Holographic Baryons as Quantum Hall Droplets

Francesco Bigazzi[a,1], Aldo L. Cotrone[a,b,2], Andrea Olzi[a,b,3] and **Jean-Loup Raymond**[b,c,4]

[a]INFN, Sezione di Firenze, Italy; [b] Dipartimento di Fisica e Astronomia, Universitá di Firenze, Italy;

[c]Université de Paris Cité, CNRS, Astroparticule et Cosmologie, F-75013 Paris, France.

(1) bigazzi@fi.infn.it, (2) cotrone@fi.infn.it, (3) andrea.olzi@fi.infn.it, (4) raymond@apc.in2p3.fr

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Abstract

We provide [1] a first-principle holographic construction of baryons as quantum Hall droplets in the Witten-Sakai-Sugimoto model [2, 3]. The baryons are described as charged D6-branes with a circular boundary on a flavor D8-brane in the Type IIA backgrounds dual to the confining and non-confining phases. The holographic description allows us to calculate precisely their properties, such as spin, mass and size.

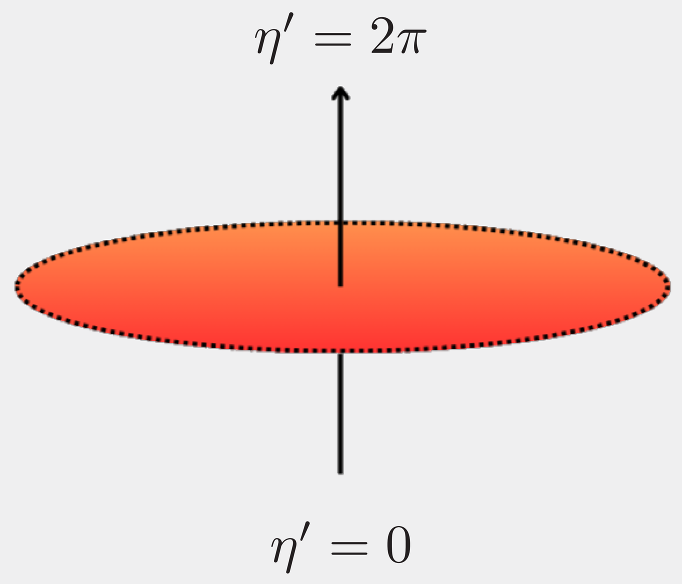
Motivation

- Low-energy baryons are well modeled by the Skyrme model, which describes them as topological composite states of mesons. The topology of these solutions trivializes for a single flavor ($N_f = 1$), and therefore the model can no longer be used.

- In the large-N limit, baryons can be viewed as pancake-shaped objects [4] charged under a 2-form symmetry, associated with a topologically conserved current:

$$j = *d\eta' .$$

- A $U(1)_N$ Chern-Simons theory lives on these sheets. This topological field theory is the one describing the fractional quantum Hall effect.



Holographic model

Witten-Sakai-Sugimoto model:

- Holographic models are based on a duality that connects a gauge theory to a gravity theory, allowing complex quantum field theory computations to be performed via simpler classical calculations in the gravity model.

- In certain string theories, there are extended objects (Dp-branes, of dimension p+1) on which a gauge theory lives, that is dual to the effective gravitational description near the brane.

- The Witten model [2] is based on the type IIA string theory solution with N stacked D4-branes wrapped on a circle $S^1_{x_4}$ of radius M_{KK}^{-1}, where M_{KK} is the Kaluza-Klein mass.

- At low energy, the effective action on the brane describes an $SU(N)$ gauge theory in 3+1 dimensions, along with a tower of massive Kaluza-Klein modes in the adjoint representation.

- The holographic dual is a gravity theory described by a 4-sphere, a cigar-like geometry in the u and x_4 directions (see figure), and 4 Minkowski dimensions:

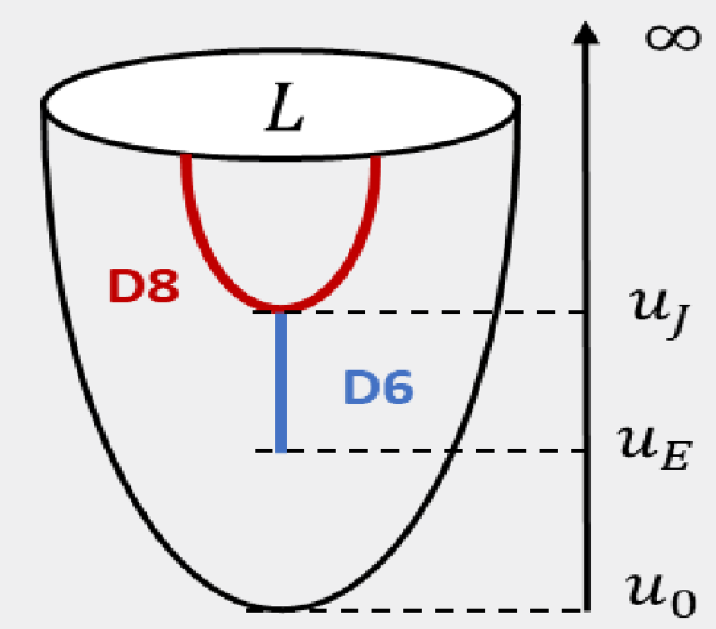


Figure 1: D6 brane dual to one-flavoured baryon in the WSS model

$$ds^2 = \left(\frac{u}{R}\right)^{3/2} \left(dx^\mu dx_\mu + f(u)\, dx_4^2\right) + \left(\frac{R}{u}\right)^{3/2} \frac{du^2}{f(u)} + R^{3/2} u^{1/2} d\Omega_4^2 ,$$

$$f(u) = 1 - \frac{u_0^3}{u^3}, \quad e^\phi = g_s \left(\frac{u}{R}\right)^{3/4}, \quad F_4 = dC_3 = \frac{2\pi N}{V_4}\omega_4 , \quad R = \pi g_s N l_s^3 .$$

- Quarks in the model are introduced by adding N_f D8-branes taking a "U" shape [3].

D6-brane properties

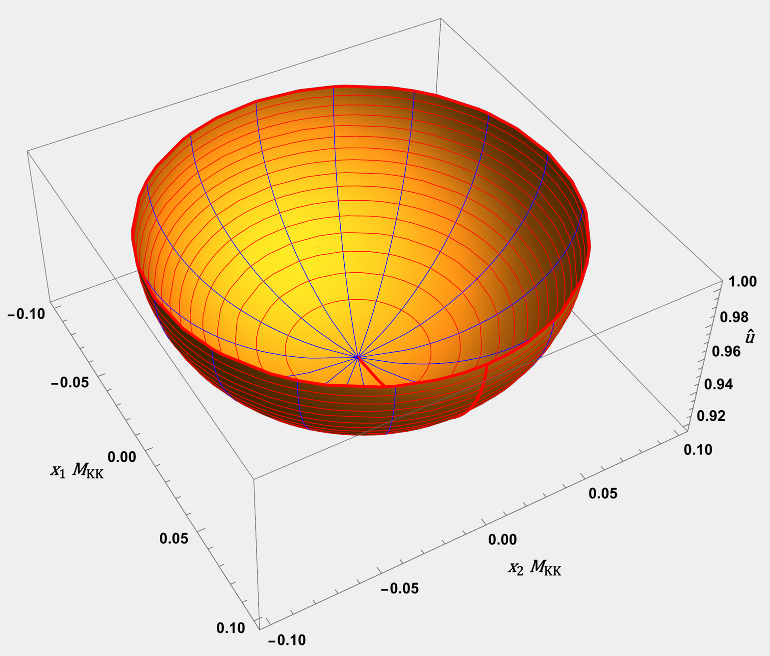


Figure 2: D6-brane dual to the baryon

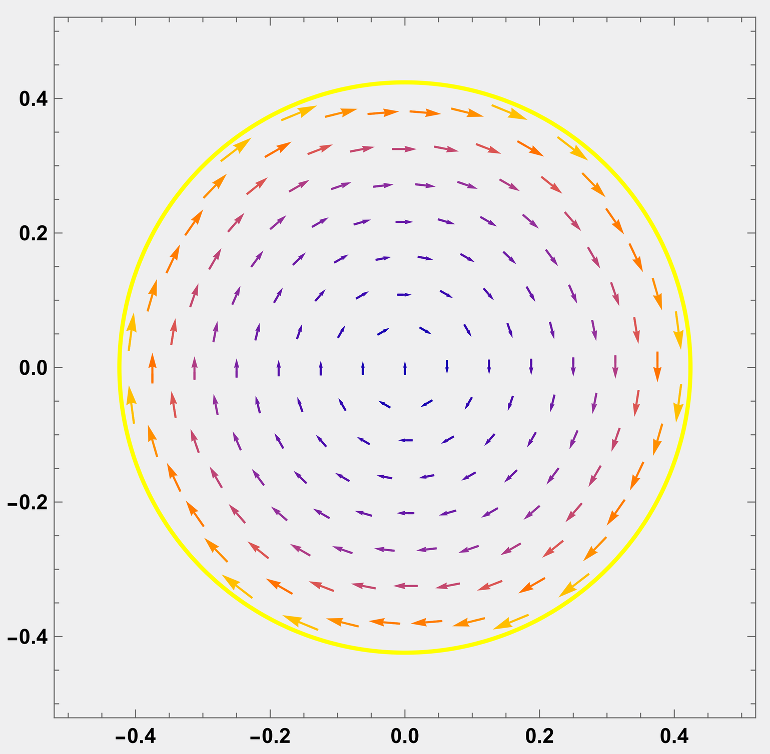


Figure 3: Gauge field vortex profile

- For $N_f = 1$, a baryon in the gauge theory is obtained in the gravity dual by adding a D6-brane with a circular intersection with the D8-brane setup at the tip of the "U".

- C_3 produces the $U(1)_N$ *Chern-Simons* theory through the Wess-Zumino term with C_3:

$$\int_{S_4 \times M_3} C_3 \wedge f \wedge f = N \int_{M_3} a \wedge f .$$

- Together with *DBI* action, the full action is [5]:

$$S_{D6} = -N \int dt \int_{u_*}^{u_J} du \left\{ \frac{(a_t\, \partial_u a_\psi - a_\psi\, \partial_u a_t)}{2} + \frac{u\, \rho(u) D(u)}{3(2\pi l_s^2)^2} \right\},$$

$$D(u) = \sqrt{\rho'(u)^2 \left(\frac{u}{R}\right)^3 + \frac{1}{f(u)} + (2\pi l_s^2)^2 \left(\frac{(\partial_u a_\psi)^2}{\rho^2} - (\partial_u a_t)^2\right)} ,$$

where (ρ, ψ) are polar coordinates in the (x_1, x_2) plane.

- $M_3 \equiv \Sigma \times \mathbb{R}_t$, and the baryon number is shown to be given by the *first Chern number* over Σ:

$$\boxed{n_B = \frac{1}{2\pi} \int_\Sigma f}.$$

- Edge modes respect $U(1)$ *Wess-Zumino-Witten algebra* relations, with prescribed topological sector, dictated by the number of quarks of this flavor.

- The winding number around the circular boundary gives the flavor number: $n_J^{(W)} = \frac{N}{2\pi} \int_{D8 \cap D6} \hat{a}$, with $\hat{a}_\psi = a_\psi - n_B$.

- Gauge connection $a_\psi(\rho)$ on the D6-brane is a *vortex* solution, with winding $N n_B$. This vortex is regularized by the *geometry* of the brane.

Physical properties

- Derived analytically *number of strings* $\boxed{q_s = N n_B}$ and *spin*, which satisfies usual anyonic relation $\boxed{J = \frac{N}{2} n_B^2}$.

- Radius analytically well estimated by rigid rotor approximation.

- Mass scales like N. Other scaling properties obtained numerically:

	Confined phase
Radius	$\sim \lambda^{-2/3} M_{KK}^{-1} b^{5/6}$
Mass	$\sim N \lambda M_{KK} b^{-1}$

where $b = u_0/u_J$, $\lambda = g_{YM}^2 N$ and M_{KK} is the Kaluza-Klein mass.

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