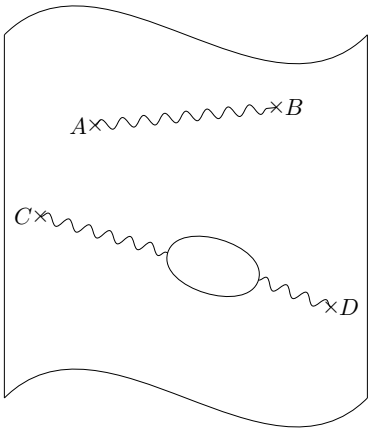
# Correspondances of QFTs across dimensions

Timothé ALEZRAA

RJP, April 2026


# What is a Quantum Field Theory

- **Fields** : Scalars, Fermions, Gauge bosons **particles** are represented by **functions** from spacetime to some representation spaces of the symmetry groups of the theory.



# What is a SUSY QFT ?

- Idea : transformation generators

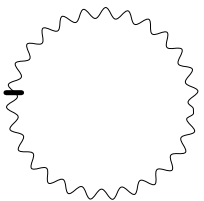
$$\mathcal{Q}_I : \text{fermions} \longleftrightarrow \text{bosons}$$

More symmetry = More structure on our theory

# String theory

- Close your eyes... imagine you are a string living in a 10-dimensional space... how peaceful must life be
- Your quantum excited states looks like actual particles living in 10d.
- Consistency enforces a choice between excitation modes, two main string theories that we call "type IIA" and "type IIB"

Open strings



Closed strings

# Compactification to lower dimensions

- Alas, we only live in 4 dimensions. The string must live on some

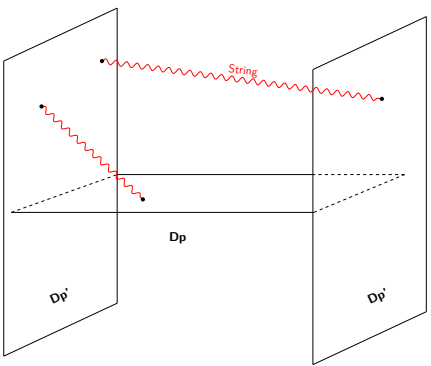
$$\text{Spacetime } = \mathbb{R}^4 \times \underbrace{X}_{\text{6d}}$$

- With $X$ a very small 6 dimensional space that we cannot see. As 4 dimensional observers, the particles that we see **effectively** depends on the choice of $X$ and the theory we start with

$$\mathcal{T}(\text{10d space} = \mathbb{R}^4 \times X) \longrightarrow \mathcal{T}'_X(\mathbb{R}^4)$$

# Stringy constructions tools to non perturbative results

- **D-branes** in string theories : D+1 dimensional objects living in 10d
- Restrict to the worldvolume of the branes $\xrightarrow[\text{Low energy}]{}$ SUSY gauge theories :



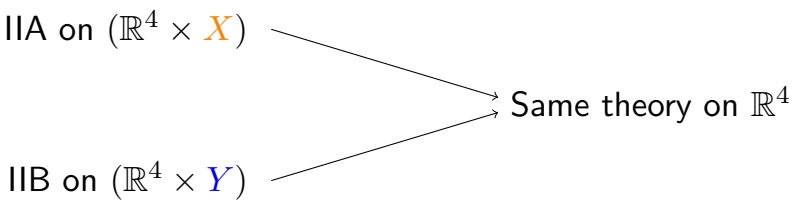

**Example**

- Vertical distance $\sim$ interaction strength

[Very active field since the 80's]

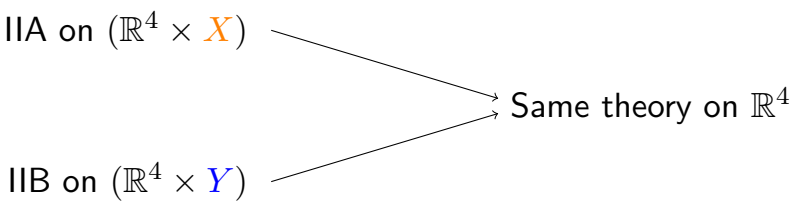
# Mirror symmetry

- Counting D-branes $\Leftrightarrow$ counting " proper " subspaces in $X \times \mathbb{R}^4$
- Something strange seems to appear



# Mirror symmetry

- Counting D-branes $\Leftrightarrow$ counting " proper " subspaces in $X \times \mathbb{R}^4$
- Something strange seems to appear




- We call $X$ and $Y$ **Mirrors**
- Whole field of research on studying these "coincidences"

# Strong-Weak duality

- Another interesting objects are supersymmetric gauge theories.

# Strong-Weak duality

- Another interesting objects are supersymmetric gauge theories.
- They come with some gauge coupling $\sim$ interaction strength $\sim g$
- Typically easier to compute observables when interaction is weak rather than strong ( but unfortunately we don't get to choose, and real life shows strong forces.)

# Strong-Weak duality

- Another interesting objects are supersymmetric gauge theories.
- They come with some gauge coupling $\sim$ interaction strength $\sim g$
- Typically easier to compute observables when interaction is weak rather than strong ( but unfortunately we don't get to choose, and real life shows strong forces.)
- But we noticed that for some theories

$$\text{computing } \langle A \rangle_g \Leftrightarrow \text{computing } \langle A' \rangle_{1/g}$$

# Strong-Weak duality

- Another interesting objects are supersymmetric gauge theories.
- They come with some gauge coupling $\sim$ interaction strength $\sim g$
- Typically easier to compute observables when interaction is weak rather than strong ( but unfortunately we don't get to choose, and real life shows strong forces.)
- But we noticed that for some theories

$$\boxed{\text{computing } \langle A \rangle_g \Leftrightarrow \text{computing } \langle A' \rangle_{1/g}}$$

- But finding such theories, proving such equalities is very hard in general.

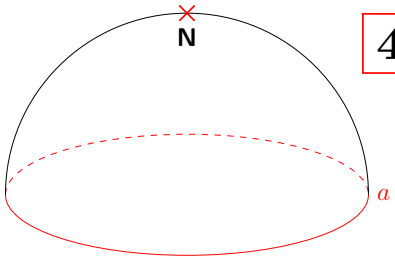
# Strong-Weak duality

- Another interesting objects are supersymmetric gauge theories.
- They come with some gauge coupling $\sim$ interaction strength $\sim g$
- Typically easier to compute observables when interaction is weak rather than strong ( but unfortunately we don't get to choose, and real life shows strong forces.)
- But we noticed that for some theories

$$\text{computing } \langle A \rangle_g \Leftrightarrow \text{computing } \langle A' \rangle_{1/g}$$

- But finding such theories, proving such equalities is very hard in general.
- My work is finding ways to compute each side independantly and looking if they are related or not

# The AGT correspondance

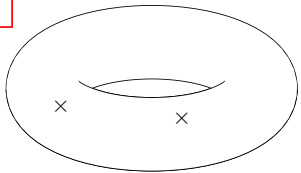



**4d**

$Z[a] \longleftrightarrow \mathcal{F}_{CFT}[a]$

**AGT**

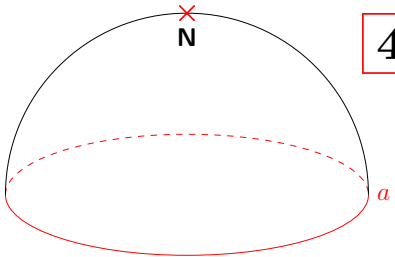
**2d**



- $g \longleftrightarrow 1/g \Leftrightarrow$ " Change of basis" of the 2d theory
- Somehow the 2d side is related to **mirror symmetry** (bc string trajectory is 2d)

[Alday,Gaiotto,Tachikawa,2009]

# The AGT correspondance

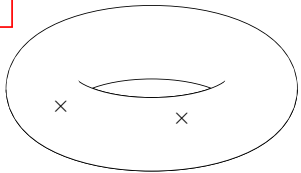



**4d**

$Z[a] \longleftrightarrow \mathcal{F}_{CFT}[a]$

**AGT**

**2d**



- $g \longleftrightarrow 1/g \Leftrightarrow$ " Change of basis" of the 2d theory
- Somehow the 2d side is related to **mirror symmetry** (bc string trajectory is 2d)
- My work : use AGT and tools developped in the mirror symmetry litterature to understand/find dualities in 4d

[Alday,Gaiotto,Tachikawa,2009]

# Conclusion

- General idea that the geometry of spacetime encodes all the algebraic properties of QFTs, like in F-theory and string theory in general
- Very hard computations in some theories might be very easy in their dual
- Connected to many open questions, research areas in maths/physics