Scattering Amplitudes

With applications to Gravitational Wave Physics

Stefano De Angelis, Rencontres de Physique des Particules 2026

The Modern Scattering Amplitudes Program

From QCD to $\mathcal{N}=4$ SYM: a historical perspective

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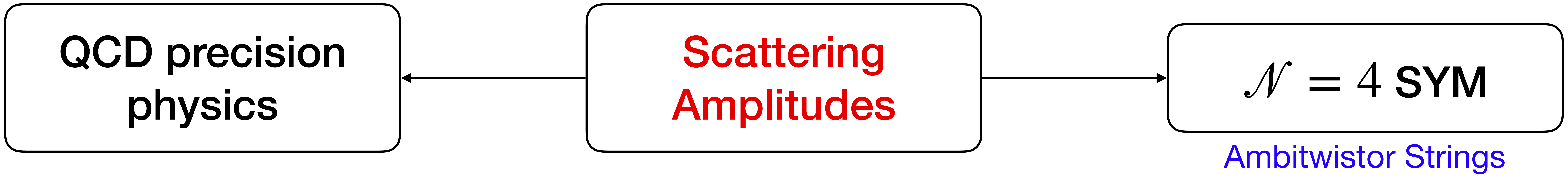

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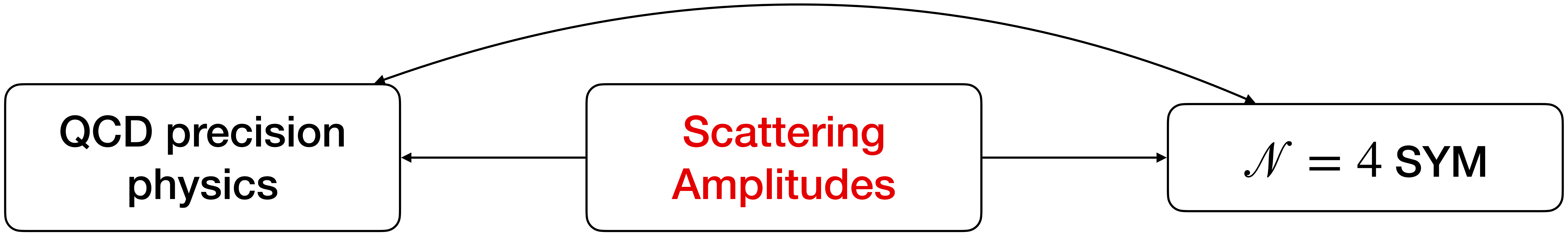
From QCD to $\mathcal{N} = 4$ SYM: a historical perspective

A. Unitarity-based construction (constructing loops from trees)
B. Simplicity and symmetries of the result (*e.g.* Parke-Taylor)
C. Principle of maximal transcendentality



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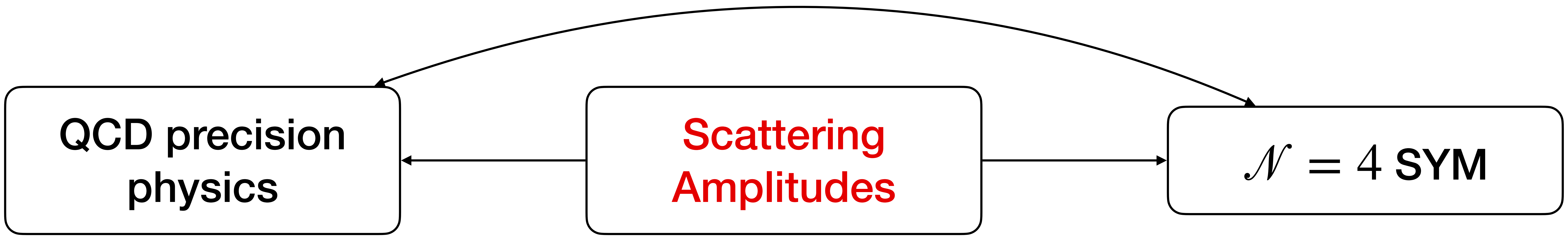

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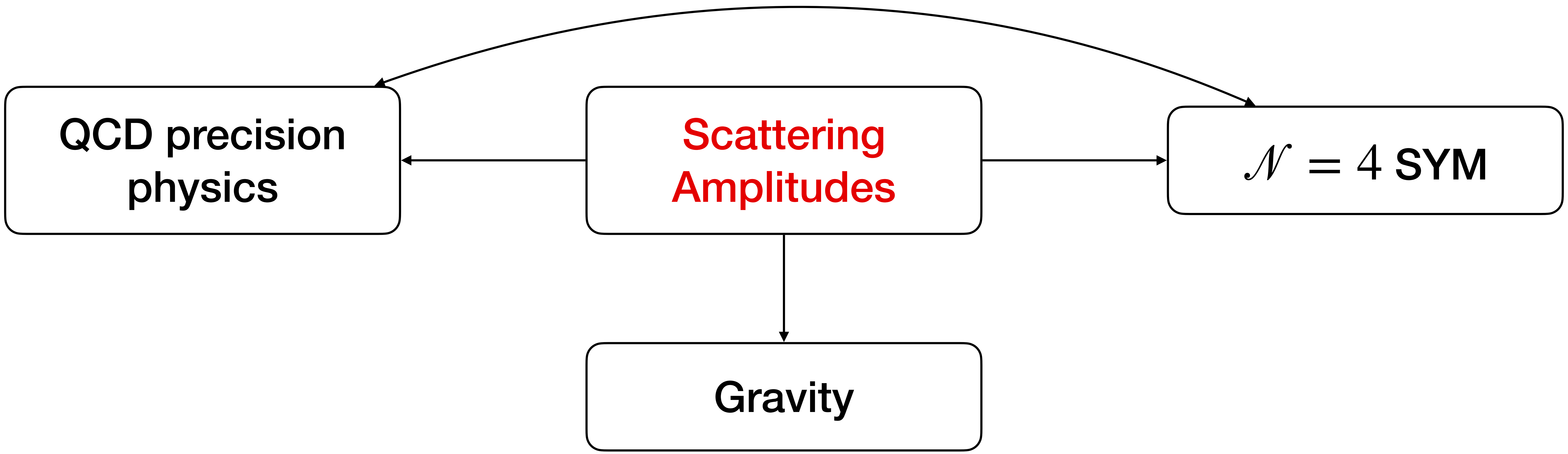

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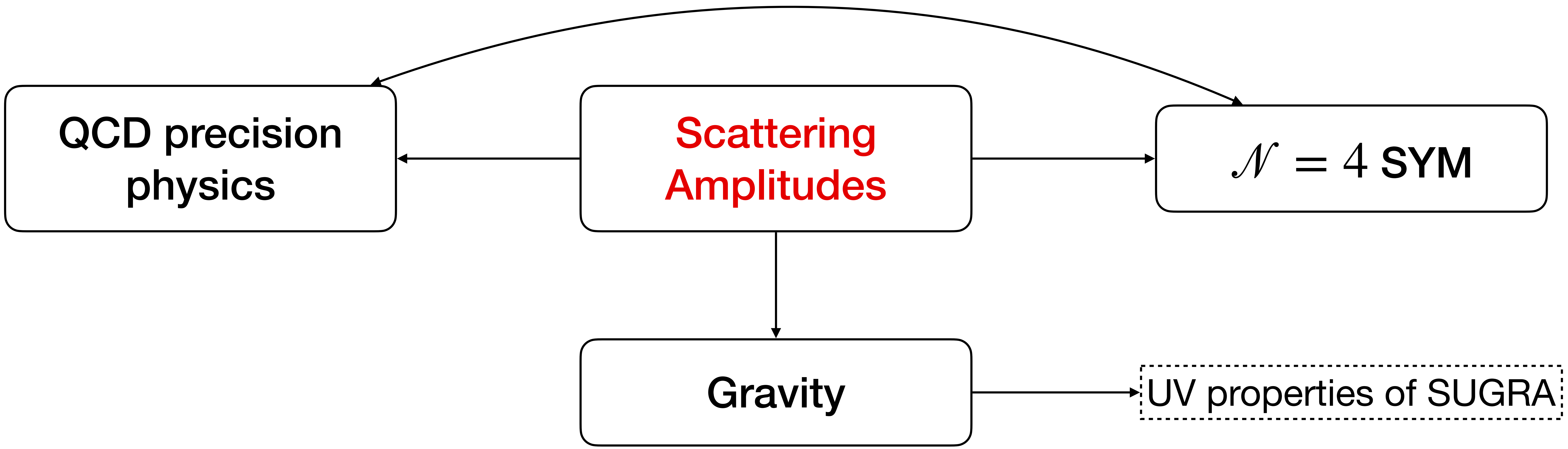

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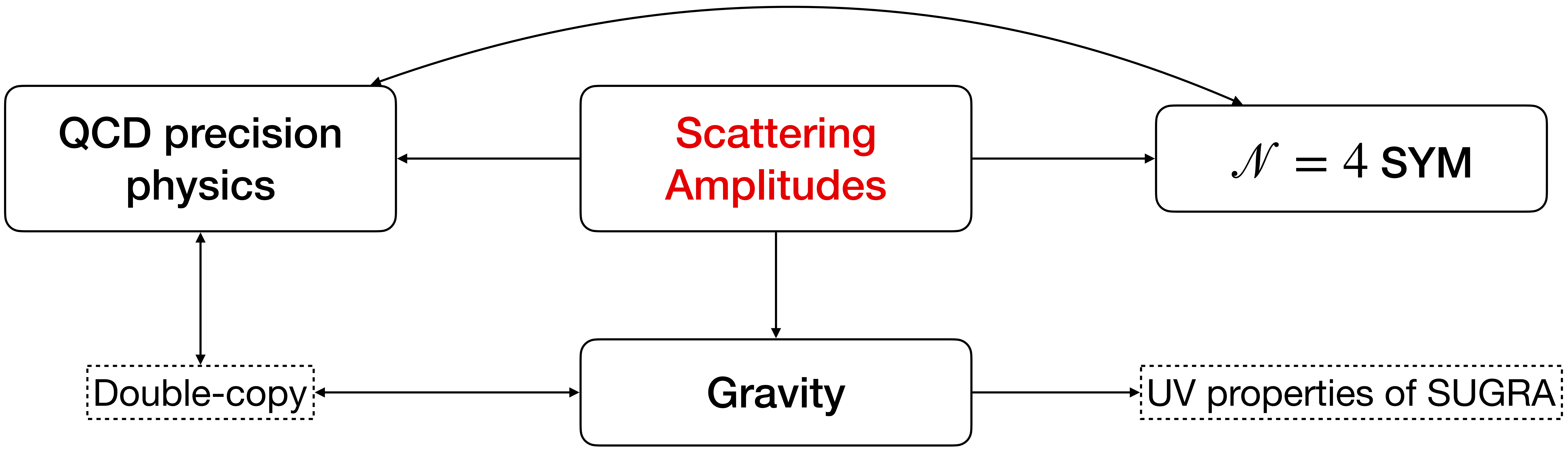

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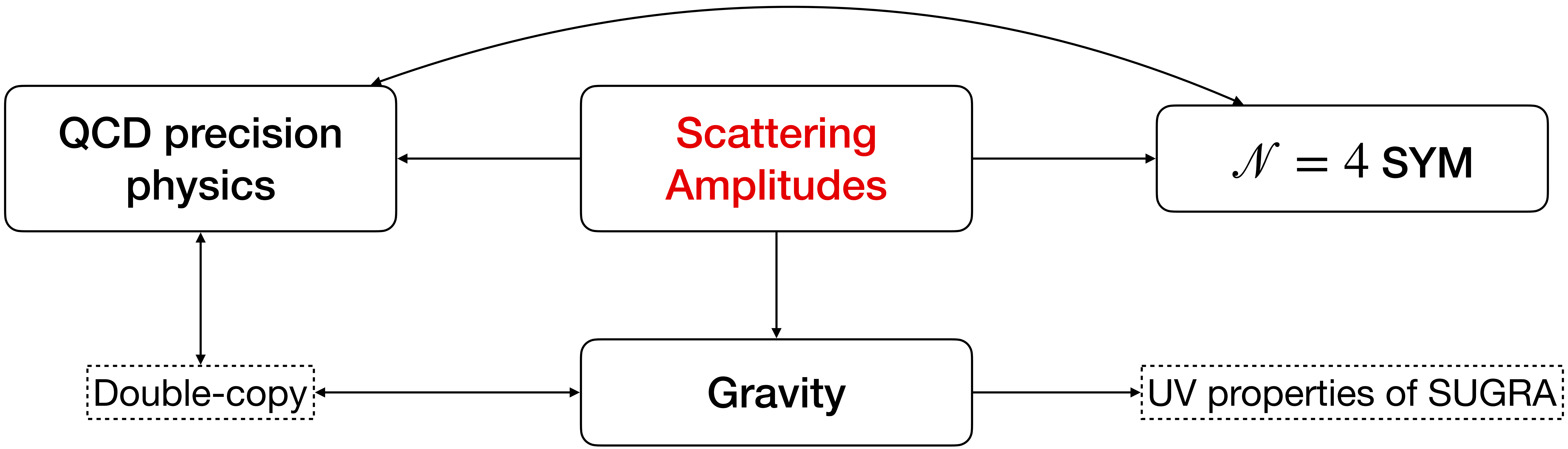

A. Colour-kinematic duality

$$\mathscr{A}_{\mathrm{YM}} = \sum_{\mathrm{graphs}} \frac{c_i \cdot n_i}{\prod_{a_i} p_{a_i}^2} ,$$

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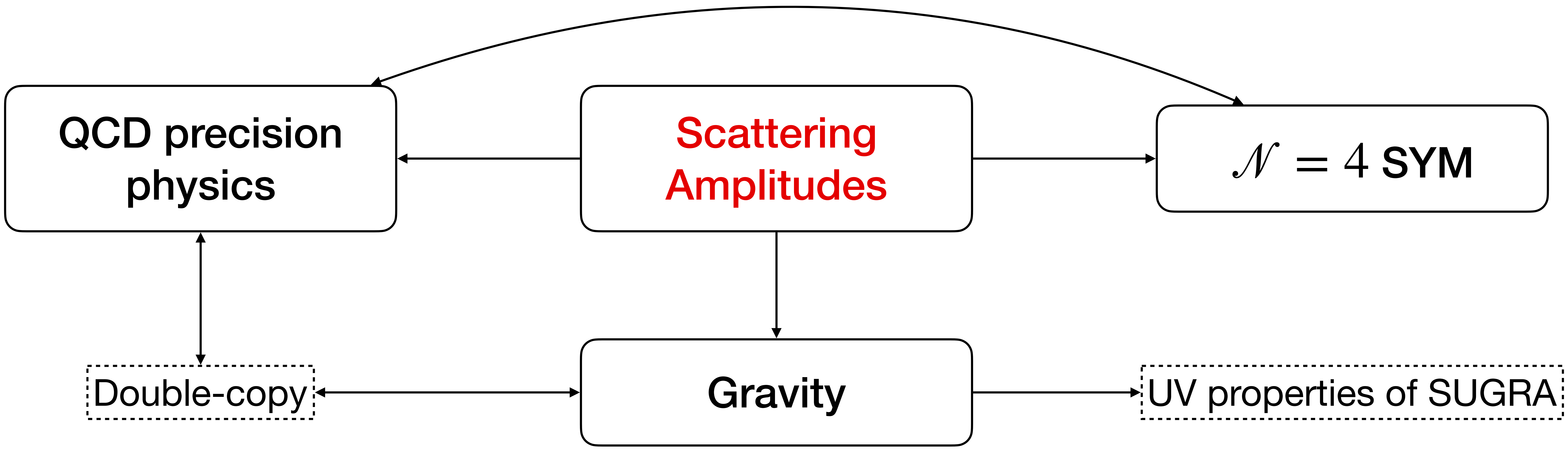

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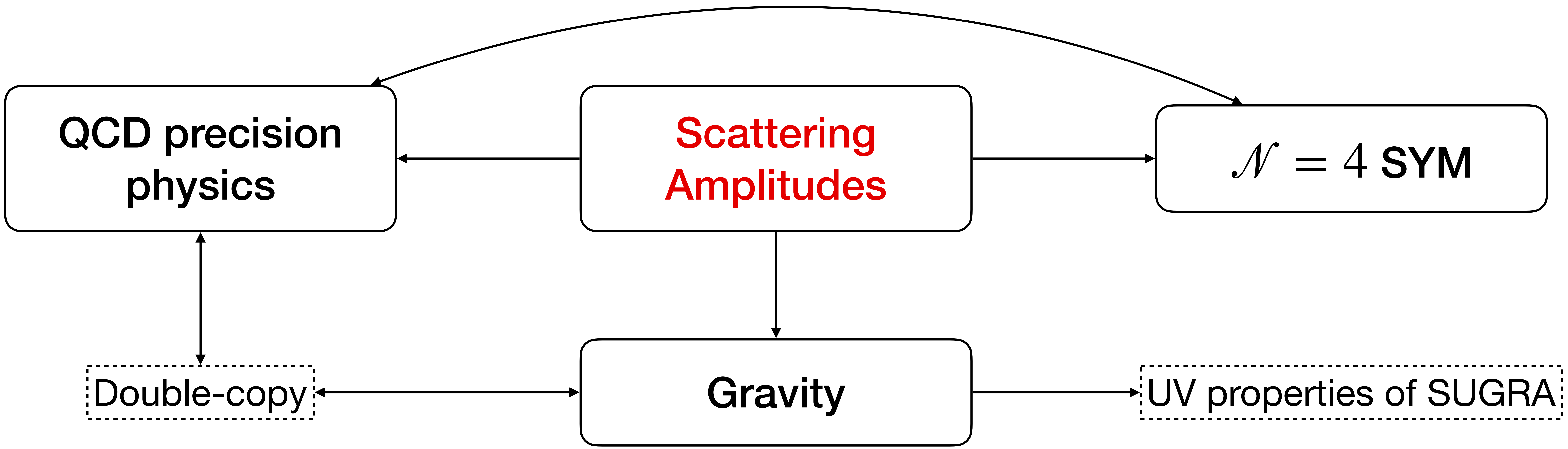

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B. $\mathrm{GR} = (\mathrm{YM})^2$

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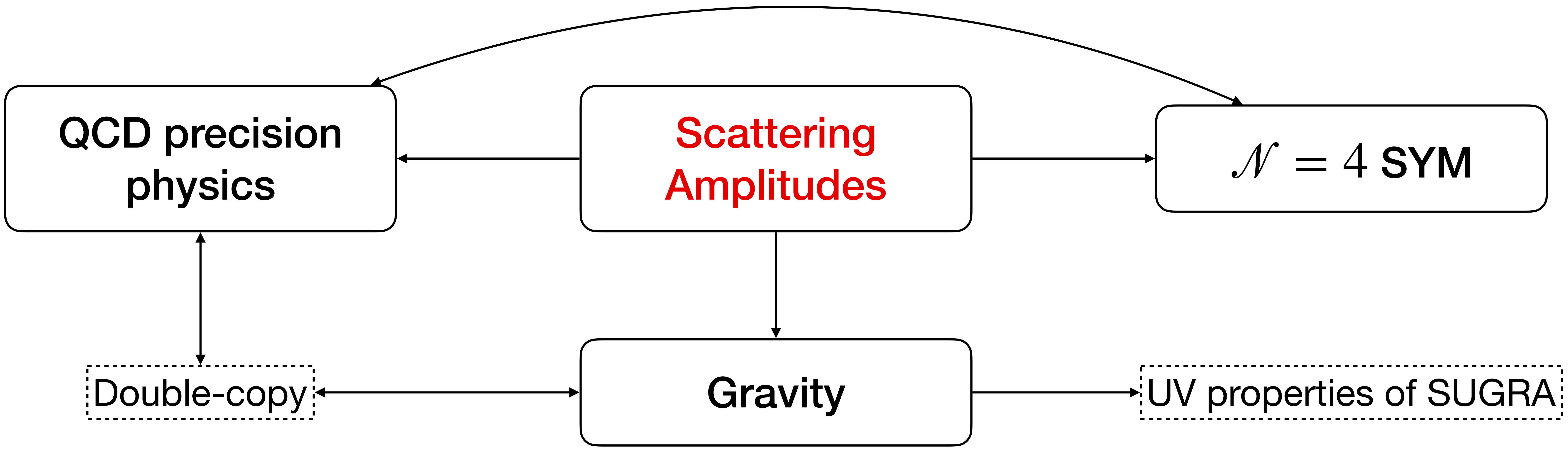

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C. Relations between a web of theories

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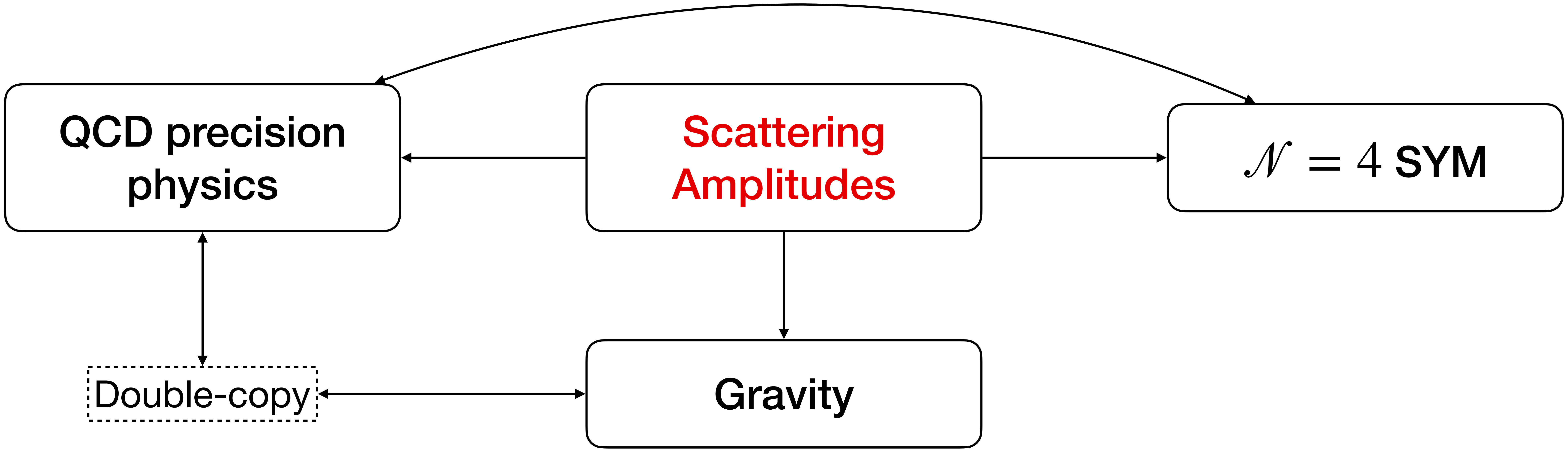

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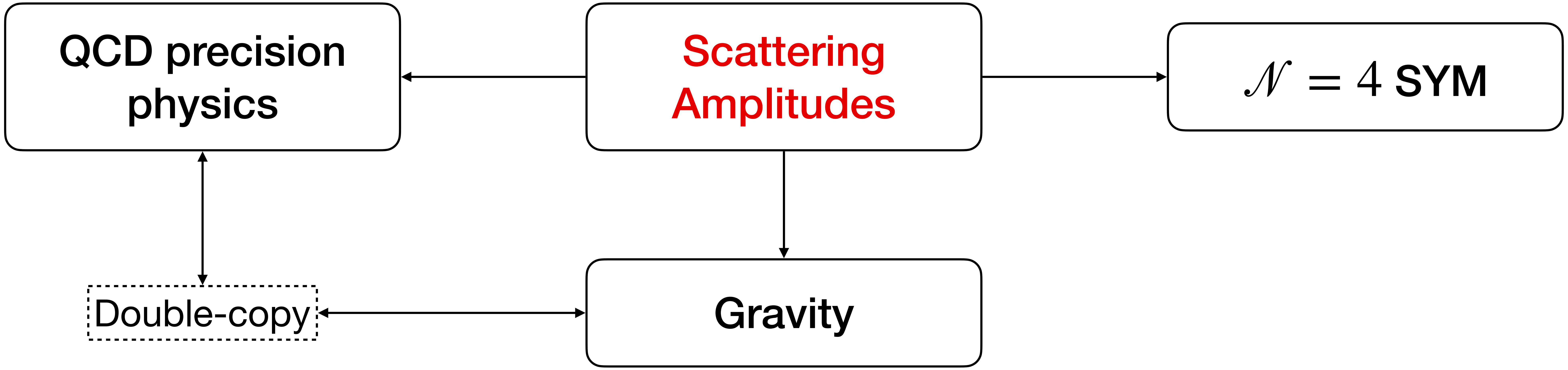

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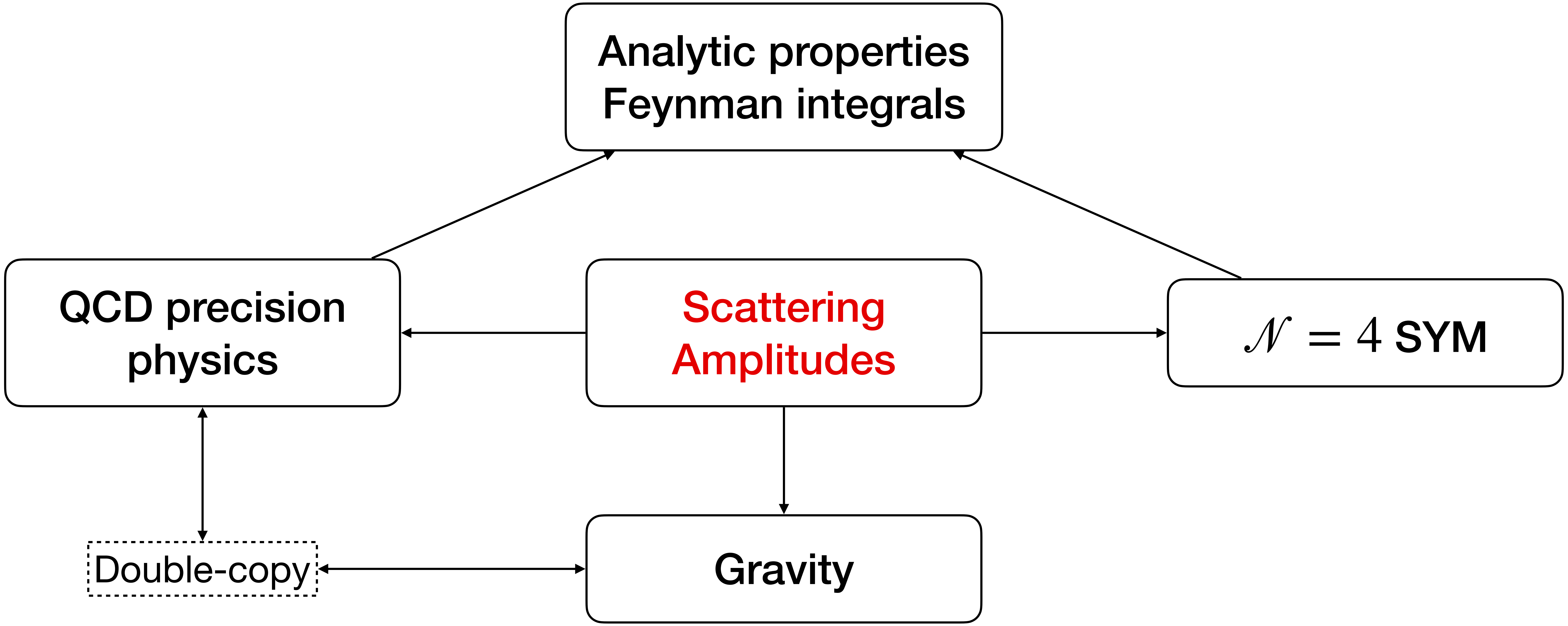

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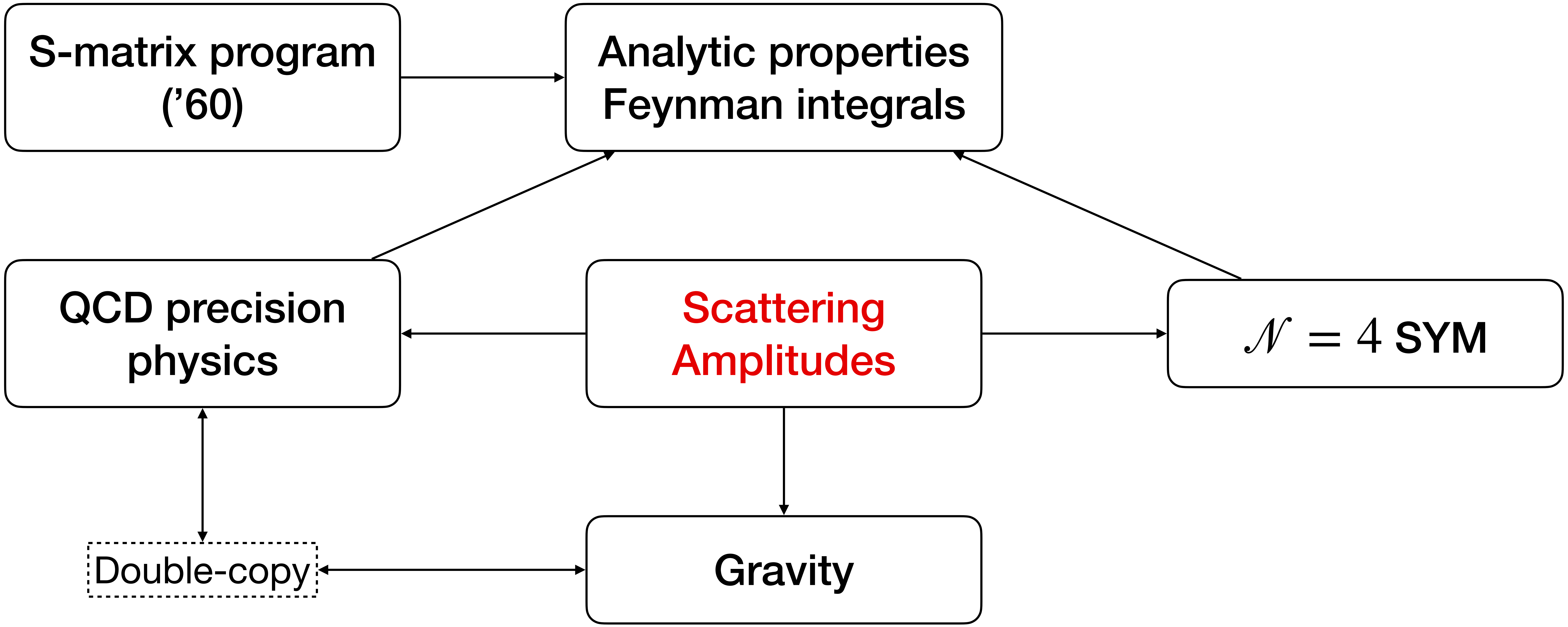

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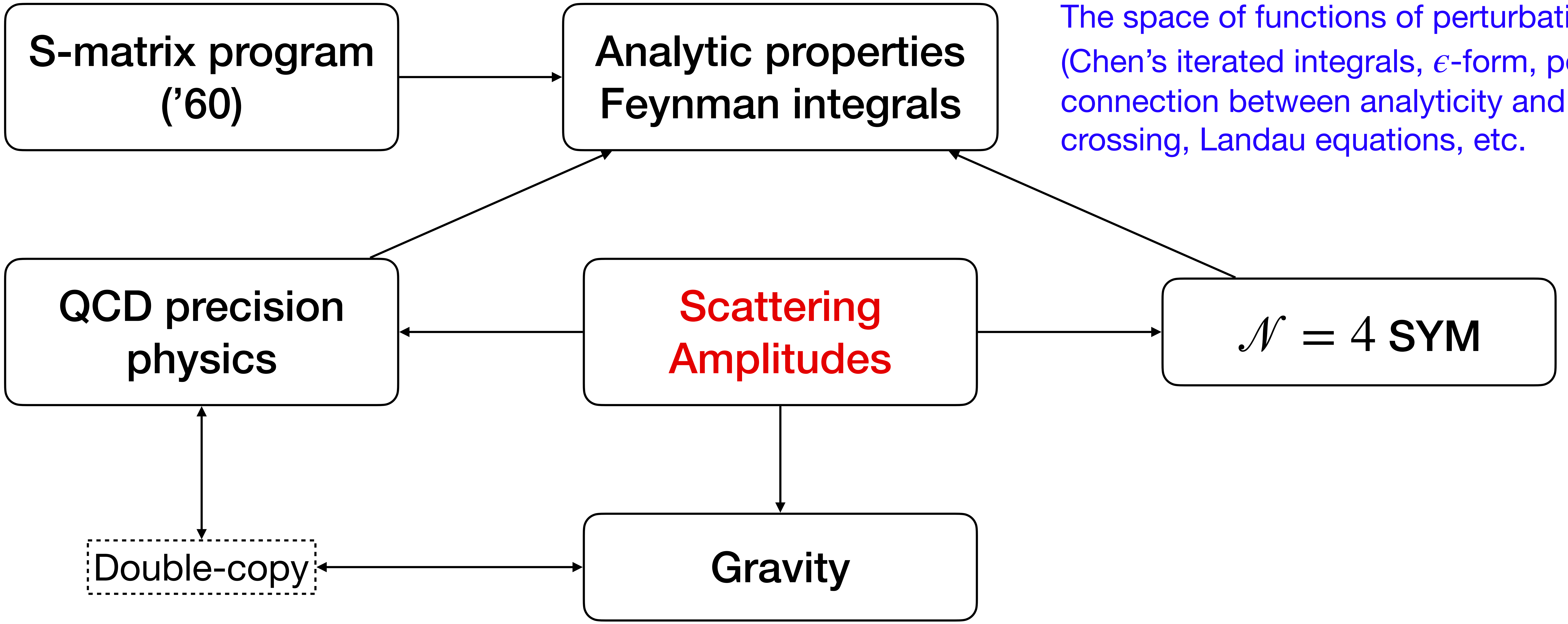

The space of functions of perturbative QFT (Chen's iterated integrals, ϵ-form, polylogs), connection between analyticity and causality, crossing, Landau equations, etc.

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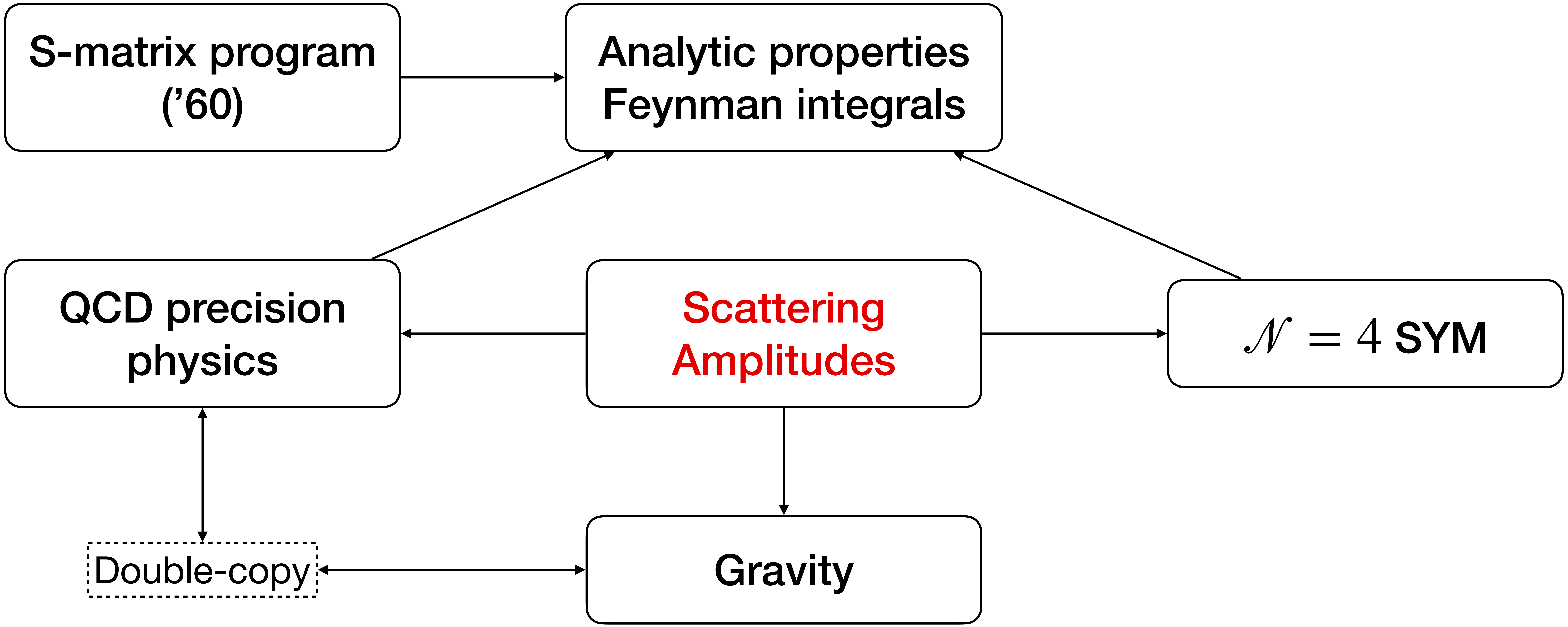

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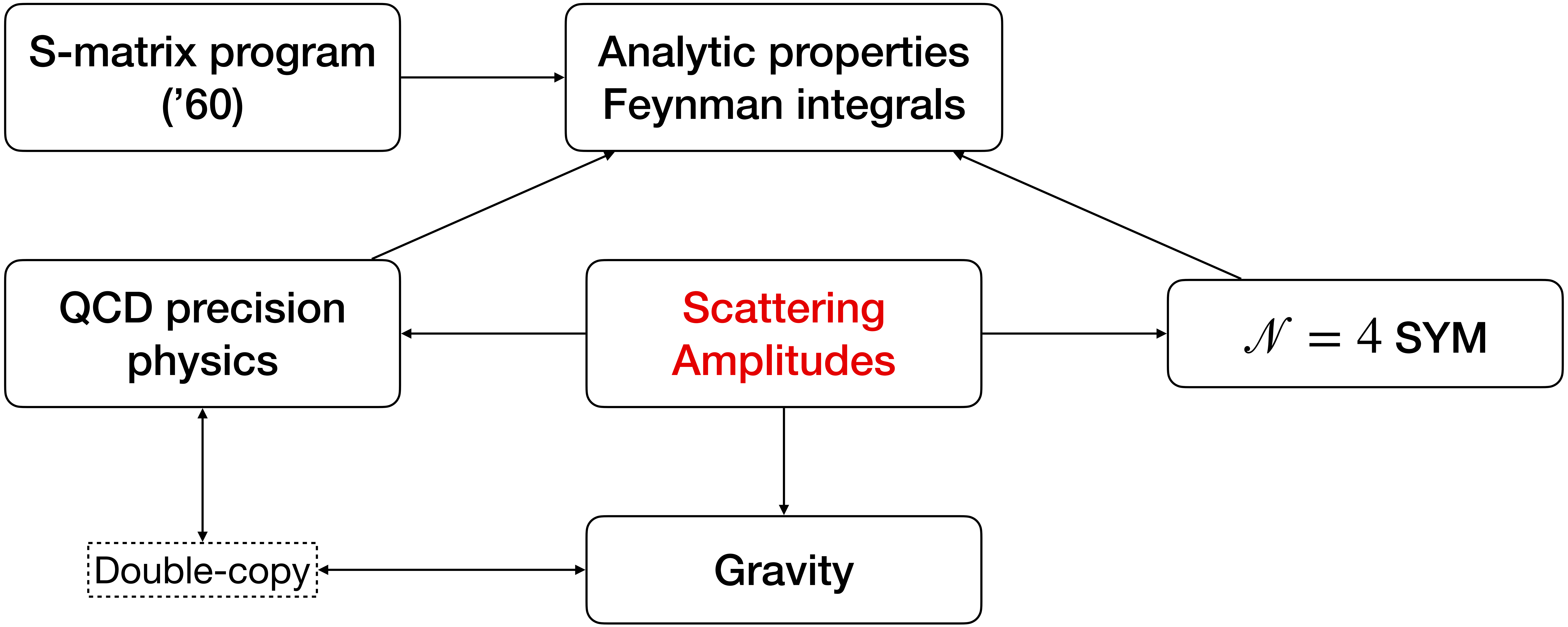

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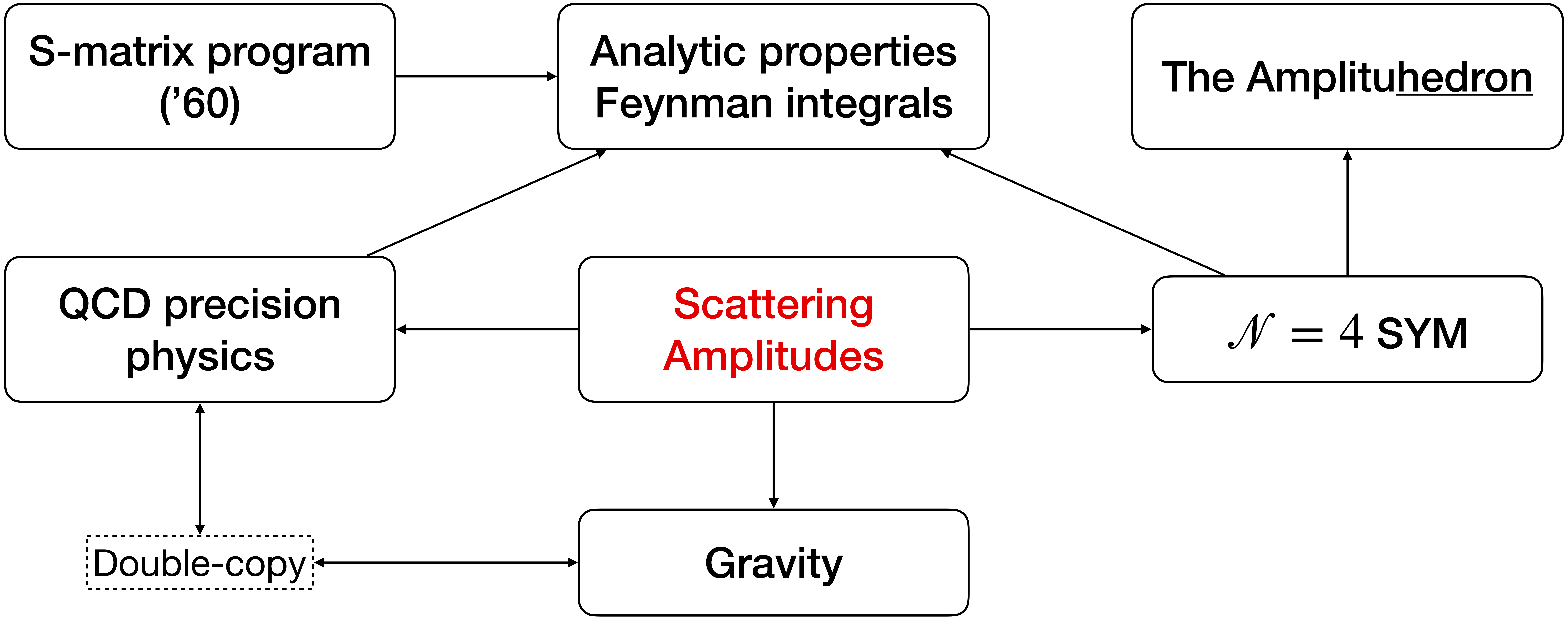

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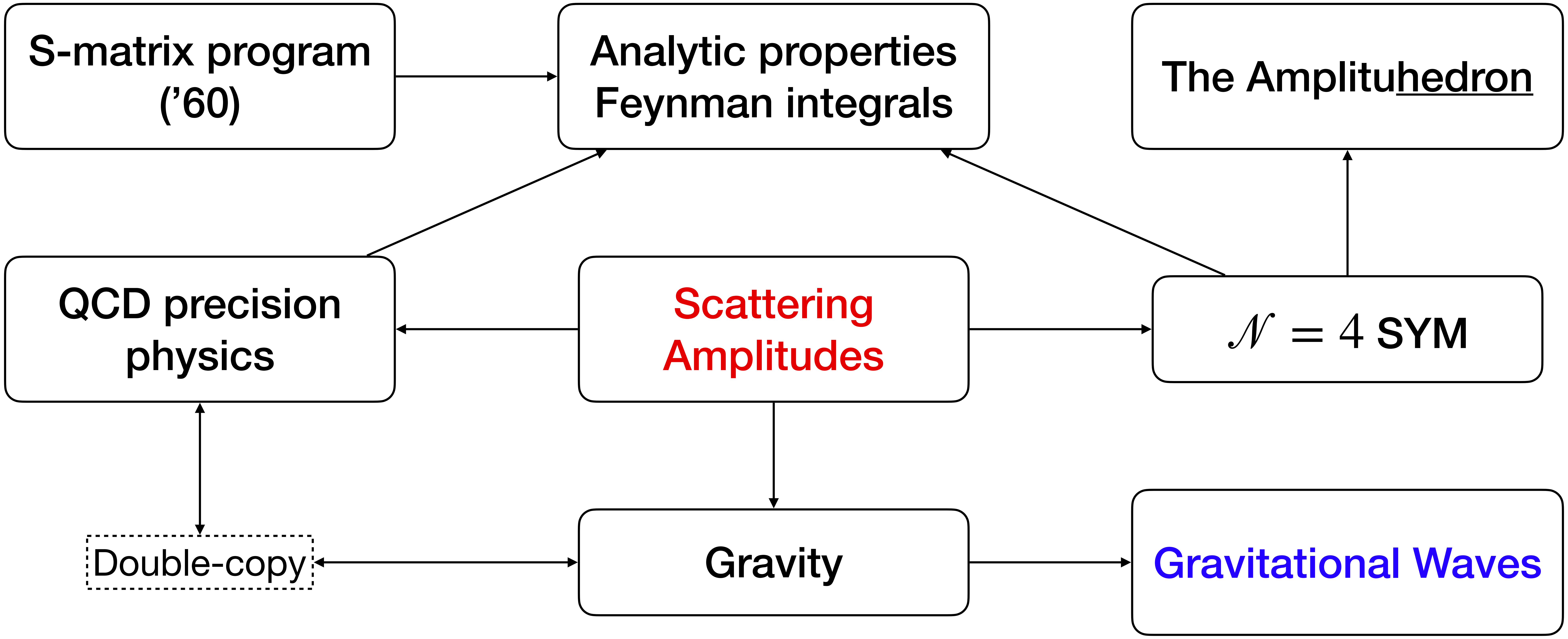

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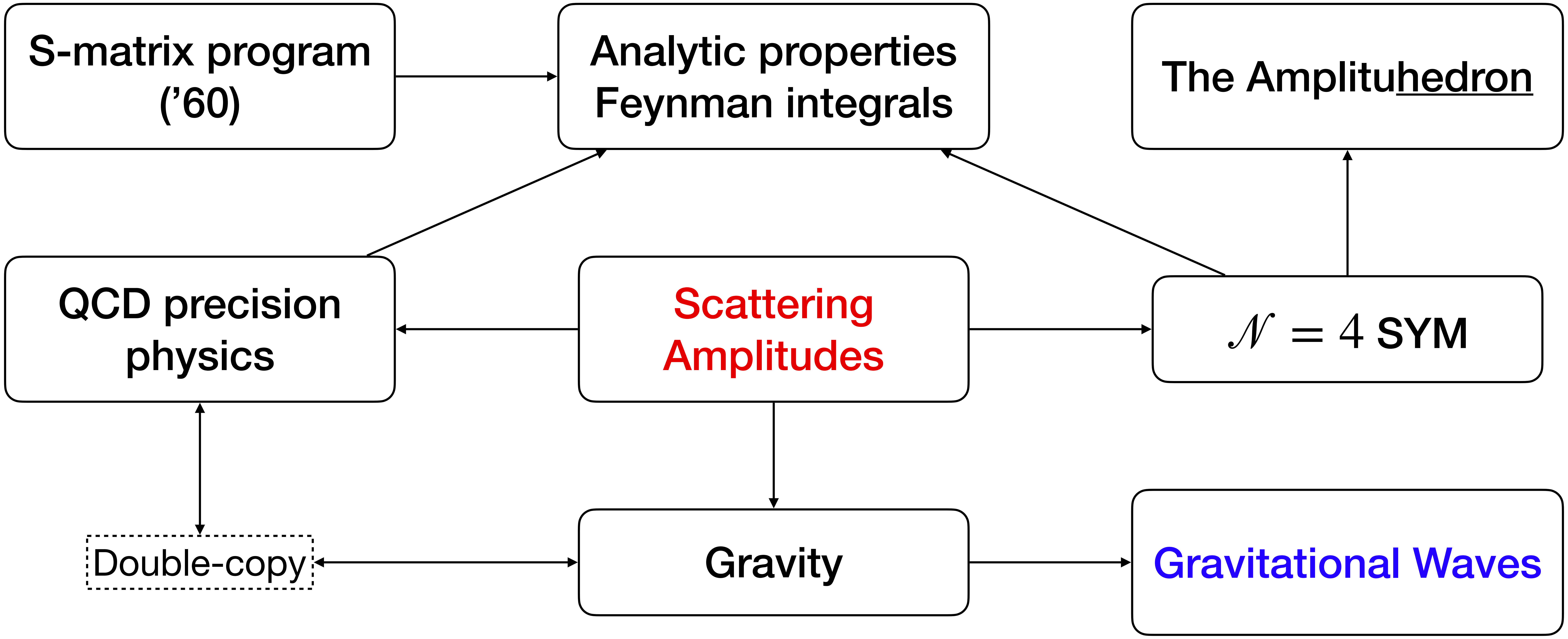

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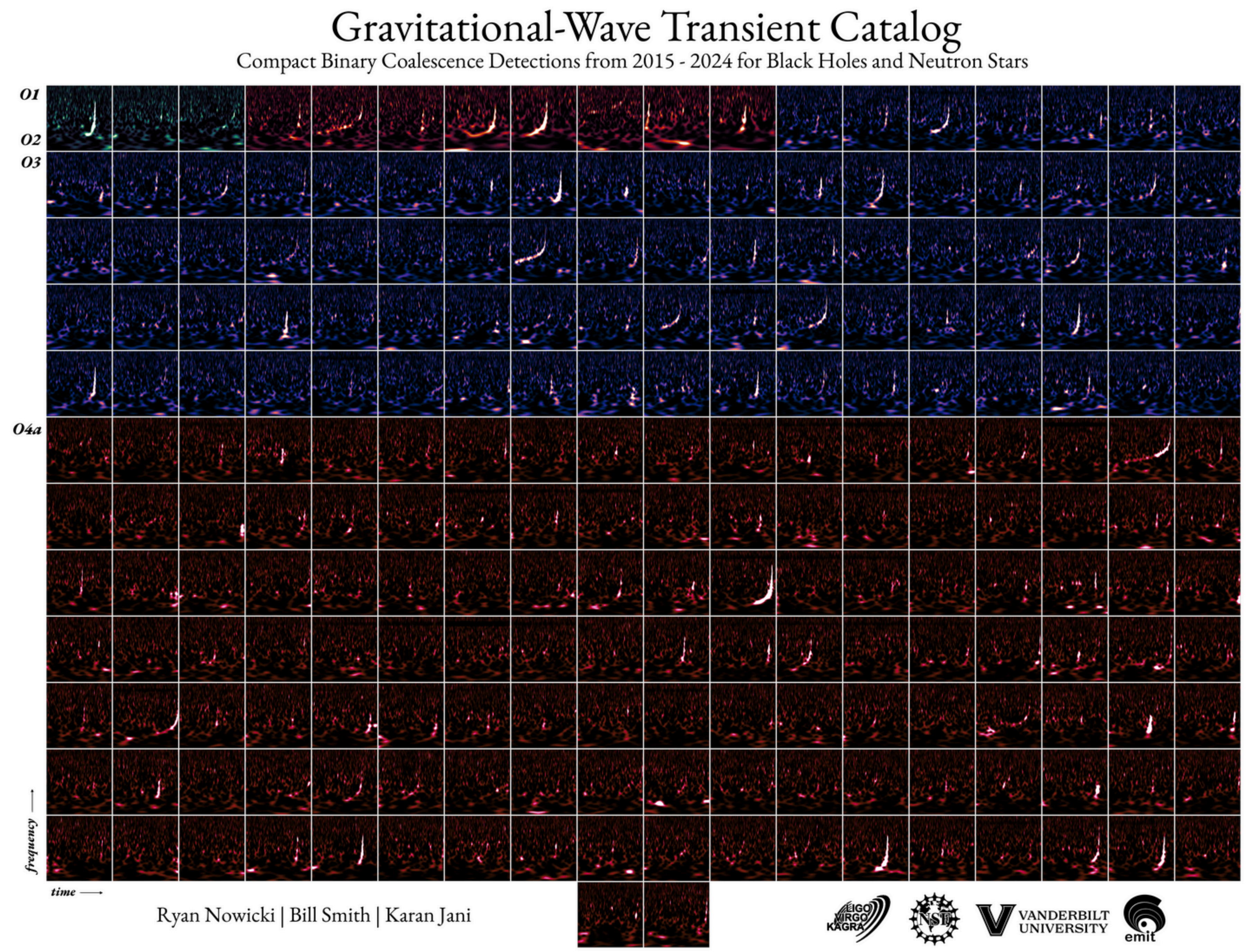
- **Gravitational Waveform from field theory**
    - Complementing, matching and improving known results from General Relativity

Gravitational-Wave Observations

Signals @ LIGO-Virgo-KAGRA; preparing for a new era of observations



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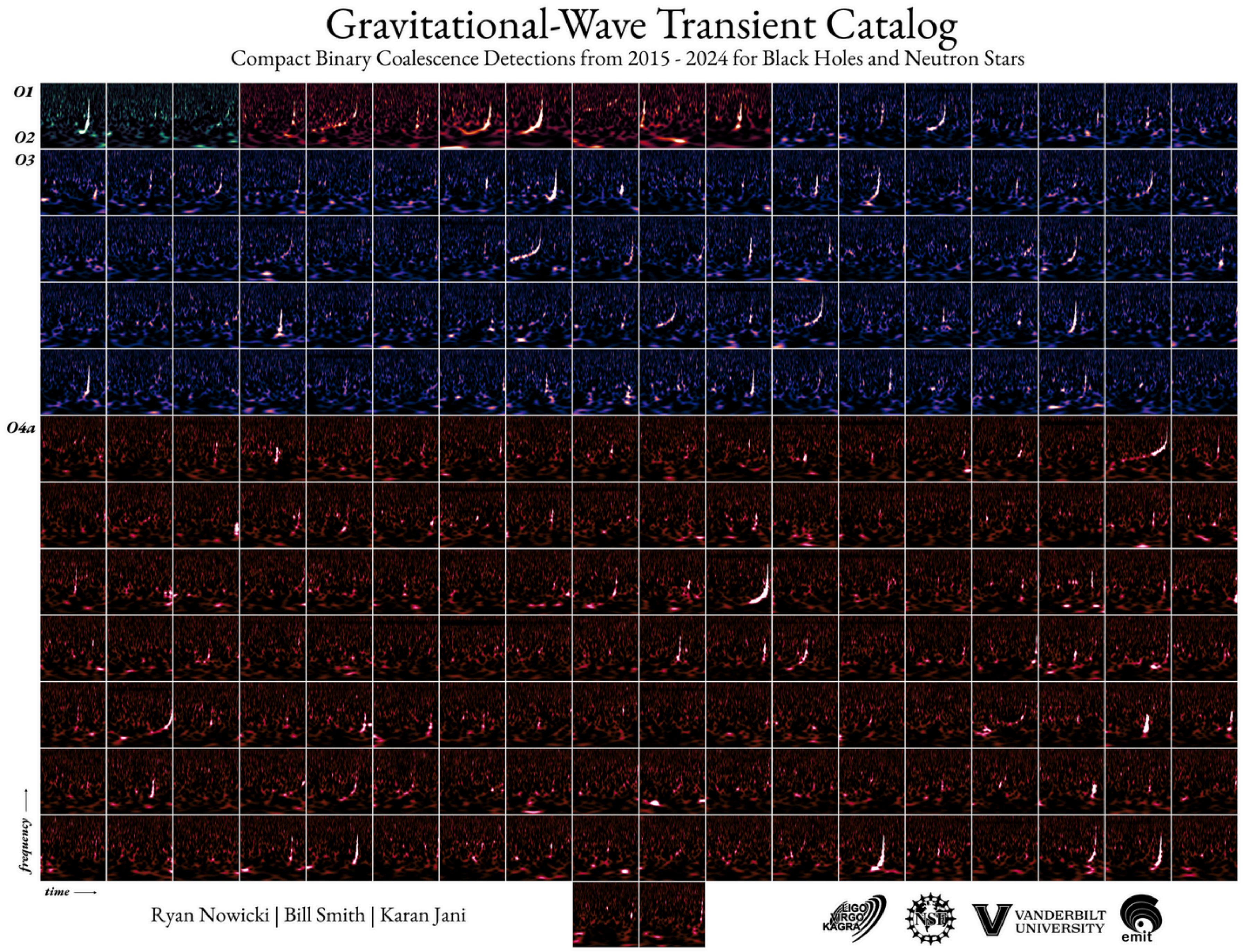

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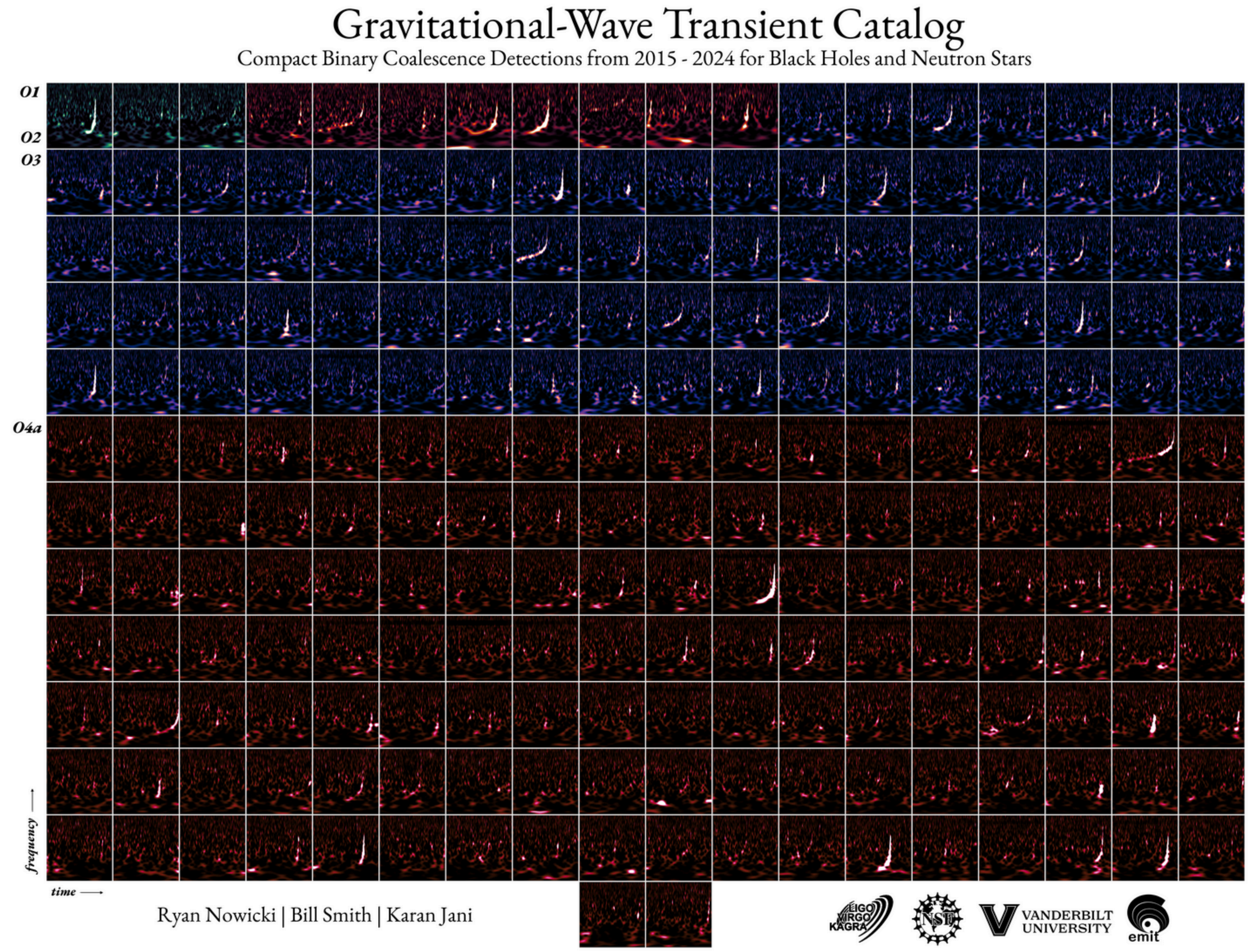

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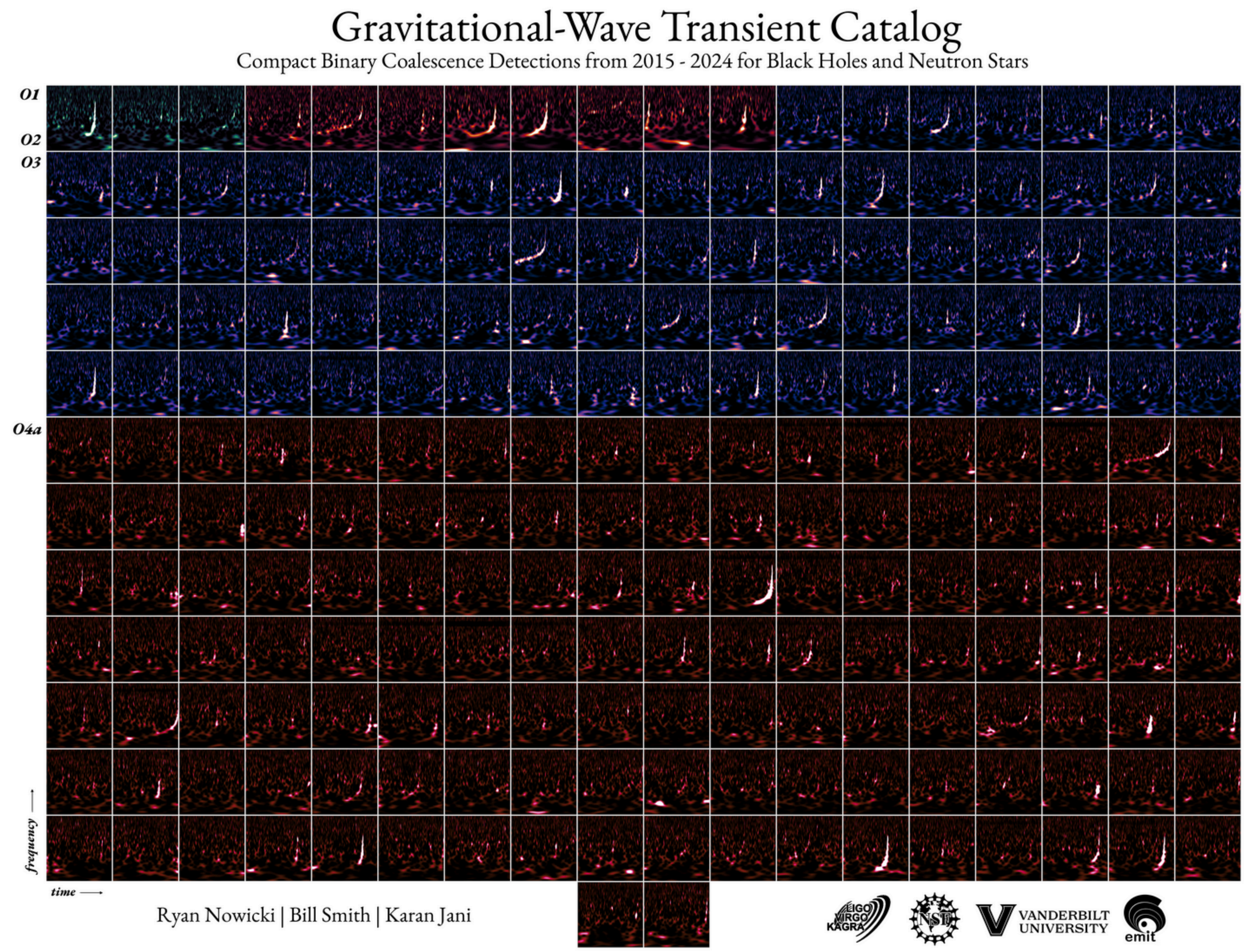

A few signals stand out: *e.g. GW231123* is the loudest event ever recorded (x20 the average signal). This feature made it possible to perform a quantitative analysis of the ringdown phase.

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Take-home message: the signals observed at LVK are **different** from those that will be observed by LISA!

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To not miss a signal, you must fill this parameter space densely enough that any physical signal overlaps with at least one template with an acceptable fitting factor.

The Effective One-Body

The inspiral, the merger and the ringdown

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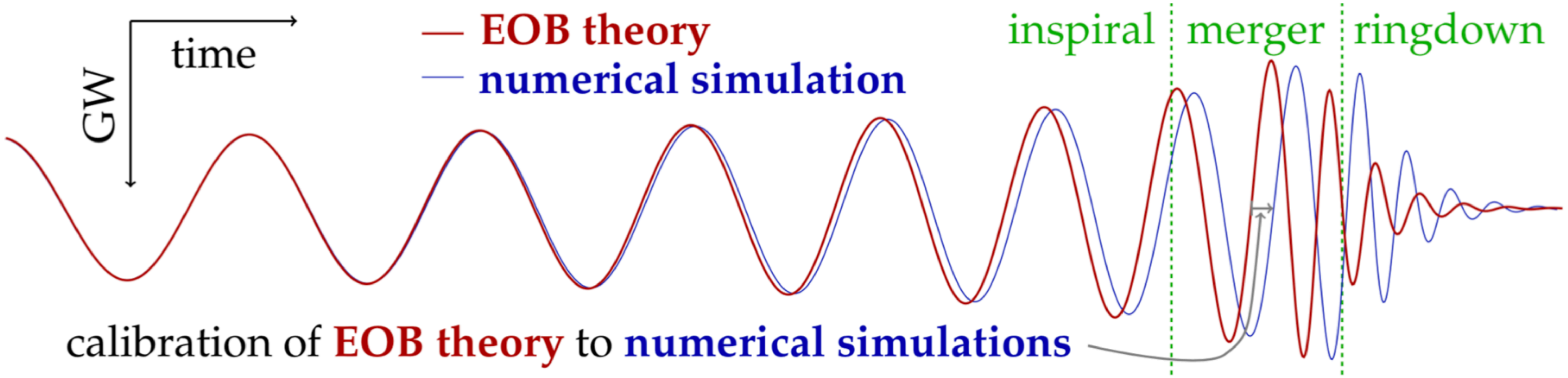
Effective One-Body framework



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- Post-Newtonian
- **Post-Minkowskian**
- Extreme-mass ratio

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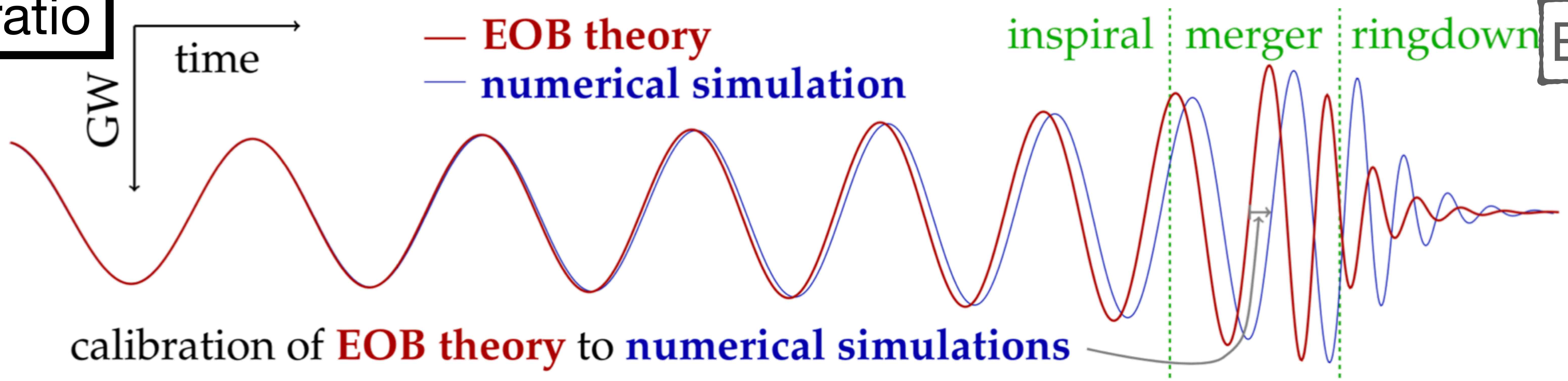

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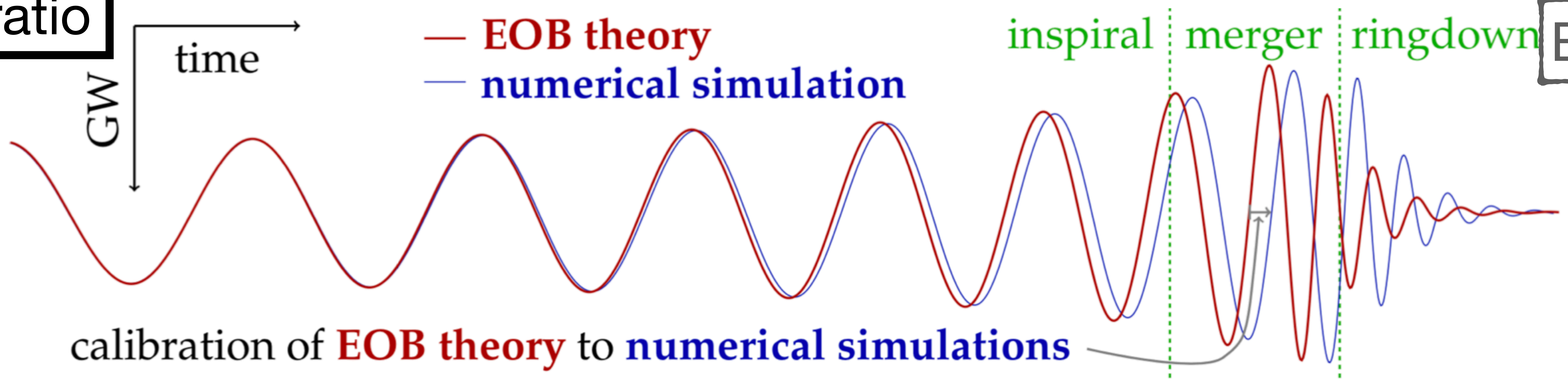

BH Perturbations

Numerical Relativity

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- The **distance of the observer from the system**, r, which is usually assumed to be the largest scale in the problem.

Analytic results: inspiral phase

Post-Newtonian and its separation of scales

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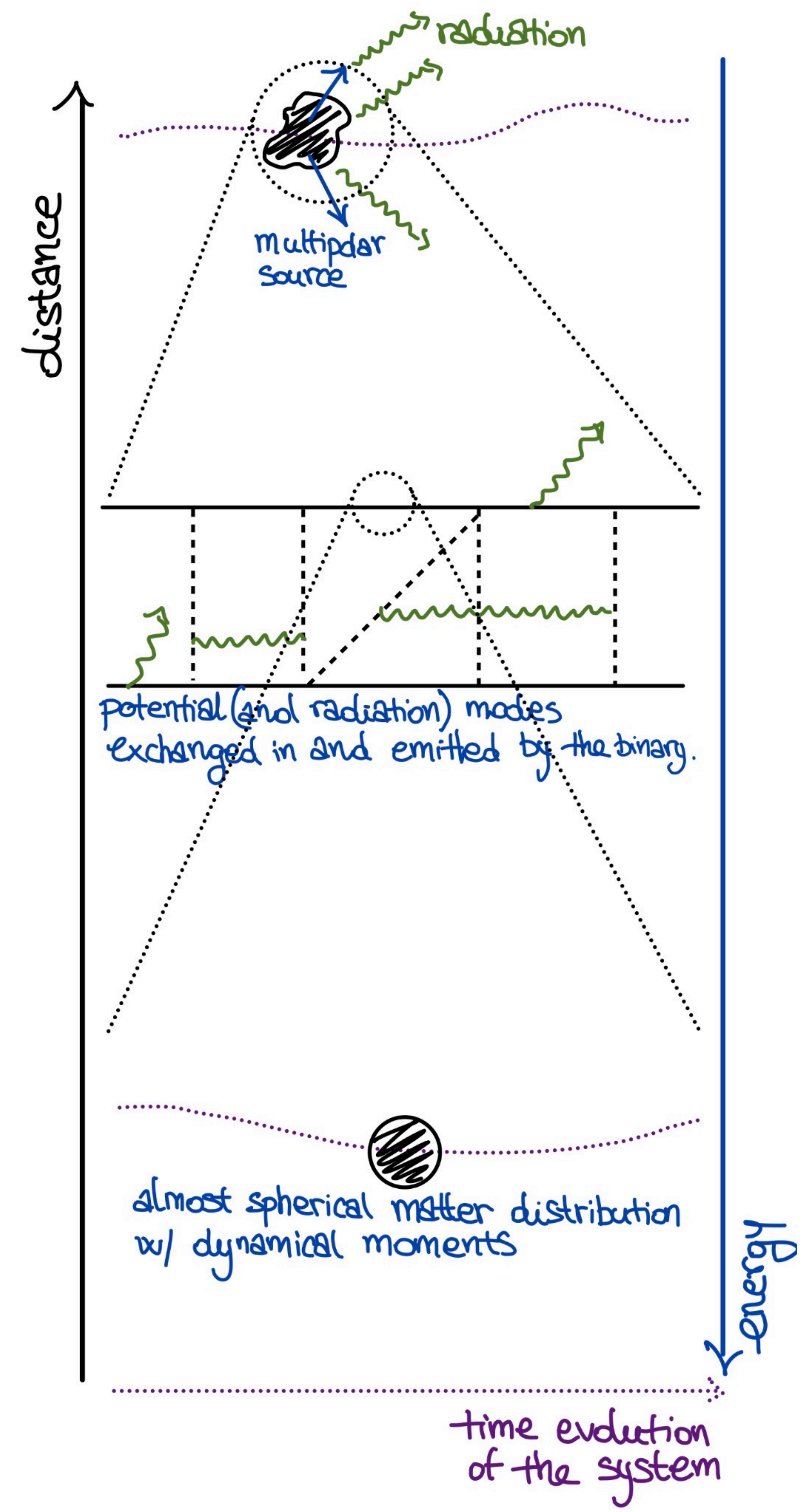

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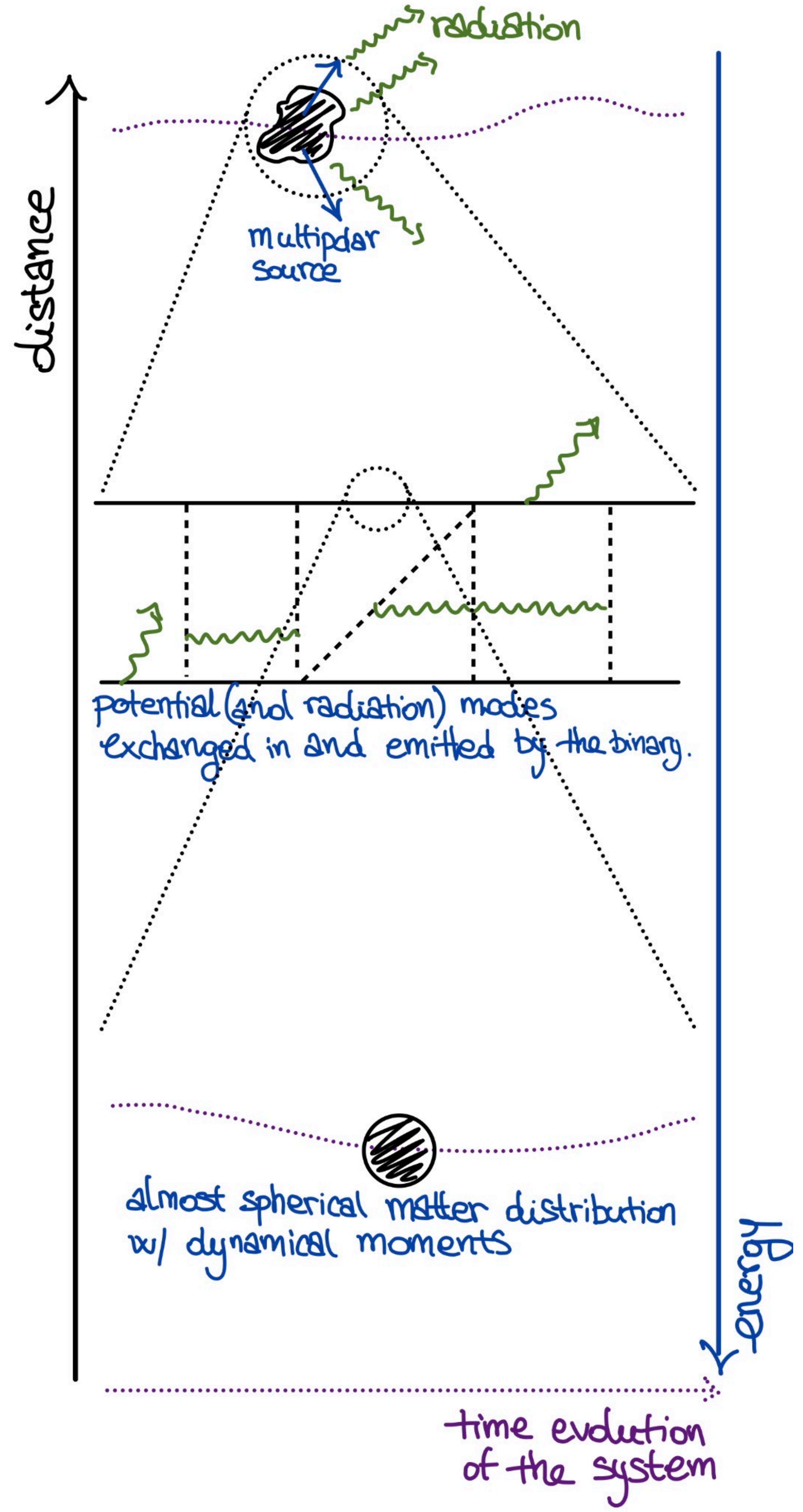

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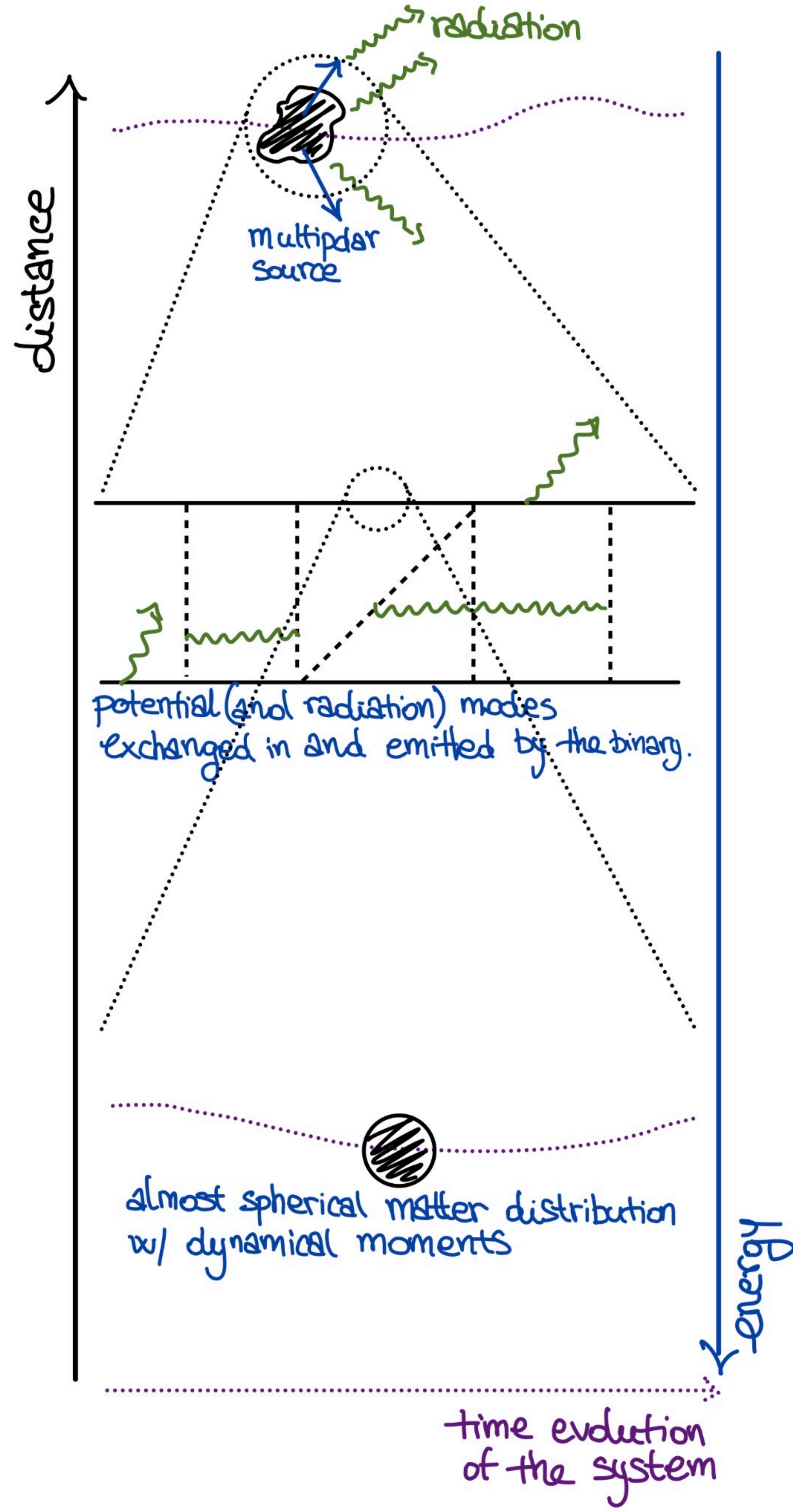

$$\lambda \sim \frac{b}{v} \gg b$$

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Relativistic velocities: the post-Minkowskian set up

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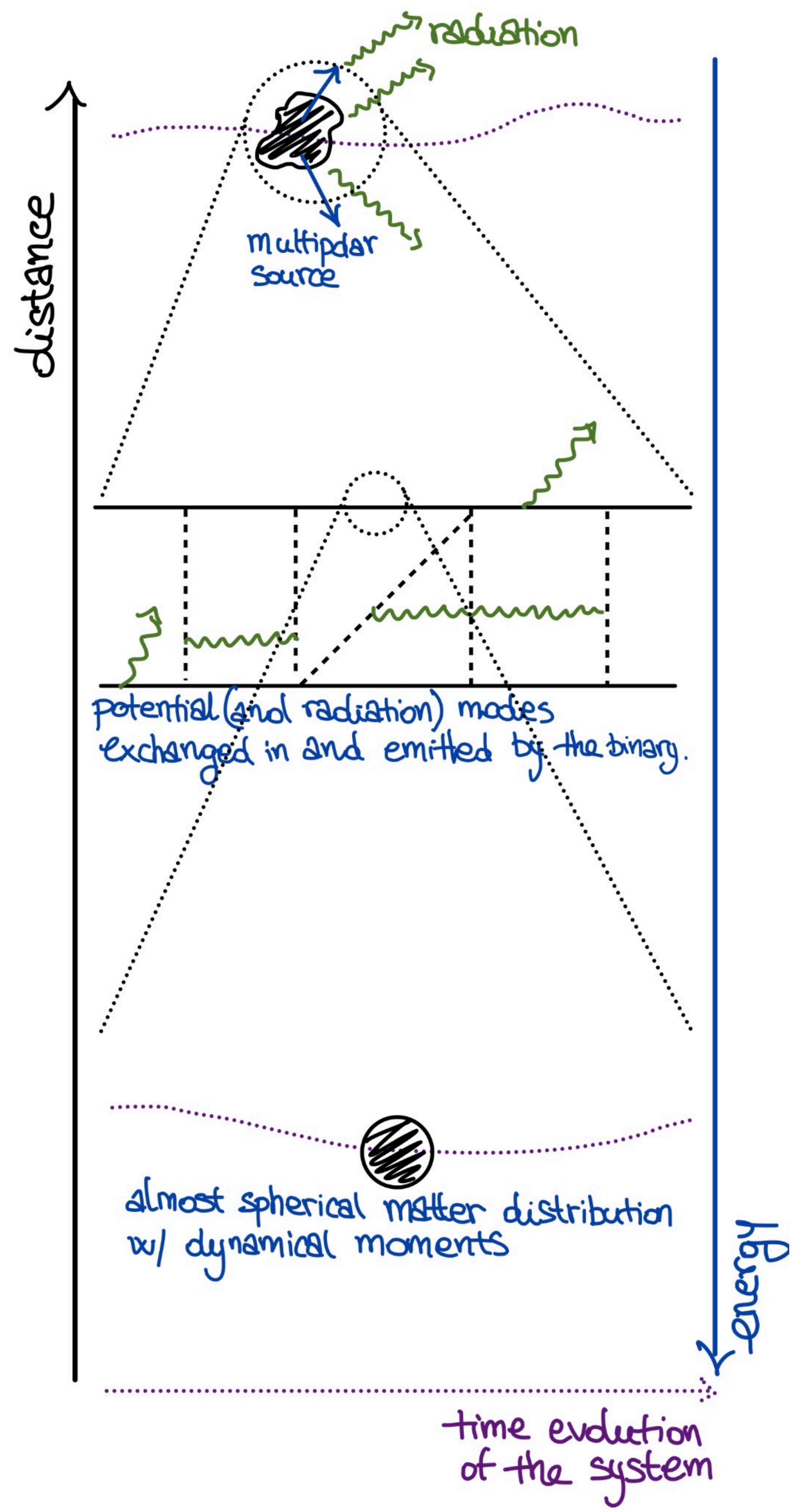

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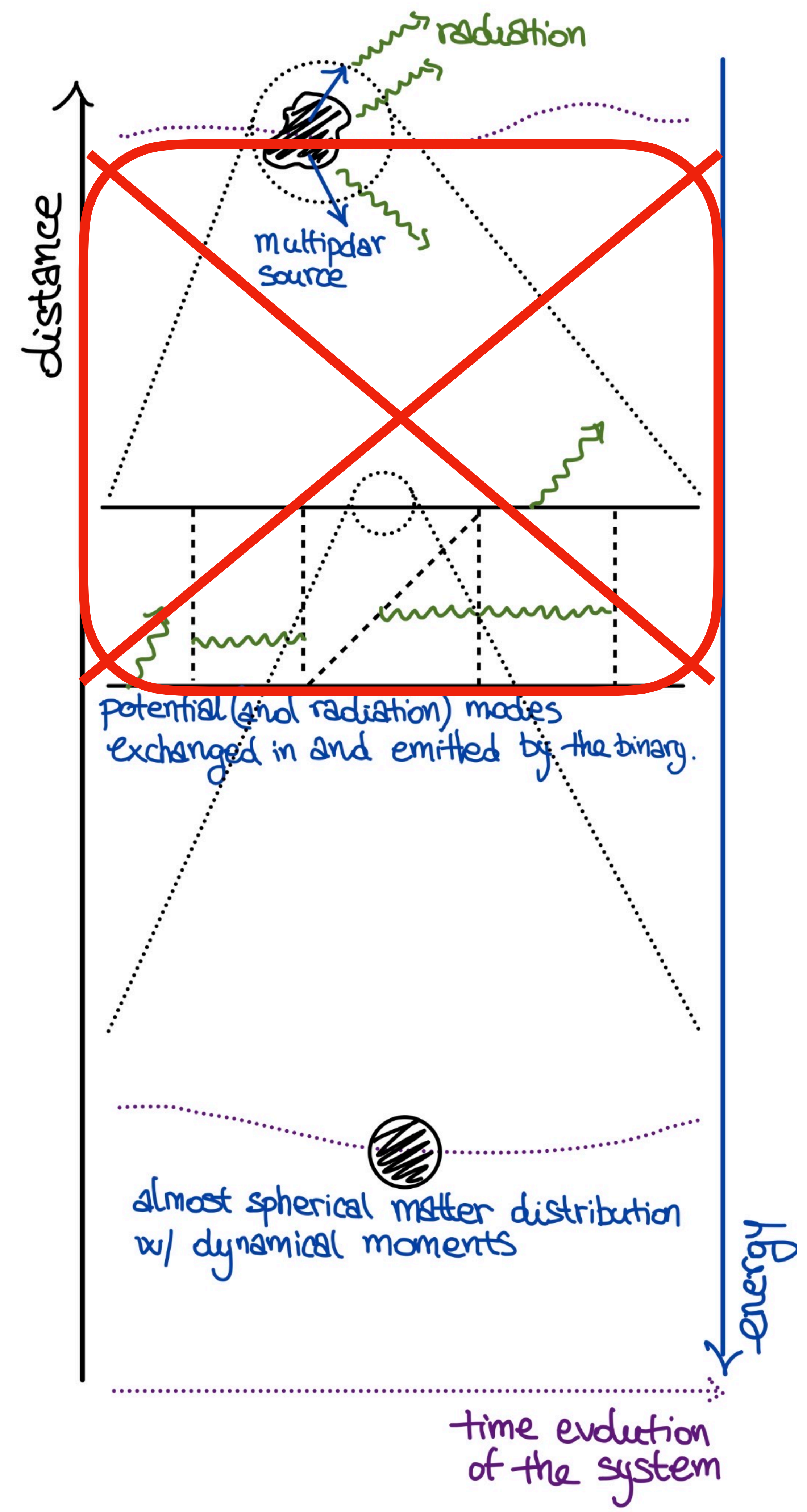

$b \gg R \gtrsim Gm$

Analytic results: inspiral phase

Relativistic velocities: the post-Minkowskian set up

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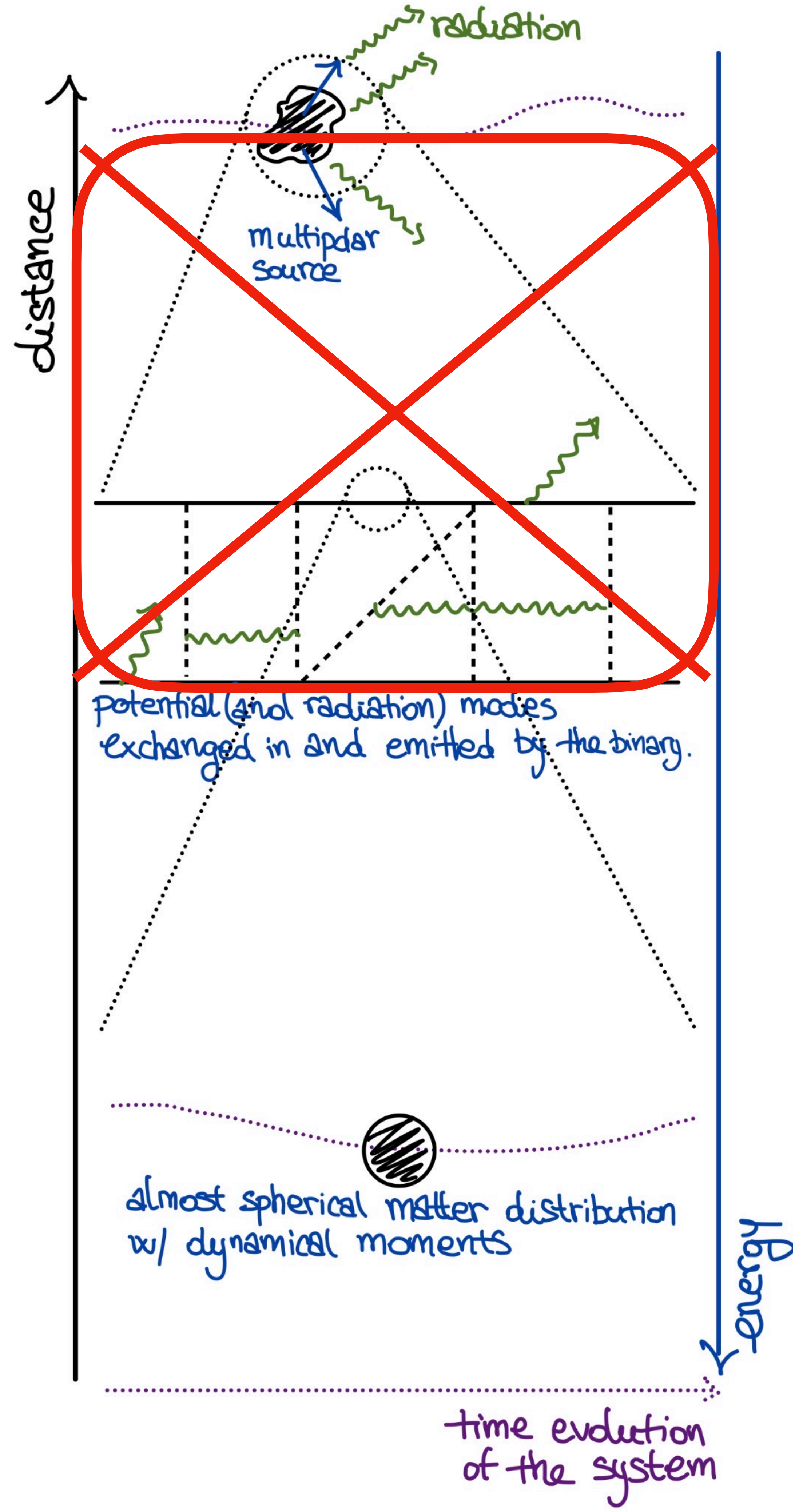
Generic velocities are typical of scattering configurations.



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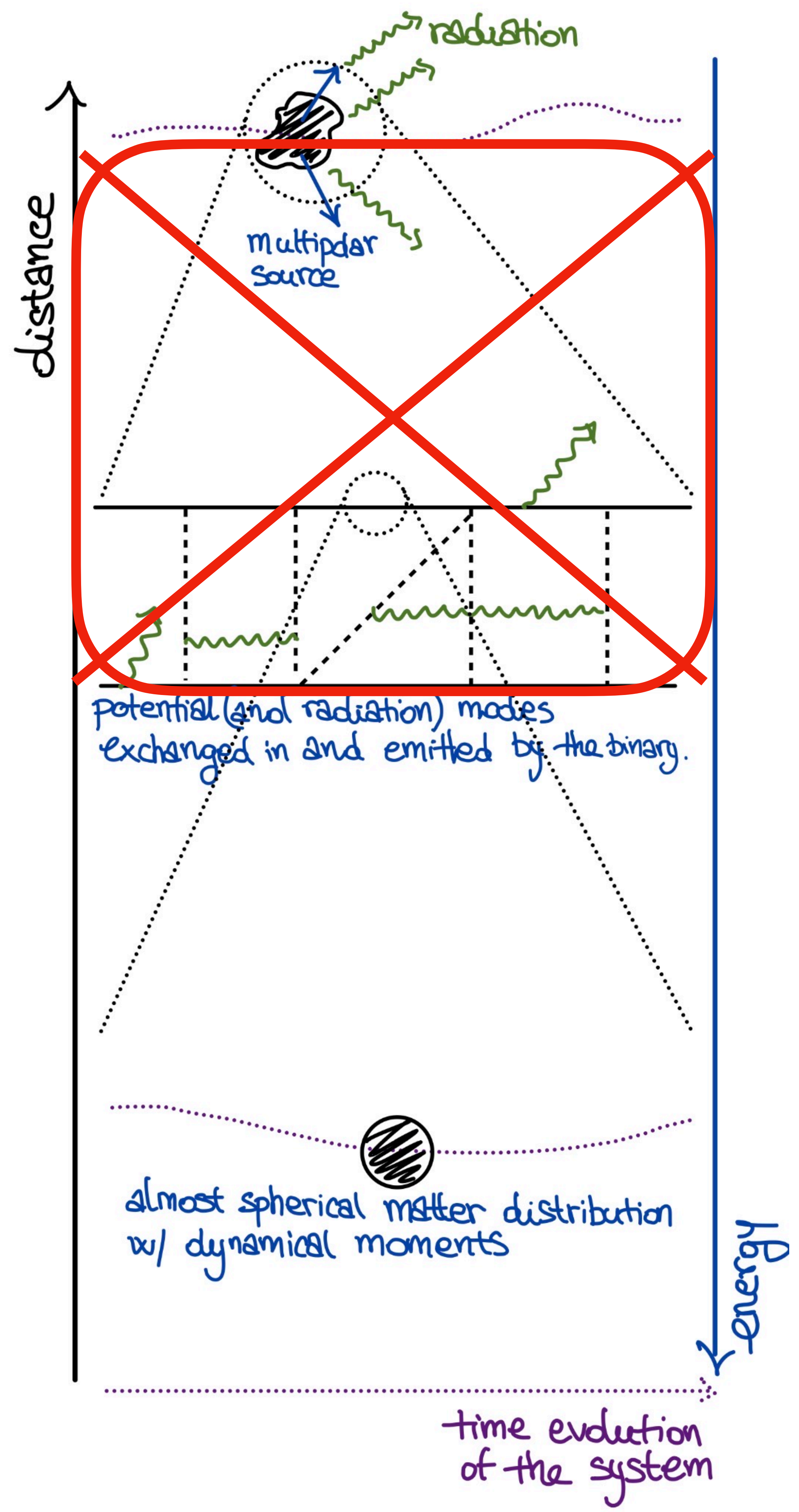

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Radiation and potential modes must be treated uniformly.

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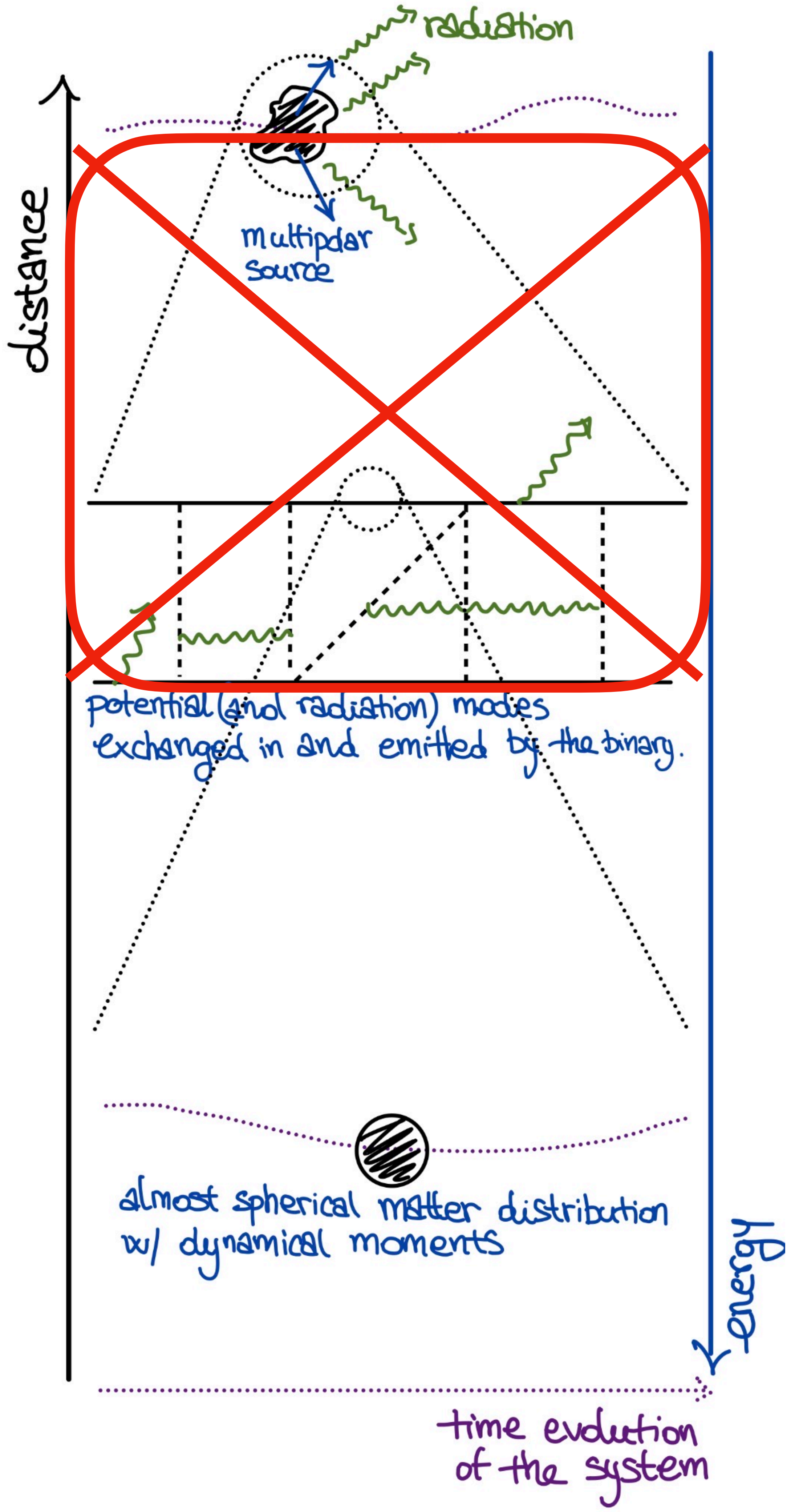

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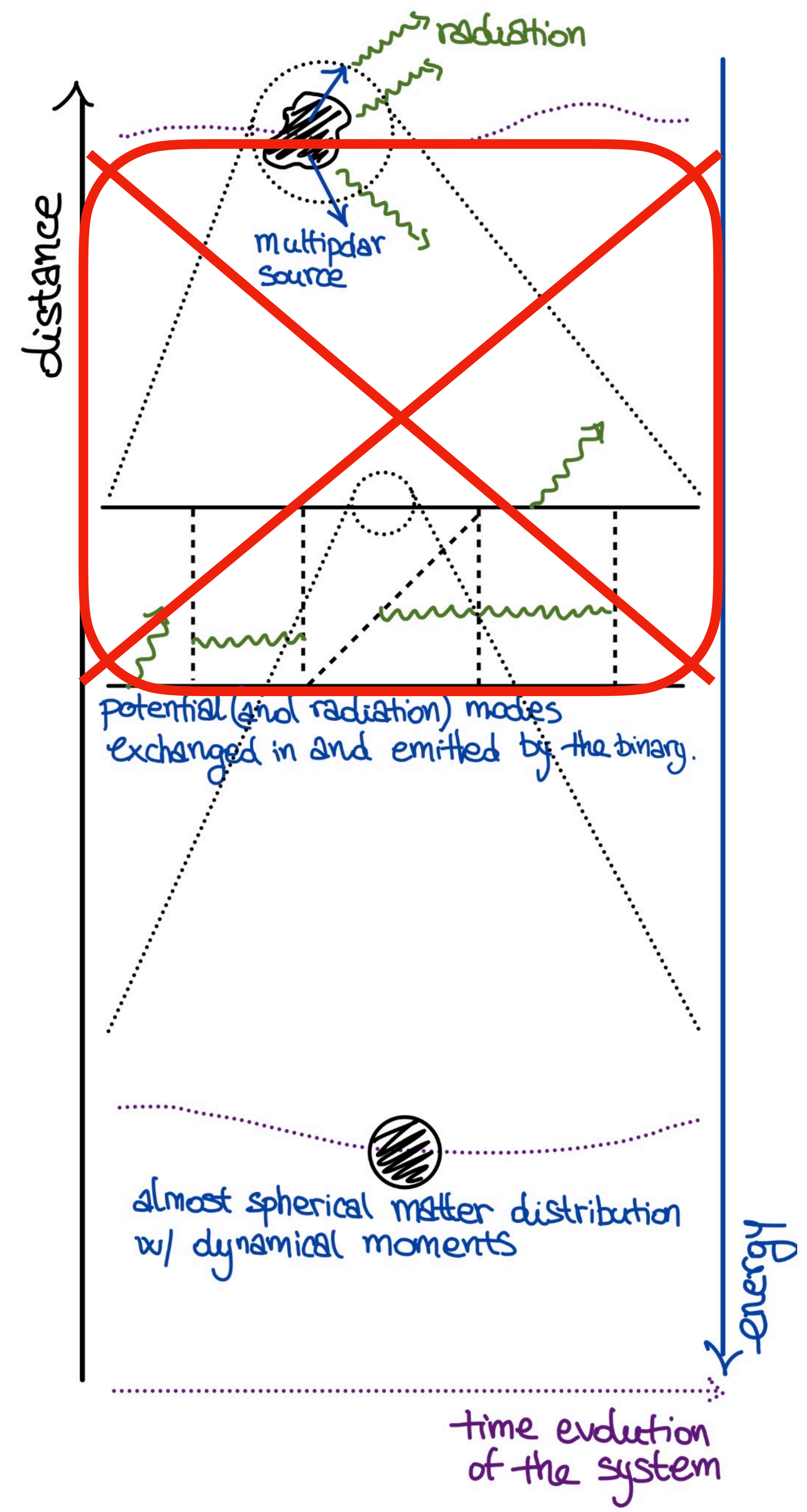

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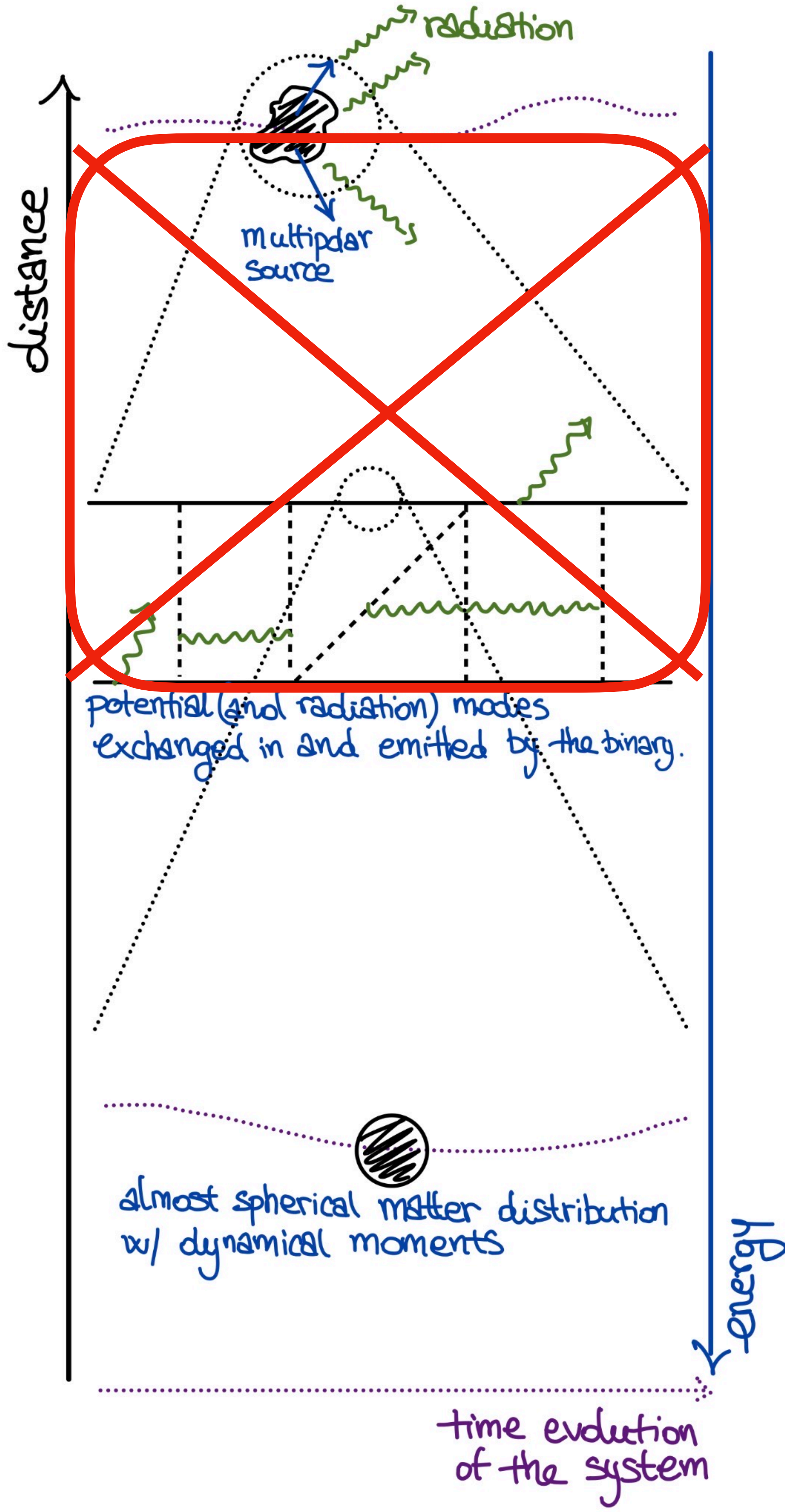

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There is separation of scales only for one of the two objects.

$$b \gtrsim Gm_1$$

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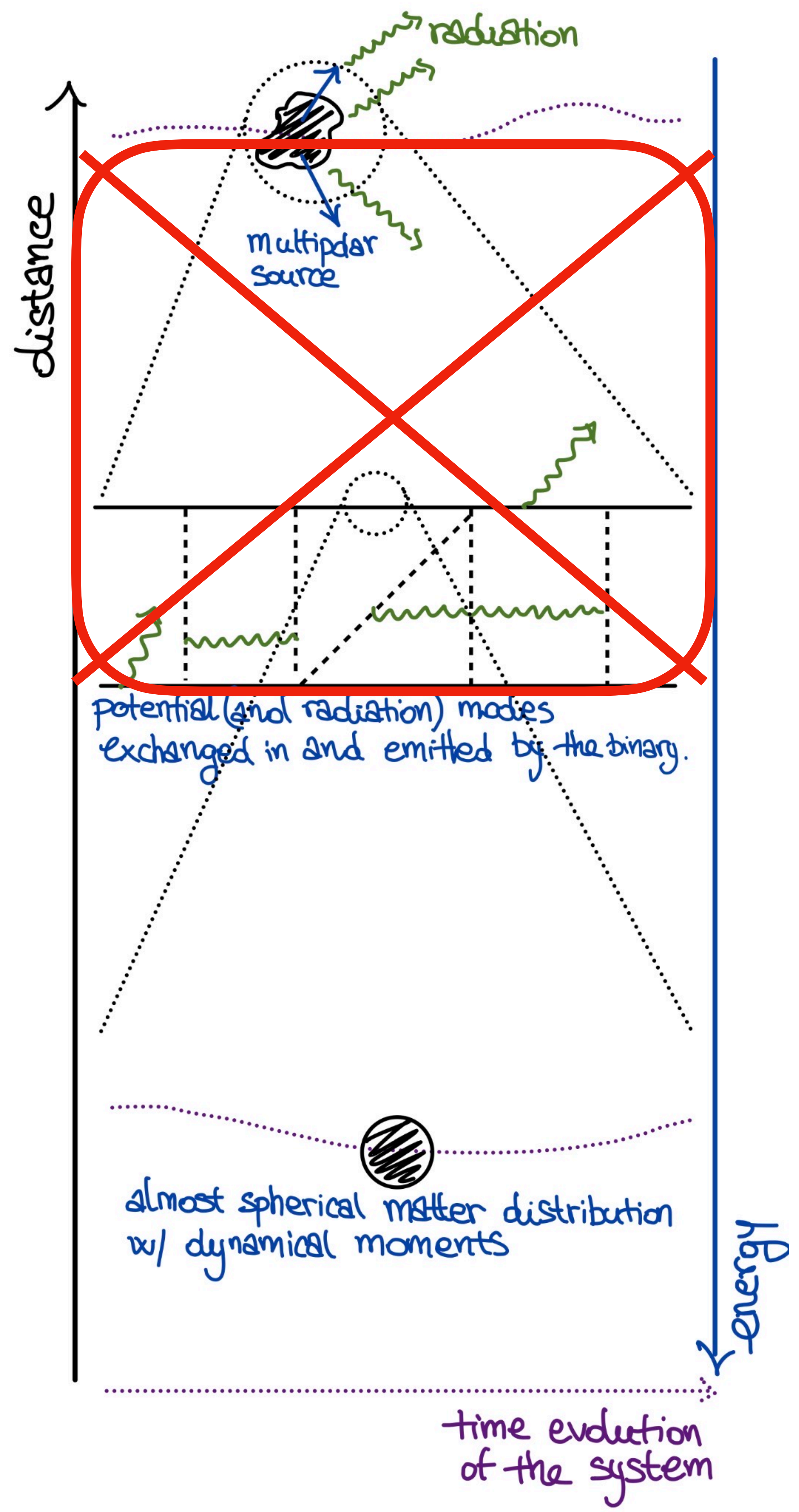

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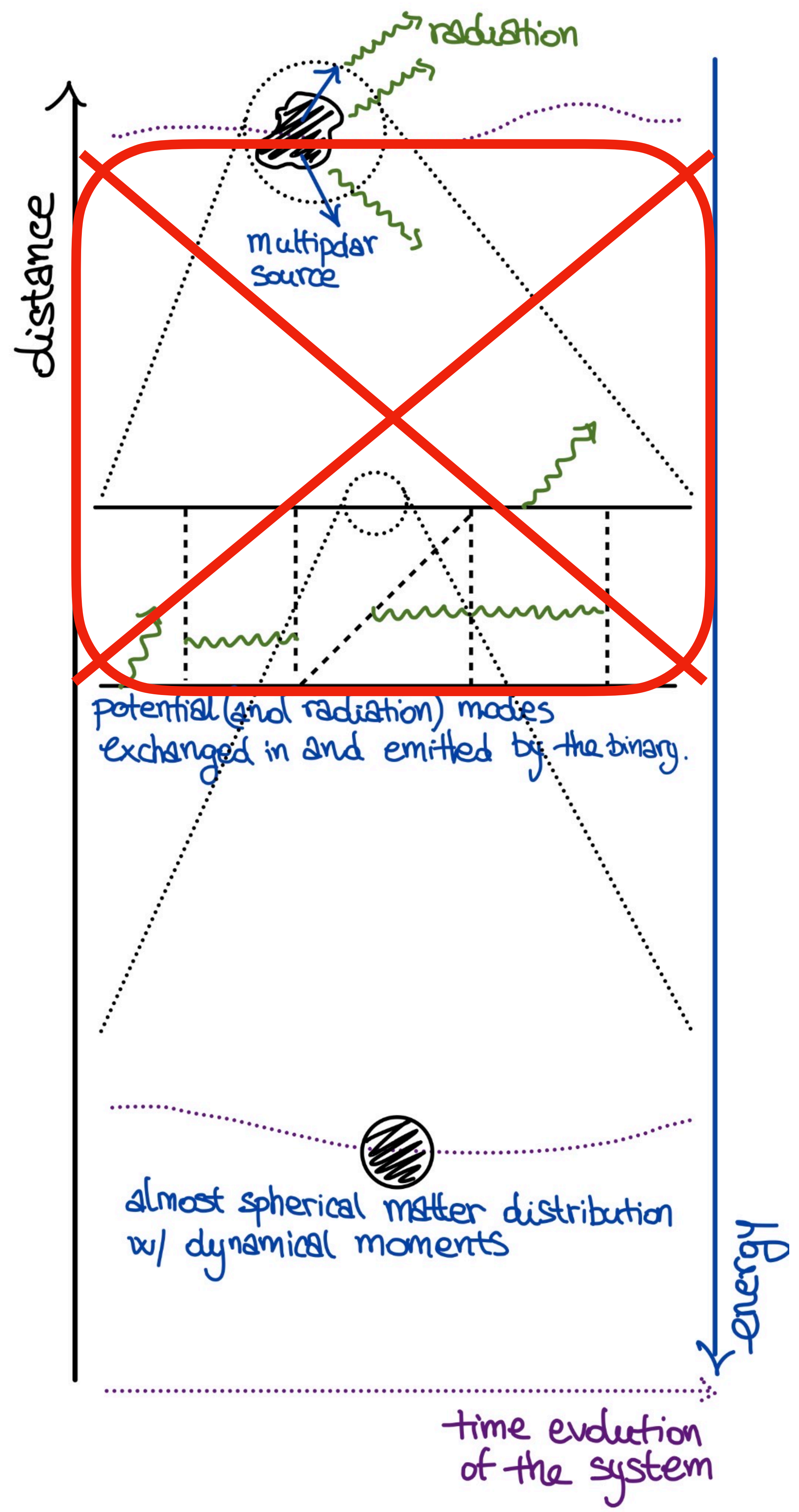

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Vantages: inherited computational techniques, properties hidden in classical dynamics (*e.g.* unitarity, analyticity).

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2. Second-order gravitational self-force

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C. **Scattering-to-bound map** and **non-local** interactions

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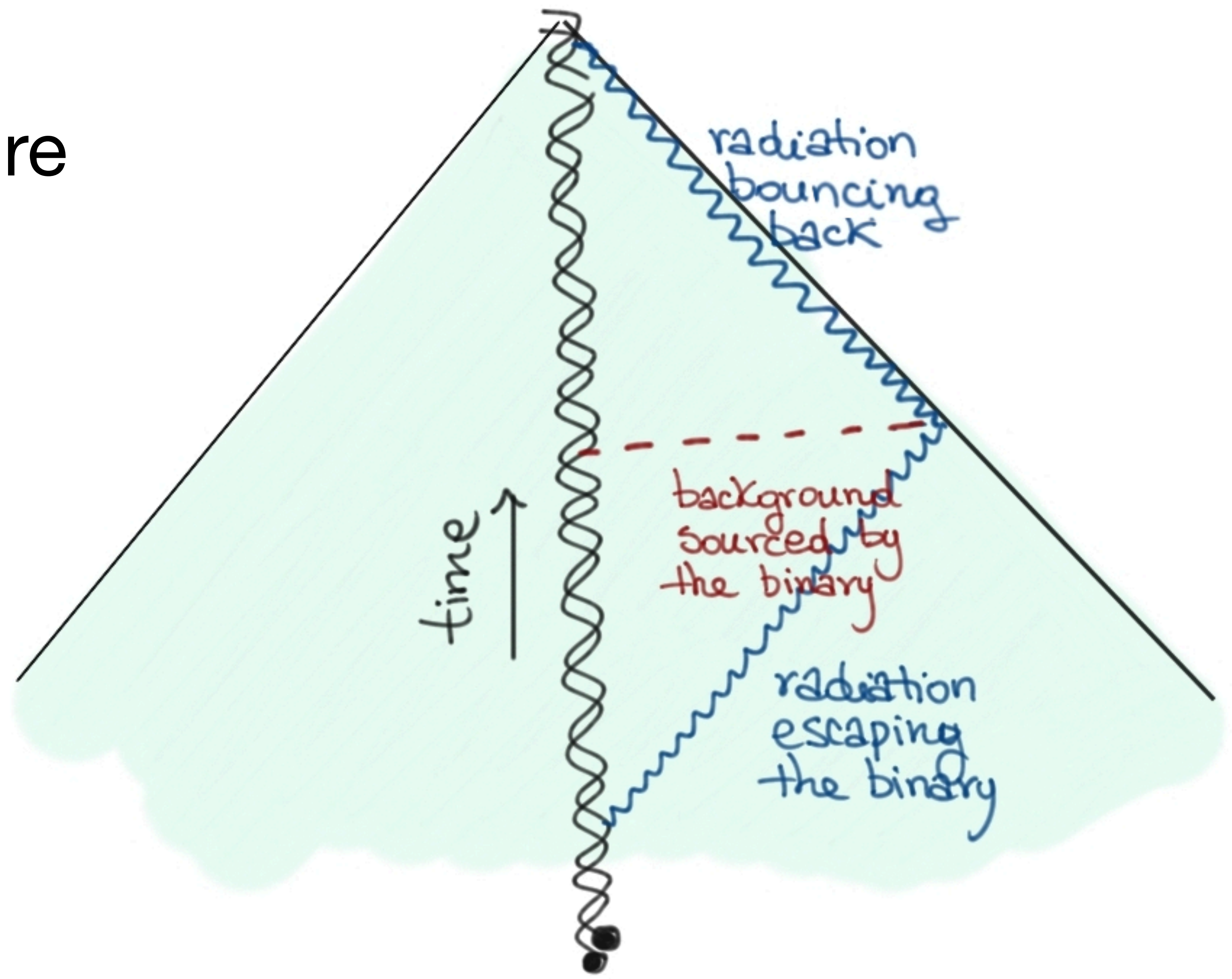

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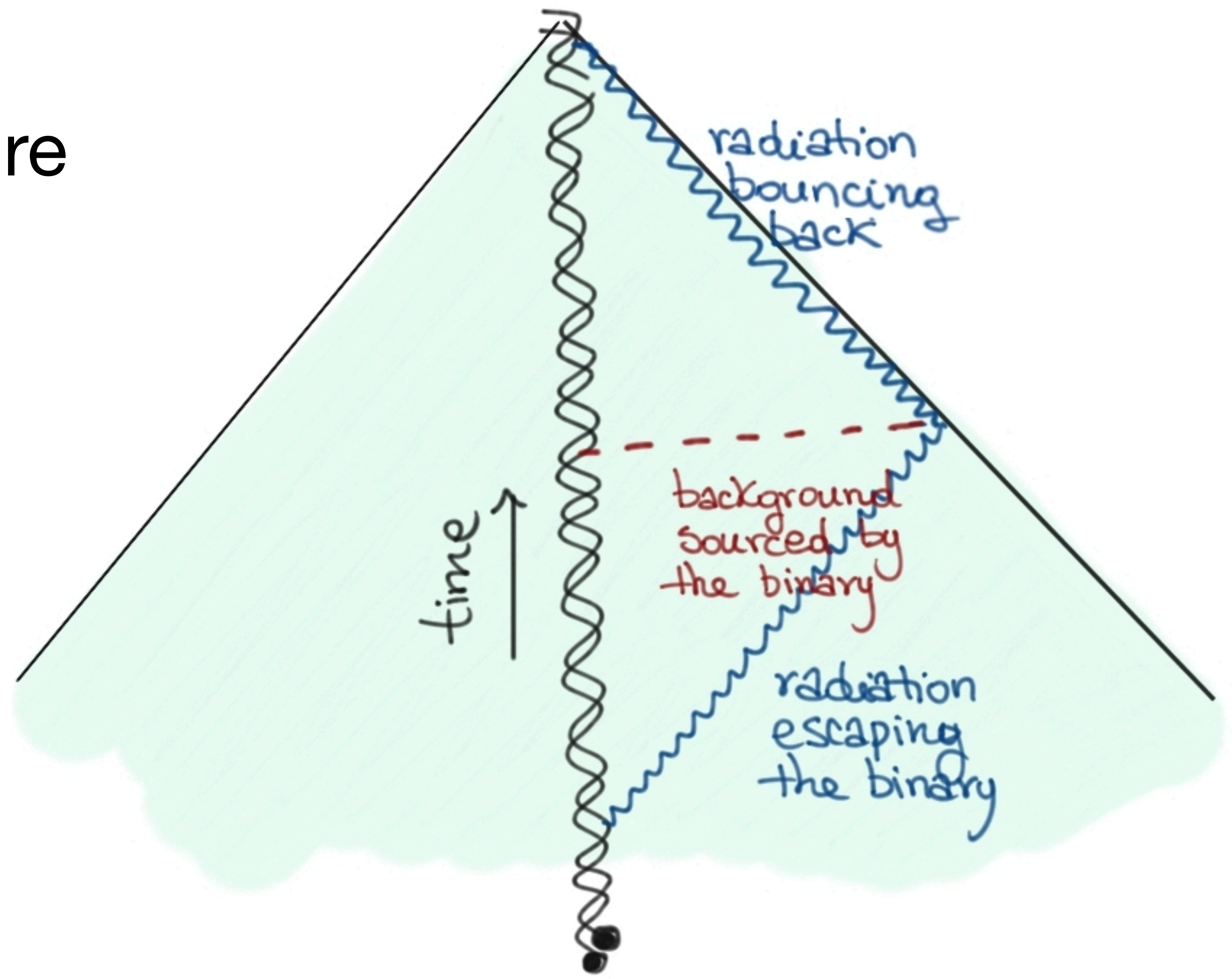
$$W \sim \ldots + \int_{-\infty}^{t} d\tau \; f(x_1(\tau), x_2(\tau))\Big|_{\text{on-shell}}$$



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A new route for a first-principle construction of templates

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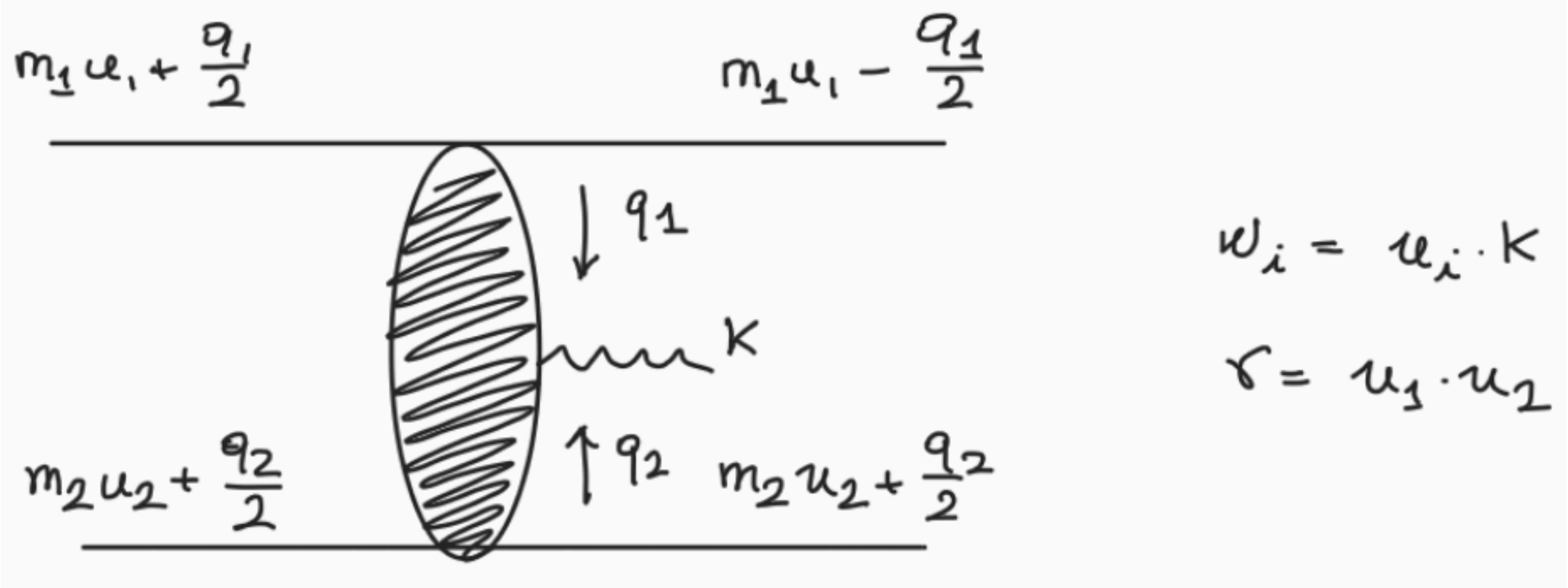

The five-point scattering amplitude

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Fourier transform to *impact parameter* space

$$\mathrm{d}\mu = \prod_{i=1}^{2} \hat{\mathrm{d}}^D q_i\, \delta(2p_i\cdot q_i + q_i^2)\, e^{ib_i\cdot q_i}$$

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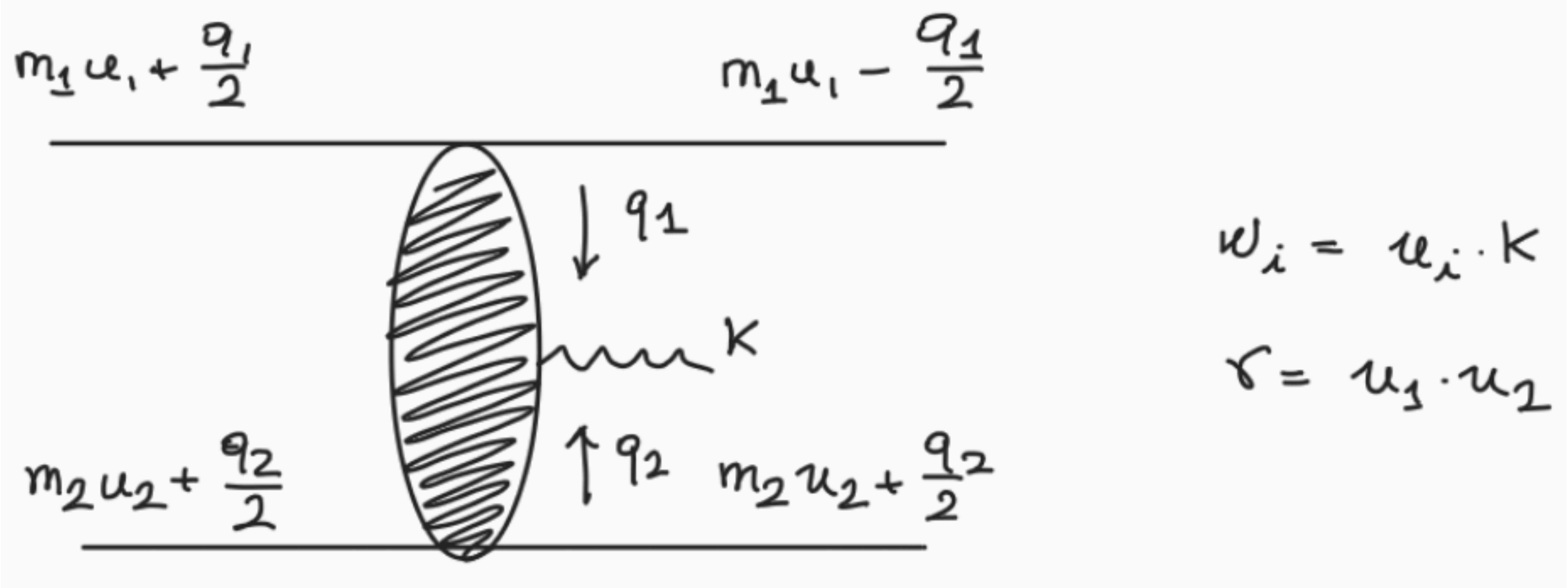

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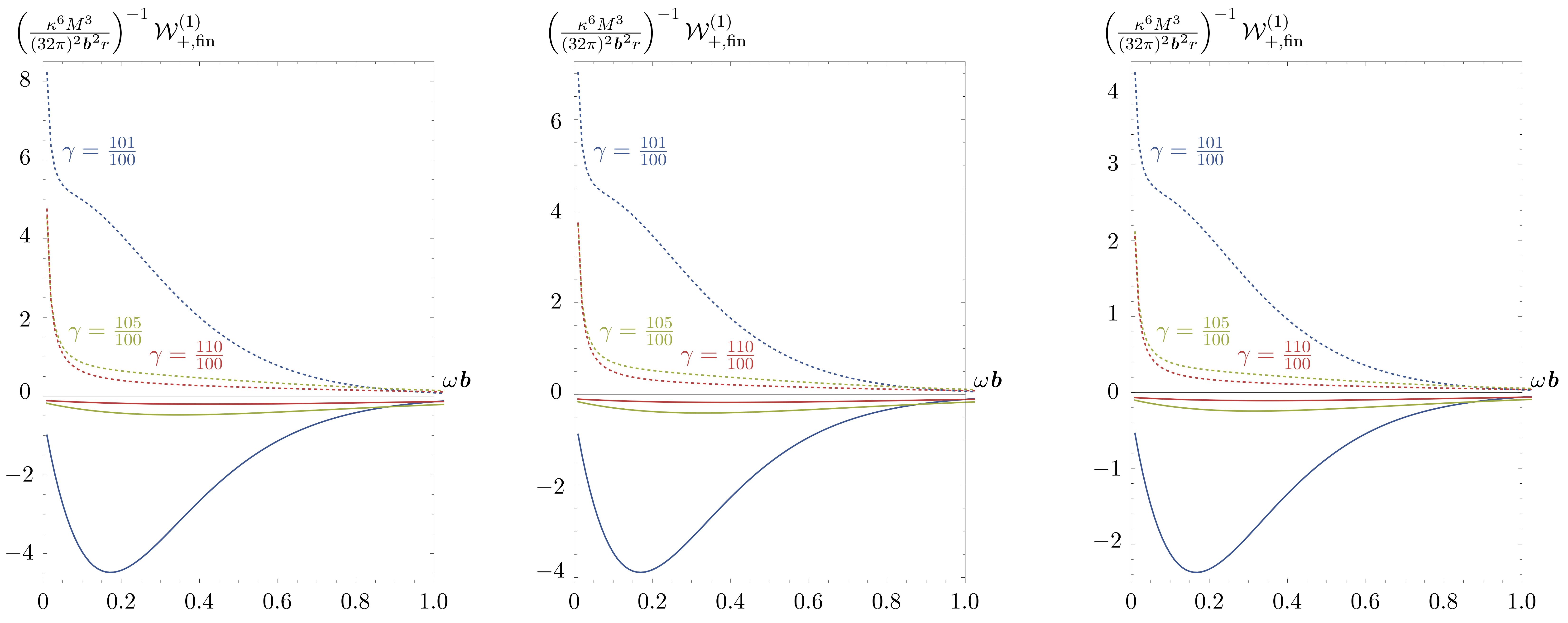

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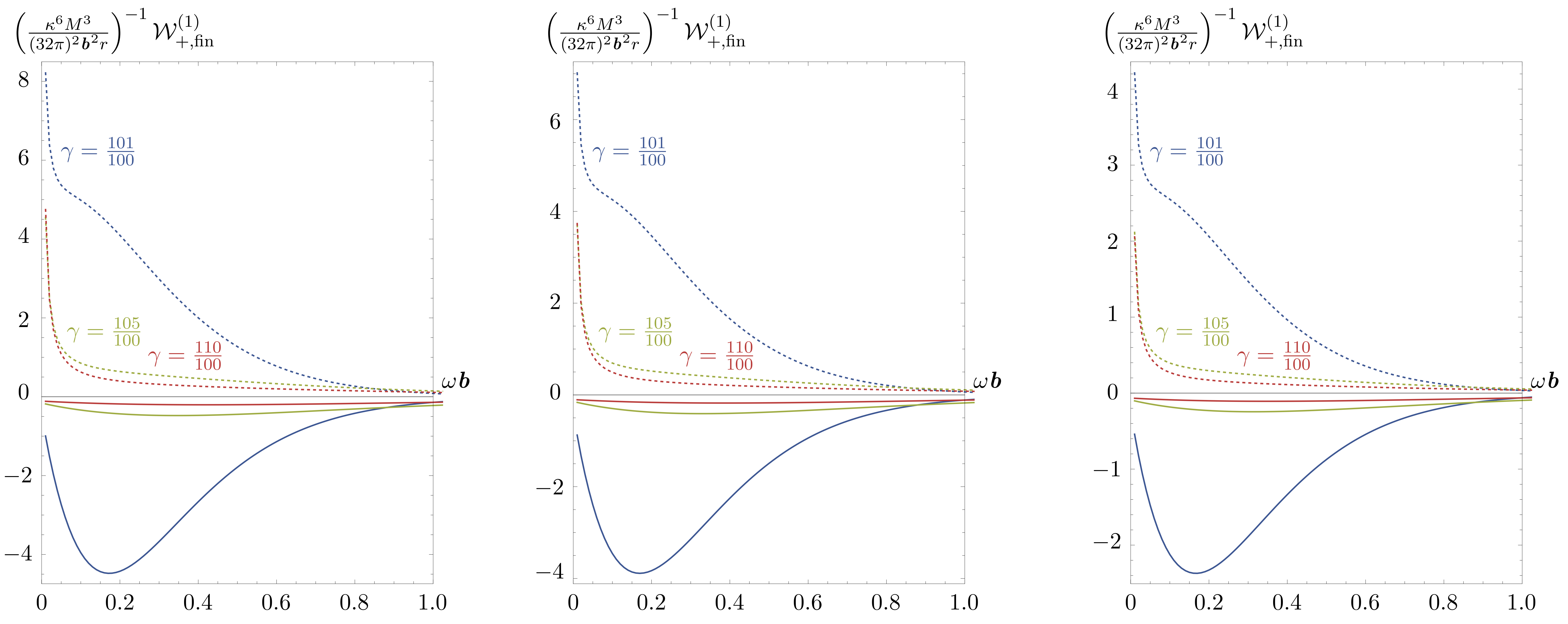

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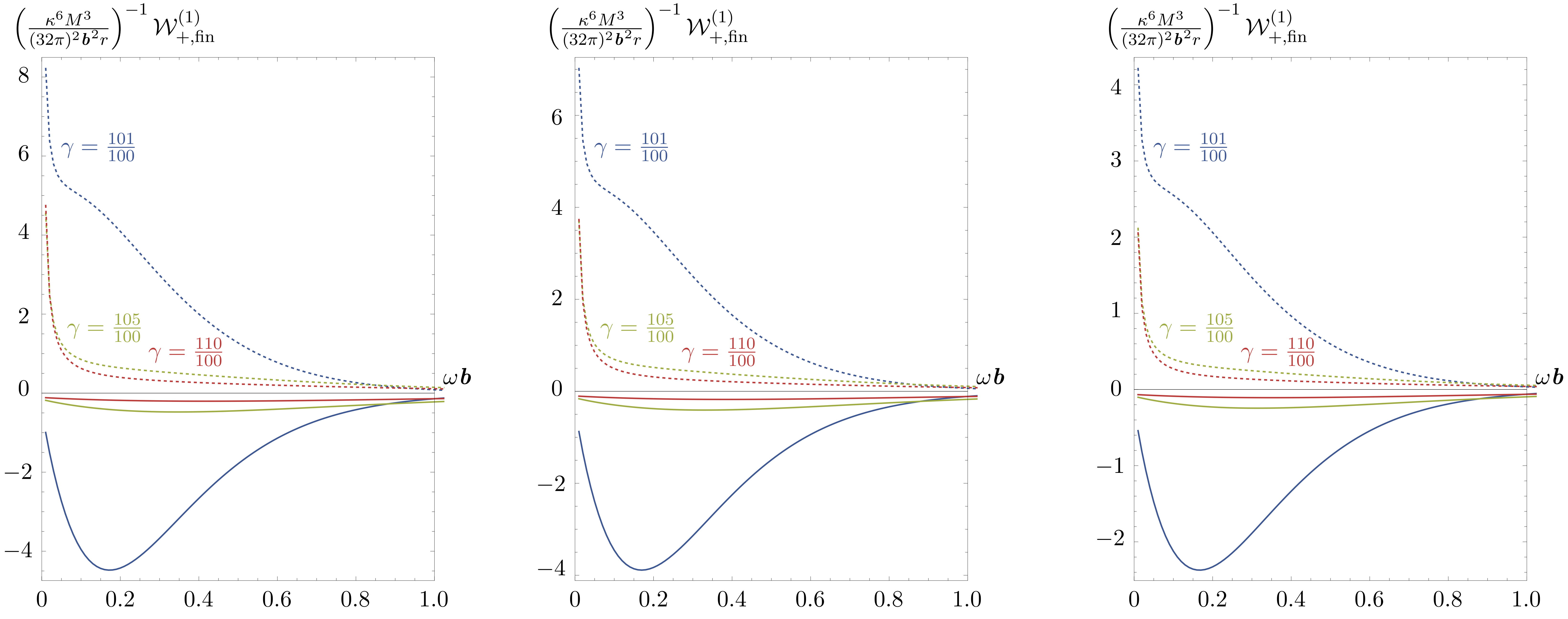

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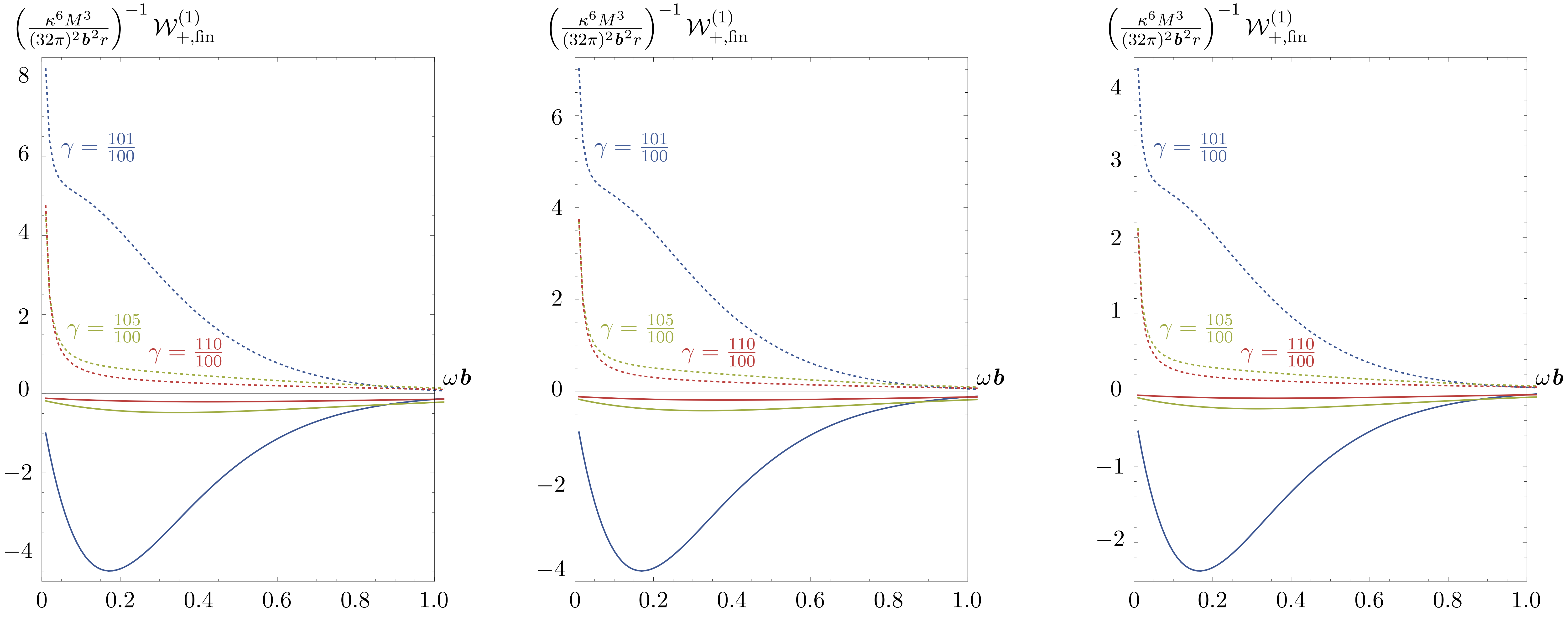

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RESUMMATION! We need to work harder to obtain first-principle **bound** waveforms!

THANK YOU!