Neutrino interaction rates at $T \sim$ MeV and their role in decoupling[1,2]

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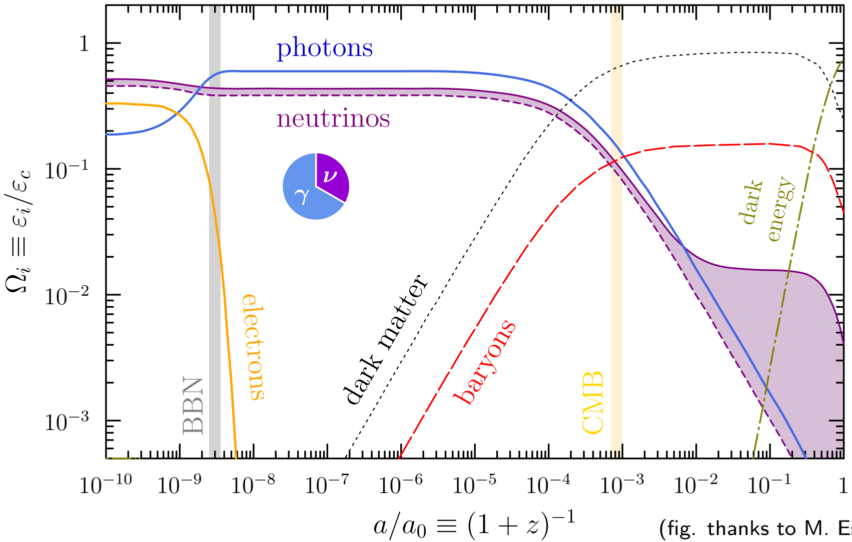
[1] based on collaboration w/ M. Escudero, M. Laine, S. Sandner

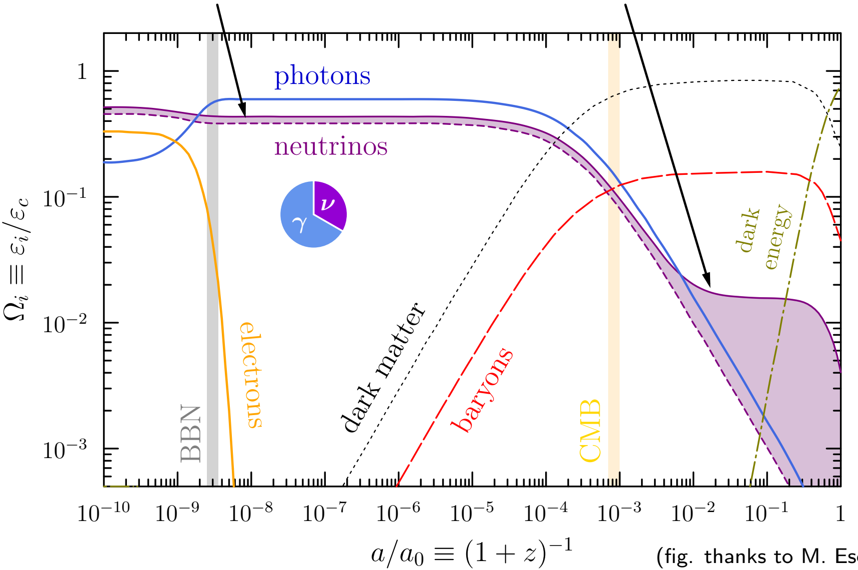
[2] supported by the ANR under grant No. 22-CE31-0018

Neutrinos are always relevant in the universe's evolution!



(fig. thanks to M. Escudero!)

$N_{\text{eff}} = 3.0 \pm 0.3$ $\sum m_\nu < 0.2$ eV



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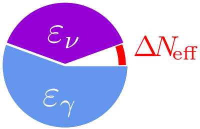
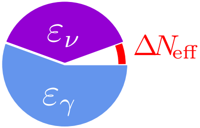
radiation energy budget:

$$\frac{\varepsilon_\nu}{\varepsilon_\gamma} \equiv \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}^{\sf SM}$$



(zeroth order approx. $N_{\rm eff} \simeq 3 =$ number of neutrino species) radiation energy budget:



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(zeroth order approx. $N_{\rm eff} \simeq 3 =$ number of neutrino species)

observation

- $N_{\rm eff}^{\rm BBN} = 2.86 \pm 0.28$ [Pisanti, *et al* (2020)]
- $N_{\rm eff}^{\rm CMB} = 2.81 \pm 0.12$ [SPT+Planck+ACT (2025)]

theory

- $N_{\rm eff}^{\rm SM} = 3.044 \pm 0.001$ $\Rightarrow$ subject of this talk

note: Simons Observatory will target ± 0.05 accuracy...

$\Rightarrow$ $\Delta N_{\rm eff}$ is a probe of BSM physics!

Why is $N_{\rm eff}$ in the SM not 3?

quantum kinetic equations: [Sigl, Raffelt (1993)]

$$\dot{\rho} \;\simeq\; -i\big[\mathcal{H}, \rho\big] + \mathcal{C}[\rho] \quad , \quad \rho_{ab} = e^{i\phi_{ab}(t)}\left\langle \frac{\hat{w}_a^\dagger \hat{w}_b}{V} \right\rangle$$

(approx/numerical solution, e.g. `FortEPiaNO` [Gariazzo, de Salas, Pastor (2019)])

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(approx/numerical solution, e.g. `FortEPiaNO` [Gariazzo, de Salas, Pastor (2019)])

- some $e^+e^- \to \nu\bar{\nu}$ heating since $T_{\mathsf{dec}} \approx m_e$ [Dicus, *et al.* (1982)] [Dolgov, *et al.* (1997)] — $\delta N_{\mathrm{eff}} \simeq +0.03$
- corrections to equation of state $P_{\mathsf{int}}(T)$ [Heckler (1994)] — $\delta N_{\mathrm{eff}} \simeq +0.01$

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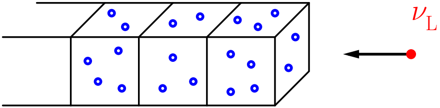
- some $e^+e^- \to \nu\bar{\nu}$ heating since $T_{\text{dec}} \approx m_e$ — $\delta N_{\text{eff}} \simeq +0.034$
  [Dicus, *et al.* (1982)] [Dolgov, *et al.* (1997)]
- corrections to equation of state $P_{\text{int}}(T)$ — $\delta N_{\text{eff}} \simeq +0.009$
  [Heckler (1994)] [Akita, Yamaguchi (2020)]
- neutrino oscillations — $\delta N_{\text{eff}} \simeq +0.001$
  [de Salas, Pastor (2016)] [Froustey, *et al.* (2020)]
- QED corrections to interaction rates — $\delta N_{\text{eff}} =$??
  [Bennet, *et al.* (2020)] [Cielo, *et al.* (2023)]

start with simpler problem: interaction rate for "tagged" particle



calculate the rate in thermal field theory Bödeker, *et al.* [1510.06742]

$$\Gamma = \frac{1}{2\omega}\mathrm{Tr}\left[\not{\mathcal{K}}\,\mathrm{Im}\,\Sigma(\mathcal{K})\right]\,, \quad \mathcal{K} = (\omega, \boldsymbol{k})$$

$\Sigma(K) =$  fermion self-energy

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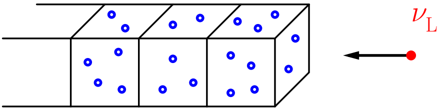

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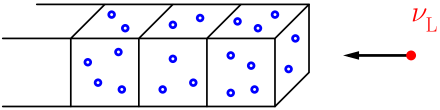
$\Rightarrow$ EFT of neutrinos, electrons, positrons and photons at $T \sim$ MeV

$$L \; = \frac{G_\mathrm{F}}{4\sqrt{2}} \Big\{ \, 2 \, \bar{\nu}_a \gamma_\mu (1 - \gamma_5) \nu_a \, \bar{\ell}_e \gamma_\mu \left[2\delta_{a,e} - 1 + 4 s_W^2 + (1 - 2\delta_{a,e}) \gamma_5 \right] \ell_e \\ + \bar{\nu}_a \gamma_\mu (1 - \gamma_5) \nu_a \, \bar{\nu}_b \gamma_\mu (1 - \gamma_5) \nu_b \Big\}$$

sum over $a, b = \{e, \mu, \tau\}$

start with simpler problem: interaction rate for "tagged" particle



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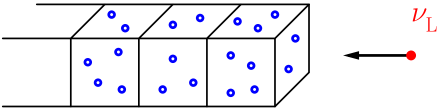
$$L \;=\; \frac{G_\mathrm{F}}{4\sqrt{2}}\Big\{\, 2\,\bar{\nu}_a\gamma_\mu(1-\gamma_5)\nu_a\,\bar{\ell}_e\gamma_\mu\left[2\delta_{a,e} - 1 + 4s_W^2 + (1-2\delta_{a,e})\gamma_5\right]\ell_e$$
$$+\,\bar{\nu}_a\gamma_\mu(1-\gamma_5)\nu_a\,\bar{\nu}_b\gamma_\mu(1-\gamma_5)\nu_b + ieC_a\bar{\nu}_a\gamma_\mu(1-\gamma_5)\nu_a\,\partial_\nu F_{\nu\mu}\,\Big\}$$

sum over $a, b = \{e, \mu, \tau\}$ 1-loop level operator!

start with simpler problem: interaction rate for "tagged" particle



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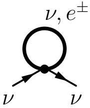
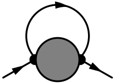
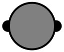
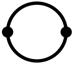
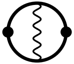
$\Sigma \;=\; aG_\mathrm{F} + bG_\mathrm{F}^2 + \ldots \;=$ 

$+$  $+\;\ldots$

 $\equiv\; \Pi^{\mu\nu}(p^0, p)$

gives QED corrections to Γ

$=$  $+$  $+$  $+$  $+$ 

the rate Γ describes the *approach* to equilibrium: GJ, Laine [2312.07015]

$$\dot{f}_{\boldsymbol{k}} \simeq -\Gamma(k)\big[\, f_{\boldsymbol{k}} - n_{\rm F}(k) \,\big]$$

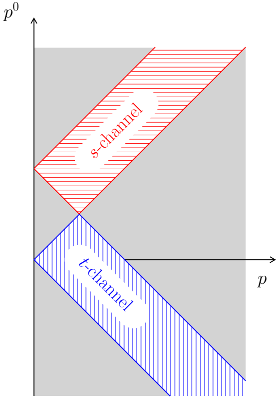
$$\Gamma = \frac{G_{\rm F}^2 k}{8\pi^2}\left(\int {\rm d}\Omega^{(t)} + \int {\rm d}\Omega^{(s)}\right)\mathcal{P}^2\left[\,1 - n_{\rm F}(k-p^0) + n_{\rm B}(p^0)\,\right]$$

$$\times \underbrace{L_{\mu\nu}(\mathcal{K},\mathcal{K}-\mathcal{P})}_{\text{leptonic tensor}}\ {\rm Im}\ \underbrace{\Pi^{\mu\nu}(p^0,p)}_{\text{self-energy}}$$

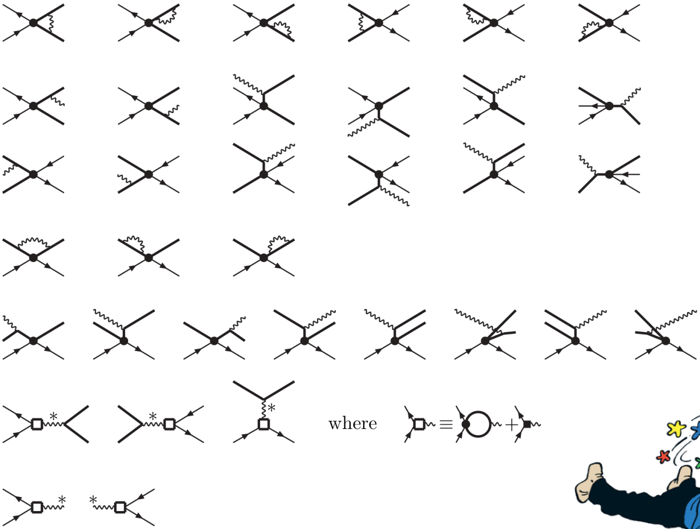
integration measures are defined by

$$\int {\rm d}\Omega^{(s)} \equiv -\frac{1}{k^3}\int_0^k {\rm d}p_- \int_k^\infty {\rm d}p_+\ p$$

$$\int {\rm d}\Omega^{(t)} \equiv \frac{1}{k^3}\int_{-\infty}^0 {\rm d}p_- \int_0^k {\rm d}p_+\ p$$



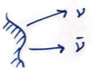
The spectral function $\operatorname{Im}\Pi_{\mu\nu}$ neatly encodes many scatterings:

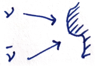


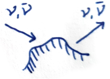
... but to obtain $N_{\rm eff}$, need to properly keep track of energy exchanges:[3]

GJ, Laine [2412.03958]

$$
\begin{aligned}
Q &\equiv \dot{\varepsilon}_{\nu+\bar\nu} \\
&= \int_{\boldsymbol{k}_\nu,\boldsymbol{q}_{\bar\nu}} (k_\nu + q_{\bar\nu})\, \Psi(\boldsymbol{k}_\nu, \boldsymbol{q}_{\bar\nu})\, (1+f_{\boldsymbol{k}_\nu})(1+f_{\boldsymbol{q}_{\bar\nu}}) \\
&- \int_{\boldsymbol{k}_\nu,\boldsymbol{q}_{\bar\nu}} (k_\nu + q_{\bar\nu})\, \widetilde{\Psi}(\boldsymbol{k}_\nu, \boldsymbol{q}_{\bar\nu})\, f_{\boldsymbol{k}_\nu} f_{\boldsymbol{q}_{\bar\nu}} \\
&+ \int_{\boldsymbol{k}_\nu,\boldsymbol{q}_\nu} (k_\nu - q_\nu)\, \Theta(\boldsymbol{q}_\nu \to \boldsymbol{k}_\nu)\, f_{\boldsymbol{q}_\nu} (1+f_{\boldsymbol{k}_\nu})
\end{aligned}
$$







the "double-differential" rates Ψ, $\widetilde{\Psi}$, Θ can be computed in thermal field theory, from the spectral function!

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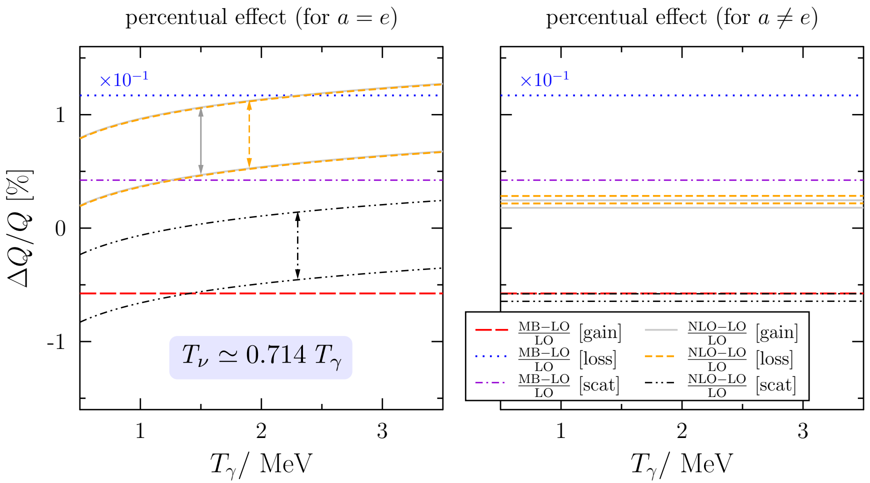
[3] tabulation & interpolation code for the e^+e^- spectral function made available in relevant kinematic domains: zenodo.14217713

Results for $\dot{\varepsilon}_{\nu+\bar{\nu}}$ at NLO



$\Rightarrow$ tiny QED corrections ($\delta N_{\text{eff}} \approx 10^{-4}$) [2312.07015] [2412.03958]

(numerically much smaller than estimated in Cielo, *et al.* [2306.05460])

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$\Rightarrow$ transfer rates ($Q = \dot{\varepsilon}_{\nu+\bar{\nu}}$ and $J = \dot{n}_{\nu+\bar{\nu}}$) enter as "coefficients"

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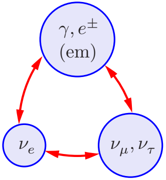
momentum-ave. approach (approx sol): Escudero [1812.05605] , [2001.04466]

$$f_{\nu_\alpha} \equiv f_{\bar\nu_\alpha} \equiv \frac{1}{e^{(q-\mu_{\nu_\alpha})/T_{\nu_\alpha}} + 1} \qquad (T_{\nu_\alpha},\, \mu_{\nu_\alpha} \text{ are effective params!})$$

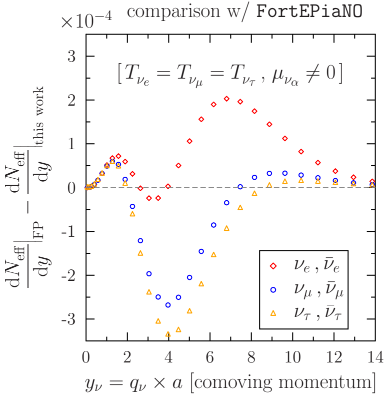
$$\frac{\mathrm{d}T_i}{\mathrm{d}t} = \frac{-3H\Big[(\varepsilon_i + P_i)\frac{\partial n_i}{\partial\mu_i} - n_i\frac{\partial\varepsilon_i}{\partial\mu_i}\Big] + Q_i\frac{\partial n_i}{\partial\mu_i} - J_i\frac{\partial\varepsilon_i}{\partial\mu_i}}{\frac{\partial n_i}{\partial\mu_i}\frac{\partial\varepsilon_i}{\partial T_i} - \frac{\partial n_i}{\partial T_i}\frac{\partial\varepsilon_i}{\partial\mu_i}}$$

$$\frac{\mathrm{d}\mu_i}{\mathrm{d}t} = \frac{3H\Big[(\varepsilon_i + P_i)\frac{\partial n_i}{\partial T_i} - n_i\frac{\partial\varepsilon_i}{\partial T_i}\Big] - Q_i\frac{\partial n_i}{\partial T_i} + J_i\frac{\partial\varepsilon_i}{\partial T_i}}{\frac{\partial n_i}{\partial\mu_i}\frac{\partial\varepsilon_i}{\partial T_i} - \frac{\partial n_i}{\partial T_i}\frac{\partial\varepsilon_i}{\partial\mu_i}}$$



Updated framework (fast & flexible):

Escudero, GJ, Laine, Sandner [2511.04747]



Part I, Standard Model[4]

- improved interaction rates
- factor ~ 10 speedup
- QED eq. of state, $\mathcal{O}(e^4)$ & $\mathcal{O}(e^5)$
- also: g_*, h_eff, $\sum m_\nu/[\Omega_\nu h^2]$

case/setup	N_eff	
FortEPiaNO	3.0439	
this work $\mu_\nu = 0$	3.0453	[0.044 %]
this work $\mu_\nu \neq 0$	3.0446	[0.020 %]
this work (w/ osc)	3.0443	[0.012 %]

$\Rightarrow$ Part II will consider various BSM implementations (stay tuned!)

[4] available in Python or Mathematica: github.com/MiguelEA/nudec_BSM

Summary

ultimate goal: describe $\nu, \bar{\nu}$ decoupling in the SM & beyond

- equilibration rate for $T \sim$ MeV [2312.07015]
- double-differential energy transfer rates [2412.03958]
- revisit momentum-averaged approach [2511.04747]

... the devil's in the details!

QED equation of state

$$P_{\mathsf{em}} = \frac{\pi^2 T_\gamma^4}{45}\left[c_0 + e^2 c_2 + e^3 c_3 + e^4\left(\tilde{c}_4 \ln\frac{\bar{\mu}}{T_\gamma} + c_4\right) + e^5\left(\tilde{c}_5 \ln\frac{\mu}{T_\gamma} + c_5\right) + \ldots\right]$$

Arnold, Zhai [hep-ph/9410360] Zhai, Kastening [hep-ph/9507380]

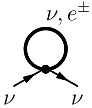
where $\bar{\mu}$ is the $\overline{\mathrm{MS}}$ renorm scale $\bar{\mu}\frac{\mathrm{d}e^2(\bar{\mu})}{\mathrm{d}\bar{\mu}} = \frac{e^4(\bar{\mu})}{6\pi^2} \Rightarrow e^2(\bar{\mu}) = \frac{6\pi^2}{\ln(\Lambda/\bar{\mu})}$

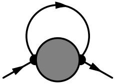
... the coefficients $c_i = c_i\left(\frac{m_e}{T_\gamma}\right)$ Escudero, GJ, Laine, Sandner [2511.04747]

	$p_{\mathsf{em}}^{(0)}$	$p_{\mathsf{em}}^{(2)\,\not{\ln}}$	$p_{\mathsf{em}}^{(2)\,\ln}$	$p_{\mathsf{em}}^{(3)}$	$p_{\mathsf{em}}^{(4,5)}$
δN_{eff}		$\approx 10^{-2}$	$\lesssim 3\times 10^{-5}$	$\approx 10^{-3}$	$\lesssim 2\times 10^{-5}$

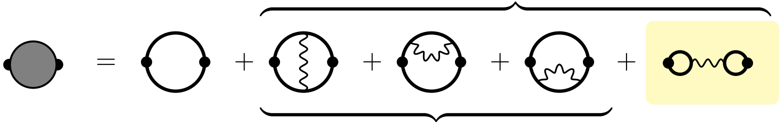
Feynman diagrams

$\Rightarrow$ address perturbatively, expansion in $G_{\rm F}$ $\left(\text{and } \alpha_{\rm em} = \dfrac{e^2}{4\pi}\right)$

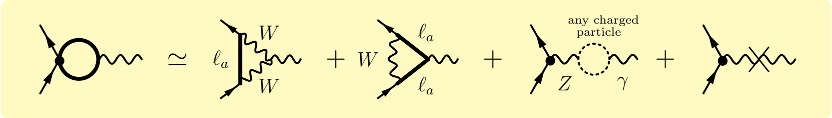
$\Sigma \quad = \quad a G_{\rm F} + b G_{\rm F}^2 + \ldots \quad =$ 

$+$  $+ \;\ldots$

QED corrections



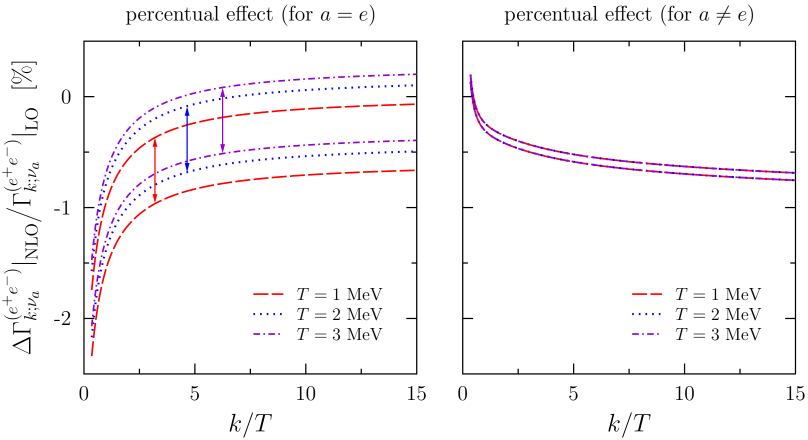
one-particle irreducible GJ [1910.07552]



more EFT details in Hill, Tomalak [1911.01493]

numerical impact of the NLO corrections

GJ, Laine [2312.07015]



⇒ we find tiny QED corrections to Γ, relatively less than ~ 1 %

Momentum ave. results