)

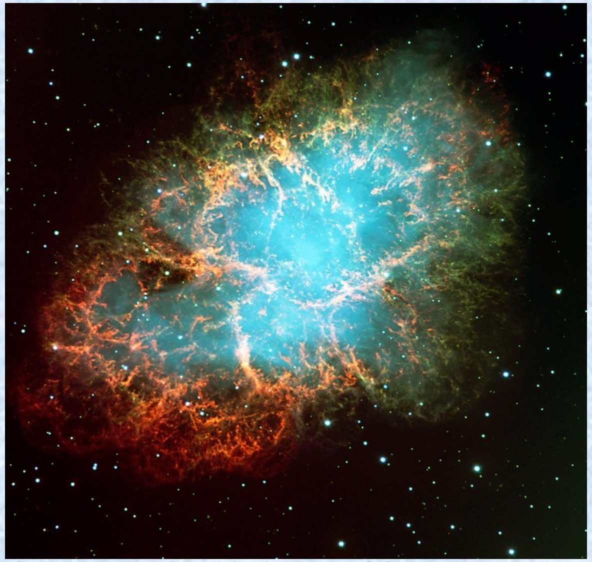






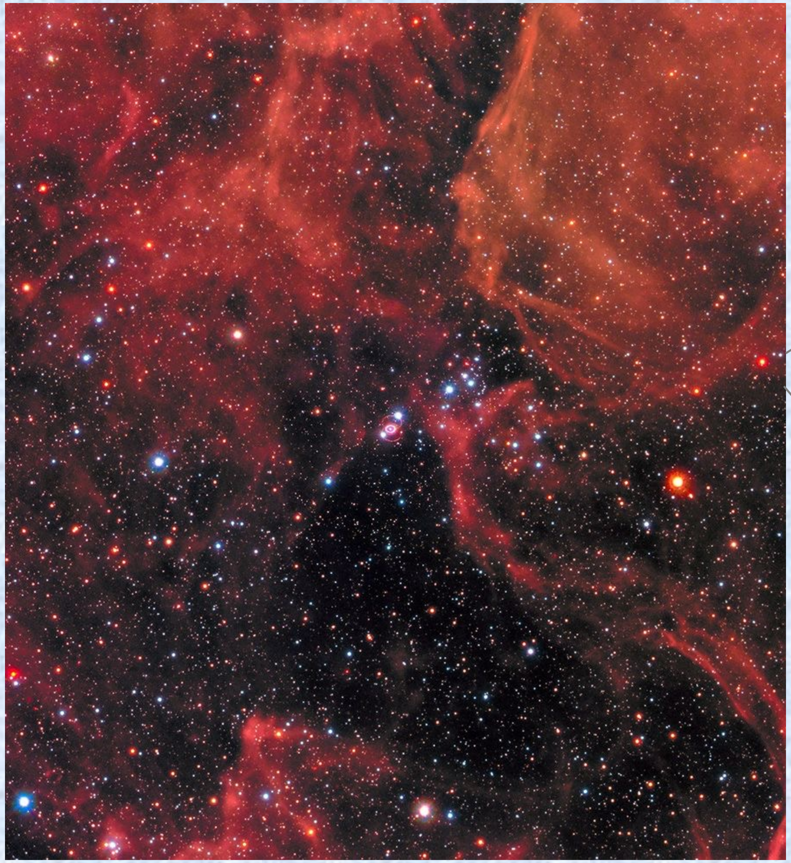
Vartanyan et al., MNRAS 482, 351 (2019)

 Core-collapse supernovae (CCSN): obsevations

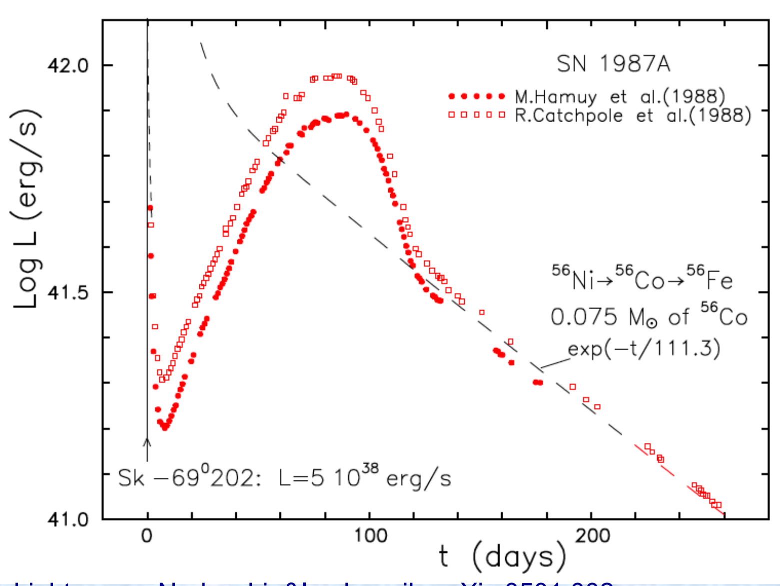


Crab Nebula (1054). Credit: ESO

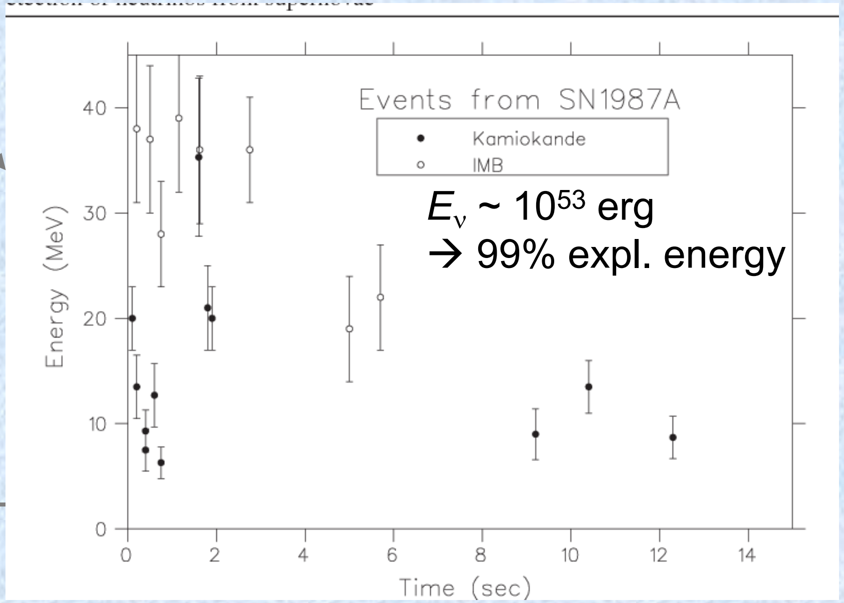
Cassiopea A. Credit: NASA, ESA, CSA, STScl



SN1987A. Credit: NASA, ESA



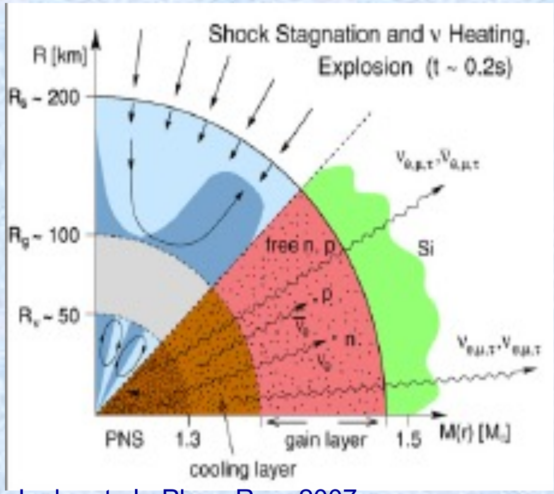
Light curve; Nadyozhin&Imshennik, arXiv:0501.002



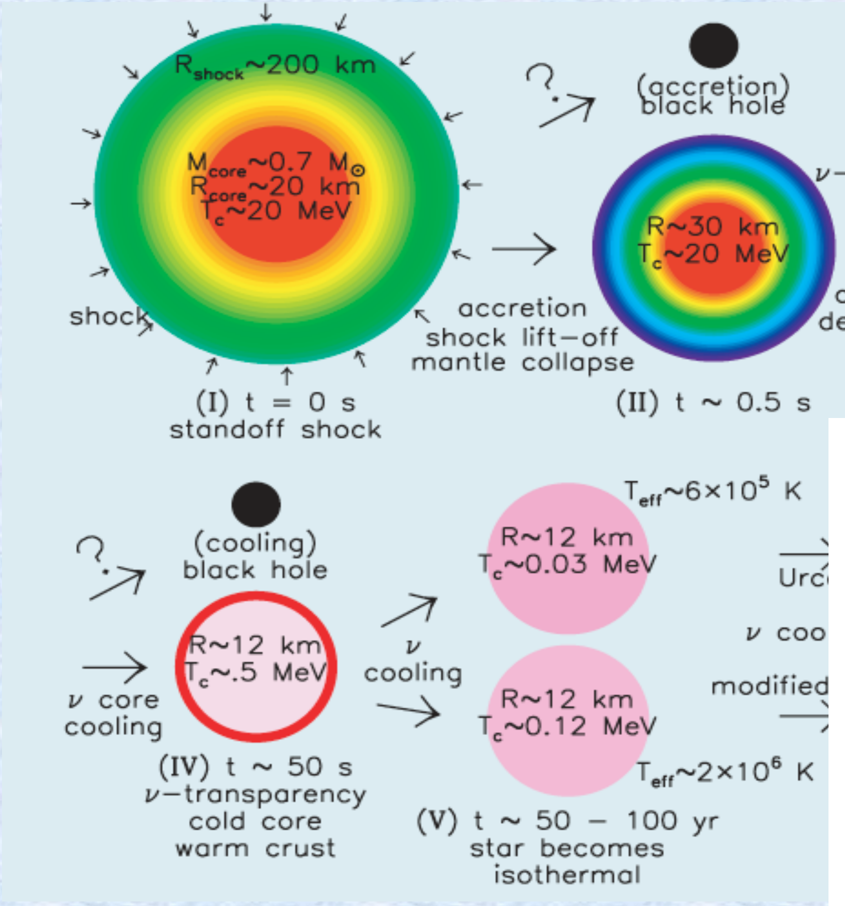
Boyd et al., J.Phys.G 29, 11 (2003)

A. F. Fantina

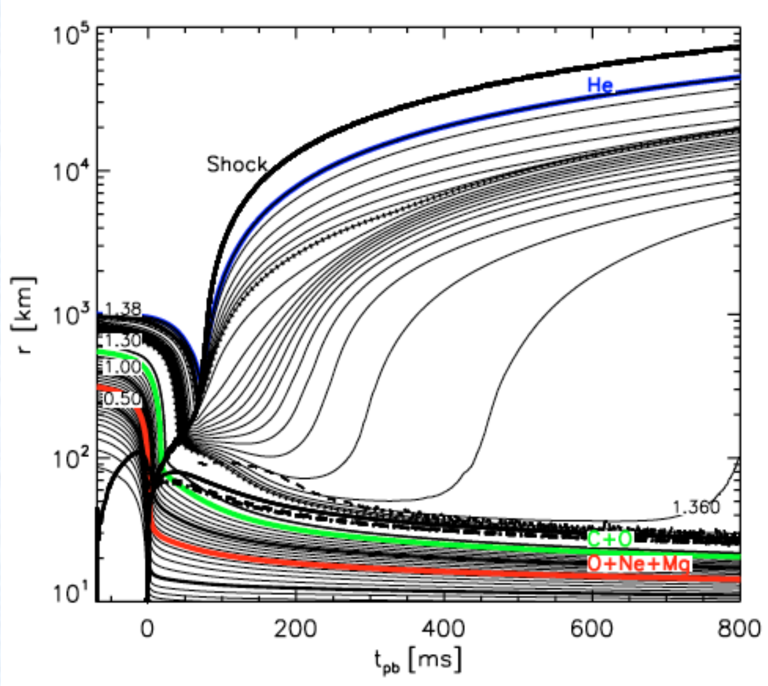
Proto-neutron stars (PNS)



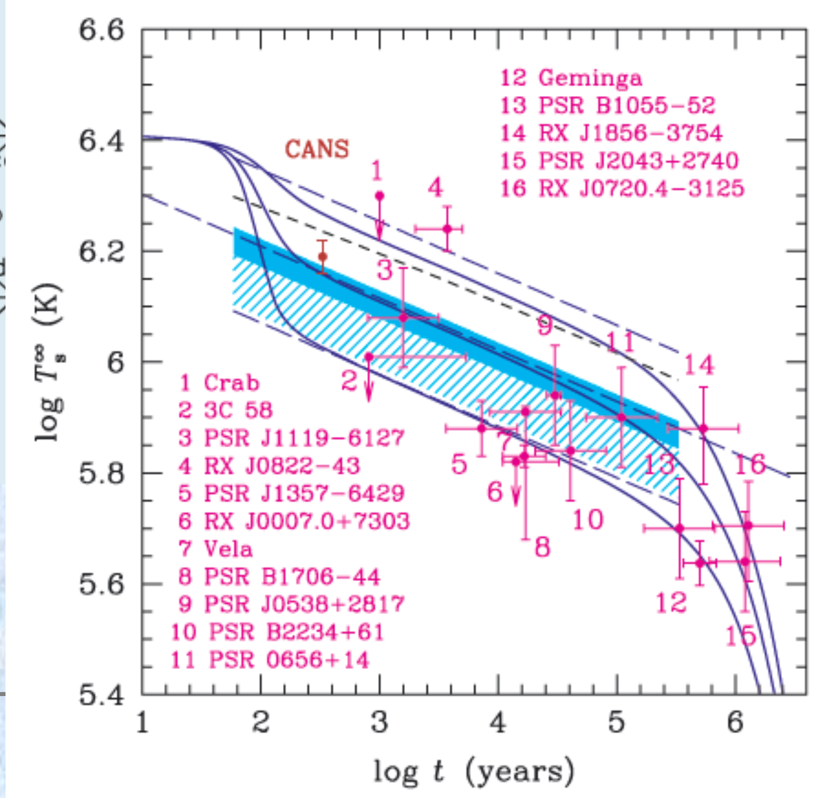
Janka et al., Phys. Rep. 2007



Lattimer & Prakash, Science 304, 536 (2004)

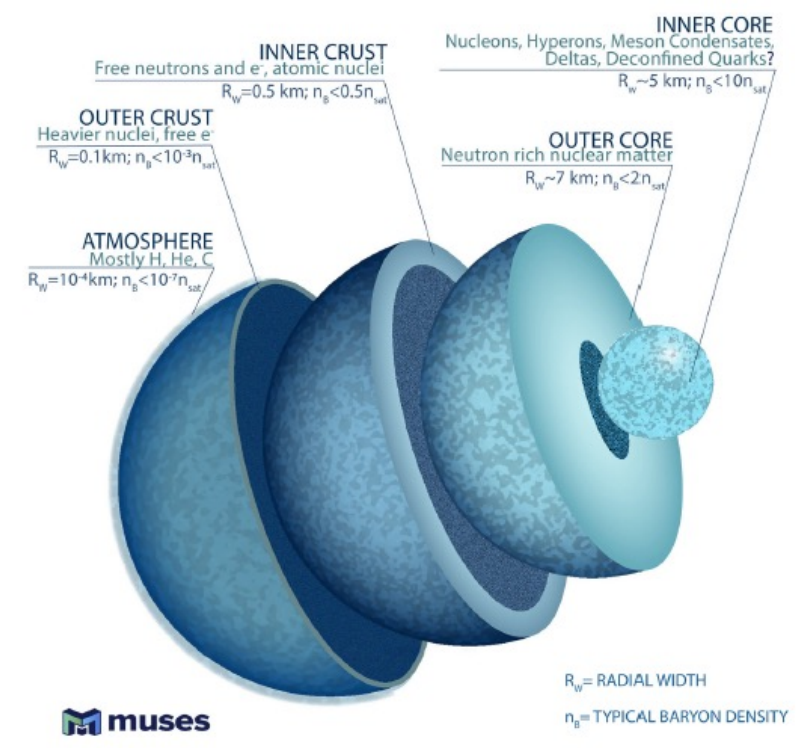


Kitaura et al., A&A 450, 345 (2006). O-Ne-Mg SN



Yakovlev et al., MNRAS 411, 1977 (2011)

Neutron-stars (NS)

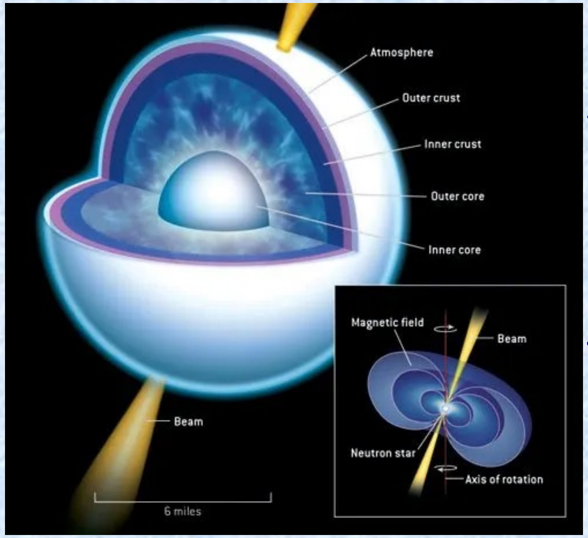


Kumar et al., Liv. Rev. Rel. 27, 3 (2024)

$$M \approx 1 - 2M_\odot$$
$$R \approx 10 \text{ km}$$
$$\bar{\rho} \approx 10^{14} - 10^{15} \text{ g cm}^{-3}$$
$$T < 10^8 \text{ K}$$

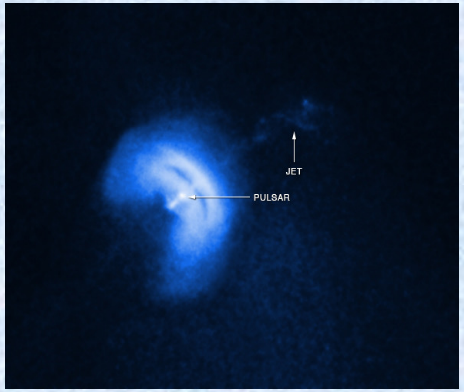
NB: $T = 0 \Leftrightarrow T << T_F$

rotation up to 716 Hz (v_{equat} ~ ¼ c)
high magnetic fields, B > 10^{12} G



Credits: https://www.astronomy.com/science/

Crab, X-ray: NASA/CXC/SAO; Optical: NASA/STScI; IR: NASA-JPL-Caltech



Vela, Credit: NASA/CXC/Univ of Toronto/ M.Durant et al

Neutron-stars (NS): fortuitous discovery

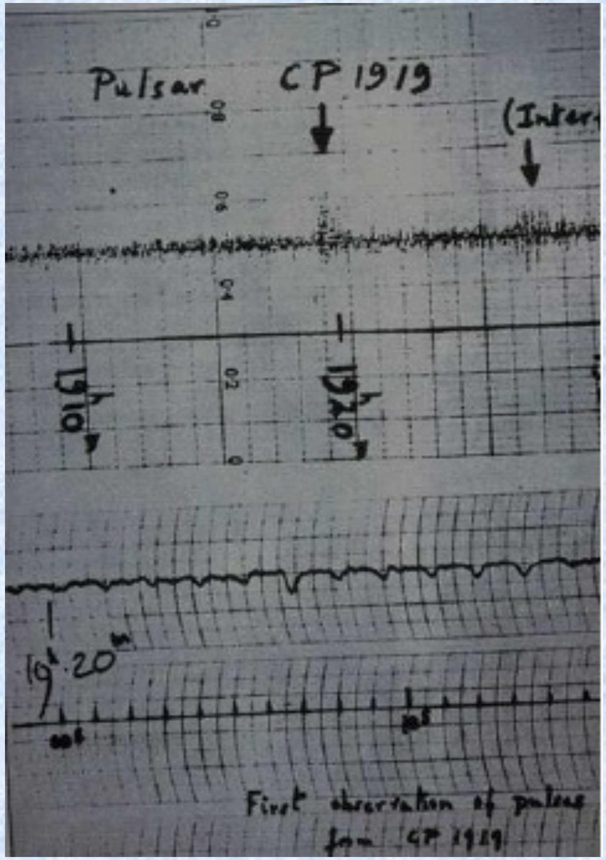
Jocelyn Bell in 1966

1967: J. Bell, PhD thesis at Cavendish Laboratory, Cambridge on radio sources. With a 3.7m diameter telescope, she discovered a very regular source, with a period of 1.3373012 s.

This source was called LGM ("Little Green Man"); a journalist from Daily Telegraph called this source "*Pulsar*" .
→ It is now known as PSR B1919+21.

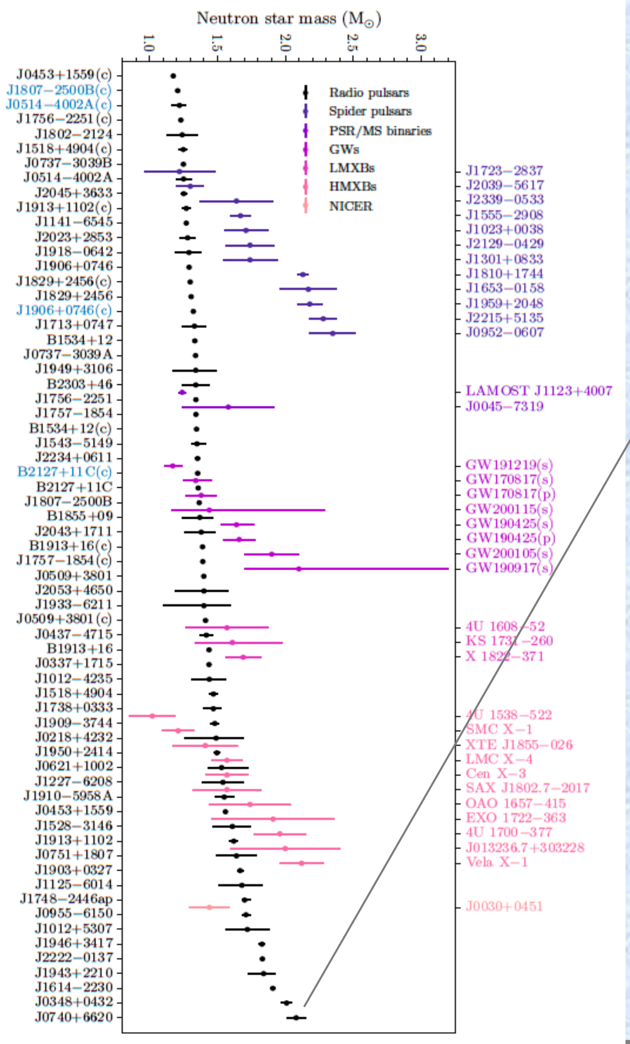
In 1974, A. Hewish (Bell's PhD supervisor) received the Nobel prize…

Now, over 3000 pulsars known!



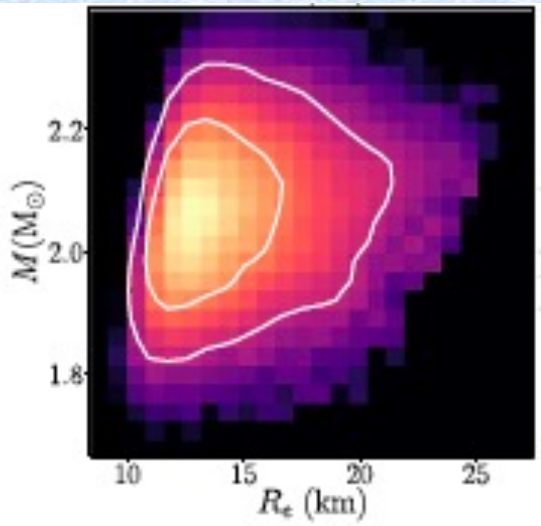
Neutron-stars (NS): observations

MASSES

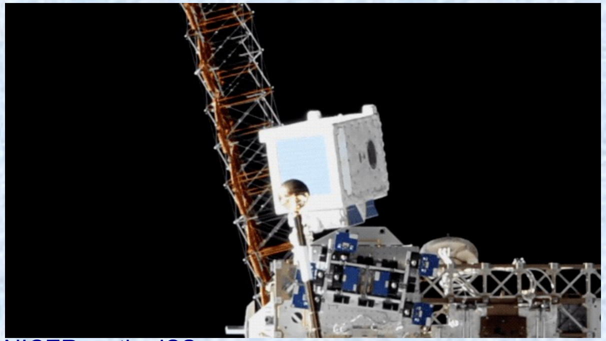


You et al., Nat. Astr. 9, 552 (2025);
https://www3.mpifr-bonn.mpg.de/staff/pfreire/NS_masses.html

RADII

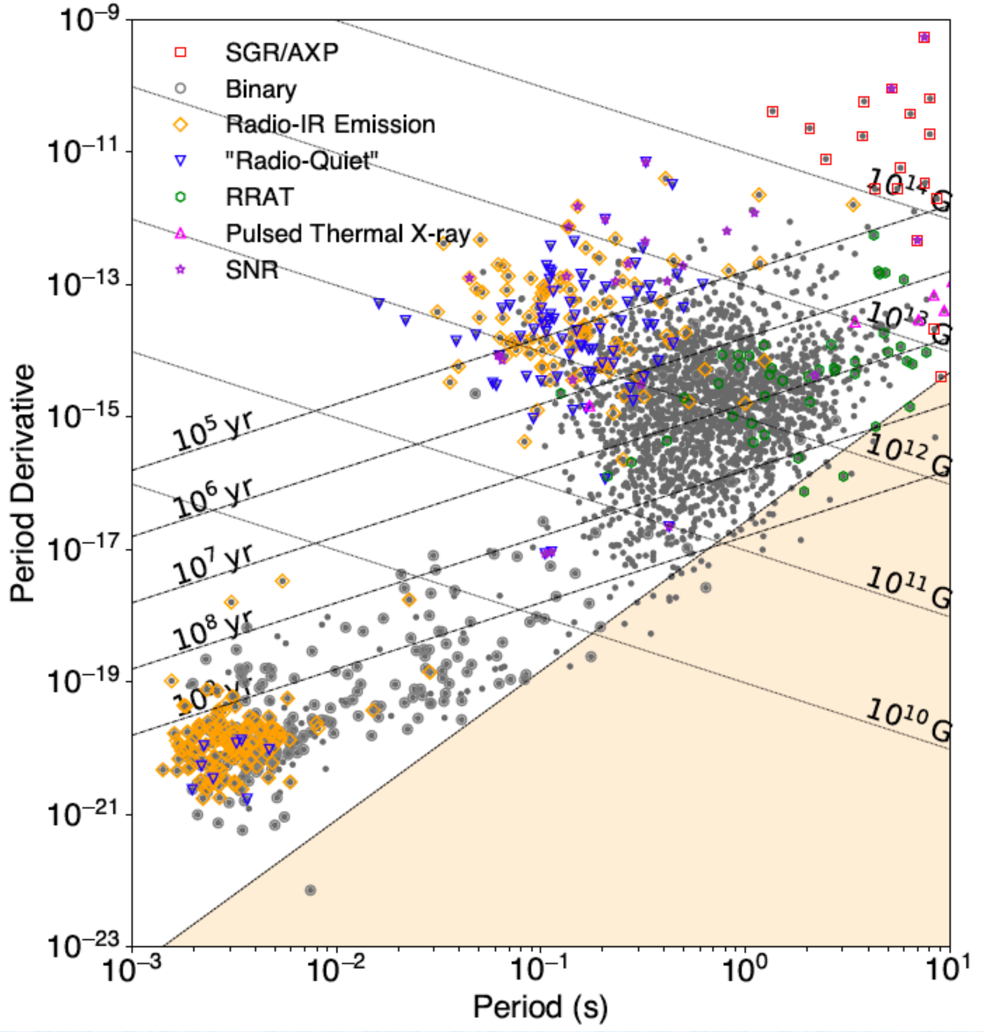


NICER observations (Miller et al., ApJ 2019, 2021 Riley et al., ApJL 2019, 2021; Rutherford et al., ApJL 2024; Mauviard et al., arXiv 2025)



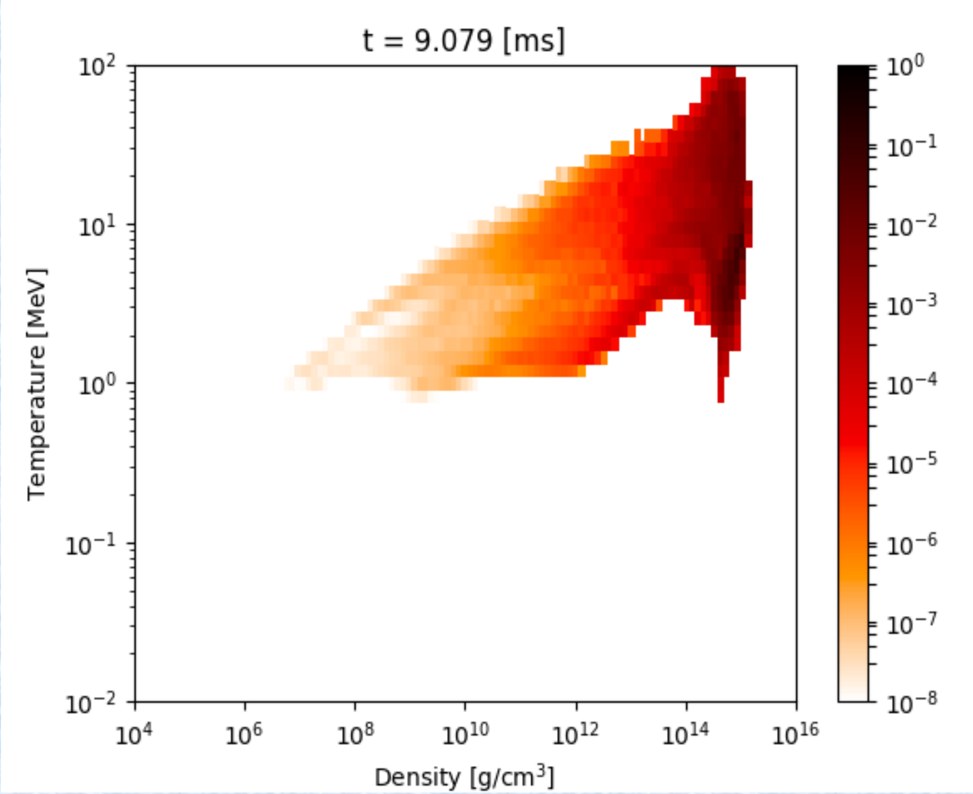
NICER on the ISS
Credits: https://heasarc.gsfc.nasa.gov/docs/nicer/

SPIN PERIOD

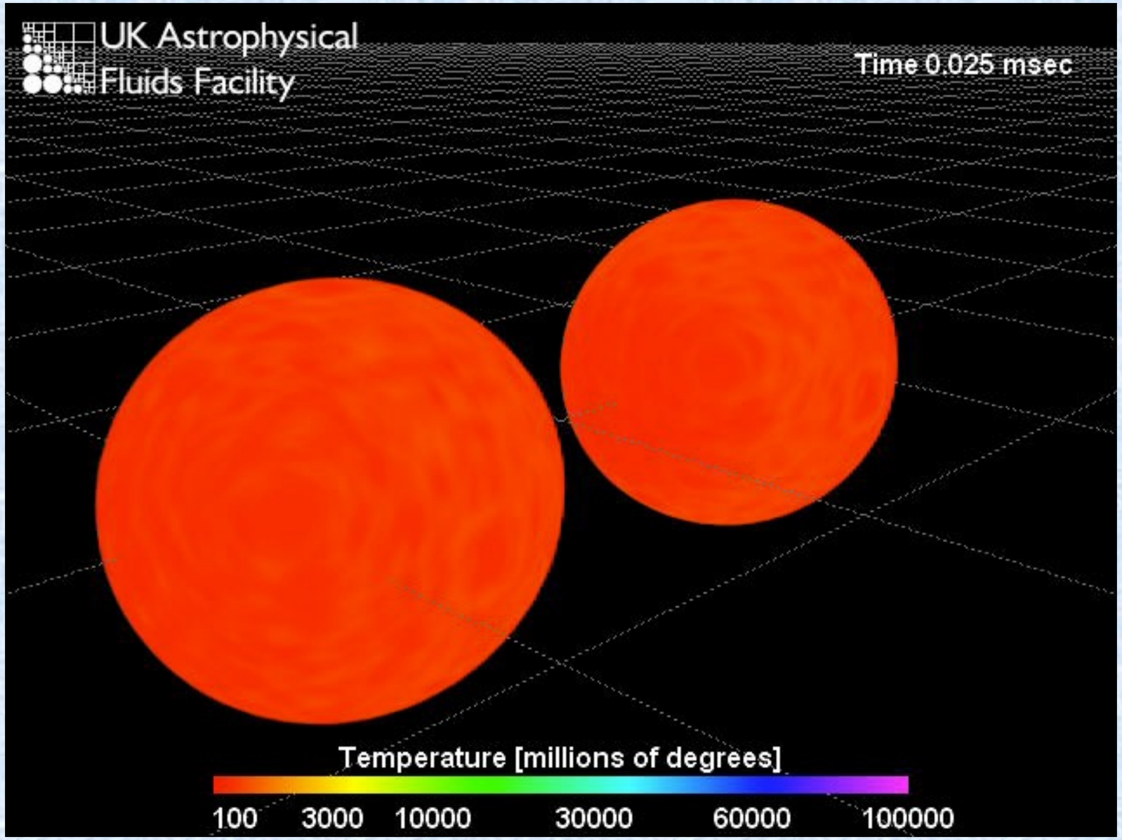


Credits: ATNF Pulsar Catalogue, https://www.atnf.csiro.au/research/pulsar/psrcat/ ; Manchester et al. AJ 129, 1993 (2005); plot generated with Python psrqpy package

Neutron-star (NS) mergers (1)

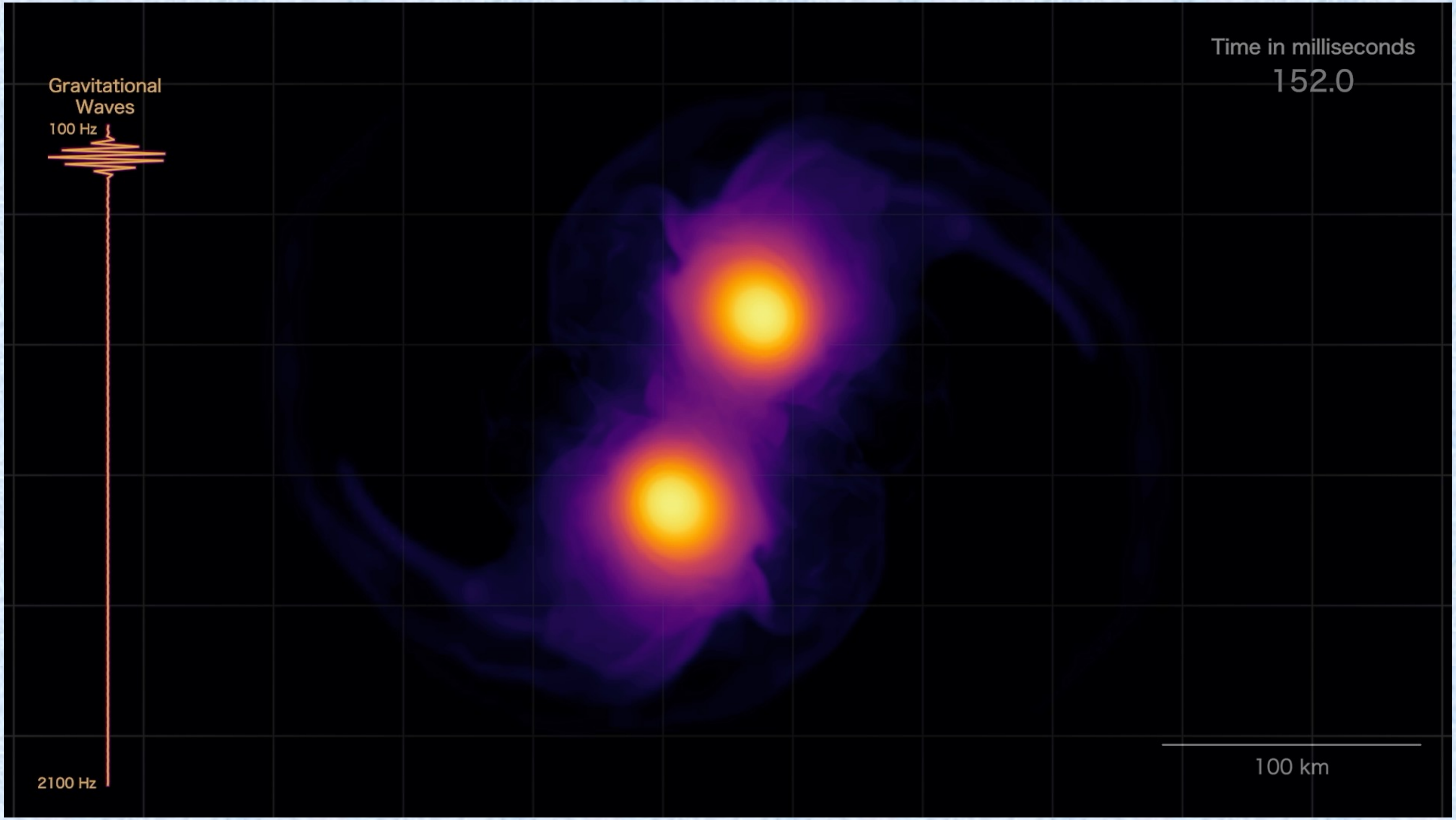


Perego et al., EPJA 55, 124 (2019)



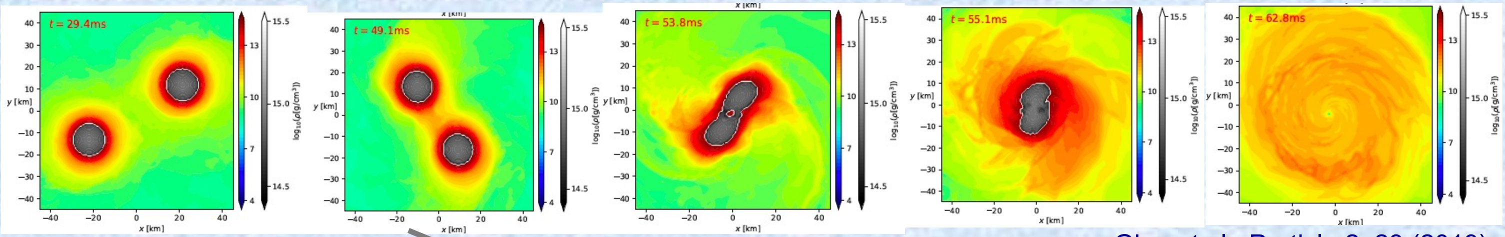
Simulation by S. Rosswog, visualisation R. West
http://www.ukaff.ac.uk/movies/nsmerger/

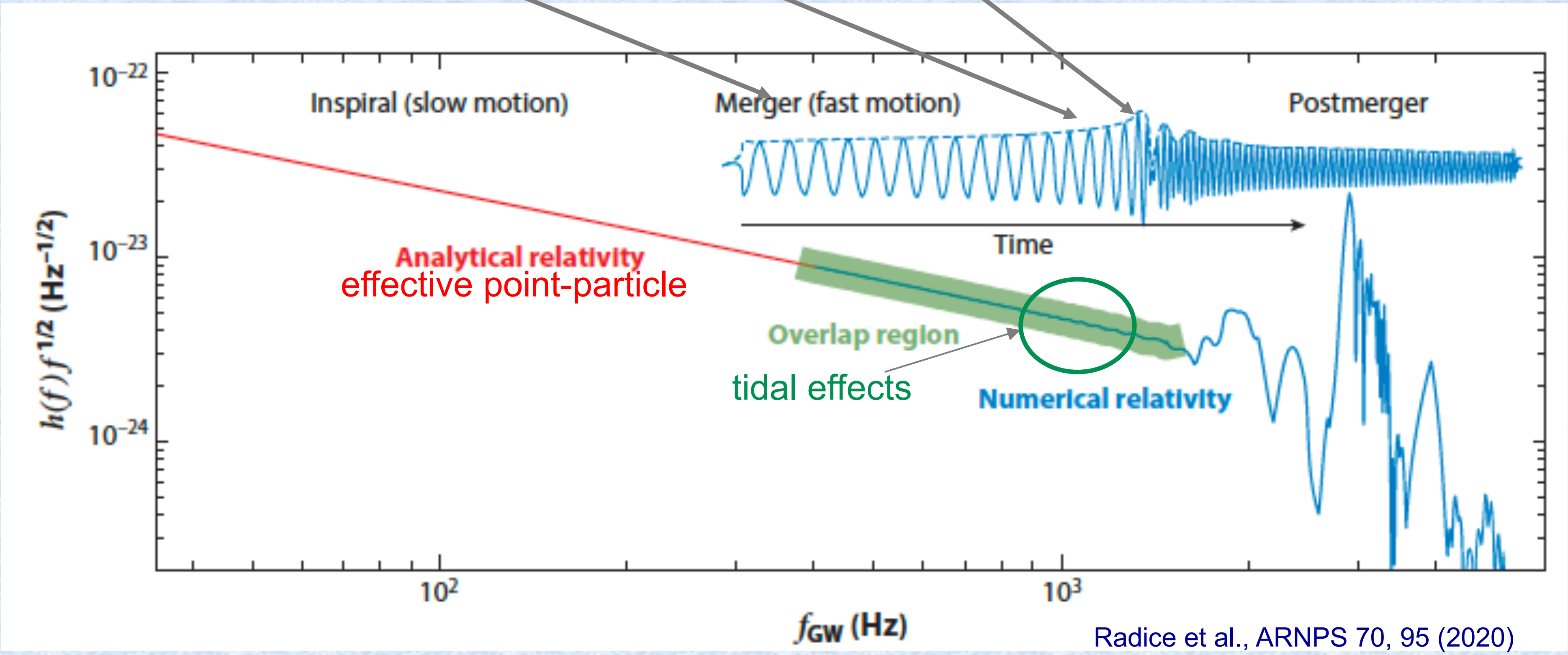
Neutron-star (NS) mergers (2)



Credit: NASA's Goddard Space Flight Center and STAG Research Centre/Peter Hammond

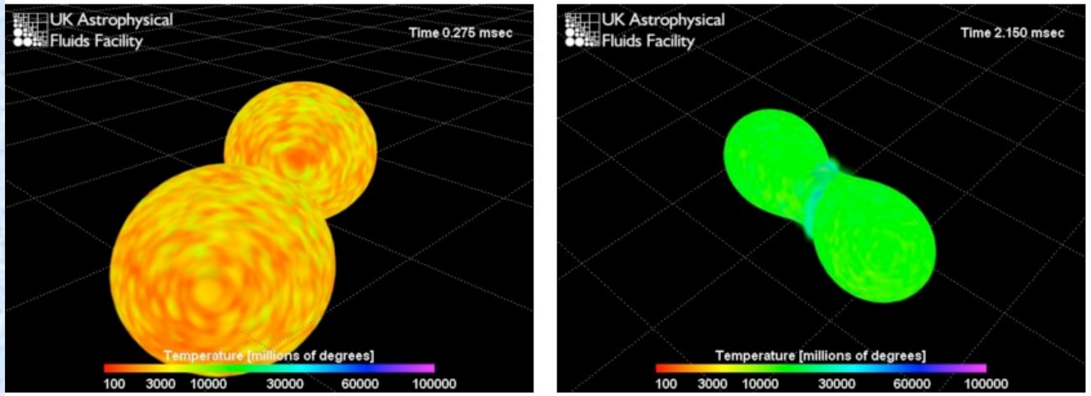
Neutron-star (NS) mergers (3)





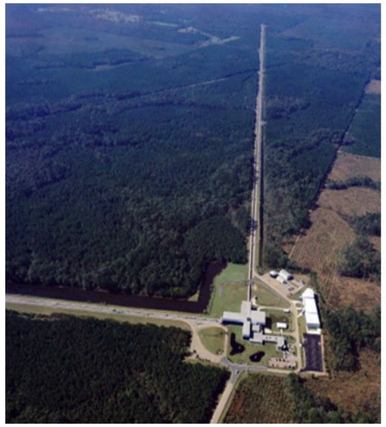
- (Late) inspiral (before merging): modification wrt point particle but still "(hydro)static"
- Temperatures not so high → like "T = 0"

NS mergers: observations



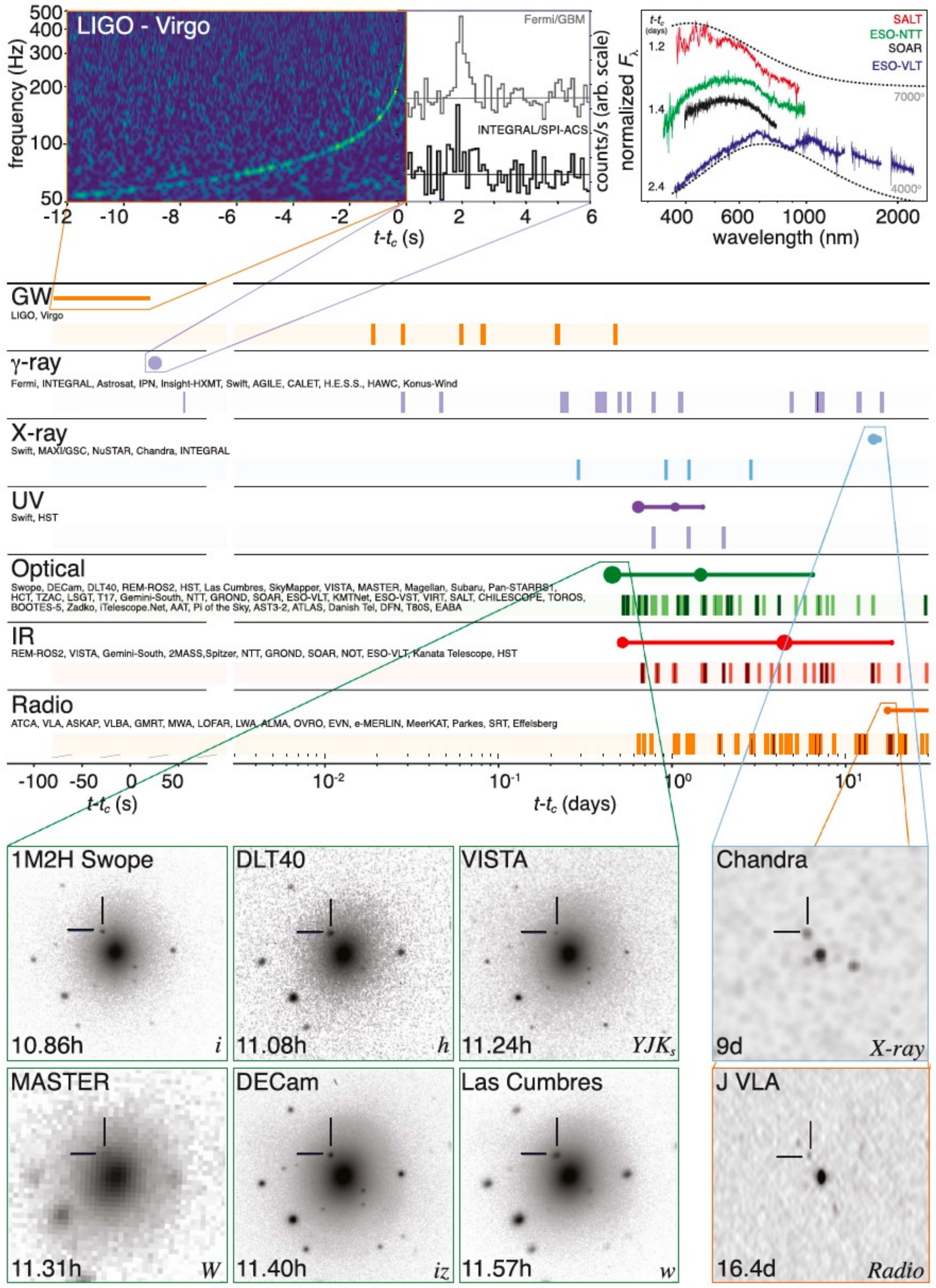
Simulation by S. Rosswog, visualisation R. West http://www.ukaff.ac.uk/movies/nsmerger/

LIGO Hanford Observatory, Washington
(Credits: C. Gray)



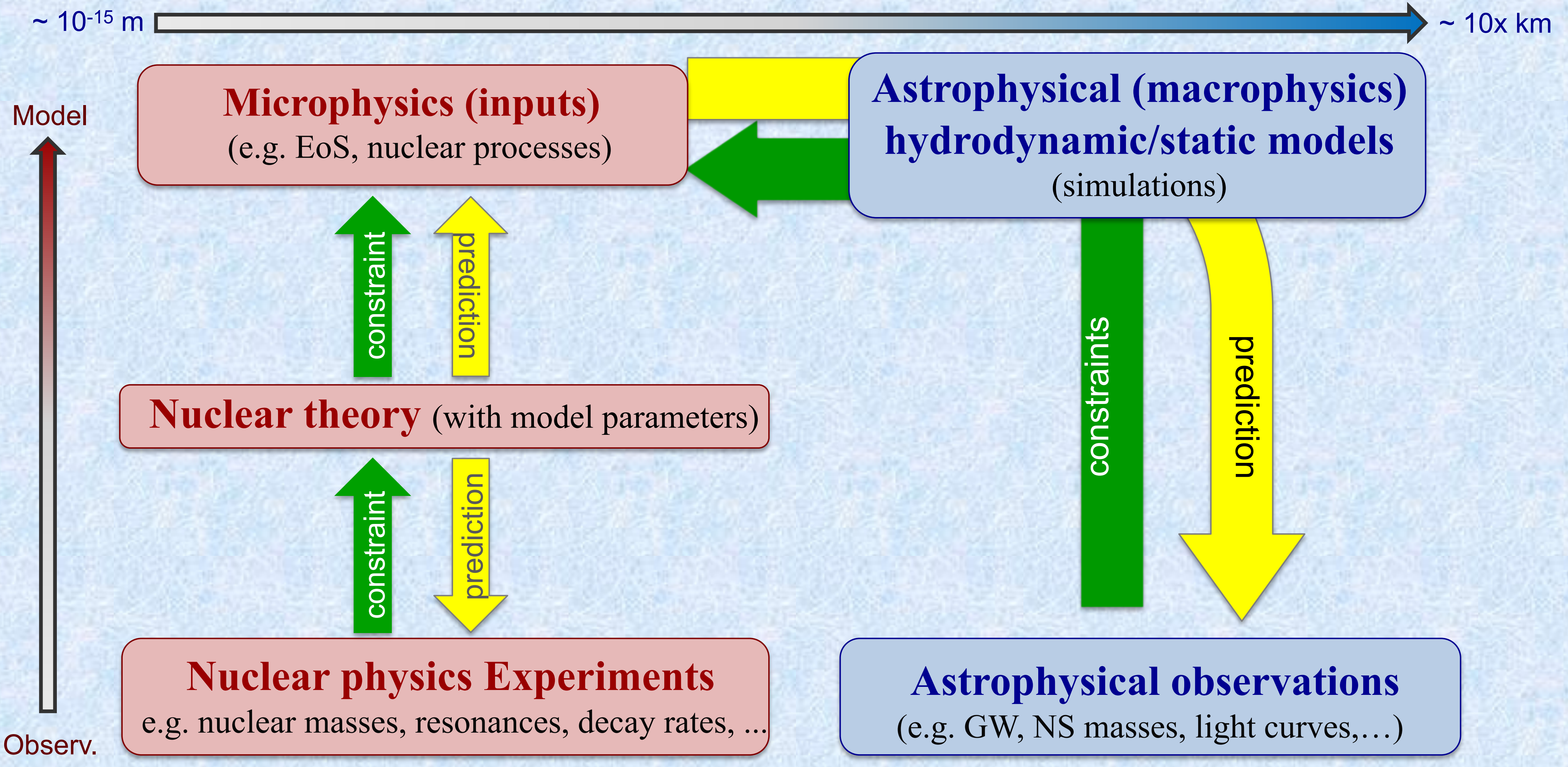
LIGO Livingston Observatory, Louisiana
(Credits: J. Giaime)

Virgo detector, Italy
(Credits: Virgo Collaboration)



Abbott et al., ApJL 848, L12 (2017)

Micro to macro through modelling and scales



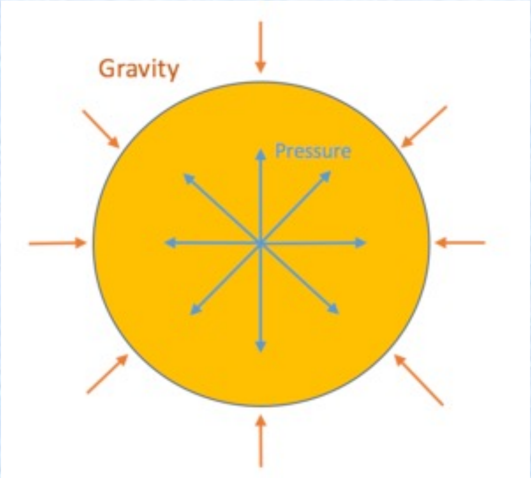
(cold) NS → static → EoS ←→ NS observables (1)

- **TOV → *M*(*R*)** (Tolmann 1939; Oppenheimer&Volkoff 1939; see also Haensel et al. Springer 2007)

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{G\rho(r)\mathcal{M}(r)}{r^2}\left[1+\frac{P(r)}{c^2\rho(r)}\right]\left[1+\frac{4\pi P(r)r^3}{c^2\mathcal{M}(r)}\right]\left[1-\frac{2G\mathcal{M}(r)}{c^2 r}\right]^{-1}$$

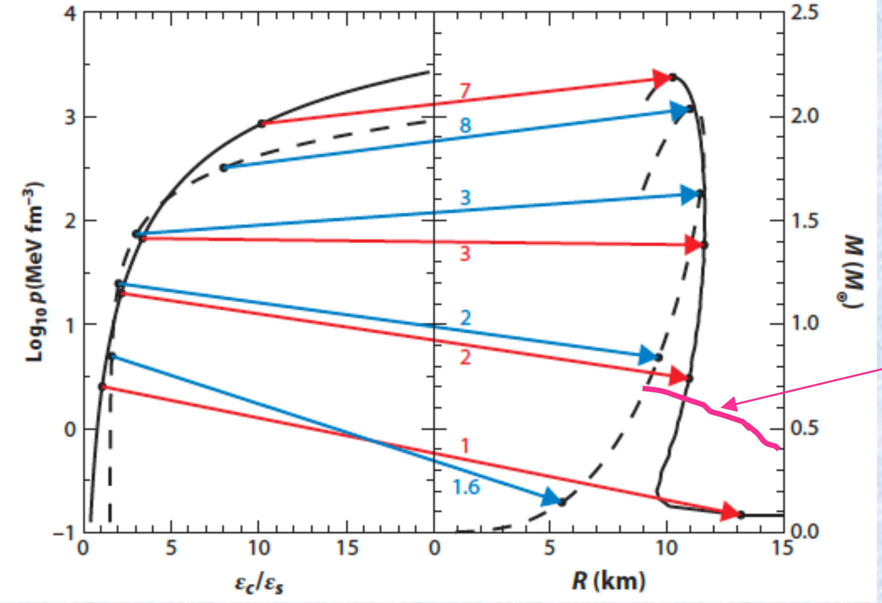
$$\mathcal{M}(r) = 4\pi\int_0^r \rho(r')r'^2\mathrm{d}r'$$

⇒ $P(\mathcal{E})$ EoS only needed!



Credits: J. Stayner, Stanford Univ.

$\forall P_c$ (or $\mathcal{E}_c$) $\rightarrow R = r(P=0),\ M = \mathcal{M}(r=R)$ → **Mass & Radius**

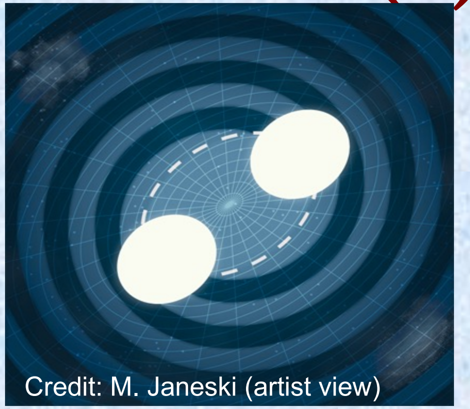


Lattimer, Annu. Rev. Part. Nucl. Sci. 62, 485 (2012)

using EoS of free n gas
→ nuclear interaction !!!
(Oppenheimer&Volkoff 1939; for a review on Mmax: Chamel et al., IJMPE 22, 1330018 (2013))

N.B.: GR in slow rotation limit w/o magnetic field !

(cold) NS → static → EoS ←→ NS observables (2)



- **NS in external, inhomogeneous gravitational field → tidal deformation**
  (Thorne & Campolattaro 1967; Hinderer, ApJ 2008; Hinderer et al., PRD 2010; Blanchet, Liv. Rev. Relat. 2014)

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$ → quadrupolar (*l*=2) momentum in response to tidal field (hp linear order, source of tidal field far away)

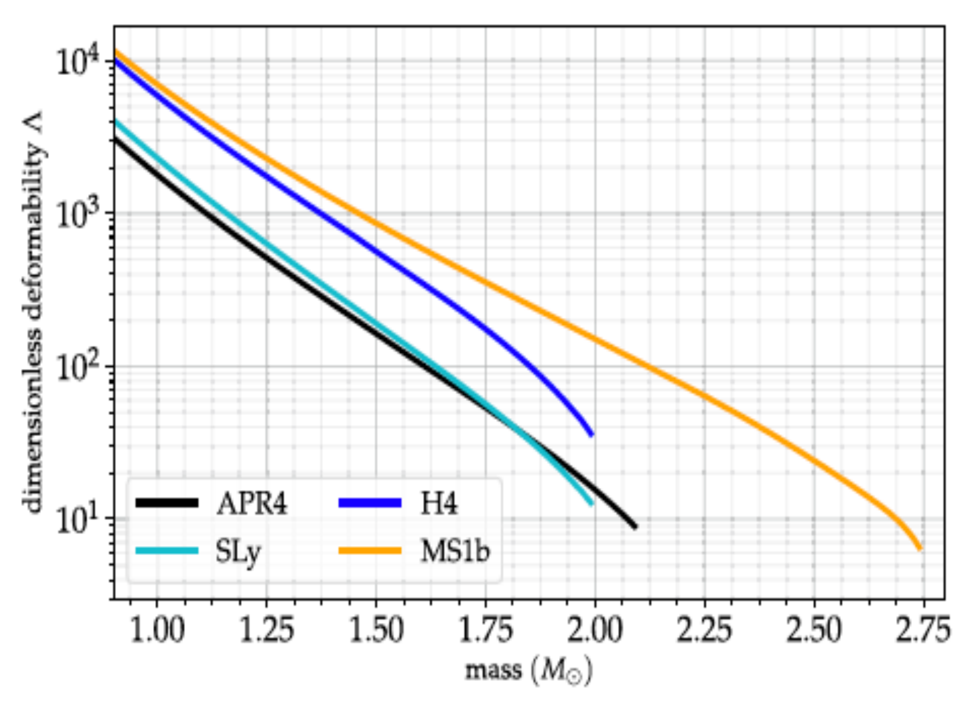
$$k_2 = \frac{3}{2}\frac{G}{R^5}\lambda$$ → tidal Love number (dimensionless)

- **Perturbation to the equilibrium metric → eq. for *y*(*r*)**

$P(r),\ \mathcal{E}(r)$

$$r\frac{dy}{dr} + y(r)^2 + F(r)y(r) + Q(r) = 0$$

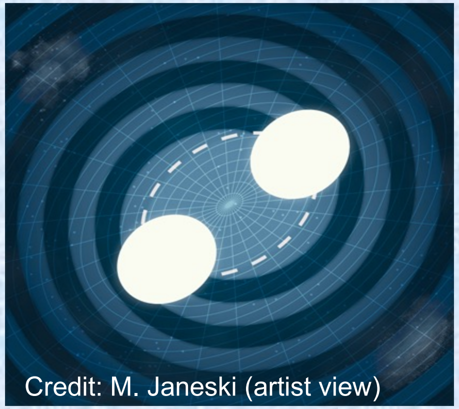
$$\to k_2 = k_2(y(r=R), M/R) \boxed{\to \Lambda = \frac{2}{3}k_2\left(\frac{Rc^2}{GM}\right)^5}$$



Dietrich et al., Gen. Rel. Gravit. 53, 27 (2021)

<u>N.B.</u>: GR in slow rotation limit w/o magnetic field !

Binary parameters and GW signal


Credit: M. Janeski (artist view)

- **Binary parameters extracted from GW signal**
  (compare *h*(*t*) with templates)

$$h(t) \propto A(t)\, \exp^{i\Phi(t)} \rightarrow \tilde{h}(f) \propto \mathcal{A} f^{-7/6} \exp^{i\phi(f)}$$

$\mathcal{A} \propto \mathcal{M}^{5/6}$ → chirp mass $\boxed{\mathcal{M} = \frac{(M_1 M_2)^{3/5}}{(M_1+M_2)^{1/5}}}$

$$\phi(f) = \phi_{\mathrm{p.p.}} + \phi_{\mathrm{tidal}}$$

In post-newtonian (→ expansion in (v/c)^2) and for non-spinning objects (Henri et al., arXiv:2005.13367; Wade et al., PRD 2014):

$$\phi(f) = 2\pi f t_c - 2\phi_0 - \frac{\pi}{4} + \frac{3}{128} \frac{(\pi G (M_1+M_2) f)^{-5/3} (M_1+M_2)^2}{M_1 M_2} \left[1 - \frac{39}{2} \tilde{\Lambda} \left(\frac{\pi G (M_1+M_2) f}{c^3} \right)^{10/3} \right]$$

- Leading mass dependence: $\frac{(M_1+M_2)^{1/3}}{M_1+M_2} \rightarrow \mathcal{M}^{-5/3}$
- Tidal effects at 5PN → $\boxed{\tilde{\Lambda} = \frac{16}{13} \frac{(M_1 + 12 M_2) M_1^4 \Lambda_1 + (M_2 + 12 M_1) M_2^4 \Lambda_2}{(M_1 + M_2)^5}}$

Flanagan & Hinderer, PRD 2008; Hinderer, ApJ 2008; Hinderer et al., PRD 2010; Blanchet, Liv. Rev. Relat. 2014; Henri et al., arXiv:2005.13367; and Hinderer et al., ASSL 457, 575 (Springer 2018), Chatziioannou, Gen. Rel. Grav. 52, 109 (2020) for a review

EoS ←→ NS observables ←→ M(R), Λ(R) ?

GR → direct correspondence
EoS ←→ NS static properties

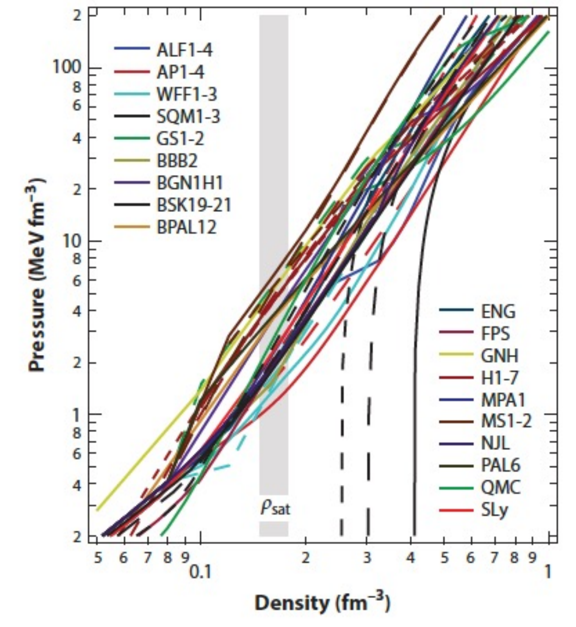
?
→ trace back to EoS
(and composition?)

<u>but</u>:

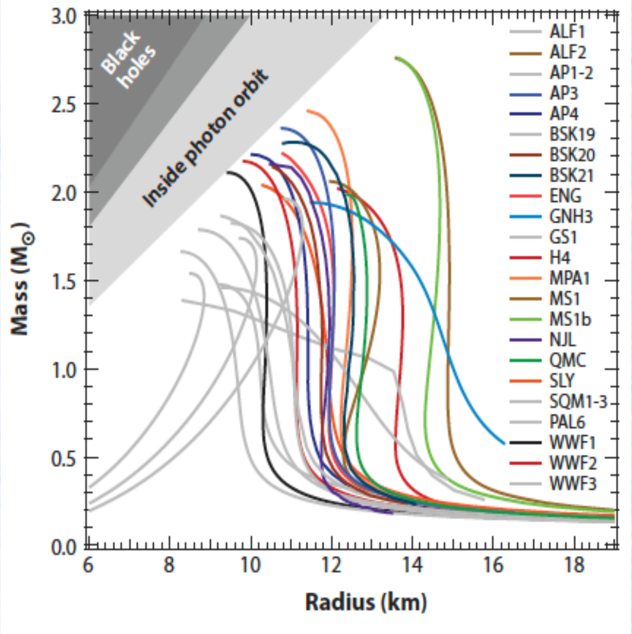
✗ EoS model dependent ! (higher uncertainties at higher ρ)

✗ <u>no</u> ab-initio dense-matter calculations
in all regimes → phenomenological models

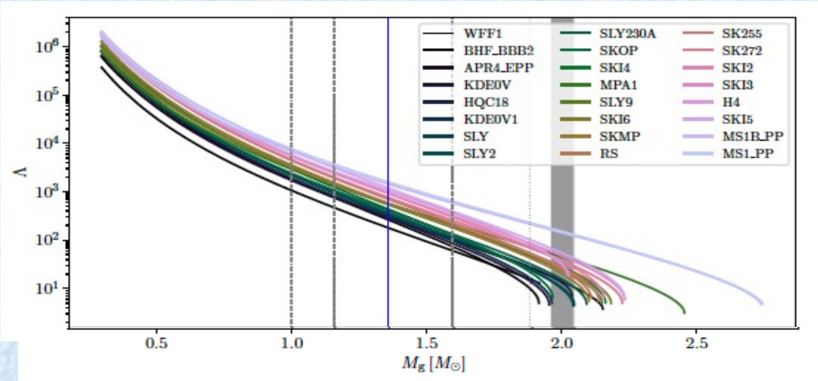
✗ composition ←→ EoS → $M(R)$?



Ozel & Freire, ARAA 54, 401 (2016)



Ozel & Freire, ARAA 54, 401 (2016)
see also Burgio & Fantina, ASSL 457, 255 (2018)



Abbott et al., Class. Quantum Grav. 37, 045006 (2020)

(see e.g. Haensel et al., “Neutron Star Structure”, Springer 2007; Oertel et al., Rev. Mod. Phys. 2017; Burgio & Fantina, ASSL Springer 2018; Blaschke & Chamel, ASSL Springer 2018 for a review)

EoS at $T = 0$: generalities (1)

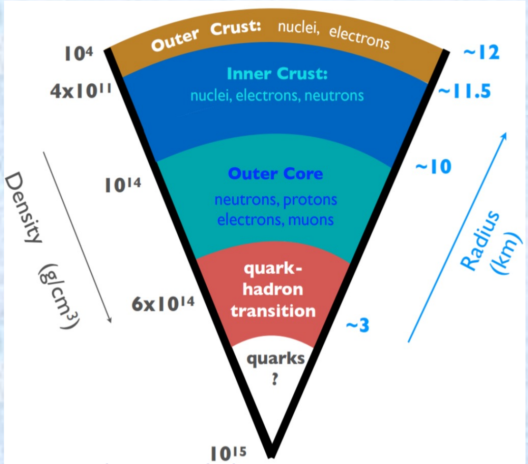


Image Credit: 3G Science White Paper

- **NS hot** (~10^{12} K → finite T important) → cool in days to ~10^9 K
  → **T = 0 → full equilibrium (strong+weak) → $T \ll T_F$**
- EoS needed to compute NS properties (M, R, etc)
- Composition input for different observables (e.g. cooling, elastic properties, oscillations, magneto-rotational evolution, …)
- <u>Different phases of matter</u> (solid crust, "liquid" core, superfluid, QGP?, …)

- <u>NB</u>: matter decouples from gravity! → when calculating EoS, gravity can be neglected (this also at T > 0)

$$ds^2 = -\left(1 - \frac{2Gm(r)}{rc^2}\right) c^2 dt^2 + \left(1 - \frac{2Gm(r)}{rc^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Variation of the metric from NS centre to surface: $\dfrac{g_{00}(r = R, m = M)}{g_{00}(r = 0, m = 0)} = 1 - \dfrac{2GM}{Rc^2} \approx 0.4 \qquad \text{if } M = 2M_\odot, R = 10\text{km}$

Variation of the metric on a nuclear scale: $\dfrac{g_{00}(r + \delta r)}{g_{00}(r)} \approx 1 + \dfrac{\delta r}{r} \approx 1 + 10^{-19}$

Haensel et al., "Neutron Star 1" (Springer 2007); Oertel et al., Rev. Mod. Phys. 89, 015007 (2017); Burgio&Fantina, ASSL 457, 255 (Springer 2018); Blaschke&Chamel, ASSL 457, 337 (Springer 2018)

EoS at T = 0: generalities (2)

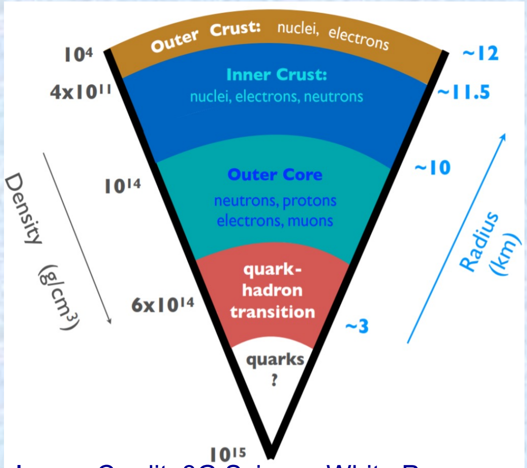


Image Credit: 3G Science White Paper

- **T = 0 → full equilibrium (cold catalysed matter)**
- EoS only density dependent $P(n_B)$ (or $P(\varepsilon)$) → one-dimensional
- **EoS at each n_B → minimise the total energy density**

- However, different matter constituents → $E(\{n_j\})$, with $n_j(r)$ → $P(\{n_j\})$
  but: conserved densities: $\mathbf{n_B, n_L, n_Q, (n_S)}$: baryon, lepton, charge, (strange)
  in the core: infinite matter → thermo limit: $n_j(r) = n_j$
- Baryons & leptons decoupled:

$$\mathcal{E} = \mathcal{E}_B + \mathcal{E}_L$$

If only baryons + e⁻: $\mathcal{E} = \mathcal{E}_B(n_n, n_p) + \mathcal{E}_e(n_e = n_p)$

If β equilibrium: 1 variable n_B as $n_B = n_n + n_p \;, \quad n_e = n_p = Y_p n_B$

$$\mu_n - \mu_p = \mu_e$$

$$P = -\left.\frac{dE}{dV}\right|_{\{N_j\},T} = n_B^2 \frac{de}{dn_B}$$

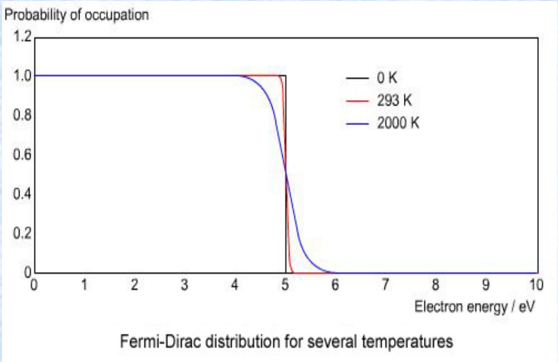
If only baryons + electrons: $P = P_B + P_e$

NB: n_B = baryon number density, $\rho c^2 = \varepsilon$ mass-energy density !

NB: Leptons are treated as free Fermi gas, but baryons interact!

Haensel et al., "Neutron Star 1" (Springer 2007); Oertel et al., Rev. Mod. Phys. 89, 015007 (2017); Burgio&Fantina, ASSL 457, 255 (Springer 2018); Blaschke&Chamel, ASSL 457, 337 (Springer 2018)

Leptons (electrons) at $T = 0$ (1)



Credit: University of Cambridge

- **Fermi gas, degenerate matter** $$f(k) = \frac{1}{1 + e^{(e(k)-\mu)/(k_B T)}}$$

$$f(k) = \Theta(k)$$

- density: $$n_e = \frac{2}{(2\pi)^3}\int d^3k f(k) = \frac{1}{\pi^2}\int_0^{\infty} dk k^2 \Theta(k) = \frac{1}{\pi^2}\int_0^{k_F} dk k^2 = \frac{k_{F,e}^3}{3\pi^2}$$

- electron energy: $$e_e(k) = \sqrt{(m_e c^2)^2 + (\hbar c k)^2}$$

- NR limit: $p_e c << m_e c^2$: $$\mathcal{E}_e = \frac{2}{(2\pi^2)^3}\int_0^{k_F} dk\ 4\pi k^2 \left(m_e c^2 + \frac{\hbar^2 c^2 k^2}{2 m_e c^2}\right) = n_e m_e c^2 + \frac{\hbar^2 c^2 k_{e,F}^5}{10\pi^2 m_e c^2}$$

$$P_e = n_e^2 \frac{d(\mathcal{E}_e/n_e)}{dn_e} = \frac{\hbar^2 c^2 k_{e,F}^5}{15\pi^2 m_e c^2} \propto n_e^{5/3}$$ → $P = \mathcal{K} n^{\gamma}$ → **Polytropic EoS**

NB: same for muons, with $\mu_\mu = \mu_e$. They appear when $\mu_\mu = m_\mu$

Phillips, “The Physics of Stars”; Weiss et al., “Cox & Giuli’s Principles of Stellar Structure” (Cambridge 2004); Haensel et al., “Neutron Star 1” (Springer 2007)

Leptons (electrons) at $T = 0$ (2)

- **Fermi gas, degenerate matter**

- UR limit: $p_e c >> m_e c^2$: $\mathcal{E}_e = \frac{2}{(2\pi^2)^3}\int_0^{k_F} dk\ 4\pi k^2 \hbar c k = \frac{\hbar c k_{e,F}^4}{4\pi^2}$

$$P_e = n_e^2 \frac{d e_e}{d n_e} = \frac{\hbar c k_{e,F} n_e}{4} \propto n_e^{4/3}$$

→ $P = \mathcal{K} n^\gamma$ → **Polytropic EoS**

e⁻ relativistic when $p_e \sim m_e c$ → $\rho_B = \frac{A}{Z} n_e m_p \approx \frac{A}{Z} n_e m_p \frac{1}{3\pi^2 \lambda_{e,C}^3} \approx 10^6 \text{ g cm}^{-3}$ $\qquad \lambda_{e,C} = \frac{m_e c^2}{\hbar c}$

- more general expressions: $\mathcal{E}_e = \frac{m_e c^2}{8\pi^2 \lambda_{e,C}^3}\left[2x^3\sqrt{1+x^2} + x\sqrt{1+x^2} - \ln\left(x + \sqrt{1+x^2}\right)\right]$ $\qquad x = \frac{\hbar c (3\pi^2 n_e)^{1/3}}{m_e c^2}$

$$P_e = \frac{m_e c^2}{8\pi^2 \lambda_{e,C}^3}\left[x\left(\frac{2}{3}x^2 - 1\right)\sqrt{1+x^2} + \ln\left(x + \sqrt{1+x^2}\right)\right]$$

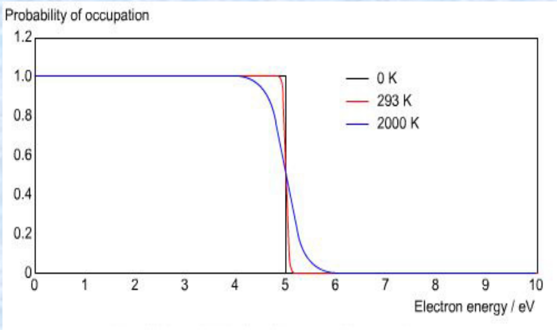
N.B.: there are corrections to this formula, e.g.:
- electron screening
- exchange energy

NB: same for muons, with $\mu_\mu = \mu_e$. They appear when $\mu_\mu = m_\mu$

Phillips, "The Physics of Stars"; Weiss et al., "Cox & Giuli's Principles of Stellar Structure" (Cambridge 2004); Haensel et al., "Neutron Star 1" (Springer 2007)

Leptons (electrons) at $T > 0$

- Fermi distribution function at $T > 0$ not step function $f(k) = \dfrac{1}{1 + e^{(e(k)-\mu)/(k_B T)}}$



Fermi-Dirac distribution for several temperatures

Credit: University of Cambridge

$$e_e = \sqrt{(m_e c^2)^2 + (\hbar c k)^2}$$ relativistic particles

$$n_e = \frac{2}{(2\pi)^3}\int d^3k f(k) = \frac{4}{\sqrt{\pi}\lambda_e^3}\left[I_{1/2}(\chi, k_BT/m_ec^2) + \frac{k_BT}{m_ec^2} I_{3/2}(\chi, k_BT/m_ec^2)\right]$$

$$\mathcal{E}_e = \frac{4}{\sqrt{\pi}}\frac{k_BT}{\lambda_e^3}\left[I_{3/2}(\chi, k_BT/m_ec^2) + \frac{k_BT}{m_ec^2} I_{5/2}(\chi, k_BT/m_ec^2)\right]$$

$$\lambda_e = \left(\frac{2\pi(\hbar c)^2}{m_ec^2k_BT}\right)^{1/2}$$

$$P_e = \frac{8}{3\sqrt{\pi}}\frac{k_BT}{\lambda_e^3}\left[I_{3/2}(\chi, k_BT/m_ec^2) + \frac{k_BT}{2m_ec^2} I_{5/2}(\chi, k_BT/m_ec^2)\right]$$

$$\chi = \frac{\mu_e - m_ec^2}{k_BT}$$

→ Fermi integral I_k → numerical resolution

$$I_k(\chi, t) \equiv \int_0^\infty \frac{x^k(1+tx/2)^{1/2}}{1+\exp(x-\chi)}dx$$

NB: for relatively small T ($T/T_F << 1$) → Sommerfeld expansion in powers of T^2

NB: same for muons

Phillips, "The Physics of Stars"; Weiss et al., "Cox & Giuli's Principles of Stellar Structure" (Cambridge 2004); Haensel et al., "Neutron Star 1" (Springer 2007)

Baryons (n,p) at $T = 0$

NN interaction not known; nuclear physics = non-perturbative regime of QCD

→ *Two different approaches*

- **Ab-initio ("microscopic") approaches**
  based on quantum many-body theory from realistic NN interactions;
  different techniques: variational methods, (D)BHF, chiral EFT, Monte-Carlo, Green's functions, …

  → usually restricted to homogeneous matter (NS core)

  see L. Jokiniemi's and P. Arthuis's lecture

- **Phenomenological approaches**
  based on effective interactions (start from Hamiltonian/Lagrangian) with parameters adjusted to reproduced nuclear properties: EDF e.g. Skyrme / Gogny, RMF, meta-models, …

  → also for inhomogeneous matter (crust+core)

  see M. Bender's lecture

see also S. Lenzi's lecture

for a review see e.g. Haensel et al. 2007 (Springer), Oertel et al., Rev. Mod. Phys. (2017), Burgio & Fantina, ASSL 457 (2018); Burgio et al., Prog. Part. Nucl. Phys. 120, 103879 (2021)

Baryons (n,p) at $T = 0$

NN interaction not known; nuclear physics = non-perturbative regime of QCD

→ *Two different approaches*

- **Ab-initio ("microscopic") approaches**
  based on quantum many-b
  realistic NN interactions;
  different techniques: variati
  chiral EFT, Monte-Carlo, Gr

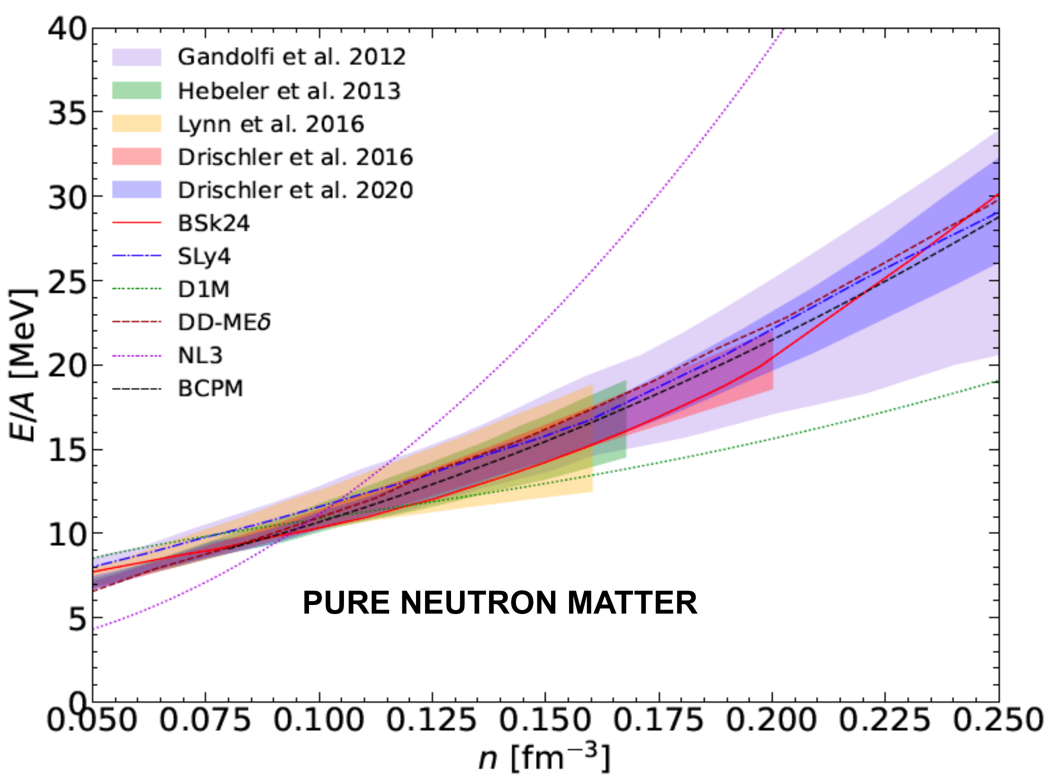
  → usually restricted to hom

  see L. Jokiniemi's and P. Ar

- **Phenomenological approaches**
  tive interactions (start from
  agrangian) with parameters
  produced nuclear properties: EDF
  Gogny, RMF, meta-models, …

  omogeneous matter (crust+core)

  . Bender's lecture



Fantina & Gulminelli, J.Phys. Conf. Ser. 2586, 012112 (2023)

for a review see e.g. Haensel et al. 2007 (Springer), Oertel et al., Rev. Mod. Phys. (2017), Burgio & Fantina, ASSL 457 (2018); Burgio et al., Prog. Part. Nucl. Phys. 120, 103879 (2021)

Baryons (n,p) at $T = 0$: mean-field approx (1)

- Fermions (see electron part), but interactions!
  → similar to electron gas but with ***"effective" masses*** and ***"effective" chemical potentials***

- Independent (quasi-)particles (non-relativistic, q = n,p)): $e_q(k) = \frac{\hbar^2 k^2}{2m_q} + U_q(k)$

  → U_q(k) from effective interactions (DFT)

- In infinite nuclear matter, $T = 0$ (transl. invariant): $\sum_q \sum_k \to \sum_q g_{s,q} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \Theta_q(k) = \sum_q g_{s,q} \int_0^{k_{F,q}} \frac{dk}{2\pi^2} k^2$

- Matter energy density: $\mathcal{E}_q = g_{s,q} \int_{k \le k_F} \frac{d^3\mathbf{k}}{(2\pi)^3} e_q(k) = \mathcal{E}_{F,q} + \underbrace{g_{s,q} \int_{k \le k_F} \frac{d^3\mathbf{k}}{(2\pi)^3} U_q(k)}_{\text{EDF} \to \text{energy functional}}$
  NB: rest-mass to be added

- Examples: *Skyrme* (0-range), *Gogny* (finite-range), *Fayans* (more flexibility than Skyrme), *BCPM* (inspired to BHF calculations), *KIDS* (based on power expansion of nuclear matter energy in terms of k_F), meta-model (based on Taylor expansion in density around saturation density)

- NB: here effects beyond mean-field not discussed — see M. Bender's lecture

for a review see e.g. Haensel et al. (Springer 2017), Oertel et al., Rev. Mod. Phys. (2017), Burgio & Fantina, ASSL 457 (2018); Burgio et al., Prog. Part. Nucl. Phys. 120, 103879 (2021); see also Ring & Schuck (Springer 2004), Chabanat et al., NPA 627, 710 (1997), Bender et al., Rev. Mod. Phys. 75, 121 (2003), Duguet (Springer 2014), Baldo & Burgio, Rep. Prog. Phys. 75, 026301 (2012), Carlson et al., Rev. Mod. Phys. 87, 1067 (2015)

Baryons (n,p) at $T = 0$: mean-field approx (2)

❖ Example of **Skyrme** EDF (Skyrme, Nucl. Phy. 9, 615 (1959)):

Matter energy (infinite matter): central term + density-dep. term + eff. mass term ~~+ s-o, grad, tensor~~

$$\mathcal{E}_{\rm Sky} = \sum_q m_q c^2 n_q + \sum_q \mathcal{E}_{F,q} + \underline{C_{1,\rm is}} n_B^2 + \sum_q \underline{C_{1,\rm is}} n_q^2$$
$$+ \underline{C_{2,\rm is}} n_B^{\alpha+2} + \sum_q \underline{C_{2,\rm iv}} n_q^\alpha n_B^2 + \underline{C_{m,\rm is}} n_B^{5/3} + \sum_q \underline{C_{m,\rm iv}} n_q^{5/3}$$

$$n_B = n_n + n_p$$

n^α terms → mimic many-body effect

$$\frac{\hbar^2}{2m_q} \to \frac{\hbar^2}{2m_q^*}$$

NB: $\mathcal{E}_{F,q} \propto n_q^{5/3}$ ⟹

$$\mathcal{E}_{\rm Sky} = \sum_q m_q n_q + \sum_q \mathcal{E}^*_{F,q} + \underline{C_{1,\rm is}} n_B^2 + \sum_q \underline{C_{1,\rm is}} n_q^2$$
$$+ \underline{C_{2,\rm is}} n_B^{\alpha+2} + \sum_q \underline{C_{2,\rm iv}} n_q^\alpha n_B^2$$

⟹ *<u>Landau (k) effective mass</u>* (momentum dependence)

⟹ $$e_q = \frac{\hbar^2 k^2}{2m_q^*} + U_q \qquad \frac{\hbar^2}{2m_q^*} = \frac{\delta E_{\rm EDF}}{\delta \tau_q} , U_q = \frac{\delta E_{\rm EDF}}{\delta n_q}$$

τ_q kin.energy dens.; no gradient in infinite matter!

Model parameters → fitted to exp.

NB: Extensions (e.g. added density-dep terms)

for a review see e.g. Haensel et al. (Springer 2017), Oertel et al., Rev. Mod. Phys. (2017), Burgio & Fantina, ASSL 457 (2018); Burgio et al., Prog. Part. Nucl. Phys. 120, 103879 (2021), Chabanat et al., NPA 627, 710 (1997), Bender et al., Rev. Mod. Phys. 75, 121 (2003), Duguet (Springer 2014)
Extensions in e.g. Lesinski et al. 2007, Bender et al. 2009, Chamel et al., 2009, Fantina et al. 2011, Margueron et al. 2012, Davesne et al. 2015.

Baryons (n,p) at $T = 0$: mean-field approx (3)

- **RMF** : - nucleons interact via field, e.g. ω (repulsion, short range) and σ (attraction, medium range)
  but others can be added, e.g. ρ needed to account for isospin!
  - meson field are replaced by average values
- Relativistic particle: $e_q(k) = \sqrt{(m_q^* c^2)^2 + (\hbar c k)^2} + V_q$
- Scalar field → effective mass: $m^* = m - \underline{g_\sigma}\sigma_0$

$$m_\sigma^2 \sigma_0 = g_\sigma \sum_q \frac{g_{s,q}}{2\pi^2} \int_0^{k_{F,q}} k^2 dk \frac{m_q^*}{\sqrt{\hbar^2 k^2 + m_q^*}}$$

→ n_s (scalar density)

- Vector field → shifts energy → V_q: $g_\omega \omega_0 (\pm g_\rho \rho)$

$\hbar = c = 1$

- Matter energy density:

$$\mathcal{E}_{\mathrm{RFM},\sigma\omega(\rho)} = \sum_q \frac{g_{s,q}}{2\pi^2} \int_0^{k_{F,q}} k^2 dk \sqrt{k^2 + m_q^{*2}} + \frac{1}{2} m_\sigma^2 \sigma_0^2 - \frac{1}{2} m_\omega^2 \omega_0^2 + \underline{g_\omega}\omega_0 n_B$$

$$\left(-\frac{1}{2} m_\rho^2 \rho_{03}^2 + \frac{1}{2} \underline{g_\rho} \rho_{03} (n_p - n_n) \right)$$

Model parameters → fitted to exp.

NB: Extensions: density-dependent (DD) couplings or non-linear (NL) self-interaction

 see e.g., Glendenning (Springer 2000) Haensel et al. (Springer 2017), Oertel et al., Rev. Mod. Phys. (2017), Burgio & Fantina, ASSL 457 (2018); Burgio et al., Prog. Part. Nucl. Phys. 120, 103879 (2021), Bender et al., Rev. Mod. Phys. 75, 121 (2003), Duguet (Springer 2014), Niksic et al., PPNP 66, 519 (2011)

Effective mass

- *Note*: Landau (NR) & Dirac (R) effective masses are different (different definition) !

- Nucleons interact in medium → mass not the same as in vacuum

  - Landau k-mass (at k_F) (momentum dep.)

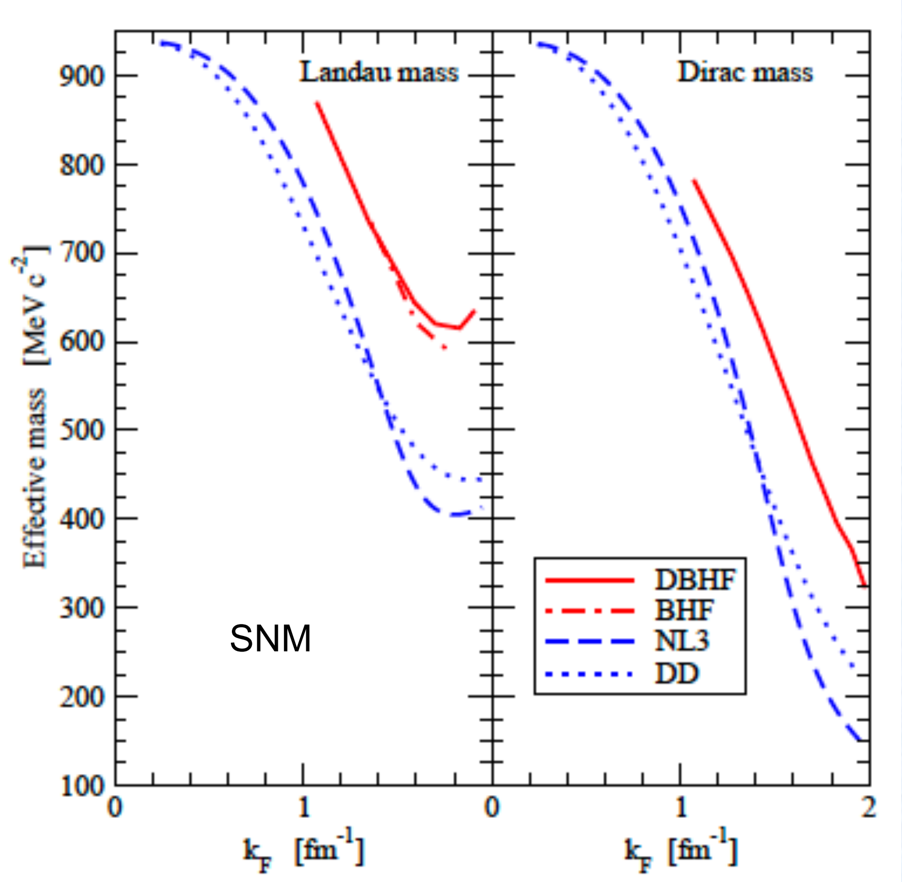
$$m_L^* = k \left(\frac{dE}{dk} \right)^{-1}$$

  - Dirac mass

$$m_D^* \quad = \quad m - g_\sigma \sigma_0 = m - \frac{g_\sigma^2}{m_\sigma^2} \sum_q \frac{g_{s,q}}{2\pi^2} \int_0^{k_F} k^2 dk \frac{m_q^*}{\sqrt{\hbar^2 k^2 + m_q^*}}$$

→ different functional dependencies and values!
→ mapping not straightforward

$$m_L^* \overset{?}{\approx} \sqrt{m_D^{*2} + (\hbar k_F)^2}$$



Van Dalen & Müther, IJMPE 19, 2077 (2010); arXiv:1004.0144

for a review see e.g. Haensel et al. (Springer 2017), Oertel et al., Rev. Mod. Phys. (2017), Burgio & Fantina, ASSL 457 (2018); Burgio et al., Prog. Part. Nucl. Phys. 120, 103879 (2021); see also Ring & Schuck (Springer 2004), Chabanat et al., NPA 627, 710 (1997), Bender et al., Rev. Mod. Phys. 75, 121 (2003), Duguet (Springer 2014), Baldo & Burgio, Rep. Prog. Phys. 75, 026301 (2012), Carlson et al., Rev. Mod. Phys. 87, 1067 (2015)

Baryons (n,p) at $T > 0$

- At $T > 0$ → Fermi function $$f(k) = \frac{1}{1 + e^{(e(k)-\mu)/(k_B T)}}$$

- In principle, same approach as before but *f(k)* → *T*-dep in occupation

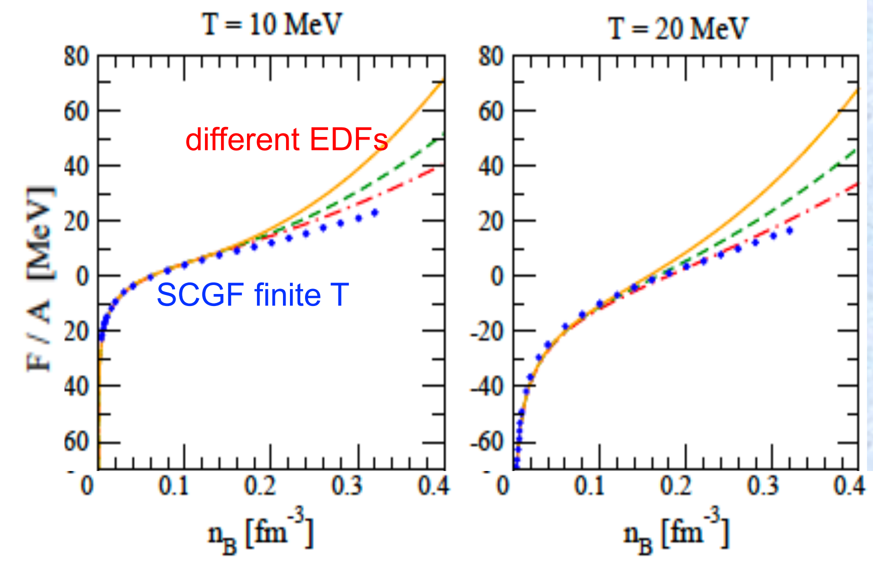
- With good approx., only kinetic part depend on *T* → *m** term!
  (beyond mean-field effect can add *T*-dep but small for *T* < 10-20 MeV)

  ❖ Non-relativistic particles

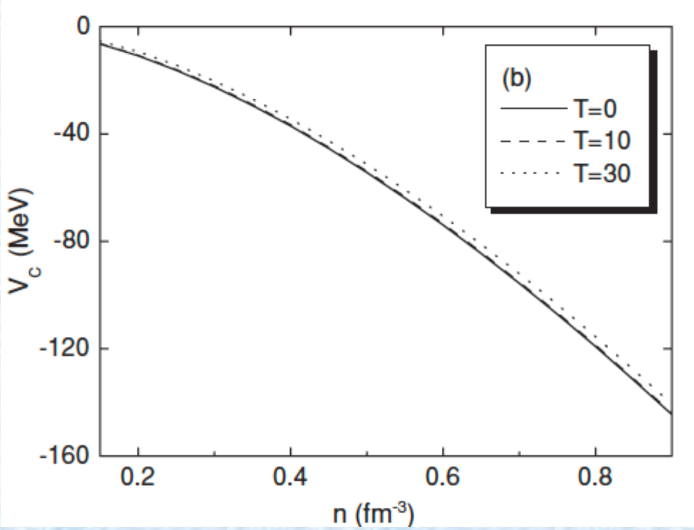
$$e_q = \frac{\hbar^2 c^2}{2m_q^*} + U_q \ , \ \tilde{\mu}_q = \mu_q - U_q$$

$$\rightarrow \mathcal{E}_{\mathrm{kin},q} = \frac{4}{\sqrt{\pi}} \frac{k_B T}{\lambda_q^3} I_{3/2}\left(\frac{\tilde{\mu}_q}{k_B T}\right)$$

caveat: what about extrapolation at much higher T
(~ 100MeV, e.g. SN, BNS mergers?)



Fantina et al., JPCS 342, 012003 (2012)

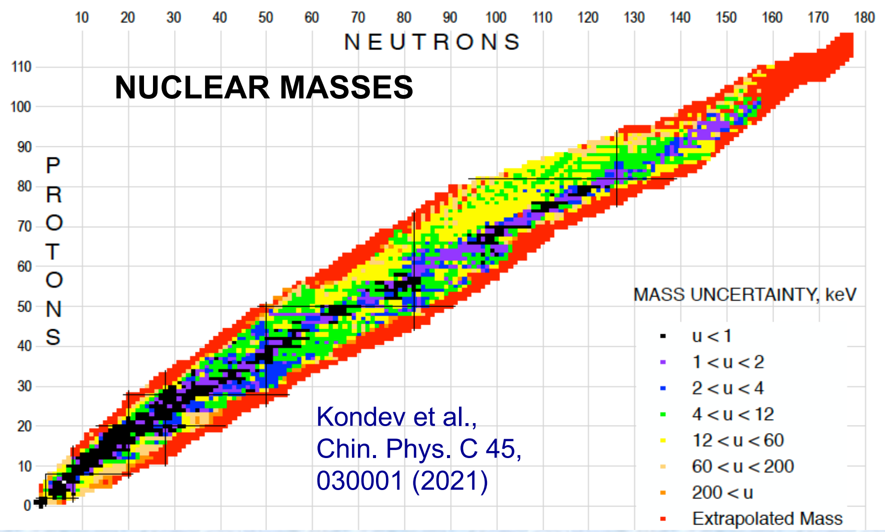


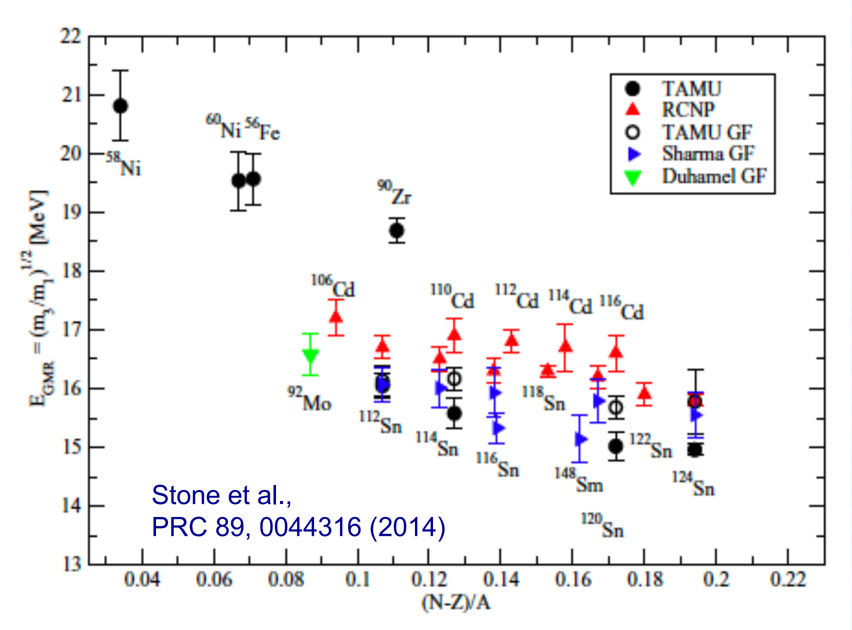
Moustakidis & Panos, PRC 79, 045806 (2009);
(see also e.g. Lu et al., PRC 2019, Shang et al., PRC 2020)

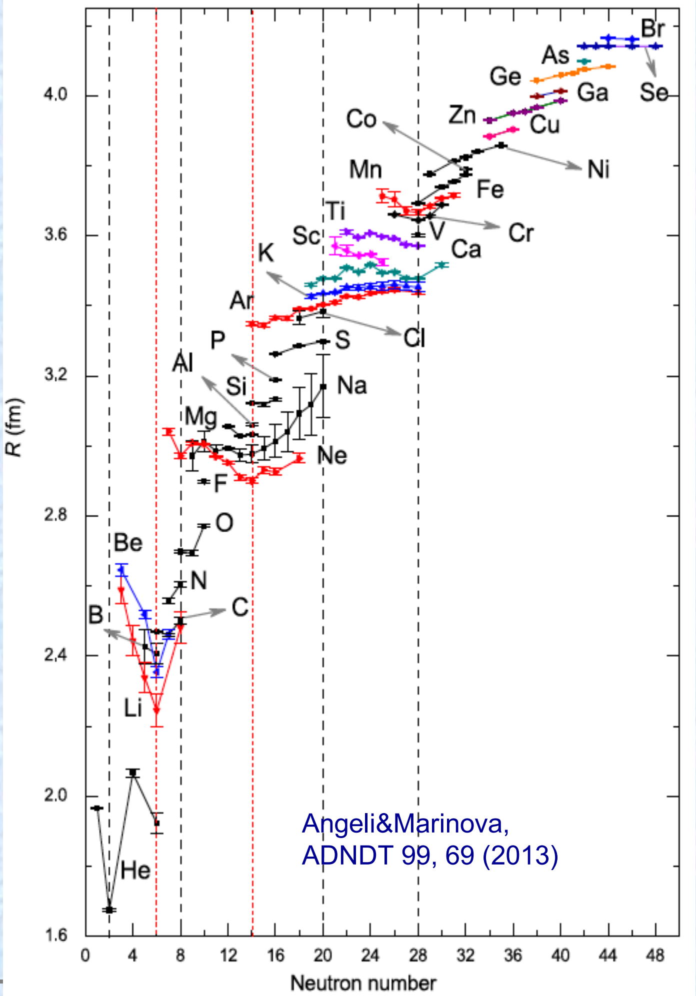
for a review see e.g. Haensel et al. (Springer 2017), Oertel et al., Rev. Mod. Phys. (2017), Burgio & Fantina, ASSL 457 (2018); Burgio et al., Prog. Part. Nucl. Phys. 120, 103879 (2021)

Connection with nuclear physics experiments (1)

- EDF (with parameters) can model nuclei (T = 0)
  → **Parameters fitted on nuclear physics experiments**







see M. Bender's lecture

for a review see e.g. Haensel et al. 2007 (Springer), Oertel et al., Rev. Mod. Phys. (2017), Burgio & Fantina, ASSL 457 (2018); Burgio et al., Prog. Part. Nucl. Phys. 120, 103879 (2021)

Connection with nuclear physics experiments (2)

- EoS of infinite matter can be Taylor expanded around saturation

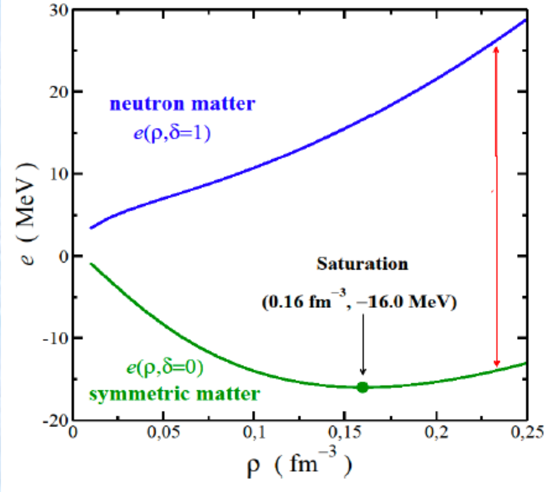
$$e(n_B,\delta) \approx e_{\rm is}(n_B) + e_{\rm iv}(n_B)\delta^2$$

but attention: kinetic part!

$$\delta = (n_n - n_p)/n$$

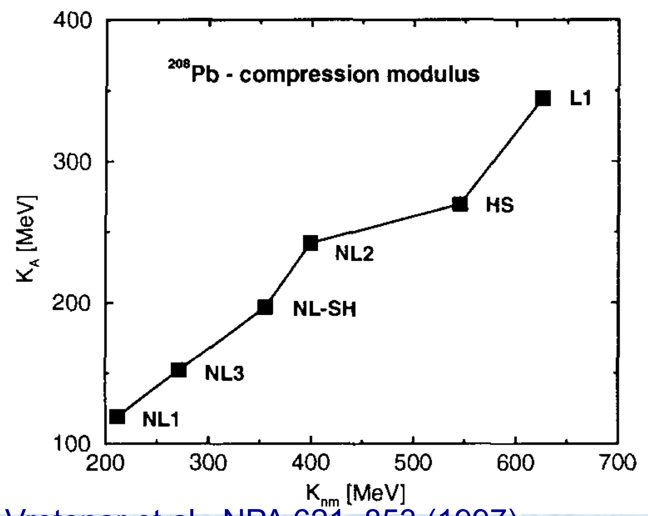
$$e_{\rm is} = E_{\rm sat} + \frac{1}{2}K_{\rm sat}x^2 + \frac{1}{6}Q_{\rm sat}x^3 + \dots$$ → $e_{\rm sat}(n_B,\delta=0)$

$$e_{\rm iv} = E_{\rm sym} + L_{\rm sym}x + \frac{1}{2}K_{\rm sym}x^2 + \frac{1}{6}Q_{\rm sym}x^3 + \dots$$ → $e_{\rm sym}(n_B)=e(n_B,\delta=1) - e(n_B,\delta=0)$

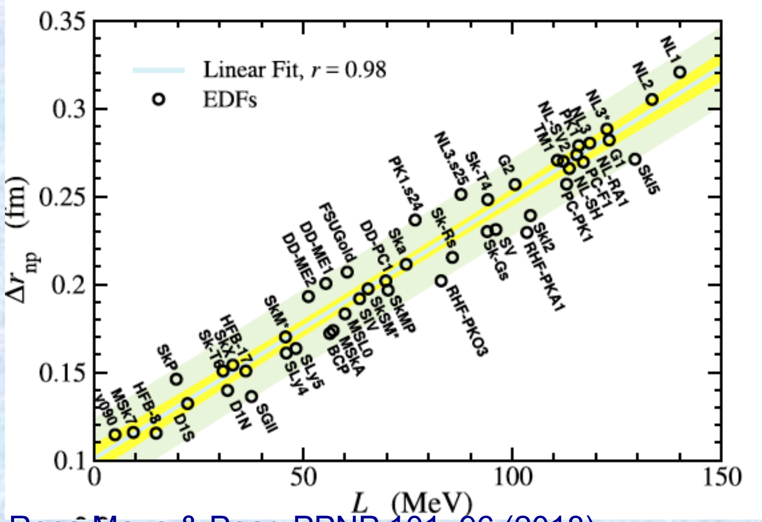


Centelles, talk@Primosten (2011)

nuclear empirical parameters = successive derivative of energy functional at $n_{\rm sat}$ → from experiments



Vretenar et al., NPA 621, 853 (1997)



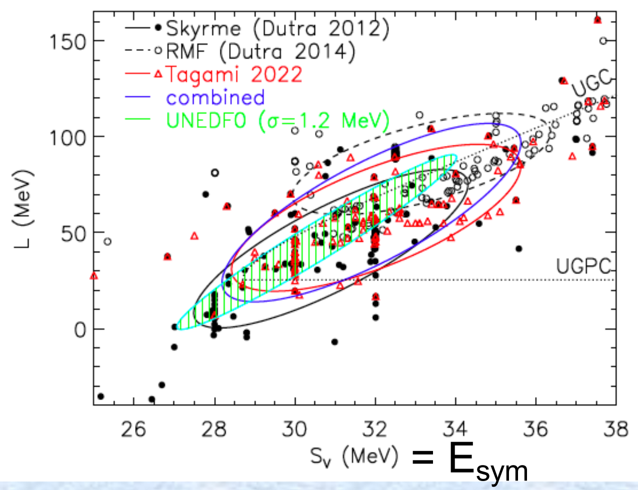
Roca Maza & Paar, PPNP 101, 96 (2018)

for a review see e.g. Haensel et al. 2007 (Springer), Oertel et al., Rev. Mod. Phys. (2017), Burgio & Fantina, ASSL 457 (2018); Burgio et al., Prog. Part. Nucl. Phys. 120, 103879 (2021)

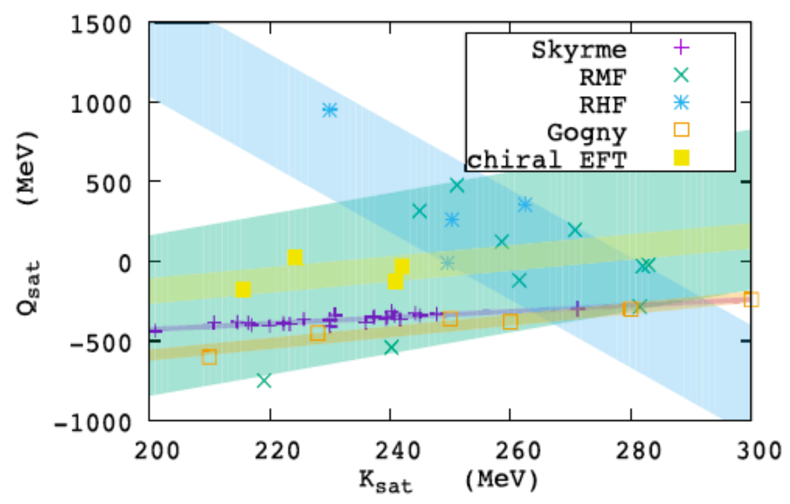
… but choice not unique!

- but: choice of the functional, data set & fitting protocol not unique
  → variety of models (even if same potential!)
  → spread at high density (extrapolation far from experimental info)

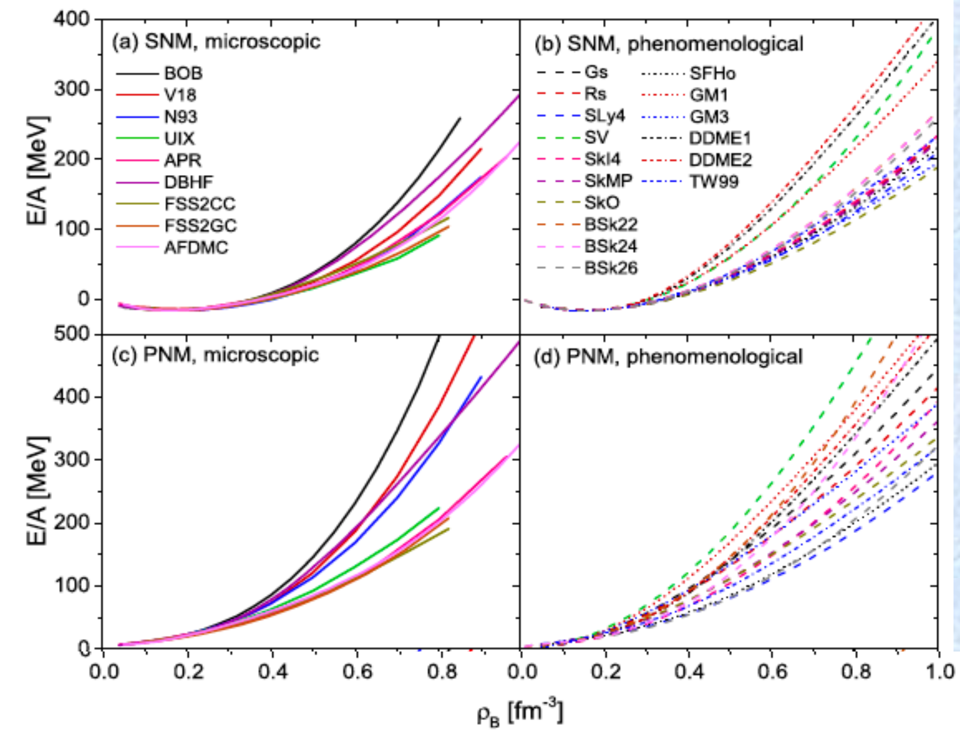
- Even for ab-initio, different many-body techniques, w/ or w/o 3BF
  → variety of models



Lattimer, Particles 6, 30 (2023)



Margueron et al., PRC 97, 025805 (2018)



Burgio et al., PPNP 120, 103879 (2021)

How this affects astro?

for a review see e.g. Haensel et al. 2007 (Springer), Oertel et al., Rev. Mod. Phys. (2017), Burgio & Fantina, ASSL 457 (2018); Burgio et al., Prog. Part. Nucl. Phys. 120, 103879 (2021)

From nuclear models to astro prediction

Two ways of investigating impact of nuclear physics in astro

1. Choose your "favourite" model(s)

- More "microscopic" model (accurately calibrated EDFs)
  → study specific model dependence
- Benchmark calculations (if simulations long)
- Calculate several nuclear inputs consistently for nuclear physics and astrophysics

✗ No parameter-space exploration

2. Statistical model exploration

- More flexible, faster computation model (EDF, meta-model-like)
  → Parameter-space exploration
- Bayes analysis
- Run millions of models and evaluate uncertainties

✗ Less microscopic

for a review see e.g. Haensel et al. 2007 (Springer), Oertel et al., Rev. Mod. Phys. (2017), Burgio & Fantina, ASSL 457 (2018); Burgio et al., Prog. Part. Nucl. Phys. 120, 103879 (2021)

NS atmosphere

atmosphere + ocean

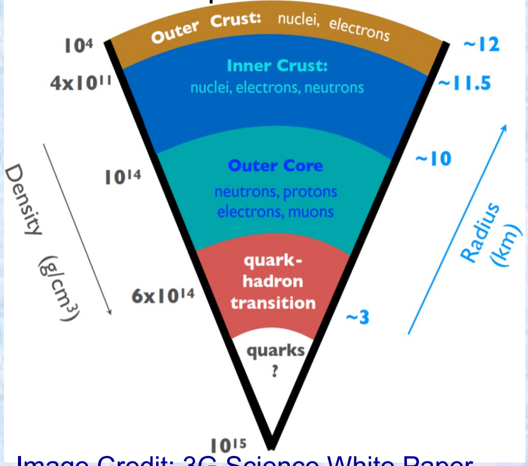
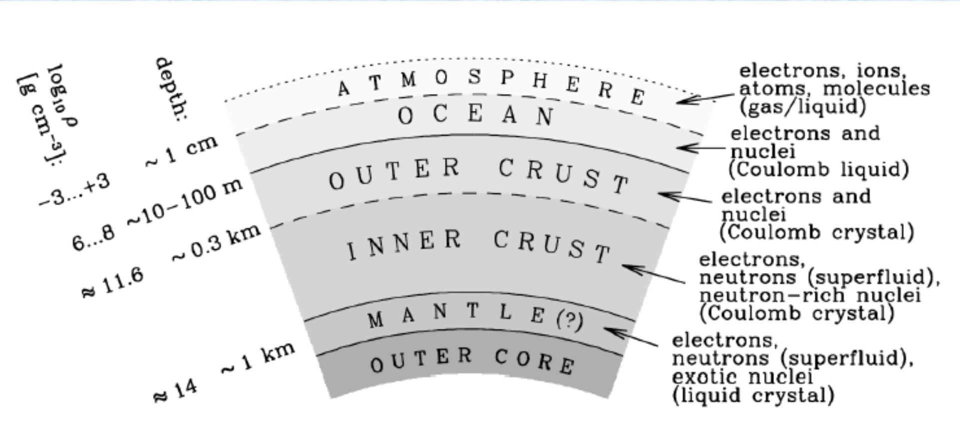
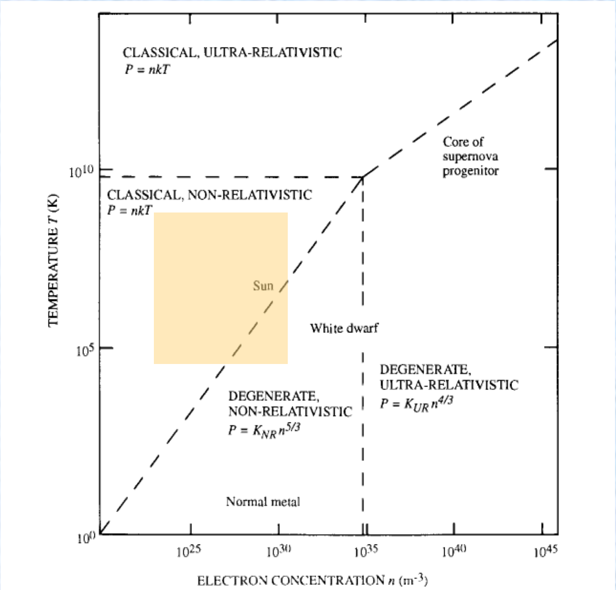


Image Credit: 3G Science White Paper



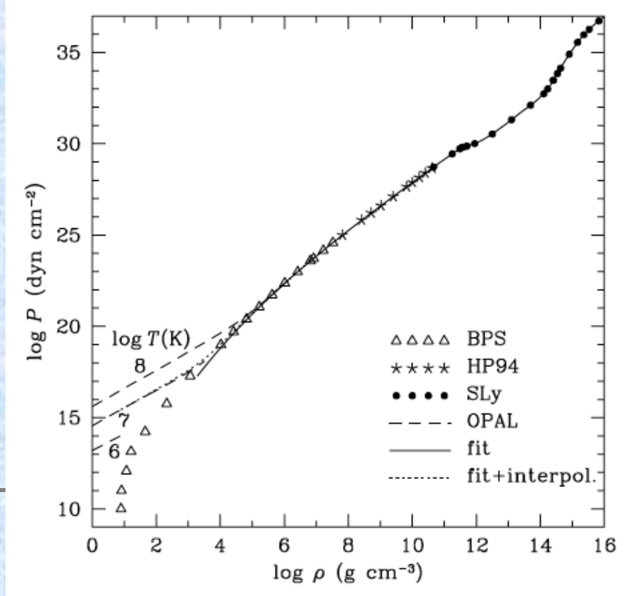
Haensel et al., Neutron Star 1 (Springer 2007)



Phillips, "The Physics of Stars"

- **Atmophsere**: (few cm) of light to intermediate-mass elements (e.g. H, He, C), e^- + ions + atoms + molecules; gas/liquid
  - Emission of thermal radiation → T_{surf}, surface gravity, etc.
  - Very low density (ρ < ~ 1 – 100 g/cm³) → EoS ideal gas $P = nRT$

<u>NB</u>: below ~ 10^6 g/cm³ EoS is T-dependent (→ T=0 approx not valid)



Haensel et al., Neutron Star 1 (Springer 2007)

Haensel et al., Neutron Star Structure 1 (Springer 2007); Nattila et al., ApJ 971, 37 (2024)

NS ocean

atmosphere + ocean

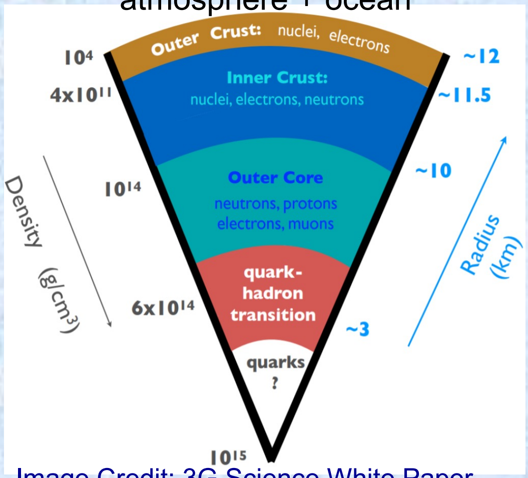
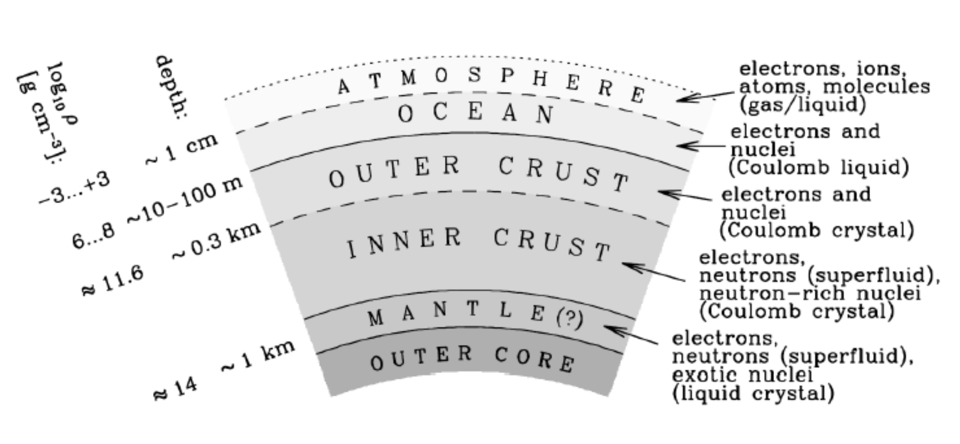
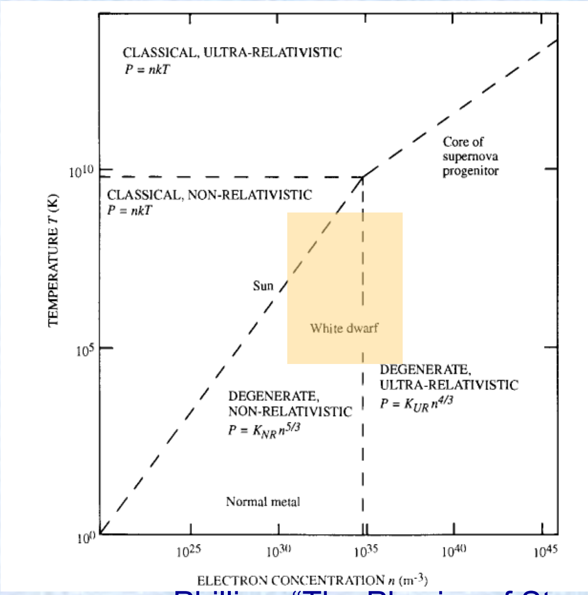


Image Credit: 3G Science White Paper



Haensel et al., Neutron Star 1 (Springer 2007)

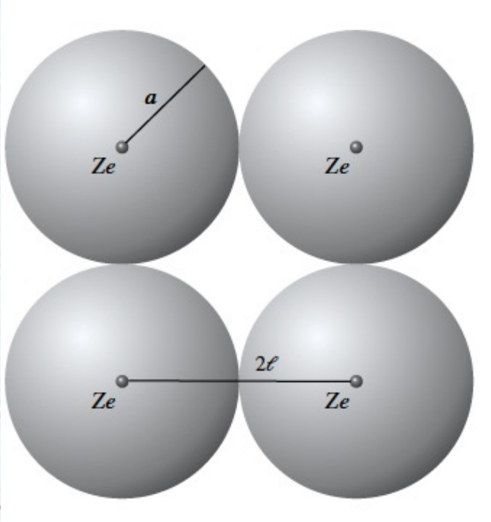


Phillips, “The Physics of Stars"

- **Ocean**: a few 10 m, ions + electrons
  - Expected to be made by Fe nuclei (at least at the bottom) + electrons
  - Low density (ρ < ~ 10^8 g/cm^3) → EoS dominated by degenerate e^- gas
  - Atoms completely ionised! When this occurs?
    → inter-atom spacing ~ atom radius (found from E_{kin} + E_{coul} = 0 when atom bound)

$$l = \left(\frac{4\pi n_N}{3}\right)^{-1/3} \approx a = \left(\frac{3\pi^2}{16}\right)^{1/3} \frac{a_0}{Z^{1/3}}$$

$$\rightarrow \rho_B \gg \rho_i = m_N n_N \approx A m_p n_N \approx 1.4\ AZ \approx 10^3 \text{ g cm}^{-3}$$



N. Chamel (2026)

Haensel et al., Neutron Star Structure 1 (Springer 2007); Nattila et al., ApJ 971, 37 (2024)

NS crust: general picture

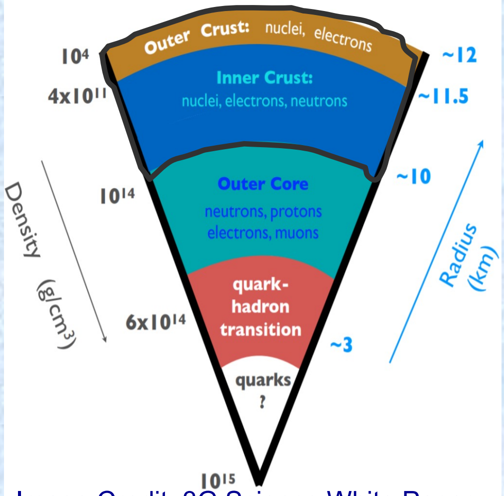
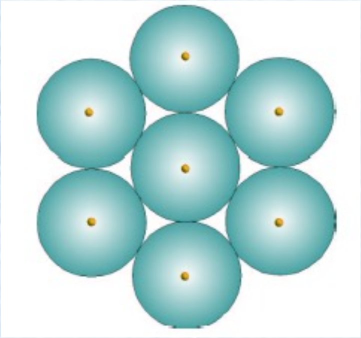
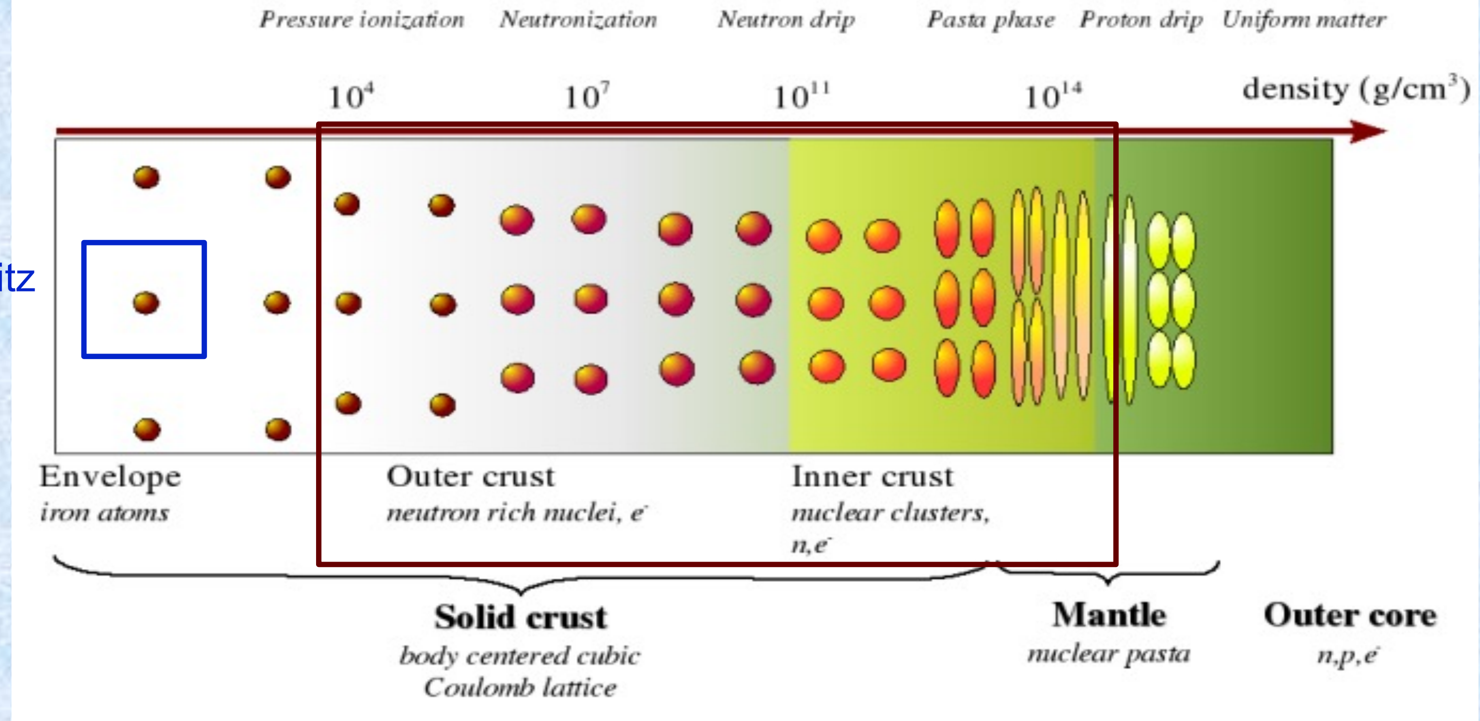


Image Credit: 3G Science White Paper





Chamel & Haensel, Liv. Rev. Relativ. 11, 10 (2008); see also Blaschke & Chamel, ASSL 457, 337 (Springer, 2018)

Haensel et al., Neutron Star Structure 1 (Springer 2007)

- To obtain the EoS → **minimisation ε (n_B)** = E_{WS}/V_{WS} = min
- One nuclear species at each density point → **one component or single nucleus approx.**
- At T = 0 → bcc lattice

see also Haensel et al., Neutron Star Structure 1 (Springer 2007)

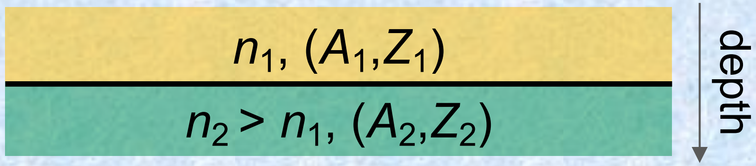
NS crust: matter neutronisation

- **With increasing density, matter becomes more neutron rich**

$$\mu_n - \mu_p = \mu_e$$ → electron Fermi energy changes with n_B

- charge neutrality: $n_e = n_p$

- Le Chatelier-Braun principle: $n_2 > n_1 \rightarrow \frac{Z_2}{A_2} > \frac{Z_1}{A_1}$



→ β eq: n/p ratio adjust → matter gets more n rich

- When does this occur? Ignoring (Coulomb) interaction and ν mass:

$$M(A,Z)c^2 + \mu_e > M(A,Z-1)c^2$$

$$\text{if } M(A,Z) \approx (A-Z)m_n + Zm_p \rightarrow \mu_e > (m_n - m_p)c^2$$

$$\text{if UR } e^- \rightarrow \mu_e \approx \hbar c(3\pi^2 n_e)^{1/3}$$

$$\rightarrow \rho_\beta = \frac{1}{3\pi^2}\frac{A}{Z}m_p\left(\frac{(m_n-m_p)c^2}{\hbar c}\right)^3 \sim 10^7 \frac{A}{Z} \text{ g cm}^{-3}$$ ⟶ first transition in NS crust

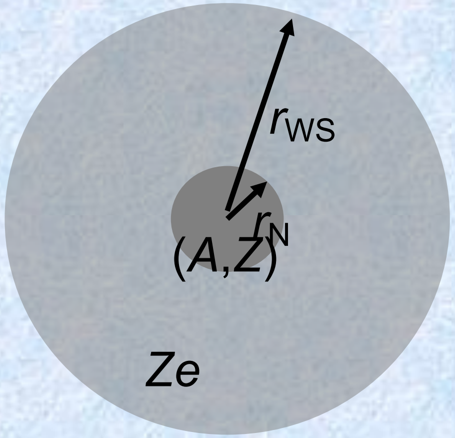
Z	N	A	$\bar{n}_{\min}$	$\bar{n}_{\max}$
26	30	56	–	4.93×10^{-9}
28	34	62	5.08×10^{-9}	1.63×10^{-7}

Pearson et al., MNRAS 481, 2994 (2018)

Chamel & Haensel, Liv. Rev. Relativ. 11, 10 (2008); Blaschke & Chamel, ASSL 457, 337 (Springer, 2018)

NS crust: Coulomb interaction

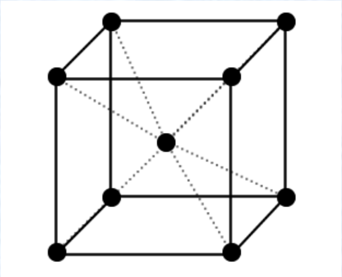
- **Gauss theorem in WS cell**



$$E_{\rm Coul} = \frac{3}{5}\frac{1}{4\pi\epsilon_0}\frac{(Ze)^2}{r_N}\left[1 + \frac{1}{2}\left(\frac{r_N}{r_{\rm WS}}\right)^3 - \frac{3}{2}\frac{r_N}{r_{\rm WS}}\right]$$

nucleus (sphere)
(Gaussian units $1/(4\pi\varepsilon_0)=1$)

Lattice energy (e-e, e-N): $$E_{\rm L} = -\frac{9}{10}\frac{(Ze)^2}{r_{\rm WS}}$$



for a bcc: ~ 0.896 ~ 9/10

Lattice pressure: $P_{\rm L} = \frac{1}{3}E_{\rm L} < 0$ → soften the EoS

Haensel et al., Neutron Star Structure 1 (Springer 2007); Chamel & Haensel, Liv. Rev. Relat. 11, 10 (2008);
Shapiro & Teukolski, “Black holes, white dwarfs, and neutron stars” (Wiley 1983)

NS outer crust

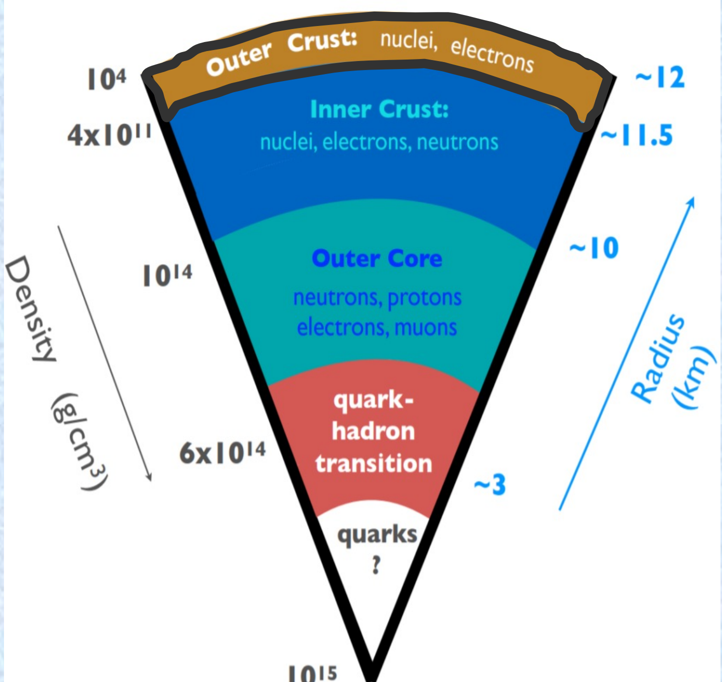
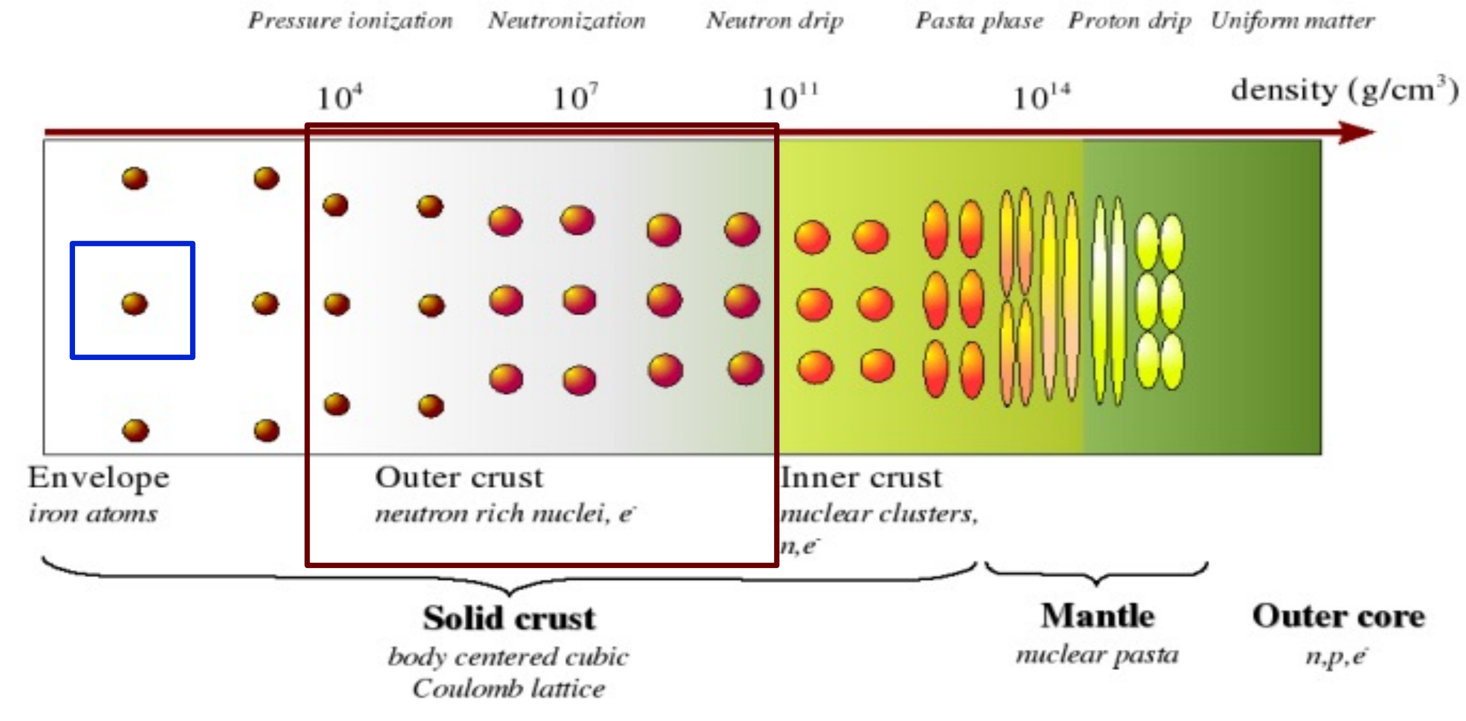


Image Credit: 3G Science White Paper



Chamel & Haensel, Liv. Rev. Relativ. 11, 10 (2008); see also Blaschke & Chamel, ASSL 457, 337 (Springer, 2018)

see also Haensel et al., Neutron Star Structure 1 (Springer 2007)

NS outer crust (1)

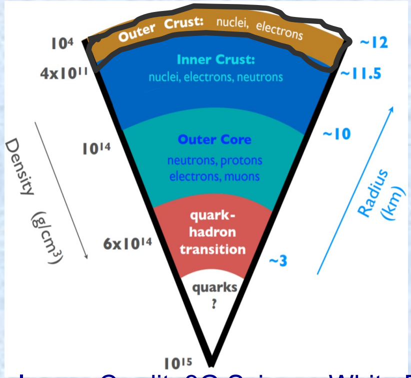
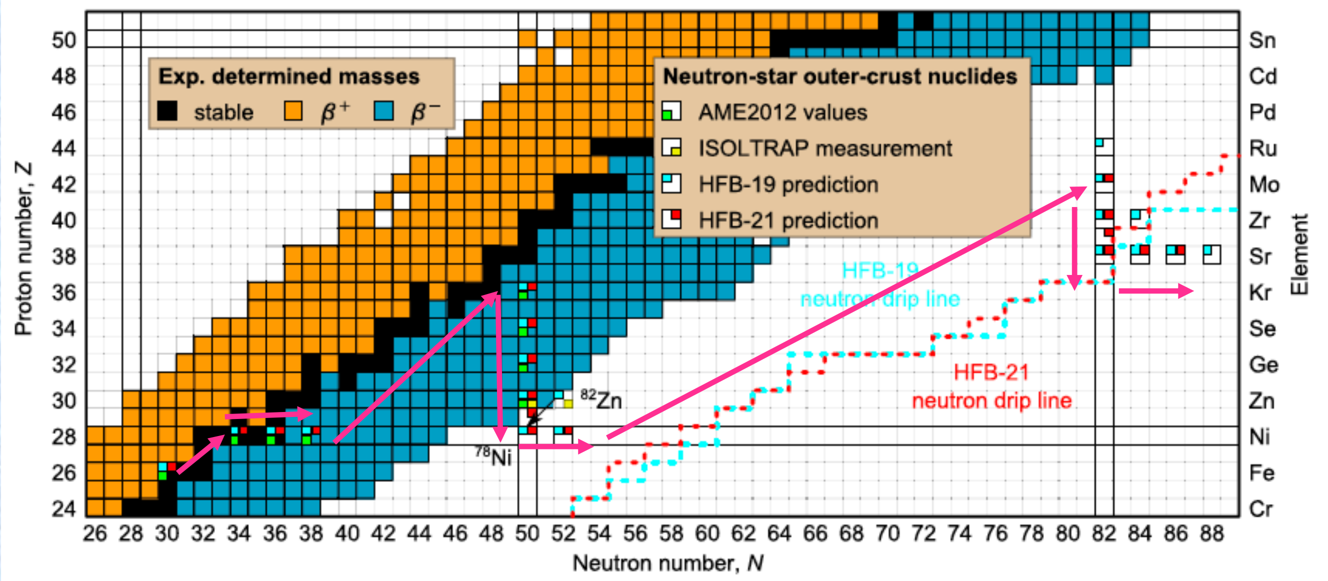


Image Credit: 3G Science White Paper

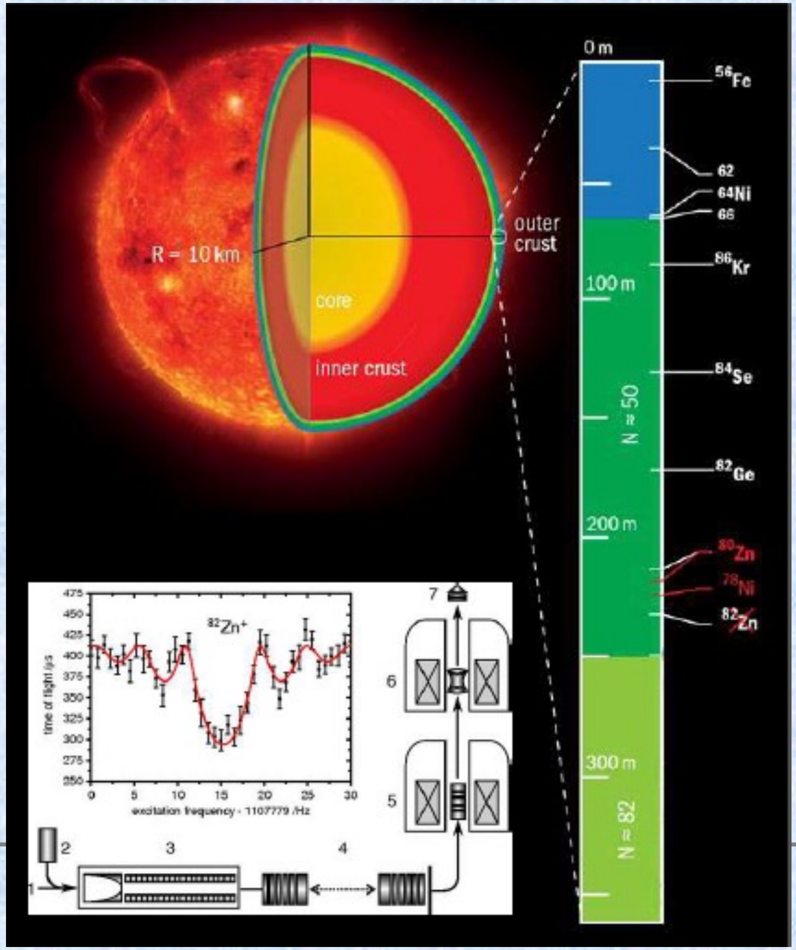
- Nuclei in bcc lattice + electrons: $\mathcal{E}_{\rm WS}(n_B) = \dfrac{M(A,Z)c^2}{V_{\rm WS}} + \mathcal{E}_e + \mathcal{E}_{\rm Coul}$

e-e and e-i int.

Only microscopic inputs are nuclear masses → Experimental or mass models



Wolf *et al.*, PRL 110, 041101 (2013) ; ISOLTRAP collaboration at CERN (^{82}Zn)



←1971

←2013

https://cerncourier.com/a/plumbing-the-depths-of-neutron-stars/

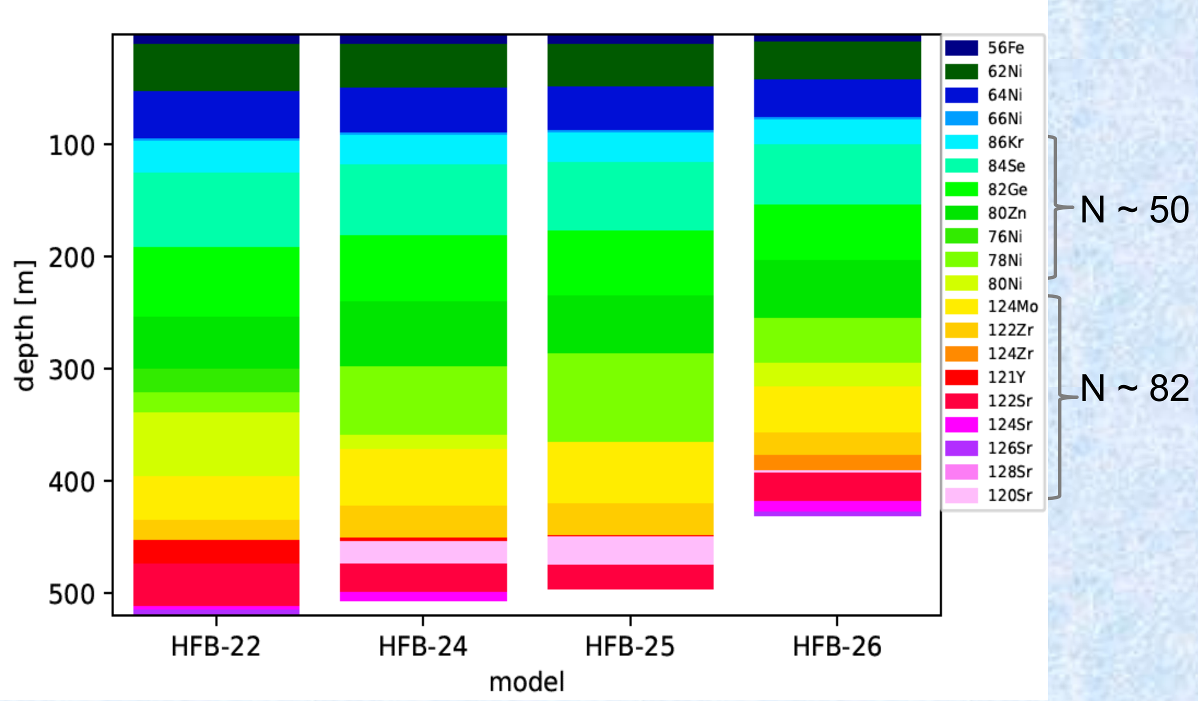
Pioneering works: Baym et al. (BPS), ApJ 170, 299 (1971); Tondeur, A&A 14, 451 (1971); review: Haensel et al., Neutron Star Structure 1 (Springer 2007); Chamel & Haensel, Liv. Rev. Relat. 11, 10 (2008); Burgio & Fantina, ASSL 457 (2018)

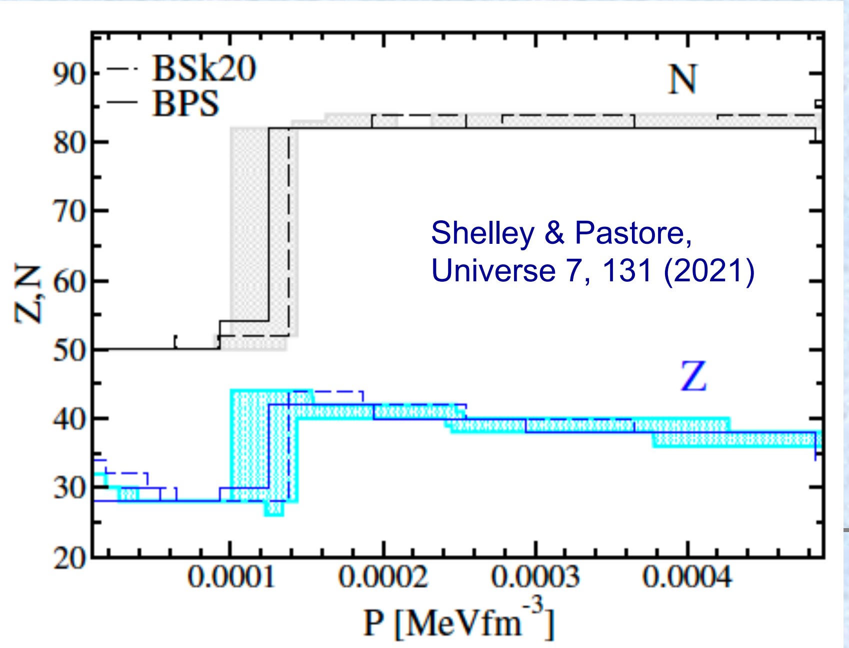
NS outer crust (2)

- Experimental info on masses → model independent results
- Beyond → nuclear mass models → model dependence

- Imprint of magic nuclei (N ~ 50, 82)
  (less on Z because of weak equilibrium
  and charge neutrality, few layers with Z=28)

- Machine learning used to fit nuclear mass data
  (e.g. Utama et al., PRC 93, 014311 (2016); Shelley & Pastore, Universe 7, 131 (2021))
  <u>but</u>: extrapolations, applicability for inner crust and core ?





Shelley & Pastore, Universe 7, 131 (2021)

Neutron drip

- At some point, nuclei become so neutron rich → n drip out of nuclei → **neutron drip**
- In nuclear physics, neutron dripline → lightest isotope such that

$$M'(A,Z) - M'(A-1,Z) > m_n$$

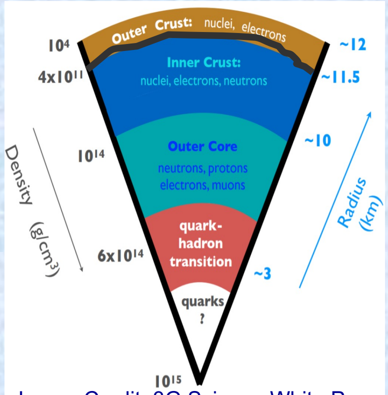
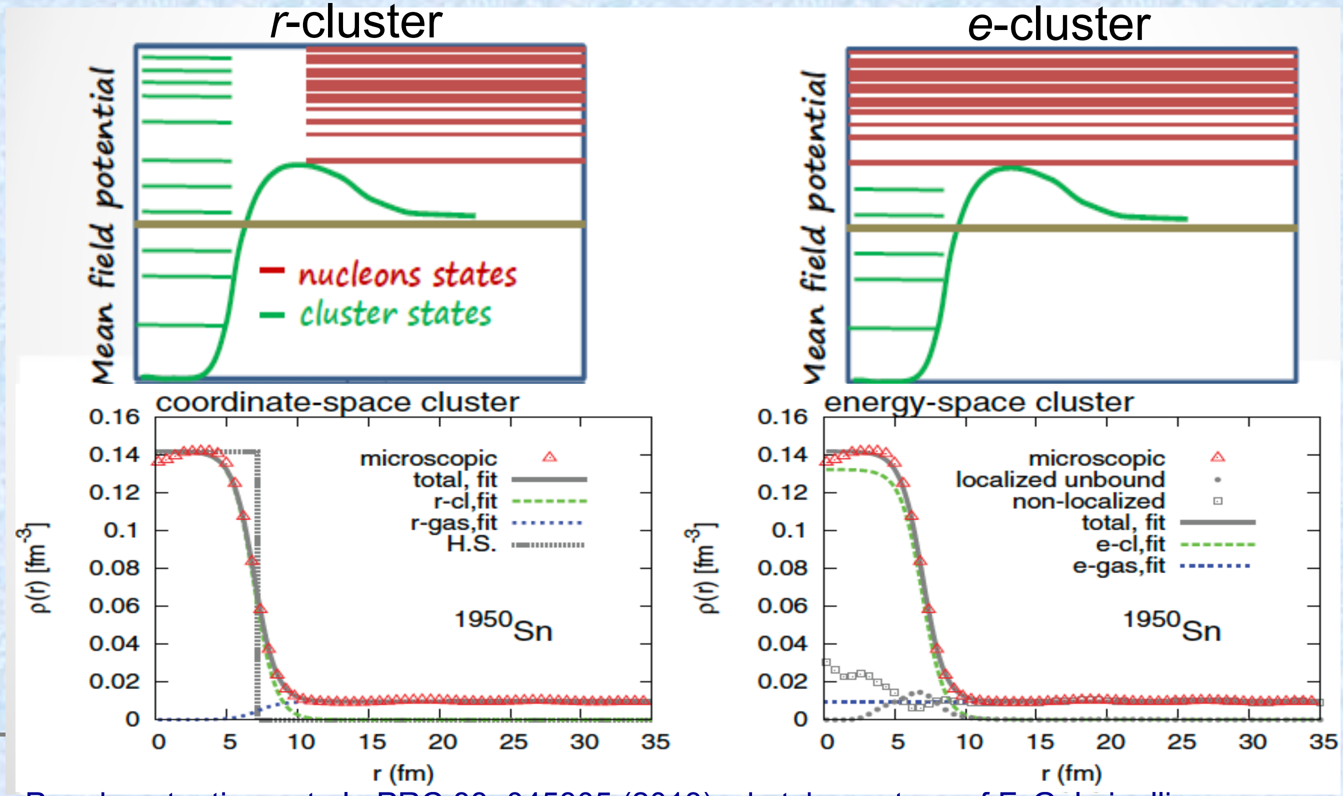


Image Credit: 3G Science White Paper

- But: nuclei (with structure effects) in a medium (background gas, lattice effects) → **dripline shifted** !
- Drip → neutrons occupy delocalised continuum states

NB: - two representations can be mapped
- neutron superfluid fraction (i.e. those not co-moving with lattice) important for e.g. glitches, elastic properties, …
→ "free" neutrons in e-cluster representation (Almirante & Urban, PRL 2025)



Papakonstantinou et al., PRC 88, 045805 (2013); sketch courtesy of F. Gulminelli

see e.g. Haensel et al., "Neutron Star 1" (Springer 2007), Chamel et al., PRC 91, 055803 (2015)