# The two-photon decay amplitude of $K_{\mathrm{L}} \to \mu^+\mu^-$ from Lattice QCD


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On behalf of the RBC/UKQCD collaboration


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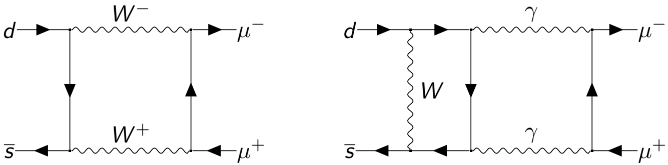
# Introduction

## $K_L \to \mu^+\mu^-$

- One-loop in the Standard Model, sensitive to UV physics.
- Precisely measured, good test for the SM [BNL E871 Collab., PRL '00]

| | B.R. $\times 10^9$ |
|---|---|
| Exp. | 6.84(11) |
| SD | 0.79(12) |
| Abs. | 6.59(5) |

- Short-distance (SD) well-known from perturbation theory [Gorbahn & Haisch '06]
- Current theory limitation: long-distance (LD) $2\gamma$ contribution
- Optical theorem: absorptive (imaginary) part inferred from $K_L \to \gamma\gamma$ [Ceccucci '17]
- Dispersive (real) part: size and interference with the SD? (target: $\sim 10\%$).



# Formalism

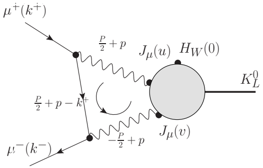
- **Lattice QCD**: Euclidean formulation of QCD on finite lattice spacing $a$ and volume $\Rightarrow$ suitable for Monte Carlo-type simulations.
- O($\alpha^2$)-leptonic kernel $\bar{L}_{\mu\nu}$ + O($G_\mathrm{F}$)-weak Hamiltonian $\mathcal{H}_\mathrm{W}$

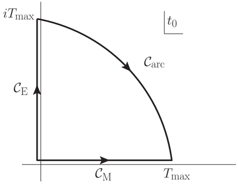
$$\mathcal{A}_{K_\mathrm{L}\mu\mu} = \int d^4u\, d^4v\, \bar{L}_{\mu\nu}(u,v)\, \langle 0|\mathrm{T}\{J_\mu(u)J_\nu(v)\mathcal{H}_\mathrm{W}(0)\}|K_\mathrm{L}\rangle\ .$$

- Wick-rotate the integrand $u^0 \to -iu^0 \Rightarrow$ the hadronic matrix element evaluated in Euclidean space. ($\mathcal{A}_{K_\mathrm{L}\mu\mu} \to \mathcal{A}^\mathrm{lat}$)
- **However** $\mathcal{A}$ is ill-defined under the above due to the presence of intermediate states with $E_n < M_K \Rightarrow$ subtraction needed

$$\mathcal{A}_{K_\mathrm{L}\mu\mu} = \mathcal{A}^\mathrm{lat} - \mathcal{A}^\mathrm{sub}\,.$$

- Evaluate both $\mathcal{A}^\mathrm{lat}$ and $\mathcal{A}^\mathrm{sub}$ on the lattice.



# Numerical implementation

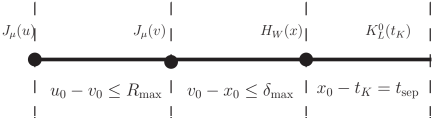
| Parameter | Value |
|---|---|
| $L^3 \times T \times L_s$ | $24^3 \times 64 \times 24$ |
| $m_\pi$ [MeV] | 142 |
| $M_K$ [Mev] | 515 |
| $a^{-1}$ [GeV] | 1.023 |

- Lattice setup: Möbius Domain Wall fermion ensemble 24ID from the RBC/UKQCD collaboration.
- $\Delta S = \pm 1$ operators considered in this work ($N_\mathrm{f} = 3$)

$$\mathcal{H}_\mathrm{W}(x) = \frac{G_\mathrm{F}}{\sqrt{2}} V_{us}^* V_{ud} (C_1 Q_1 + C_2 Q_2)\,,$$

$$Q_1 \equiv (\bar{s}_a \Gamma_\mu^L d_a)(\bar{u}_b \Gamma_\mu^L u_b) + (s \leftrightarrow d)\,, \quad Q_2 \equiv (\bar{s}_a \Gamma_\mu^L d_b)(\bar{u}_b \Gamma_\mu^L u_a) + (s \leftrightarrow d)\,.$$

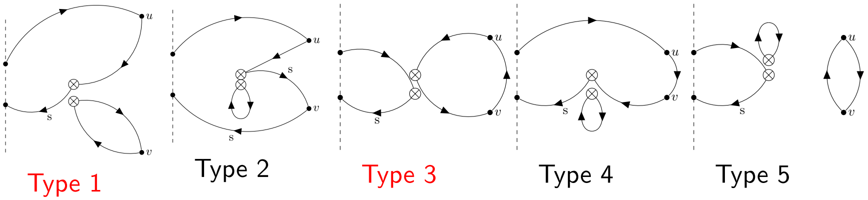
- States lighter than the kaon:
  - $\pi^0$: direct subtraction, calculate $\mathcal{A}_{\pi^0}^{\mathrm{sub}}(\delta_{\mathrm{max}})$.
  - $\pi\pi\gamma$: no exp. term expected for 24ID due to the IR cut-off, controlled by imposing $|u_0 - v_0| \leq R_\mathrm{max}$ (est. $\lesssim 10\%$ systematic effects [Chao et al, PRD '24]).
  - $3\pi$: phase-space suppressed, subdominant.



# Numerical implementation

## Wick contractions

- All non-SU$(3)_\mathrm{f}$-suppressed Wick-contractions for $\mathcal{A}^{\mathrm{lat}}(N_\mathrm{f})$
  Dashed line: $K_\mathrm{L}(t_K)$, crosses: $\mathcal{H}_\mathrm{W}(x)$, solid dots: $J_\mu(u)$ and $J_\nu(v)$

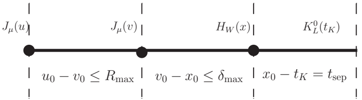



- Well-established numerical strategies with state-of-the-art noise-reduction techniques.

- **BUT** not a complete calculation. Counterterms needed for renormalization!

# Results

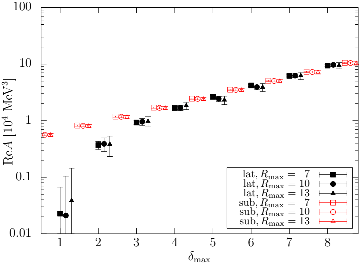
## Reconstruction of physical contributions



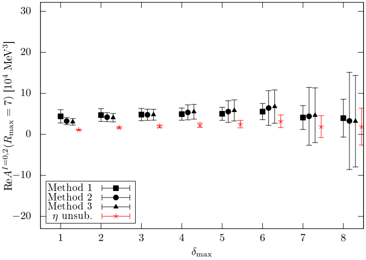

$$\mathcal{A}_n^{\rm sub} \propto e^{(M_K - M_n)\delta_{\max}}/(M_K - M_n)$$

- Successful removal of the unphysical, exponentially growing $\pi^0$.
- Slow convergence of $K_{\rm L} \to \eta \to \gamma\gamma$

  $\Rightarrow$ 3 methods to reconstruct the physical contribution.
- $\eta$: Dominant source of statistical noise.




$\mathcal{A}^{\rm lat}$ vs. $\mathcal{A}^{\rm sub}_{\pi^0}$.

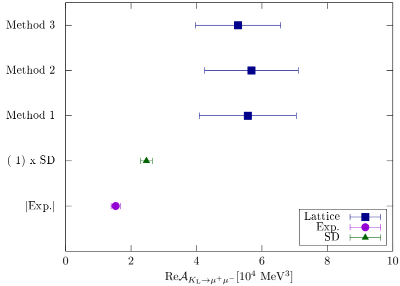


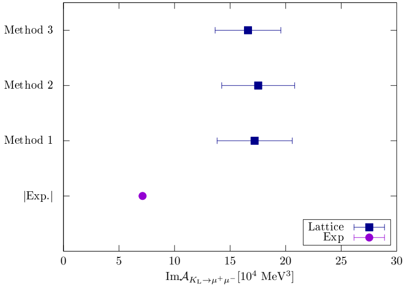

Before ('unsub.') vs. after the $\eta$-reconstruction.

# Results

## Combined results

- LD2$\gamma$ contribution from the lattice, $\langle \mu^+\mu^- | \mathcal{H}_{\mathrm{W}}(0) | K_{\mathrm{L}} \rangle = \frac{G_{\mathrm{F}} e^4}{\sqrt{2}} |V_{us}||V_{ud}| \mathcal{A}$.




- Consistent results from three different treatments of the $\eta$
- The numbers obtained are in the right ballpark with reasonably-sized errors.
- **NOT** final as extra counter terms are needed for renormalization!

# Conclusions and outlook

### Conclusions

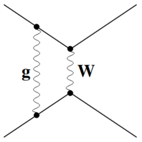
- Developed a lattice-QCD formalism for the complex $2\gamma$, $K_{\mathrm{L}} \to \mu^+\mu^-$ decay amplitude, expected to provide up to 10% precision.
- Numerical strategies for the most computationally-demanding have been developed and successfully applied to a $24^3 \times 64$ lattice at $1/a = 1.023$ GeV.
- Counter terms still needed to complete the current calculation.

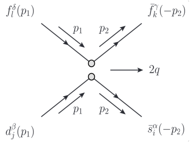
### Outlook

- Theoretical development and implementation of the counter terms underway.
- Running on $48^3 \times 96$ ($1/a = 1.73$ GeV) and $64^3 \times 128$ ($1/a = 2.38$ GeV) physical pion mass ensembles starting soon.

# Back-up slides

# Flavor physics on the lattice




- Effective weak Hamiltonian: four-quark operators after integrating out the heavy degrees of freedom.
- Regularization-independent (RI) scheme: correspond the Green's function to the tree-level result [Martinelli et al, Nucl. Phys. B '95]
- Non-perturbative renormalization: matching the lattice and continuum operators at the renormalization scale $\mu$

$$\sum_{jkl} C_j^{\mathrm{cont}}(\mu) R_{jk}^{\mathrm{cont}\leftarrow\mathrm{RI}} Z_{kl}^{\mathrm{RI}\leftarrow\mathrm{lat}}(\mu, a) Q_l' \stackrel{!}{=} \sum_i C_i^{\mathrm{lat}}(\mu) Q_i \,.$$

# Numerical implementation

## Some technical details

- Wilson coefficients non-perturbatively matched to $\overline{\mathrm{MS}}$ [RBC, PRD '23].

  Hierarchy between the current-current operator and the QCD+EW penguins:

$$C_1 = -0.312(14) - i1.34(33) \times 10^{-5},$$
$$C_2 = 0.718(14) + i1.68(33) \times 10^{-5},$$
$$C_3 = 0.018(14) + i7.5(3.2) \times 10^{-6}, \quad \text{others} \lesssim 0.02\,.$$

- Available coordinate-space data from other projects ($N_{\mathrm{cnfg}} = 120$):
  - Coulomb-gauge-fixed wall sources for the kaon.
  - Point-source propagators at 512 source positions.

- Low-mode deflated (z-)Möbius accelerated Domain Wall Fermion, with up to 2000 low modes available.

- Main numerical cost: Type 3 and Type 5.