RPP2026 @ LUPM
11 mars

Vue du Panthéon à Paris
Jean-Baptiste-Camille Corot (1796-1875)

Gravitational waves from flavoured SU(2) early-universe phase transitions

Anna Chrysostomou (LPTHE), Alan S. Cornell (UJ), Aldo Deandrea (IP2I), Luc Darmé (IP2I), Thibault Demartini (CEA)

arXiv: 2512.02148

Initiative Physique des Infinis
Alliance Sorbonne Université

LPTHE
LABORATOIRE DE PHYSIQUE THEORIQUE ET HAUTES ENERGIES

Overview

- ➔ Particle physics in the GW era: a search pipeline
- ➔ The first-order phase transition in our $SM+SU(2)_f$
- ➔ Energy budget and hydrodynamics
- ➔ Can we detect a GW signal from this new SU(2) model?

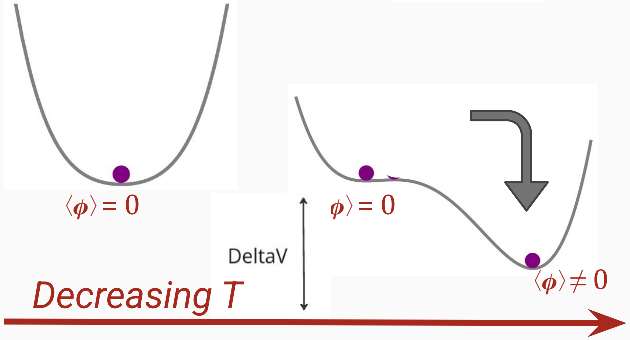
↳**Focus**: for which PT parameters do we obtain observable signals?

How to source gravitational waves?

BSM phase transition → bubble expansion in plasma → collisions + sound waves + turbulence → gravitational waves

$$V(\phi) = a\phi^2 + b\phi^3 + c\phi^4$$

$(a = b = 0)$ $(a, b < 0)$



potential barrier between symmetric (vev= 0) & broken (vev≠0) phase

> cubic term from thermal contribution ~$(\varphi^\dagger\varphi)^{3/2}$

> SM: large top quark Yukawa coupling strongly modifies Higgs thermal potential, weakening EWPT

> SM: lattice simulations confirm smooth crossover for $m_H \gtrsim 80$ GeV

Find degenerate min{$V_{eff}(\mu,T)$}

Is the PT first order ?

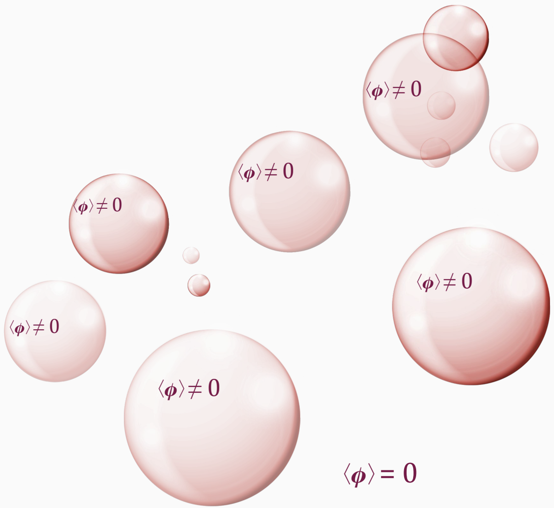
$$\frac{\phi_c}{T_c} \geq 1$$

⇨ *observable signal is sourced by new physics*

(sphaleron decoupling)

How to source gravitational waves?

BSM phase transition → bubble expansion in plasma → collisions + sound waves + turbulence → gravitational waves



Energy released by going from the false to the true vacuum is transferred to the bubble wall -> expand with v~c.
> Trigger extremely large-scale perturbation of the plasma => GWs
We use relativistic hydrodynamics + some particle physics to understand the interactions wall/plasma

$$\int_{t_c}^{t_n} dt \frac{\Gamma(t) P_f(t)}{H(t)^3} \approx \int_{T_N}^{T_c} \frac{dT}{T} \frac{\Gamma(T)}{H(T)^4} = 1$$

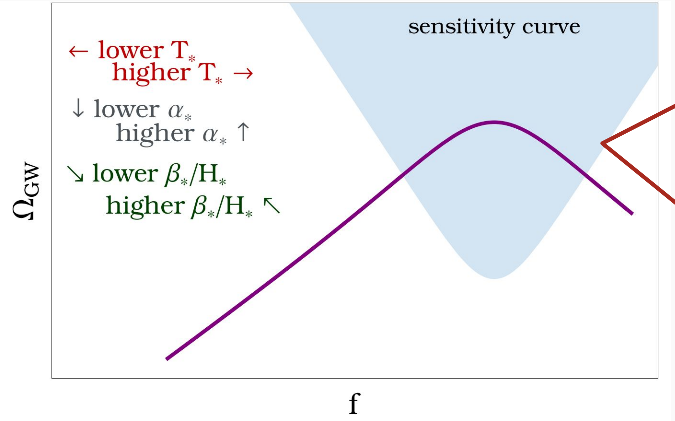
$$\Gamma(T) \simeq T^4 \left(\frac{S_3(T)}{2\pi T}\right)^{3/2} \exp\left(-\frac{S_3(T)}{T}\right)$$

$$H^2(T) = \frac{1}{3M_{\text{Pl}}^2}\left(\frac{\pi^2}{30} g_* T^4 + \Delta V(T)\right)$$

S_3 = O(3)-symmetric tunnelling ("bounce" solution)

How to source gravitational waves?

BSM phase transition → bubble expansion in plasma → collisions + sound waves + turbulence → gravitational waves



$$\Omega_{bpl} = \Omega_{peak} \left(\frac{f}{f_{peak}}^{n_1}\right) \left[1 + \left(\frac{f}{f_{peak}}^{n_1}\right)^{\Delta}\right]^{(n_1 - n_2)/\Delta}$$

For sound waves (dom. contribution) :

$n_1 = 3$, $n_2 = -4$

$\Delta = 2$ *(smoothening parameter)*

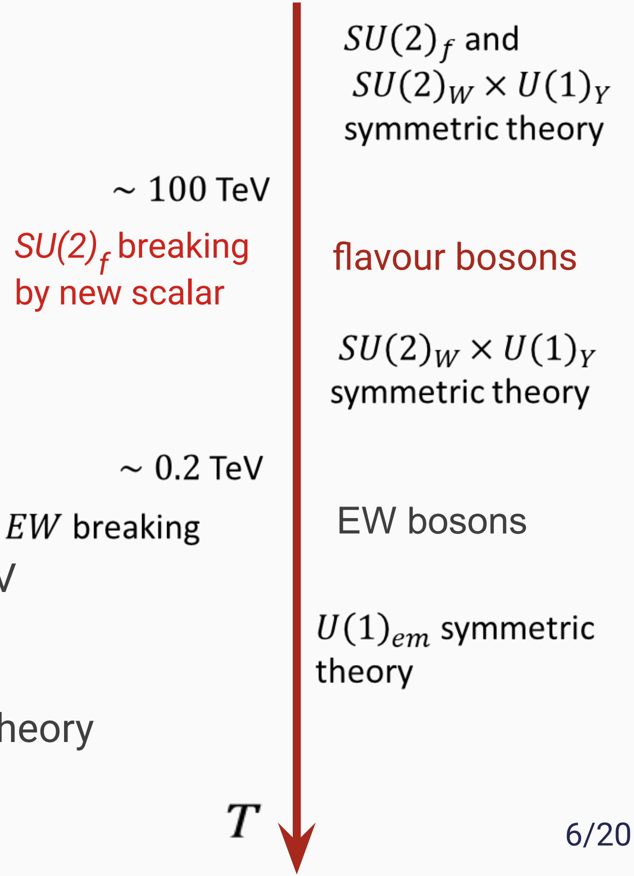
Our BSM framework: SM+SU$(2)_f$

Add a new SU(2) gauge group in the SM, acting on flavour space

3 new « W-like » gauge bosons carrying a « flavour-charge »

$$M_{V_1}^2 = M_{V_2}^2 = M_{V_3}^2 = \frac{g_f}{2}\sum_i v_\phi^2$$

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$SU(2)_f$	*DoF*
Φ	1	1	0	2	1
S	3	1	2/3	2	$3 \times 2 \times 2 = 12$
V	1	1	0	3	$3 \times 3 = 9$



To break the flavour gauge symmetries, we need the new scalar VEV

This $\langle\phi\rangle \neq 0$ appears in the early universe at temperatures close to the VEV

Flavour constraints point towards 100 TeV for complete flavourful theory

L. Darmé, A. Deandrea, and F. Mahmoudi, JHEP 05 (2024)
arXiv: 2307.09595

Thermal resummation: DR approach to IR sensitivities of light bosons

Step-by-step approach to decouple all thermal degrees of freedom

1. RGE from μ_{ini} to μ_{hard}
2. Match 4d to 3d at « hard scale »

   $\mu_{hard} \sim \pi T$ (thermal mass of fermions + transverse gauge bosons)
3. Run gT in the 3d theory
4. Decouple remaining bosonic modes, except scalar field ϕ triggering the PT

Implement using DRalgo

Up to NNLO matching in some cases !

4D theory – μini = 50 TeV

Decoupling of the towers of thermal modes

"hard" μ4d $\sim \pi T$

3D theory

"soft" μ3d $\sim gT$

Scalars + temporal (longitudinal) components of gauge bosons

Symmetry breaking scale

"ultrasoft" μlow $\sim \frac{g^2}{\pi} T$

Only the lightest scalar → corresponds to the effective potential

T

L. Gould and T.V. I. Tenkanen, JHEP 06 (2021)
arXiv: 2104.04399
A. Ekstedt, P. Schicho, T. V. I. Tenkanen, CPC 288 (2023)
arXiv: 2205.08815

Thermal resummation: DR approach to IR sensitivities of light bosons

$$\lambda_\phi^{3\mathrm{d}} = T\left[\lambda_\phi + \frac{1}{(16\pi)^2}\left(g_f^4\,(6-9L_b) + 72 g_f^2 \lambda_\phi L_b - 48 L_b\left(\lambda_{\phi s}^2 + 4\lambda_\phi^4\right)\right)\right]$$

$$L_b = 2\gamma + \log\frac{\mu^2}{(4\pi T)^2}$$

$$\lambda_\phi^{\mathrm{US}} = \lambda_\phi^{3\mathrm{d}} - \frac{3}{32\pi}\left[\frac{\left(\lambda_{\phi A_0}^{3\mathrm{d}}\right)^2}{\sqrt{m_{D,f}}}\right]$$

$$v_\phi \equiv \langle\phi\rangle_{T_c} \sim \frac{\kappa_3}{\lambda_\phi^{\mathrm{US}}}\,, \qquad \kappa_3 \sim \frac{(g_f^{\mathrm{US}})^3}{16\pi}$$

★ *to enhance the barrier, decrease λ^{US} or increase g^{US}*

★ *but at some point, g^4 matching corrections to λ^{US} begin to dominate, such that $\langle\phi\rangle_{Tc} \propto 1/g^{US}$*

4D theory – μini = 50 TeV

Decoupling of the towers of thermal modes

"hard" μ**4d** $\sim \pi T$

3D theory

"soft" μ**3d** $\sim gT$

integrated out hard scale

Symmetry breaking scale

Scalars + temporal (longitudinal) components of gauge bosons

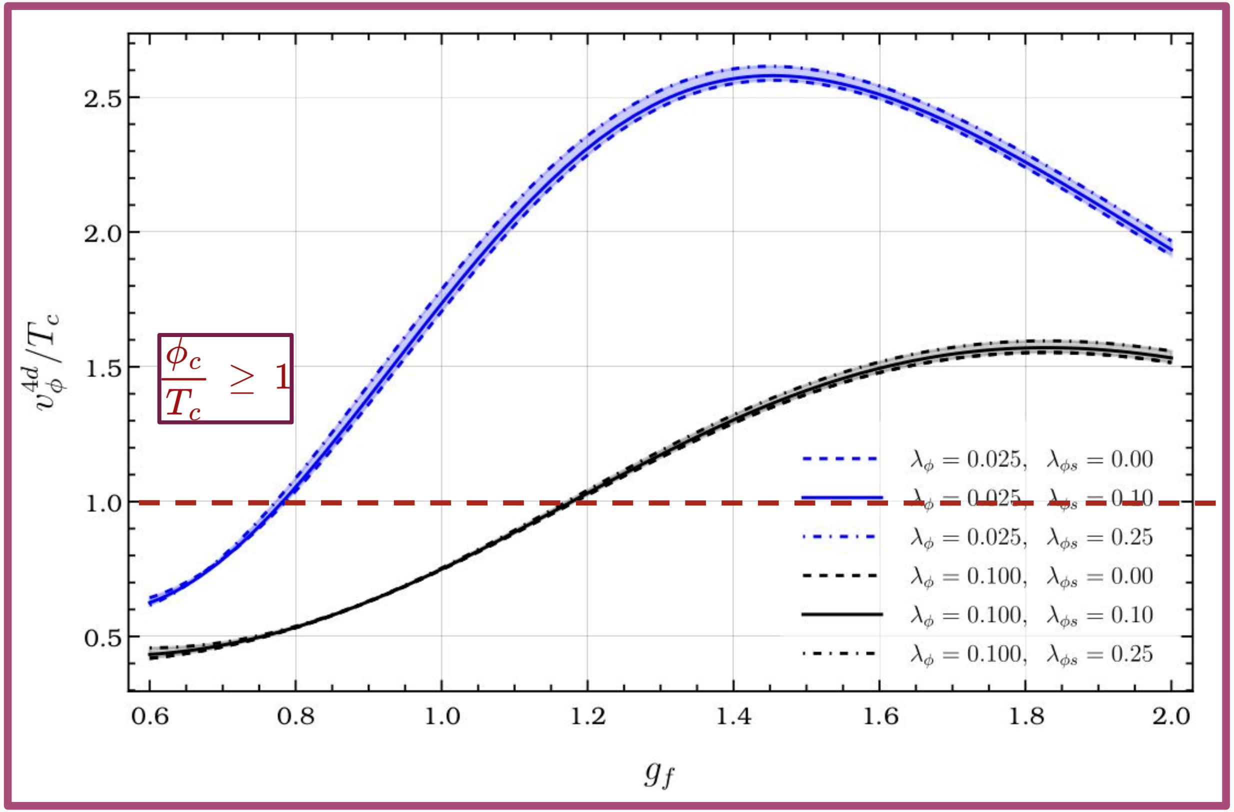
"ultrasoft" μ**low** $\sim \frac{g^2}{\pi}T$

integrated out soft scale

Only the lightest scalar → corresponds to the effective potential

T

Phase transitions case study: breaking a new SU(2) flavour symmetry



★ *order-one gauge couplings are always required for a FOPT*

★ *model favours quartic interactions of the order ~0.005*

★ *coupling between symmetry-breaking scalar ϕ & scalar LQ strengthens FOPT*

$$v_\phi \equiv \langle \phi \rangle_{T_c} \sim \frac{\kappa_3}{\lambda_\phi^{\mathrm{US}}}, \qquad \kappa_3 \sim \frac{(g_f^{\mathrm{US}})^3}{16\pi}$$

From Lagrangian to GWs: mapping microphysics to macroscopic phenomena

model → **action** → **bubble dynamics** → **GW vs SNR**

Build $V_{eff}(\mu,T)$

Determine field content, dof, etc.
Potential: zero-T + finite-T
Find degenerate min{$V_{eff}(\mu,T)$}

Compute PT parameters

Compute Euclidean / 3d action
Extract phase transition parameters:
> Transition temperature $T_* \approx T_N$
> PT strength α_N
> Inverse of PT duration β/H_N

Energy budget & hydrodynamics

Model wall-plasma interactions &
plasma hydrodynamics around wall
Compute energy budget parameters:
> Efficiency parameter κ
> Scalar field energy density ρ_φ
> Friction/out-of-equilibrium
> Bubble wall speed v_w

GW spectrum vs sensitivity of detectors

Compute energy density of GWs
Compare GW power spectrum
against detector sensitivity curve

From Lagrangian ... pic phenomena

latent heat released by PT, normalised against the radiation density

$$\alpha_N = \frac{1}{\rho_R}\left[\Delta V_{\rm eff}(\phi,T) - \frac{T}{4}\Delta\frac{dV_{\rm eff}(\phi,T)}{d\ln T}\right]_{T=T_N} \qquad \rho_R = \frac{\pi^2}{30}g_* T_N^4$$

inverse phase transition duration relative to the Hubble rate at T_N

$$\frac{\beta}{\mathcal{H}*} = T\,\frac{dS_3(T)}{dT}\bigg|_{T=T_N}$$

$$h^2\Omega_{\rm GW}(f; H_*, \alpha, \beta, v_w)$$

$\Omega_{\rm peak}$, $f_{\rm peak}$

model → **action** → **bubble dynamics** → **GW vs SNR**

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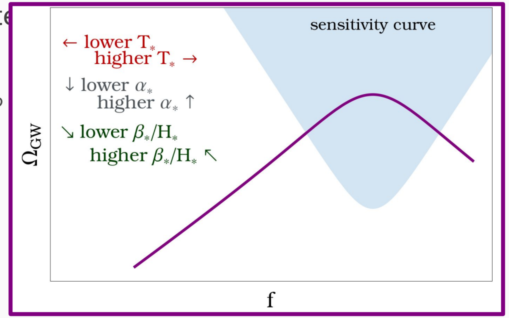
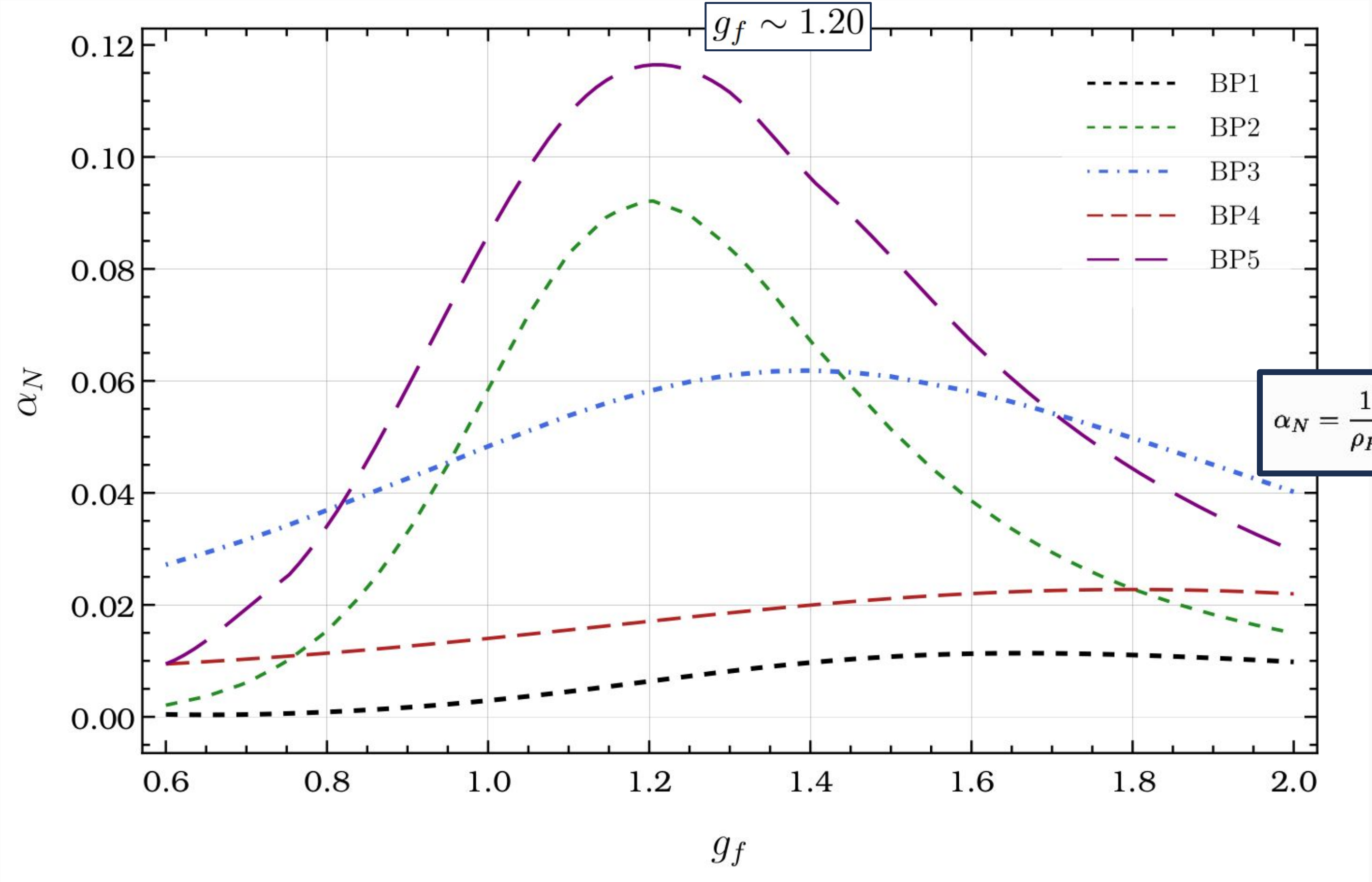
flavour gauge bosons interacting with plasma

GW spectrum vs sensitivity of detectors

Compute energy density of GWs
Compare GW power spectrum



Considering the energy budget: FOPT strength

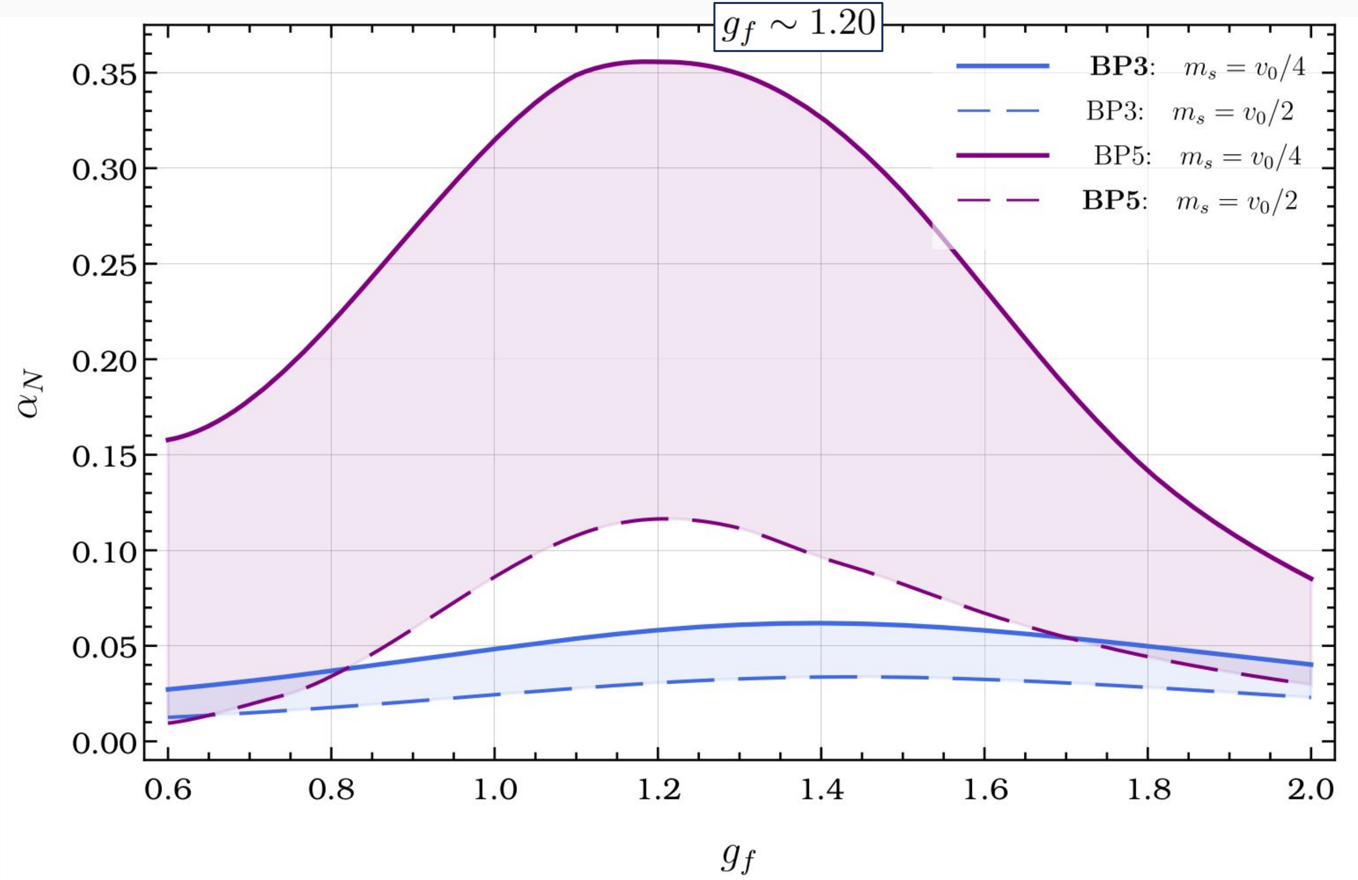


	m_S/v_0	λ_ϕ	$\lambda_{\phi s}$	g_f
BP1	1.00	0.025	0.00	1.5
BP2	1.00	0.005	0.01	1.0
BP3	0.25	0.025	1.00	1.0
BP4	0.10	0.100	1.50	1.5
BP5	0.50	0.005	0.25	1.5

$$\alpha_N = \frac{1}{\rho_R}\left[\Delta V_{\text{eff}}(\phi, T) - \frac{T}{4}\Delta\frac{dV_{\text{eff}}(\phi, T)}{d\ln T}\right]_{T=T_N}$$

$$\rho_R = \frac{\pi^2}{30} g_* T_N^4$$

Considering the energy budget: FOPT strength



	m_S/v_0	λ_ϕ	$\lambda_{\phi s}$	g_f
BP3	0.25	0.025	1.00	1.0
BP5	0.50	0.005	0.25	1.5

★ *model favours quartic interactions of the order ~0.005*

★ *coupling between symmetry-breaking scalar ϕ & scalar LQ strengthens FOPT*

★ *decreasing by 2 the mass of the LQ with respect to the reference scale introduces a corresponding linear increase in FOPT strength*

Hydrodynamics – at equilibrium!

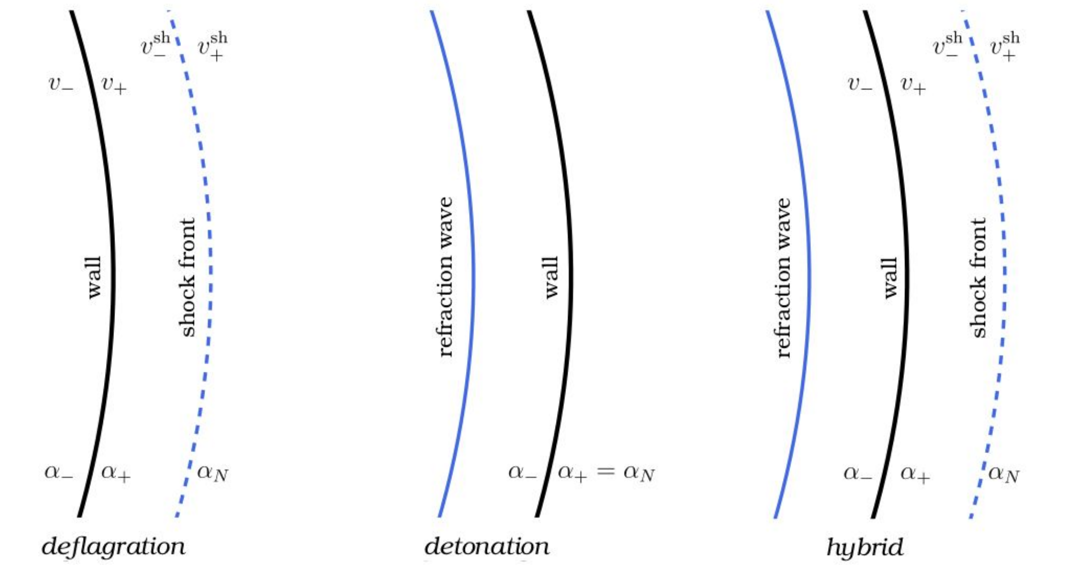
ONLY when the system around the wall has reached local thermal equilibrium
=> capture the hydrodynamics of the bubble wall velocity and the plasma fluid velocity:

★ *steady-state (constant v)*
★ *runaway (accelerating v) solutions*

$$\partial^\mu T_{\mu\nu} = \partial^\mu T^{\phi}_{\mu\nu} + \partial^\mu T^{\text{eq}}_{\mu\nu} = 0$$

$$T^{\phi}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}\partial_\rho\phi\partial^\rho\phi - V_{\text{eff}}(\phi, T)\right]$$

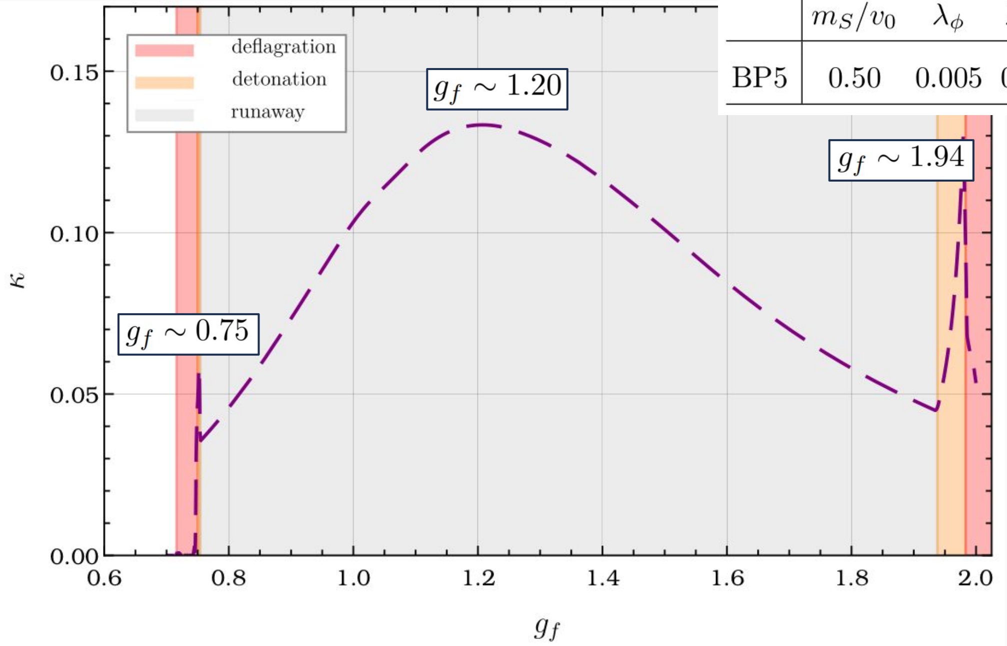
$$T^{\text{eq}}_{\mu\nu} = w u_\mu u_\nu - p g_{\mu\nu}$$



$$w_+ v_+^2 \gamma_+^2 + p_+ = w_- v_-^2 \gamma_-^2 + p_-, \qquad w_+ v_+ \gamma_+^2 = w_- v_- \gamma_-^2$$

Regime	v_w	v_+	v_-	Features of the expansion
Detonation	$v_w > c_s$	$v_+ = v_w$	$v_- < v_w$	Supersonic: rarefaction wave behind wall
Deflagration	$v_w < c_s$	$v_+ > 0$	$v_- = v_w$	Subsonic: shock wave in front of the wall
Hybrid	$v_w > c_s$	$c_s > v_+ > 0$	$v_- = c_s$	Sonic: rarefaction and shock wave
Runaway	$v_w \to 1$	$v_+ \approx 0$	$v_- \approx 0$	No steady-state, walls accelerate indefinitely

Considering the energy budget: efficiency



	m_S/v_0	λ_ϕ	$\lambda_{\phi s}$	g_f
BP5	0.50	0.005	0.25	1.5

$$T^{\rm pl}_{\mu\nu} = T^{\rm eq}_{\mu\nu} + T^{\rm out}_{\mu\nu}\,,$$
$$T^{\rm eq}_{\mu\nu} = \sum_i \int \frac{d^3p}{(2\pi)^3 E_i} p_\mu p_\nu f_i^{\rm eq}(p_\mu, x)\,,$$
$$T^{\rm out}_{\mu\nu} = \sum_i \int \frac{d^3p}{(2\pi)^3 E_i} p_\mu p_\nu \delta f_i(p_\mu, x)\,.$$

pressure from the out-of-equilibrium contributions,

$$P^{m_f}_{\rm out} = \gamma_w v_w \frac{9 m_D^2 T_N}{32\pi L_w} \int_0^1 \frac{1-x}{x} dx$$

$$x \equiv \phi(z)/\phi_N$$

treatment of gauge bosons as classical fields breaks down when the particle wavelength approaches that of the wall thickness
=> IR cutoff

$$\lambda_f \ll L_w \quad \Rightarrow \quad m_f(v_\phi) L_w \gg 1$$

$$x_{\rm IR} = \frac{1}{m_f(v_\phi) L_w}$$

$$\phi(z) = \frac{1}{2}\phi_N \left[\tanh\left(\frac{z}{L_w}\right) + 1\right]$$

$$\kappa = \frac{3}{\alpha_N \rho_R v_w^3} \int_{c_s}^{v_w} w\,\xi^2 \frac{v^2}{1-v^2}\, d\xi$$

$$\xi = r/t.$$

$$\alpha_N = \frac{1}{\rho_R}\left[\Delta V_{\rm eff}(\phi, T) - \frac{T}{4}\Delta \frac{dV_{\rm eff}(\phi,T)}{d\ln T}\right]_{T=T_N}$$

GW spectra from FOPTs - based on simulations [LISA]

Caprini *et al.* JCAP 03 (2020)

$$h^2\Omega_{\rm sw}(f) = 2.59\times10^{-6}\left[\left(\frac{g_*}{100}\right)^{-1/3}\right]\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2\left(\frac{\beta}{H_N}\right)^{-1}\max(v_w,c_s)\; S_{\rm sw}(f,v_w)$$

$$S_{\rm sw}(f) = \left(\frac{f}{f_{\rm sw}^{peak}}\right)^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2}$$

$$f_{\rm sw}^{\rm peak} = 8.9\times10^{-6}\ {\rm Hz}\left[\left(\frac{g_*}{100}\right)^{1/6}\left(\frac{T_N}{100\ {\rm GeV}}\right)\right]\frac{1}{\max(v_w,c_s)}\left(\frac{\beta}{H_N}\right)$$

★ *sound waves dominate (we checked)*
★ *for large amplitude, we want strong phase transition with a swift onset*
★ *small quartic and coupling between symmetry-breaking scalar ϕ & scalar LQ s*

	m_S/v_0	λ_ϕ	$\lambda_{\phi s}$	g_f	T_c [TeV]	T_N [TeV]	α_N	β/H_N
BP1	1.00	0.025	0.00	1.5	21.28	19.06	0.011	1886
BP2	1.00	0.005	0.01	1.0	16.95	12.34	0.055	1123
BP3	0.25	0.025	1.00	1.0	23.61	19.50	0.048	1339
BP4	0.10	0.100	1.50	1.5	25.76	24.09	0.021	2850
BP5	0.50	0.005	0.25	1.5	17.37	13.76	0.085	884

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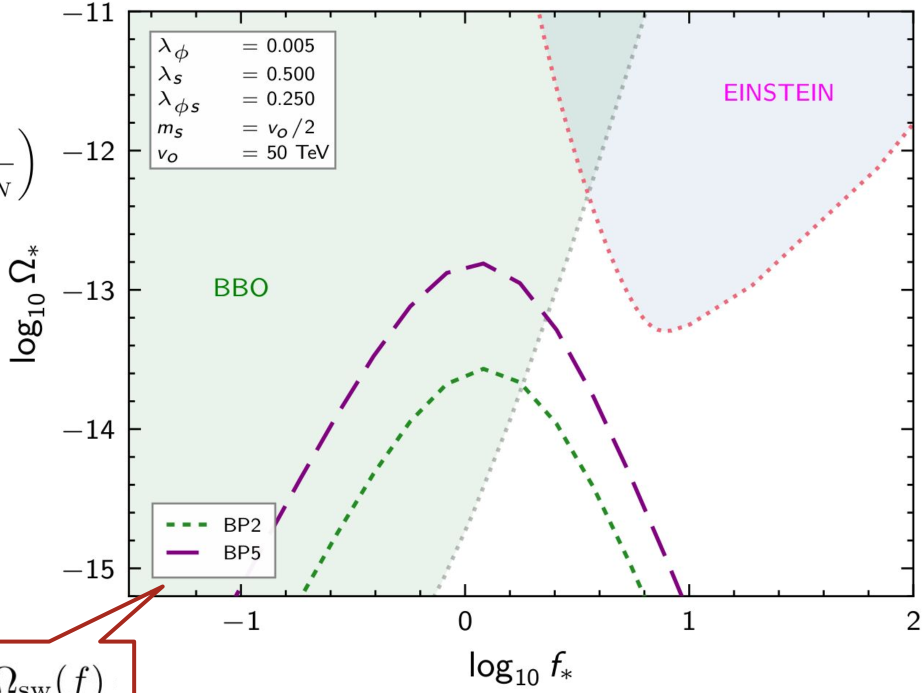
	m_S/v_0	λ_ϕ	$\lambda_{\phi s}$	g_f
BP2	1.00	0.005	0.01	1.0
BP5	0.50	0.005	0.25	1.5

★ *GW spectrum peaks in BBO when vev = 50 TeV*



$h^2\Omega_{\rm sw}(f)$

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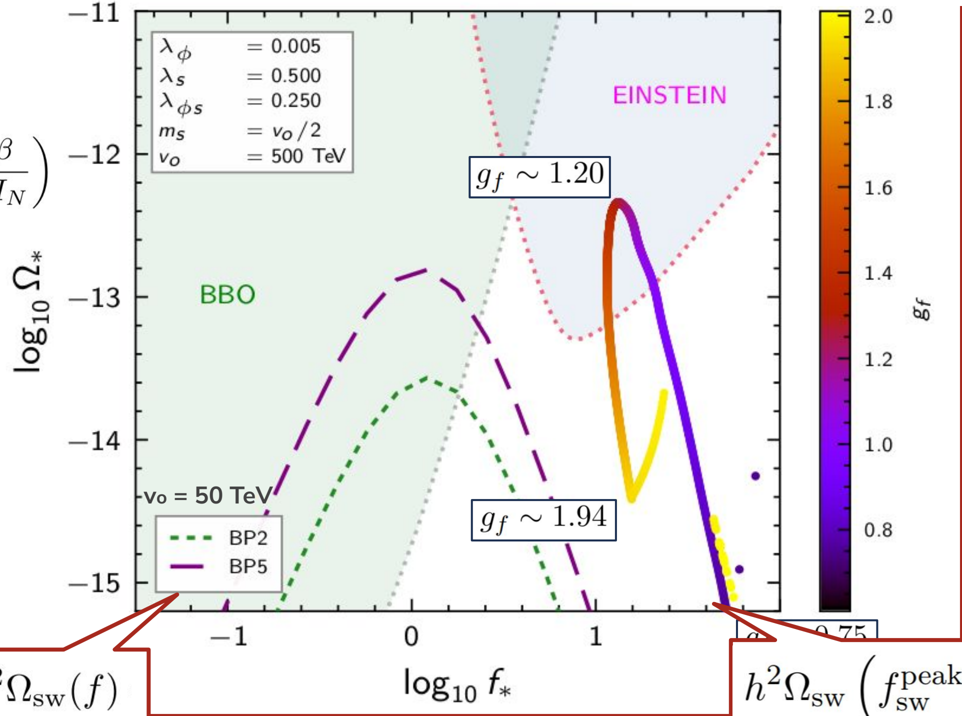
★ *setting the reference scale to 500 TeV, we plot the peak frequency and amplitude for BP5, varying g_f => Einstein predicted to be sensitive to the signal*



$h^2\Omega_{\rm sw}(f)$

$h^2\Omega_{\rm sw}\left(f_{\rm sw}^{\rm peak}\right)$

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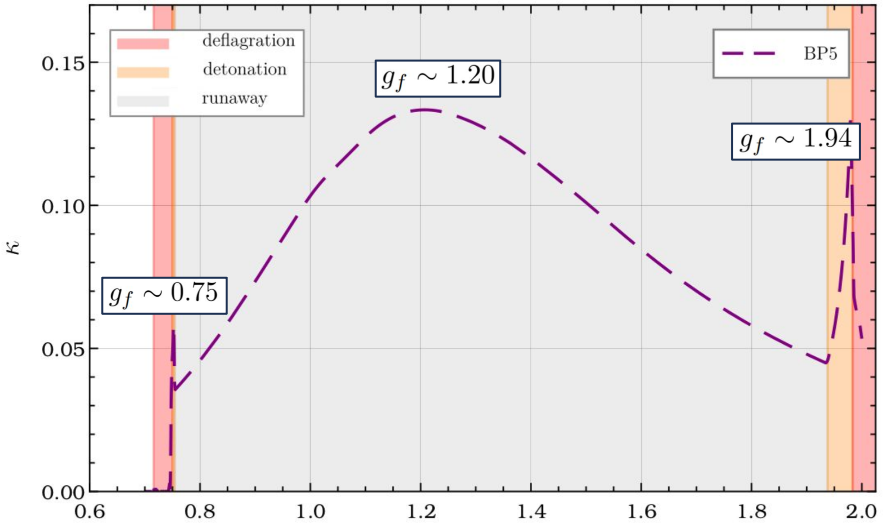
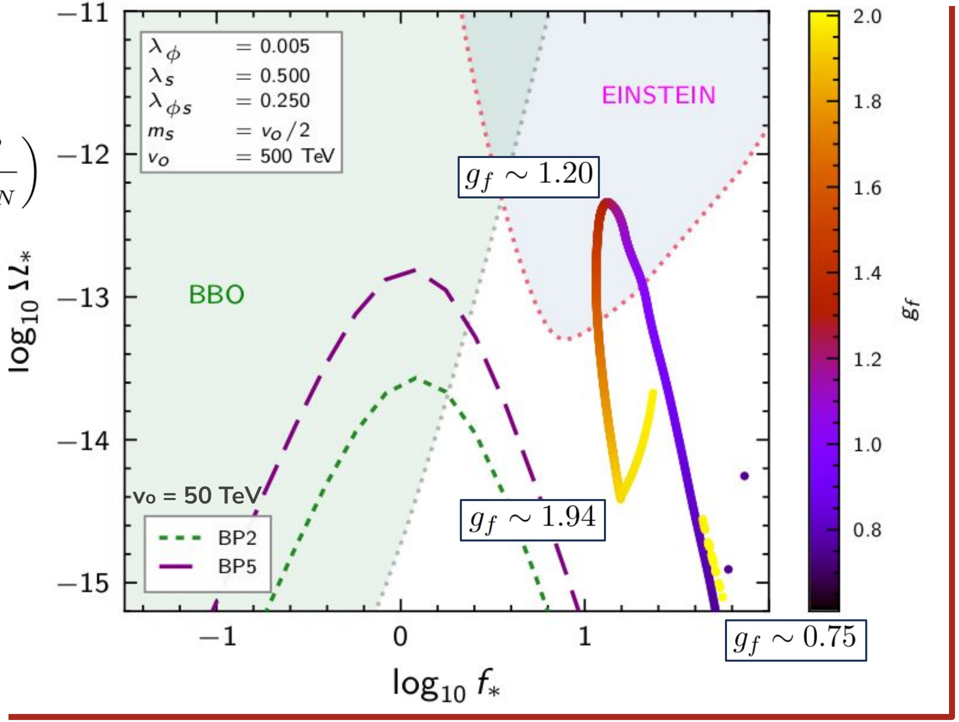




Conclusions

- ➔ Flavour models with SU(2) horizontal gauge symmetry naturally produce moderate FOPTs, driven by additional DoFs near the symmetry-breaking scale (scalar +LQ)
  - □ can be detectable, probe flavour scale up to $\sim 10^7$ GeV
- ➔ Here: state-of-the-art thermal calculations (DRalgo, FindBounce) + plasma friction
- ➔ $g_f \sim O(1)$ are required; for $g_f \lesssim 0.7$, a FOPT is typically not realised
- ➔ Stronger FOPTs occur for small quartic couplings; more generic with additional DoF (LQs)
- ➔ For strong FOPT ($\alpha \gtrsim 0.1$), recent simulations favour runaway walls, which we adopt
- ➔ Future GW detectors - demonstrably ET and BBO - probe parameter space inaccessible to collider experiments => powerful complementary BSM search tool
- ➔ Ongoing work: complete flavour model, improve wall-plasma modelling

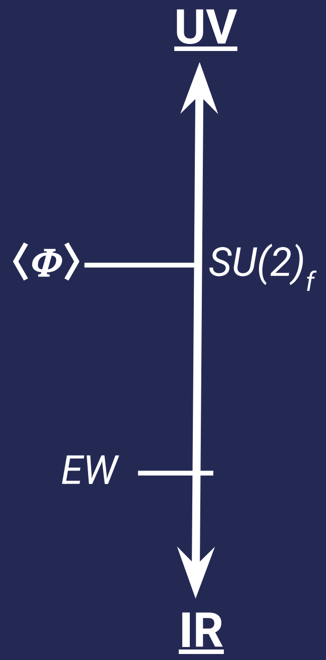
Thanks!

Any questions?

Or write to us at
chrysostomou@lpthe.jussieu.fr

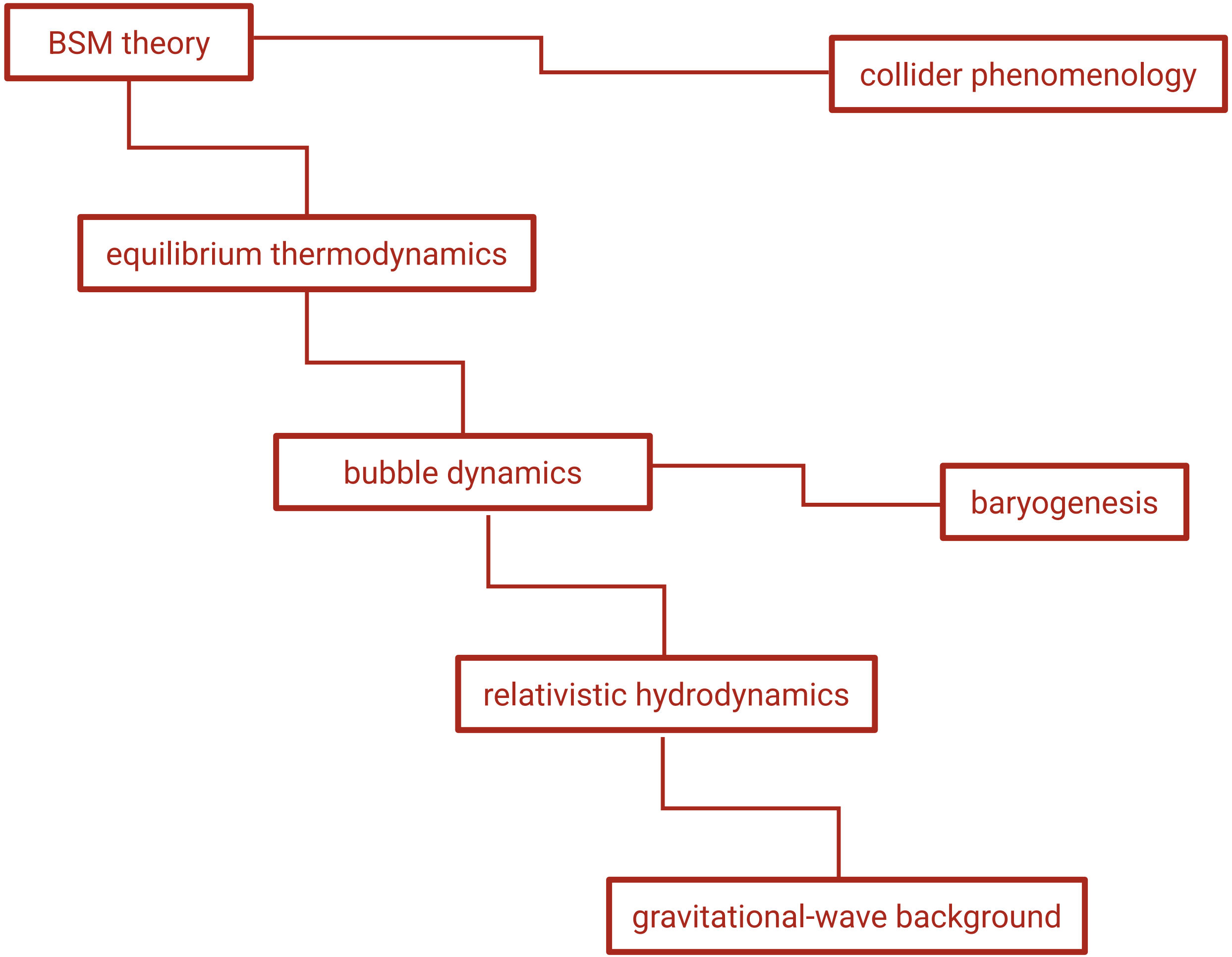
Vue du Panthéon à Paris
Jean-Baptiste-Camille Corot (1796-1875)

Backup slides



Pedagogical intro

1. A seminal work and a very useful conceptual introduction to finite temperature quantum field theory:
https://arxiv.org/abs/hep-ph/9901312
(note: this is one of the standard references cited in the literature, with most people relying on the imaginary ti
formalism)
2. A very nice and modern set of lecture notes that introduce you to dimensional reduction:
https://arxiv.org/abs/2307.00068
3. The focus of these notes is on the dynamics that follow the phase transition, so they serve as a helpful link for
the final steps of the "GW detection pipeline":
https://arxiv.org/abs/1705.01783, https://arxiv.org/abs/1512.06239


BSM theory
collider phenomenology
equilibrium thermodynamics
bubble dynamics
baryogenesis
relativistic hydrodynamics
gravitational-wave background

BSM theory

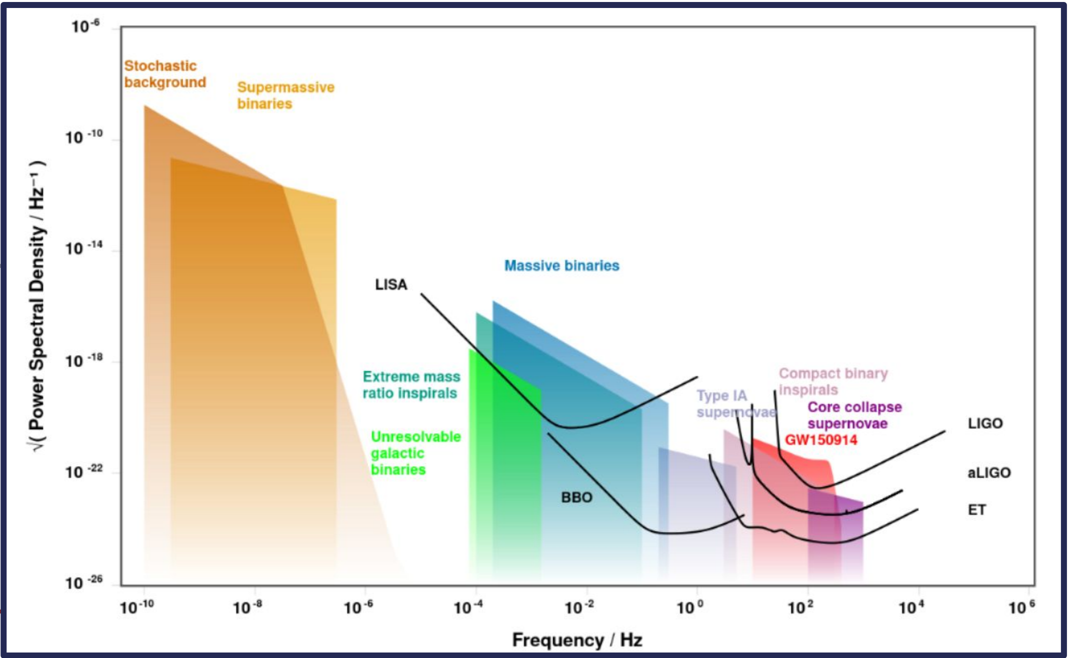
collider phenomenology

Our case study:
introduce to the SM an extra horizontal $SU(2)_f$ of flavour

equilibrium thermodynamics

We calculate phase transition parameters from $V(\phi, T)_{eff}$ using perturbation theory,
but IR sensitivities at high-T require resummation
⇒ « dimensional reduction » to encode IR physics of the high-T plasma into a 3d EFT

1st-order PT proceeds by the nucleation of bubbles of the scalar field driving the transition
⇒ expanding nucleated bubbles of the scalar field expand & interact with surrounding plasma
⇒ interactions between bubbles & excited plasma source GWs (dominant: sound waves)



C. Moore, R. Cole, & C. Berry's GWplotter

gravitational-wave background

Procedure

P. Athron et al. Prog. Part. Nucl. Phys. 135 (2024)
arXiv: 2305.02357

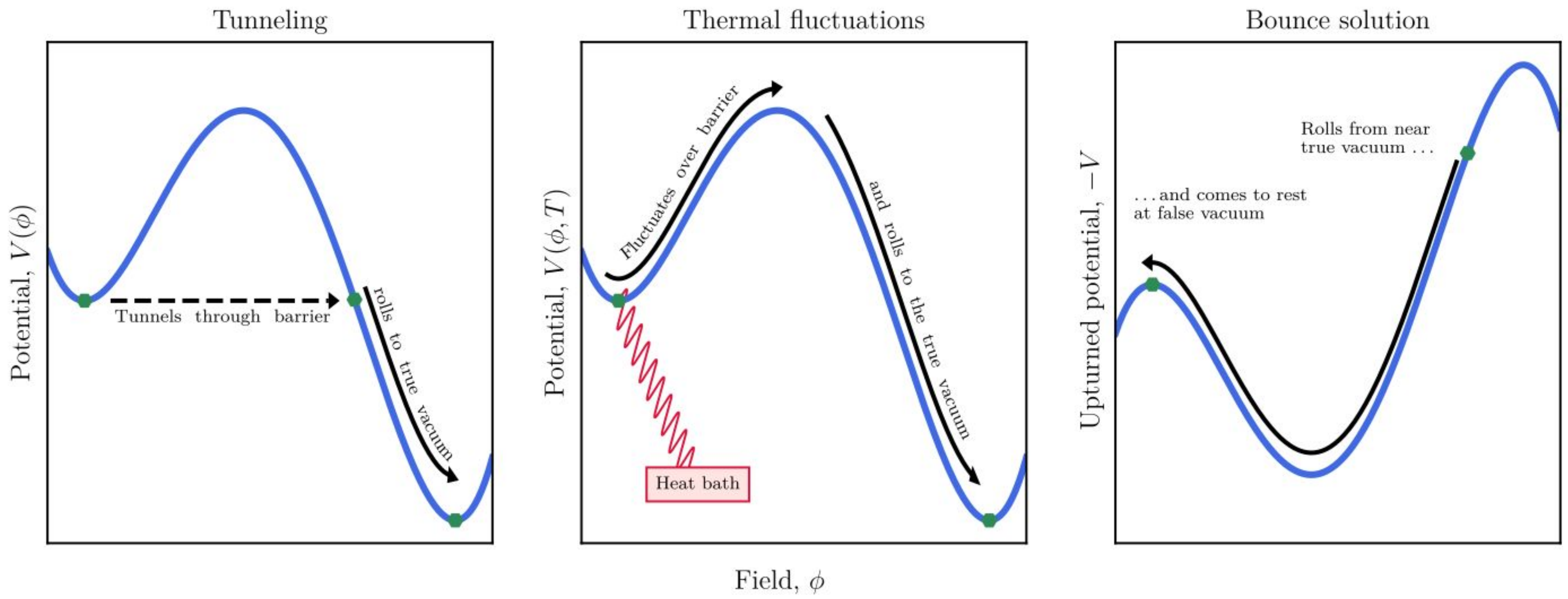
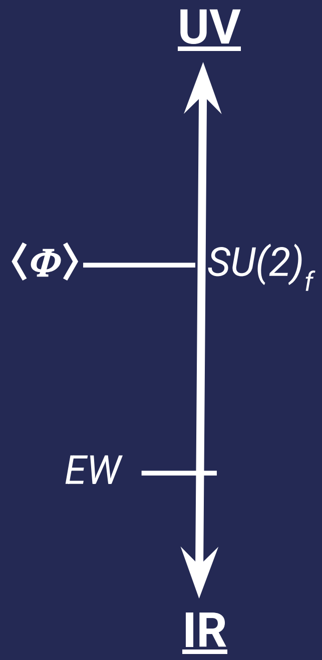
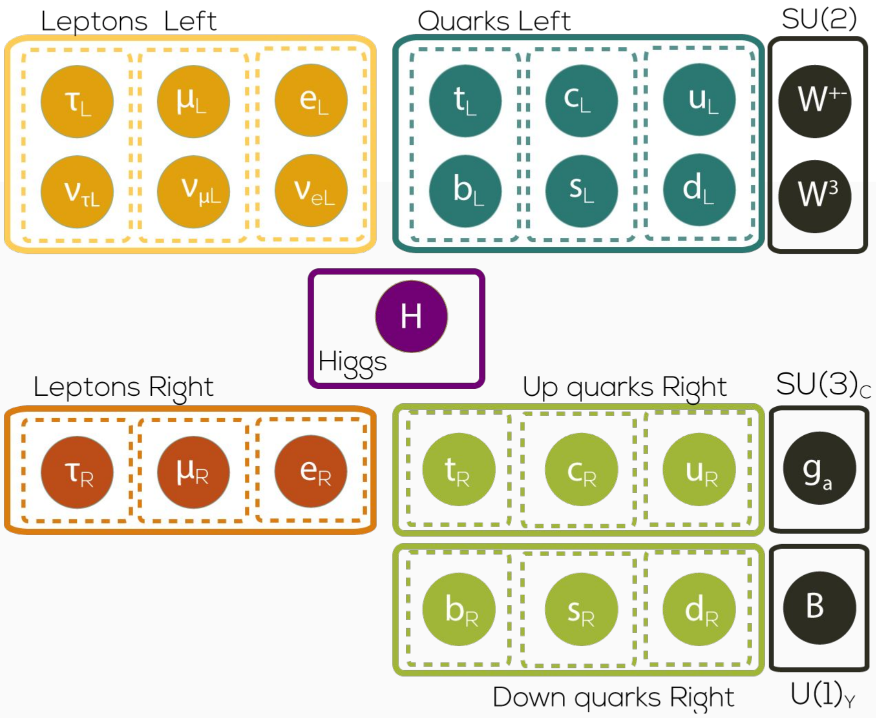


Fig. 9. The field may transition from the false vacuum by (left) tunneling through the barrier and then rolling into the true vacuum or (center) thermal fluctuations over the barrier. The transition probability in both cases involves the bounce solution (right), where we solve for the initial field position from which the field rolls down the upturned potential, along the barrier, and comes to a rest at the false vacuum.

Breaking a new « horizontal gauge symmetry » in the flavour sector



Horizontal flavour gauge group



The SM has a large global $U(3)^5$ symmetry group

→broken by the Yukawa interactions

$$\mathcal{L}_Y = -Y^d_{ij}\,\overline{Q^I_{Li}}\,\phi\, d^I_{Rj} - Y^u_{ij}\,\overline{Q^I_{Li}}\,\epsilon\,\phi^*\, u^I_{Rj} + \text{h.c.},$$

We can gauge a subset of this group ?

→U(1) case: Frogatt-Nielsen constructions, $L_\mu - L_\tau$, flavons, etc...

The non-abelian case has been sparsely studied.

→In any case the new gauge coupling is a free parameter

Horizontal flavour gauge group

L. Darmé, A. Deandrea, and F. Mahmoudi, JHEP 05 (2024)
arXiv: 2307.09595

Add a new SU(2) gauge group in the SM, acting on flavour space

- The « charged » SM fermion is part of a doublet
- Only the $SU(2)_f{}^2 \times U(1)_Y$ mixed anomaly is non-zero

$$\mathcal{A} = ([C(Q_i) - C(L_i)] - [2C(u_{R,i}) - C(d_{R,i}) - C(e_{Ri})])$$

Gauge boson masses are free parameters!

- We push to multi-TeV scale, with upper bound on g_f enforced by perturbativity limit
- Even with a large VEV, small gauge couplings (required by flavour constraints) imply light new states
  - For instance: left-handed scenario with $(12)_\ell (12)_{Q_L}$ interactions
    - → Reduce the number of fundamental fermions
    - → Couples both to LH leptons and LH quarks

3 new « W-like » gauge bosons carrying a « flavour-charge »

$$M_{V_1}^2 = M_{V_2}^2 = M_{V_3}^2 = \frac{g_f}{2} \sum_i v_\phi^2$$

+ *rotation matrices to mass basis: V_{uL}, V_{dL}, …*

Masses and textures

The presence of $SU(2)_f$ implies that the fermion mass matrices have a structure: let us focus on a left-handed model with Q_i, L_i

→We introduce δY_i , a $SU(2)_f$ spurion

→In the most generic case, this does not distinguish first and second generation

$$\delta Y_i = (\delta Y, 0)$$

$$L \supset y_d^\alpha \delta Y_i\, \bar{Q}^i \cdot H\, d_{R,\alpha} + \tilde{y}_d^\alpha \delta Y^{\dagger,i} \epsilon_{ij} \bar{Q}^j \cdot H\, d_{R,\alpha} + Y_{3,d} \bar{Q}_3 \cdot H\, b_R$$

α are generation indices but NOT gauge indices

i,j are $SU(2)_f$ gauge indices

We use the $U(3)_f$ global reparametrisation for $d_{R,\alpha}$

$$L \supset \delta Y(\, \bar{Q}^1 \cdot H\, (y_d^\alpha d_{R,\alpha}) - \delta Y\, (\bar{Q}^2 \cdot H\, (\tilde{y}_d^\alpha d_{R,\alpha}) + Y_{3,d} \bar{Q}_3 \cdot H\, b_R$$

Arranging $\delta Y \ll Y_3$ still leads to the same mass scale for first and second generation

Masses and textures

How can we generate a hierarchy between 1st and 2nd generation ?

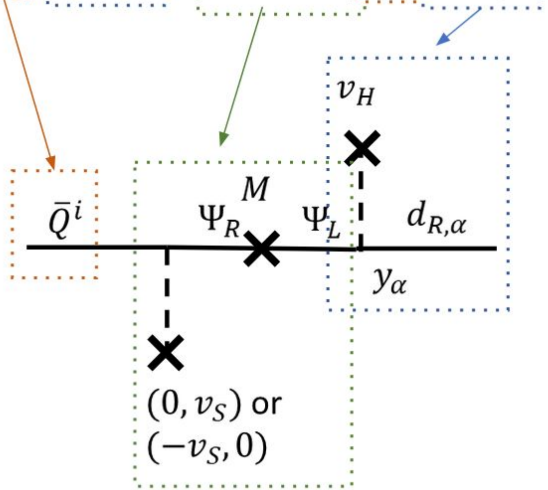
→ Standard approach: add another U(1) factor distinguishing 1st and 2nd

and therefore a new spurion…

→ We take a step back and realise that y_d^α and $\tilde{y}_d^\alpha$ are not necessarily independent parameters

→ Let's consider a simple model with a $SU(2)_f$ breaking scalar S_i and a VL quark

$$y_d^\alpha \delta Y_i \bar{Q}^i \cdot H\, d_{R,\alpha} + \tilde{y}_d^\alpha \delta Y^{\dagger,i} \epsilon_{ij} \bar{Q}^j \cdot H\, d_{R,\alpha}$$



A simple VL-driven example

$$L \supset \delta Y(\bar{Q}^1 \cdot H\,(y_d^\alpha d_{R,\alpha}) - \delta Y\,(\bar{Q}^2 \cdot H\,(\tilde{y}_d^\alpha d_{R,\alpha}) + Y_{3,d} \bar{Q}_3 \cdot H\, b_R$$

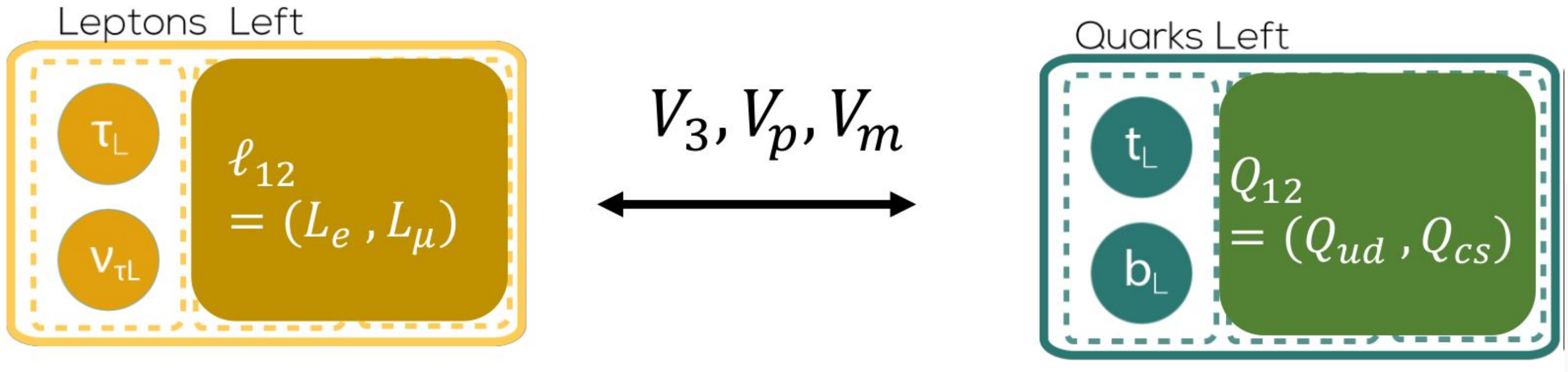
Leads to $y_d^\alpha \propto \tilde{y}_d^\alpha$

→ The down-quark mass matrix is only rank 2

$$L \supset \delta Y(\bar{\tilde{Q}}^2 \cdot H\,(y_d^\alpha d_{R,\alpha}) + Y_{3,d} \bar{Q}_3 \cdot H\, b_R$$

□ Repeat for the third generation

The « flavour-transfer » mechanism



rather than break flavour, the new gauge bosons transfer flavour from one fermionic sector to another

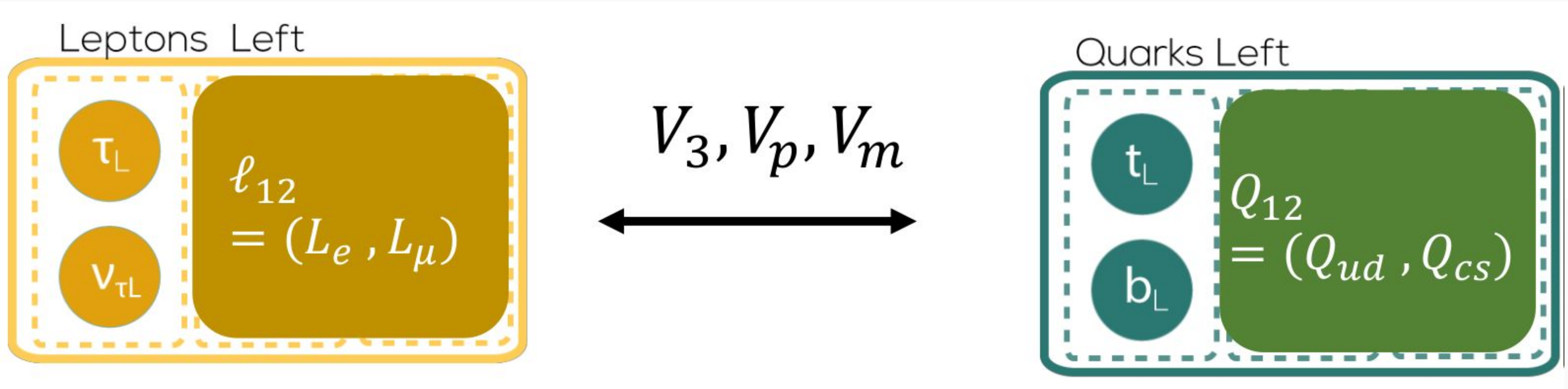
A flavour-violating transition ΔF_f in one fermionic sector is pairwise related to $\Delta F'_f$ in another

Four-fermion operators arising from flavour gauge boson exchanges satisfy $\Delta F_f + \Delta F'_f = 0$

➔ Ensures overall balance in the flavour structure.

➔ Only the $SU(2)_f \times SU(2)_f \times U(1)_Y$ mixed anomaly is non-zero

The « flavour-transfer » mechanism



rather than break flavour, the new gauge bosons transfer flavour from one fermionic sector to another

$$V_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad V_p = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad V_m = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The corresponding generators in flavour space

Building the finite-temperature effective potential: Truncated Full Dressing (TFD) vs Dimensional Reduction (DR)

TFD: $V_{eff}(\mu, T) \rightarrow V_{eff}(\mu + \pi T, T)$

DR: $V_{4deff}(\mu, T) \rightarrow V_{3deff}(\mu_3, T)$

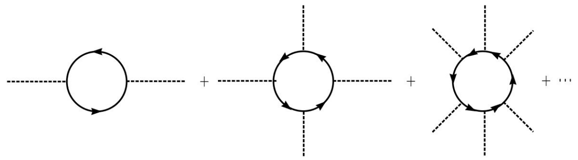
Thermal corrections : TFD vs DR

M. Quiros, ICTP HEPAC Summer School (1998)
arXiv: hep-ph/9901312
D. Curtin, P. Meade, and H. Ramani, EPJC 78 (2018)
arXiv: 1612.00466

How to compute the effective thermal potential ?

- ➔ Describe the correlation functions a QFT in a thermal bath, Green's functions can be computed by compactifying time along the imaginary time direction
- ➔ Stability of the vacuum be estimated from this quantity (equivalent to free energy in thermodynamics)

Stay in 4d, every loop comes with an infinite sum from the modes in along the imaginary time direction



Standard approach - TFD

Compactify 4d theory onto circle of radius $\beta = T^{-1}$. Phenomena on length scales $L >> \beta$ do not "feel" compact dimension, and thus can be described by a purely 3d EFT. Integrate out the heavy & $n > 0$ modes and match the 4d theory to a 3d theory.

Modern "EFT-like" approach - DR
partially automated through DRalgo

Effective potential: Truncated Full Dressing (Parwani)

$$V_{\text{tree}}(\phi) = -\frac{1}{2}\mu_\phi^2\phi^2 + \frac{1}{4}\lambda_\phi\phi^4 + \frac{1}{2}\mu_s^2|s|^2 + \frac{1}{4}\lambda_s|s|^4 + \frac{1}{2}\lambda_{\phi s}\phi^2|s|^2$$

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$SU(2)_f$	DoF
Φ	1	1	0	2	1
S	3	1	2/3	2	$3\times 2\times 2 = 12$
V	1	1	0	3	$3\times 3 = 9$

$$V(\phi, T) = V_{tree}(\phi) + V_{CW}(\phi) + V_T(\phi, T)$$

$$V_{\text{CW}}(\phi) = \sum_{i=\phi,\chi,f,s} \pm\frac{n_i}{64\pi^2}m_i^4\left[\log\left\{\frac{m_i^2}{\mu^2}\right\} - C_i\right]$$

Gives the usual log-like
Coleman Weinberg terms

$$V_T(\phi, T) = \sum_i \frac{n_i T^4}{2\pi^2} J_B\left(\frac{m_i^2}{T^2}\right) + \sum_i \frac{n_i T^4}{2\pi^2} J_F\left(\frac{m_i^2}{T^2}\right)$$

$$J_{B/F}(a) = \pm\int_o^\infty dy y^2 \log\left[1 \mp e^{-\sqrt{y^2+a}}\right]$$

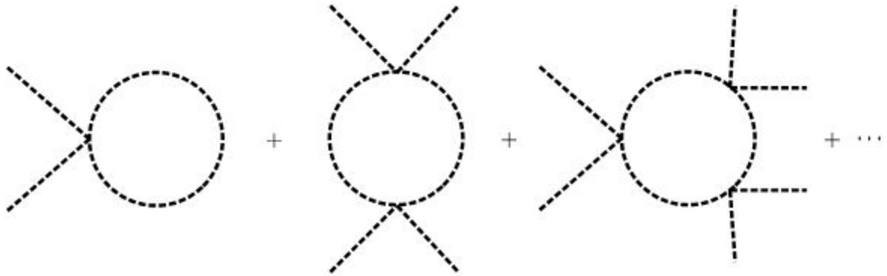
Effective potential: Truncated Full Dressing (Parwani)

We consider the simplest model of one self-interacting real scalar field, described by the lagrangian

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi - V_0(\phi)$$

with a tree-level potential

$$V_0 = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$$

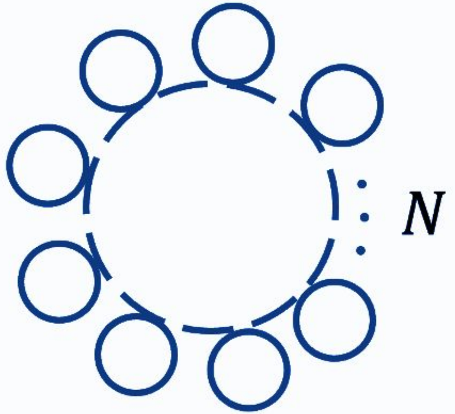


The one-loop correction to the tree-level potential should be computed as the sum of all 1PI diagrams with a single loop and zero external momenta.

Effective potential: Truncated Full Dressing (Parwani)

$$m^2(\phi) = m^2_{\text{tree}}(\phi) + \Pi(\phi, T)$$

$\sim g^2 T^2$



$$V_{\text{CW}}(\phi) = \sum_{i=\phi,\chi,f,s} \pm \frac{n_i}{64\pi^2} m_i^4 \left[\log\left\{\frac{m_i^2}{\mu^2}\right\} - C_i\right]$$

$$V_T(\phi, T) = \sum_i \frac{n_i T^4}{2\pi^2} J_B\left(\frac{m_i^2}{T^2}\right) + \sum_i \frac{n_i T^4}{2\pi^2} J_F\left(\frac{m_i^2}{T^2}\right)$$

$$\pi_\phi = \pi_\chi = \frac{\lambda_\phi}{2} + \frac{9}{48} g_f^2$$

$$\pi_{\text{f}}^L = \frac{3}{2} g_f^2$$

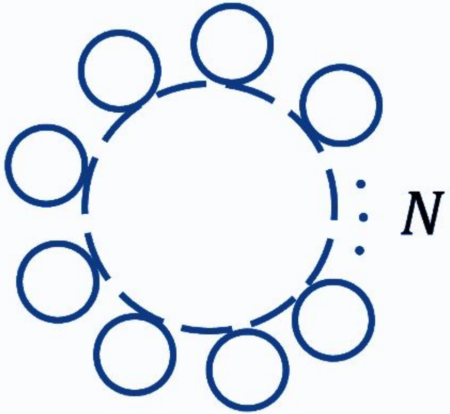
Effective potential: Truncated Full Dressing (Parwani)

$$m^2(\phi) = m^2_{\text{tree}}(\phi) + \Pi(\phi, T)$$

$\sim g^2\, T^2$

$$V_{\text{CW}}(\phi) = \sum_{i=\phi,\chi,f,s} \pm \frac{n_i}{64\pi^2} m_i^4 \left[\log\left\{\frac{m_i^2}{\mu^2}\right\} - C_i\right]$$

$$V_T(\phi, T) = \sum_i \frac{n_i T^4}{2\pi^2} J_B\left(\frac{m_i^2}{T^2}\right) + \sum_i \frac{n_i T^4}{2\pi^2} J_F\left(\frac{m_i^2}{T^2}\right)$$



$$\pi_\phi = \pi_\chi = \frac{\lambda_\phi}{2} + \frac{9}{48} g_f^2$$

$$\pi_{\text{f}}^L = \frac{3}{2} g_f^2$$

Temperature corrections in “Parwani” method

$$V_T(\phi, T) = \sum_i \frac{n_i T^4}{2\pi^2} J_B\left(\frac{m_i^2}{T^2}\right) + \sum_i \frac{n_i T^4}{2\pi^2} J_F\left(\frac{m_i^2}{T^2}\right)$$

$$J_{B/F}(a) = \pm \int_o^\infty dy y^2 \log\left[1 \mp e^{-\sqrt{y^2+a}}\right]$$

$$J_{B,F}^{low}(a) \approx -\sqrt{\frac{\pi}{2}} a^{3/4} e^{-\sqrt{a}} \left(1 + \frac{15}{8} a^{-1/2} + \frac{105}{128} a^{-1}\right)$$

$$J_B^{high}(a) \approx -\frac{\pi^4}{45} + \frac{\pi^2}{12} a - \frac{\pi}{6} a^{3/2} - \frac{a^2}{32}\left(\log(a) - c_B\right)$$

$$J_B(a) \approx e^{-\left(\frac{a}{6.3}\right)^4} J_B^{high}(a) + \left(1 - e^{-\left(\frac{a}{6.3}\right)^4}\right) J_B^{low}(a)$$

Effective potential: Truncated Full Dressing (Parwani)

D. Croon , O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, JHEP 04 (2021)
arXiv: 2009.10080

Multiple sources of theoretical uncertainty :

- ➔ Nonperturbativity (IR modes at high T) [Linde 1980]
- ➔ Inconsistencies (non-negligible Im{V}) [Weinberg & Wu 1987; Weinberg 1992]
- ➔ higher-order perturbative corrections [Arnold & Espinosa 1992]
- ➔ gauge dependence [Laine 1994]
- ➔ renormalisation scale dependence [Farakos *et al.* 1994]

Uncertainties

D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, G. White, JHEP 04 (2021)
arXiv: 2009.10080

$\Delta\Omega_{\rm GW}/\Omega_{\rm GW}$	4d approach	3d approach
RG scale dependence	$\mathcal{O}(10^2 - 10^3)$	$\mathcal{O}(10^0 - 10^1)$
Gauge dependence	$\mathcal{O}(10^1)$	$\mathcal{O}(10^{-3})$
High-T approximation	$\mathcal{O}(10^{-1} - 10^0)$	$\mathcal{O}(10^0 - 10^2)$
Higher loop orders	unknown	$\mathcal{O}(10^0 - 10^1)$
Nucleation corrections	unknown	$\mathcal{O}(10^{-1} - 10^0)$
Nonperturbative corrections	unknown	unknown

Sources of theoretical uncertainty and relative importance quantified by the parameter ΔΩGW /ΩGW over the range M = {580 − 700} GeV in the SMEFT. Although we do not have reliable estimates for the uncertainties of the 4d approach due to higher loop orders and nucleation corrections, they are expected to be much larger than the corresponding uncertainties of the 3d approach

Thermal resummation: DR approach to IR sensitivities of light bosons

L. Gould and T.V. I. Tenkanen, JHEP 06 (2021)
arXiv: 2104.04399

How to compute the effective thermal potential ?

- ➔ Describe the correlation functions of a QFT in a thermal bath, Green's functions can be computed by compactifying time along the imaginary direction: $t \to i\beta$ for $\beta = T^{-1}$
- ➔ Phenomena on length scales $L \gg \beta$ do not "feel" compact dimension, and thus can be described by a purely 3d EFT

$$\underbrace{\left(\frac{g}{4\pi}\right)^2 \pi T}_{\text{"ultrasoft"}} \ll \underbrace{\left(\frac{g}{4\pi}\right)^{3/2}}_{\text{supersoft}} \ll \underbrace{\left(\frac{g}{4\pi}\right) \pi T}_{\text{"soft"}} \ll \underbrace{\left(\frac{g}{4\pi}\right)^{1/2}}_{\text{semisoft}} \pi T \ll \underbrace{\pi T}_{\text{"hard"}}$$

- ➔ At sufficiently high T, the infinite tower of non-zero modes can then be integrated out, leaving only the purely spatial (static) zero modes which live at the soft scale.
- ➔ DR approach resums the leading IR-divergent contributions captured by daisy resummation, while additionally correctly resumming large logs that spoil the perturbation theory

$\Rightarrow V_{eff}$ remains perturbative at high T

Calculation of propagators depends on the chosen path C in the complex plane going from an initial arbitrary time t to t - $i\beta$.
Momentum of propagators replaced with Euclidean momentum or Matsubara frequencies,

$$p_\mu = (i\omega_n, \vec{k}),$$

$$\omega_n = \begin{cases} 2\pi n T & \text{bosons} \\ 2\pi(n+1)T & \text{fermions} \end{cases}$$

$$n_B(E_p, T) \equiv \left(e^{E_p/T} - 1\right)^{-1}$$
$$n_F(E_p, T) \equiv \left(e^{E_p/T} + 1\right)^{-1}$$
$$E_p = \sqrt{p^2 + m^2}$$

Replace integrals over p^0 with Matsubara sums:

$$\int \frac{dp^0}{2\pi} \to \frac{1}{\beta} \sum_n$$

Using DRalgo

4D parameters, g lambda, mssquared ...

V4d V3d, full (with the tower of modes)

Labelling for the scalar fields: (Phi_i,S) = (f_r1,f_r2,f_i1,f_i2,S)

PerformDRhard[]

pi T

V3d, cut → "Soft", Temporal model A0 + Gauge Ai + zero mode Scalar fields

3D parameters, g3d, lambda3d, mssquared3d ...

PerformDRsoft[{5}] → Tells DRAlgo to further integrate the heavy scalar singlet S in the model

g T

V3dUS → "Ultra Soft", Gauge Ai + zero mode Scalar field doing the PT

mulow

3D US parameters, g3dUS, lambda3dUS ...

→ LO, NLO and NNLO, using standard loop techniques in 3D

$$= \frac{1}{4}\left(2\, m\phi sq3dUS\, \phi^2 + \lambda\phi 3dUS\, \phi^4\right) - \frac{3\left(g3dUS^2\, \phi^2\right)^{3/2} + 12\left(m\phi sq3dUS + \lambda\phi 3dUS\, \phi^2\right)^{3/2} + 4\left(m\phi sq3dUS + 3\,\lambda\phi 3dUS\, \phi^2\right)^{3/2}}{48\,\pi}$$

Power counting

To illustrate next-to-leading order dimensional reduction, we consider a schematic model with scalar mass parameter μ^2, scalar quartic coupling λ, and gauge coupling g. Given the power counting $\mu^2 \sim g^2T^2$, $\lambda \sim g^2$, the matching of the mass parameter is

$$\bar{\mu}_3^2 = \underbrace{\boxed{\overset{\text{tree-level}}{\mu^2} + \overset{\text{1-loop}}{\#g^2T^2}}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\overset{\text{1-loop}}{\#g^2\mu^2} + \overset{\text{2-loop}}{\#g^4T^2}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^6)$$

$$+ \underbrace{\boxed{\overset{\text{1-loop}}{\#g^2m_{\text{D}}}}}_{\mathcal{O}(g^3)} + \underbrace{\boxed{\overset{\text{2-loop}}{\#g^4}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^5)\ , \tag{1.3}$$

where the first line (with even powers of g) results from the first step, and the second line (with odd power of g) from second step of the dimensional reduction. In practice, *full* $\mathcal{O}(g^4)$ contributions are included. Going to higher orders, requires a three-loop computation for both steps of the dimensional reduction. The situation is similar for the coupling:

Power counting

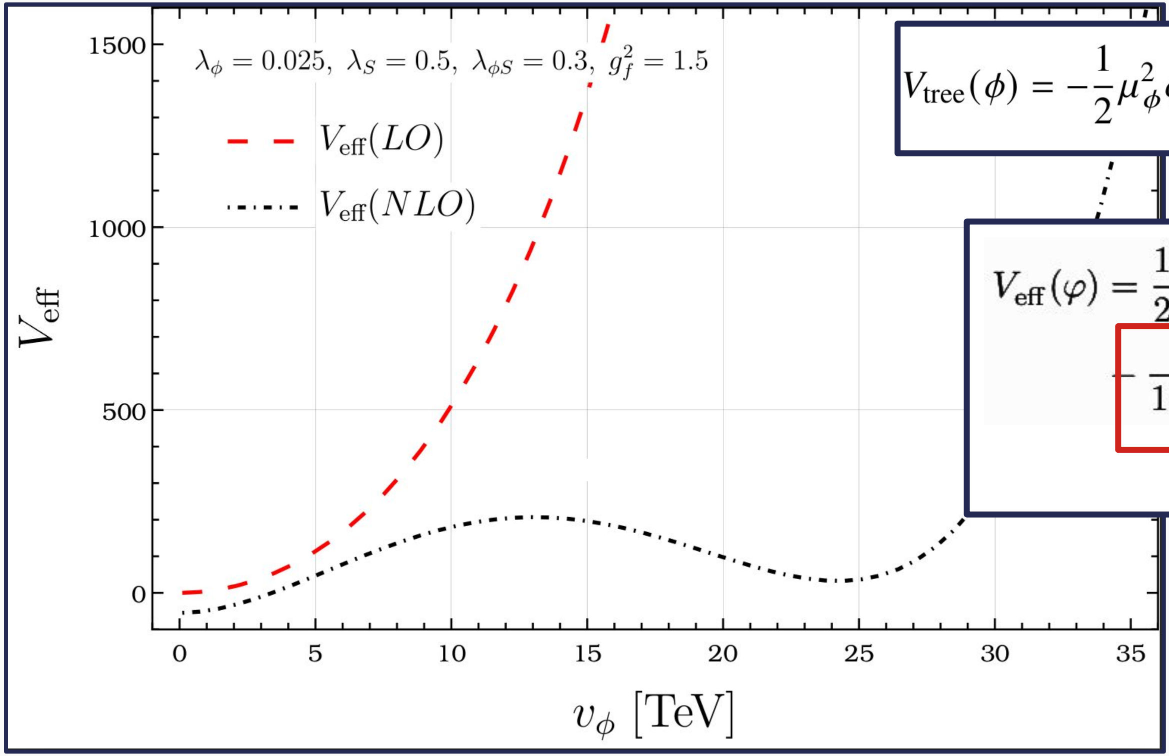
$$\bar{\lambda}_3 = \underbrace{\boxed{\begin{array}{c}\text{tree-level}\\ T\lambda\end{array}}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\begin{array}{c}\text{1-loop}\\ \#g^4\end{array}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^6)$$

$$+ \underbrace{\boxed{\begin{array}{c}\text{1-loop}\\ \#\frac{g^4}{m_\mathrm{D}}\end{array}}}_{\mathcal{O}(g^3)} + \underbrace{\boxed{\begin{array}{c}\text{2-loop}\\ \#\frac{g^6}{m_\mathrm{D}^2}\end{array}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^5)\,.$$

$$V^{\mathrm{3d}}_{\mathrm{eff}} = \underbrace{V^{\mathrm{3d}}_{\mathrm{tree}}}_{\mathcal{O}(g^2)} + \underbrace{V^{\mathrm{3d}}_{\text{1-loop}}}_{\mathcal{O}(g^3)} + \underbrace{V^{\mathrm{3d}}_{\text{2-loop}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^5)\,.$$

Effective potential: LO vs NLO

BM: $\lambda_\phi = 0.025,\ \lambda_S = 0.5,\ \lambda_{\phi S} = 0.3,\ M_S = 50\ \text{TeV}$



$$V_{\text{tree}}(\phi) = -\frac{1}{2}\mu_\phi^2\phi^2 + \frac{1}{4}\lambda_\phi\phi^4 + \frac{1}{2}\mu_s^2|s|^2 + \frac{1}{4}\lambda_s|s|^4 + \frac{1}{2}\lambda_{\phi s}\phi^2|s|^2$$

$$V_{\text{eff}}(\varphi) = \frac{1}{2}m_{\phi,3}^2\varphi^2 + \frac{1}{4}\lambda_{\phi,3}\varphi^4 - \frac{1}{16\pi}g_3^3\varphi^3 - \frac{1}{12\pi}\left(3(m_D^2 + \frac{1}{2}h_3\varphi^2\right)^{\frac{3}{2}}$$

vector boson term

temporal scalar term

$$V_{\text{eff}}^{3d}\,T \simeq V_{\text{eff}}^{4d} \quad \text{to } \mathcal{O}(g^3)$$

$$\phi^{3d}\sqrt{T} \simeq \phi^{4d} \quad \text{to } \mathcal{O}(g^2)$$

J. Gould and T.V.I. Tenkanen, JHEP 06 (2021)