Basic concepts in Nuclear physics

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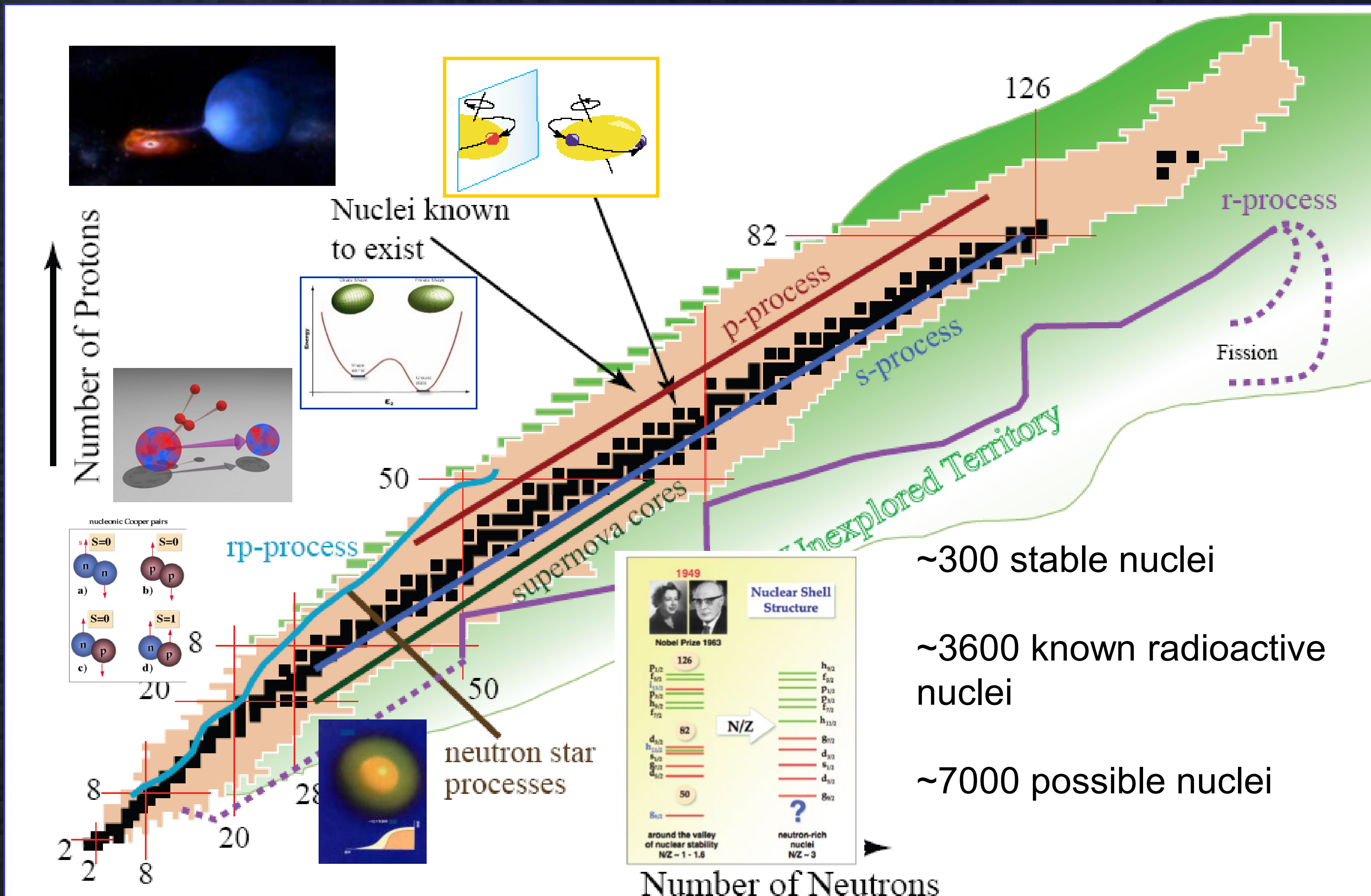
New perspectives in Nuclear Physics

These are exciting times in Nuclear Physics:

Existing and planned accelerator facilities worldwide, and new detector systems with increased sensitivity and resolving power not only will allow us to answer some important questions we have today, but most likely will open up a window to new and unexpected phenomena.

New developments in theory and computer power are shaping a path to a predictive theory of nuclei and reactions.

Nuclear Physics
The new frontier: exotic nuclei



~300 stable nuclei

~3600 known radioactive nuclei

~7000 possible nuclei

Some fundamental questions

- How does the nuclear chart emerge from the underlying interactions?
- How do regular patterns and collective behaviour emerge in complex nuclei?
- How do the underlying symmetries manifest in collective motion or single-particle behaviour and how these modes compete?
- How does the nuclear force depend on isospin?
- What are the limits of existence for bound nuclei?
- Which are the properties of exotic nuclei at the limits of binding?
- What’s new? collective motion, shapes, decay modes?
- How does the shell structure evolve across the nuclear landscape?
- Is it possible to achieve a unified description of all nuclei?

Nuclear Structure

At the low excitation-energy regime, nuclear states can be resolved.

Thus, quantum effects are predominant: the nucleus is an optimal laboratory, a quantum system formed by two different types of fermions that strongly interact.

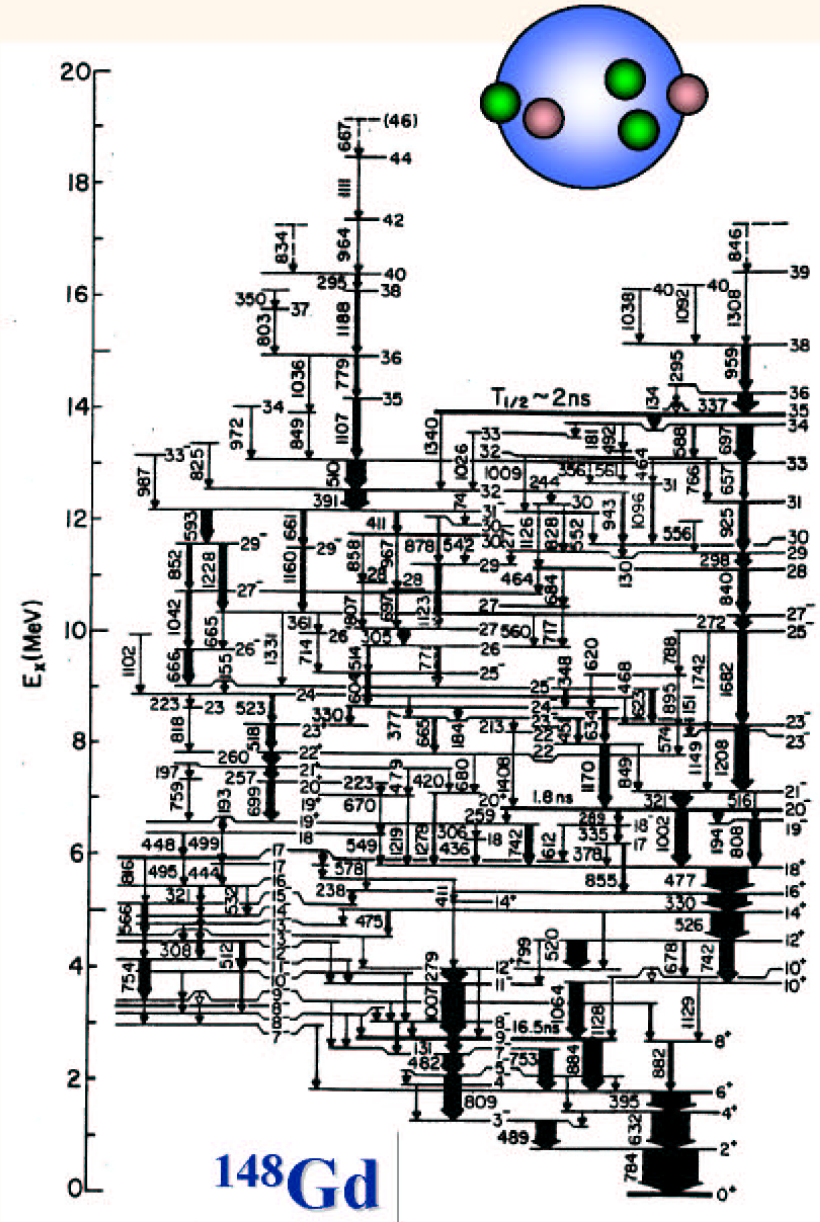
At high excitation energy, the level density increases rapidly and the dynamics is governed by statistical mechanics.

Theoretical models are able to describe only part of the excited states that are observed: the rich phenomena of nuclei arises from the number and type of its constituents and the interaction among them.

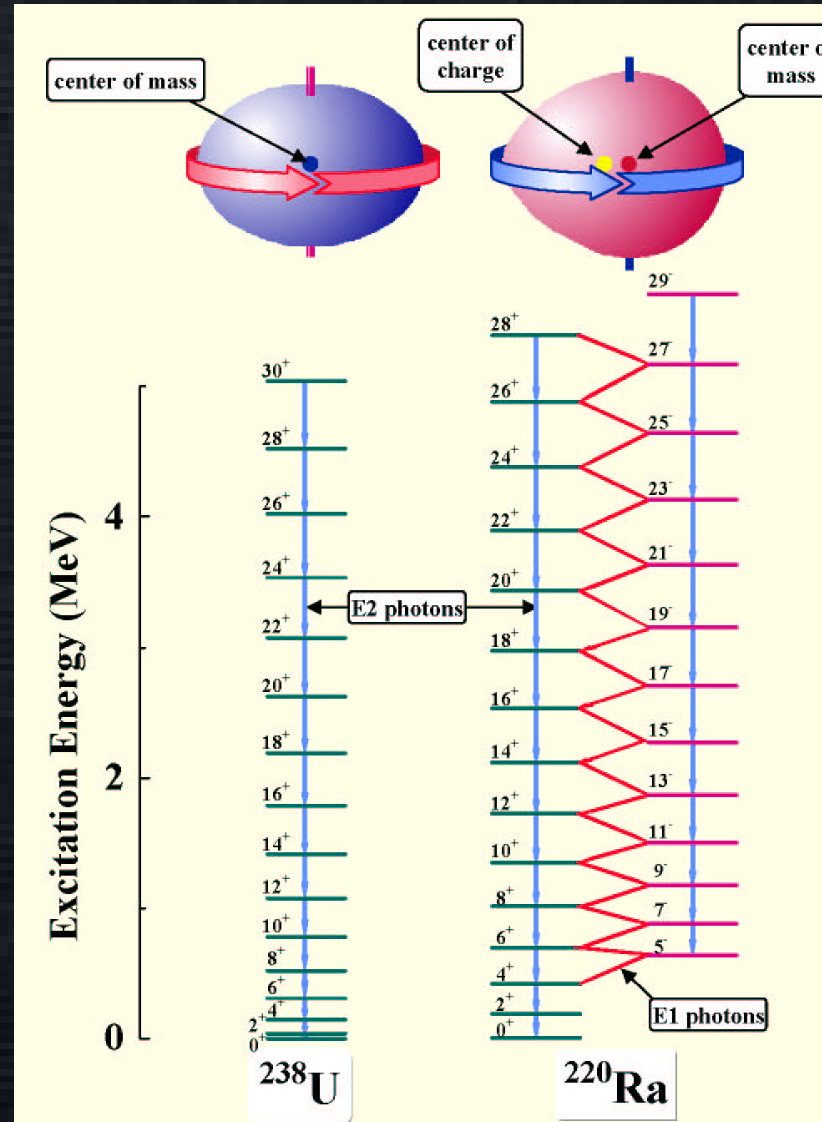
The number of nucleons is large enough to prevent an exact microscopic calculation but small enough to allow a statistical treatment.

We have to deal with macroscopic and microscopic models.

Noncollective excitations



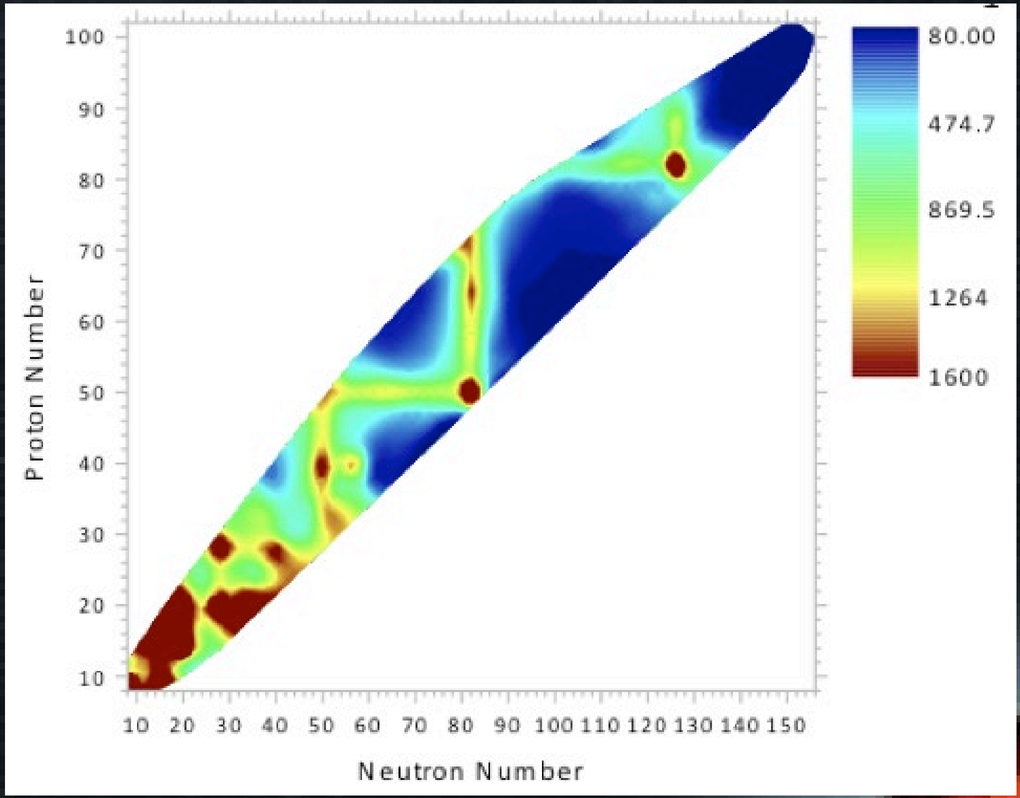
Collective excitations



Regular patterns

Nuclear excitations often show regular patterns.
They indicate a reduction of the number of degrees of freedom.
Abrupt changes indicate phase transitions or the emergence of new phenomena.

First 2^+ state excitation energies (in keV) for even-even nuclei



The Nucleon-Nucleon Force

The strong interaction in the nucleus

In the structure of nuclei, the full complexity of the strong interaction does not usually come into play, owing to the fact that nuclei are relatively weakly bound systems.

The energy required to remove a nucleon from a nucleus S_n ~ 5-10 MeV
Average kinetic energy of the nucleons in the nucleus E_k ~ 25 MeV.

These energies are small compared with the rest energies not only of the nucleons themselves (Mc^2 ~ 1000 MeV), but also of the lightest of the hadrons, the $\boldsymbol{\pi}$ mesons ($m_\pi c^2$ ~140 MeV).

In the analysis of nuclear bound states and reactions at not too high energies, it is therefore a good first approximation to regard the nucleus as composed of a definite number of nucleons moving with nonrelativistic velocities (v/c ~ 0.1).

The virtual presence of other particles may then be approximately taken into account in terms of forces acting between the nucleons.

Range of the Nuclear Force

The range of an interaction is related to the mass of the exchanged particle

The Heisenberg Uncertainty Principle establishes: $\Delta E \, \Delta t \approx \hbar$

A particle can only create another particle of mass m for a time $t \approx \hbar/mc^2$ during which interval the particle can travel at most $x = ct$

Taking ct as an estimate of the range R, gives: $R \approx \hbar/mc$

This yields a range $R \approx 1.4$ fm for pion exchange

The Pauli principle precludes the scattering of pairs of particles toward occupied orbits -> single-particle motion with a long mean free path of ~ 7 fm

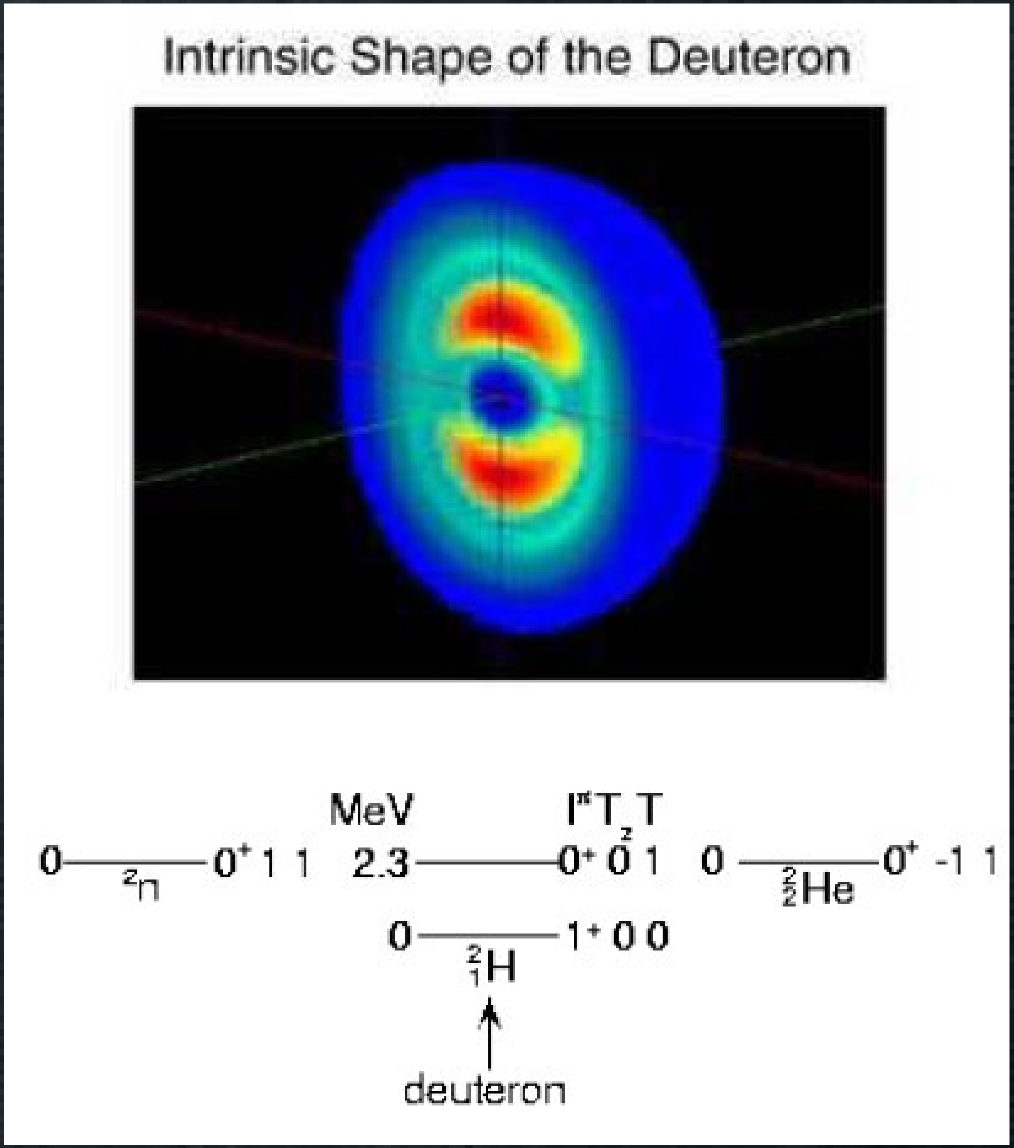
How can we study the nuclear force?

Two main methods:

1. in a system where two nucleons are bound together (in analogy to the levels of the hydrogen atom from which the Coulomb potential can be deduced)

2. in scattering processes between nucleons: (in analogy to Rutherford scattering for charged particles that can tell us about the Coulomb potential)

The Deuteron

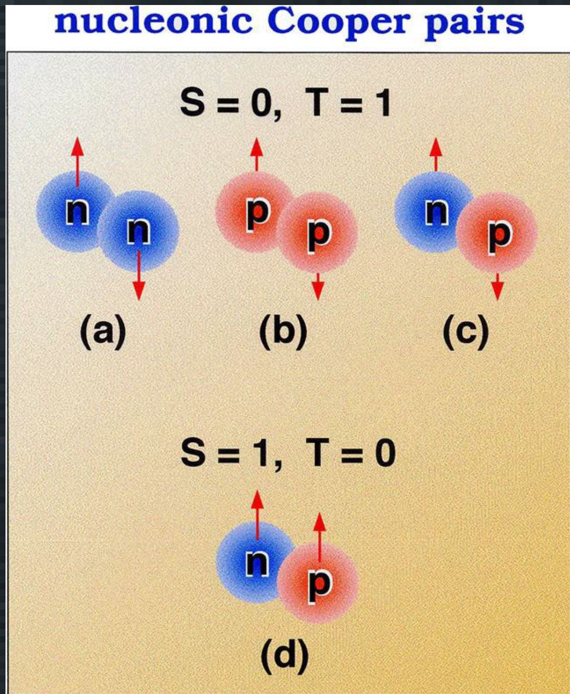


The deuteron consists of a bound proton-neutron system

Its ground-state is the only state which is bound, at -2.23 MeV. The first excited state is unbound

The ground state has spin and parity $I^{\pi} = 1^+$

Spin dependence of the force



The spin of the bound state of the deuteron is J=1.
The lowest energy state is then,

$$L = 0$$

$$\vec{J} = \vec{L} + \vec{S} = 1$$
$$\Rightarrow \vec{S} = 1$$

The state with S=0 is higher in energy and is unbound by only 60 keV!

We can therefore conclude that the force between a proton and a neutron is SPIN DEPENDENT

Non central potential

The magnetic moment of the deuteron (measured) is $\mu_d = 0.8574 \frac{e\hbar}{2M_p}$

Since it is in an L=0 state, then only the spin magnetic moments should be present: $\mu_{s,\pi} + \mu_{s,\nu} = 0.8797 \frac{e\hbar}{2M_p}$

The difference suggests that there is some orbital current (L≠0).

The measured electric quadrupole moment is: $Q_d = 0.282 efm^2$

The wave function of the deuteron is then $\Psi = a_0\varphi_{1s} + a_2\varphi_{1d}$ with $|a_0|^2 = 0.96, |a_2|^2 = 0.04$

L≠1 for parity constrains

$S = 1$ in both cases

This implies that the (orbital) angular momentum is not conserved!

Nucleon-nucleon scattering

To characterize the interaction between two particles we study the scattering between them

Cross section

$$\Psi_{\text{scat}}(\mathbf{r}) \overset{\mathbf{r}\to\infty}{\longrightarrow} e^{ikz} + f(\theta,\varphi)\frac{e^{ikr}}{r}$$

$$\sigma_k(\theta,\varphi) = |f(\theta,\varphi)|^2$$

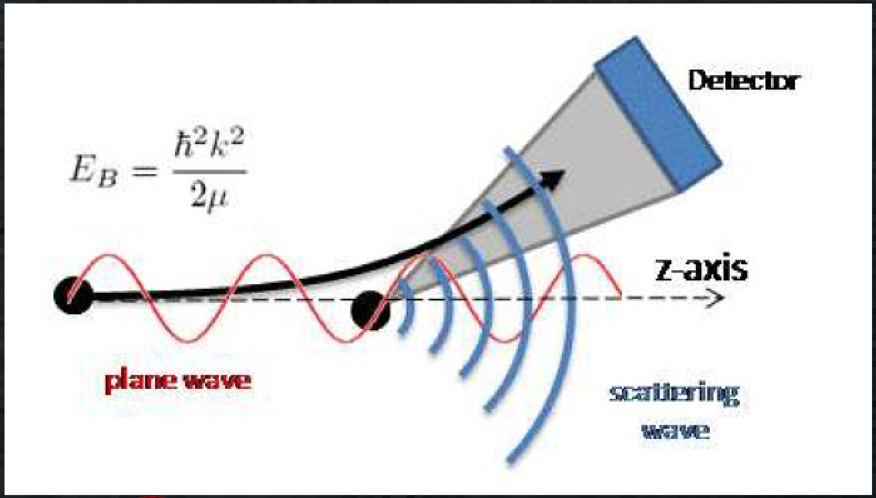
Taking into account the spherical symmetry of the problem

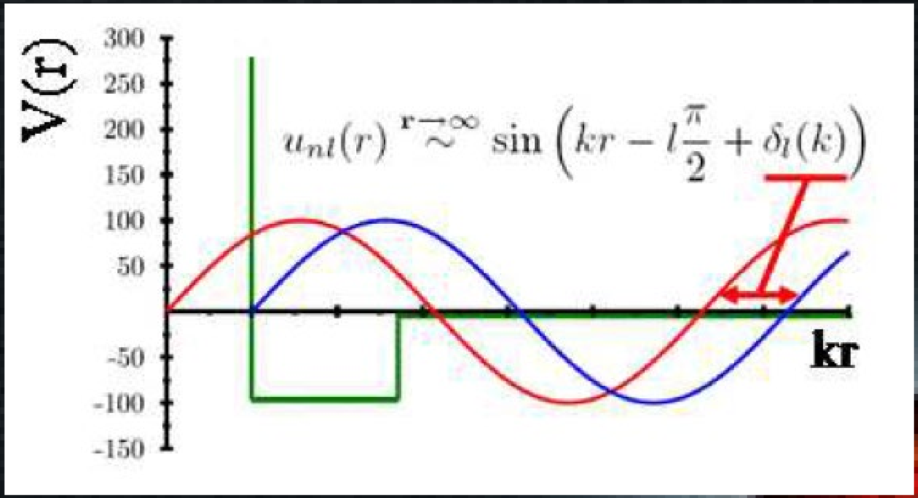
$$\Psi_{\text{scat}}(\mathbf{r}) = \sum_{nlm} c_{nlm}\frac{u_{nl}(r)}{r}Y_{lm}(\theta,\varphi)$$

$$u_{nl}(r) \overset{\mathbf{r}\to\infty}{\sim} \sin\left(kr - l\frac{\pi}{2} + \delta_l(k)\right)$$

and

$$\sigma(k) = \frac{4\pi}{k^2}\sum_l (2l+1)\sin^2[\delta_l(k)]$$





Central term from the N-N data

How can we get the nuclear force from the nucleon-nucleon interaction?

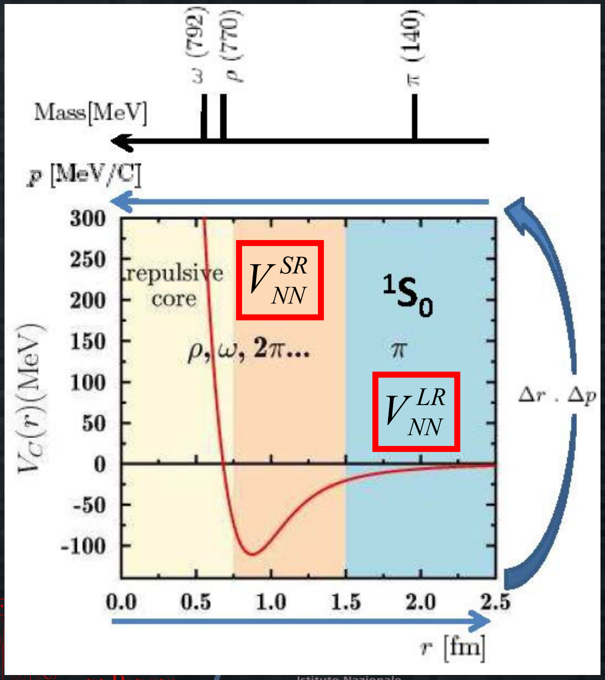
- fit the experimental data (~30 parameters)
- different parametrizations agree in the low-energy (long range) regime, but differ in the high energy part (short range)
- They are not unique.

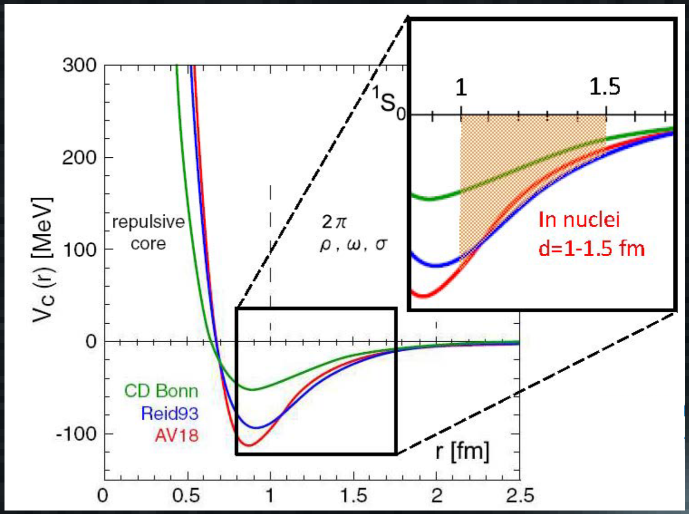
$$\boxed{V_{NN} = V_{NN}^{EM} + V_{NN}^{LR} + V_{NN}^{SR}}$$

$\boxed{V_{NN}^{EM}}$ Electromagnetic

$\boxed{V_{NN}^{LR}}$ Long-range

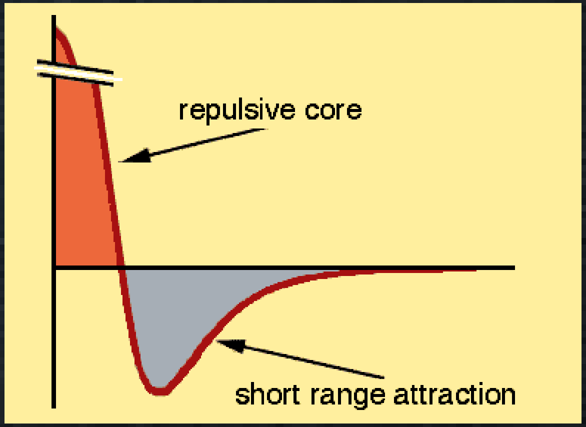
$\boxed{V_{NN}^{SR}}$ Short-range





Properties of the N-N force

- The force is spin dependent
- The force is (nearly) charge symmetric
- The force is (nearly) charge independent
- The force has a non-central component
- The force depends on the relative velocity or momentum of the nucleons
- The force has a repulsive core
- Exchange model: force mediated by pion exchange



Radius of nucleon: ~ 1 fm

Radius of hard core: ~ 0.2 fm

Nucleon mean free path: ~ 7 fm

Volume of the hard core is only ~ 2% of the nuclear volume

Nuclear Models

The Uniqueness of Nuclei

The nucleus is a unique ensemble of strongly interacting fermions: the nucleons

Its large, yet finite, number of constituents controls the physics.

Both single-particle (out-of-phase) and collective (in-phase) excitation modes occur

Single-particle and collective motions

- ❖ Adiabatic approximation: identify fast and slow modes
- ❖ Molecules: electronic motion fastest, vibrations are 10^2 times slower, rotations are 10^6 times slower
- ❖ These different motions have very different time scales, thus the wavefunction separates into a product
- ❖ In nuclei the timescales are much closer
- ❖ Collective and single-particle modes can perhaps be separated but they will interfere!

Some types of Nuclear Excitation

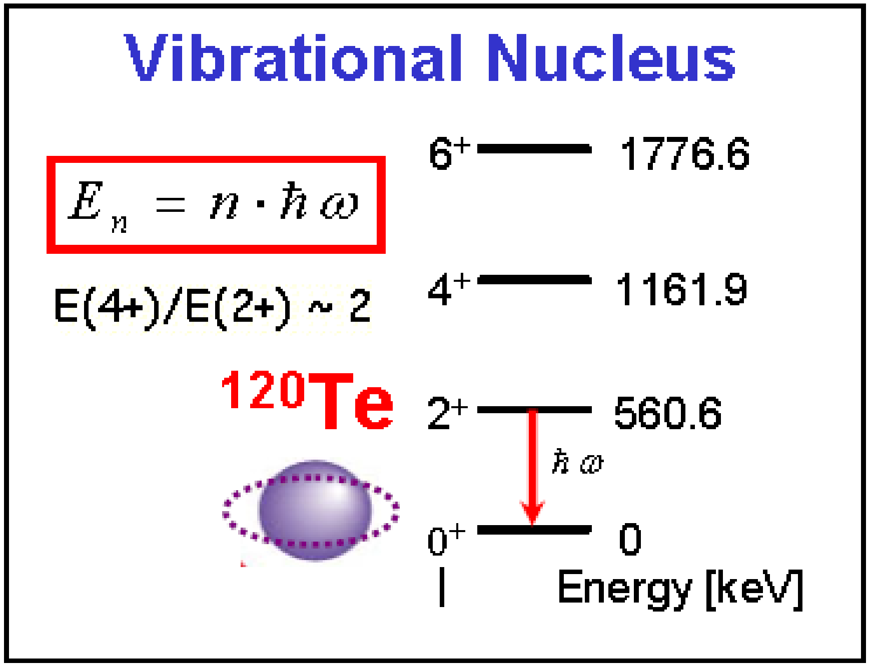
- All even-even nuclei have a ground state with $J^{\pi} = 0^{+}$, a consequence of nuclear pairing.

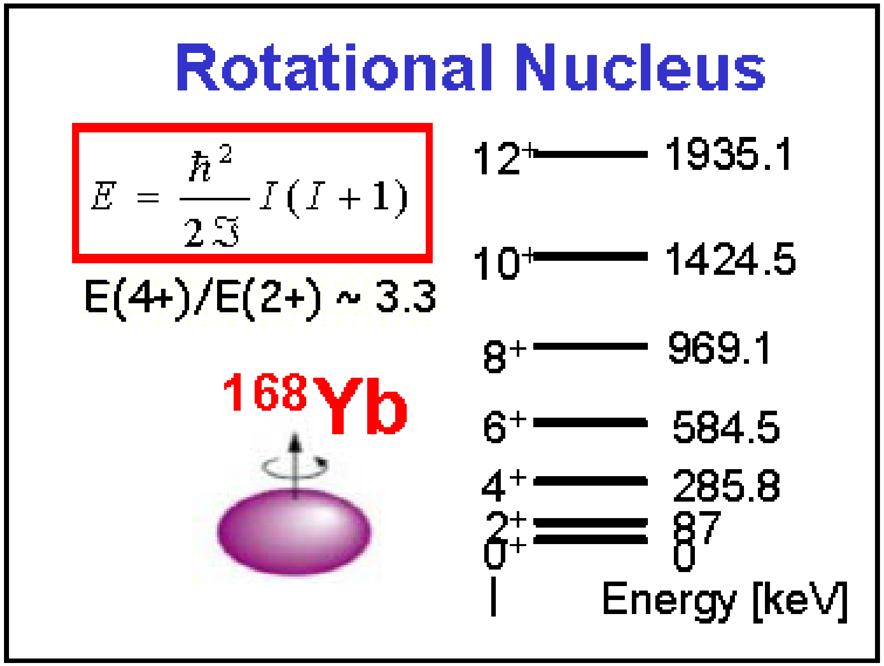
- Closed-shell nuclei are spherical and excited nuclear states can only be formed by breaking pairs of nucleons or by vibrations.

- For odd-mass nuclei (near closed shells) the low-lying excited states map out the single-particle spectrum of states around the Fermi level.

- Deformed nuclei exhibit regular rotational bands: with quadrupole or octupole shapes, etc.

Collective modes





The choice of the model

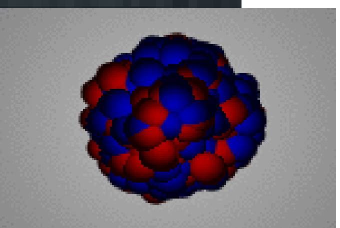
In nuclei, the force is not well known and complicated.

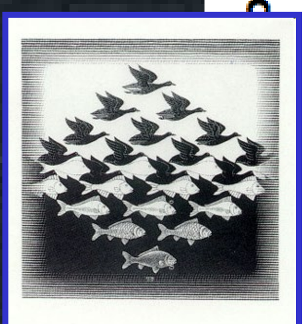
We are dealing with a many-body problem of great complexity.

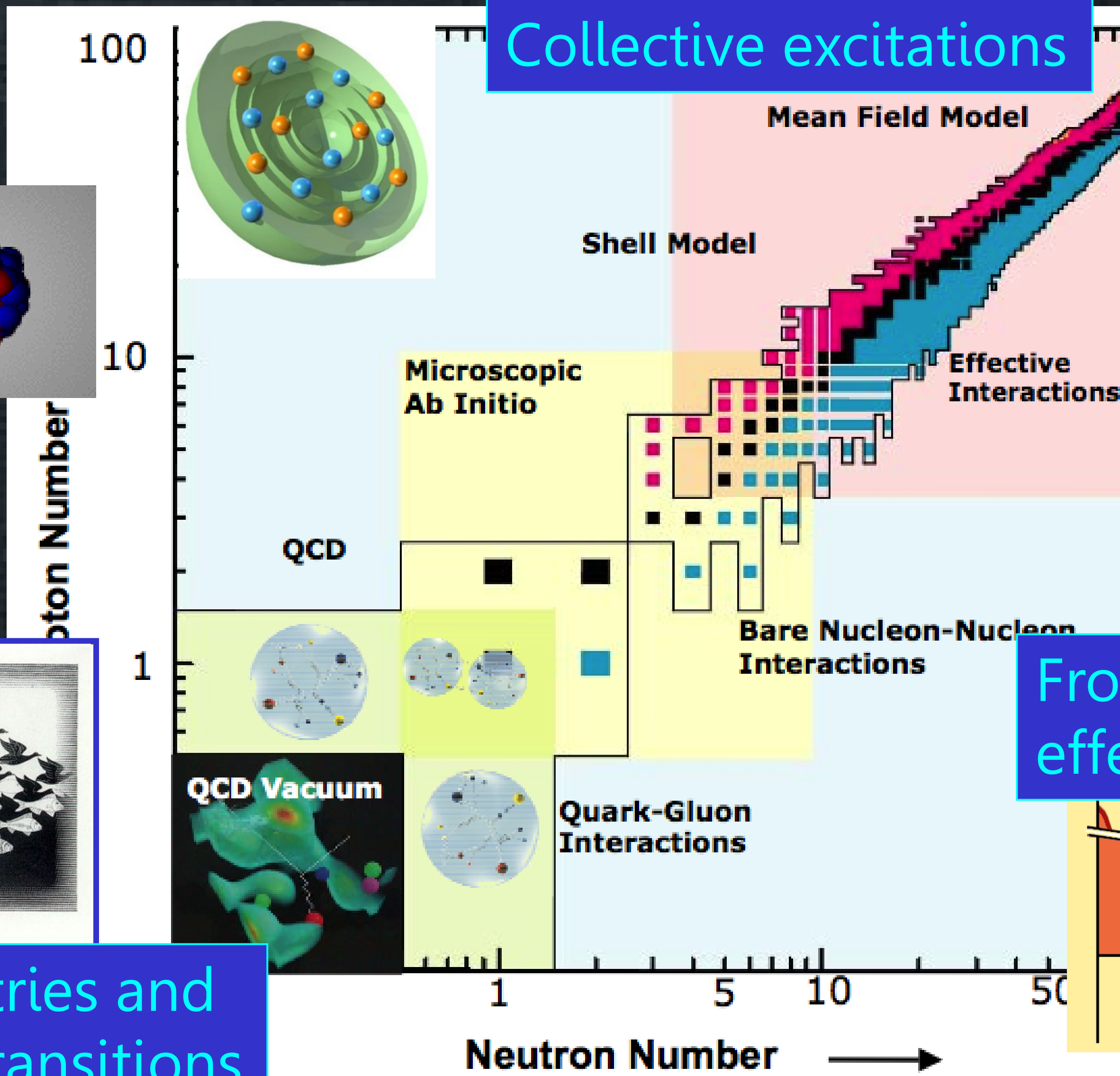
A nuclear model is simply a way of looking at the nucleus that gives a physical insight into a range of its properties as wide as possible.

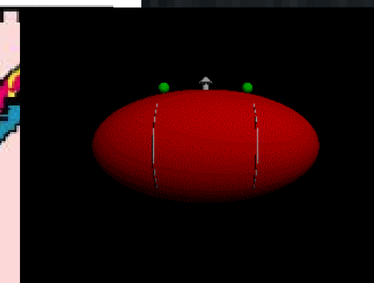
The usefulness of a model is tested by its ability to provide predictions that can be verified experimentally in the laboratory.

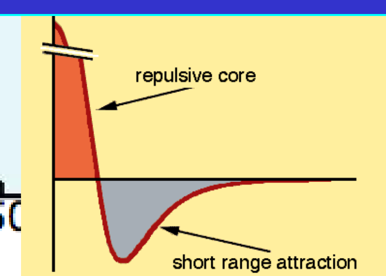
Theoretical models and methods











Collective excitations

From bare NN to effective interactions

Symmetries and phase transitions

Collective models

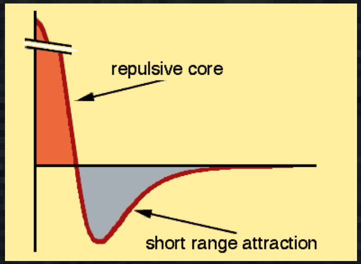
The liquid drop model

Why a liquid drop?

Phenomenological model to understand binding energies. Bethe and Bacher, 1936; von Weizsäcker, 1935

Consider a liquid drop

- Ignore gravity
- Intermolecular force repulsive at short distances, attractive at intermediate distances and negligible at large distances
- ➔ constant density.



Analogy with nucleus

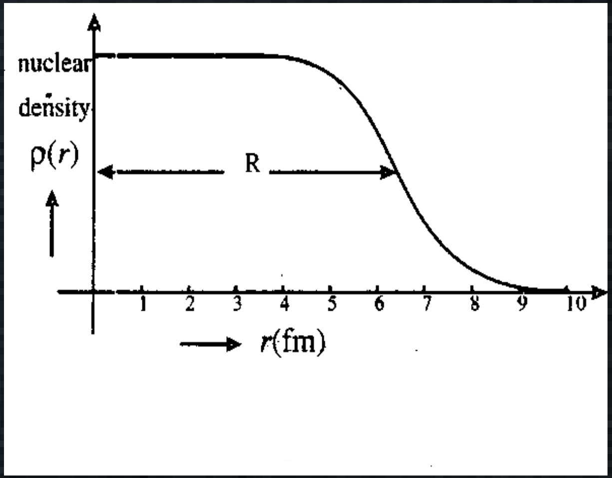
- Nucleus has constant density
- From nucleon-nucleon scattering experiments:
- Nuclear force has short range repulsion and attractive at intermediate distances.
- Assume charge independence of nuclear force, neutrons and protons have the same strong interactions.

However, the mean free path of nucleons is about the nuclear size, while for a liquid drop, collisions are much more frequent.
The average distance between nucleons is 2.4 fm, not that of the minimum of the attractive potential (0.7 fm)

Saturation properties

Nuclear forces exhibit saturation properties.
The binding energy is proportional to A:

$$\boxed{B(N,Z) \propto A}$$



In a non-saturated system the binding energy is proportional to the number of total interactions: $\boxed{A(A-1)/2}$

The nucleus presents low compressibility and therefore a well-defined surface.

$$\boxed{\frac{B(N,Z)}{A} \approx 8\mathrm{MeV}}$$

This is rather independent of A and Z

→ nuclear force is independent of the charge

Semi-empirical mass formula (Bethe-Weiszäcker)

surface　　　　Coulomb

$$B(N,Z) = \boxed{a_v A} - \boxed{a_s A^{2/3}} - a_a \frac{(N-Z)^2}{A} - a_c \frac{Z(Z-1)}{A^{1/3}} - \frac{\delta}{A^{1/2}}$$

volume　　　　symmetry　　　　pairing

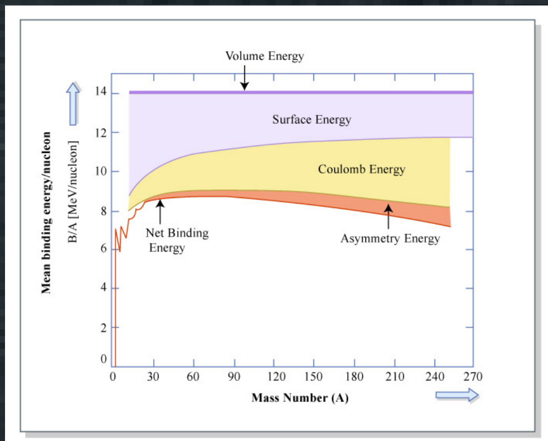
a_s = 17.23 MeV

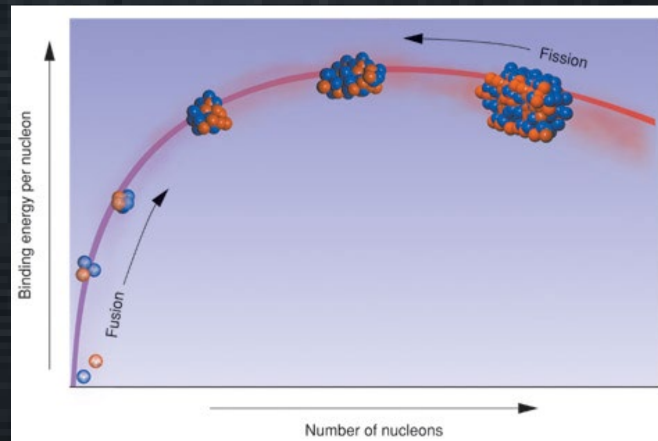
a_c=0.697 MeV

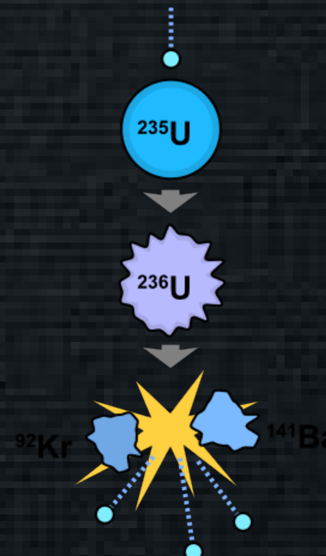
a_v = 15.56 MeV

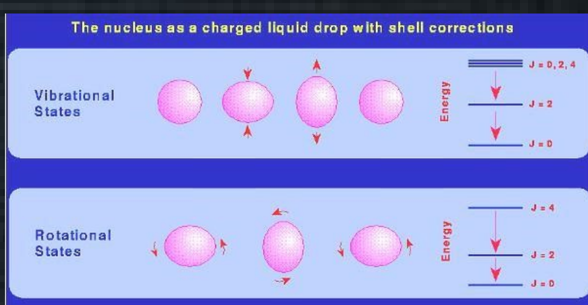
a_a = 23.28 MeV

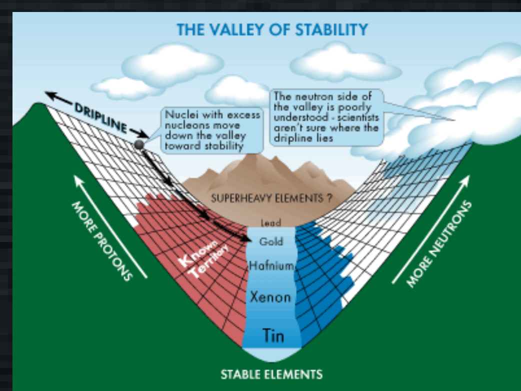
The semiempirical mass formula

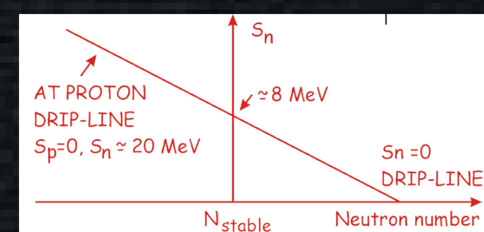








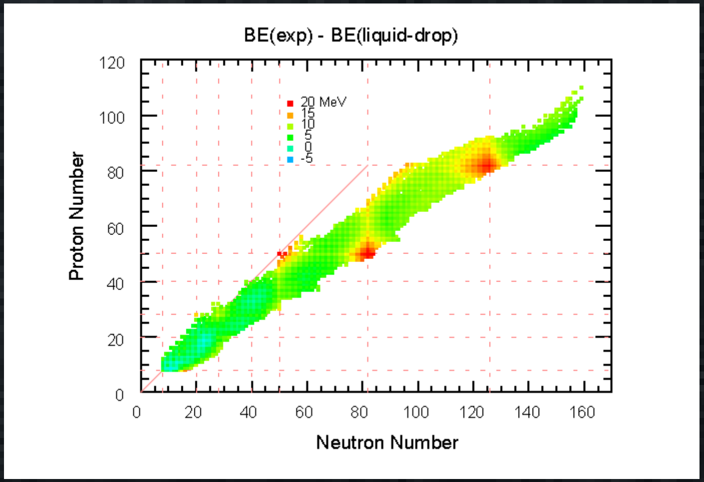




Pros and cons

Pros

•Fit not too bad (good to ~ 1%)
• Deviations are interesting ➔ shell effects
• Coulomb term agrees with calculation
• Asymmetry term ➔ Fermi gas
• Explains the valley of stability
• Explains energetics of radioactive decays, fission and fusion.



Shortcomings

• magic numbers for N and Z
• spin & parity of nuclei do not fit into a drop model
• magnetic moments of nuclei are incompatible with drops
• value of nuclear density is unpredicted
• values of the SEMF coefficients, except Coulomb and Asymmetry, are completely empirical

Nuclear deformation

The nuclear surface

To describe **vibrations of the nuclear surface or deformation**, we consider an expansion of the surface in spherical harmonics with time-dependent shape parameters as coefficients.
The length of the radius vector pointing from the origin to the surface is:

$$\boxed{R(\theta,\varphi,t) = R_0\left(1+\sum_{\lambda=0}^{\infty}\sum_{\mu=-\lambda}^{\lambda}\alpha^*_{\lambda\mu}(t)\,Y_{\lambda\mu}(\theta,\varphi)\right)}$$

Where R_0 is the radius of a sphere with the same volume

The radius has to be a real function:

$$R(\theta,\varphi,t) = R^*(\theta,\varphi,t)$$

$$Y^*_{\lambda\mu}(\theta,\varphi) = (-1)^{\mu}\,Y_{\lambda-\mu}(\theta,\varphi)$$

$$\alpha^*_{\lambda\mu} = (-1)^{\mu}\,\alpha_{\lambda-\mu}$$

The radius has to be a scalar under rotations : $R'(\theta',\varphi') = R(\theta,\varphi)$

Then the $\alpha_{\lambda\mu}$ have to transform as a tensor of rank λ

To hold parity transformations the $\alpha_{\lambda\mu}$ have to change sign like the spherical harmonics: $(-1)^{\lambda}$

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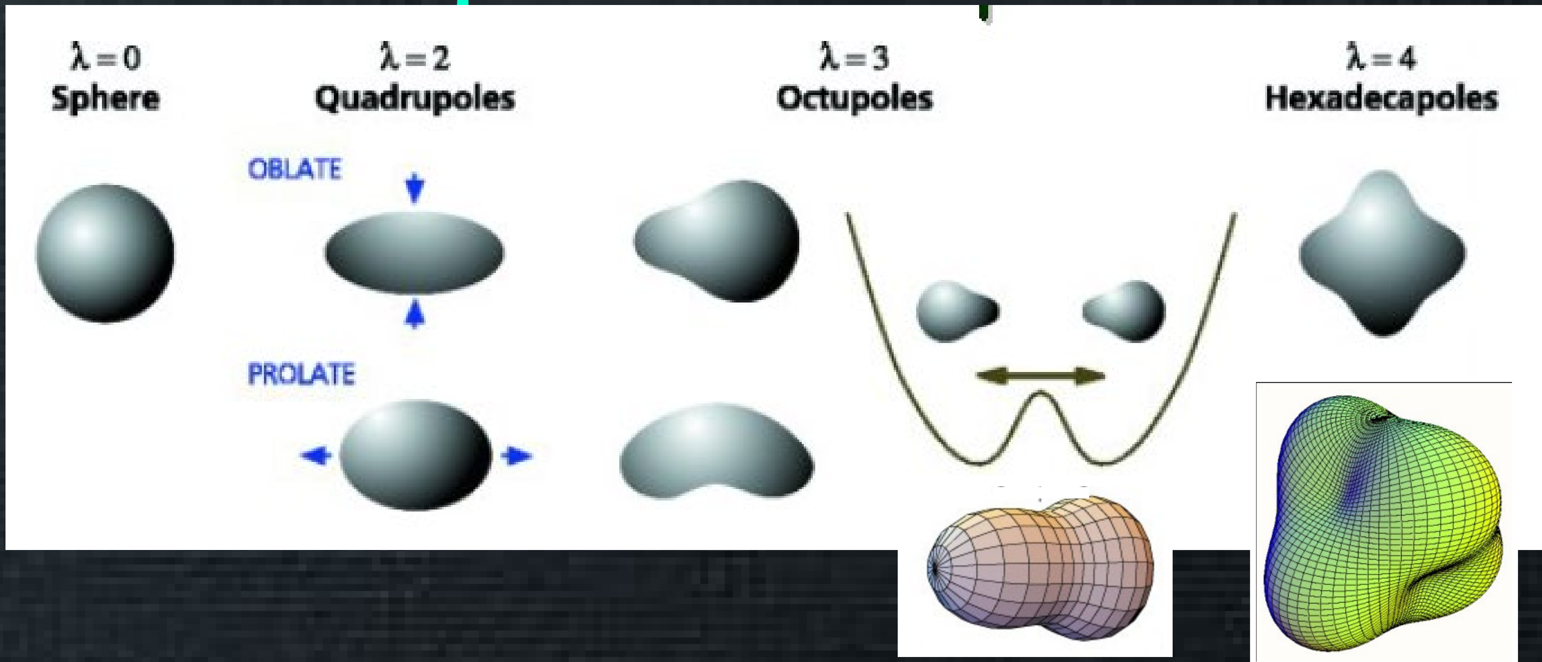
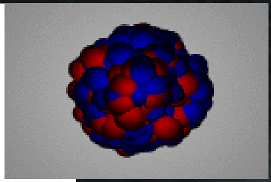
Multipole deformations

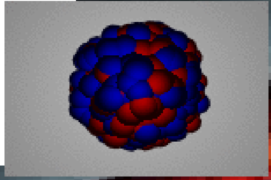


$\boxed{\lambda = 0}$

$$Y_{00}(\theta,\varphi) = \frac{1}{\sqrt{4\pi}}$$

is constant, so a non vanishing α_{00} implies a change of the radius. This excitation is called *breathing mode* and is found at high energy (Monopole Giant Resonance).



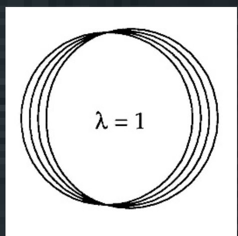


In the following we will consider that there is no change in the density, then it can be shown that (up to second order)

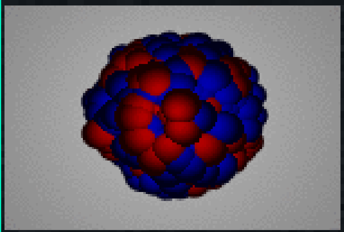
$$\alpha_{00} = -\frac{1}{\sqrt{4\pi}} \sum_{\lambda\mu} \left|\alpha_{\lambda\mu}\right|^2$$

Multipole deformations

$\lambda = 1$

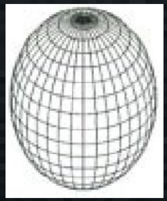


Dipole mode at low energy corresponds to a shift of the centre of mass and therefore violates conservation of momentum. (Centre of mass displacement can't be the result of internal forces).
At high excitation energy it can describe giant dipole resonances where protons oscillate against neutrons



$\lambda = 2$

Quadrupole excitations result to be the most important modes of collective excitations in nuclei

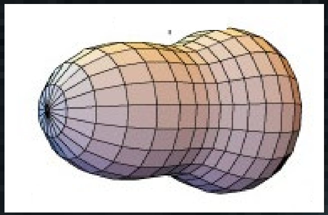


Prolate

Oblate

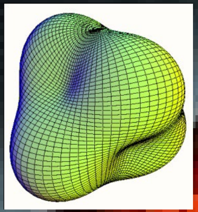
$\lambda = 3$

Octupole modes correspond to asymmetric shapes like pears (μ=0), bananas (μ=1) or peanuts (μ=2,3).



$\lambda = 4$

Pure hexadecapole deformations contribute together with quadrupole modes in many cases.

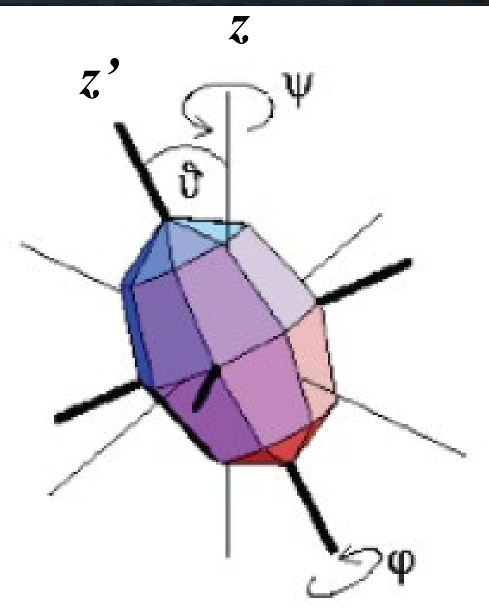


Quadrupole deformation

For the pure quadrupole deformation, the surface is described as:

$$\boxed{R(\theta,\varphi,t) = R_0\left(1 + \sum_{\mu=-2}^{2} \alpha^*_{2\mu}(t)\, Y_{2\mu}(\theta,\varphi)\right)}$$

In the laboratory Cartesian framework



α_{20} describes the stretching of the z axis with respect to x and y

$\alpha_{2\pm2}$ $\alpha_{2\pm1}$ describe complex deformations

The problem with this representation is that in the laboratory frame the deformation and orientation of the nucleus are mixed.
It is therefore better to **separate these degrees of freedom by passing to the intrinsic frame**

In the intrinsic frame: Principal axes

Here we still have 5 independent parameters:

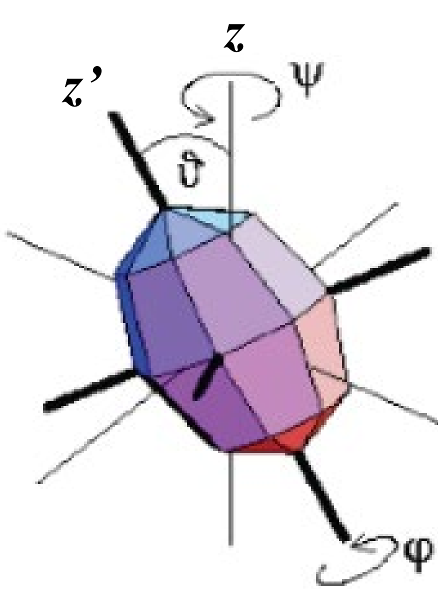
$$\alpha'_{20} \equiv a_0$$
$$\alpha'_{2\pm1} = 0$$
$$\alpha'_{2\pm2} \equiv a_2$$

And the 3 Euler angles: ψ, φ, ϑ. *We call x', y' and z' the coordinates in the intrinsic frame*

But now these parameters have a more clear geometrical interpretation:

a_0 Indicates the stretching of the z′ axis with respect to x′ and y′ axes

a_2 Determines the difference in length between x′ and y′ axes



The three Euler angles determine the orientation of the principal axis of the nucleus with respect to the laboratory frame.

The advantage is that in this way vibration and rotation are separated.

Vibration and deformation are given by a_0 and a_2 while rotations are accounted for by changes of the Euler angles.

The Bohr's β and γ parameters

Bohr introduced two new parameters to describe the deformation,
they are related to the previous ones (always in the intrinsic frame!)

$$a_0 = \beta \cos\gamma \qquad a_2 = \frac{1}{\sqrt{2}} \beta \sin\gamma$$

The 1/sqrt(2) factor has been chosen such that:

$$\sum_{\mu=-2}^{2} \left|\alpha_{2\mu}\right|^2 = \sum_{\mu=-2}^{2} \left|\alpha'_{2\mu}\right|^2 = a_0^2 + 2a_2^2 = \beta^2$$

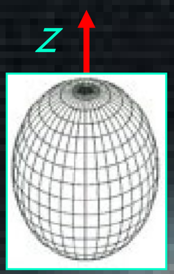
This is rotationally invariant and therefore has the same value in the intrinsic and the laboratory frame

The meaning of these parameters in terms of deformation is:

β Measures the total deformation (deformation parameter)

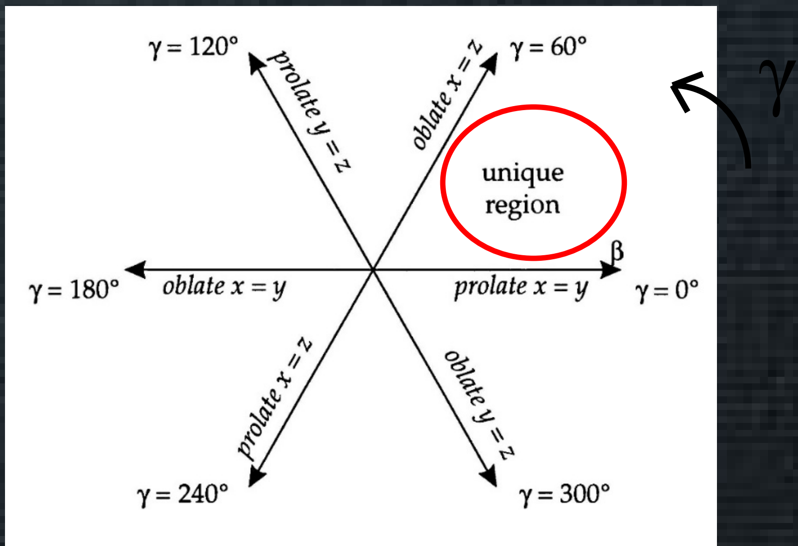
γ Measures the triaxial degree of deformation:

$\gamma = 0$ Means prolate deformation with the *z* axis as the symmetry axis



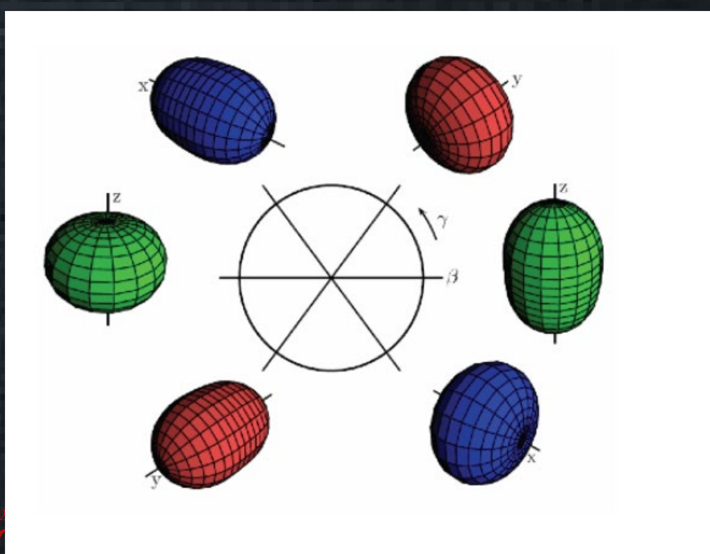
$\gamma = 60^\circ$ Means oblate deformation with the *y* axis as the symmetry axis

Lund convention

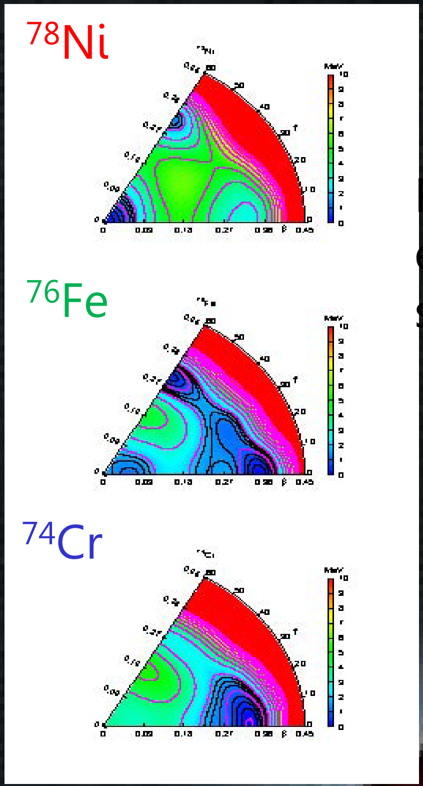


Identical nuclear shapes repeat in the plane, only the naming of the angles changes.

There are six equivalent regions, each of them contains all possible quadrupole deformed shapes.



$\beta = 0$ spherical
$\beta > 0$ prolate
$\beta < 0$ oblate



Projected energy surface

-2 p

-2 p

Description of the surface

In the intrinsic frame, the surface can now be described as:

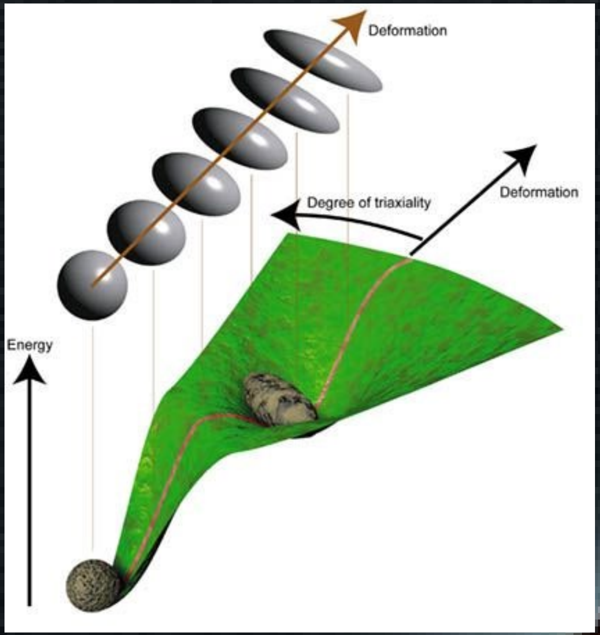
$$R(\theta',\varphi') = R_0\left(1+\sqrt{\frac{5}{16\pi}}\beta\left\{\cos\gamma(3\cos^2\theta'-1)+\sqrt{3}\sin\gamma\sin^2\theta'\cos 2\varphi'\right\}\right)$$

The deformations along the principal axes are:

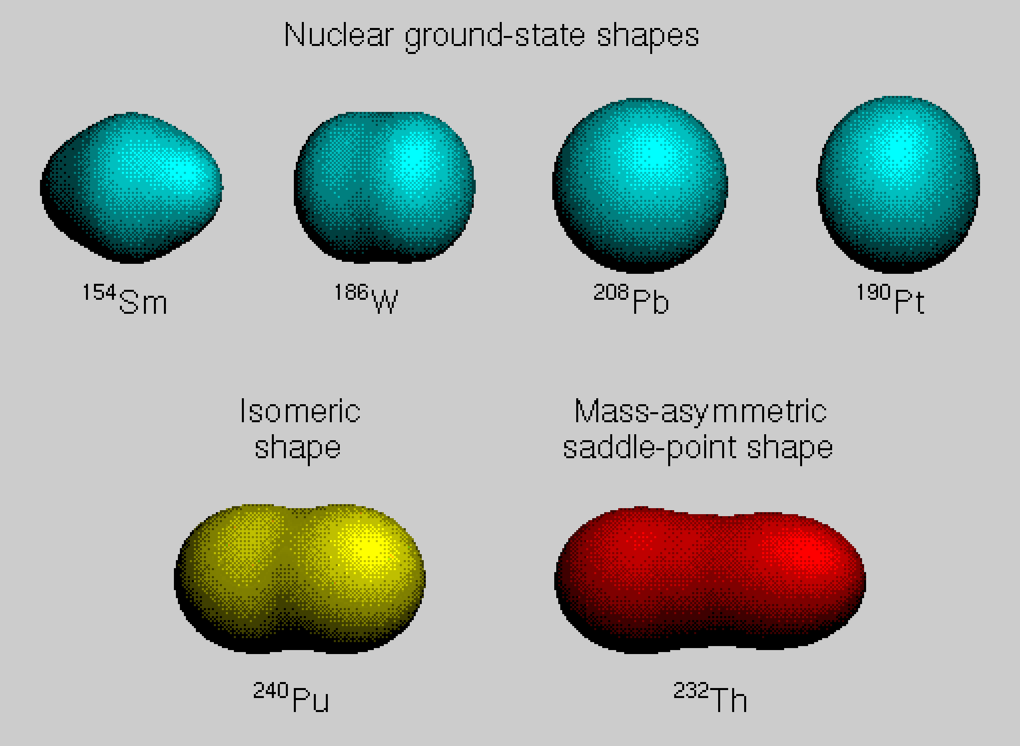
$$\delta R_{x'} = R\left(\frac{\pi}{2},0\right) - R_0 = \sqrt{\frac{5}{4\pi}}\beta\, R_0 \cos(\gamma - \frac{2\pi}{3})$$

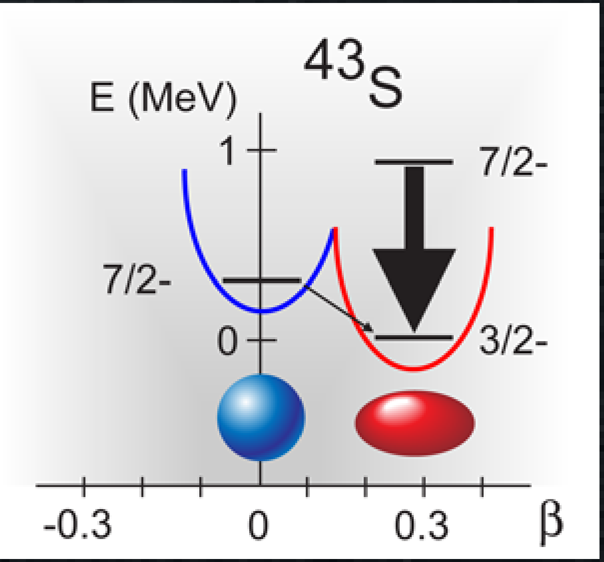
$$\delta R_{y'} = R\left(\frac{\pi}{2},\frac{\pi}{2}\right) - R_0 = \sqrt{\frac{5}{4\pi}}\beta\, R_0 \cos(\gamma + \frac{2\pi}{3})$$

$$\delta R_{z'} = R(0,0) - R_0 = \sqrt{\frac{5}{4\pi}}\beta\, R_0 \cos(\gamma)$$



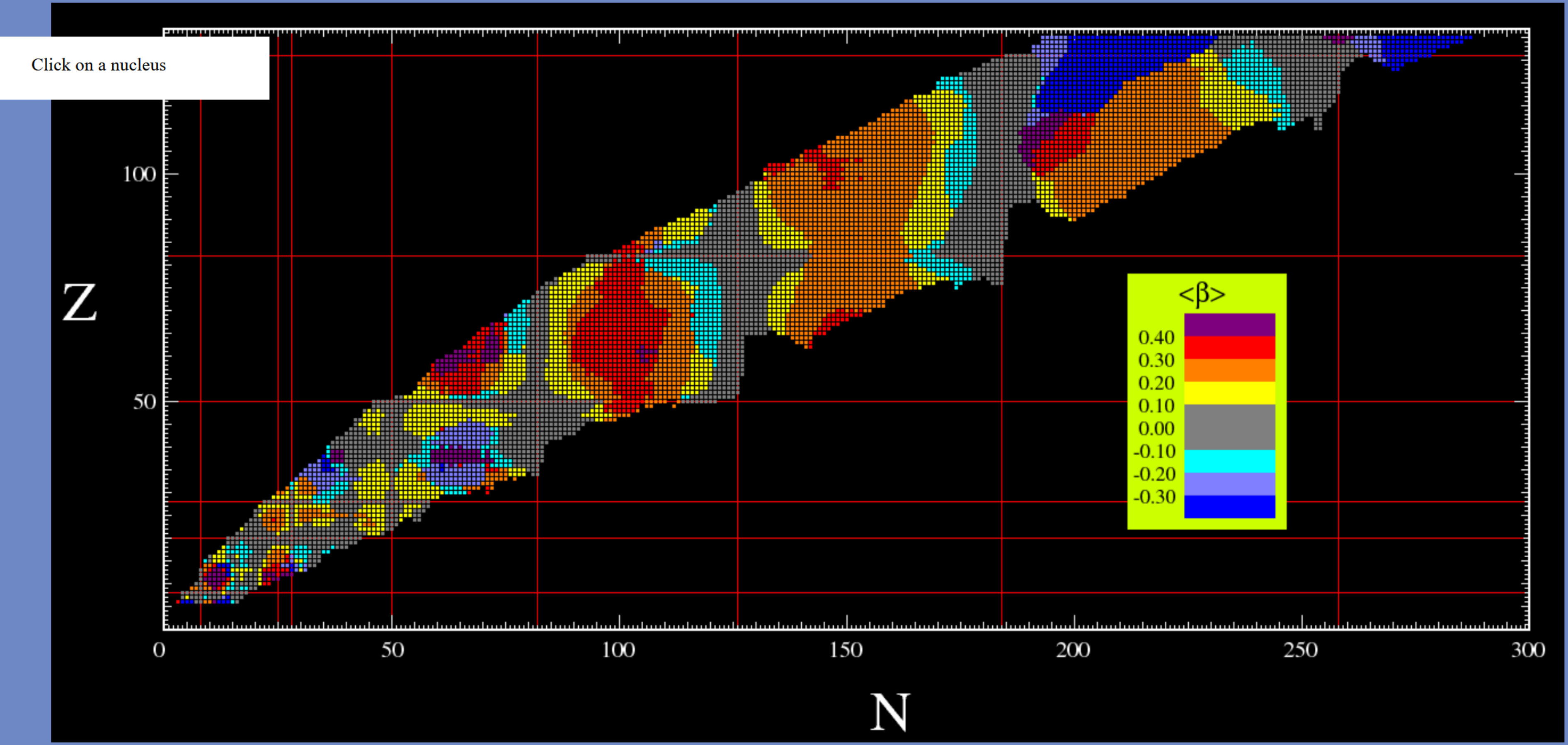
Deduced deformations





Shape coexistence

HFB results with Gogny force



S. Hilaire and M. Girod ((CEA)

Nuclear Rotations

A quantum rotor

In classical mechanics, the energy of a rigid (triaxial) rotor is

$$E_{rot} = \sum_{i=1}^{3} \frac{1}{2} \Im_i \omega_i^{\,2} = \sum_{i=1}^{3} J_i^{\,2} / 2\Im_i$$

where $\Im_i$ is the moment of inertia with respect to the *i*th principal axis

A quantum system cannot rotate around a symmetry axis.
Then:
- a spherical nucleus does not rotate
- an axially symmetric system cannot rotate around the symmetry axis

If *z* is the symmetry axis, the energy results:

$$E_{rot} = \frac{\hbar^2}{2\Im}\left(J_x^{\,2} + J_y^{\,2}\right)$$

Notation

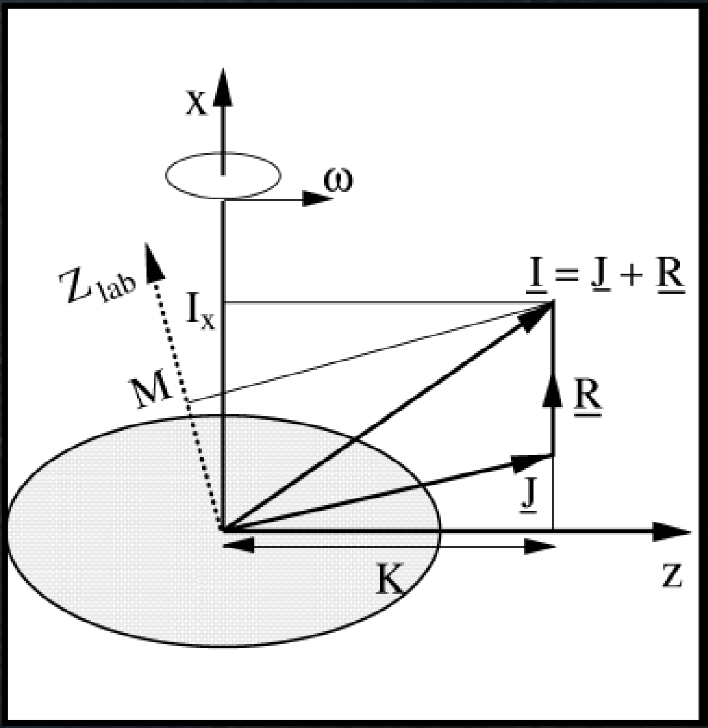
We will in general work in the intrinsic frame.

To simplify notation we will omit the " prime " and will call the axes *x, y, z.*

The laboratory axes will be called x_{lab}, y_{lab}, z_{lab}.

Conserved quantum numbers

$$\boxed{\vec{I} = \vec{R} + \vec{J}}$$



In the body-fixed reference system *x,y,z,* we choose *x* as the rotational axis.
A deformed nucleus rotates with angular momentum ***R*** around an axis (*x*) perpendicular to its symmetry axis.

It may have an intrinsic angular momentum *J* (from particles or vibrations).
J is not a conserved quantity in the intrinsic frame, but it couples to ***R*** to give the total angular momentum ***I*** of the nucleus.

I is a constant of the motion.
The projection of ***I*** on the z_{lab} axis of the laboratory frame is called *M*, while its projection on the z axis of the intrinsic frame is called *K.*
M and *K* are good quantum numbers.

Rotational motion in nuclei

Provided that the collective rotation is slow compared to the single-particle motion (adiabatic condition), the intrinsic and rotational degrees of freedom can be decoupled:
The nucleus will have an equilibrium deformed shape and therefore the Hamiltonian can be separated into intrinsic and rotational components.

$$\boxed{H = H_{int} + H_{rot}}$$

with eigenvalues

$$\boxed{E = E_{int} + E_{rot}}$$

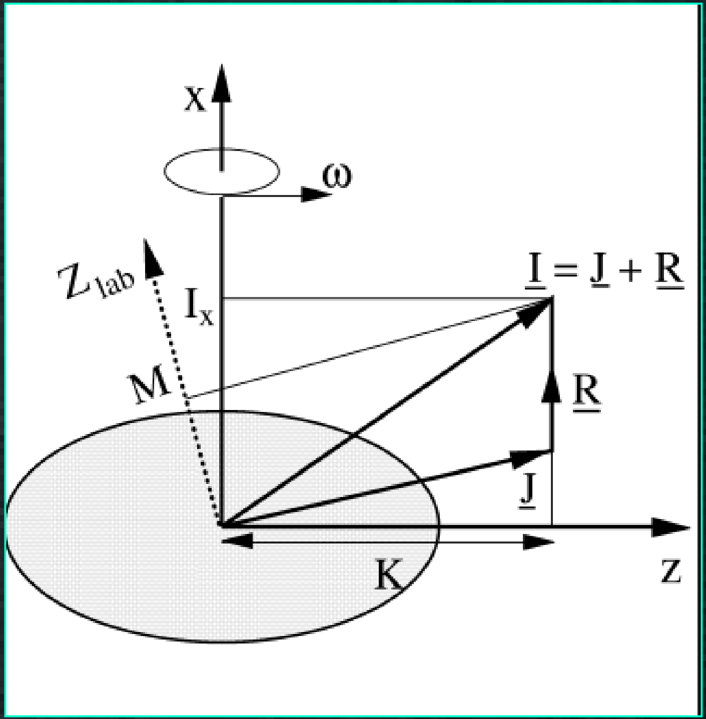
and eigenfunctions

$$\boxed{\Psi = \Phi_{int}\,\Xi_{rot}(\theta,\varphi,\psi)}$$

The rotational energy will be

$$\boxed{E_{rot} = \frac{\hbar^2}{2\Im}R(R+1)}$$

Rotational wave function



The projections on the symmetry axis *z*

$$\boxed{I_z = J_z = K}$$

are conserved quantities for the rotation about an axis perpendicular to it (x or y). The intrinsic wavefunction can be then characterized by *K*. The complete wavefunction can be characterized by the 3 q.n.

$$\boxed{I^2, I_{z_{lab}} = M, I_z = K}$$

$$\boxed{\Psi_{IMK} = \sqrt{\frac{2I+1}{8\pi^2}}\Phi_K D^I_{MK}(\theta,\varphi,\psi)}$$

where D^I_{MK} are the rotational matrices and (θ, φ, ψ) are the Euler angles

A rotational band

$$\Psi_{IMK} = \sqrt{\frac{2I+1}{8\pi^2}}\Phi_K D^I_{MK}(\theta,\varphi,\psi)$$

The value of *K* is determined by the intrinsic motion and is assumed to have a constant value along a rotational band.

Thus, the intrinsic wave function Φ_K should remain constant within a band. This is in practice approximately valid only in the low-spin region.

At higher spins or excitation energy, deviations from this picture occur because of the coupling of particle degrees of freedom with the collective motion.

Rotations of an axial symmetric rotor

For $K = 0$ the rotational matrices reduce to the spherical harmonics

$$\Psi_{r,IMK=0} = \sqrt{\frac{1}{2}}\,\Phi_{r,K=0}\;Y_{IM}$$

where the q.n. r is connected to the reflexion invariance of the axial system with respect to the plane perpendicular to the symmetry axis.

The values of r (signature) are

$$r = (-1)^I$$

with $I = 0, 2, 4,$…for K = 0 and r = +1
$I = 1, 3, 5,$…for K = 0 and r = -1

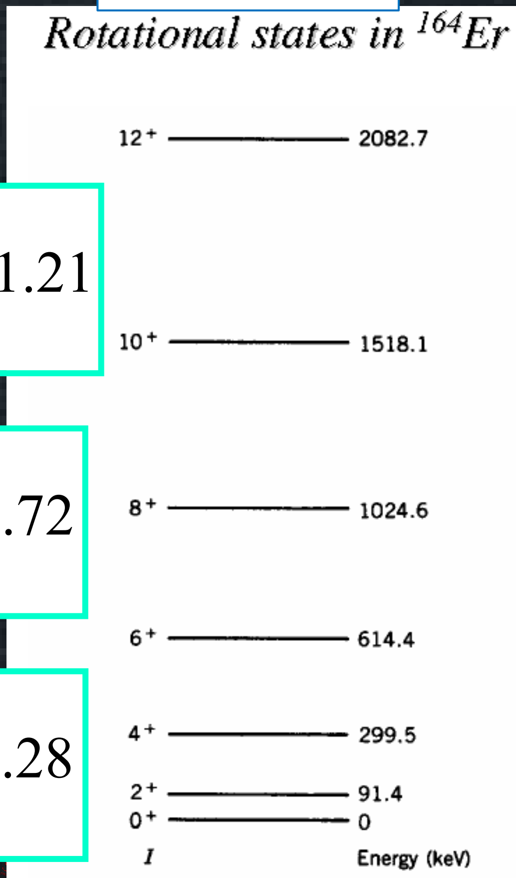
These two sequences form rotational bands where the states within each band are separated by $\Delta I = 2$

Example: a quadrupole rotational band

$$E_{rot}(I) = \frac{\hbar^2}{2\Im} R(R+1)$$

For an even-even nucleus with K=0, r=+1

Experiment



$$\frac{E_{8^+}}{E_{2^+}} = 11.21$$

$$\frac{E_{6^+}}{E_{2^+}} = 6.72$$

$$\frac{E_{4^+}}{E_{2^+}} = 3.28$$

Theory

$$E(0^+) = 0$$

$$E(2^+) = 6\left(\frac{\hbar^2}{2\Im}\right)$$

$$E(4^+) = 20\left(\frac{\hbar^2}{2\Im}\right)$$

$$E(6^+) = 42\left(\frac{\hbar^2}{2\Im}\right)$$

$$E(8^+) = 72\left(\frac{\hbar^2}{2\Im}\right)$$

$$\frac{E_{4^+}}{E_{2^+}} = \frac{20}{6} = 3.3$$

$$\frac{E_{6^+}}{E_{2^+}} = \frac{42}{6} = 7$$

$$\frac{E_{8^+}}{E_{2^+}} = \frac{72}{6} = 12$$

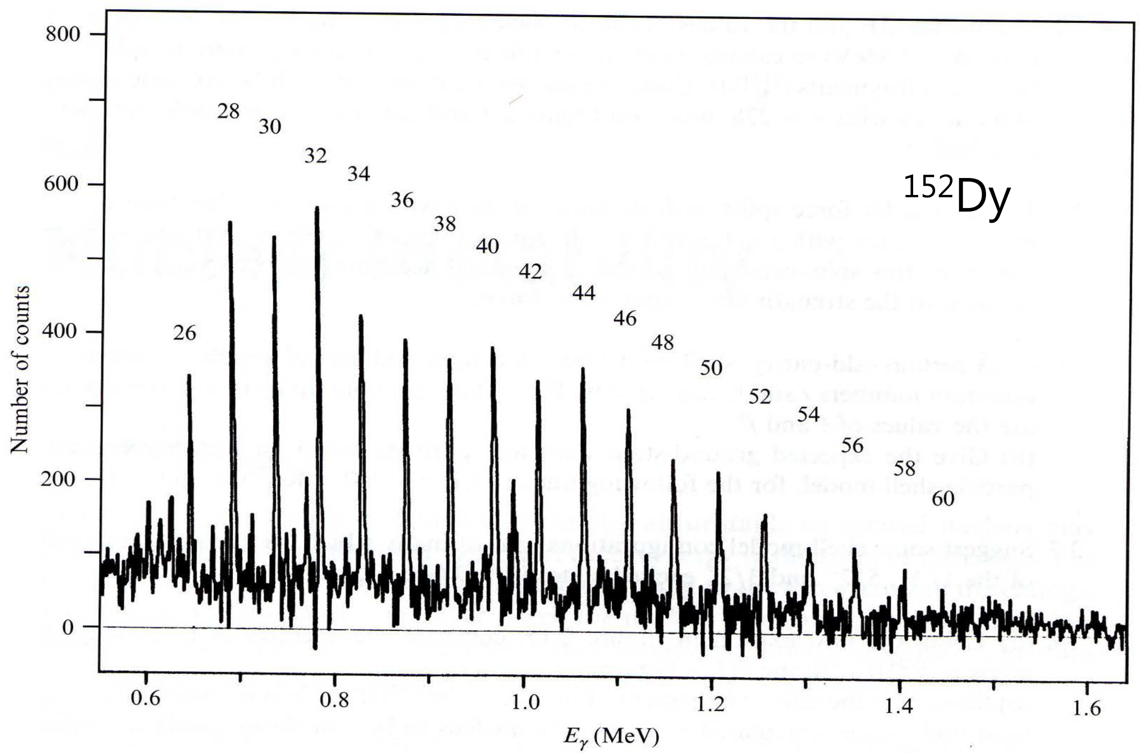
The agreement is very good in heavy nuclei, in medium and light nuclei, the regular behavior is observed at low spin.

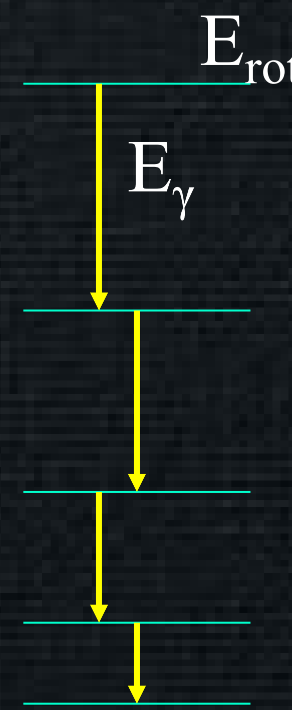
Gamma-ray spectrum of a rotor

Excitation energy $$E_{rot} = \frac{I(I+1)\,\hbar^2}{2\mathfrak{J}}$$

transition energy $$\Delta E_{rot}(I+2 \rightarrow I) = \frac{(2I+3)\hbar^2}{\mathfrak{J}} = E_\gamma$$

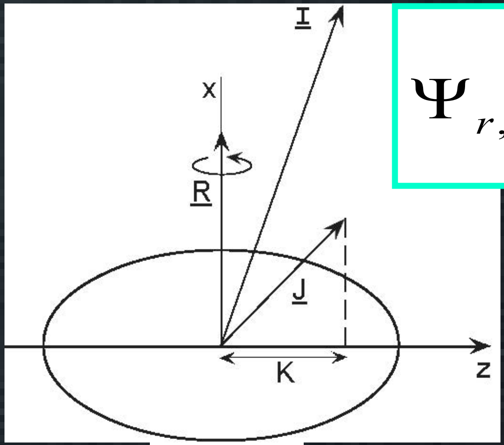
difference between consecutive transition energies $$\Delta E_\gamma = \frac{4\hbar^2}{\mathfrak{J}}$$





The case of $K \neq 0$

For $K \neq 0$ the total nuclear wavefunction is antisymmetrized:



$$\Psi_{r,IMK} = \sqrt{\frac{2I+1}{16\pi^2}}\left[\Phi_{r,K} D^I_{MK} + (-1)^{I+K}\Phi_{r,\hat{K}} D^I_{M-K}\right]$$

$$r = (-1)^{I+K}$$

The signature arises from the rotational symmetries (of π) of the w.f.

and $\Phi_{r,\hat{K}}$ corresponds to a projection of the angular momentum –K.
It is obtained from $\Phi_{r,K}$ by a rotation of π around the x axes.

The consequence of rotational symmetry for K ≠ 0 is that the two intrinsic states are degenerate and constitute e single sequence of rotational states with spins:

$$I = K, K+1, K+2, \ldots.$$

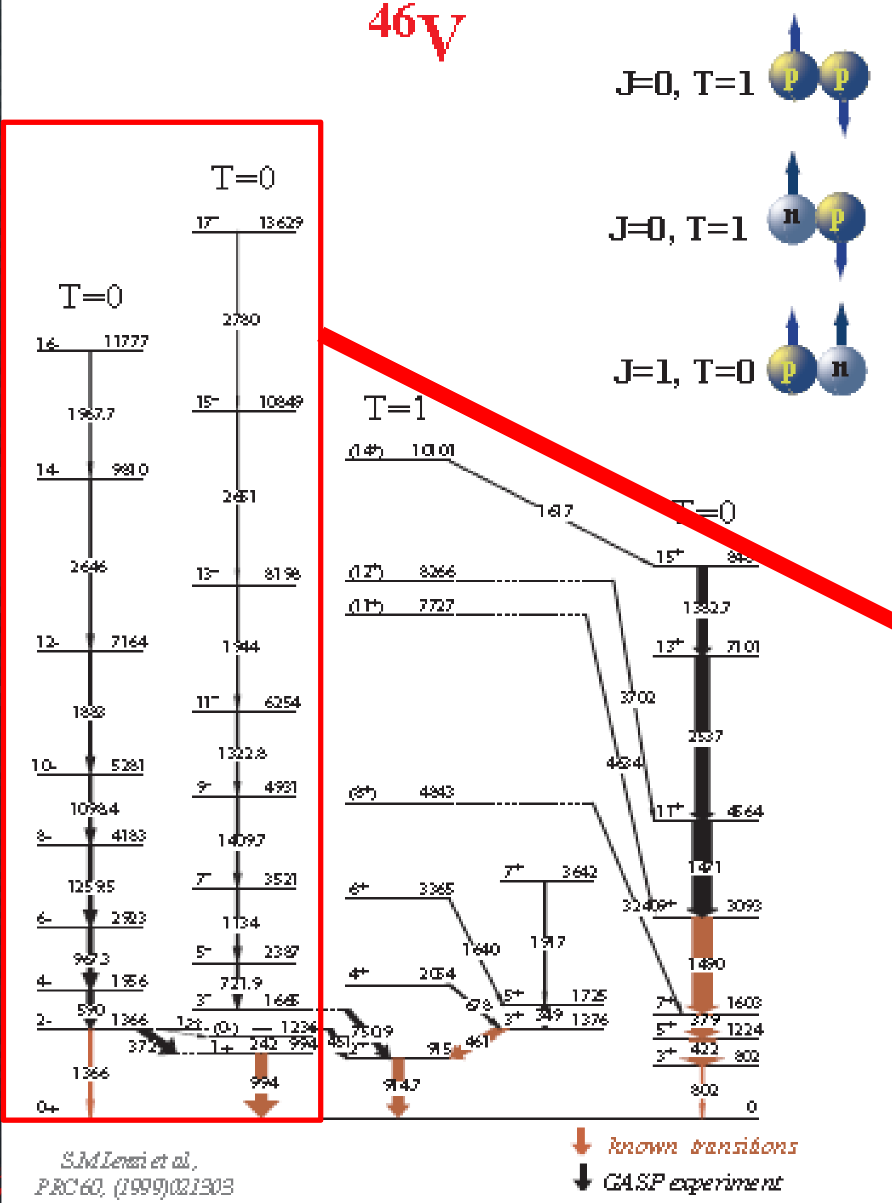
The case of K≠0

The phase factor $r = (-1)^{I+K}$ *signature* changes sign within a band for alternate spin values.

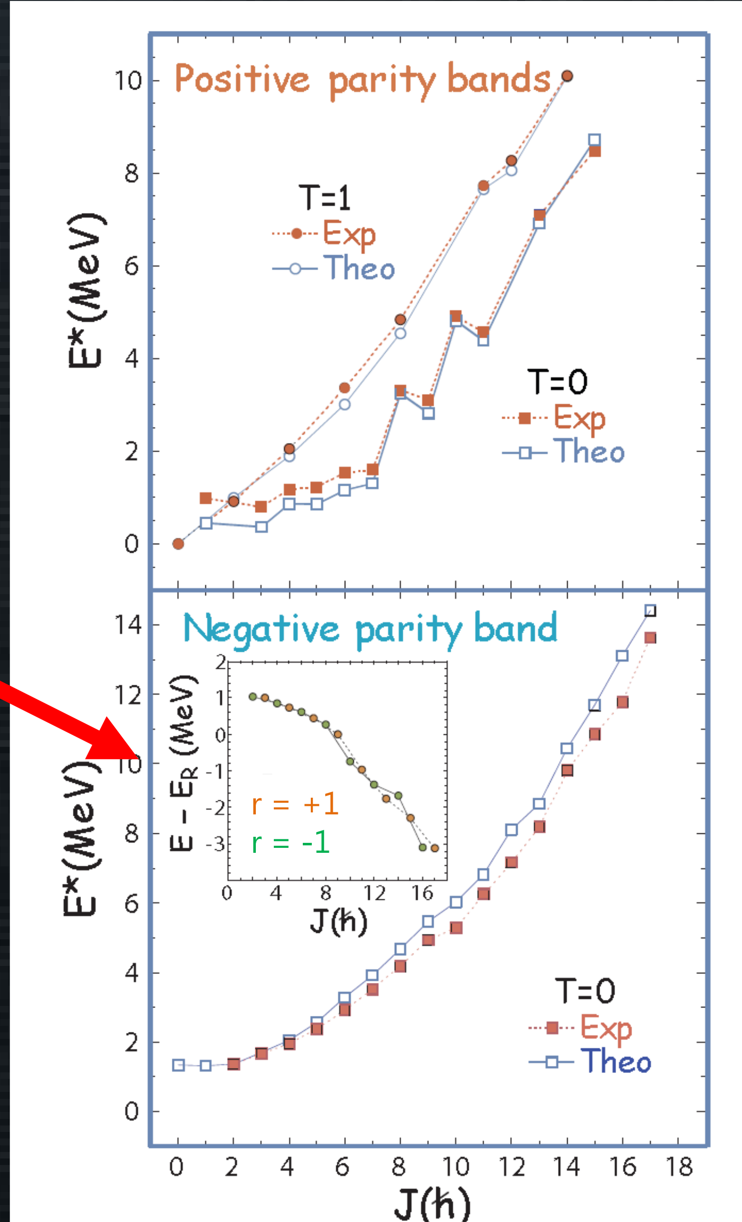
A rotational band can be considered as composed of two sequences with opposite signature:

I = K, K+2, K+4,…… and *I = K+1, K+3, K+5 ….*

Proton-neutron pairing correlations in the odd-odd N = Z nucleus ^{46}V



S.M.Lenzi et al., PRC 60, (1999)021303



Particle-rotation coupling

The rotational hamiltonian is given by:

$$\boxed{H_{rot} = \frac{\hbar^2}{2\Im}\vec{R}^2 = \frac{\hbar^2}{2\Im}[\vec{I} - \vec{J}]^2 = \frac{\hbar^2}{2\Im}[\vec{I}\cdot\vec{I} + \vec{J}\cdot\vec{J} - 2\vec{I}\cdot\vec{J}]}$$

where $\vec{I}\cdot\vec{J}$ couples the degree of freedom of the particles to the rotational motion of the core, analogue to the classical Coriolis force.

Coriolis coupling

The rotational Hamiltonian can be expanded:

$$\vec{R}^2 = (\vec{I} - \vec{J})^2 = I^2 + J^2 - 2\vec{I}\cdot\vec{J}$$
$$= I^2 + J^2 - 2K^2 - (I_+J_- + I_-J_+)$$

where $I_\pm = I_x \pm iI_y, J_\pm = J_x \pm iJ_y$ $J_z = I_z = \pm K$

The coupling term $I_+J_- + I_-J_+$

corresponds to the Coriolis force and couples *J* to *R*

Rotation and coupling

$$E_{rot} = \frac{\hbar^2}{2\Im}\left(I^2 + J^2 - 2K^2 - (I_+J_- + I_-J_+)\right)$$

If $2K^2 >> (I_+J_- + I_-J_+)$

We can neglect the Coriolis term. In this limit *K* is a constant of motion, and since *J* is also constant,

$$E_{rot} = E_K + \frac{\hbar^2}{2\Im} I(I+1)$$

$$I = K, K+1, K+2, \ldots$$

where E_K is the energy of the lowest member of the rotational band having $I = K$: the bandhead.

Electromagnetic properties

Electric multipole operator

The electric multipole operator in the limit of long wavelengths (low transition energies) is given by

$$\hat{Q}_{\lambda\mu} = e\int_V \rho_p(\vec{r})r^\lambda Y_{\lambda\mu}(\theta,\varphi)d^3r$$

$$\hat{Q}_{\lambda\mu} = \rho_p e\int_{4\pi} d\cos\theta\, d\varphi\, Y_{\lambda\mu}(\theta,\varphi)\frac{1}{\lambda+3}R^{\lambda+3}(\theta,\varphi)$$

Up to first order we have:

$$\hat{Q}_{\lambda\mu} = \frac{3e}{4\pi}ZR_0^\lambda\hat{\alpha}_{\lambda\mu}$$

In the intrinsic frame in first order:

$$\hat{Q}^{\text{int}}_{\lambda\mu} = \frac{3e}{4\pi}ZR_0^\lambda\hat{\alpha}'_{\lambda\mu}$$

Laboratory and intrinsic Q

$$\widehat{Q}_{\lambda\mu}^{lab} = \sum_{\mu'} D_{\mu\mu'}^{\lambda} \widehat{Q}_{\lambda\mu'}^{\text{int}}$$

If we restrict to a pure quadrupole K band, the Q_2 matrix elements are:

$$< I_1 K n_\beta n_\gamma \| \widehat{Q}_2^{lab} \| I_2 K n_\beta n_\gamma >=$$
$$= \sqrt{\frac{5}{16\pi}} Q_0(n_\beta, n_\gamma) \sqrt{(2I_1+1)(2I_2+1)} (-1)^{I_1-K} \begin{pmatrix} I_1 & 2 & I_2 \\ -K & 0 & K \end{pmatrix}$$

where **Q_0 is the intrinsic quadrupole moment** that depends on the quantum numbers that characterize the band (vibrations, s. particle)

For fixed β and $\gamma = 0$:

$$Q_0 = \sqrt{\frac{16\pi}{5}} \frac{3}{4\pi} Z e R_0^2 \beta$$

Not an observable!

Prolate deformation $Q_0 > 0$ Oblate deformation $Q_0 < 0$

Quadrupole moments and transitions

The spectroscopic quadrupole moment is given by the diagonal matrix element:

$$Q_2 = \sqrt{\frac{16\pi}{5}} < IK \| \hat{Q}_2^{lab} \| IK > = Q_0 \frac{3K^2 - I(I+1)}{(2I+3)(I+1)}$$

This is an observable!

If K=0

Prolate deformation $Q_2 < 0$

Oblate deformation $Q_2 > 0$

The reduced transition probability is given by:

$$B(E2, I_1 \to I_2) = \frac{1}{2I_1+1} |< I_2 K \| \hat{Q}_2 \| I_1 K >|^2$$

$K \neq 1/2, 1$

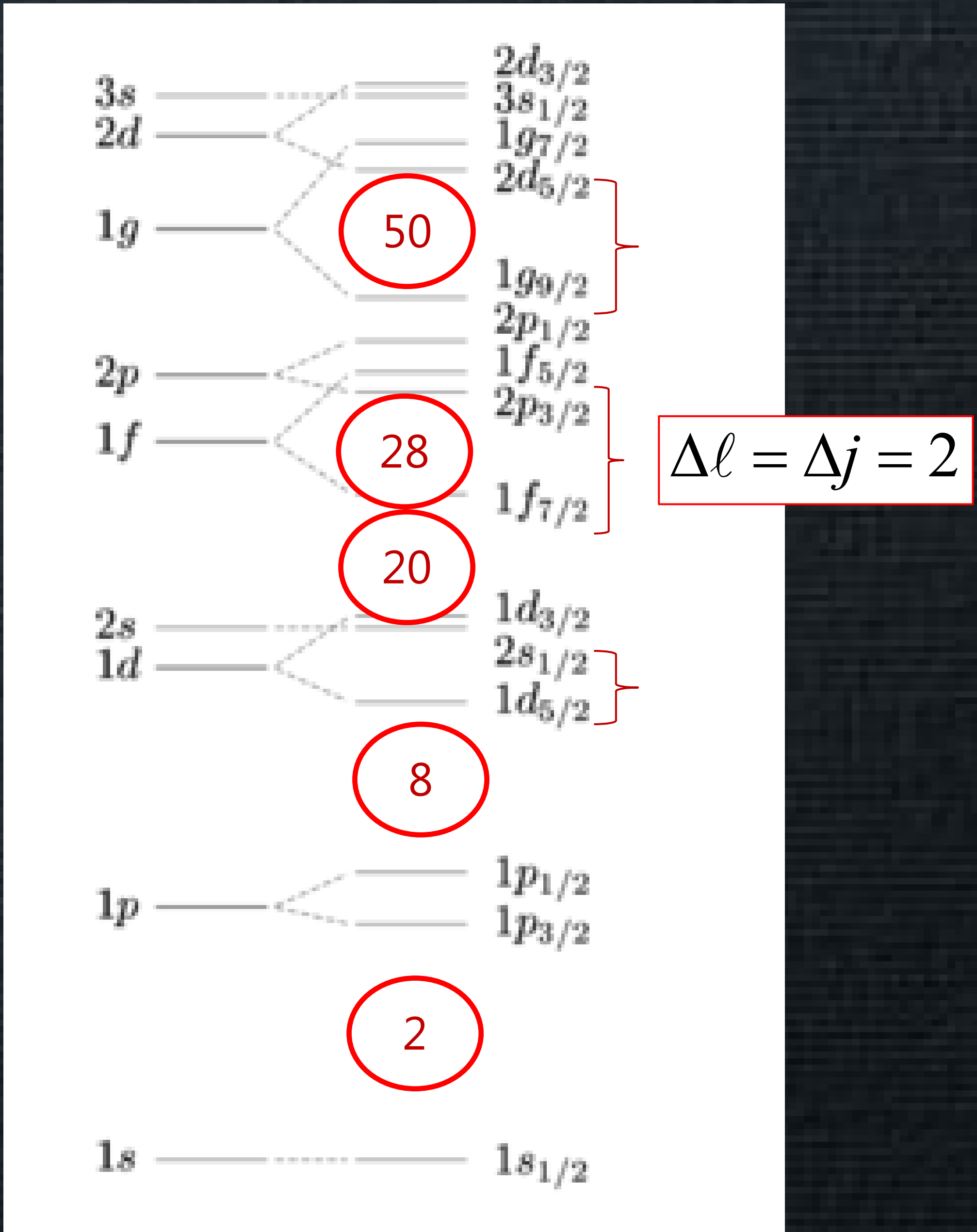
$$B(E2, I+2 \to I) = \frac{5}{16\pi} e^2 |< I+2 K 2 0 | I K >|^2 Q_0^2$$

If K=0

$$B(E2, I+2 \to I) = Q_0^2 \frac{5}{16\pi} \frac{3}{2} \frac{(I+1)(I+2)}{(2I+3)(2I+5)}$$

By measuring Q_2 or B(E2) we can obtain Q_0 and thus the degree of deformation

Candidates for quadrupole deformation

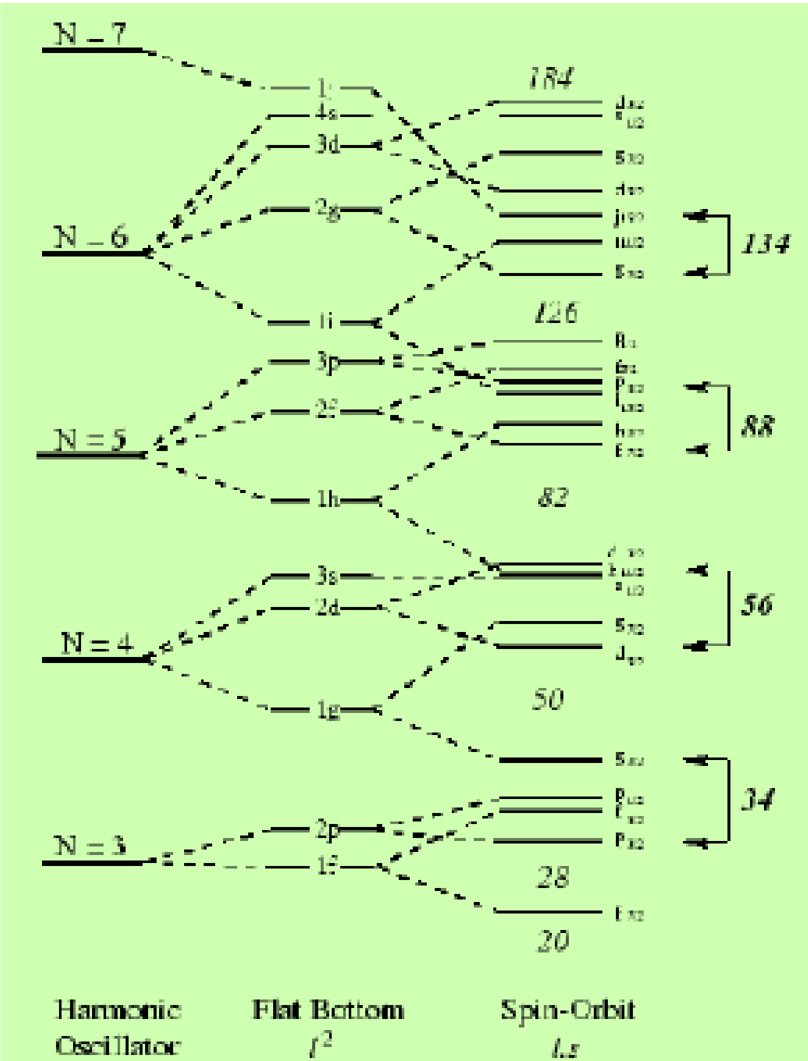


Quadrupole correlations occur between orbitals that differ in

$$\Delta\ell = \Delta j = 2$$

Nuclei with both, protons and neutrons filling these orbitals are the best candidates to show quadrupole effects

Candidates for octupole deformation



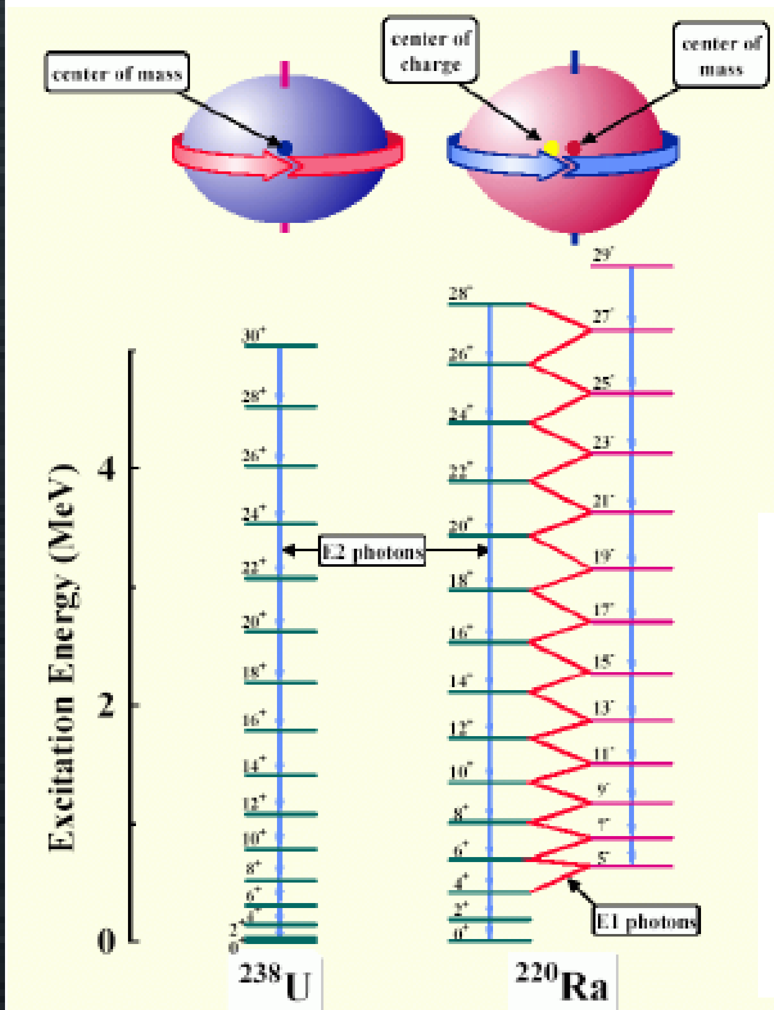
Octupole correlations occur between orbitals that differ in

$$\boxed{\Delta\ell = \Delta j = 3}$$

For octupole deformation, the "magic numbers" occur at 34, 56, 88 and 134

Nuclei with both, protons and neutrons close to these numbers are the best candidates to show octupole effects

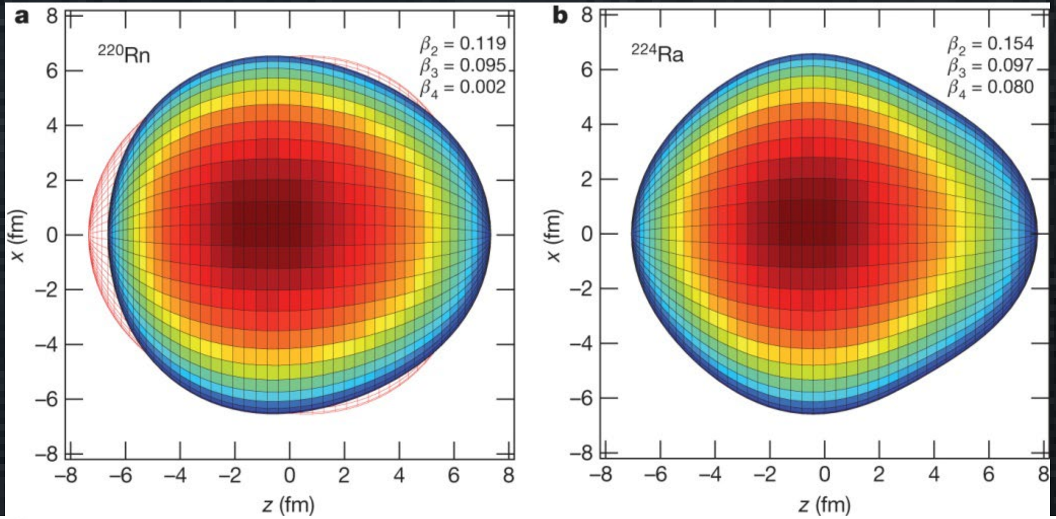
Electric Dipole Moment

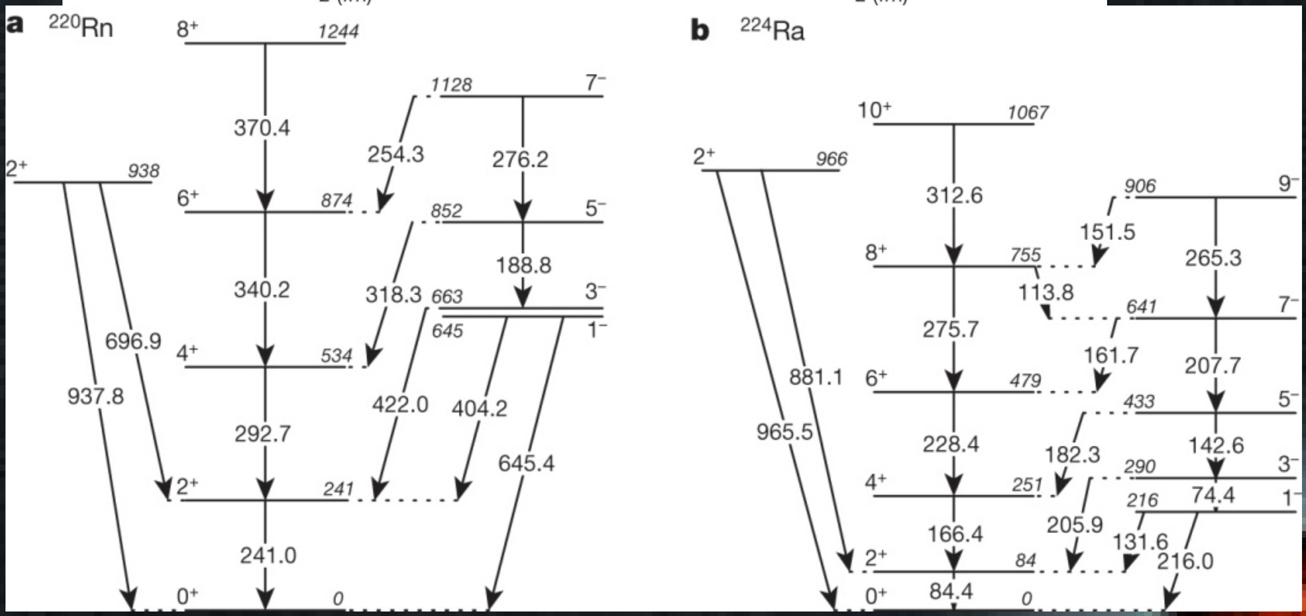


In an octupole deformed nucleus, the centre of mass and the centre of charge tend to separate and a finite dipole moment is created

Octupole deformation of ^{220}Rn and

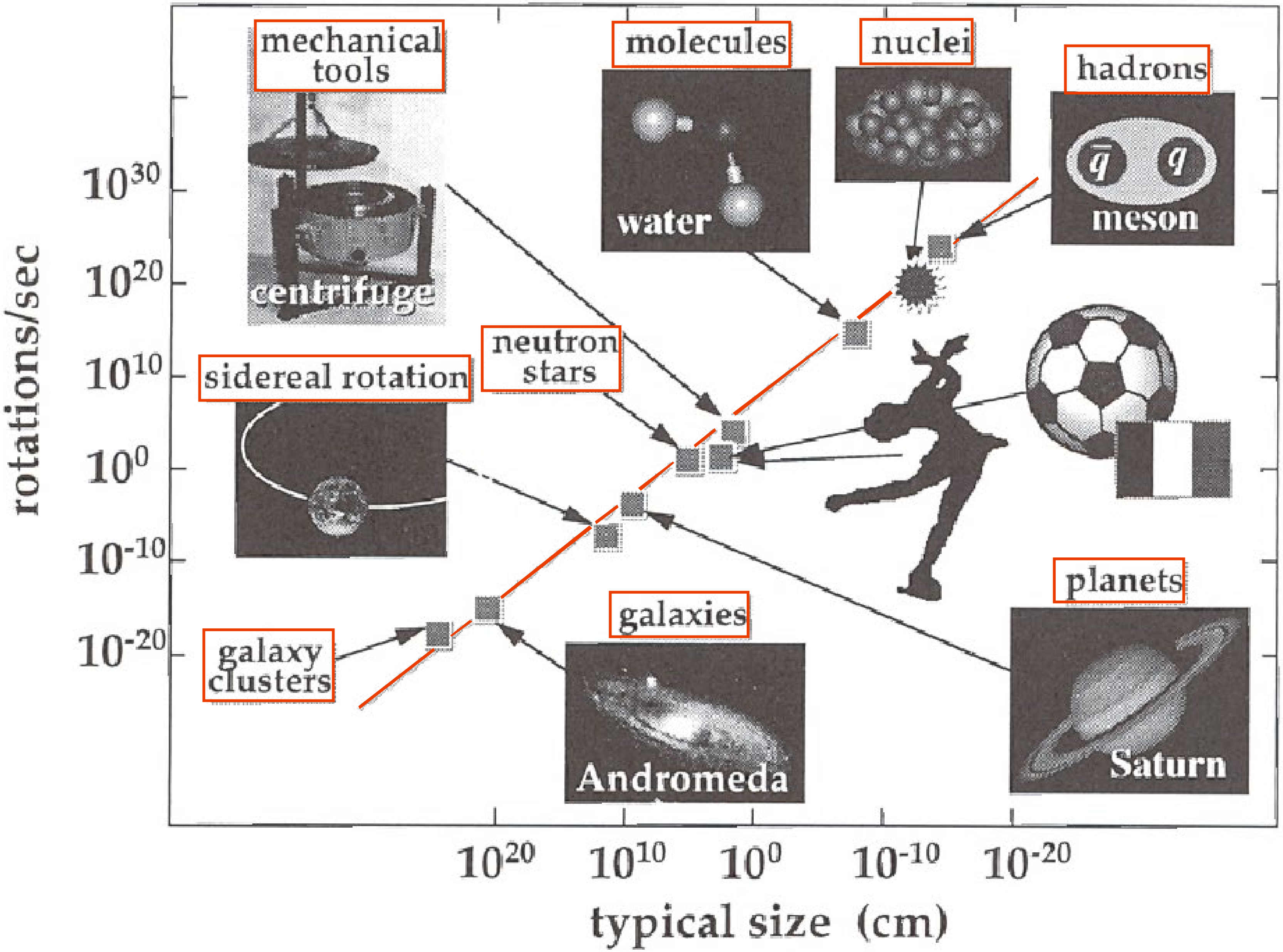
Coulomb excitation experiments were performed using accelerated beams of heavy, radioactive ions
at ISOLDE in CERN.
The data on ^{220}Rn and ^{224}Ra show clear evidence for stronger octupole deformation in ^{224}Ra.





LP Gaffney *et al.* *Nature* **497**, 199-204 (2013) doi:10.1038/nature12073

rotations in the universe



Take-away message

Nuclei tend to deform far from closed shells: quadrupole (oblate, prolate, triaxial), octupole

A deformed nucleus rotates with a spectrum characterized by excitation energies proportional to the square of the rotational angular momentum.

Pure rotations may be coupled to single particle degrees of freedom at high spin: Coriolis-antipairing

A rotational band can be constructed on an excited bandhead that can have an arbitrary structure provided it is deformed

The rotational model is a macroscopic model that does not give any information about the intrinsic structure

Microscopic models: The shell model

The non-interactive shell model

The nuclear system

The atomic nucleus is a strongly interacting quantum many-body system

- Strongly interacting means non perturbative
- Quantum: Schrödinger equation needs to be solved
- Many-body: *A* fermions in 3-D space with spin and isospin degrees of freedom

In principle, in coordinate representation the *A*-body wave function could be written as

$$\boxed{\Psi_i(\vec{r}_1,\vec{s}_1,\vec{t}_1,\vec{r}_2,\vec{s}_2,\vec{t}_2,\ldots,\vec{r}_A,\vec{s}_A,\vec{t}_A)}$$

that is complex and antisymmetric.

However, in practice, this kind of approach can be done only to few of the lightest nuclei!

The full solution

Let us assume that we have found a solution for

$$\boxed{\Psi(\vec{r}_1,\vec{s}_1,\vec{t}_1,\vec{r}_2,\vec{s}_2,\vec{t}_2,\ldots,\vec{r}_A,\vec{s}_A,\vec{t}_A)}$$

that is an eigenfunction of the Schrödinger equation

$$\boxed{H\Psi = E\Psi}$$

Now, one can calculate any observable from ψ.

Let us first check that our wave function is properly normalized to unity:

$$\boxed{1 = <\Psi|\Psi> = \int \Psi^*(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A)\Psi(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A)d\vec{r}_1 d\vec{r}_2\ldots d\vec{r}_A}$$

Take ^{40}Ca (A=40), a box from -5 fm to 5 fm with mesh spacing of Δr = 1 fm (11 spatial points towards each direction)

The integral becomes a sum of 11x11x…x11 = 11^{3x40} = 9 x 10^{124} terms. Even with a very powerful computer (~100 Petaflops) the sum of all these terms would take ~ 10^{100} years!

A simpler solution

One possible approximation is to assume that the many-body wave function is a product of one-body wave functions (independent particle motion in a mean field)

$$\Psi_{1,2,\ldots.A}(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A) = \prod_i \varphi_i(\vec{r}_i)$$

$$H_0 = \sum_{i=1}^{A} h_0(i)$$

This reduces to

$$H_0\Psi = E_0\Psi \qquad h_0(i)\varphi_i(\vec{r}_i) = \varepsilon_i\varphi_i(\vec{r}_i) \qquad E_0 = \sum_{i=1}^{A}\varepsilon_i$$

Now checking the wave-function normalization in ^{40}Ca would require a summation of about 40 x 11^3 ~ 5 x 10^4 terms.
A significant reduction compared to 9 x 10^{124}!

This wave function is not antisymmetric! Therefore one should use Slater determinants

$$\Psi(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A) = \frac{1}{\sqrt{A!}}\begin{vmatrix} \varphi_1(\vec{r}_1) & \ldots & \varphi_A(\vec{r}_1) \\ \vdots & \vdots & \vdots \\ \varphi_1(\vec{r}_A) & \ldots & \varphi_A(\vec{r}_A) \end{vmatrix}$$

Justification of the mean-field

At first sight, the mean field seems a too simplified picture compared to a fully interacting system. However, there are reasons why it works relatively well.

The nucleon mean free path inside the nucleus is of the same order of magnitude as the nuclear radius. This means that inside the nucleus the system resembles a non-interacting Fermi gas.

In other words, on average, nucleons return back to their independent particle picture before the next collision takes place.

The applicability of the shell model has actually a profound meaning

ELSEVIER

Nuclear Physics A649 (1999) 45c

NUCLEAR PHYSICS A

Why are nuclei described by independent particle motion ?

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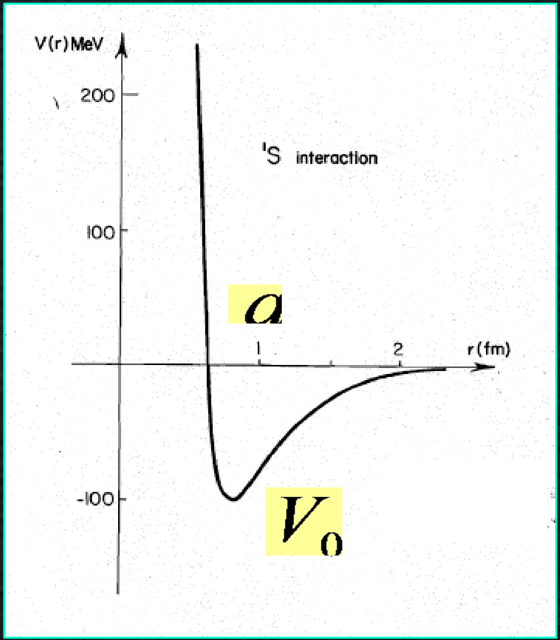


"Quantality Parameter"

$$\Lambda = \frac{\hbar^2 / M a^2}{V_0}$$

The "quantality" parameter measures the strength of the two-body attraction, V_0, expressed in units of the quantal kinetic energy associated with a localization of a constituent particle of mass *M* within the distance *a* corresponding to the radius of the force at maximum attraction.

In other words, it is the ratio between the zero-point kinetic energy of the confined particle to its potential

The nucleus: a Fermi Liquid

Constituents	M	V_0 [eV]	a [cm]	Λ	T=0 matter
^{3}He	3	$9{\cdot}10^{-4}$	$2.9{\cdot}10^{-8}$	0.21	liquid
^{4}He	4	$9{\cdot}10^{-4}$	$2.9{\cdot}10^{-8}$	0.16	liquid
H_2	2	$3{\cdot}10^{-3}$	$3.3{\cdot}10^{-8}$	0.07	solid
Ne	20	$3{\cdot}10^{-3}$	$3.1{\cdot}10^{-8}$	0.007	solid
nuclei	1	$1{\cdot}10^{8}$	$9{\cdot}10^{-14}$	0.4	liquid

$$\Lambda = \frac{\hbar^2 / Ma^2}{V_0}$$

For small Λ **(<0.1)** the quantal effect is small and the ground state of the many body system will be, as in classical mechanics, a configuration in which each particle finds a static optimal position with respect to its nearest neighbours. If Λ is big enough **(>>0.1)** the ground state may be a quantum liquid in which the individual particles are delocalized and the low-energy excitations have infinite mean free path.

Maria Goeppert Mayer

She was student of Max Born.

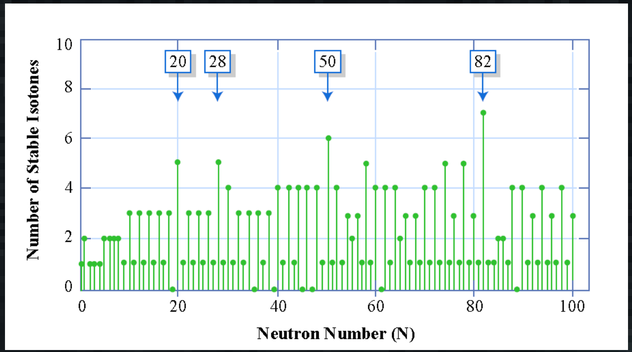
Her thesis (1930) regarded the problem of emission of two quanta. The same year she married the American chemist Joseph Mayer.

But the ‘Anti-Nepotism Rules’ did not allow more than one family member to work in the same faculty.

For three decades, Maria followed her husband: John Hopkins Univ. (Baltimore) and Columbia Univ. (New York) until they came to Chicago.

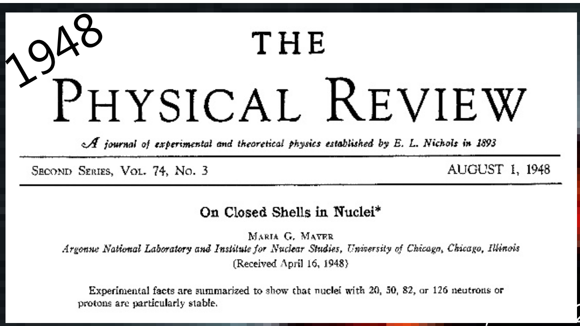
In Columbia University:

- with Urial ^{235}U enrichment program (failed)
- with Teller: «Opacity program» (hydrogen bomb)
- meets Enrico Fermi



1946- University of Chicago: Volonteer researcher. Argonne National Lab offers Maria a regular appointment as a Senior Physicist (half time)

with Teller: cosmological model of the origin of the chemical elements→ magic numbers


1948

THE PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 74, No. 3 AUGUST 1, 1948

On Closed Shells in Nuclei*

MARIA G. MAYER

Argonne National Laboratory and Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

(Received April 16, 1948)

Experimental facts are summarized to show that nuclei with 20, 50, 82, or 126 neutrons or protons are particularly stable.

Magic numbers: the spin-orbit coupling

«One day, as Fermi was leaving my office he asked:
Is there any indication of spin-orbit coupling?
This explained everything! and Fermi taught it in his class in nuclear physics»

1949

On Closed Shells in Nuclei. II

MARIA GOEPPERT MAYER
Argonne National Laboratory and Department of Physics, University of Chicago, Chicago, Illinois

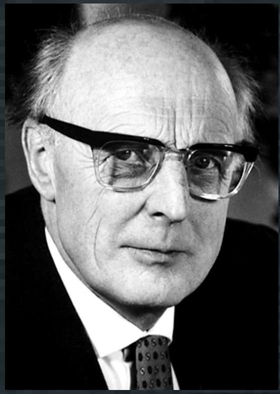
February 4, 1949

THE spins and magnetic moments of the even-odd nuclei have been used by Feenberg[1,2] and Nordheim[3] to determine the angular momentum of the eigenfunction of the odd particle. The tabulations given by them indicate that spin orbit coupling favors the state of higher total angular momentum. If strong spin-orbit coupling, increasing with angular momentum, is assumed, a level assignment different from either Feenberg or Nordheim is obtained. This assignment encounters a very few contradictions with experimental facts and requires no major crossing of the levels from those of a square well potential. The magic numbers 50, 82, and 126 occur at the place of the spin-orbit splitting of levels of high angular momentum.

Experimental evidences (MGP, Phys. Rev. 78 (1950) 16):

- J=0 in the g.s. of even-even nuclei (pairing)
- J=j of the unpaired particle in the g.s. of odd nuclei
  There are two main exceptions: 23Na, 55Mn (Nilsson-explained)
- Parity, magnetic moments, isomerism, beta-decay…

The shell model: the beginning



1949

On the "Magic Numbers" in Nuclear Structure

OTTO HAXEL
Max Planck Institut, Göttingen
J. HANS D. JENSEN
Institut f. theor. Physik, Heidelberg
AND
HANS E. SUESS
Inst. f. phys. Chemie, Hamburg

April 18, 1949

A SIMPLE explanation of the "magic numbers" 14, 28, 50, 82, 126 follows at once from the oscillator model of the nucleus,[1] if one assumes that the spin-orbit coupling in the Yukawa field theory of nuclear forces leads to a strong splitting of a term with angular momentum l into two distinct terms $j=l\pm\frac{1}{2}$.

First article refused: «it is not really physics but rather playing with numbers»

TABLE I. Classification of nuclear states.

1 Oscil-lator-quan-tum num-ber r	2 Multi-plicity	3 Sum of all multi-plicities	4 Orbital momen-tum l	5 Total angular momen-tum j	6 l_j-symbol	7 Multi-plicities	8 Magic num-bers
1	2	2	0	1/2	$s_{1/2}$	2	
2			1	3/2	$p_{3/2}$	4	
	6	8		1/2	$p_{1/2}$	2	
3			2	5/2	$d_{5/2}$	6	14
				3/2	$d_{3/2}$	4	
	12	20	0	1/2	$s_{1/2}$	2	
4			3	7/2	$f_{7/2}$	8	28
				5/2	$f_{5/2}$	6	
			1	3/2	$p_{3/2}$	4	
	20	40		1/2	$p_{1/2}$	2	
5			4	9/2	$g_{9/2}$	10	50
				7/2	$g_{7/2}$	8	
			2	5/2	$d_{5/2}$	6	
				3/2	$d_{3/2}$	4	
	30	70	0	1/2	$s_{1/2}$	2	
6			5	11/2	$h_{11/2}$	12	82
				9/2	$h_{9/2}$	10	
			3	7/2	$f_{7/2}$	8	
				5/2	$f_{5/2}$	6	
			1	3/2	$p_{3/2}$	4	
	42	112		1/2	$p_{1/2}$	2	
7			6	13/2	$i_{13/2}$	14	126
				11/2	$i_{11/2}$	12	
			4	9/2	$g_{9/2}$	10	

Shell model, based on the independent-particle model (IPM) arrives when Bohr's «compound-nucleus model» was very successful (resonances, fission).

Pauli principle guarantees the IPM at low excitation energy

Flowers, Elliot
Correlations in the spherically symmetric mean field by configuration mixing

⟷

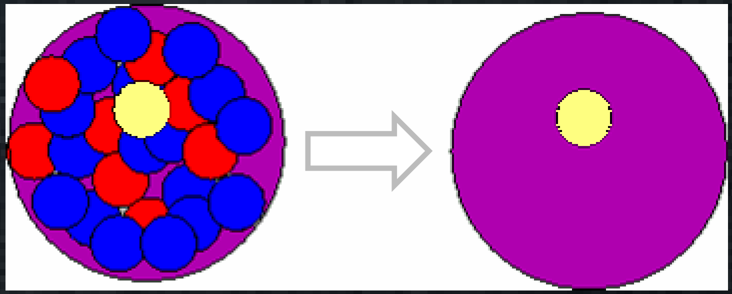
Bohr, Mottelsson, Nilsson
Introduce correlations *ab initio* considering a deformed mean field

Nuclear Potentials

There are two approaches:

1. An empirical form of the potential is assumed, e.g. square well, harmonic oscillator, Woods-Saxon
2. The mean field is generated self-consistently from the nucleon-nucleon interaction (Hartree Fock)

N nucleons in a nucleus



A nucleon in the Mean Field of N-1 nucleons

- Assumption: ignore detailed two-body interactions
- Each particle moves in a state independent of the other particles
- The Mean Field is the average smoothed-out interaction with all the other particles
- An individual nucleon only experiences a central force

The one-body potential

This is an independent particle model where the nucleus is described in terms of non-interacting particles in the orbits of a spherical symmetric (central) potential U(r) which is itself produced by all the nucleons.

Then, the resulting orbit energies are mass dependent.

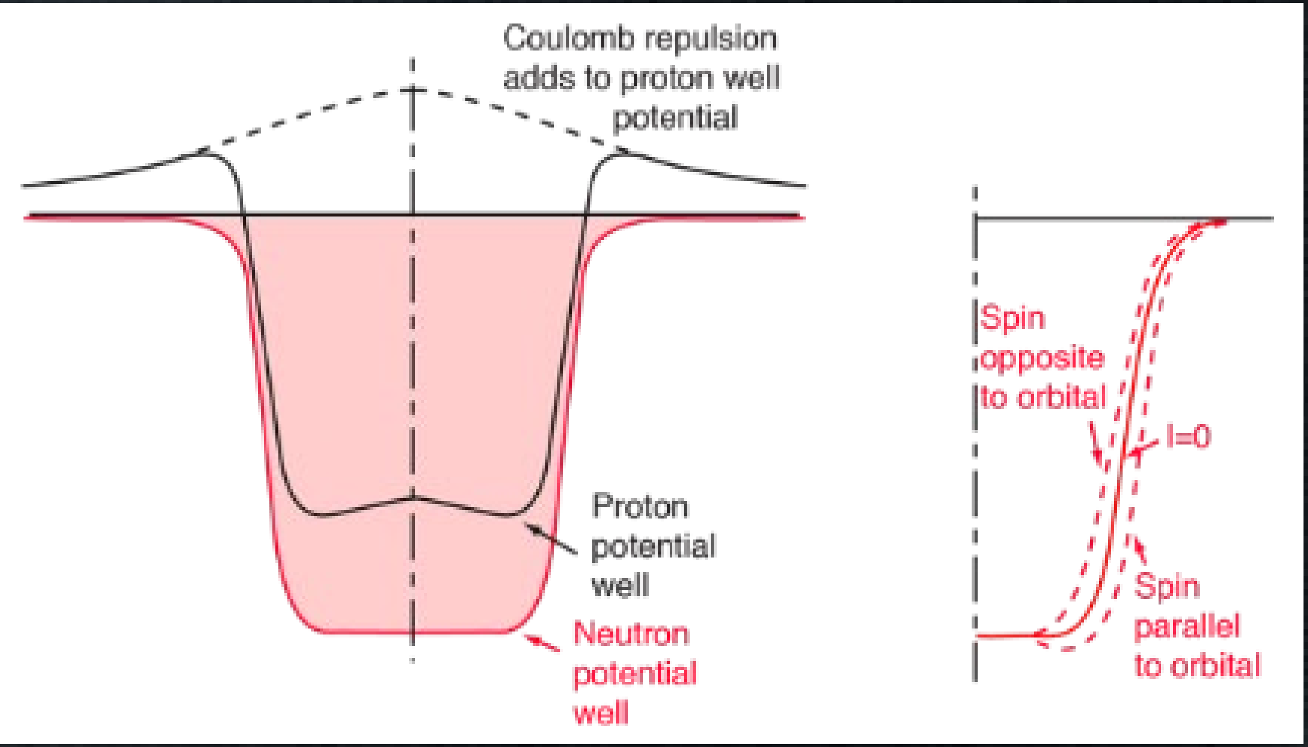
This model is applicable to nuclei with one single nucleon outside closed shell.

When more valence nucleons are considered, we have to include the residual interaction between these nucleons.

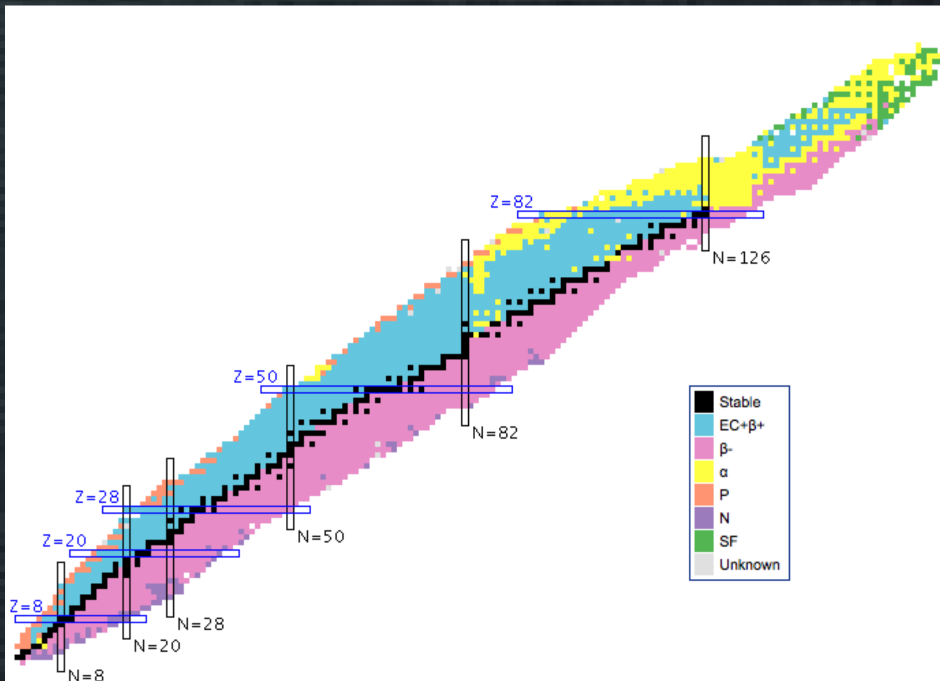
The simplest potentials are the square well and the harmonic oscillator, the more realistic one is the Woods-Saxon potential (no analytical solutions).

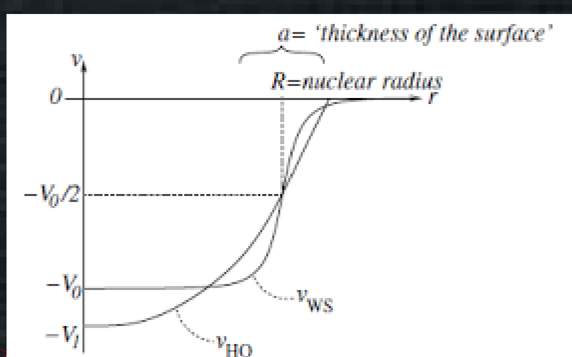
Woods-Saxon potential

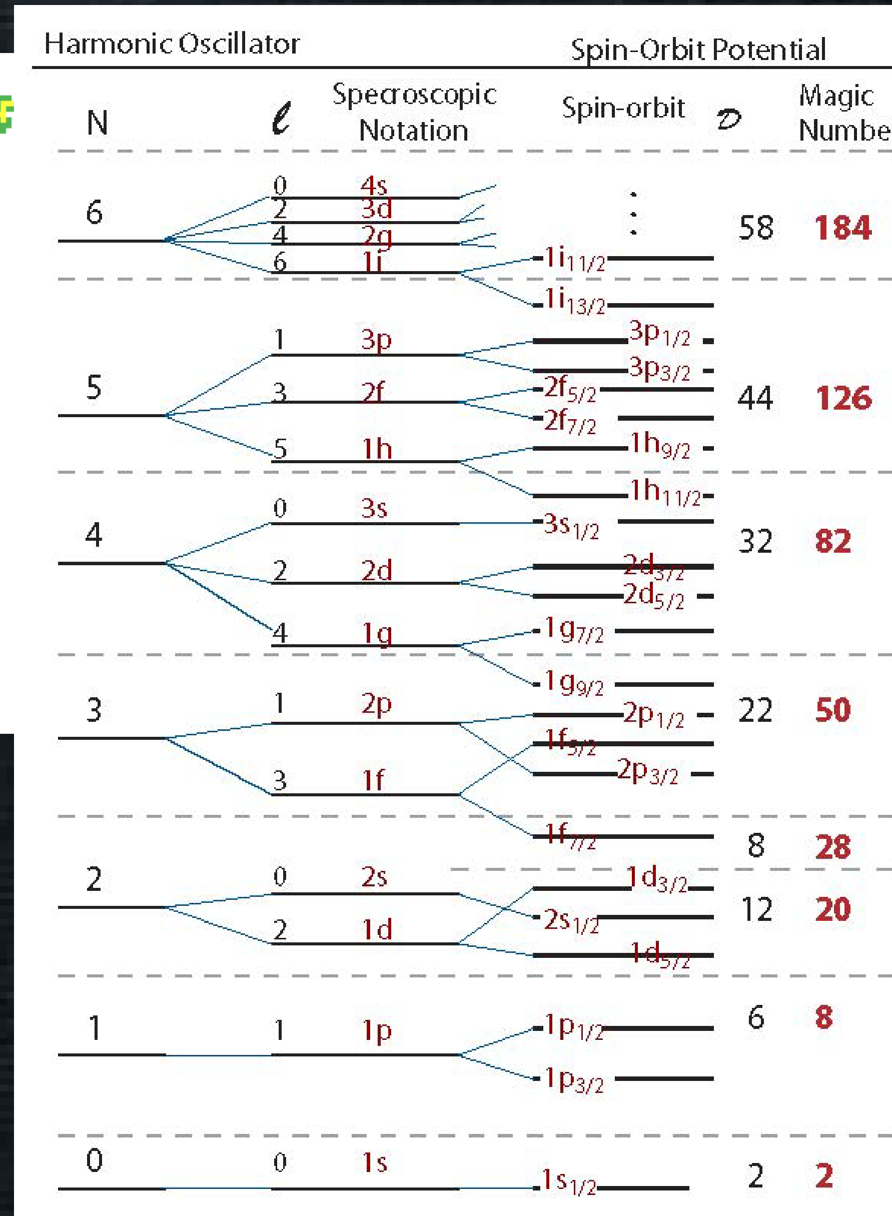
- The Woods-Saxon potential is considered to be the most realistic nuclear potential
- For real nuclei, the Coulomb potential has to be added.



The H.O. potential+ $\ell.\ell$ + $\ell.s$

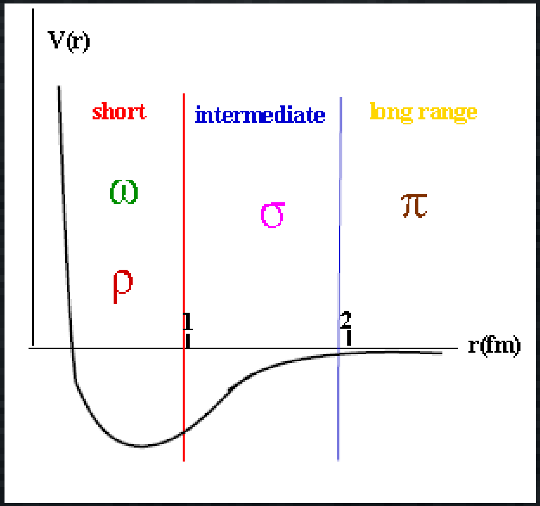






Independent Particle Motion and correlations

At low energy, the nucleus do manifest itself as a system of independent particles in many cases, and when it does not, it is due to the medium-range correlations that produce strong configuration mixing and not to the short-range repulsion.



The Interacting Shell Model

The Interacting Shell Model

It is an approximation to the exact solution of the nuclear A-body problem using effective interactions in restricted model spaces.

The only way to obtain a tractable problem is to define a new reference “vacuum”.

The many-body hamiltonian

We want to solve the Schrödinger equation $$H|\psi\rangle = \left(\sum_i T_i + \sum_{i<j} V_{ij} \right) |\psi\rangle = E|\psi\rangle$$

Now, we add and subtract a one-body potential U

$$H|\Psi\rangle = \left(\sum_i (T_i + U_i) + \sum_{i<j} V_{ij} - \sum_i U_i \right) |\Psi\rangle = E|\Psi\rangle$$

We express the Hamiltonian as

$$H|\Psi\rangle = (H_0 + H_1)|\Psi\rangle = E|\Psi\rangle$$

with the mean-field Hamiltonian

$$H_0 = \sum_i h_{0i} \qquad h_0|\varphi_i\rangle = (T_i + U_i)|\varphi_i\rangle = \varepsilon_i|\varphi_i\rangle$$

and the residual term is $$H_1 = \sum_{i<j} V_{ij} - \sum_i U_i$$

Where the one-body potential *U* can be optimized with a Hartree-Fock potential

The interacting shell model

We want to solve the eigenvalue problem

$$H|\Psi\rangle = (H_0 + H_1)|\Psi\rangle = E|\Psi\rangle$$

and E is the true energy of the system.

However, we will work in a "model space", not in the full Hilbert space. We thus need to construct an effective Hamiltonian H_{eff} acting in the model space, such that

$$H_{eff}|\Psi'\rangle = E|\Psi'\rangle$$

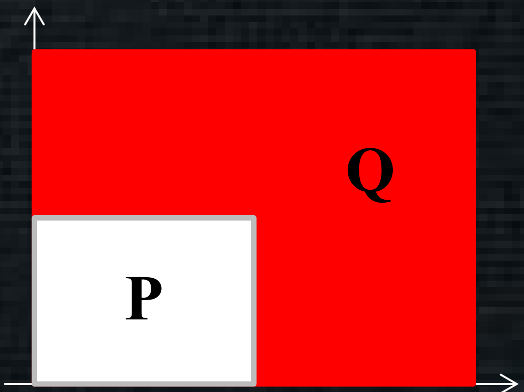
Reducing the model space

Reduce the Schrödinger equation of the A-body system to a secular equation acting only in the selected subspace, with the condition

$$\langle \psi' | H_{eff} | \psi' \rangle = E$$

we divide the full Hilbert space into a model space P and an excluded space Q which is achieved using projection operators

$$P = \sum_{i=1}^{d} |\psi_i\rangle\langle\psi_i| \qquad Q = \sum_{i=d+1}^{\infty} |\psi_i\rangle\langle\psi_i|$$



$$P^2 = P, \quad Q^2 = Q, \quad PQ = QP = 0$$

The effective interaction

Hilbert space → Valence space

$$V_{NN} \Rightarrow V^{eff}$$

$$\hat{H}\,|\Psi\rangle = E|\Psi\rangle \Rightarrow \hat{H}^{eff}\,|\Psi_{eff}\rangle = E|\Psi_{eff}\rangle$$

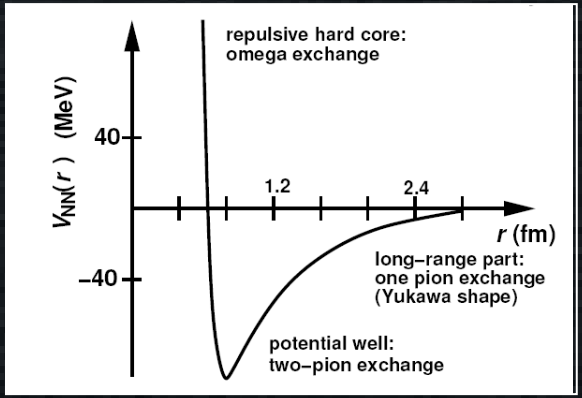
$$\langle\Psi|\hat{O}|\Psi\rangle \Rightarrow \langle\Psi_{eff}|\hat{O}_{eff}|\Psi_{eff}\rangle$$

- *microscopic* effective interaction (*realistic* interaction)
- *empirical* interaction (fitted to the data)
- *schematic* interaction (delta-force, etc)

Deriving a realistic effective interaction

All modern NN potentials fit equally well ($\chi^2/N_{data} \sim 1$) the deuteron properties and the NN scattering data up to the inelastic threshold: CD-Bonn, Argonne V18 , Nijm I, Nijm II, N^3LO potentials,...

However, these potentials cannot be used directly in the derivation of V_{eff} due to the strong short-range repulsion, but a renormalization procedure is needed.



Many-body methods:

- Brueckner G-matrix
- $V_{low\text{-}K}$: smooth low-momentum potential that is used in a perturbative approach to derive V_{eff}
- Etc…

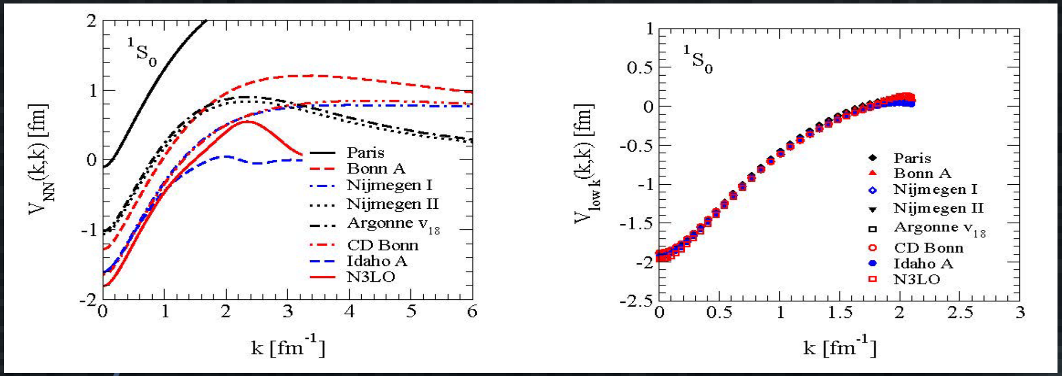
The $V_{low\text{-}k}$

Inspired by the effective field theory and renormalization group for low-energy systems

Realistic V_{NN} potential → Construct $V_{low\text{-}k}$ integrating out the high-momentum part of V_{NN} → preserves the physics of V_{NN} up to a cutoff momentum Λ

Features of $V_{low\text{-}k}$

- \- eliminates sources of non-perturbative behavior
- \- real potential in the *k*-space
- \- gives an approximately unique representation of the NN potentials for $\Lambda \cong 2\ \mathrm{fm}^{-1}$ $E_{Lab} \cong 350$ MeV



The model space

The success of the independent particle model strongly suggests that the very singular free NN interaction can be regularized in the nuclear medium.

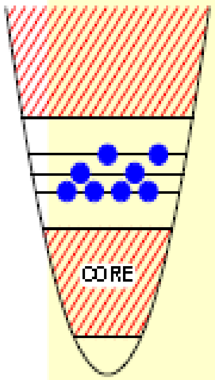
For a given number of protons and neutrons the mean field orbitals can be grouped in three blocks:

- **Inert core**: orbits that are always fully occupied
- **Valence space**: orbits around the Fermi surface that contain the physical degrees of freedom relevant to a given property. The distribution of the valence particles among these orbitals is governed by the interaction
- **External space**: all the remaining orbits that are always empty

Shell model approach

Calculations Ab Initio ☞QCD

- Realistic NN interactions
- 3-body forces



- define valence space
- $H_{\rm eff}\Psi_{\rm eff} = E\Psi_{\rm eff}$

⇝ INTERACTIONS (monopole corrections)

- build and diagonalize Hamiltonian matrix

⇝ CODES

Weak processes:
- β decays
- $\beta\beta$ decays

$$[T_{1/2}^{0\nu}(0^+ \to 0^+)]^{-1} = G_{0\nu}|M^{0\nu}|^2\langle m_\nu\rangle^2$$

☞ASTROPHYSICS

☞PARTICLE PHYSICS

Collective excitations:
- deformation, superdeformation
- superfluidity
- symmetries

Shell evolution far from stability:
- Shell quenching
- New magic numbers

☞ASTROPHYSICS

F. Nowacki

Eigenvalues and eigenfunctions

We solve the Schrödinger equation in the model space

$$H\,|\Psi\rangle = E|\Psi\rangle$$

$$H = H_0 + H_{res} = \sum_{i=1}^{A} h_0(i) + H_{res}$$

$$E = E_0 + E_{res}$$

The solutions of the central s.p. Hamiltonian are

$$h_0|\varphi_i\rangle = (T_i + U_i)|\varphi_i\rangle = \varepsilon_i|\varphi_i\rangle$$

A-particles wave functions

$$\psi_k^0(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A) = \frac{1}{\sqrt{A!}}\det\begin{pmatrix} \varphi_1(\vec{r}_1) & \varphi_1(\vec{r}_2) & \ldots & \varphi_1(\vec{r}_A) \\ \vdots & & \ddots & \vdots \\ \varphi_A(\vec{r}_1) & \varphi_A(\vec{r}_2) & \ldots & \varphi_A(\vec{r}_A) \end{pmatrix}$$

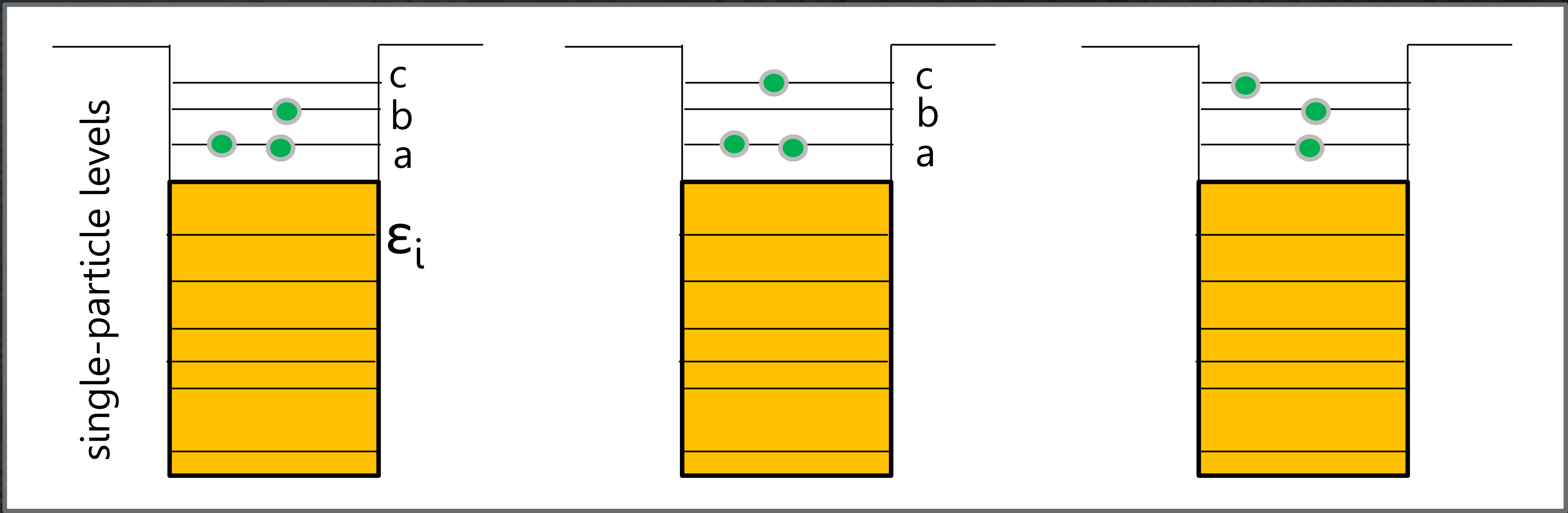
$$E_k^0 = \sum_{i=1}^{A} \varepsilon_i$$

$$H_0|\psi_i^0\rangle = E_i^0|\psi_i^0\rangle$$

$$|\Psi\rangle = \sum_{k=1}^{n} a_k|\psi_k^0\rangle$$

The final solution will be expanded in terms of Slater determinants of the s.p. solutions

The concept of “configuration”



configuration 1

$$\psi_1^0(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A)$$

configuration 2

$$\psi_2^0(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A)$$

configuration 3

$$\psi_3^0(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A)$$

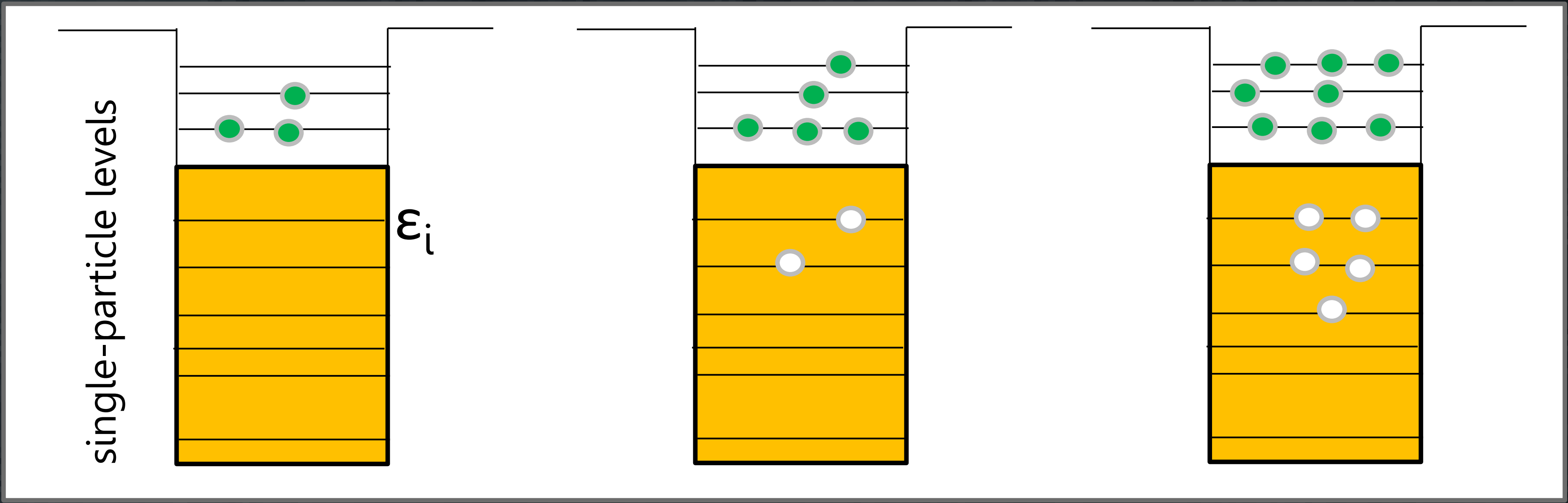
occupation numbers

orbit a: 2
orbit b: 1
orbit c: 0

orbit a: 2
orbit b: 0
orbit c: 1

orbit a: 1
orbit b: 1
orbit c: 1

The concept of "particle-hole configuration"



configuration 1

$$\psi_1^0(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A)$$

3p-0h

configuration 2

$$\psi_2^0(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A)$$

5p-2h

configuration 3

$$\psi_3^0(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A)$$

8p-5h

The "correlated" wave function

The correlated wave function of the *p*-th state in the A-particle system will be a linear combination of the different configurations represented by the slater determinants ψ_k^0

$$H\left|\Psi_p\right\rangle = E_p\left|\Psi_p\right\rangle$$

$$\left|\Psi_p\right\rangle = \sum_{k=1}^{n} a_{pk}\left|\psi_k^0\right\rangle$$

The probability of each configuration *k* in the *p*-th state will be $\boldsymbol{a_{pk}^2}$

The normalization of the wave function imposes that

$$\sum_{k=1}^{n} a_{pk}^2 = 1$$

The general case

If the basis set is defined as $\left|\psi_k^0\right\rangle (k=1,\dots,n)$

$$\psi_k^0(\vec{r}_1,\vec{r}_2,\dots,\vec{r}_A)=\frac{1}{\sqrt{A!}}\det\begin{pmatrix}\varphi_1(\vec{r}_1) & \varphi_1(\vec{r}_2) & \dots & \varphi_1(\vec{r}_A)\\ \vdots & & \ddots & \vdots\\ \varphi_A(\vec{r}_1) & \varphi_A(\vec{r}_2) & \dots & \varphi_A(\vec{r}_A)\end{pmatrix}$$

The total wave function can be expanded as: $\left|\Psi_p\right\rangle=\sum_{k=1}^{n}a_{kp}\left|\psi_k^0\right\rangle$

The coefficients a_{kp} have to be determined by solving the Schrödinger equation

$$H\left|\Psi_p\right\rangle=E_p\left|\Psi_p\right\rangle \Rightarrow$$

$$(H_0+H_{res})\sum_{k=1}^{n}a_{kp}\left|\psi_k^0\right\rangle=E_p\sum_{k=1}^{n}a_{kp}\left|\psi_k^0\right\rangle$$

or

$$\sum_{k=1}^{n}\left\langle\psi_l^0\right|H_0+H_{res}\left|\psi_k^0\right\rangle a_{kp}=E_p a_{lp}$$

Configuration mixing

If the non-diagonal matrix elements are of the order of the unperturbed energy differences

$$|H_{ij}| \approx |E_i^0 - E_j^0|$$

large configuration mixing will result and the final energy eigenvalues E_p will be very different from the unperturbed ones.

On the contrary, if the non-diagonal matrix elements are small, these energy shifts will be small and we can use perturbation theory to solve the problem

$$|H_{ij}| << |E_i^0 - E_j^0|$$

Shell model basis

The basis states have good angular momentum (coupling all j values to J), good parity and good isospin.

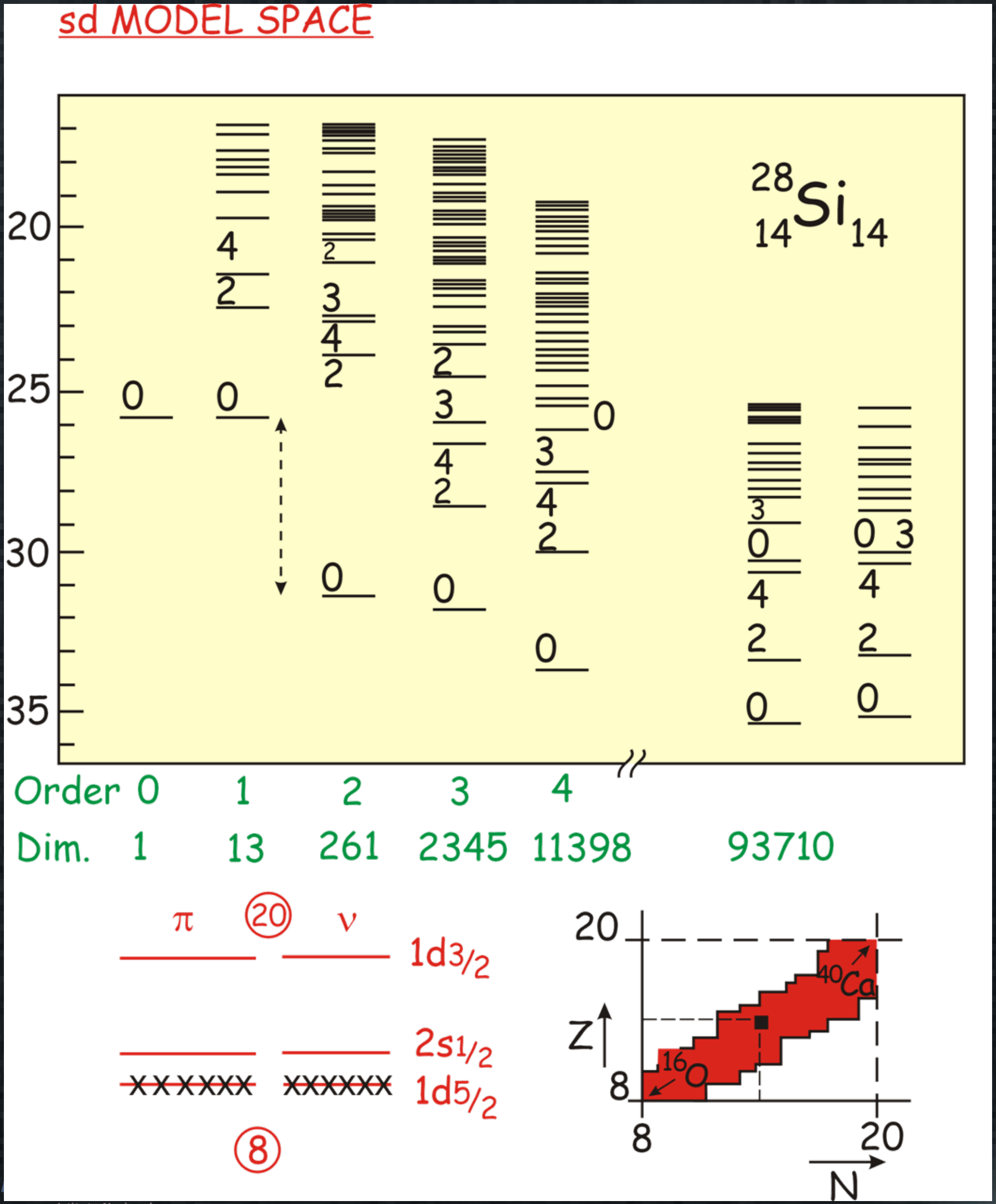
The number of basis states can be estimated approximately as :
(Ω: shell degeneracy, n: valence particles)

$$N_{basis} = \binom{\Omega_p}{n_p}\binom{\Omega_n}{n_n}$$

For example, for ^{60}Zn with 10 valence protons and 10 valence neutrons in the *fp* shell

$$N_{basis} = \binom{20}{10}\binom{20}{10} \approx 3.4 \cdot 10^{10}$$

An example



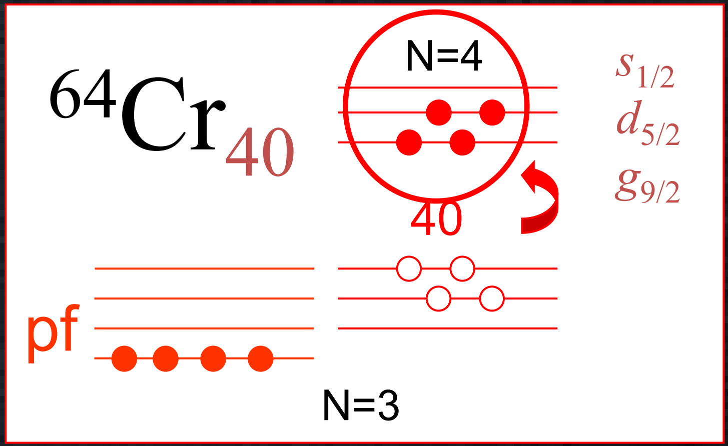
K. Heyde

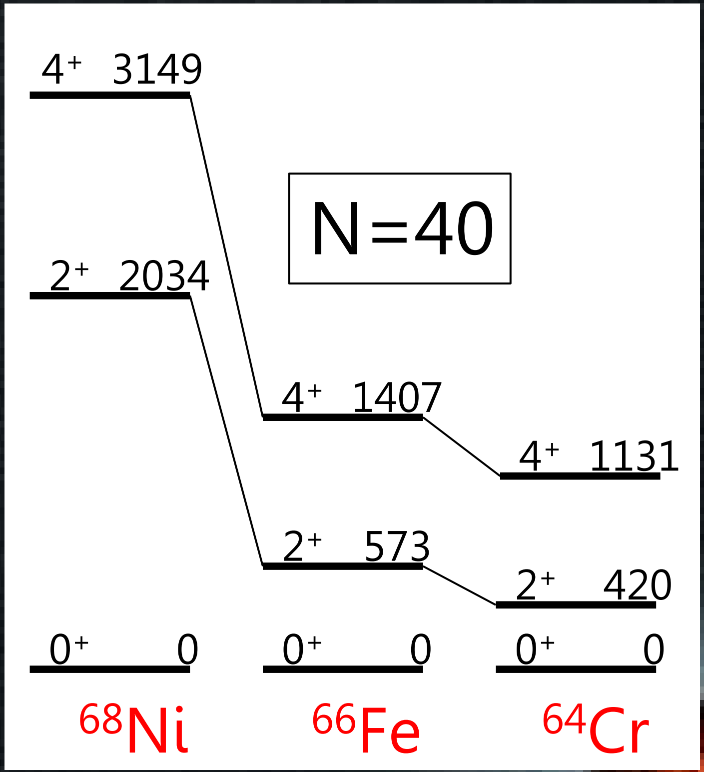
Can the Shell Model reproduce deformed and rotational nuclei?

The island of inversion at N=40

At N=40 the h.o. shell gap vanishes for very neutron rich nuclei.

Deformed intruder configurations fall below the spherical ones

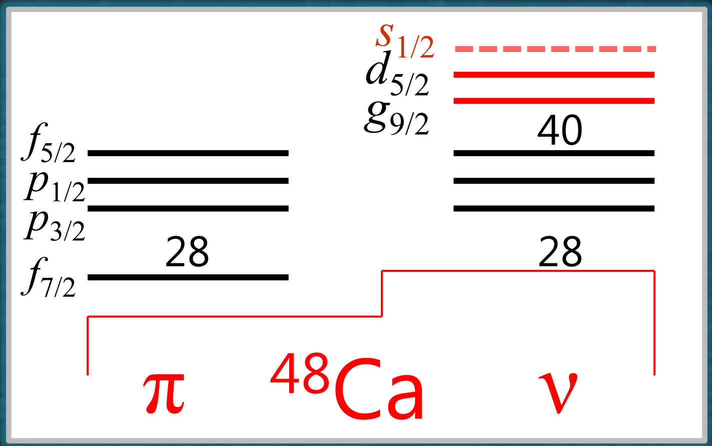




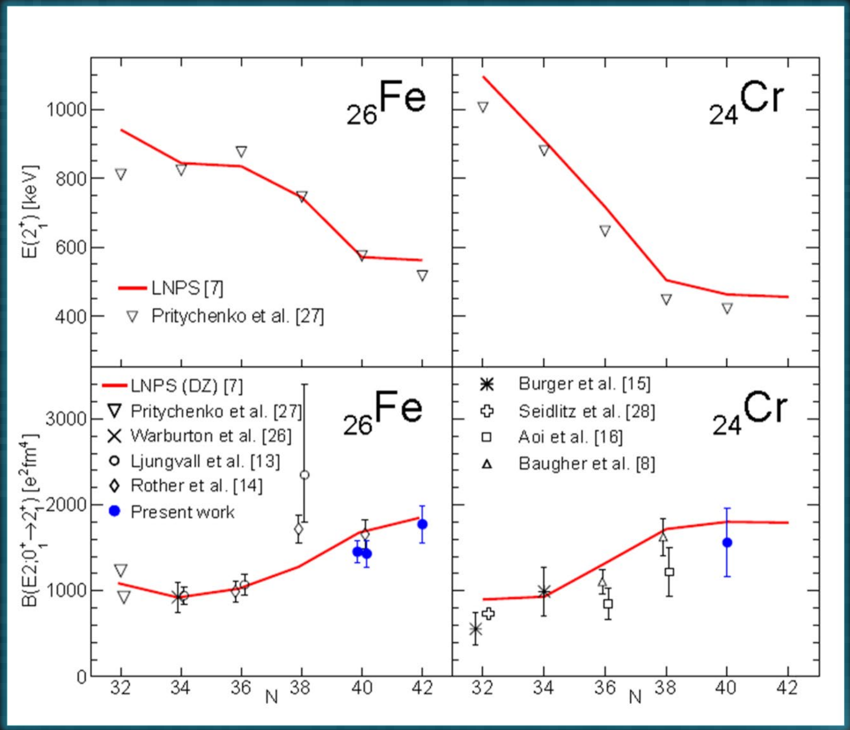
Deformation and SM in the fpgd space

LNPS interaction: renormalized realistic interaction
+ monopole corrections

^{48}Ca core
protons: full *pf* shell
neutrons: $p_{3/2}$, $f_{5/2}$, $p_{1/2}$, $g_{9/2}$, $d_{5/2}$



Lenzi, Nowacki, Poves, Sieja
(LNPS), PRC 82, 054301 (2010)



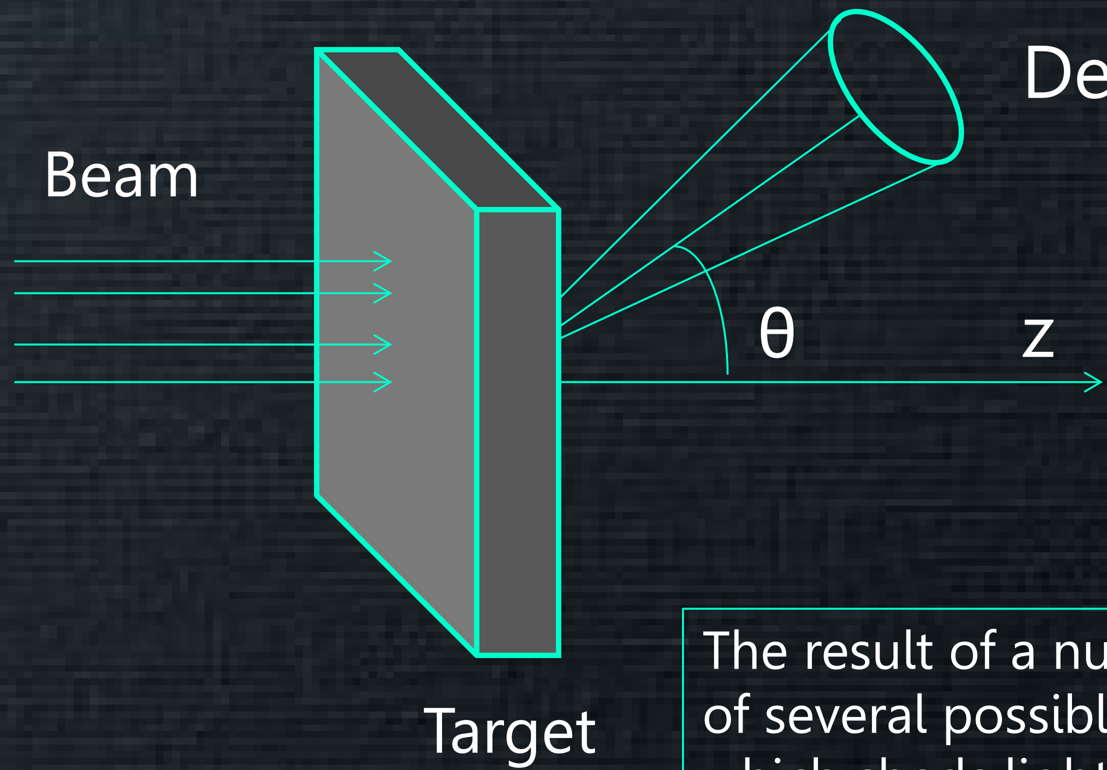
H. L. Crawford et al., PRL 110, 242701 (2013)
T. Baugher et al., PRC 86, 011305(R) (2012)

Other effective interactions:
$V_{low\ k}$: L. Coraggio et al., PRC 89, 024319 (2014).
A3DA: Tsunoda et al., PRC 89, 031301 (2014).

Experimental methods

Probing the nuclear behaviour

To probe their properties of a nucleus, physicists need to disturb the system.



A nuclear collision leaves, in general, nuclei in excited states

The result of a nuclear collision may be one of several possible nuclear reactions, each of which sheds light on a particular aspect of nuclear structure or nuclear behaviour.

Probability of a nuclear reaction

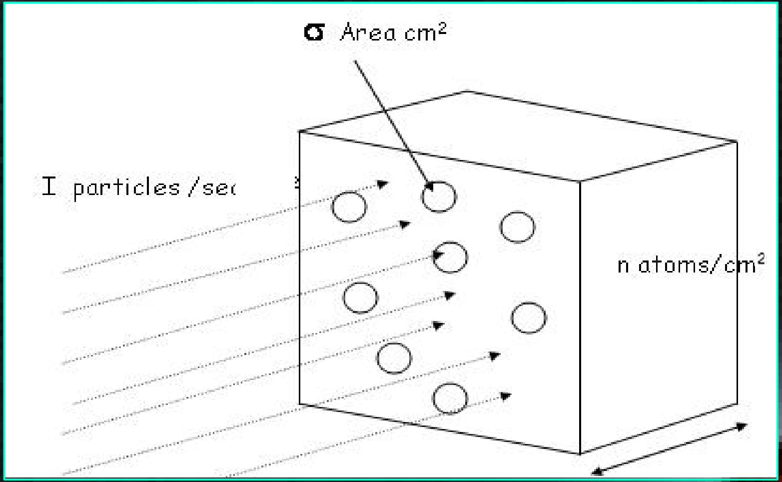
Consider an ion beam that impinges on a target. The total number of reactions (N) that will take place can be calculated with this formula:

$$N = I \cdot t \cdot n_t \cdot \sigma$$

where:

I: **intensity of the incident flux**, measured in particles per second (s^{-1}).
t: **bombarding time**, measured in seconds (s).
n_t: **areal density**: atoms per square cm (cm^{-2}).
σ: scaling factor that represents the probability that the reaction takes place

$[\sigma] = cm^2$
the units of σ are those of an area and therefore is called "*cross section*".

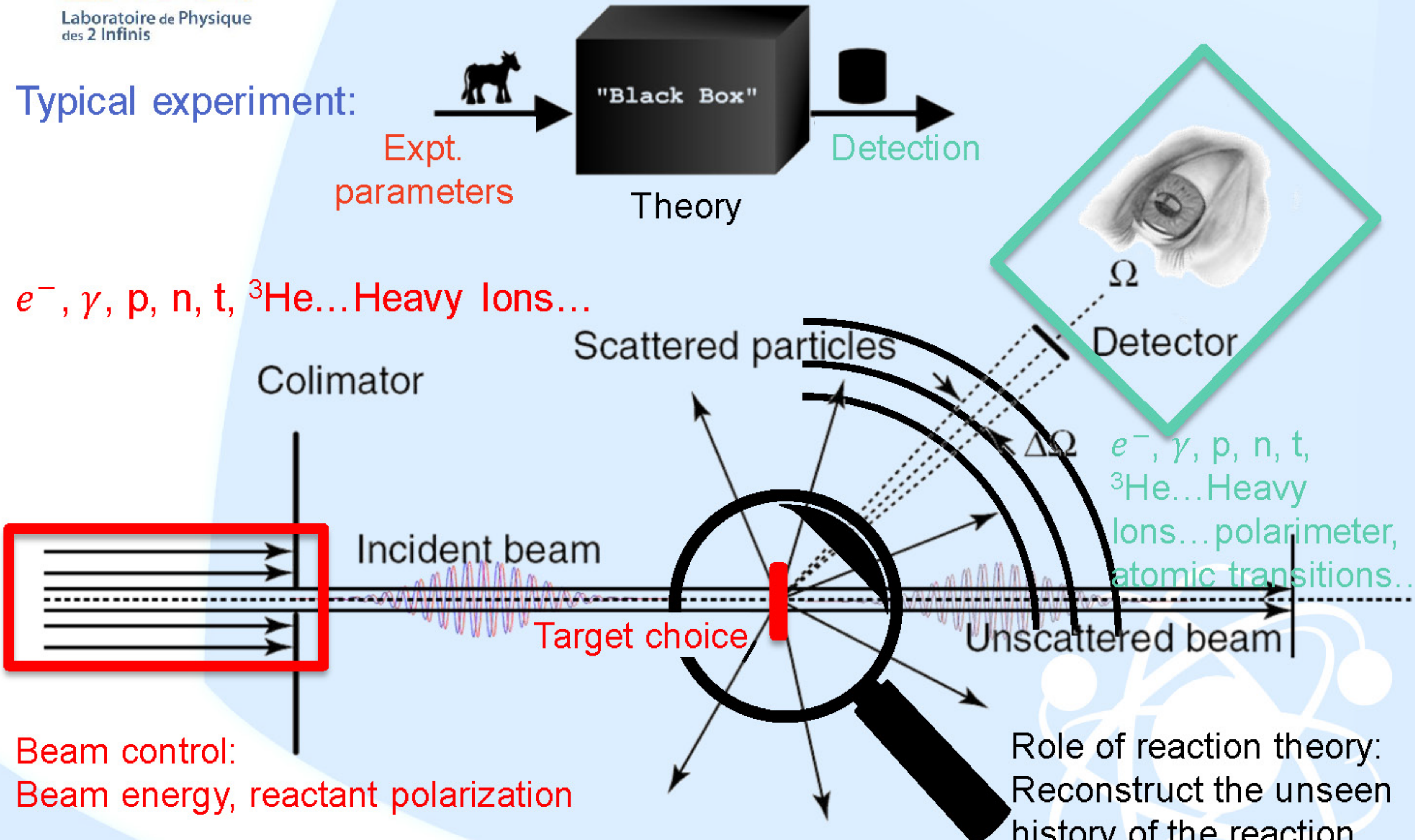


This is the "effective" area that a single target nucleus presents to the incoming beam for a specific reaction to occur.

SCHEMATIC VIEW OF A NUCLEAR REACTION

Typical experiment:

e^-, γ, p, n, t, ^{3}He…Heavy Ions…



Beam control:
Beam energy, reactant polarization

Role of reaction theory:
Reconstruct the unseen
history of the reaction

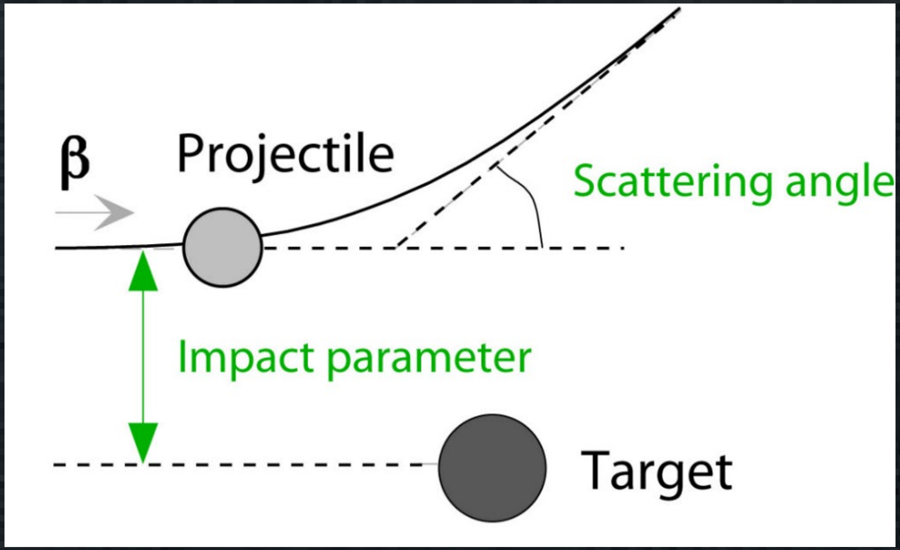
Courtesy G. Hupin

Some useful nuclear reactions

- Coulomb excitation and inelastic reactions (no exchange of matter)
- transfer reactions (exchange of nucleons)
- multi-nucleon transfer and deep inelastic
- fusion-evaporation
- fusion-fission reactions
- fission
- fragmentation
  * projectile fragmentation
  * target fragmentation

Coulomb excitation

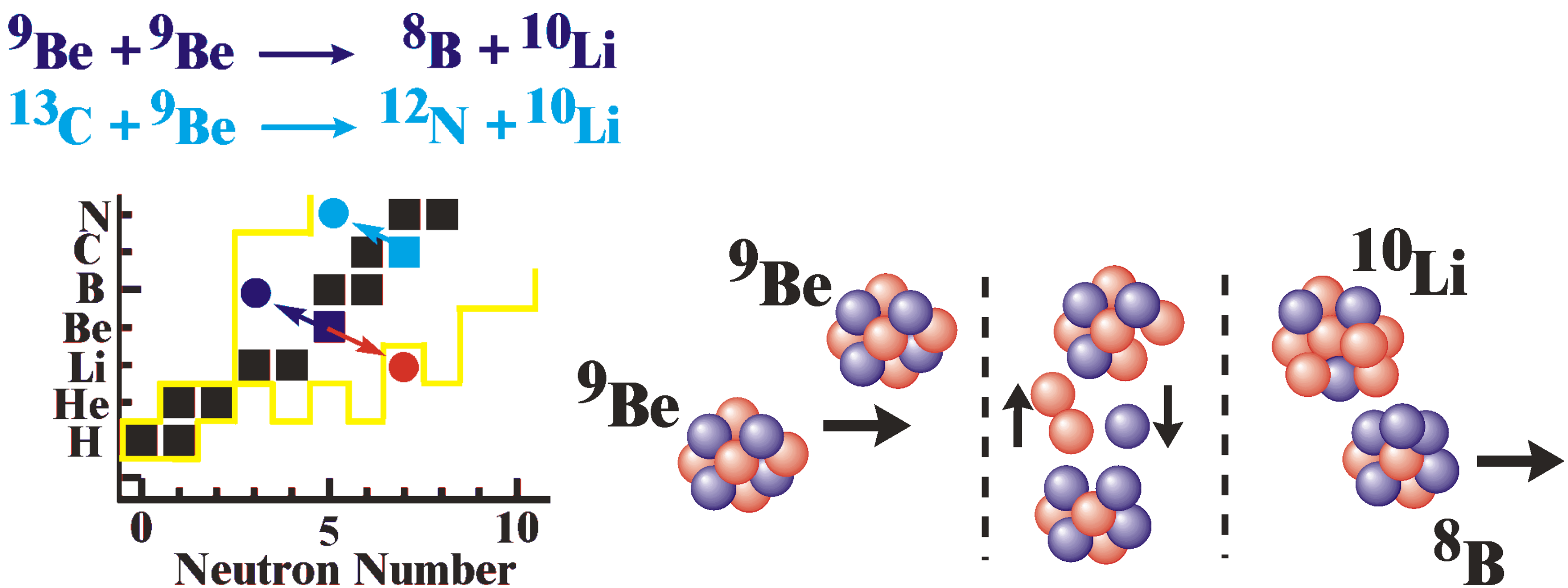
A projectile is excited in the Coulomb field of a heavy target impinging on it at "safe" sub-barrier energies so that the nuclear field is not at play



This method is particularly useful for investigating collectivity in nuclei, as collective excitations are often connected by electric quadrupole transitions.
These reactions populate excited states with low angular momentum and excitation energy.

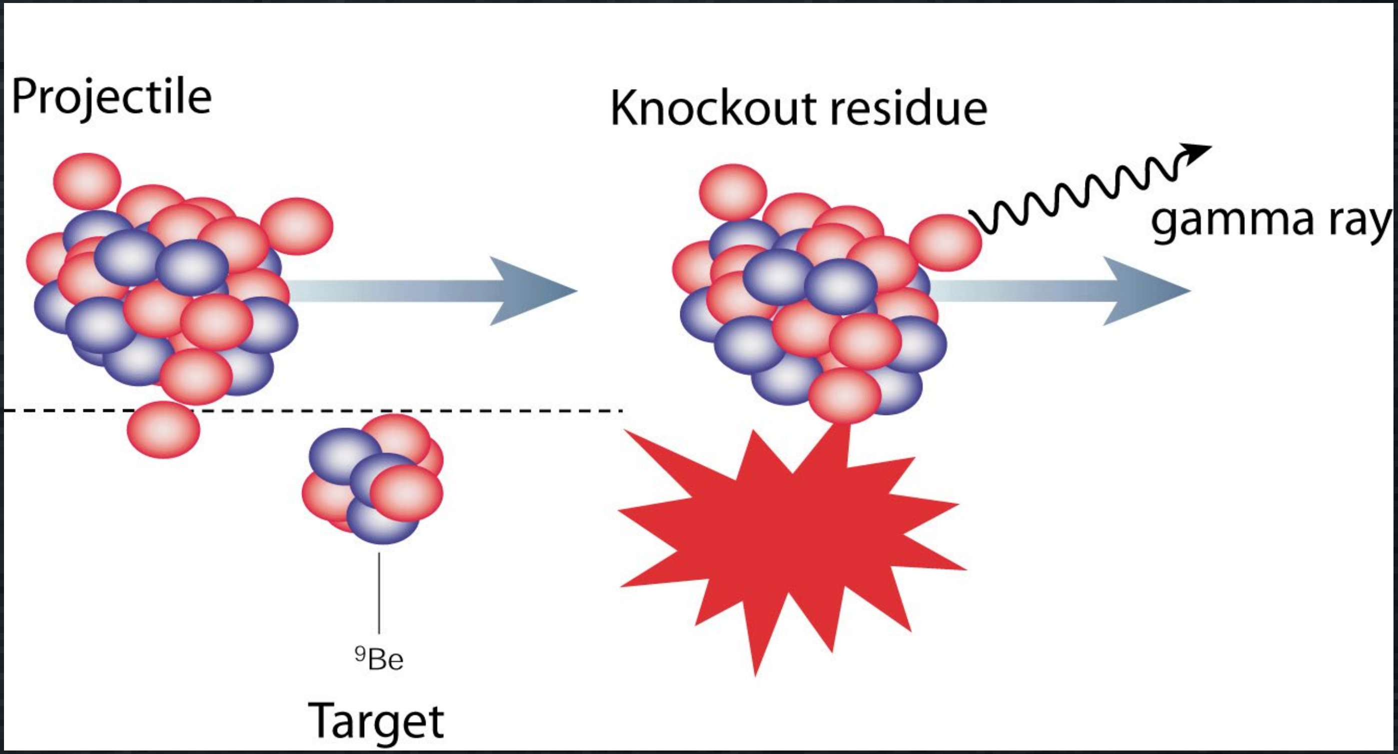
Transfer Reactions

The interacting nuclei exchange few nucleons. These are peripheral reactions and populate excited states with low angular momentum and excitation energy

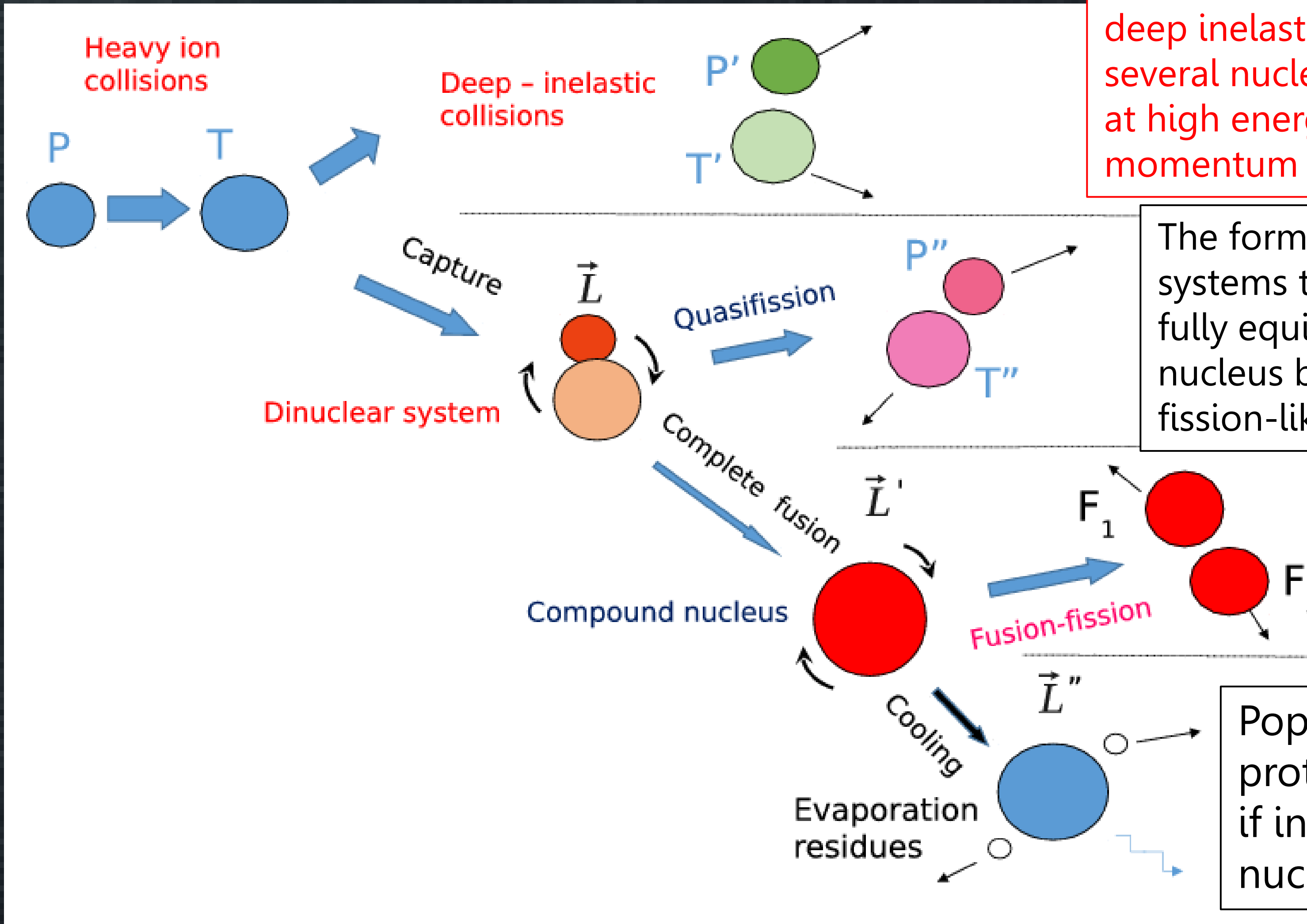


Knock-out Reactions

In knockout reactions, a nucleon is removed from the projectile nucleus upon peripheral collision with a light reaction target (typically ^{9}Be).
The momentum of the knocked-out nucleon and that of the final residue are related by momentum conservation.



Deep-inelastic, fusion and fission



Multinucleon transfer and deep inelastic: nuclei exchange several nucleons and are excited at high energy and angular momentum

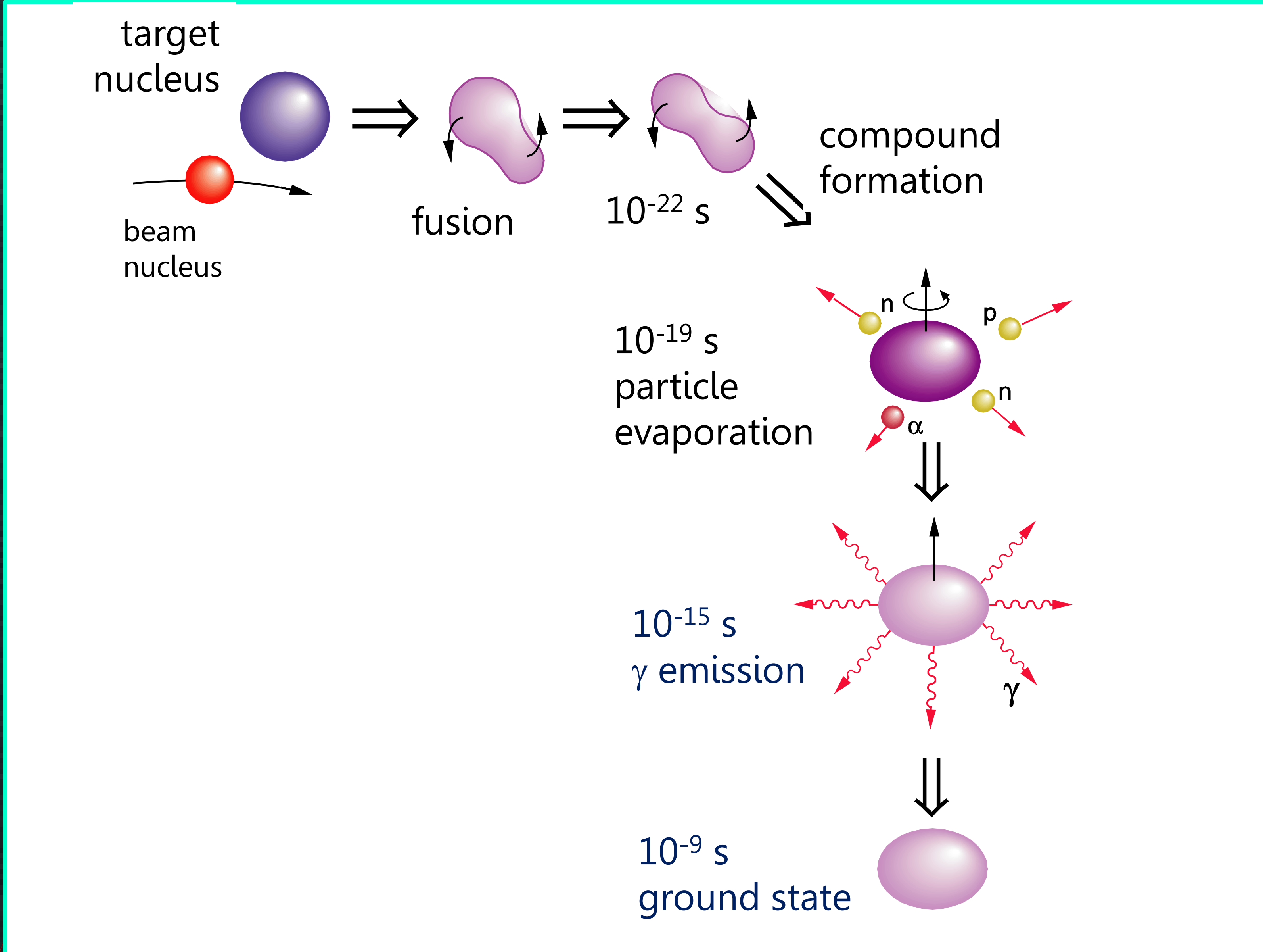
The formation of composite systems that do not reach the fully equilibrated compound nucleus before splitting in two fission-like fragments

Involve mainly heavy nuclei

Populate mainly proton-rich nuclei if induced by stable nuclei

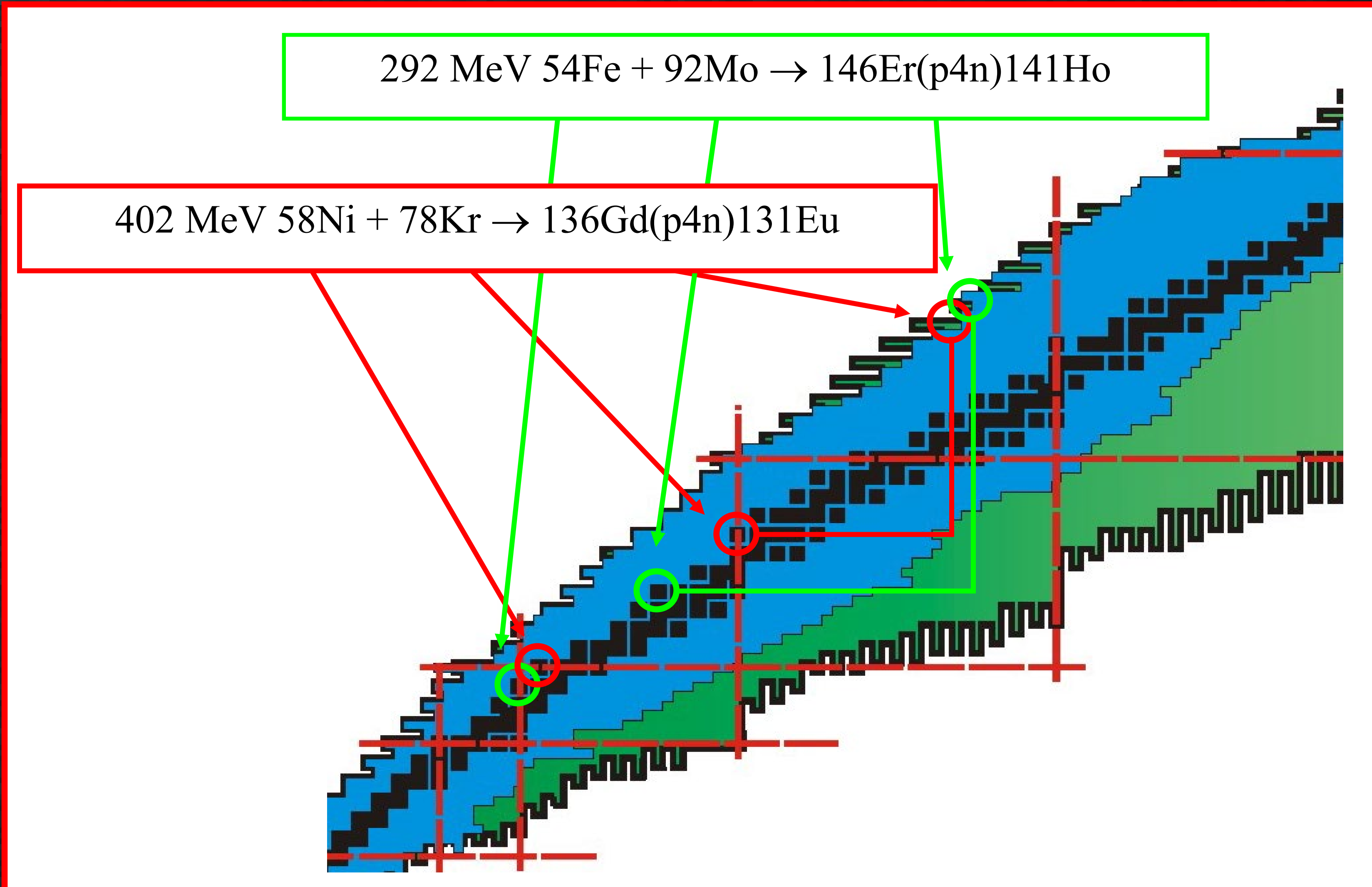
In all cases, nuclei are formed in an excited state from where decay to the g.s.

Fusion-evaporation dynamics



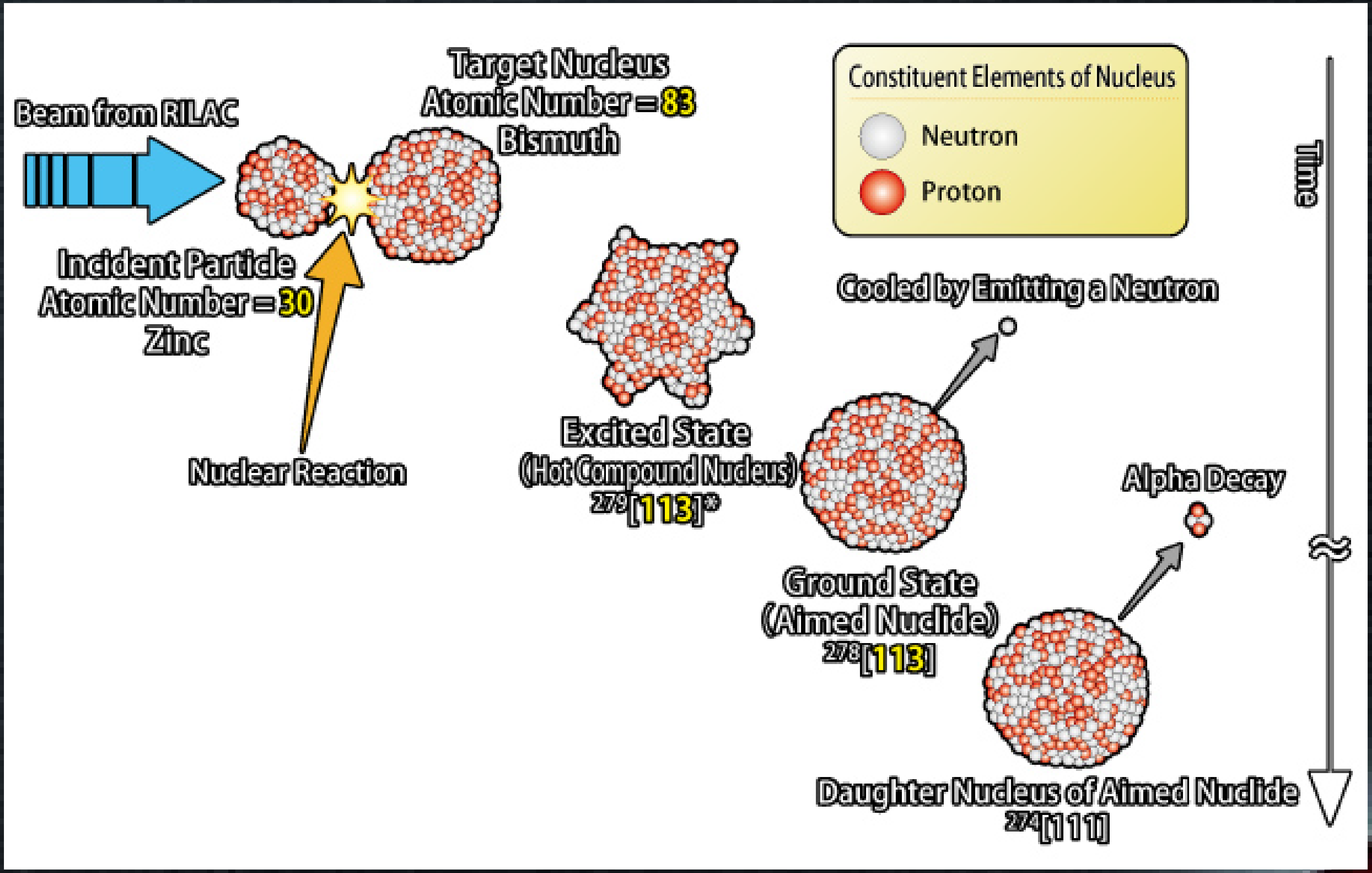
Fusion-evaporation reactions

create proton-rich systems at high angular momentum

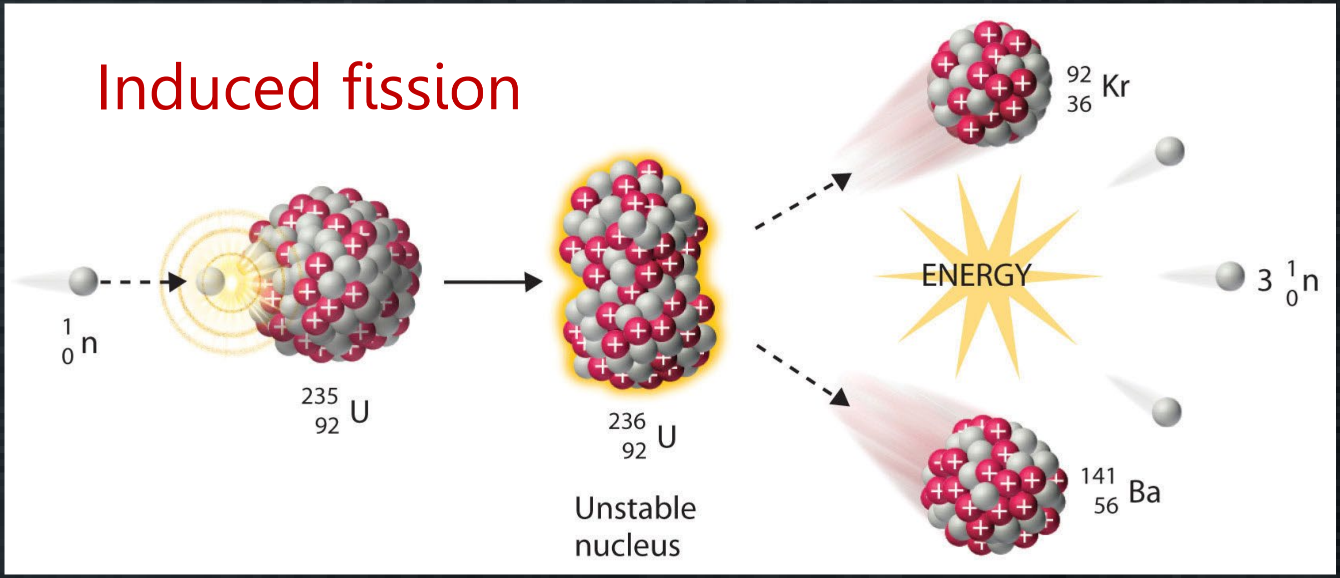


Production of superheavy nuclei

Example: Element Z=113 Nihonium (Japan/Nihon: The land of rising sun)



Fission

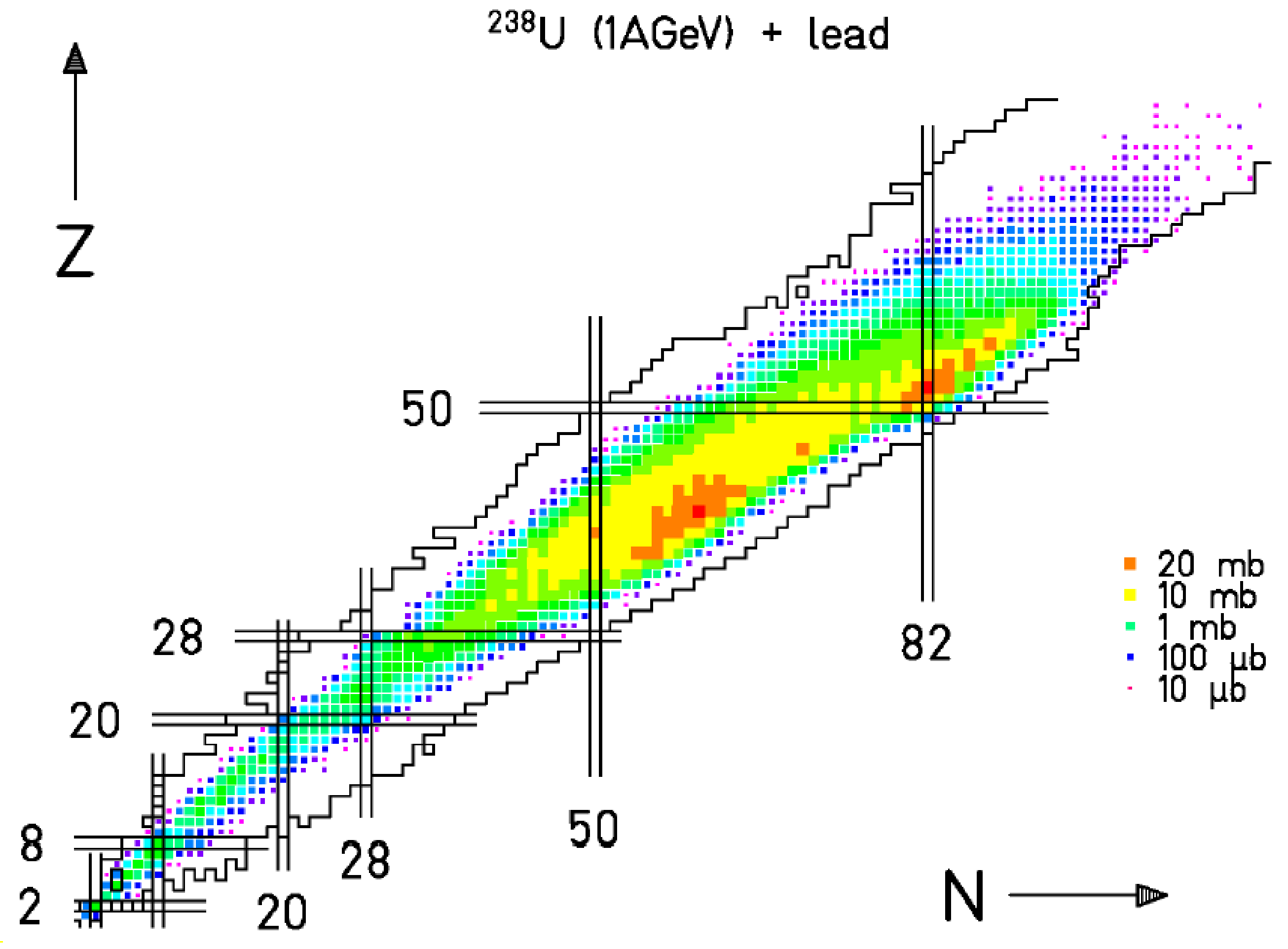


spontaneous fission

Increases with the mass. It is observed above A~232. An example is ^{252}Cf (half-life 2.645 years, with a branching ratio of about 3.1% of SF)

Fission of heavy nuclei

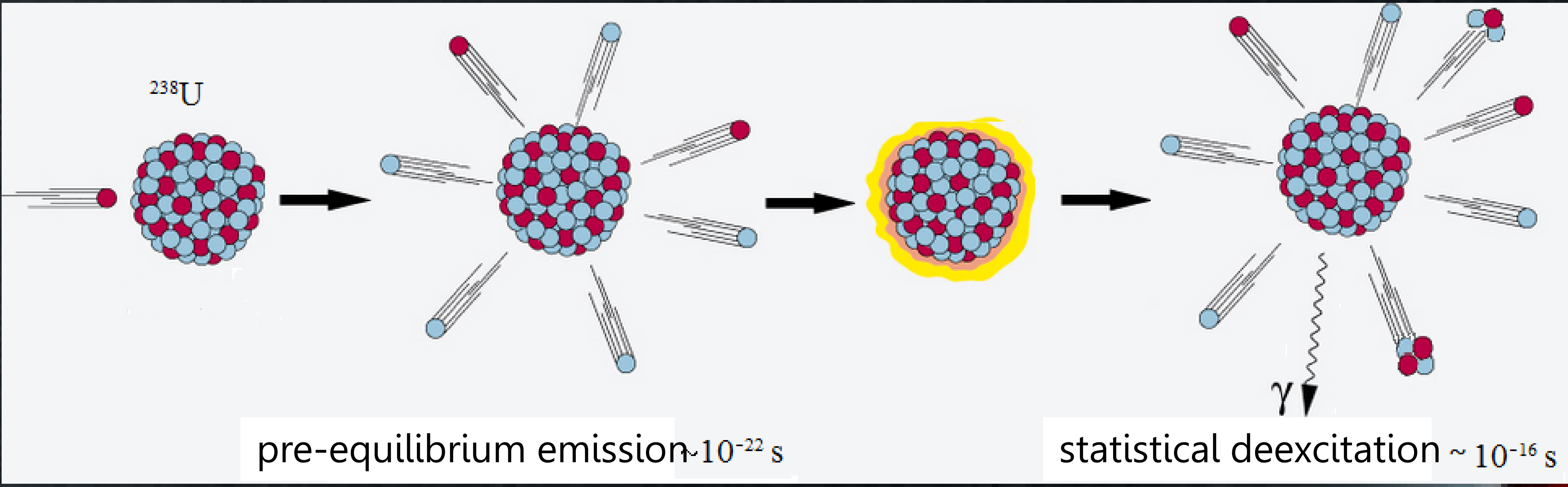
creates neutron-rich systems



K.H. Schmidt *et al.,* Model predictions of the fission-product yields for 238U (2001)

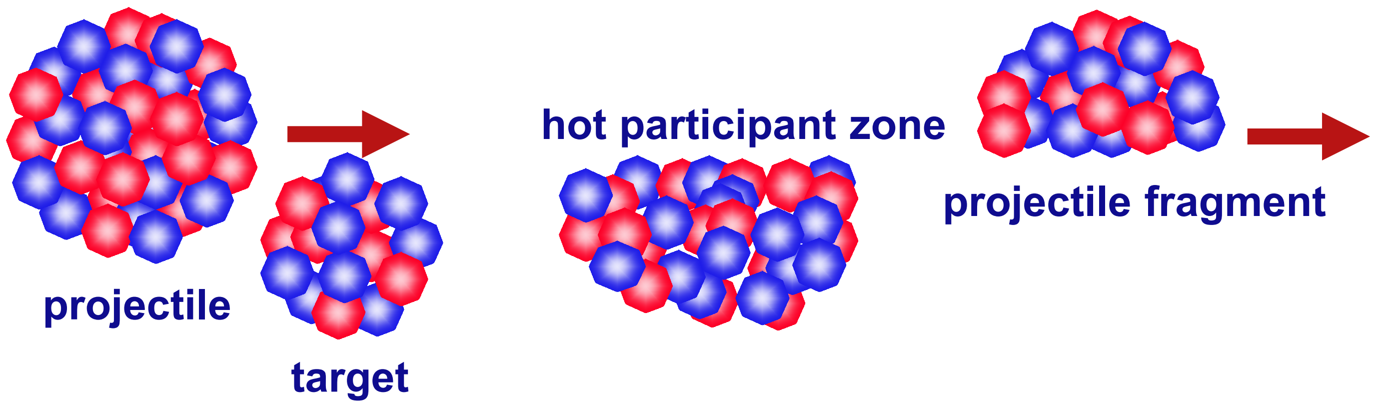
Target fragmentation

A very energetic light projectile is bombarded on a heavy target.
Protons, neutrons, etc. are emitted (pre-equilibrium and statistical emission)

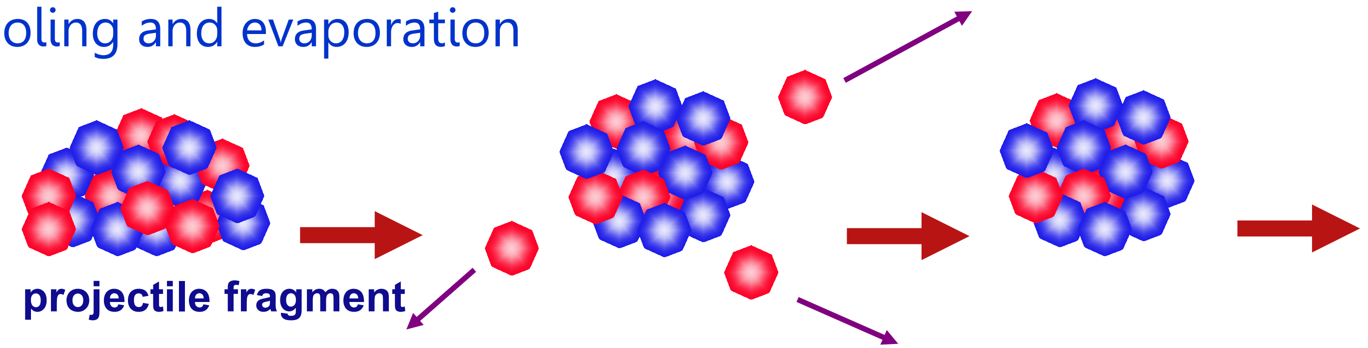


Projectile fragmentation

In inverse kinematics (projectile heavier than target)

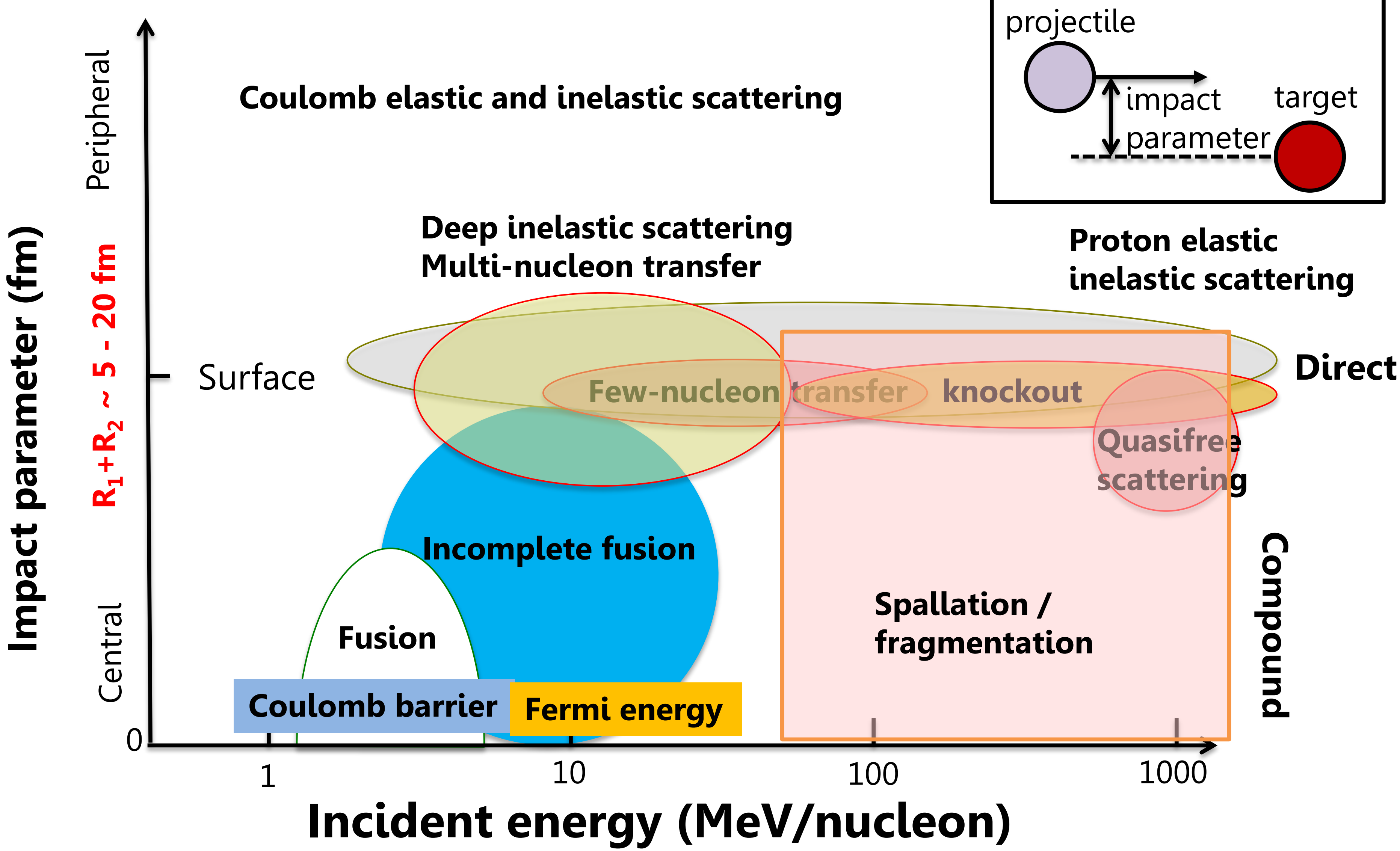


Cooling and evaporation

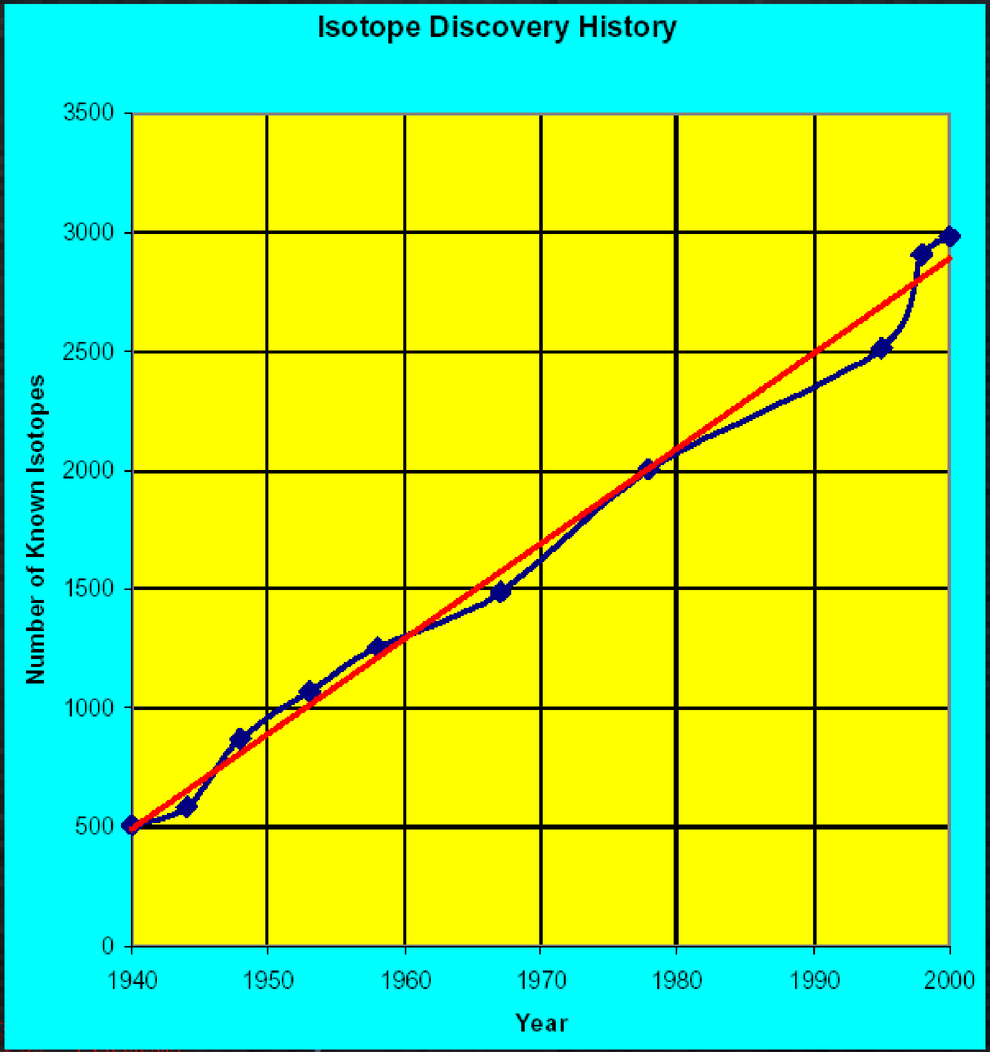


Nuclear Reactions

A. Obertelli @ Euroschool



Discovery History



Today around 3000 isotopes have been observed

Only 284 are stable

Other ~3000 have still to be produced

Press Release

Previous Index Next

June 8, 2010

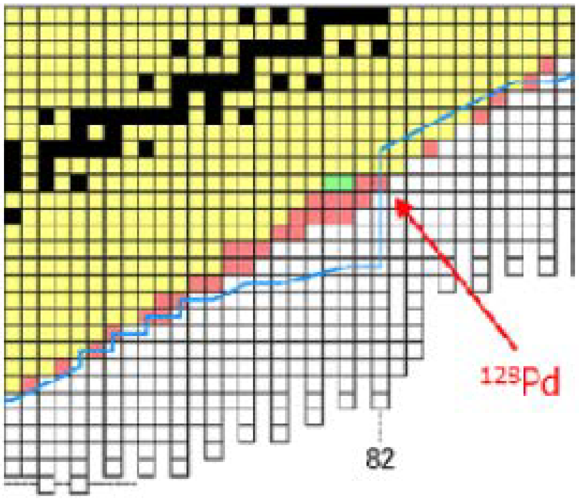
Like 0 Tweet

Scientists discover 45 new radioisotopes in 4 days

The world's most powerful beam of heavy ions has enabled Japanese scientists and their international collaborators to uncover 45 new neutron-rich radioisotopes in a region of the nuclear chart never before explored. In only four days, a team of researchers at the RIKEN Nishina Center for Accelerator-Based Science (RNC) have identified more new radioisotopes than the world's scientists discover in an average year.

Radioactive isotopes (RI) or radioisotopes, unstable chemical elements with either more or fewer neutrons than their stable counterparts, open a door onto a world of nuclear physics where standard laws break down and novel phenomena emerge.

The RNC's Radioactive Isotope Beam Factory (RIBF) was created to explore this world, boasting an RI beam intensity found nowhere else in the world. Accelerated to 70% the speed of light using RIBF's Superconducting Ring Cyclotron, uranium-238 nuclei are smashed into beryllium and lead targets to produce an array of exotic radioisotopes believed to play a central role in the origins of elements in our universe.



Newly-discovered radioisotopes (red) on nuclear chart

BIGRIPS at RIKEN (Japan)

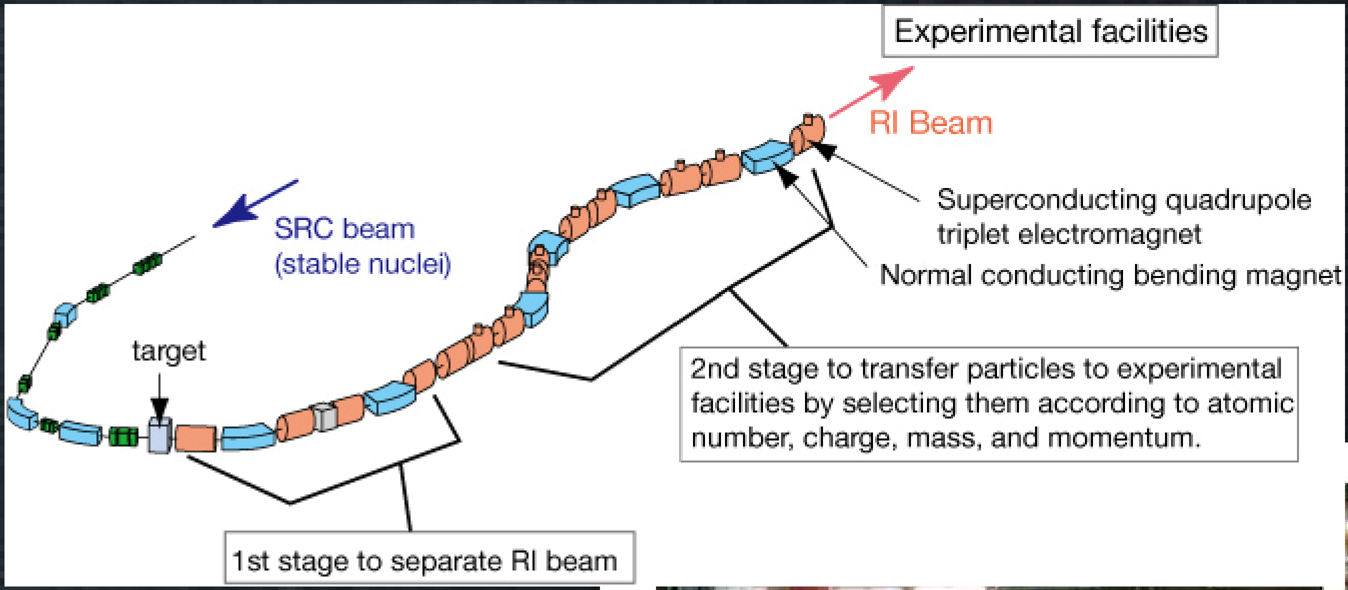
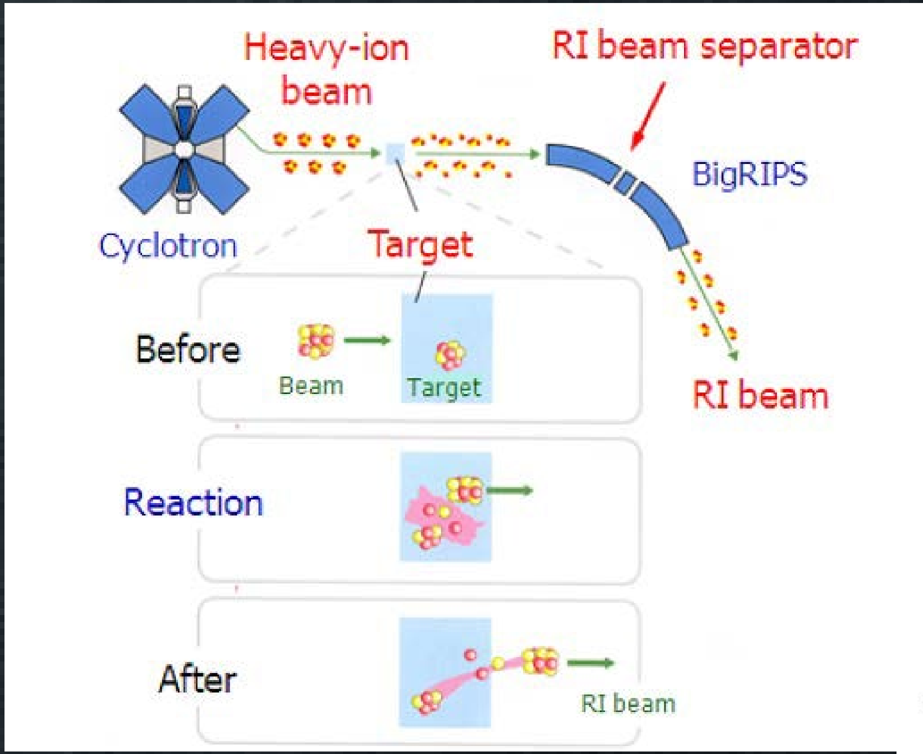


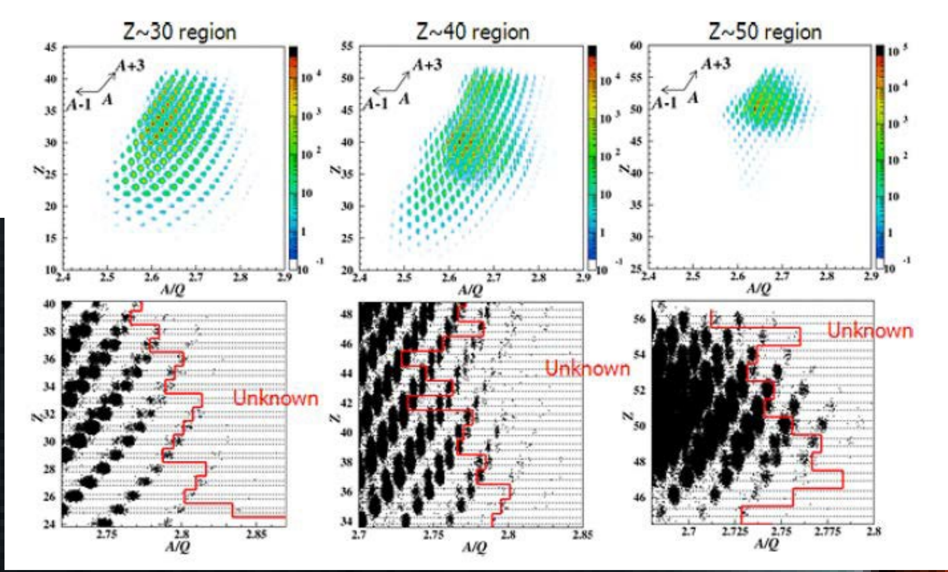
Figure 4: Photographs of the first stage (left) and second stage (right) of the BigRIPS separator

Selecting the isotopes of interest



Production of radioisotopes using heavy-ion beam and radioactive ion beam separator

Three different settings of the BigRIPS separator, each targeting new radioisotopes in the region with atomic numbers around 30, 40, and 50, respectively



Production of radioactive beams

Technical issues:

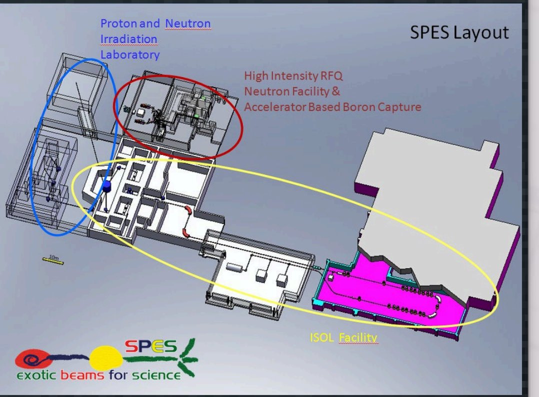
- Efficiency: Produce enough quantity of nuclei in order to be able to study them
- Selectivity: Purity of the beam
- Velocity: Necessity of reacceleration
- Time: Short lifetimes into play

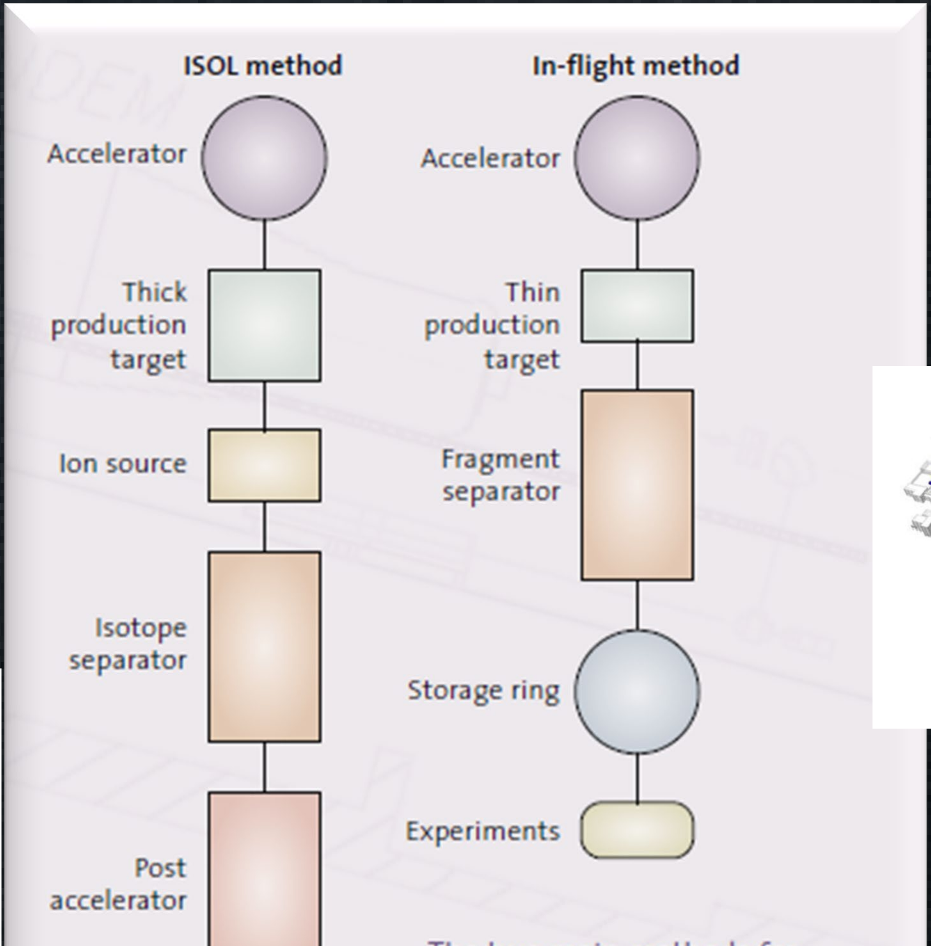
There are two main methods to produce exotic beams

- ISOL (Isotope Separation On Line)
- In Flight or Fragmentation

There are some facilities that provide exotic beams in the world and several new facilities under construction

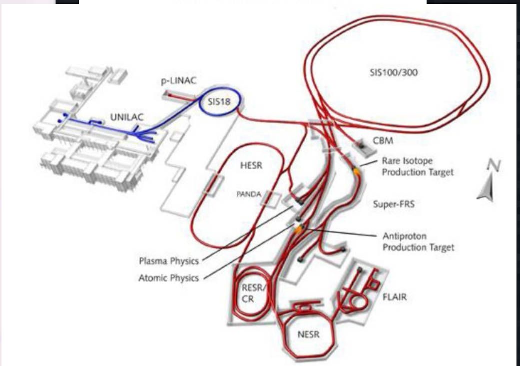
Radioactive beams production

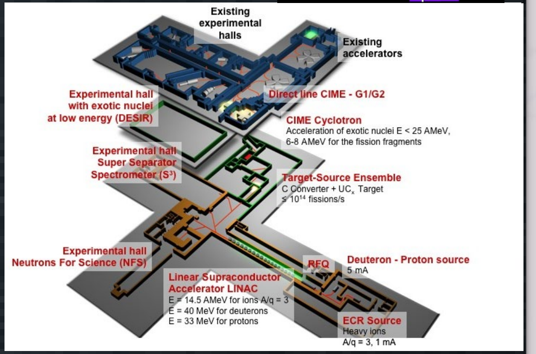




The two main methods for producing radioactive ion beams

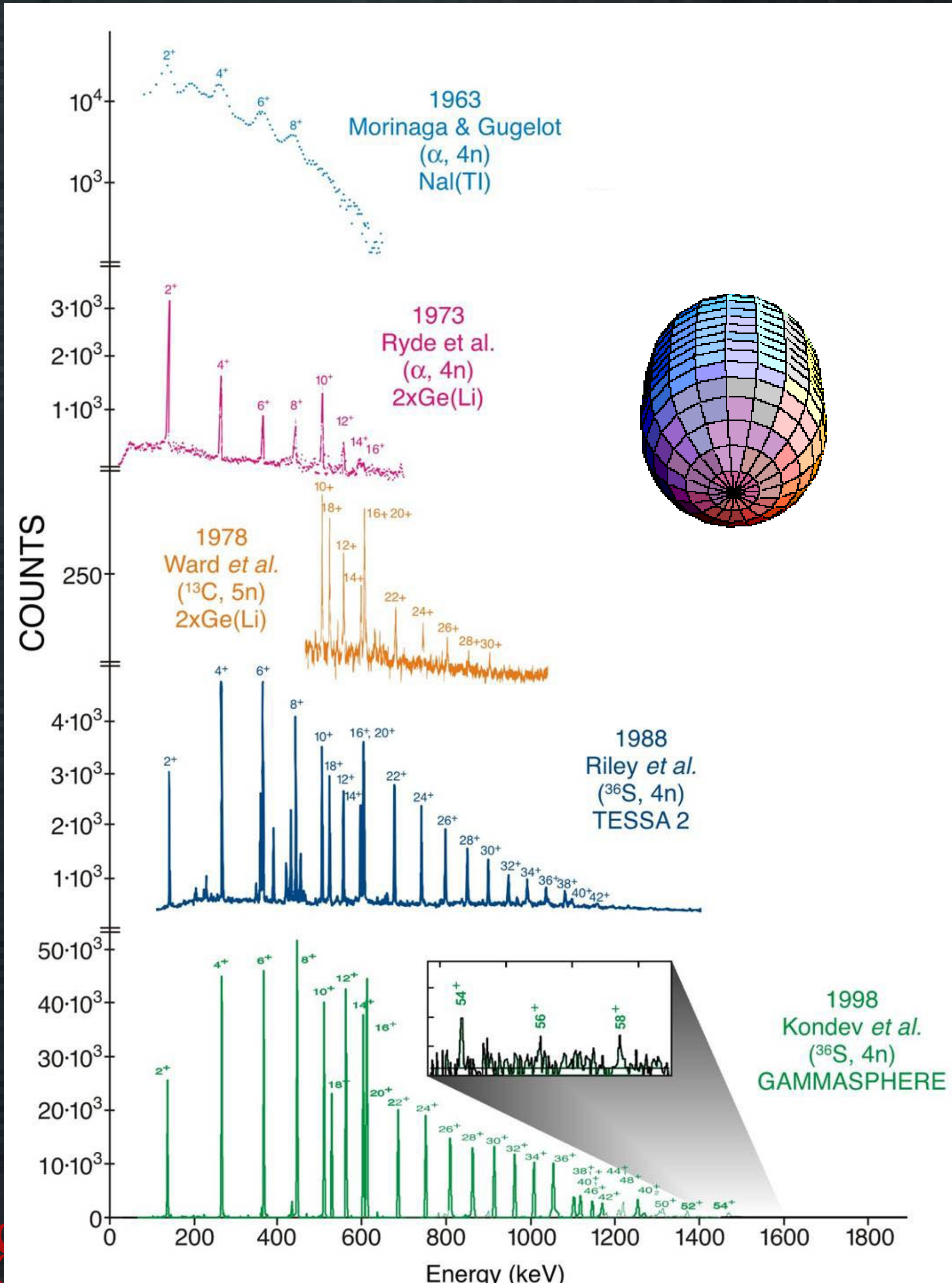
Facility for Antiproton and Ion Research





Introduction to detectors for nuclear-structure studies

Gamma spectroscopy



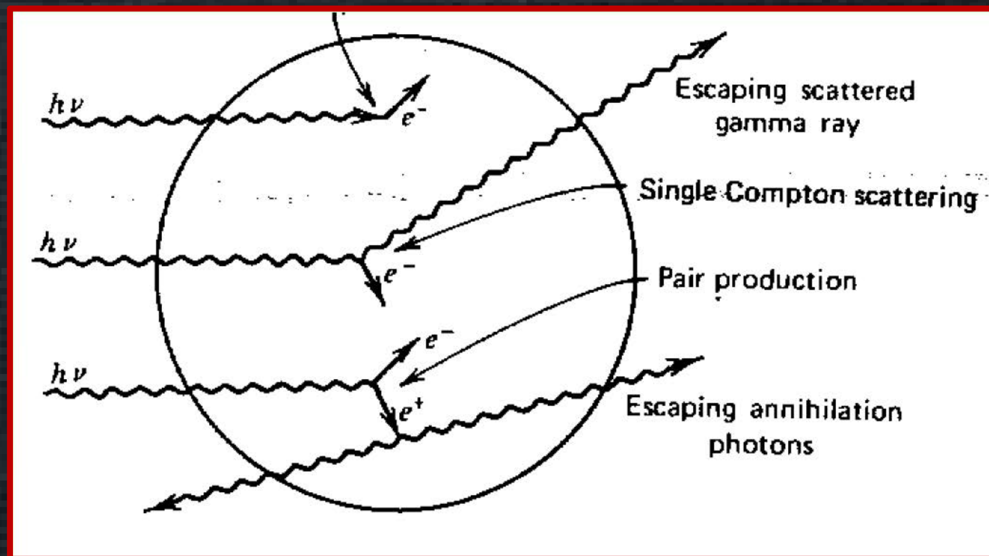
<u>Spectroscopic history</u> of a rotational sequence in ^{156}Dy during four decades

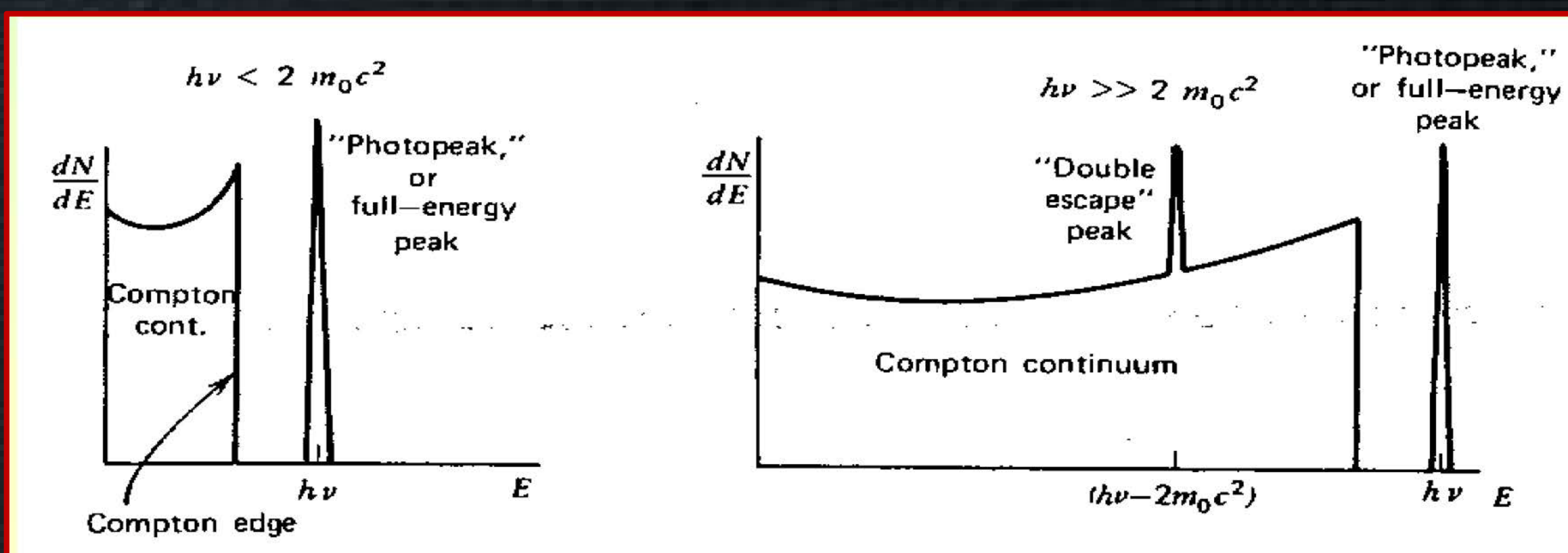
A bare Ge detector

photoelectric

Compton

pair production

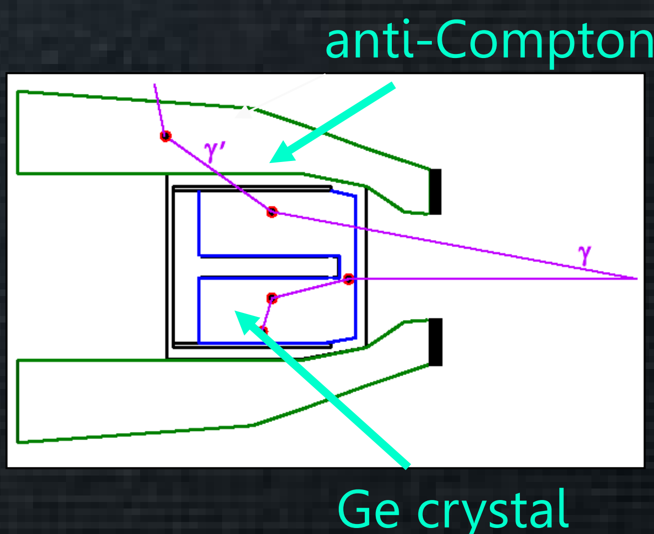


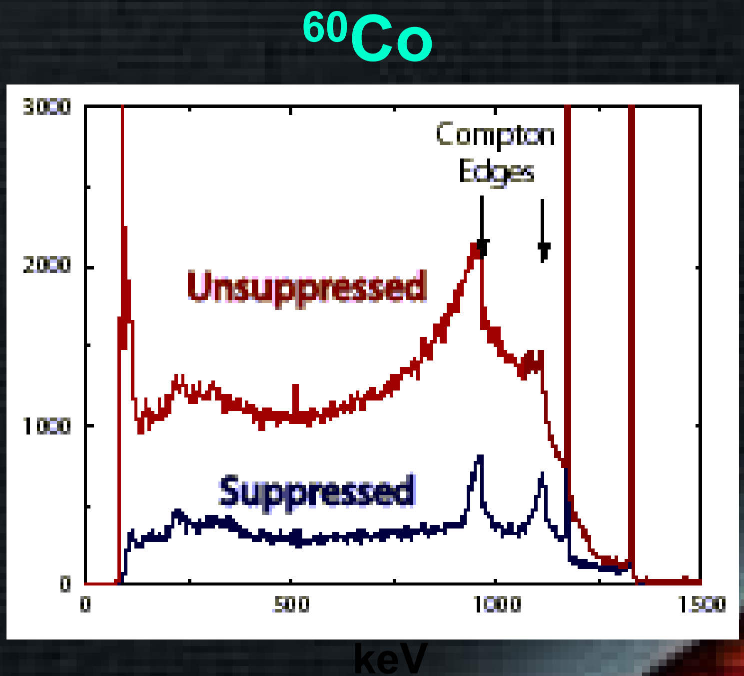


Anti-Compton or escaped suppressed Ge-detectors

...a big improvement

- Single Ge crystal has limited volume
- To improve the response function the anti-Compton shield is needed
- For large volume crystals Peak/Total ratio improves from ~ 30% to ~ 60%
- Drawback: part of the solid angle is used by the shielding detector





Improving resolving power

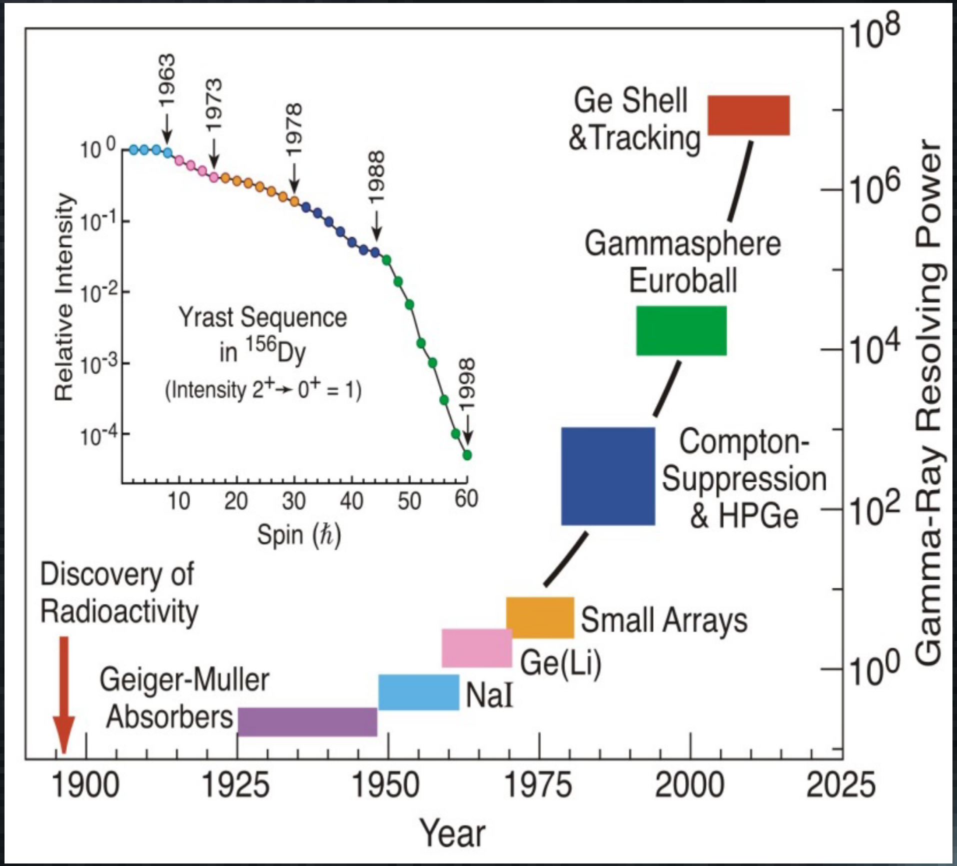
Energy resolution → Ge detectors
P/T ratio → anticompton

and in order to face the physics of high spins or high energy:

Doppler broadening → small solid angle

Efficiency → many detectors

High-fold coincidences → many detectors, segmentation and tracking



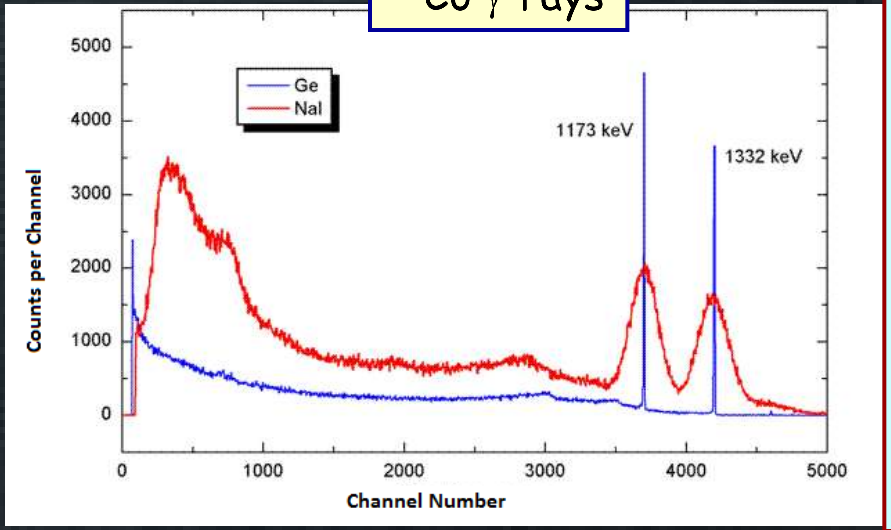
Scintillator detectors

Modern gamma spectrometers are based on Ge detectors and also on scintillators.

Scintillators have worst resolution but have other advantages as: efficiency, timing.
NaI, BaF_2, BGO, $LaBr_3$

Properties of scintillators

^{60}Co γ-rays



- Scintillators have higher efficiency (high Z material, larger sizes)
- Scintillators offer better timing (< 1 ns) Ge: 5-10 ns
- Ge detectors are very expensive and fragile
- Ge detectors need cooling $\rightarrow$ complex infrastructure

ΔE = 2 keV Ge-detectors
ΔE = 70 keV NaI detectors

Cristallo	Densitá	ΔT (FWHM)	$\Delta E/E$	Vol Max.
BaF_2	4.88	~ 0.4 ns	$\sim 11\%$	> 4 dm^3
NaI	3.67	~ 1.5 ns	$\sim 7\%$	> 4 dm^3
BGO	7.13	~ 5.0 ns	$\sim 15\%$	> 4 dm^3
HpGe	5.32	~ 3.0 ns	$\sim 0.2\%$	< 0.2 dm^3
LaBr3(Ce)	**5.29**		**~ 3%**	**most recent material**

With low intensity beams

With the present days radioactive beam facilities

Higher beam currents

one can return to use the
high-resolution Ge detectors
(of lower efficiency)
with the proper segmentation to
reduce the Doppler broadening
(AGATA, EXOGAM, SEGA, MINIBALL,...)

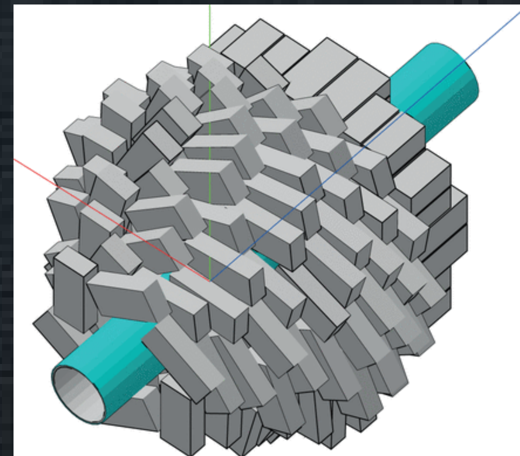
but....

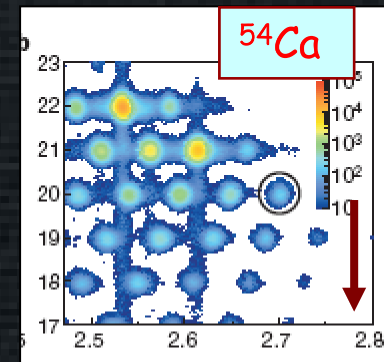
Even at the most powerful facilities there will be always
<u>very exotic nuclear beams with very low-intensity</u>

Need of modern arrays of scintillators detectors
(CAESAR at MSU, DALI2 at RIKEN,...)

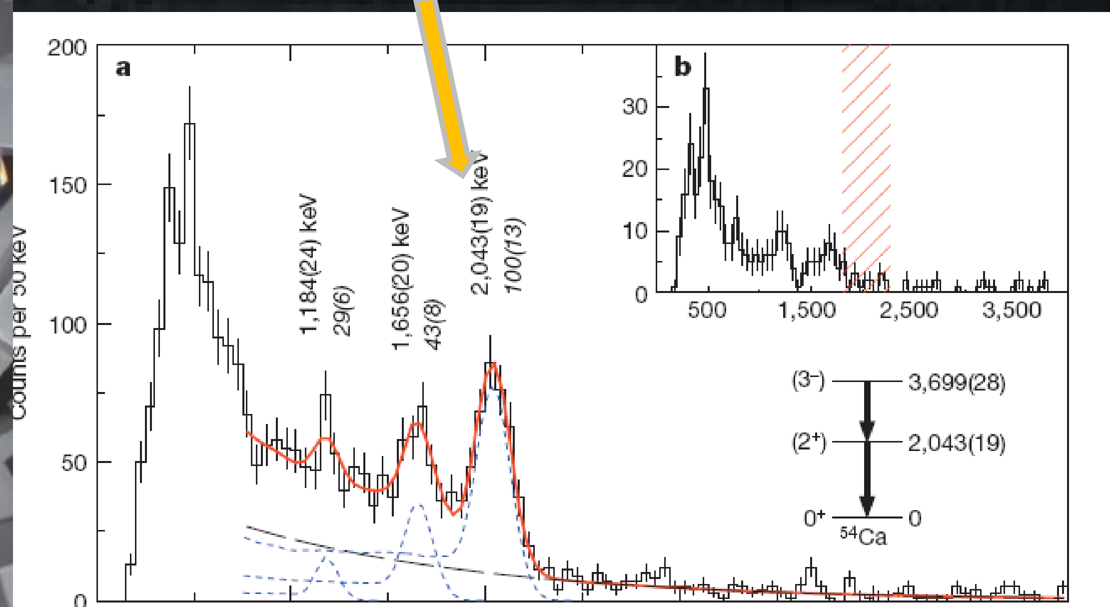
New magic number in RIKEN

DALI2 scintillator array at RIKEN
186 NaI(Tl) crystals
ΔE ~ 8% at 662 keV.
Eff. ~ 30% at 1 MeV.





the first 2^+ state in ^{54}Ca

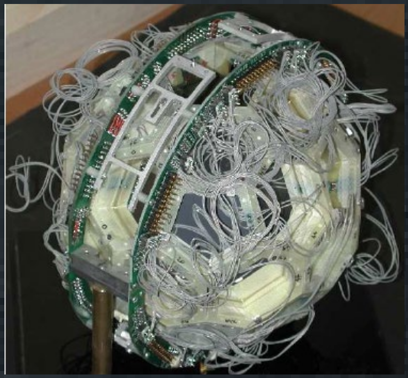


Ancillary detectors

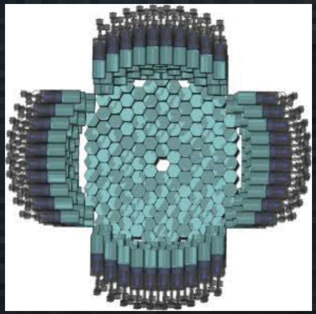
Proton-rich nuclei in fusion-evaporation

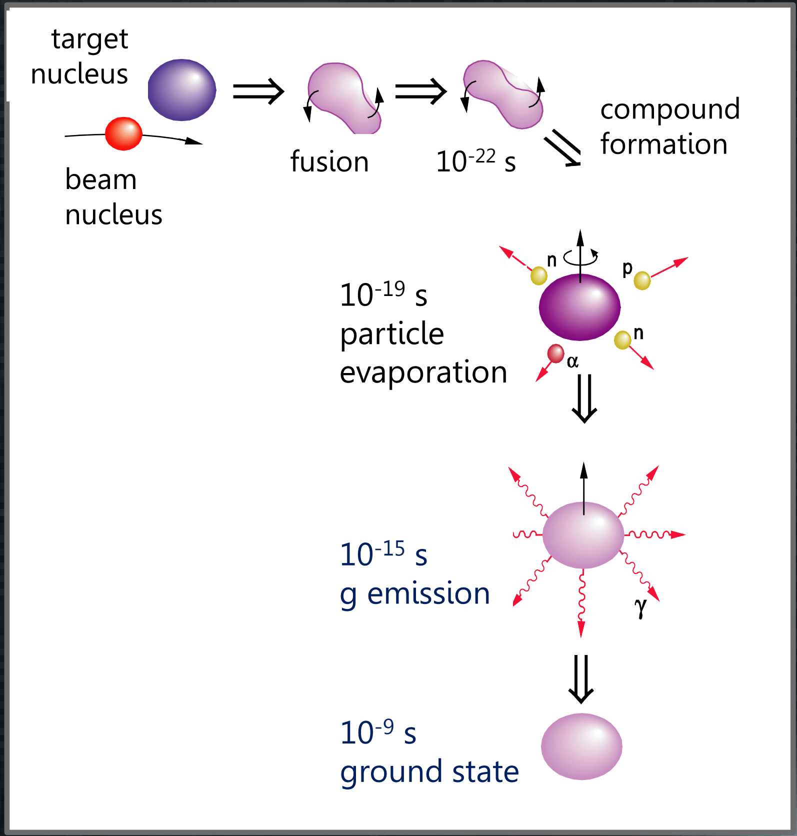
In fusion-evaporation reactions many different residues are produced. The nuclei of interest are identified by the particles emitted from the compound nucleus

charged particles

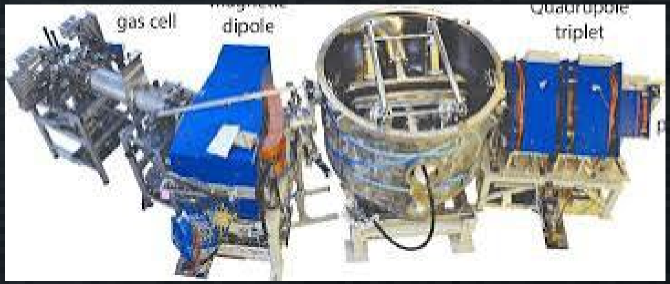


neutrons



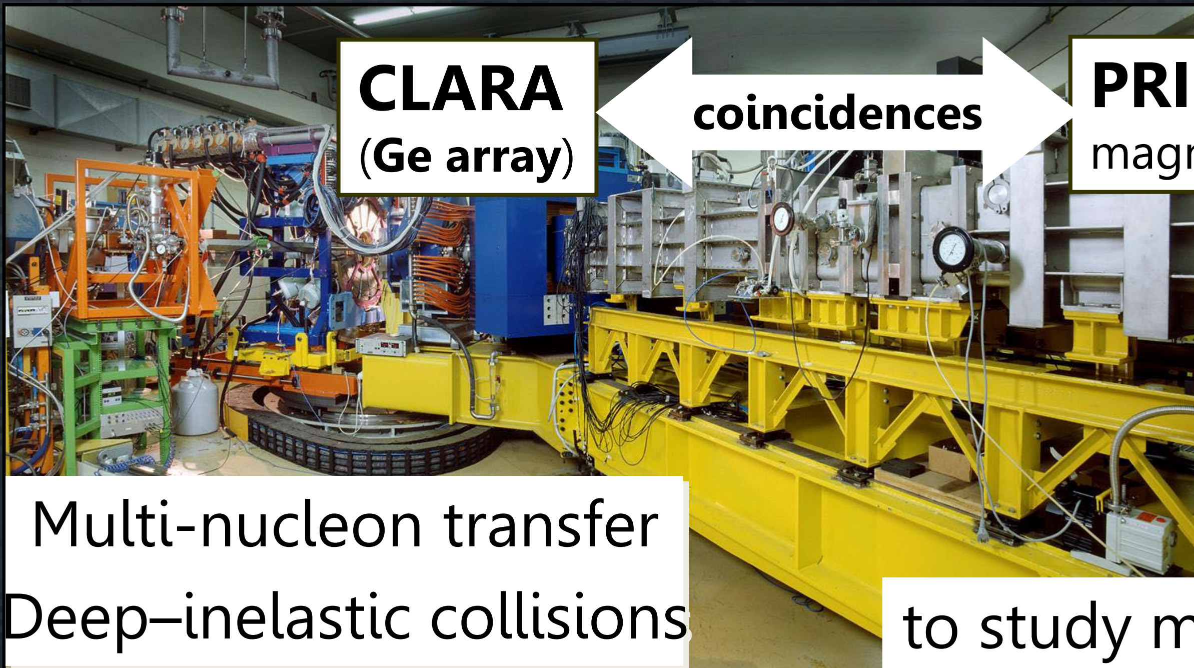


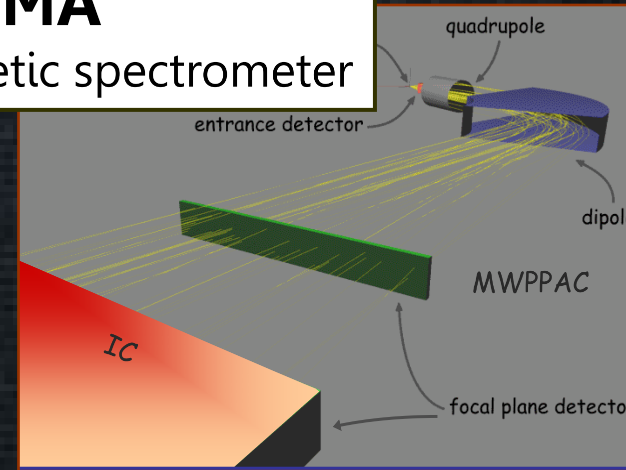
Mass and magnetic spectrometers



We need high selectivity

CLARA (**Ge array**) ⟷ **coincidences** ⟷ **PRISMA** magnetic spectrometer





Multi-nucleon transfer

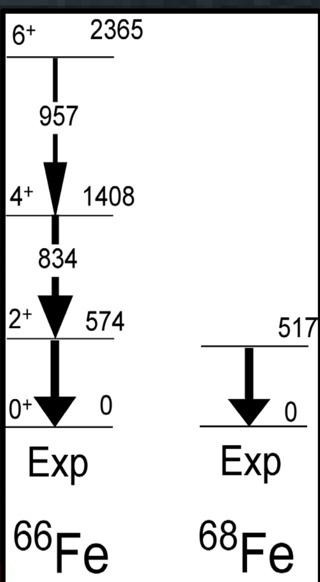
Deep–inelastic collisions

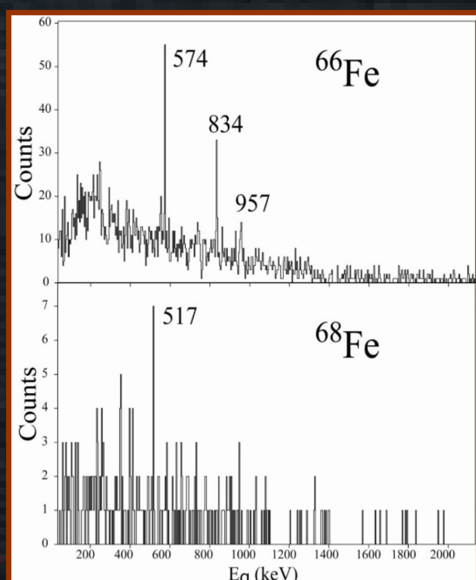
to study moderately n-rich nuclei

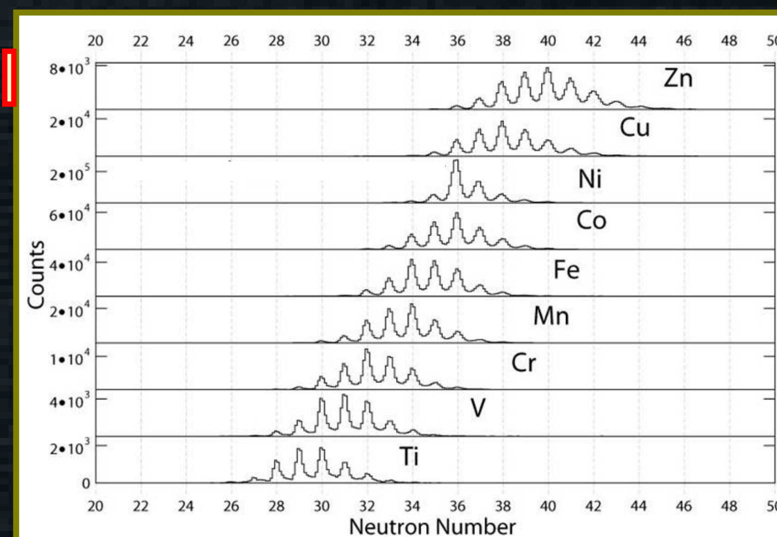
γ in coincidence

^{70}Zn (460 MeV) + ^{238}U

Projectile-like fragment detected







v/c range:
4-10%

Segmentation and tracking

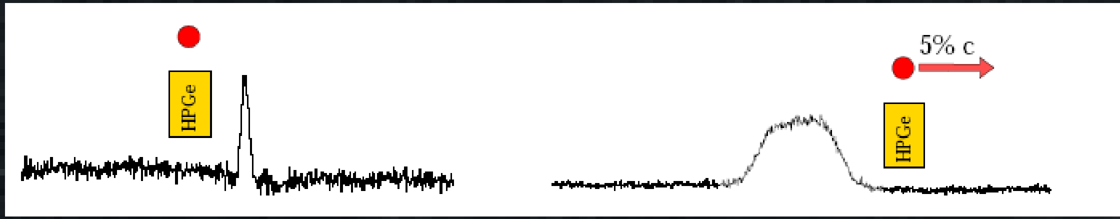
Arrays for high-energy beams

With the advent of radioactive-beam facilities it was clear that a new generation of highly efficient γ-detector arrays made of Ge-detectors was needed to make optimum use of these low intensity beams.

To increase the efficiency, the detectors have to be placed close to the target

Often, with radioactive beams one has to deal with high velocities of the recoils emitting gammas (especially in **inverse kinematics** reactions)

→ Doppler broadening



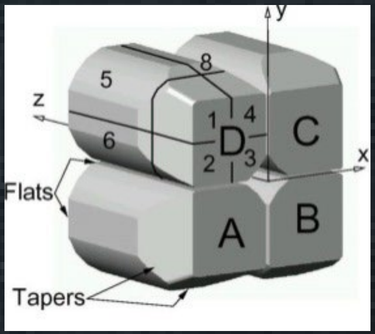
→ the detectors should subtend a small angle ➡ { high granularity, segmentation }

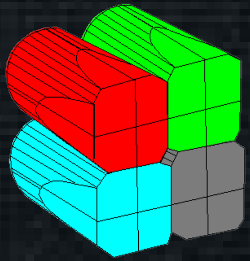
Improving energy resolution

The energy resolution ΔE is a combination of several factors, mostly arising from Doppler effects:

- Intrinsic resolution of the detectors → Ge detectors
- Doppler broadening arising from the opening angle of the detectors.
- Doppler broadening arising from the angular spread of the recoils.
- Doppler broadening arising from the velocity (energy) variation of the recoils.

How to achieve high efficiency and good energy resolution?





Arrays of segmented germanium detectors

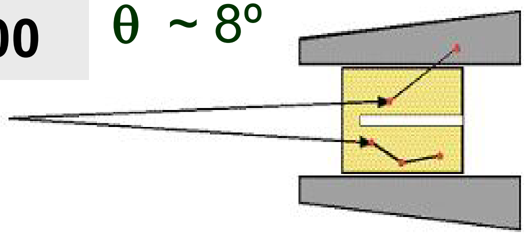
Granularity is absolutely necessary to reduce, to a reasonably small value, the Doppler-broadening of γ-rays emitted by nuclei with very high velocities.

The best solution is gamma-ray tracking

Motivation of gamma-ray tracking

ε_{ph}	~ 10%
N_{det}	~ 100

Ω ~ 40%
θ ~ 8°

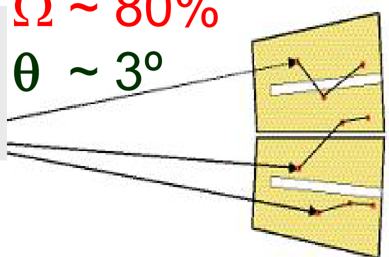


- 50% of solid angle taken by the AC shields
- large opening angle ➔ poor energy resolution at high recoil velocity

Ge Sphere

ε_{ph}	~ 50%
N_{det}	~ 1000

Ω ~ 80%
θ ~ 3°

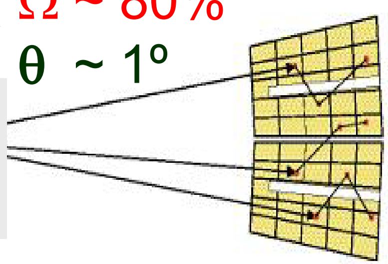


- too many detectors needed to avoid summing effects
- opening angle still too big for very high recoil velocity

Tracking Array

ε_{ph}	~ 50%
N_{det}	~ 100

Ω ~ 80%
θ ~ 1°



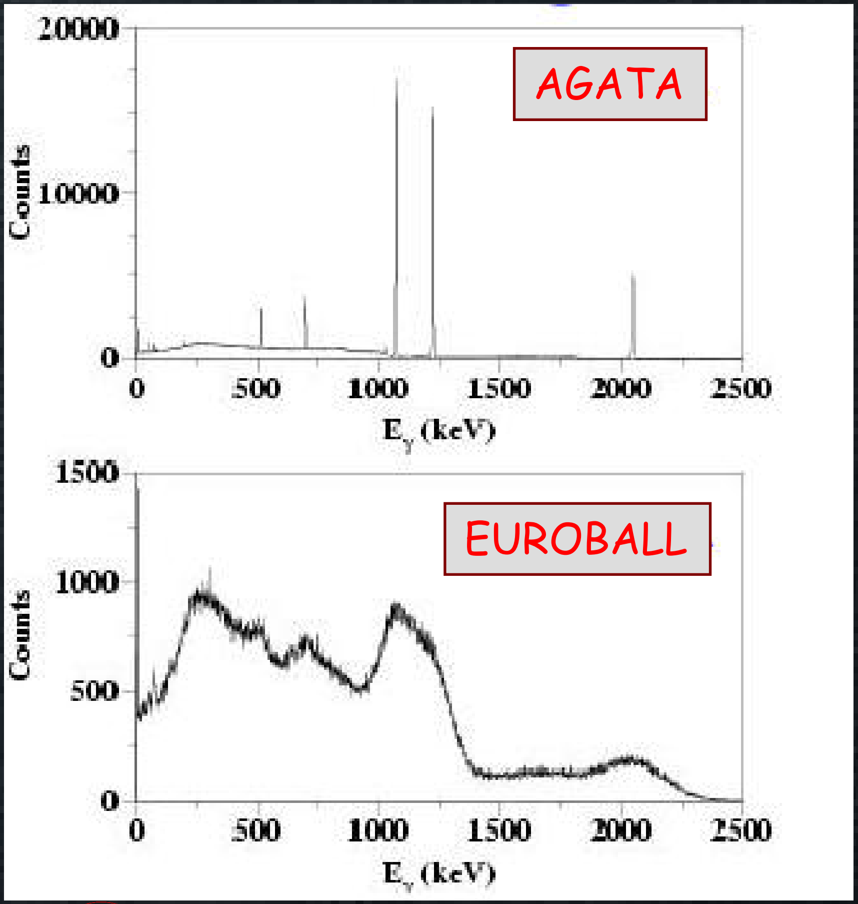
Smarter use of Ge detectors

- segmented detectors
- digital electronics
- time stamping of events
- analysis of pulse shapes
- tracking of γ-rays

Pulse-shape Analysis
Gamma-ray Tracking ➔ θ_{eff} ~ 1°, N_{eff} ~ 10000

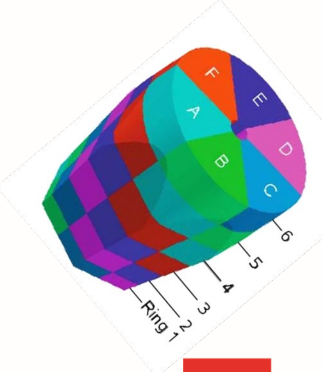
Tracking advantage

Simulated γ-ray spectra comparing the response of AGATA with that of a conventional Ge-array assuming a fragmentation reaction at 100 MeV/A.



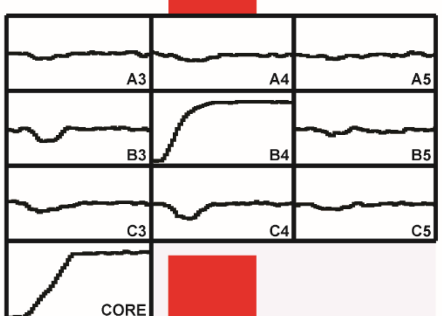
The enormous gain in resolving power is impressive!

The AGATA concept

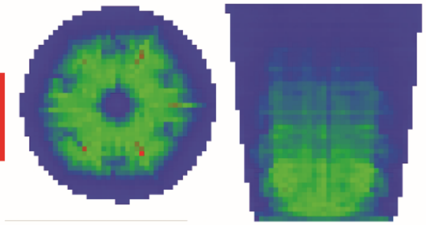


electrically segmented HPGe detector

Digital electronics to record and process segment signals



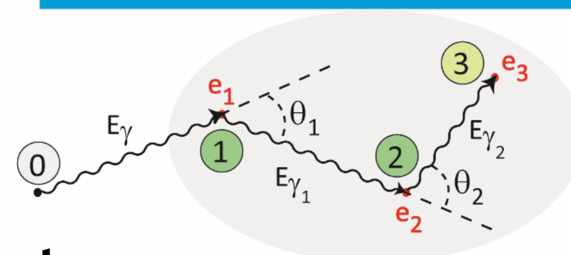
Deconvolution of the recorded waves through comparison with reference signals



Identification of the individual interaction points

$(x,y,z,E,t)_i$

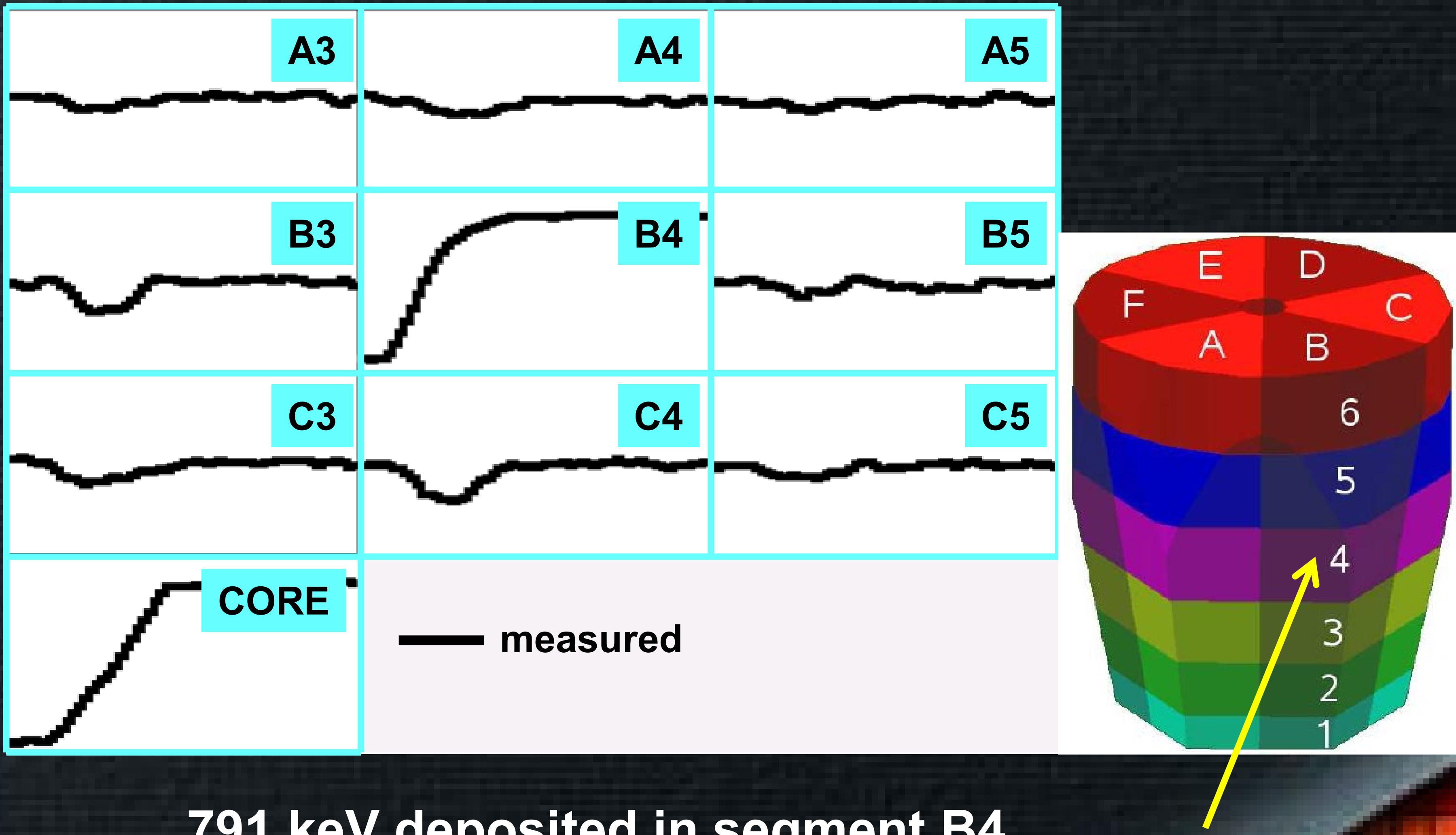
Reconstruction of tracks evaluating permutations of interaction points



gamma first interaction point within few mm

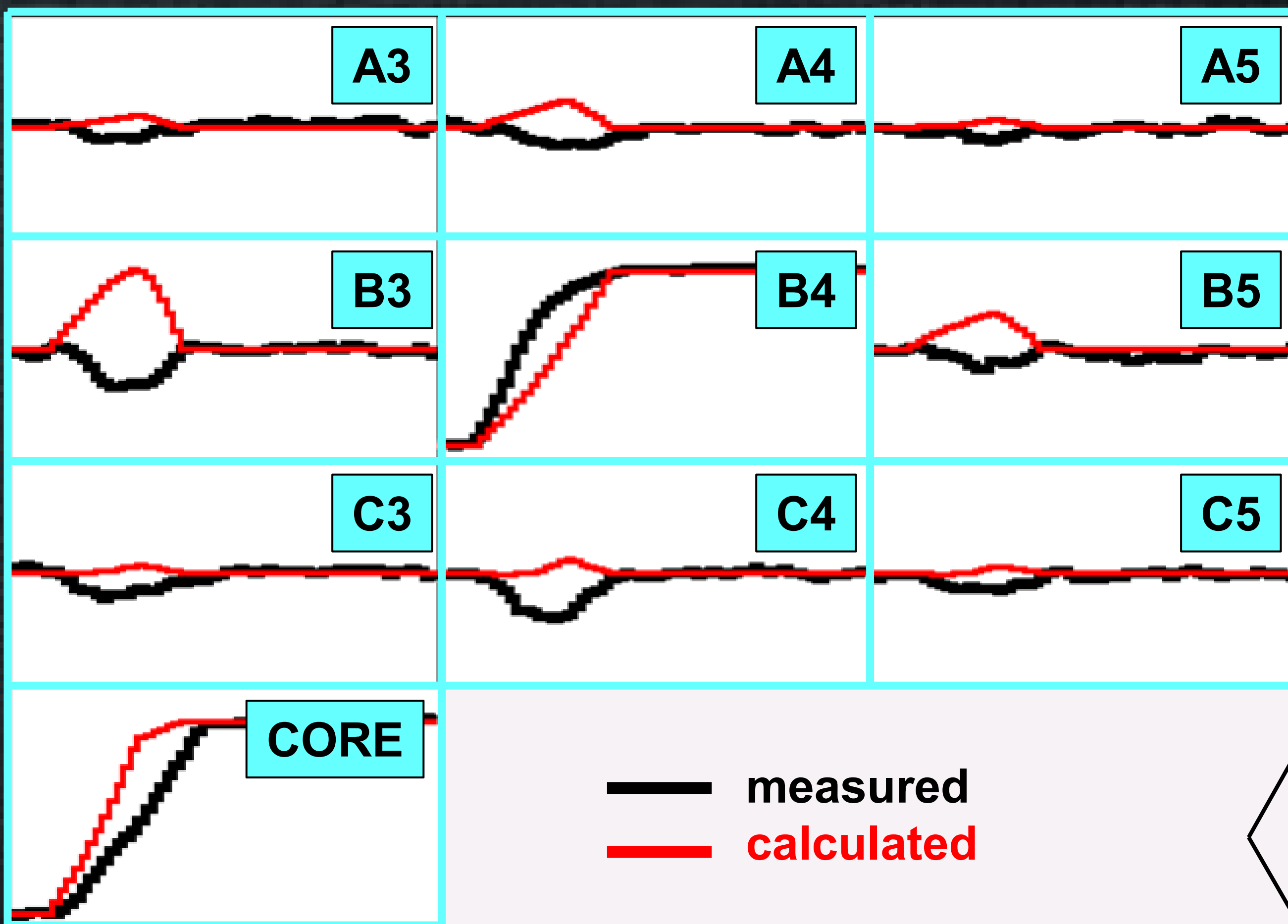
Energy and direction of γ-rays

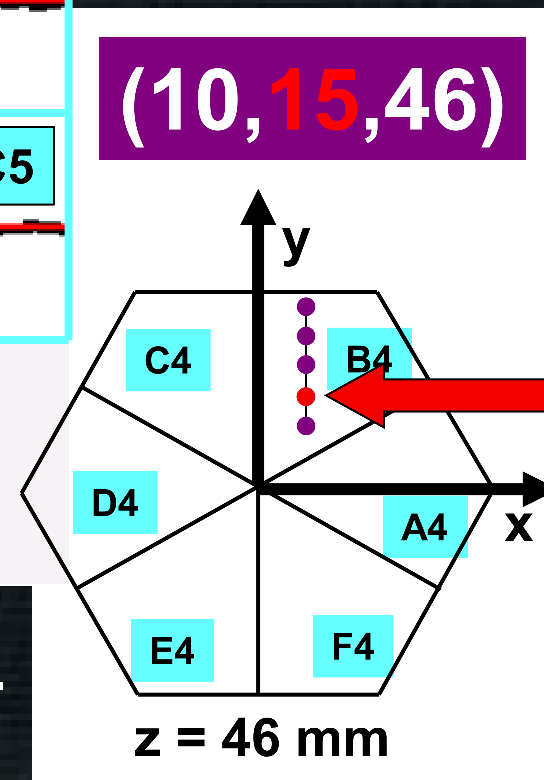
The pulse-shape analysis process



791 keV deposited in segment B4

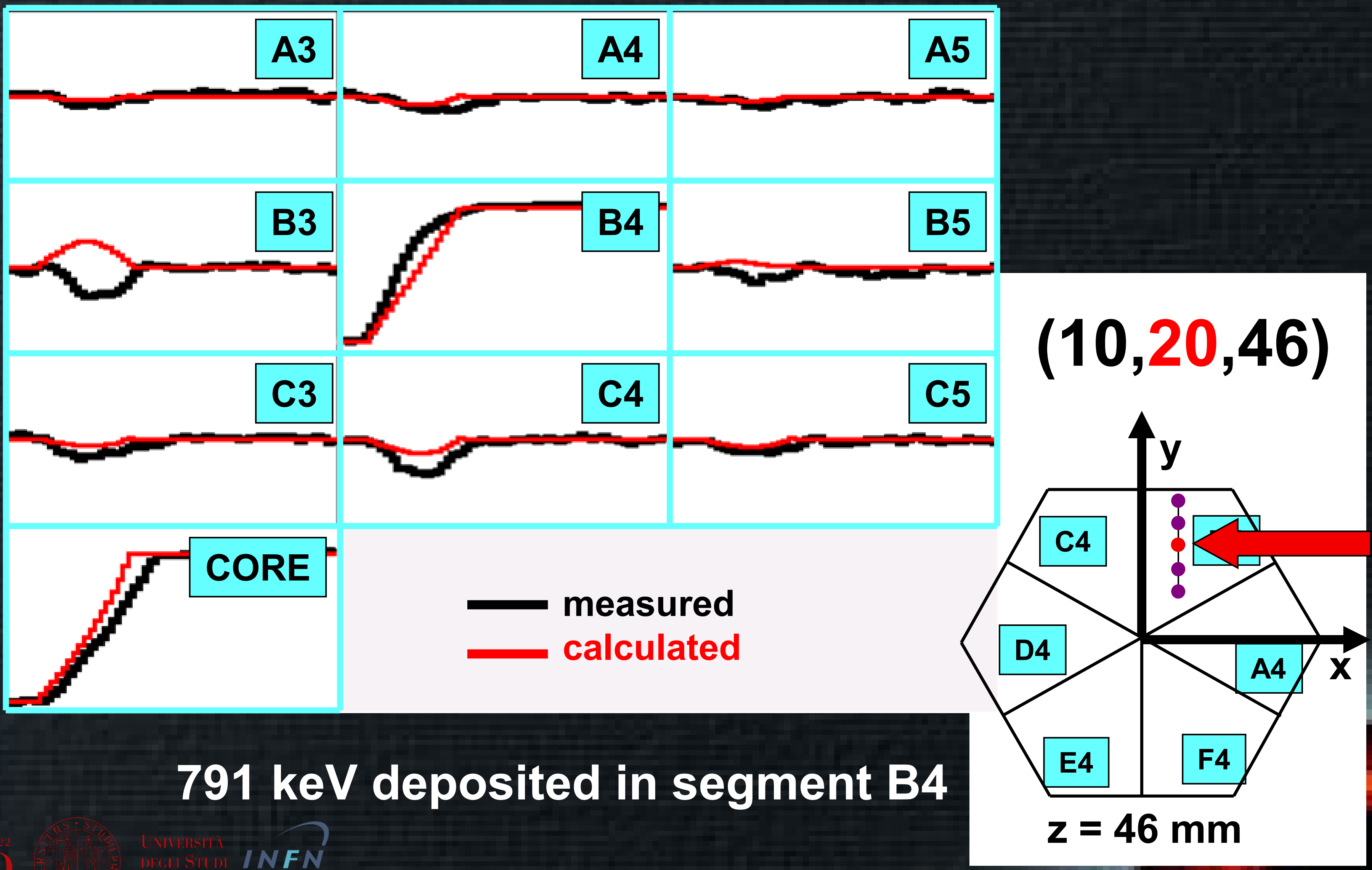
The pulse-shape analysis process





791 keV deposited in segment B4

The pulse-shape analysis process



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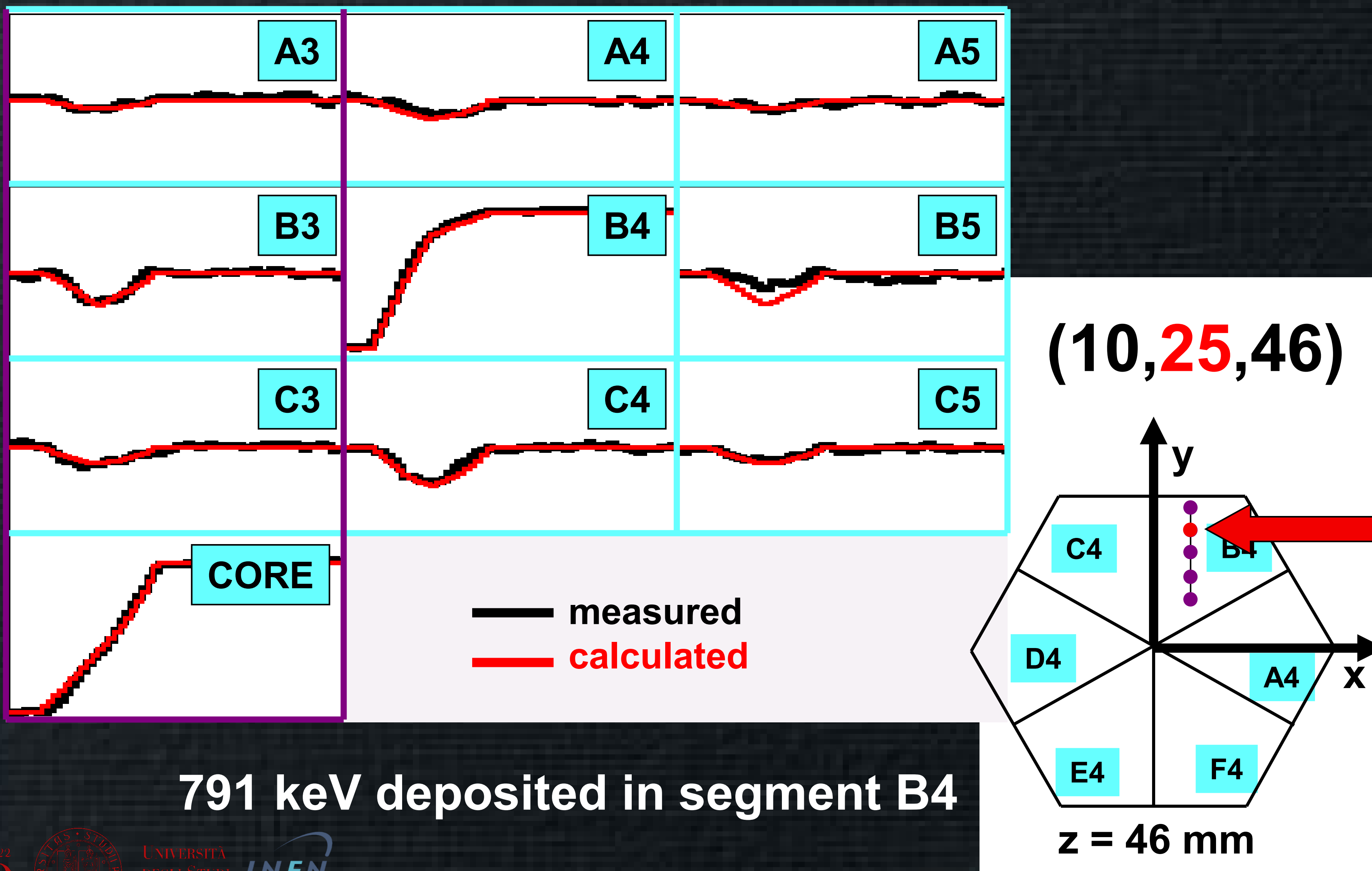

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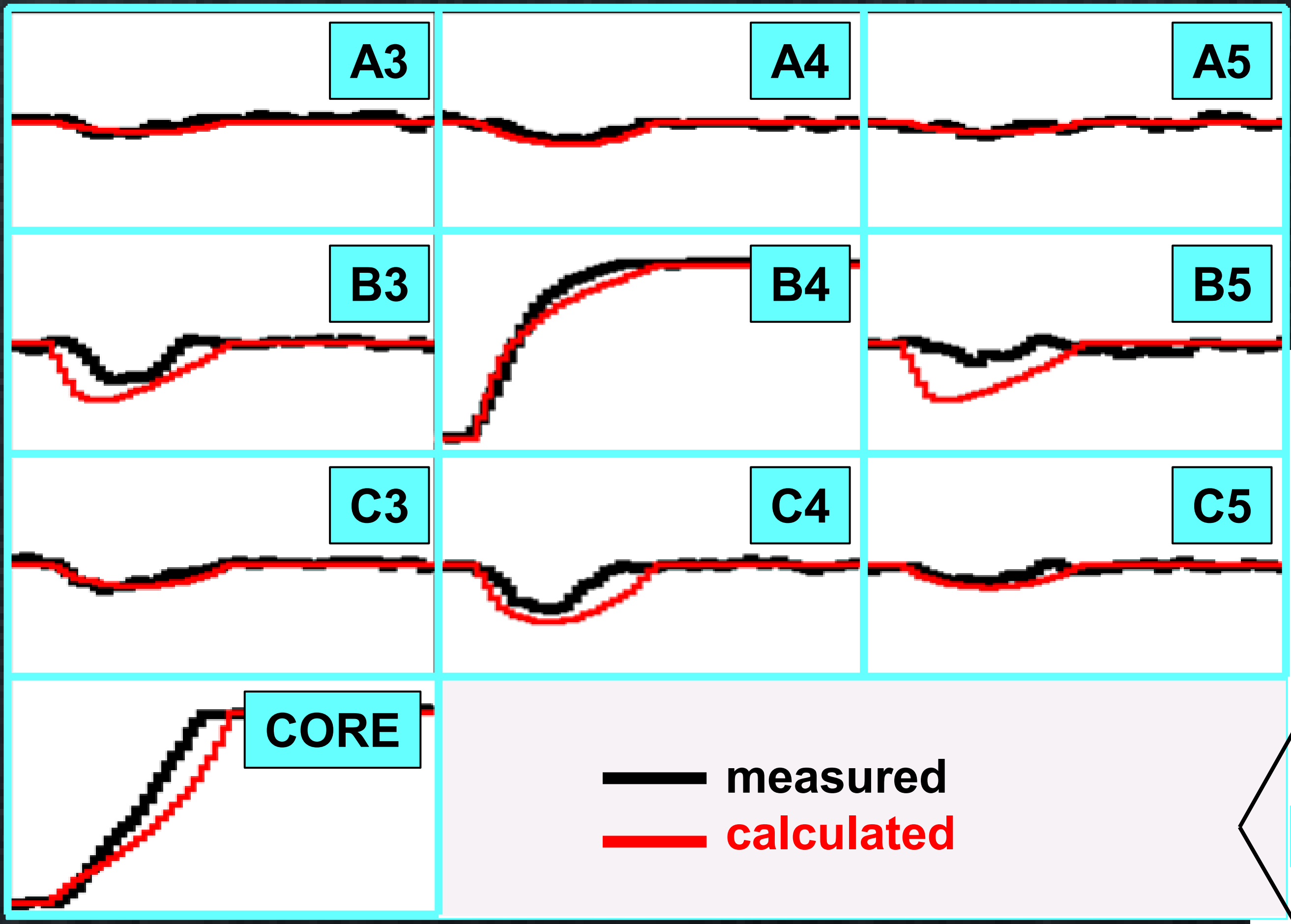
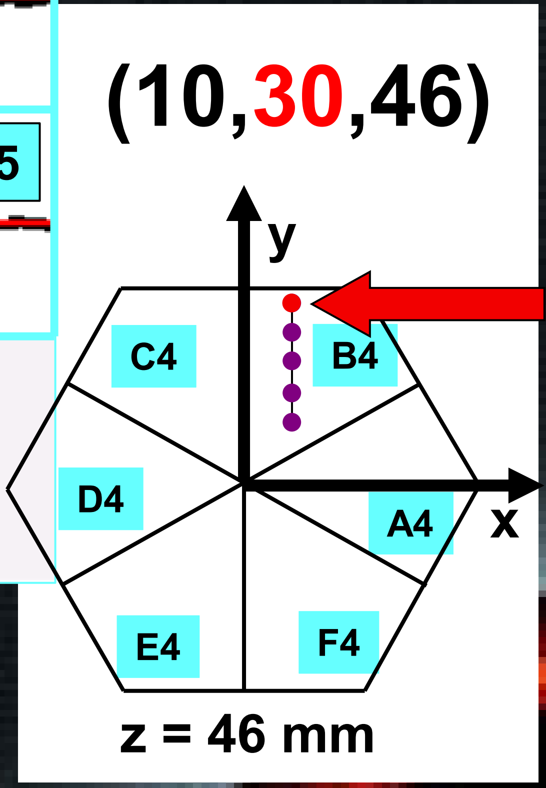
The pulse-shape analysis process





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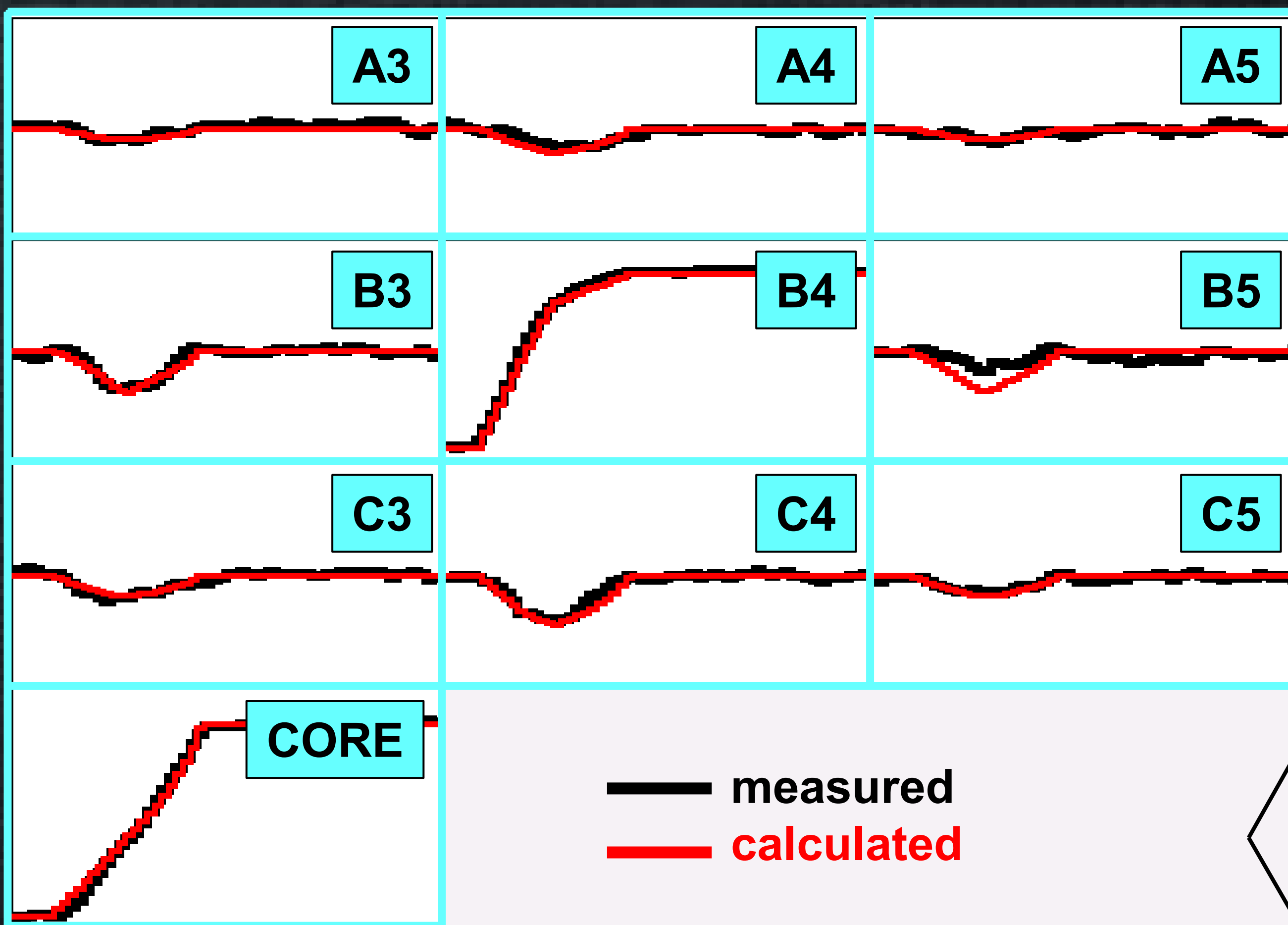
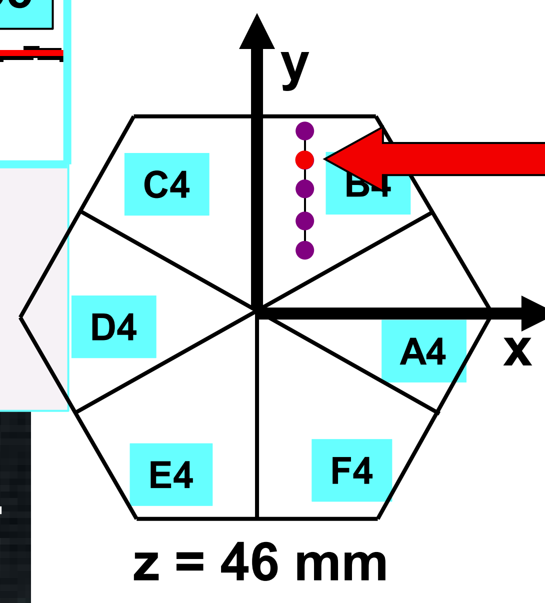
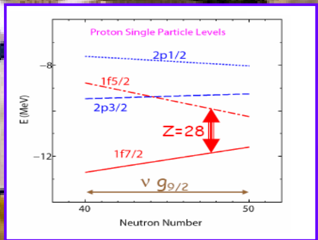


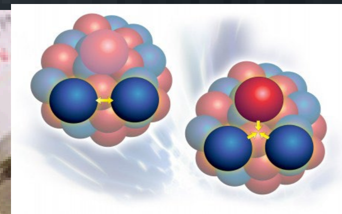
Result of *Grid Search* Algorithm

(10,25,46)

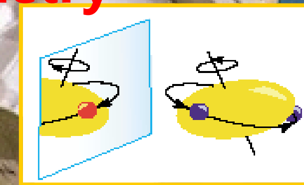


791 keV deposited in segment B4





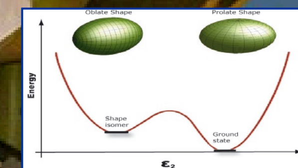
Isospin symmetry breaking

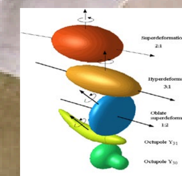


Three-body forces

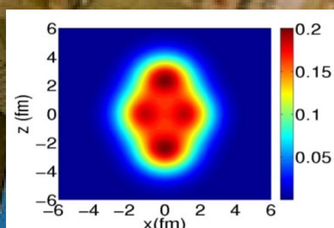
Shell evolution far from stability

Nuclear shapes and coexistence





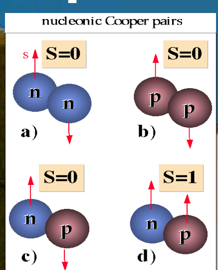
clusterization

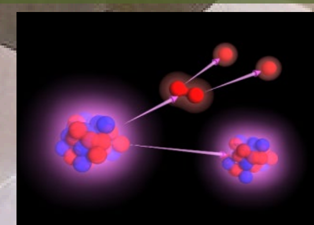


High-resolution gamma-ray spectroscopy is an optimum tool to study nuclear structure properties and investigate how they emerge from fundamental interactions.

p-n pairing

Super heavy elements





Nuclear Astrophysics

Coupling to the continuum

AGATA and GRETA are the ideal tools for high resolution spectroscopy

Thank you for your attention!

Enyoy the School!

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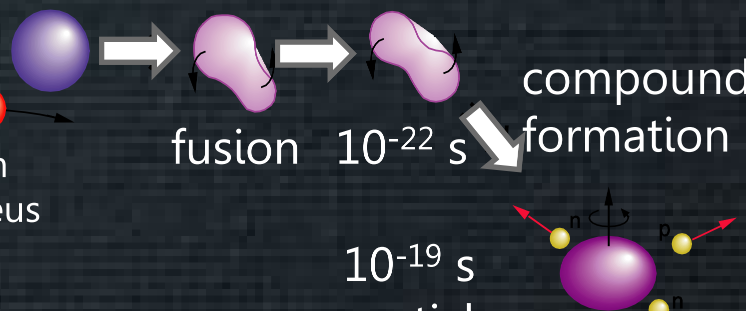
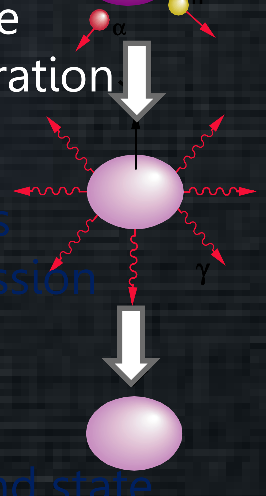
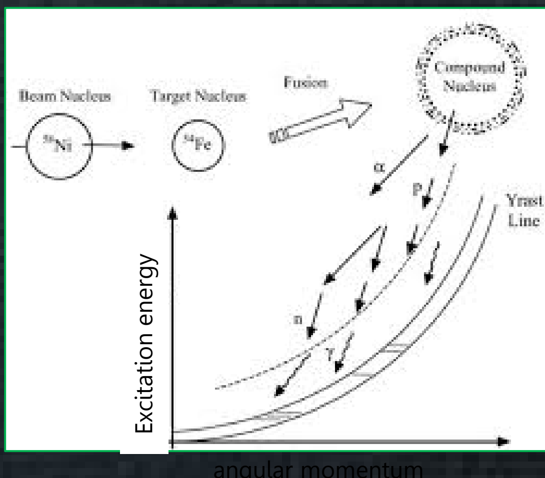
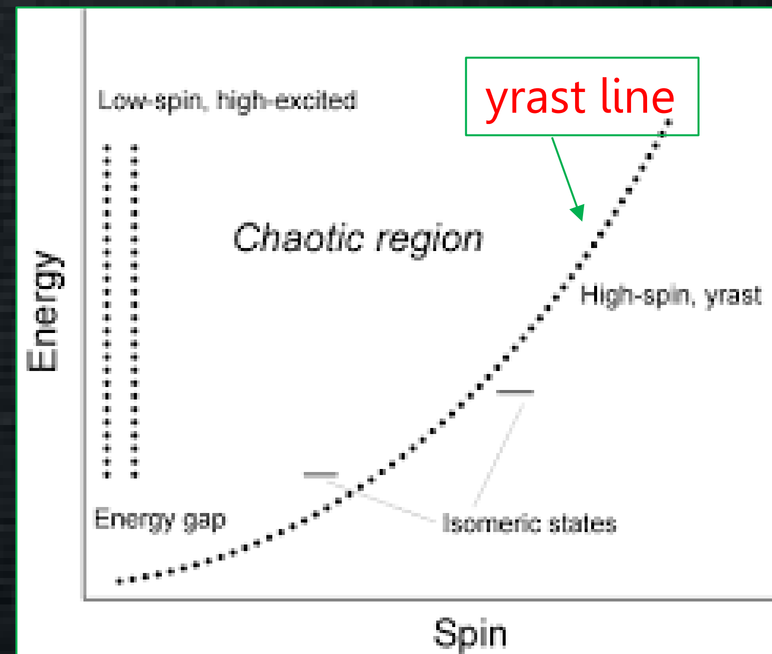
Backup material

The yrast line





The yrast line is the line that marks the lowest excitation energy for each angular momentum or
the maximum angular momentum for each excitation energy
(yrast (Swedish) means whirlingest)

The matrix equation

The eigenvalue equation becomes a matrix equation:

$$\boxed{[H][A] = [E][A]}$$

This forms a secular equation for the eigenvalues E_p:

$$\boxed{\begin{vmatrix} H_{11} - E_p & H_{12} & \ldots H_{1n} \\ H_{21} & H_{22} - E_p & \ldots H_{2n} \\ \vdots & \vdots & \vdots \\ H_{n1} \quad \ldots & \ldots & H_{nn} - E_p \end{vmatrix} = 0}$$

This is a *n*th degree equation for the *n*-roots E_p (p=1,2,...n)

The solutions

Once we have the energies E_p, we can use

$$\sum_{k=1}^{n} E_k^0 \delta_{lk} a_{kp} + \sum_{k=1}^{n} \langle \psi_l^0 | H_{res} | \psi_k^0 \rangle a_{kp} = E_p a_{lp}$$

to obtain the coefficients a_{kp}. Using the orthonormalization: $$\sum_{k=1}^{n} a_{kp} a_{kp'} = \delta_{pp'}$$

we can then write $$\sum_{l,k=1}^{n} a_{lp'} E_k^0 \delta_{lk} a_{kp} + \sum_{l,k=1}^{n} a_{lp'} \langle \psi_l^0 | H_{res} | \psi_k^0 \rangle a_{kp} = E_p \delta_{pp'}$$

Which is a matrix equation of the form $$[A]^{-1}[H][A] = [E]$$

This equation indicates a similarity transformation to a new basis that makes [H] diagonal

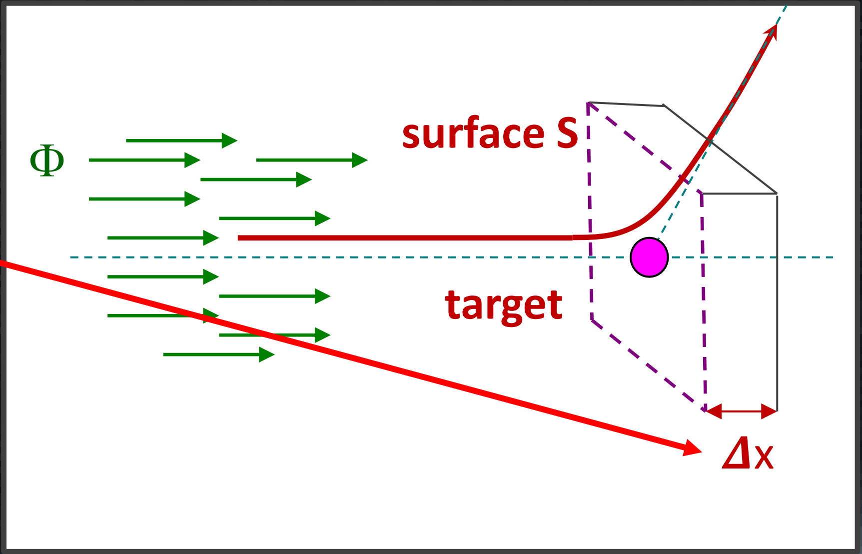
If n is large this process needs a high-speed computer

The cross section

Consider an experiment in which a beam of N_i particles impinges on a target with N_t particles, with a relative velocity v.

The flux of incident particles, i.e. the number of particles per unit surface and time is:

$$\Phi = \frac{N_i}{\Delta S \cdot \Delta t} = \frac{N_i \Delta x}{\Delta S \cdot \Delta x \cdot \Delta t} = \frac{N_i v}{V} = n_i v$$



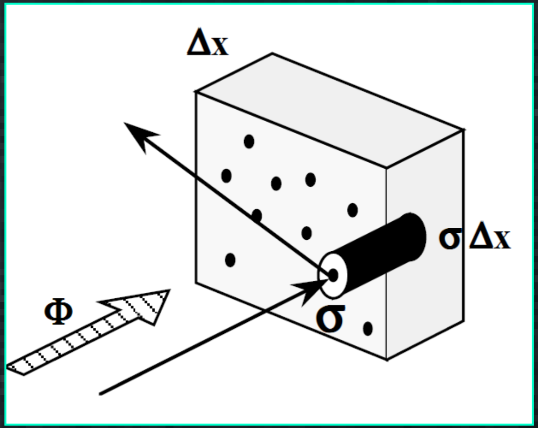
The number of particles in the target per unit surface is:

$$\frac{N_t}{\Delta S} = \frac{N_t \Delta x}{V} = n_t \Delta x$$

where n_i and n_t are the number of particles per unit volume

The cross section (cont.)

The incident flux is attenuated by the interaction with the target.
The attenuation is proportional to the density of particles n_t and the thickness Δx of the target.



$$\Delta\Phi = -\Phi\sigma\Delta x \underline{n_t}$$

volume density

surface density

The fraction of flux removed due to the interaction with the target is proportional to the number of particles in the target in a volume ***σΔ***x

$$-\frac{\Delta\Phi}{\Phi} = n_t\sigma\Delta x$$

***σ* is the cross section of the process and has the dimensions of a surface**: it represents the interaction area between the incident and target particles

When passing through the target, the incident flux is atenuated with the law

$$\Phi(x) = \Phi_0 e^{-n_t\sigma x}$$