

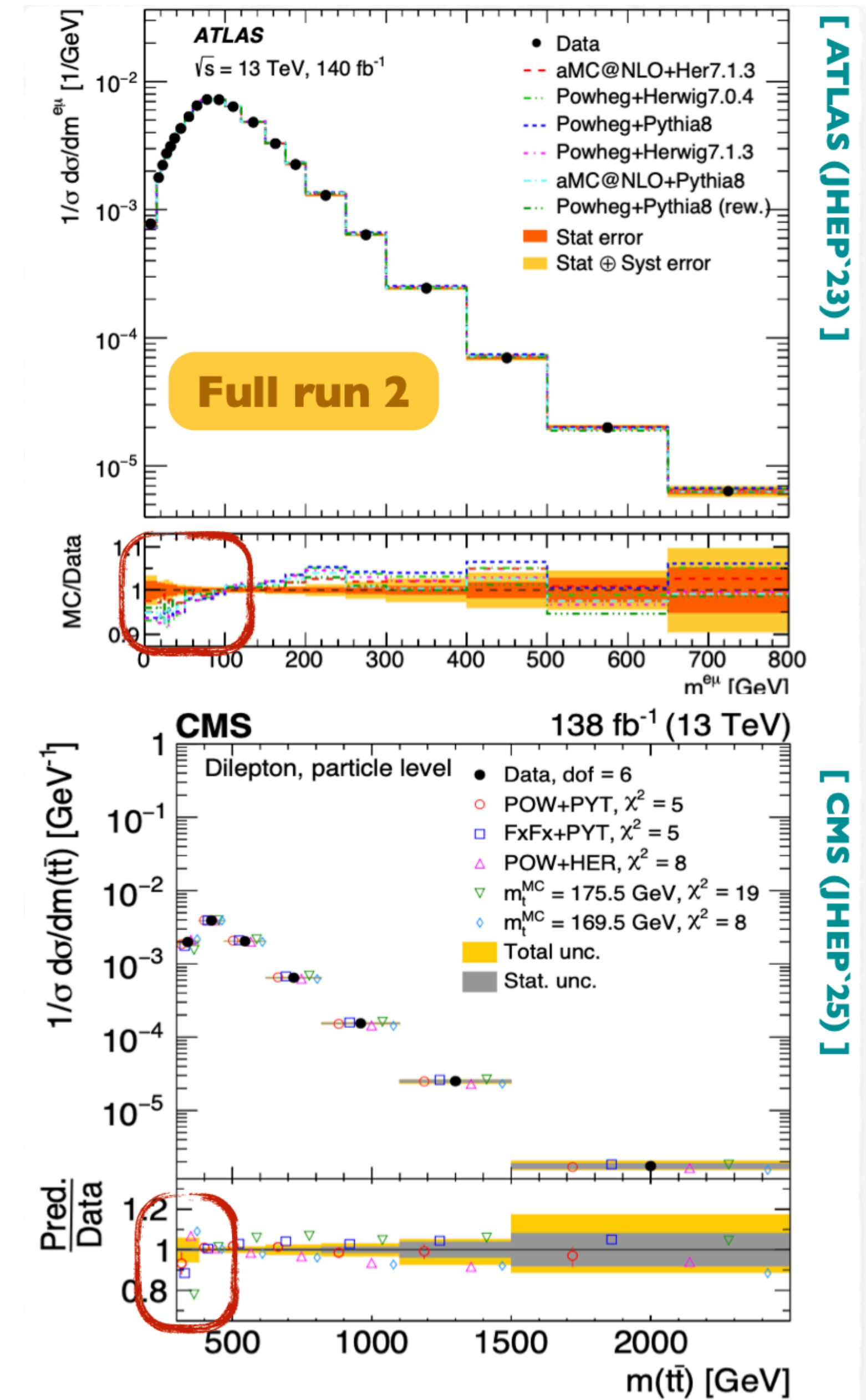
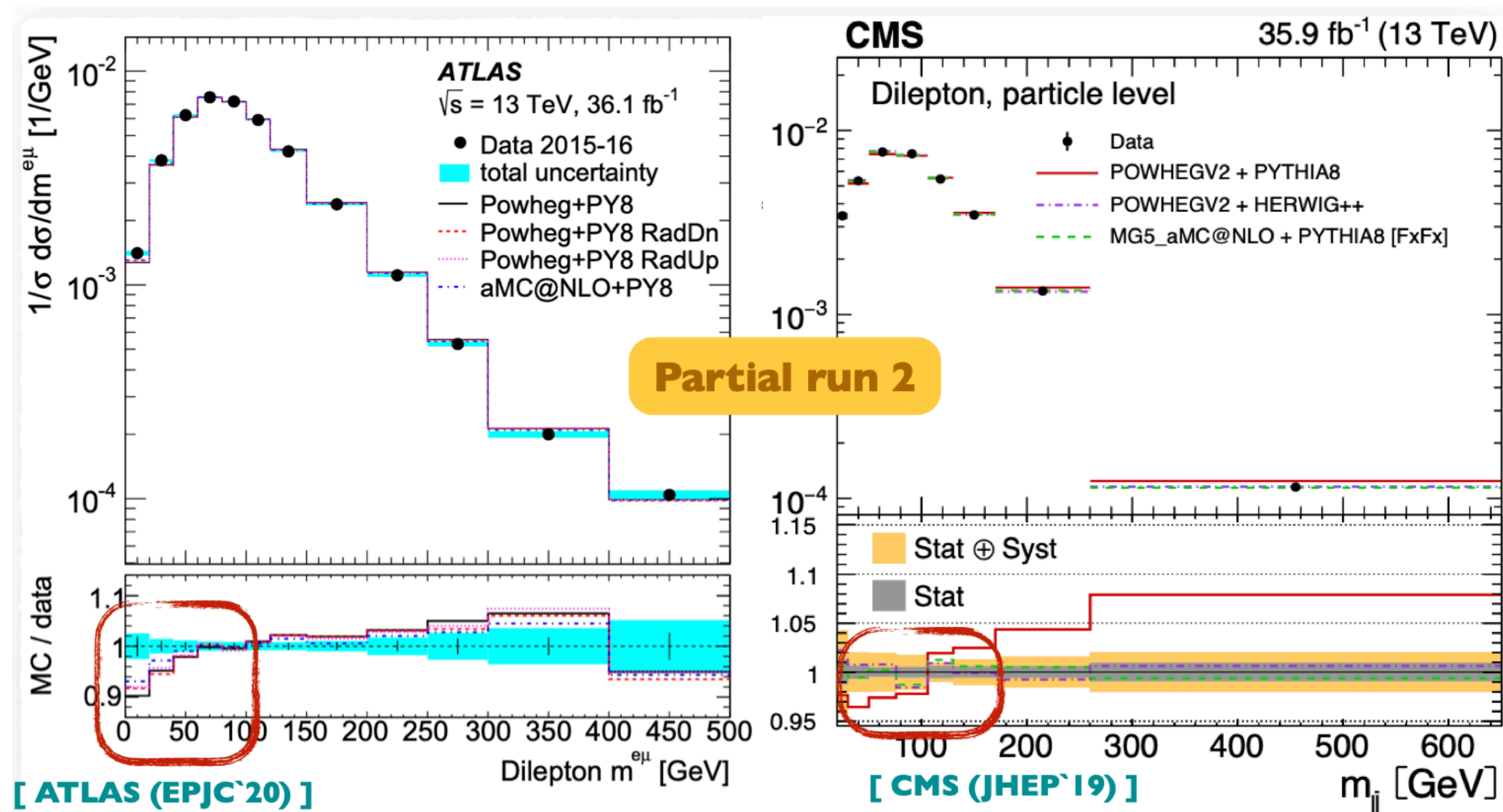
Toponium's physics and the space left for BSM physics

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2512.03220

Historical context and motivations



- Toponium's prediction : beginning of the 90's
 Production ~ 1% of total production top-antitop
- Experimental excess of 3σ : 2019-2020
- Experimental excess of 5σ : 2025

Why Green's functions formalism?

1- Fixed order expansion breaks down

Hierarchy of scales :

$$m_t \gg m_t v \gg m_t v^2$$

v : relative velocity of the quarks

At 1 loop* :

$$\mathcal{M}_1 \sim g_s^2 C_F \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} \cdot \frac{1}{(p_1 - k)^2 - m_t^2 + i\epsilon} \cdot \frac{1}{(p_2 + k)^2 - m_t^2 + i\epsilon} \cdot \mathcal{M}_0$$

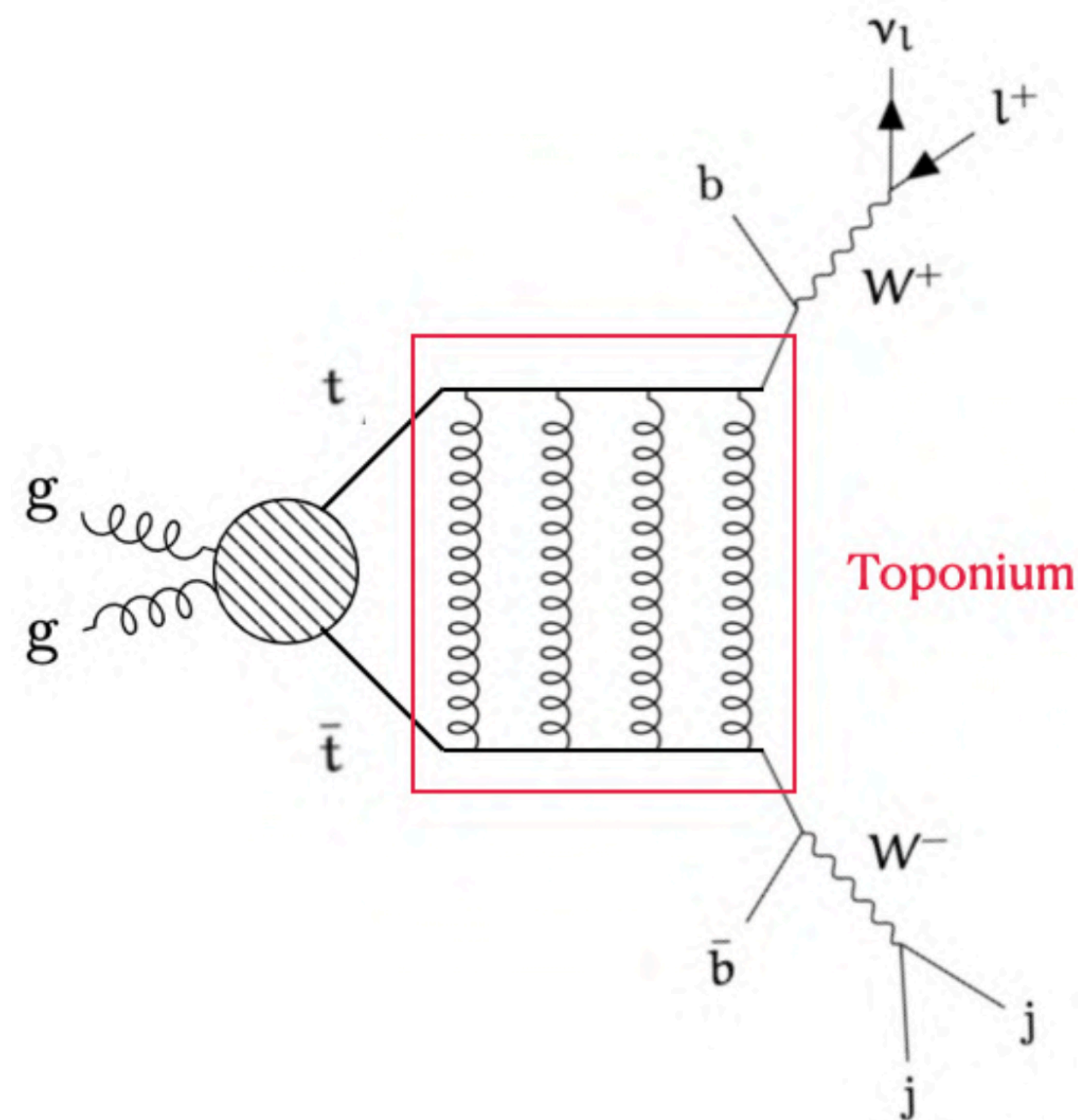
In the soft approximation :

$$k^0 \sim m_t v^2, \quad |\vec{k}|^2 \sim m_t^2 v^2$$

Yielding
$$\mathcal{M}_1 \propto \mathcal{M}_0 \frac{\pi \alpha_s C_F}{v} \longrightarrow \sigma_1 \propto \sigma_0 \frac{\pi \alpha_s C_F}{v}$$

Resuming all orders :

$$\sigma \sim \sigma_0 \left(1 + \frac{\alpha_s}{v} + \left(\frac{\alpha_s}{v} \right)^2 + \dots \right)$$



*this is true for the scattering state

Why Green's functions formalism?

2 - Perturbation theory still viable

Hierarchy of scales :

$$m_t \gg m_t v \gg m_t v^2$$

→ *Non relativistic dynamics (NRQCD)*

Top quark width :

$$\Gamma_t \sim m_t v^2$$

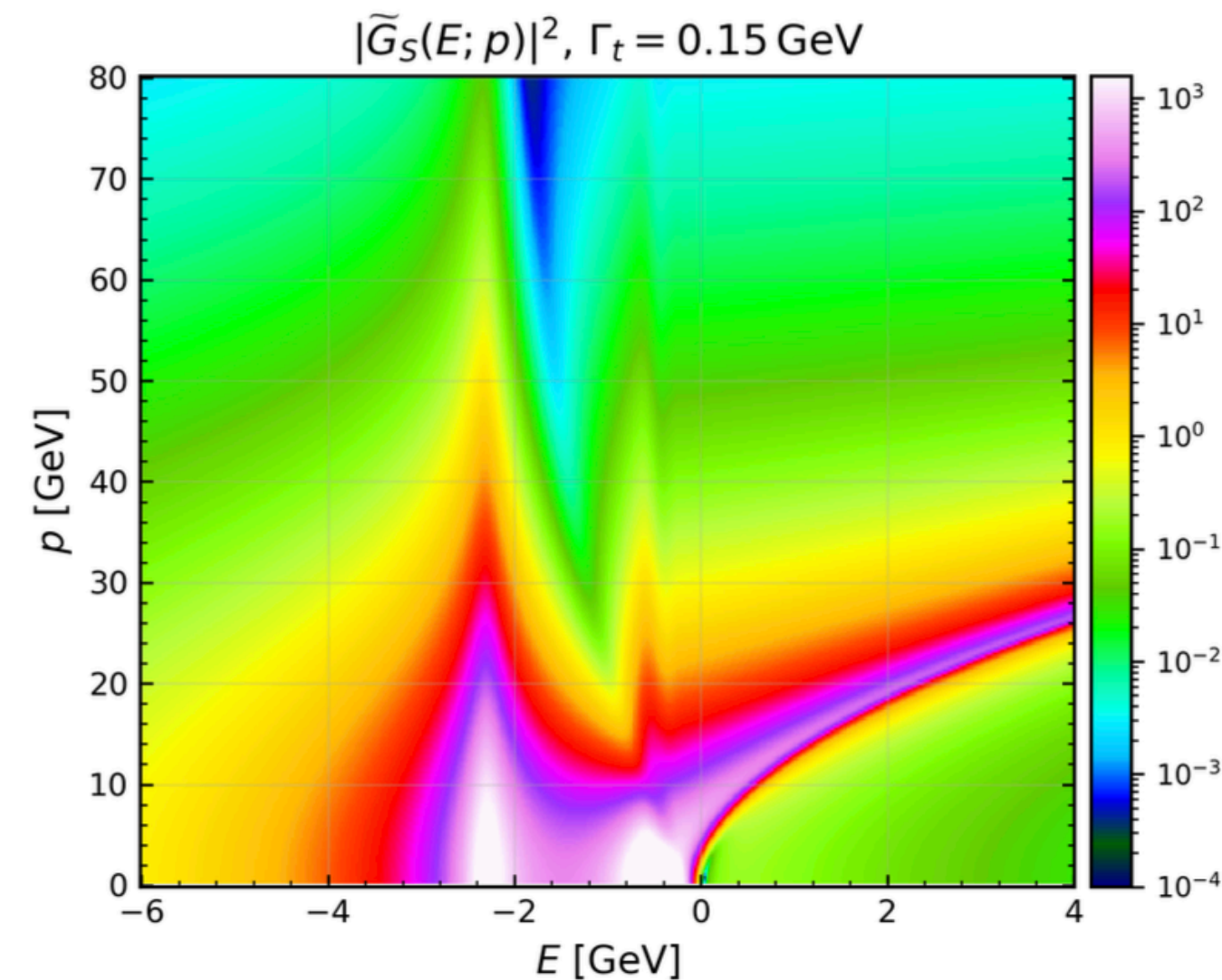
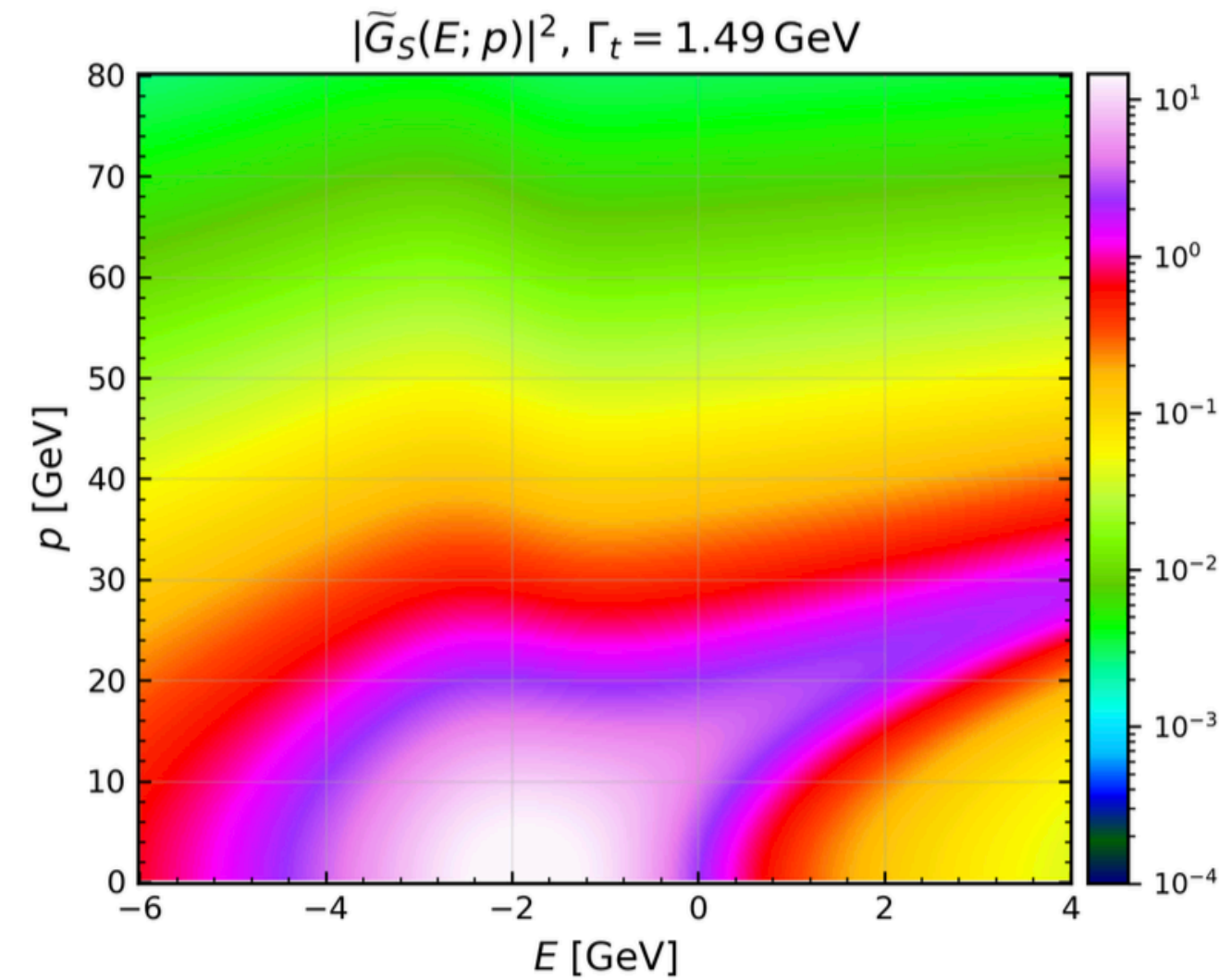
→ *Infrared cutoff*

Spectral representation :

$$G(E; \vec{r}, \vec{r}') = \sum_n \frac{\psi_n(\vec{r}) \psi_n^*(\vec{r}')}{E_n - E - i\Gamma_t} + \int_0^\infty dE' \frac{\rho(E'; \vec{r}, \vec{r}')}{E' - E - i\Gamma_t}$$

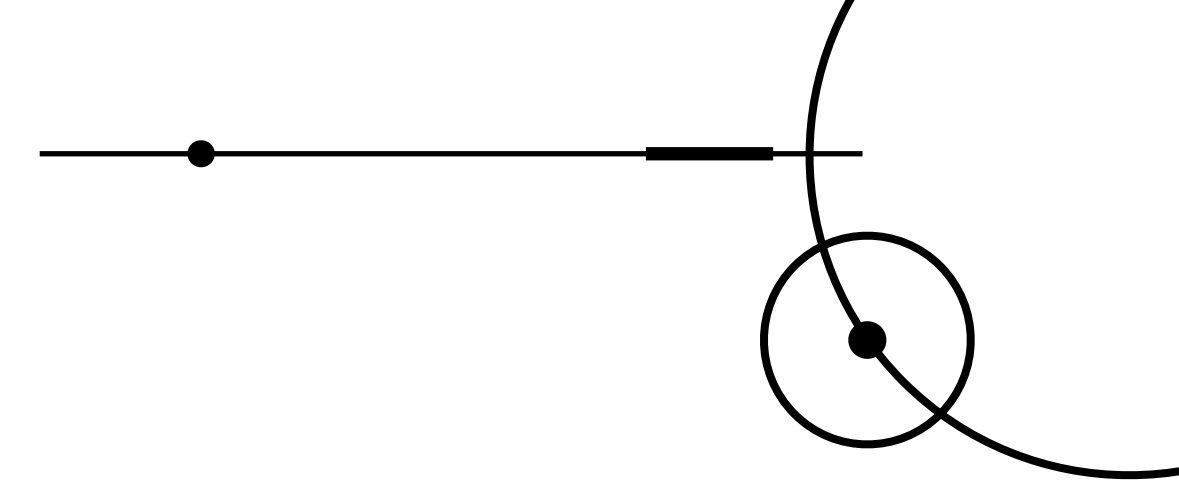
$$\Gamma_t > \Delta E_n$$

→ *Poles not resolved*

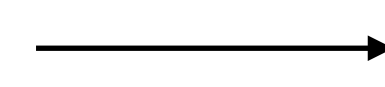


Why Green's functions formalism?

3 - Green's functions contains everything we need



$$[H - E - i\Gamma_t] G(E; \vec{r}, \vec{r}') = -\delta^{(3)}(\vec{r} - \vec{r}')$$

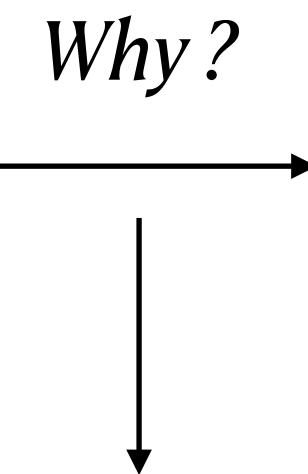


$$G = \frac{1}{E + i\Gamma_t - H} \quad G_0 = \frac{1}{E + i\Gamma_t - H_0} \quad (1)$$

With $H = H_0 + V(\vec{r}) = -\frac{\vec{\nabla}^2}{2m_t} + V(\vec{r})$

Equation (1) yields the operator equation

$$\begin{aligned} G &= G_0 + G_0 V G \\ &= G_0 + G_0 V G_0 + G_0 V G_0 V G_0 + \dots \end{aligned}$$



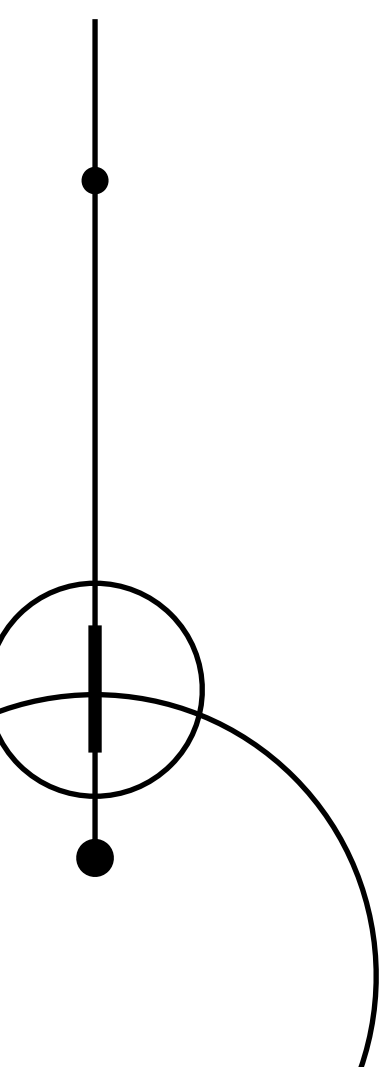
Go to momentum space

$$\tilde{G}(E; \vec{p}, \vec{p}') \equiv \langle \vec{p} | G | \vec{p}' \rangle$$

Momentum space is the QFT language (and experimentalist !)

Ex:

$$\sigma_{tot}(E) \propto \text{Im } G(E; \vec{0}, \vec{0}), \quad 2 \text{Im } G(E, \vec{0}, \vec{0}) = \int \frac{d^3 p}{(2\pi)^3} |\tilde{G}_S(E; p)|^2 \Gamma_{t\bar{t}}(E; p)$$



Why Green's functions formalism?

3 - Green's functions contains everything we need

$$G = G_0 + G_0 V G \quad \xrightarrow{\langle \vec{p} | \cdot | \vec{p}' \rangle} \quad \tilde{G}_l(E; p) = \tilde{G}_0(E; p) + \tilde{G}_0(E; p) \int_0^\infty dq \tilde{V}_l(p, q) \tilde{G}_l(E; q)$$

For S and P wave :

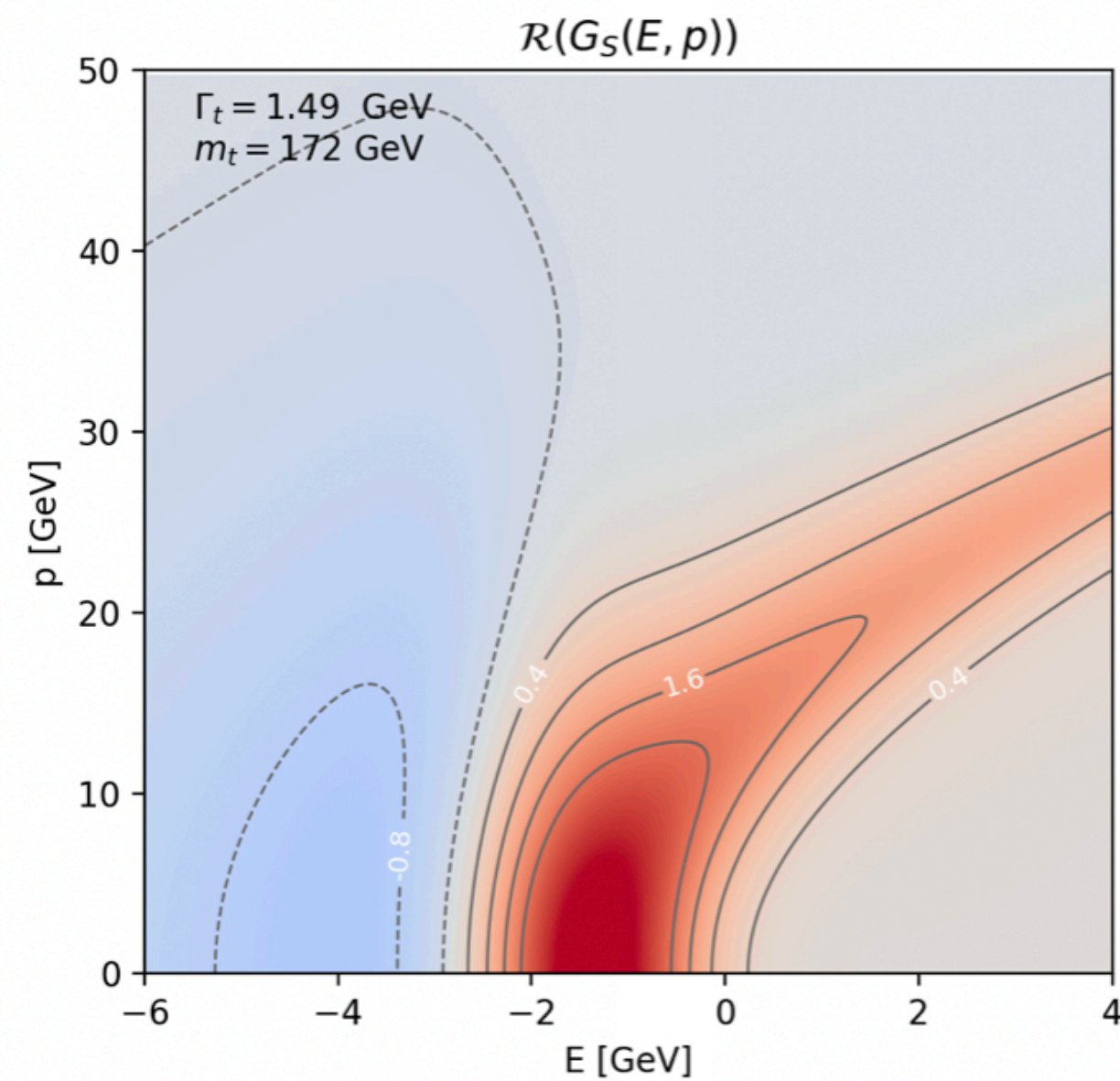
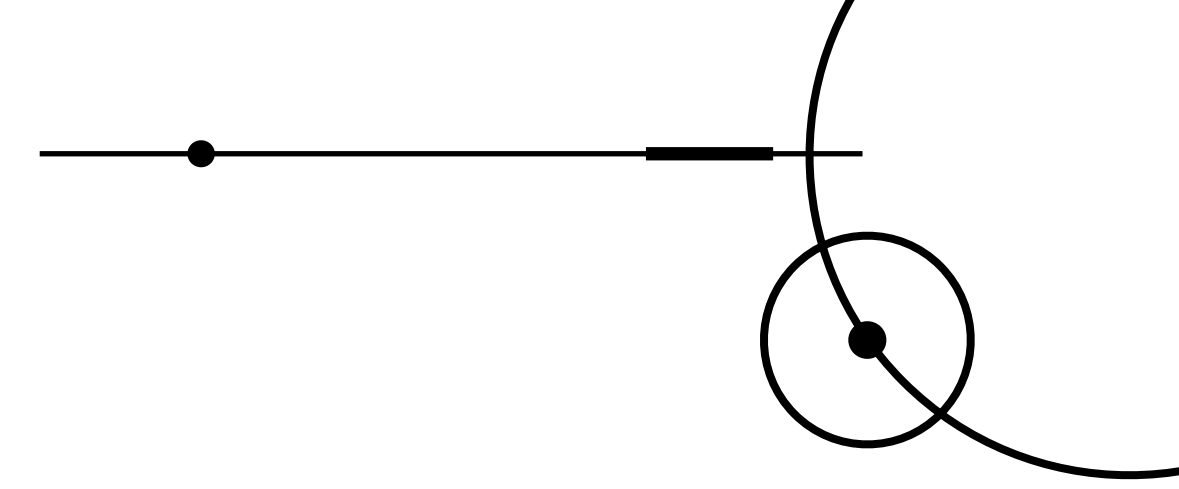
$$\tilde{G}_S(E; p) = \tilde{G}_0(E; p) + \tilde{G}_0(E; p) \int_0^\infty dq \hat{V}_S(p, q) \tilde{G}_S(E; q)$$

$$\tilde{G}_P(E; p) = \tilde{G}_0(E; p) + \tilde{G}_0(E; p) \int_0^\infty dq \hat{V}_P(p, q) \tilde{G}_P(E; q)$$

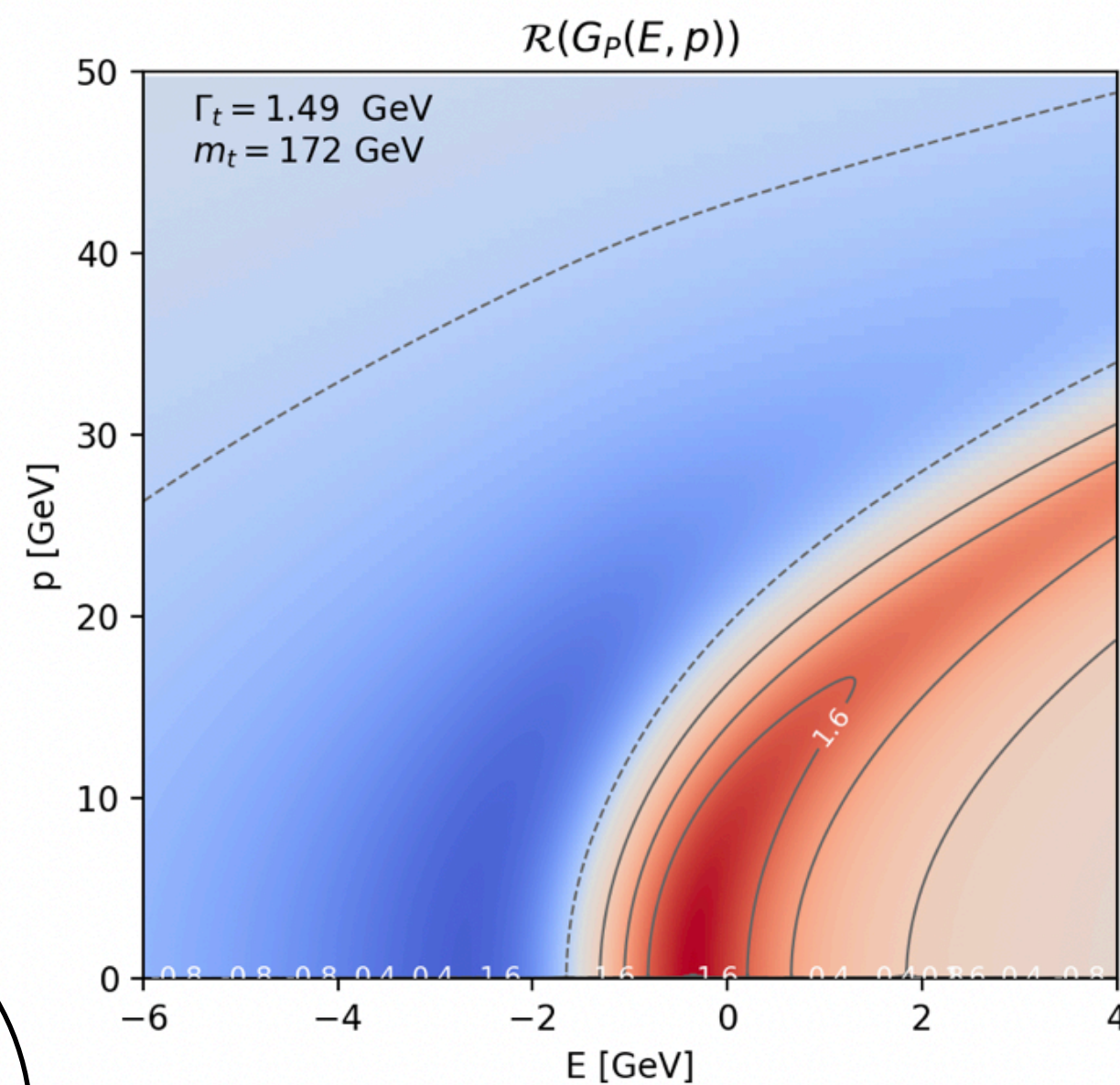
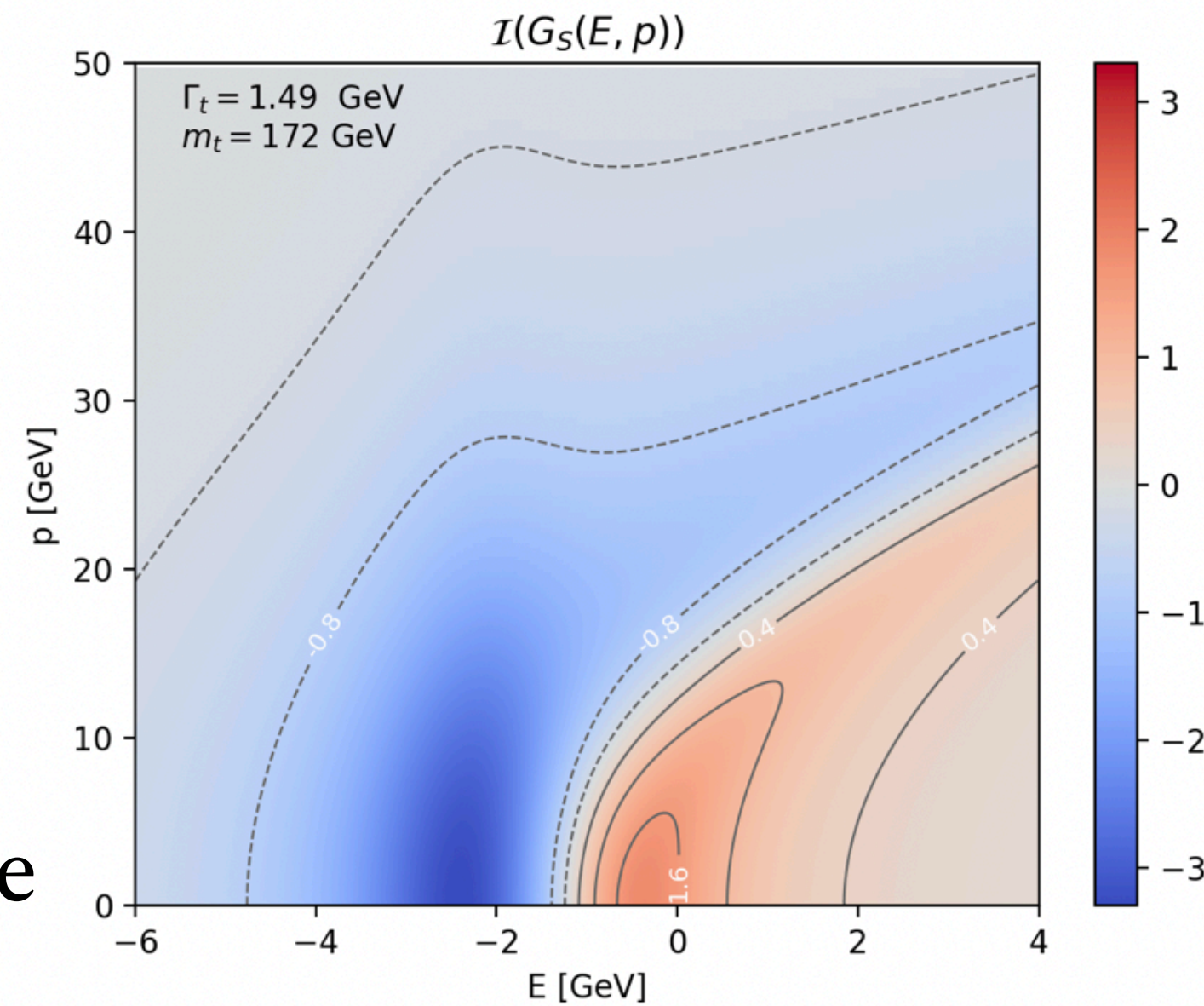
$$\hat{V}_S(p, q) = \frac{q^2}{4\pi^2} \int_{-1}^1 d \cos \theta \tilde{V} \left(\sqrt{p^2 + q^2 - 2pq \cos \theta} \right)$$

$$\hat{V}_P(p, q) = \frac{q^3}{4\pi^2 p} \int_{-1}^1 d \cos \theta \cos \theta \tilde{V} \left(\sqrt{p^2 + q^2 - 2pq \cos \theta} \right)$$

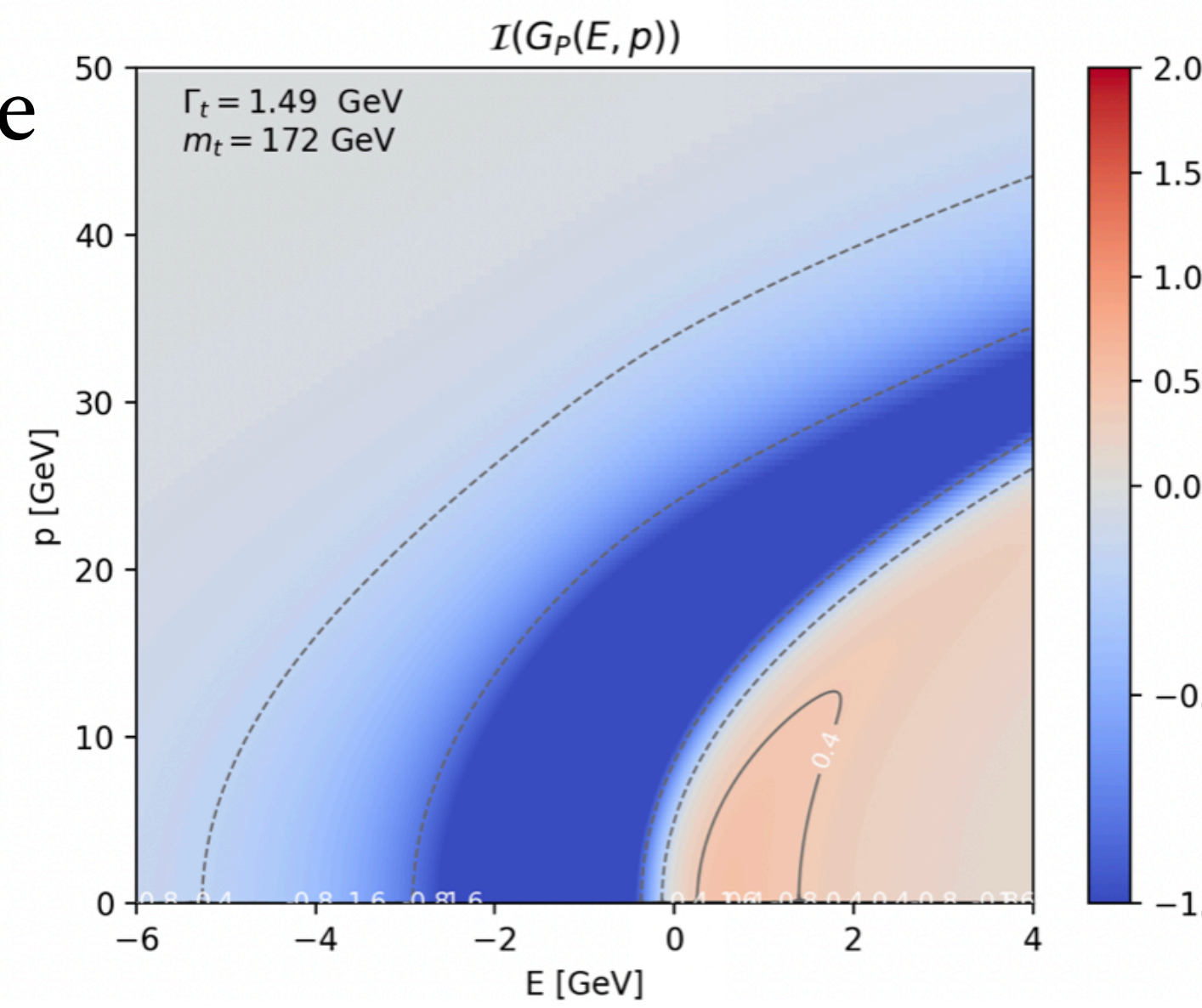
Benchmark: Coulomb potential



S wave



P wave



p : relative momentum

$$E = \sqrt{s} - 2m_t$$

$$\tilde{G}_S(E; p) \sim \frac{R_n(p)}{E - E_n + i\Gamma_t}$$

$$= R_n(p) \left[\frac{E - E_n}{(E - E_n)^2 - \Gamma_t^2} - \frac{i\Gamma_t}{(E - E_n)^2 - \Gamma_t^2} \right]$$

Im G : absorptive part

Re G : dispersive part

Simulating events at LHC

MADGRAPH5_aMC@NLO

Simulation and production of events

Pythia8

Simulation and production of events

Creation of partonic shower

MadAnalysis5

Analysis of results

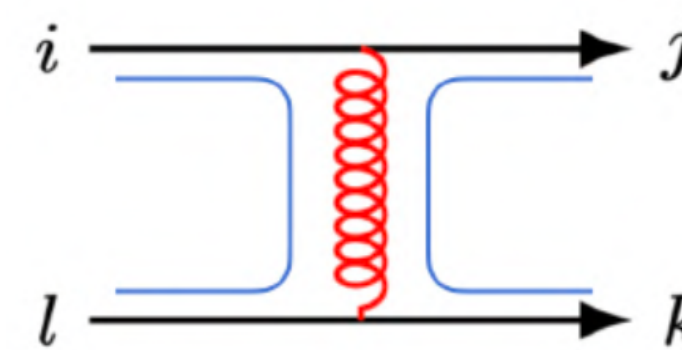
Fastjet (antikt algorithm)

Reconstruction of hadronized objects

Including toponium's effects in Madgraph

$$|\mathcal{M}|^2 \rightarrow |\mathcal{M}|^2 \left| \frac{\tilde{G}(E, p)}{\tilde{G}_0(E, p)} \right|^2$$

Singlet state

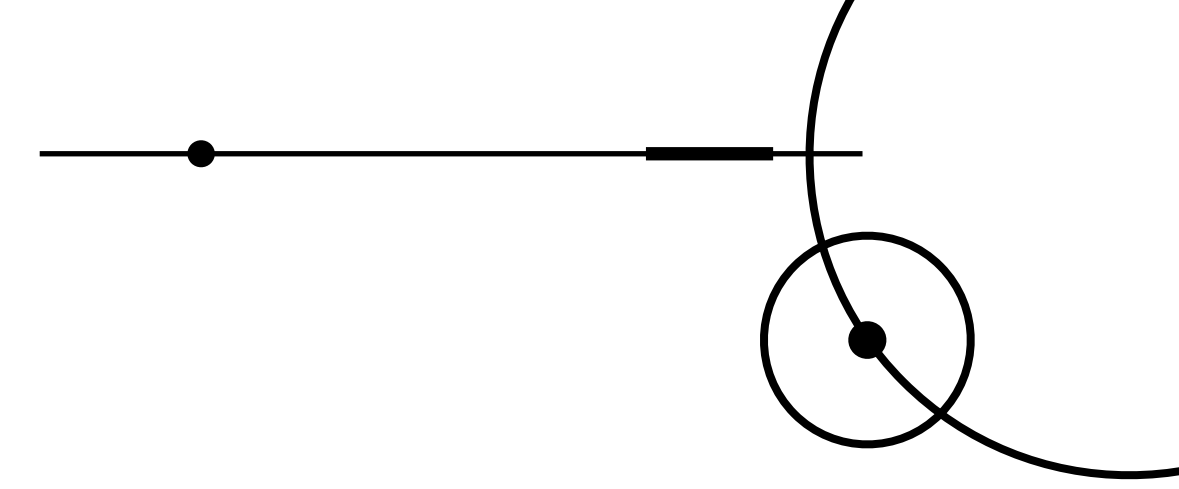


Projection

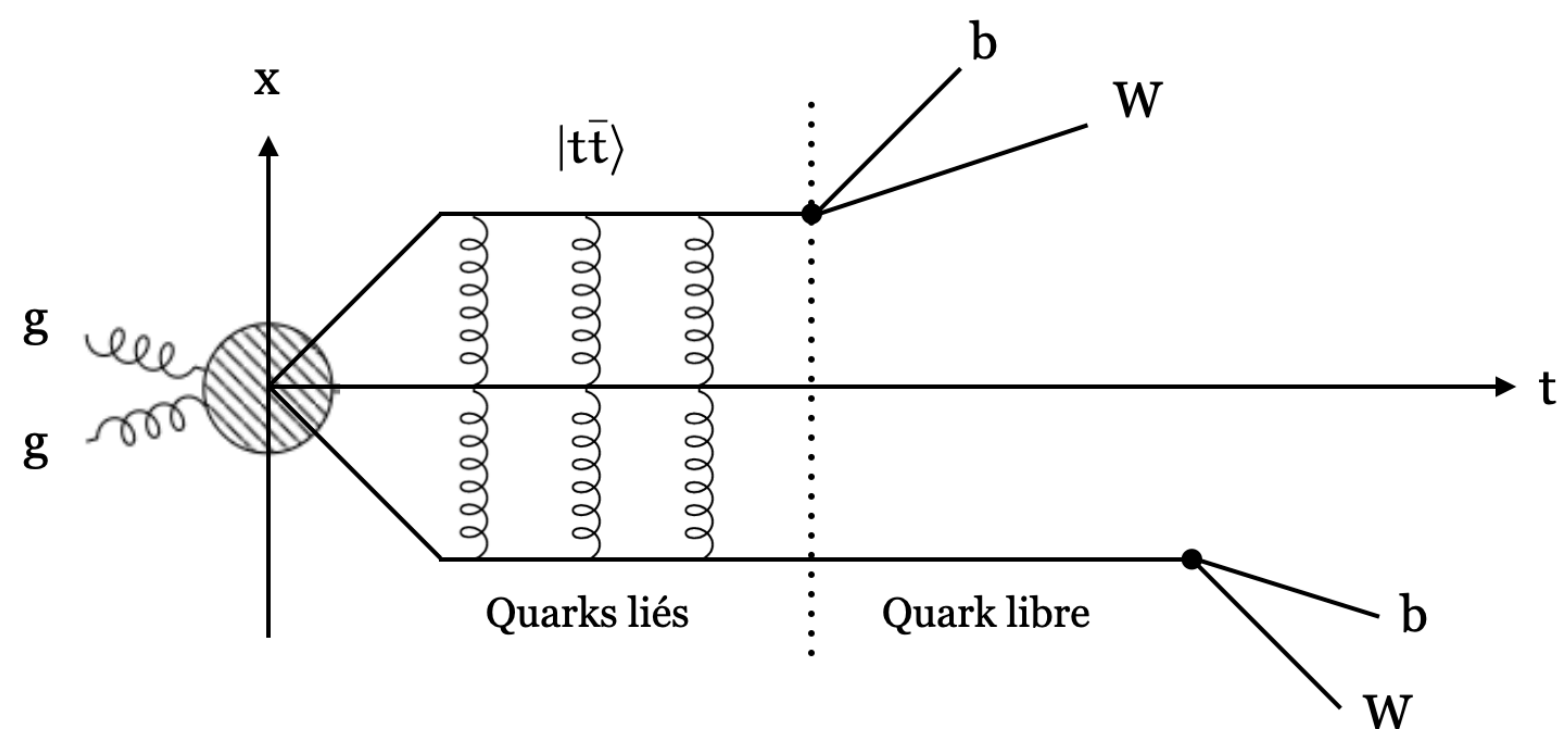
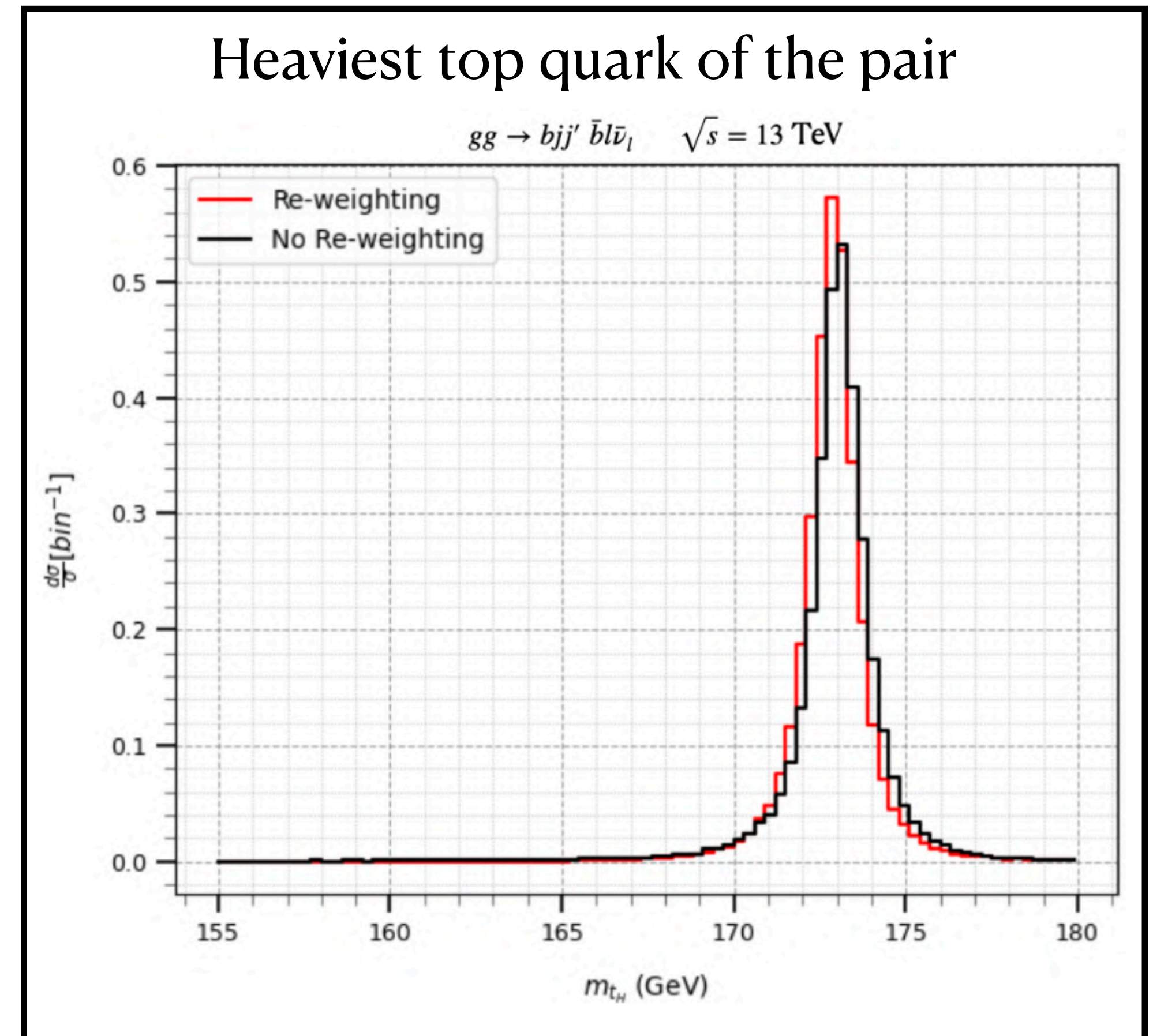
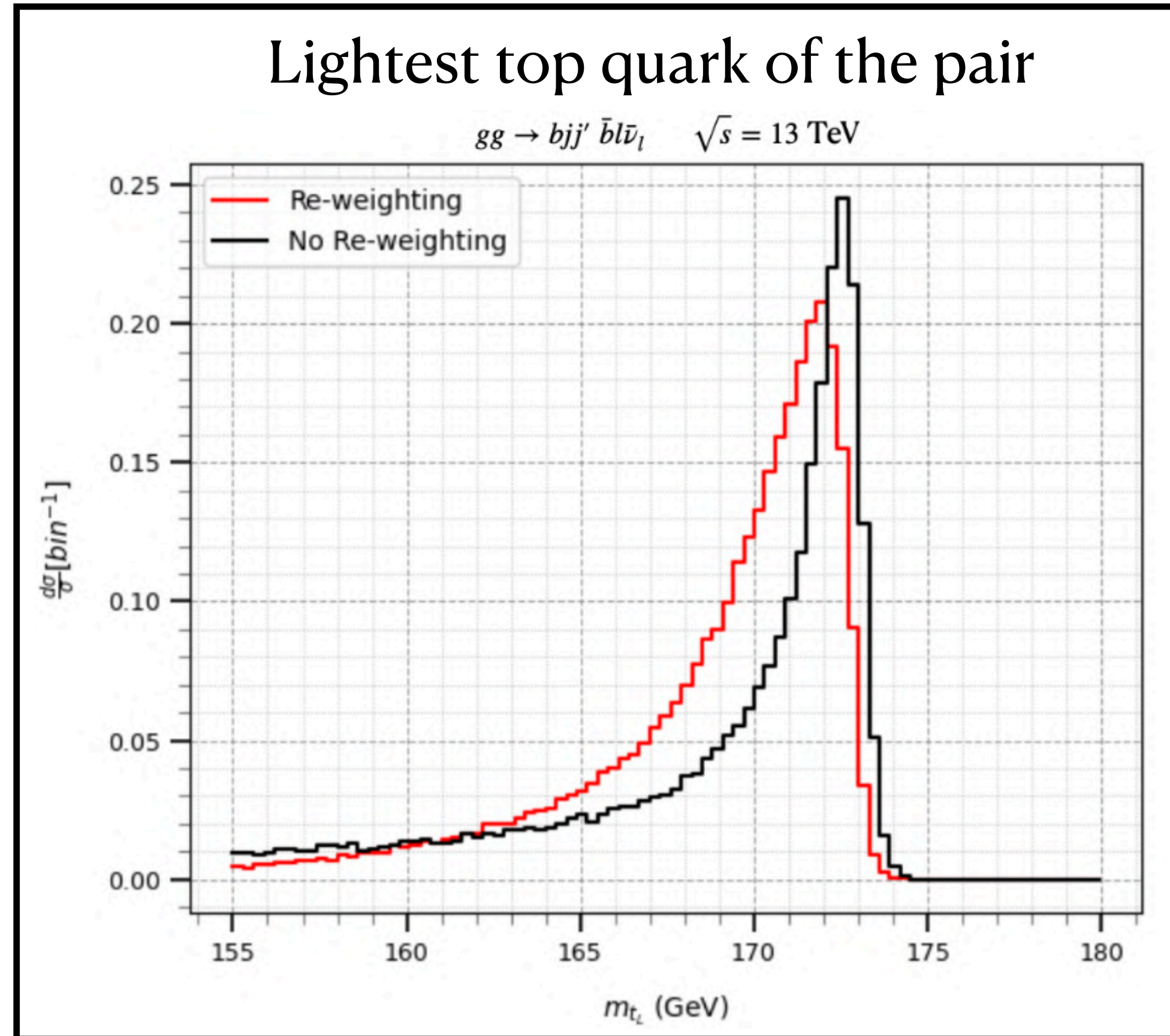
$$V_{\text{QCD}} \delta_{kj} \propto \frac{1}{2} \left(\delta_{il} \delta_{kj} - \frac{1}{N_c} \delta_{ij} \delta_{kl} \right) \delta_{kj}$$

$$= \frac{1}{2} \delta_{li} \left(N_c - \frac{1}{N_c} \right) > 0$$

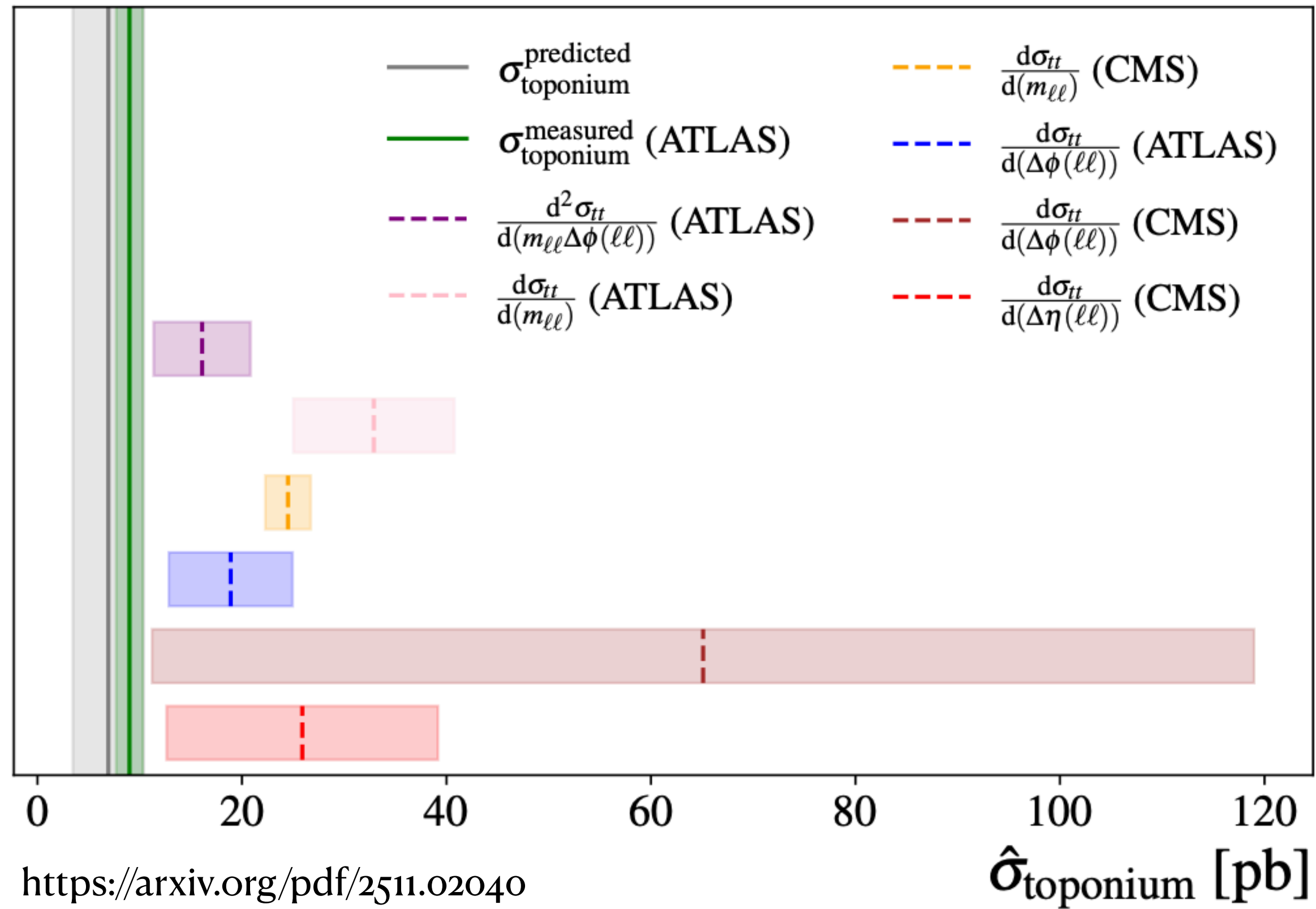
Impact on distributions



$$|\mathcal{M}|^2 \rightarrow |\mathcal{M}|^2 \left| \frac{\tilde{G}(E, p)}{\tilde{G}_0(E, p)} \right|^2$$



Toponium and BSM



- Missing higher-order QCD corrections in the conventional NNLO baseline
- NNLO effects in the top quark decay treatment in the LHC
- New physics ?

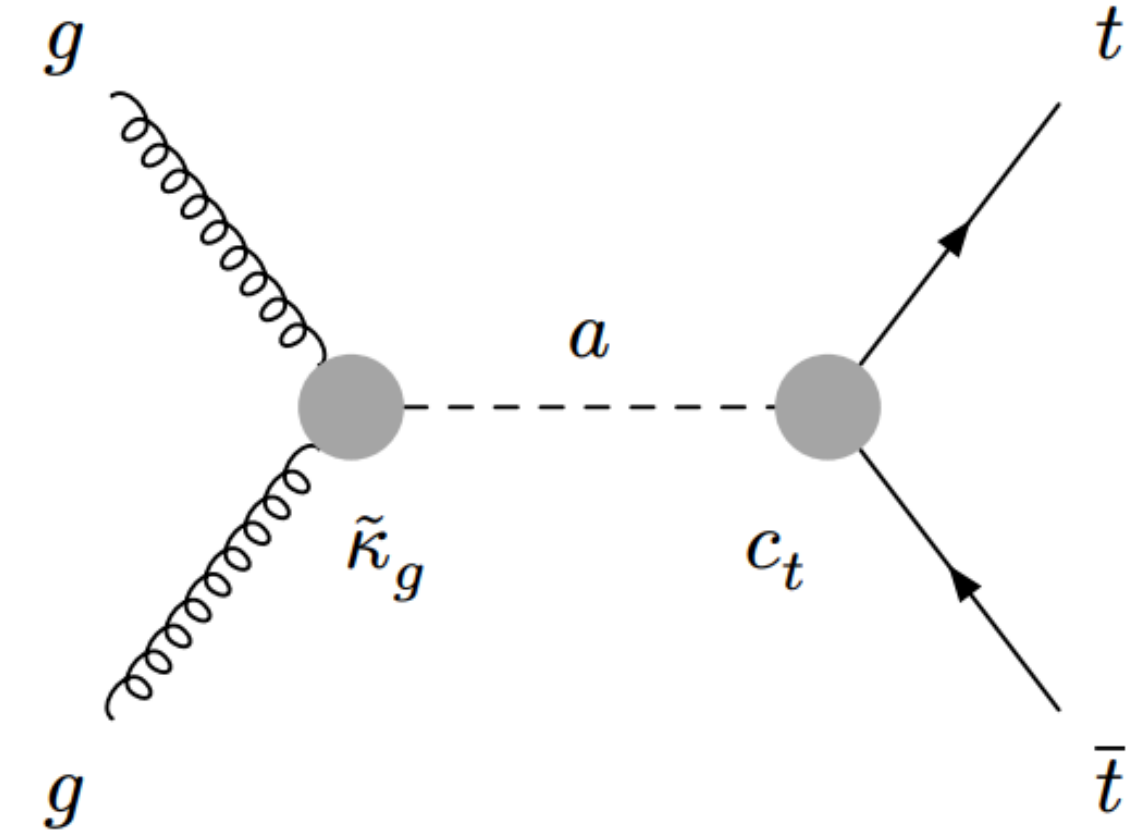
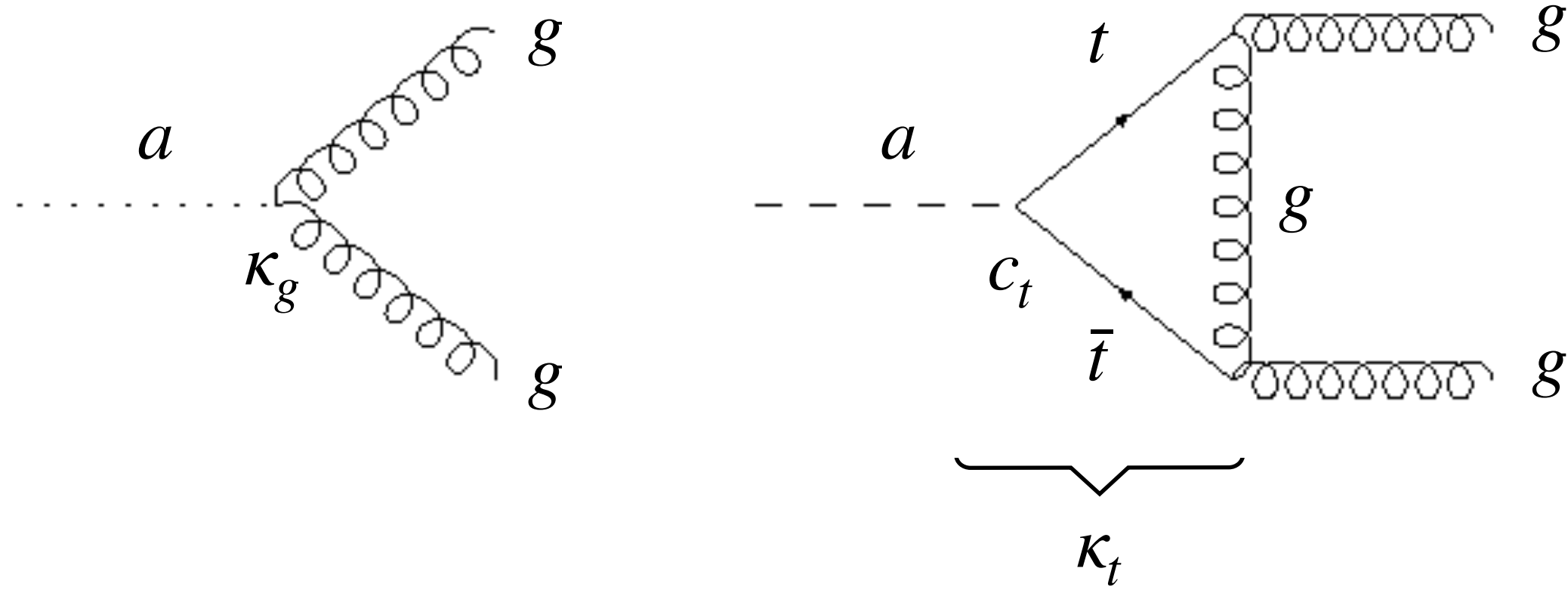
Toponium and BSM: the model

New pseudo-scalar a only coupling to gluons and top quarks:

$$\mathcal{L}_a = \frac{1}{2} \partial_\mu a \partial^\mu a - \frac{1}{2} M_a^2 a^2 + \frac{\alpha_s}{8\pi v} \tilde{\kappa}_g a G_{\mu\nu}^a \tilde{G}^{a\mu\nu} + 2c_t a \bar{t} i \gamma^5 t$$

v : Higgs VeV

With $\tilde{\kappa}_g = \kappa_g + \kappa_t$



Fixed order : good modelling far from threshold

$$\sigma_{QCD} = \sigma_{pQCD} + \sigma_{NRQCD}[\mathcal{M}_{QCD}]$$

$$\sigma_{BSM} = \sigma_{pBSM} + \sigma_{NRQCD}[\mathcal{M}_{BSM}]$$

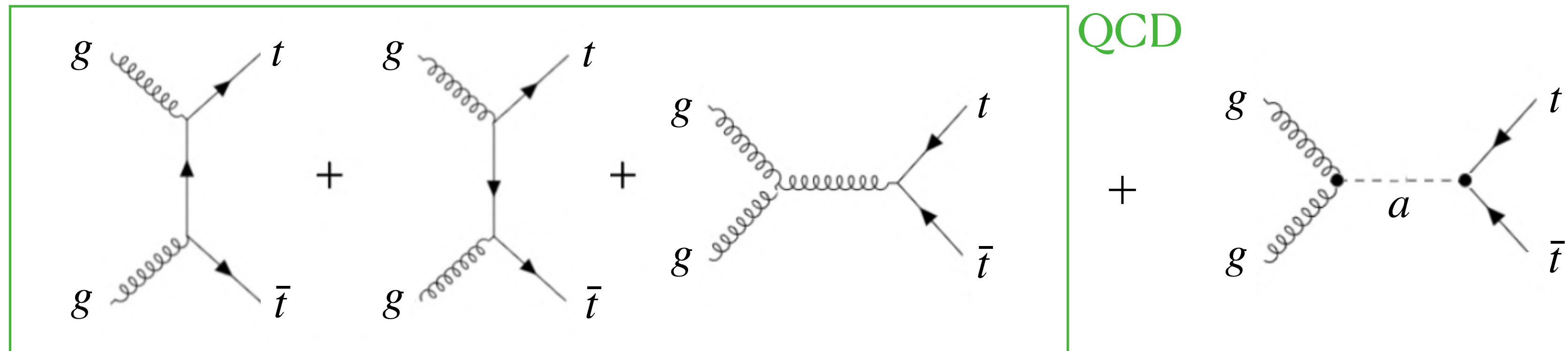
Toponium's contribution :
good modelling near threshold

Toponium and BSM: the model

$$\sigma_{QCD} = \sigma_{pQCD} + \sigma_{NRQCD}[\mathcal{M}_{QCD}]$$

$$\sigma_{BSM} = \sigma_{pBSM} + \sigma_{NRQCD}[\mathcal{M}_{BSM}]$$

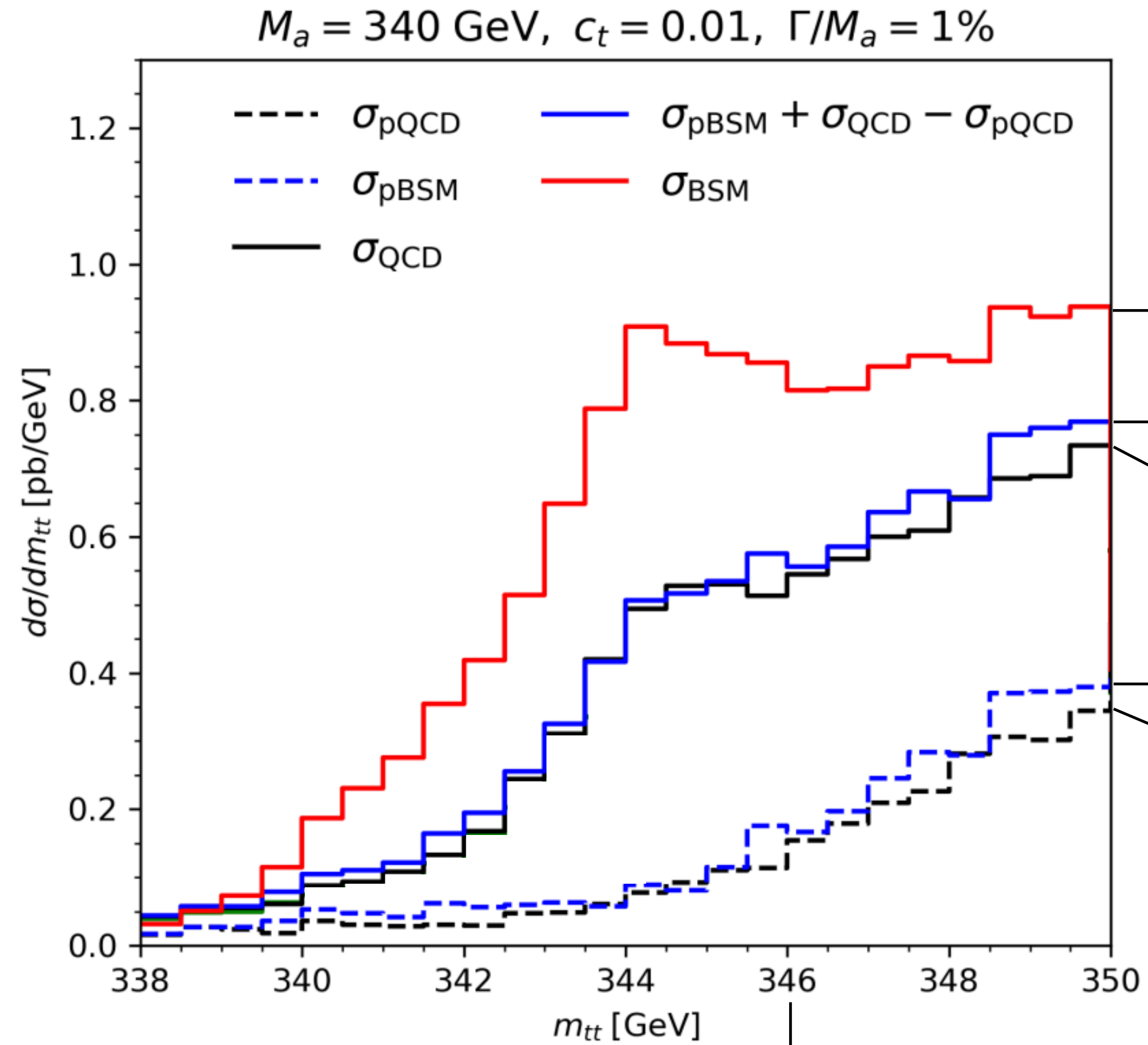
$$\text{With } \mathcal{M}_{BSM} = \mathcal{M}_{QCD} + \mathcal{M}_a$$



$$\sigma_{NRQCD}[\mathcal{M}_{BSM}] = \int_{threshold} d\Phi \left| \mathcal{M}_{BSM} \cdot \frac{\tilde{G}(E, p^*)}{\tilde{G}_0(E, p^*)} \right|^2$$

$$\sigma_{NRQCD}[\mathcal{M}_{QCD}] = \int_{threshold} d\Phi \left| \mathcal{M}_{QCD} \cdot \frac{\tilde{G}(E, p^*)}{\tilde{G}_0(E, p^*)} \right|^2$$

Toponium and BSM: the results



Allow slight off-shellness + resums
the Coulomb ladder

$$\sigma_{pBSM} + \sigma_{NRQCD}[\mathcal{M}_{BSM}]$$

$$\sigma_{pBSM} + \sigma_{NRQCD}[\mathcal{M}_{QCD}]$$

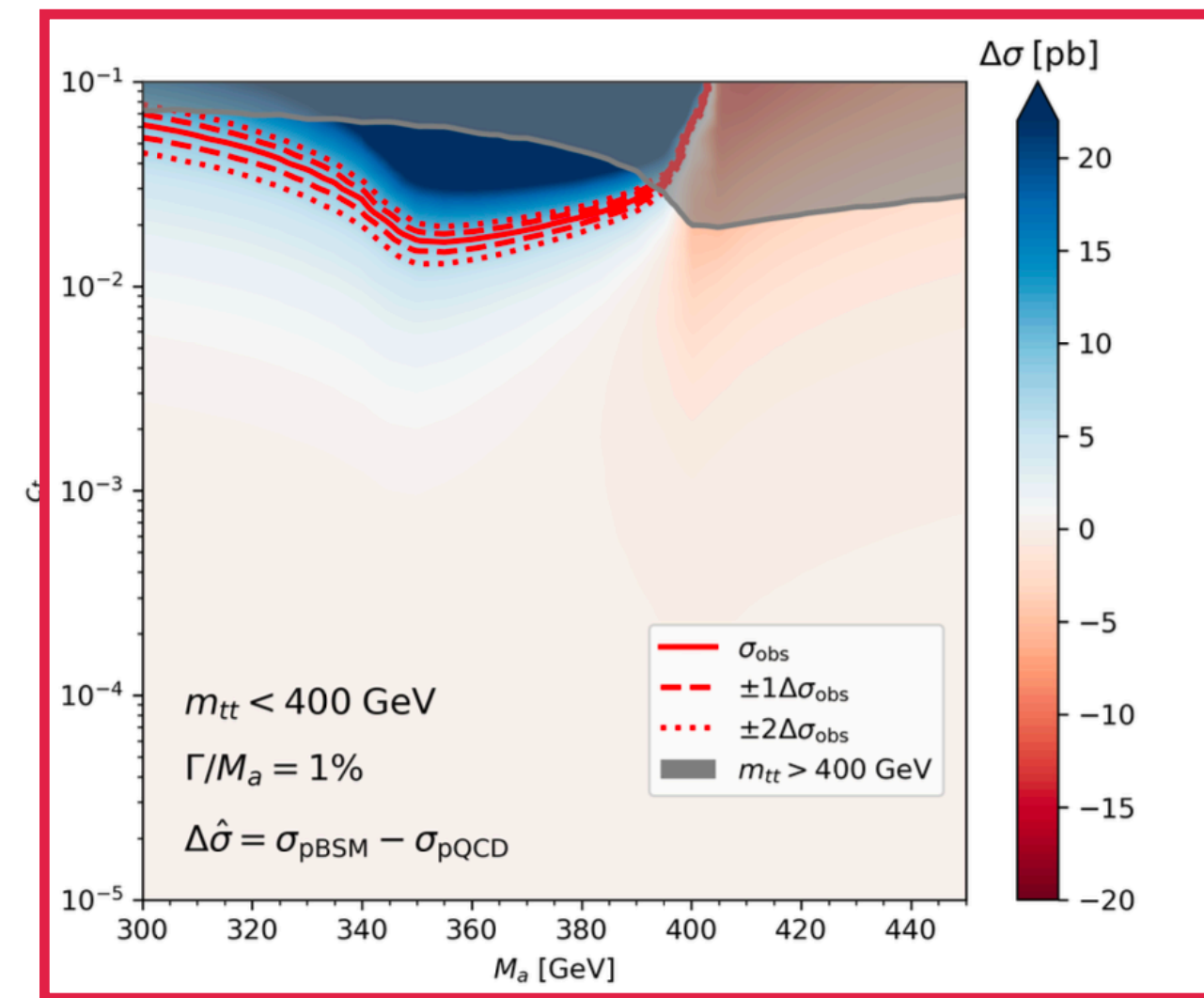
$$\sigma_{QCD} = \sigma_{pQCD} + \sigma_{NRQCD}[\mathcal{M}_{QCD}]$$

σ_{pBSM} : BSM perturbation theory

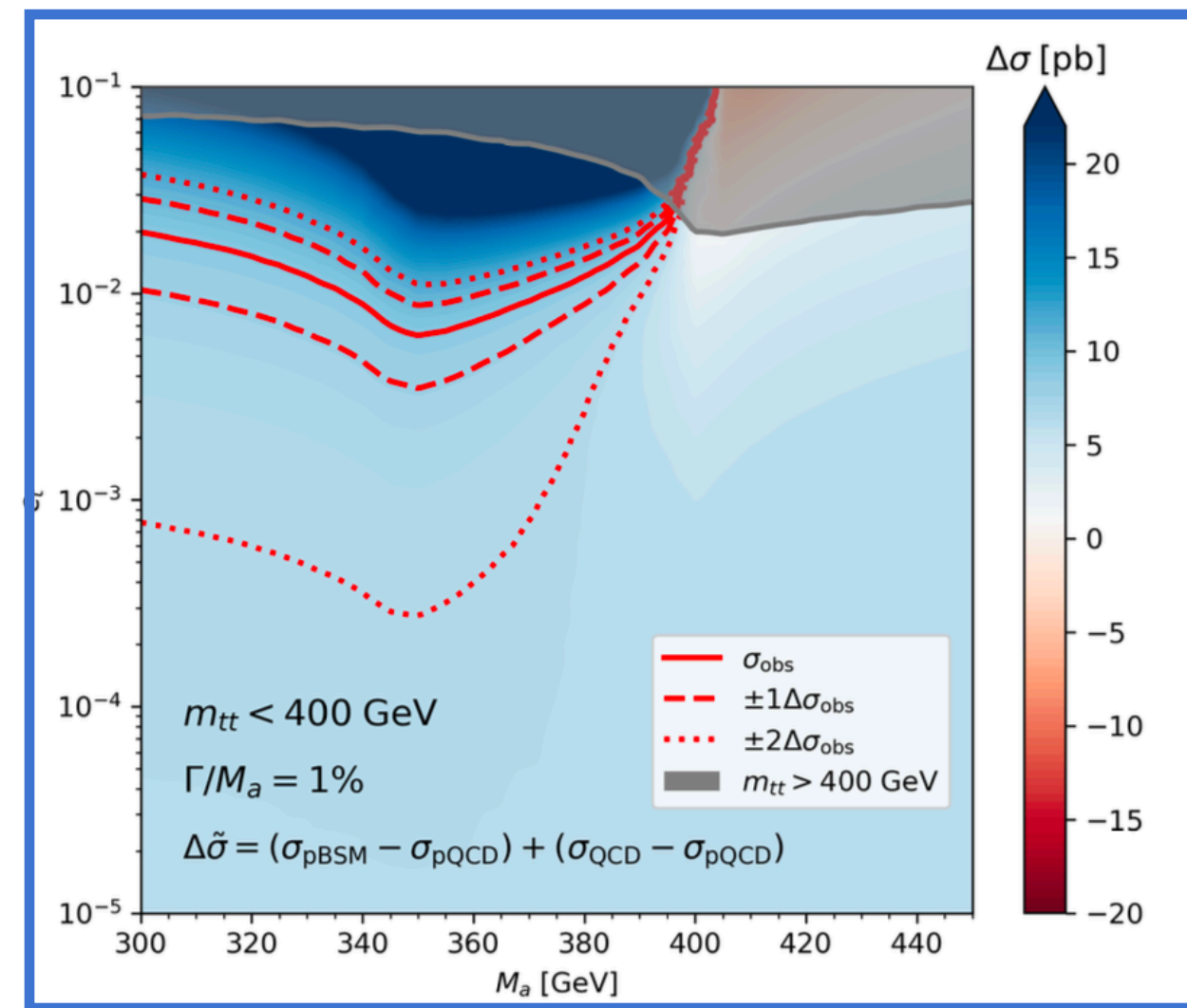
σ_{pQCD} : SM perturbation theory

$2m_t$

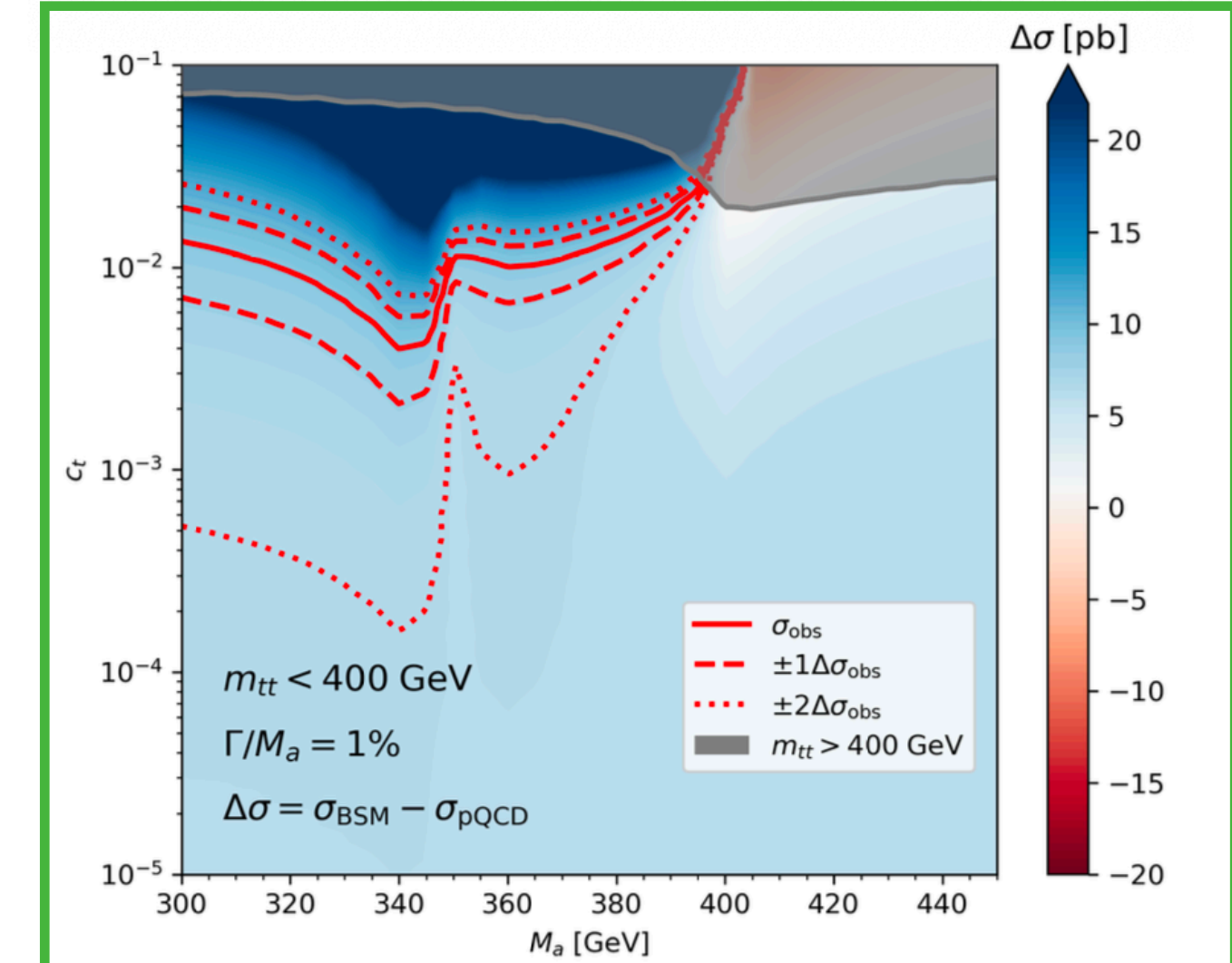
Toponium and BSM: the results



SM perturbation theory + BSM



(SM toponium) + BSM

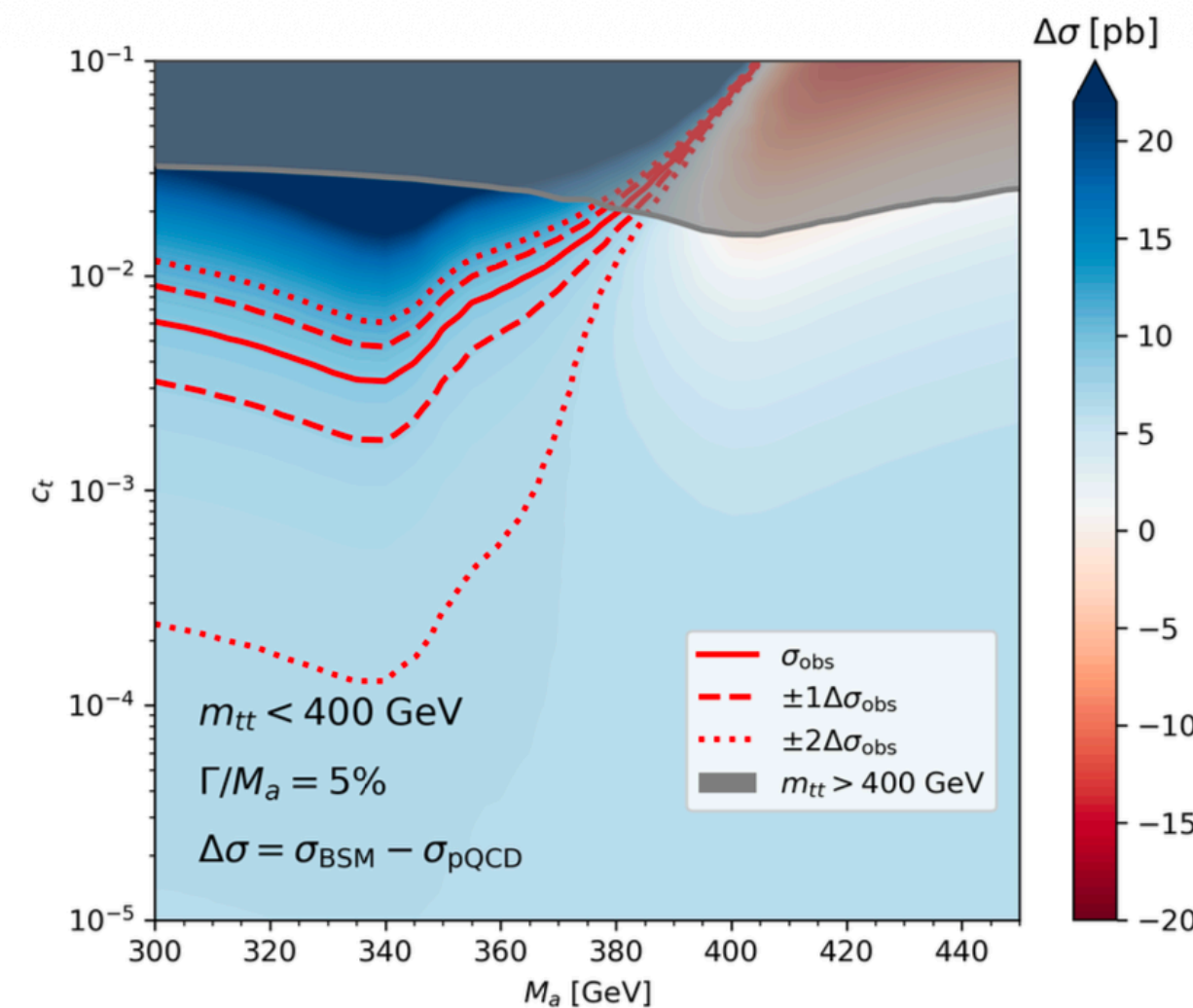
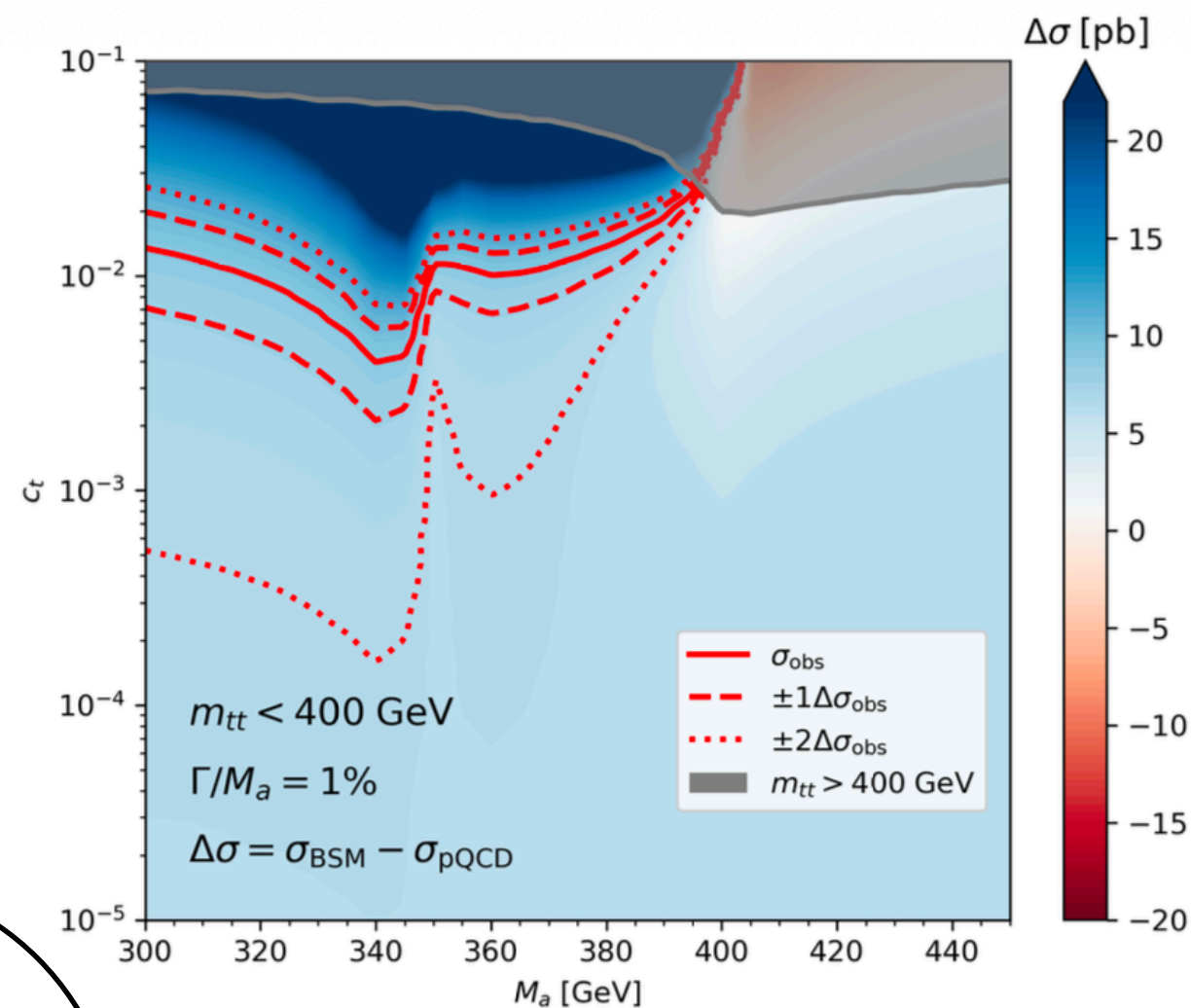
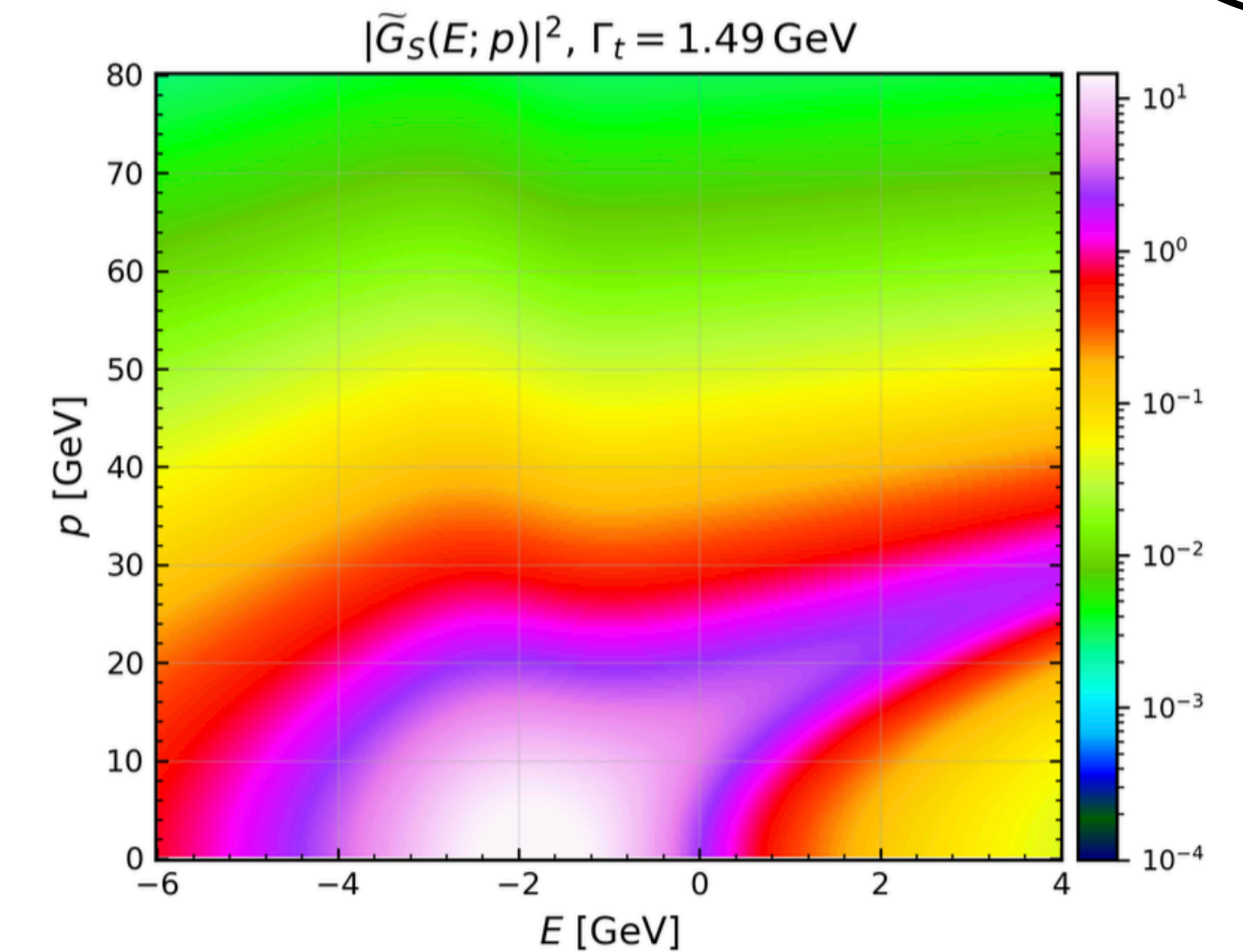


(SM + BSM) toponium

General conclusions

Green's functions in momentum space are an ideal NRQCD framework

- They include perturbative effects and can be associated with observables
- They can be used to model pseudo bound states effects : toponium
- P wave's analytical results are under work



BSM studies concludes that :

- SM toponium effects can explain the ATLAS excess on their own, but the data are compatible with additional pseudo-scalar contributions
- Deriving robust constraints on top-philic pseudo-scalars requires correctly treating BSM-modified toponium production

Thank you!

