Quantum (and classical) detection of gravitational waves: scope and limitations

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Based on: C. Beadle, PB, R.T. D'Agnolo, S.A.R. Ellis, arXiv:2604.XXXXX

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Question (and conclusion)

Q: Will we ever be able to detect primordial-background gravitational-wave signals at very high frequencies?

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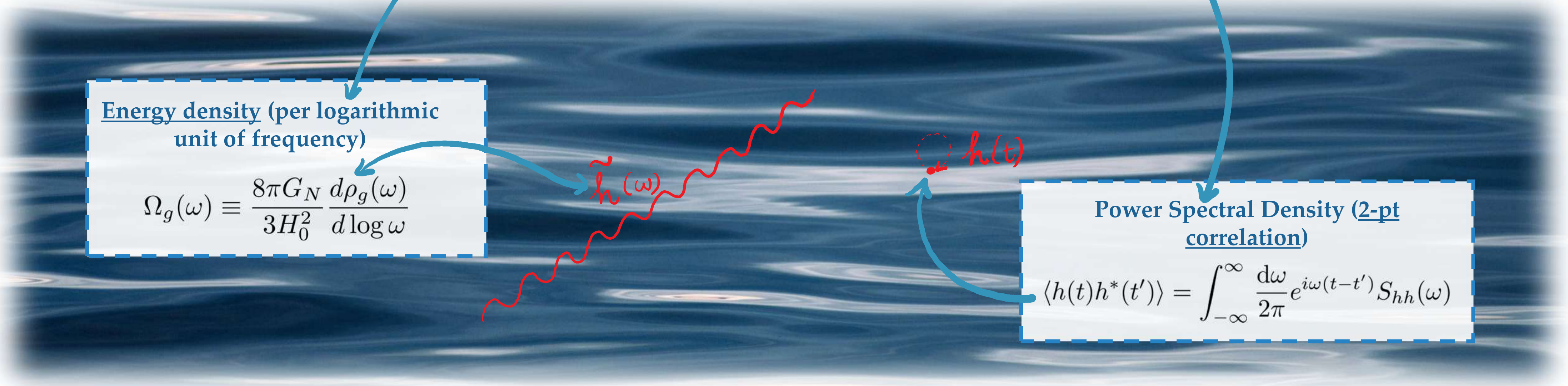
DENIED

In fact, **heuristics** tell us that [S.A.R. Ellis, R.T. D'Agnolo, arXiv:2412.17897]

Sensitivity ⟷ **Energy conversion efficiency**

Energy density and 2-point correlation

$$\boldsymbol{\Omega_g(\omega) \propto \omega^3 S_{hh}(\omega)}$$



Energy density (per logarithmic unit of frequency)

$$\Omega_g(\omega) \equiv \frac{8\pi G_N}{3H_0^2}\frac{d\rho_g(\omega)}{d\log\omega}$$

Power Spectral Density (2-pt correlation)

$$\langle h(t)h^*(t')\rangle = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}e^{i\omega(t-t')}S_{hh}(\omega)$$

$$\frac{\Omega_g(\omega)}{\Omega_g(\omega_{\rm ref})} \stackrel{!}{=} 1 \quad \stackrel{S_{hh}^{\rm min}\sim(U^{\rm det})^{-1}}{\Longrightarrow} \quad U^{\rm det} = \left(\frac{\omega}{\omega_{\rm ref}}\right)^3 U^{\rm det}_{\rm ref}$$

Main shortcoming: the minimal detectable strain

- **Minimal detectable strain:**

$$\mathrm{SNR} = \left(t_{\mathrm{int}} \int \frac{\mathrm{d}\omega}{2\pi} \frac{\Omega_g(\omega)^2}{\Omega_n(\omega)^2} \right)^{\frac{1}{2}} \simeq 1$$

- **Bound on minimal signal strength for detection of GWs coming from primordial backgrounds** [M. Kawasaki *et al.*, Phys. Rev. Lett. 82, 4168 (1999) & Phys. Rev. D 62, 023506 (2000), M. Maggiore, *Physics Reports* 331 (2000), 283-367]

Reduced Hubble parameter → h^2_{eff}

$$\int d\log\omega \, h^2_{\mathrm{eff}} \Omega_g(\omega) \lesssim 5 \times 10^{-6} \Delta N_{\mathrm{eff}}$$

ΔN_{eff} ← **Uncertainty on the number of neutrino species**

BSM signals and sources

- **Which high frequencies?**

Characteristic wavelength → $\begin{cases} \lambda_* \leq H_*^{-1} & \text{(Process occurring at temperature } T_*\text{)} \\ \omega_0 = a(t_*)/a(t_0)\,\omega_* & \text{(Redshift of gravitons)} \end{cases}$

Signal at GUT scale:

 # d.o.f. → g_*

$$\omega_0 \gtrsim 100\ \mathrm{MHz}\left(\frac{T_*}{10^{15}\ \mathrm{GeV}}\right)\left(\frac{g_*(T_*)}{100}\right)^{1/6}$$

- **What could have produced GW stochastic backgrounds?**

Vacuum fluctuations, phase transitions, cosmic strings, domain walls,…

[M. Maggiore, *Gravitational waves* (Oxford University Press, 2007); Aggarwal *et al.*, *Living Rev. Rel.* **24**, 4 (2021)]

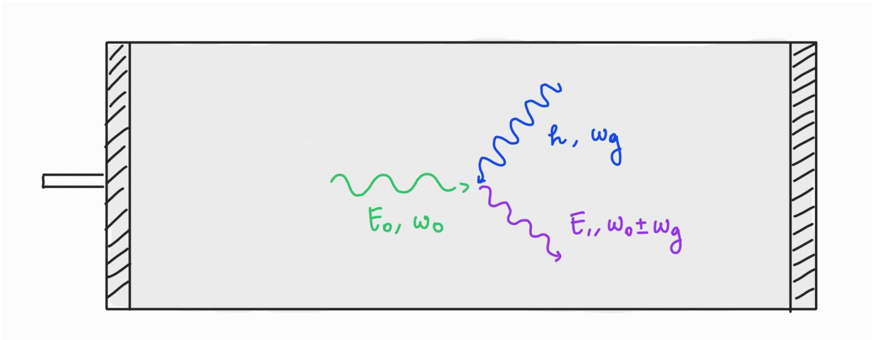
Now, all of this was based on heursitics…

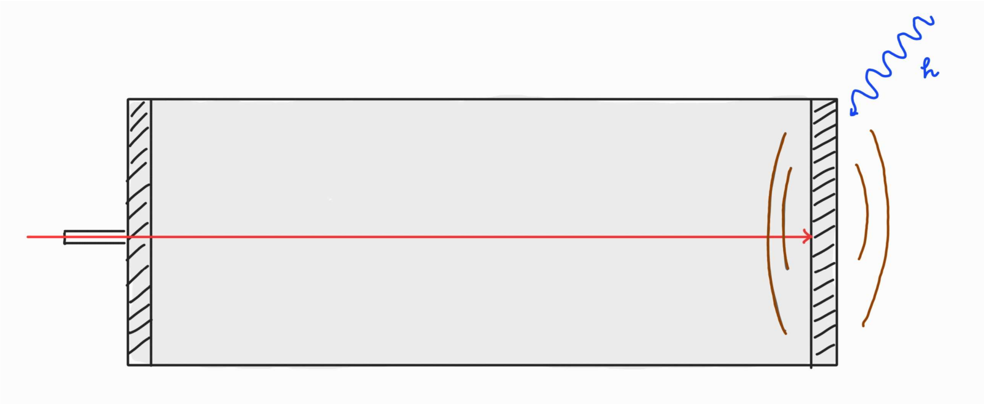
A new question

Now, all of this was based on heursitics…

Q: Can we analytically all of this, starting from an ab initio computation in quantum mechanics and taking into account all form factors?

Two toy models to describe (almost) any detector





EM resonators

- Large static magnetic field
- **Readout:** $\omega_1 \approx \omega_g$
- **MADMAX** [arXiv:2409.06462], **CAST** [arXiv:1705.02290], **IAXO** [*Eur. Phys. J. C* **79** (2019) 1032]

- Transition mode 0 (loaded) → 1 (readout)
- **Readout:** $\omega_1 = \omega_0 \pm \omega_g$
- **MAGO** [*Phys. Rev. D* **108** (2023) 084058]

Resonant EM microwave cavities [*Physical Review* D **105** 116011 (2022)], **Lumped LC resonators** [*Phys. Rev. Lett.* **129** (2022) 041101]

Mechanical resonators

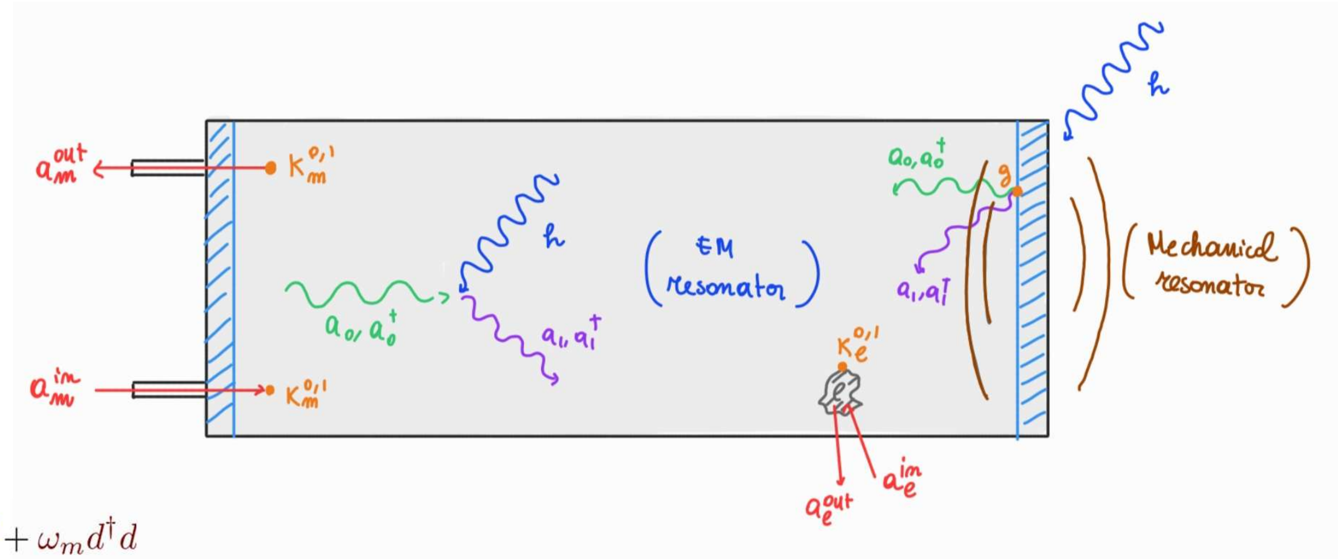
- Like test masses. Their position is measured through an **EM readout**

- **Interferometers** (**LVK**, **Holometer**), **Optomechanical sensors** (**levitating sphere** [A. Arvanitaki, A. A. Gercai, *Phys. Rev. Lett.* **110** (2013) 071105]), **Weber bars** (**AURIGA** [M. Cerdonio *et al.*, *Classical and Quantum Gravity* **14** (1997) 1491]), **Magnetic Weber bars** [V. Domcke, S.A.R. Ellis, N. L. Rodd, *Phys. Rev. Lett. 134, 231401*]

Quantum mechanical set-up

Prototypical system

$$H(t) = H_0(t) + H_{G+OM}(t) + H_R(t)$$



Free $$H_0(t) = \sum_{n,r} \Delta_n a^\dagger_{n,r} a_{n,r} + \int_V d^3x |B_0|^2 + \omega_m d^\dagger d$$

Int. $$H_{G+OM}(t) = \int_V \mathrm{d}^3x h_{\mu\nu} T^{\mu\nu} + gxX_1 = h(t) \left\{ \sum_{jj'} C_{jj'} a^\dagger_j(t) a_{j'}(t) + \sum_j \left[D_j a_j(t) + D^*_j a^\dagger_j(t) \right] \right\} + gxX_1$$

Back-action

Readout $$H_R(t) = \sum_{j=0}^{1} \sum_l \int \mathrm{d}\omega \left\{ \omega\, b^\dagger_l(\omega) b_l(\omega) + i g^j_l \left[b^\dagger_l(\omega) a_j(t) - b_l(\omega) a^\dagger_j(t) \right] \right\}$$

Thermal baths

Procedure

- **Input-output formalism** [Beckey *et al.*, arXiv:2311.07270]

$$X_n = \frac{a_n + a_n^\dagger}{\sqrt{2}}, \quad Y_n = -\frac{i(a_n - a_n^\dagger)}{\sqrt{2}}$$

  - EoMs

$$\begin{cases} \dot{X}_n(t) = \Delta_n Y_n(t) \mp \frac{1}{2}\sum_k \tilde{k}_j^n X_j(t) + \sum_\Lambda \sqrt{\kappa_\Lambda^n} X_\Lambda^{\substack{\text{in}\\\text{out}}}(t) + F_X^n(t) \\ \dot{Y}_n(t) = -\Delta_n X_n(t) \mp \frac{1}{2}\sum_k \tilde{k}_j^n Y_j(t) + \sum_\Lambda \sqrt{\kappa_\Lambda^n} Y_\Lambda^{\substack{\text{in}\\\text{out}}}(t) + F_Y^n(t) \\ \dot{x}(t) = \frac{p(t)}{M} \\ \dot{p}(t) = -M\omega_m^2 x(t) - \gamma_m p(t) - g X_1(t) + F_m(t) \end{cases}$$

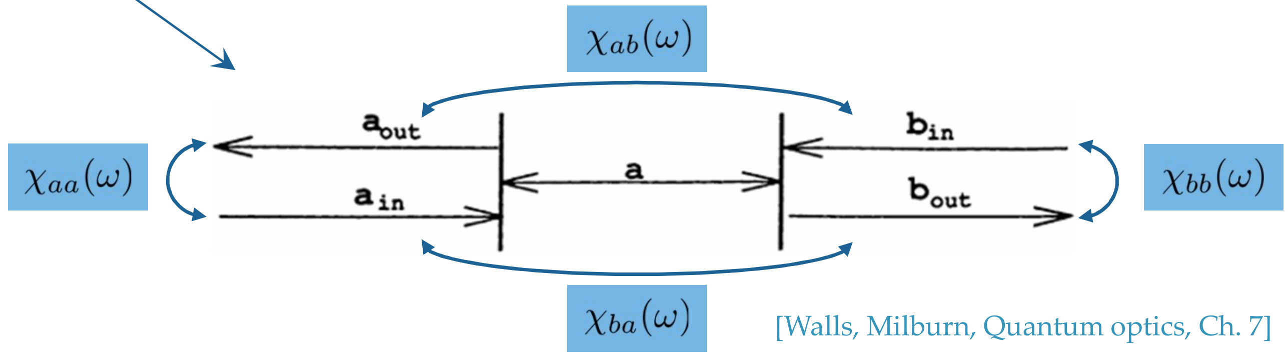
EM

Mech.

  - Input-output relations

$$\begin{cases} X_\Lambda^{\text{out}} = X_m^{\text{in}} - \sum_j \sqrt{\kappa_\Lambda^j} X_j \\ Y_\Lambda^{\text{out}} = Y_m^{\text{in}} - \sum_j \sqrt{\kappa_\Lambda^j} Y_j \end{cases}$$

Transfer functions



[Walls, Milburn, Quantum optics, Ch. 7]

Power Spectral Density and Minimal Detectable Strain

- Quadrature PSD:

$$S^{\text{out}}_{Y_m Y_m}(\omega) = \sum_{\Lambda}\left[|\chi_{Y_m Y_\Lambda}(\omega)|^2 S^{\text{in}}_{Y_\Lambda Y_\Lambda}(\omega) + |\chi_{Y_m X_\Lambda}(\omega)|^2 S^{\text{in}}_{X_\Lambda X_\Lambda}(\omega)\right]$$
$$+ \sum_{\Lambda}\left[\chi_{Y_m X_\Lambda}(\omega)\chi_{Y_m Y_\Lambda}(\omega)^* S^{\text{in}}_{Y_\Lambda X_\Lambda}(\omega) + \chi_{Y_m Y_\Lambda}(\omega)\chi_{Y_m X_\Lambda}(\omega)^* S^{\text{in}}_{X_\Lambda Y_\Lambda}(\omega)\right]$$
$$+ \sum_{I} |\chi_{Y_m F_I}(\omega)|^2 S_{F_I F_I}(\omega) \supset S_{hh}(\omega)$$

Noise

Signal

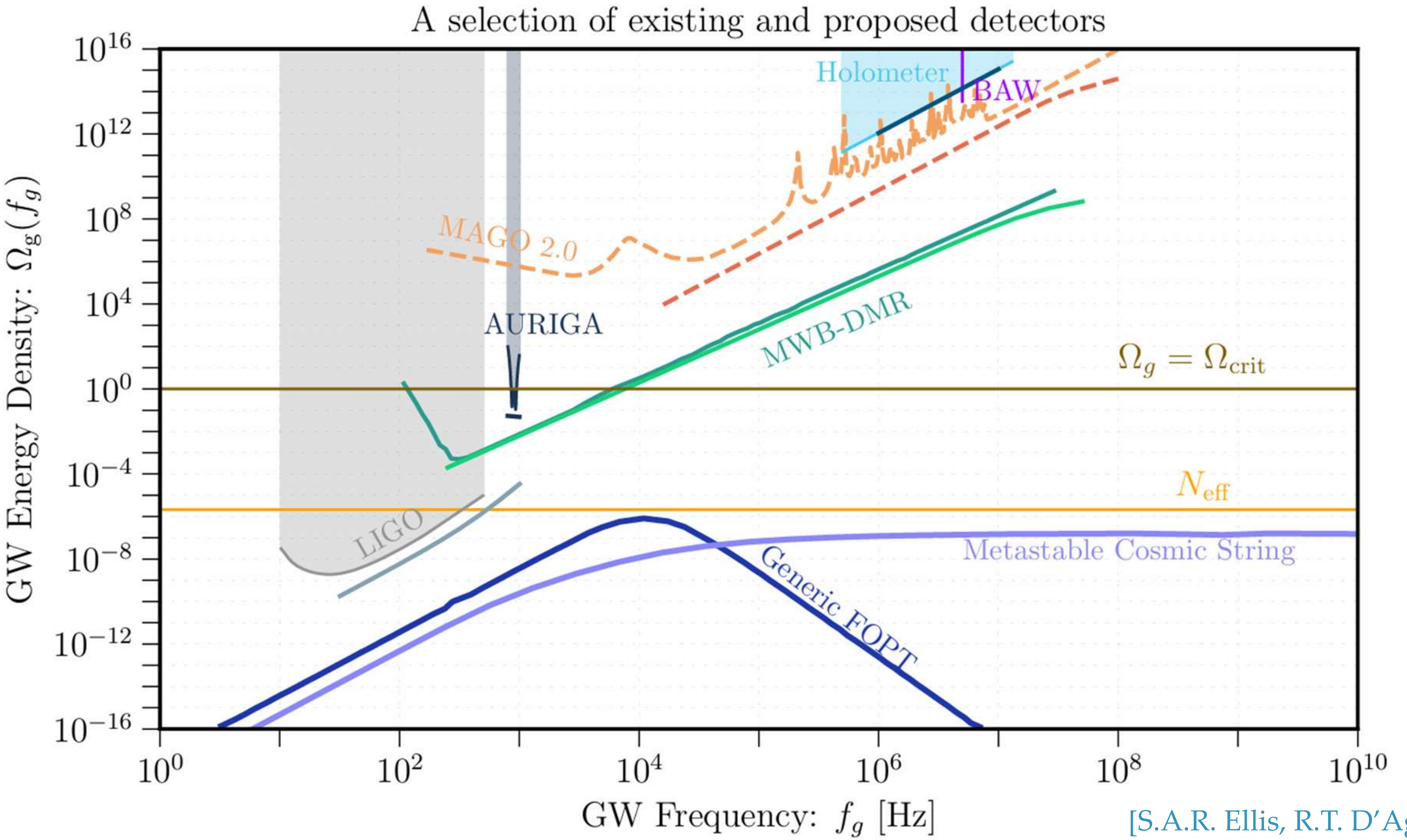
h-PSD ↔ GW energy density

$$\Omega_g(\omega) = \frac{\omega^3 S_{hh}(\omega)}{3\pi H_0^2}$$

SNR $\simeq 1$ →

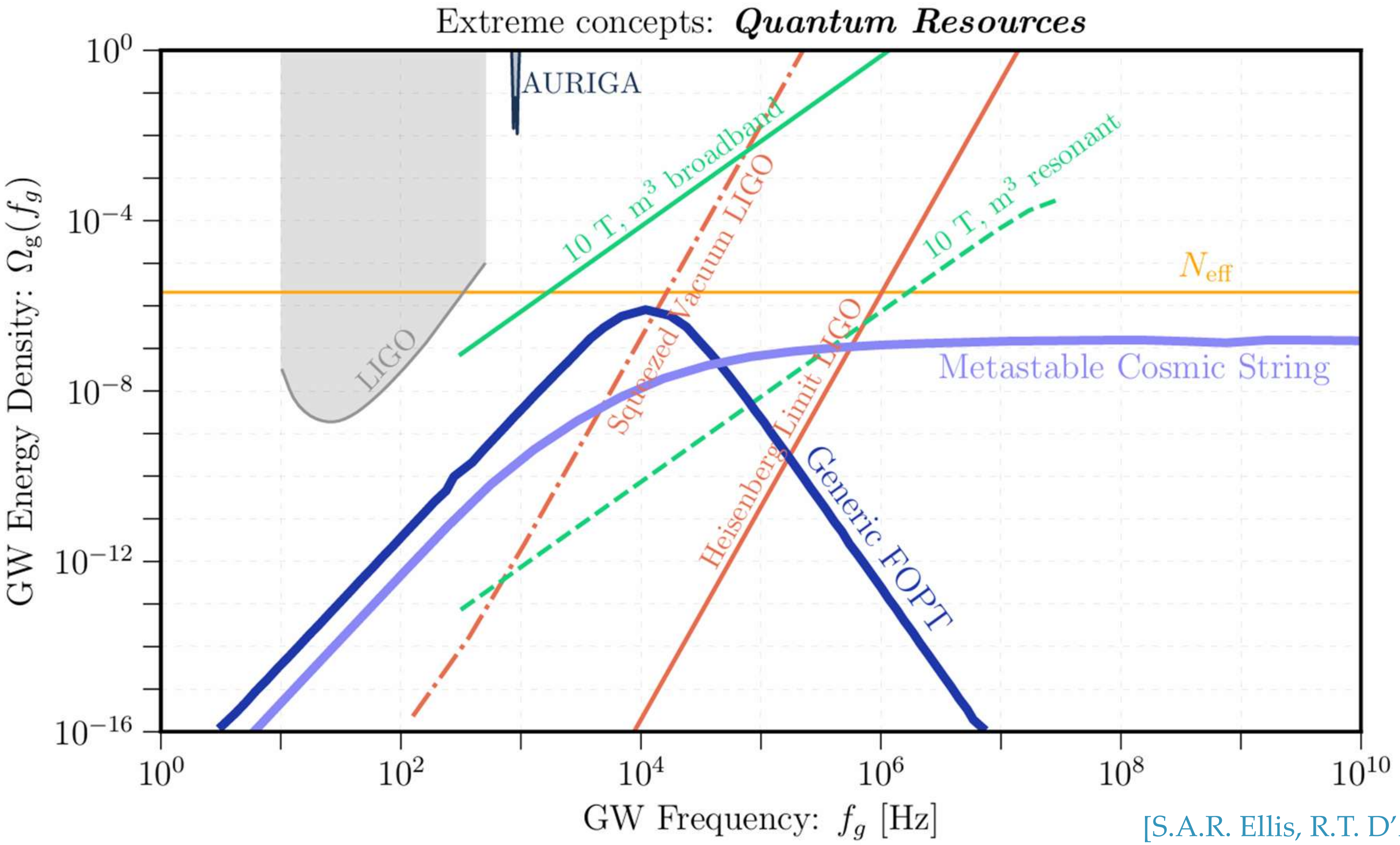
$$(\Omega_g(\omega))_{\min} = \frac{\omega^3}{3\pi H_0^2}\left[t_{\text{int}}\int_{\omega-\Delta\omega/2}^{\omega+\Delta\omega/2}\frac{d\omega'}{2\pi}\left(\frac{|\chi_{Y_m h}(\omega')|^2}{S^{\text{out}}_{Y_m Y_m}(\omega')|_{\text{NOISE}}}\right)^2\right]^{-\frac{1}{2}}$$

Classical heuristics



[S.A.R. Ellis, R.T. D'Agnolo, arXiv:2412.17897]

Quantum heuristics



[S.A.R. Ellis, R.T. D'Agnolo, arXiv:2412.17897]

Conclusions (again!) and outlook

- The novel analysis based on the input-output formalism is going to give **analytic bounds (including form factors)** on GW detection and accurately tell us what we can expect from present and near-future GW detectors

- **However we expect the precise quantum computations to confirm the conclusions drawn from the heuristics**

Upshot: **High-frequency gravitational waves coming from primordial backgrounds might remain out of the experimental reach of current detectors**