Earth rotation impact for long-duration signals

ET France
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- *Motivations*
- *Mismatch study*
- *SNR study*
- *Conclusion - Discussion*

Motivations

$$h(t) = h_+(t)F_+(\theta,\phi,\psi) + h_\times F_\times(\theta,\phi,\psi)$$

ET-Xylophone

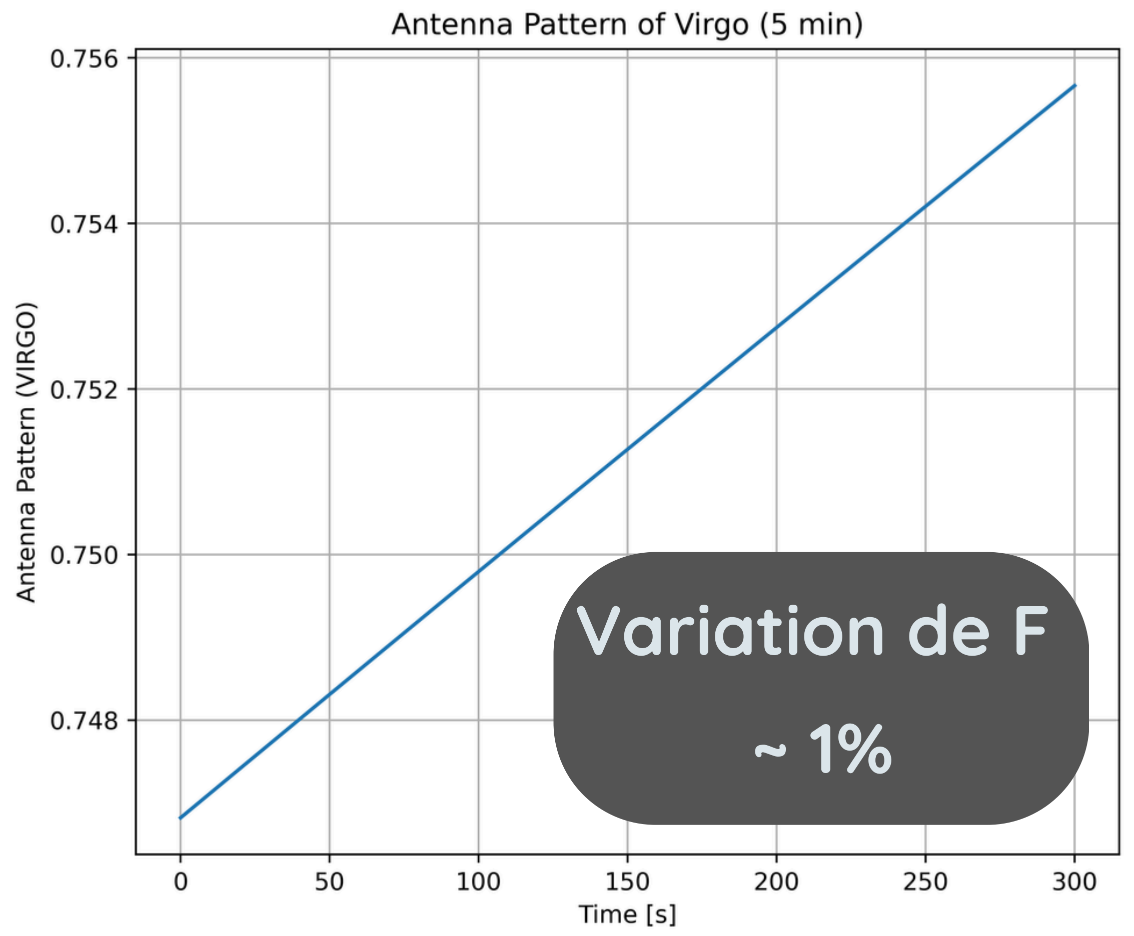
Signal duration

- Virgo : ~5 min
- ET : ~1-2h

Random example

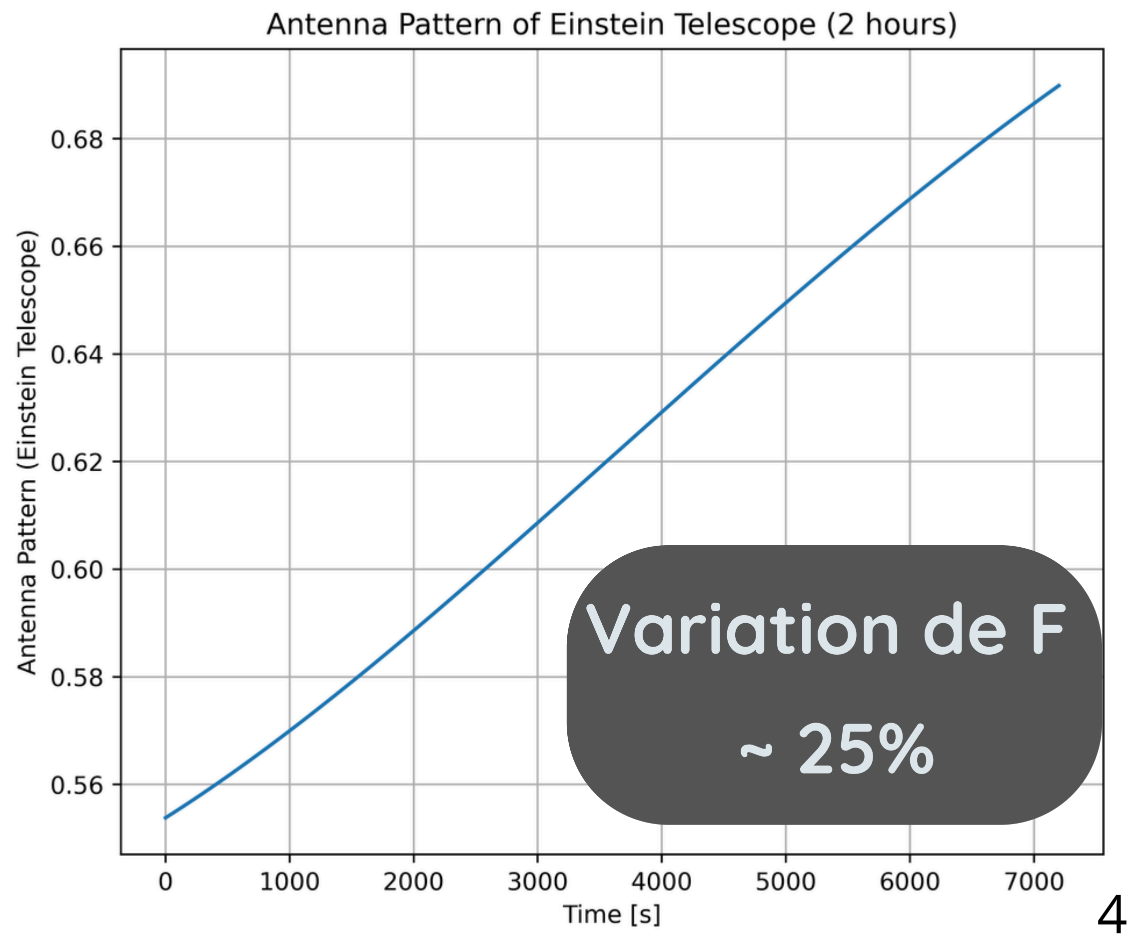
RA = 3.45 [rad]

DEC = -0.408 [rad]



Variation de F

~ 1%



Variation de F

~ 25%

Incoming signal
(Naturally modulated by F(t))

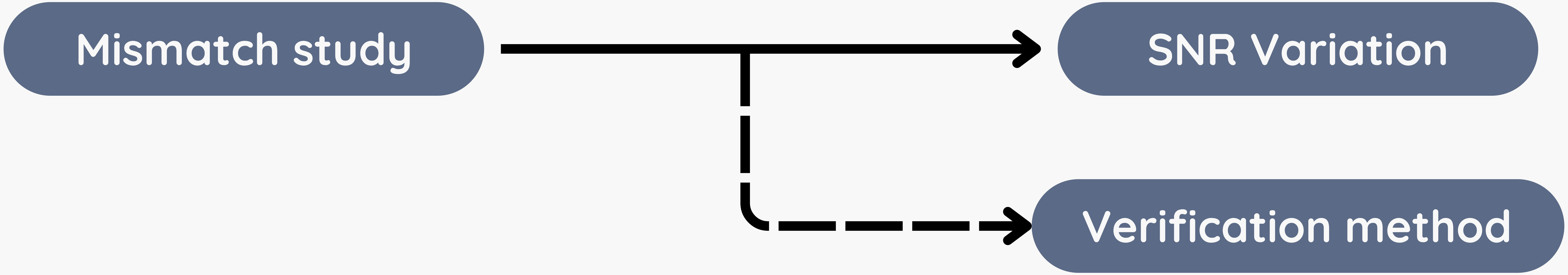
Optimal template
=
Time-modulated template
F(t)h(t)
(Complexifies the bank)

"Simple" template
=
Non time-modulated template
F(0)h(t)
(Virgo-like case)

Which one do we choose ?

How much Match and SNR do we loose if taking F(0)h(t) ?

$$\frac{\Delta\rho}{\rho} := \frac{\rho_{mod} - \rho_{const}}{\rho_{mod}} \approx 1 - \mathcal{M}[F(t)h(t), F(0)h(t)]$$



For each (RA, DEC) point in the sky

Generating h(t) (+ and x)

Computation of F(t) and F(0) (+ and x)

Computation of F(t)h(t) and F(0)h(t)

Mismatch computation between F(t)h(t) and F(0)h(t)

Coded in Python (with PyCBC)

$$\mathcal{M}(s,T) = \max_{\Delta t, \Delta \phi} \frac{< s, T >}{||s||.||T||}$$

Simple case

M1 = M2 = 1.4 $M_\odot$

f_low = 5 Hz

WF model: IMRPhenomD

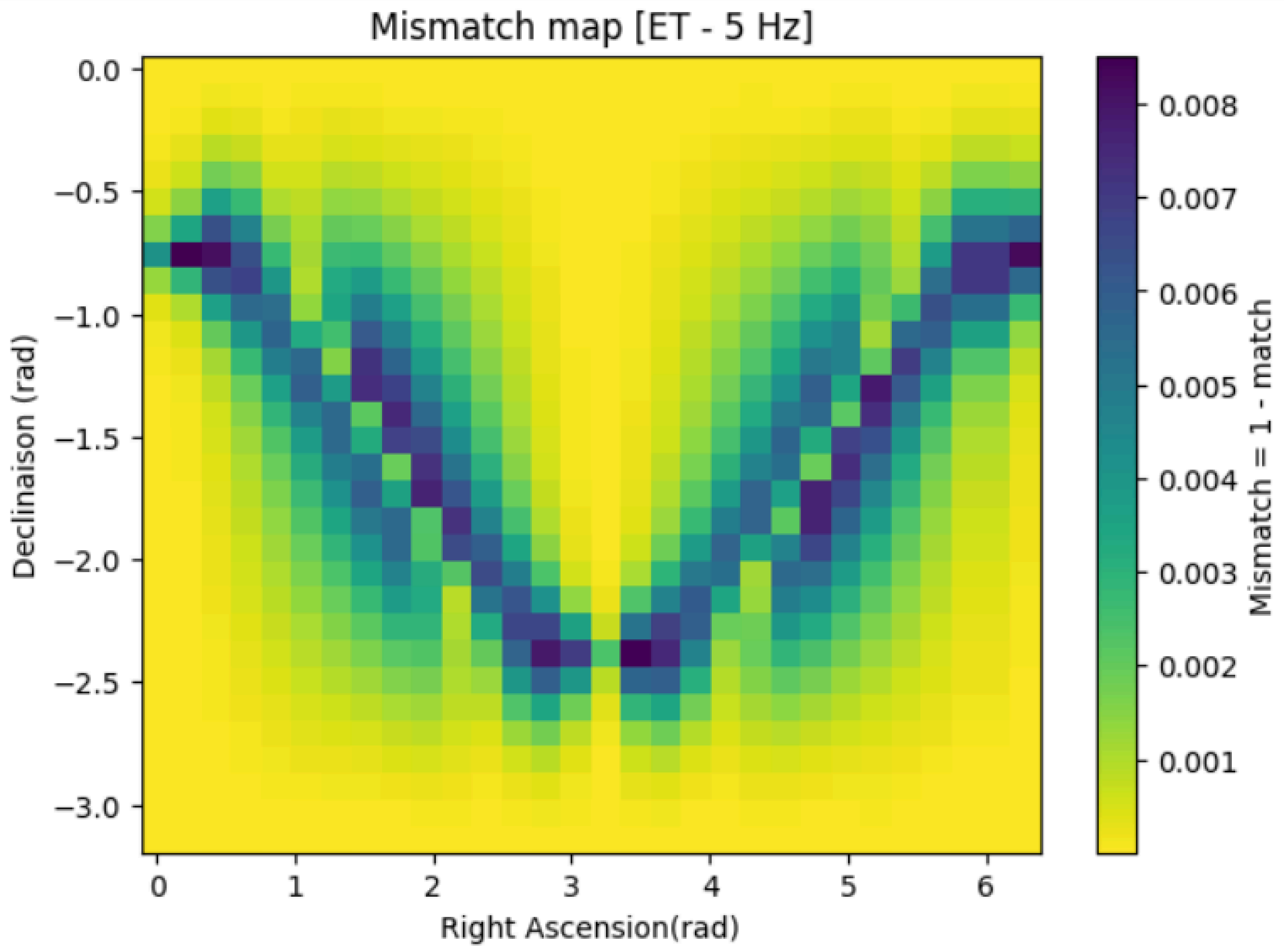
Distance = 100 Mpc

No spin

Inclination = 0°

PSI = 0 *

*Equal contribution of F+ and Fx in the Match



Max ~ 0.8%

Earth rotation can be neglected here

Slow computation

Semi-analytic study through SPA (M. Pillas)

Development of a semi-analytical formula to compute the mismatch

$$h_+(t) = A(t)\ (1 + \cos^2(\imath))\ \cos(\Phi(t))$$

$$h_\times(t) = A(t)\ 2\ \cos(\imath)\ \sin(\Phi(t))$$

$$A(t) = \frac{4}{D}\left(\frac{G\mathcal{M}}{c^2}\right)^{5/3}\left(\frac{\pi f_{\rm gw}(t)}{c}\right)^{2/3}$$

$$f_{\rm gw}(t_c - t) = \frac{1}{\pi}\left(\frac{G\mathcal{M}}{c^3}\right)^{-5/8}\left(\frac{5}{256}\frac{1}{(t_c - t)}\right)^{3/8}$$

$$\Phi(t) = -2\left(\frac{5G\mathcal{M}}{c^3}\right)^{-5/8}(t_c - t)^{5/8} + \phi_c$$

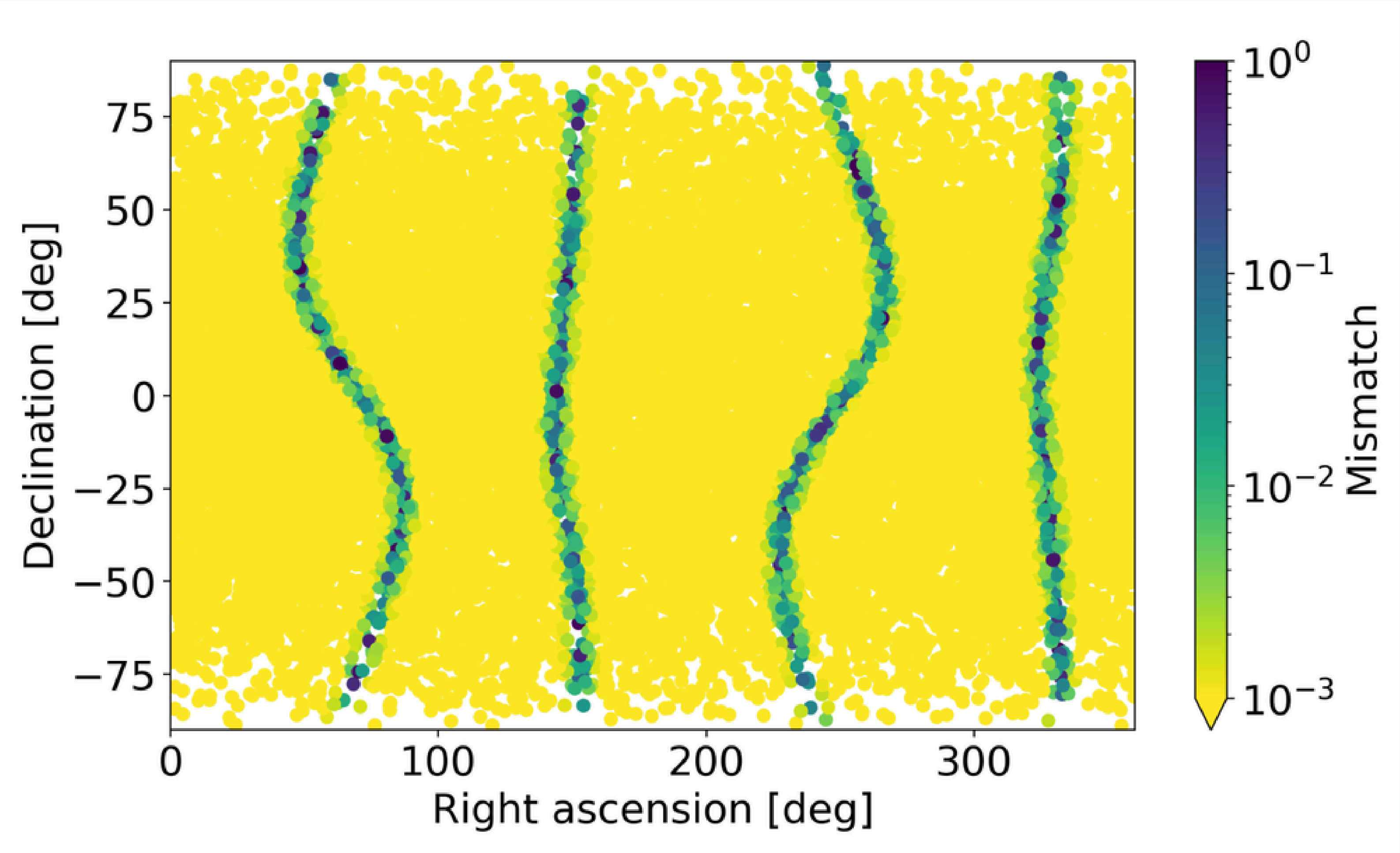
$$m(h_{\rm rot}, h_{\rm const}) = \max_{\Delta t_c, \Delta\phi_c} \frac{\Re \int \frac{f^{-7/3}|B(t_{\rm rot}(f))| e^{-i\theta(t_{\rm rot}(f))} e^{i(2\pi f \Delta t_c - \Delta\phi_c)} e^{i\phi_0}}{S_n(f)} df}{\sqrt{\int \frac{f^{-7/3}|B(t_{\rm rot}(f))|^2}{S_n(f)} df \ \int \frac{f^{-7/3}}{S_n(f)} df}}$$

With:

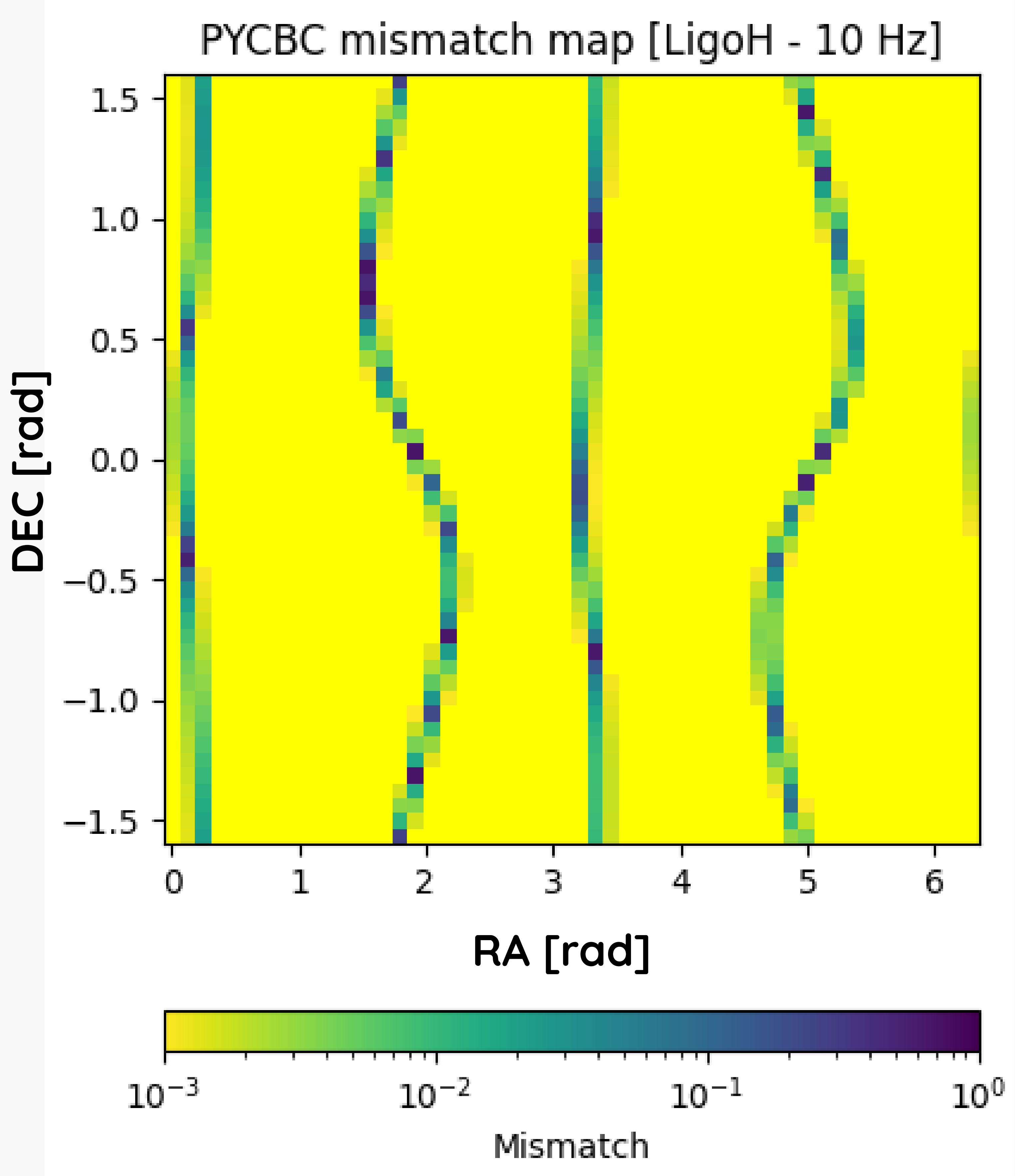
$$B(t(f)) = \sqrt{F_+^2(t_{\rm rot}(f))(1+\cos^2(\imath))^2 + F_\times^2(t_{\rm rot}(f))\ 4\ \cos^2(\imath)}$$

$$\theta(t(f)) = \arctan\left(\frac{-F_\times(t_{\rm rot}(f))\ 2\ \cos(\imath)}{F_+(t_{\rm rot}(f))(1+\cos^2(\imath))}\right)$$

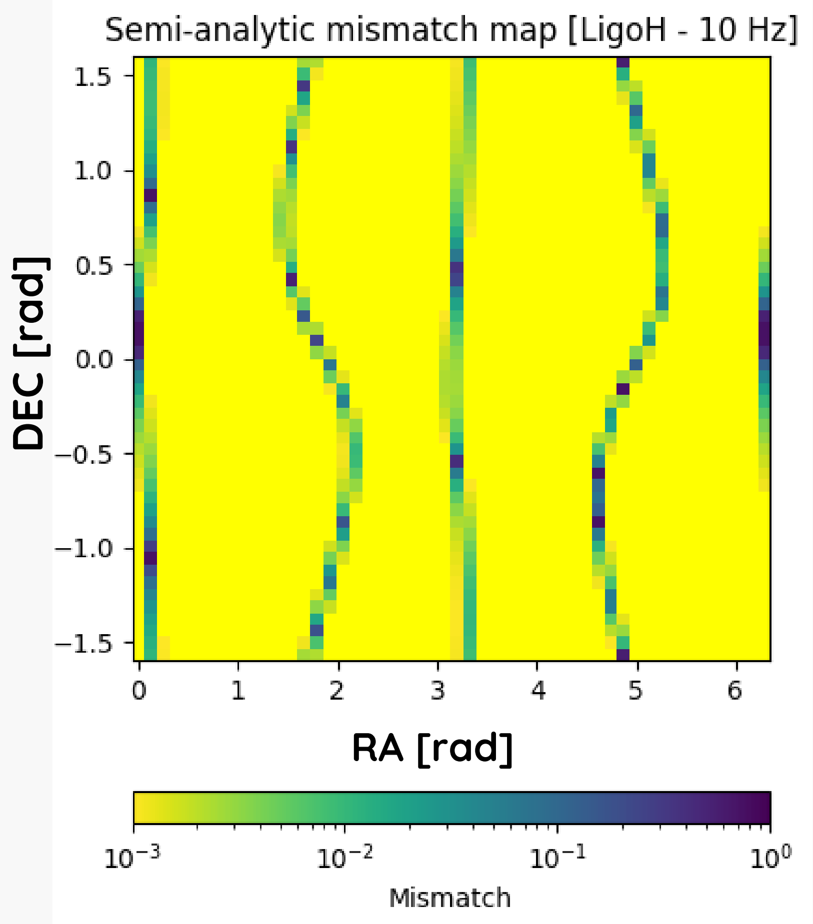
Comparison with our method

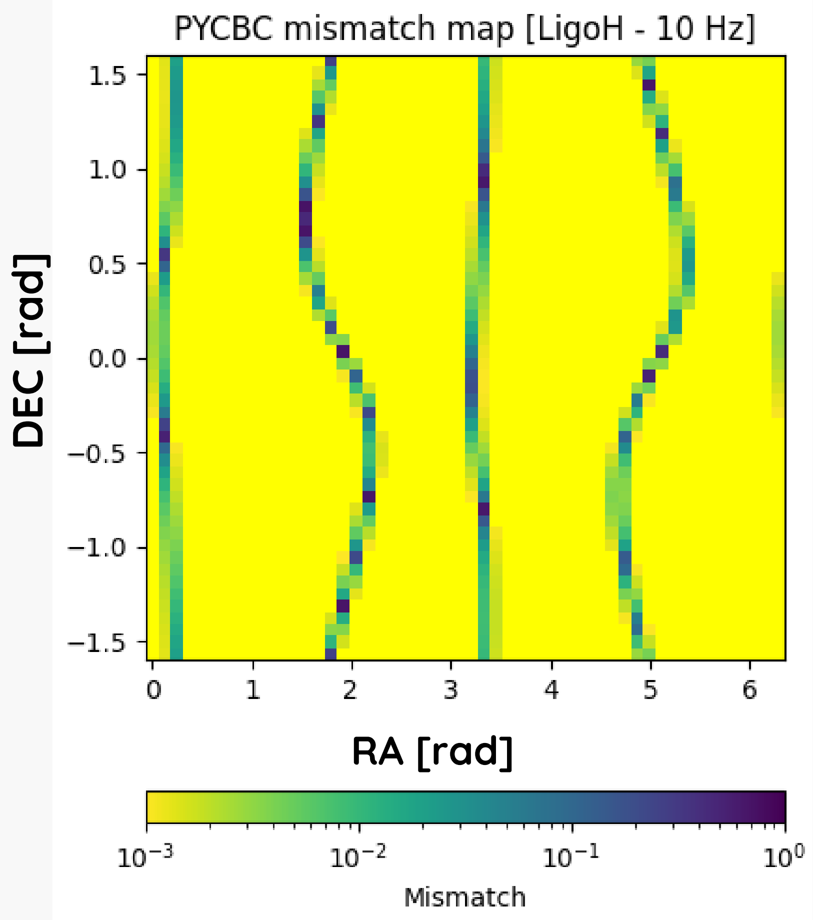


LigoH, f_low = 10 Hz, inclination = 90°
M_chirp = 1.2 M_sol

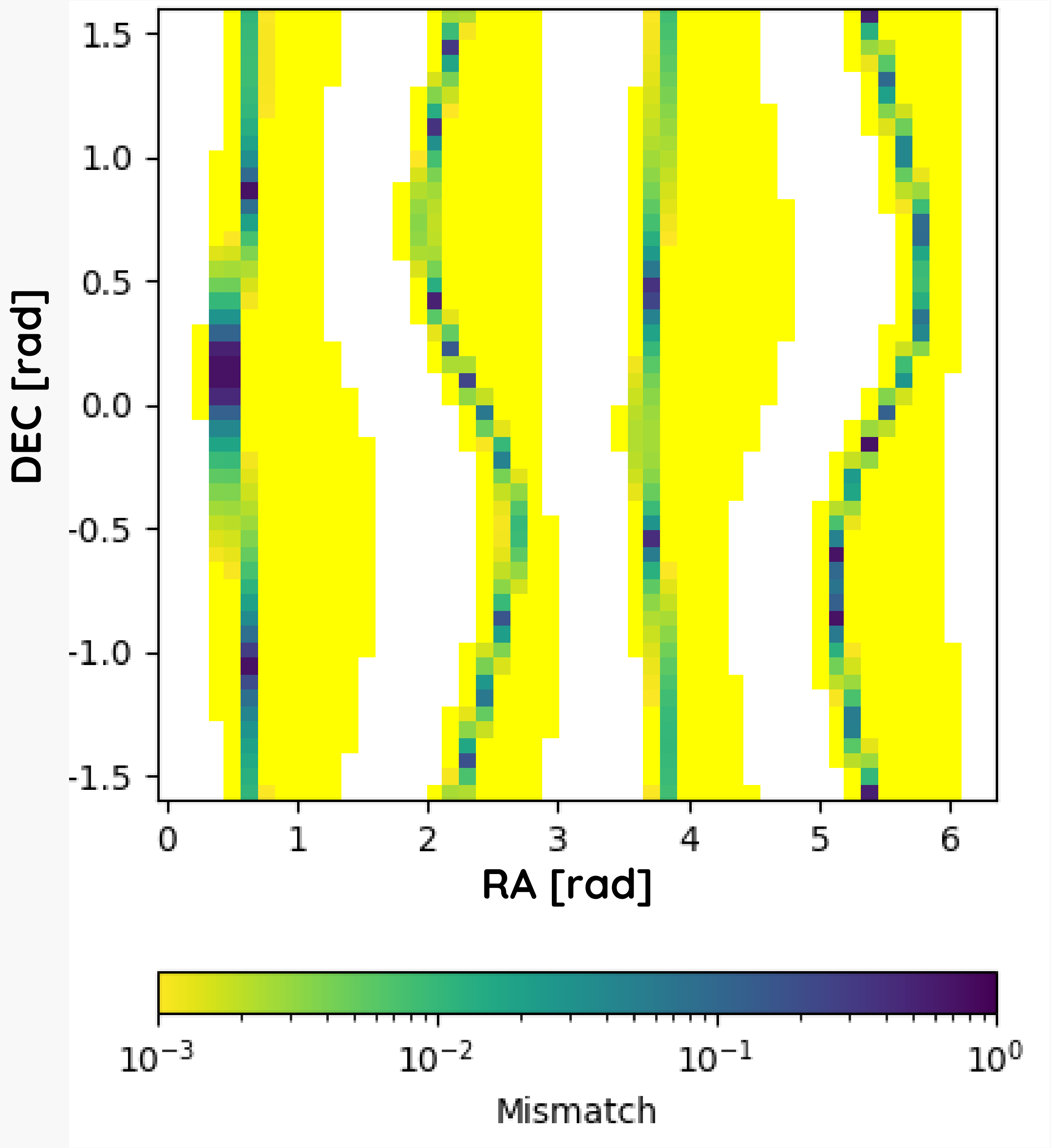


Differences between SA and PyCBC method

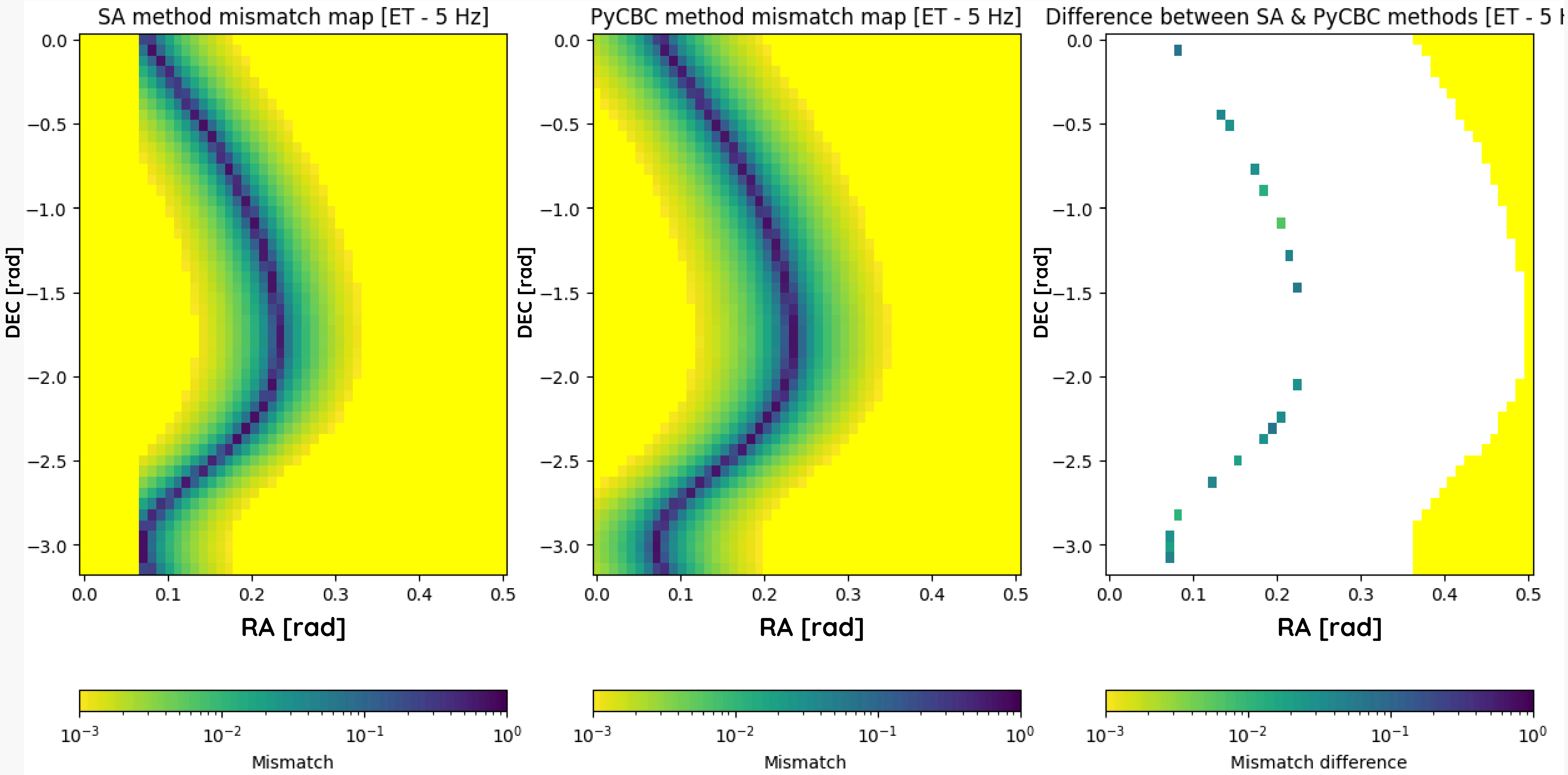




SA - PyCBC



Due to the low map resolution



Zooming in removes discrepencies

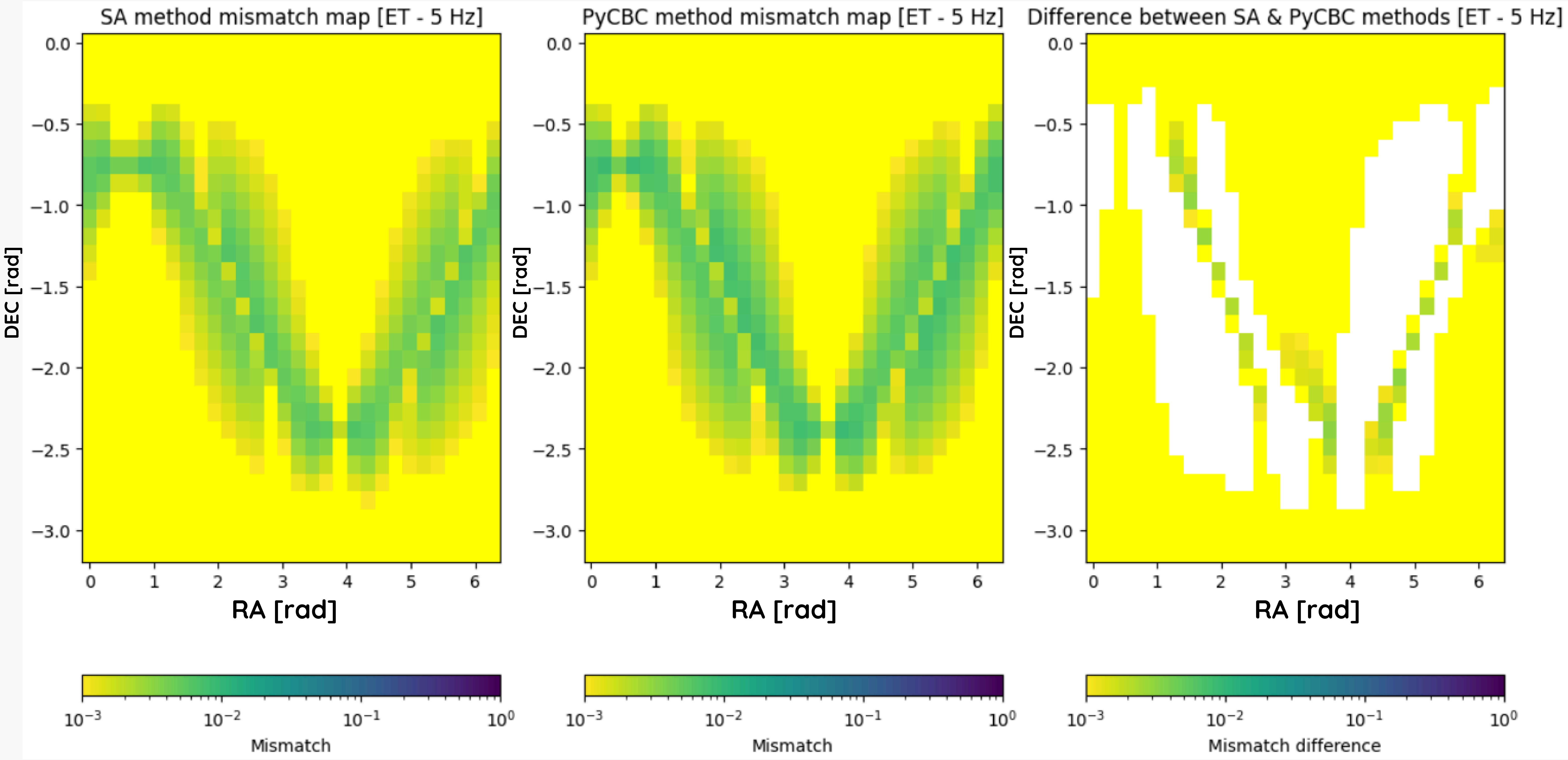
Simulation of the same physics

ET Xylo, f_{low} = 5 Hz, ι = 0°, Ψ = 0°, M_{Chirp} = 1.2$M_{\odot}$

SA Method (Max = 0.006)

PyCBC method (Max = 0.008)

Differences



SA method much faster than the PyCBC one

Will use SA in priority

&

Keep use of PyCBC when possible to highlight eventual differences

Global study of the Mismatch Monte-Carlo simulation

Sky position

$$\alpha, \cos(\delta) \sim \mathcal{U}[0, 2\pi], \mathcal{U}[0, 1]$$

Polarisation

$$\Psi \sim \mathcal{U}[0, 2\pi]$$

Inclination

$$\cos(\iota) \sim \mathcal{U}[0, 1]$$

Individual masses

$$M_i \sim \mathcal{N}(1.35,\ 0.1)$$

(Low) frequency cutoff

$$f_{low} = 5\, Hz$$

Coalescence time

$$t_c \sim \mathcal{U}[0, 86400]$$

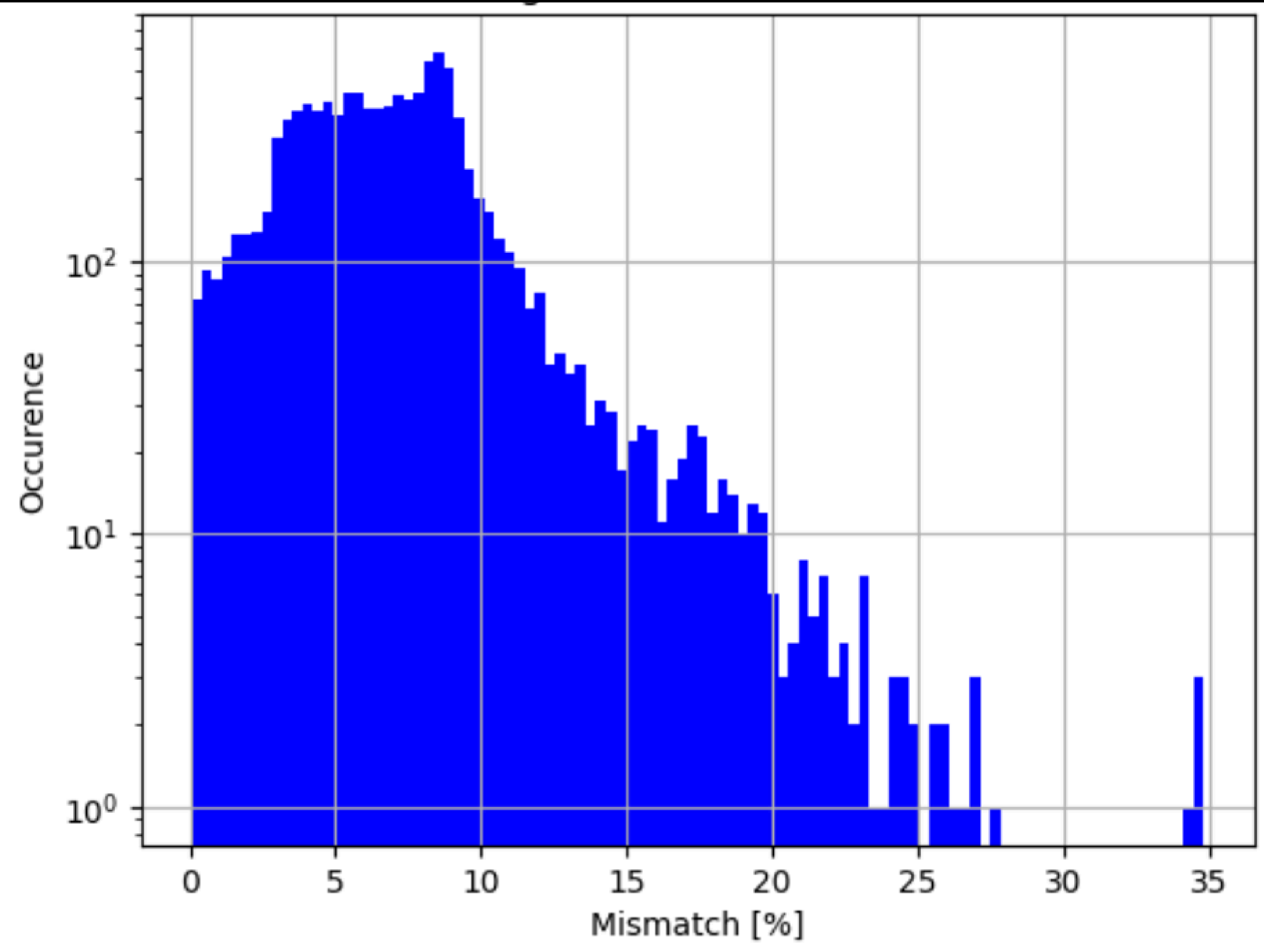
Redshift

$$p(z) \propto \frac{dV_c}{dz} \mathcal{R}(z)$$

Tidal and spin neglected

(10.000 sources)

Mismatch histogram



Up to 40% Mismatch

~13% of sources have Mismatch > 10%

~70% of sources have Mismatch > 5%

~90% of sources have Mismatch > 3%

Global study of SNR

Match Filtering Output (complex)

$$z(t_s) = 4 \int_0^{\infty} \mathrm{d}f \, \frac{\tilde{h}_{\mathrm{data}}(f) \tilde{h}^*_{\mathrm{template}}(f, t_s = 0, \phi_0 = 0)}{S_n(f)} \, e^{2\pi i f t_s}$$

Optimal SNR

$$\rho^2_{\mathrm{opt}} = 4 \int_0^{\infty} \mathrm{d}f \, \frac{|\tilde{h}(f)|^2}{S_n(f)}$$

SNR (Time Serie)

$$\rho(t_s) = \frac{|z(t_s)|}{\rho_{\mathrm{opt}}}$$

(Template's optimal SNR)

SNR (Time Serie)

$$\rho(t_s) = \frac{|z(t_s)|}{\rho_{opt}}$$

(Template's optimal SNR)

SNR

$$\rho = \max_{t_s} \rho(t_s)$$

Total SNR of ET

$$\mathrm{SNR}_{tot}^2 = \rho_1^2 + \rho_2^2 + \rho_3^2$$

$$\rho_{const}$$

$$\rho[F(t)h(t), F(0)h(t)]$$

$$\rho_{mod}$$

$$\rho[F(t)h(t), F(t)h(t)] = \rho_{opt}[F(t)h(t)]$$

Match Filtering Output (complexe)

$$\rho_{const} = \frac{< h, T >}{||T||}$$

Cauchy-Schwarz

$$| < h, T > | \leq ||h||.||T||$$

We necessarily lose SNR

$$\rho_{const} \leq ||h|| = \rho_{opt,mod}$$

Let's do a Monte-Carlo !

Sky position

$$\alpha, \cos(\delta) \sim \mathcal{U}[0, 2\pi], \mathcal{U}[0, 1]$$

Polarisation

$$\Psi \sim \mathcal{U}[0, 2\pi]$$

Inclination

$$\cos(\iota) \sim \mathcal{U}[0, 1]$$

Individual masses

$$M_i \sim \mathcal{N}(1.35,\ 0.1)$$

(Low) frequency cutoff

$$f_{low} = 5\,Hz$$

Coalescence time

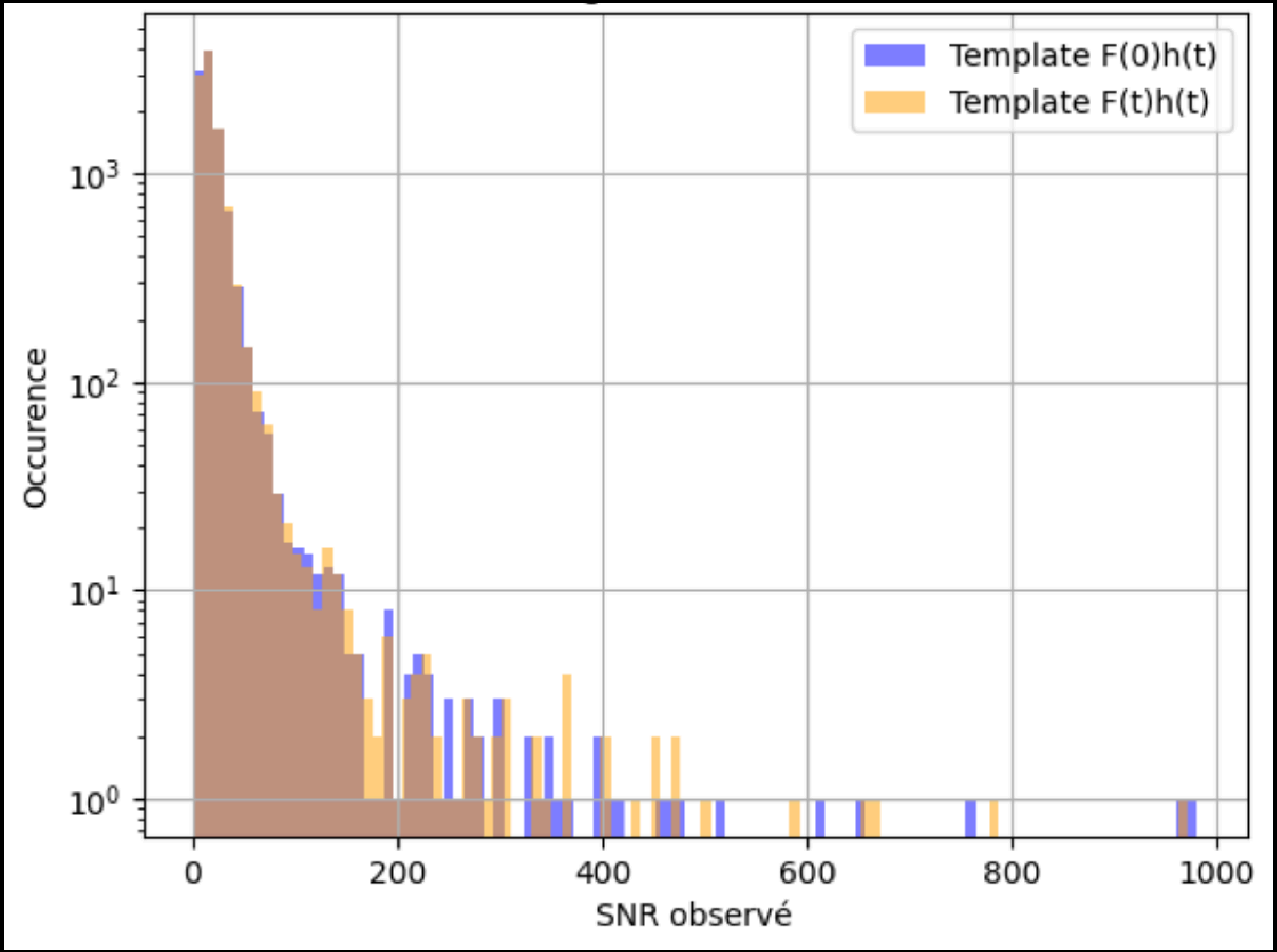
$$t_c \sim \mathcal{U}[0, 86400]$$

Redshift

$$p(z) \propto \frac{dV_c}{dz}\mathcal{R}(z)$$

Tidal and spin neglected

SNR histogram



(10.000 sources)

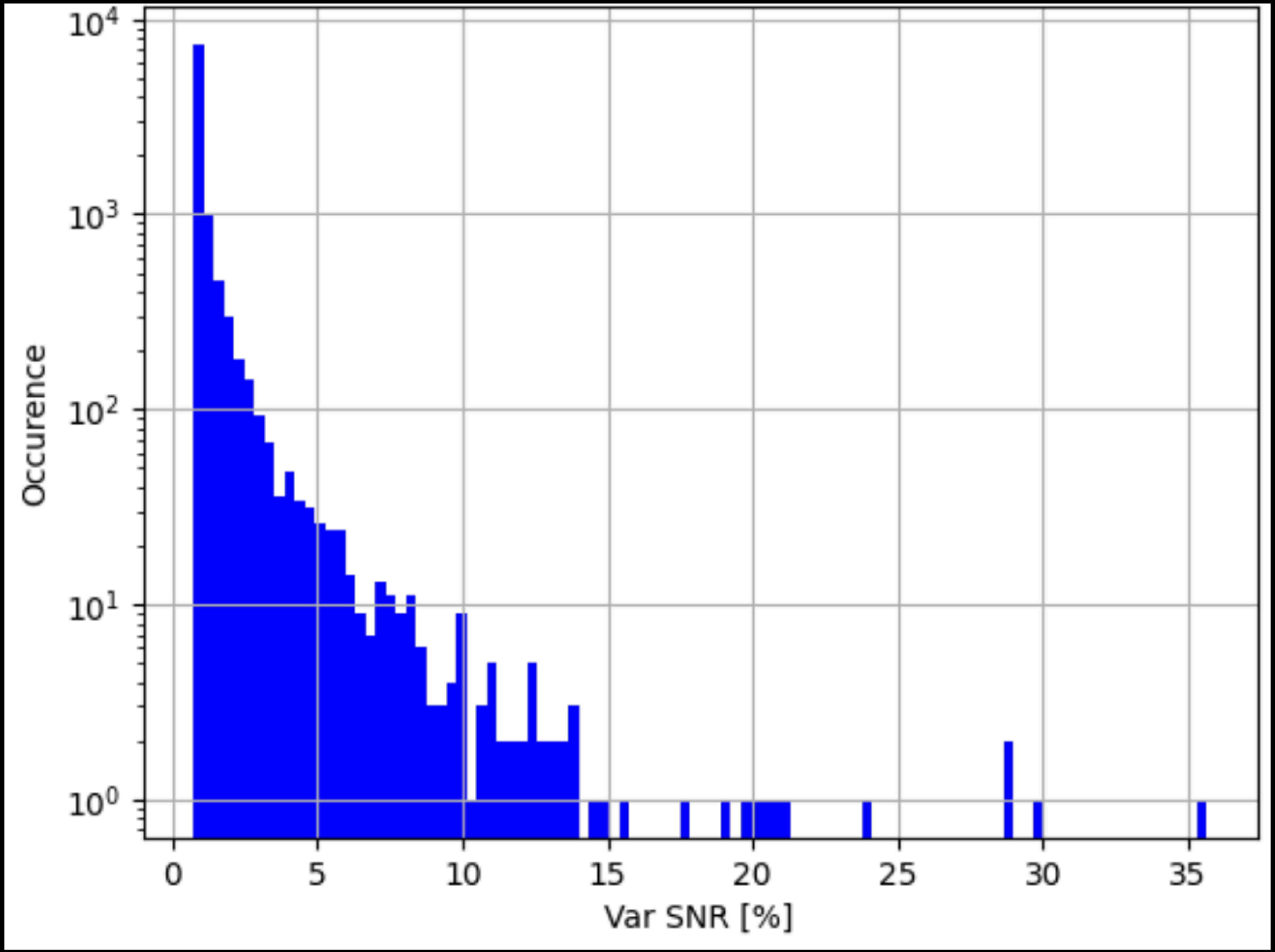
SNR between 0 and 1000

Mean SNR ~24

~75% of sources have 8 < SNR < 55

More optimistic than MDC

SNR variations histogram



(10.000 sources
9.200 detected)

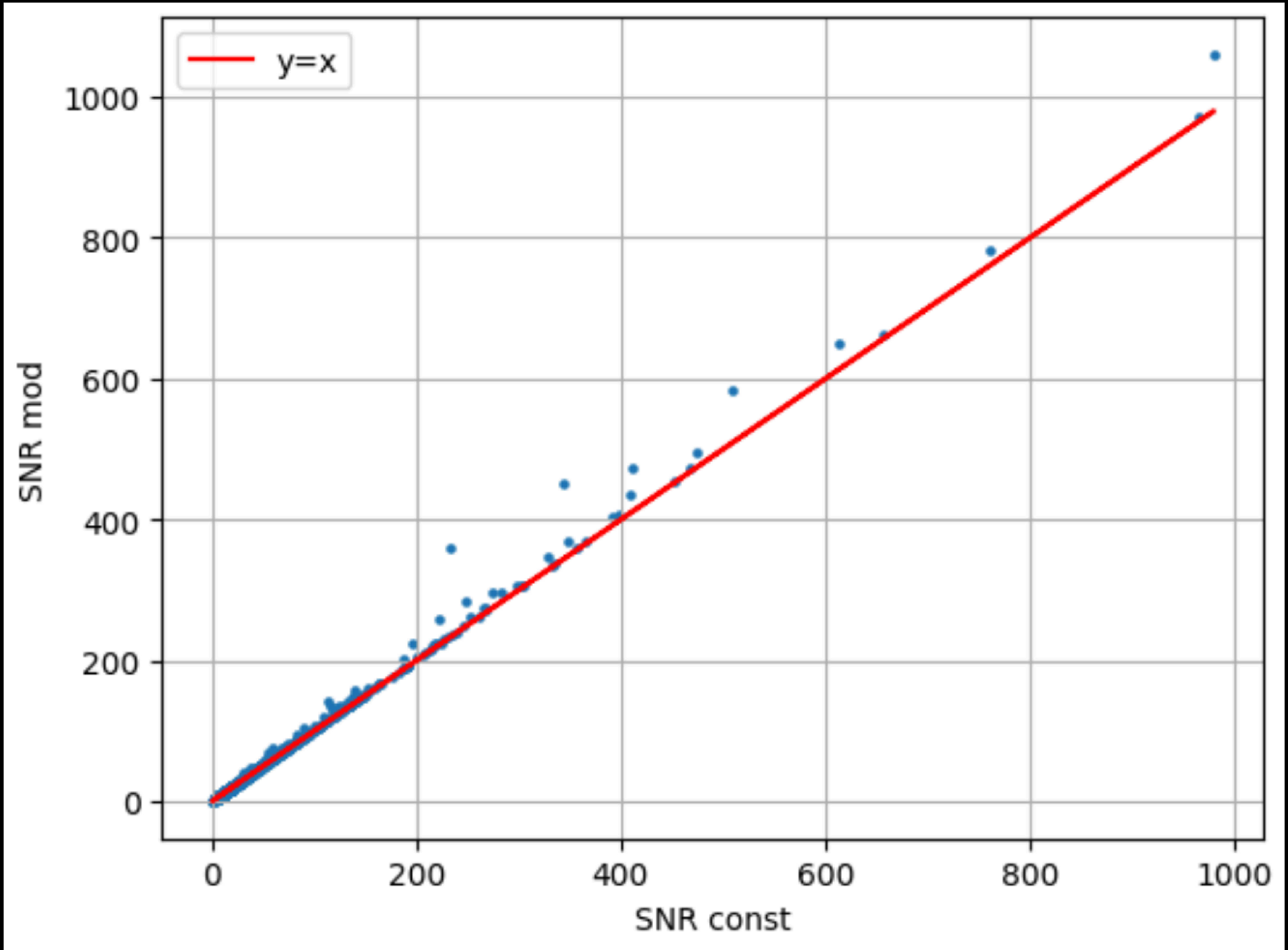
Variation < 40%

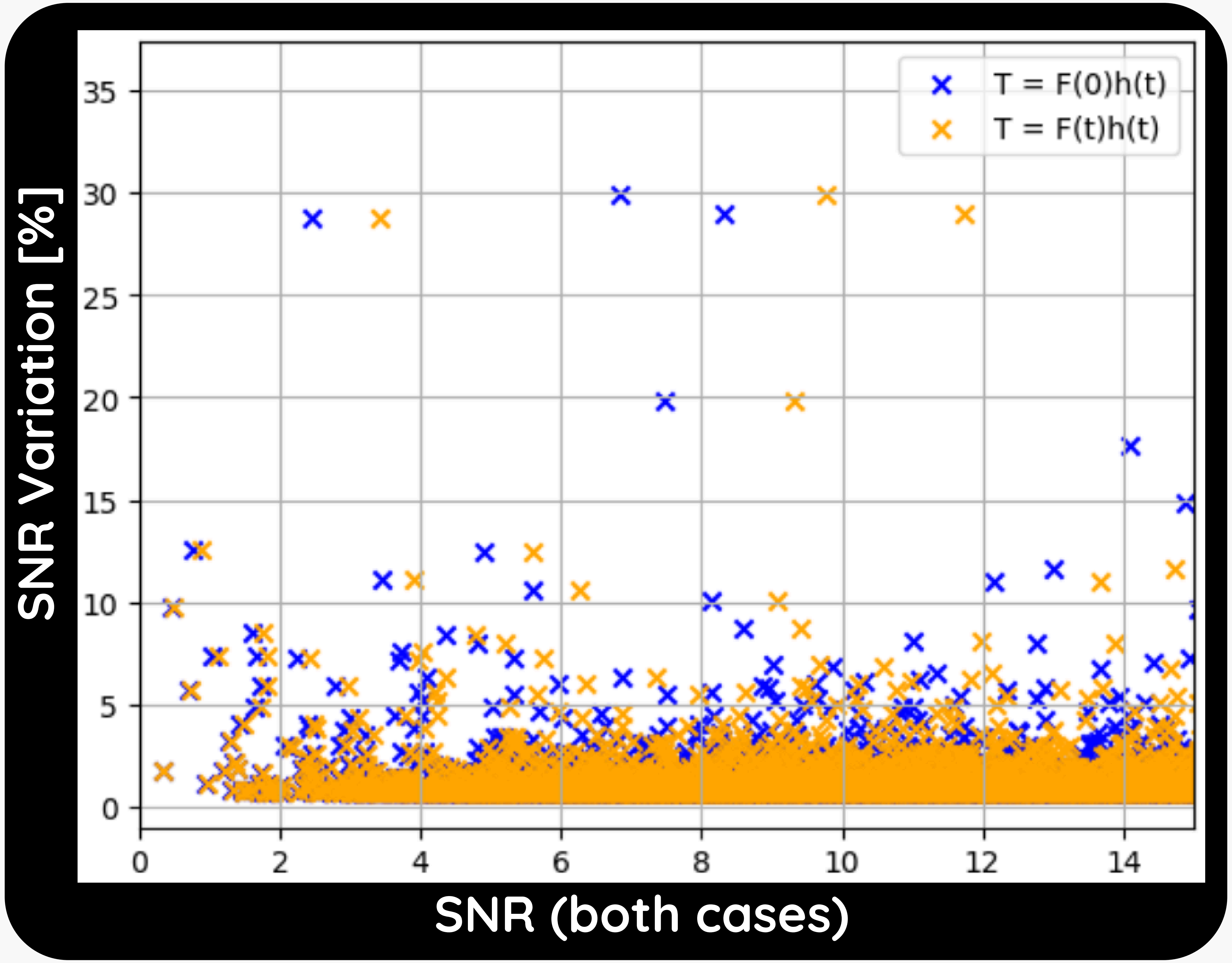
0.6% have variation > 10%

3% have variation > 5%

7.5% have variation > 3%

ρ_{const} vs ρ_{mod}



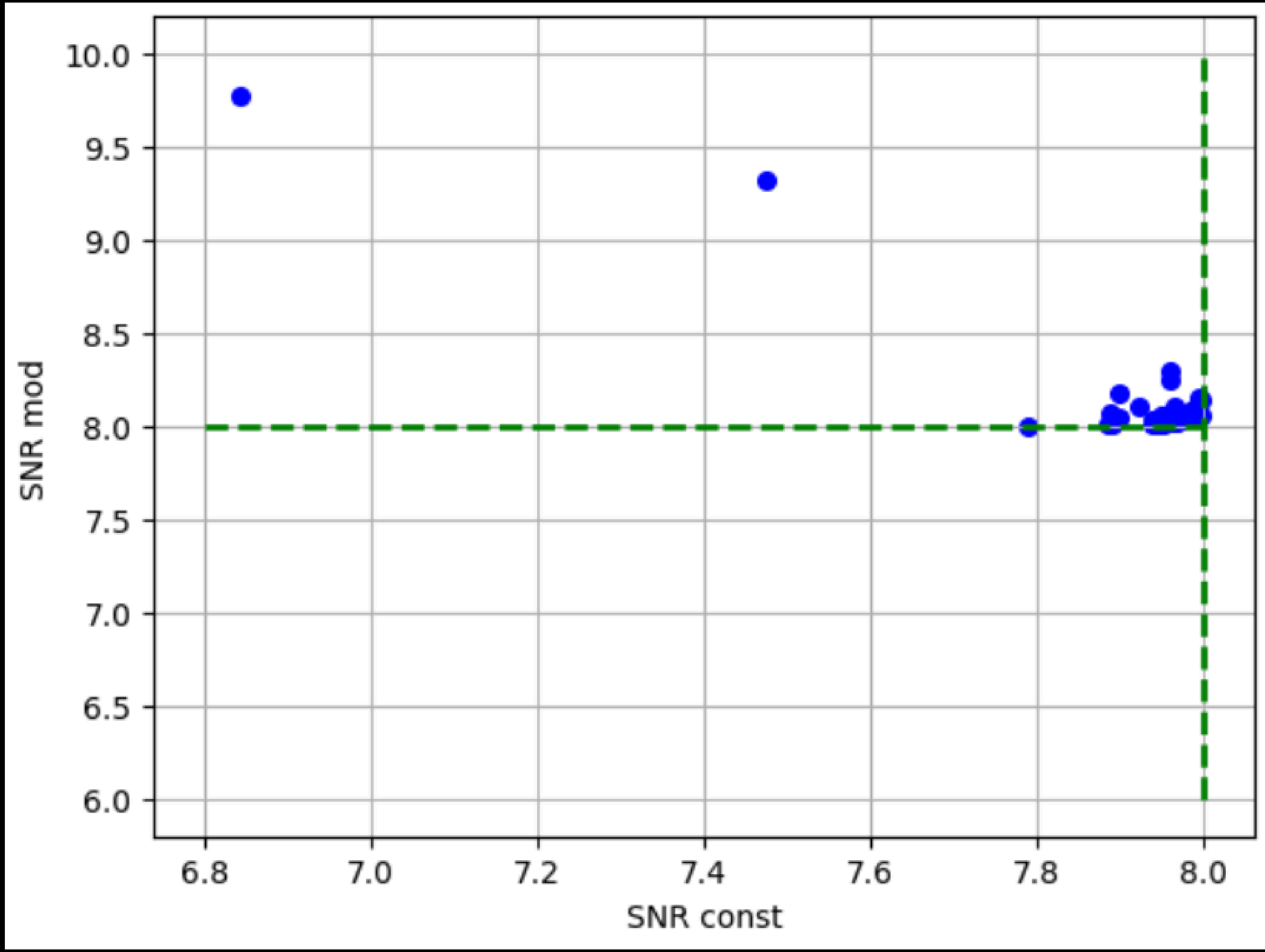


(10.000 sources
9.200 detected)

45 sources with

SNR_const < 8 BUT SNR_mod > 8

Recovered SNR stay around SNR = 8



Statistics !

Assuming 70.000 BNS/year

~ 400 recovered sources taking Earth Rotation into account

Hence ~ 0.6% of ALL detected sources

Conclusion

MC simulation show that
Mismatch is heavily impacted

Earth rotation-induced SNR variation can be neglected for f_{low} > 5Hz

Taking it into account would complexify the Match Filtering for <1% of all detected sources

Thank you !

Backup

$$\frac{\rho_{mod} - \rho_{const}}{\rho_{mod}} = 1 - \frac{\rho_{const}}{\rho_{mod}}$$

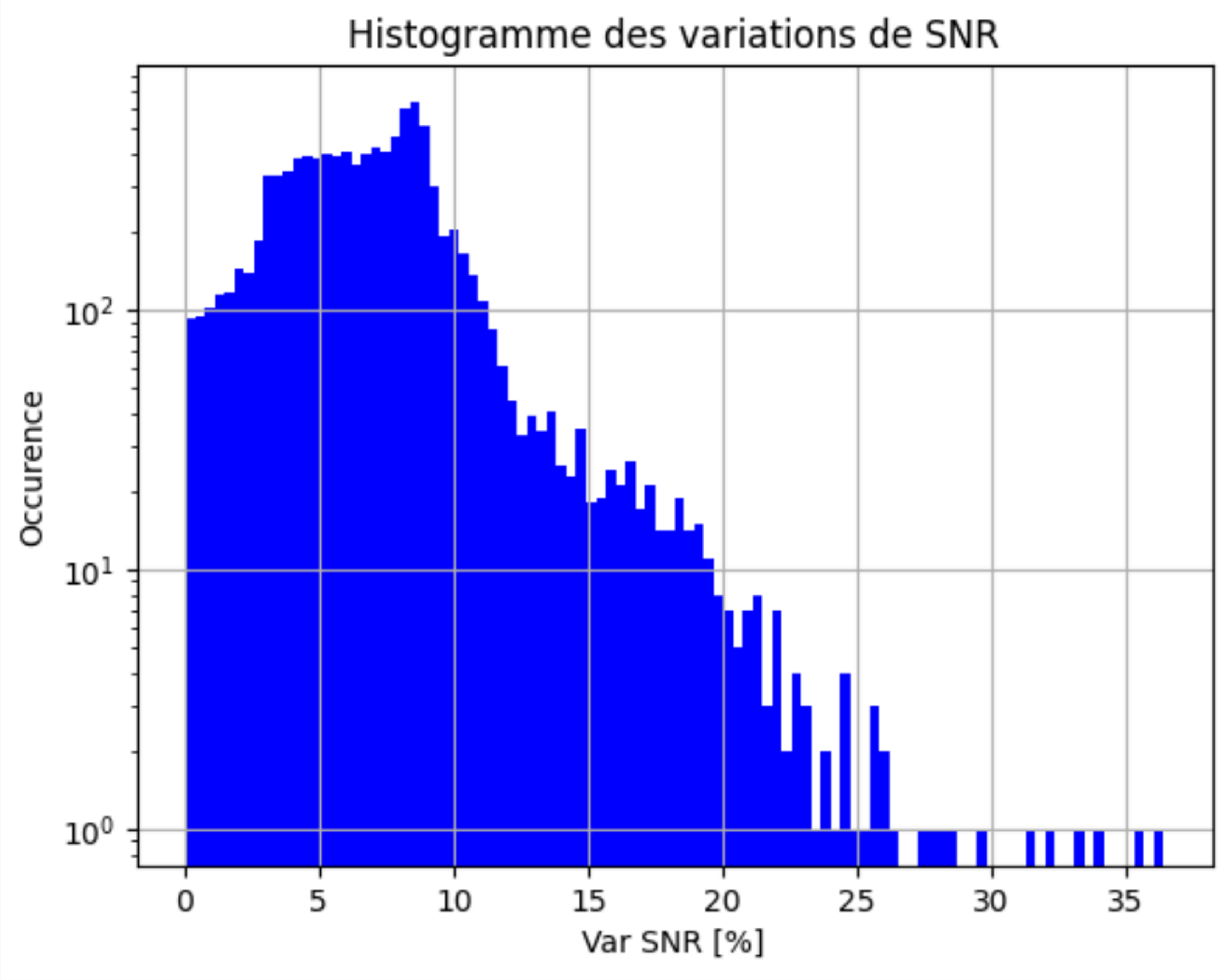
$$\frac{\rho_{mod} - \rho_{const}}{\rho_{mod}} = 1 - \frac{\rho_{const}}{||s||}$$

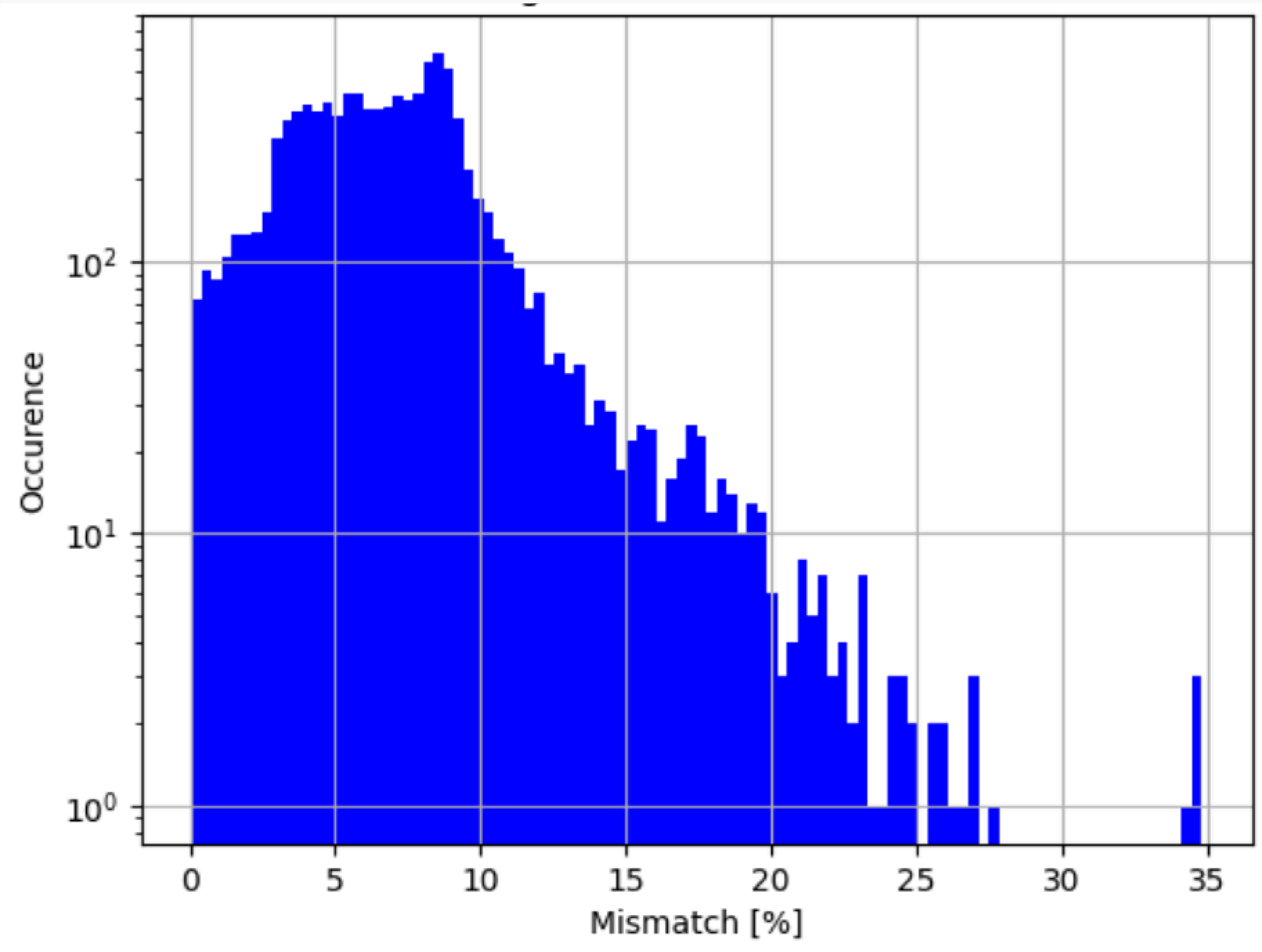
$$\mathcal{M}(s,T) = \max_{\Delta t, \Delta \phi} \frac{< s, T >}{||s||.||T||}$$

$$\mathcal{M}(s,T) \approx \frac{< s, T >}{||s||.||T||}$$

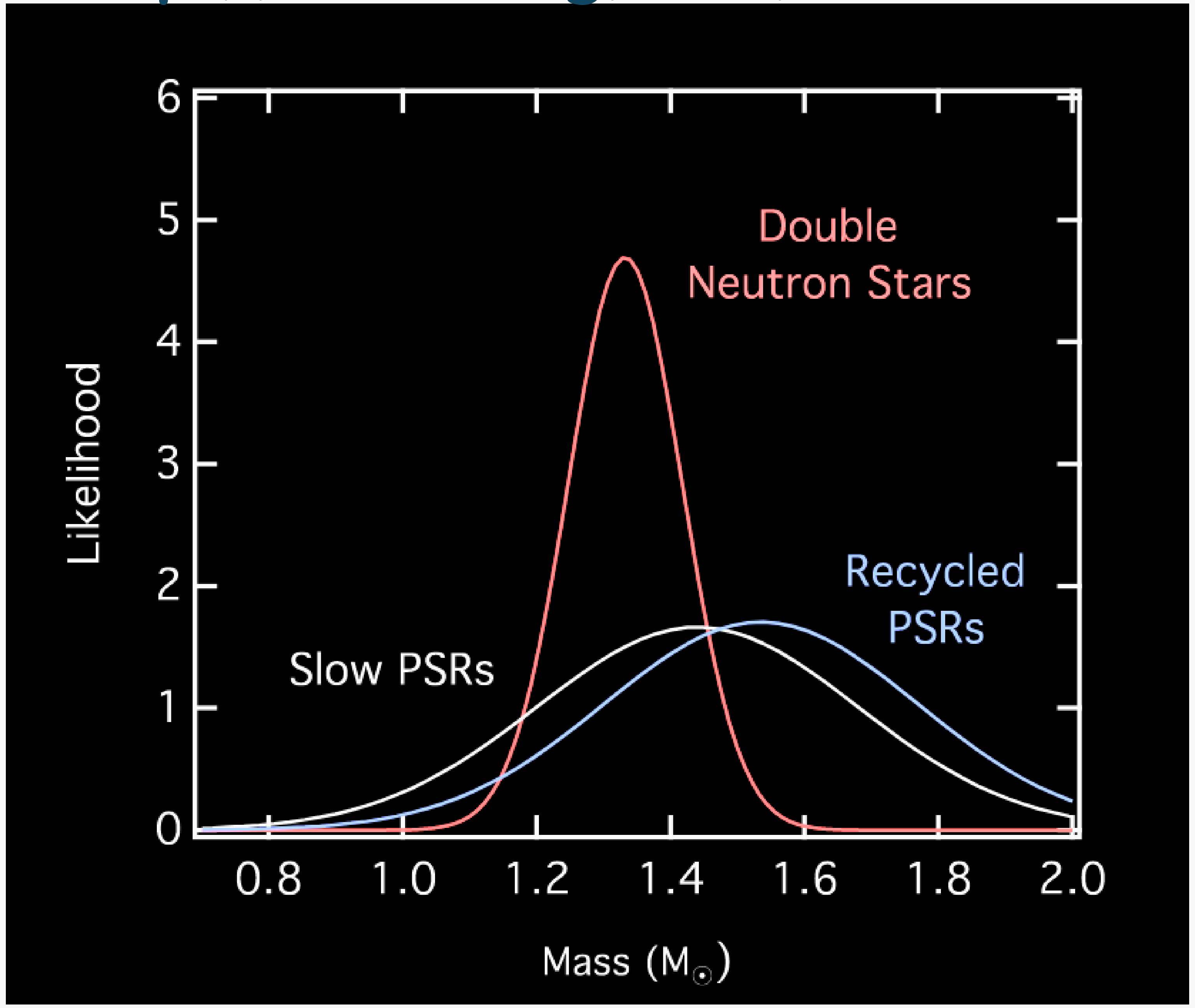
$$\mathcal{M}(s,T) \approx \frac{\rho_{const}}{||s||}$$

$$\rho_{const} = \frac{< h, T >}{||T||}$$


Histogramme des variations de SNR
Occurence
Var SNR [%]
10^0
10^1
10^2
0
5
10
15
20
25
30
35


Occurence
Mismatch [%]
10^0
10^1
10^2
0
5
10
15
20
25
30
35

http://arxiv.org/abs/1603.02698



http://arxiv.org/abs/1403.0007

$$\psi(z) = 0.015 \frac{(1+z)^{2.7}}{1+[(1+z)/2.9]^{5.6}} \, M_\odot \, \mathrm{year}^{-1} \, \mathrm{Mpc}^{-3}.$$

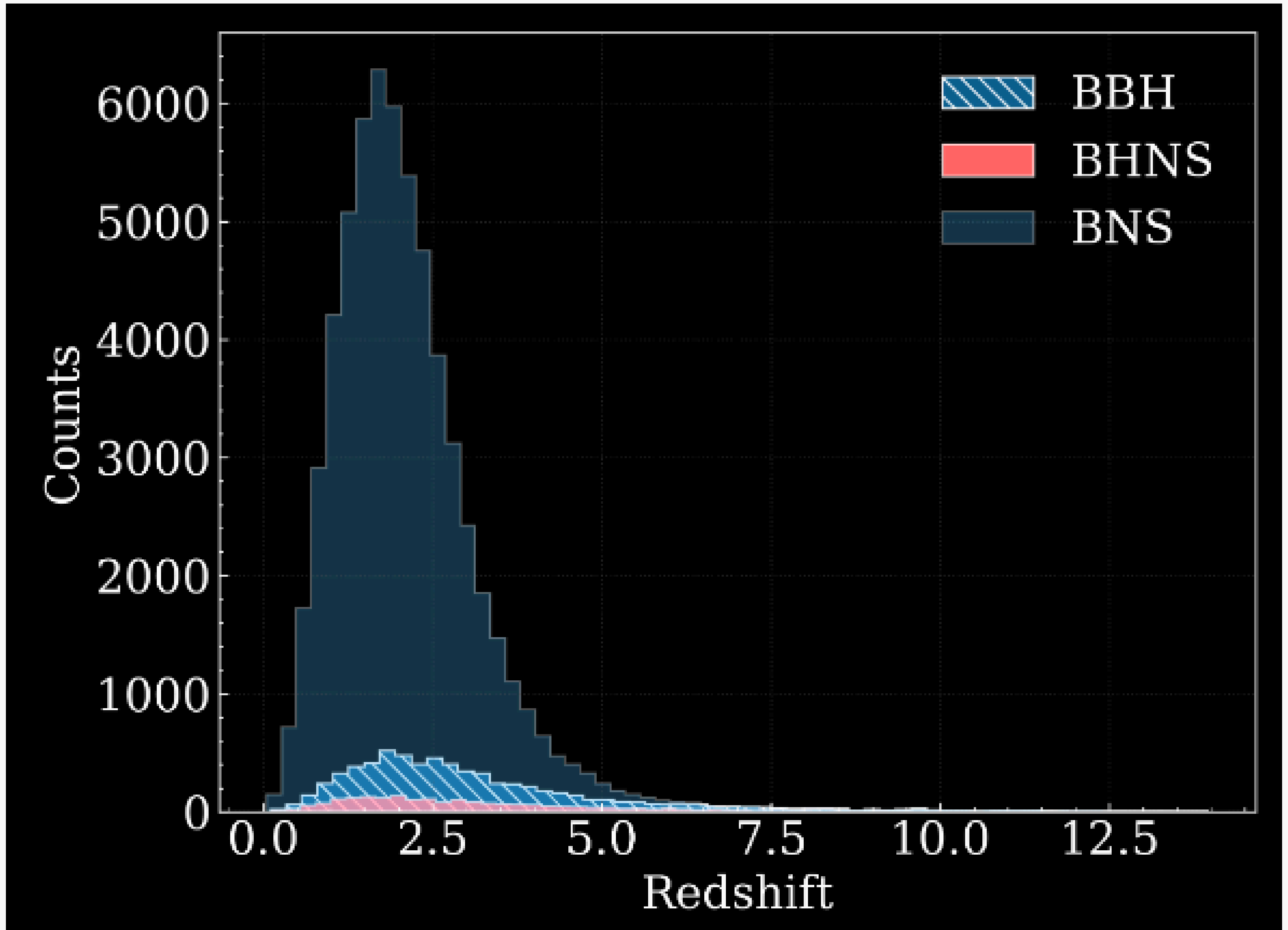


FIG. 10. Histogram of redshifts for the sources in our dataset. BNSs are shown in blue, BHNSs in green, and BBHs in red.

- 61031 BNSs: 19% have SNR > 8, 7% have SNR > 12.

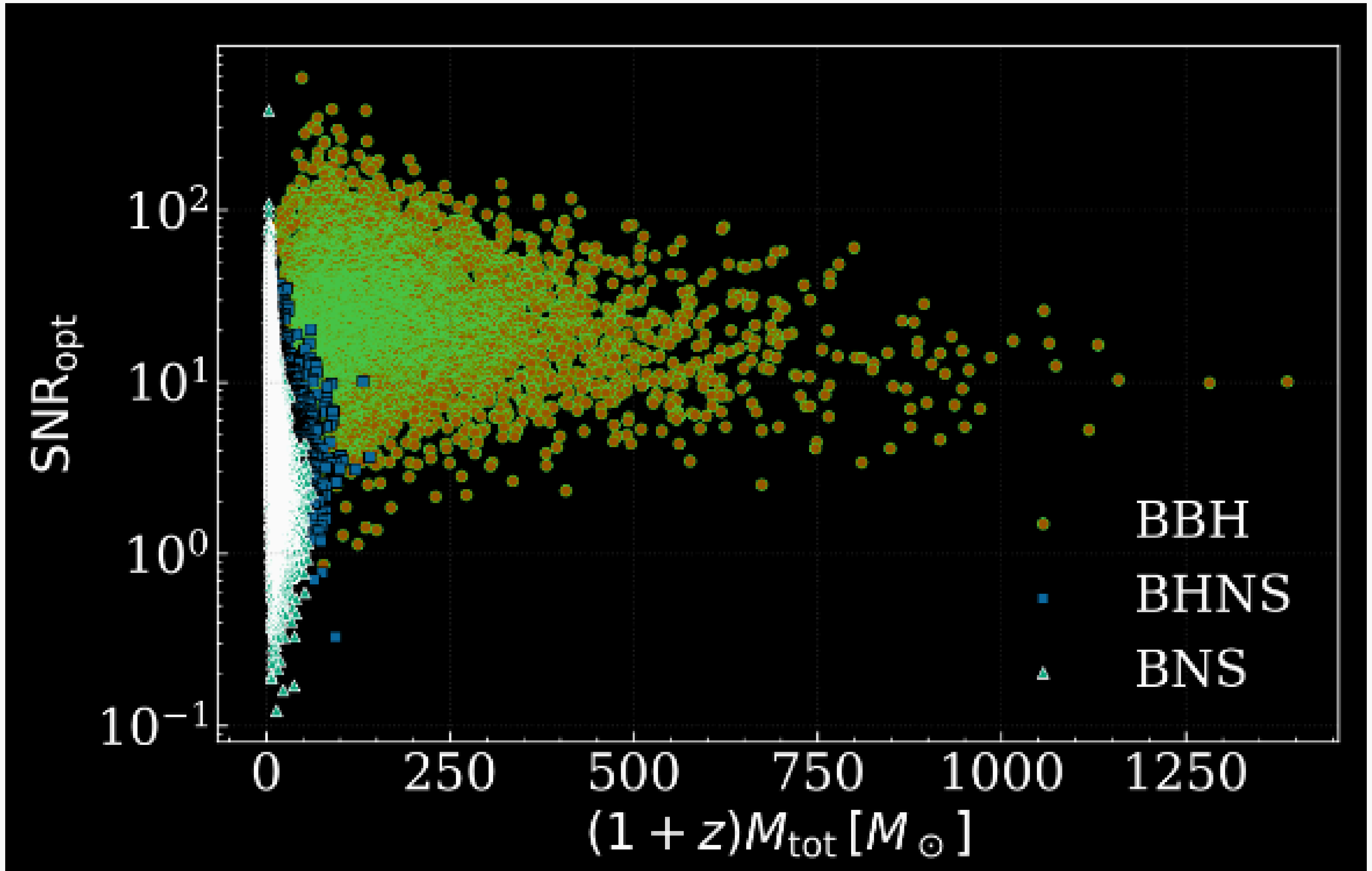
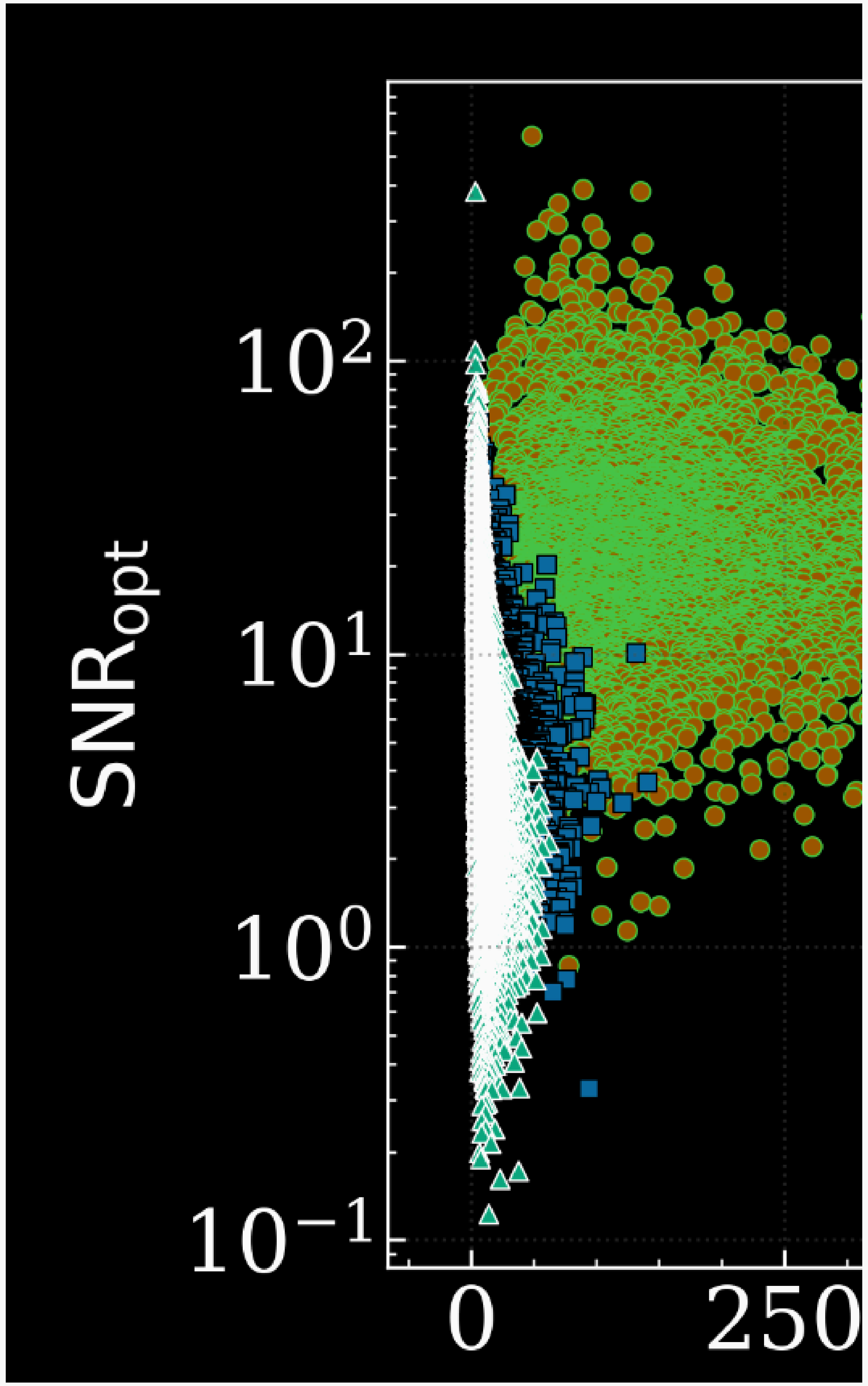
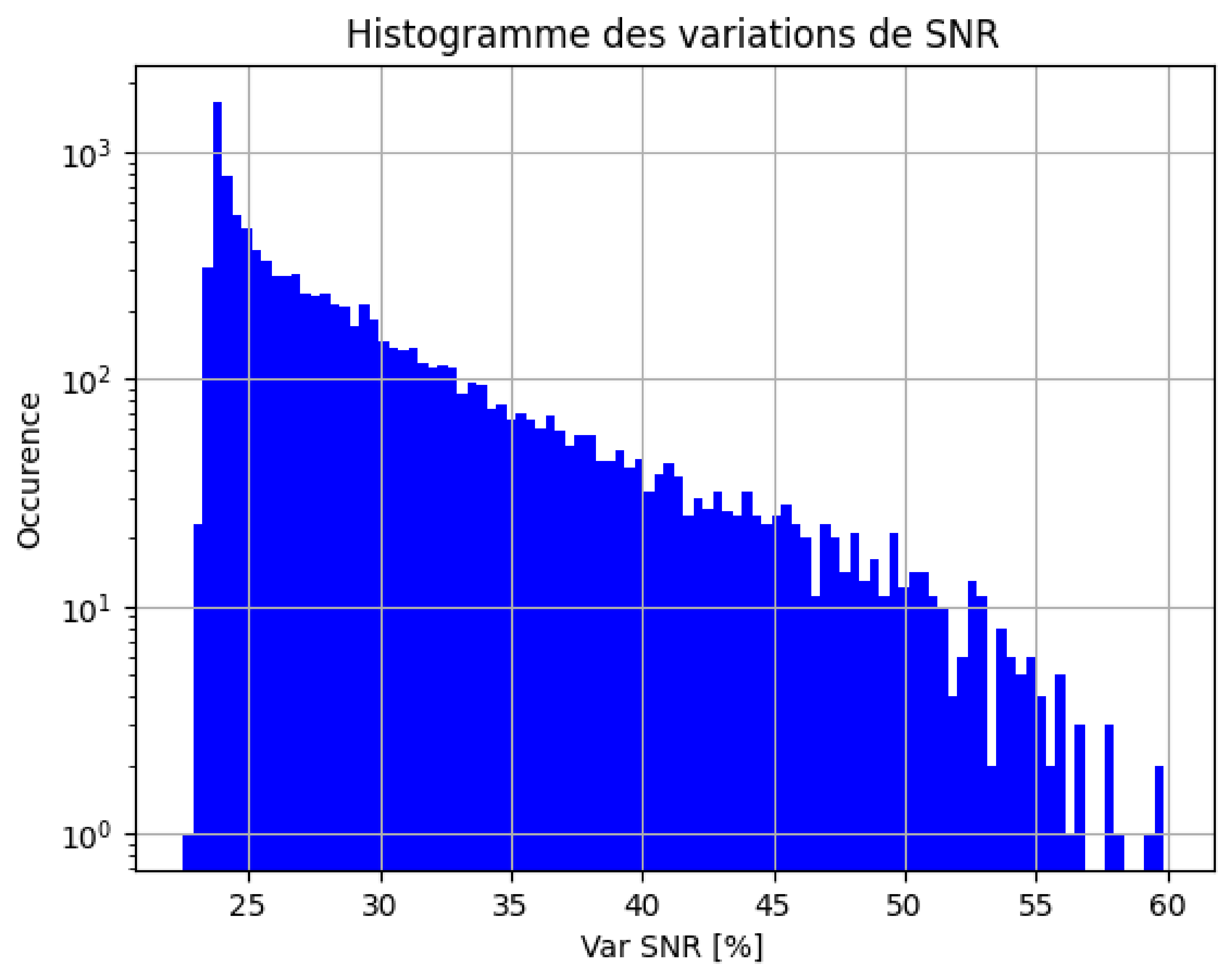
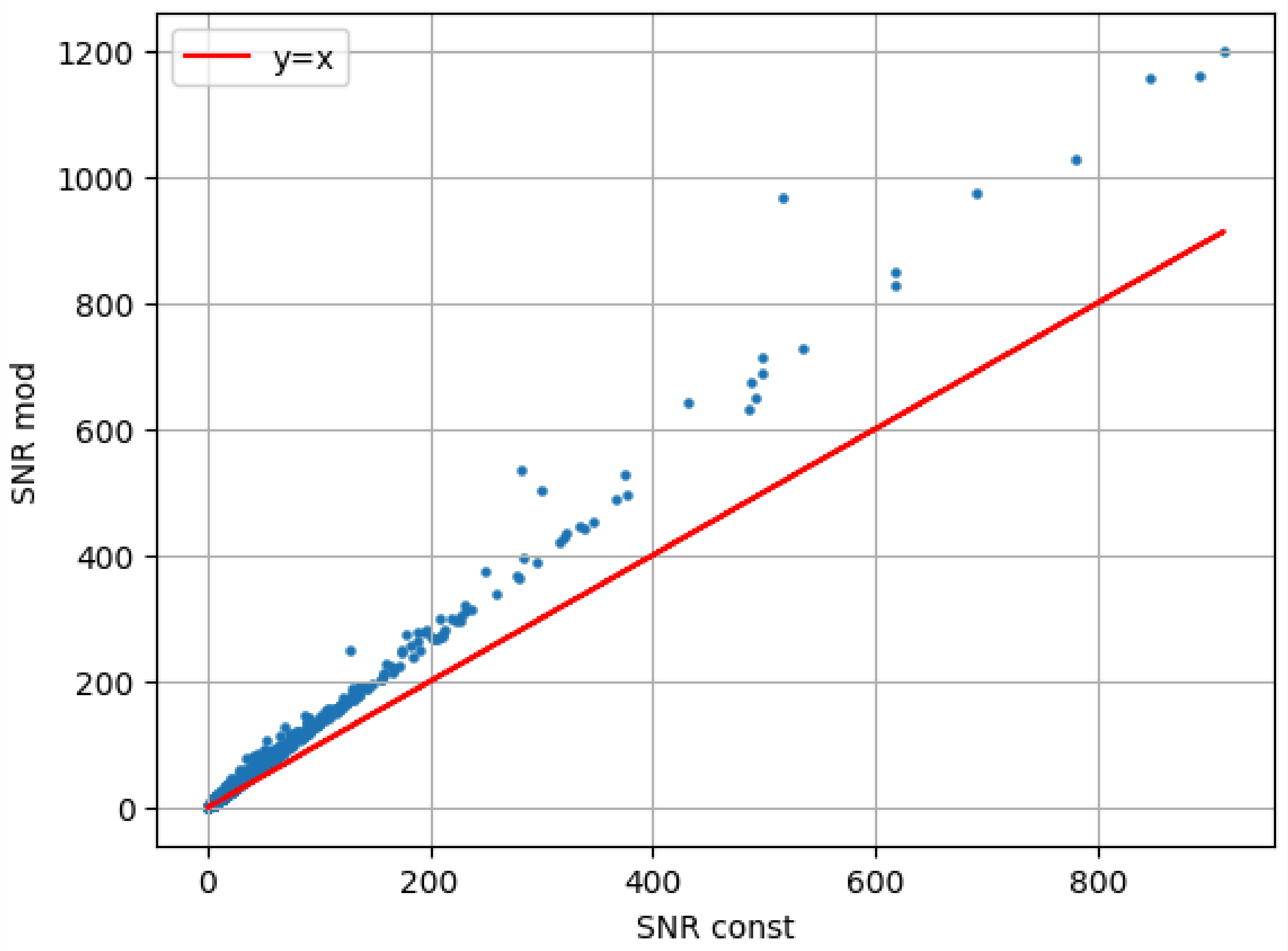
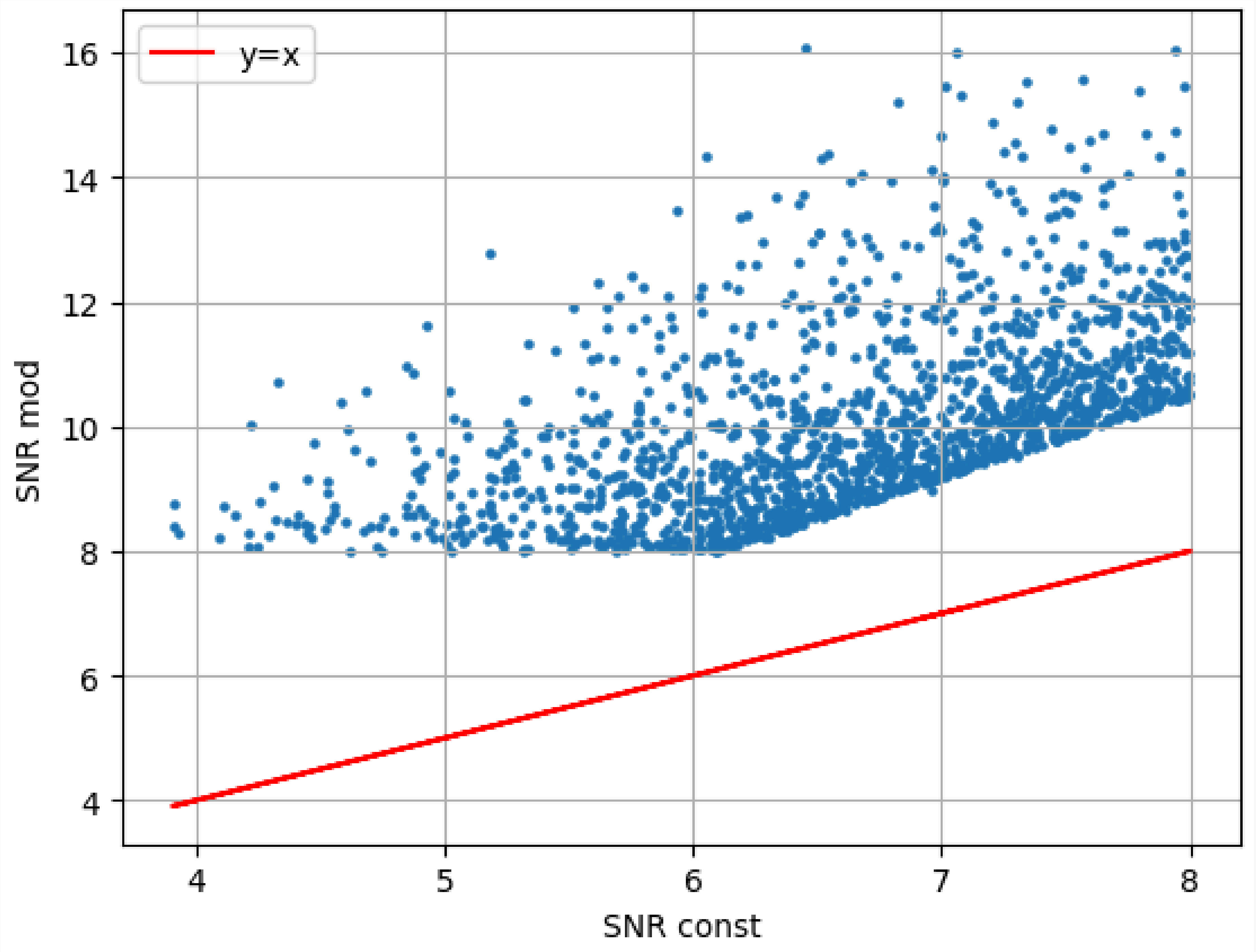


FIG. 13. Optimal signal-to-noise ratio (ρ_{opt}) as a function of the observed total mass $M_{z,tot}$.



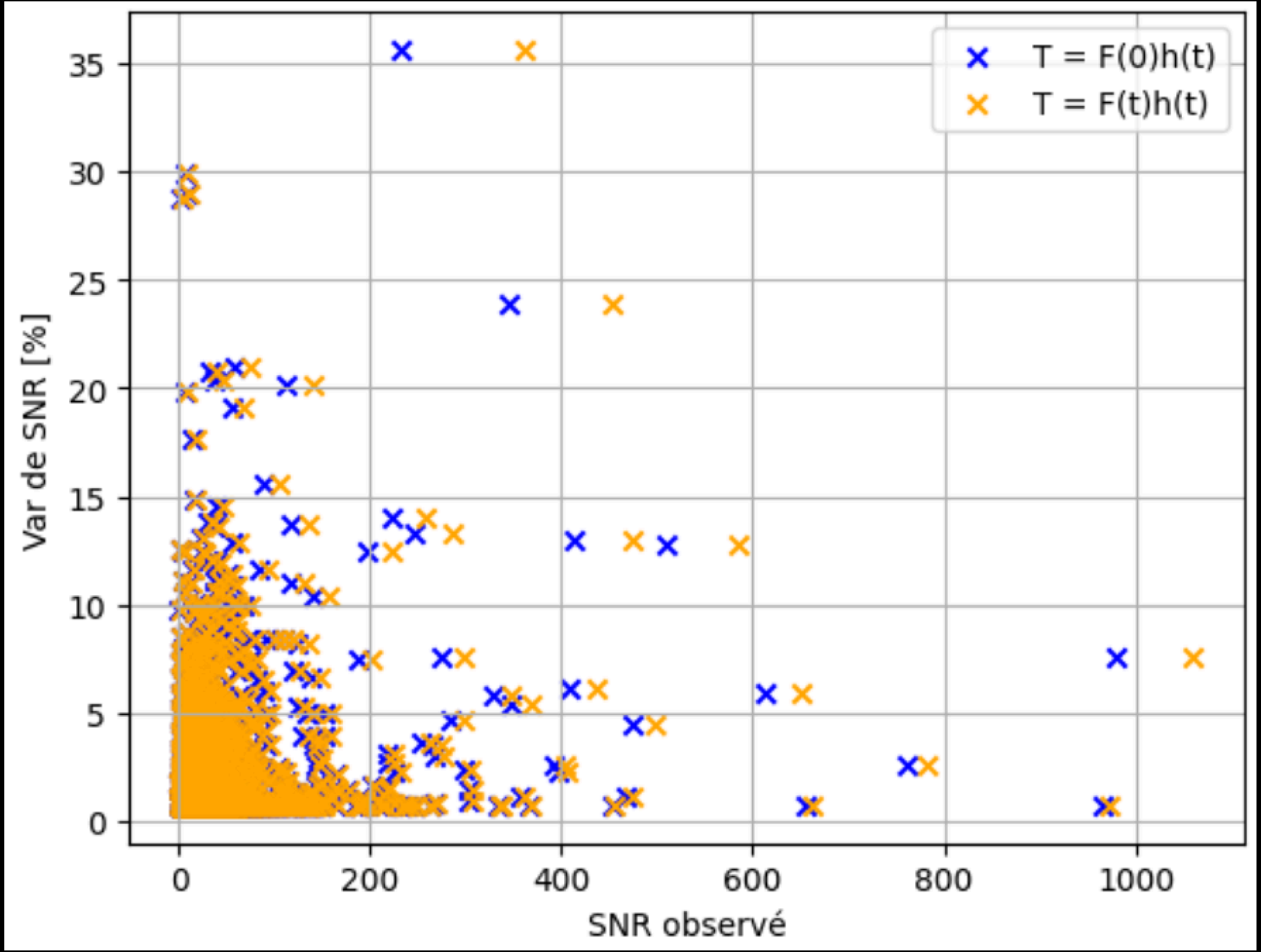

Histogramme des variations de SNR
Occurence
10^3
10^2
10^1
10^0
25
30
35
40
45
50
55
60
Var SNR [%]


y=x
SNR mod
1200
1000
800
600
400
200
0
0
200
400
600
800
SNR const

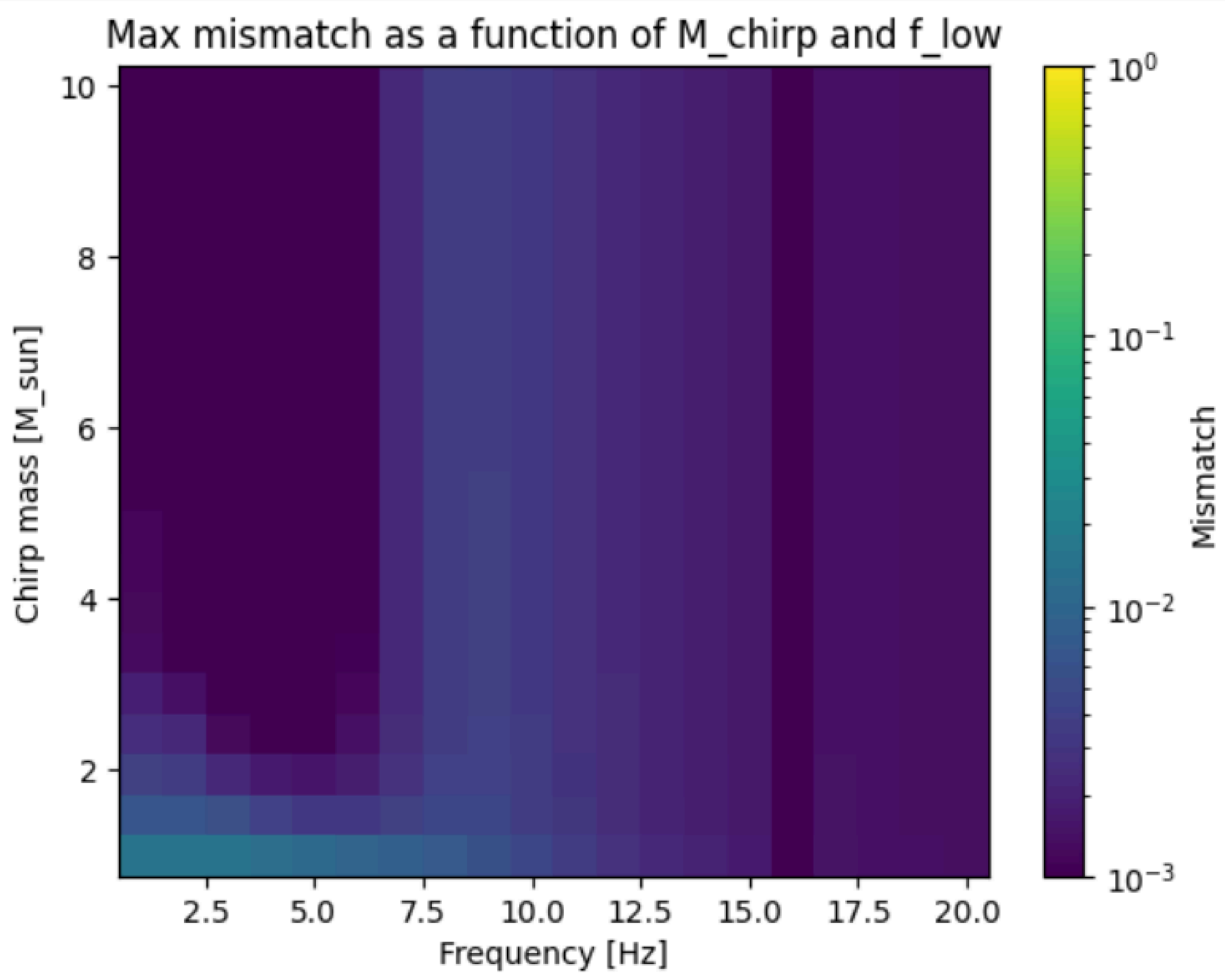


If f_{low} = 2Hz

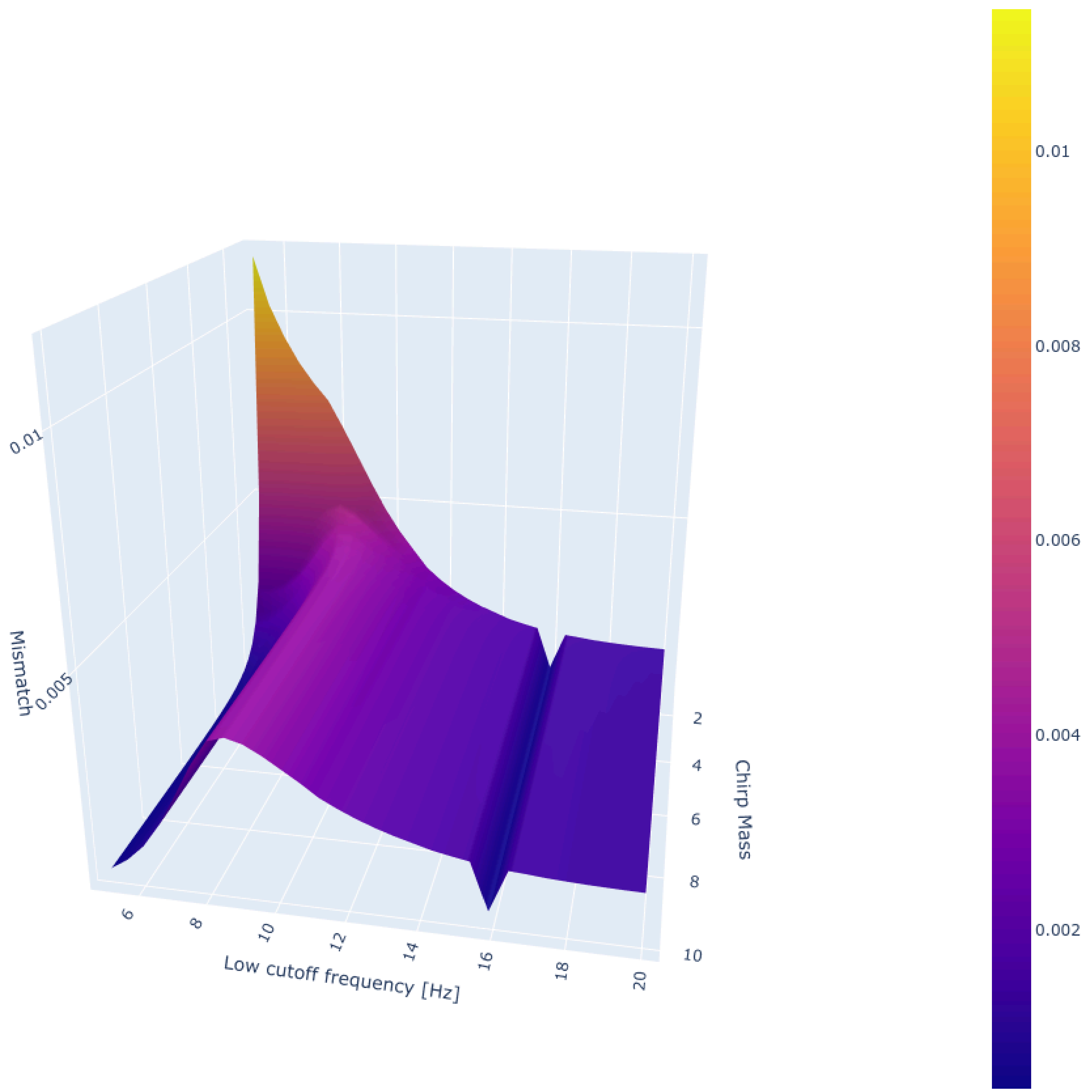
~ 25% of ALL detected sources can be retrieved



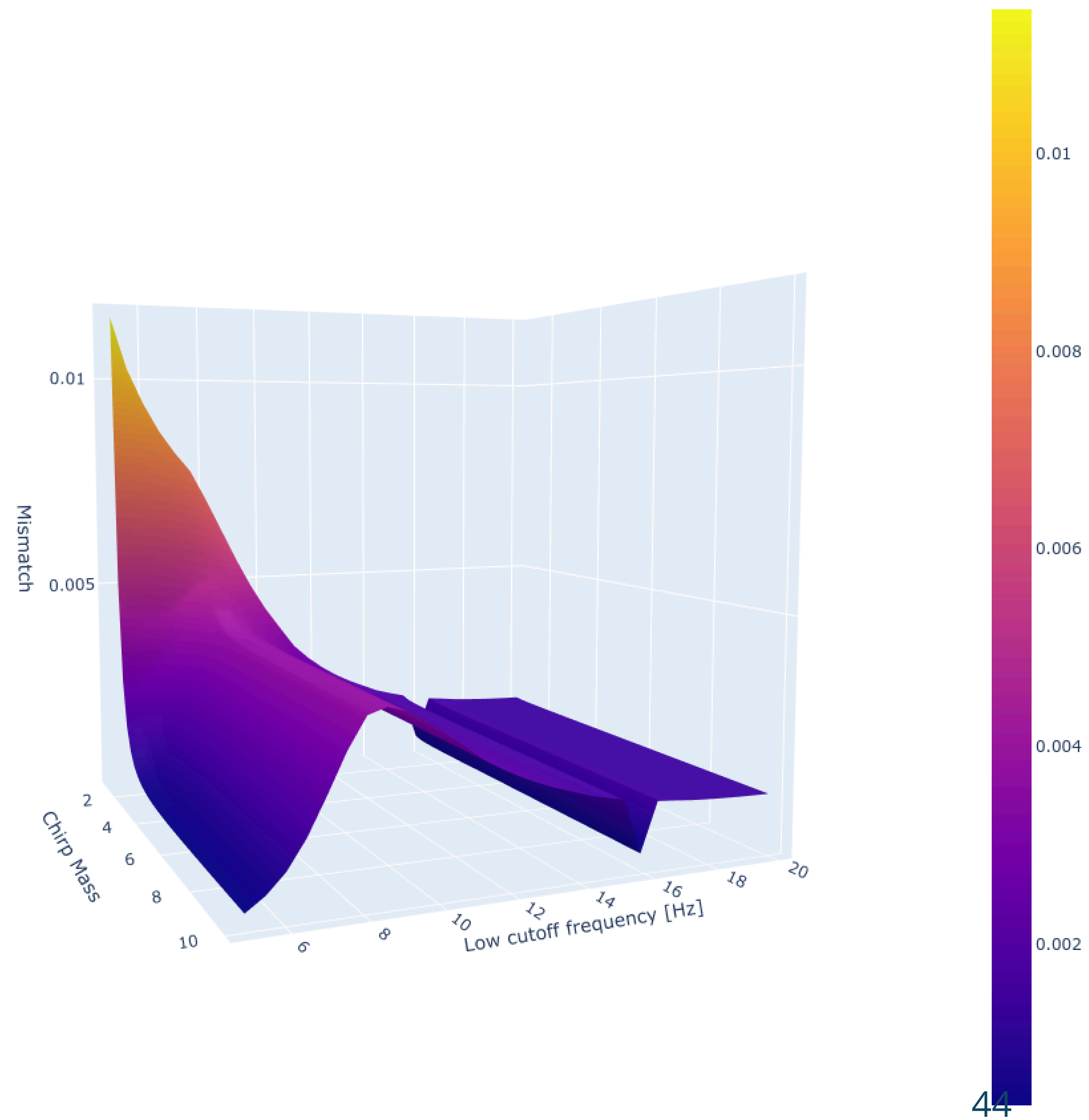
T = F(0)h(t)
T = F(t)h(t)
Var de SNR [%]
SNR observé
0
5
10
15
20
25
30
35
0
200
400
600
800
1000


Max mismatch as a function of M_chirp and f_low
Chirp mass [M_sun]
10
8
6
4
2
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
Frequency [Hz]
Mismatch
10^0
10^{-1}
10^{-2}
10^{-3}

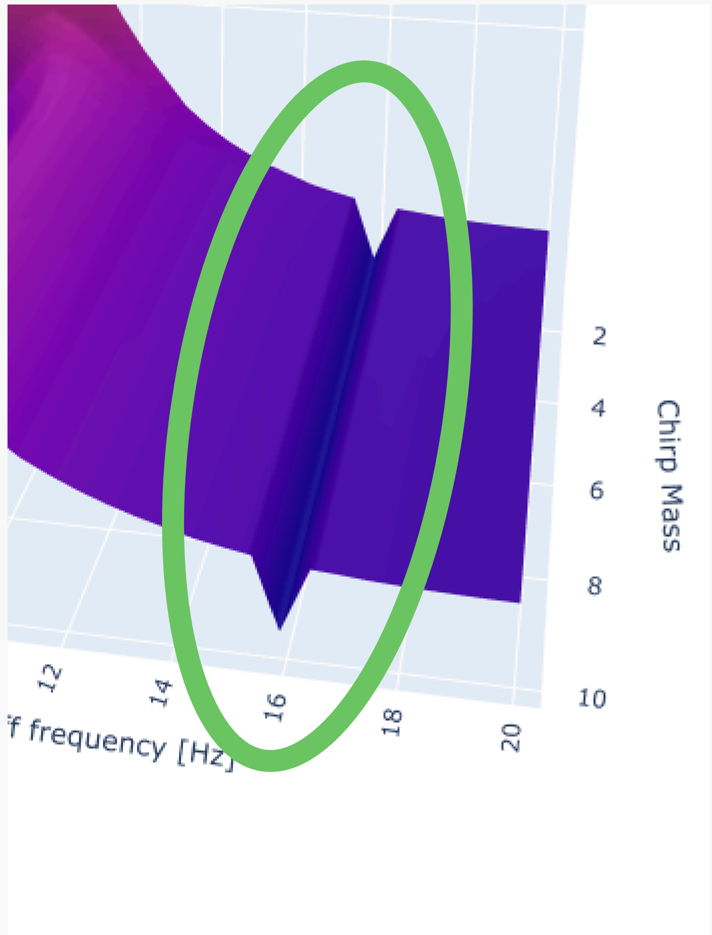
Mismatch as a function of M_chirp and f_low



Mismatch as a function of M_chirp and f_low

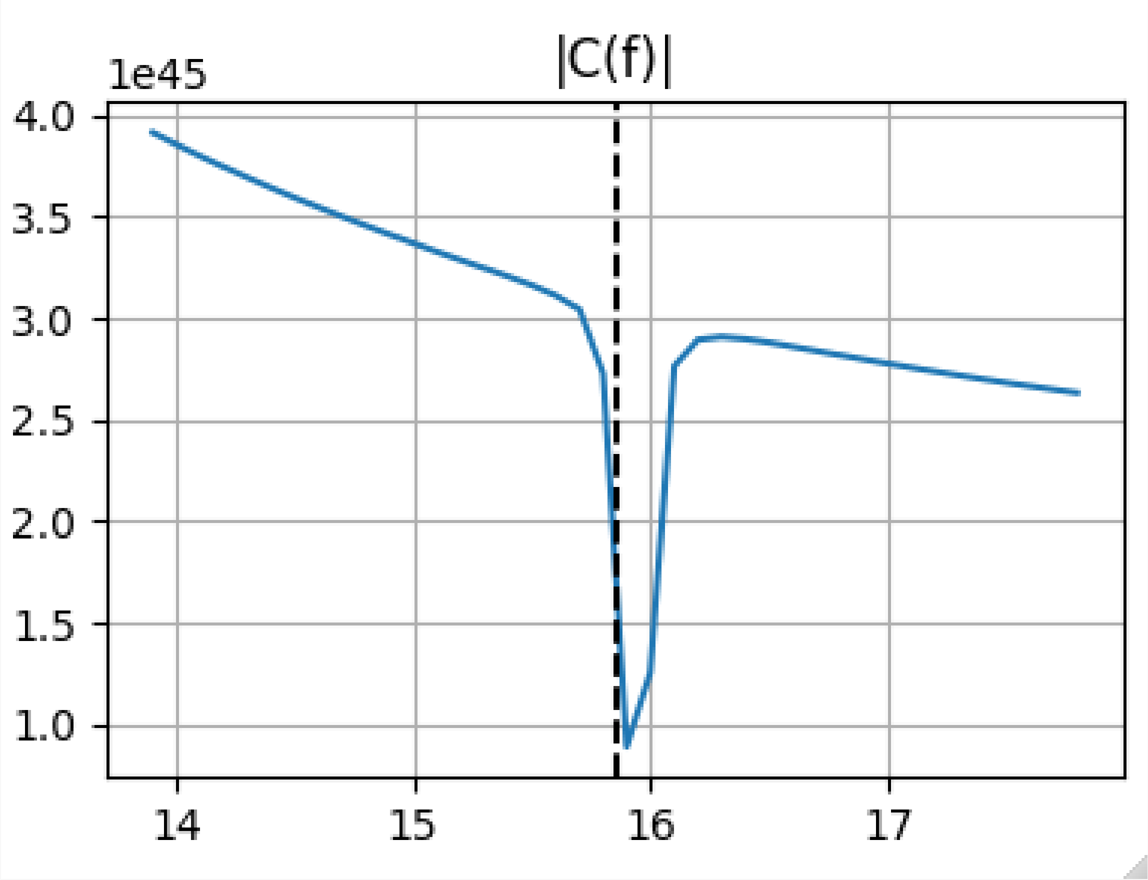


Caused by the PSD

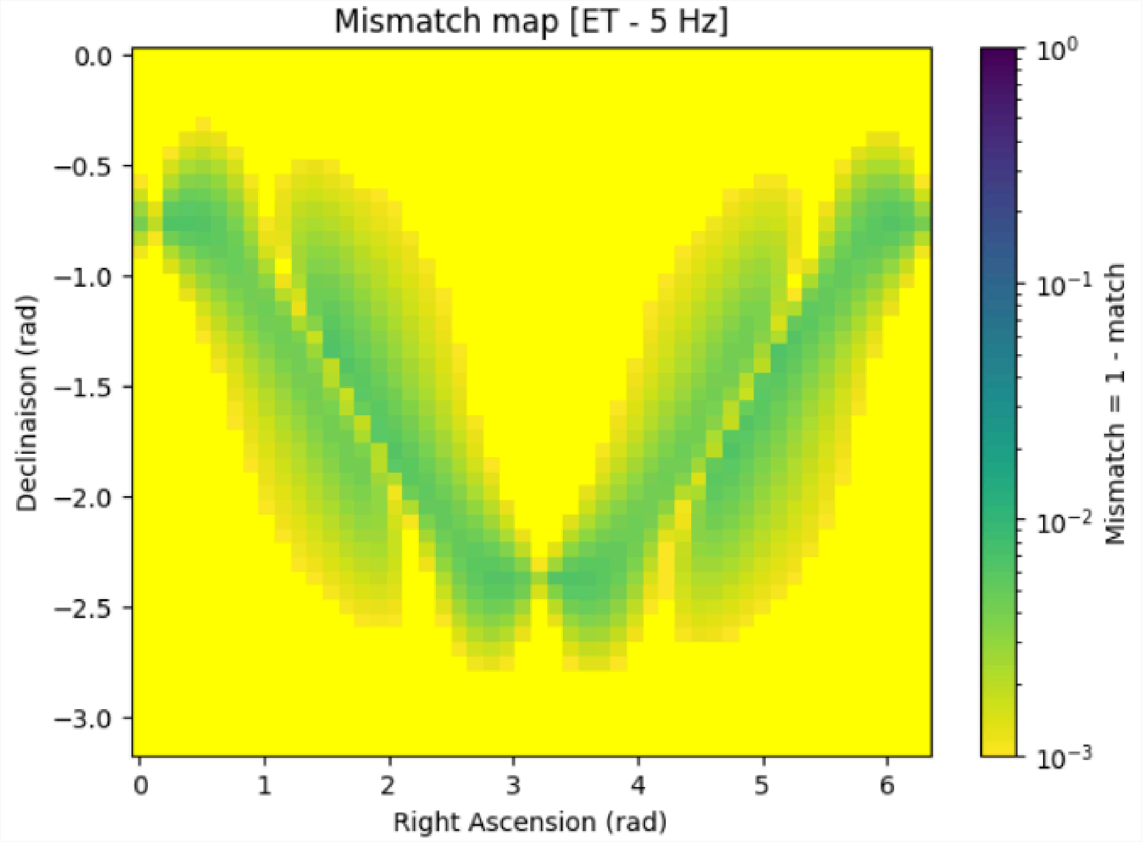


$$m(h_{\mathrm{rot}}, h_{\mathrm{const}}) = \max_{\Delta t_c, \Delta\phi_c} \frac{\Re \int \frac{f^{-7/3}|B(t_{\mathrm{rot}}(f))| e^{-i\theta(t_{\mathrm{rot}}(f))} e^{i(2\pi f \Delta t_c - \Delta\phi_c)} e^{i\phi_0}}{S_n(f)} df}{\sqrt{\int \frac{f^{-7/3}|B(t_{\mathrm{rot}}(f))|^2}{S_n(f)} df \int \frac{f^{-7/3}}{S_n(f)} df}}$$

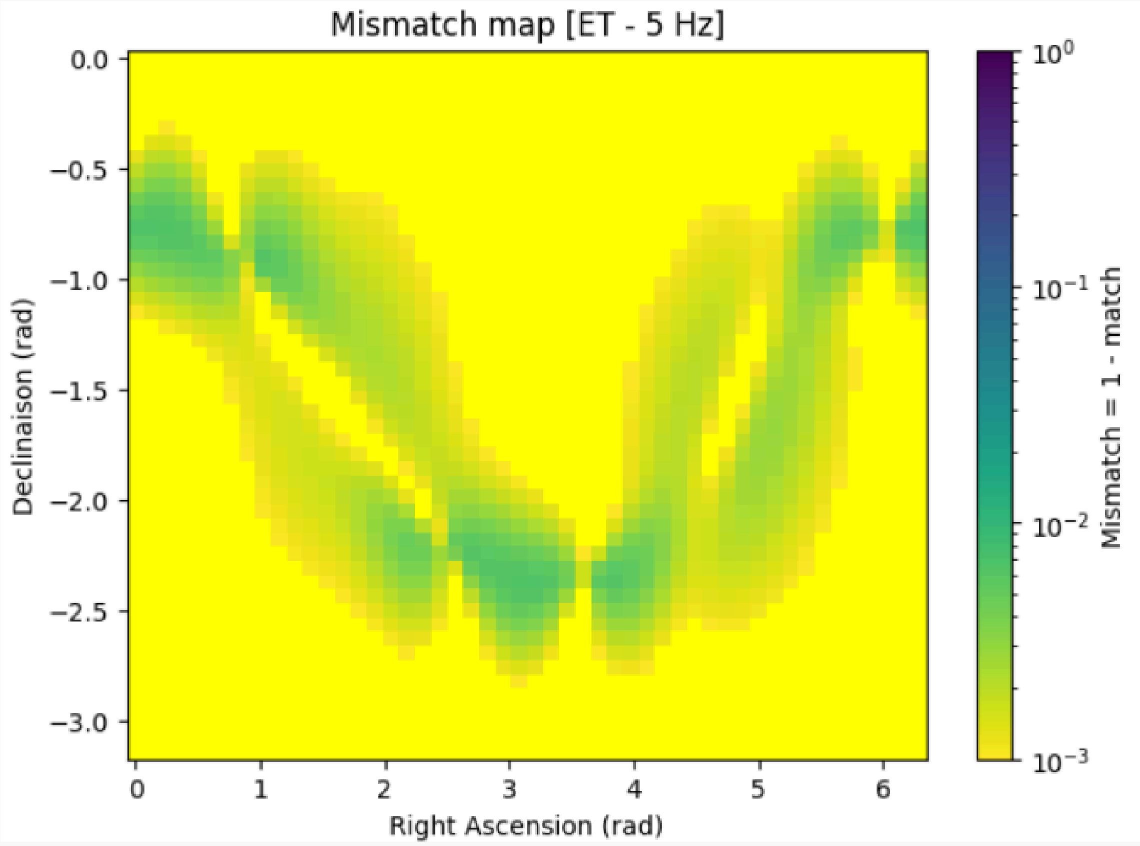
Modulus of numerator's integrand



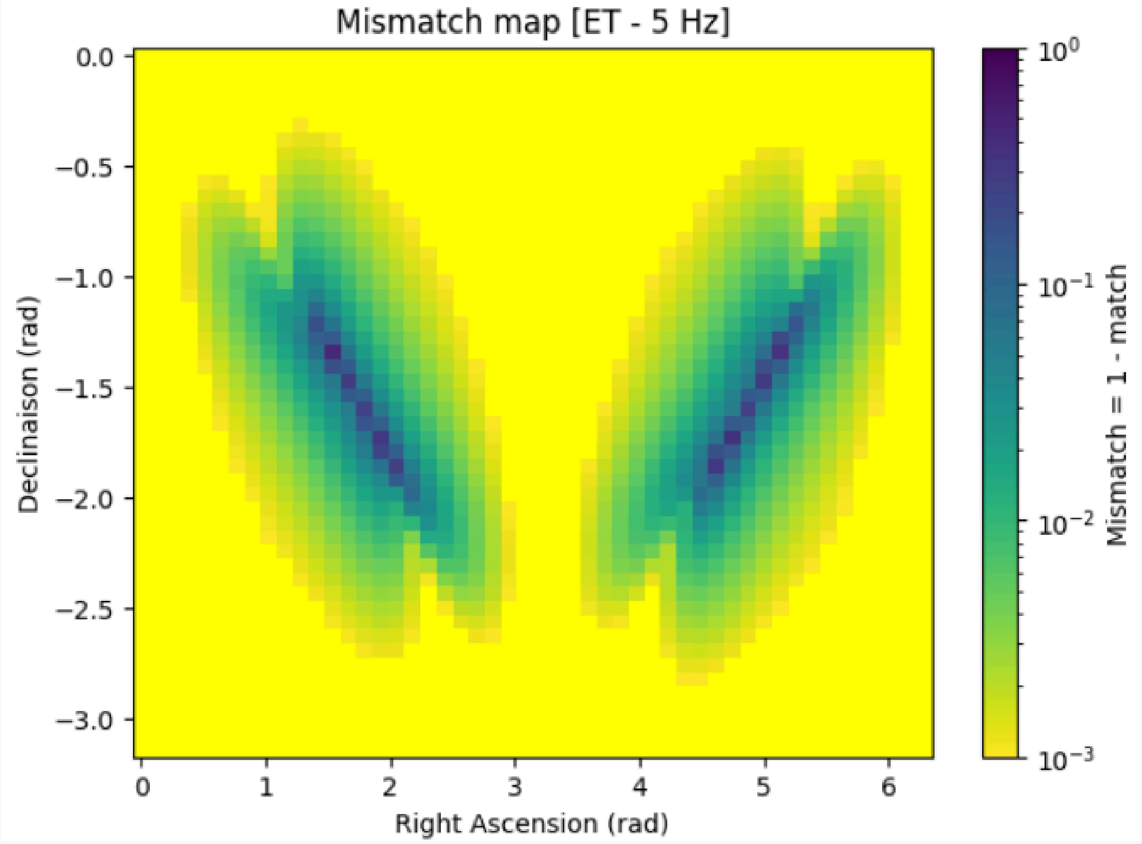
Physical interpretation yet to be given (if not an error)



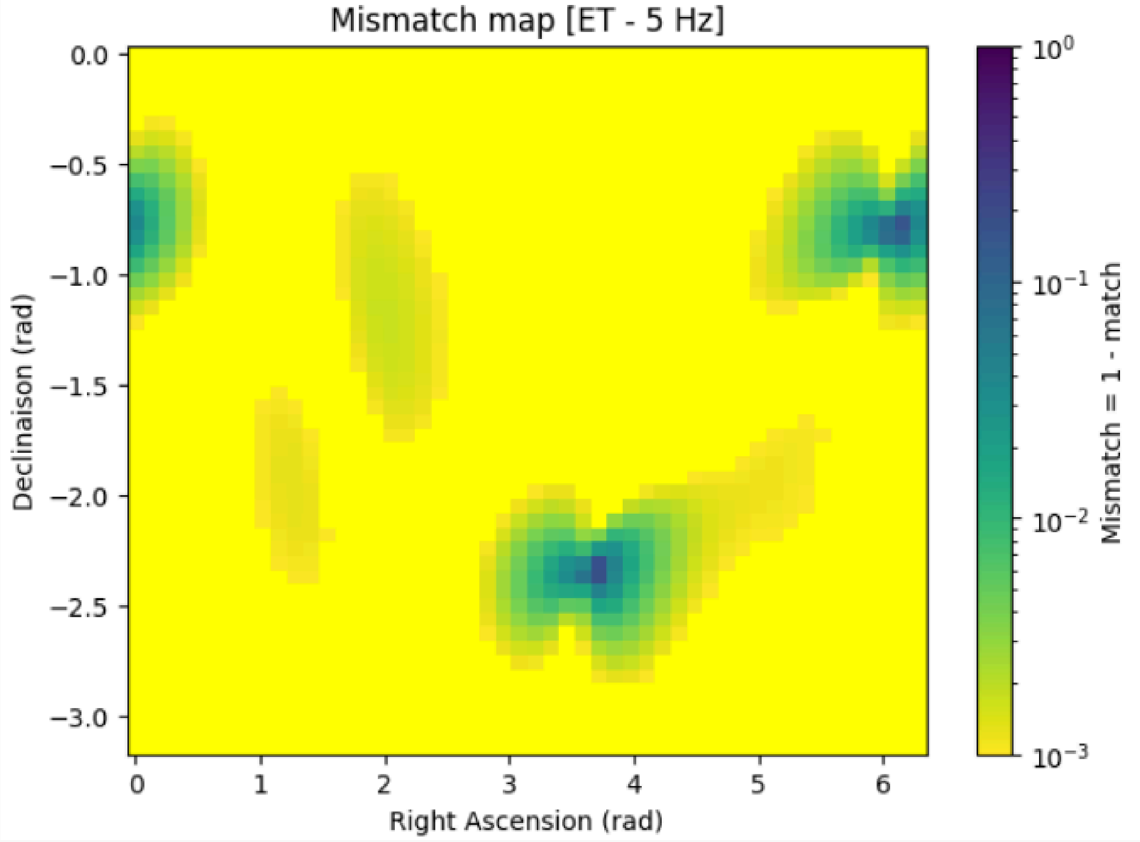
PSI = 0°, incl = 0°



PSI = 30°, incl = 0°



PSI = 0°, incl = 90°



PSI = 30°, incl = 90°

Development of equations using Marion's formalism

$$\mathcal{F}(\alpha,\delta,\psi,t) = F_+(\alpha,\delta,\psi,t) + iF_\times(\alpha,\delta,\psi,t)$$

$$\begin{cases} h_{mod}(\alpha,\delta,\psi,t) = f^{-\frac{7}{6}}(t)(F_+(\alpha,\delta,\psi,t) + iF_\times(\alpha,\delta,\psi,t)) = f^{-\frac{7}{6}}\mathcal{F}(\alpha,\delta,\psi,t) \\ h_{const}(\alpha,\delta,\psi) = f^{-\frac{7}{6}}(t)(F_+(\alpha,\delta,\psi,0) + iF_\times(\alpha,\delta,\psi,0) = f^{-\frac{7}{6}}\mathcal{F}(\alpha,\delta,\psi,0) \end{cases}$$

For 3 detectors (ET-Xylophone)
&
inclination = 0°

$$\mathcal{M}(\alpha,\delta,\psi) \propto \max_{\Delta\tau} \int_{f_{low}}^{f_{high}} \frac{f^{-\frac{7}{6}}}{S_n(f)} \Re([\mathcal{F}_{ET}(\alpha,\delta,\psi,0)][\mathcal{F}^*_{ET}(\alpha,\delta,\psi,t)])\, df$$

$$\mathcal{L}(\psi) := \Re([\mathcal{F}_{ET}(\alpha,\delta,\psi,0)][\mathcal{F}^*_{ET}(\alpha,\delta,\psi,t)])$$

$$\begin{aligned}\mathcal{L}(\psi) &= \sqrt{A + \mathcal{B}\sin(4\psi+\Phi_1) + \mathcal{C}\sin(8\psi+\Phi_2)} + \sqrt{A - \mathcal{B}\sin(4\psi+\Phi_1) + \mathcal{C}\sin(8\psi+\Phi_2)} \\ &= f_+(\psi) + f_\times(\psi)\end{aligned}$$

Match has a non-trivial polarisation angle depedency

For 3 detectors (ET-Xylophone)
&
any inclination angle (iota)

$$\begin{aligned}\mathcal{F}_{ET}(\alpha,\delta,\psi,t) &= [1+\cos^2(\iota)]F_{+,ET}(\alpha,\delta,\psi,t) + i[2\cos(\iota)]F_{\times,ET}(\alpha,\delta,\psi,t)\\ &= \eta F_{+,ET}(\alpha,\delta,\psi,t) + i\xi F_{\times,ET}(\alpha,\delta,\psi,t)\end{aligned}$$

$$\mathcal{L}(\psi) = \eta^2 f_+(\psi) + \xi^2 f_\times(\psi)$$

Match has a non-trivial
polarisation angle depedency

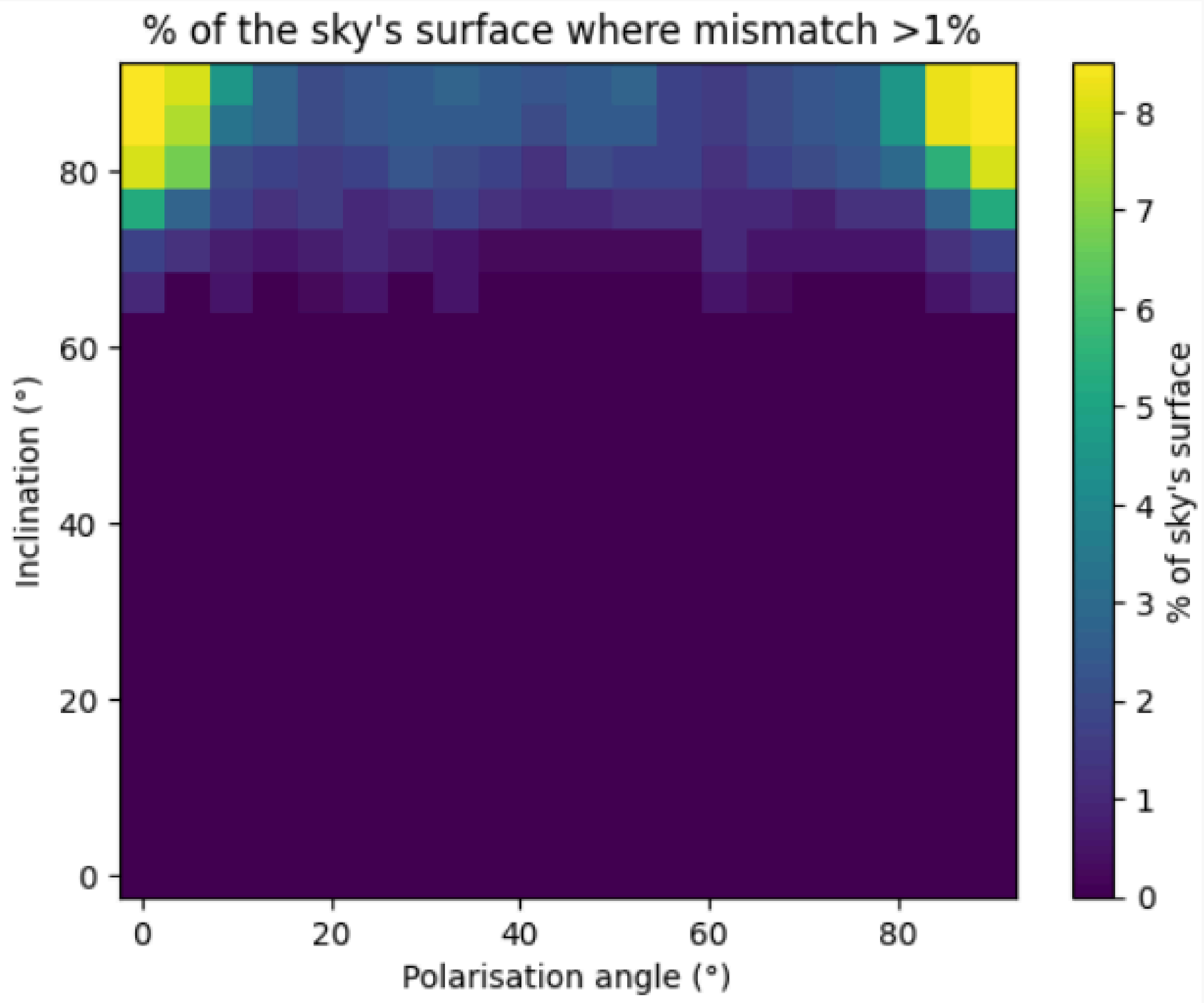
Studying the L function gives us the "geometric" behavior of the match

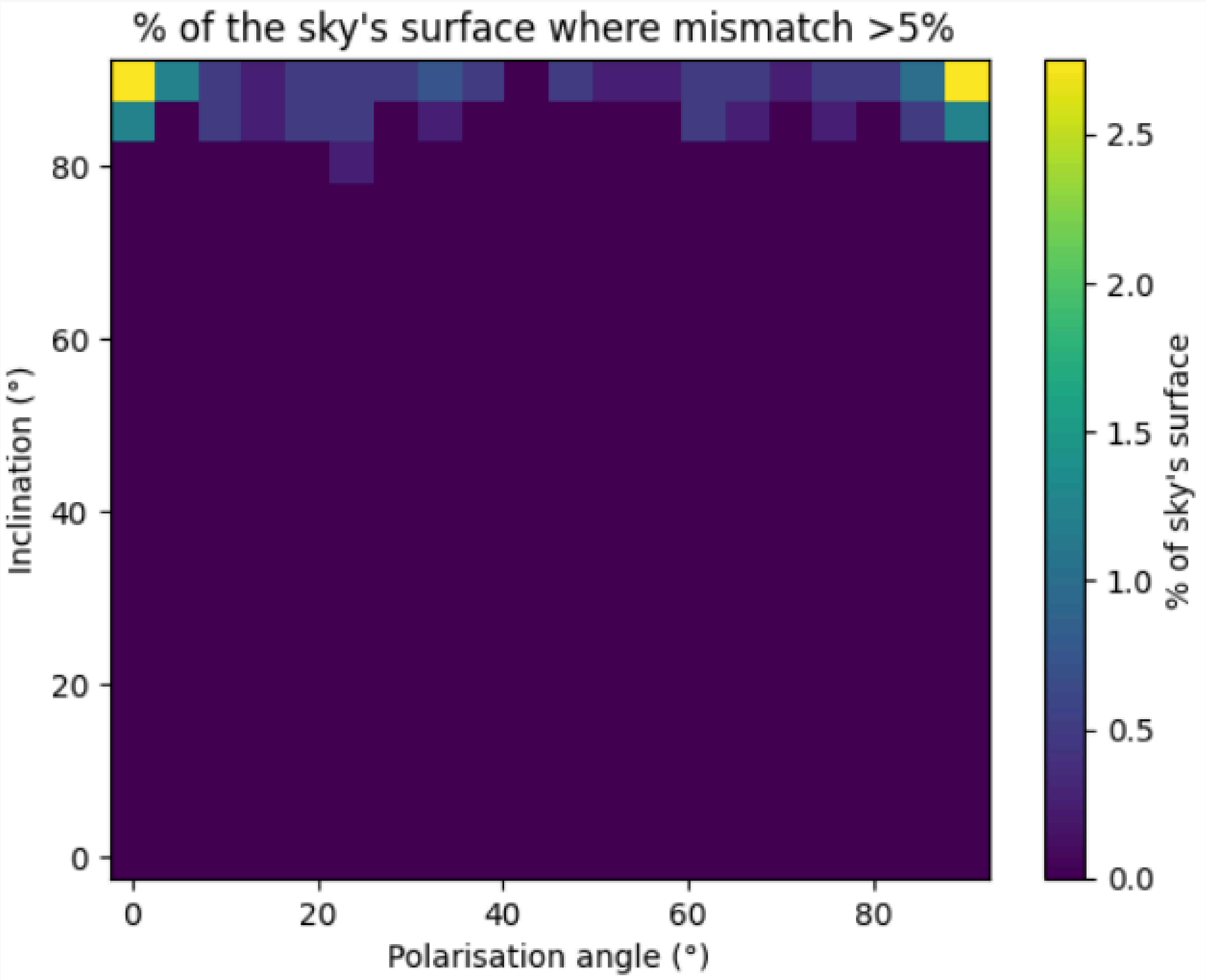
$$\mathcal{L}(\psi) = \eta^2 f_+(\psi) + \xi^2 f_\times(\psi)$$

Extendable to any network of L-shaped detectors

$$F_{+,\text{net}}(\alpha,\delta,\psi,t) = \sqrt{\sum_{i=1}^{n} F_{+,i}^2(\alpha,\delta,\psi,t)}$$
$$F_{\times,\text{net}}(\alpha,\delta,\psi,t) = \sqrt{\sum_{i=1}^{n} F_{\times,i}^2(\alpha,\delta,\psi,t)}$$

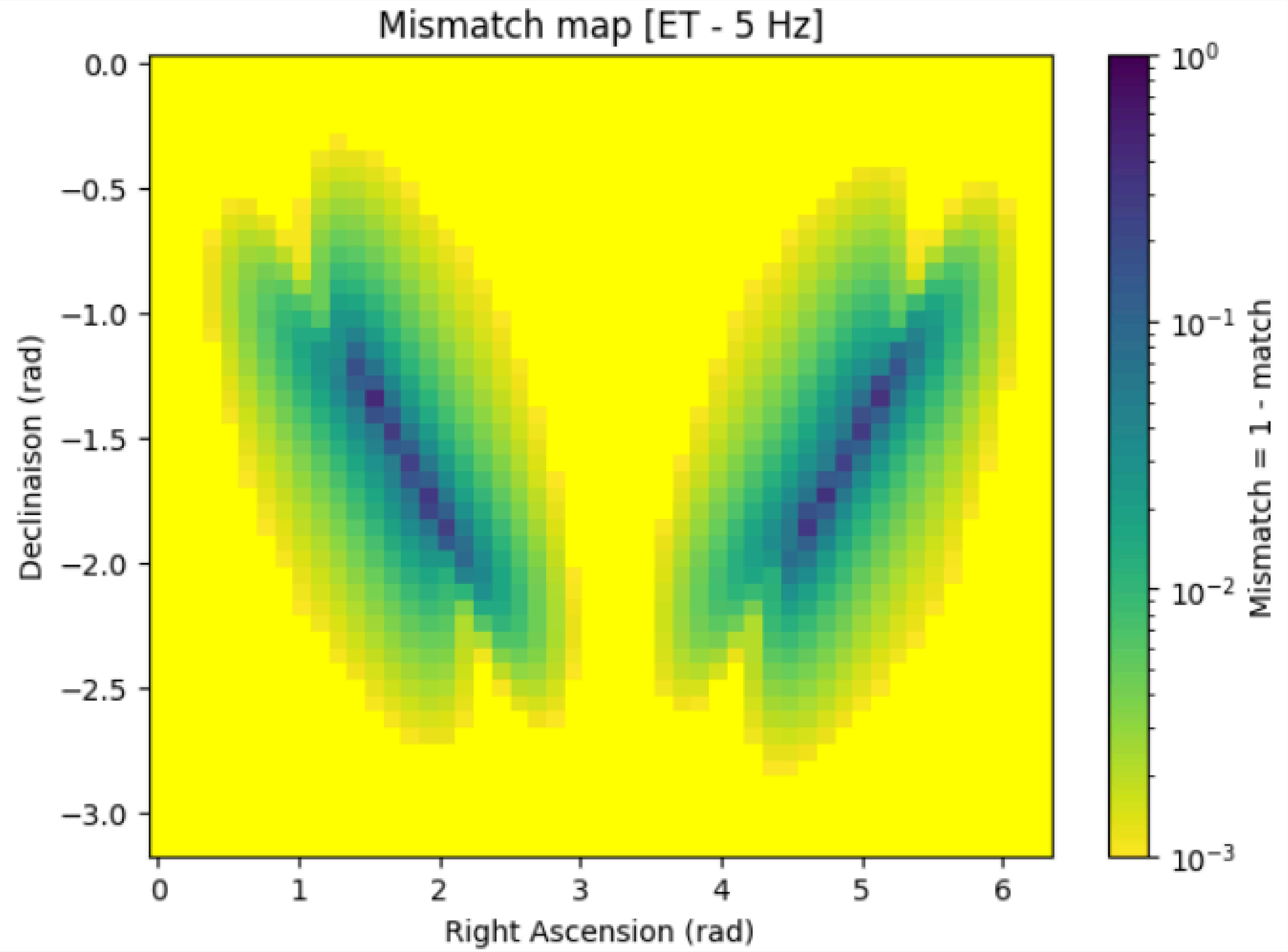
Areas where mismatch >1% and 5%





Confirmed !

Inclination = 90°, PSI = 90°



Max mismatch ~ 50%