Revisiting extremely high energy Bremsstrahlung in matter

From a QCD mystery to a QED discovery

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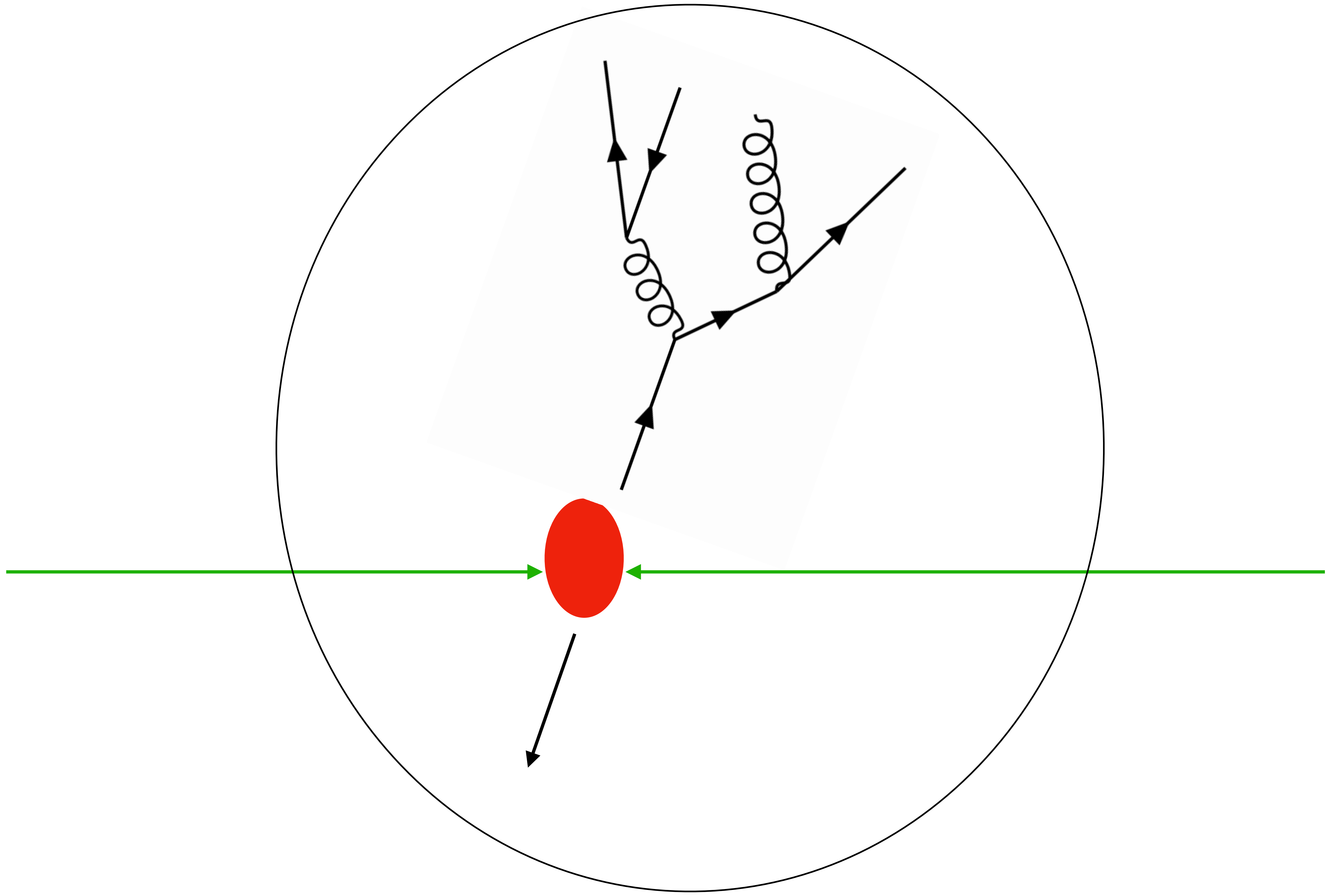
Rencontres de Physique des Particules 2026

Based on work with:

Peter Arnold, Shahin Iqbal and Joshua Bautista
JHEP03(2026)015

Particle showers

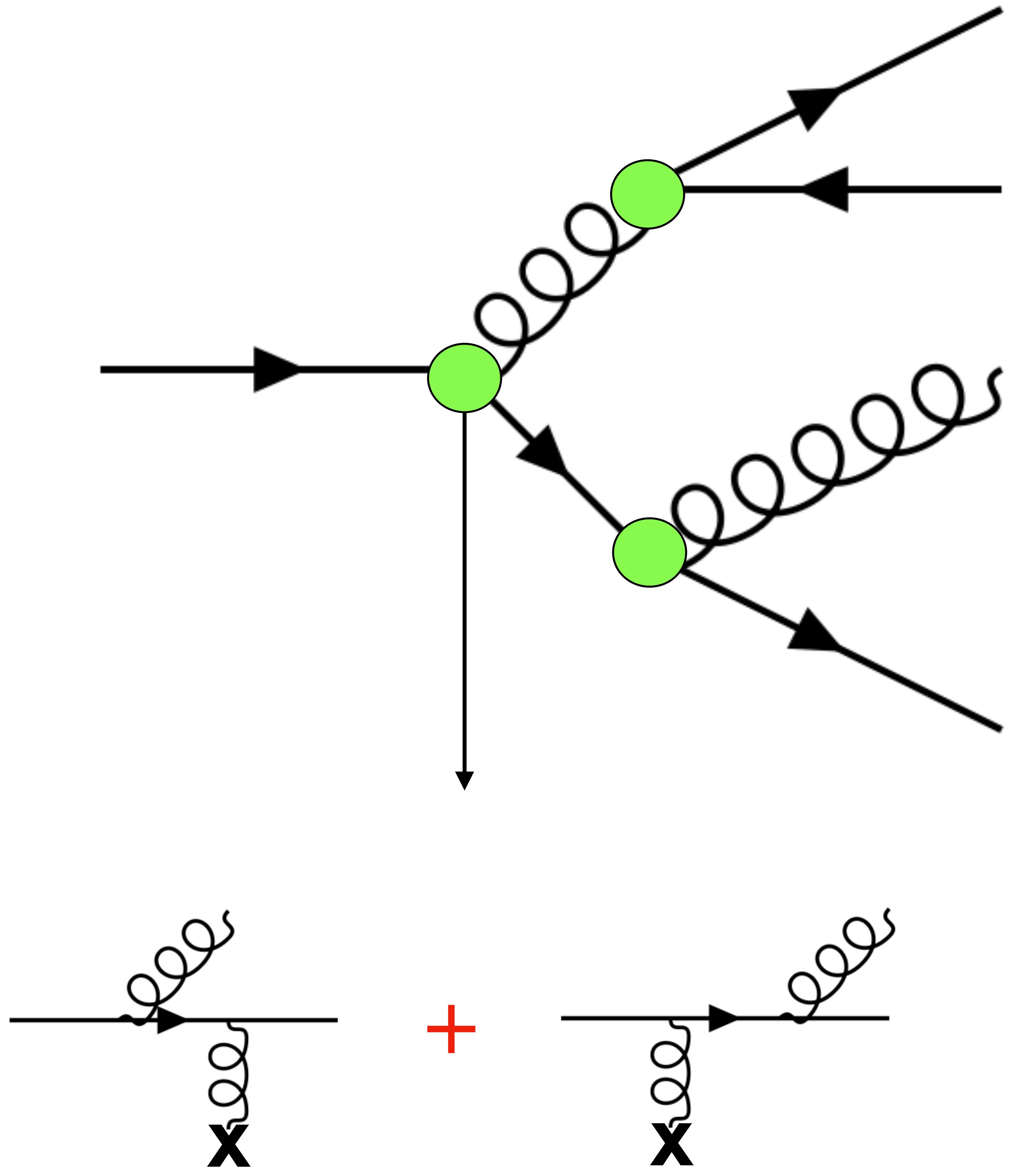
- Cosmic ray EM shower
- Shower in EM calorimeter
- A QCD shower in QGP



In-medium showers

Beithe-Hitler rate

- Each collision with the medium offers a chance for emission.
- Prob of splitting ~ α per collision
- The green region is the quantum mechanical duration of the splitting known as the formation time.



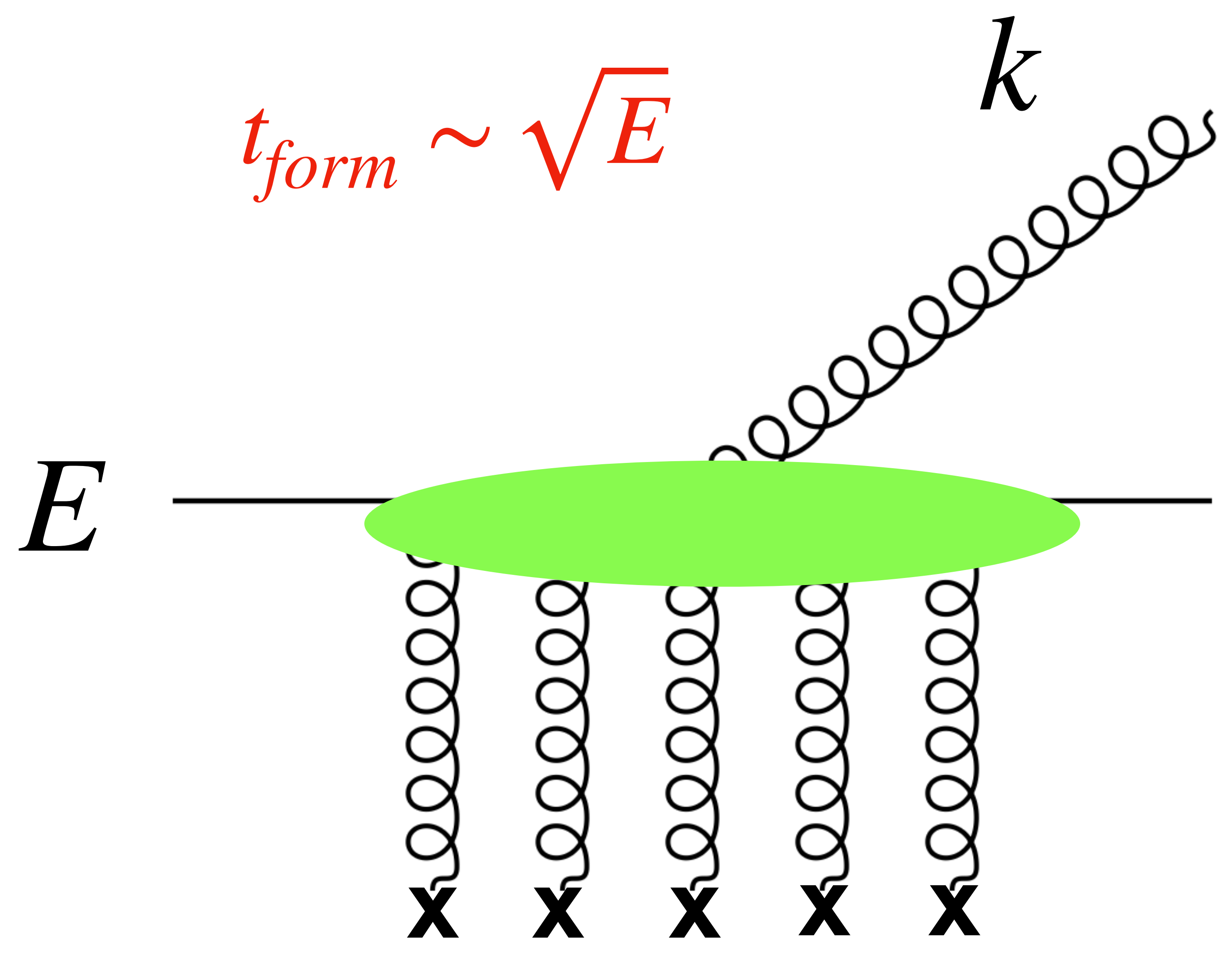
In-medium showers

LPM effect

- When $t_{form} \gg \tau_{scatt}$, many scatterings take place within one formation time. This is know as the Landau-Pomeranchuk & Migdal (LPM) effect.

- Prob of splitting ~ α per formation time.

- This reduction in the rate was figured out in the 1950's for QED (LPM) and for QCD (1990s) by BDMPS-Z.

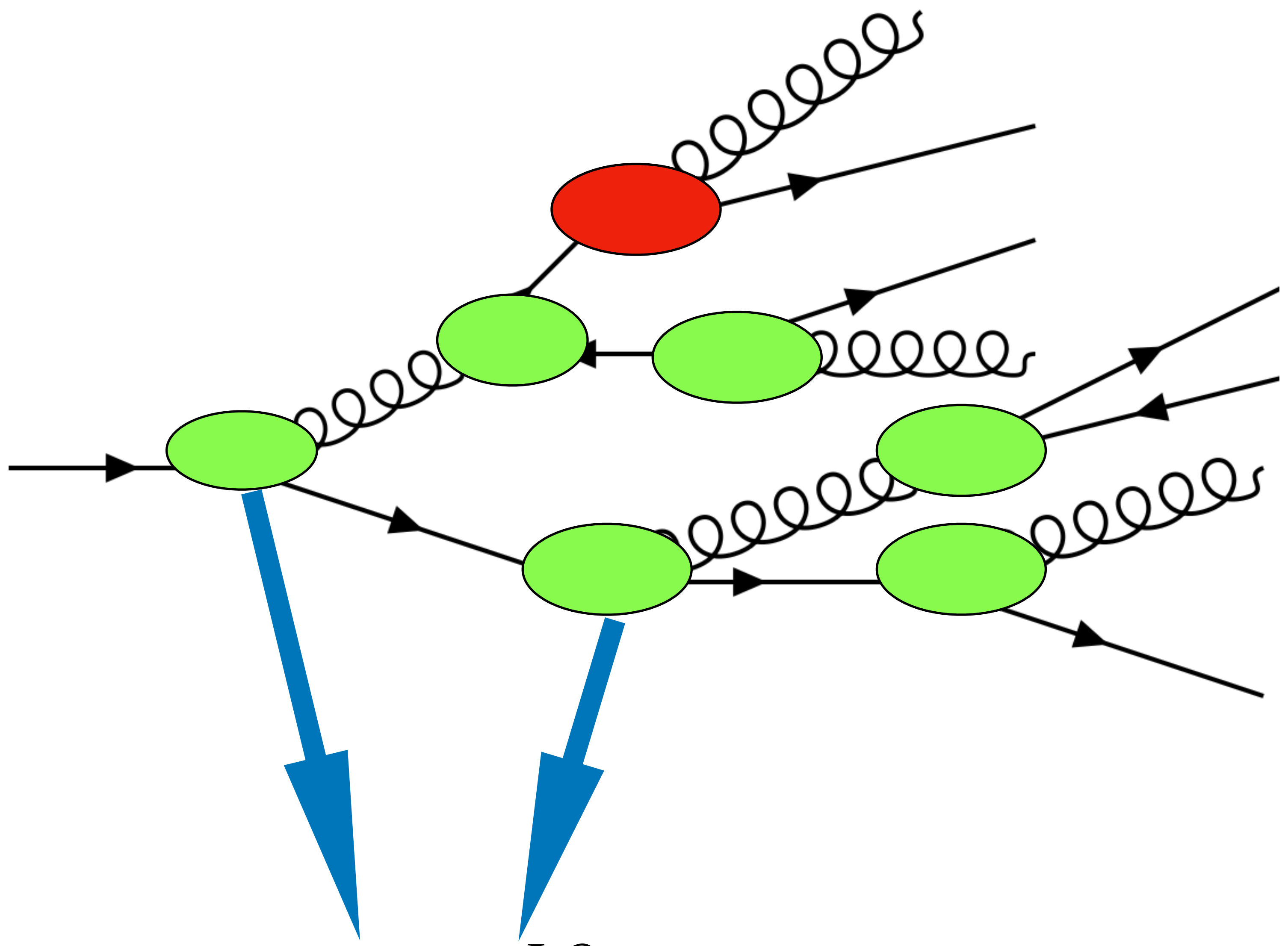


I will call the rate for splitting accounting for this effect $\left[\frac{d\Gamma}{dk}\right]^{LO}$.

Shower type

Weakly or strongly coupled ?

- Can we use the LPM splitting rate to model shower development ?



$$\left[\frac{d\Gamma}{dk}\right]^{LO}$$

LPM splitting rate

One may statistically model shower development by treating high-energy particles classically between splittings, and rolling dice based on the splitting rates to decide when and how each particle splits.

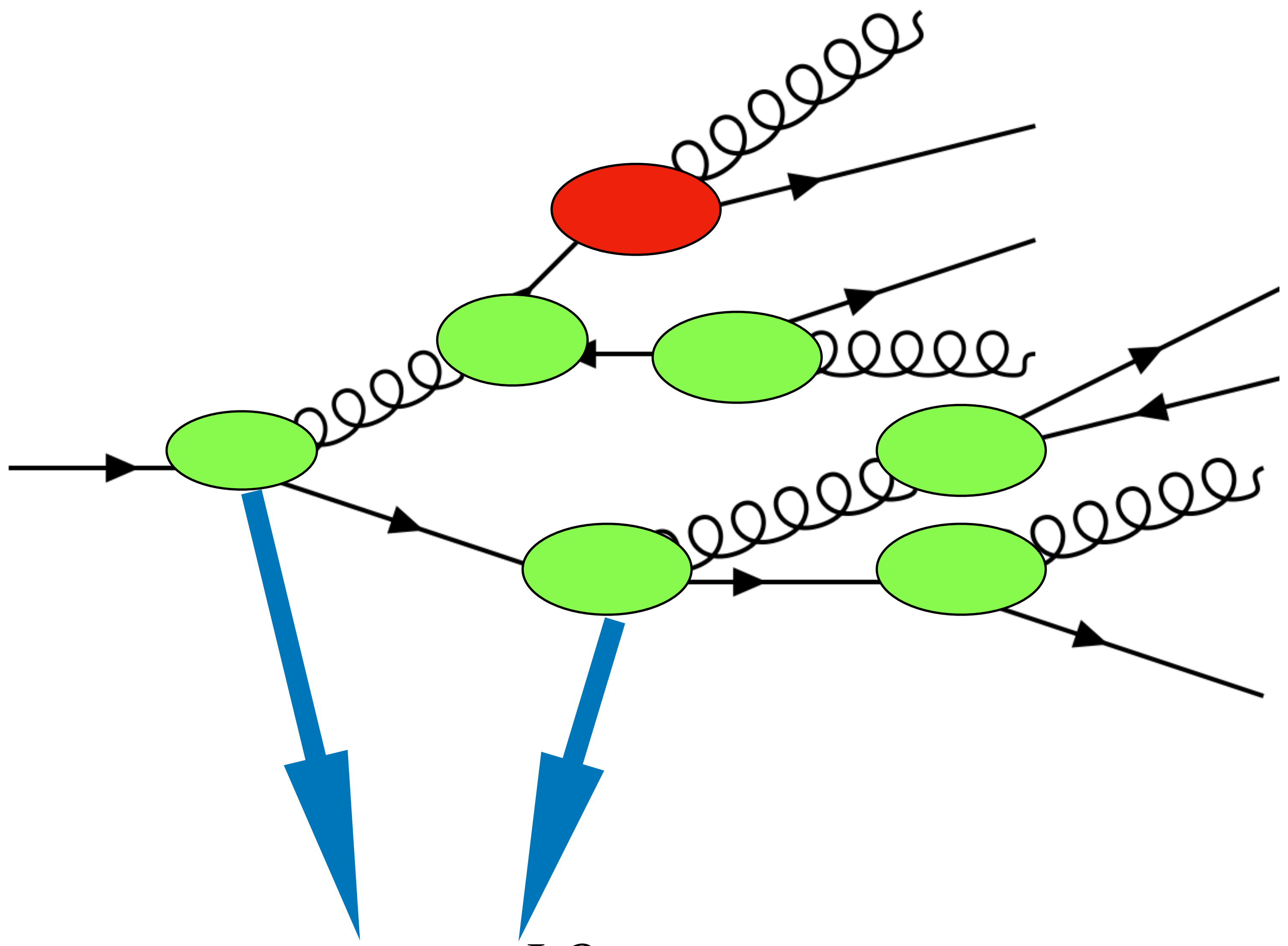
We call this a weakly coupled shower .

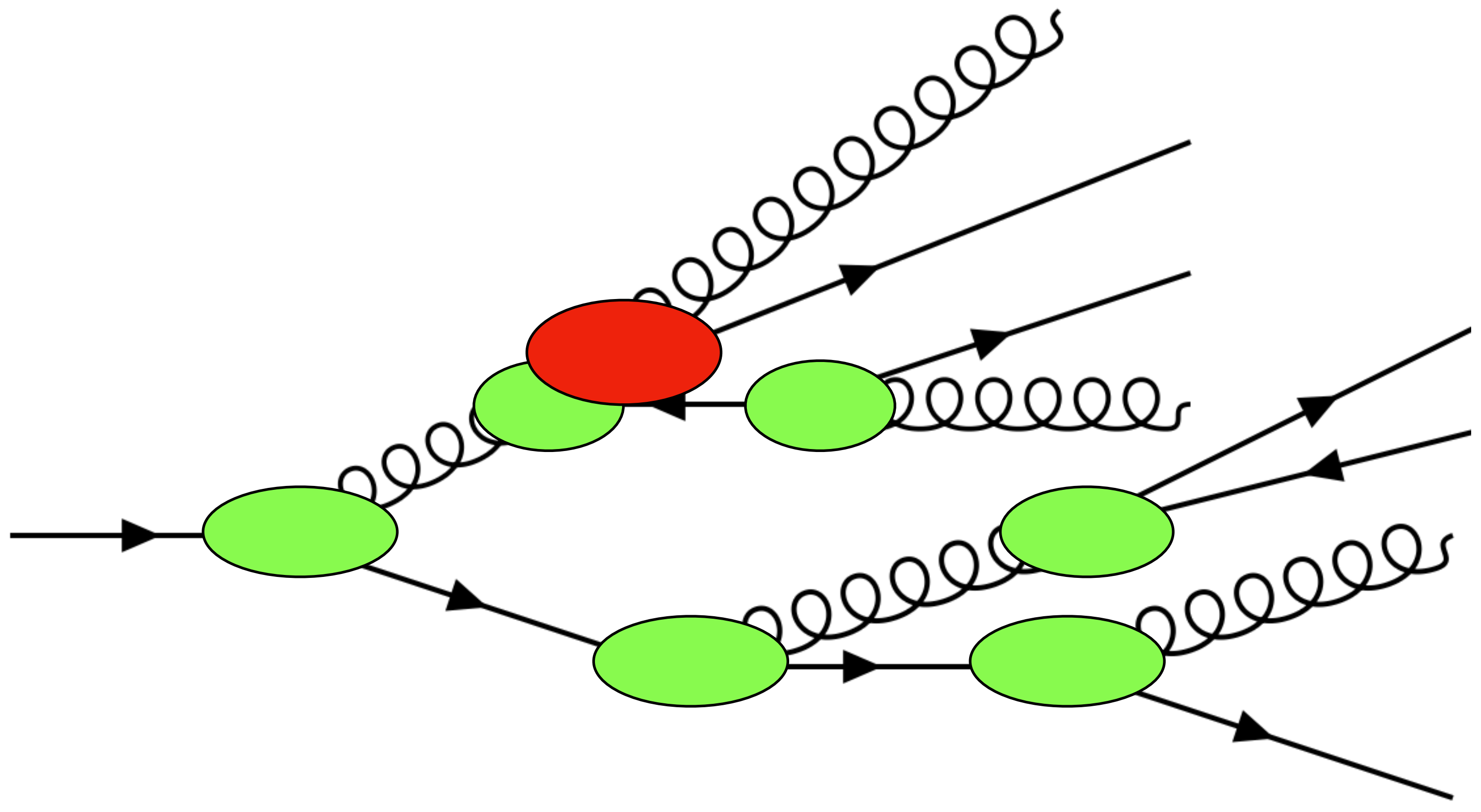
Shower type

Weakly or strongly coupled ?

- Or can splittings overlap?

Formation time grows with energy.



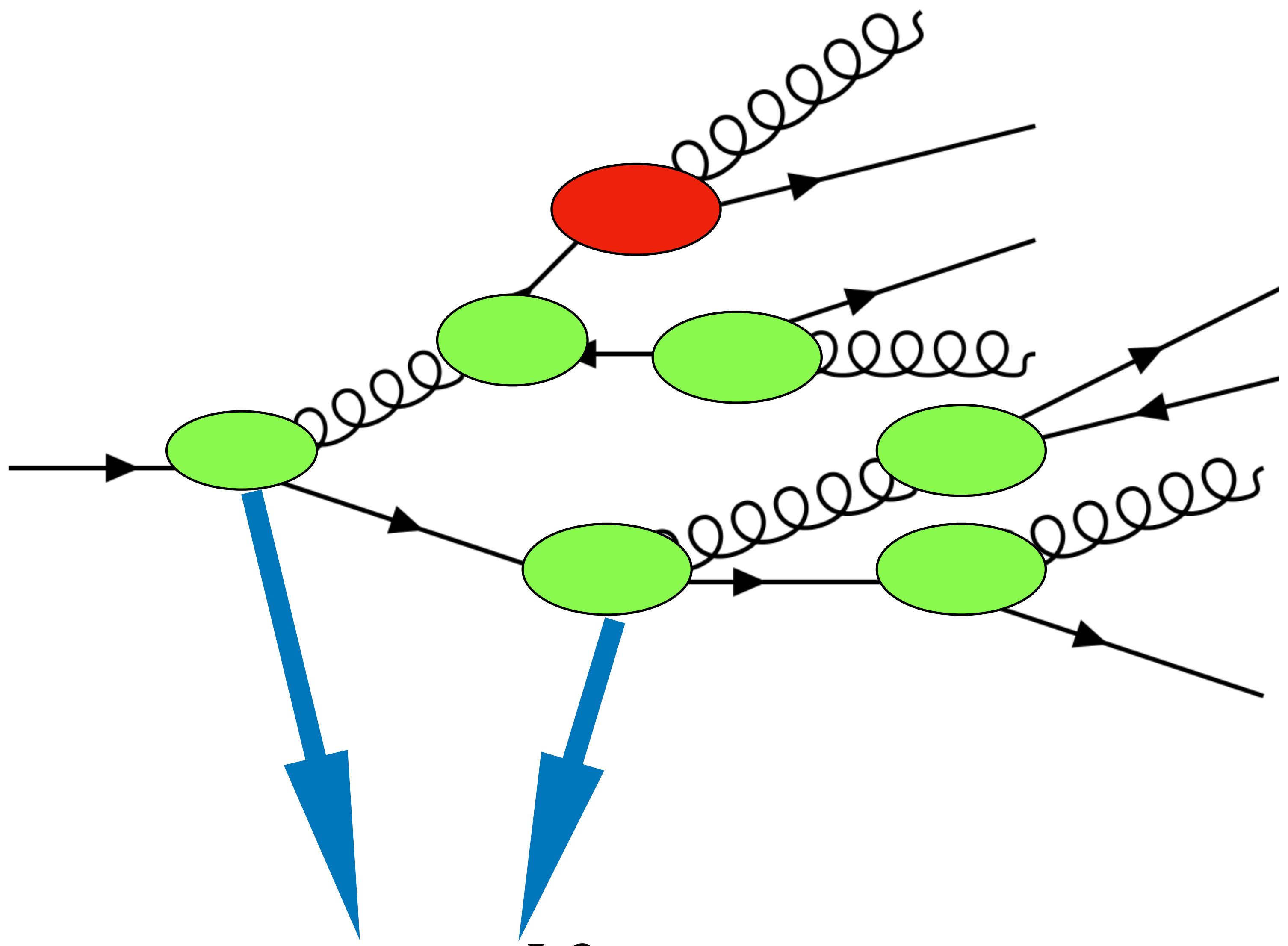


$$\left[\frac{d\Gamma}{dk}\right]^{LO}$$

LPM splitting rate

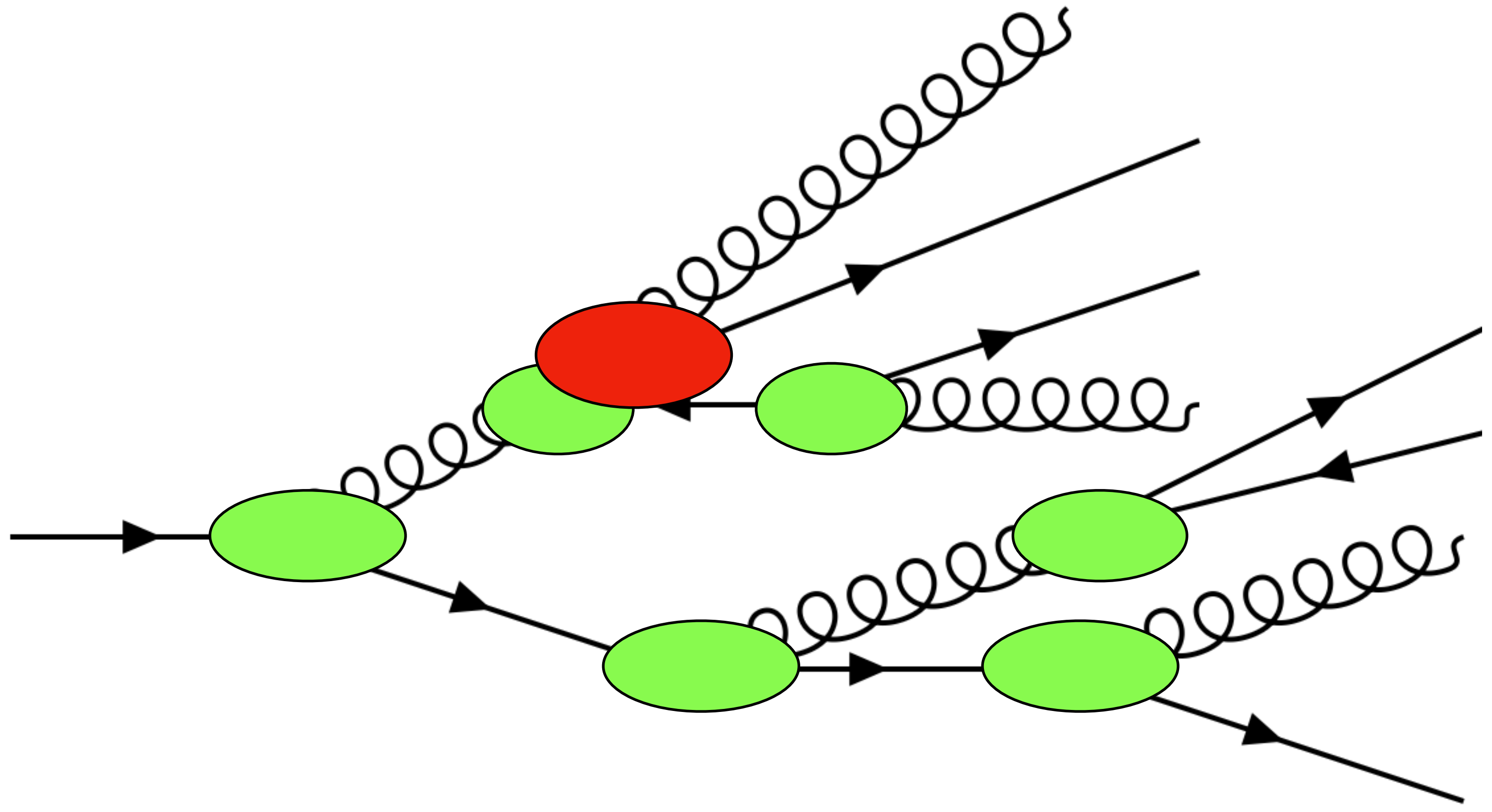
Shower type

Weakly or strongly coupled ?

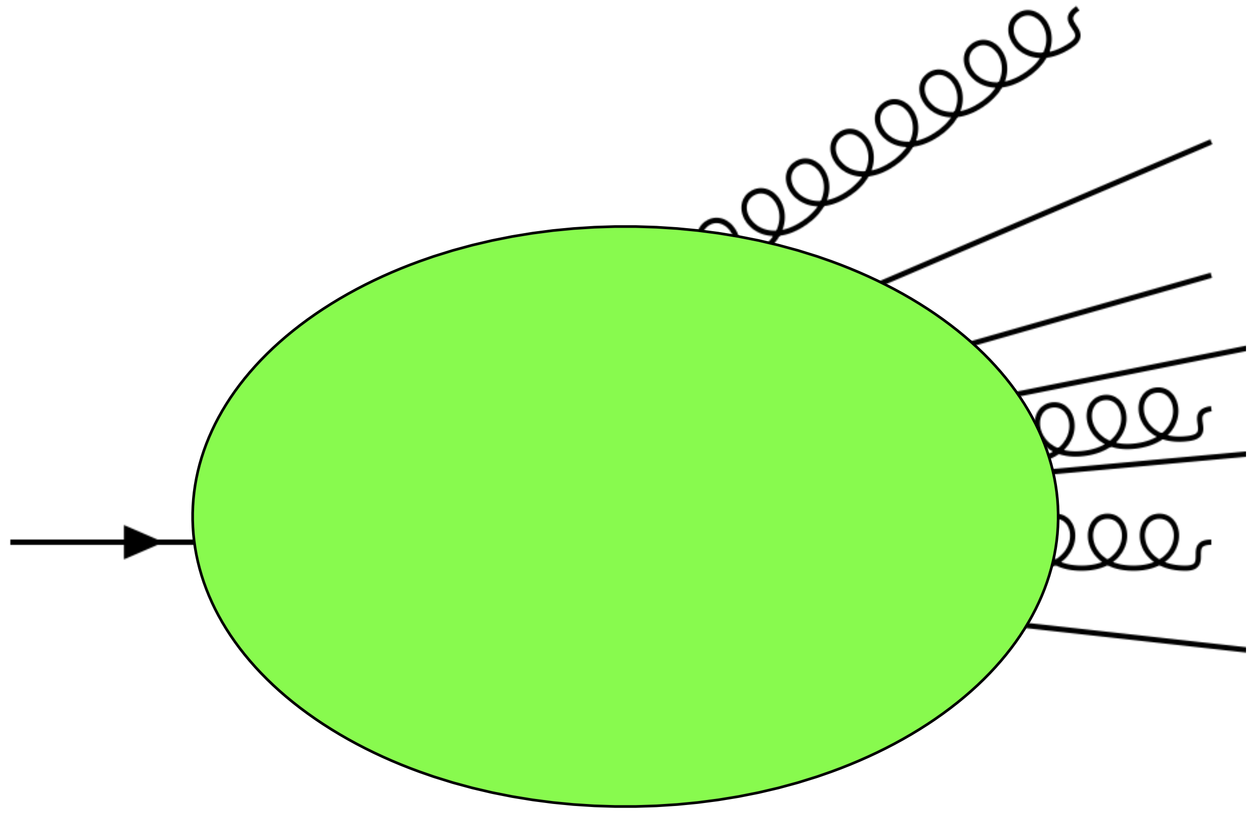


$$\left[\frac{d\Gamma}{dk}\right]^{LO}$$

LPM splitting rate



If formation times are large compared to times between splittings, one may not treat different splittings as quantum mechanically independent.

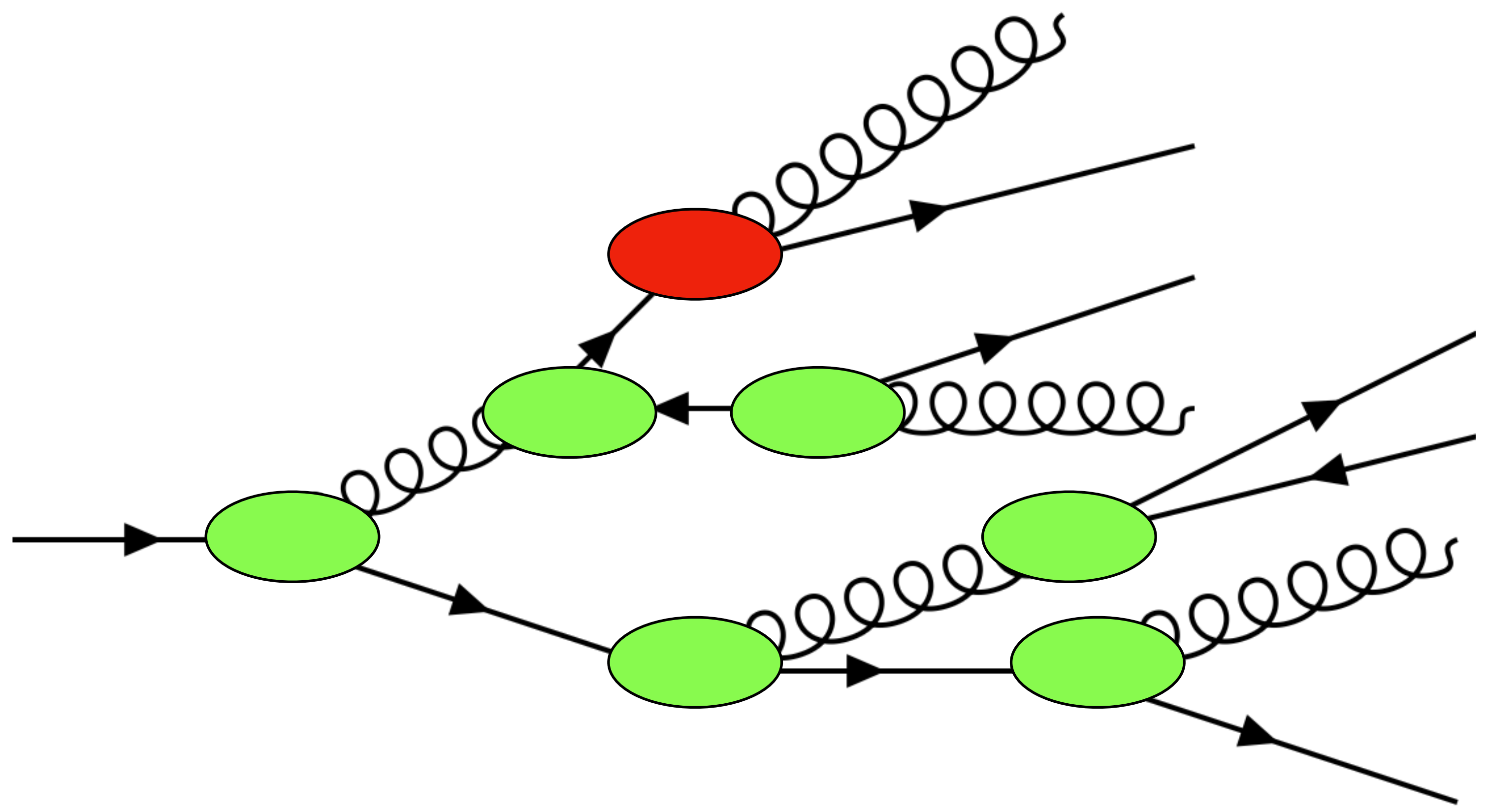


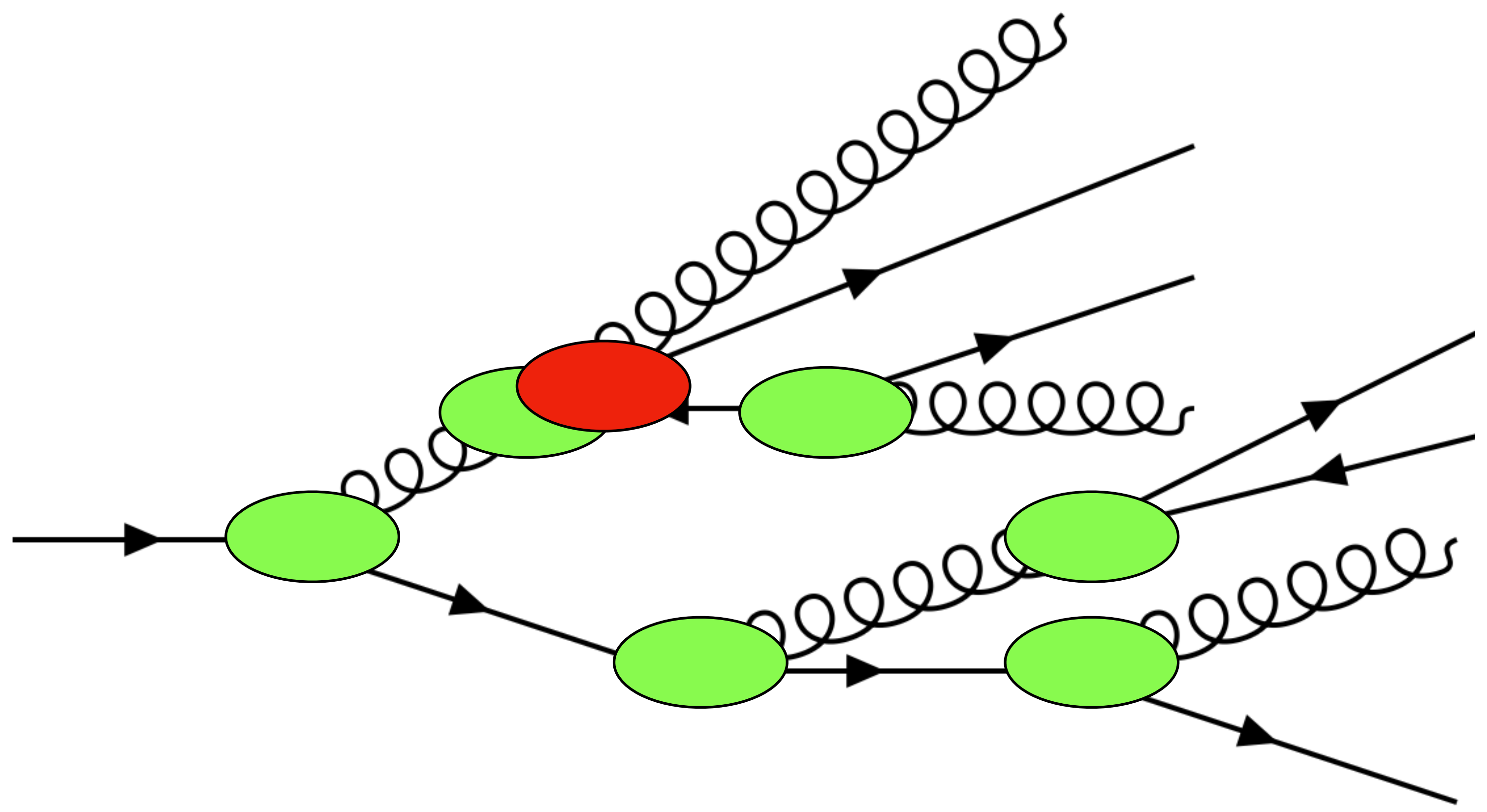
Resort to ADS/CFT

Our approach

a theoretical test

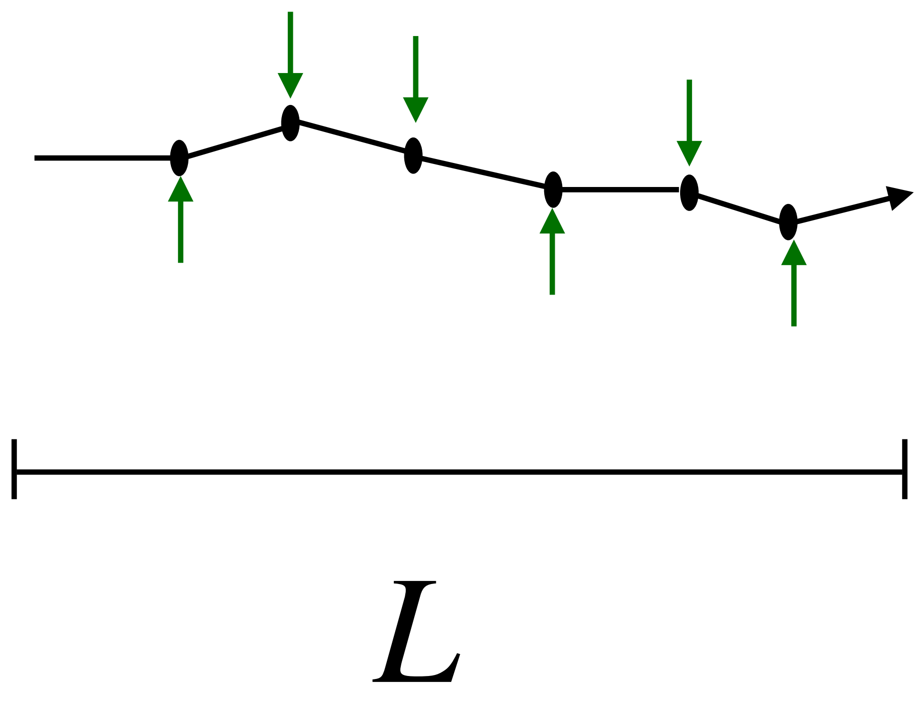
- Treat α as small, but calculate the correction to the weakly coupled picture. For reasonable values of α, how large are these corrections ?





Our approach
Assumptions

- The medium is static, homogeneous and large enough to stop the shower.
- Start with a single high energy particle that is very close to on-shell.
- Use the $\hat{q}$ approximation for elastic scattering with the medium $\langle \Delta p_\perp^2 \rangle = \hat{q} L$.



In that case, $t_{form} \sim \sqrt{E/\hat{q}}$.

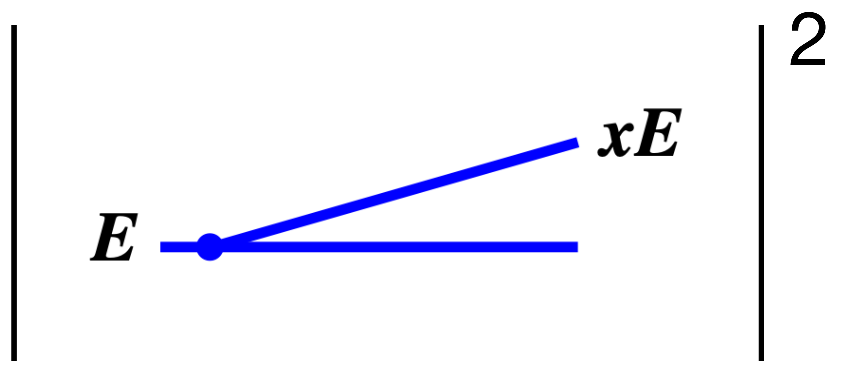
- The large N_c limit [with $N_f = 0$ (pure gluons) or $N_f \gg N_c$ (many quark flavors)].
- Integrate over $p_\perp$.

Rate Diagrams

Leading Order

- Brem or pair production

LPM/BDMPS-Z rate =



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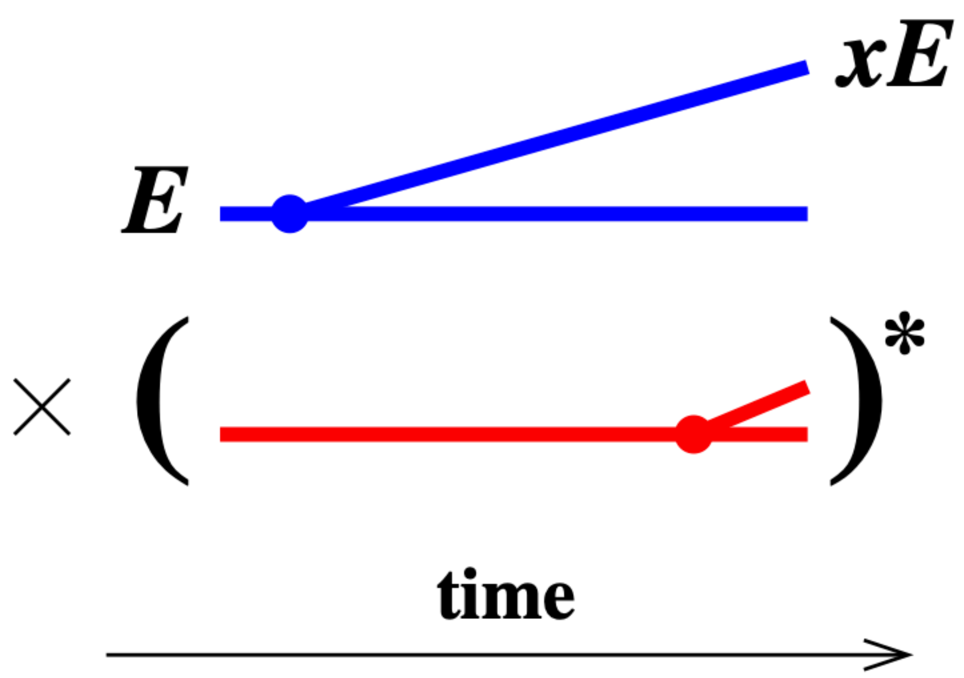
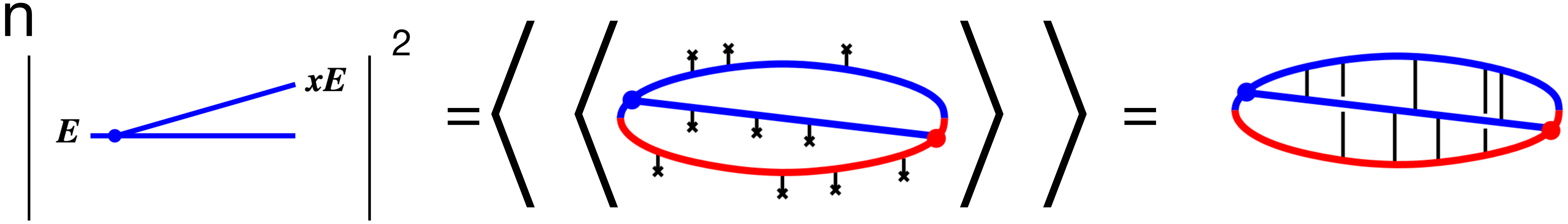


- The solid lines represents (a quark or a gluon)

Rate Diagrams

Leading Order

- Brem or pair production

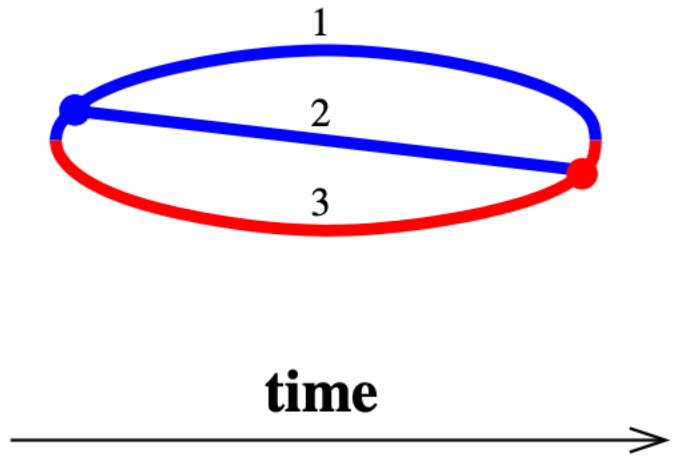
  LPM/BDMPS-Z rate = 

- When computing a rate in a random background field, one should average over that randomness.

- After this average, the interactions of the particles with the background fields, and their correlations, may be replaced with a "potential energy" term in the QM problem.

The LO calculation

Step 1: High energy approximation



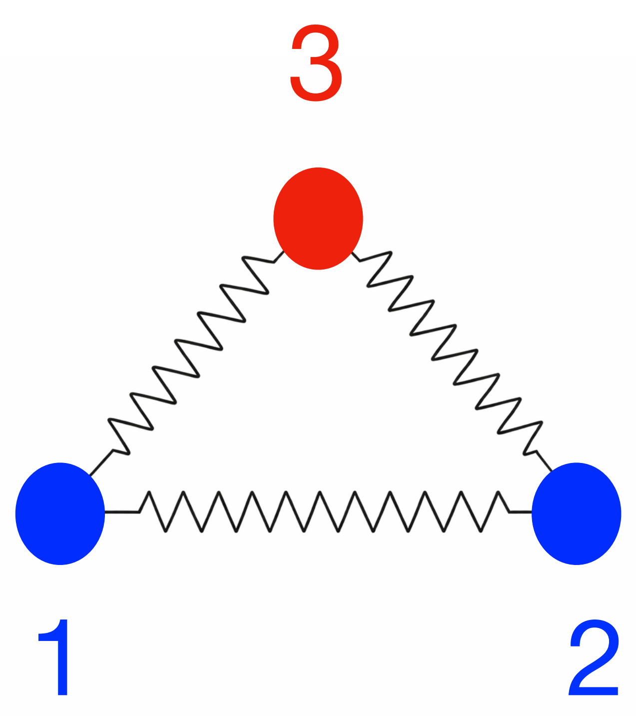
- Let's ignore medium interactions for now, and look at the particles 1,2 and 3.
- Re-interpret the diagram as *three* particles evolving forward in time.Translate calculation into a 3-particle "QM" problem.
- For particles (1,2) in the amplitude $e^{-iE_n t}$ and for particle 3 in the conjugate amplitude $e^{+iE_n t}$.
- Define a hamiltonian *H* for the total evolution e^{-iHt}, with *H*= $E_1 + E_2 - E_3$.
- Use $E = \sqrt{p_z^2 + p_\perp^2} \simeq |p_z| + \frac{p_\perp^2}{2p_z}$, to get $H \simeq \frac{p_{\perp 1}^2}{2|p_{z1}|} + \frac{p_{\perp 2}^2}{2|p_{z2}|} - \frac{p_{\perp 3}^2}{2|p_{z3}|}$.
- This looks like a 2-d QM kinetic energy term $p_{zi} \longrightarrow M_i$, except $M_3 < 0$.

The LO calculation

Step 2: the "potential energy"

- In the $\hat{q}$ approximation, the potential takes the form

$$V(\Delta b) = -\frac{i}{4}\hat{q}(\Delta b)^2$$



- For $g \to gg$,

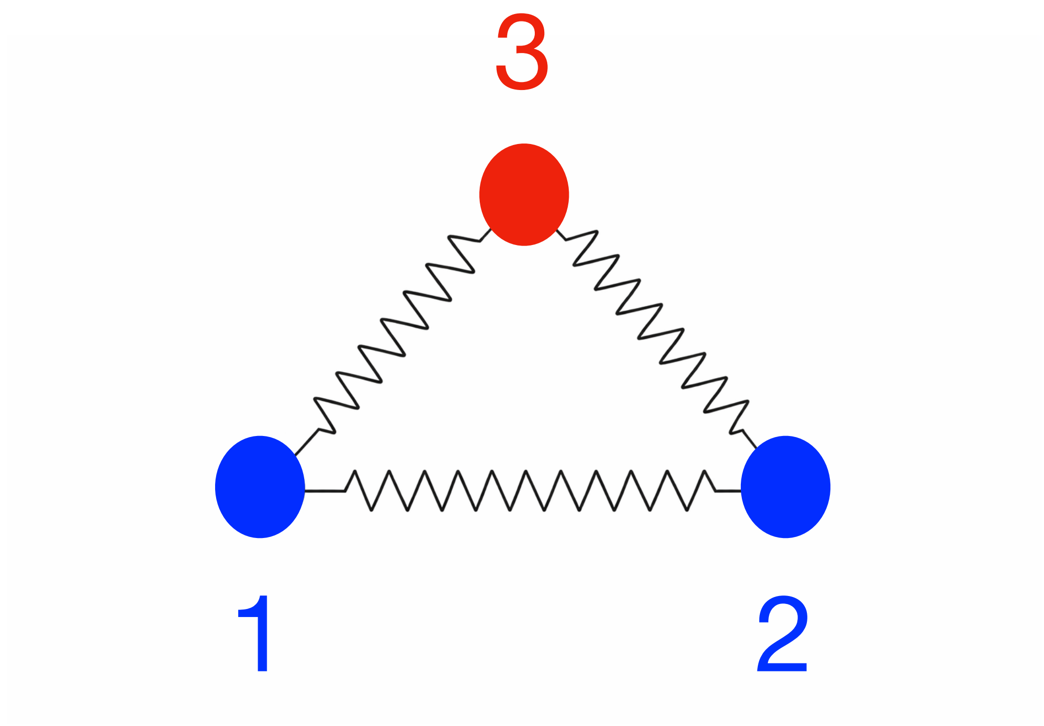
$$V(b_1, b_2, b_3) = V(b_{12}) + V(b_{23}) + V(b_{31}) = -\frac{i\hat{q}_A}{8}\left[(b_2 - b_1)^2 + (b_3 - b_2)^2 + (b_1 - b_3)^2\right]$$

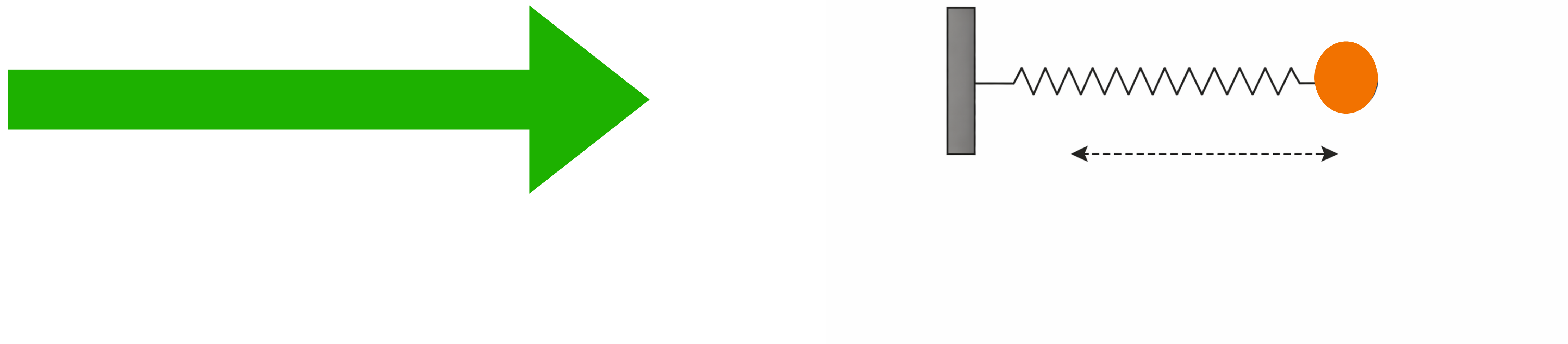
- This decomposition into 2-body potentials is valid for (i) weak coupling, (ii) for any coupling in the $\hat{q}$ approximation (iii) and any coupling in the large N_c limit.

The LO calculation

Step 3: Reduce N to N-2

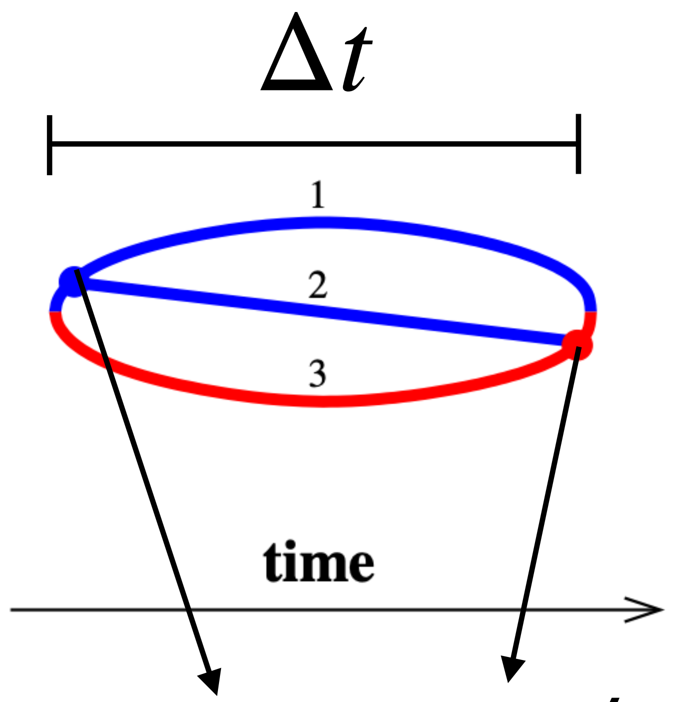
- We can use (i) 2d translation invariance and (ii) 3d rotational invariance, to reduce the *3-particle* harmonic oscillator problem to an effective *1-particle* problem.
- $H = \dfrac{P_\perp^2}{2M} + \dfrac{M\Omega_0^2 B^2}{2}$, with a complex frequency Ω_0.
- $P_\perp = x_j p_{\perp i} - x_i p_{\perp j}$ and $B = \dfrac{b_i - b_j}{x_i + x_j}$, which satisfies $[P_\perp, B] = -i$.





The LO calculation

Putting it all together



- Zakharov's form of the calculation
- $$\frac{d\Gamma}{dx} = \alpha_s\, P(x)\, 2\, Re \int_0^\infty d(\Delta t)\ \nabla_B \cdot \nabla_{B'} \langle B, \Delta t \,|\, B', 0 \rangle \Big|_{B=0, B'=0}$$

DGLAP splitting function

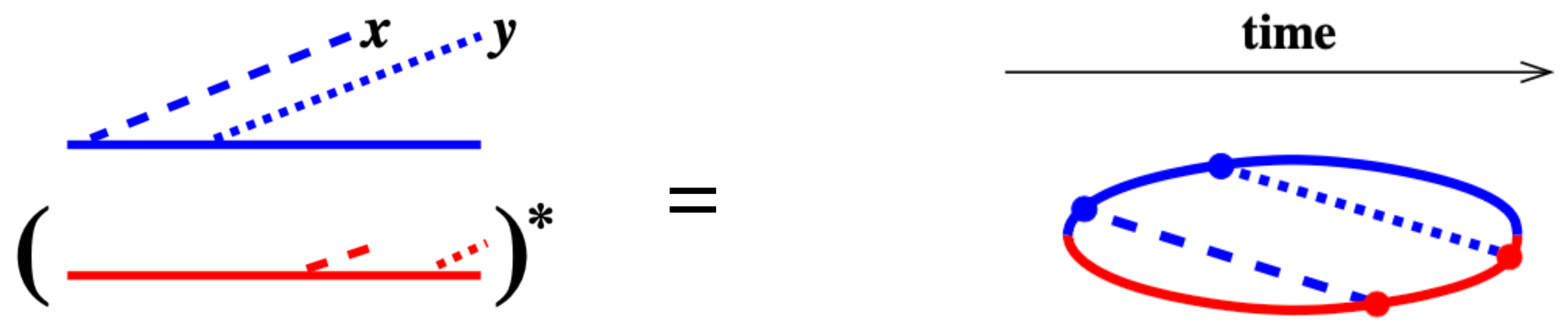
the $\hat{q}$ approximation →HO propagator

- The result is simple $\frac{d\Gamma}{dx} = \frac{\alpha_s}{\pi} P(x) Re\{i\Omega_0\}$, with complex $\Omega_0 = \sqrt{\frac{-i\hat{q}_A}{2x(1-x)E}}$

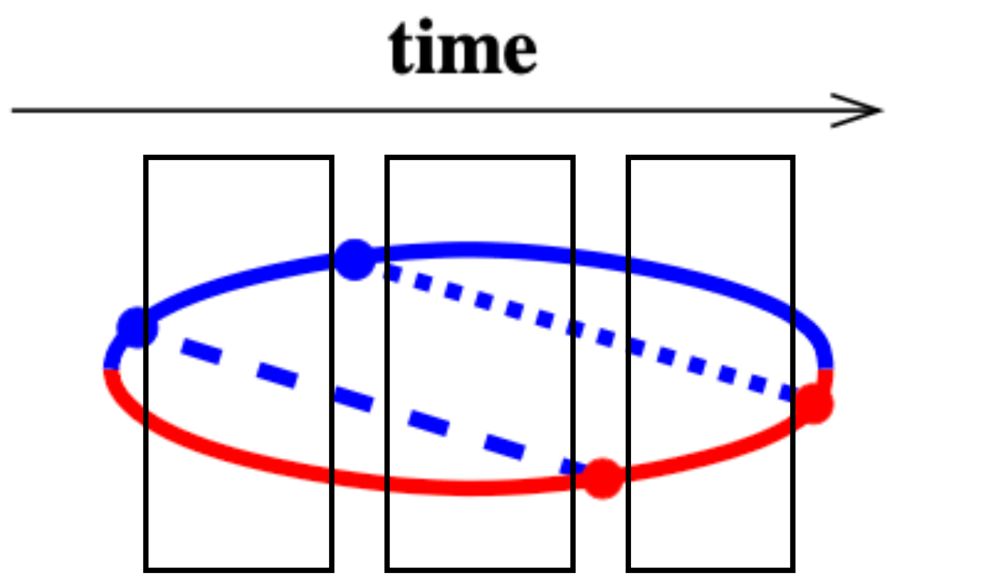
The NLO calculation

Overlapping formation times

- Let's consider an example of such a diagram in Zakharov's description.



- As we saw before, we put together QFT matrix elements for each vertex, then evolve in between with QM propagators.



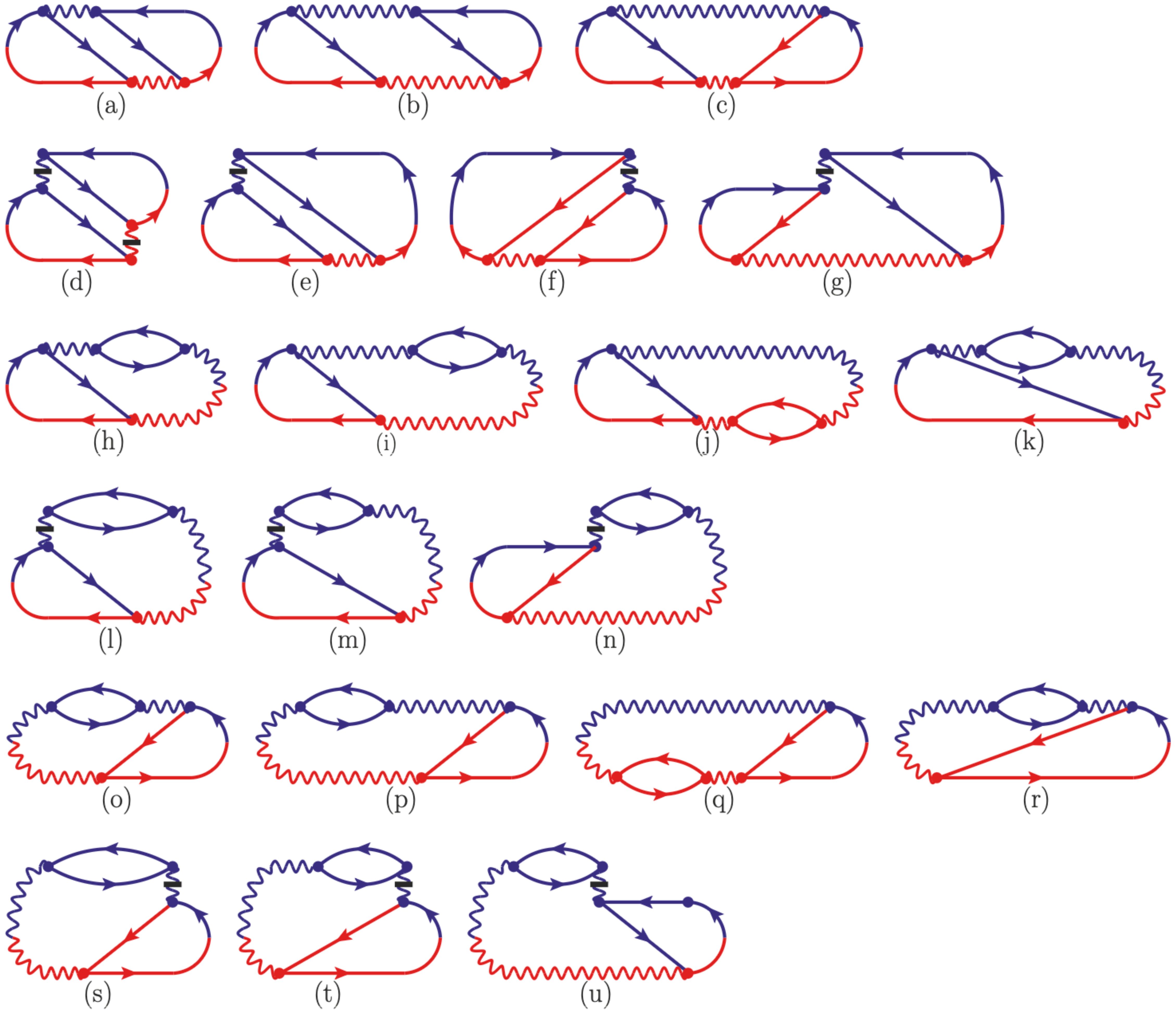
3-particle 4-particle 3-particle

Using symmetries of the problem→ 1-particle 2-particle 1-particle

The NLO calculation

Time-ordered diagrams (Real & virtual)

- For large-N_f QCD



QED result

Overlap corrections $= -99\,\%\,N_f\alpha(\mu)$
($N_f \gg 1$, many flavors)

[P.Arnold & S.Iqbal 2019]

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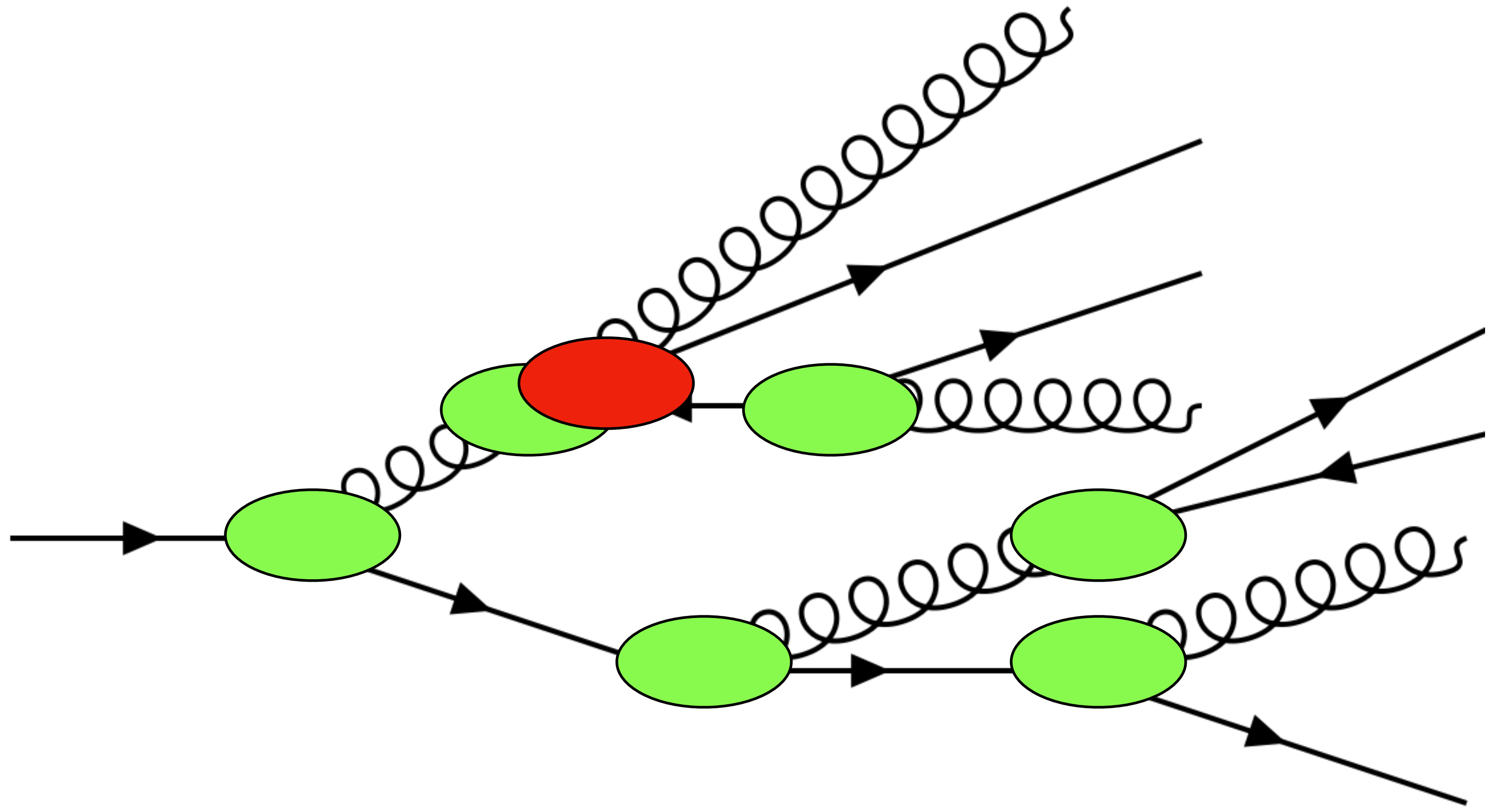

QCD result

Overlap corrections $= -1\,\%\,N_c\alpha_s(\mu)$
(pure gluons)

[P.Arnold & **OE** & S.Iqbal]

Phys.Rev.Lett. 131 (2023) 16, 162302

Overlap corrections $= -0.4\,\%\,N_f\alpha_s(\mu)$
($N_f \gg N_c \gg 1$, many quarks)

[P.Arnold & **OE** & S.Iqbal]

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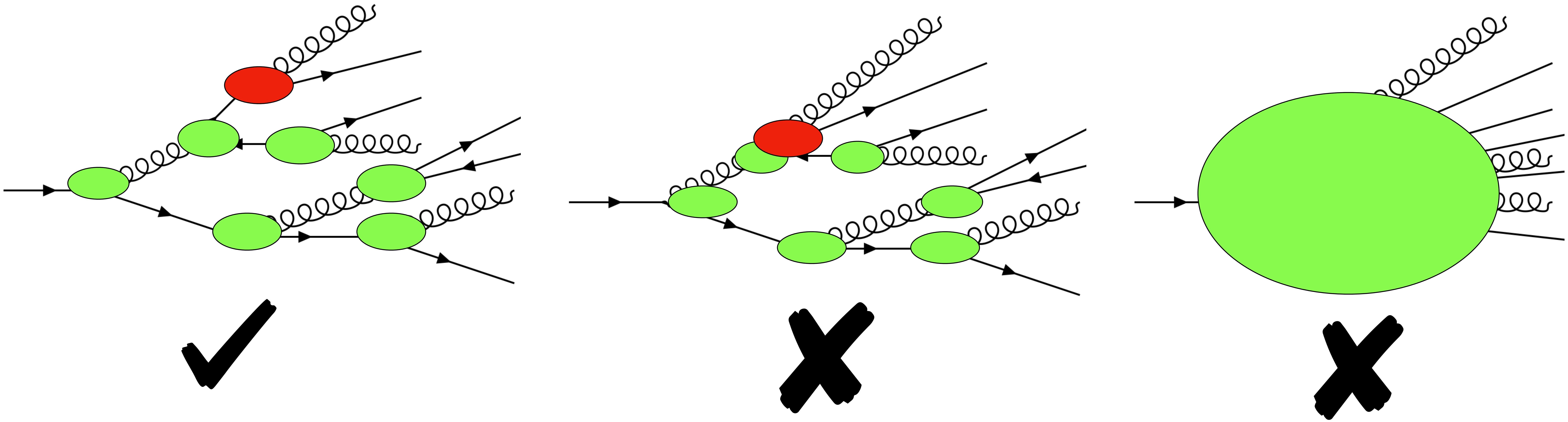
Comments

QCD: Weakly coupled shower

- Using our simplifying assumptions and theoretical observables :

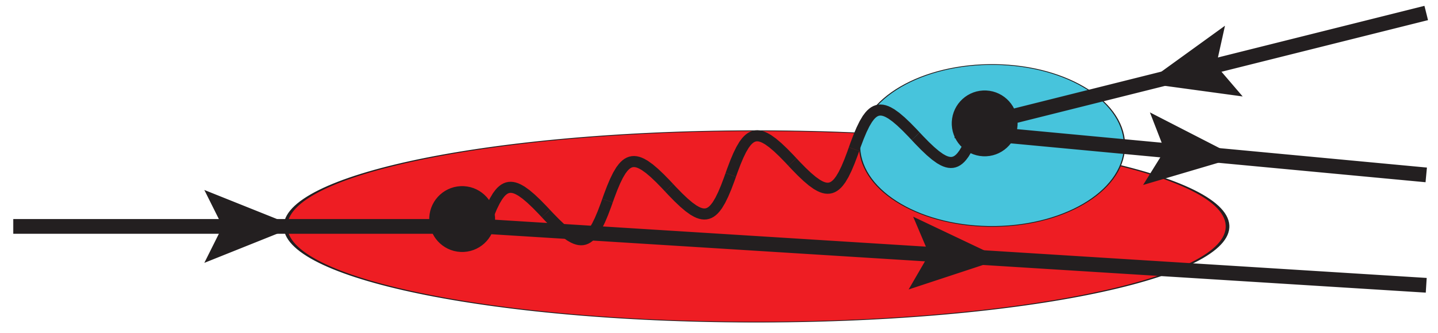


- Provided one treats $\hat{q}$ as a parameter that can be measured at one scale and then evolved to other scales (known for LL).

Comments

Why QED is different than QCD ?

- Large corrections coming from overlapping soft bremsstrahlung with a subsequent pair production.
- $t_{form}^{e\to e\gamma} \sim \sqrt{\dfrac{E}{x_\gamma\,\hat{q}}}$ in the limit $x_\gamma \to 0.$

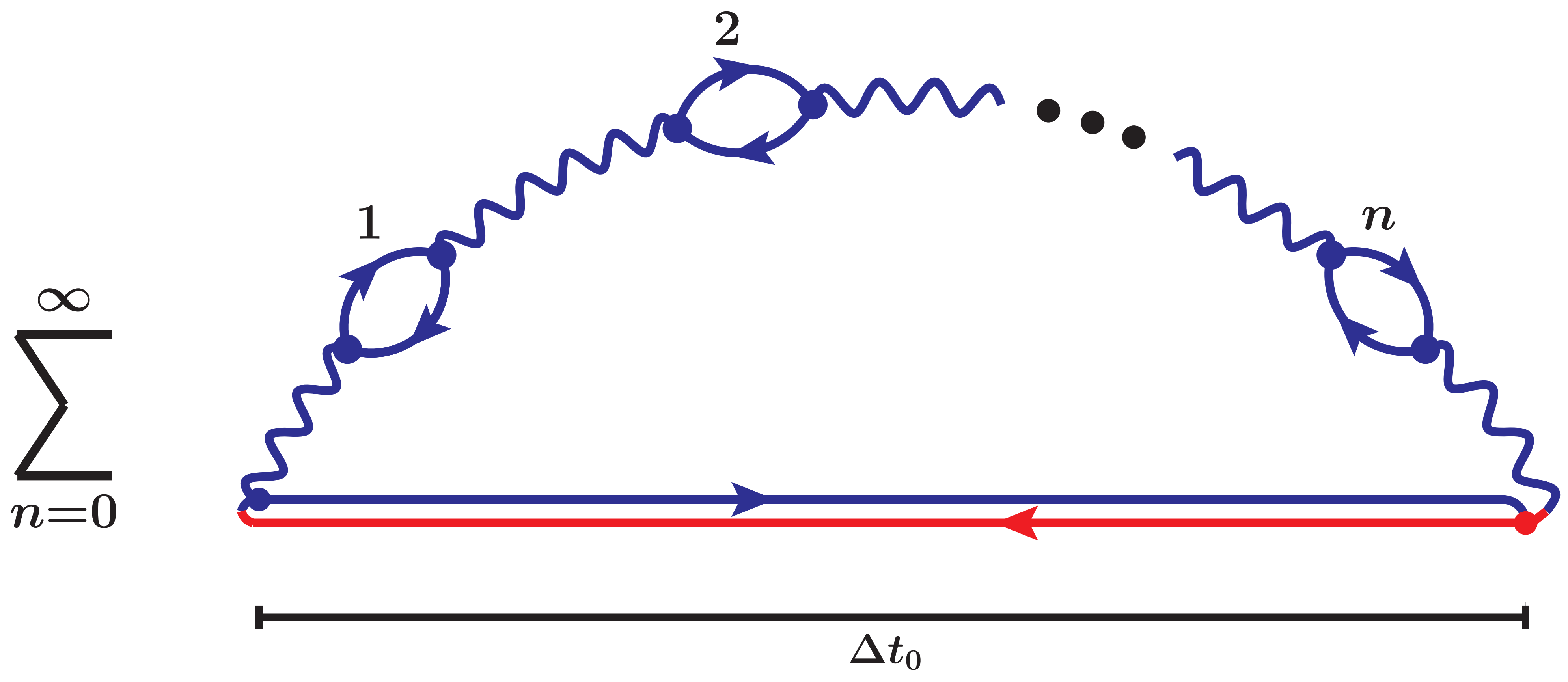


- The soft e^+e^- can be easily scattered by the QED medium, which destroys the coherence of the brem.
- Soft gluons already interact easily with the QCD medium and so overlap does not make the same qualitative change.

Comments

QED

- Our theoretical observable depends on the NLO/LO ratio
- $$\frac{\delta\left[\frac{d\Gamma}{dx_\gamma}\right]_{e\to e}}{\left[\frac{d\Gamma}{dx_\gamma}\right]_{e\to e}} \sim \frac{N_f\,\alpha}{x_\gamma}$$ in the limit $x_\gamma \to 0.$
- A precise calculation of the rate for $x_\gamma \lesssim N_f\,\alpha$ in QED would require resuming higher-order effects.

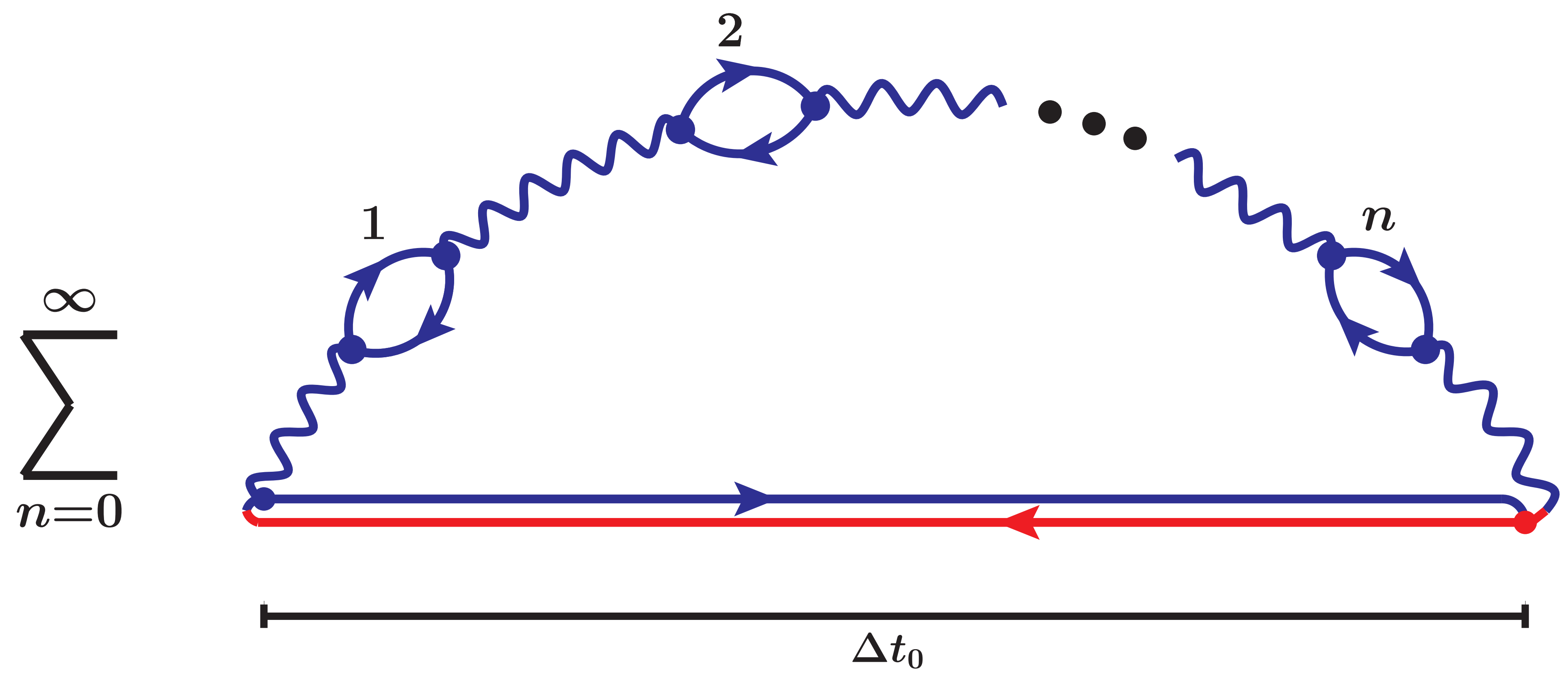


The cost of adding a loop is $\frac{N_f\,\alpha}{x_\gamma}$.

Bubble Resummation

Resumed rate

- In the soft photon limit, the rate factorizes into:



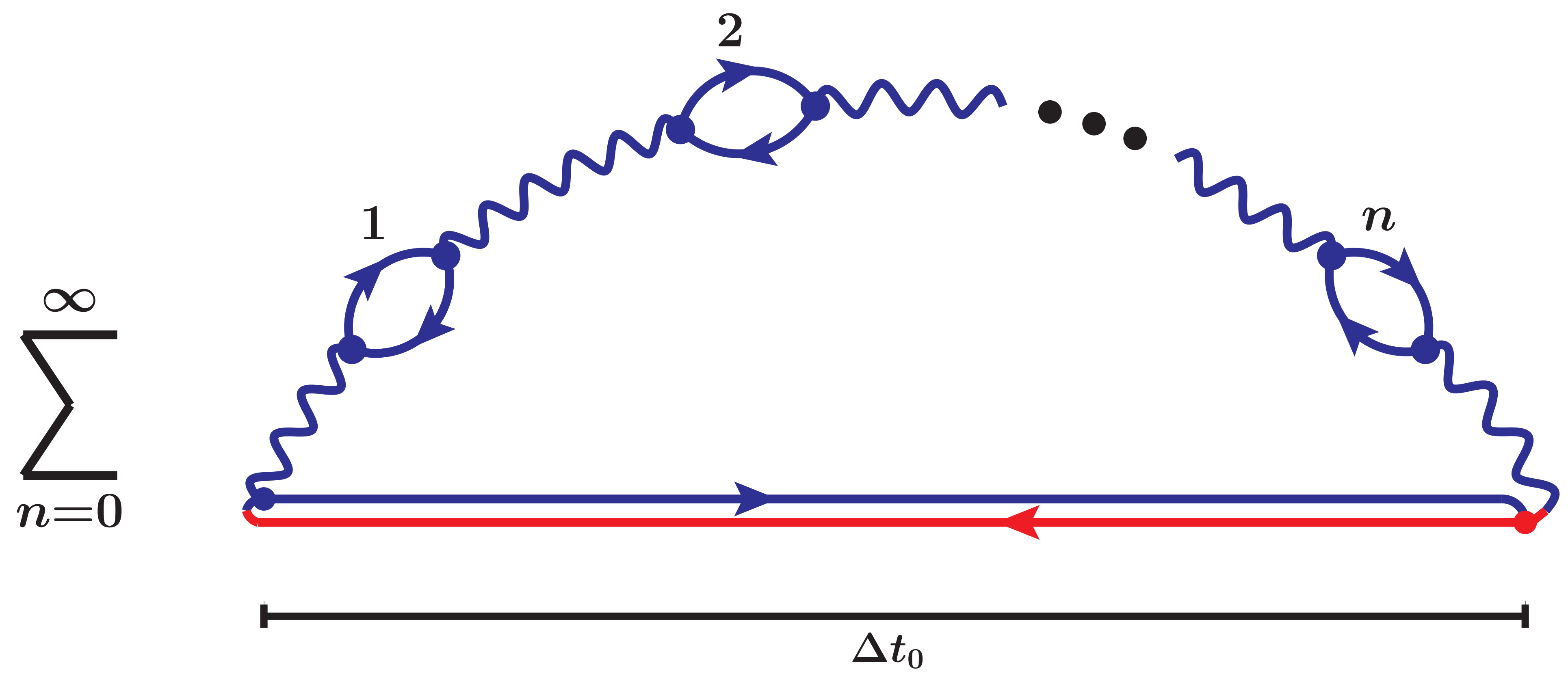
$$\left[\frac{d\Gamma}{dx_\gamma}\right]_{n\geq 0} \simeq 2Re\int_0^\infty d(\Delta t_0)\left[\frac{dg}{dx_\gamma d\Delta t_0}\right]_{brem} e^{-g_{pair}\Delta t_0}$$

Note $\Gamma_{pair} = 2Re\, g_{pair}$

Bubble Resummation

Resumed rate

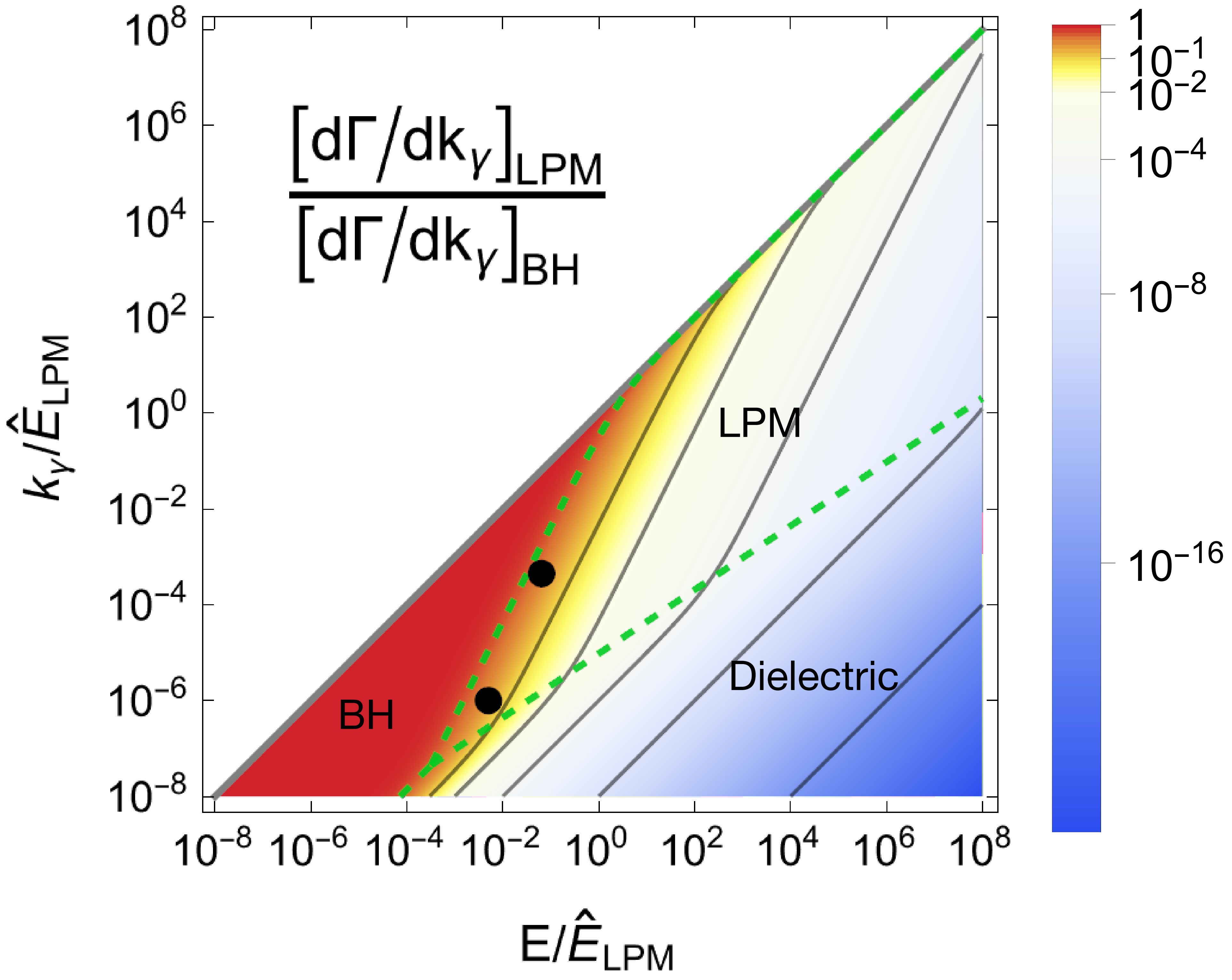
- In the soft photon limit, the resumed LPM rate becomes



$$\left[\frac{d\Gamma}{dx_\gamma}\right]_{\mathrm{LPM+}} = \left[\frac{d\Gamma}{dx_\gamma}\right]_{\mathrm{LPM}} + \delta\left[\frac{d\Gamma}{dx_\gamma}\right]$$

Results

LPM rate



Here $\alpha = 1/137$ and $m_\gamma/m = 10^{-4}$ (roughly the value for Lead or Gold).

Given the scaling of the axes, the plots are independent of the value of E_{LPM}.

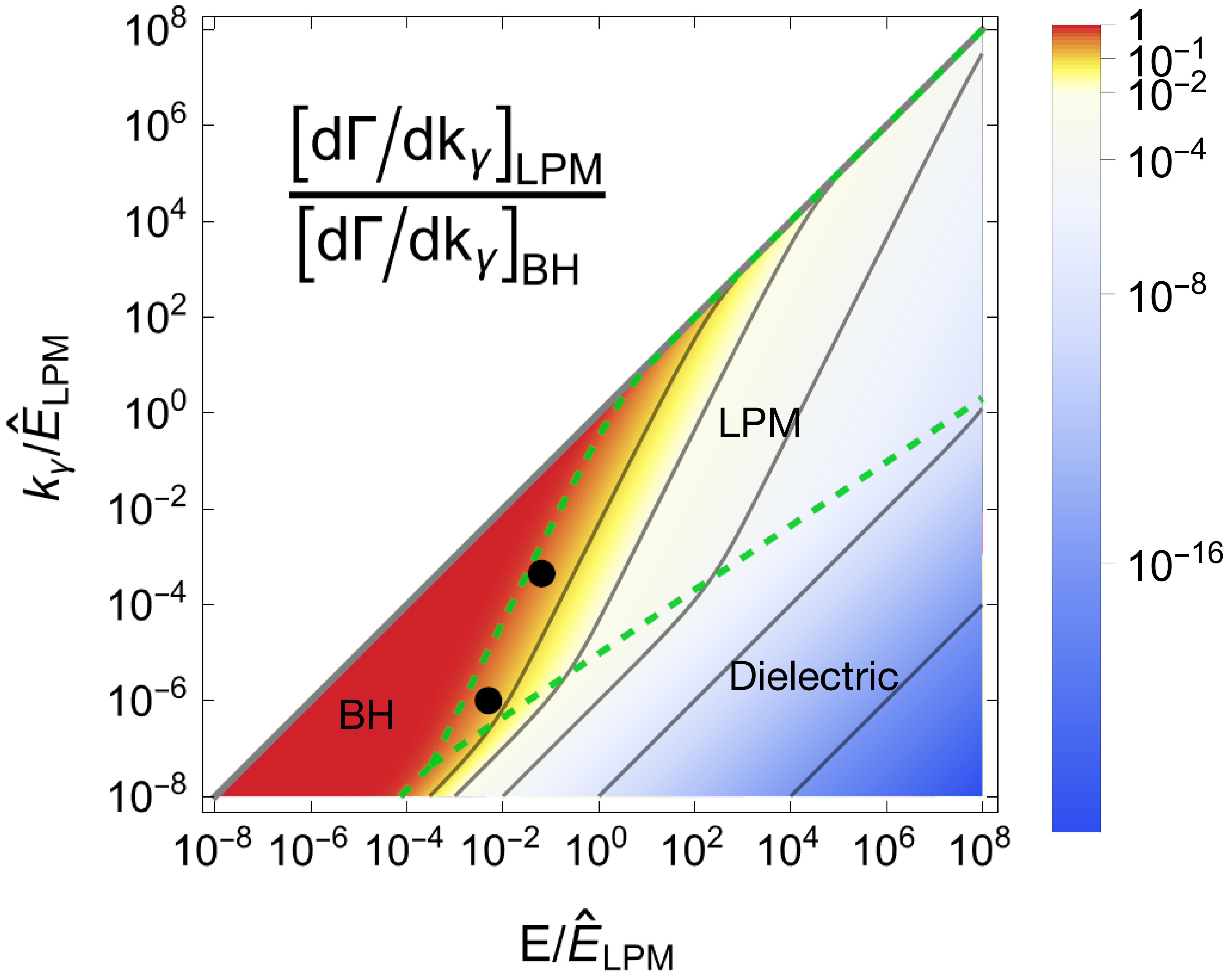
The value of m_γ/m only affects the location of the dielectric-effect regions

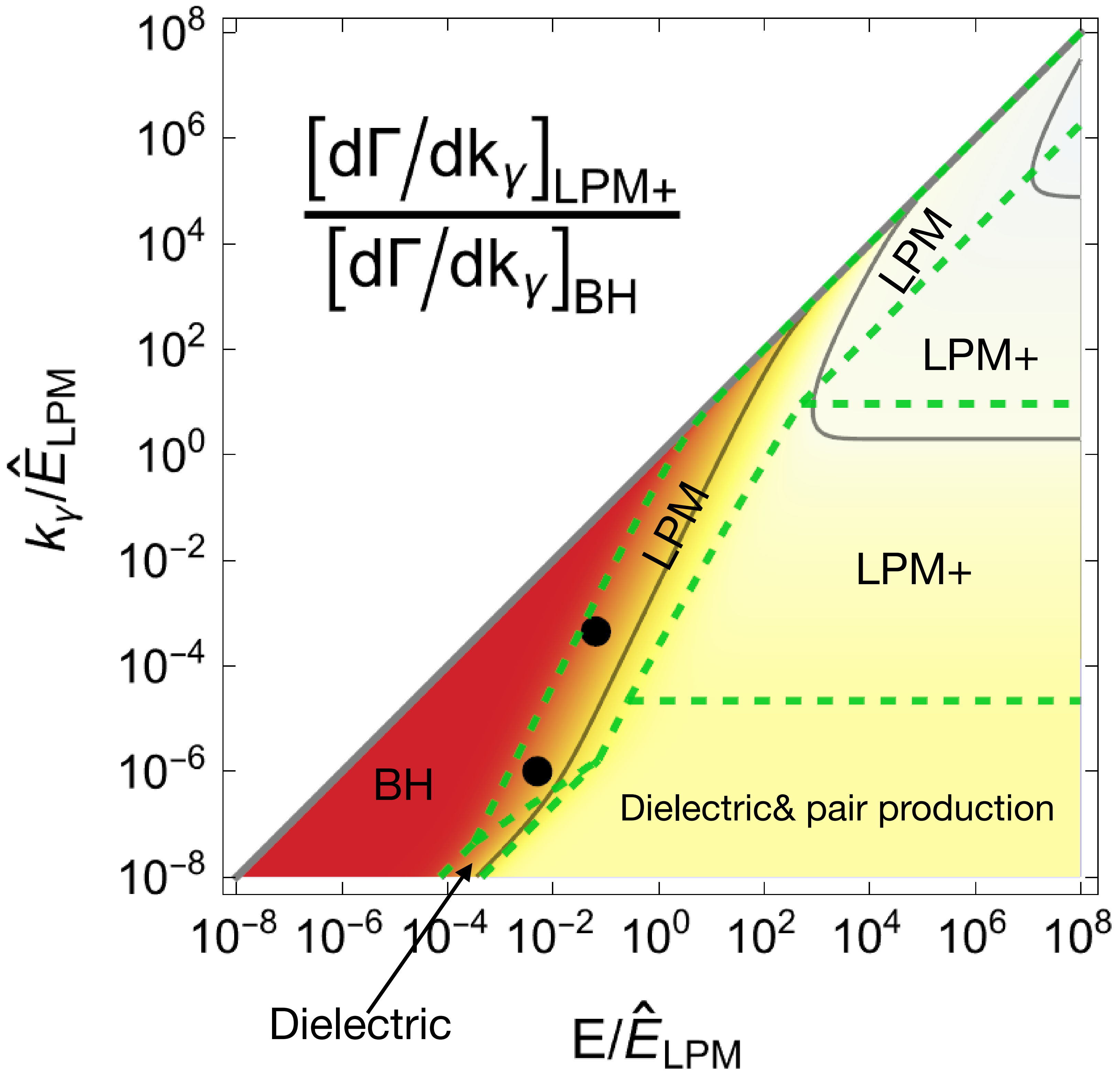
The two black dots indicate 2 examples each of (k_γ, E) from the range of values tested by two dedicated LPM experiments: $(5\,MeV, 25\,GeV)$ in Gold and $(2\,GeV, 287\,GeV)$ in Iridium, for which $\hat{E}_{LPM} \simeq 2E_{LPM} \simeq 5.0$ and $4.5\,TeV$ respectively.
[Klien, Reviews of Modern Physics, Vol. 71, No. 5, October 1999]

E_{LPM} is an energy scale determined by properties of the medium and is, for example, about $E_{LPM} = 2.5\,TeV$ for Gold.

Results

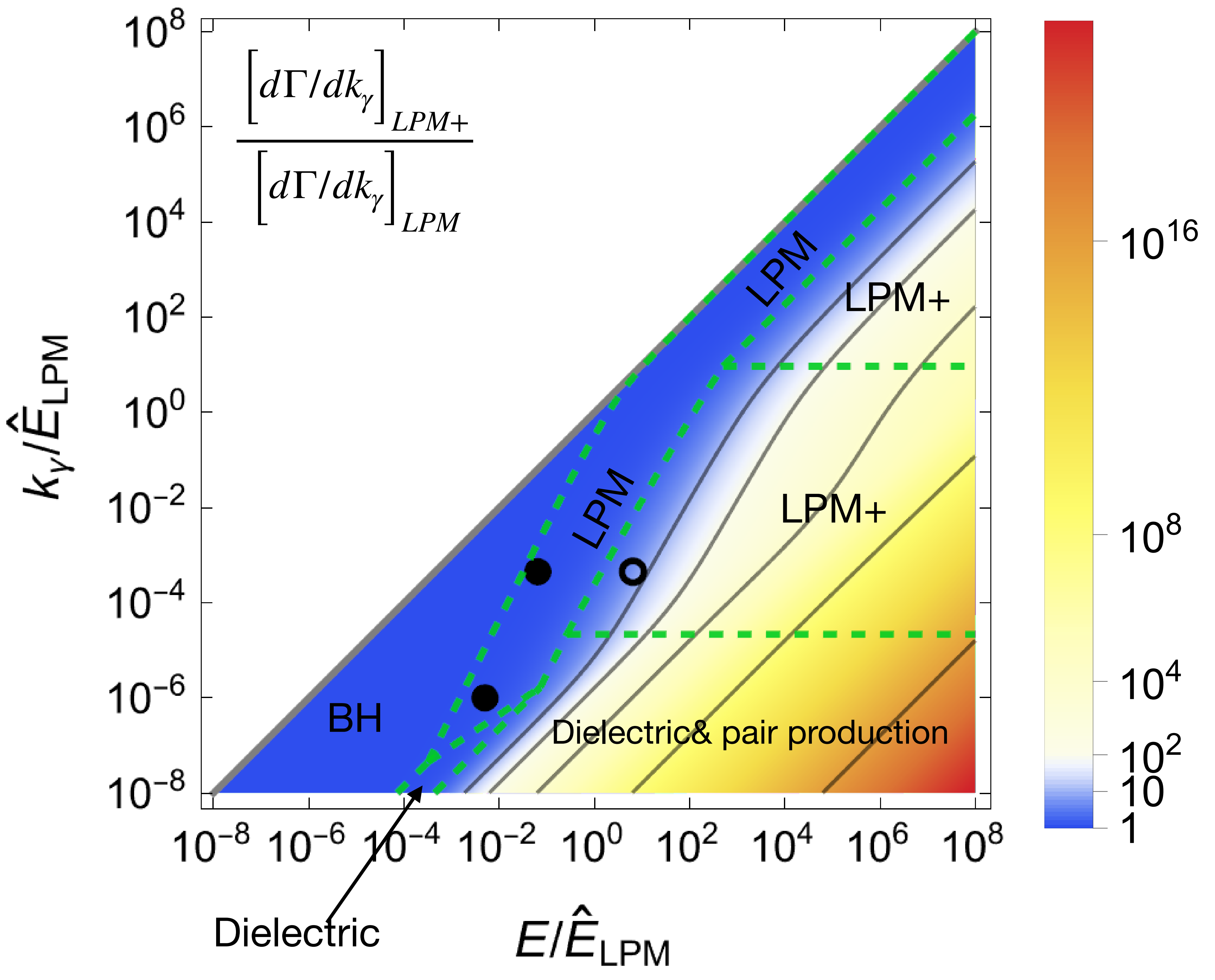
LPM rate





Results

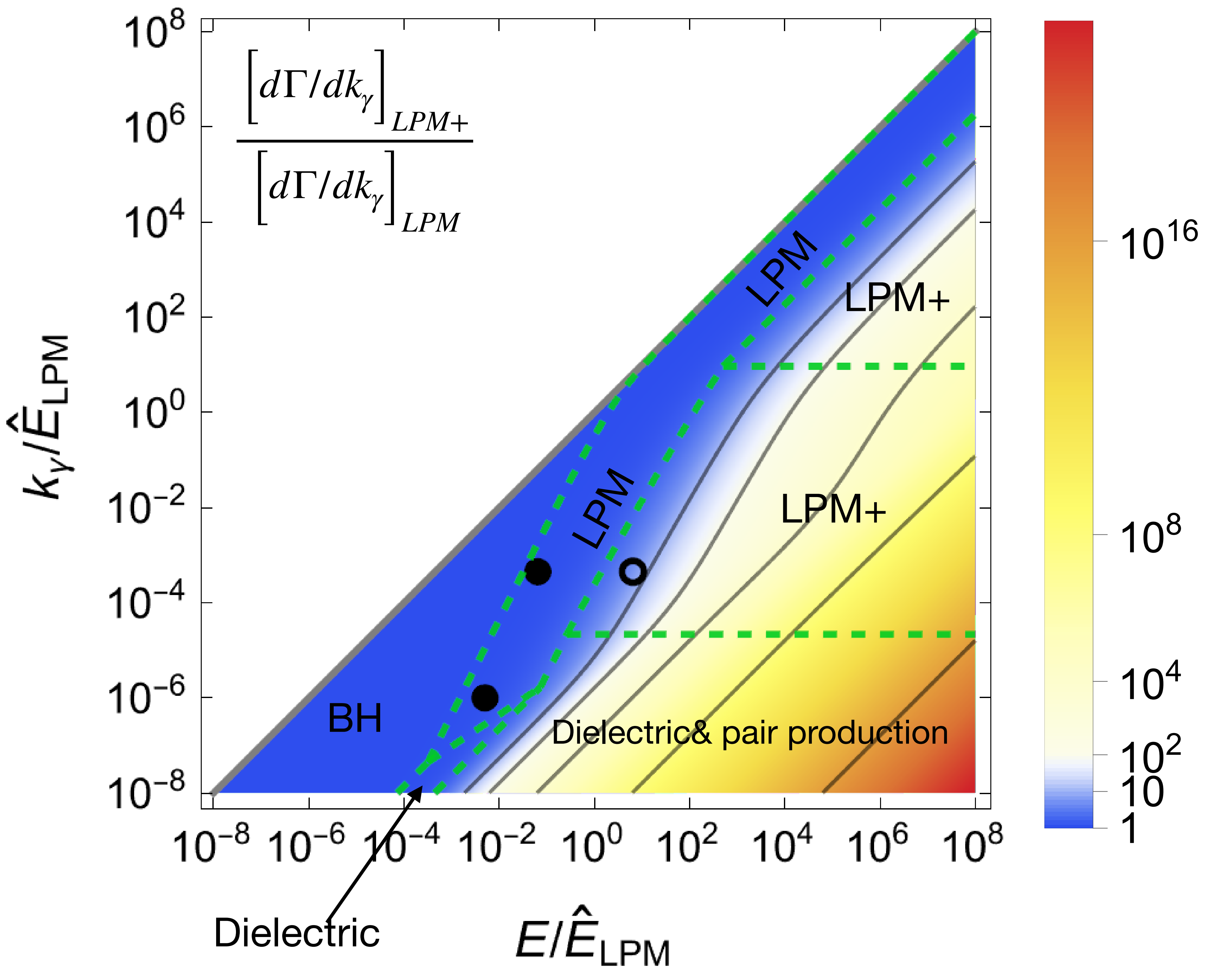
LPM vs. LPM+



Log-log contour plot of the LPM+ /LPM ratio of the LPM+rate (which includes overlapping pair production) to the LPM rate (which does not).

Results

LPM vs. LPM+



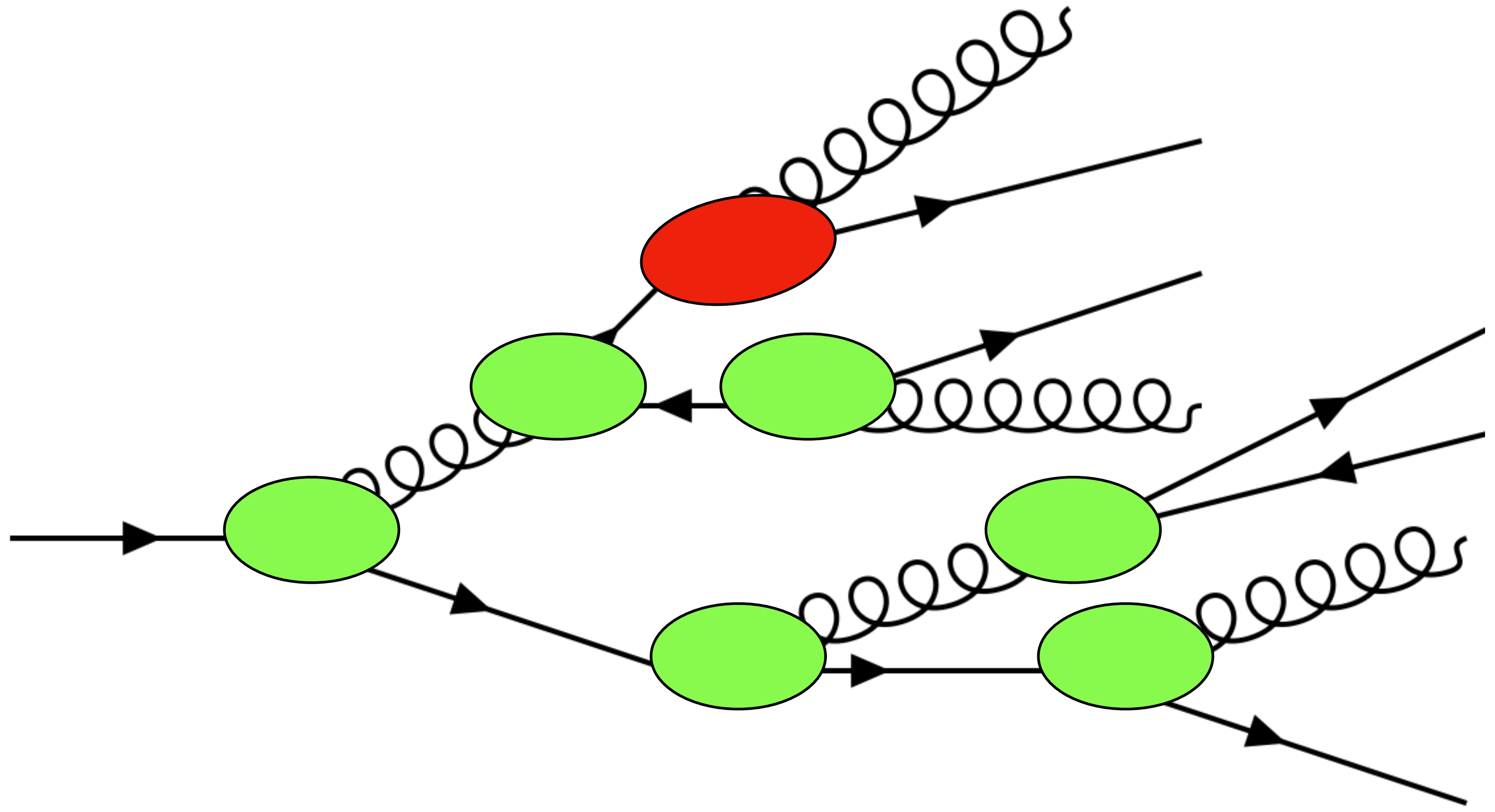
Log-log contour plot of the LPM+ /LPM ratio of the LPM+rate (which includes overlapping pair production) to the LPM rate (which does not).

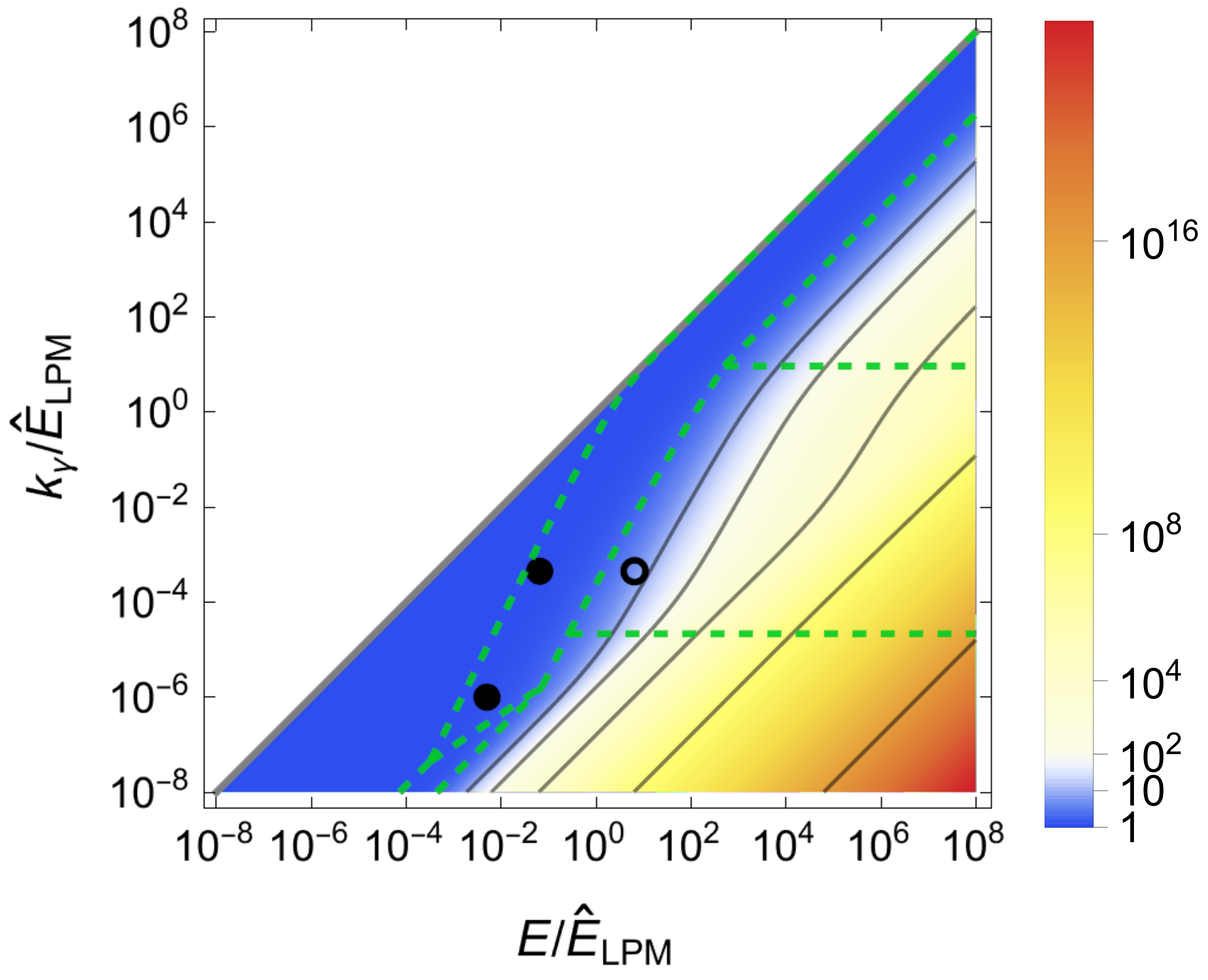
If a~50 TeV proton beam became available (FCC-hh accelerator), and if the concept of the CERN LPM experiment could be scaled up by a factor of 100 in electron energy, that might open the possibility of exploring the small open circle where the LPM correction is significant.

Summary and Conclusion

- QCD showers are weakly coupled. Provided one treats $\hat{q}$ as a parameter that can be measured at one scale and then evolved to other scales (known for LL).

- QED, on the other hand, gets large modifications from overlapping bremsstrahlung with pair production, significantly increasing the bremsstrahlung rate compared to the LPM calculation.





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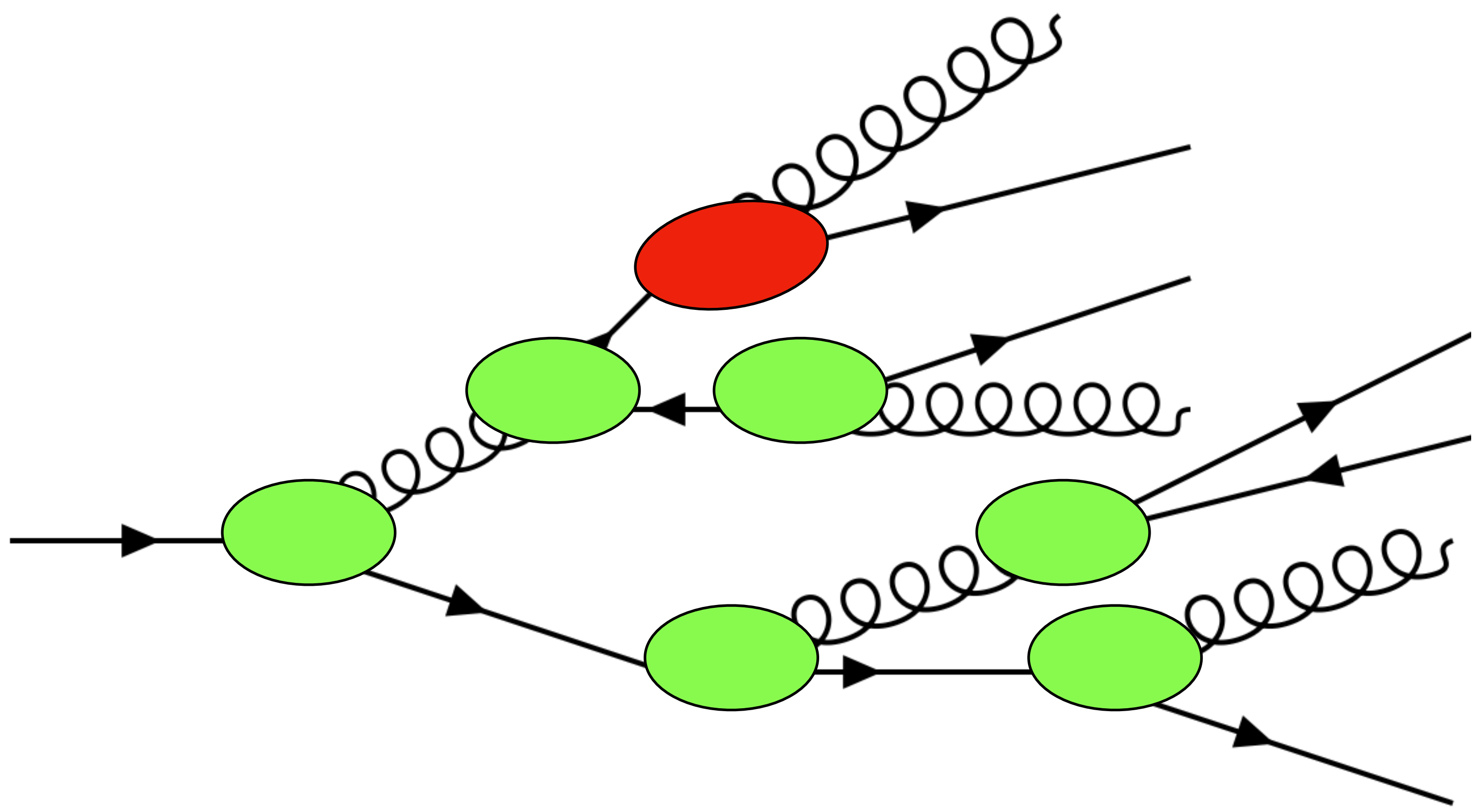
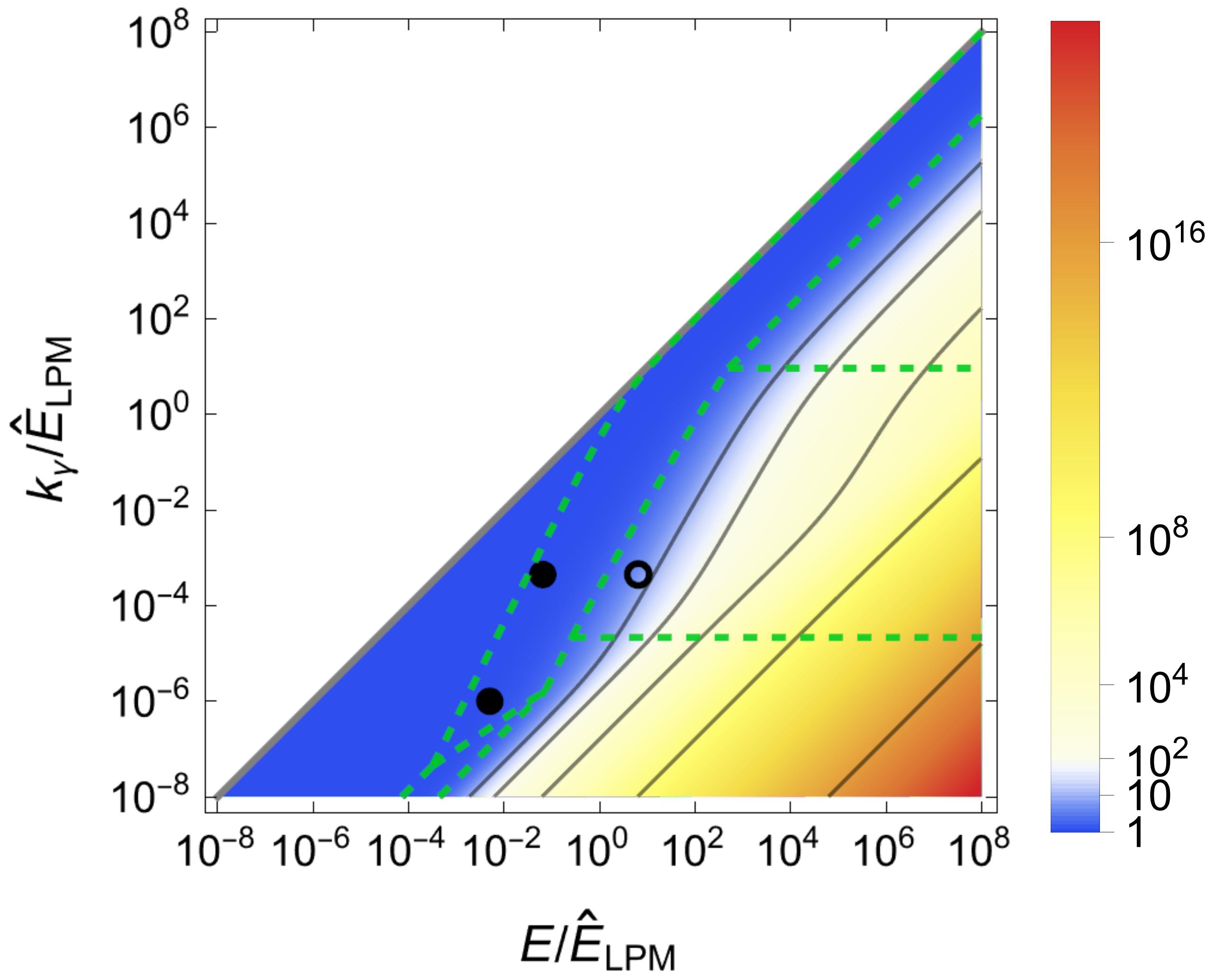


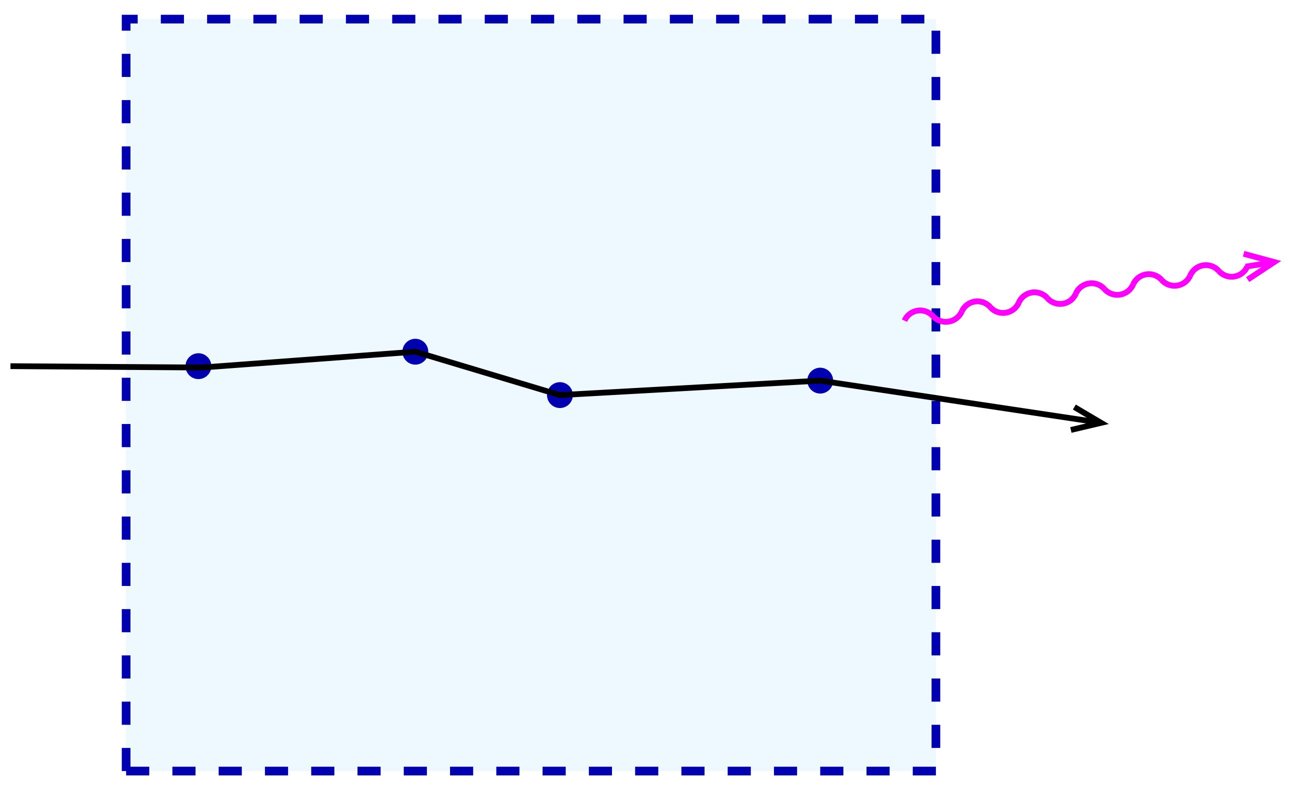


THANKS FOR LISTENING :)

A qualitative picture

The rest frame of the medium

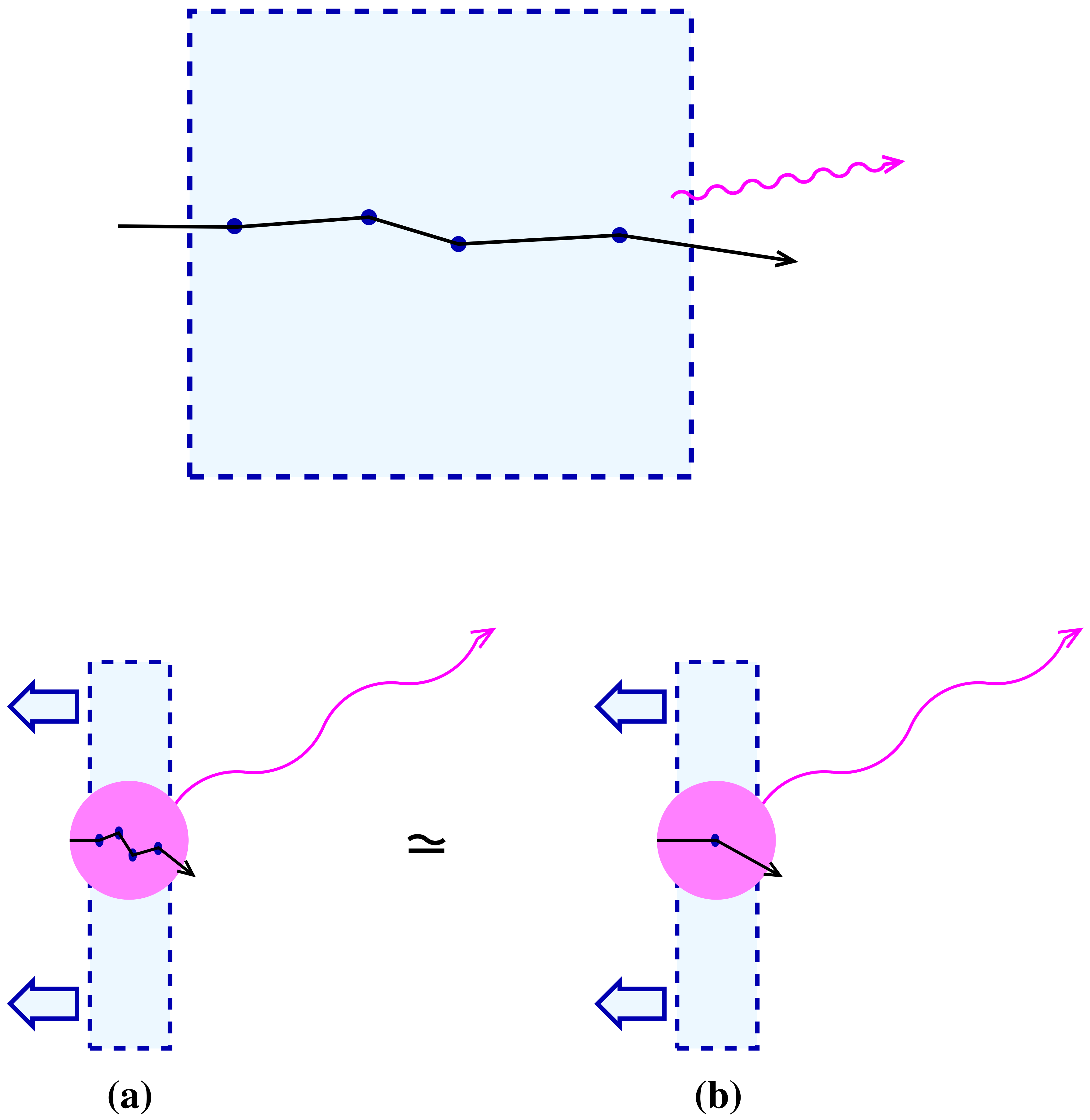
- Consider several elastic collisions in a row in the rest frame of the medium.
- Deflection angles are almost always very small at high energy, and so the bremsstrahlung radiation is nearly collinear



A qualitative picture

A boost to the right

- Boosting to a frame that is moving to the right at close to the speed of light, say the center of momentum frame of the two daughters $e\gamma$ of the bremsstrahlung $e \to e\gamma$.
- The medium is extremely Lorentz contracted, the electron and bremsstrahlung photon have extremely less energy, and so the photon has extremely longer wavelength.
- The Lorentz-expanded photon wavelength may be large compared to the Lorentz-contracted elastic mean free path.
- A single photon cannot effectively resolve details smaller than its wavelength.

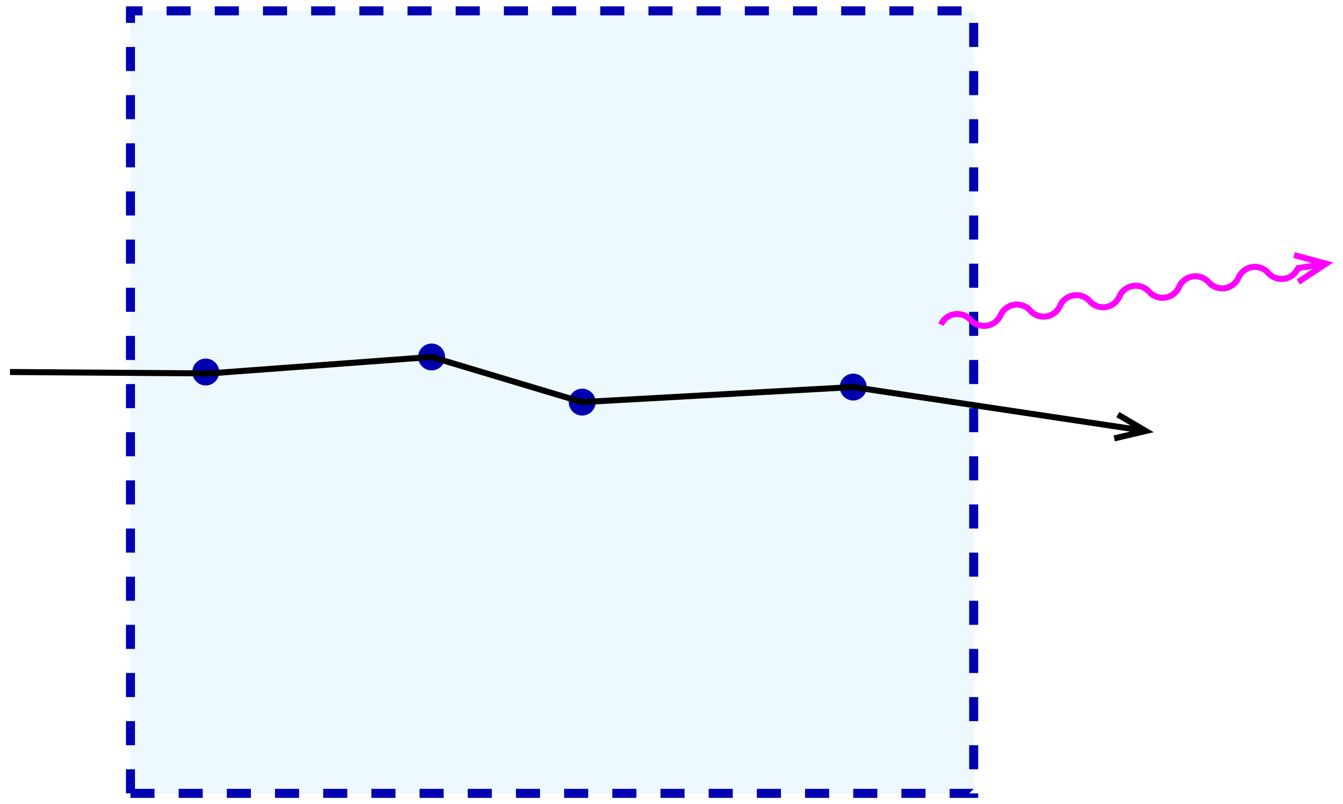


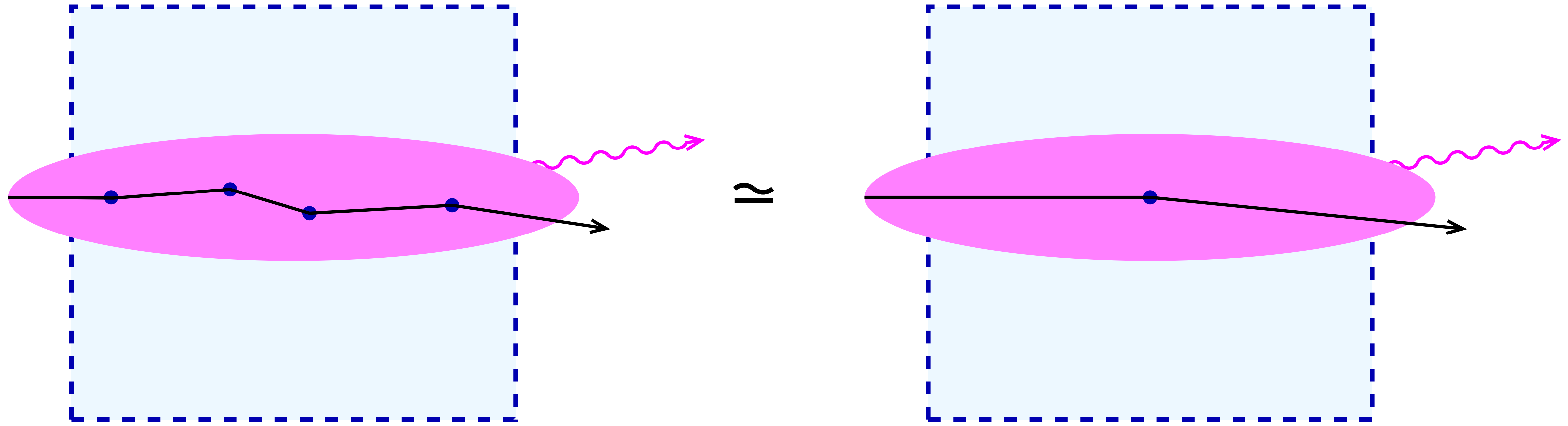
A qualitative picture

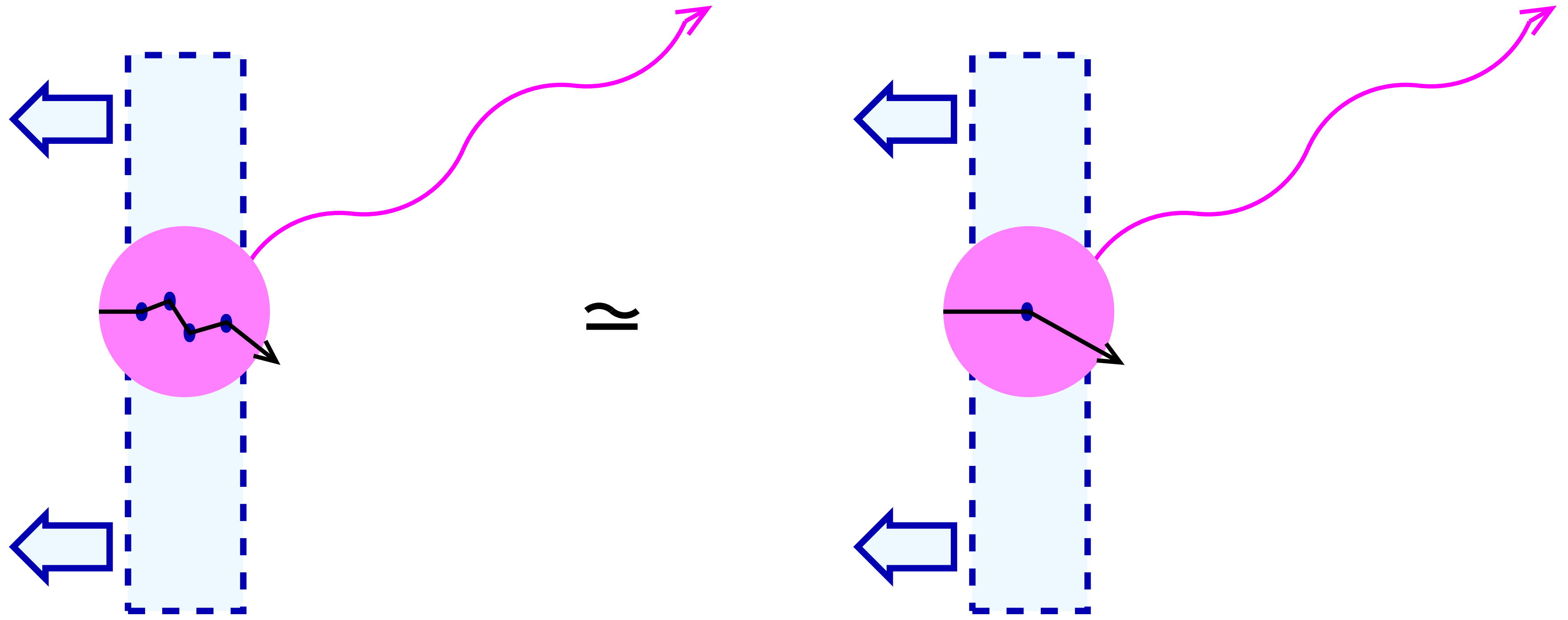
Boost back

- Boosting back to the rest frame of the medium, the unresolved region (magenta circle) becomes long and stretched. This length corresponds to the “formation time/length” t_{form} for the bremsstrahlung photon.

- Multiple collisions with the medium do not give independent chances for bremsstrahlung but instead give only one chance.



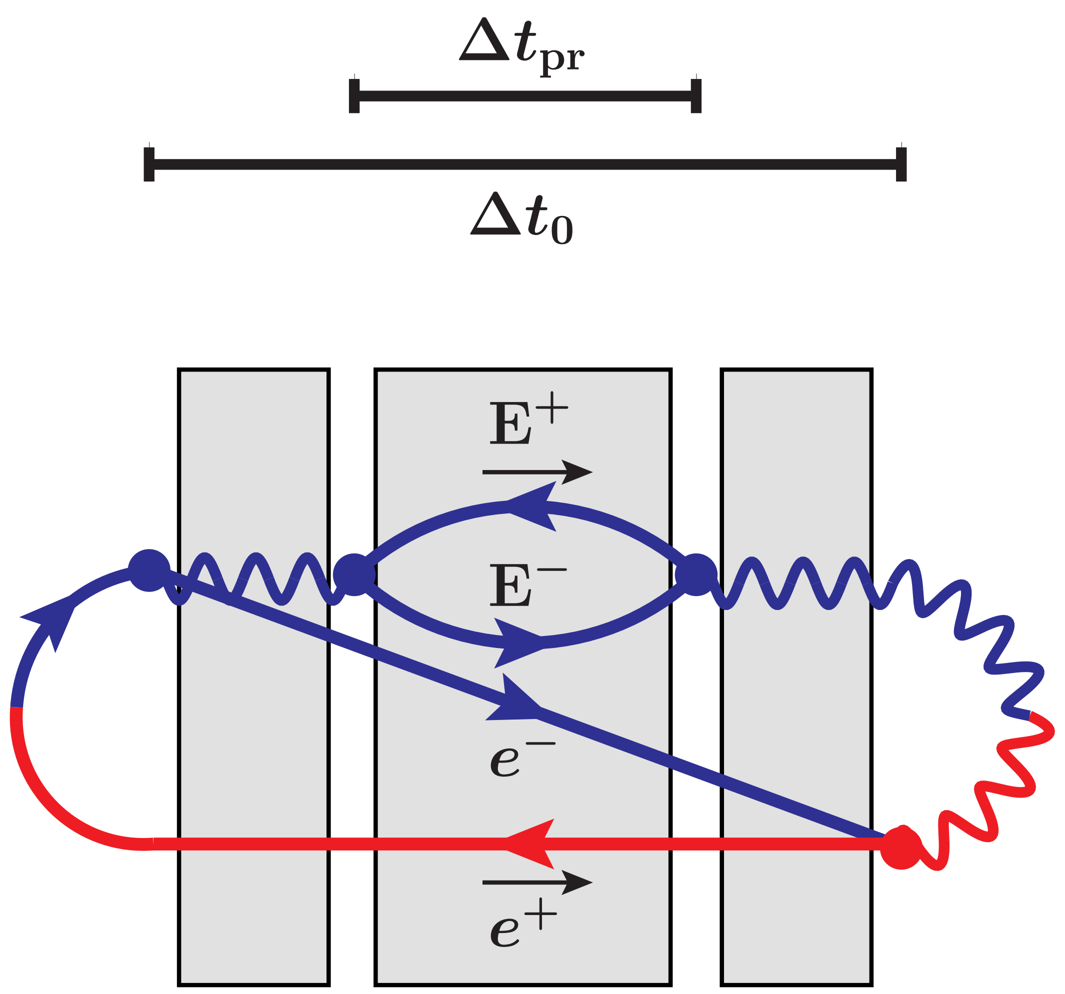




(a) (b)

Bubble Resummation

The soft photon approximation $x_\gamma \ll 1$



- In the soft photon limit:

$$p_\perp^{e\to e\gamma} \sim \left(\hat{q}\, t_{form}^{e\to e\gamma}\right)^{1/2} \sim \left(\hat{q}\, E/x_\gamma\right)^{1/4} \quad \Longrightarrow \quad |b_{e^-} - b_{e^+}| \sim \left(\hat{q}E/x_\gamma\right)^{-1/4}$$

$$p_\perp^{\gamma\to e^-e^+} \sim \left(\hat{q}\, t_{form}^{\gamma\to e^-e^+}\right)^{1/2} \sim \left(\hat{q}\, x_\gamma\, E\right)^{1/4} \quad \Longrightarrow \quad |b_{\mathrm{E}^-} - b_{\mathrm{E}^+}| \sim \left(\hat{q}\, x_\gamma\, E\right)^{-1/4}$$

- Because of this, a good approximation to the 4-body potential is

$$V_4(b_{e^-}, b_{e^+}, b_{\mathrm{E}^-}, b_{\mathrm{E}^+}) = -\frac{i\hat{q}}{4}(b_{\mathrm{E}^-} - b_{\mathrm{E}^+})^2 - \frac{i\hat{q}}{4}(b_{e^-} - b_{e^+})^2$$

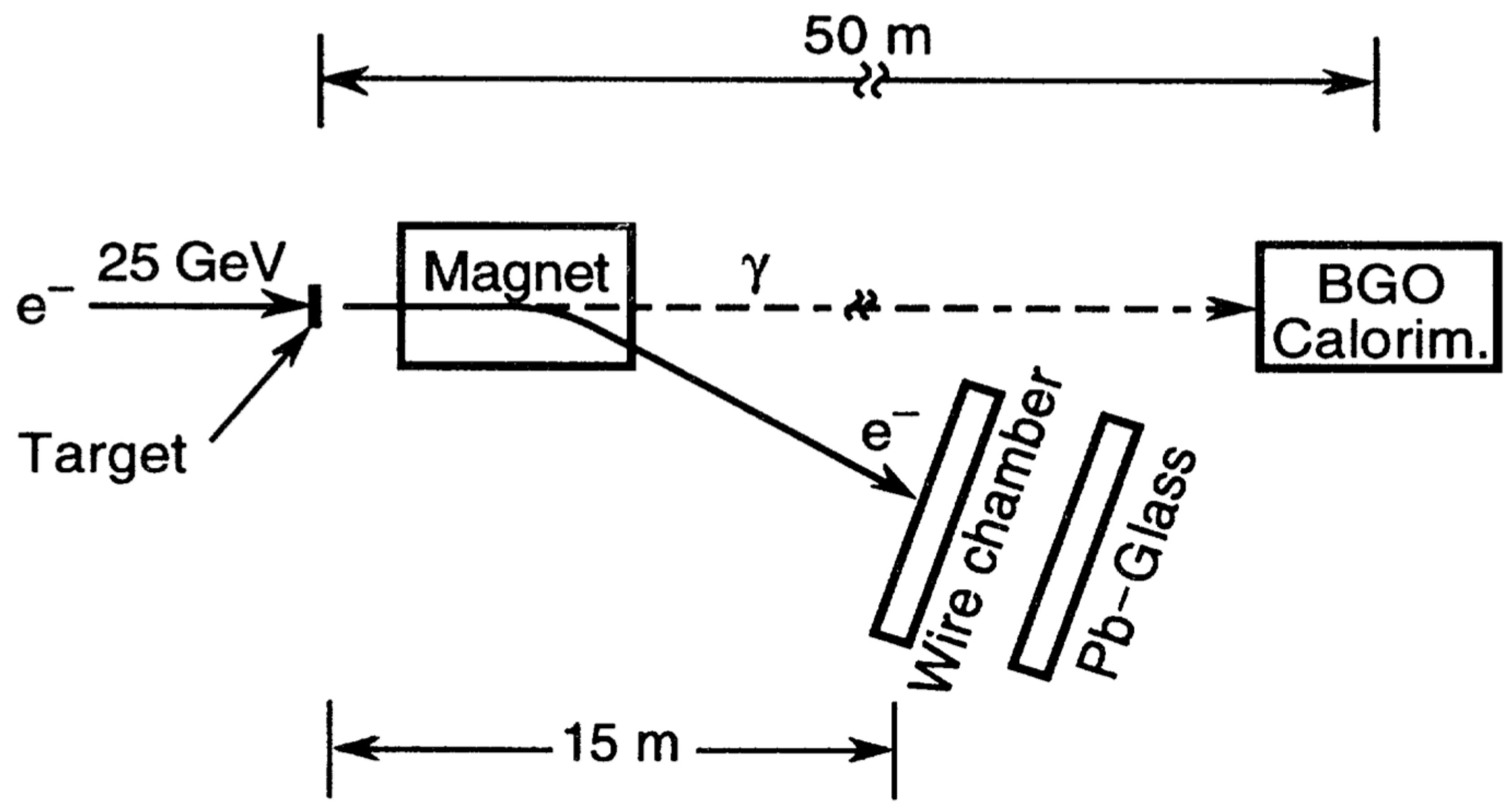
The first LPM experiment

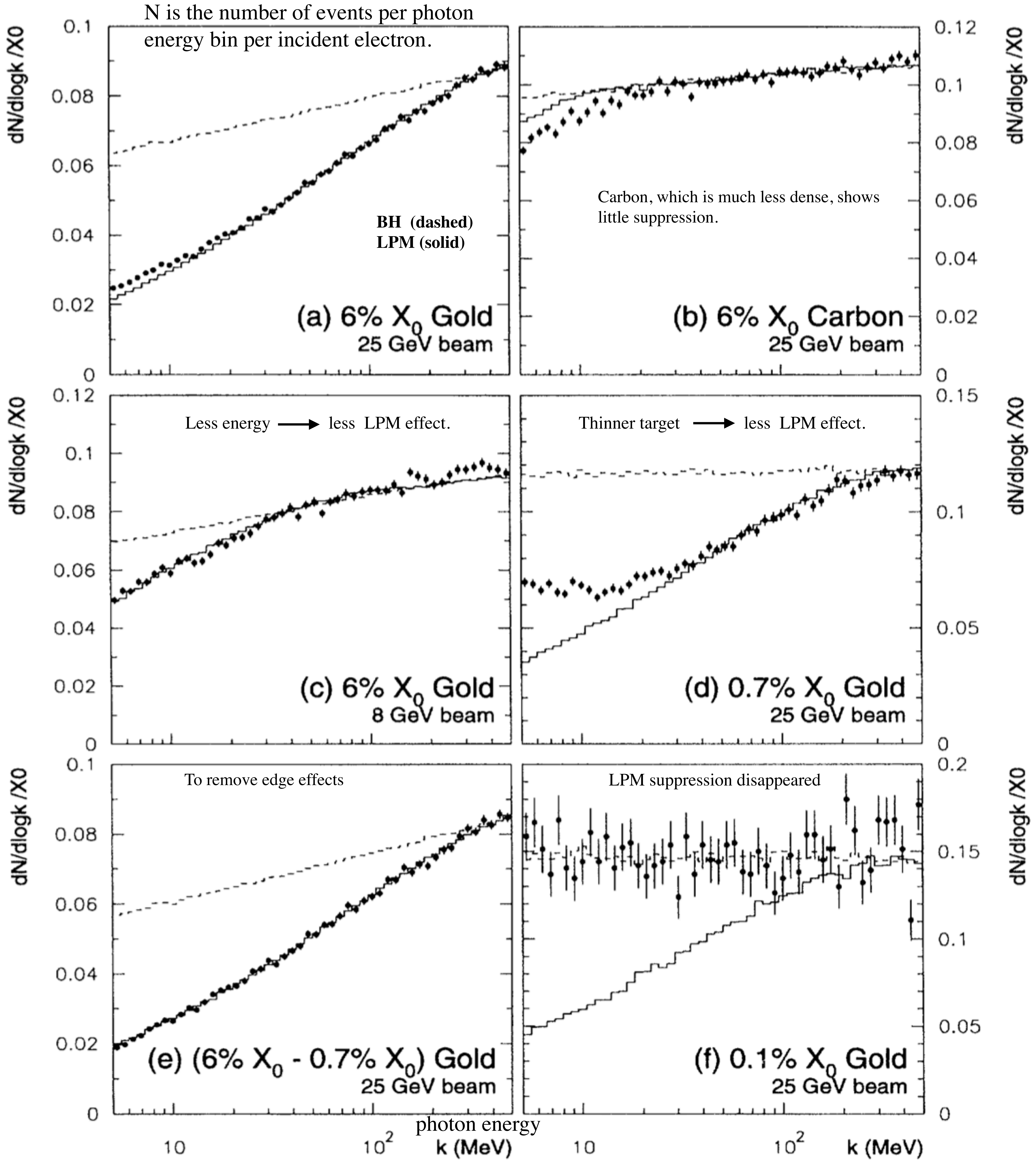
SLAC [Anthony *et al.*, PRL.75.1949]

They sent 8 and 25 GeV electrons through thin gold targets and measured the bremsstrahlung spectrum.

At low photon energies, the radiation rate was suppressed compared to Bethe–Heitler.

The suppression matches Migdal's LPM prediction to within 5%.





The second LPM experiment

CERN [Hansen *et al.*, PRL 91, 014801]

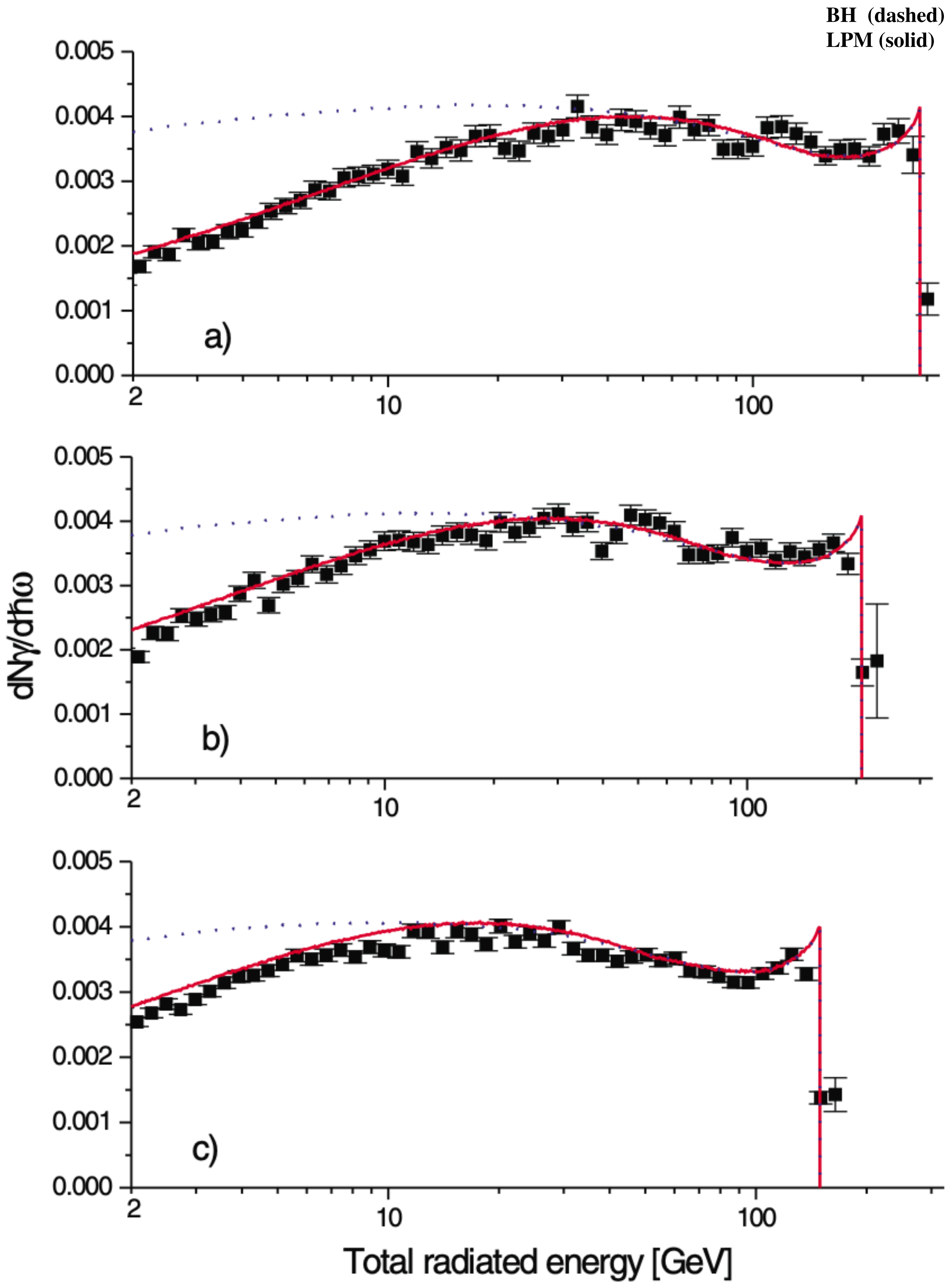
Bremsstrahlung spectrum (a) 287, (b) 207, and (c) 149 GeV electrons on 0.128 mm Ir (4.36 % X_0).

- $t_{form}^{e\to e\gamma} \sim \sqrt{\dfrac{E}{x_\gamma \hat{q}}}$ in the limit $x_\gamma \to 0$.

$$t_{form} \gg \tau_{el}$$

$$k_\gamma \ll \frac{E^2}{E_{LPM}}$$

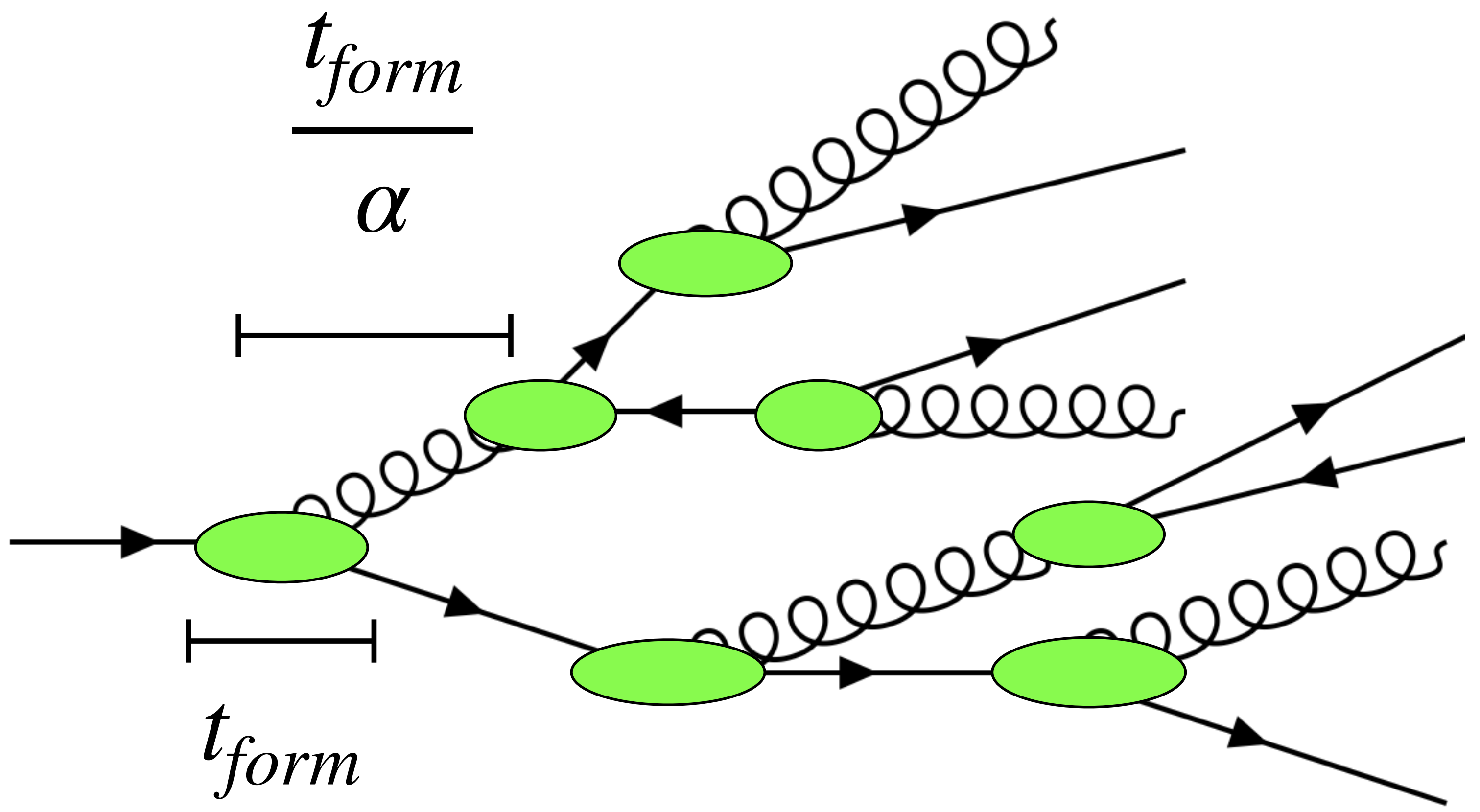
The LPM effect is found at photon energies approximately proportional to E^2.



Hierarchy of scale

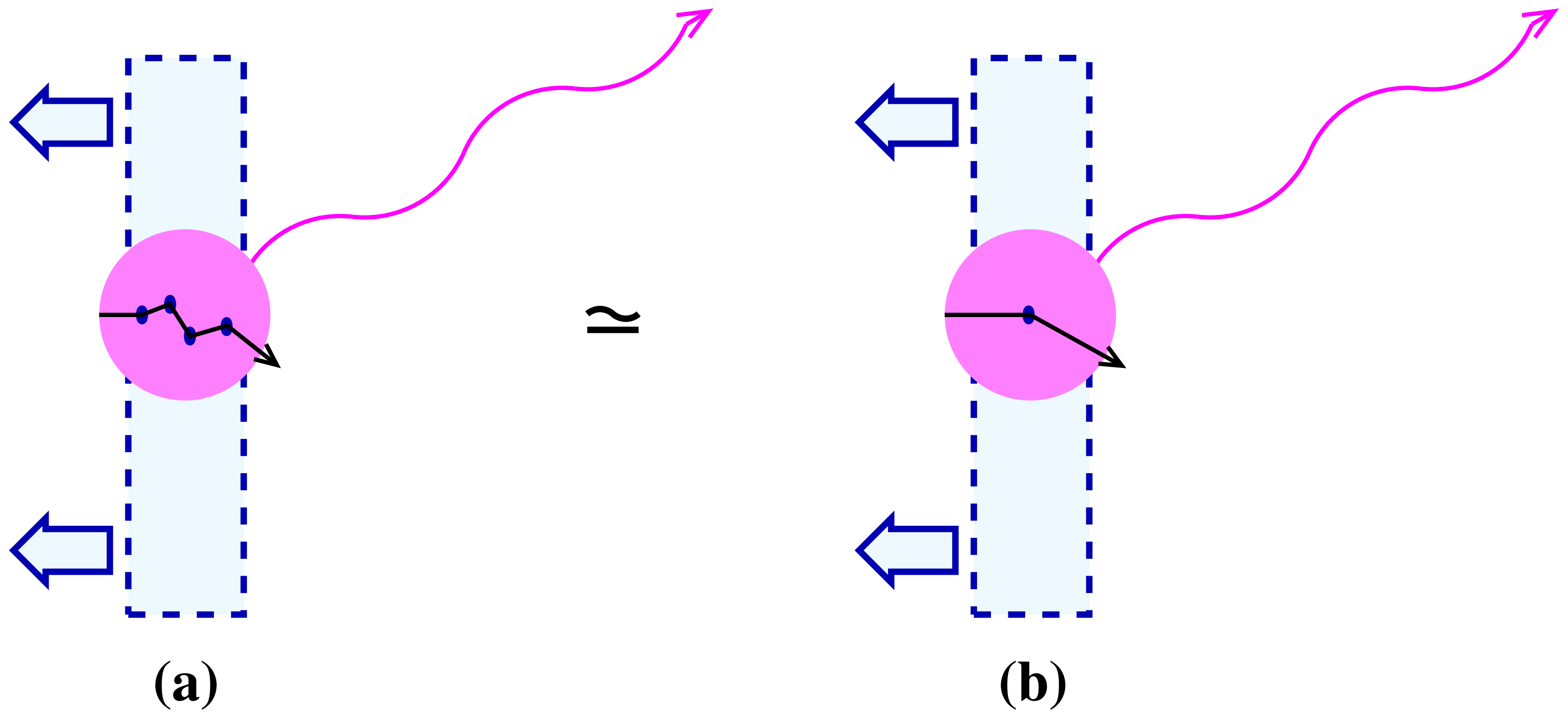
Weakly or strongly coupled ?

- Prob of splitting ~ α per formation time. The prob of overlapping splitting is $\alpha(\mu)$, where μ is the scale at which splittings happen. Here it depends on E of the parent not T of the QGP. However, it turns out the prop of splitting is accompanied by a large double log .



The LPM formation time

- The bremsstrahlung is insensitive to details of the collisions that fit within a wavelength.



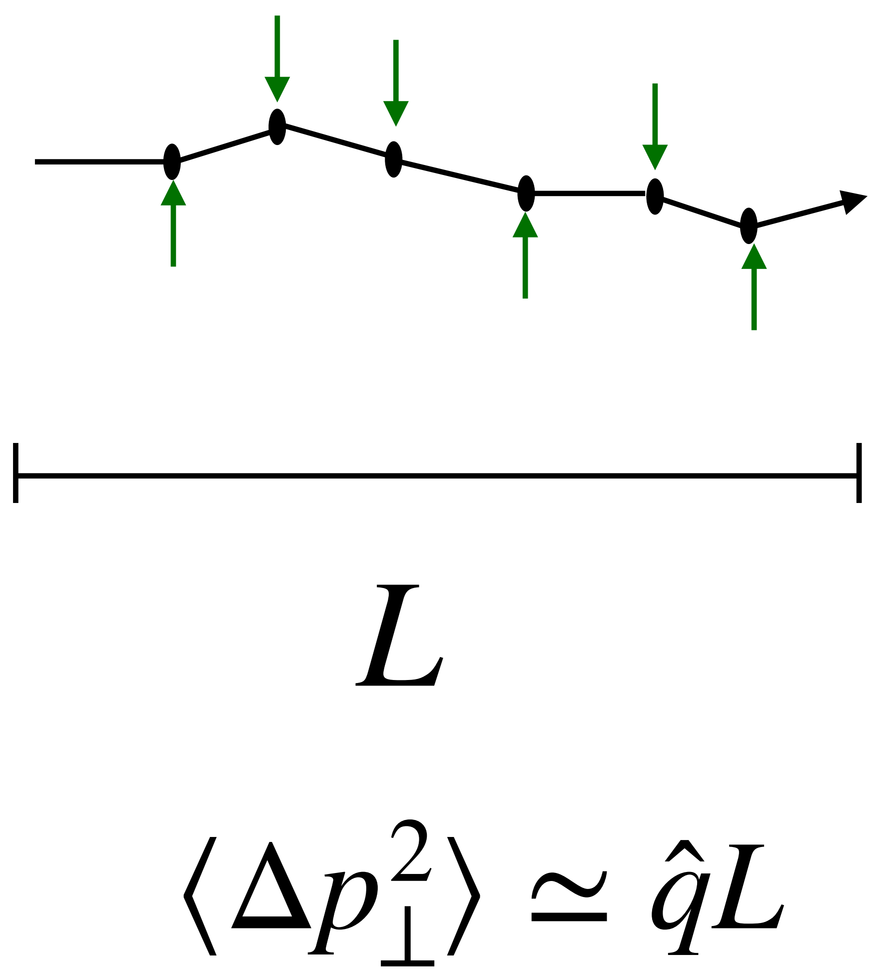
- This could be written as $|\Delta x_\mu k^\mu| \lesssim 1$ (true in any frame).
- The formation time will be given as
$$1 \sim |\Delta x_\mu k^\mu| \simeq k_\gamma t_{form}(1 - \cos\theta_{\gamma e}) \sim k_\gamma t_{form}\theta_{\gamma e}^2 \longrightarrow t_{form} \sim \frac{1}{k_\gamma \theta_{\gamma e}^2}$$
- In QED, $\theta_{\gamma e} \sim \theta_{scatt} \sim \dfrac{\Delta p_\perp}{p_z}$ → $\dfrac{\sqrt{\hat{q} t_{form}}}{E}$ for LPM, → $\dfrac{m}{E}$ for BH

What do we mean by a “potential” ?

The $\hat{q}$ approximation

- Random walk in the $p_\perp$ space, which can be described by a diffusion eq for the probability distribution $\Psi(p_\perp, t)$.

$$\partial_t \Psi(\boldsymbol{p}_\perp, t) = -\Gamma_{\rm el}\Psi(\boldsymbol{p}_\perp, t) + \int d^2q_\perp \frac{d\Gamma_{\rm el}}{d^2q_\perp}\,\Psi(\boldsymbol{p}_\perp - \boldsymbol{q}_\perp, t)$$



- Fourier transforming to position space:

$$i\partial_t \Psi(\boldsymbol{b}, t) = V(\boldsymbol{b})\,\Psi(\boldsymbol{b}, t) = -i\int d^2q_\perp \frac{d\Gamma_{\rm el}}{d^2q_\perp}\left(1 - e^{i\boldsymbol{q}_\perp\cdot\boldsymbol{b}}\right)\Psi(\boldsymbol{b}, t)$$

- Taking the small-***b*** limit, one finds

$$V(b) = -\frac{i}{4}\hat{q}b^2 \qquad \hat{q} = \int d^2q_\perp \frac{d\Gamma_{\rm el}}{d^2q_\perp}\, q_\perp^2$$

The harmonic oscillator approximation, but with an imaginary spring constant.

Disagreement with Galitsky and Gurevich

us

$$\left[\frac{d\Gamma}{dx_\gamma}\right]_{BH} \sim \alpha P_{e\to\gamma} \times \frac{1}{\tau_{el}} \qquad \text{with } \tau_{el} \sim \frac{m^2}{\hat{q}}$$

$$\left[\frac{d\Gamma}{dx_\gamma}\right]_{LPM} \sim \alpha P_{e\to\gamma} \times \frac{1}{t_{form}^{LPM}} \qquad \text{with } t_{form}^{LPM} \sim \sqrt{\frac{E^2}{k_\gamma \hat{q}}}$$

$$\frac{\left[d\Gamma/dx_\gamma\right]_{LPM}}{\left[d\Gamma/dx_\gamma\right]_{BH}} \sim \frac{\tau_{el}}{t_{form}^{LPM}} \sim \sqrt{\frac{k_\gamma m^4}{\hat{q}E^2}}$$

Galitsky & Gurevich

$$\frac{\left[d\Gamma/dx_\gamma\right]_{LPM}}{\left[d\Gamma/dx_\gamma\right]_{BH}} \sim \frac{t_{form}^{LPM}}{t_{form}^{BH}} \sim \sqrt{\frac{k_\gamma m^4}{\hat{q}E^2}}$$

$$\text{with } t_{form}^{BH} \sim \frac{E^2}{k_\gamma m^2}$$

Both versions give the same ordinary LPM brem suppression, but they suggest completely different behavior when overlapping pair production becomes important and reduces the LPM formation time.