Quantum thermodynamics and spin polarization in a boost-invariant fluid of Dirac fermions

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In collaboration with D. Roselli

In preparation...

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MARIE CURIE ACTIONS

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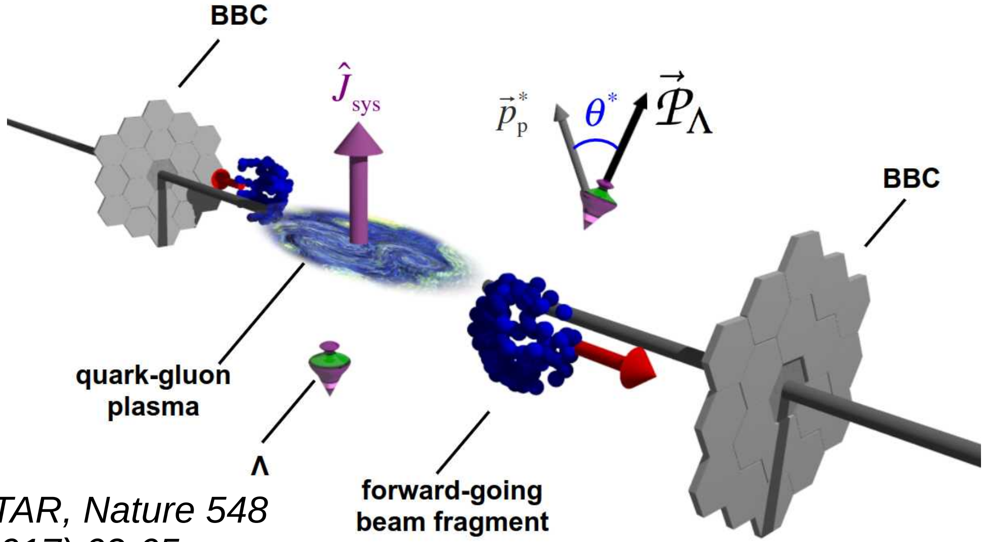
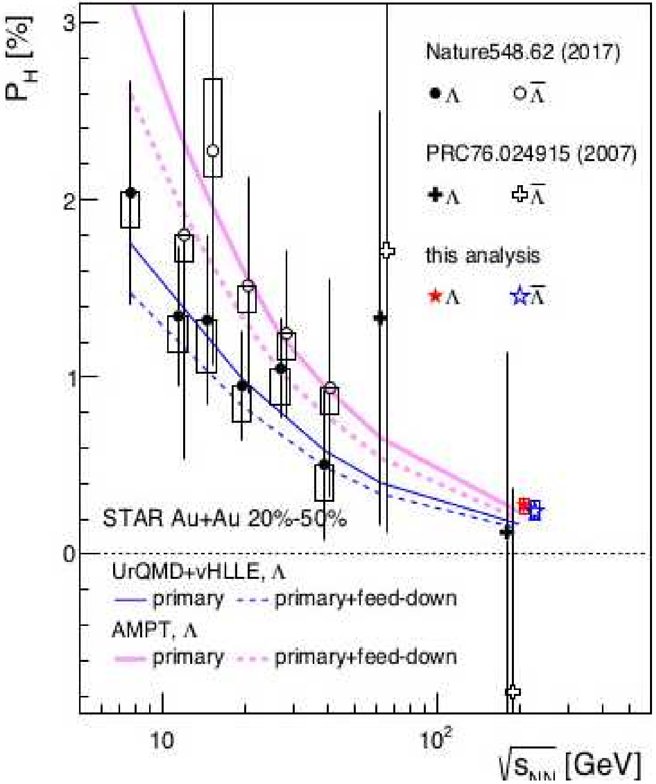
Spin in heavy ion collisions

Spin of particles can be measured in heavy ion collisions. Polarization of Λ particles can be related to the "angular velocity" of the QGP

$$S^{\mu}(p) = -\frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_{\sigma}\frac{\int d\Sigma \cdot p\, n_F(1-n_F)\varpi_{\nu\rho}}{\int d\Sigma \cdot p\, n_F}$$



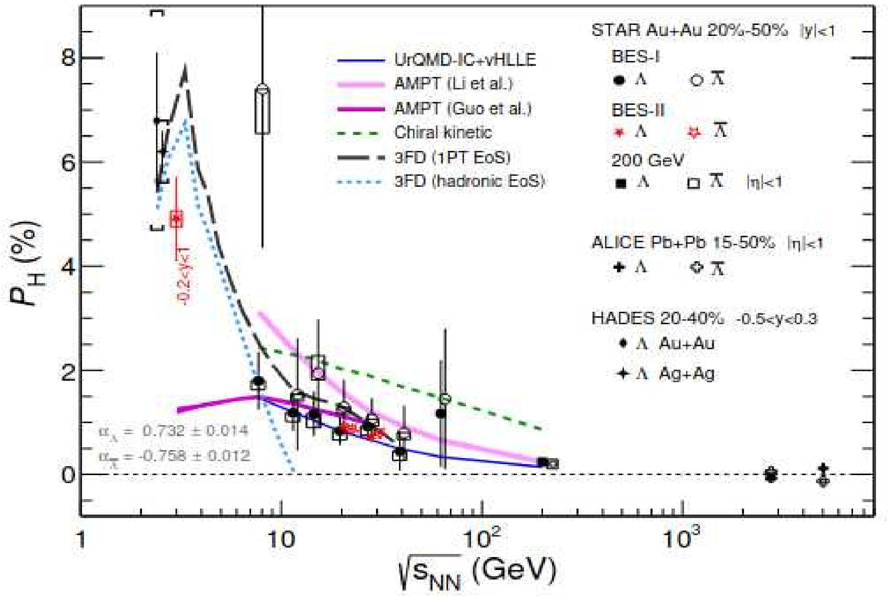
STAR, Nature 548 (2017) 62-65

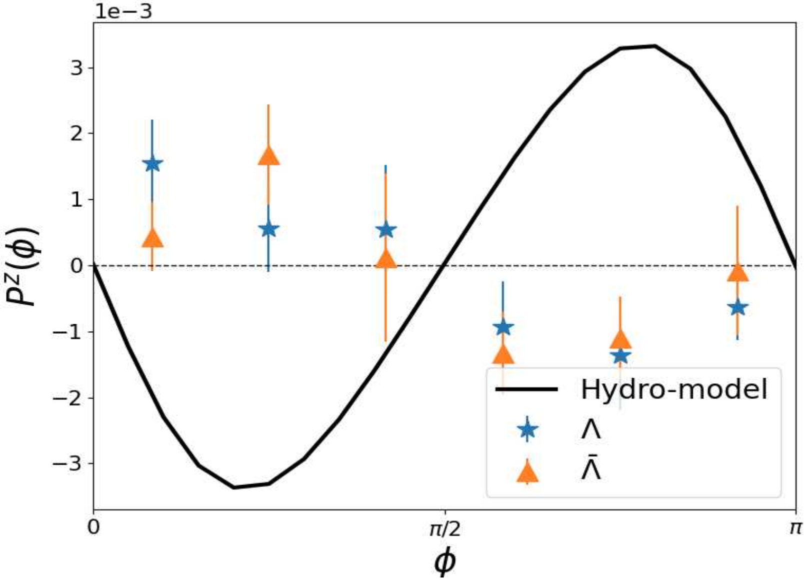


T. Niida, Nucl.phys.A,2019

Rotation-induced polarization cannot explain momentum-dependent measurements.

Becattini, Buzzegoli, Niida, Pu, Tang Wang Int.J.Mod.Phys.E 33 (2024) 06, 2430006





Other sources of spin polarization have been discovered: shear induced polarization, spin hall effect. They alleviate the problem but don't solve it entirely.

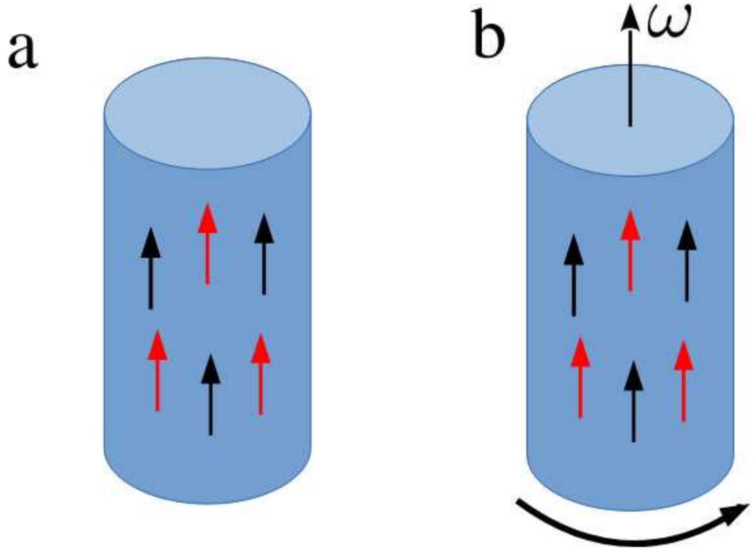
We know hydrodynamics works well for QGP physics. Maybe including the spin tensor in the thermo-hydrodynamic picture can accommodate the discepancy?

Spin Hydrodynamics

The spin tensor is related to the total angular momentum: $\widehat{\mathcal{J}}^{\lambda,\mu\nu} = x^\mu \widehat{T}^{\lambda\nu} - x^\nu \widehat{T}^{\lambda\mu} + \widehat{\mathcal{S}}^{\lambda,\mu\nu}$

Spin hydrodynamics equations:

$$\partial_\mu T^{\mu\nu} = 0$$
$$\partial_\mu \mathcal{S}^{\mu,\rho\sigma} = T^{\sigma\rho} - T^{\rho\sigma}$$



Becattini, Florkowski, Speranza Phys.Lett.B 789 (2019) 419-425

To describe spin tensor dynamics independently, a new thermodynamic variable is introduced, the spin potential $\mathfrak{S}^{\mu\nu}$. Can describe polarization without rotation.

An out-of-equilibrium system with finite temperature and spin potential can be described by the density operator

$$\widehat{\rho} = \frac{1}{Z} \exp\left(- \int_{\Sigma(\tau)} \mathrm{d}\Sigma n_\mu \widehat{T}^{\mu\nu} \beta_\nu - n_\mu \widehat{\mathcal{S}}^{\mu,\nu\rho} \mathfrak{S}_{\nu\rho} \right)$$

Studying a relativistic system with a finite **spin potential is very challenging**.

Thermodynamic relations are modified by the presence of the spin potential, but there is ambiguity regarding such modifications.

It's not very clear how to **test the existence of a spin potential** in heavy ion collisions.

Our goal: Compute exactly expectation values for a phenomenologically motivated yet simplified setup: a longitudinally boost-invariant fluid of Dirac fermions.

$$\beta^\mu = \frac{u^\mu}{T(\tau)}\,, \qquad u^\mu = \frac{1}{\tau}(t,0,0,z)\,, \qquad \mathfrak{S}_{\rho\sigma} = \frac{1}{2}\mathfrak{S}(\tau)\left(\delta^1_\rho\delta^2_\sigma - \delta^2_\rho\delta^1_\sigma\right).$$

Boost invariance is an approximation often employed in heavy ion collisions, and it is particularly appropriate for very high energy central collisions.

We study a gas of non-interacting Dirac fermions

The energy momentum and spin tensor are:
M.Buzzegoli, A.Palermo, Phys.Rev.Lett. 133 (2024) 26, 262301

$$\widehat{T}^{\mu\nu}_{\mathrm{C}}(x) = \frac{i}{2}\bar{\psi}(x)\gamma^{\mu}(x)D^{\nu}\psi(x) - \frac{i}{2}D^{\nu}\bar{\psi}(x)\gamma^{\mu}(x)\psi(x)$$

$$\widehat{S}^{\mu,\nu\lambda}_{\mathrm{C}}(x) = \frac{1}{2}\epsilon^{\rho\mu\nu\lambda}(x)\bar{\psi}(x)\gamma_{\rho}(x)\gamma^{5}\psi(x)$$

Expectation values are constrained by symmetries:

$$\langle\widehat{T}^{\mu\nu}(x)\rangle = \mathcal{E}(\tau)\hat{\tau}^{\mu}\hat{\tau}^{\nu} + \mathcal{P}_{\mathrm{T}}(\tau)\left(\hat{x}^{\mu}\hat{x}^{\nu} + \hat{y}^{\mu}\hat{y}^{\nu}\right) + \mathcal{P}_{\mathrm{L}}(\tau)\hat{\eta}^{\mu}\hat{\eta}^{\nu} + \frac{\mathcal{T}(\tau)}{2}\left(\hat{y}^{\mu}\hat{x}^{\nu} - \hat{x}^{\mu}\hat{y}^{\nu}\right)$$

$$\langle\widehat{S}^{\mu,\lambda\nu}(x)\rangle = \epsilon^{\mu\lambda\nu\rho}\mathcal{S}(\tau)\hat{\eta}_{\rho}$$

$$\frac{1}{\tau}\frac{\partial}{\partial\tau}\left(\tau\mathcal{S}(\tau)\right) = \mathcal{T}(\tau)$$

We solve the Dirac equation in Milne coordinates. Milne and Minkowski modes are related

$$\widehat{A}_r(\mathbf{p}_{\mathrm{T}}, \mu) = \int_{-\infty}^{+\infty} \mathrm{d}\vartheta \sqrt{\frac{m_{\mathrm{T}} \cosh\vartheta}{2\pi}}\, e^{-i\mu\vartheta} \widehat{a}_r(p)$$

The density operator

$$\widehat{\rho} = \frac{1}{Z} \exp\left\{ -\tau \int \mathrm{d}^2\mathrm{x}_{\mathrm{T}} \mathrm{d}\eta \left[\frac{\widehat{T}^{\tau\tau}}{T(\tau)} - \mathfrak{S}(\tau) \widehat{S}^{\tau,xy} \right] \right\} = \frac{1}{Z} \int \mathrm{d}^3\mathfrak{p}\, \Phi^T(\mathfrak{p}) \mathcal{H}_{eff} \Phi(\mathfrak{p})$$

$$\Phi^T(\mathfrak{p}) \equiv \left(\widehat{A}_+(\mathfrak{p}), \widehat{A}_-(\mathfrak{p}), \widehat{B}_+^\dagger(-\mathfrak{p}), \widehat{B}_-^\dagger(-\mathfrak{p}) \right)$$

Our task is then to find the appropriate **Bogoliubov transformation** to compute exact expectation values.

In the case with **vanishing spin potential**, we recover known results:

$$n_{\mathrm{F}}^{\mp}(\tau, \mathfrak{p}) = \left\{\exp\left[\frac{\varepsilon(\tau, \mathfrak{p})}{T(\tau)} \mp \zeta(\tau)\right] + 1\right\}^{-1}$$

$$\mathcal{E}(\tau) = \frac{2}{(2\pi)^3 \tau} \int \mathrm{d}^3\mathfrak{p} \sqrt{m_{\mathrm{T}}^2 + \frac{\mu^2}{\tau^2}} \left[n_{\mathrm{F}}^{-}(\varepsilon(\tau)) + n_{\mathrm{F}}^{+}(\varepsilon(\tau))\right]$$

$$\mathcal{P}_L = \mathcal{P}_T$$

Spin polarization effects are described by the spin density matrix, which in this case is isotropic

$$\Theta_{sr} = \frac{\int \mathrm{d}\mu \, \langle \widehat{A}_s^{\dagger}(\mathfrak{p}) \widehat{A}_r(\mathfrak{p}) \rangle}{\int \mathrm{d}\mu \, \sum_t \langle \widehat{A}_t^{\dagger}(\mathfrak{p}) \widehat{A}_t(\mathfrak{p}) \rangle} = \frac{1}{2}\delta_{rs}$$

No shear-induced polarization in a boost invariant fluid!

When a **non-vanishing spin potential** in included, diagonalization is much harder.

Spin polarization can be computed analitically only in special cases:

Longitudinally moving particle: $\mathrm{p_T}=0$

$$\Theta_{rs} = \frac{\delta_{rs}}{2} + \frac{\sigma^z_{rs}}{2} \sinh(\mathfrak{S}/2) \frac{\int_0^\infty \mathrm{d}\mu \left[\cosh(\beta m_\mathrm{L}(\mu)) + \cosh(\mathfrak{S}/2)\right]^{-1}}{\tau m^2 \int_0^\infty \mathrm{d}\mu \, \frac{\mathfrak{h}(\tau,\mu)}{m_\mathrm{L}(\mu)} \frac{\left[e^{-\beta m_\mathrm{L}(\mu)} + \cosh(\mathfrak{S}/2)\right]}{\left[\cosh(\beta m_\mathrm{L}(\mu)) + \cosh(\mathfrak{S}/2)\right]}}$$

Negligible temperature effects:

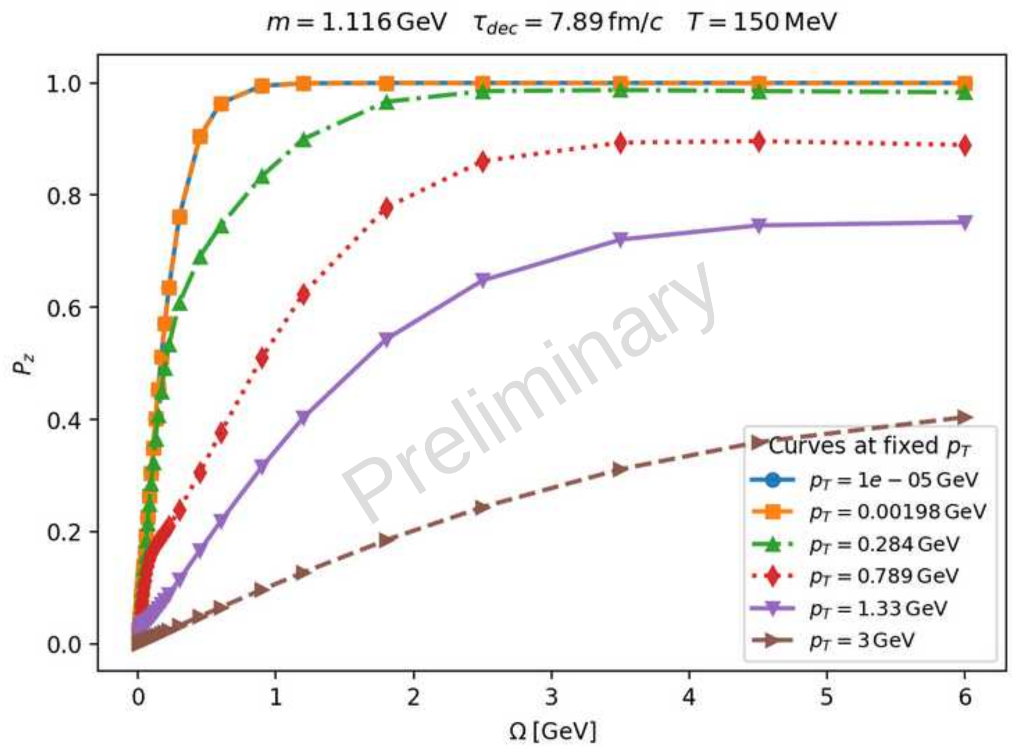
$$\Theta_{rs} = \frac{\delta_{rs}}{2} + \frac{\sigma^z_{rs}}{2} \frac{m}{m_T} \tanh(\mathfrak{S}/4)$$

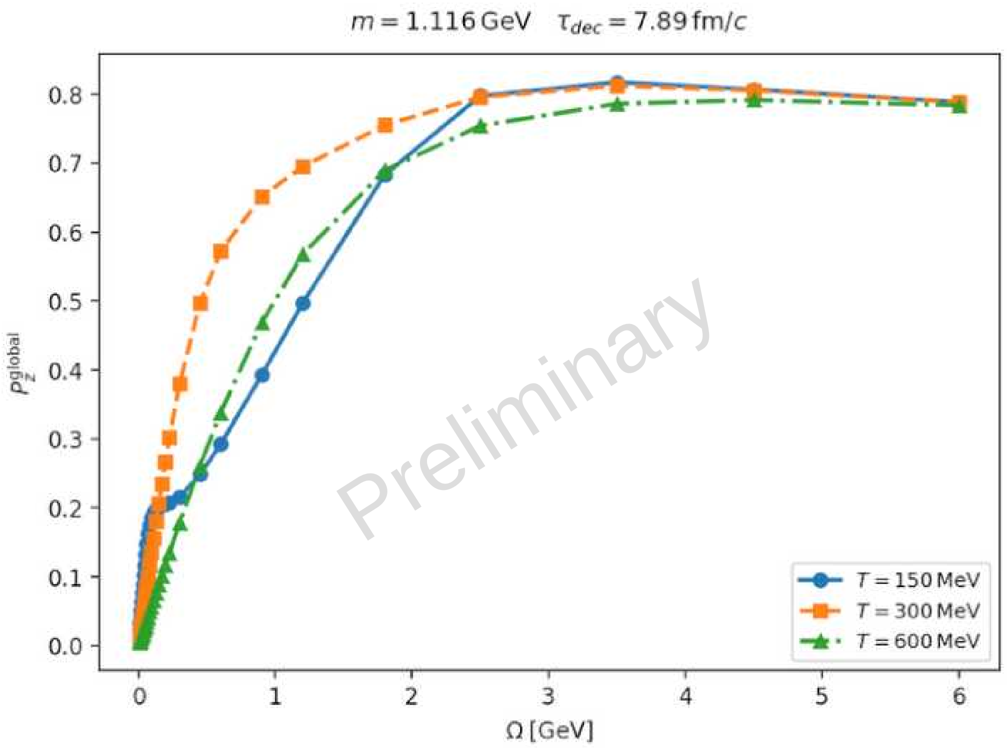
In all other cases formulae become too complicated. Exact diagonalization can be performed numerically.

Preliminary results on spin polarization. Vanishing transverse polarization

$$P_x = P_y = 0$$

Longitudinal polarization $\mathfrak{S} = \Omega/T$





Conclusions

We have studied an exactly solvable non equilibrium system: a boost invariant fluid constituting of Dirac fermions

In the absence of a spin potential, no polarization is possible.

We are able to compute exactly polarization for massive particles. Upcoming: pressure, energy density, spin tensor and spin torque.

Going forward: study thermodynamic relations with a finite spin potential.

Experimentally: The study of spin polarization in central collisions could tell us more about the importance of spin potential in relativistic heavy ion collisions.

Thank you for your attention!

We use Milne coordinates

$$\tau = \sqrt{t^2 - z^2}, \qquad \eta = \frac{1}{2}\ln\frac{t+z}{t-z}, \qquad \mathrm{d}s^2 = \mathrm{d}\tau^2 - \mathrm{d}\mathbf{x}_\mathrm{T}^2 - \tau^2\mathrm{d}\eta^2$$

Solution of the Dirac equation

$$\psi\left(\tau, \mathbf{x}_\mathrm{T}, \eta\right) = \frac{1}{\left(2\pi\right)^{3/2}}\sum_{r=\pm}\int \mathrm{d}^3\mathfrak{p}\left[e^{i\mathfrak{p}\cdot\mathfrak{r}}U_r(\tau,\mathfrak{p})\widehat{A}_r(\mathfrak{p}) + e^{-i\mathfrak{p}\cdot\mathfrak{r}}V_r(\tau,\mathfrak{p})\widehat{B}_r^\dagger(\mathfrak{p})\right]$$

$$U_r\left(\tau,\mathfrak{p}\right) = -\frac{i\sqrt{\pi}}{2}e^{-i\pi\nu/2}\mathrm{H}_\nu^{(2)}(m_\mathrm{T}\tau)\,u_r(\mathbf{p}_\mathrm{T}) \qquad V_r\left(\tau,\mathfrak{p}\right) = \frac{i\sqrt{\pi}}{2}e^{i\pi\bar{\nu}/2}\mathrm{H}_{\bar{\nu}}^{(1)}(m_\mathrm{T}\tau)\,v_r(\mathbf{p}_\mathrm{T})$$

$$\nu \equiv \frac{1}{2}\gamma^3\gamma^0 - i\mu\mathbb{I}$$

Milne and Minkowski modes are related

$$\widehat{A}_r(\mathbf{p}_\mathrm{T},\mu) = \int_{-\infty}^{+\infty}\mathrm{d}\vartheta\sqrt{\frac{m_\mathrm{T}\cosh\vartheta}{2\pi}}\,e^{-i\mu\vartheta}\widehat{a}_r(p)$$