Investigating Quarkonium physics
(at low q_T)

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Coming next

- **Part I: Motivation** and **framework**

- **Part II: TMD shape function at the EIC**

  see: Boer, Bor, **LM**, Pisano, Yuan, *JHEP 08* (2023) 🔗

  LM, Boer, Bor, *Phys.Rev.D 112* (2025) 🔗

  Echevarría, et al., *JHEP 09* (2024) 🔗

  Echevarría, et al., *2510.11809* (2026) 🔗

- **Part III**: Quarkonium associated with jets

  Boer, Nanako, **LM**, Pisano, *work in progress*

A few words about: Quarkonia

Quarkonia are bound states of a heavy quark-antiquark pair ($Q\overline{Q}$)

Characteristics: **Large** mass $\boldsymbol{M}$ **Small** relative velocity $\boldsymbol{v}$

Described by: **NRQCD EFT** Bodwin et al., *PRD 51* (1994) Fragmentation functions Kang et al., *PRD 90* (2014) CSM Baier, Ruckl, *Z.Phys.C 19* (1983) …and more

Short-distance scale

(perturbative)

Hard scattering that originates the heavy quarks

Expanded in coupling constants

Long-distance scale

(non-perturbative)

Describes the hadronization into the quarkonium

$$D_{k\to\mathcal{Q}}(z;\mu_0^2) \approx \sum_n d_{k\to n}(z;m_Q,\mu_0^2)\ \langle\mathcal{O}_{\mathcal{Q}}[n]\rangle$$

$\langle\mathcal{O}_{\mathcal{Q}}[n]\rangle$ LDME (universal parameters)

A wannabe review of Quarkonium physics

Doubts on the best model to describe quarkonium formation

Most used: **non-relativistic QCD** approach but the size of CO LDME/CS LDME is unclear

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In a collinear framework computations currently reach **NLO** precision

$pp \to J/\psi + W$ Butenschoen, Kniehl, *PRL 130* (2023)

$ep \to J/\psi$ Qiu, et al., *Chin.Phys.Lett 38* (2021)

Reviews

Lansberg, *Phys.Rept. 889* (2020)

Brambilla, et al., *EPJC 71* (2021)

Automated tools describe quarkonium production at **LO** precision

HelacOnia

Shao, *J.CPC 198* (2016)

MadSons package for MadGraph5

(in development the inclusion states with $L = 1$)

Colpani-Serri et al., *JHEP 02* (2026)

LM, Shao, Simon, *work in progress*

Resummation of large logs

in BFKL towards "true" NLL

in TMDs also NLL with some NLL'

Fucilla, et al., *2601.04142* (2026)

LM, et al., *Phys.Rev.D 112* (2025)

Echevarría, et al., *2510.11809* (2026)

The twist-2 TMD table

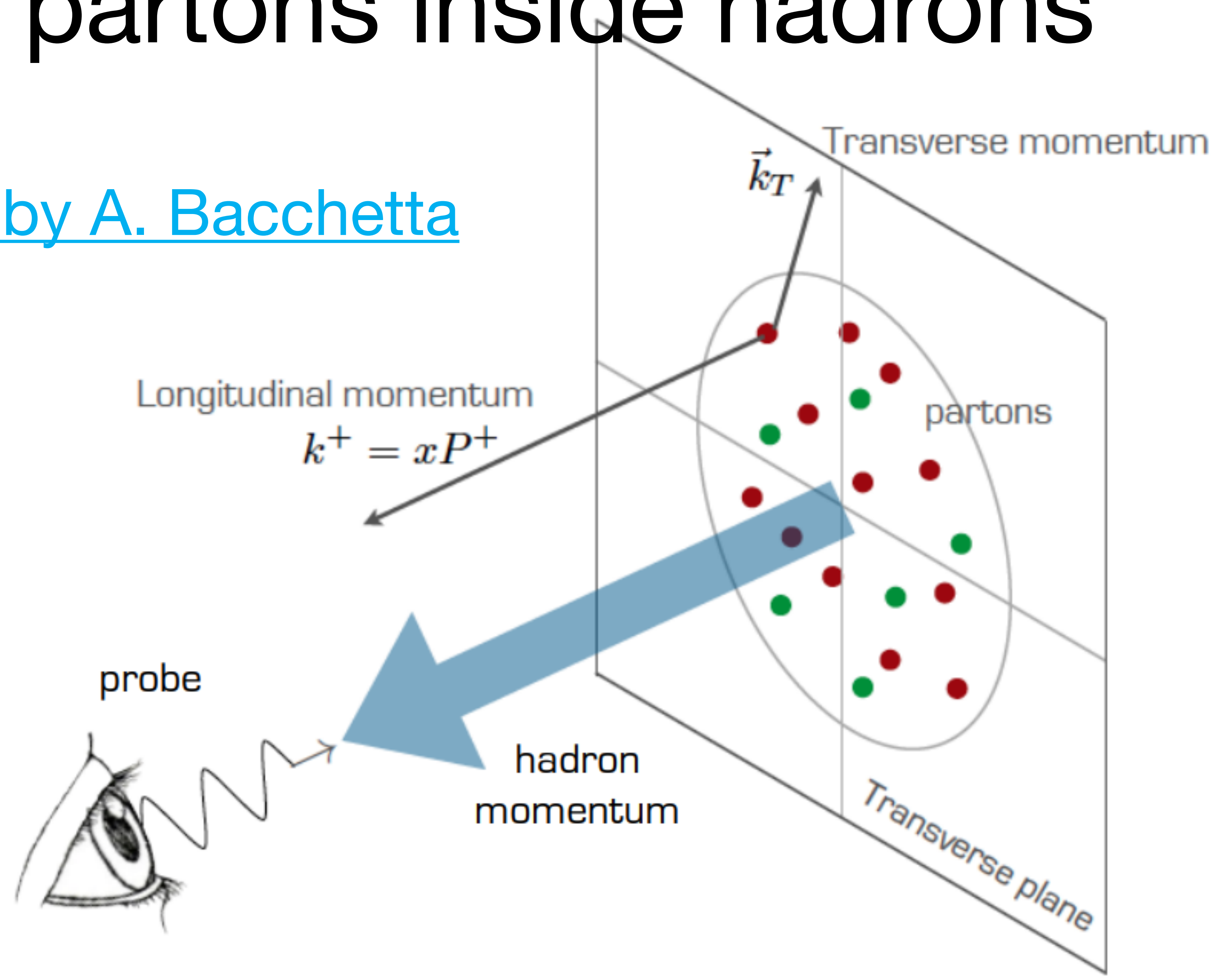
TMD handbook, *2304.03302* (2023)

(transverse momentum dependent)

TMD distributions describe the 3D momenta distribution of partons inside hadrons

quark polar. / proton polar.	Unpolarized	Circular	Linear
Unpolarized	f_1		$h_1^\perp$
Longitudinal		g_{1L}	$h_{1L}^\perp$
Transverse	$f_{1T}^\perp$	g_{1T}	h_1 , $h_{1T}^\perp$

picture by A. Bacchetta



also collinear | T-Even | T-Odd

gluon polar. / proton polar.	Unpolarized	Circular	Linear
Unpolarized	f_1		$h_1^\perp$
Longitudinal		g_{1L}	$h_{1L}^\perp$
Transverse	$f_{1T}^\perp$	g_{1T}	h_1 , $h_{1T}^\perp$

A few words about: TMDs

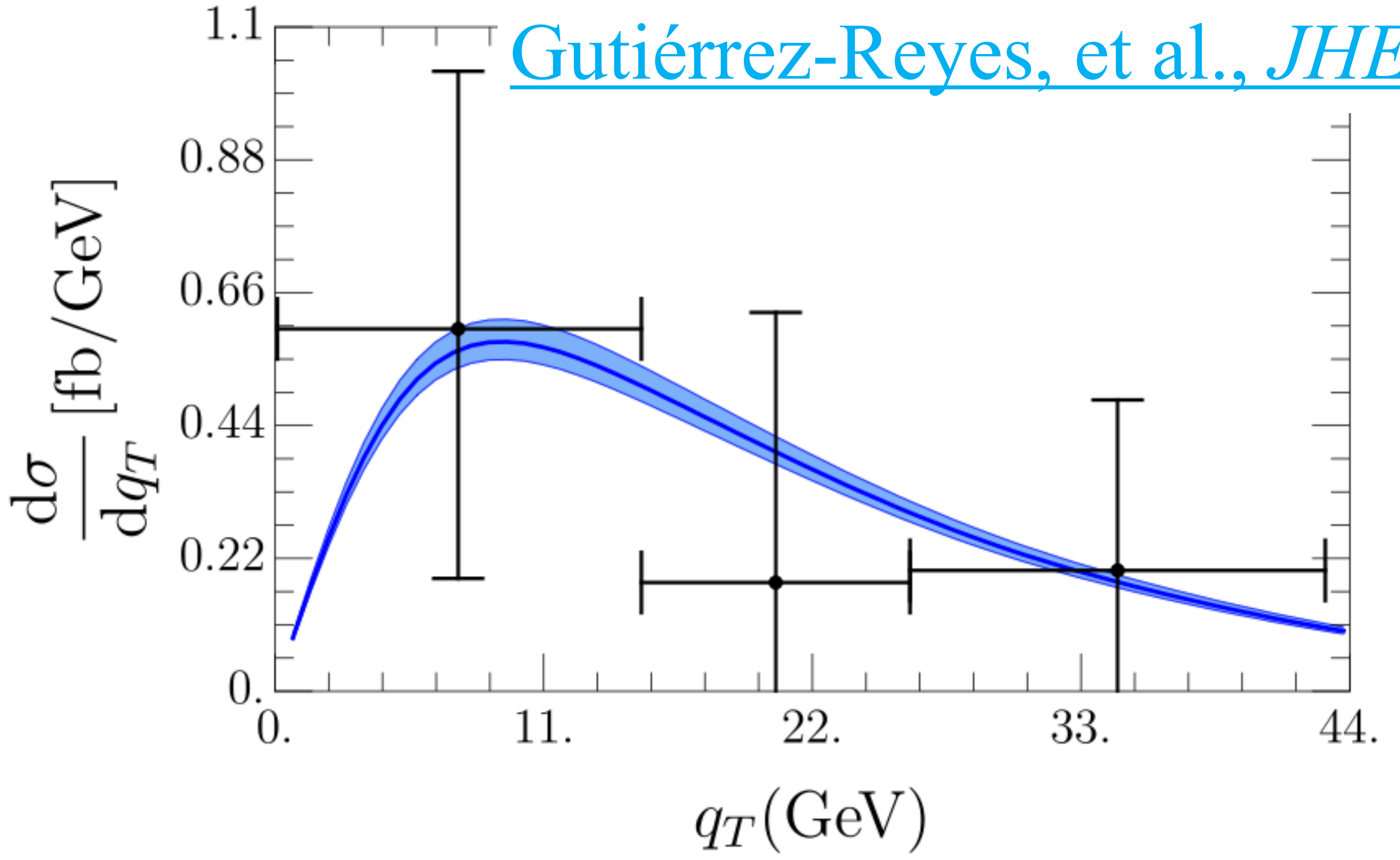
TMD handbook, *2304.03302* (2023)

(transverse momentum dependent)

TMD distributions are needed to describe observables at low q_T

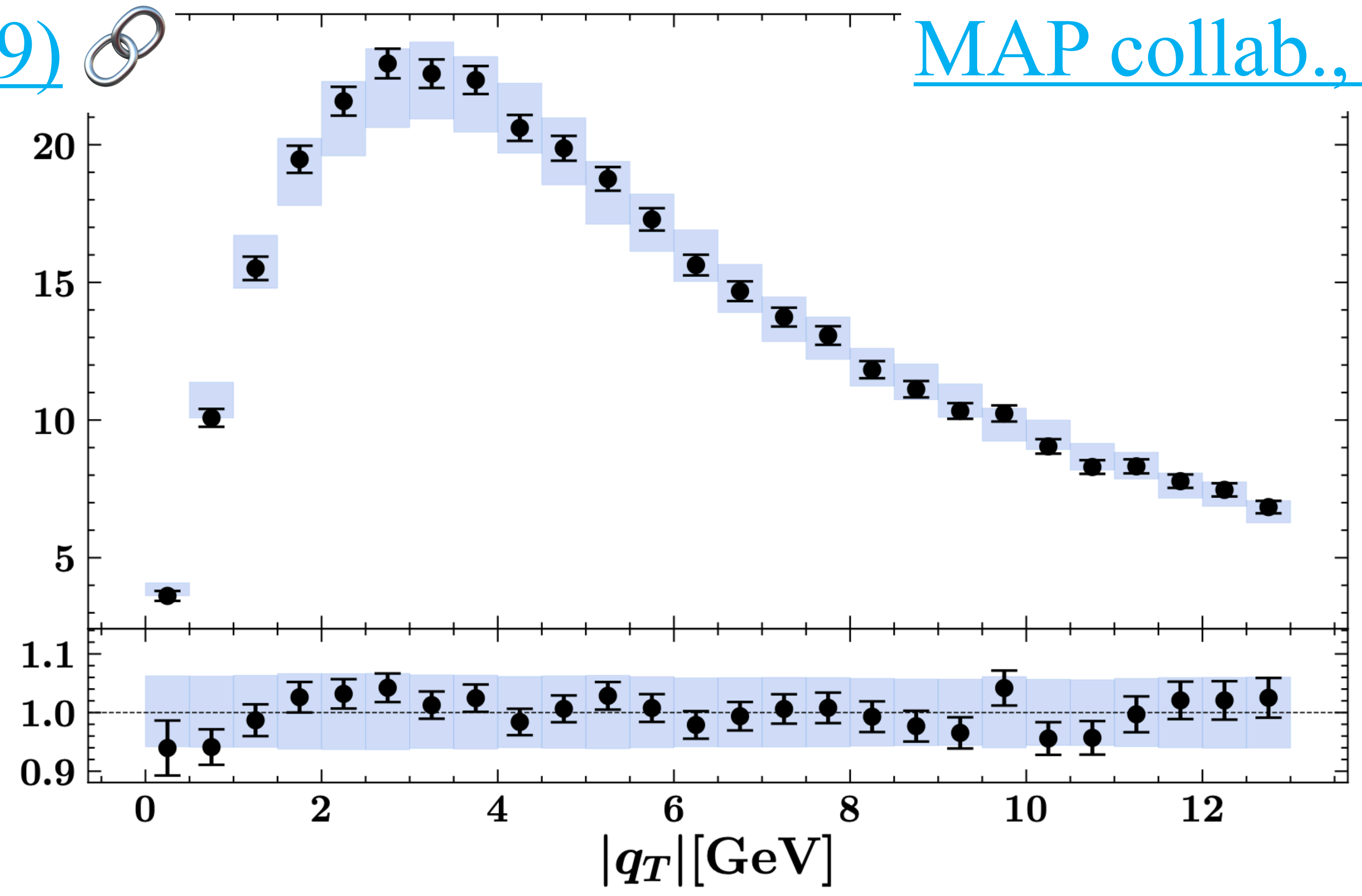
$pp \to H + X$

Gutiérrez-Reyes, et al., *JHEP 11* (2019)

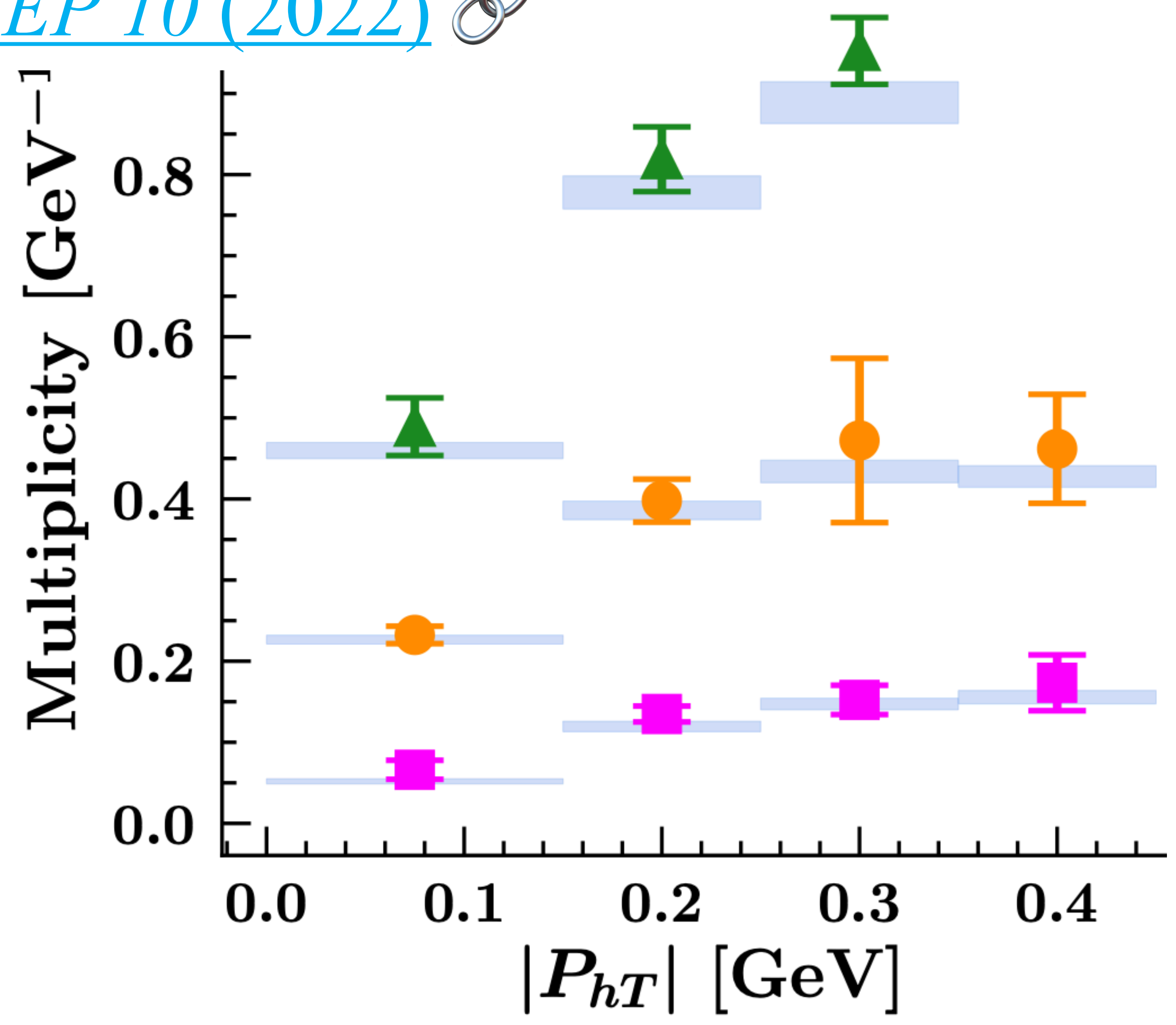


$p\bar{p} \to Z + X$

MAP collab., *JHEP 10* (2022)

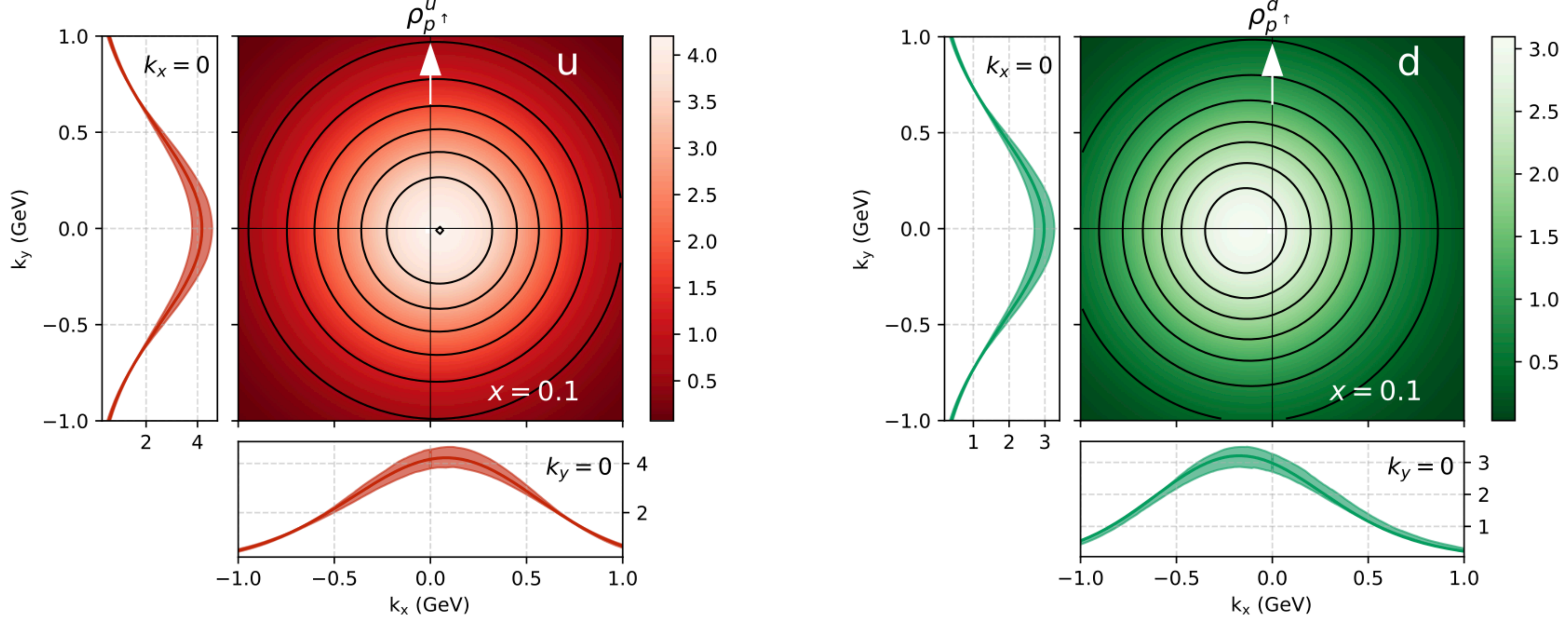


$ep \to \pi^+ + X$



Access to multidimensional tomography of hadrons

(with quarks)



Bacchetta, et al., *Phys.Lett.B 827* (2022)

Using quarkonia to access gluon TMDs

Investigating the gluonic sector requires identifying new observables sensitive to gluons

→ Quarkonium has been identified as a good candidate!

Boer, Pisano, *Phys.Rev.D 86* (2012) — $pp \to \eta_Q + X$

den Dunnen, et al., *Phys.Rev.Lett. 112* (2014) — $pp \to J/\psi + \gamma + X$

Scarpa, et al., *Eur.Phys.J.C 80* (2020) — $pp \to J/\psi + J/\psi + X$

Quarkonium physics pre-2019:

TMD effects from initial state solely → Quarkonium hadronisation is assumed to be collinear

Example SIDIS: $$\int \mathrm{d}x\,\mathrm{d}z\,\mathrm{d}q_T\; f_a(x, q_T)\,\hat{\sigma}_{a,n}\,\langle \mathcal{O}_{\mathcal{Q}}[n]\rangle\,\delta(z-1)$$

Quarkonium physics post-2019:

Echevarría, *JHEP 10* (2019)   Mehen, Fleming, Makris, *JHEP 04* (2020)

Recognition of TMD effects also in the quarkonium hadronisation process

Example SIDIS: $$\int \mathrm{d}x\,\mathrm{d}z\,\mathrm{d}q_T\; \mathscr{C}\left[f_a \Delta_{\mathcal{Q}}^{[n]}\right](x, z, q_T)\,\hat{\sigma}_{a,n}$$

$\langle \mathcal{O}_{\mathcal{Q}}[n]\rangle\,\delta(1-z)$ LDME → $\Delta_{\mathcal{Q}}^{[n]}(z, k_T)$ TMD shape function

The TMD shape function

The gluon radiation is organized into different modes:

collinear X_n | anticollinear $X_{\bar{n}}$ (absent in ep) | soft X_s

These modes enter in the operator defintion

$$\Delta^{[m]}_{\mathcal{Q}}(b_T) \sim \mathrm{Tr}\langle 0 \,|\, [(S_v S_n S_{\bar{n}})^\dagger \chi^\dagger \Gamma^c_m \psi](b_T)\ [a^\dagger_{\mathcal{Q}} a_{\mathcal{Q}}]\ [(S_v S_n S_{\bar{n}}) \psi^\dagger \Gamma^c_m \chi](0) \,|\, 0 \rangle / \sqrt{S(b_T)}$$

Fourier conjugate of q_T

Soft radiation (Wilson line) along

n ($\bar{n}$): incoming proton(s)

v: outgoing quarkonium

from the completeness

$\Sigma_{X_s} | X_s \rangle \langle X_s | = 1$

The TMD shape function

The gluon radiation is organized into different modes:

collinear X_n | anticollinear $X_{\bar{n}}$ (absent in ep) | soft X_s

These m

$\Delta^{[m]}_{\mathcal{Q}}(b_T)$

Fourier conju

TMD quantities (including the shape function) should match their collinear counterparts at large q_T

$$\Delta^{[n]}_{\mathcal{Q}}(q_T \gg \Lambda_{\mathrm{QCD}}) = \sum_{n'} C_{nn'} \otimes \langle \mathcal{O}_{\mathcal{Q}}[n'] \rangle$$

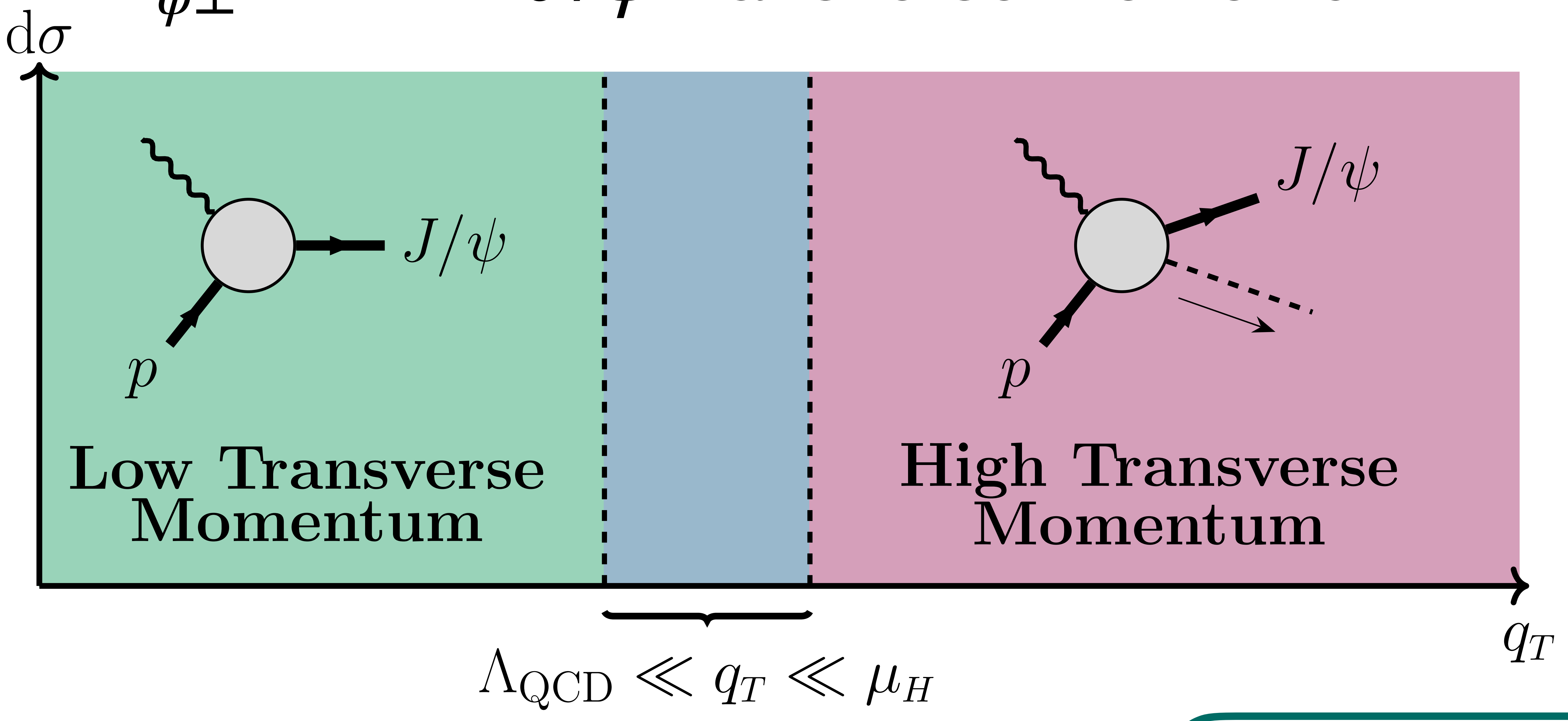
Determine the TMD shape function

Boer, Bor, **LM**, Pisano, Yuan, *JHEP 08* (2023)

q_T → photon transverse momentum

$P_{\psi\perp}$ → J/ψ transverse momentum

$$q_T = \frac{P_{\psi\perp}}{\hat{z}}$$



TMD factorization $q_T \ll \mu$

↓

Overlapping at $\Lambda_{\text{QCD}} \ll q_T \ll \mu$

↑

Collinear factorization $q_T \gg \Lambda_{\text{QCD}}$

If the two factorization describe the same dynamics
Then they should match!

TMD shape function perturbative tail

In SIDIS ($e\,p \to J/\psi + X$) the cross section at small q_T is parameterized as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}\mathbf{P}^2_{\psi\perp}\,\mathrm{d}\phi_\psi} \propto \left\{ \left[1+(1-y)^2\right] \mathcal{F}_{UUT} + 4(1-y)\,\mathcal{F}_{UUL} + 4(1-y)\,\cos 2\phi_\psi\,\mathcal{F}_{UU}^{\cos 2\phi_\psi} \right\}$$

unpol. gluon TMD *lin. pol. gluons TMD*

Comparing TMD and collinear framework at $\Lambda_{\mathrm{QCD}} \ll q_T \ll \mu_H$

$$\mathcal{F}_{UU}^{\cos 2\phi}\big|_{\mathrm{TMD}} = F_{UU}^{\cos 2\phi}\big|_{\mathrm{coll}} \;\Rightarrow\; \Delta_{h,\psi}^{[n]} = \delta^{(2)}(k_T^2)\,\langle \mathcal{O}_\psi[n]\rangle\,\delta(1-z)$$

[Boer, Bor, **LM**, Pisano, Yuan, *JHEP 08* (2023)](#)

$$\left.\begin{aligned} \mathcal{F}_{UUT}\big|_{\mathrm{TMD}} &\neq F_{UUT}\big|_{\mathrm{coll}} \\ \mathcal{F}_{UUL}\big|_{\mathrm{TMD}} &\neq F_{UUL}\big|_{\mathrm{coll}} \end{aligned}\right\} \;\Rightarrow\; \Delta_\psi^{[n]} = -\frac{\alpha_s}{2\pi^2 k_T^2} C_A \left(1 + \log\frac{M_\psi^2}{M_\psi^2 + Q^2}\right) \langle \mathcal{O}_\psi[n]\rangle\,\delta(1-z)$$

Interpretation of the result

$$\Delta_{\psi}^{[n]} = -\frac{\alpha_s}{2\pi^2 k_T^2} C_A \left(1 + \log \frac{M_\psi^2}{M_\psi^2 + Q^2} \right) \langle \mathcal{O}_\psi[n] \rangle \, \delta(1-z)$$

Boer, Bor, **LM**, Pisano, Yuan, *JHEP 08* (2023)

- The presence of a mass suppresses the $\log q_T/Q$ term

Sun, et al., *Phys.Rev.D 88* (2013)

- The constant "1" was also find in other works $(pp \to J/\psi + X)$

→ Hint of the TMD shape function universality

Echevarría, et al., *2510.11809* (2026)

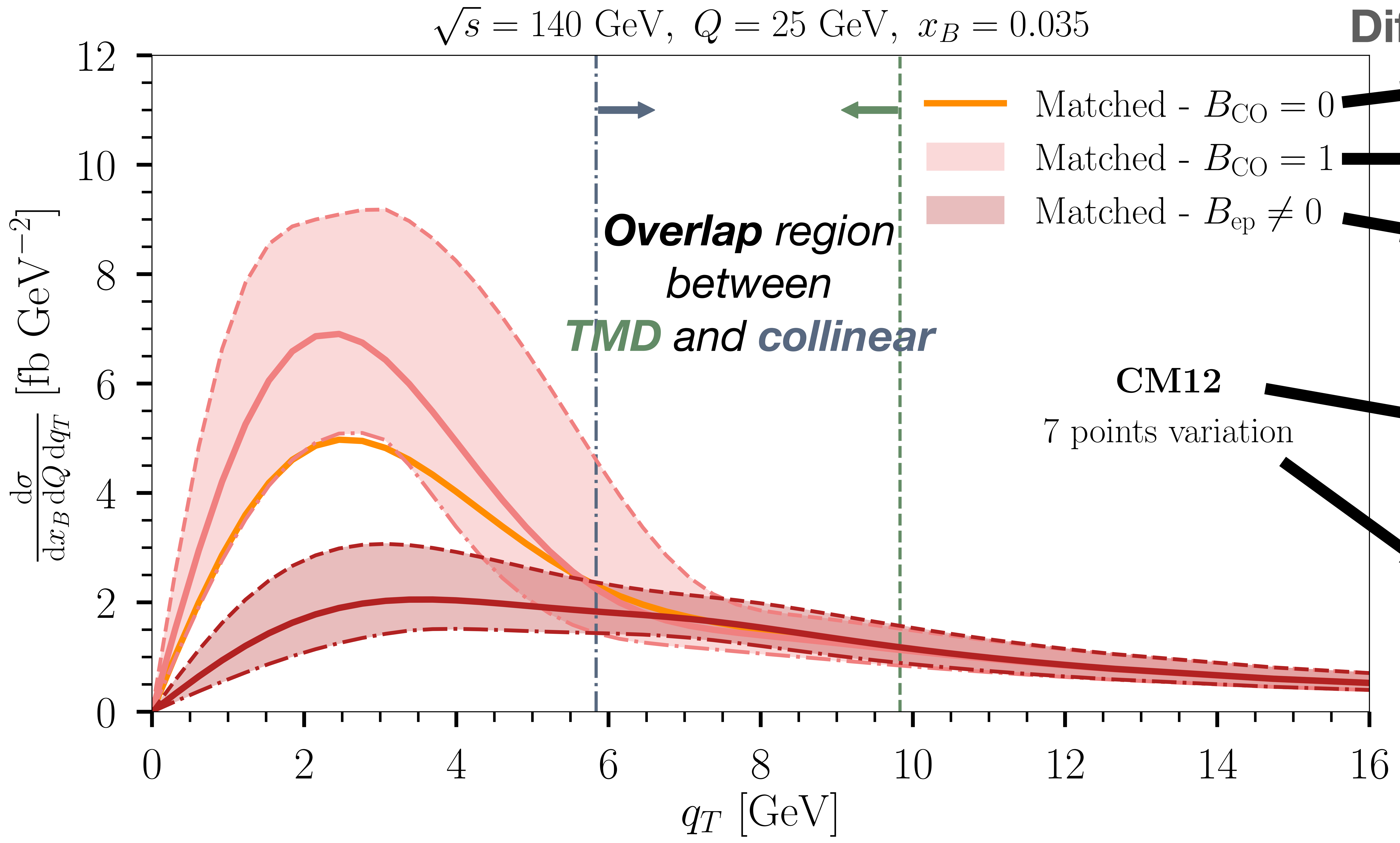
- The log of hard scale corresponds to the rapidity scale regulator

- The dominant TMD contribution is still constrained in the $z \to 1$ region

Application to the differential cross section

[LM, Boer, Bor, *Phys.Rev.D 112* (2025)](#)

This curve is obtained by matching TMD and collinear curves



Different TMDShF assumptions

- No TMD shape function
- $\Delta \propto 1$ **enhancement**
- $\Delta \propto 1 + \log\left[M_\psi^2/\left(M_\psi^2 + Q^2\right)\right]$ **suppression**

CM12 → LDME set from [Chao, et al., *Phys.Rev.Lett. 108* (2012)](#)

7 points variation → Number of combinations of scale variation taken

Other observables to probe gluon TMDs

The $z \to 1$ constraint would force TMD studies in a complicated region

→ large diffractive signal at $z \sim 1$ may wash-out TMD contributions

Subleading q_T/M_ψ terms might be crucial to use SIDIS data to fit gluon TMDs effectively

Other processes can be used that do not present the same z constraint

Examples (not fully comprehensive):

$ep^{(\uparrow)} \to J/\psi + jet + X$

- X D'Alesio, et al., *PRD 100* (2019)
- Y **LM**, Yuan, *PRD 110* (2024)

$pp^{(\uparrow)} \to \mathcal{Q} + X$

- ?? Sun, et al., *Phys.Rev.D 88* (2013)
- X **LM**, et al., *PRD 102* (2020)
- X Nanako, **LM**, Pisano, *PRD 110* (2024)
- ?? Saleev, Shilyaev, *Mod.Phys.Lett.A* (2025)

$pp^{(\uparrow)} \to \mathcal{Q} + j\,(\gamma) + X$

- X den Dunnen, et al., *PRL 112* (2014)
- Y Boer, Nanako, **LM**, Pisano, *in progress*

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Associated quarkonium production

Description of quarkonium + jet production at the LHC in the TMD framework

Boer, Nanako, **LM**, Pisano, *in progress*

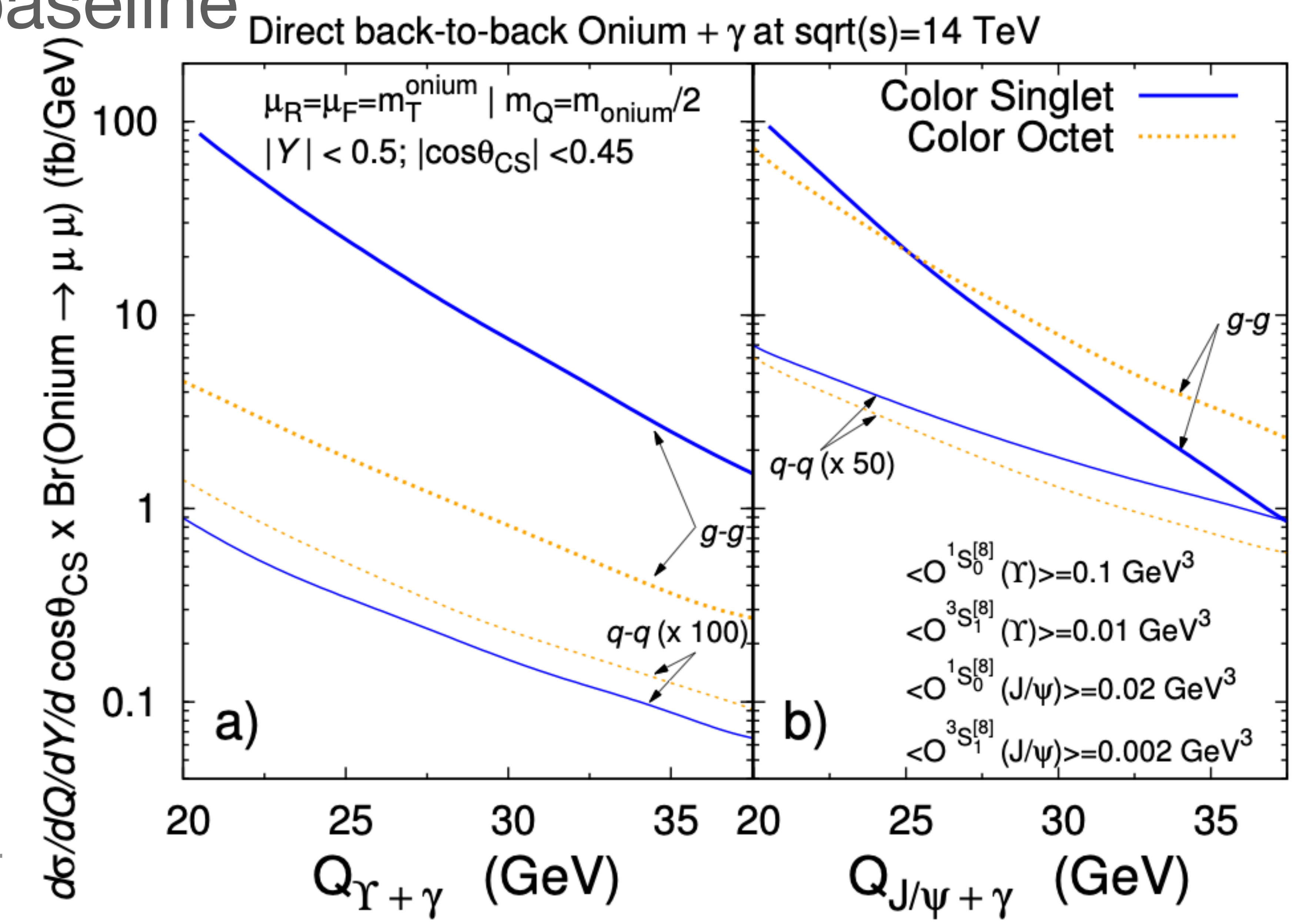
$$\mathrm{d}\sigma_{UU} \sim F_0 + F_2 \mathscr{C}_{fh} \cos(2\phi) + F_4 \mathscr{C}_{hh} \cos(4\phi)$$

$\phi \equiv \phi_\psi - \phi_q$

$$\mathrm{d}\sigma_{UT} \sim F_0 \sin(\phi_S) + F_2 \left[\mathscr{C}_{fh} \sin(2\phi + \phi_S) + \mathscr{C}_{fh_T} \sin(2\phi - \phi_S)\right] + F_4 \left[\mathscr{C}_{hh} \sin(4\phi - \phi_S) + \mathscr{C}_{hh_T} \sin(4\phi + \phi_S)\right]$$

It is important to understand the relevance of the CO mechanism vs CS

→ we use $\mathcal{Q} + \gamma$ as our baseline



den Dunnen, et al., *Phys.Rev.Lett. 112* (2014)

to be revised
(and corrected ?)

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Description of quarkonium + jet production at the LHC in the TMD framework

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It is important to understand the relevance of the CO mechanism vs CS

→ we use $\mathcal{Q} + \gamma$ as our baseline

Summary

- **Quarkonia to investigate perturbative physics**

  Used to explore gluonic content of protons

- **Understanding of emergent properties of protons via their multidimensional distributions**

  Gluon TMDs are still in the shadow nowadays

- **TMD factorization implies an extension of hadronization models**

  TMD shape function

- **New ideas to them to use quarkonium observables**

  To be applied at both EIC (electron-ion collider) and (future) LHC

Investigating Quarkonium physics (at small q_T)

Back-up slides

TMD and collinear cross sections

$q_T \ll \mu_H$

Bacchetta, Boer, Pisano, Taels, *EPJC 80* (2020)

$$\mathrm{d}\sigma|_{\mathrm{TMD}} \propto \left[1+(1-y)^2\right] \mathcal{F}_{UUT} + 4(1-y)\,\mathcal{F}_{UUL} + (1-y)\cos 2\phi\, \mathcal{F}_{UU}^{\cos 2\phi})$$

Involves the convolutions: $\mathcal{C}\left[f_1^g \Delta^{[n]}\right](x, q_T)$ $\quad\mathcal{C}\left[w\, h_1^{\perp g} \Delta_h^{[n]}\right](x, q_T)$

$q_T \gg \Lambda_{\mathrm{QCD}}$

Lepton tensor from

Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, *JHEP 02* (2007)

$$\mathrm{d}\hat{\sigma}^{a\,[n]}|_{\mathrm{coll}} \propto \int \frac{\mathrm{d}\hat{x}}{\hat{x}} \frac{\mathrm{d}\hat{z}}{\hat{z}} \frac{L^{\mu\nu}}{Q^4} H_\mu^{a\,[n]} H_\nu^{*\,a\,[n]} f_a(x_B/\hat{x})\ \delta(\hat{x}', \hat{z}) \rightarrow \hat{x}' = \frac{x_B}{\hat{x}} \frac{M_\psi^2 + Q^2}{Q^2}$$

small q_T limit obtained by expanding the delta: Boer, D'Alesio, Murgia, Pisano, Taels, *JHEP 09* (2020)

$$\delta(\hat{x}', \hat{z}) \sim \frac{M_\psi^2 + Q^2}{M_\psi^2/\hat{z} + Q^2} \frac{\hat{z}}{(1-\hat{z})_+} \delta(1-\hat{x}') + \log \frac{M_\psi^2 + Q^2}{\boldsymbol{q}_T^2} \delta(1-\hat{x}')\,\delta(1-\hat{z}) + \frac{\hat{x}'}{(1-\hat{x})_+} \delta(1-\hat{z})$$

subleading

only for continuous functions

Are cross sections continuous?

Boer, Bor, **LM**, Pisano, Yuan, *JHEP 08* (2023)

Delta expansion relies on the hard amplitude to be continuous, but…

Delta expansion applicable ✓

Delta expansion not applicable ✗

$$F_{UU}(\hat{x}', \hat{z}) = F^{(0)}_{UU}(\hat{x}', \hat{z}) + \sum_{k=1} \left(\frac{1-\hat{z}}{1-\hat{x}'}\right)^k F^{(k)}_{UU}(\hat{x}', \hat{z})$$

Continuous functions of $\hat{x}'$ and $\hat{z}$

Poles up to 2^{nd} order are found in $F^{(k)}_{UUT}$ and $F^{(k)}_{UUL}$ for $\gamma^* g$ processes

$$\delta(\hat{x}', \hat{z}) \sim \frac{\hat{x}'}{(1-\hat{x})_+}\delta(1-\hat{z}) + \log\frac{M_\psi^2 + Q^2}{q_T^2}\delta(1-\hat{x}')\,\delta(1-\hat{z})$$

An effective substitution takes place

$$\log\frac{M_\psi^2 + Q^2}{q_T^2} \to \frac{1}{2}\left(\log\frac{M_\psi^2 + Q^2}{q_T^2} - 1 - \log\frac{M_\psi^2}{M_\psi^2 + Q^2}\right)$$

TMDShF tail from eikonal gluons

$$S_g(v_1, v_2) = \frac{v_1 \cdot v_2}{(v_1 \cdot p_g)(v_2 \cdot p_g)}$$

$$\mathrm{d}\sigma_1 \propto \int_{\frac{-p_{g\perp}^2}{M_\psi^2+Q^2}}^{1} \frac{\mathrm{d}x_g}{x_g} \left[2\, S_g(p_a, P_\psi) + S_g(P_\psi, P_\psi) \right] \propto \frac{1}{2} \left(\log \frac{M_\psi^2 + Q^2}{q_T^2} - \log \frac{M_\psi^2}{M_\psi^2 + Q^2} - 1 \right)$$

TMD-PDF large logs

Something else

Relation to quark-pair Fragmentation Function?

Kang, Ma, Qiu, Sterman, *PRD 90* (2014) & *PRD 91* (2015)

Ma, Qiu, Sterman, Zhang, *PRL 113* (2014)

$$\mathrm{d}\sigma[\mathcal{Q}] = \int \mathrm{d}\xi_i\, \mathrm{d}\xi_j\, \mathrm{d}z\ f_i\, f_j\, \mathrm{d}\hat{\sigma}_{i+j\to f+X}\, D_{f\to\mathcal{Q}}(z) + \int \mathrm{d}\xi_i\, \mathrm{d}\xi_j\, \mathrm{d}z\ f_i\, f_j\, \mathrm{d}\hat{\sigma}_{i+j\to QQ+X}\, \boldsymbol{D_{QQ\to\mathcal{Q}}}(z)$$

The W term and resummation of logs

A few details about the resummation in b_T space

expansion at $b_T^2 \ll \Lambda_{\mathrm{QCD}}^{-2}$

$$W^{g,[n]}(x,z,b_T;\mu_b,\mu) = \sum_{a'} C_{ga'} \otimes f_1^{a'}(x;\mu_b) \sum_{n'} C_{nn'} \otimes \langle \mathcal{O}_\psi[n'] \rangle(z;\mu_b)\, e^{-S_{\mathrm{pert}}(\mu_b,\,\mu)}$$

- $\langle \mathcal{O}_\psi[n'] \rangle(\mu_b) \approx \langle \mathcal{O}_\psi[n'] \rangle(\mu_\psi)$ for the dominant LDME considered here
- $S_{\mathrm{pert}}(\mu_b, \mu)$ evaluated at **NLL** accuracy

In line with Ayabat, Rogers, *PRD 83 (2011)*

For all b_T : $b_T \to b_*$ and $\mu_b \to \mu'_{b_*}$

$S_{\mathrm{NP}}(b_T)$

$$W^{g,[n]}(b_T,\mu_b) = W^{g,[n]}(b_*,\mu'_{b_*}) \exp\left[-\left(A_{\mathrm{NP}} \log\frac{\mu}{\mu_{\mathrm{NP}}} + B_{\mathrm{NP}}\right) + g_\psi\right]$$

Sudakov at NLL

$$S_{\rm pert}^{\rm NLL}(C_1\,\mu_b^2,\ C_2\,\mu^2) =$$

$$-\frac{4}{\beta_0}\left[A^{(1)}\log\left(\frac{\mu^2}{\mu_b^2}\right)-\left(B^{(1)}+A^{(1)}\mathrm{L}_\mu\right)\log\left(\frac{\mathrm{L}_\mu}{\mathrm{L}_{\mu_b}}\right)\right]-\frac{16}{\beta_0^2}A^{(2)}\left[\log\left(\frac{\mathrm{L}_\mu}{\mathrm{L}_{\mu_b}}\right)-\frac{1}{\mathrm{L}_{\mu_b}}\log\left(\frac{\mu^2}{\mu_b^2}\right)\right]$$

$$\mathrm{L}_\eta = \log\frac{\eta^2}{\Lambda_{\rm QCD}^2} \qquad\qquad \beta_0 = 11 - 2n_f/3 = 23/3$$

$$A^{(1)} = A_g^{(1)} = \frac{C_A}{2} \qquad\qquad A^{(2)} = A_g^{(2)} = \frac{C_A}{4}\left[C_A\left(\frac{67}{18}-\frac{\pi^2}{6}\right)-\frac{5}{9}n_f+\beta_0\log C_1\right]$$

$$B^{(1)} = B_g^{(1)} + B_{\rm CO}^{(1)} = -\frac{C_A}{2}\left[\frac{\beta_0}{6}+1+\log\frac{M_\psi^2}{M_\psi^2+Q^2}+2\,\log\frac{C_2}{C_1}\right]$$

Matching TMD and fixed-order

Inverse-error weighting method first introduced in: Echevarría, et al., *Phys,Lett.B 781* (2018)

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{dPS}}\right|_{q_T \ll \mu} = W + \mathcal{O}\left(\frac{q_T^2}{\mu^2}\right) + \mathcal{O}\left(\frac{m^2}{\mu^2}\right) \qquad \left.\frac{\mathrm{d}\sigma}{\mathrm{dPS}}\right|_{q_T \gg \Lambda_{\mathrm{QCD}}} = \mathrm{FO} + \mathcal{O}\left(\frac{m^2}{q_T^2}\right)$$

$$\Delta W = \left(\frac{q_T^2}{\mu^2} + \frac{m^2}{\mu^2}\right)\frac{\mathrm{d}\sigma}{\mathrm{dPS}} \qquad \Delta\mathrm{FO} = \frac{1}{2}\frac{m^2}{q_T^2}\left(2 + \log\frac{M_\psi^2 + Q^2}{M_\psi^2} + \log\frac{\mu^2 + q_T^2}{q_T^2}\right)\frac{\mathrm{d}\sigma}{\mathrm{dPS}}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{dPS}} \approx \omega_1\, W + \omega_2\, \mathrm{FO}$$

$$\omega_1(q_T) = \frac{\Delta W^{-2}(q_T)}{\Delta W^{-2}(q_T) + \Delta \mathrm{FO}^{-2}(q_T)} \qquad \omega_2(q_T) = \frac{\Delta \mathrm{FO}^{-2}(q_T)}{\Delta W^{-2}(q_T) + \Delta \mathrm{FO}^{-2}(q_T)}$$

$$\Delta\sigma = \frac{\Delta W\, \Delta \mathrm{FO}}{\sqrt{\Delta W^2 + \Delta \mathrm{FO}^2}}$$

Inverse-error weighting applied

At **intermediate q_T** the most reliable expression combines **TMD** and **collinear** **factorizations** (avoiding double counting)

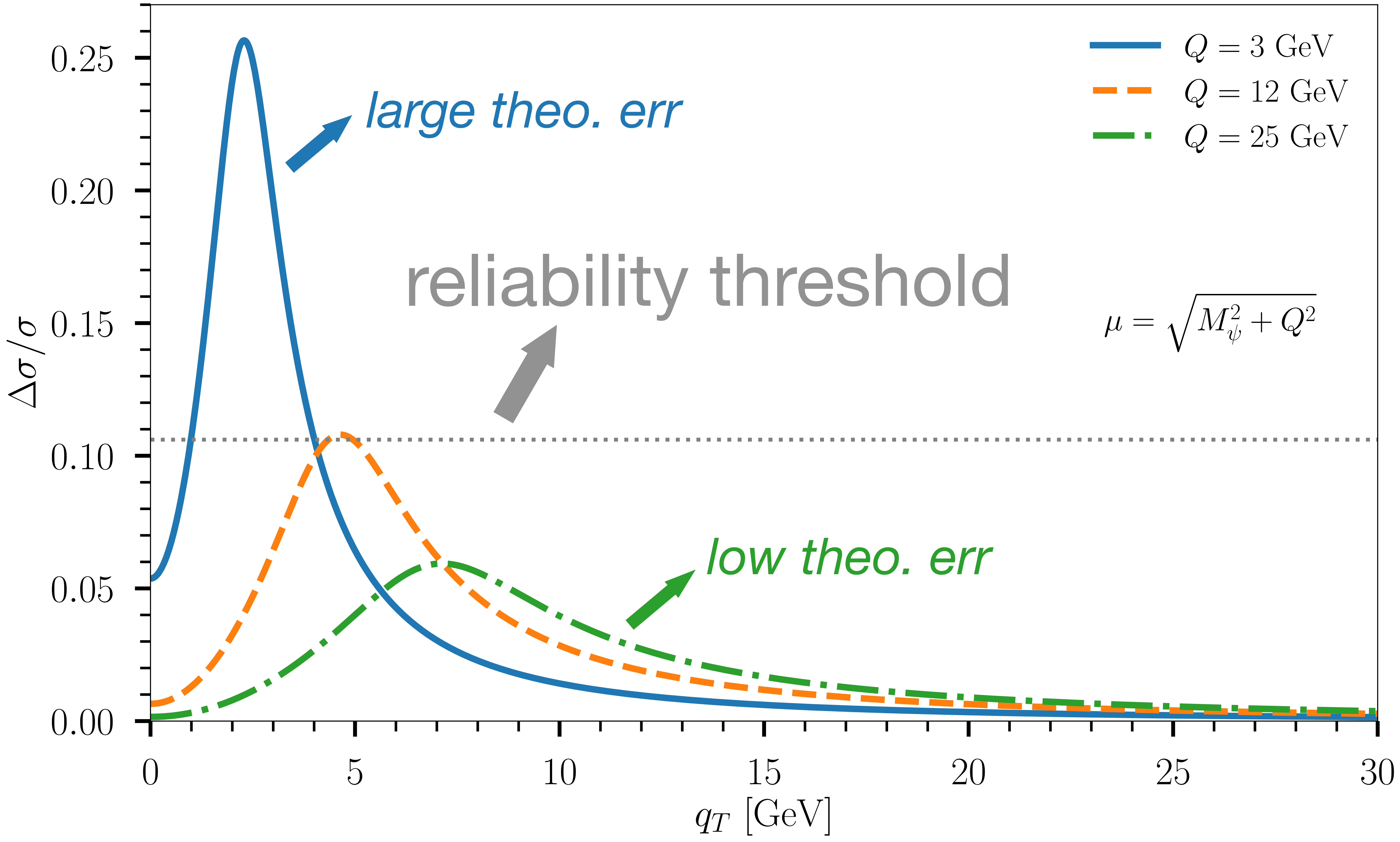
Echevarría, et al., *Phys.Lett.B 781* (2018)

The **Inverse error weighting** utilises the theory error of each factorization

$$\frac{\mathrm{d}\sigma}{\mathrm{dPS}} \approx \omega_1\, W \;+\; \omega_2\, \mathrm{FO}$$

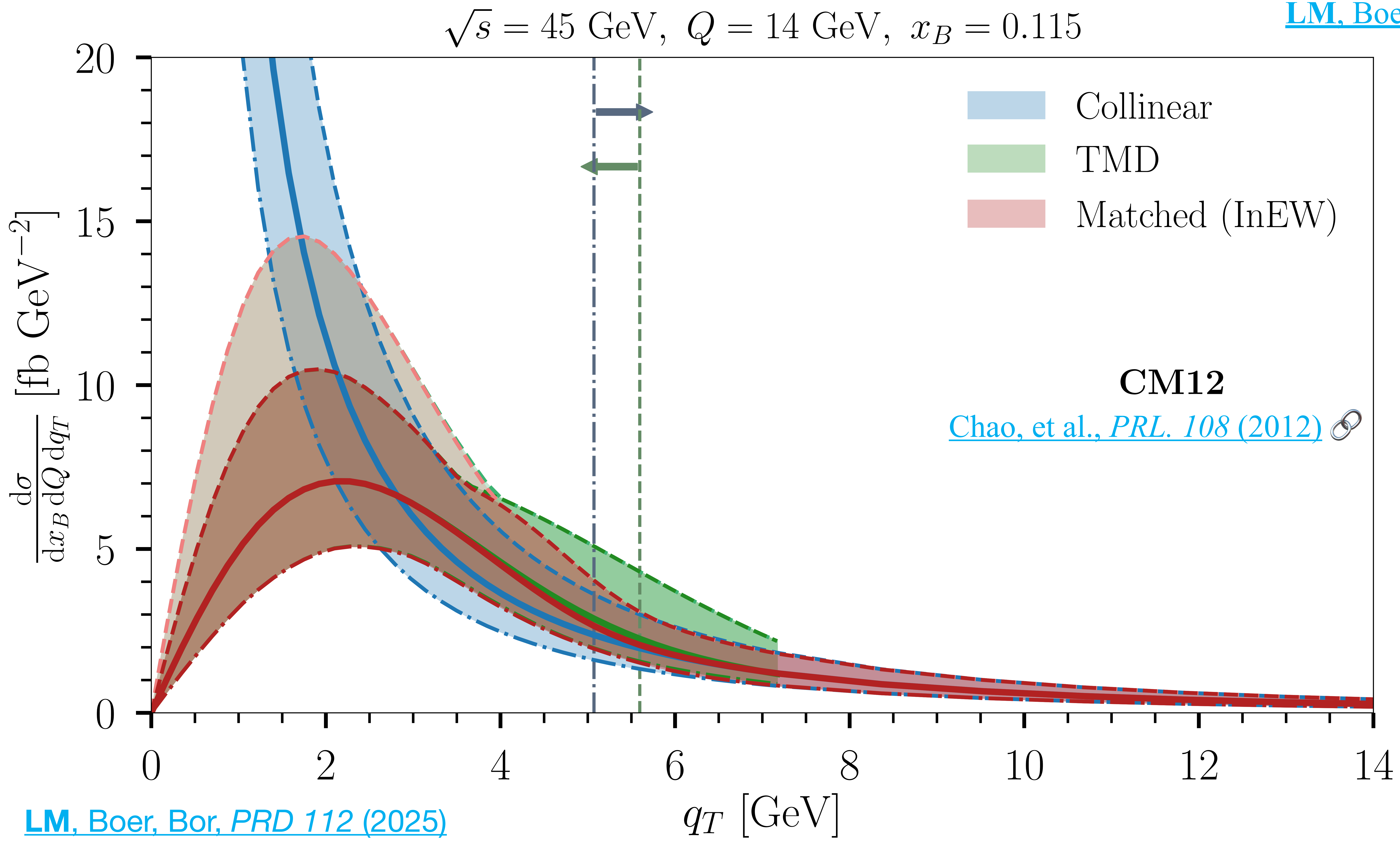
$$\omega_1(q_T) = \frac{\Delta W^{-2}(q_T)}{\Delta W^{-2}(q_T) + \Delta \mathrm{FO}^{-2}(\mathrm{q_T})}$$

$$\omega_2(q_T) = \frac{\Delta \mathrm{FO}^{-2}(q_T)}{\Delta W^{-2}(q_T) + \Delta \mathrm{FO}^{-2}(q_T)}$$



Isotropic distribution ($B_{ep} \neq 0$ - $\sqrt{s}$ = 45 GeV - Q = 14 GeV)

LM, Boer, Bor, *Phys.Rev.D 112* (2025)



Other fix variables:

$y \approx 0.85$

$W^2 \approx 1500$ GeV^2

LM, Boer, Bor, *PRD 112* (2025)

Non-relativistic QCD approach

It makes use of the relative velocity (v) expansion

$$d\sigma_{\mathcal{Q}}(P_T) \approx \sum_n d\hat{\sigma}_{Q\overline{Q}[n]}(P_T)\ \langle \mathcal{O}_{\mathcal{Q}}[n] \rangle$$

Bodwin, Braaten, Lepage, *PRD 51* (1997)

Quarkonia are produced through a general colour/quantum number configuration n

➡ The transition $n \to \mathcal{Q}$ goes via NRQCD operators

The hard amplitude (and matching coefficient) follows standard perturbation theory

Operator contributions are ordered in powers of v

➡ Scaling rule:

	${}^1S_0^1$	${}^3S_1^1$	${}^1S_0^8$	${}^3S_1^8$	${}^1P_1^1$	${}^3P_0^1$	${}^3P_1^1$	${}^3P_2^1$	${}^1P_1^8$	${}^3P_0^8$	${}^3P_1^8$	${}^3P_2^8$
η_c	1		v^4	v^3					v^4			
J/ψ		1	v^3	v^4						v^4	v^4	v^4
h_c			v^2		v^2							
χ_{c0}				v^2		v^2						
χ_{c1}				v^2			v^2					
χ_{c2}				v^2				v^2				

Schuler *9702230* (1997)

NRQCD symmetries (e.g. between J/ψ and η_c)   LDME mixing

NRQCD: advantages and issues

Generalisation of the CS mechanism with additional contributions with different P_T shapes

Solution of the P-wave infrared safety: $\langle \mathcal{O}[P] \rangle^{\text{ren.}} \sim \langle \mathcal{O}[P] \rangle^{\text{bare}} + Z \langle \mathcal{O}[S] \rangle^{\text{bare}}$

$$|\mathcal{M}[P^{(1)}]|^2 \sim \frac{|\mathcal{M}[S^{(8)}]|^2}{(\hat{s} - M^2)^4}$$

LDME extractions depends tremendously on the dataset chosen

LDME SET	$\langle \mathcal{O}_\psi[^3S_1^{(1)}] \rangle$ GeV^3	$\langle \mathcal{O}_\psi[^1S_0^{(8)}] \rangle$ 10^{-2} GeV^3	$\langle \mathcal{O}_\psi[^3S_1^{(8)}] \rangle$ 10^{-2} GeV^3	$\langle \mathcal{O}_\psi[^3P_0^{(8)}] \rangle / m_c^2$ 10^{-2} GeV^3
Butenschoen and Kniehl, *PRL 106* (2011)	1.32	4.50 (72)	0.312 (93)	−0.54 (16)
Chao et al., *PRL 108* (2012)	1.16	8.90 (98)	0.30 (12)	0.56 (21)
Sharma and Vitev, *PRC 87* (2013)	1.2	1.80 (87)	0.13 (13)	1.80 (87)
Bodwin et al., *PRL 113* (2014)		9.9 (2.2)	1.1 (1.0)	0.49 (44)
Brambilla et al., *PRD 105* (2022)	0.576	−4.8 (1.5)	1.70 (18)	3.00 (34)

The NRQFT projector

Bodwin, Braaten, Lepage, *PRD 51* (1994)

The amplitude includes a bound-state of two elementary particles in the final state

Projection into **colour**, **spin** and **orbital angular momentum**

$$\mathrm{d}\hat{\sigma}(\{\mathcal{Q}[n]\}_i) \sim \left| \Sigma_{J_z,L_z,S_z,\lambda_Q,\lambda_{\bar{Q}'}} \langle L, L_z; S, S_z \,|\, J, J_z \rangle \, \mathbb{P}_C \left[\mathbb{P}_L \, \mathbb{P}_S \, \mathcal{A}(\{Q\bar{Q}'\}_i) \right]_{q=0} \right|^2$$

For each onia i in the final state:

$$\mathbb{P}_{C=1} = \frac{\delta_{i\bar{i}}}{\sqrt{N_c}}$$

(only for QCD states)

$$\mathbb{P}_{C=8} = \sqrt{2} T^{c_i}_{i\bar{i}}$$

$$\mathbb{P}_{S=0} = \frac{\bar{v}_{\lambda_{\bar{Q}'}}(k_{\bar{i}}) \, \gamma_5 \, u_{\lambda_{Q'}}(k_i)}{2(2m_Q m_{\bar{Q}'})^{1/2}}$$

$$\mathbb{P}_{S=1} = \frac{\bar{v}_{\lambda_{\bar{Q}'}}(k_{\bar{i}}) \, \not{\varepsilon}^*_{S_z} \, u_{\lambda_{Q'}}(k_i)}{2(2m_Q m_{\bar{Q}'})^{1/2}}$$

Colpani-Serri et al., *JHEP 02* (2026)

$$\mathbb{P}_L = \left(\varepsilon^{*\,\alpha}_{L_z} \frac{\mathrm{d}}{\mathrm{d}q^\alpha} \right)^L$$

LM, Shao, Simon, *work in progress*

P-wave: dual variables - examples

Dual numbers $x = x_0 + \epsilon\, x_1$ & $y = y_0 + \epsilon\, y_1$

Sum

$$x + y = x_0 + y_0 + \epsilon\left(x_1 + y_1\right)$$

```
function add_DualVariable(a, b) result(res)
   type(Dual), intent(in) :: a, b
   type(Dual) :: res
   integer::i
   do i=0,2**npwave-1
   res%comp(i) = a%comp(i) + b%comp(i)
   enddo
end function add_DualVariable
```

Multiplication

$$x \cdot y = x_0 \cdot y_0 + \epsilon\left(x_1 \cdot y_0 + x_0 \cdot y_1\right)$$

```
function multiply_DualVariable(a, b) result(res)
   type(Dual),intent(in)::a,b
   type(Dual)::res
   integer::i,j

   do i=0,2**npwave-1
      j=i
      do
         res%comp(i) = res%comp(i) + a%comp(j)*b%comp(i - j)
      if (j.eq.0) exit
      j = iand(j - 1, i)
      enddo
   enddo
end function multiply_DualVariable
```

Exponential

$$x^n = x_0^n + \epsilon\left(n\, x_0^{n-1} x_1\right)$$

```
interface operator(**)
   module procedure power_DualVariable_int
   module procedure power_DualVariable_real
end interface operator(**)
```

P-wave: dual variables - examples II

Dual numbers for n derivatives $x = x_0 + \epsilon_1 x_1 + \epsilon_2 x_2 + \ldots + \epsilon_n x_n$

The variable x can be stored in an array of size 2^n and dimensions $(0 : 2^n - 1)$

The i^{th} derivative corresponds to the position 2^{i-1}:

$$x = x_0 + \epsilon_1 x_1 + \epsilon_2 x_2 + \epsilon_3 x_3 \Rightarrow x = (x_0,\, x_1,\, x_2,\, 0,\, x_3,\, 0,\, 0,\, 0)$$

→ In the example the 0s are the mixed derivative ($\epsilon_i \epsilon_j$ with $i \neq j$)

In our case the propagation of the derivative is initiated by the momenta

$$k_{Q_i} = \frac{P_{\mathcal{Q}_i}}{2} + q_i \qquad k_{\overline{Q}'_i} = \frac{P_{\mathcal{Q}_i}}{2} - q_i \quad \longrightarrow \quad k^\mu_{Q_i} = \frac{P^\mu_{\mathcal{Q}_i}}{2} + \epsilon_i \varepsilon^{*\,\mu}_{iL_z} \qquad k^\mu_{\overline{Q}'_i} = \frac{P^\mu_{\mathcal{Q}_i}}{2} - \epsilon_i \varepsilon^{*\,\mu}_{iL_z}$$