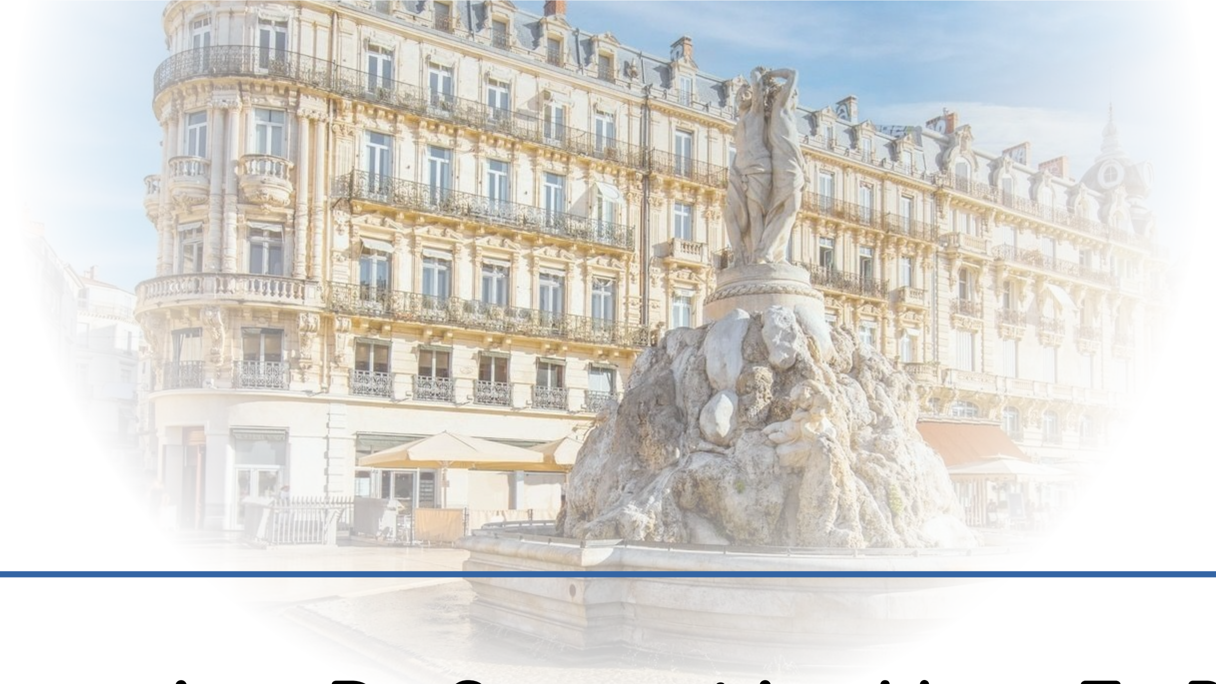

Chiral Dynamics : Do Symmetries Have To Break ?

In collaboration with A. Pastor Gutiérrez and Haolin Li

$$\mathcal{L} = \sum \bar{\psi} \not{D} \psi - \frac{1}{4} G^{a}_{\mu\nu} G^{a\mu\nu}$$

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$$\psi \in$$

	$SU(3)_c$	Flavor
q_L	$\square$	3 $\begin{matrix} u_L \\ d_L \\ s_L \end{matrix}$
q_R	$\square$	3 $\begin{matrix} u_R \\ d_R \\ s_R \end{matrix}$

$$\mathcal{L} = \sum \bar{\psi} \not{D} \psi - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$\rightarrow SU(3)_L \times SU(3)_R \times U(1)$

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$\rightarrow SU(3)_L \times SU(3)_R \times U(1)$

What happen in the IR ?

$$\mathcal{L} = \sum \bar{\psi} \not{D} \psi - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$$\psi \in$$

	$SU(3)_c$	Flavor
q_L	$\square$	3 u_L, d_L, s_L
q_R	$\square$	3 u_R, d_R, s_R

$\rightarrow SU(3)_L \times SU(3)_R \times U(1)$

What happen in the IR ?

- **Confinement :** colorless physical states / gluon mass gap / linear quark potential

$$\mathcal{L} = \sum \bar{\psi} \not{D} \psi - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

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$\rightarrow SU(3)_L \times SU(3)_R \times U(1)$

What happen in the IR ?

- Confinement : colorless physical states / gluon mass gap / linear quark potential
- Condensation : emergence of a condensate $\langle \bar{q} q \rangle \neq 0$

$$\mathcal{L} = \sum \bar{\psi} \not{D} \psi - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$\psi \in$

	$SU(N_c)$	Flavor
q_L	$\square$	N_f
q_R	$\square$	N_f

$$\mathcal{L} = \sum \bar{\psi} \not{D} \psi - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$$\rightarrow SU(N_f)_L \times SU(N_f)_R \times U(1)$$

$\psi \in$

	$SU(N_c)$	Flavor
q_L	$\square$	N_f
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What happen in the IR ?

- Confinement
- Condensation

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What happen in the IR ?

- Confinement
- Condensation

- Lattice
- Functional Method

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$\psi \in$

	$SU(N_c)$	Flavor
q_L	$\square$	N_f
q_R	$\square$	N_f

$\rightarrow SU(N_f)_L \times SU(N_f)_R \times U(1)$

What happen in the IR ?

- Confinement
- Condensation

- Lattice
- Functional Method
- Vafa-Witten
- 't Hooft Matching

$$\mathcal{L} = \sum \bar{\psi} \not{D} \psi - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

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$\psi \in$

	$SU(5)$	Flavor
ψ_L	$\square$	1
χ_L	$\begin{array}{\|c\|}\hline \\ \hline \\ \hline\end{array}$	1

$$\mathcal{L} = \sum \bar{\psi} \not{D} \psi - \frac{1}{4} G^{a}_{\mu\nu} G^{a\mu\nu}$$

$\rightarrow U(1)$

$\psi \in$

	$SU(5)$	Flavor
ψ_L	$\square$	1
χ_L	$\begin{array}{c}\square\\[-3pt]\square\end{array}$	1

$$\mathcal{L} = \sum \bar{\psi} \not{D} \psi - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$\psi \in$

	$SU(5)$	Flavor
ψ_L	$\square$	1
χ_L	$\begin{array}{c}\square\\[-4pt]\square\end{array}$	1

$\rightarrow U(1)$

GUT $\rightarrow$

$$\mathcal{L} = \sum \bar{\psi} \not{D} \psi - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$\psi \in$

	$SU(5)$	Flavor
ψ_L	$\square$	1
χ_L	$\boxminus$	1

$\rightarrow U(1)$

Top → Down

GUT $\rightarrow$

$$\mathcal{L} = \sum \bar{\psi} \not{D} \psi - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$\rightarrow U(1)$

$\psi \in$

	$SU(5)$	Flavor
ψ_L	$\square$	1
χ_L	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1

Top → Down

GUT $\rightarrow$

Strong CP
Neutrino masses
Baryo/Leptogenesis
Dark Matter
Flavor Anomalies

$$\mathcal{L} = \sum \bar{\psi} \not{D} \psi - \frac{1}{4} G^{a}_{\mu\nu} G^{a\mu\nu}$$

$\rightarrow U(1)$

$\psi \in$

	$SU(5)$	Flavor
ψ_L	$\square$	1
χ_L	$\begin{matrix}\square\\ \square\end{matrix}$	1

<u>Top → Down</u>

<u>Bottom → Up</u>

GUT →

Strong CP
Neutrino masses
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Dark Matter
Flavor Anomalies

String Theory

$$\mathcal{L} = \sum \bar{\psi} \not{D} \psi - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$\psi \in$

	$SU(5)$	Flavor
ψ_L	$\square$	1
χ_L	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1

$$\mathcal{L} = \sum \bar{\psi} \not{D} \psi - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$\psi \in$

	$SU(N_c)$	Flavor
ψ_L	$\square$	$N_c - 4$
χ_L	$\begin{array}{c}\square\\[-4pt]\square\end{array}$	1

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$$\rightarrow SU(N_c - 4) \times U(1)$$

What happen in the IR ?

Stable Vacuua ?

$$\mathcal{L} = \sum \bar{\psi} \not{D} \psi - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$\psi \in$

	$SU(N_c)$	Flavor
ψ_L	$\square$	$N_c - 4$
χ_L	$\boxminus$	1

→ $SU(N_c - 4) \times U(1)$

What happen in the IR ?

Stable Vacuua ?
Confinement ?

$$\mathcal{L} = \sum \bar{\psi} \not{D} \psi - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$\psi \in$

	$SU(N_c)$	Flavor
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What happen in the IR ?

Stable Vacuua ?
Confinement ?
Condensation ?
Breaking Pattern ?

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What happen in the IR ?

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- Lattice
- Functional Method
- Vafa-Witten
- 't Hooft Matching
- Large N

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$\mathcal{B} = \chi\psi\psi$

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$$\mathcal{B} = \chi\psi\psi$$

$$m_{\mathcal{B}} = 0 \ ?$$

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What happen in the IR ?

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Functional Method

$$\mathcal{B} = \chi\psi\psi$$

$$m_{\mathcal{B}} = 0 \ ?$$

Previous Analysis

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→ **Without Confinement**

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Without Confinement

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Without Confinement

	$SU(N_c)$	Flavor
ψ_L	$\square$	$N_c + 4$
χ_L	$\square\square$	1

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Bars-Yankielowicz Model

Previous Analysis

- Color-dependent IR
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} **Without Confinement**

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ψ_L	$\square$	$N_c + 4$
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Bars-Yankielowicz Model

- $\mathcal{B} = \chi\psi\psi \quad , \quad m_{\mathcal{B}} = 0$?

Previous Analysis

→

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- At low N_c : $\langle \psi \chi \rangle \neq 0$
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} **Without Confinement**

→

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Bars-Yankielowicz Model

- $\mathcal{B} = \chi\psi\psi$, $m_{\mathcal{B}} = 0$?
- Confinement + Chiral Symmetry Breaking

Functional Methods

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$$\longrightarrow \quad \mathcal{Z}[J] = \int \mathcal{D}\varphi \;\; e^{-S[\varphi]+J\varphi}$$

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Functional Methods

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→ $$\mathcal{W}\left[J\right] = \ln \mathcal{Z}\left[J\right]$$

$$\langle \phi\left(x_1\right)\ldots\phi\left(x_n\right)\rangle_c = \frac{\partial^n}{\partial J\left(x_1\right)\ldots\partial J\left(x_n\right)}\mathcal{W}\left[J\right]$$

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- **1-PI Effective Action :**

Functional Methods

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- **1-PI Effective Action :** $$\Gamma\left[\phi\right] = \sup_J\left\{\int J\phi - \mathcal{W}\left[J\right]\right\}$$

Functional Methods

→ $$\mathcal{Z}[J] = \int \mathcal{D}\varphi \;\; e^{-S[\varphi]+J\varphi}$$

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→ **1-PI Effective Action :** $$\Gamma\left[\phi\right] = \sum_n \frac{1}{n!} \int_{x_1 \dots\, x_n} \Gamma^{(n)}\left[\phi = 0\right] \phi\left(x_1\right) \dots \phi\left(x_n\right)$$

Functional Methods

- $$\mathcal{Z}[J] = \int \mathcal{D}\varphi \ \ e^{-S[\varphi]+J\varphi}$$

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- **Solved**

Functional Methods

→ $$\mathcal{Z}[J] = \int \mathcal{D}\varphi \;\; e^{-S[\varphi]+J\varphi}$$

→ $$\mathcal{W}[J] = \ln \mathcal{Z}[J]$$ $$\langle \phi(x_1)\dots\phi(x_n)\rangle_c = \frac{\partial^n}{\partial J(x_1)\dots\partial J(x_n)}\mathcal{W}[J]$$

→ **1-PI Effective Action :** $$\Gamma[\phi] = \sum_n \frac{1}{n!}\int_{x_1\dots x_n} \Gamma^{(n)}[\phi=0]\,\phi(x_1)\dots\phi(x_n)$$

→ **Solved :** $\Gamma^{(n)}$

Functional Methods

- $$\mathcal{Z}[J] = \int \mathcal{D}\varphi \;\; e^{-S[\varphi]+J\varphi}$$
- $$\mathcal{W}\left[J\right] = \ln \mathcal{Z}\left[J\right]$$ $$\langle \phi\left(x_1\right) \ldots \phi\left(x_n\right) \rangle_c = \frac{\partial^n}{\partial J\left(x_1\right) \ldots \partial J\left(x_n\right)} \mathcal{W}\left[J\right]$$
- **1-PI Effective Action :** $$\Gamma\left[\phi\right] = \sum_n \frac{1}{n!} \int_{x_1 \ldots\, x_n} \Gamma^{(n)}\left[\phi = 0\right] \phi\left(x_1\right) \ldots \phi\left(x_n\right)$$
- **Solved :** $\Gamma^{(n)}$ → **Masses**

Functional Methods

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- $$\mathcal{W}\left[J\right] = \ln \mathcal{Z}\left[J\right]$$ $$\langle \phi\left(x_1\right)\ldots\phi\left(x_n\right)\rangle_c = \frac{\partial^n}{\partial J\left(x_1\right)\ldots\partial J\left(x_n\right)}\mathcal{W}\left[J\right]$$
- **1-PI Effective Action :** $$\Gamma\left[\phi\right] = \sum_n \frac{1}{n!}\int_{x_1\ldots\, x_n} \Gamma^{(n)}\left[\phi = 0\right]\phi\left(x_1\right)\ldots\phi\left(x_n\right)$$
- **Solved :** $\Gamma^{(n)}$
  - → **Masses**
  - → **Symmetry Breaking**

Functional Methods

→ $$\mathcal{Z}[J] = \int \mathcal{D}\varphi \;\; e^{-S[\varphi]+J\varphi}$$

→ $$\mathcal{W}\left[J\right] = \ln \mathcal{Z}\left[J\right] \qquad \langle \phi\left(x_1\right)\ldots\phi\left(x_n\right)\rangle_c = \frac{\partial^n}{\partial J\left(x_1\right)\ldots\partial J\left(x_n\right)}\mathcal{W}\left[J\right]$$

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→ **Solved :** $\Gamma^{(n)}$

→ **Masses**

→ **Symmetry Breaking**

→ **Confinement**

Functional Methods

→ $$\mathcal{Z}[J] = \int \mathcal{D}\varphi \;\; e^{-S[\varphi]+J\varphi}$$

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→ **Solved :** $\Gamma^{(n)}$

→ **Masses**

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→ ...

How To Diagnose $\chi\mathrm{SB}$?

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$$\rightarrow \qquad \Gamma\left[\bar{\psi}, \psi, A_\mu\right] \supset \int \lambda_{\mathrm{4F}}\, \bar{\psi}\psi\bar{\psi}\psi$$

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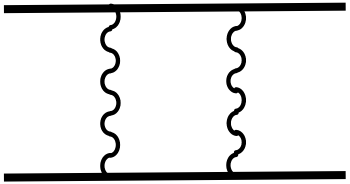

How To Diagnose χSB?

→ $$\Gamma\left[\bar{\psi}, \psi, A_\mu\right] \supset \int \lambda_{4\mathrm{F}}\, \bar{\psi}\psi\bar{\psi}\psi$$

→ Divergent 4F flow signal χSB

How To Diagnose $\chi\mathrm{SB}$?

→ $$\Gamma\left[\bar{\psi},\psi,A_\mu\right]\supset\int\lambda_{4\mathrm{F}}\,\bar{\psi}\psi\bar{\psi}\psi$$

→ **Divergent 4F flow signal $\chi\mathrm{SB}$**

→ $$(\psi_1\psi_2)\,(\psi_1\psi_2)\Longrightarrow\langle\psi_1\psi_2\rangle\neq0$$

How To Diagnose χSB ?

→ $$\Gamma\left[\bar{\psi},\psi,A_\mu\right] \supset \int \lambda_{4F}\,\bar{\psi}\psi\bar{\psi}\psi$$

→ Divergent 4F flow signal χSB

→ $$(\psi_1\psi_2)\,(\psi_1\psi_2) \Longrightarrow \langle\psi_1\psi_2\rangle \neq 0$$

→ $\alpha_{\rm crit}$: If reached, a condensate emerges

How to bootstrap Confinement ?

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→ Absence of colored asymptotic states

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- Mass Gap : $\Gamma^{(2)}_{AA}\,(p=0) = m^2{}_{\text{gap}} > 0$

→

How to bootstrap Confinement ?

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$$e^{-\Gamma[\phi]} = \int \mathcal{D}\varphi \;\; e^{-S[\phi+\varphi]+\frac{\partial \Gamma}{\partial \phi}\varphi}$$

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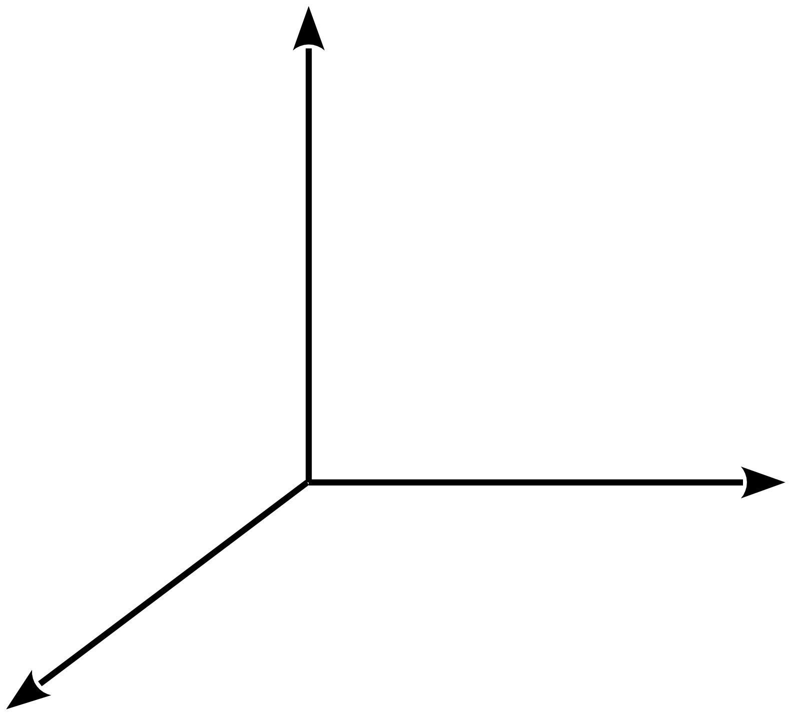
Integro-Differential Equation HARD

fRG functional Renormalization Group



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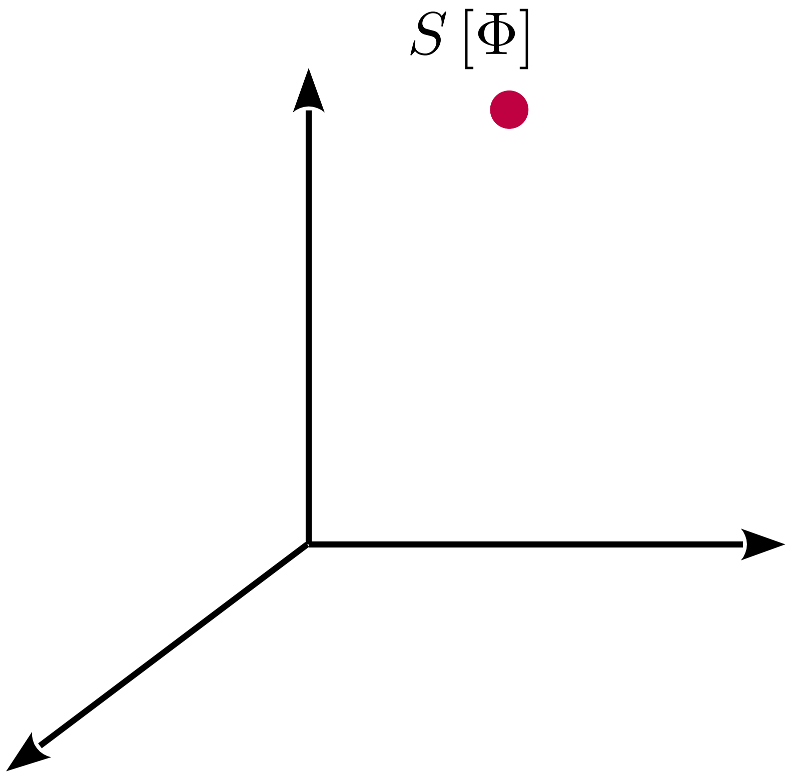

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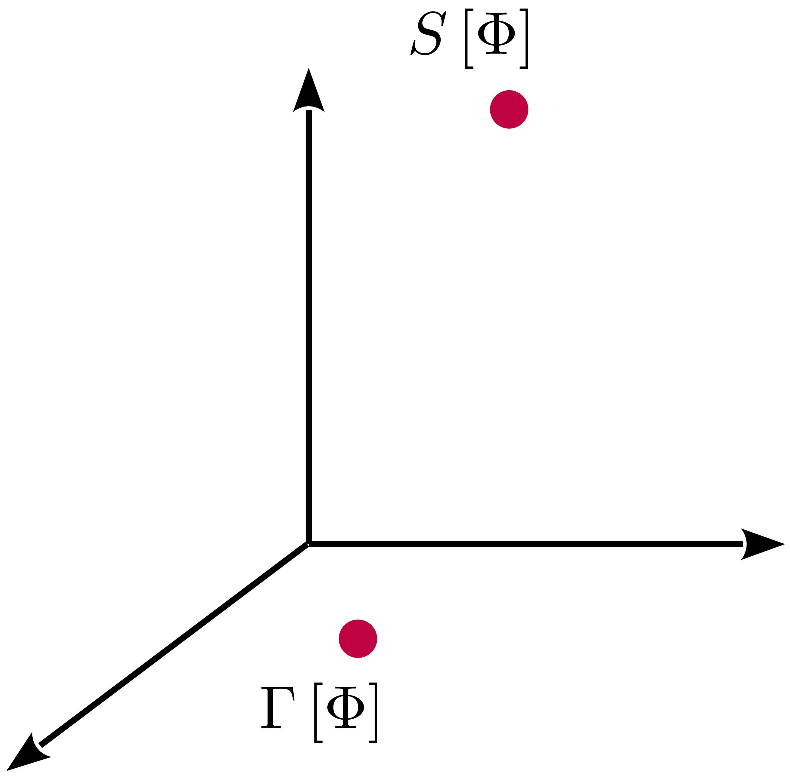

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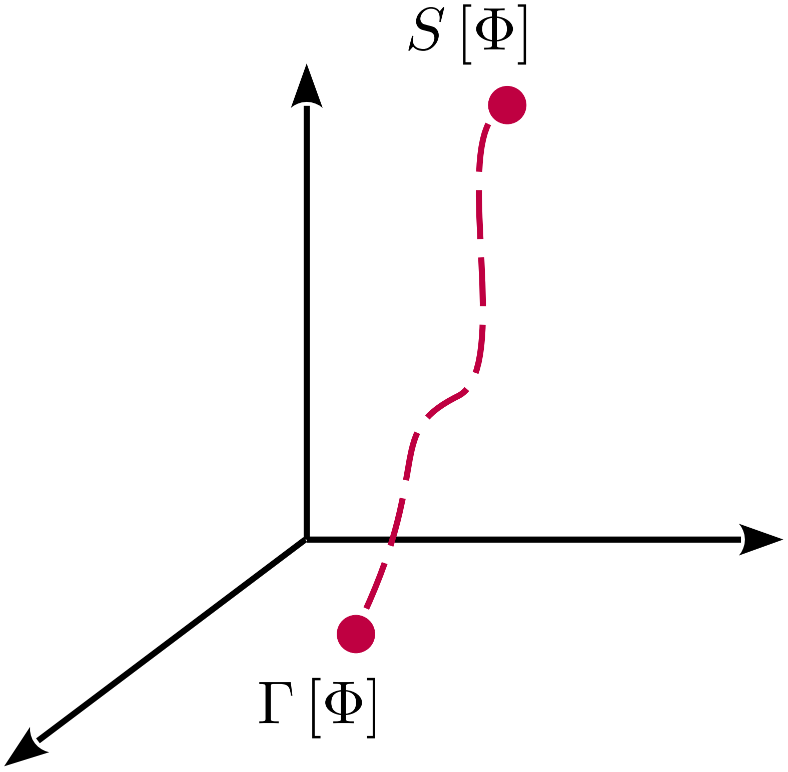

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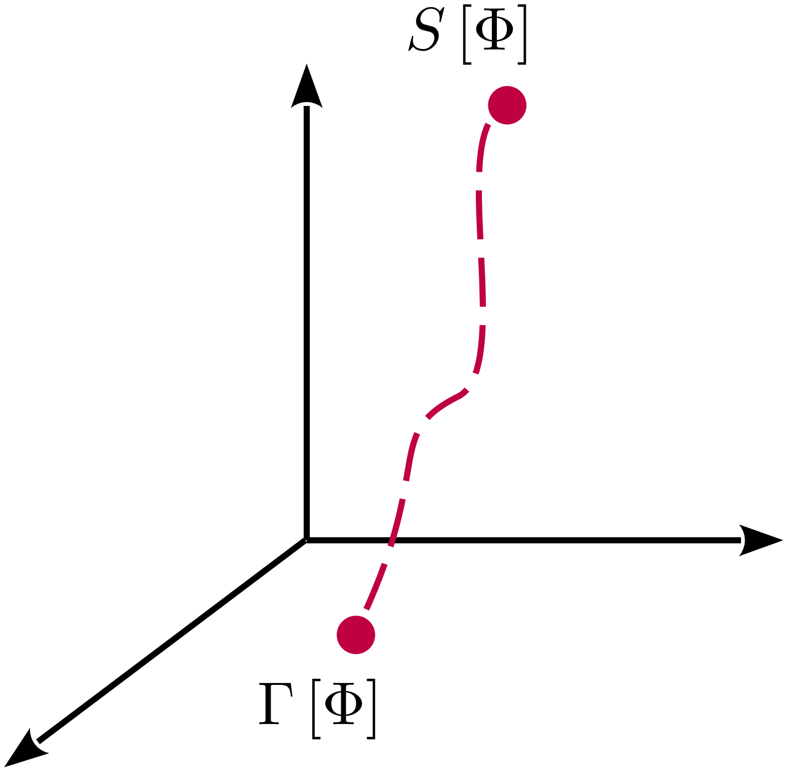

$$\mathcal{Z}_k[J] = \int_{\varphi(p),\, p>k} \mathcal{D}\varphi \quad e^{S+J\varphi}$$

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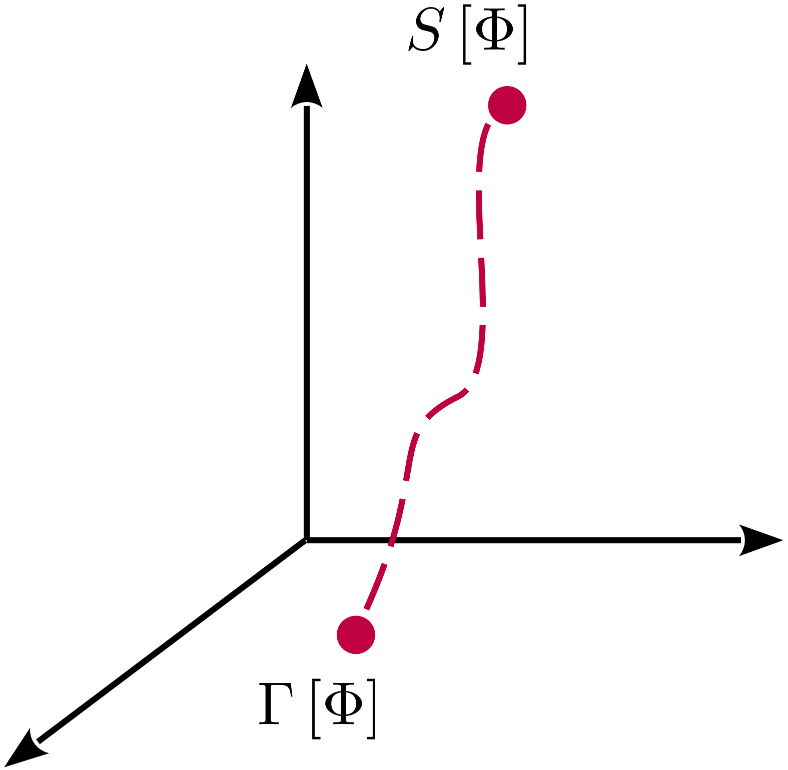

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$$\underbrace{\hphantom{\varphi(p),\, p>k}}_{\int \mathcal{D}\varphi e^{-\frac{1}{2}\phi(p) R_k(p) \phi(-p)}}$$

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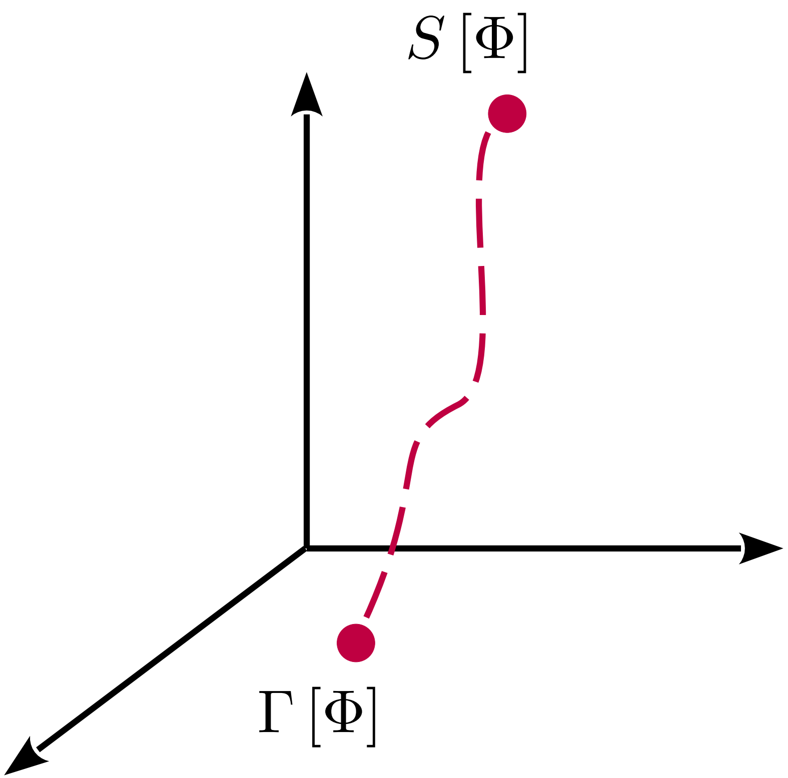

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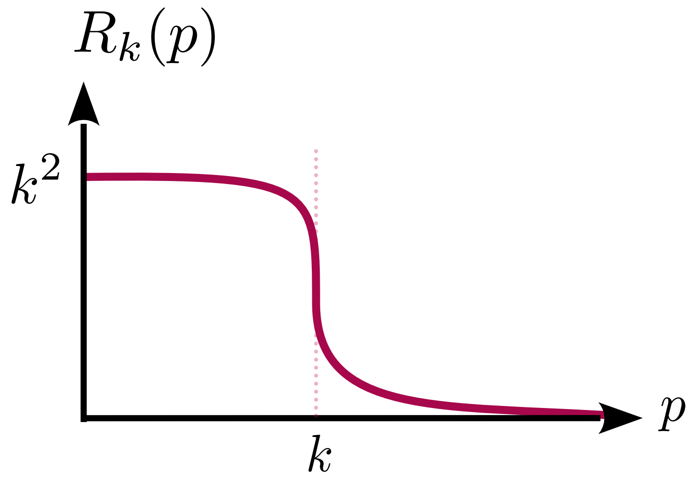

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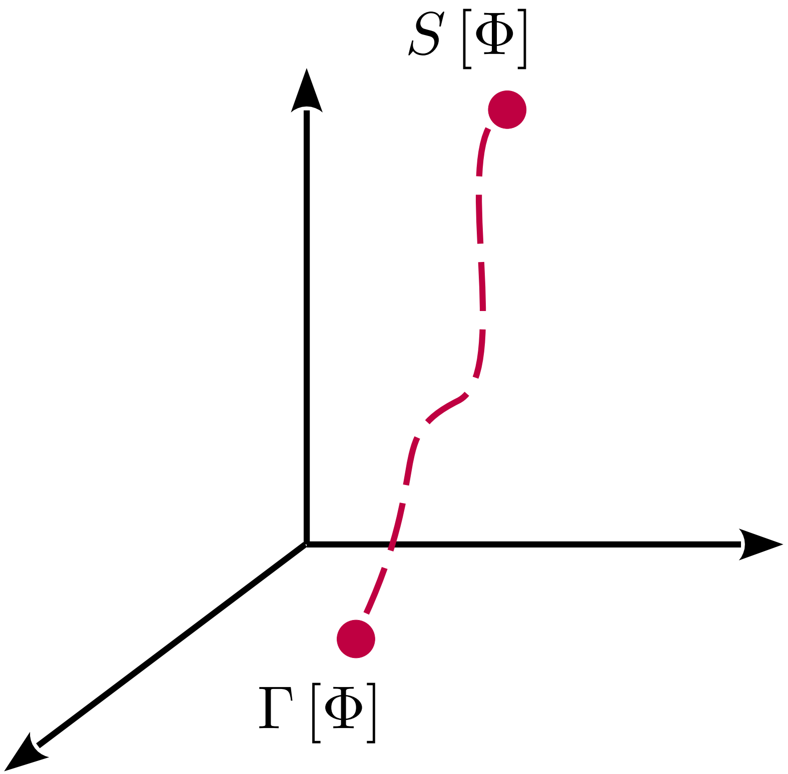

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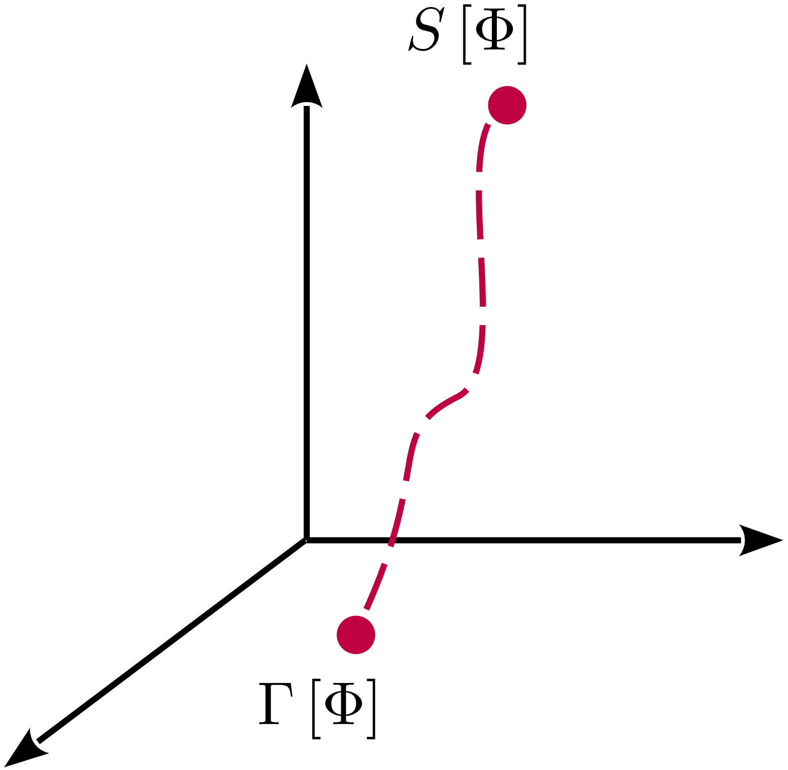

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$$\downarrow$$

$$\mathcal{W}_k[J]$$

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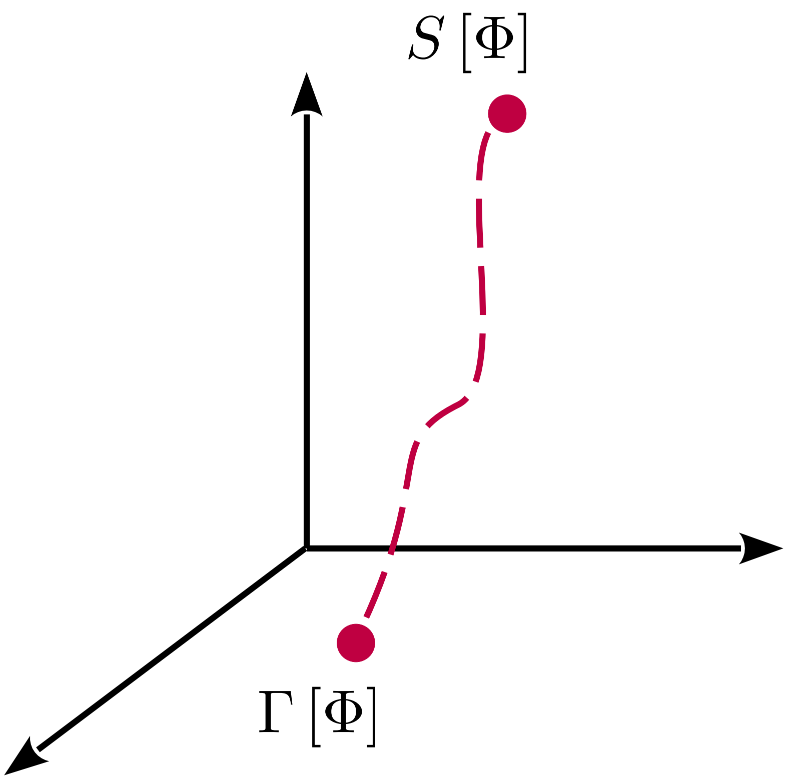

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$$\Gamma_k[\phi]$$

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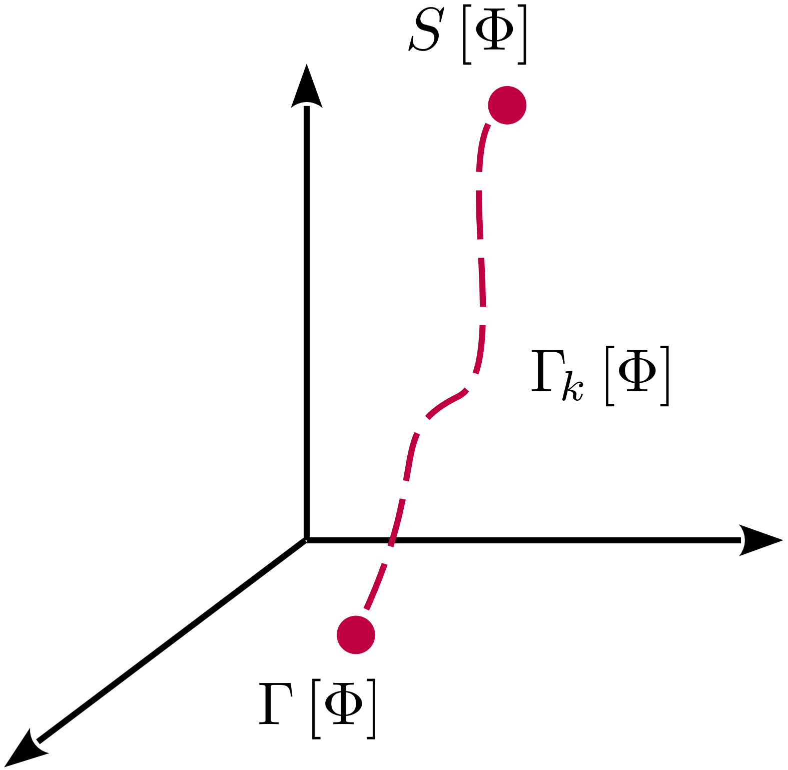

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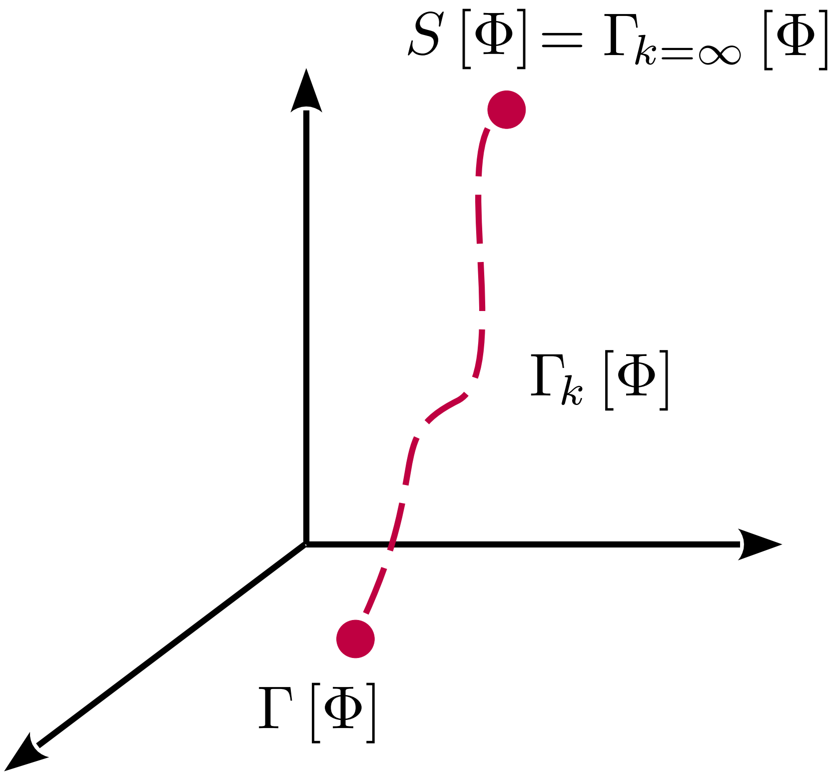

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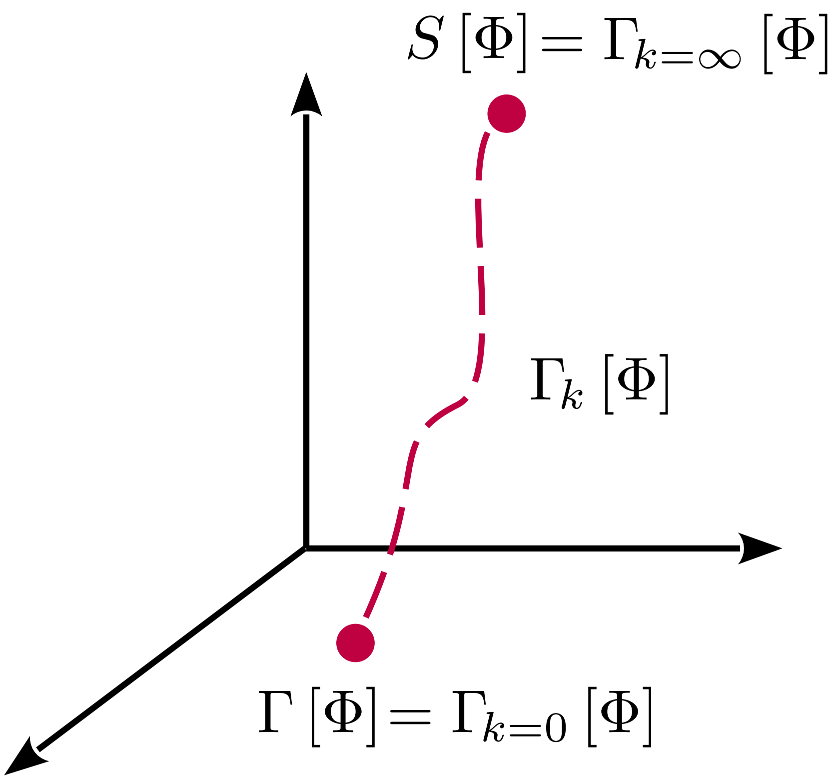

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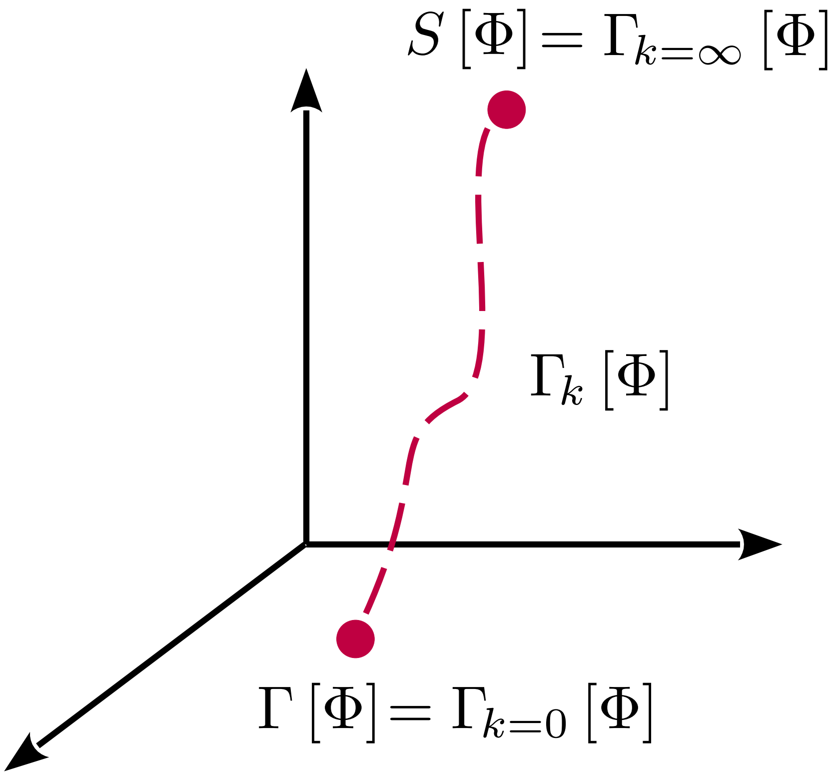

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$$\boxed{\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \frac{\partial_t R_k}{\Gamma^{(2)} + R_k}}$$

$$t = \ln \frac{k}{\Lambda_{\mathrm{UV}}}$$

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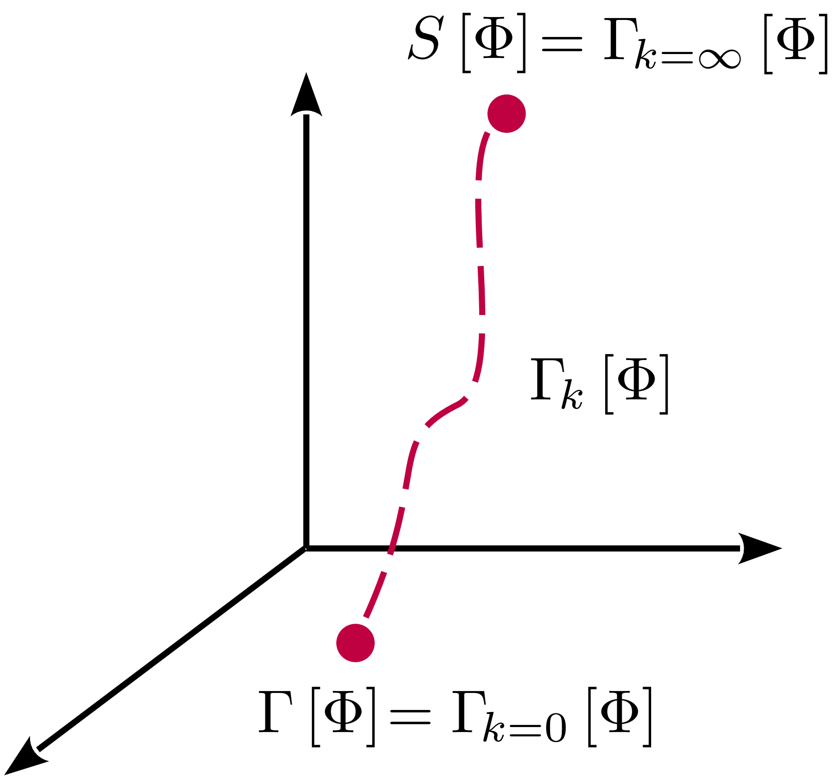

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« Simple » Differential Equation

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Infinite tower of differential equations ! In practice, $\Gamma^{(n)} = 0, \ \ \forall n > N_{\mathrm{max}}$

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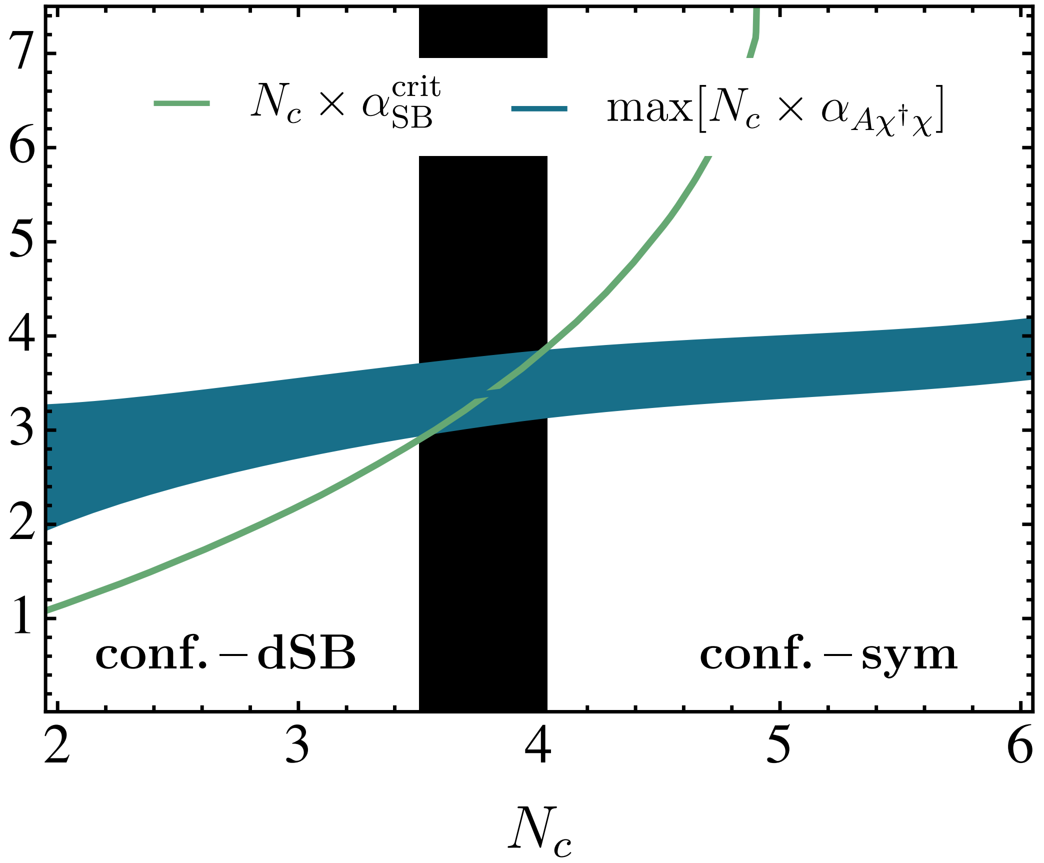
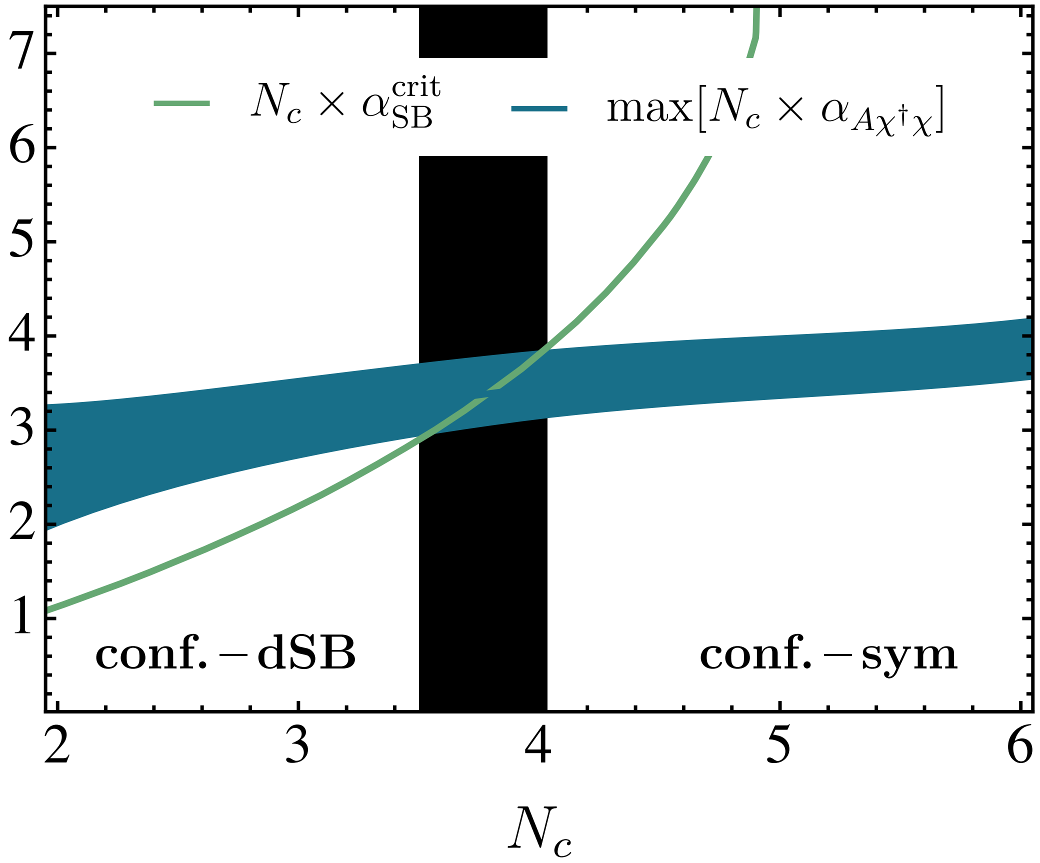
- We compute : α_{crit}
- We analyse for the presence of scaling solutions
- We compute : $\max\left[\alpha\right]$

Results !



$N_c \times \alpha_{\mathrm{SB}}^{\mathrm{crit}}$
$\max[N_c \times \alpha_{A\chi^\dagger\chi}]$
conf. − dSB
conf. − sym
1
2
3
4
5
6
7
2
3
4
5
6
N_c

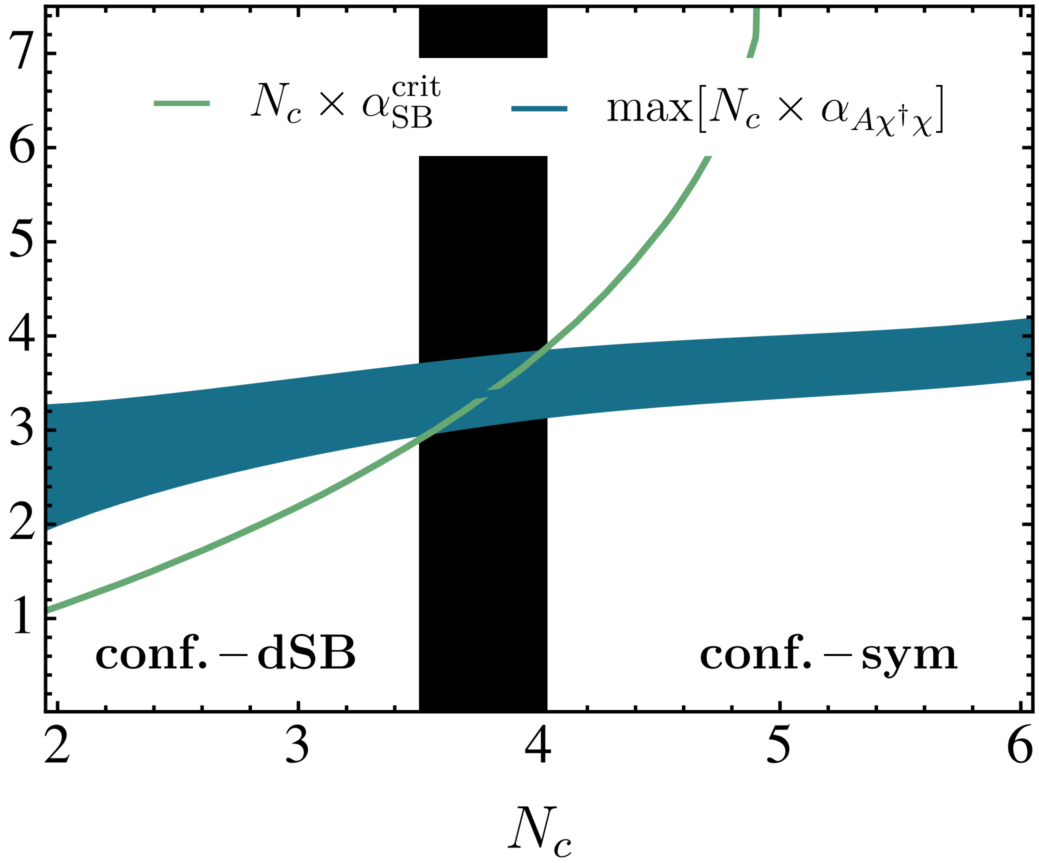
Results !



→ $N_c < 4 : \langle \chi\chi \rangle \neq 0$

- Symmetry Breaking
- Confining

Results !



→ $N_c < 4 : \langle \chi\chi \rangle \neq 0$

- Symmetry Breaking
- Confining

→ $N_c > 4 : \langle \chi\chi \rangle = 0$

- Symmetry Preserving
- Confining

Conclusion

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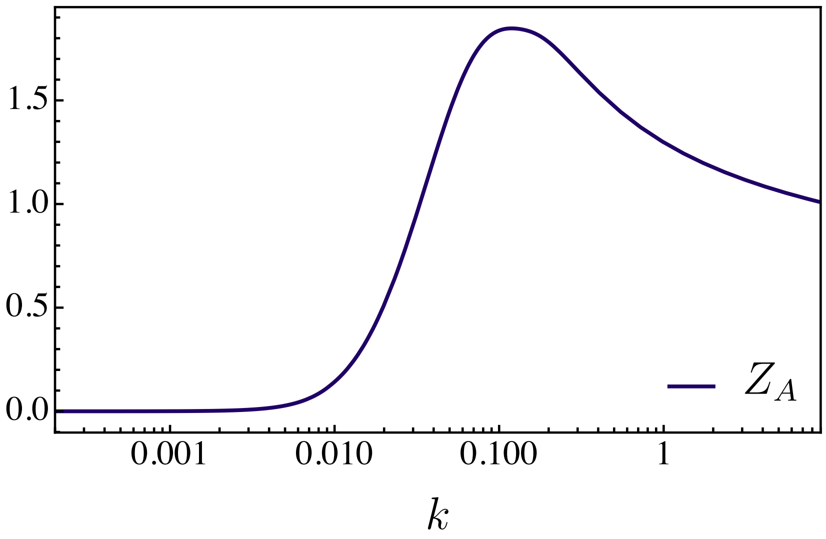
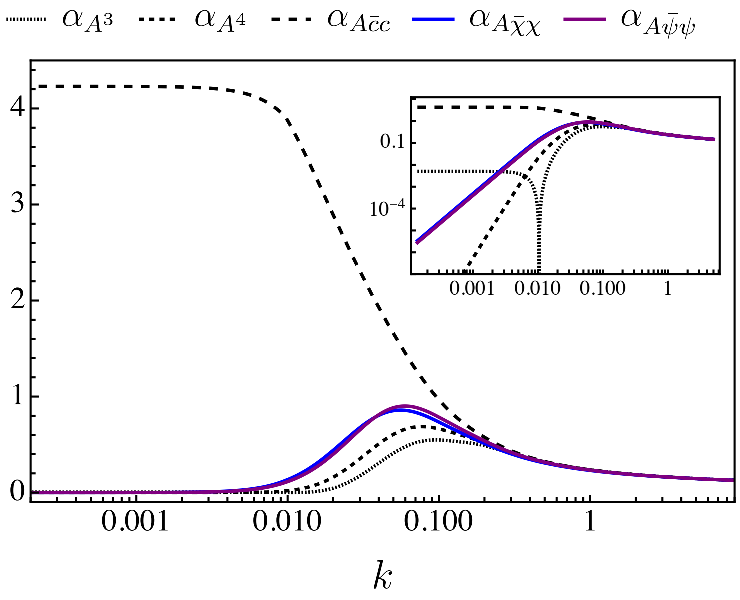
- fRG allows for straightforward and systematic improvements.

- Chiral gauge theories have long posed a major challenge for QFT
- We clarified a path towards their study from first principles
- A vast landscape of gauge-fermion theories is now open, allowing the exploration of novel dynamical phenomena.

Results !

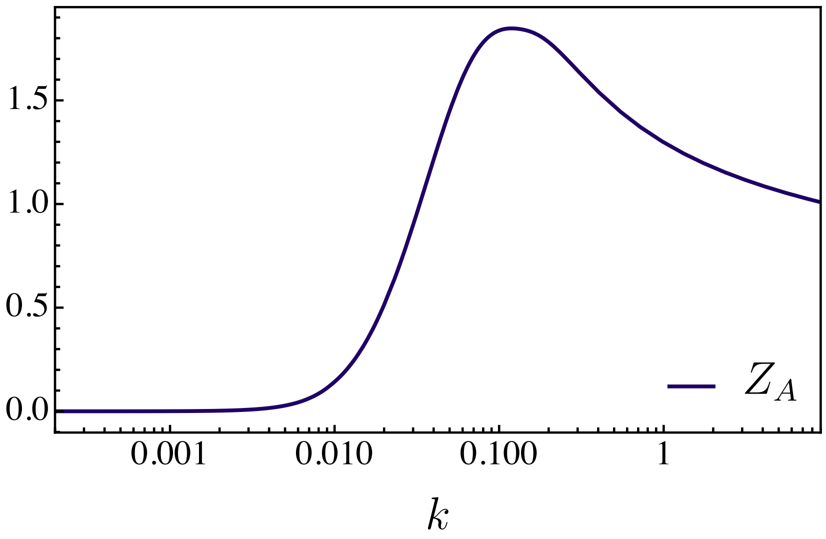
Bars-Yankielowicz $Nc = 4$

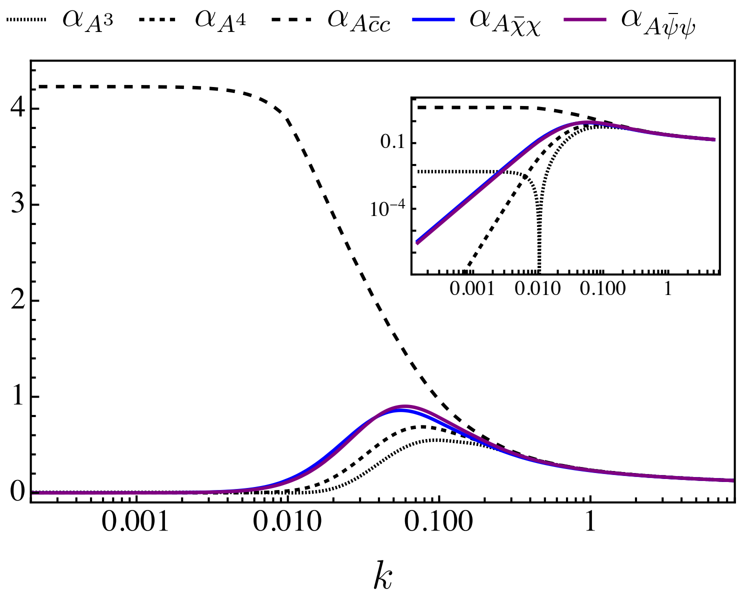




Results !

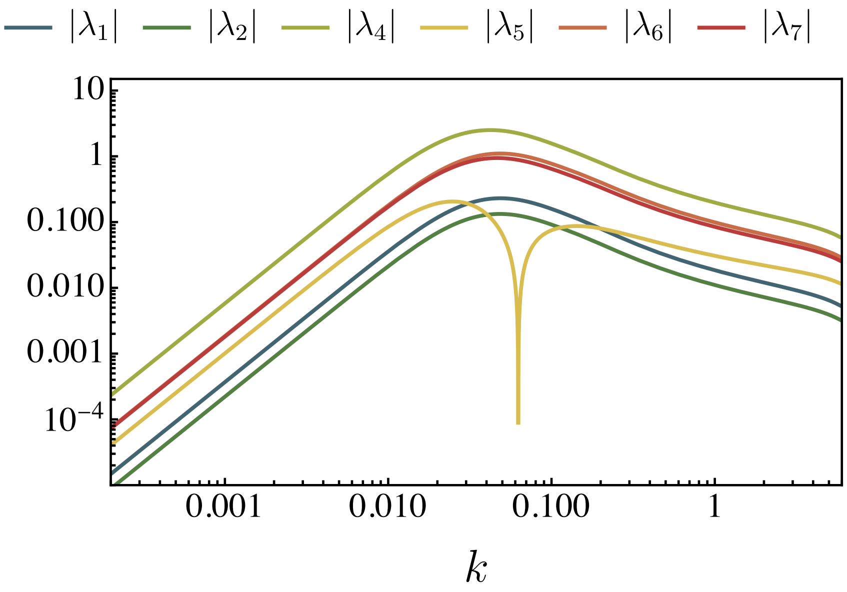
Bars-Yankielowicz $Nc = 4$ → Confinement !





Results !

Bars-Yankielowicz $Nc = 4$ → Confinement !

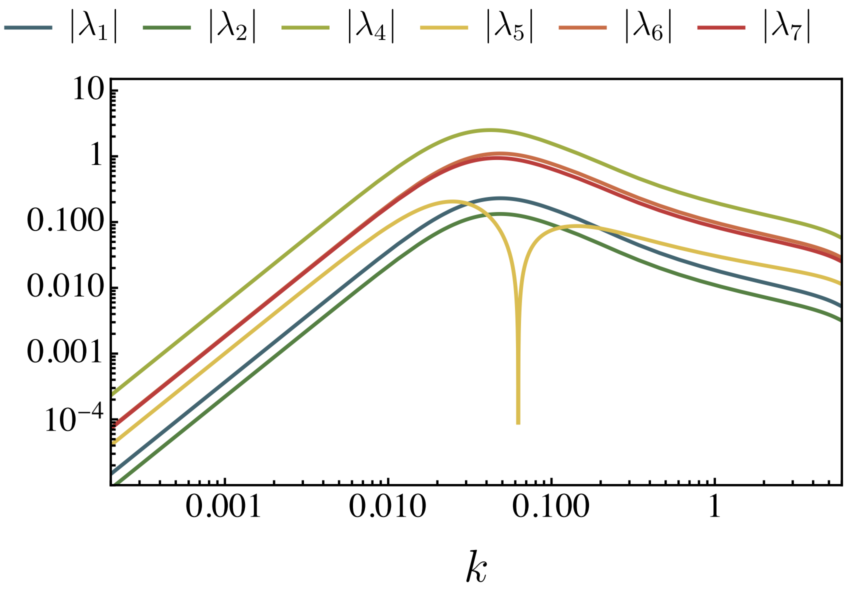


Results !

Bars-Yankielowicz $Nc = 4$

→ Confinement !

→ Without Symmetry Breaking !



Uncertainties

Pertubation Theory

- Renormalization scale :

$$\mu \in [Q/2\ ;\ 2Q]$$

- Scheme dependence

- Pertubation Theory :

$$A \sim \sum a_n \alpha^n$$

$$\Delta_n \sim a_n \alpha^n$$

fRG

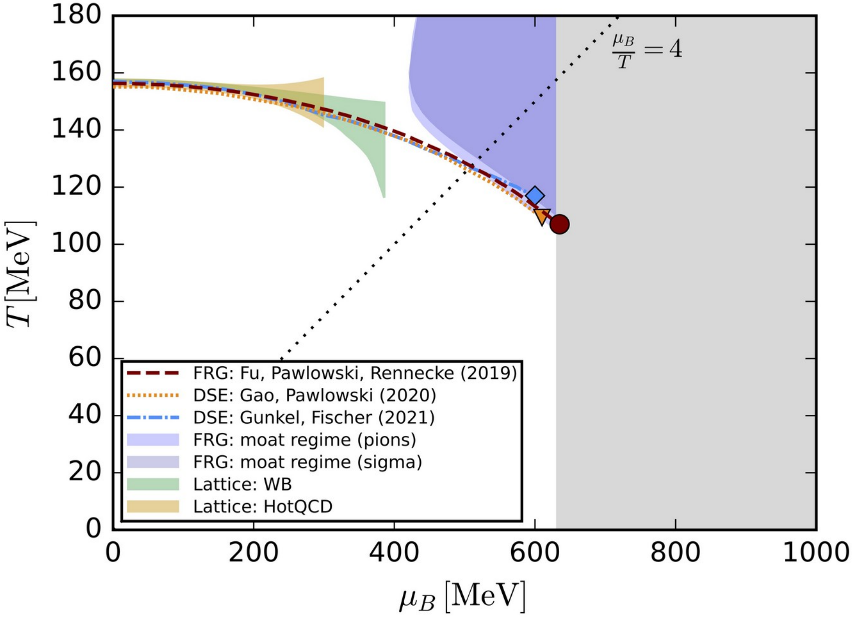
- Λ_{UV}

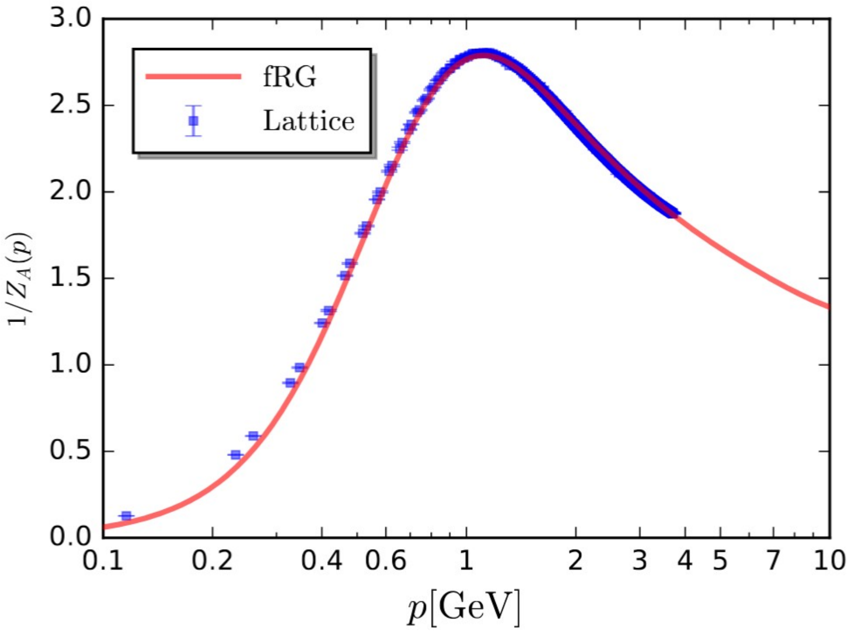
- Regulator dependence R_k

- Vertex Expansion ?

$$\Gamma = \sum_n \Gamma^{(n)} ?$$

Power of Functional Methods





Results

$$\mathcal{O}^1 = \left(\overline{\psi}^{f_1 i_1}\gamma^\mu P_L \psi_{f_1 i_1}\right)\left(\overline{\psi}^{f_2 i_2}\gamma^\mu P_L \psi_{f_2 i_2}\right),$$

$$\mathcal{O}^2 = \left(\overline{\psi}^{f_1 i_1}\gamma^\mu P_L \psi_{f_2 i_1}\right)\left(\overline{\psi}^{f_2 i_2}\gamma^\mu P_L \psi_{f_1 i_2}\right),$$

$$\mathcal{O}^3 = \left(\overline{\chi}^{f_1 a_1}\gamma^\mu P_L \chi_{f_1 a_2}\right)\left(\overline{\chi}^{f_2 a_2}\gamma^\mu P_L \chi_{f_2 a_1}\right),$$

$$\mathcal{O}^4 = \left(\overline{\chi}^{f_1 a_1}\gamma^\mu P_L \chi_{f_1 a_1}\right)\left(\overline{\chi}^{f_2 a_2}\gamma^\mu P_L \chi_{f_2 a_2}\right),$$

$$\mathcal{O}^5 = \left(\overline{\chi}^{f_1 a_1}\gamma^\mu P_L T^{a_2}_{a_1} \chi_{f_1 a_2}\right)\left(\overline{\chi}^{f_2 a_3}\gamma^\mu P_L T^{a_4}_{a_3} \chi_{f_2 a_4}\right),$$

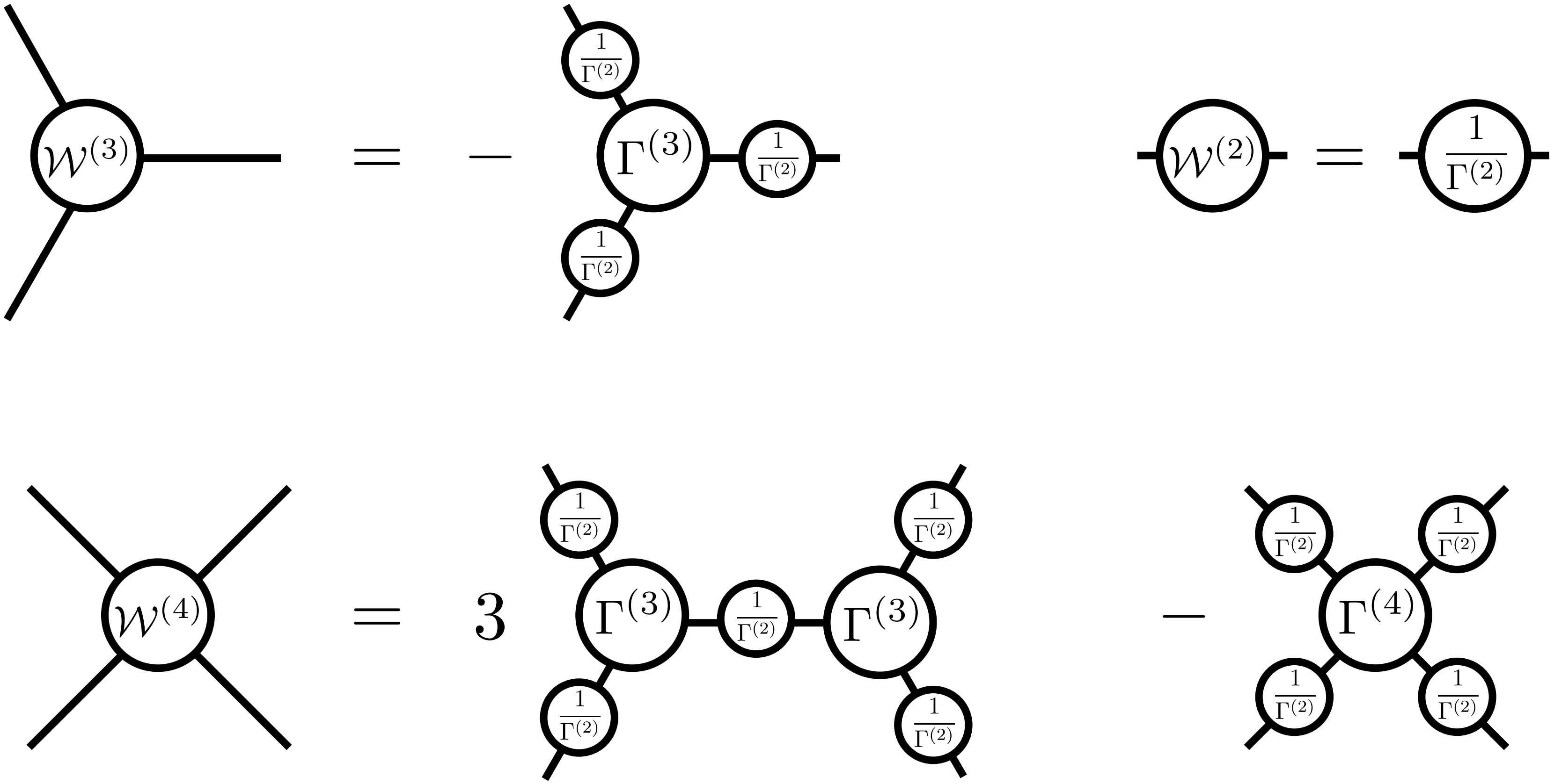
$$\mathcal{O}^6 = \left(\overline{\psi}^{f_1 i_1}\gamma^\mu P_L \psi_{f_1 i_1}\right)\left(\overline{\chi}^{f_2 a_2}\gamma^\mu P_L \chi_{f_2 a_2}\right),$$

$$\mathcal{O}^7 = \left(\overline{\psi}^{f_1 i_1}\gamma^\mu P_L T^{i_2}_{i_1} \psi_{f_1 i_2}\right)\left(\overline{\chi}^{f_2 a_3}\gamma^\mu P_L T^{a_4}_{a_3} \chi_{f_2 a_4}\right).$$

$$\psi \longleftrightarrow \bar{F}$$

$$\chi \longleftrightarrow A$$

How To Diagnose χSB?



Chiral Dynamic

- Vacuum $\Rightarrow G/H$

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- H - Anomalies $\neq 0$

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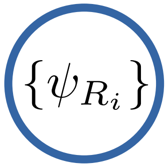
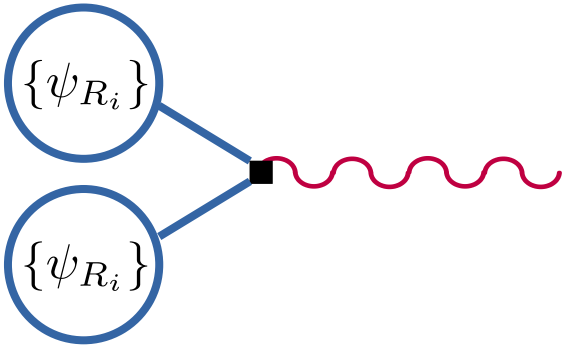
$\longrightarrow \quad \mathcal{B} = \{\psi_{R_i}\} \qquad m_{\mathcal{B}} = 0$

Chiral Dynamic

- Vacuum $\Rightarrow G/H$
- H- Anomalies $\neq 0$

→ $\mathcal{B} =$ 

$m_{\mathcal{B}} = 0$

→ 

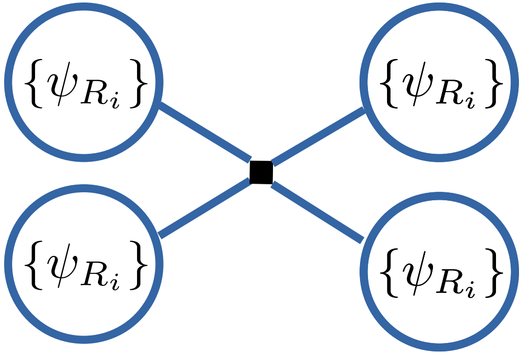
$\rho_\mu = \bar{\psi}\sigma_\mu\psi$

Chiral Dynamic

- Vacuum $\Rightarrow G/H$
- H- Anomalies $\neq 0$

→ $\mathcal{B} = \{\psi_{R_i}\}$ $\quad\quad m_{\mathcal{B}} = 0$

→



How To Diagnose χSB?

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- $U(1)_V \times U(1)_A$

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$$\sigma \to \sigma + y\frac{\bar{\psi}\psi}{\sqrt{2}m^2} \qquad \pi \to \pi + i\, y\frac{\bar{\psi}\gamma^5\psi}{\sqrt{2}m^2}$$

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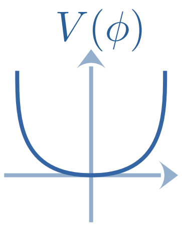

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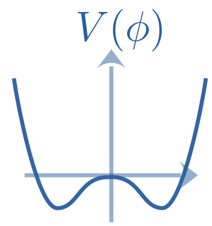

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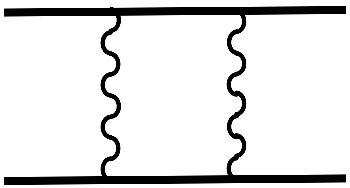

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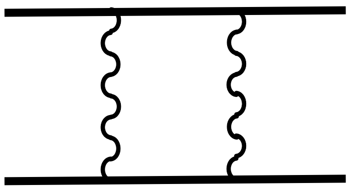

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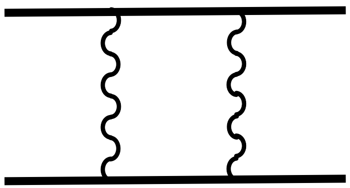

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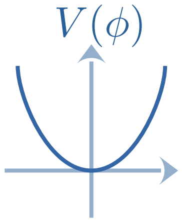

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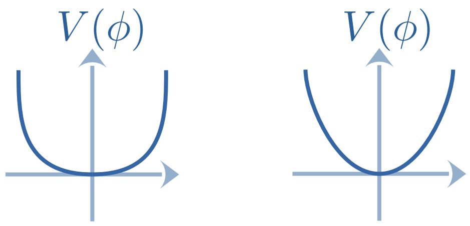

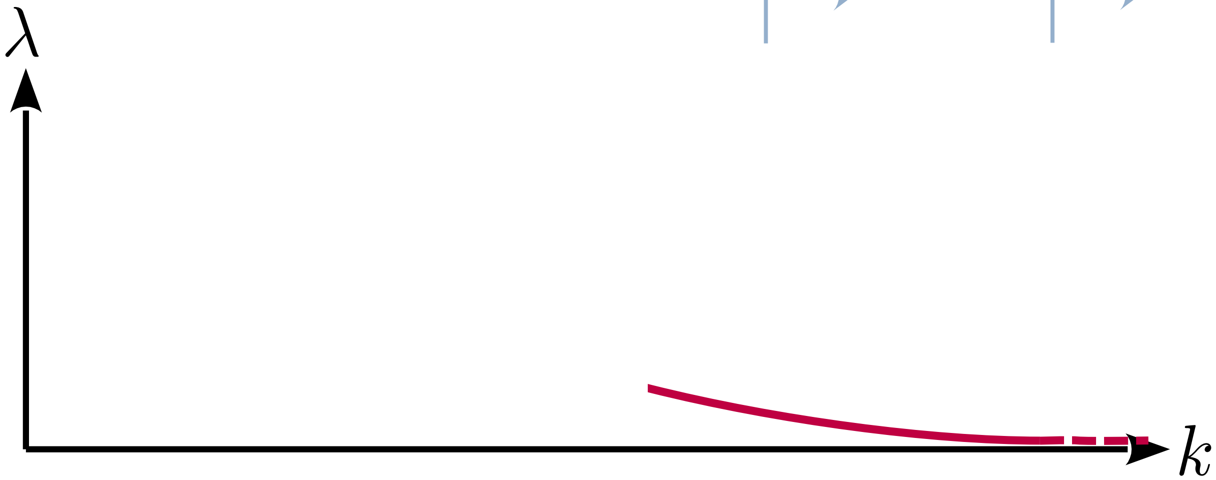

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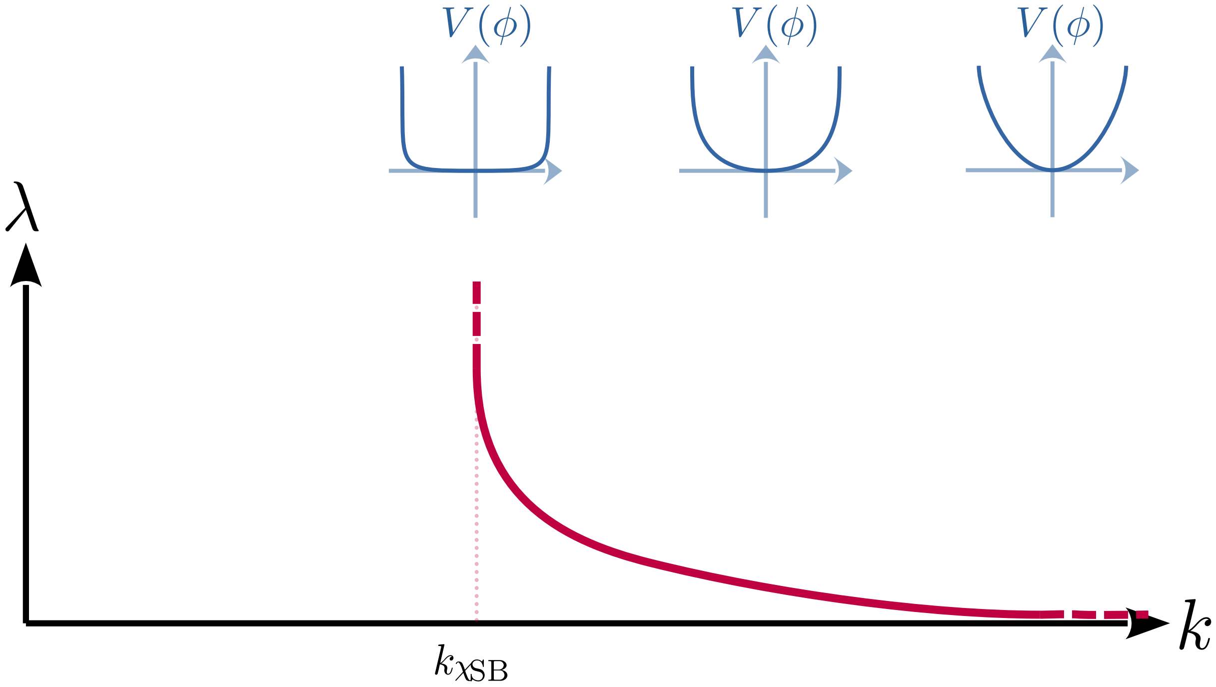

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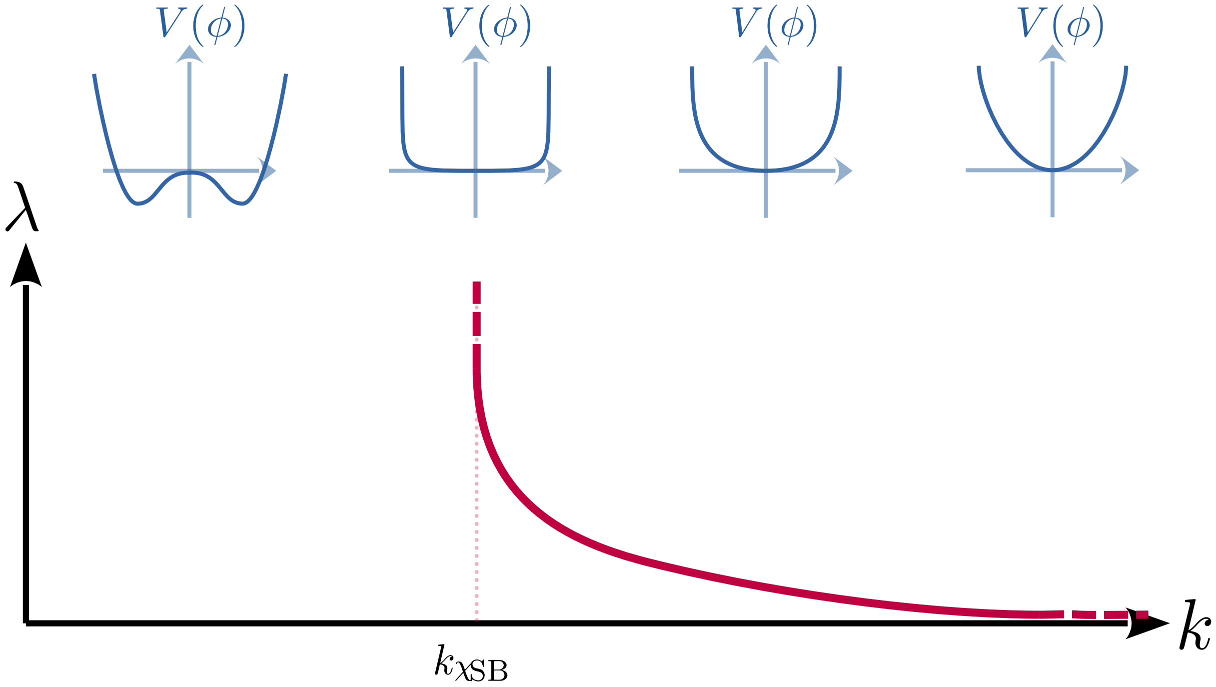

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→ Does one of the λ_i diverge ?

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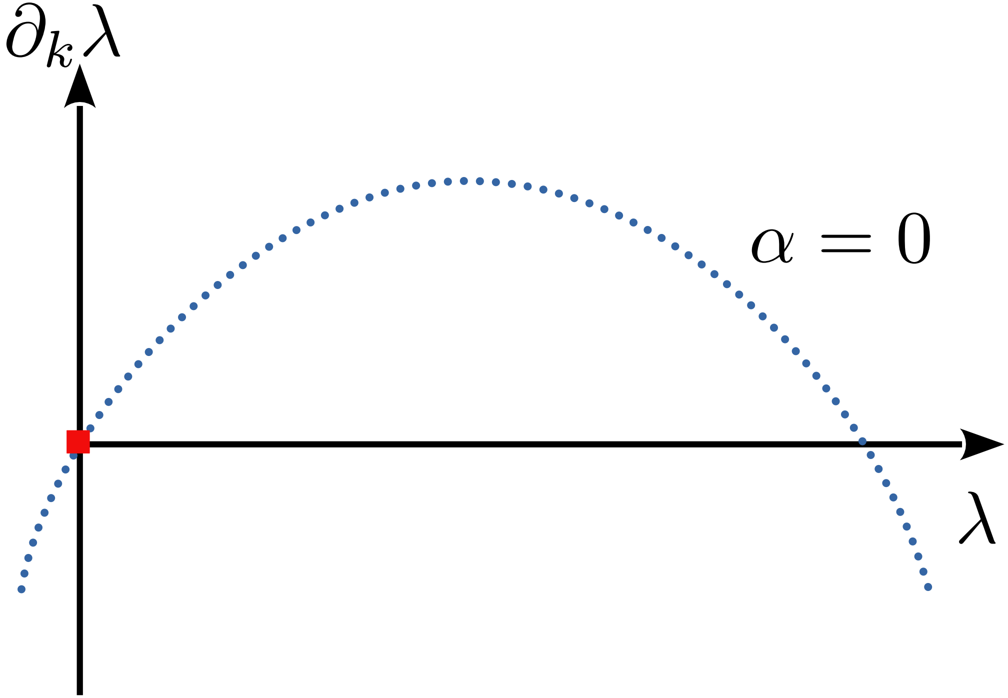

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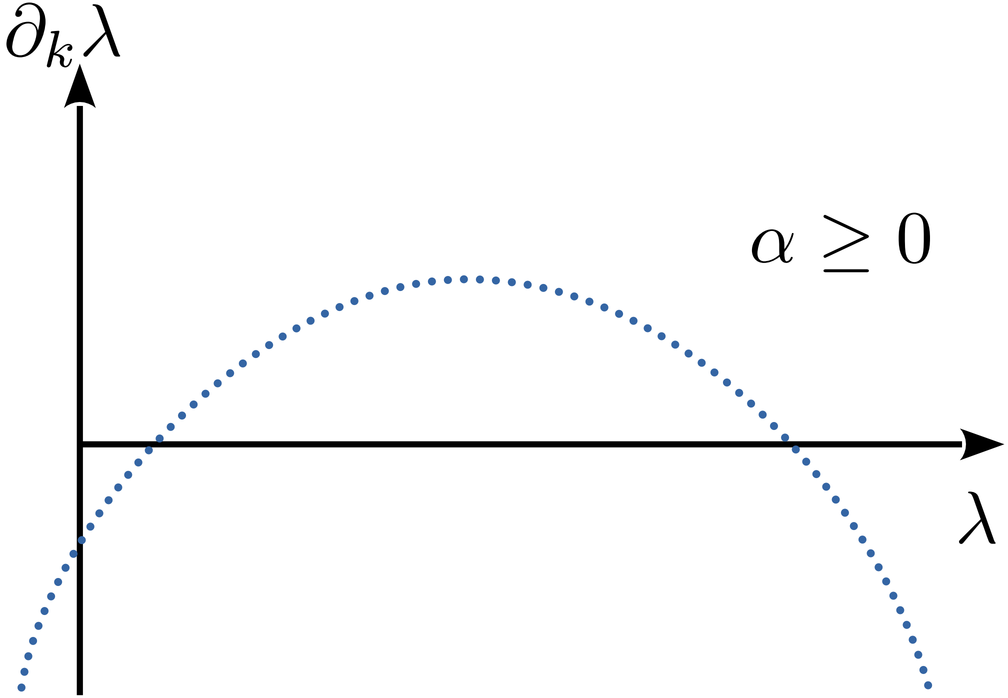

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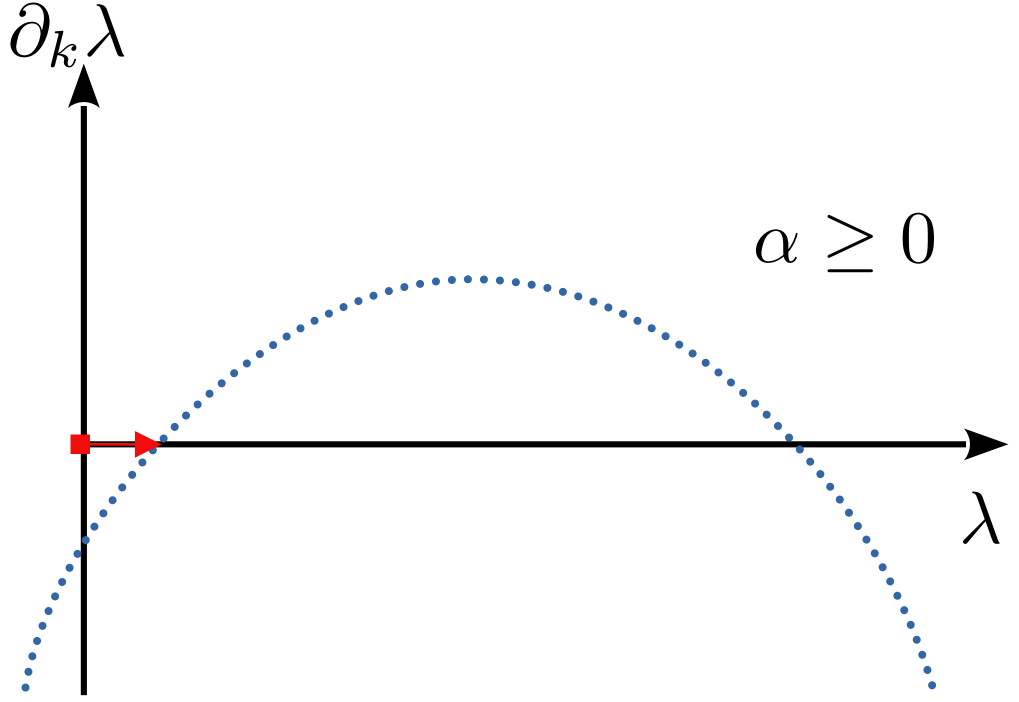

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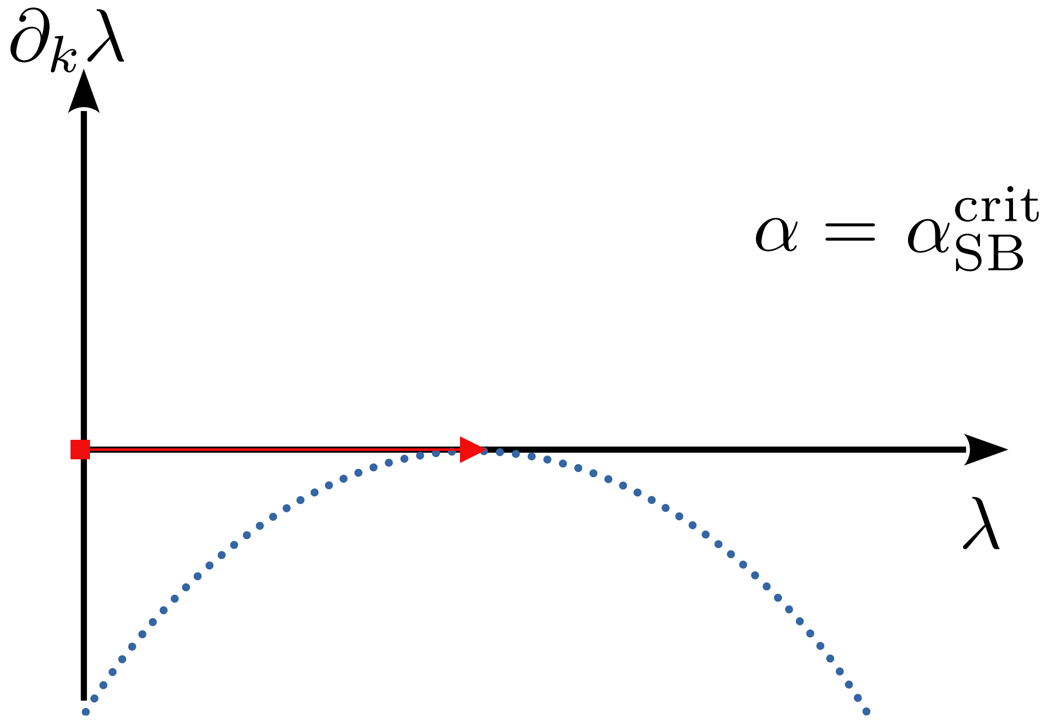

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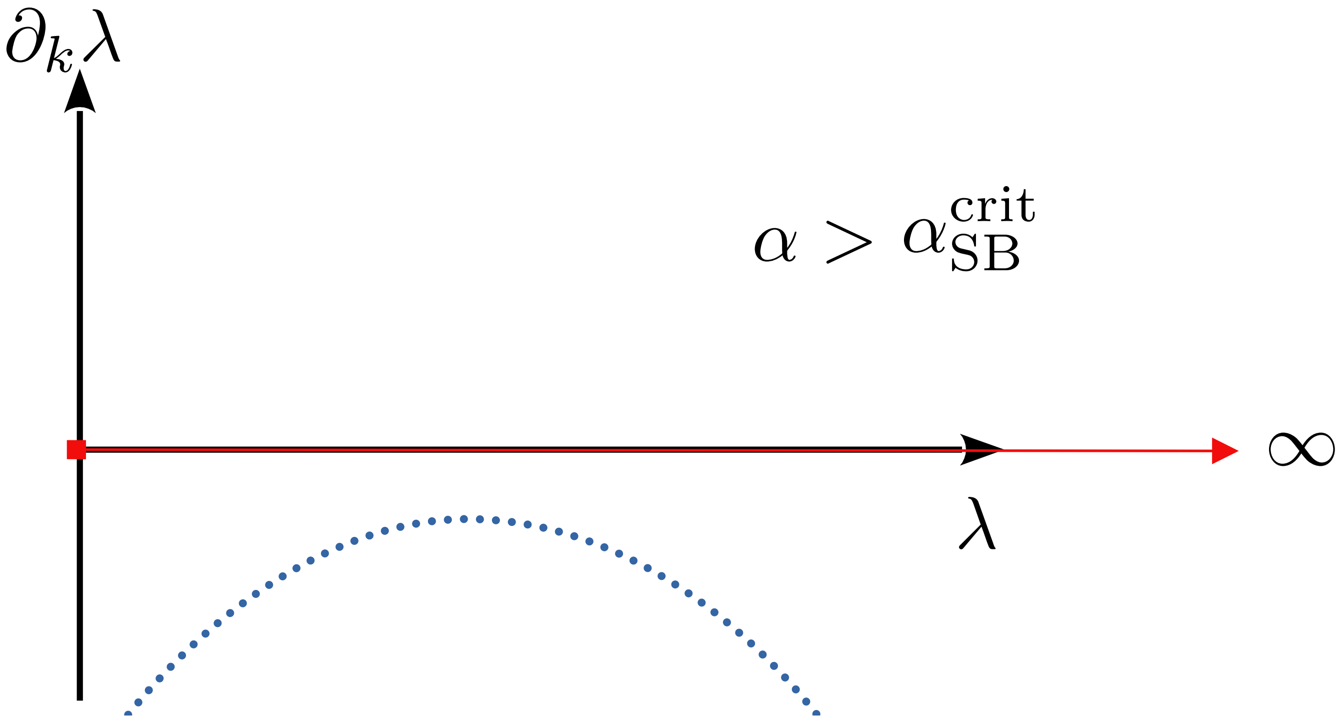

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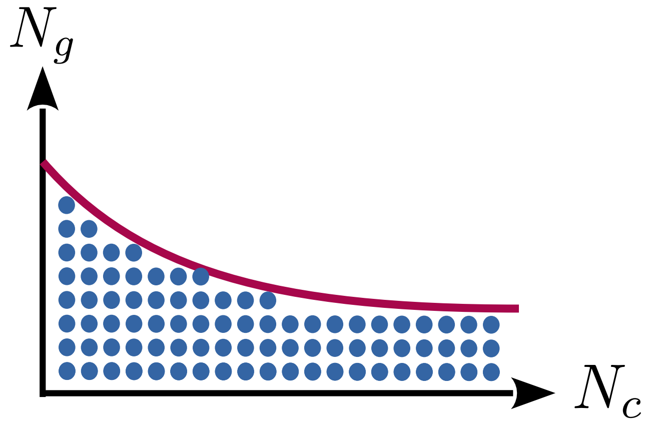

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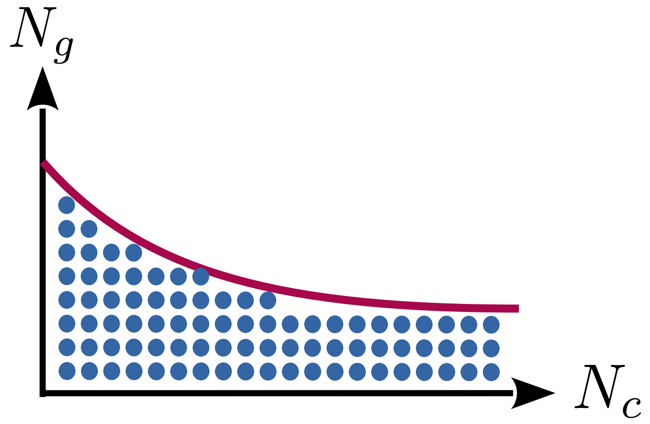

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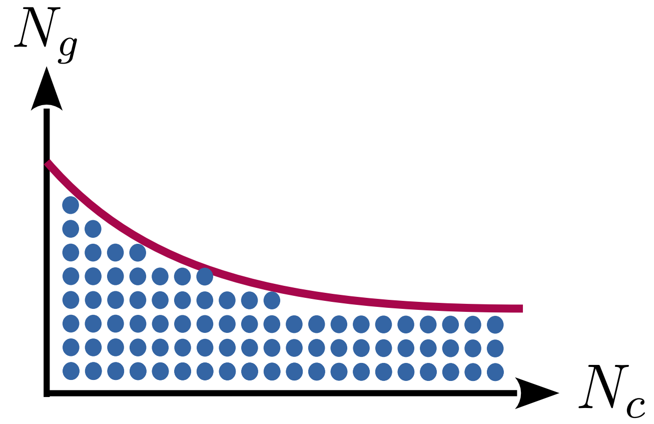

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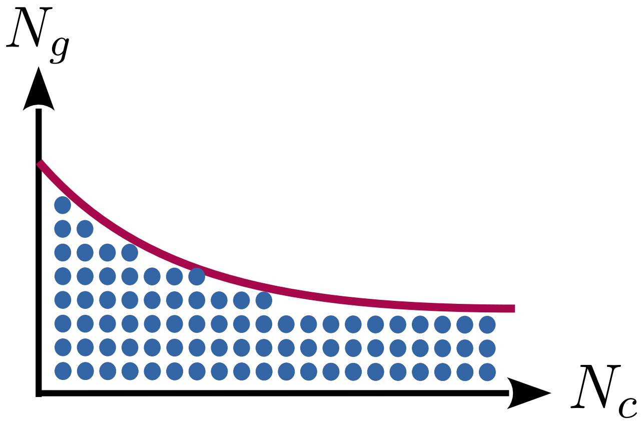

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We use 2, 3, 4-loops β - function

Results

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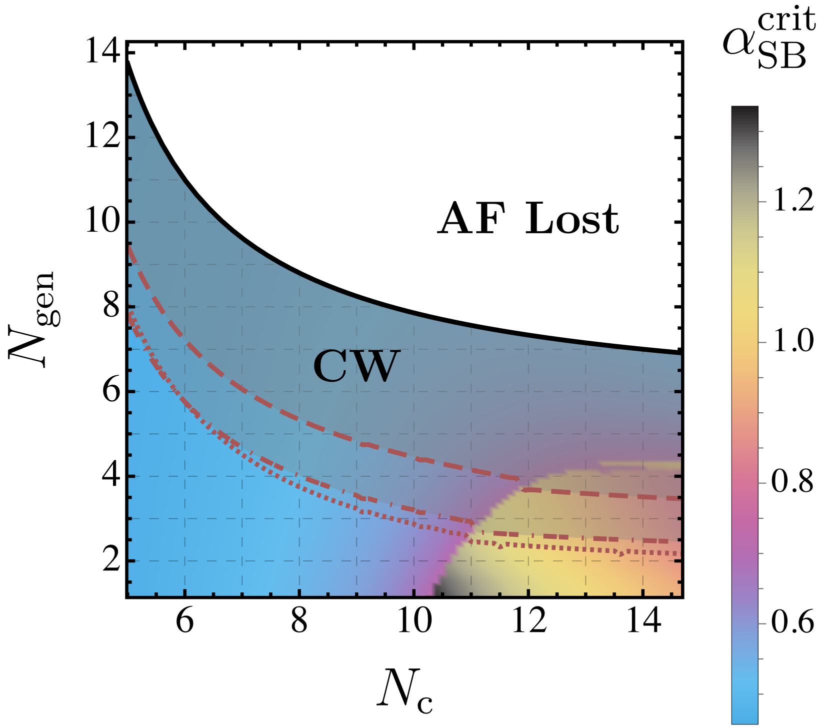
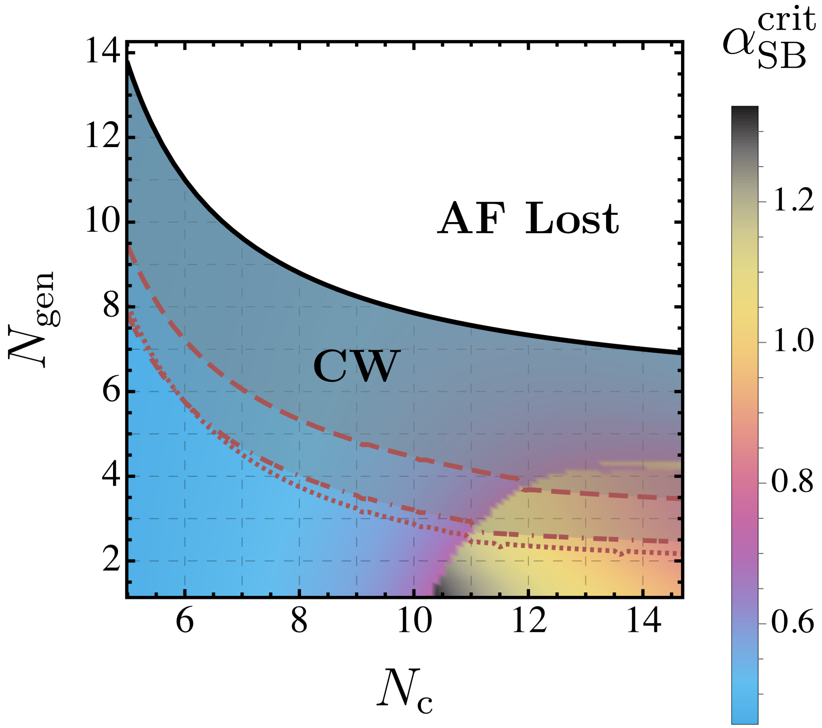


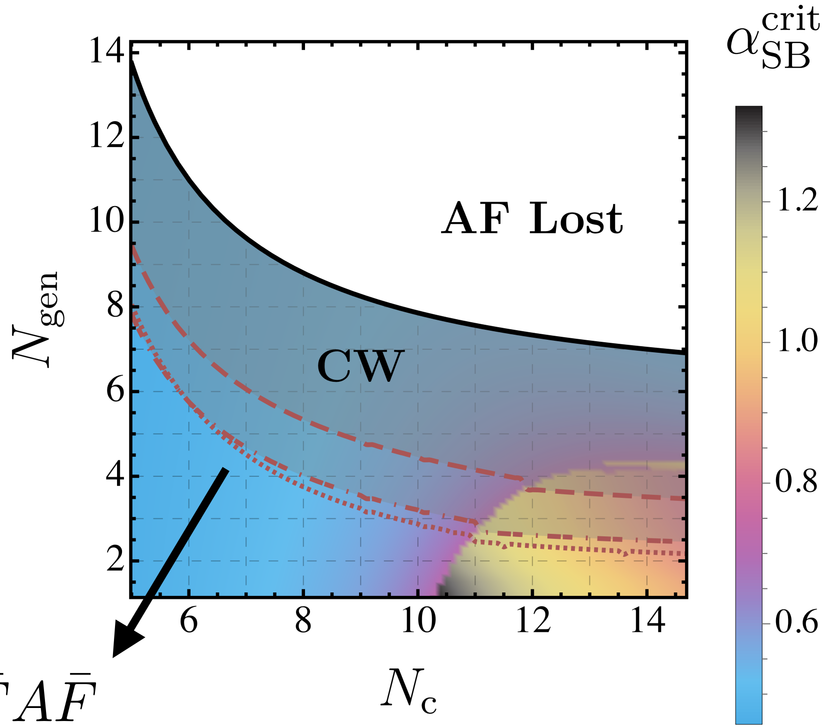
AF Lost
CW
$N_{\rm gen}$
$N_{\rm c}$
$\alpha_{\rm SB}^{\rm crit}$
14
12
10
8
6
4
2
6
8
10
12
14
1.2
1.0
0.8
0.6

Results



→ IR Conformal !

Results



$A\bar{F}A\bar{F}$

$\langle A\bar{F}\rangle \neq 0$

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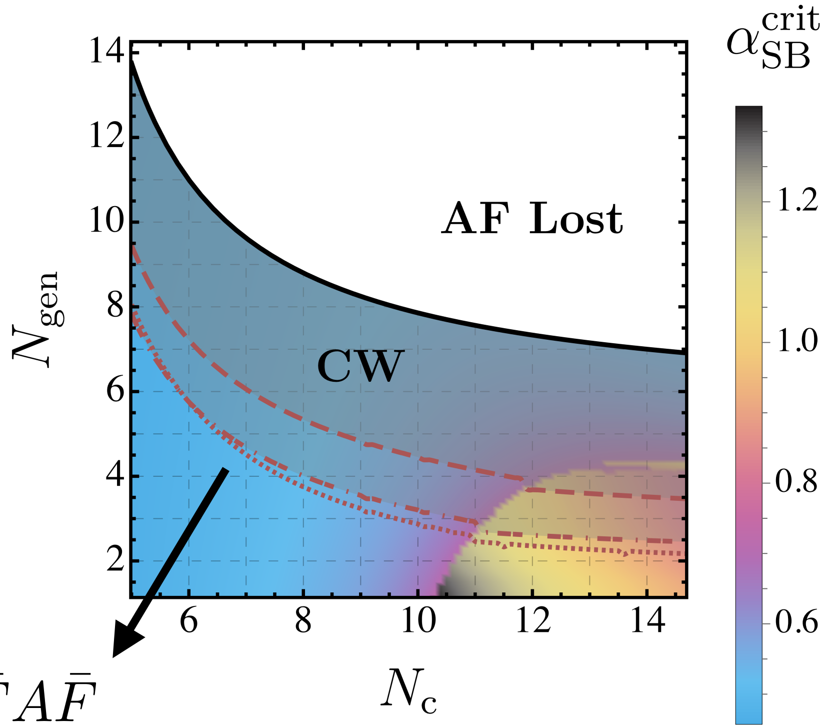
→ Clear signal for χSB

Results



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→ 2nd region for $\chi\mathrm{SB}$