No room for monopole dark matter

based on

arXiv:2509.21924

w/ F. Brümmer, T. Fischer and M. Frigerio

ULB

PHYS TH

Monopole dark matter (?)

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Unavoidable

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Naturally stable (topology)

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$\Rightarrow$

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[Murayama, Shu, 2010;
Khoze, Ro, 2014;
Graesser, Osi`nski 2020;
Yang, Zhou, Bian, 2023]

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This talk ✗

The 't Hooft-Polyakov model

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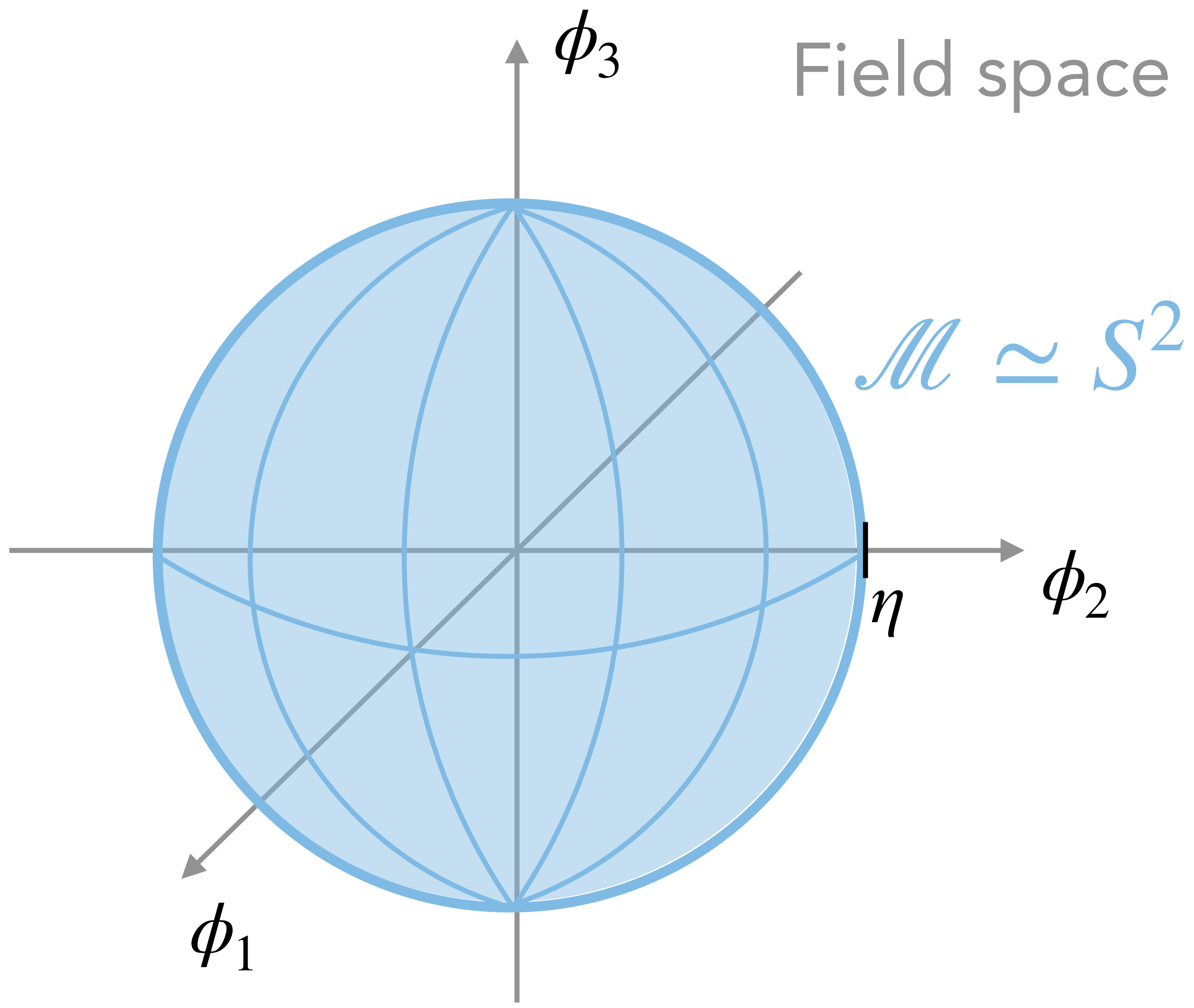

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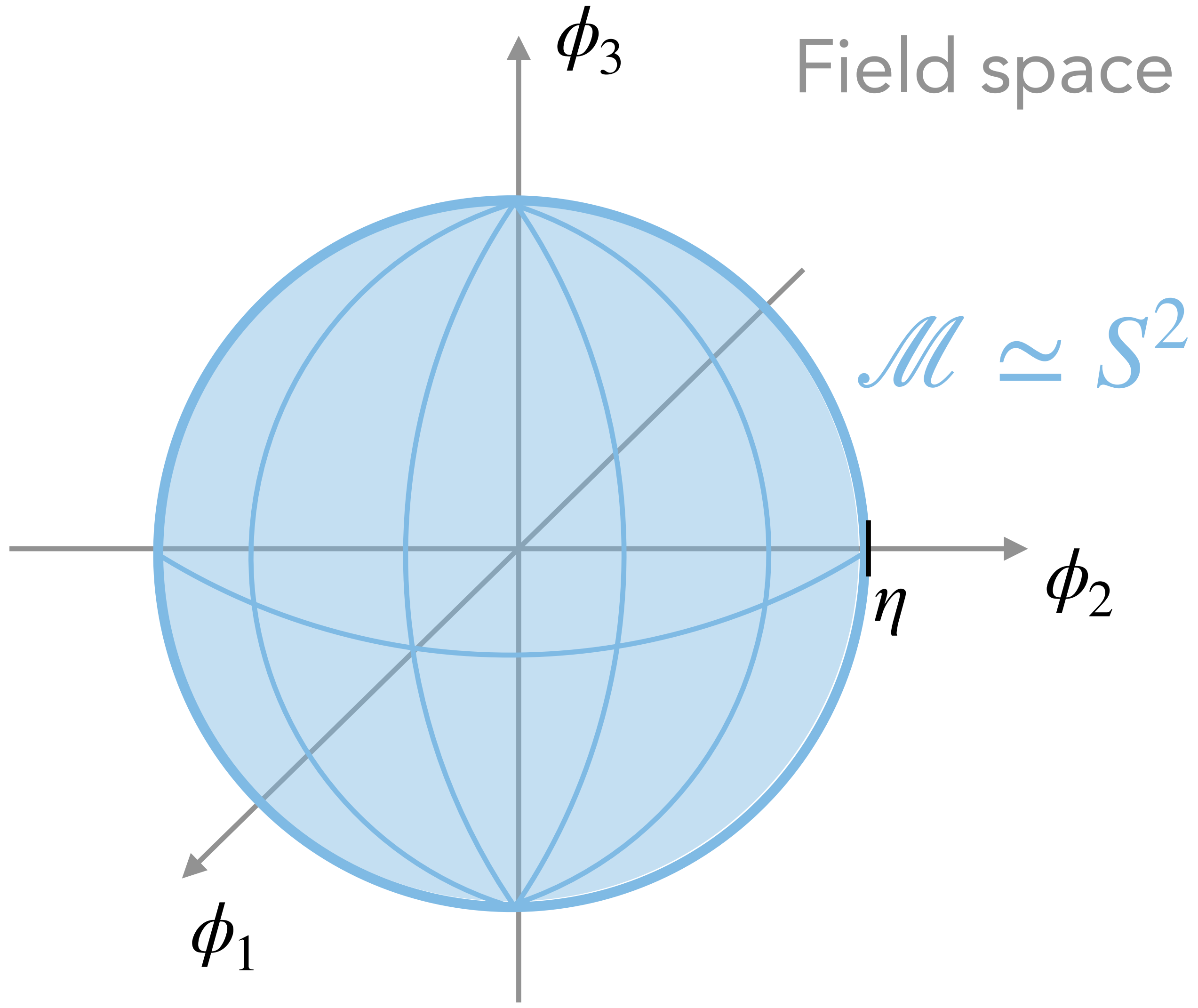

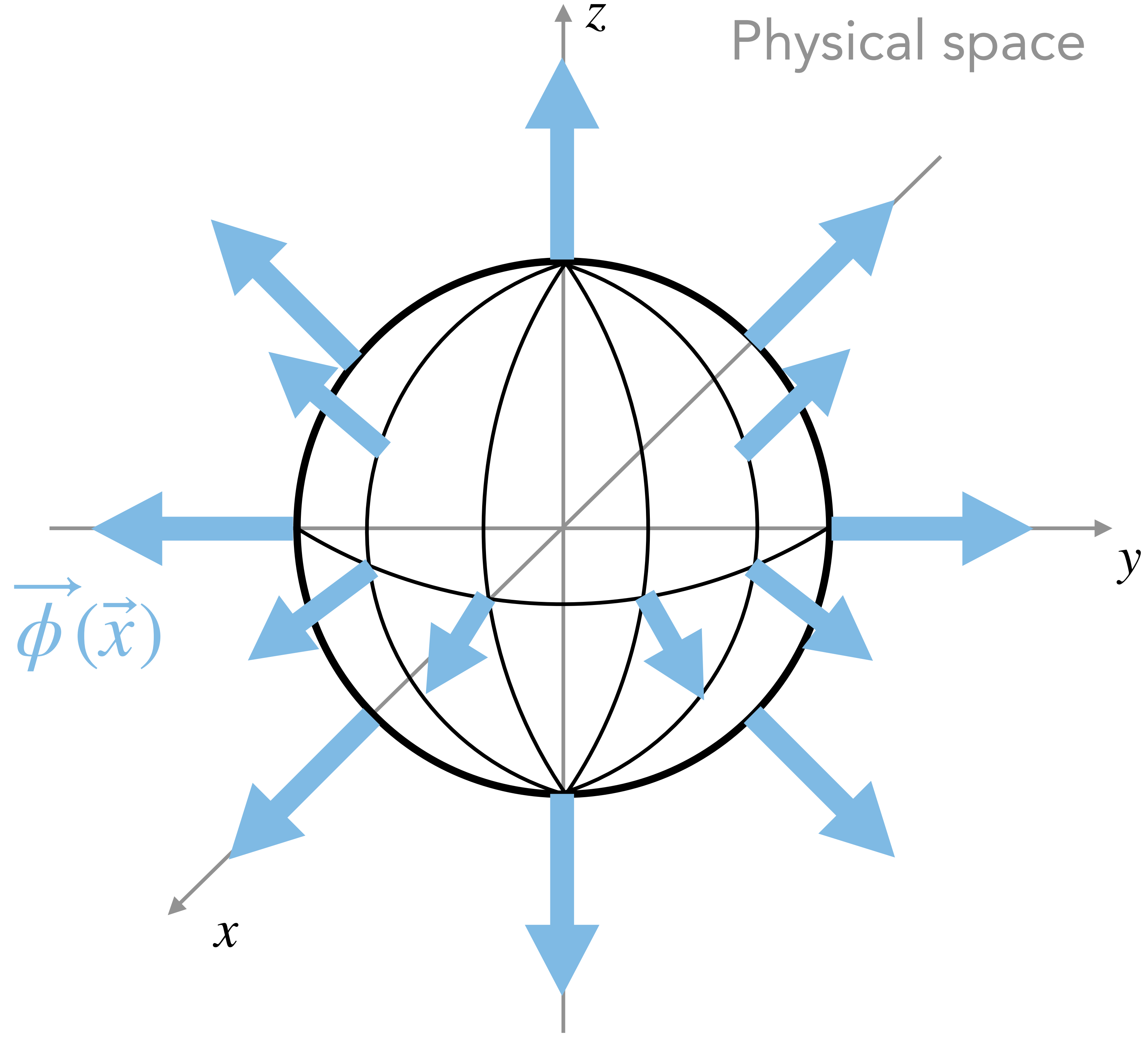

['t Hooft, 1974]
[Polyakov, 1974]

$$\begin{cases} \lim_{r\to\infty} \vec{\phi}(r) \sim \eta\frac{\vec{x}}{r} \\ \\ \end{cases}$$

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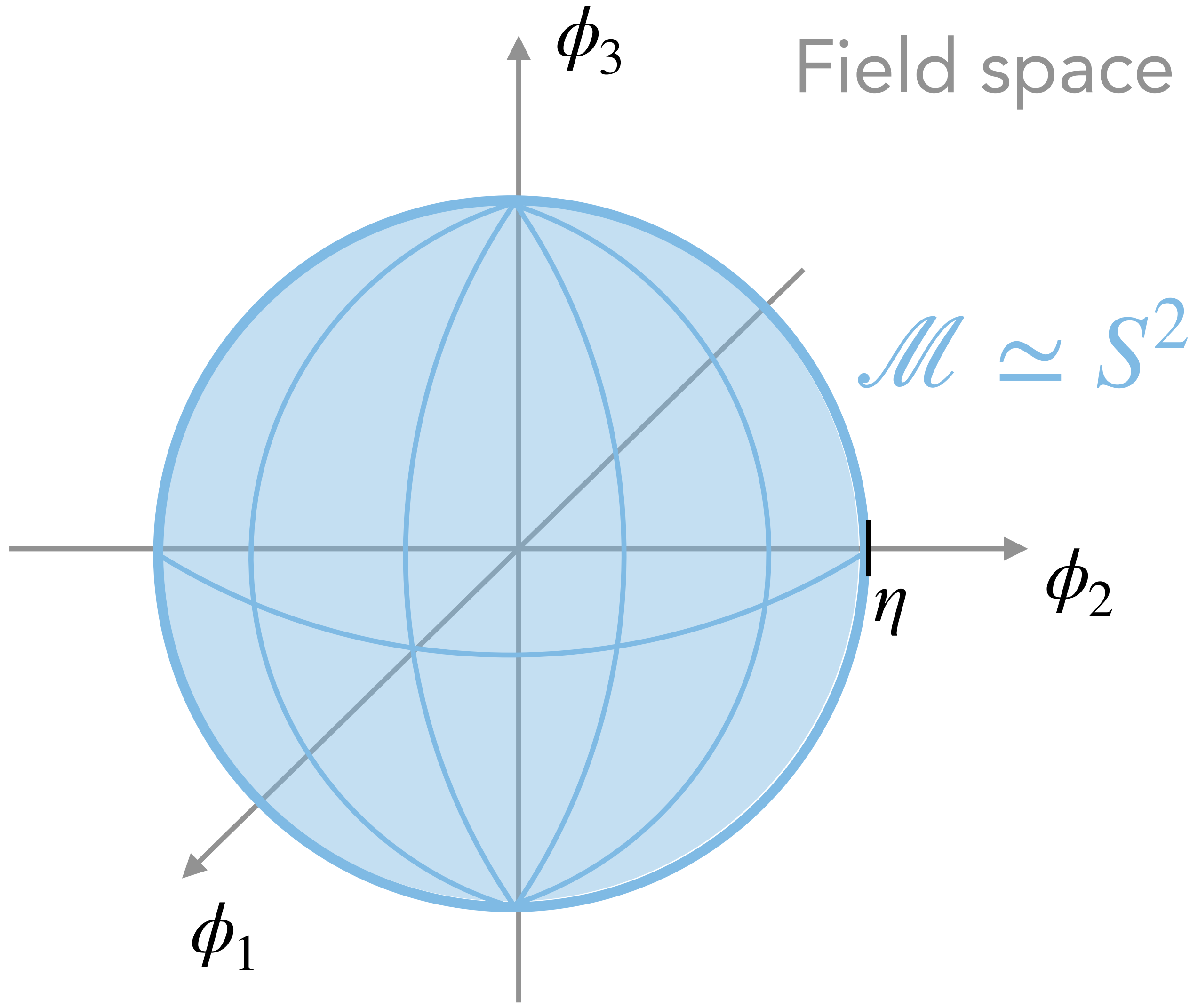

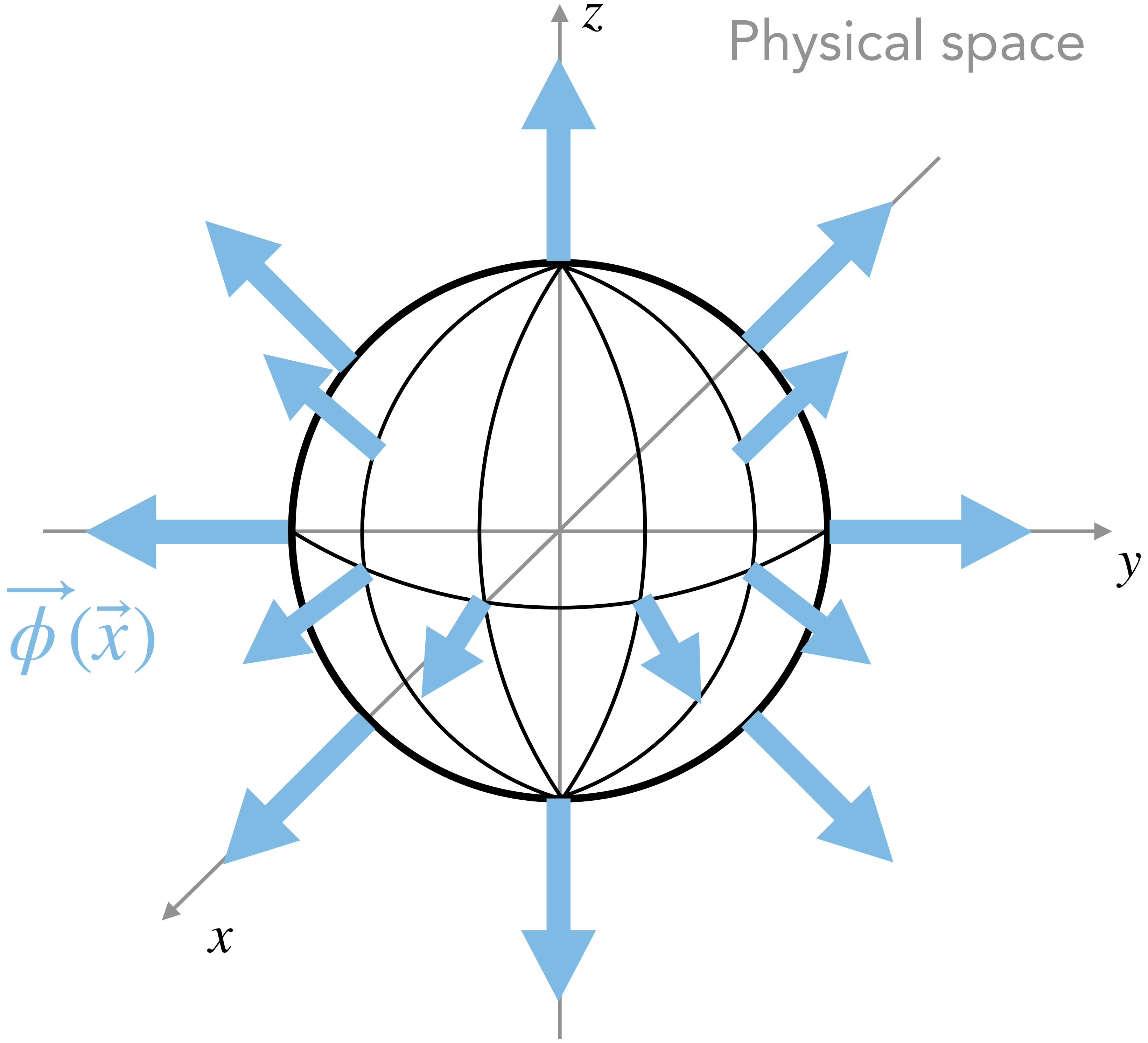

['t Hooft, 1974]
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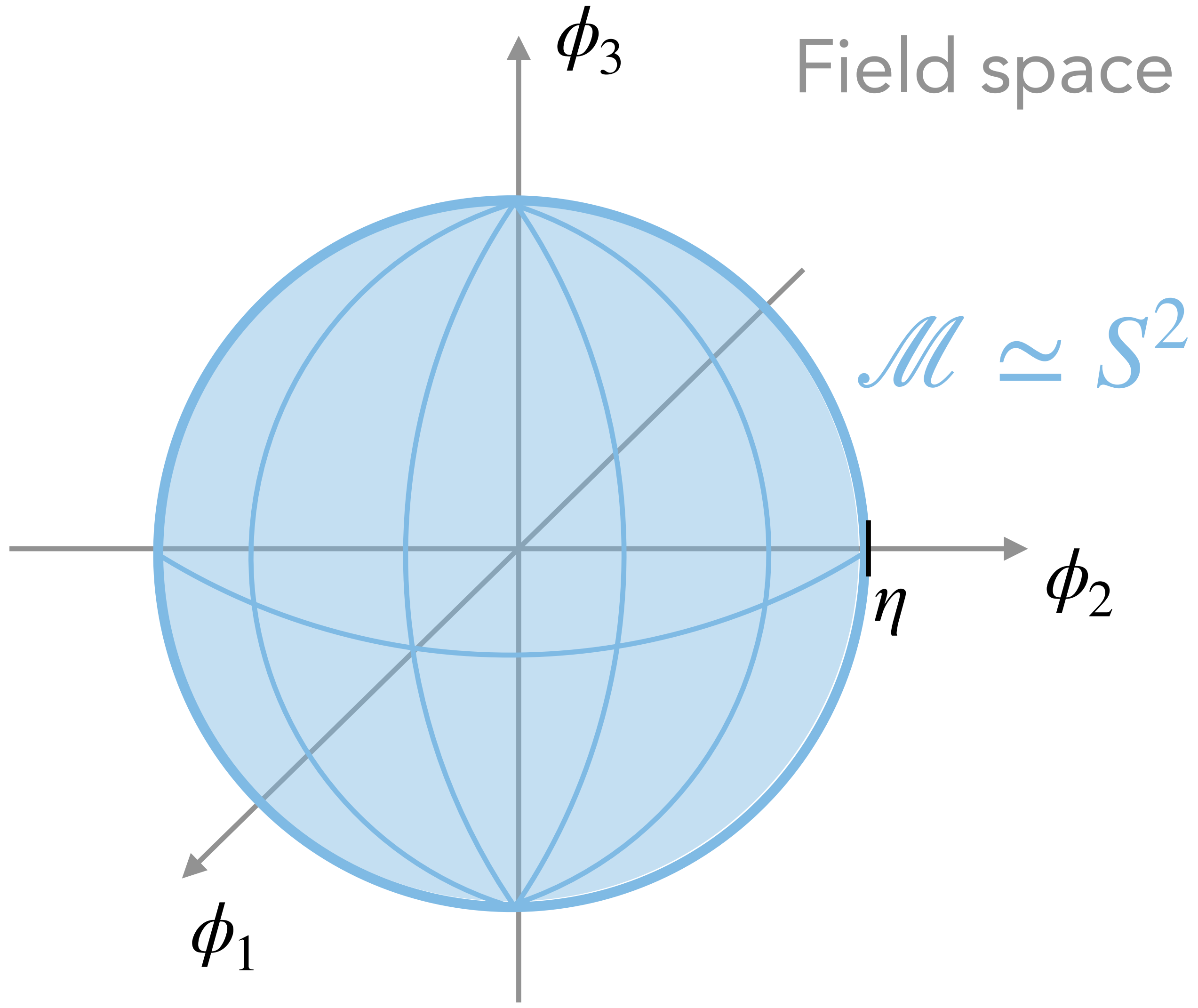

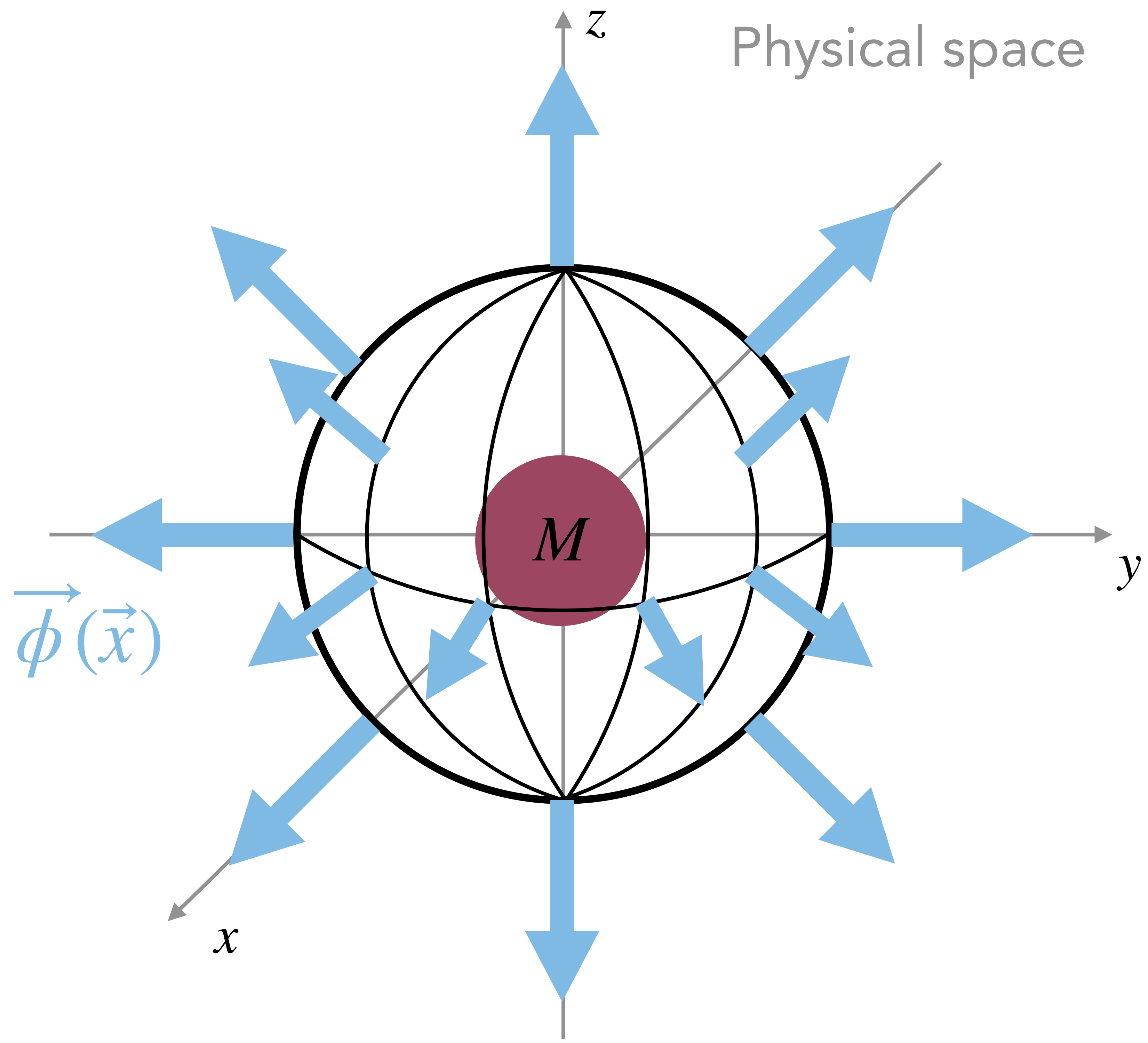

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The 't Hooft-Polyakov model

Stable relics

$$SO(3)_D \to SO(2)_D$$

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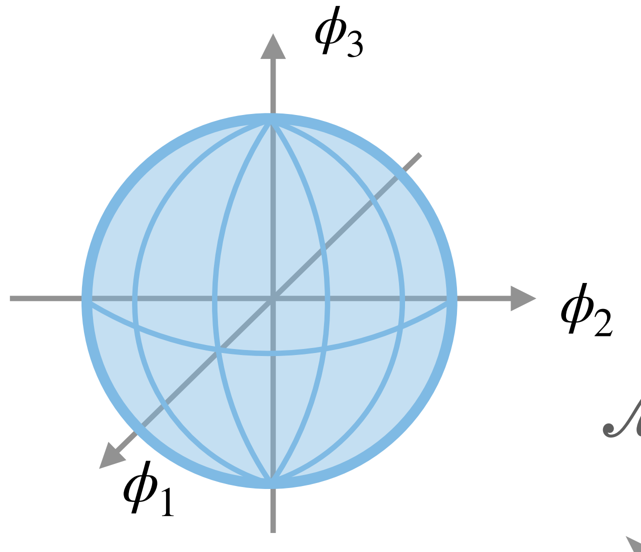


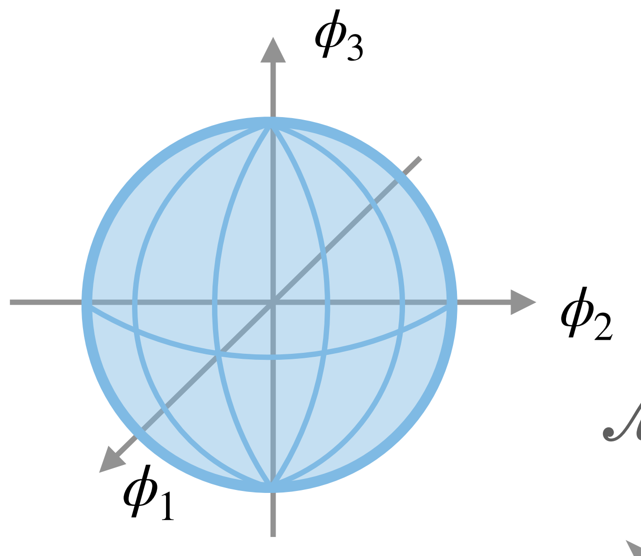
$\mathscr{M} \simeq S^2$

$$\mathrm{SO(3)_D} \to \mathrm{SO(2)_D}$$

Monopoles
topologically stable

The ’t Hooft-Polyakov model

Stable relics



$\mathrm{SO}(3)_\mathrm{D} \to \mathrm{SO}(2)_\mathrm{D}$

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Higgs mechanism

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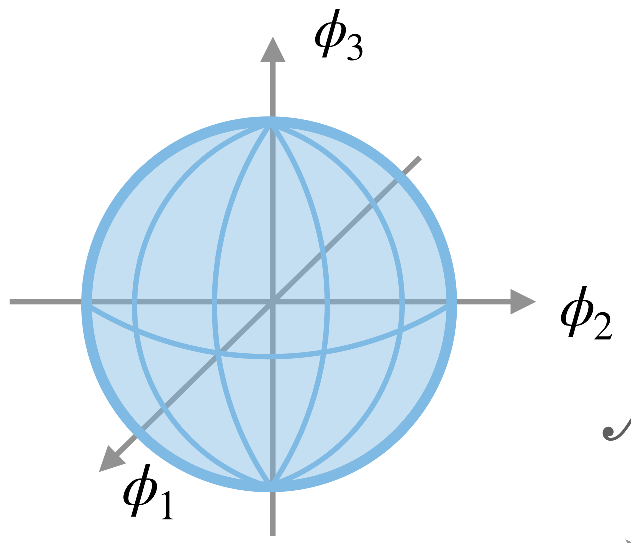

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Massive $W'^{\pm}$
lightest particles w/ electric charge

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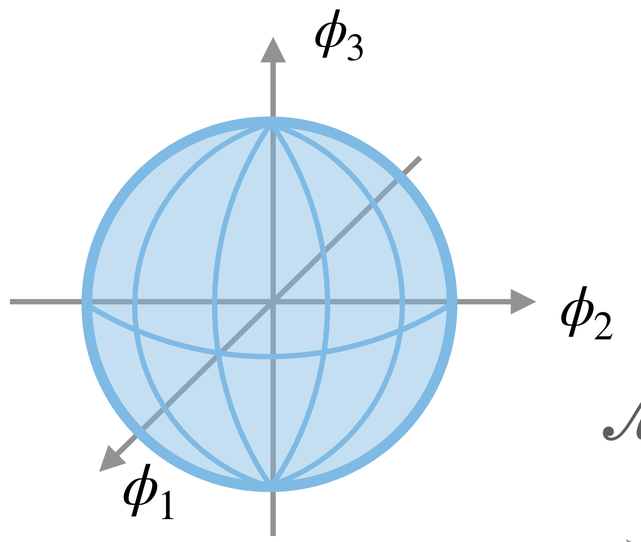

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Higgs mechanism

Monopoles
topologically stable

Massive $W'^{\pm}$
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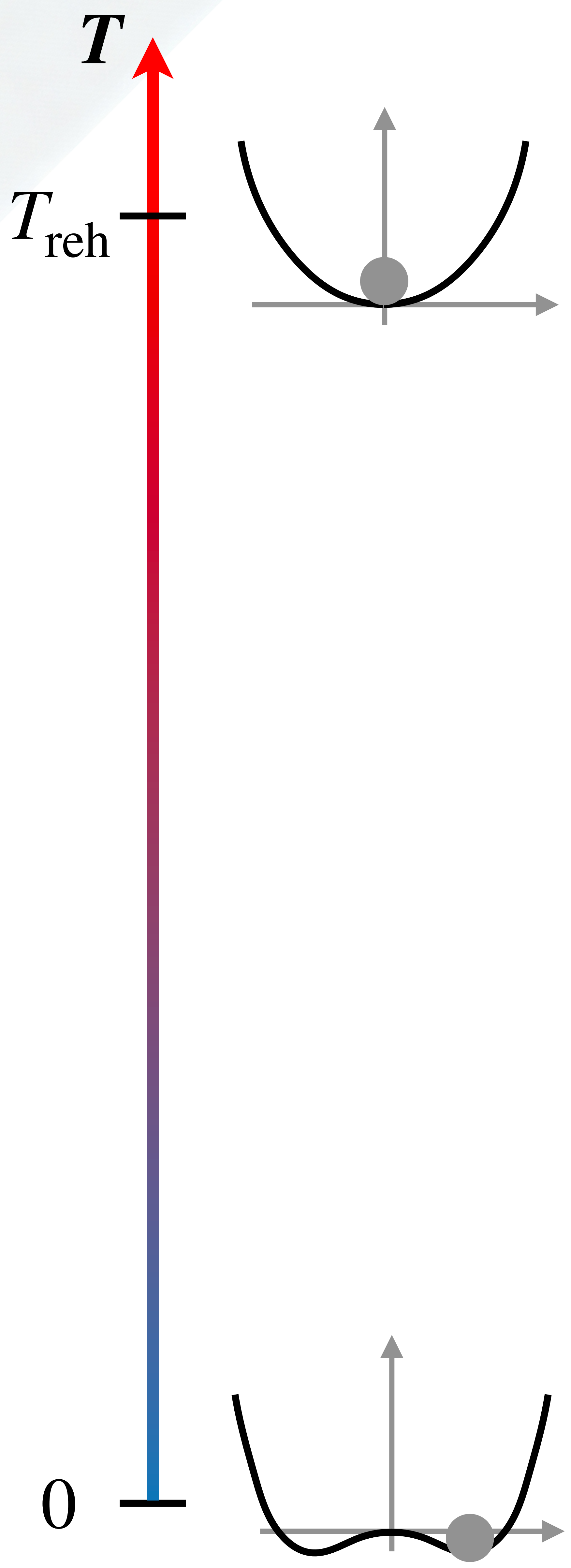
$$\Omega_M h^2 > \Omega_{W'} h^2$$

Monopole production



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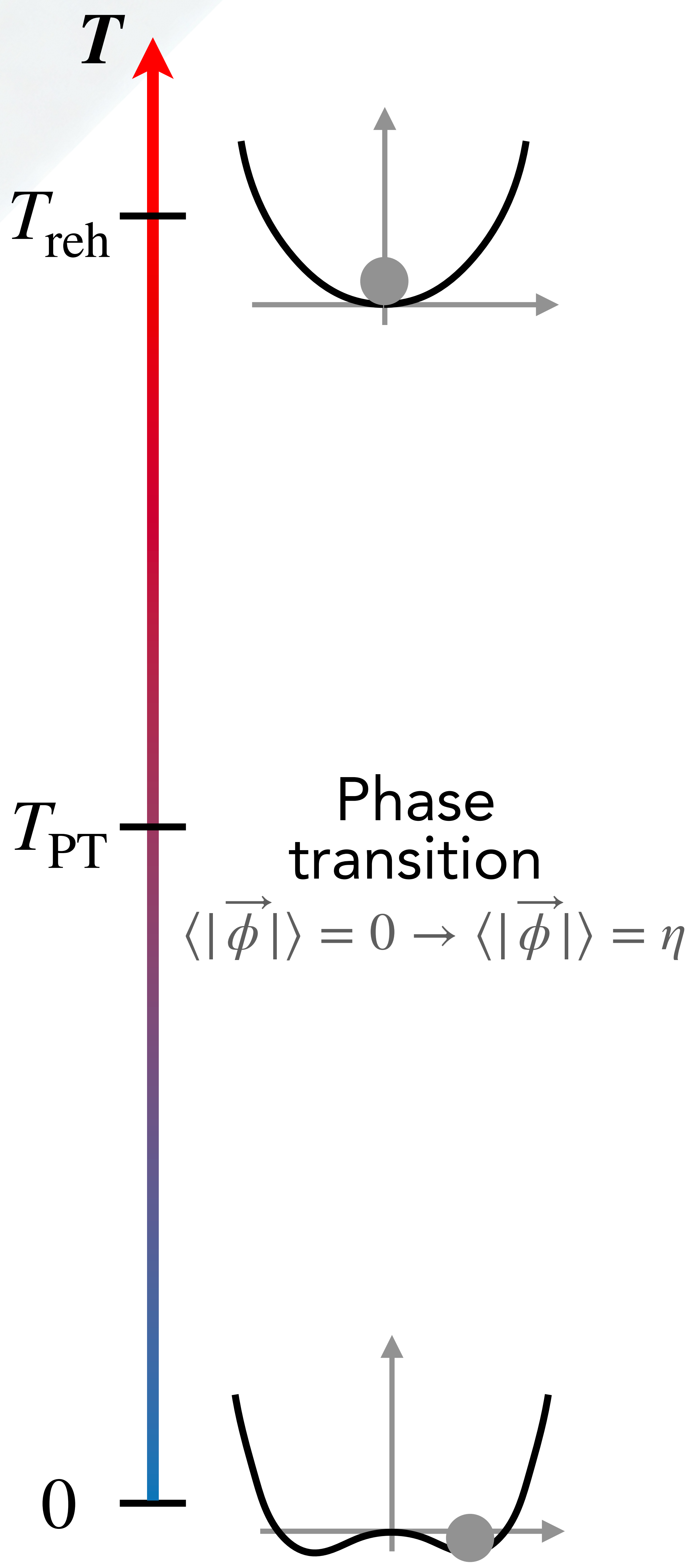

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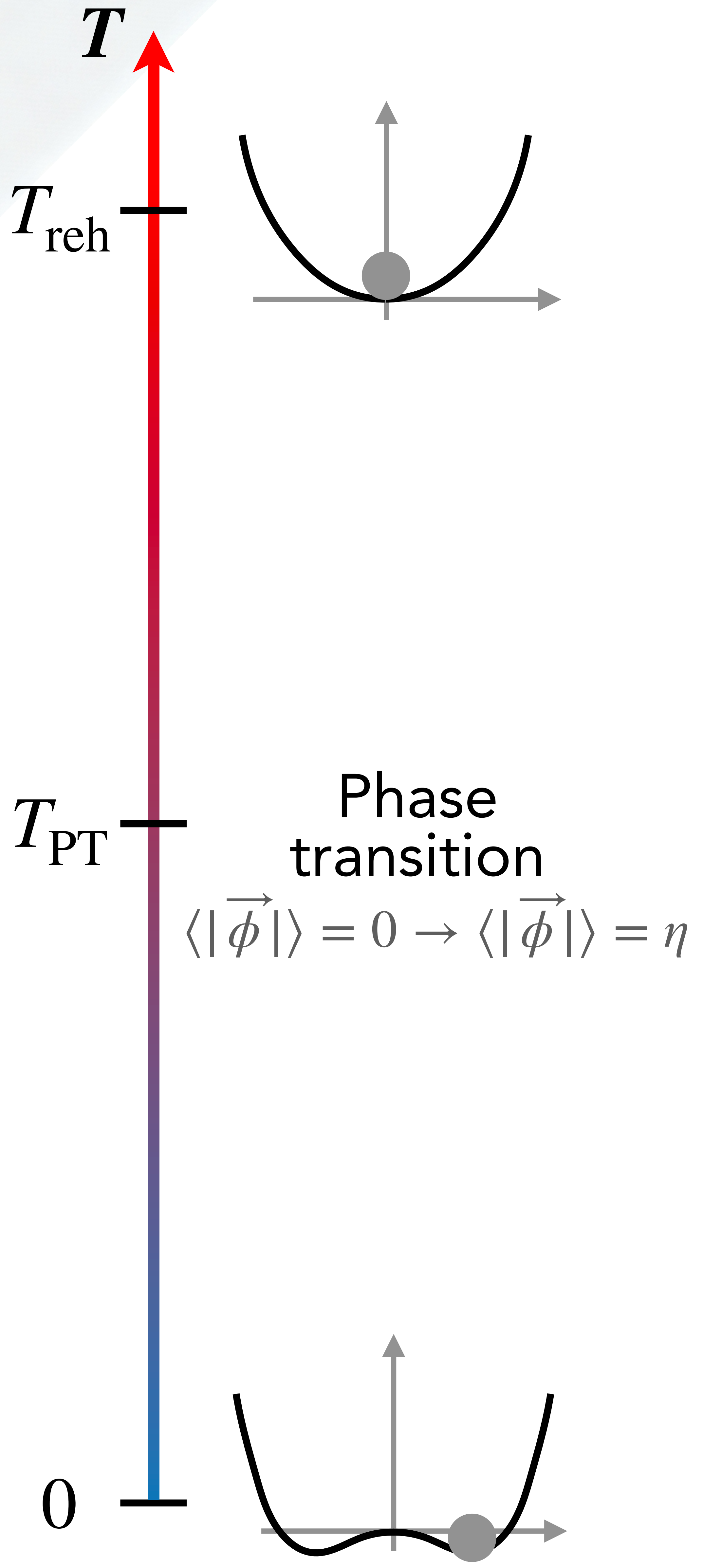

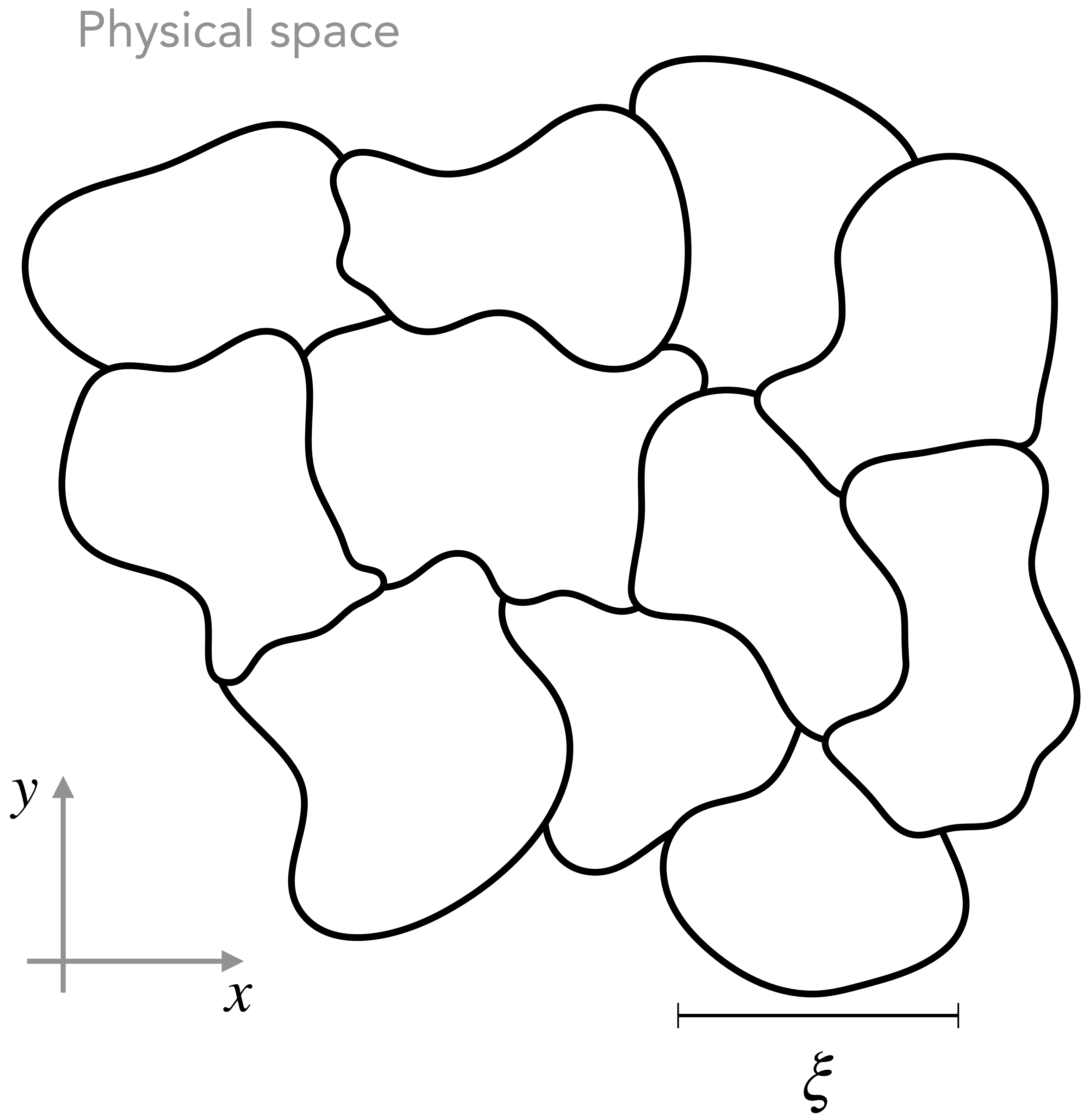

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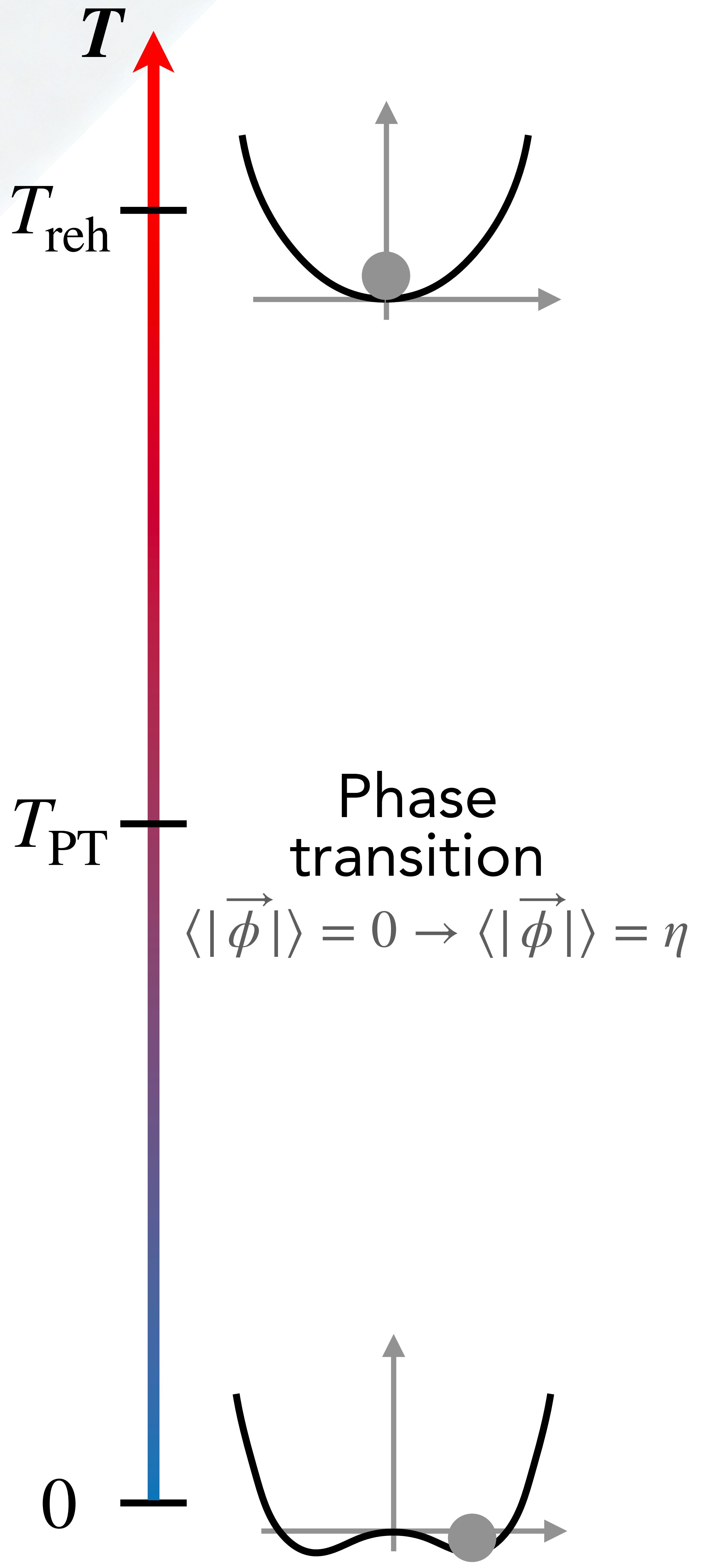

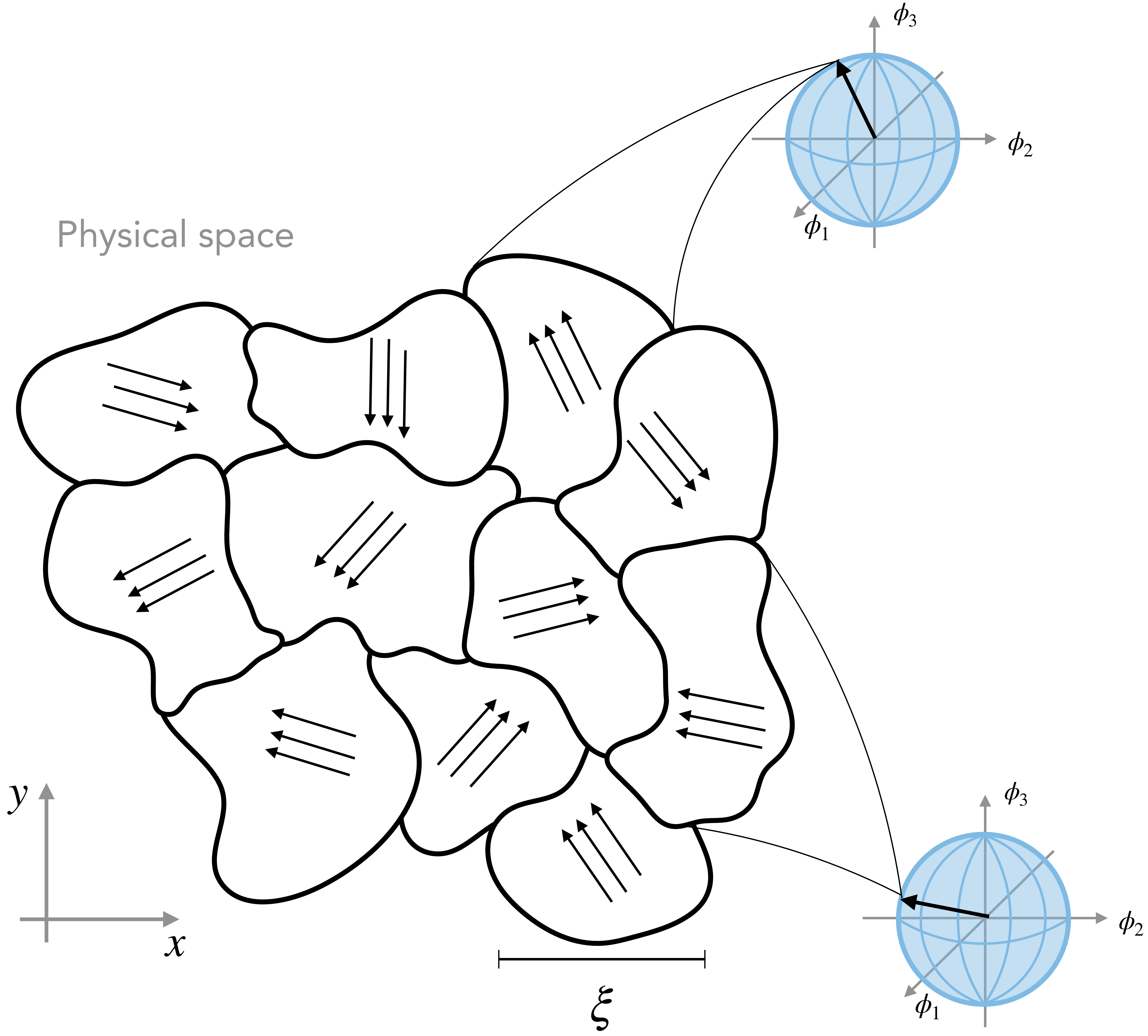

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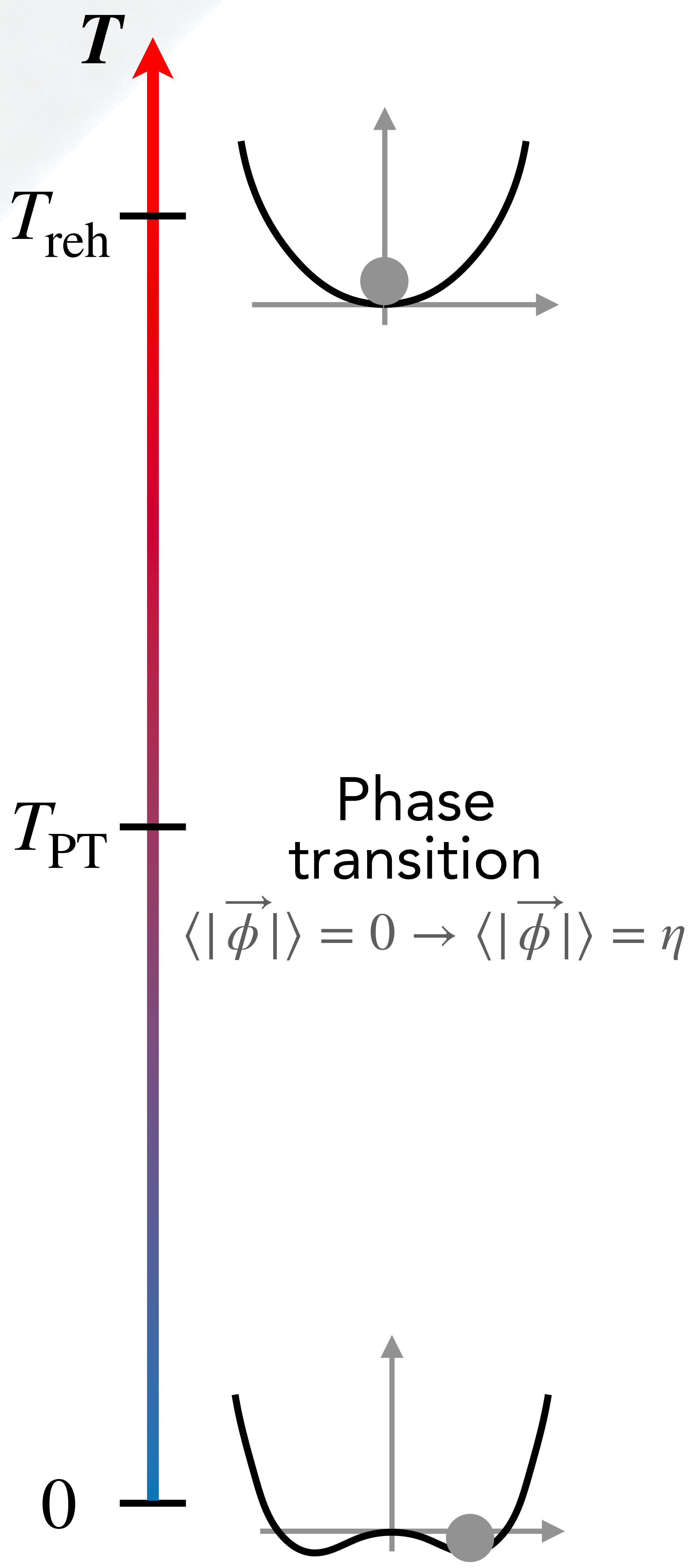

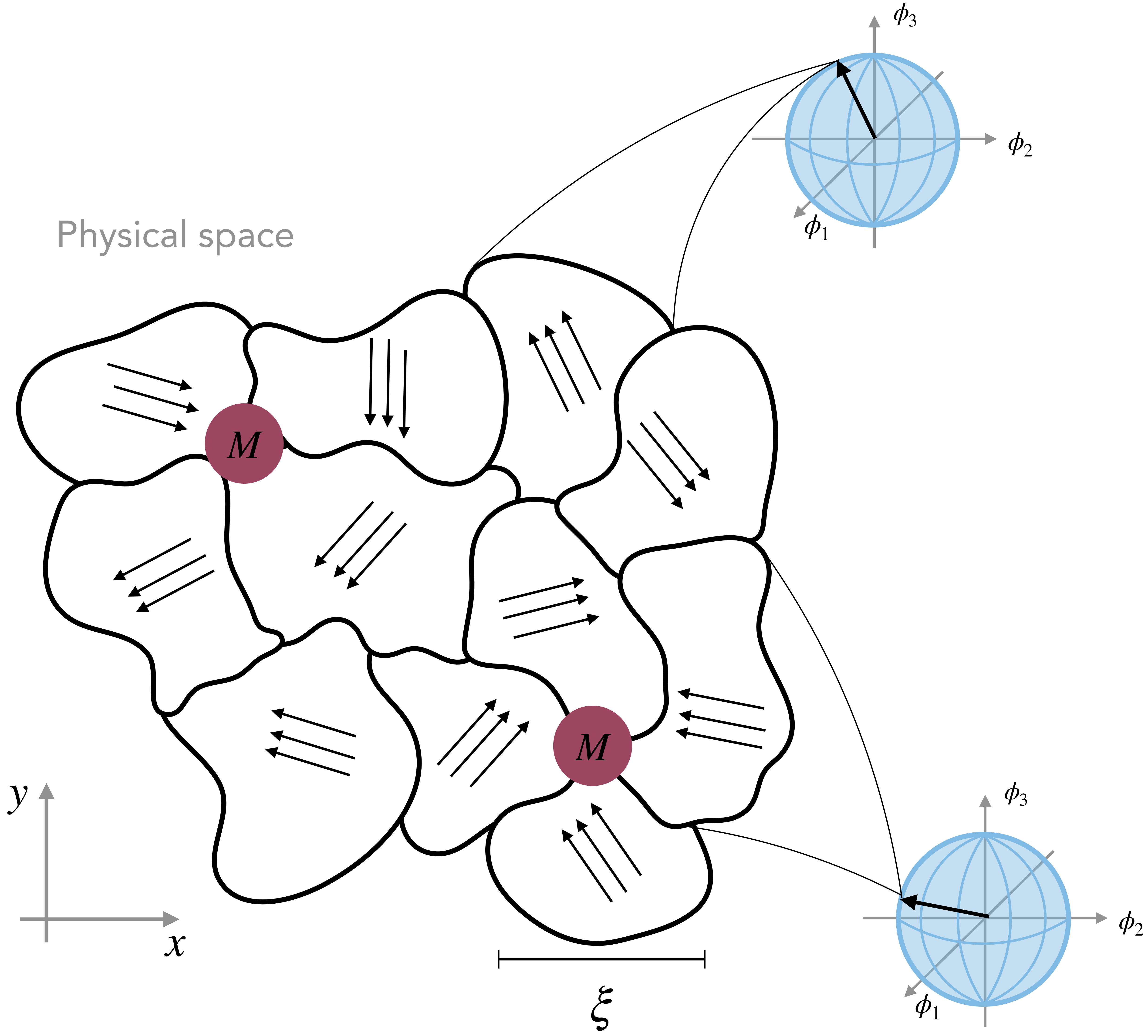

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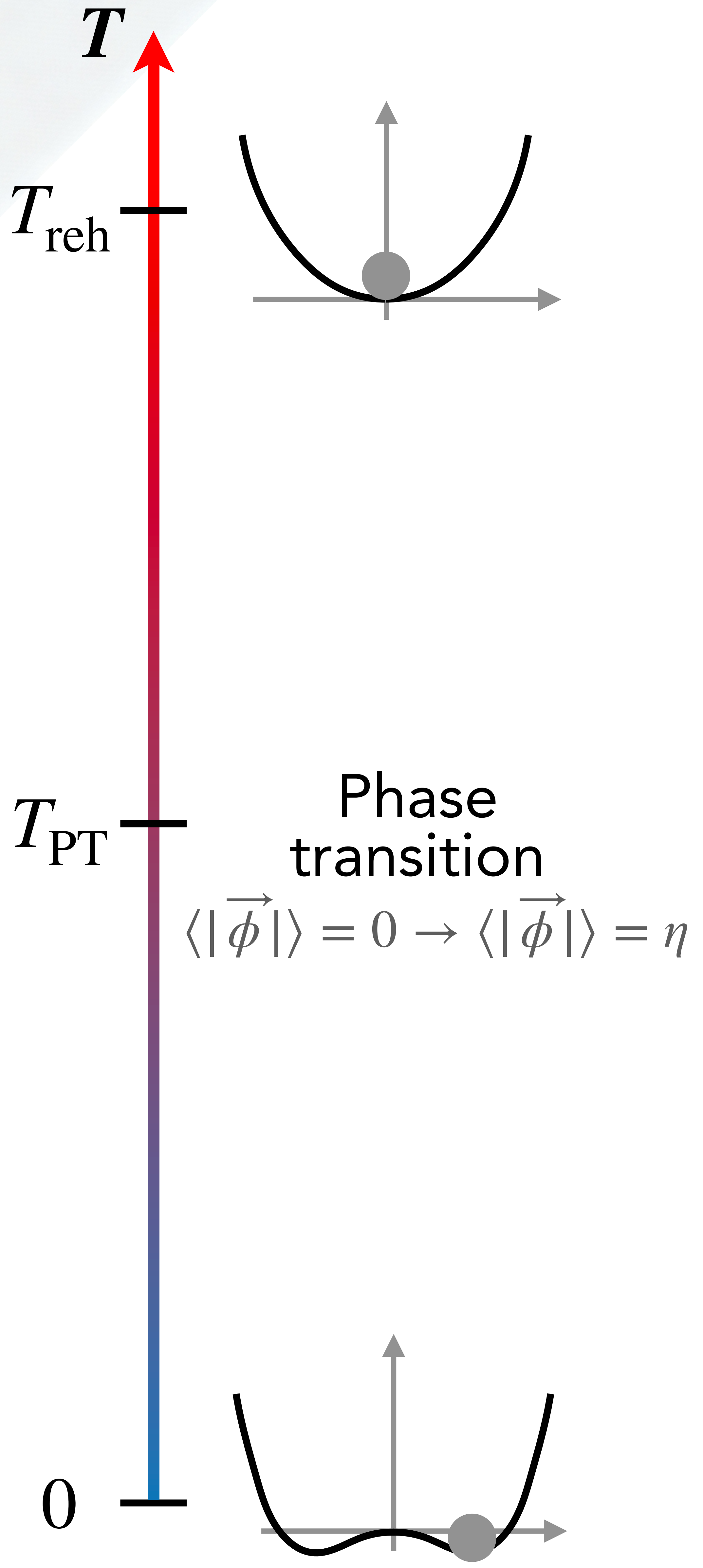
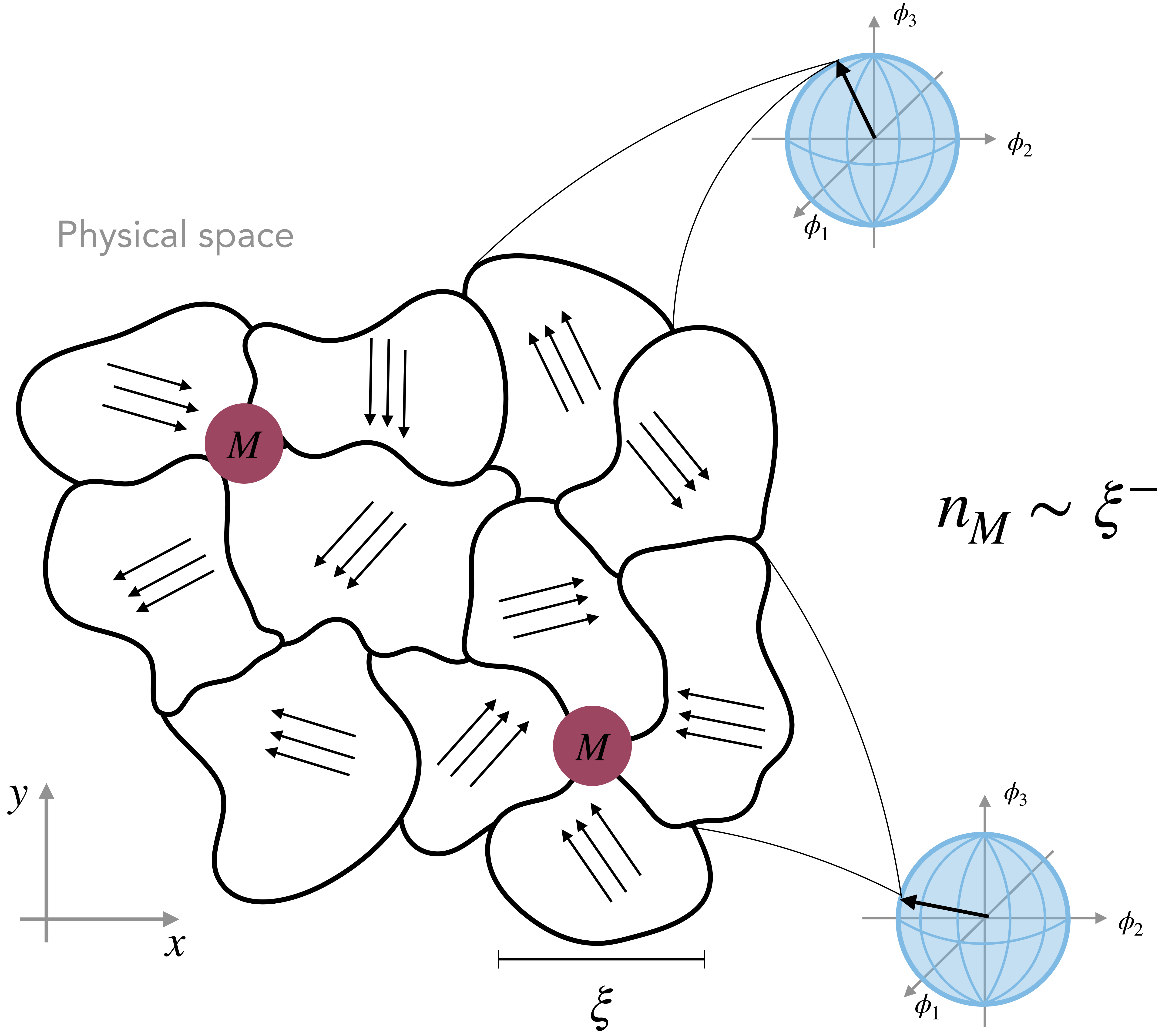
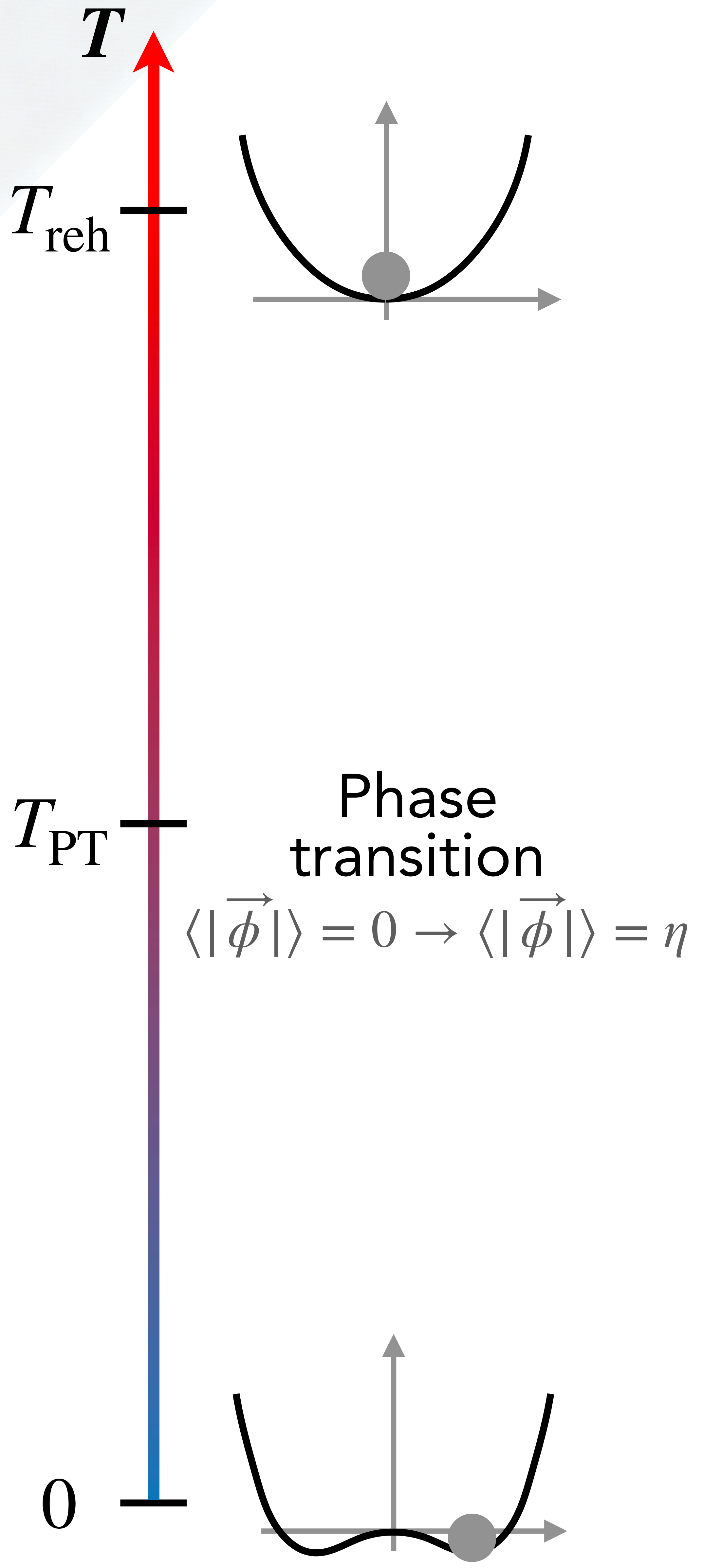


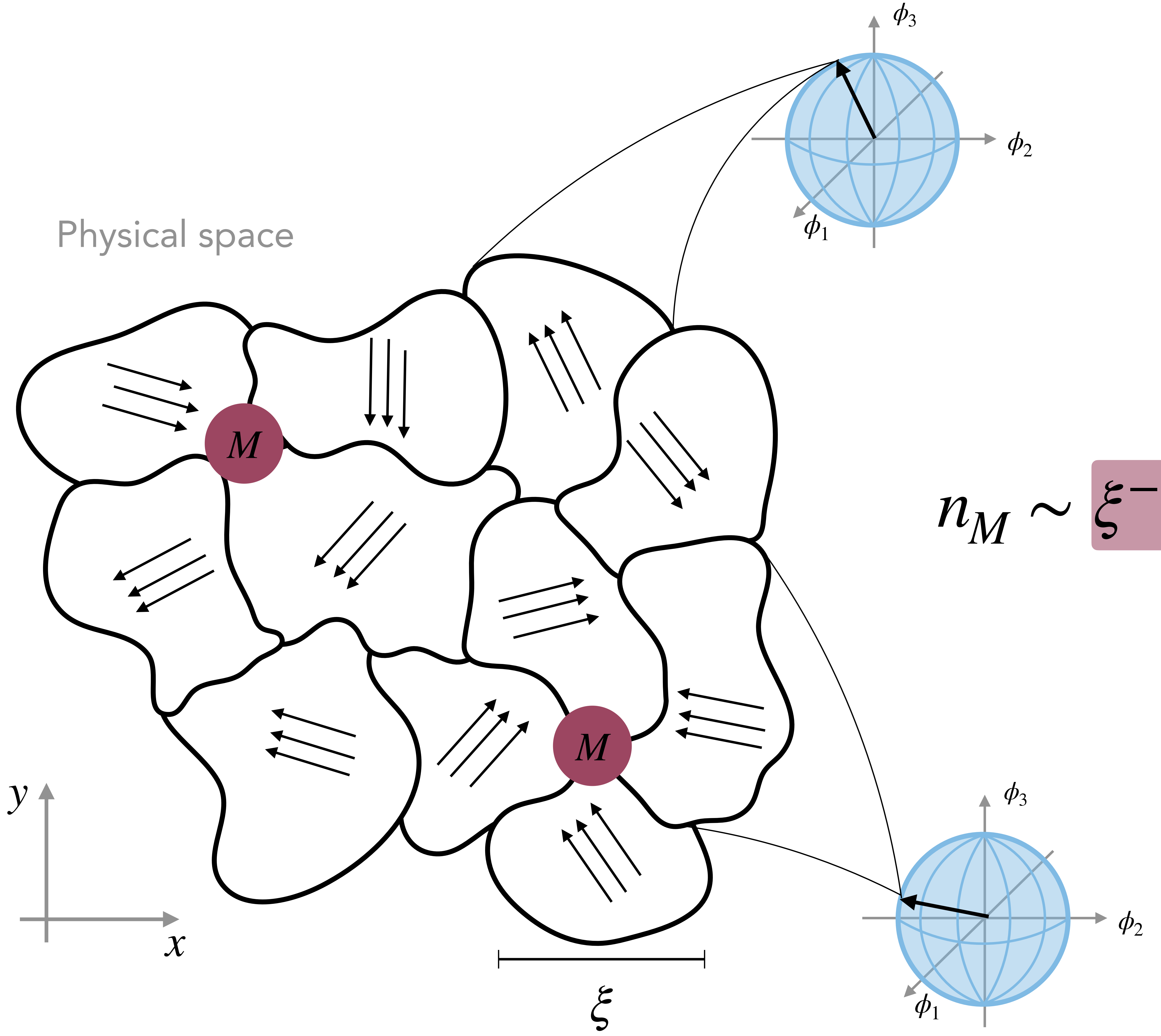


$$n_M \sim \xi^{-3}$$

[Kibble, 1976]

Monopole production





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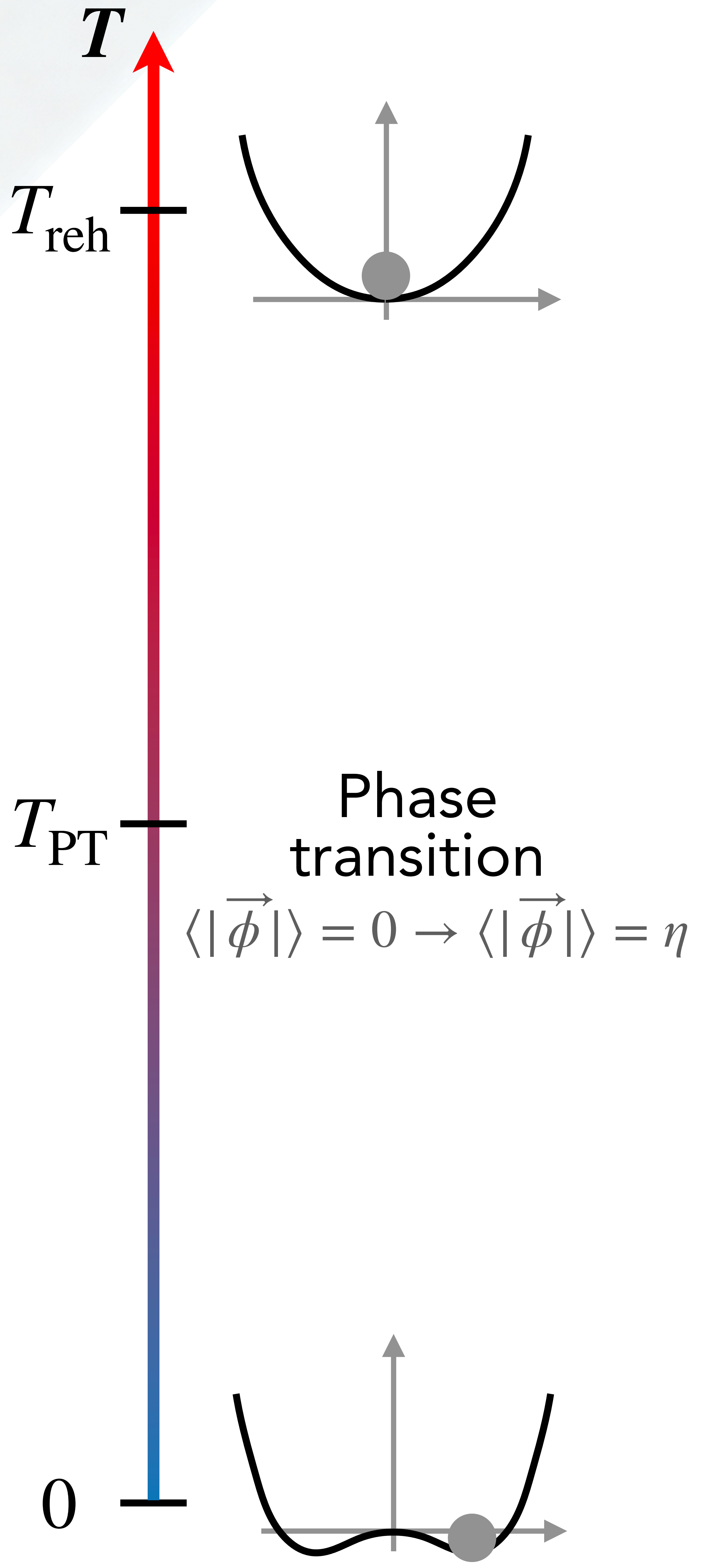
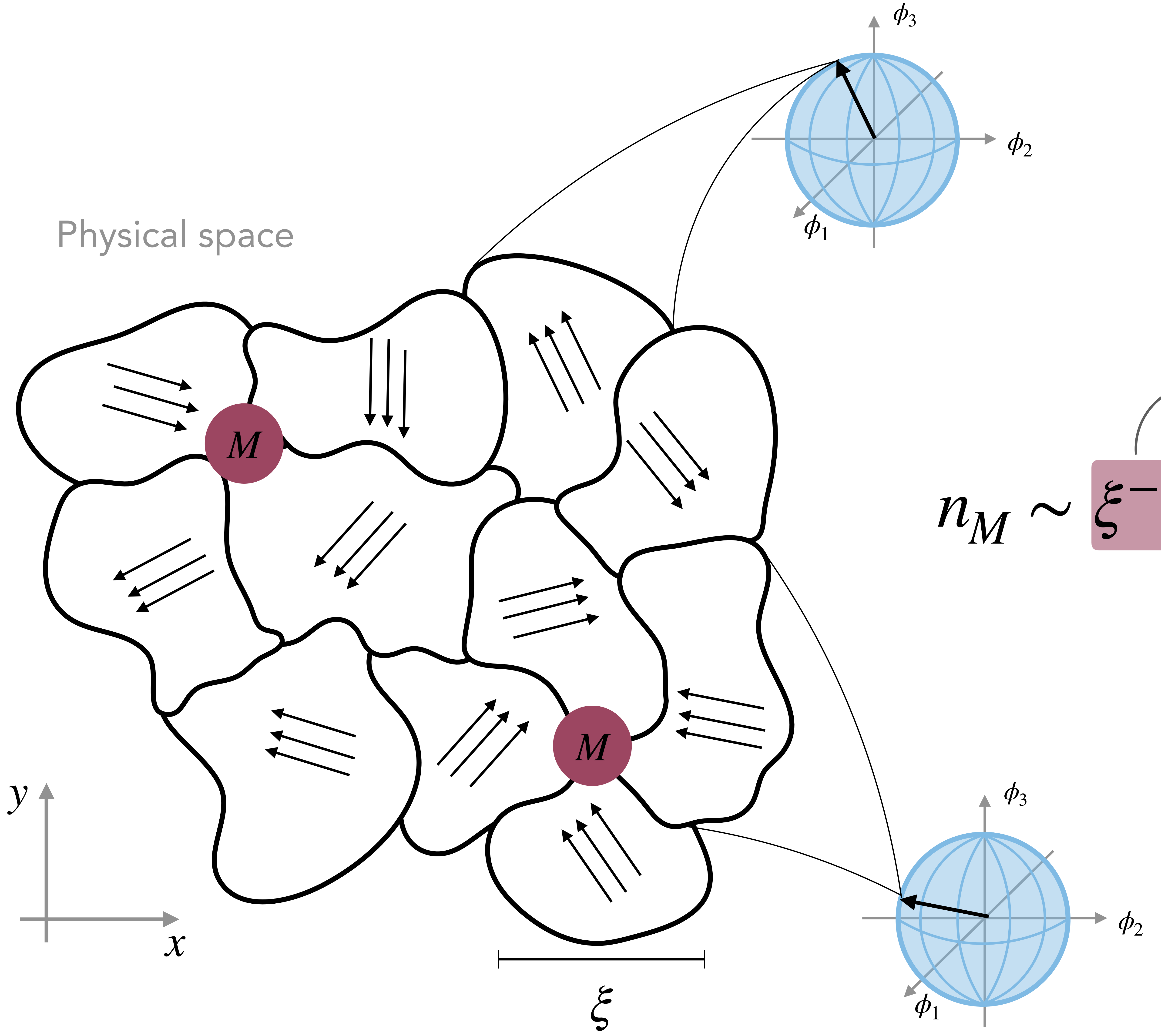




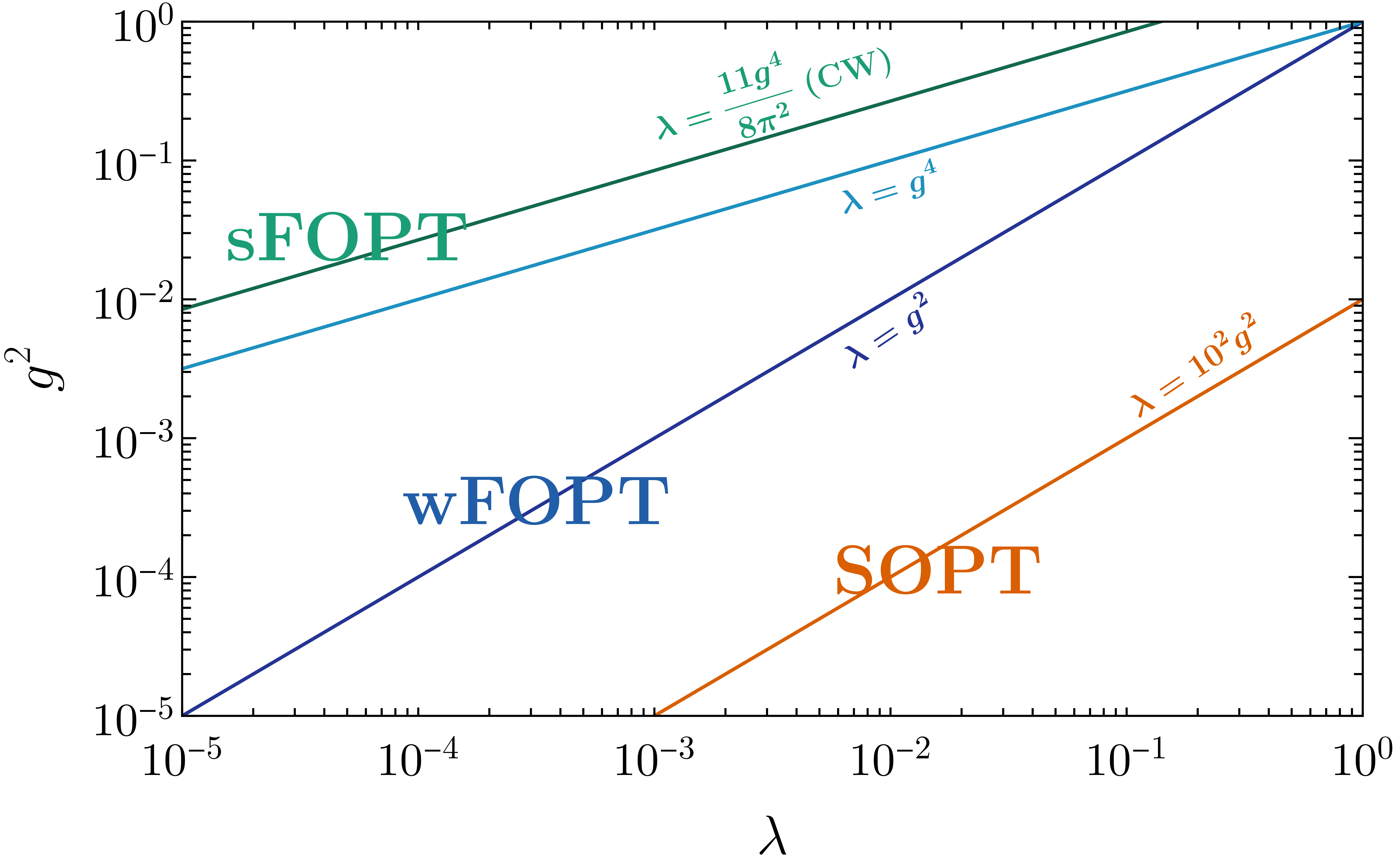
$$n_M \sim \xi^{-3}$$

Depends on the nature of the PT

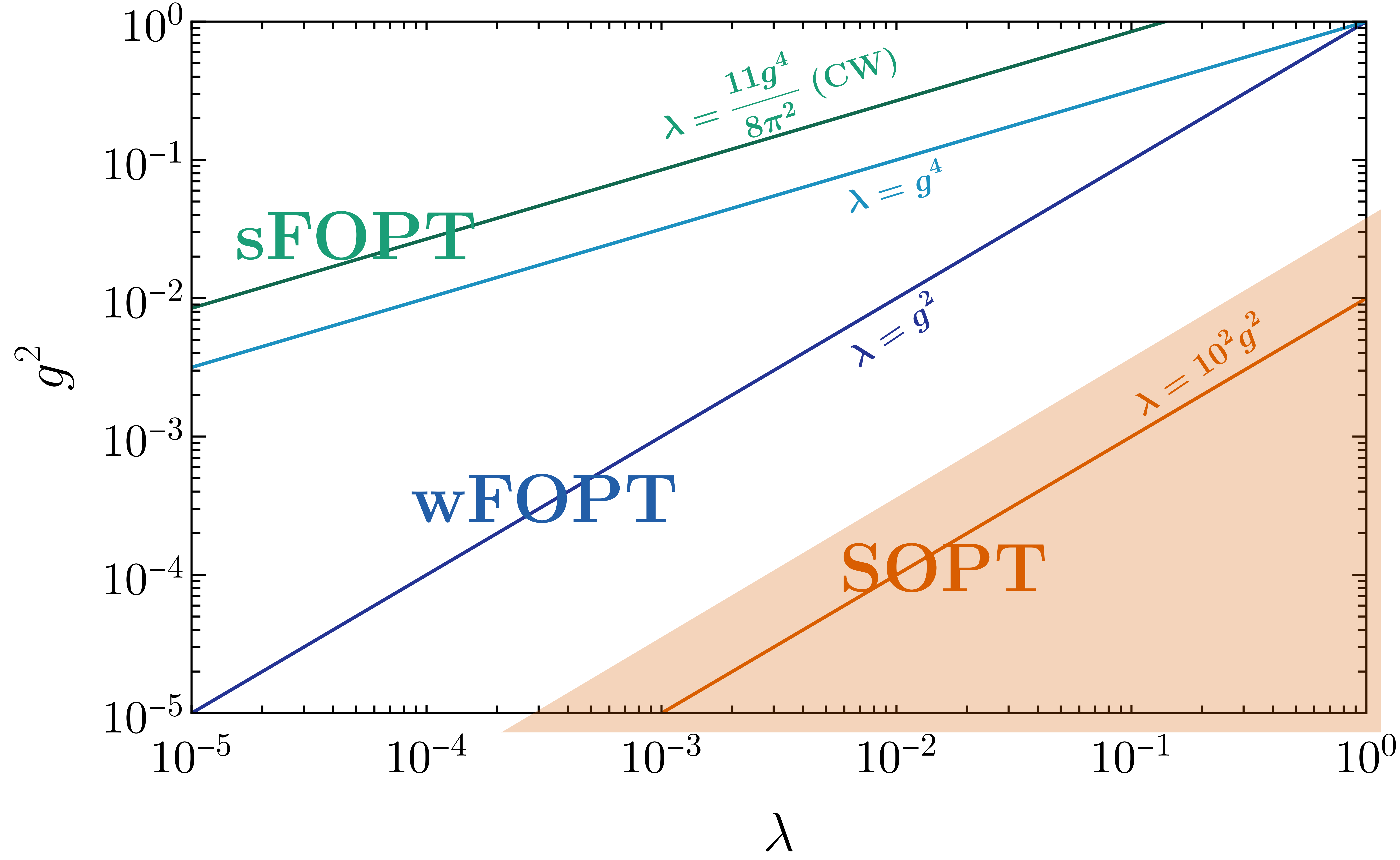
[Kibble, 1976]

The 't Hooft-Polyakov model

Phase diagram



Second-order phase transitions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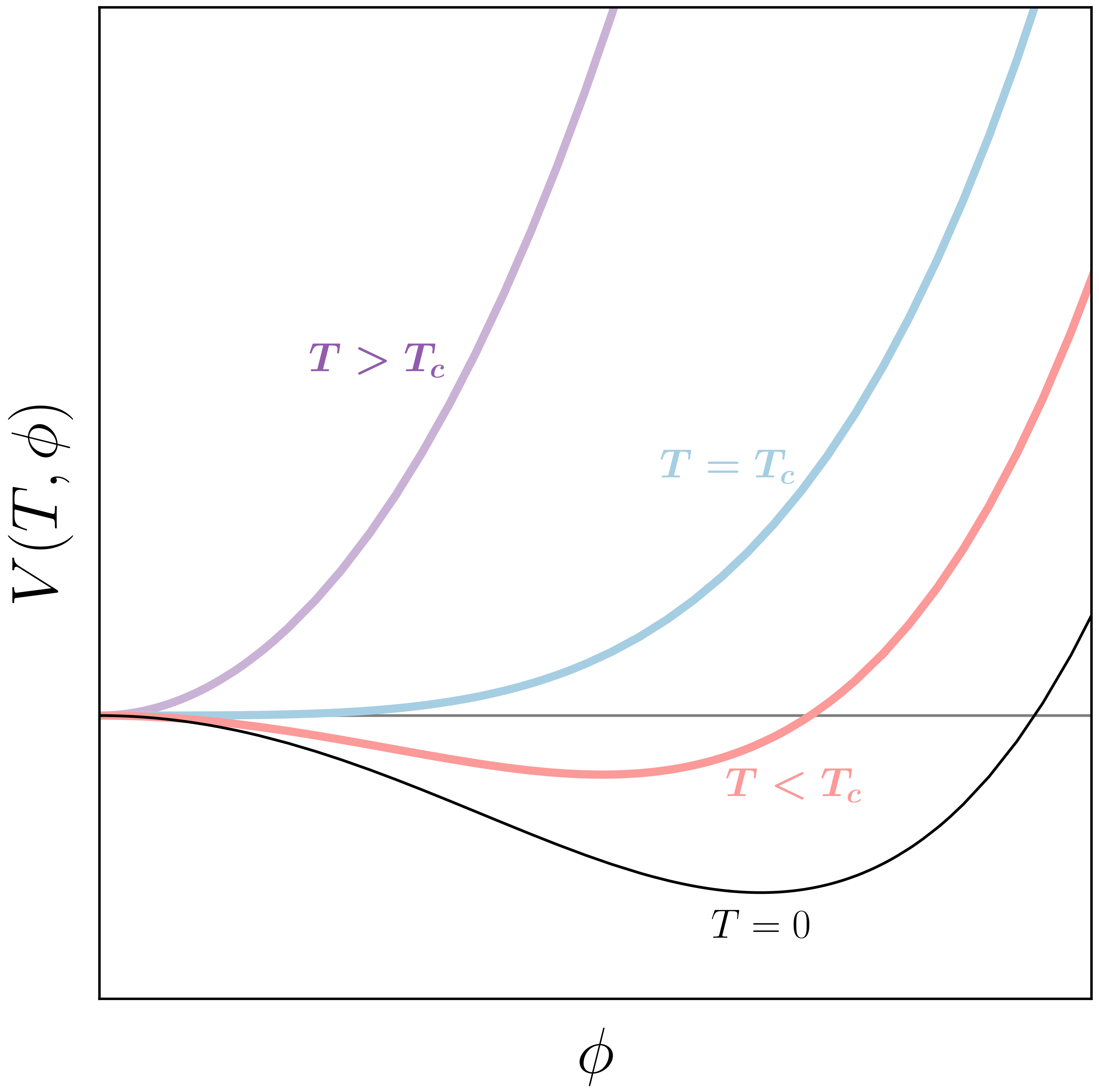
10^{-3}
10^{-4}
10^{-5}

Correlation length

The Kibble-Zurek mechanism



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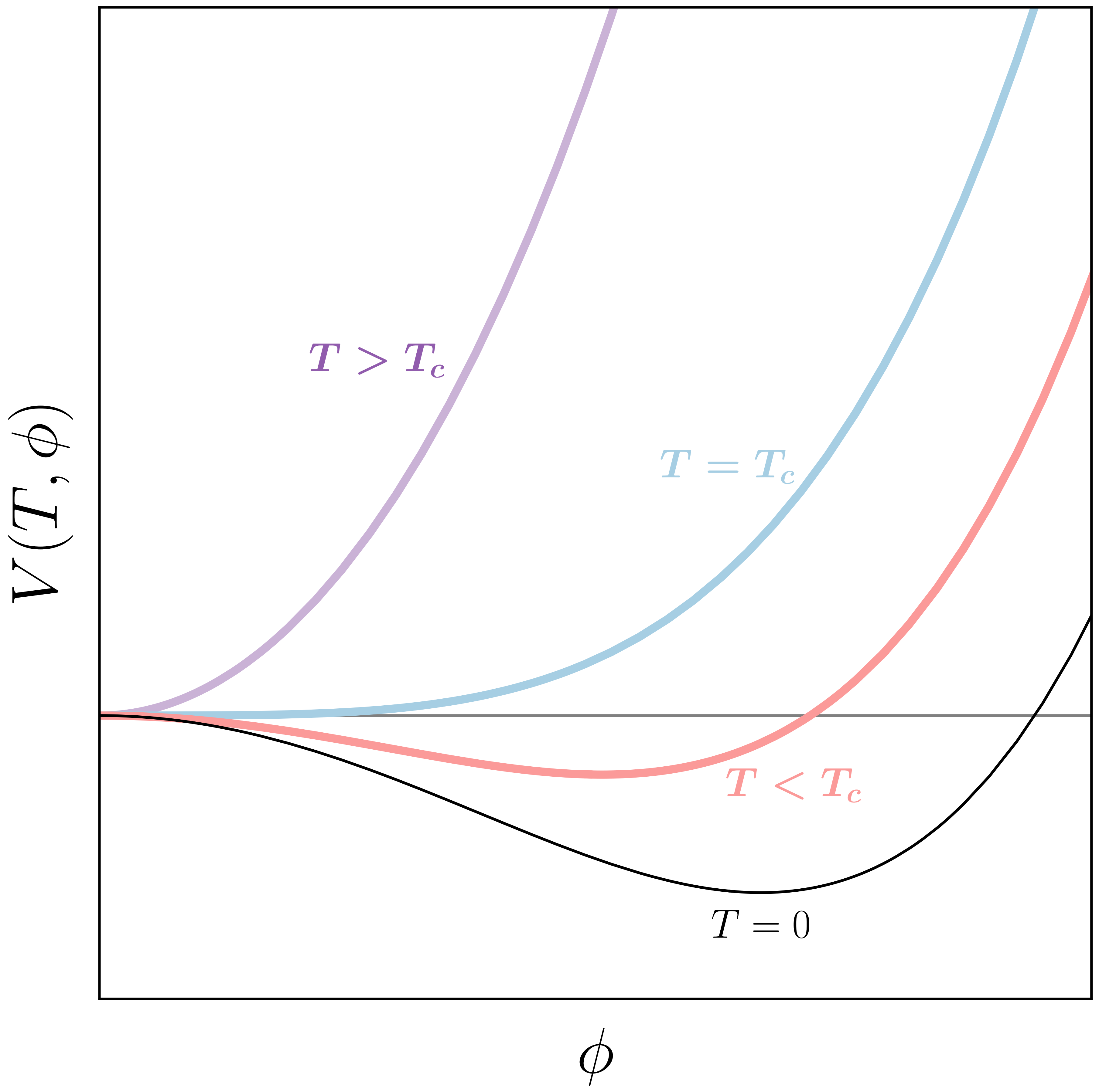


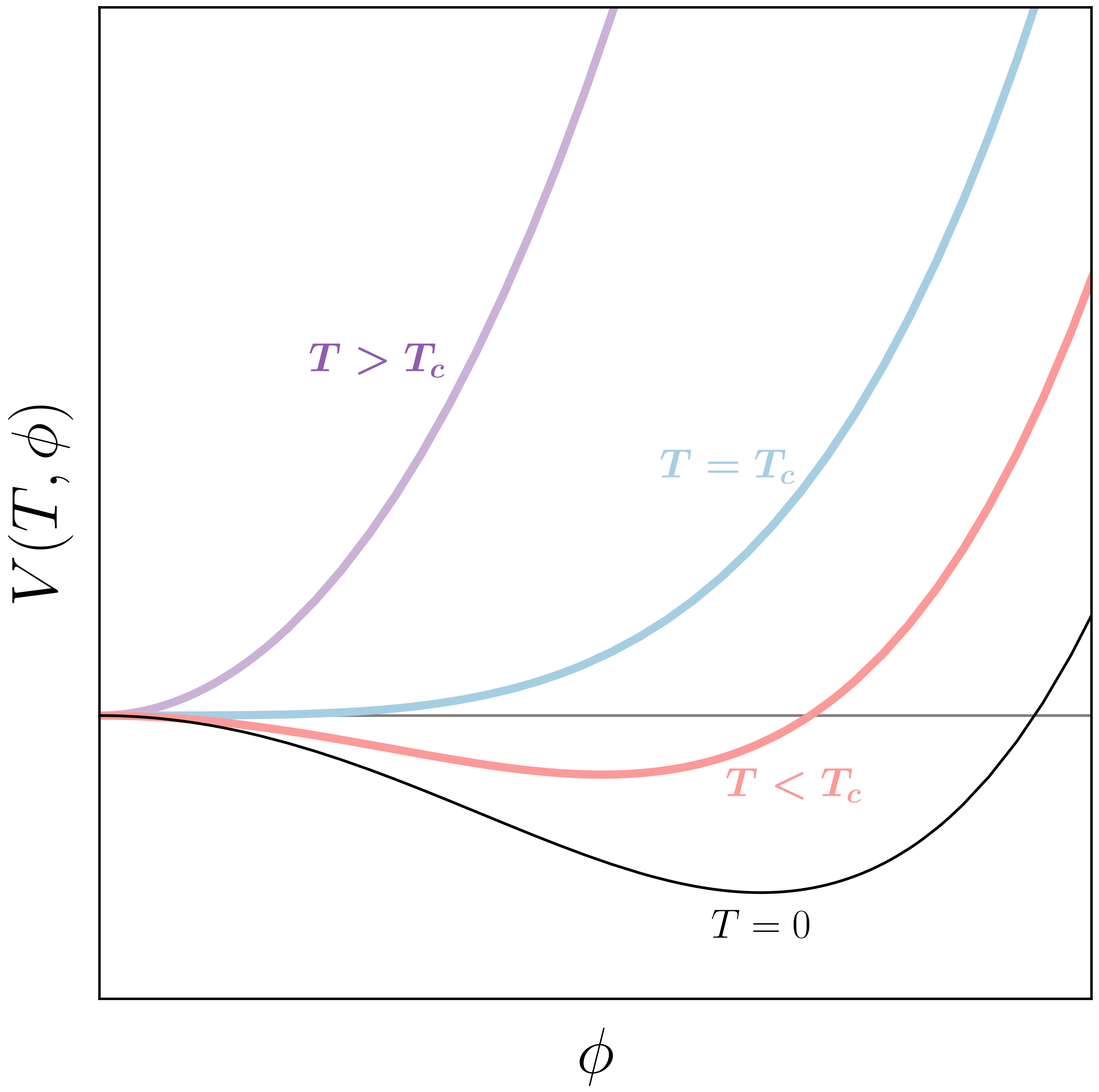
Relaxation time

Correlation length

$$\tau(t) \sim \xi(t) \sim \frac{1}{m_\phi} |\epsilon(t)|^{-\nu},$$

Correlation length

The Kibble-Zurek mechanism



Relaxation time

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$$\tau(t) \sim \xi(t) \sim \frac{1}{m_\phi}|\epsilon(t)|^{-\nu}, \qquad \epsilon(t) \equiv \frac{T(t) - T_c}{T_c}$$

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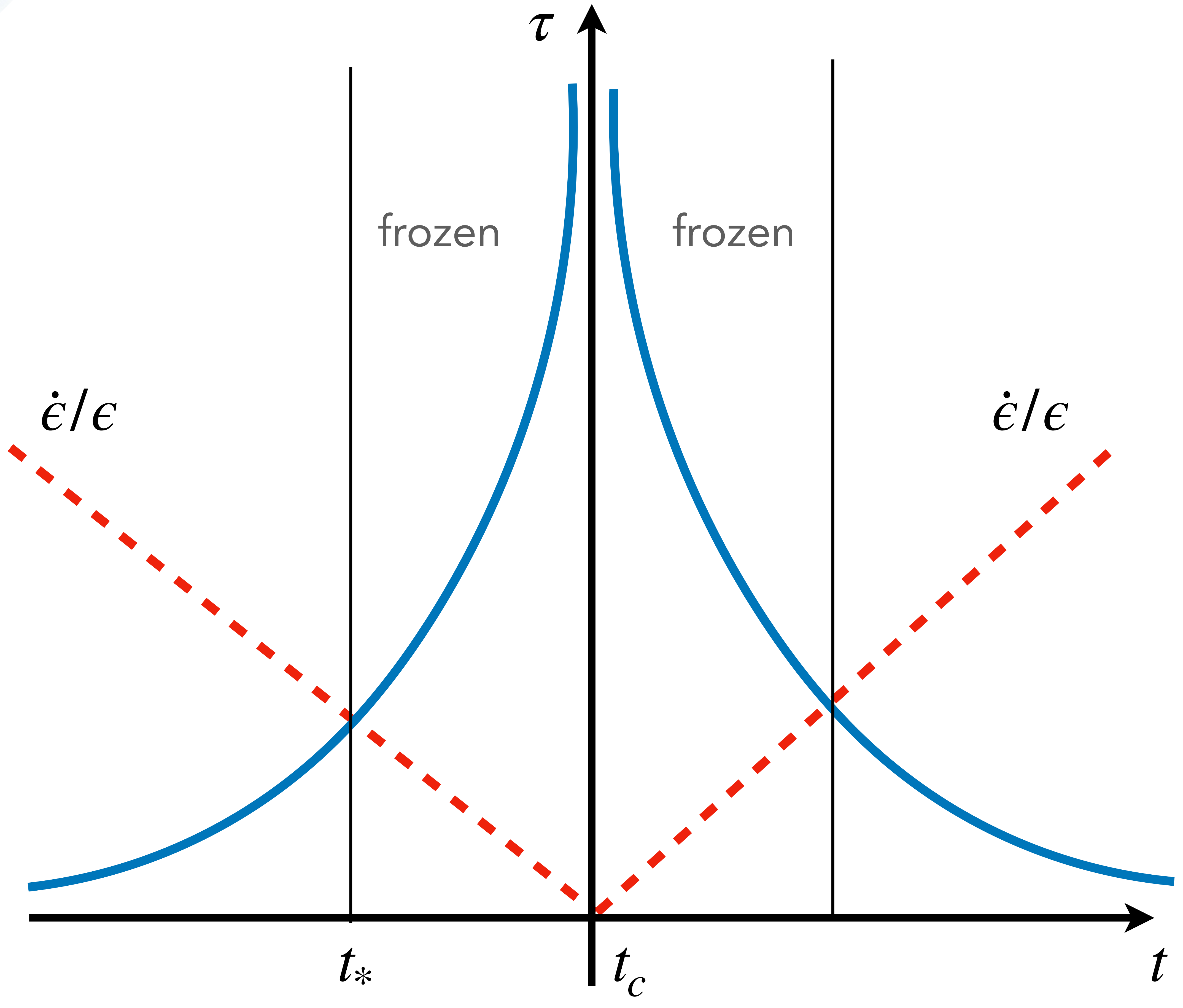

Reproduced from [Del Campo, Zurek, 2013]

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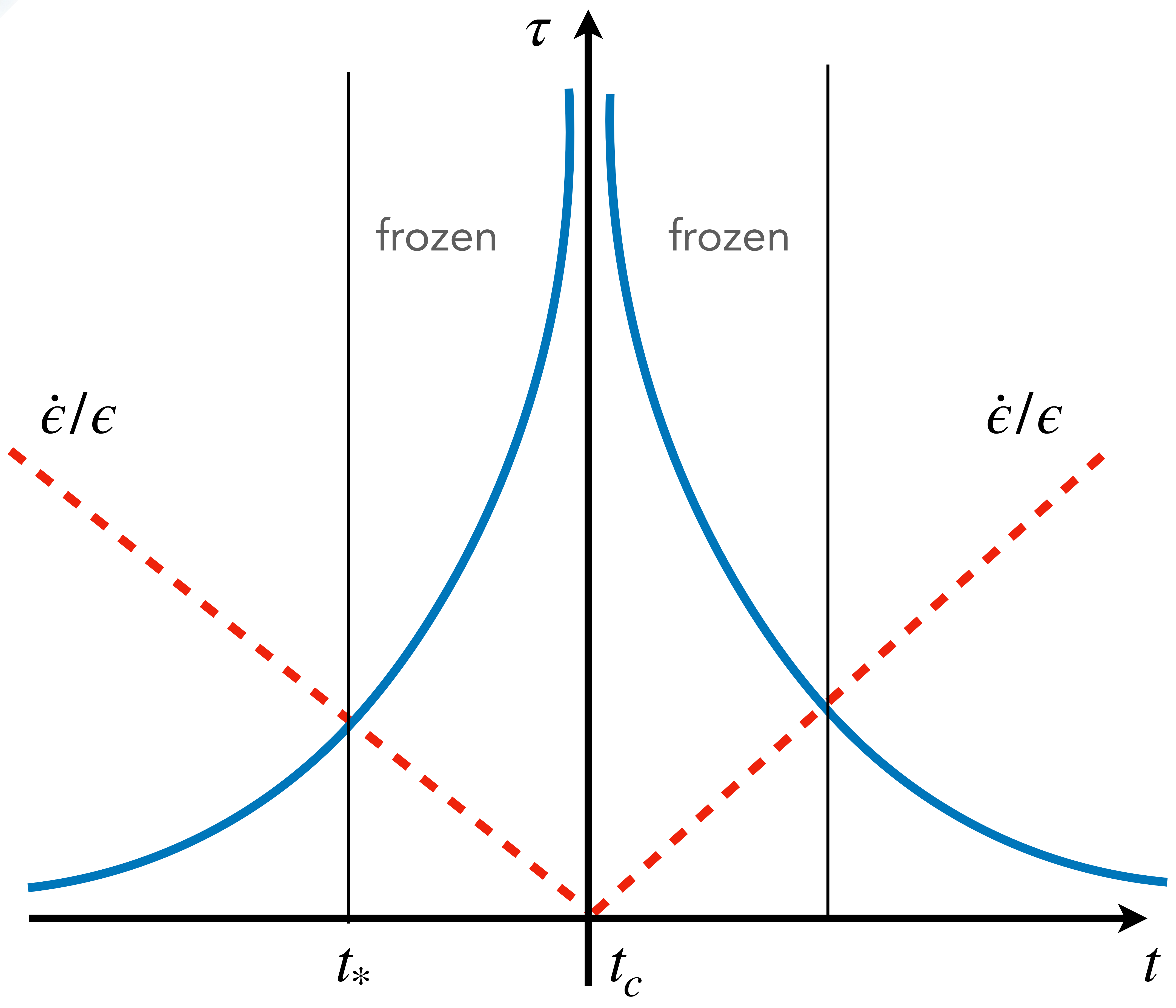

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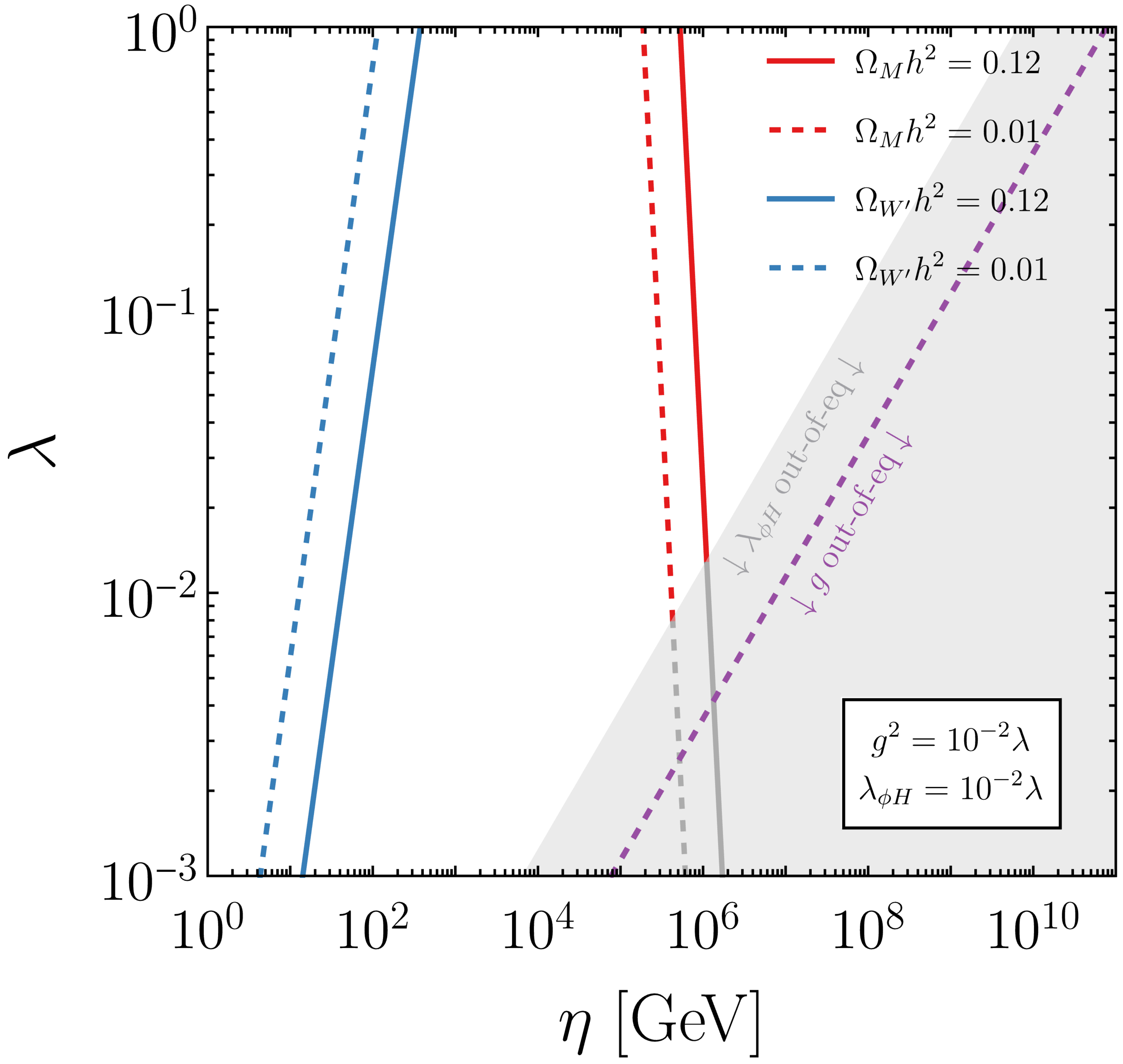
$$\boxed{\xi \approx \xi(t_*)}$$

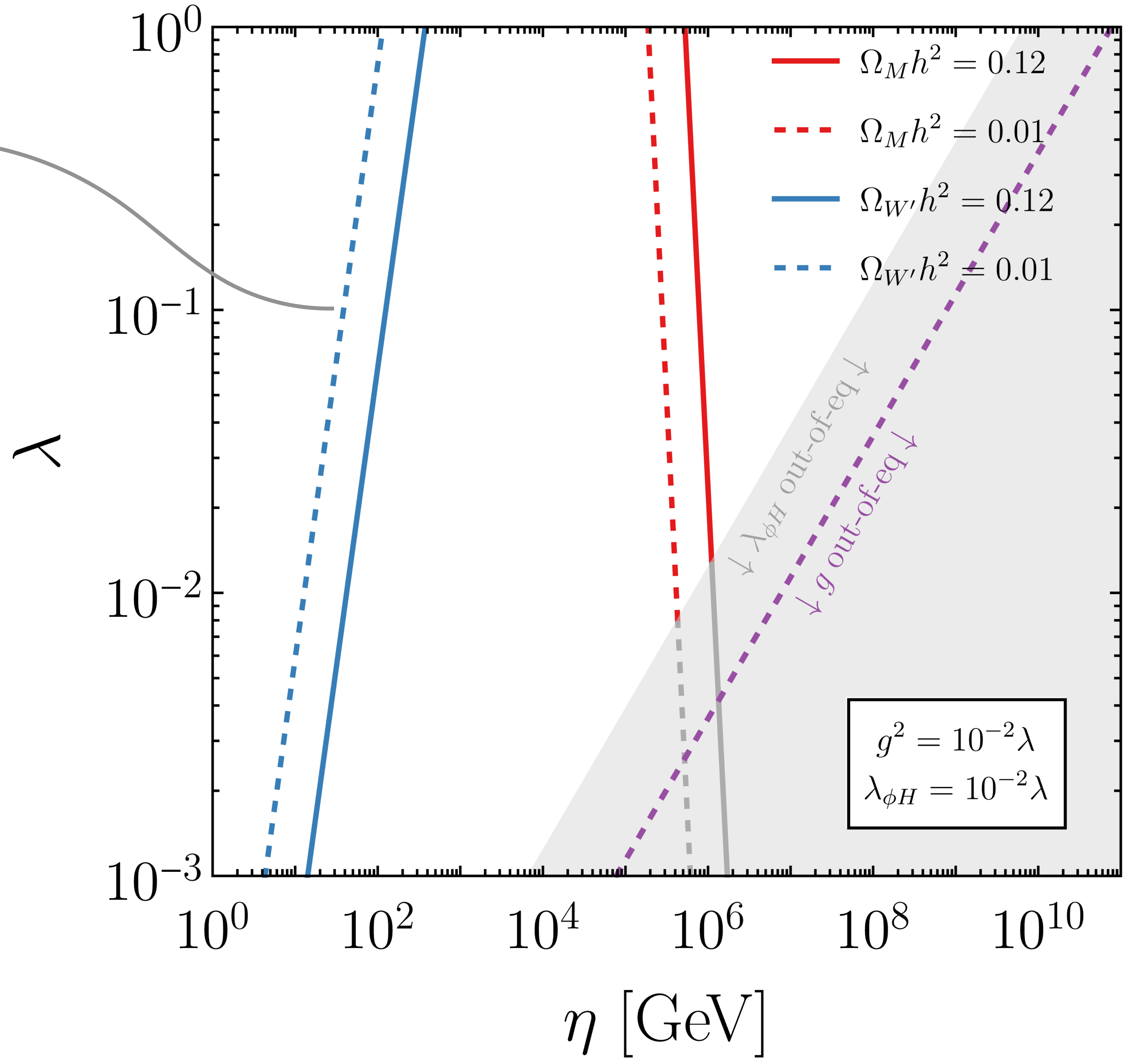
[Zurek, 1985]
[Murayama, Shu, 2009]

Monopole relic density



Monopole relic density

Freeze-out
$W'^+W'^- \leftrightarrow \gamma'\gamma'$



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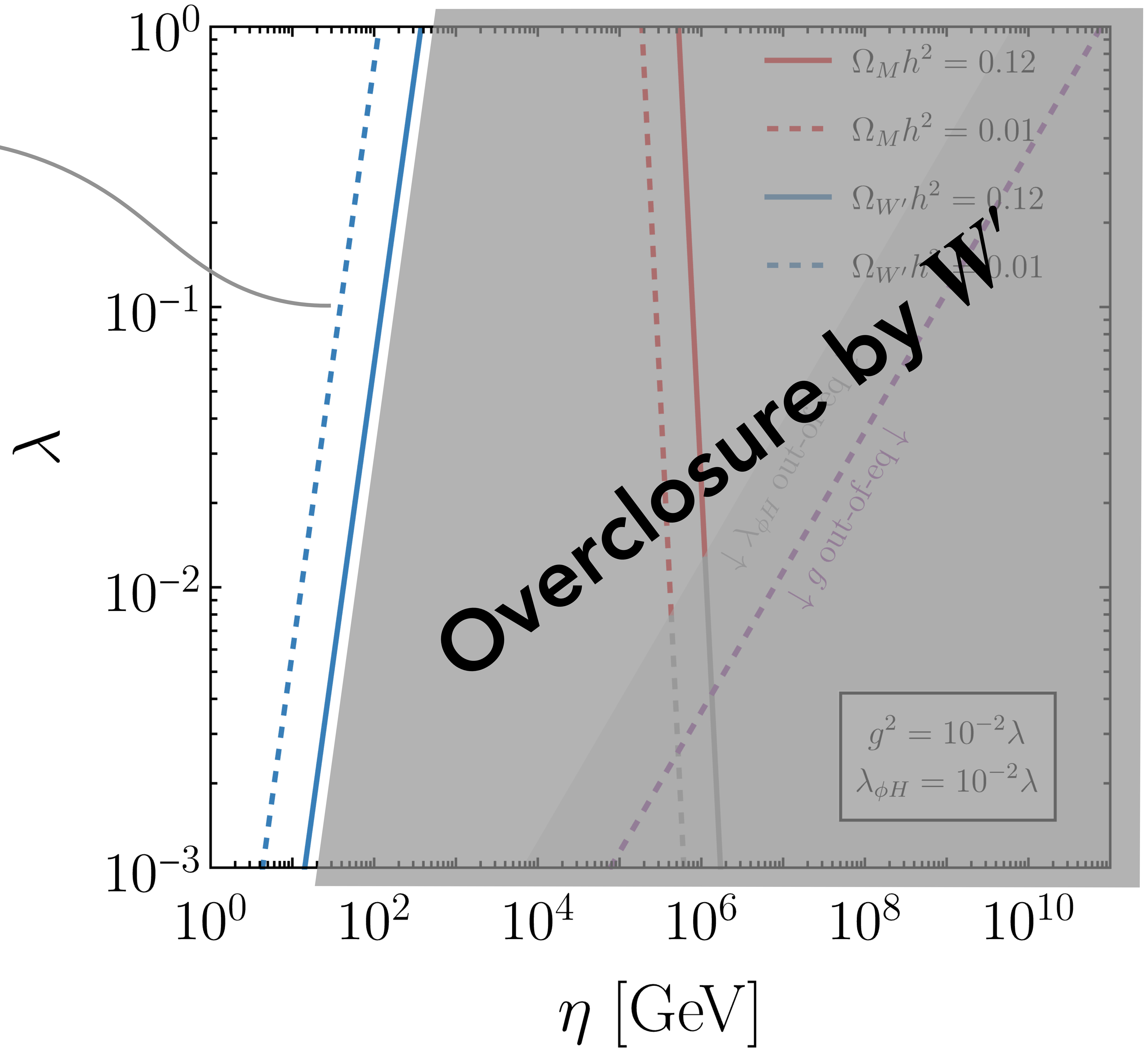

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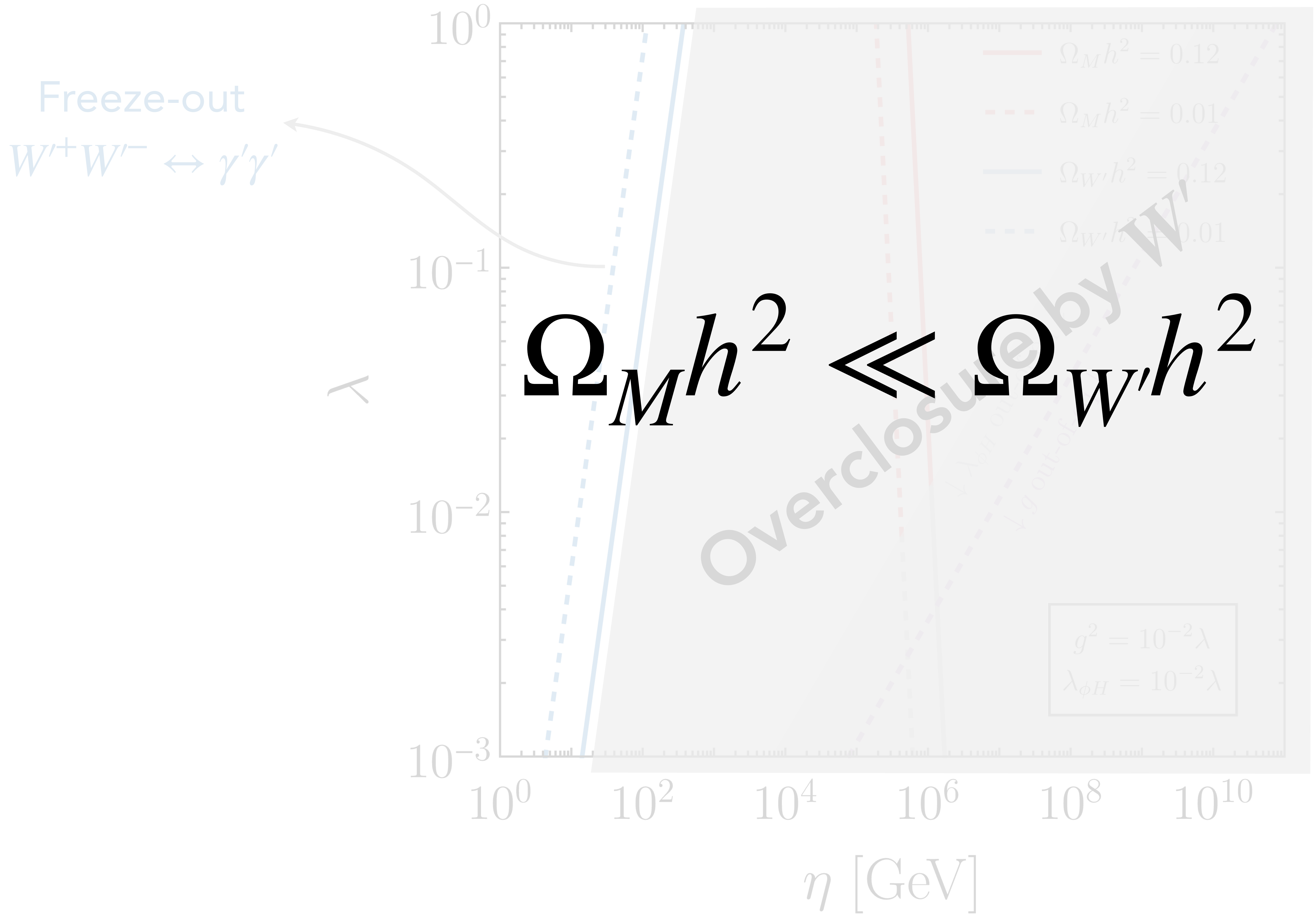
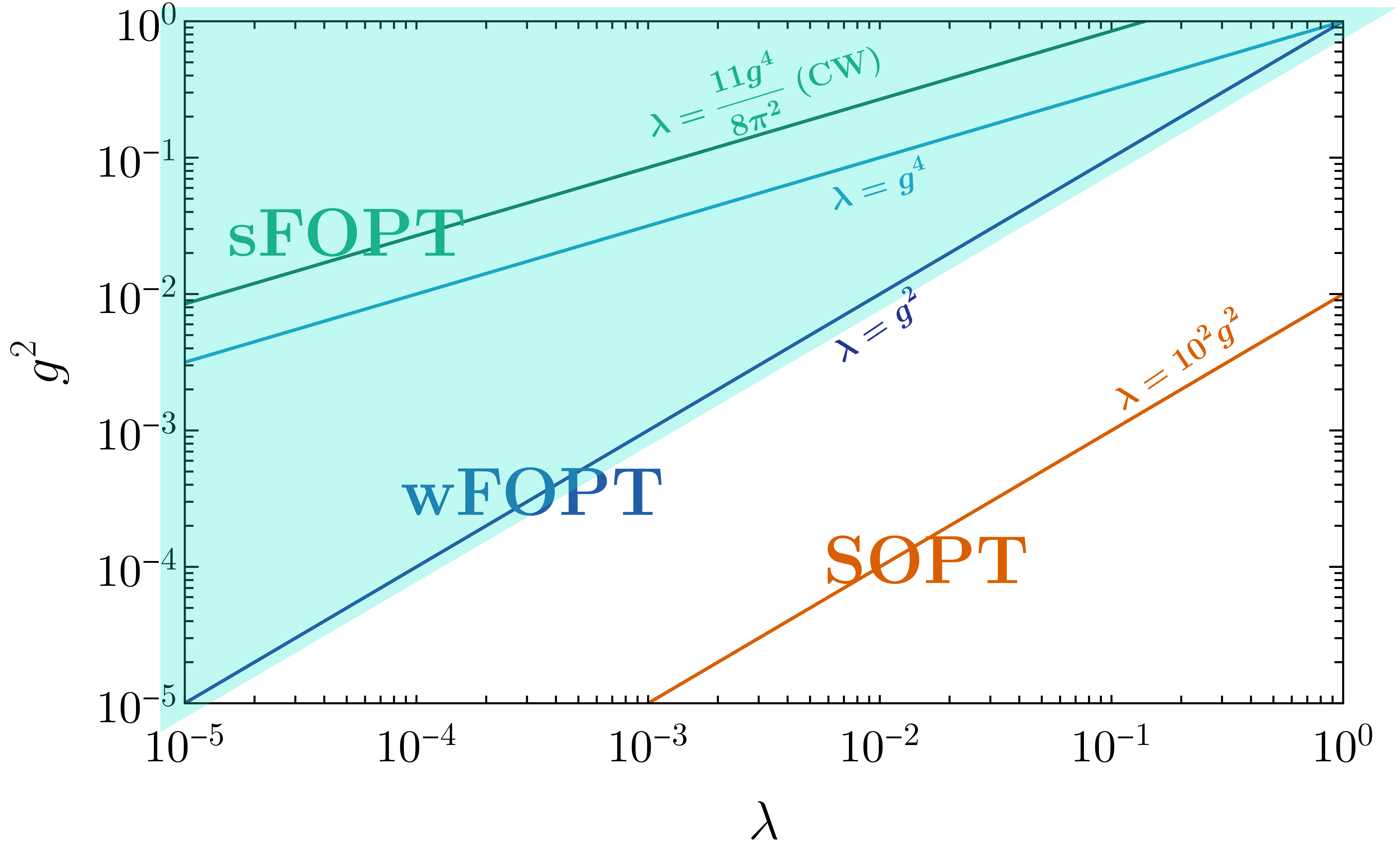


$$\Omega_M h^2 \ll \Omega_{W'} h^2$$

First-order phase transitions



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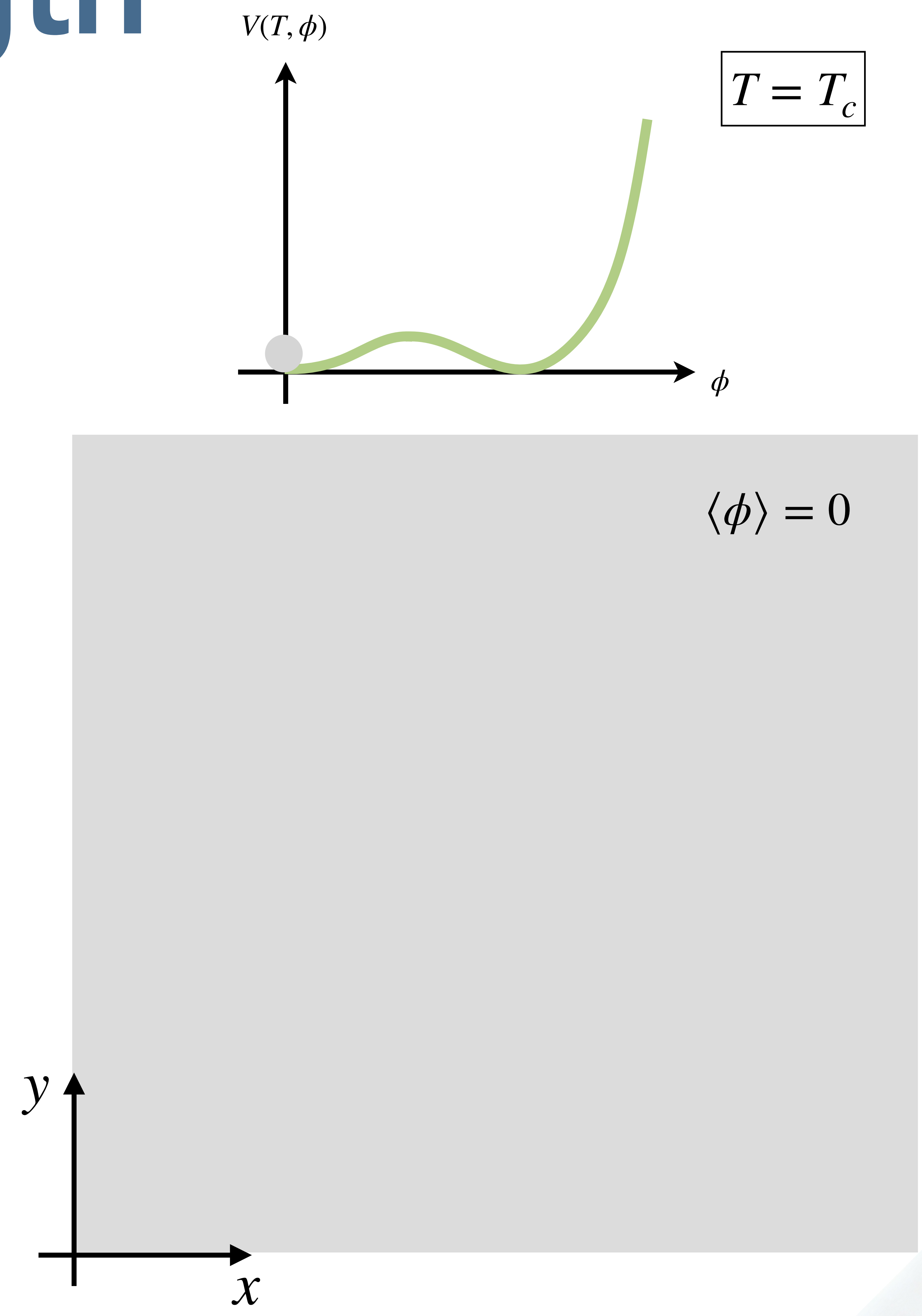
Correlation length



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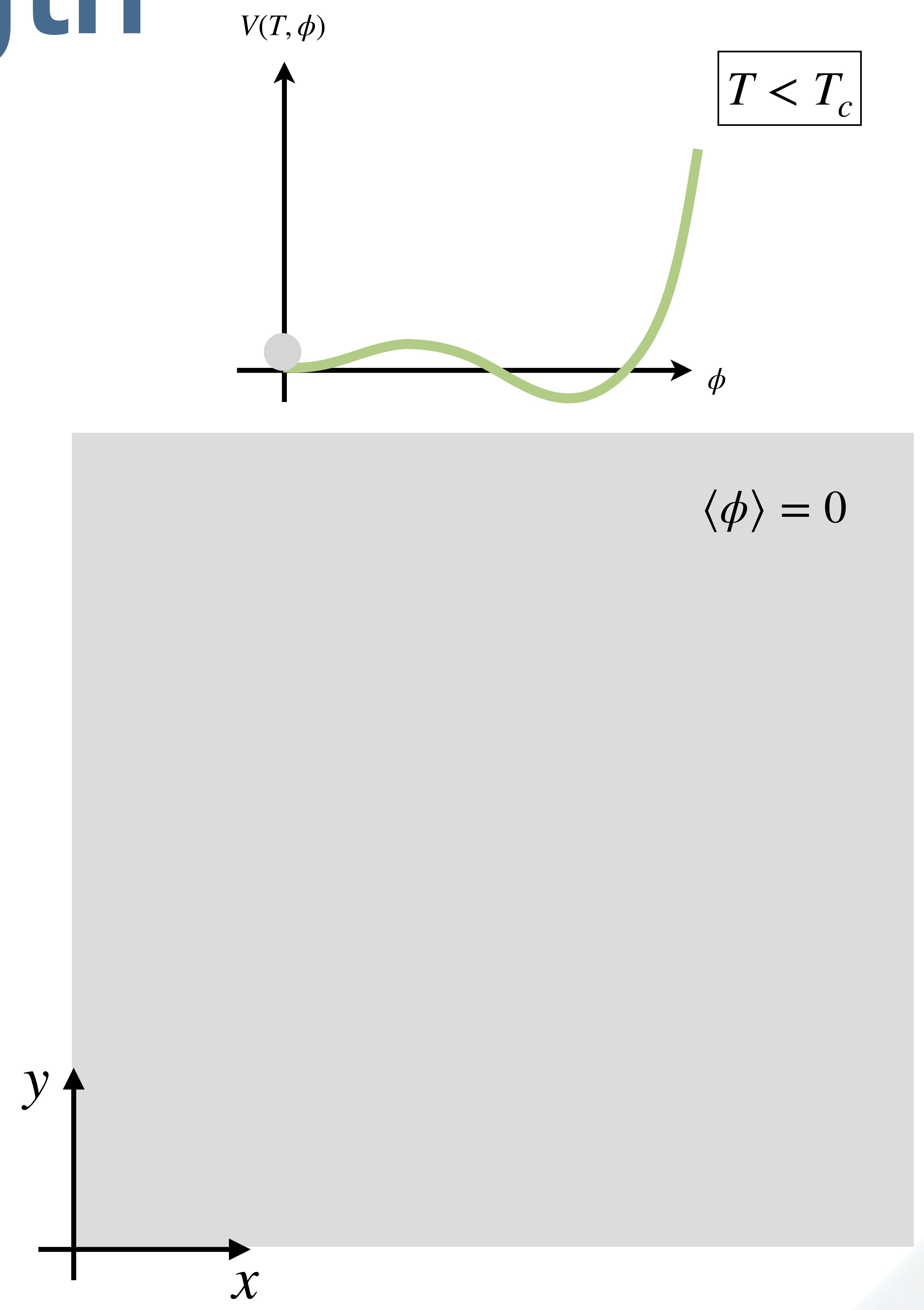
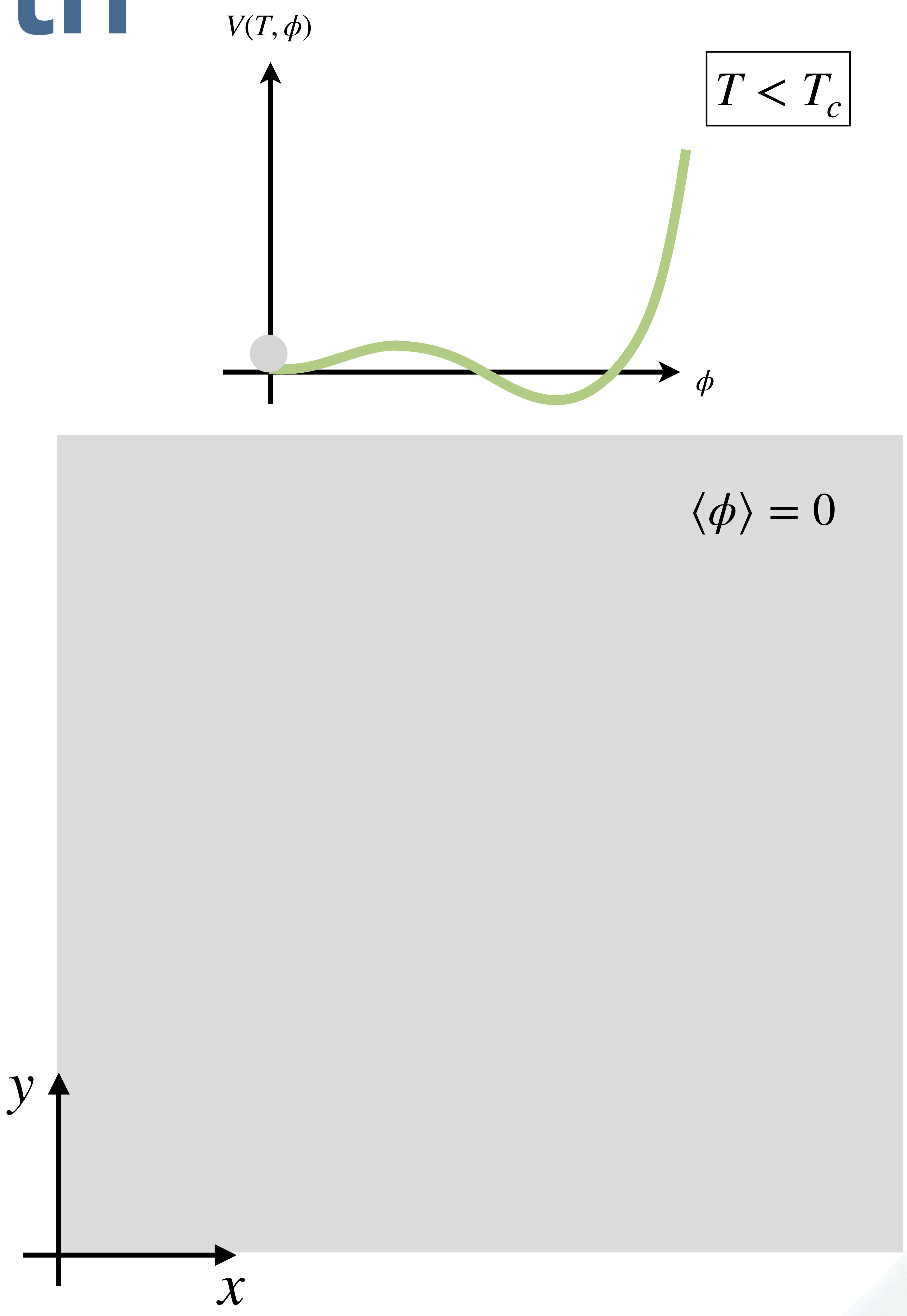


Correlation length

Tunneling rate: $\Gamma \approx T^4 e^{-S_3/T}$ [Linde, 1980]



Correlation length

Model dependent

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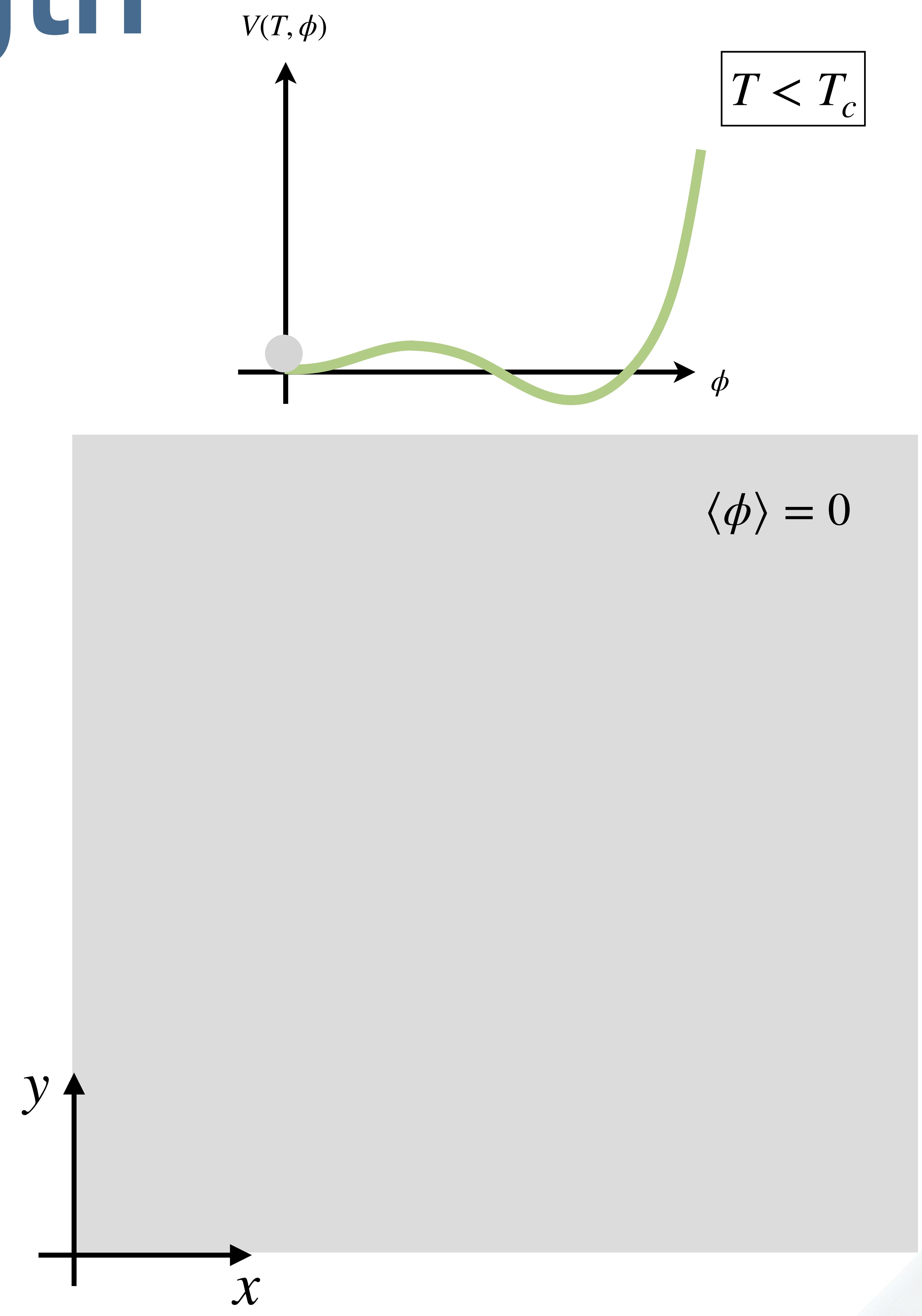

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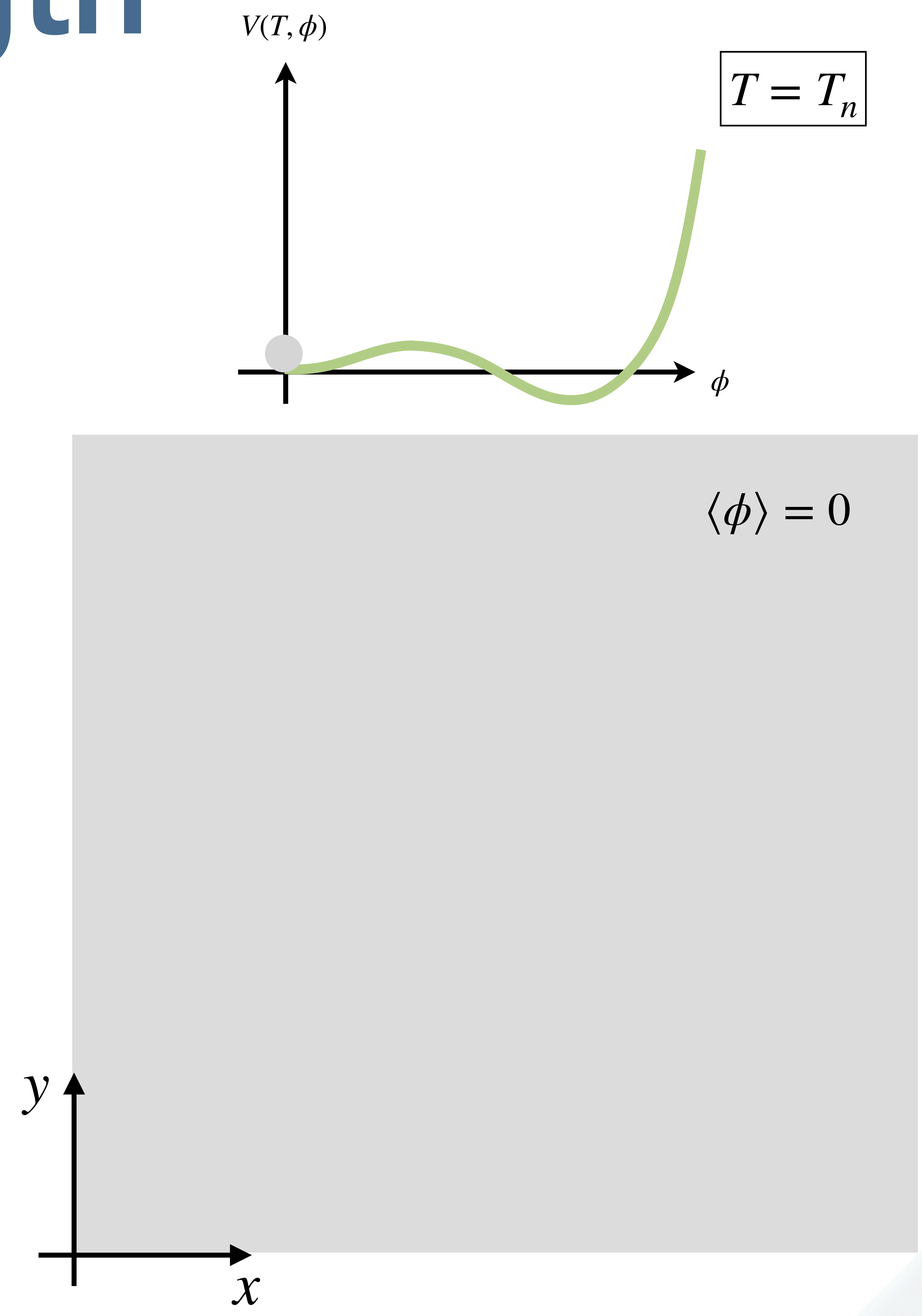

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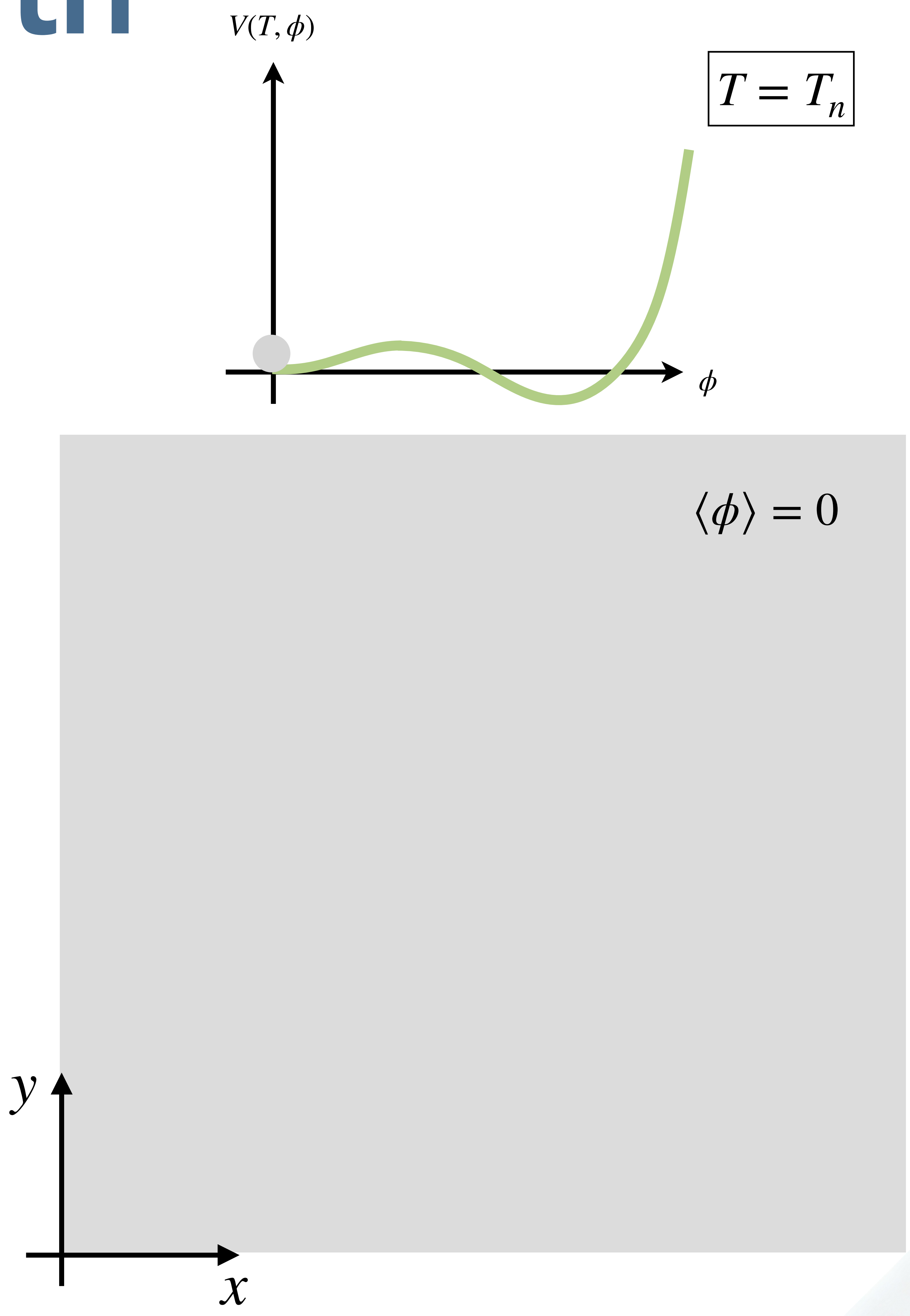
Model dependent

T_n : $\Gamma(T_n) \approx H(T_n)^4$



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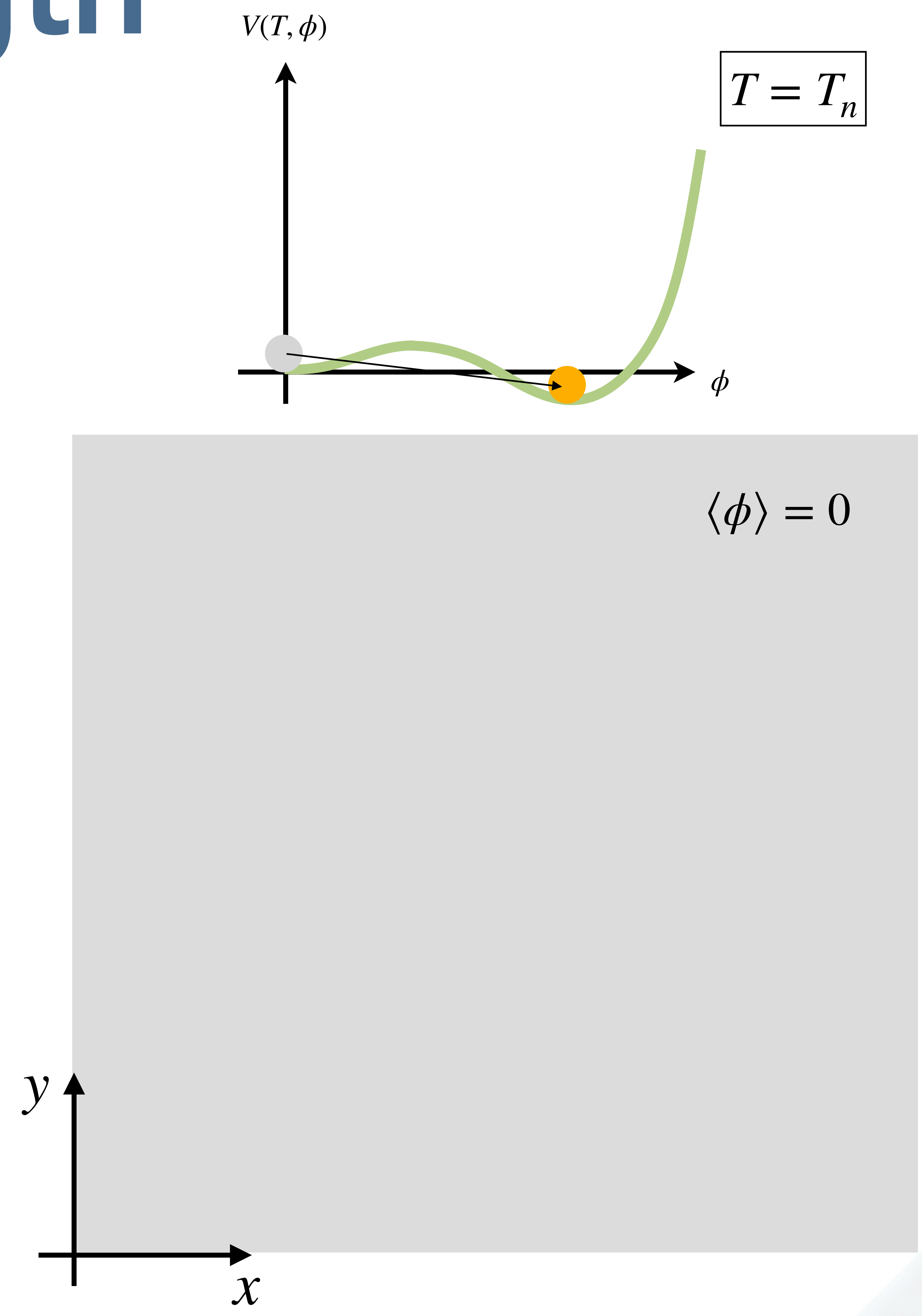

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Tunneling rate: $\Gamma \approx T^4 e^{-S_3/T}$ → Model dependent

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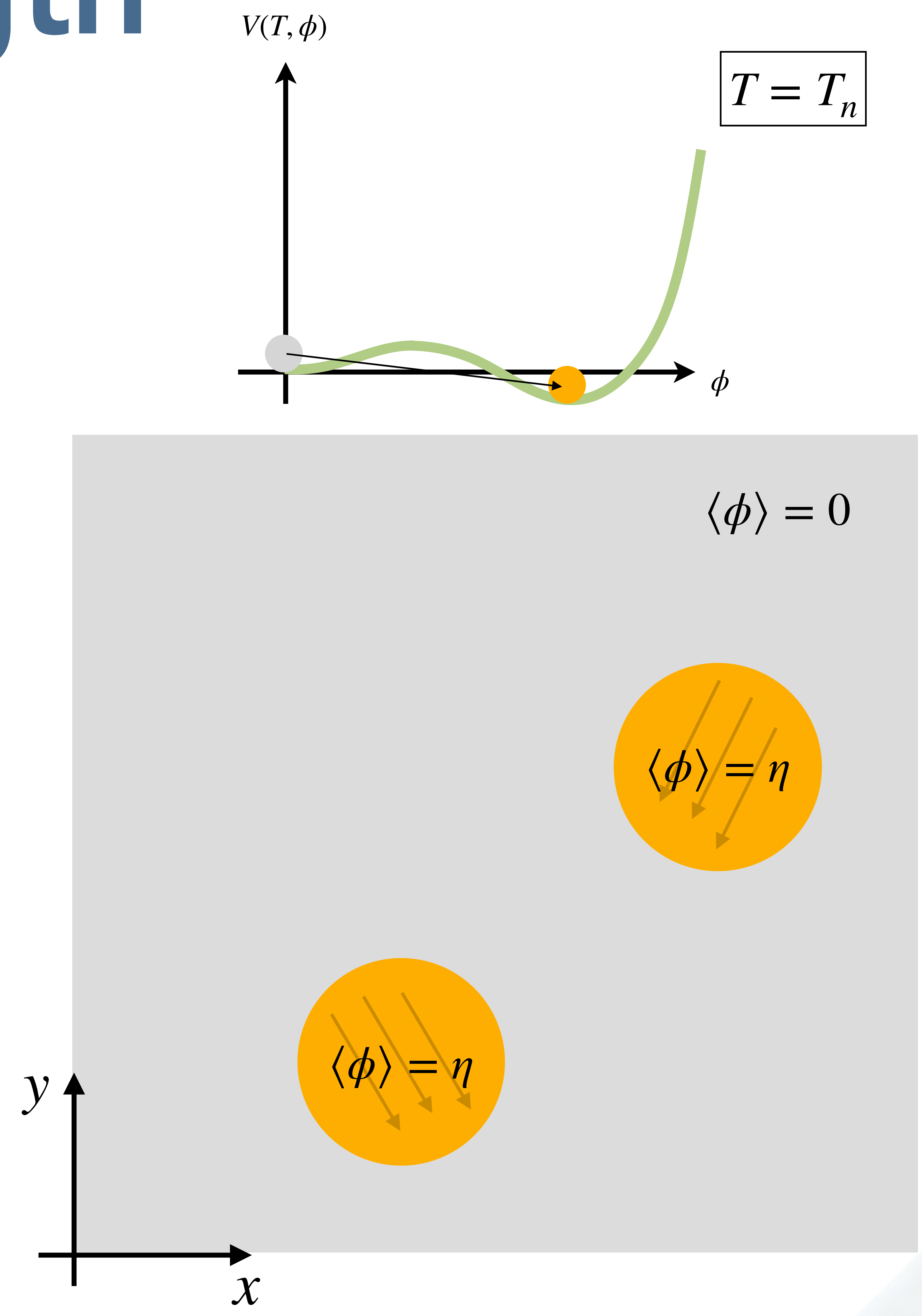

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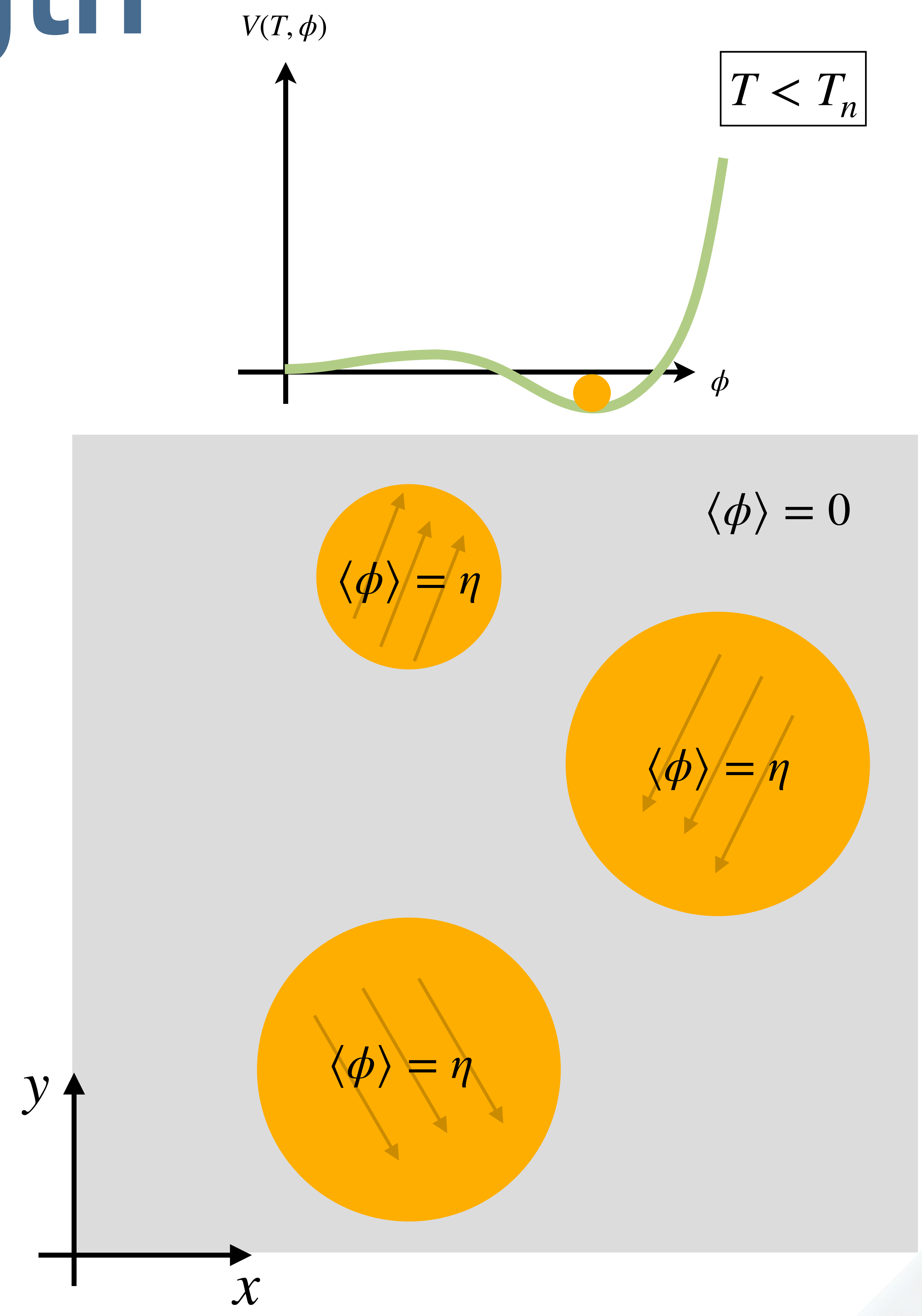
Tunneling rate: $\Gamma \approx T^4 e^{-S_3/T}$ [Linde, 1980]

S_3 → Model dependent

$$T_n : \quad \Gamma(T_n) \approx H(T_n)^4$$

$$\mathscr{P}_{\langle\phi\rangle=0}(T) \approx \exp - \left[\int_T^{T_c} dT' \frac{\Gamma(T')}{T'^4 H(T')} \left(\int_T^{T'} v_b \frac{dT''}{H(T'')} \right)^3 \right]$$ [Guth et al., 1980]



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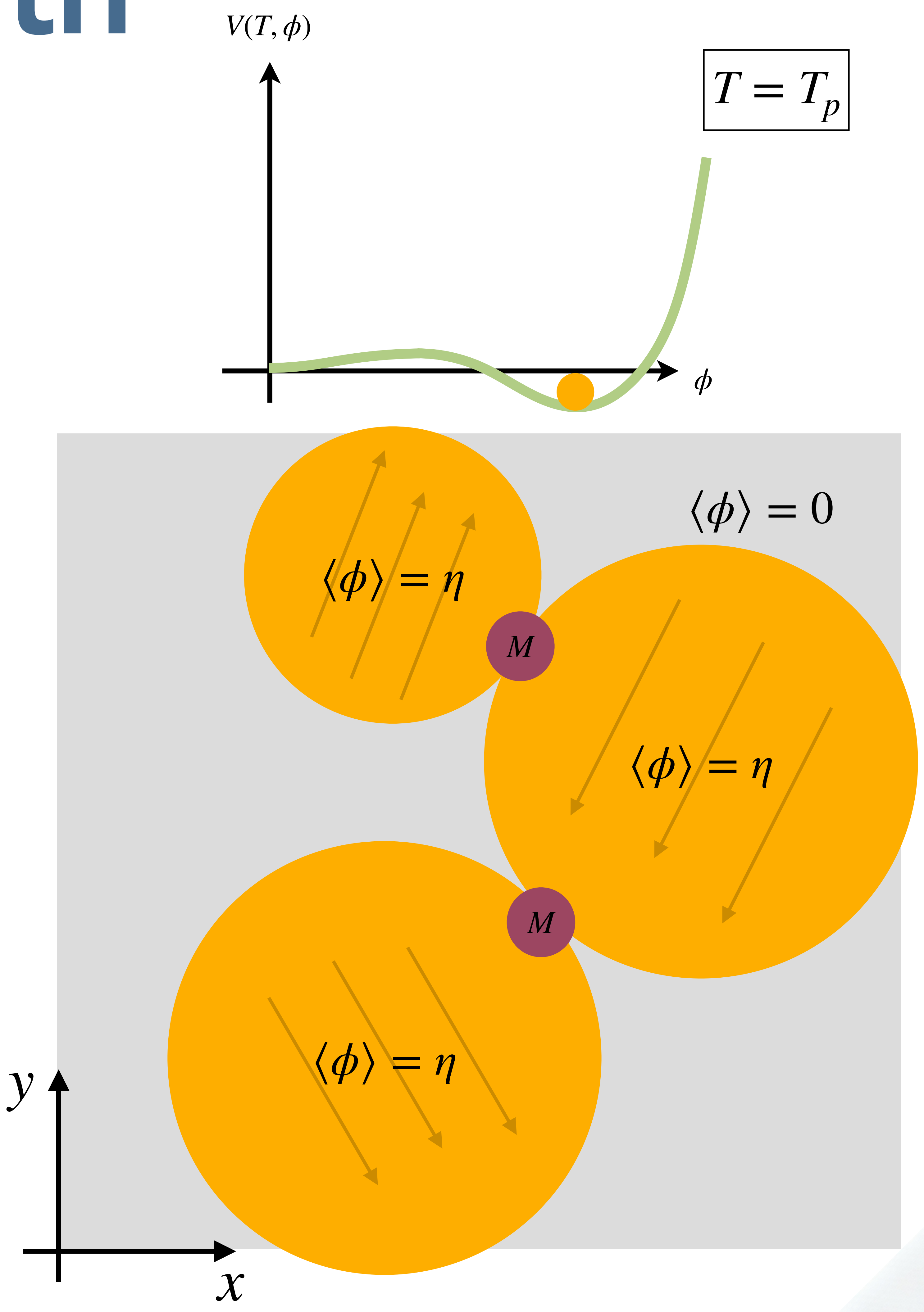
$$T_p : \quad \mathscr{P}_{\langle\phi\rangle=0}(T_p) = 0.71$$



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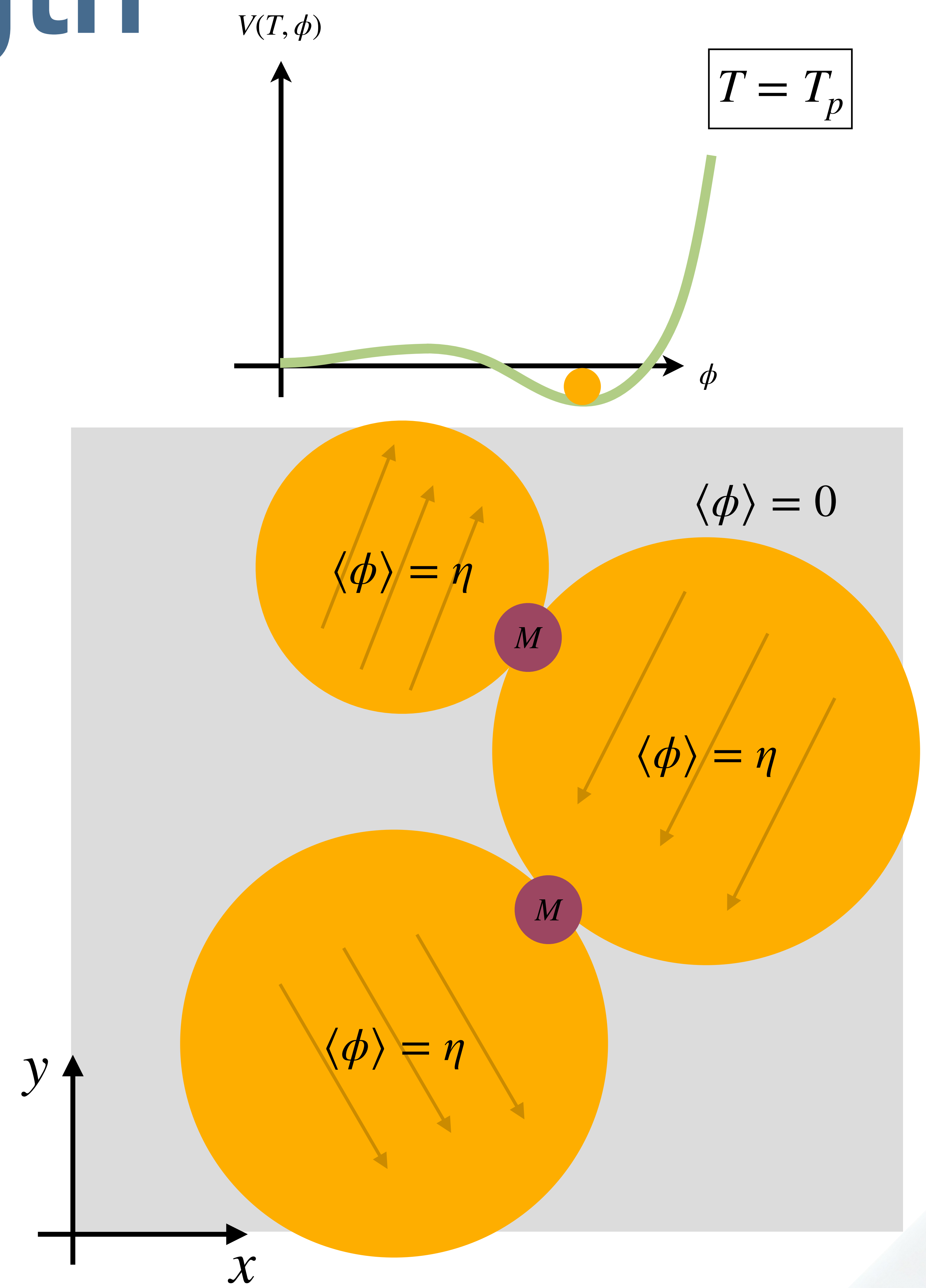
$$T_p: \quad \mathscr{P}_{\langle\phi\rangle=0}(T_p) = 0.71$$

$$\xi \approx R(T_p)$$



Weakly first-order phase transition

$$T_p \lesssim T_c$$

Weakly first-order phase transition

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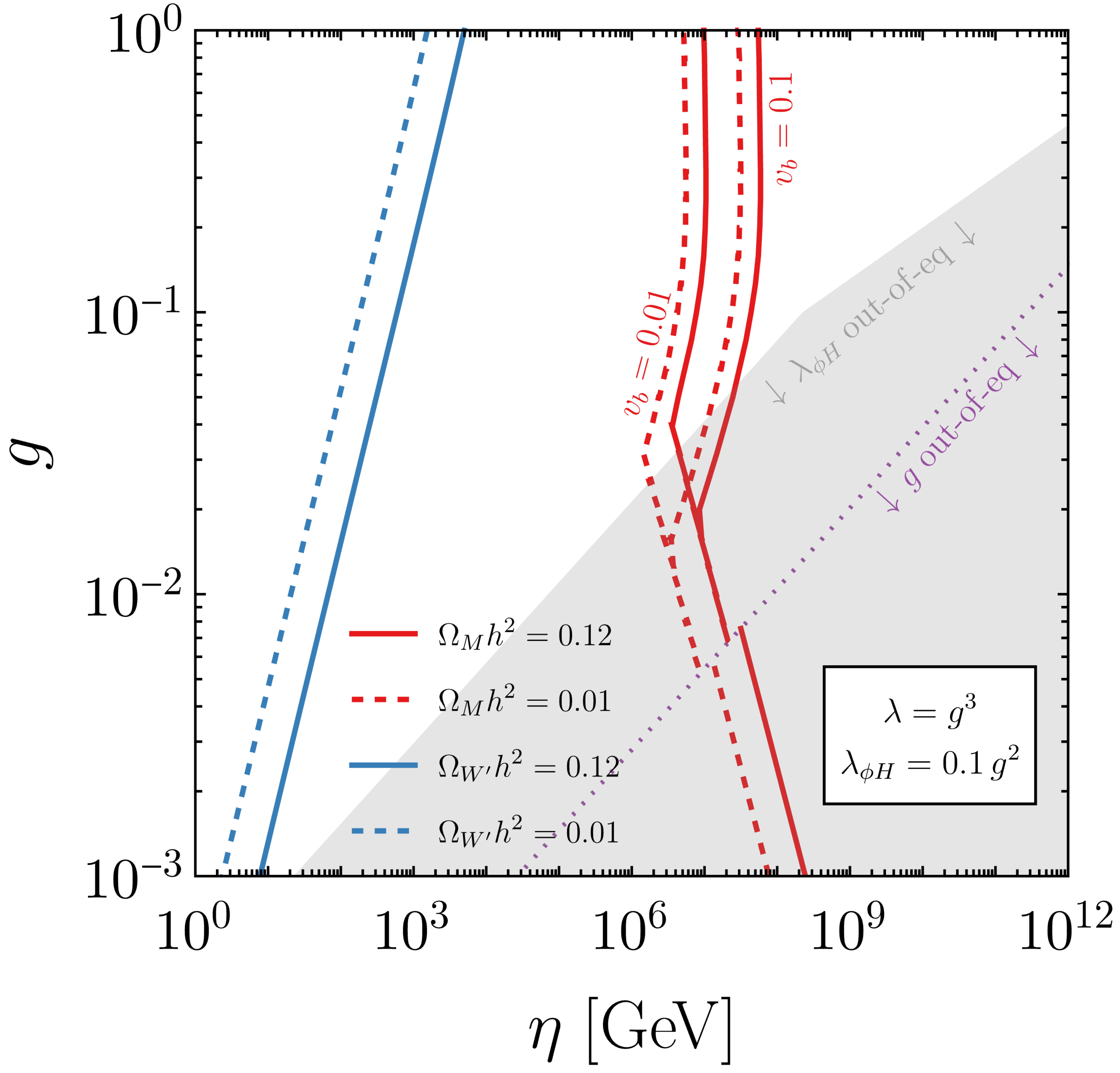
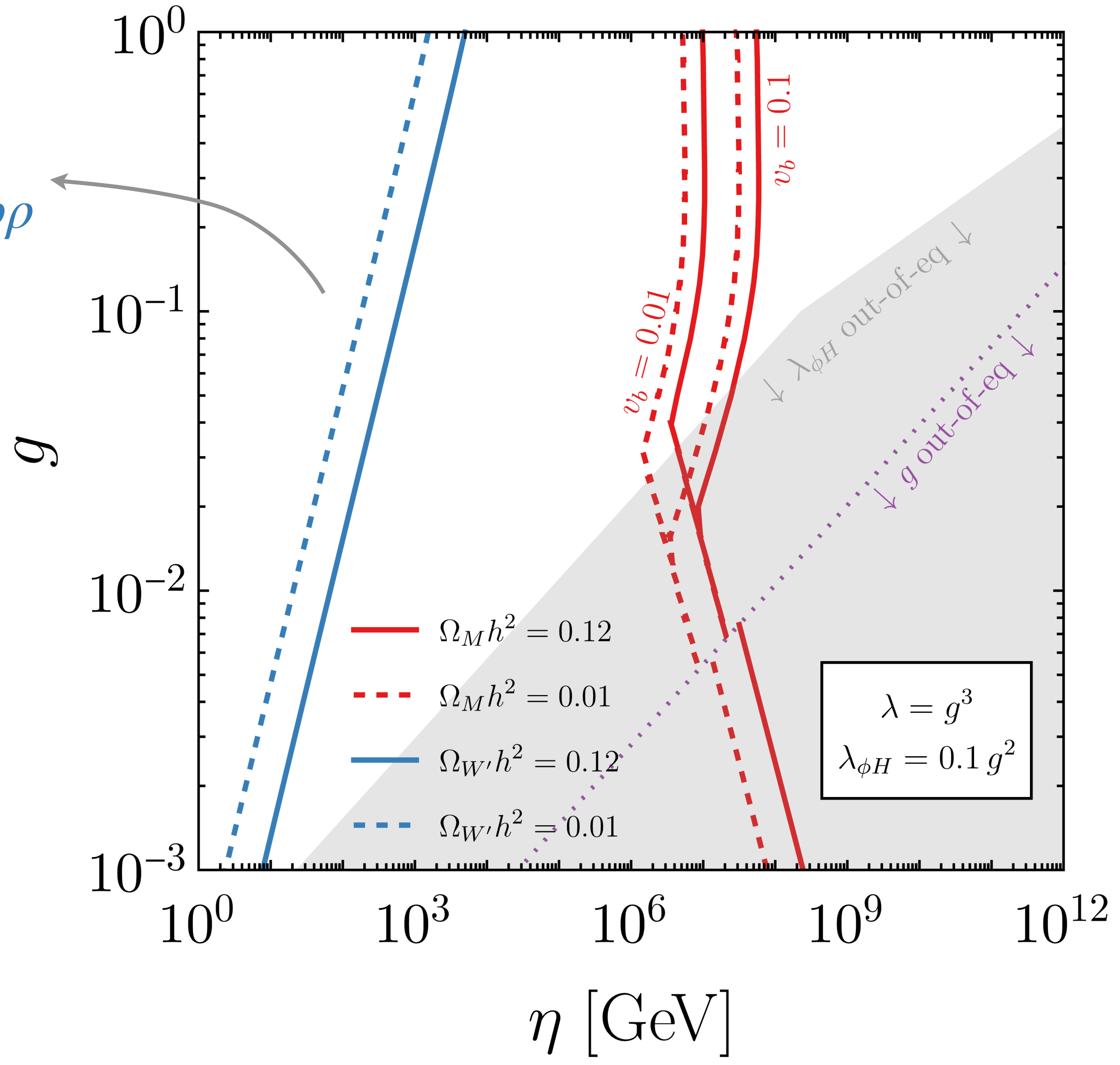


Weakly first-order phase transition

$T_p \lesssim T_c$

Freeze-out

$W'^+W'^- \leftrightarrow \gamma'\gamma', \rho\rho$



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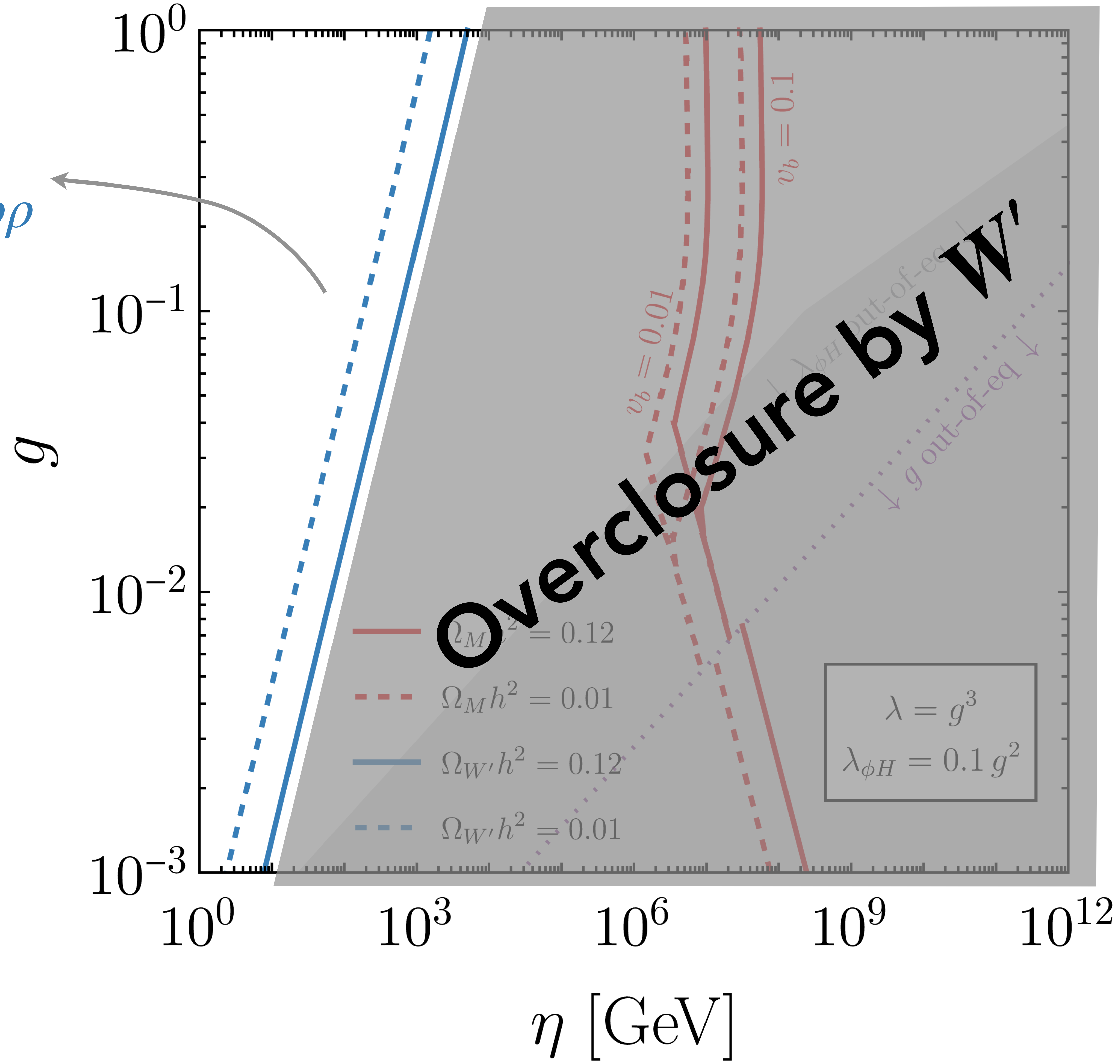

Weakly first-order phase transition

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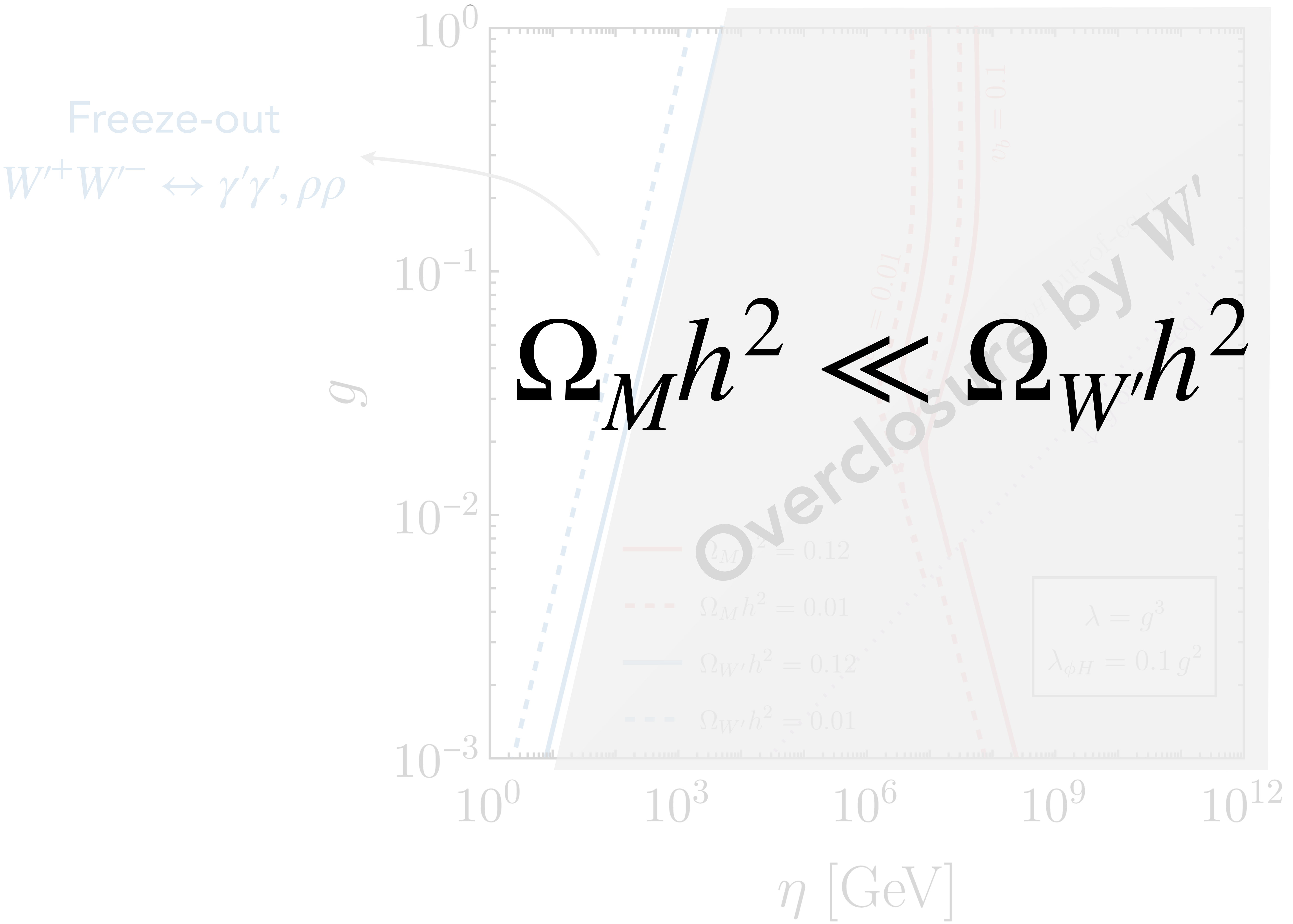


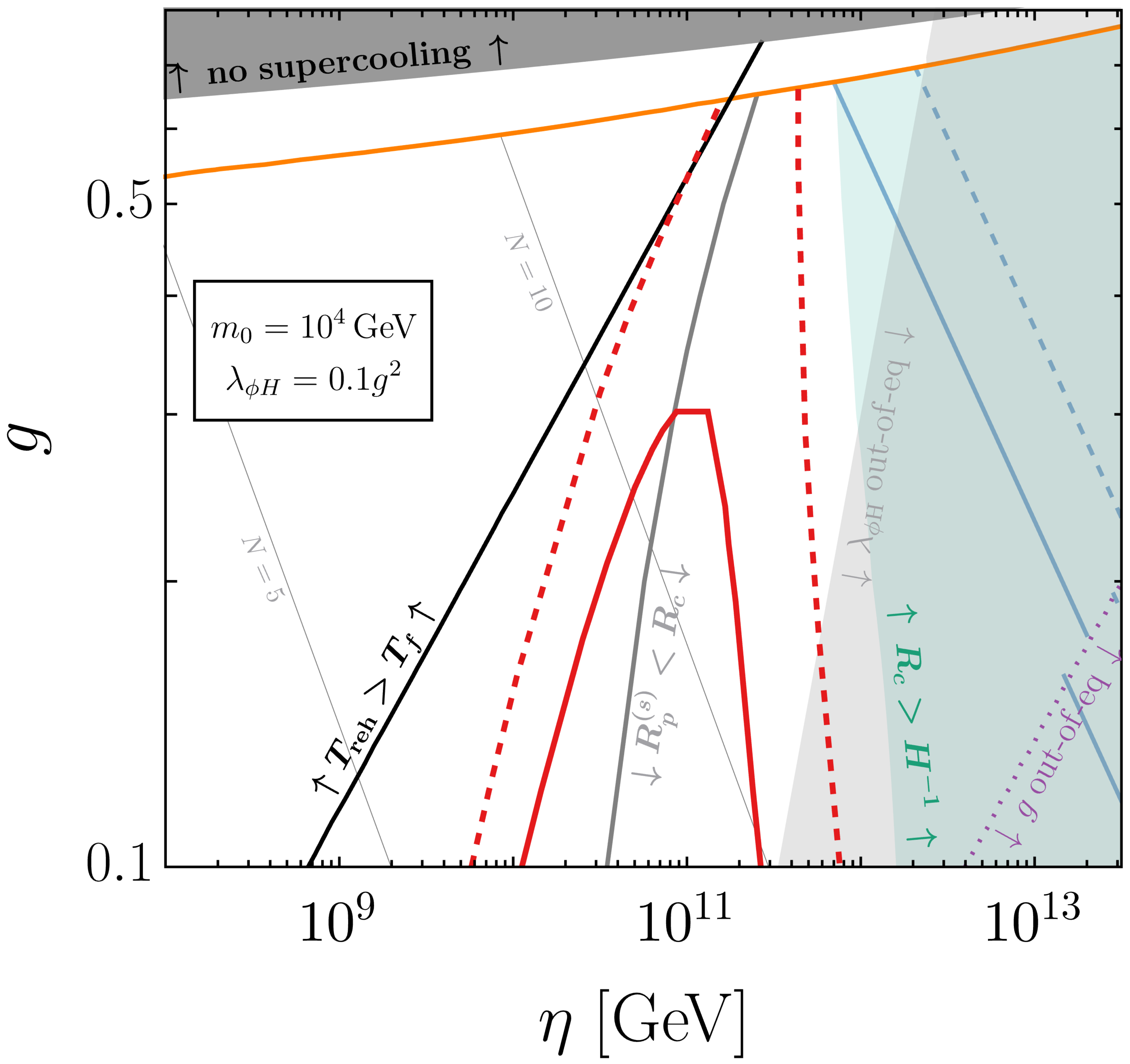
$$\Omega_M h^2 \ll \Omega_{W'} h^2$$

Strongly first-order phase transition

$$T_p \ll T_c$$

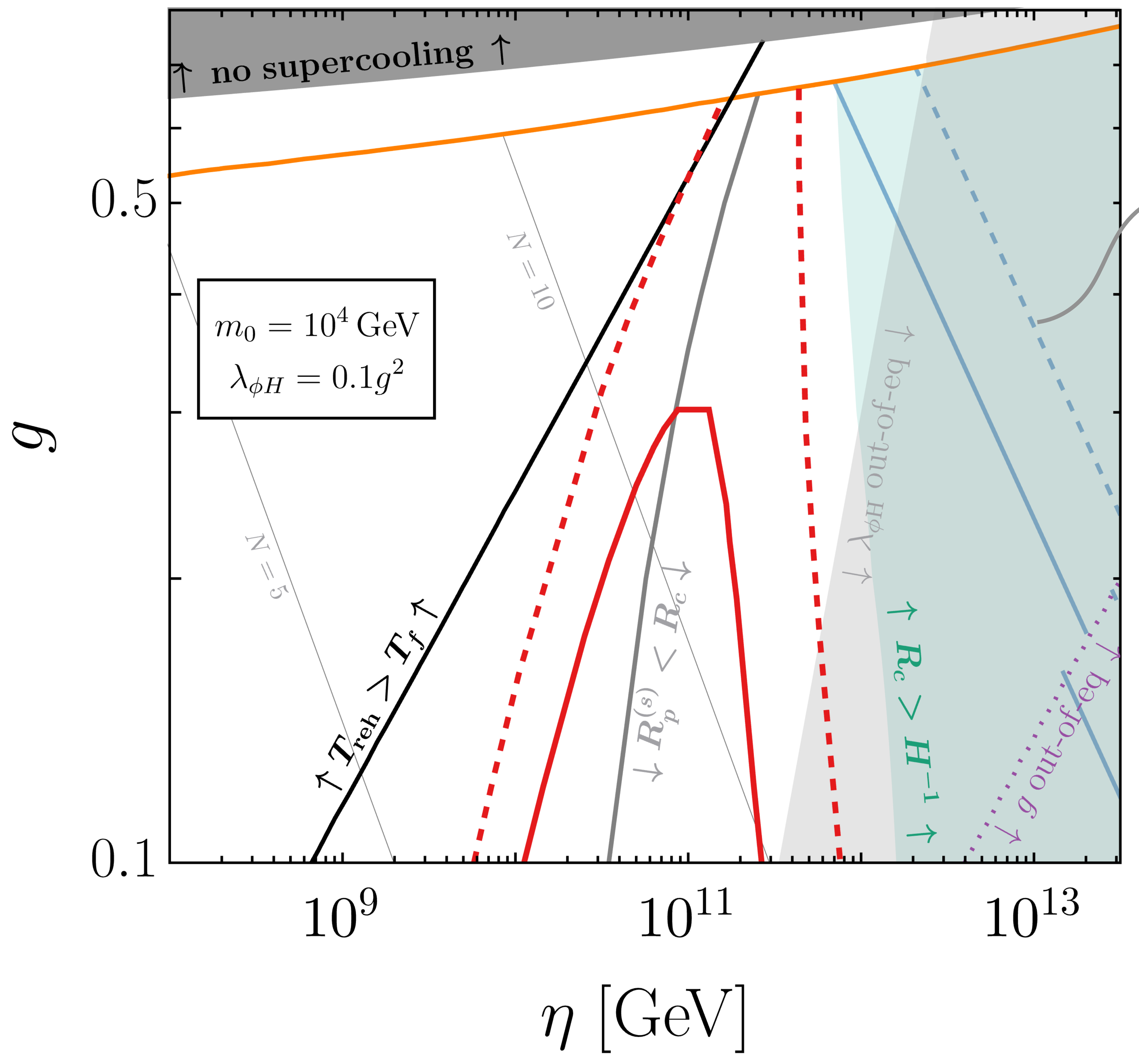
Strongly first-order phase transition

$T_p \ll T_c$



Strongly first-order phase transition

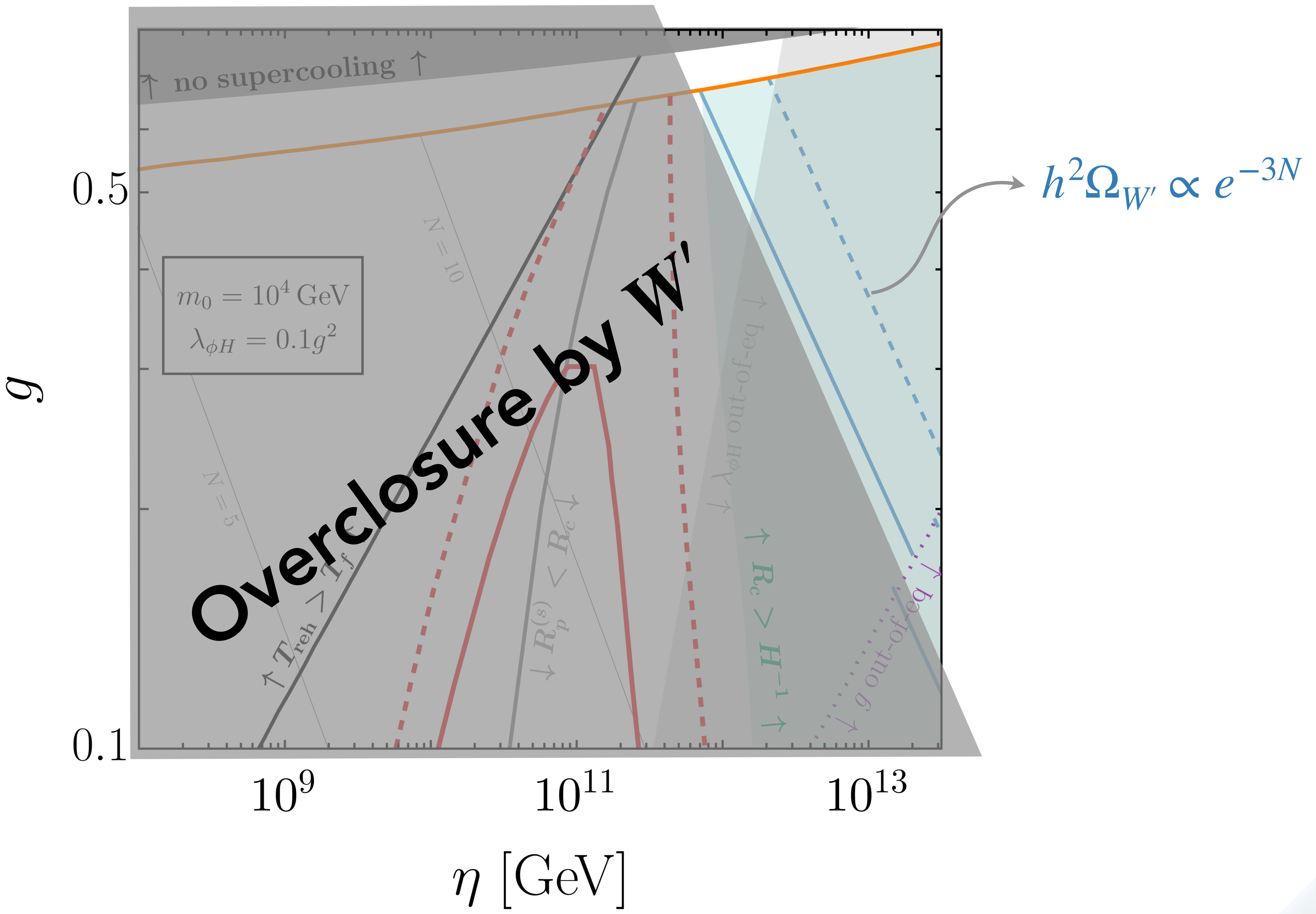
$$T_p \ll T_c$$



$h^2\Omega_{W'} \propto e^{-3N}$

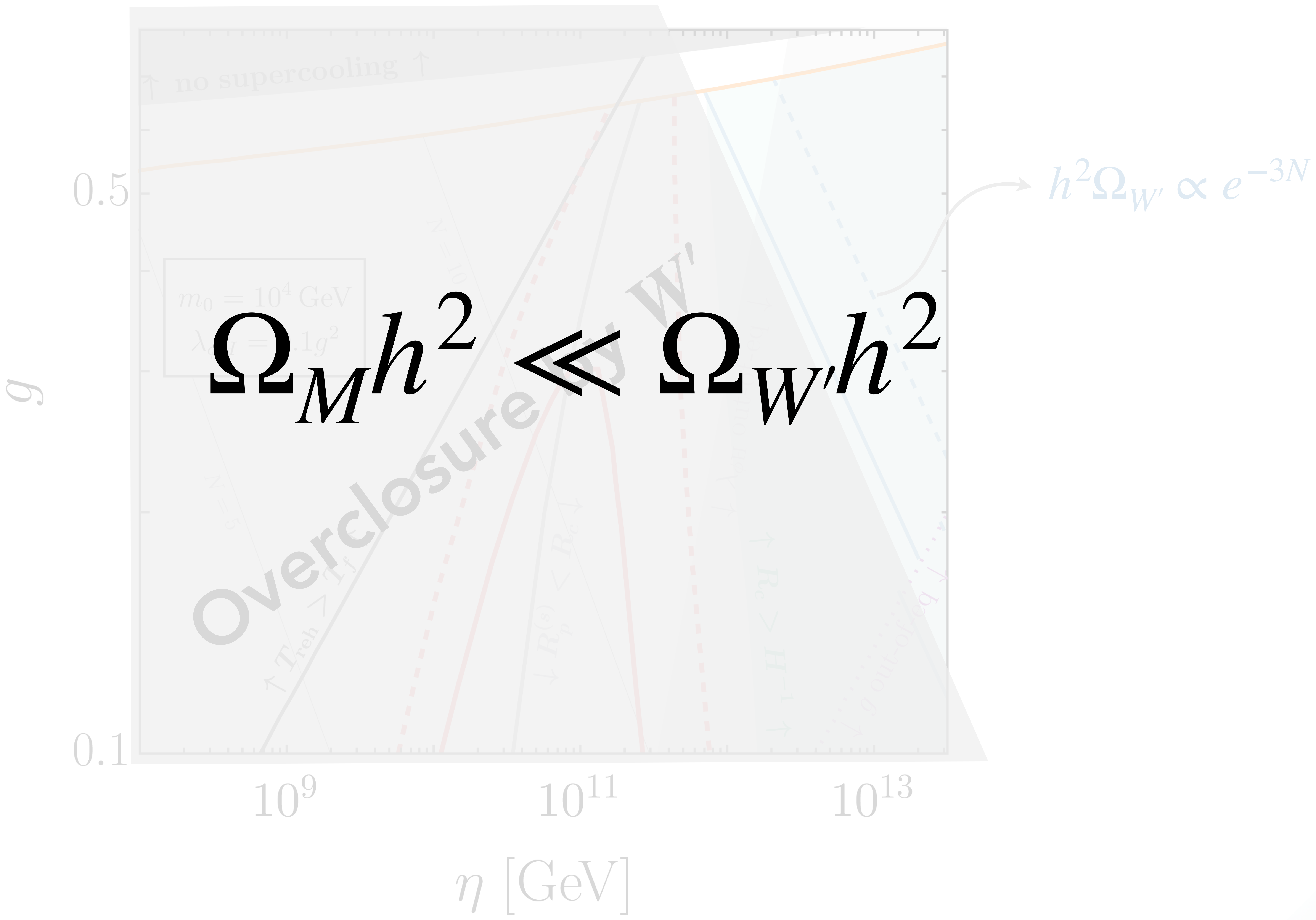
Strongly first-order phase transition

$T_p \ll T_c$



Strongly first-order phase transition

$$T_p \ll T_c$$



$$\Omega_M h^2 \ll \Omega_{W'} h^2$$

Summary and future prospects

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No darkness for the poor, old monopole

(at least in the minimal model)

Summary and future prospects

No darkness for the poor, old monopole

(at least in the minimal model)

Effects of gravity on the tunnelling process?

Interplay w/ the electroweak phase transition?

Alternative scalar reps?

Charged fermions?

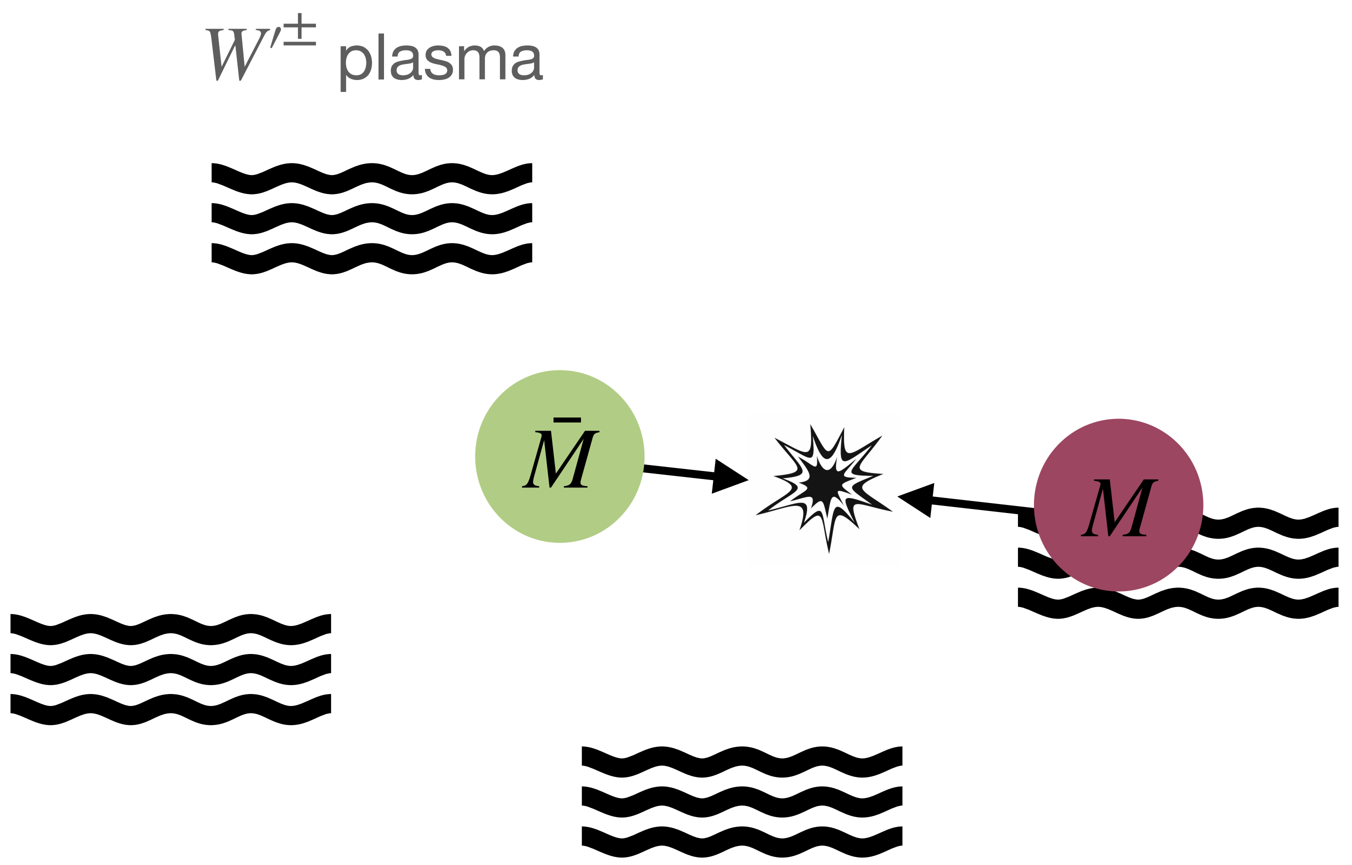
[F. Brümmer, GF, T. Fischer and M. Frigerio, *to appear*]

Thank you for your attention!

giacomo.ferrante@ulb.be

Backup Slides

Monopole annihilation



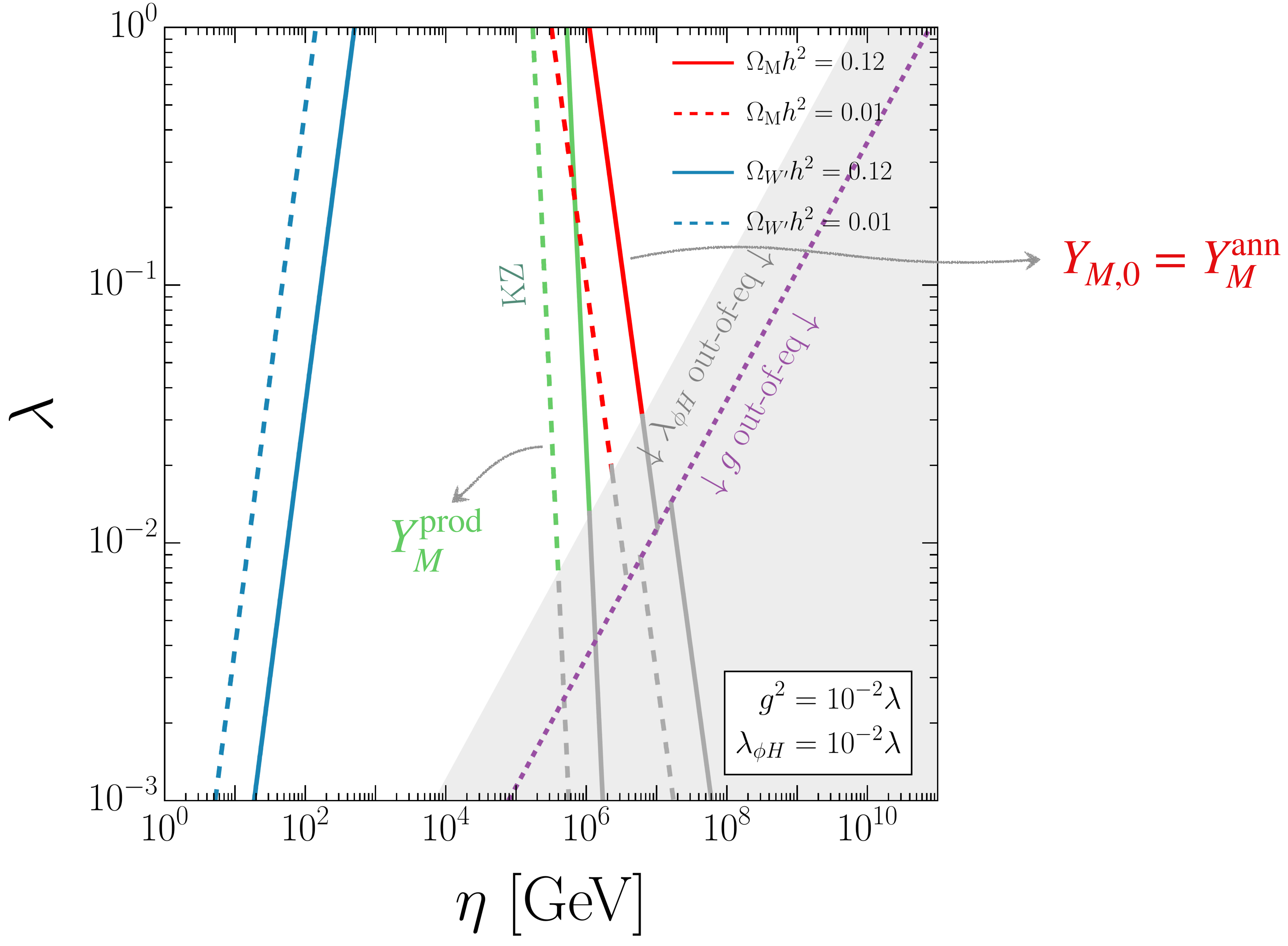
$$Y_M^{\text{prod}} \sim \frac{\xi^{-3}}{s(T_{\text{PT}})}$$

$$Y_M^{\text{ann}} \sim q_M^{-2} \frac{\bar{T}}{M_{\text{Pl}}} \sim g^2 \frac{m_{W'}}{M_{\text{Pl}}}$$

$$Y_{M,0} = \min \left[Y_M^{\text{ann}},\, Y_M^{\text{prod}} \right]$$

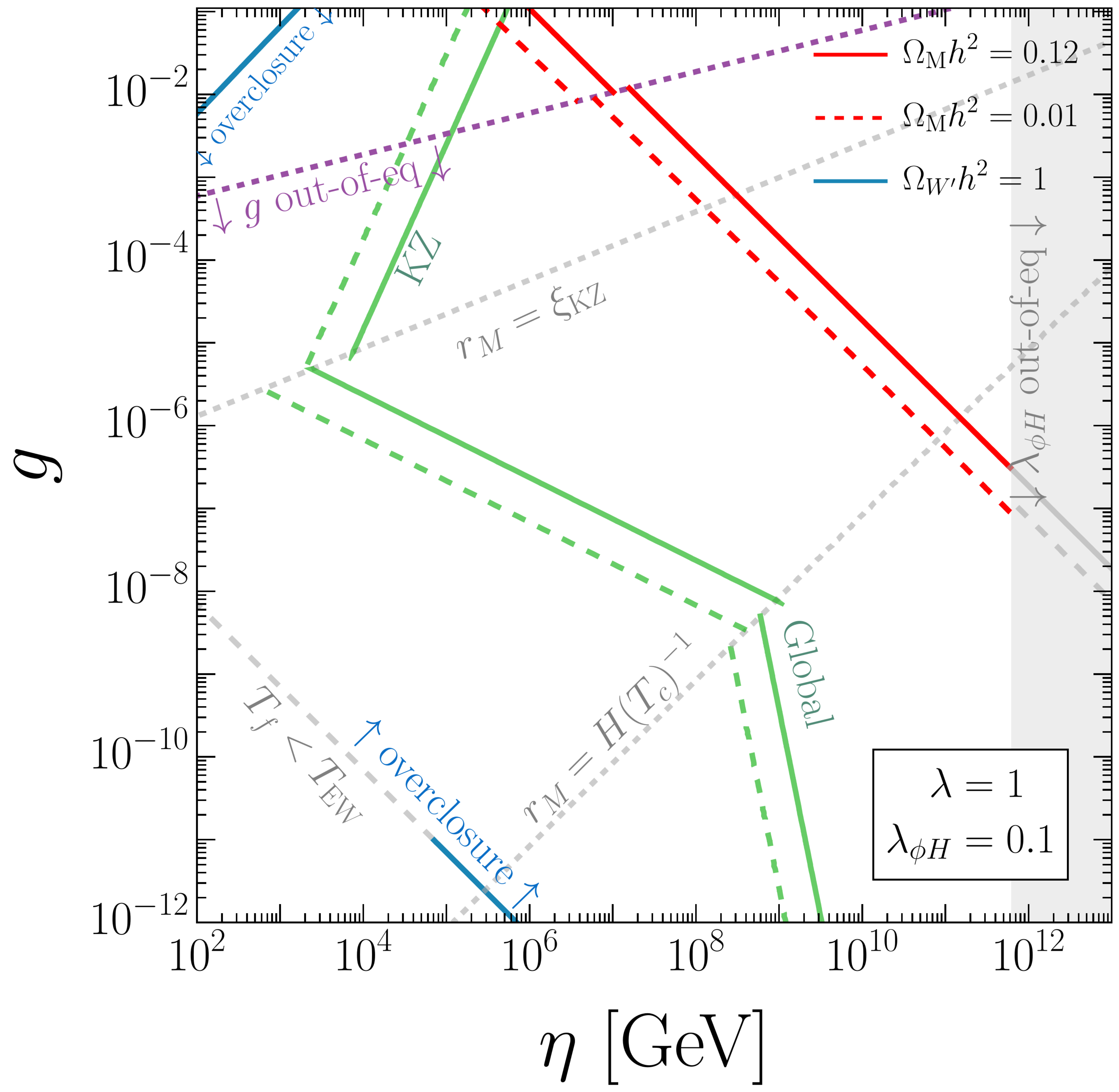
Second order phase transition

Monopole abundance



Second order phase transition

Global monopoles



$$\xi_{KZ} < r_M < H(T_c)^{-1} : \qquad Y_M \approx \frac{1}{4}\gamma_*^{-1}(r_M T_c)^{-3} .$$

$$r_M > H(T_c)^{-1} : \qquad Y_M \approx \frac{\zeta}{4}(r_M M_{Pl})^{-3/2}\gamma_*^{-1/4} .$$

[Barriola, Vilenkin, 1989]

First order phase transition

$$T_{\rm eq}: \quad \rho_r = \rho_v \quad \Longrightarrow \quad T_{\rm eq} = \left(\frac{30}{g_*\pi^2}\Delta V\right)^{1/4} \sim g\eta$$

$$\Gamma \approx T^4 e^{-S_3/T} \approx C\, e^{\beta(t-t_p)} \quad \Longrightarrow \quad \beta \equiv \left.\frac{d\log\Gamma}{dt}\right|_{t_e} = -H(T_e)T_e \left.\frac{d\log\Gamma}{dT}\right|_{T_e}$$

$$\mathcal{P}_f(T) = \exp\left[-\frac{4\pi}{3}\int_T^{T_c} dT' \frac{\Gamma(T')}{T'^4 H(T')}\left(\int_T^{T'} v_b \frac{dT''}{H(T'')}\right)^3\right] \equiv e^{-I(T)} \qquad I(T) \approx \frac{8\pi}{3} v_b^3 \frac{\Gamma(T)}{\beta^4}$$

$$\frac{dn_b(t)}{dR} = \frac{\Gamma[t_i(t,R)]\mathcal{P}_f[t_i(t,R)]}{v_b}\left[\frac{a[t_i(t,R)]}{a(t)}\right]^4 \qquad n_b(t) = \frac{\Gamma(t)}{\beta I(t)}\left[1 - \mathscr{P}_f(t)\right] \qquad R_b(T_p) \approx v_b \beta^{-1}$$

$$R_c^2 = \frac{3\lambda_{\rm eff}(T)}{3\lambda_{\rm eff}(T)m^2(T) - \delta^2(T) + \delta(T)\sqrt{\delta^2(T) - 4\lambda_{\rm eff}(T)m^2(T)}}$$

First order phase transition

Bounce action

$$V_{\text{eff}}(\phi, T) \simeq \frac{m_T^2}{2}\phi^2 - \frac{\delta_T}{3}\phi^3 + \frac{\lambda_T}{4}\phi^4$$

$$S_3 = \begin{cases} \dfrac{m_T^3}{\delta_T^2}\dfrac{2\pi}{3(\kappa - 2/9)^2}F(\kappa), & \kappa > 0 \\[2ex] \dfrac{m_T^3}{\delta_T^2}\dfrac{27\pi}{2}\left[\dfrac{1 + \exp\left(-1/\sqrt{|\kappa|}\right)}{1 + 9|\kappa|/2}\right], & \kappa < 0 \end{cases}$$

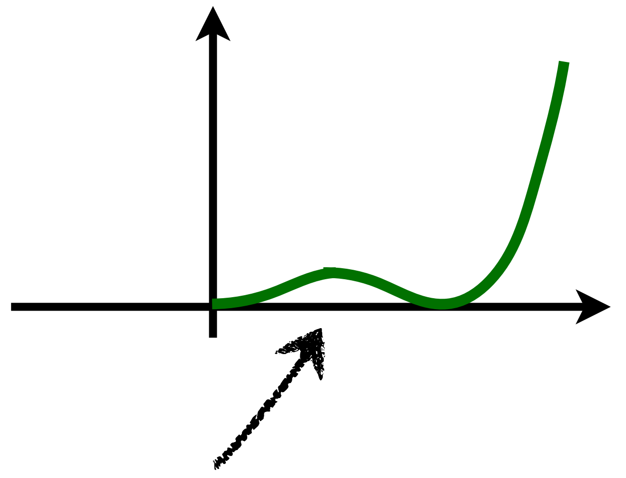
[Adams, 1993]
[Levi et al., 2022]

[Levi et al., 2022]
[Salvio, 2023]

$$\kappa \equiv \lambda_T \frac{m_T^2}{\delta_T^2}$$

Weakly first-order phase transition

$T_p \approx T_c$



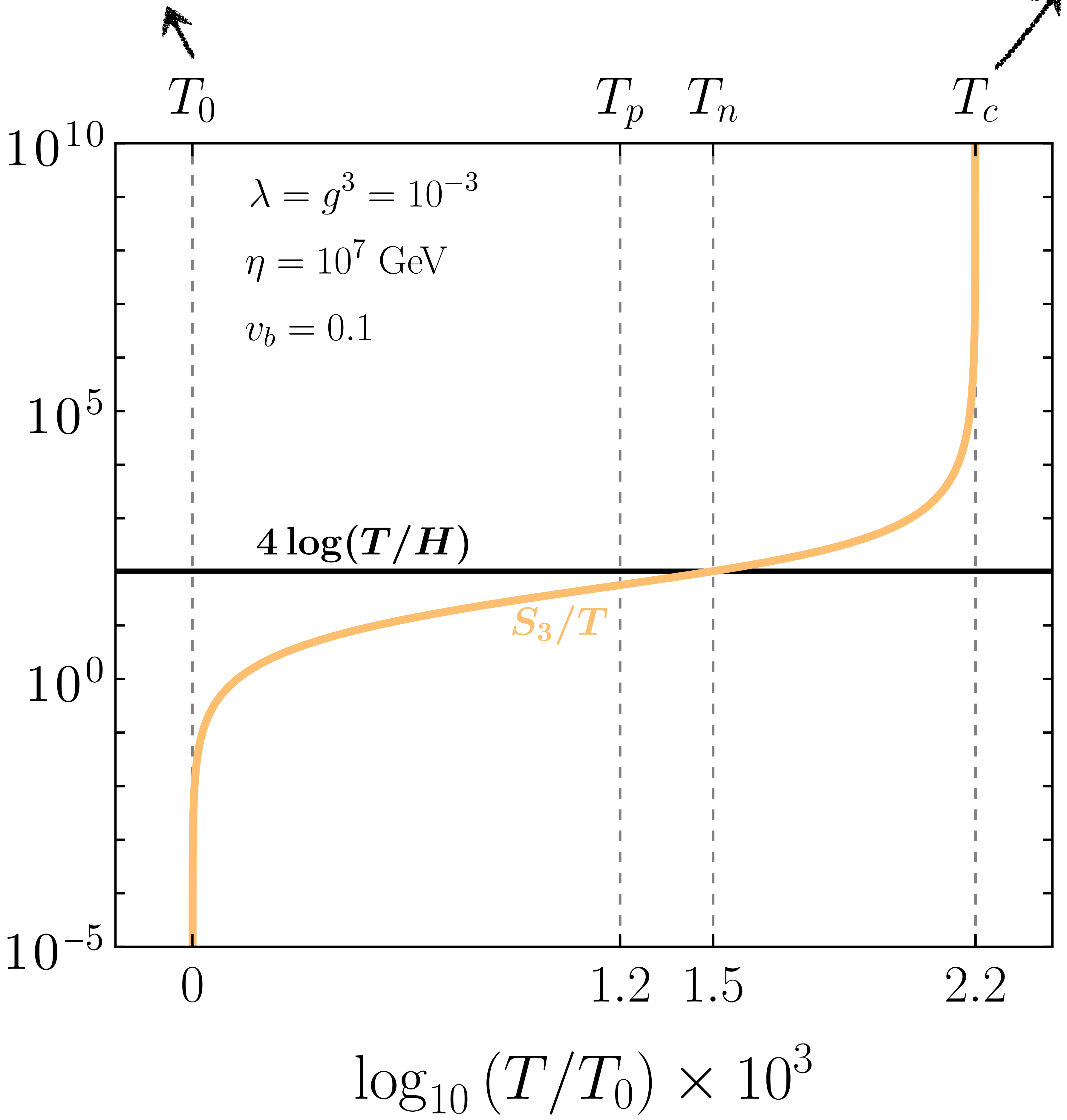
$$\lambda = g^3$$

$m^2(T_0) = 0$

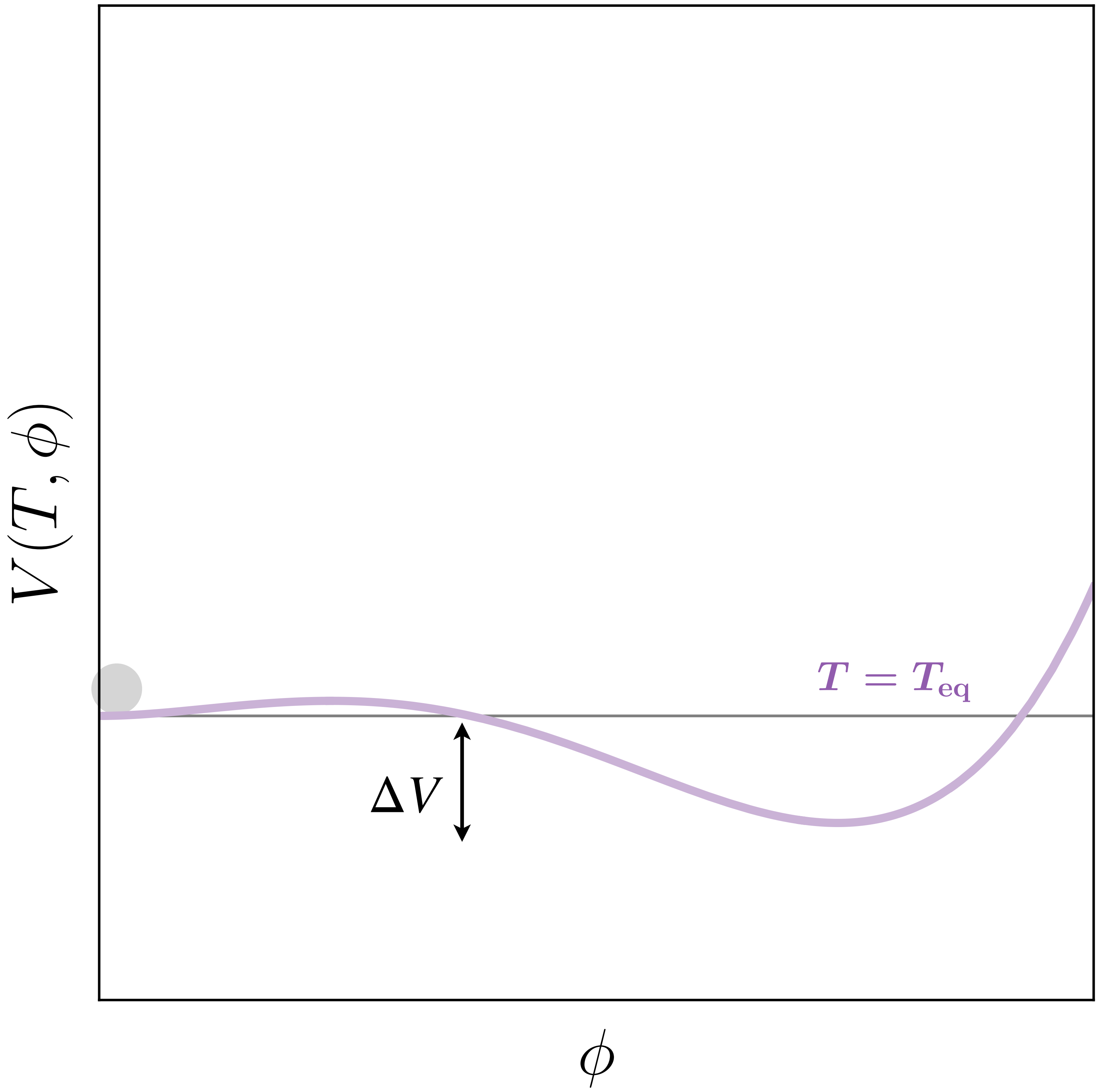
[Dolan, Jackiw, 1974]
[Arnold, Espinosa, 1994]

$$V(T,\phi) \approx \frac{1}{2}\underbrace{\left(\frac{g^2}{2}T^2 - \lambda\eta^2\right)}_{\equiv m^2(T)}\phi^2 - \frac{g^3}{2\pi}T|\phi|^3 + \frac{\lambda}{4}(\phi^2)^2$$

$$T_c \approx (1+g)T_0 \quad \Longrightarrow \quad T_0 \lesssim T_p \lesssim T_c$$



Strongly first-order phase transition



$$\begin{cases} \rho_r \propto a^{-4} \propto T^4 \\ \rho_v = \Delta V \propto \text{"}const.\text{"} \end{cases}$$

$T_{\rm eq}: \quad \rho_r = \rho_v$ ⟶ Thermal inflation

$$N = \log \frac{T_{\rm eq}}{T_p}$$

Strongly first-order phase transition

Classically scale invariant model

$T_p \ll T_c$

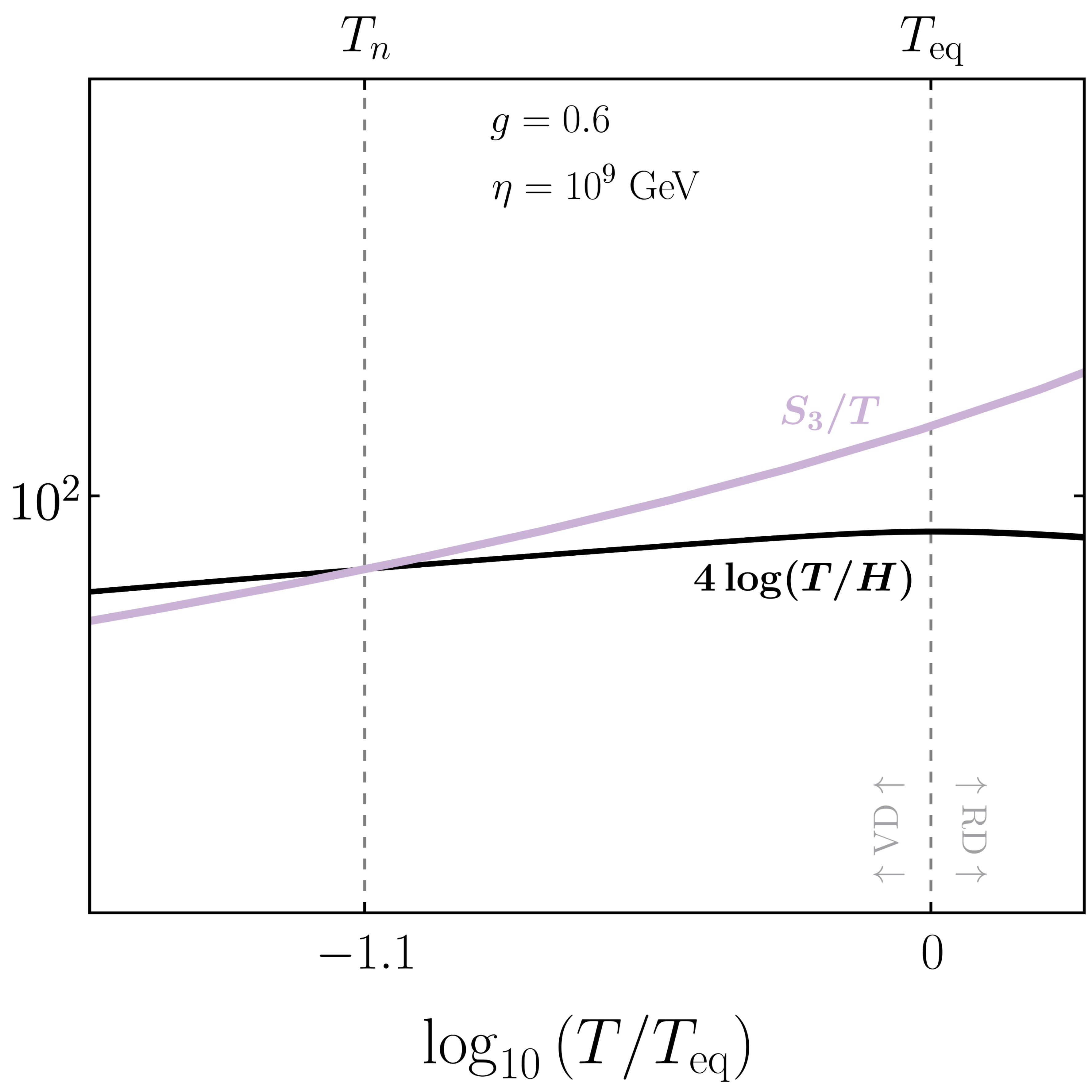
$\rho_r(T_{\rm eq}) = \rho_v$

[Salvio, 2023]

$$V(T,\phi) \approx \underbrace{\frac{g^2}{4}}_{\equiv m^2(T)} T^2\phi^2 - \frac{g^3}{2\pi}T|\phi|^3 + \frac{3g^4}{16\pi^2}\log\left(\frac{\eta}{T}\right)(\phi^2)^2$$

[Hambye et al., 2018]

$$N \approx \log\frac{T_{\rm eq}}{T_p} \implies h^2\Omega_{W'} \propto e^{-3N}, \quad \text{if } T_{\rm reh} < T_{\rm fo}$$



$T_p \ll T_c$

Strongly first-order phase transition

Classically scale invariant model

[Salvio, 2023]

$$V(T,\phi) \approx \underbrace{\frac{g^2}{4}T^2}_{\equiv m^2(T)}\phi^2 - \frac{g^3}{2\pi}T|\phi|^3 + \frac{3g^4}{16\pi^2}\log\left(\frac{\eta}{T}\right)(\phi^2)^2$$

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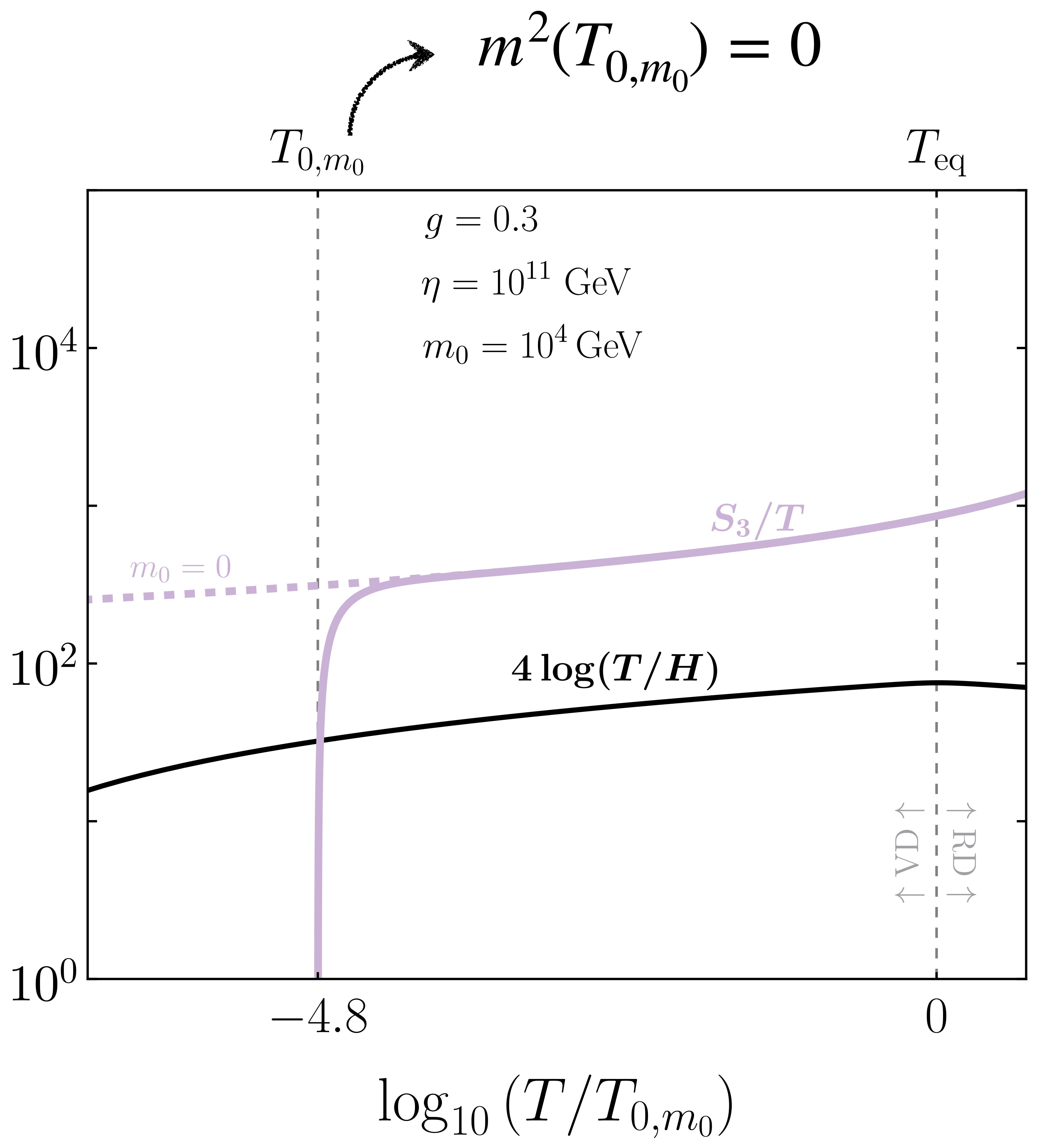
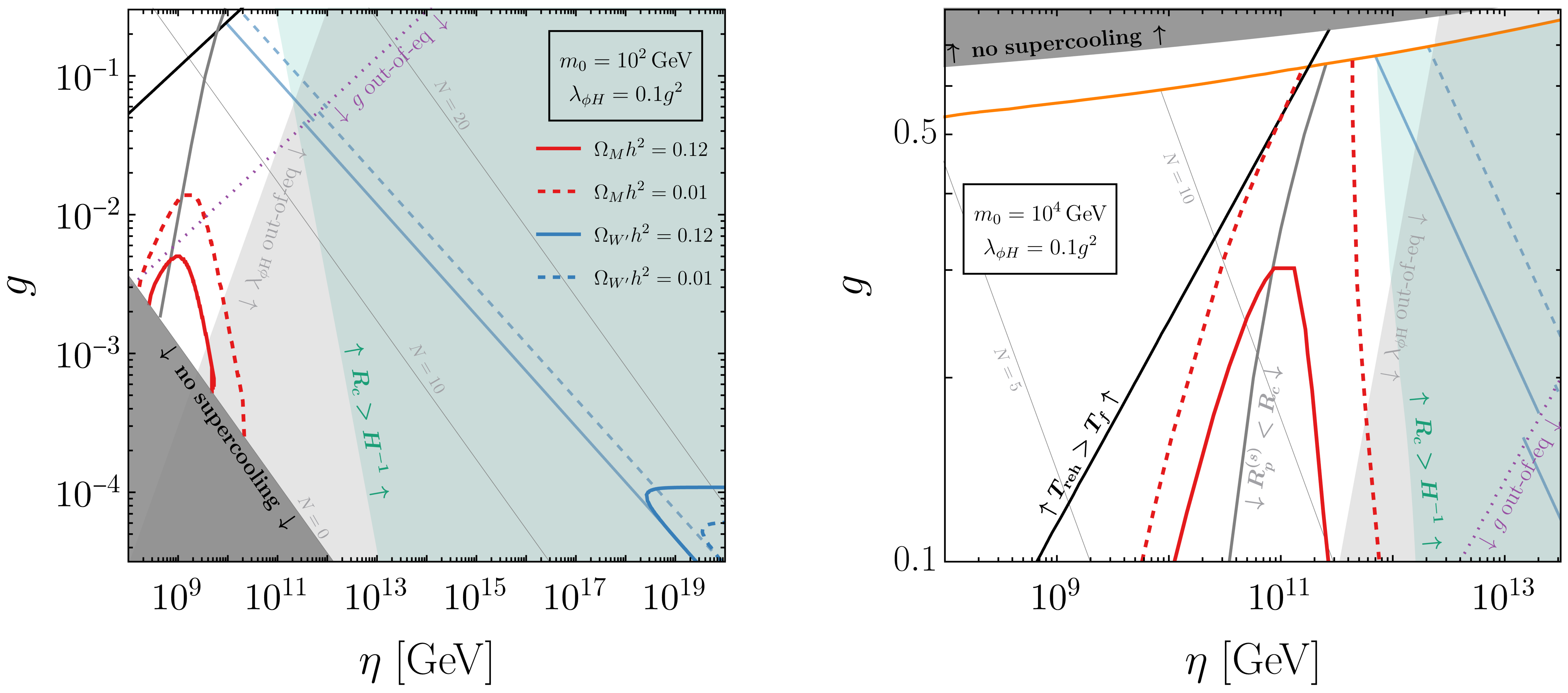
$$N \approx \log\frac{T_{\rm eq}}{T_p} \implies h^2\Omega_{W'} \propto e^{-3N}, \quad \text{if } T_{\rm reh} < T_{\rm fo}$$

$$m^2(T) \to m^2(T) - m_0^2 \implies T_p \approx T_{0,m_0} \lll T_{\rm eq}$$



Supercooled phase transition



W' freeze-out

$\lambda > g^2: \; W'W' \to \gamma'\gamma'$ $$\langle \sigma v \rangle_{W'^+W'^- \to \gamma'\gamma'} \simeq \frac{19 g^4}{72 \pi m_{W'}^2}$$

$\lambda < g^2: W'W' \to \gamma'\gamma', \rho\rho$ $$\langle \sigma v \rangle_{W'^+W'^- \to \rho\rho} \simeq \frac{11 g^4}{144 \pi m_{W'}^2} - \frac{14 g^2 \lambda}{192 \pi m_{W'}^2} + \frac{31 \lambda^2}{2304 \pi m_{W'}^2} \; + \; \langle \sigma v \rangle_{W'^+W'^- \to \gamma'\gamma'} \simeq \frac{19 g^4}{72 \pi m_{W'}^2}$$