Does the Bohm (Hillas) limit determine the maximum energy of shock-accelerated cosmic rays?

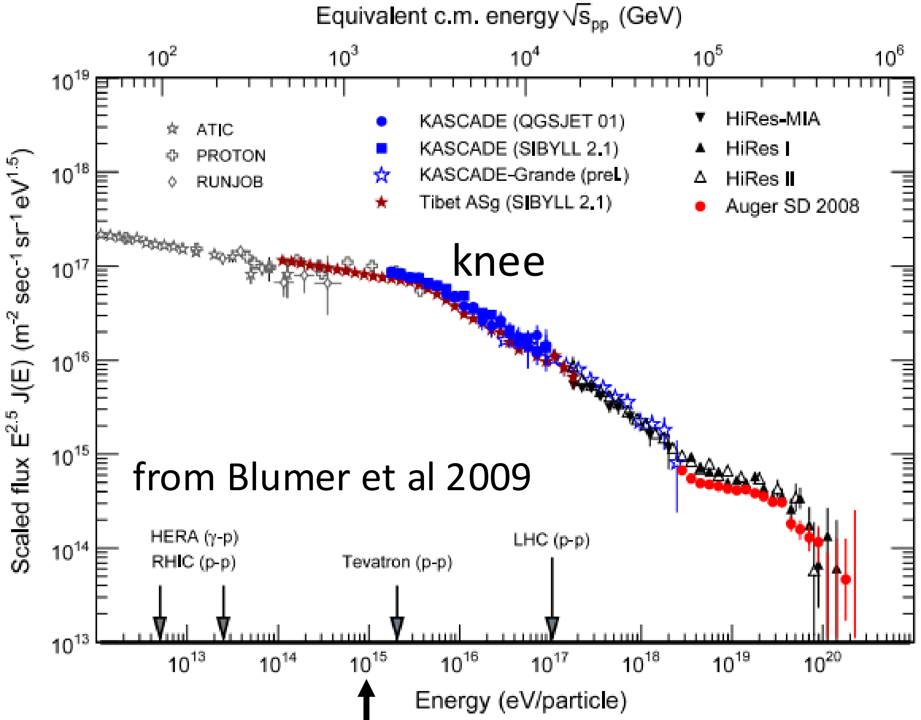
a sequel to
Cosmic ray acceleration and transport with magnetic mirroring
by
Tony Bell, James Matthews, Andrew Taylor, Gwenael Giacinti

Cassiopeia A, the brightest (extra-solar) radio source in the sky

http://hubblesite.org/newscenter/archive/releases/2006/30/image/a/format/xlarge_web/

Galactic CR acceleration

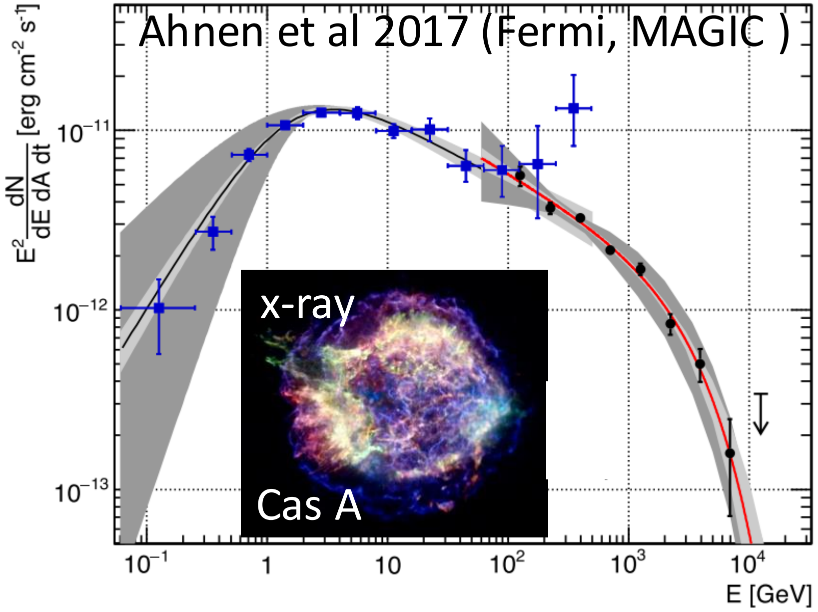
where/what are the Pevatrons?



knee

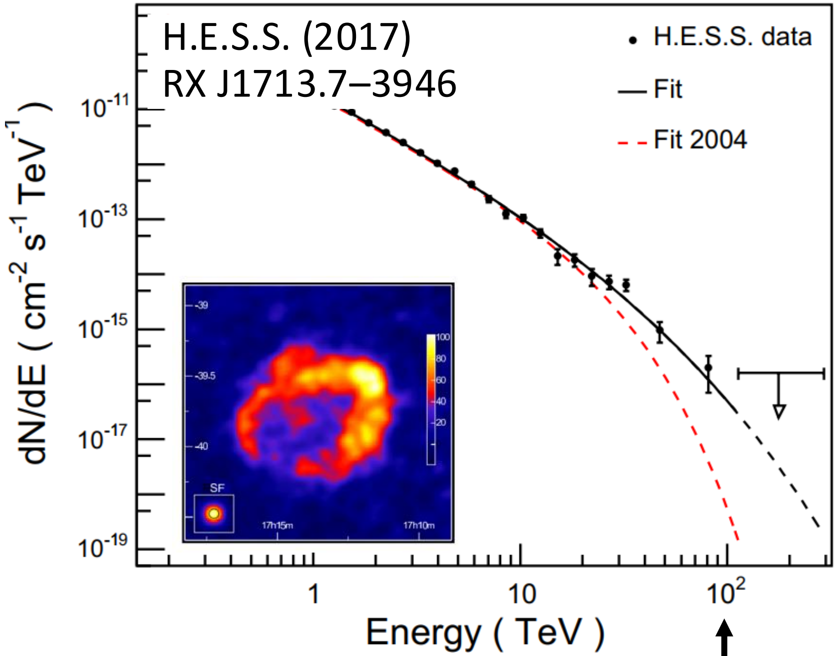
from Blumer et al 2009

1PeV



Ahnen et al 2017 (Fermi, MAGIC)

10TeV

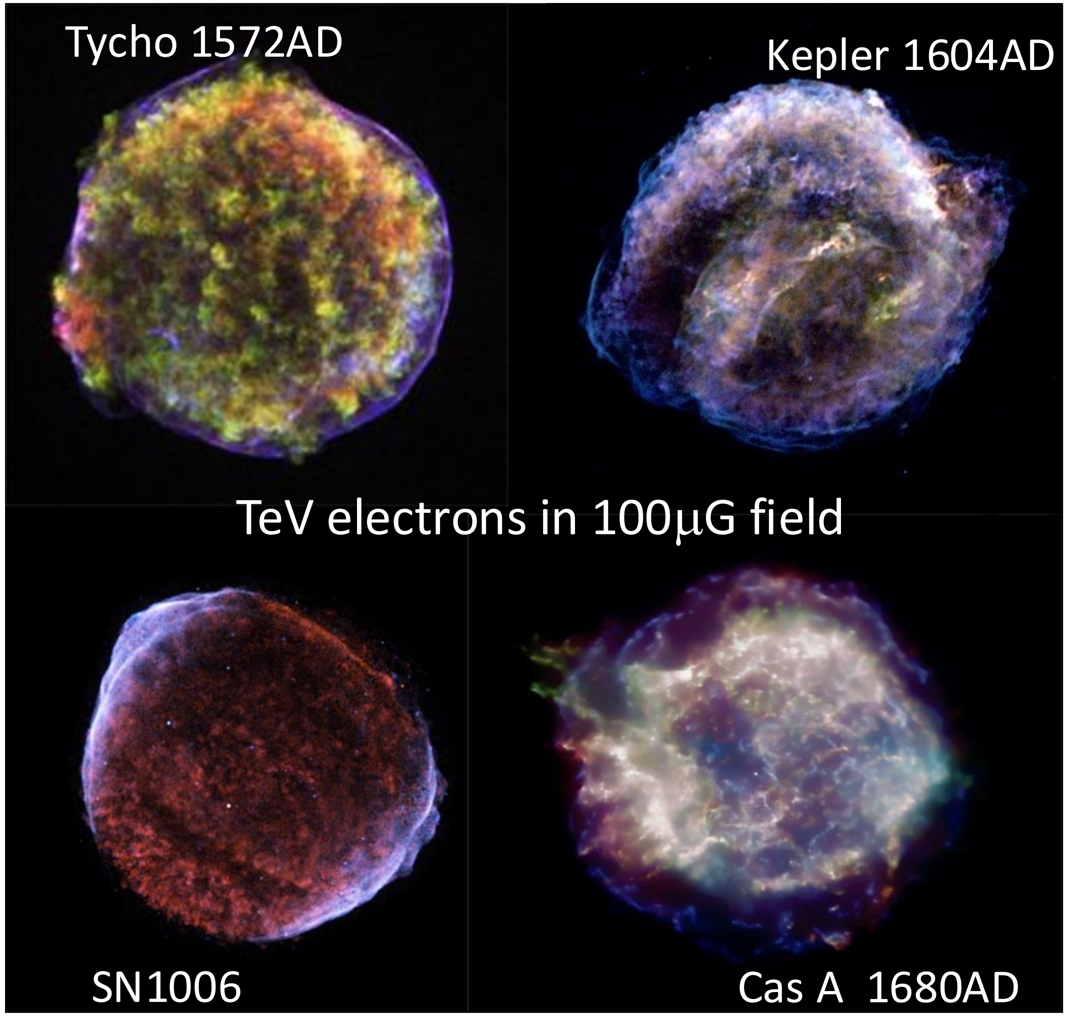


H.E.S.S. (2017)
RX J1713.7–3946

100TeV

YOUNG SNR CANDIDATES

Historical shell supernova remnants



TeV electrons in 100μG field

Chandra observations (x-ray)

Radius, R = 2-10pc

Present velocity = 5000-7000 km s^{-1}

Mean velocity, R/t = 6000-10000 km s^{-1}

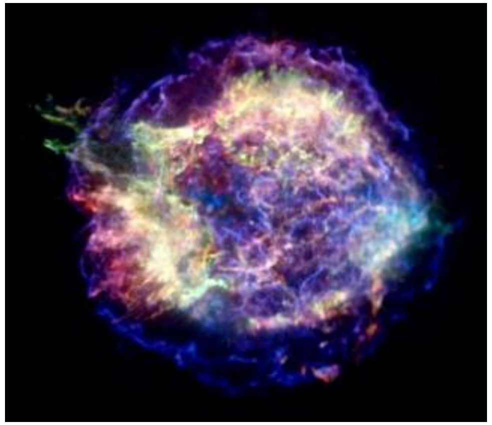
NASA/CXC/Rutgers/
J.Hughes et al.

NASA/CXC/Rutgers/
J.Warren & J.Hughes et al.

NASA/CXC/NCSU/
S.Reynolds et al.

NASA/CXC/MIT/UMass Amherst/
M.D.Stage et al.

Hillas/Bohm limit for Cas A

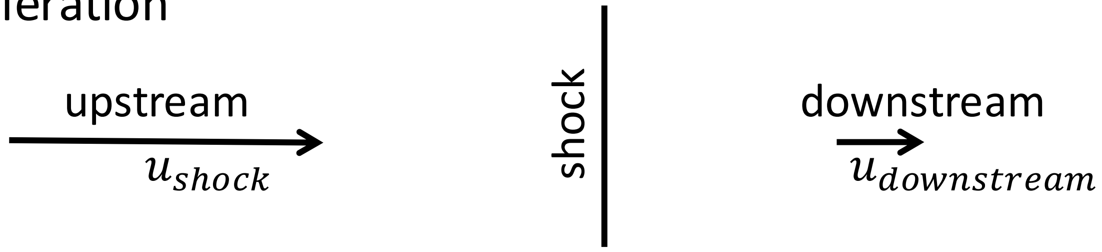


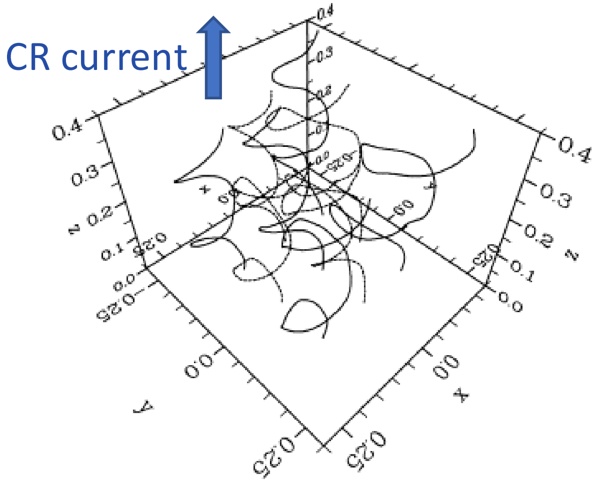
Maximum CR energy for protons (in eV) $\varepsilon = uBR$

$u = 5000$ kms^{-1}
$B = 500\ \mu$G (Völk, Berezhko & Ksenofontov, 2005) $\Longrightarrow$ $\varepsilon = 13$PeV
$R = 1.7$ pc

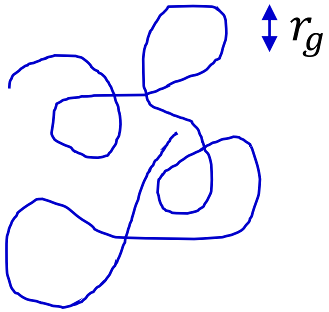
Bohm diffusion falls short of Hillas limit

Diffusive shock acceleration





assume Bohm diffusion $D_{Bohm} = cr_g = \frac{c\epsilon}{B}$



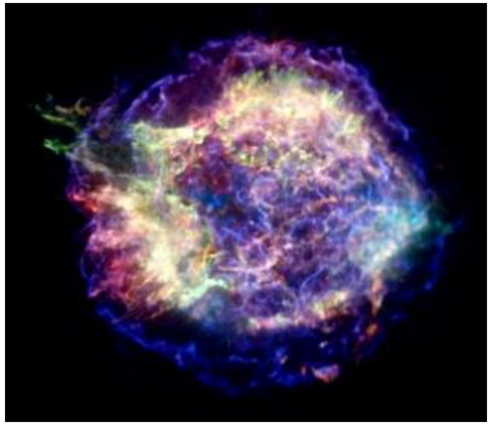
mean free path ~ r_g

magnetic field turbulent on scale of gyroradius

Maximum energy $\epsilon_{eV} = \frac{u_{upstream} B_{shock} R}{4} \left(1 + \frac{4B_{upstream}}{B_{downstream}}\right)^{-1}$

Lagage & Cesarsky (1983)

Hillas/Bohm limit for Cas A



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$R = 1.7$ pc

$\Longrightarrow$ $\varepsilon = 13$PeV

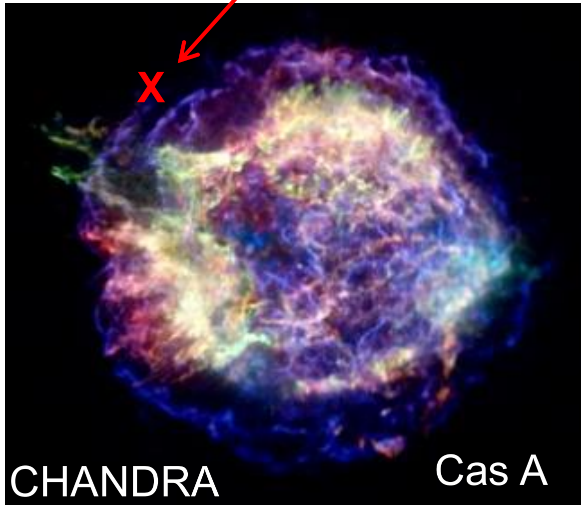
$\varepsilon \sim 2$PeV after Lagage & Cesarsky

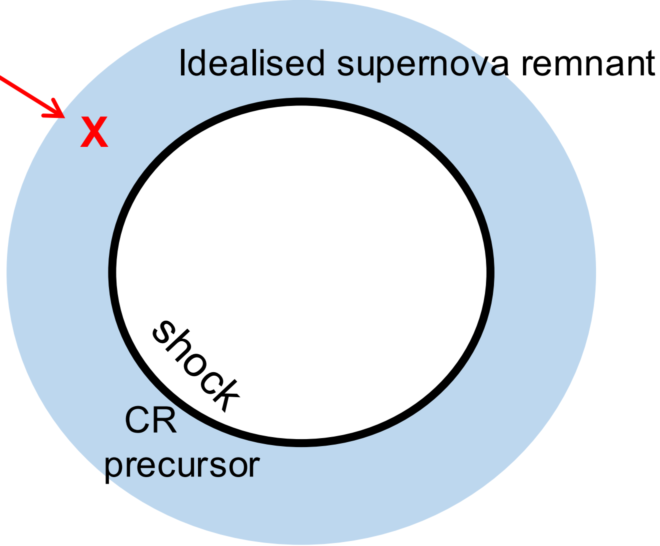
Two likely problems with magnetic field

1) localised to shock
2) structured on scale smaller than PeV proton Larmor radius

Escaping cosmic rays generate their own magnetic field

Magnetic field grows ahead of shock

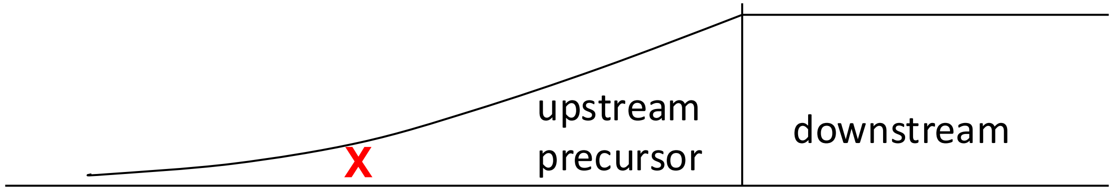



Idealised supernova remnant

X

shock

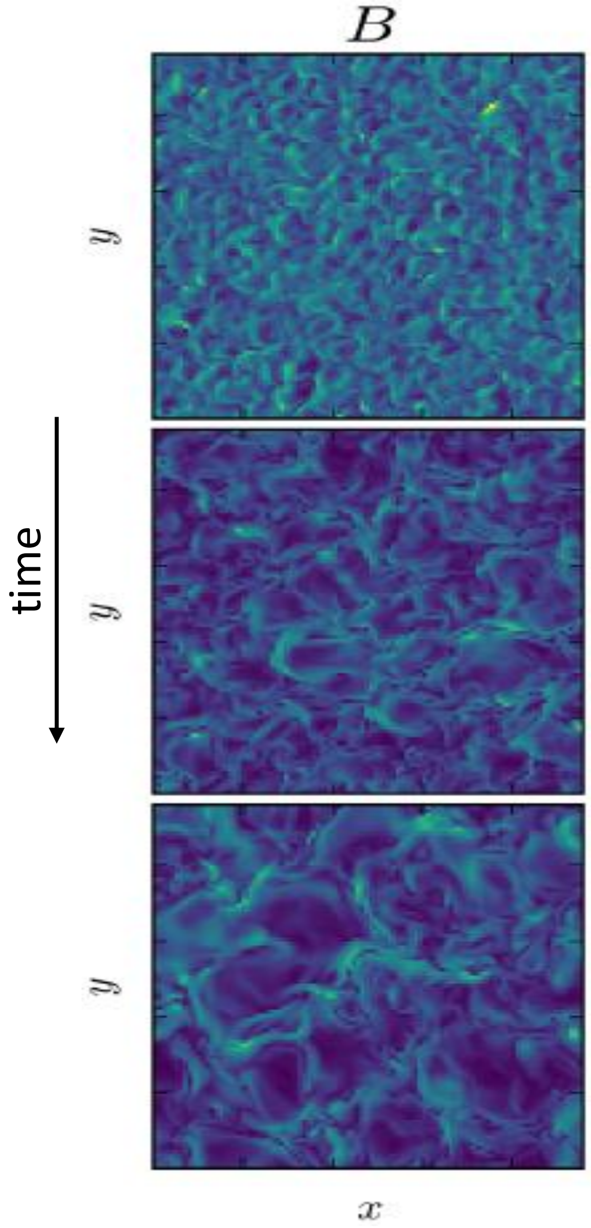
CR precursor


X

upstream precursor

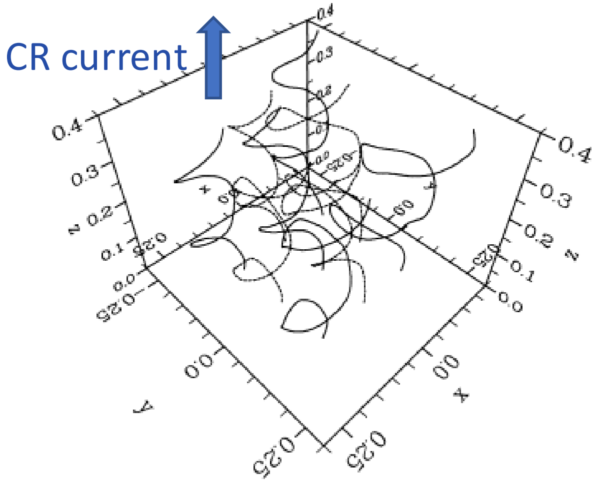
downstream

Shock



Matthews et al 2017

Saturation magnetic field amplification looks promising



Growth requires
1) Tension in field line less than driving jxB force
2) CR Larmor greater than spatial scale of turbulence

$$\frac{B^2}{2\mu_0} = \eta_{CR} \frac{u_{shock}}{c} \frac{1}{2} \rho {u_{shock}}^2$$

Fraction of energy going into CR

Amplifies B by 100x
Saturates by growing to Larmor scale

Problem is the growth rate

Zirakashvili & Ptuskin 2008; Bell et al 2013

$$E_{\max} = 230\, n_{\mathrm{e}}^{1/2} u_7^2 R_{\mathrm{pc}}\ \mathrm{TeV}$$

density
expansion velocity
radius

WHAT ARE THE POSSIBLE SOLUTIONS?

Look beyond middle-aged SNR

- Very young SNR eg SN1987a $E_{\mathrm{max}} = 230\, n_{\mathrm{e}}^{1/2} u_7^2 R_{\mathrm{pc}}\ \mathrm{TeV}$
- Stellar clusters: lots of SN, strong winds, large B, bubble termination shocks
- Galactic centre

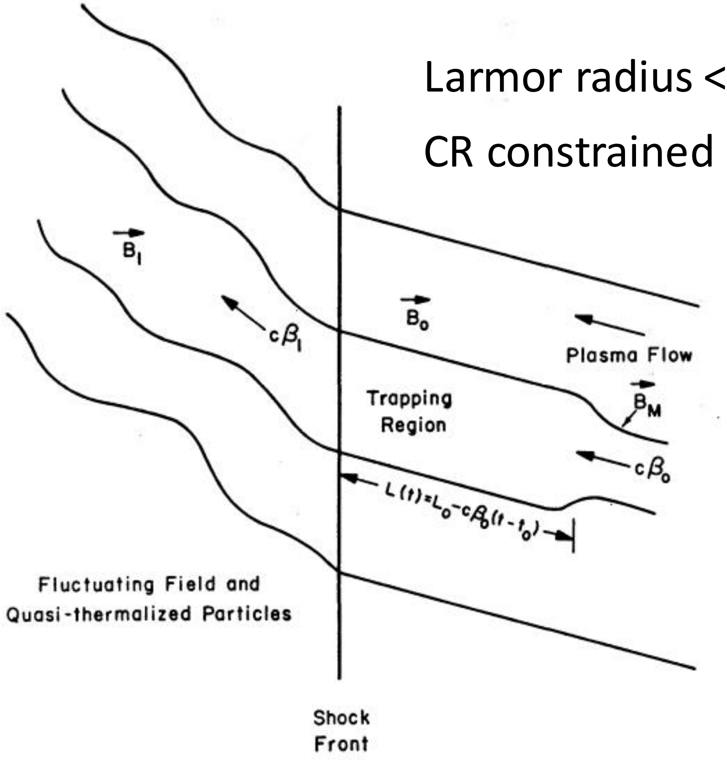
Try an alternative solution:

mirror-dominated shock acceleration (MDSA)

Alternative strategy: can we reach Hillas limit using mirrors?

A MODEL OF FERMI ACCELERATION AT SHOCK FRONTS WITH AN APPLICATION TO THE EARTH'S BOW SHOCK*

J. R. Jokipii†
California Institute of Technology, Pasadena, California
Received June 18, 1965



Larmor radius << scalelength

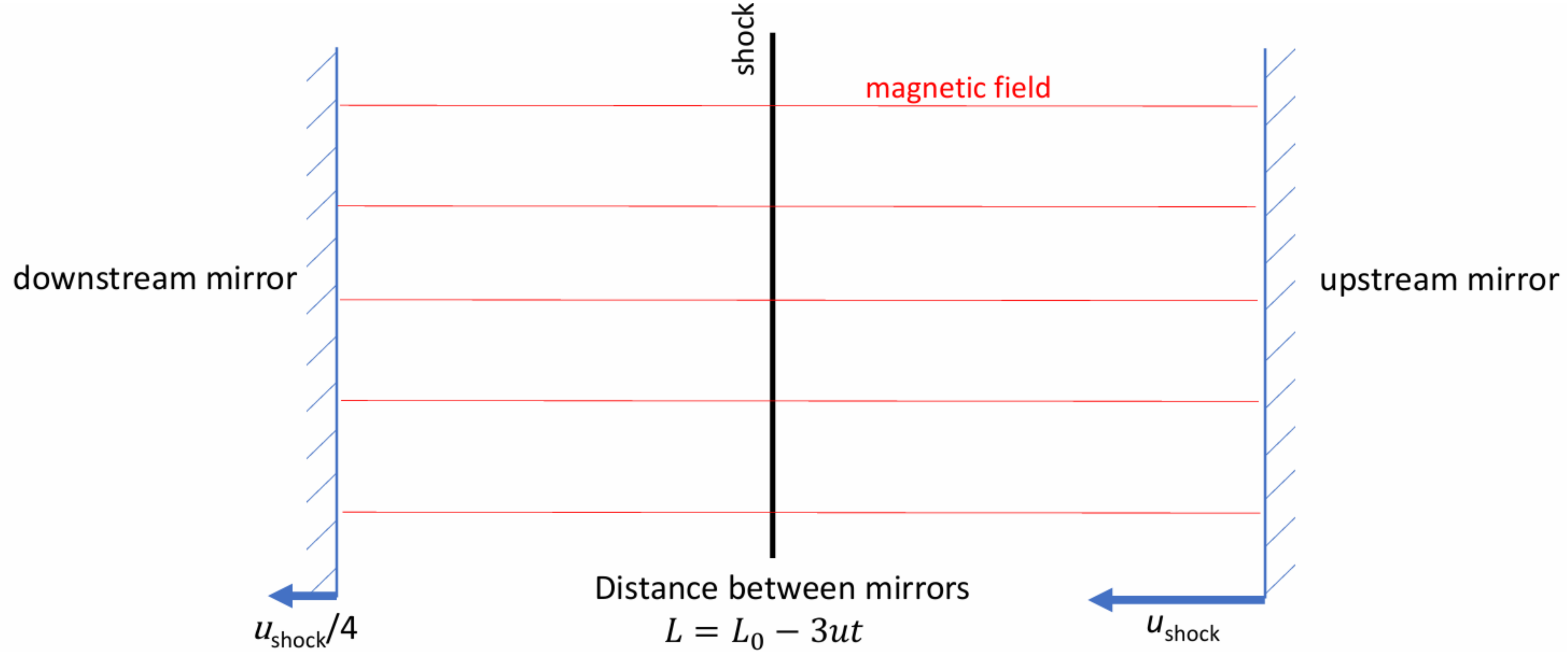
CR constrained to field lines

Fig. 1.—Schematic diagram illustrating the assumed magnetic field configuration and relevant parameters.

† Randy Jokipii
(1939-2022)

Refer to this as mirror-dominated shock acceleration (MDSA)

Idealised mirror-shock acceleration



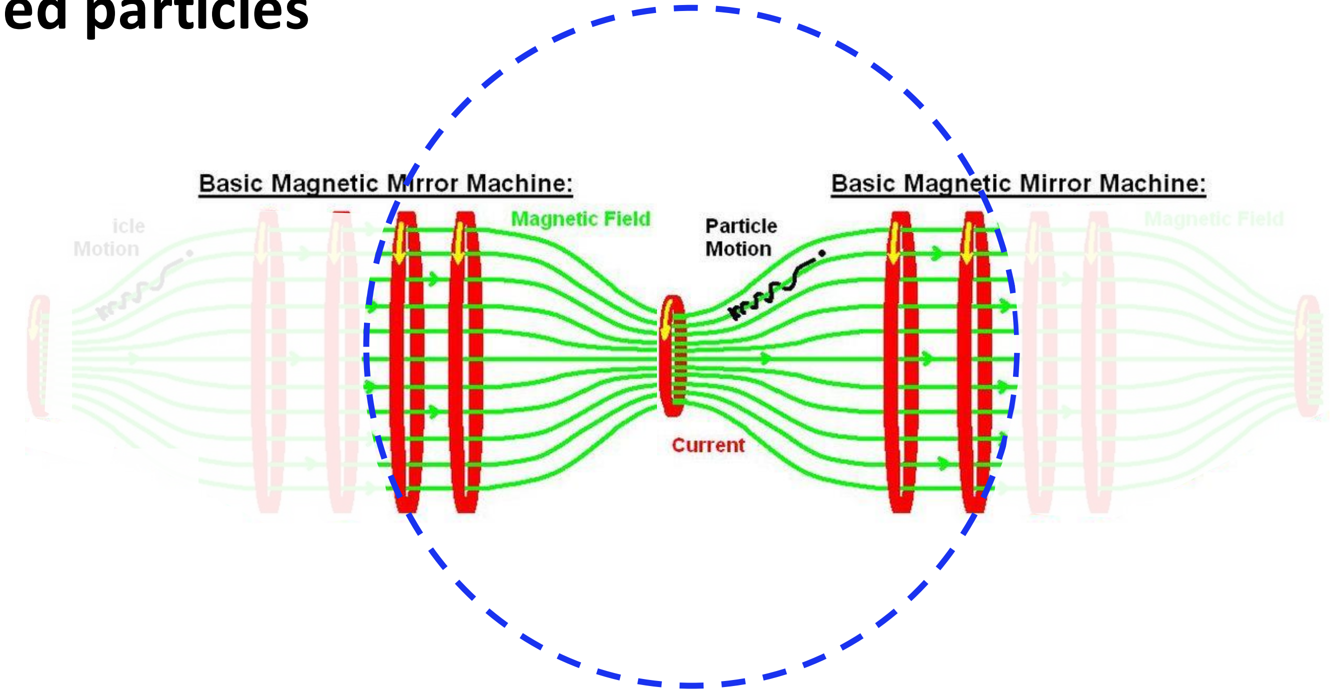
Perfect reflection between mirrors increase parallel momentum

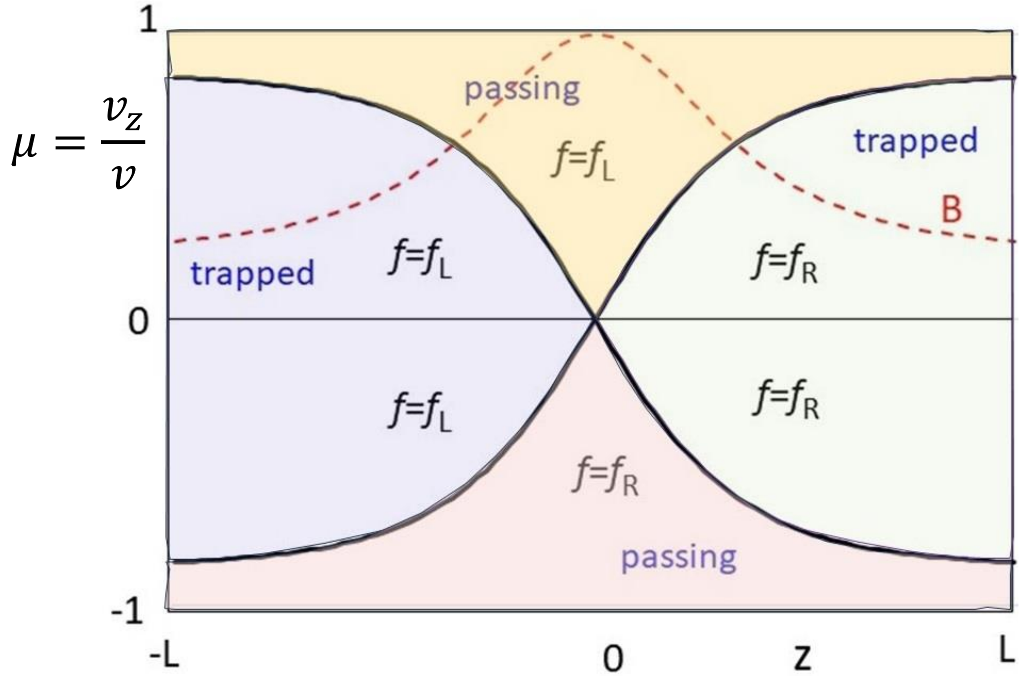
Perpendicular momentum is constant

Energy increases only by small multiple

Jokipii (1966): need scattering by magnetic fluctuations or shock interaction

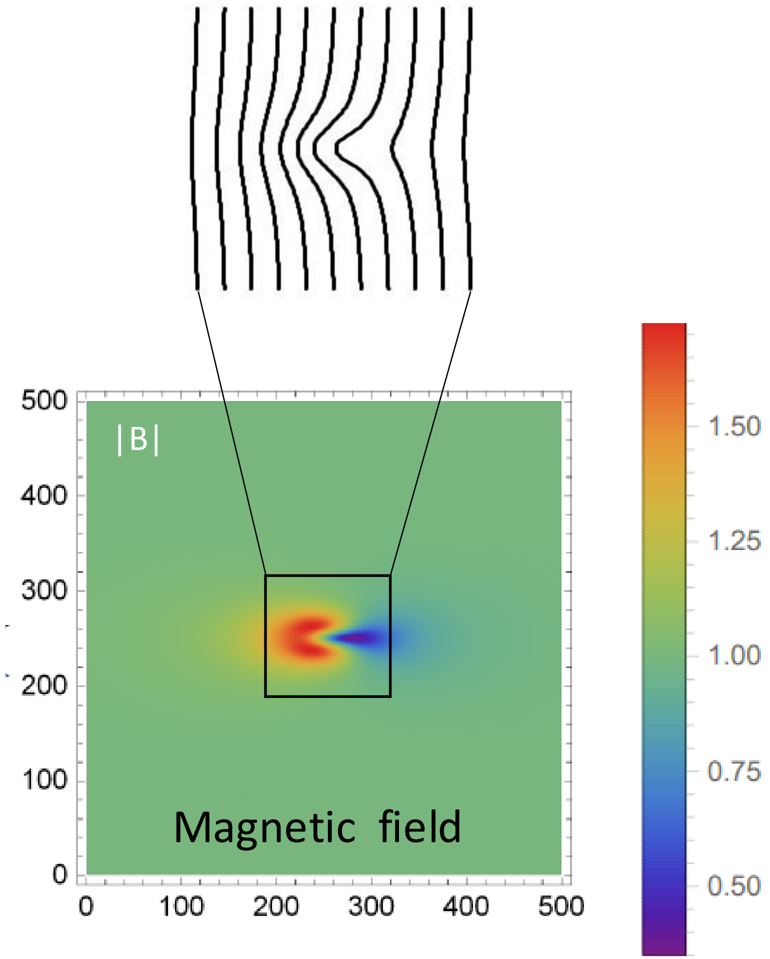
Passing and trapped particles at a single mirror





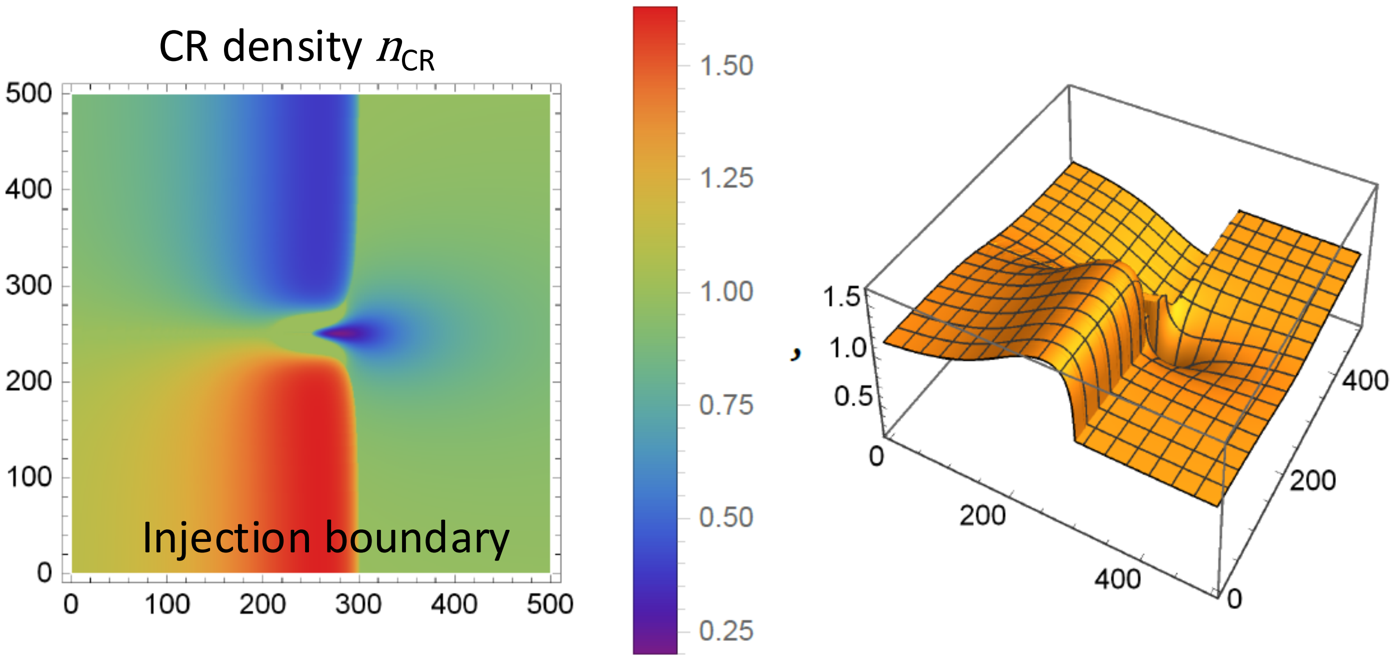
Mirrors do not have to look like mirrors

Magnetic field lines



Magnetic field

CR density n_{CR}



Injection boundary

VLASOV-FOKKER-PLANCK SIMULATIONS (VFP)

VFP modelling in the limit of small Larmor radius

$$\frac{\partial f}{\partial t} + c\mu\frac{\partial f}{\partial x} - \frac{c}{2B}\frac{\partial B}{\partial x}(1-\mu^2)\frac{\partial f}{\partial \mu} = 0$$

$$\mu = \frac{p_x}{p}$$

Mirror term

Vlasov equation for transport along a field line, neglecting cross-field drifts

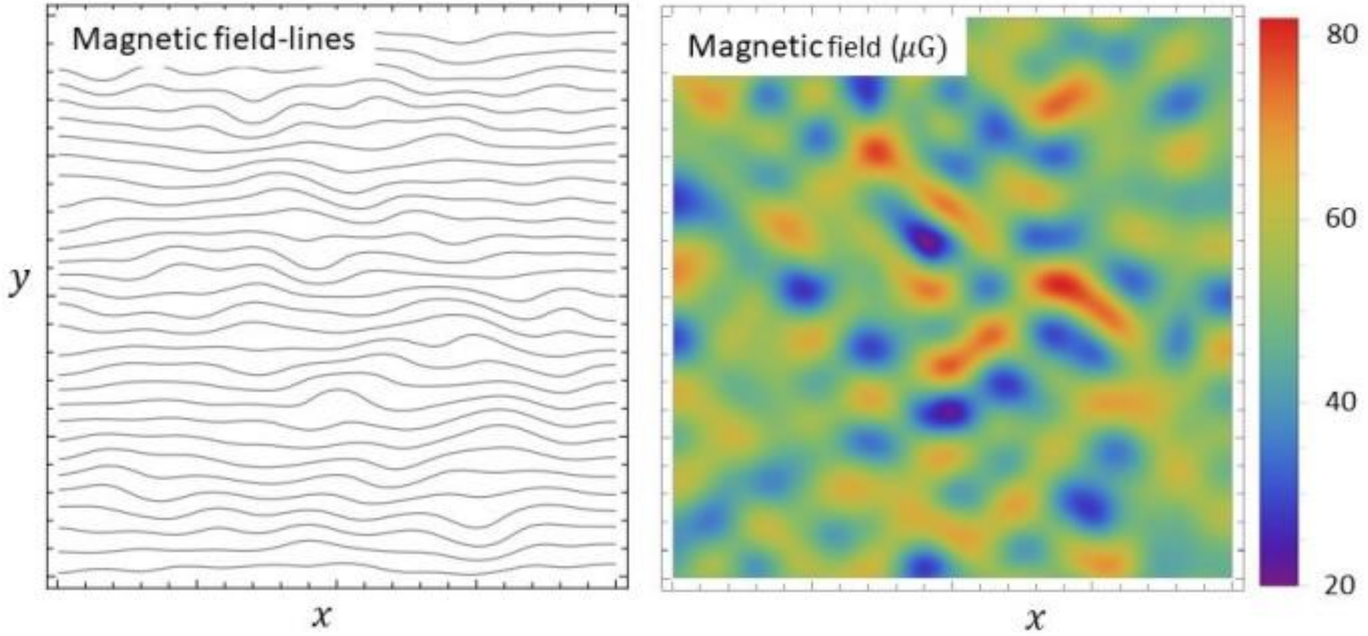
$$f(\mathbf{r},\mu) = \sum_n f_n(\mathbf{r})P_n(\mu)$$

Equation for evolution of coefficients

$$\frac{\partial f_n}{\partial t} = -\frac{n}{2n-1}\mathrm{v}\mathbf{b}.\nabla f_{n-1} - \frac{n+1}{2n+3}\mathrm{v}\mathbf{b}.\nabla f_{n+1} - \frac{n(n+1)}{2}\nu f_n + \frac{\mathrm{v}}{2|\mathbf{B}|}\mathbf{b}.\nabla|\mathbf{B}|\left(\frac{(n+1)(n+2)}{2n+3}f_{n+1} - \frac{n(n-1)}{2n-1}f_{n-1}\right)$$

b is local unit vector in direction of magnetic field

Solve VFP equation for mono-energetic CR in (x,y) plane with B_z =0



Computational box 1.7parsec square

Mean field is 50 μG

CR energy notionally 1 PeV

Initialise with CR at left-hand boundary

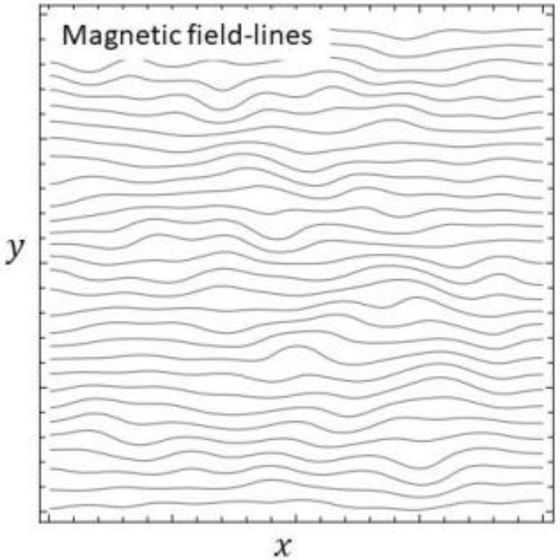
Free escape at right-hand boundary

Periodic boundary condition in vertical direction

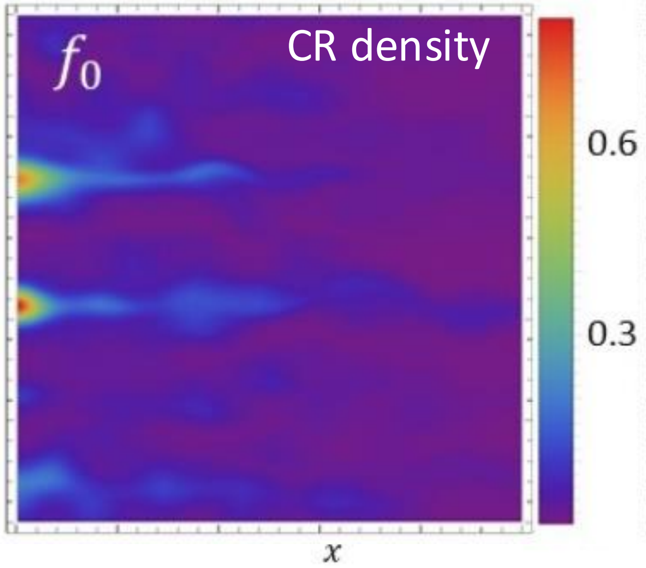
Reference calculation

Angular scattering time is 100 Larmor gyration times

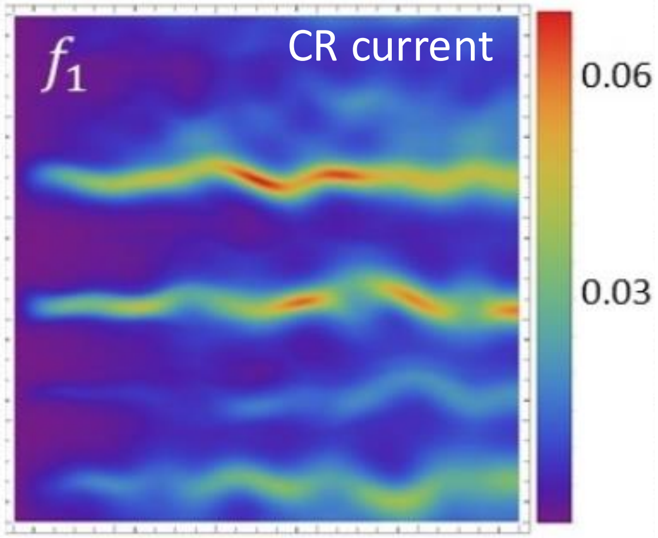
After 50 years



Magnetic field with wiggles



CR trapped by mirroring

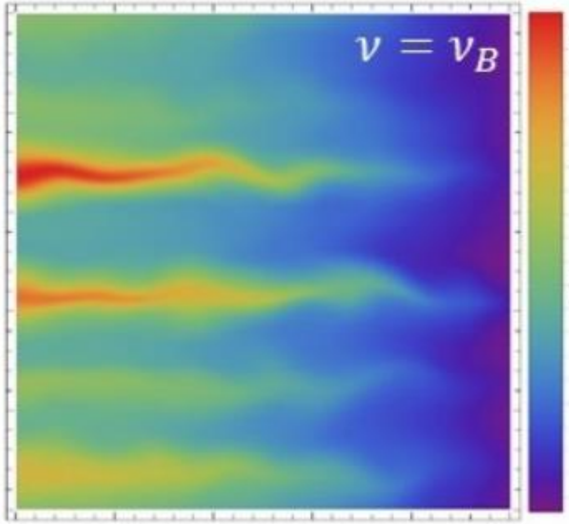


Weak currents along B

Vary the level of angular scattering (same B)

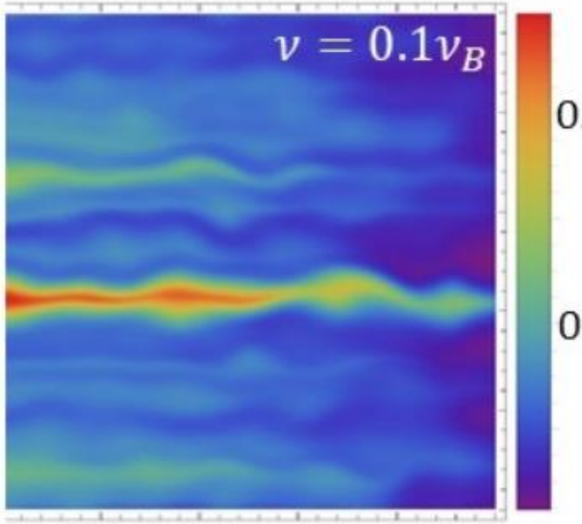
Bohm scattering

mfp ~Larmor radius



Scattering stops

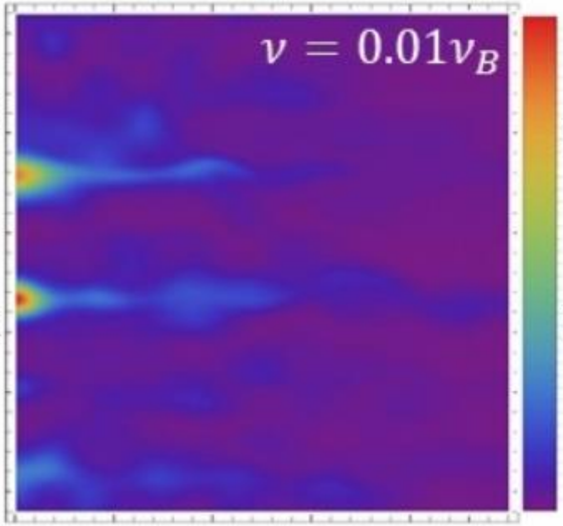
CR escaping



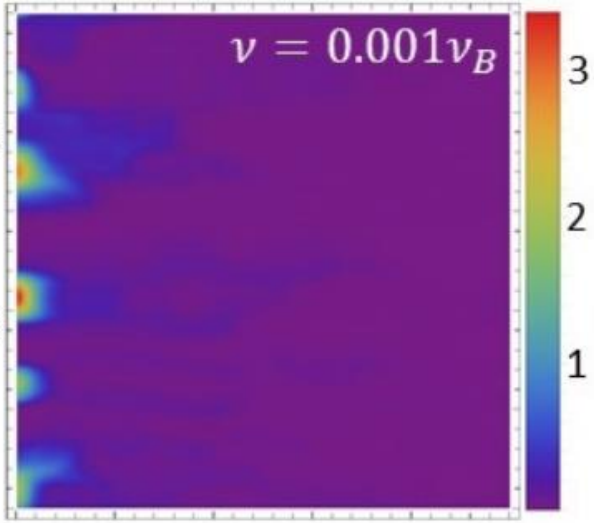
CR unconfined

Neither scattering nor mirrors
stop CR escaping
through RH boundary

Reference parameters



Negligible scattering



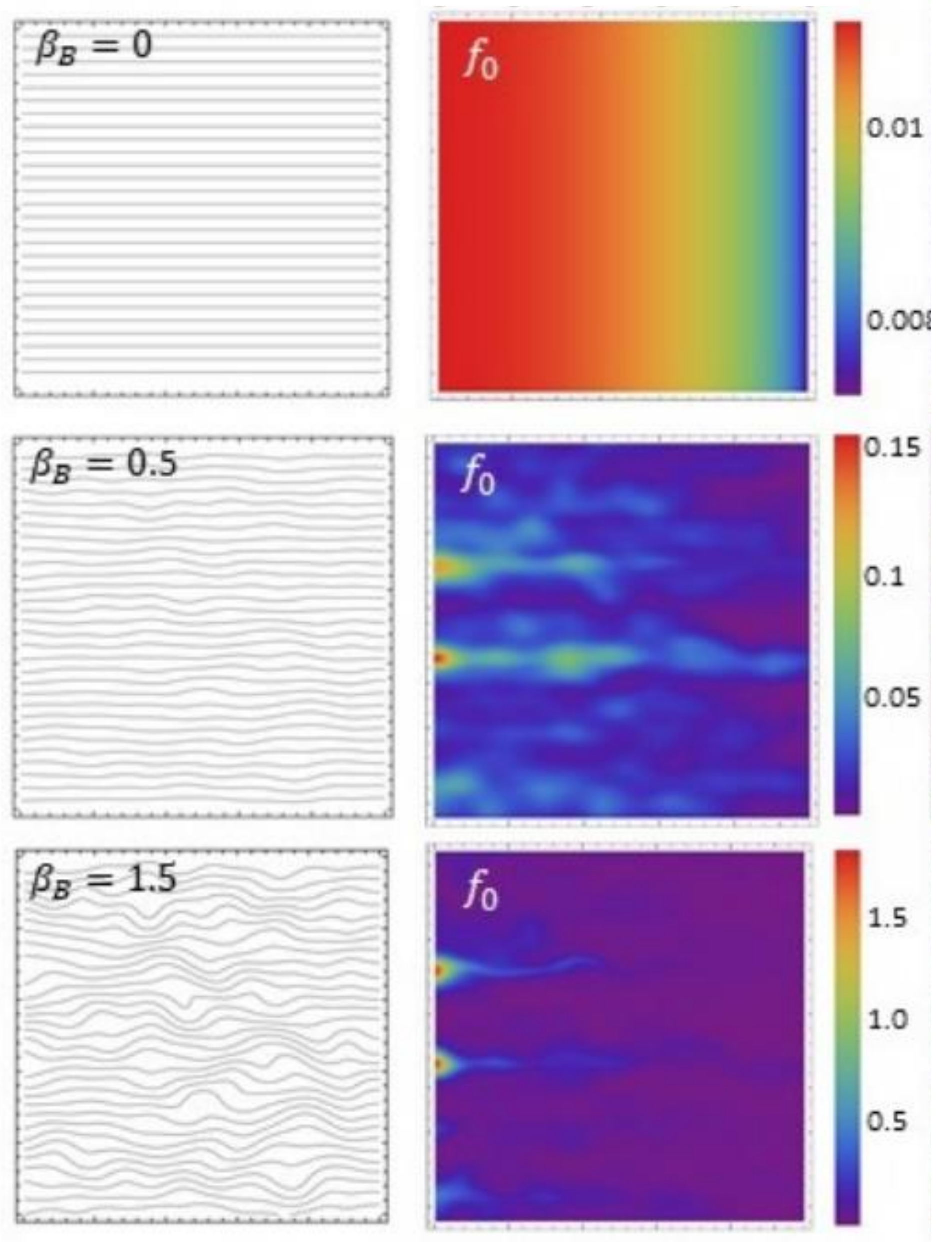
Strong trapping of some CR

Vary amplitude of magnetic field fluctuations (same $\nu = 0.01\nu_B$)

Uniform magnetic field
Diffusive escape
Large diffusion coefficient

Reduce fluctuations by half
Still get mirroring

Increase fluctuations by half
Strong trapping



For details of VFP calculation
Bell, Matthews, Taylor, Giacinti (2025)

PARTICLE MODEL

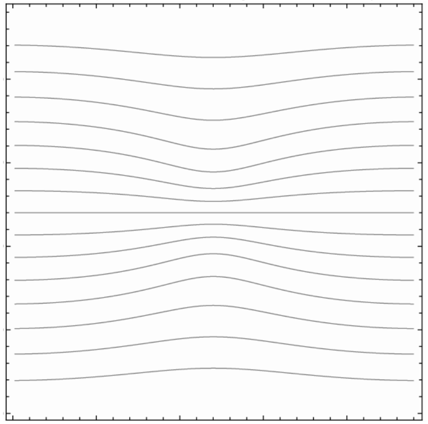
CR acceleration by shock advancing into a mirror

No scattering

Standard mirror in background uniform field B_0=5μG

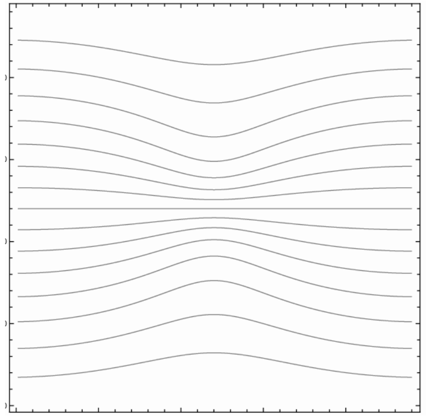
Magnetic field lines
In cylindrical geometry

Mirror field with peak
B_{mirror}=20μG



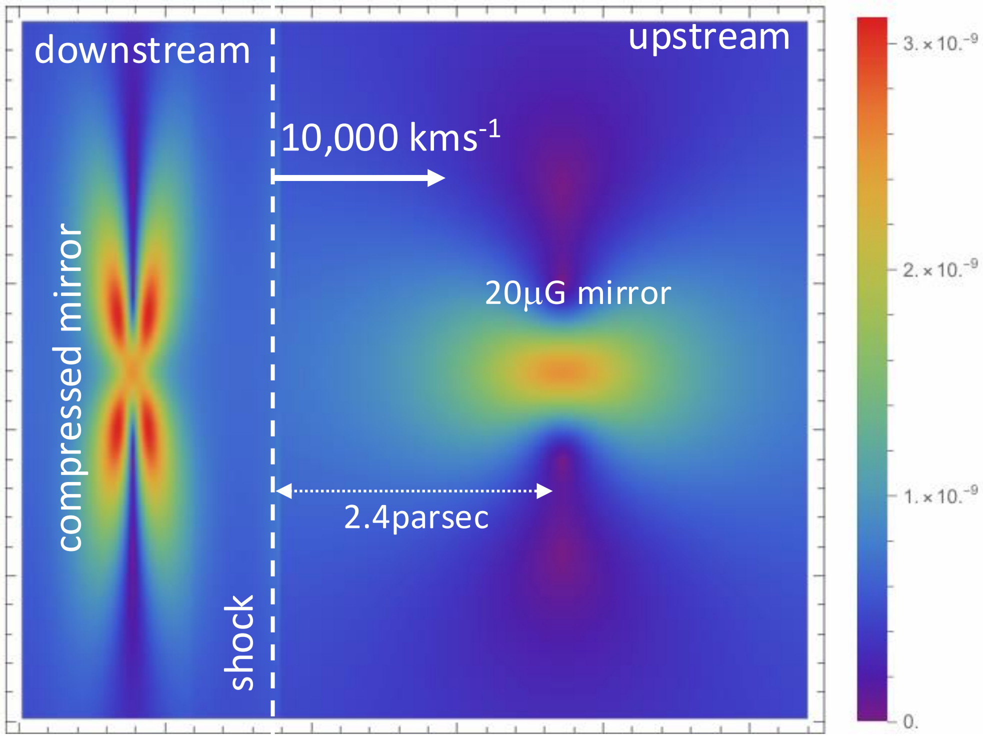
Magnetic field lines
In cylindrical geometry

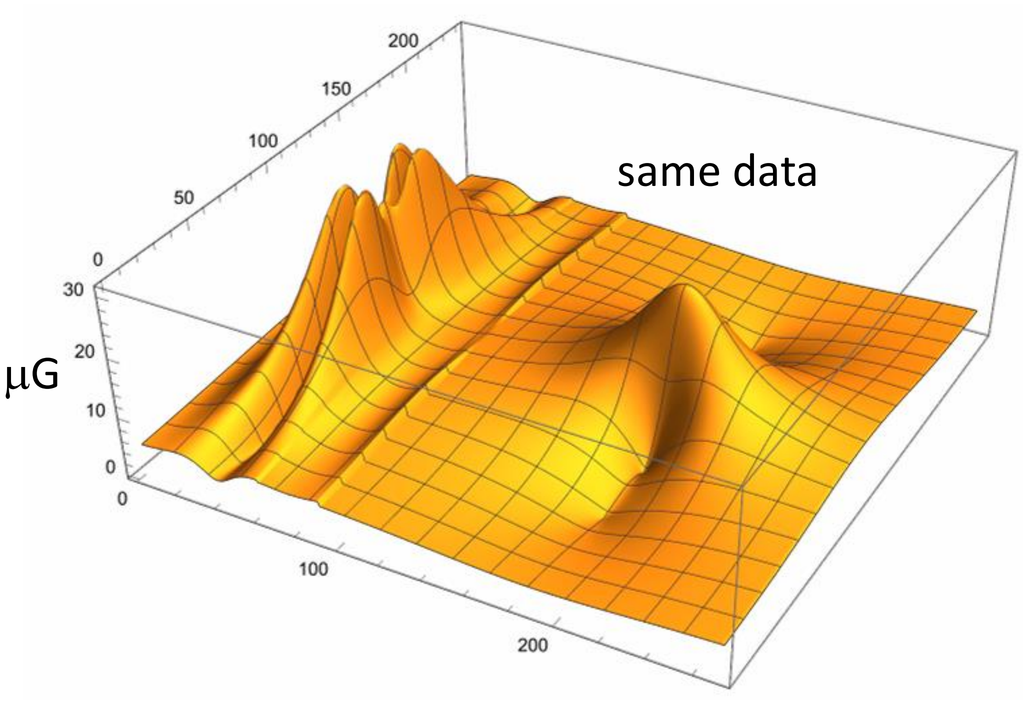
Mirror field with peak
B_{mirror}=50μG



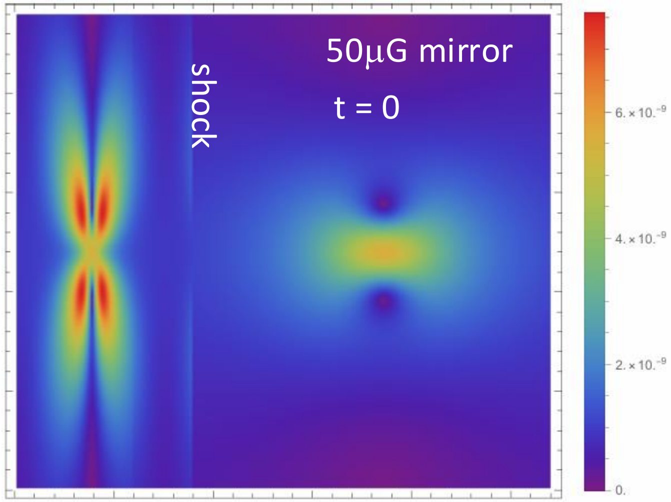
Magnitude of magnetic field

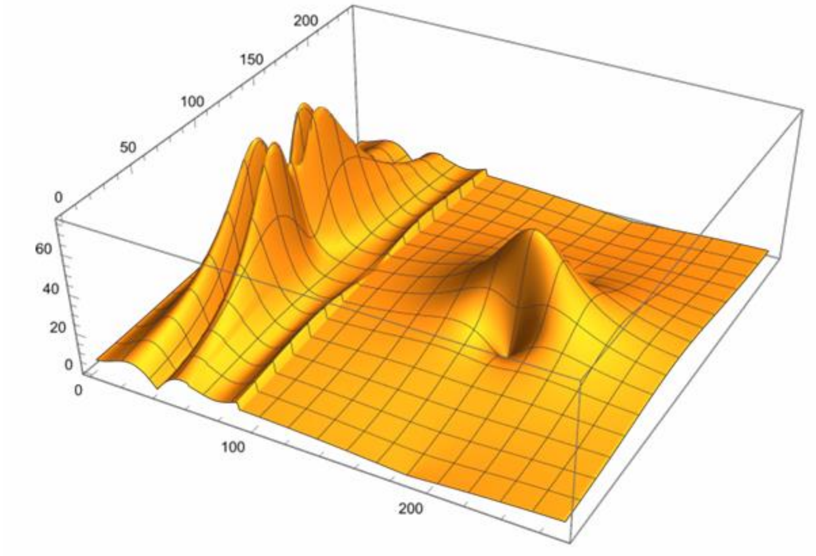
20μG mirror in parallel (θ=0°) 5μG uniform background

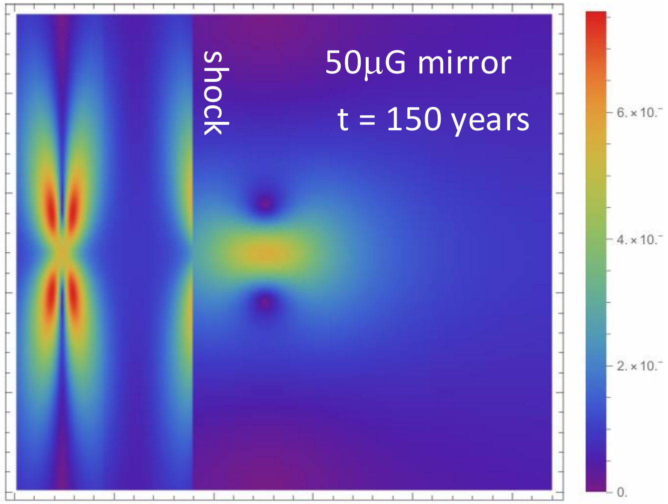


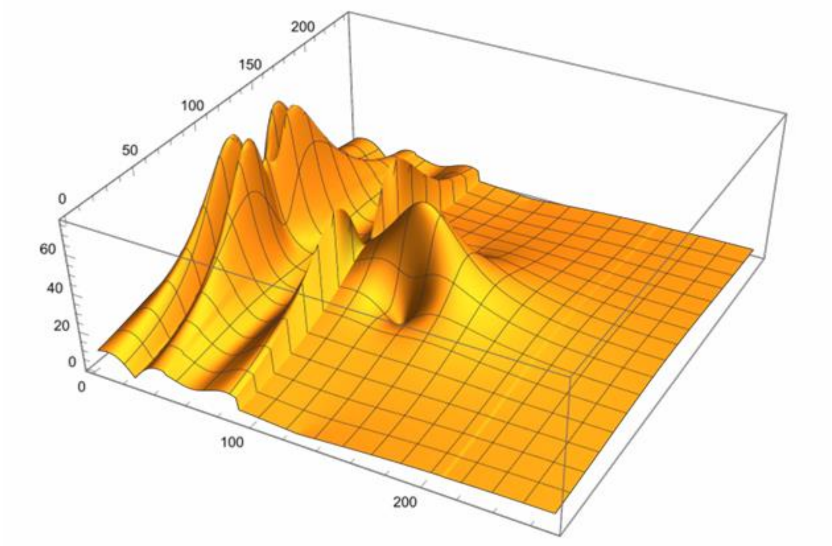


Mirror flows into shock



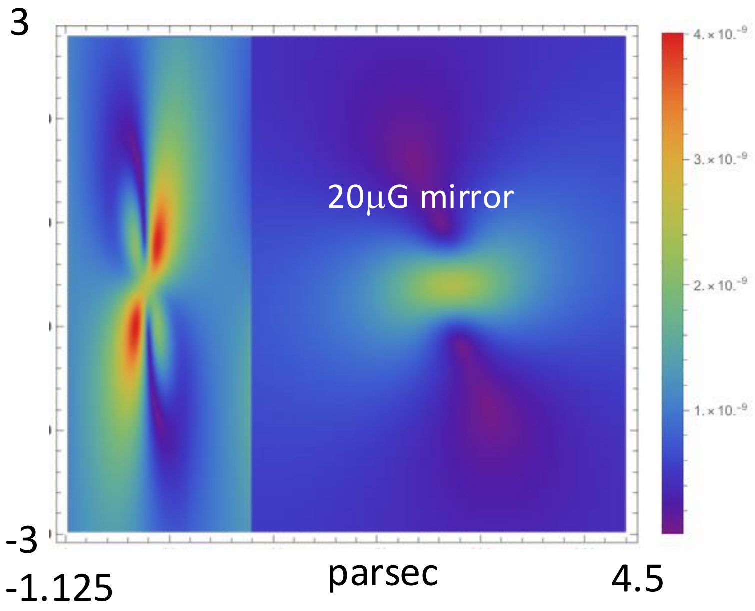


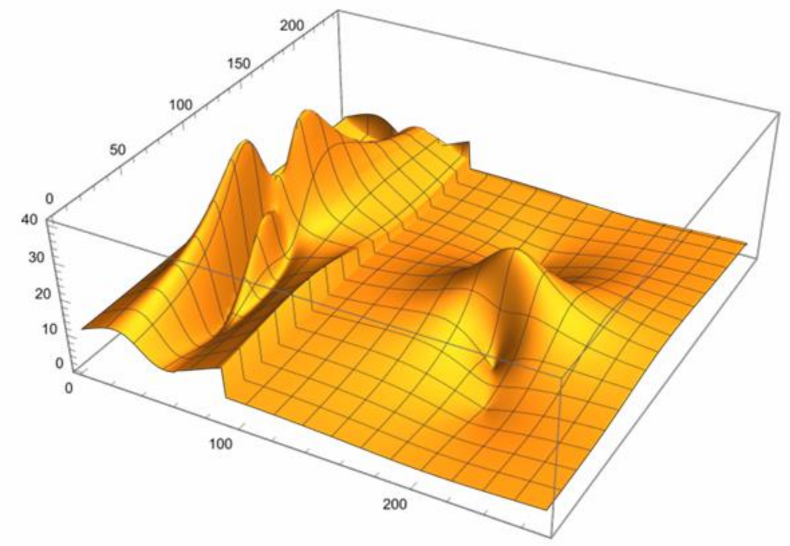




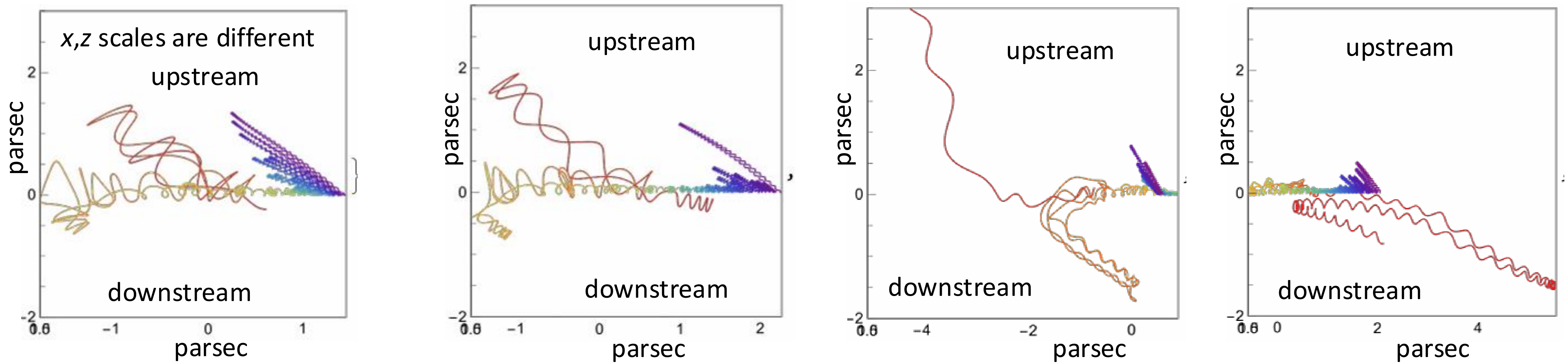
300,000 particles injected at random positions with energy 100TeV, followed for 300year

20μG mirror in parallel 5μG uniform background tilted at 30degrees (θ=30^o)





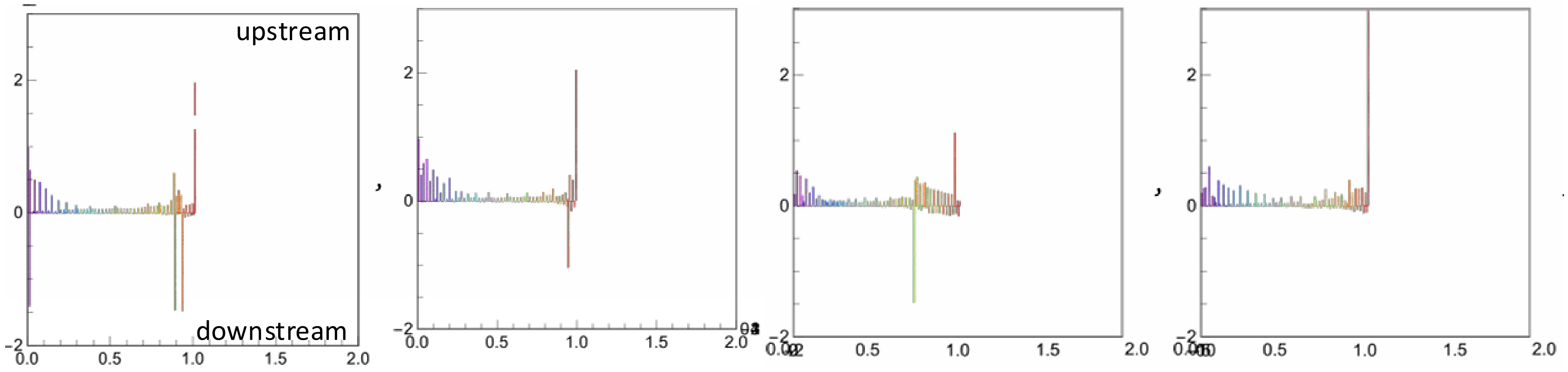
Some trajectories of CR accelerated beyond 1PeV, colour-coded by energy



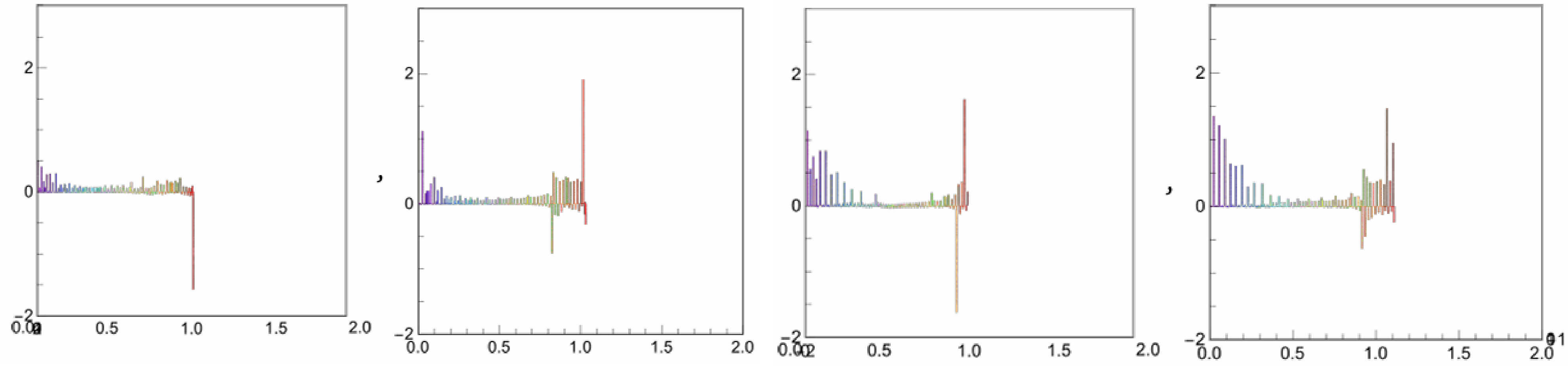
Sample trajectories: distance from shock (vertical) versus log of energy gain (horizontal)

All these reach 1PeV (just)

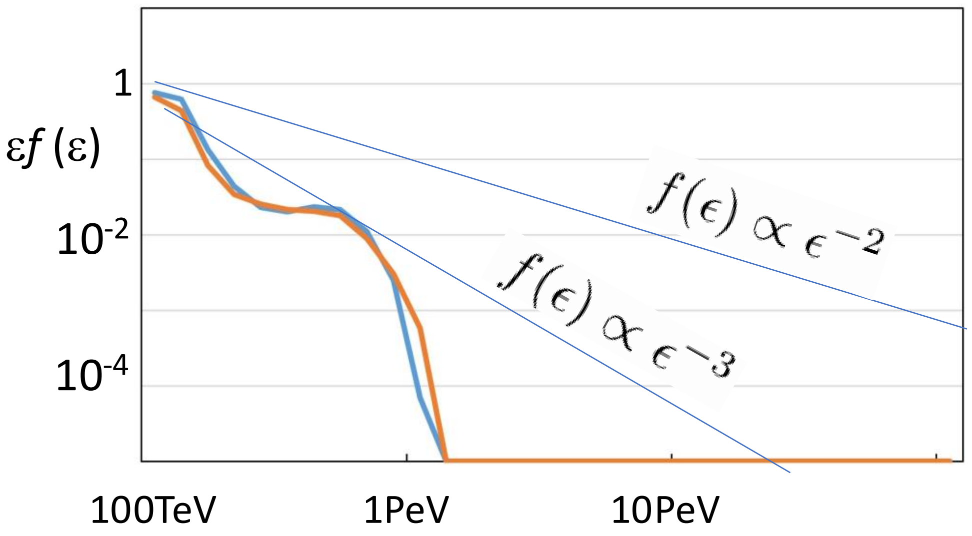
distance from shock in parsec

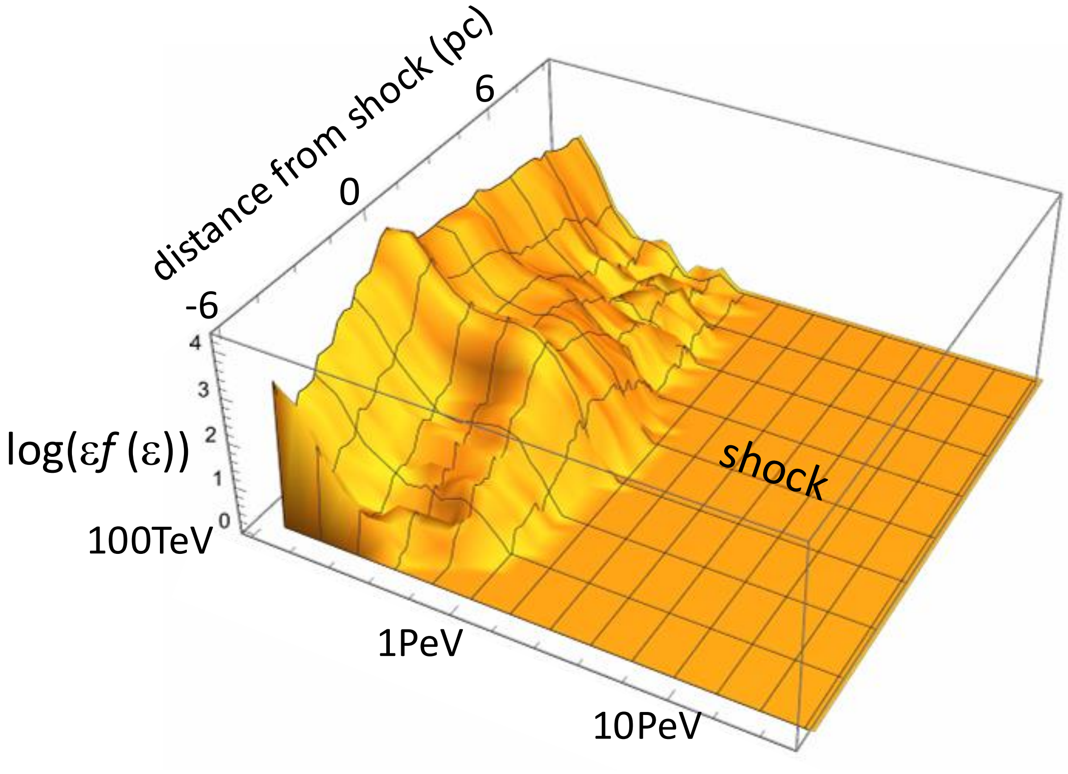


Horizontal axis: $\log_{10}$ (local rest-frame energy/100TeV)



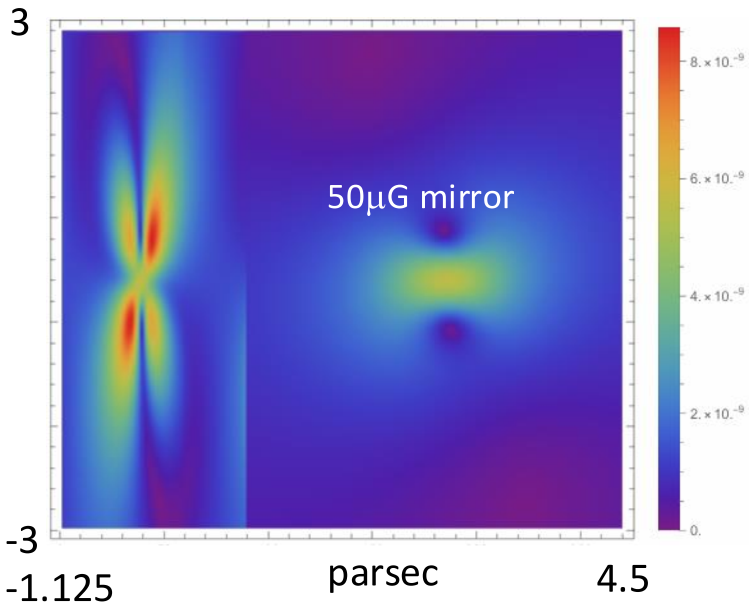
Spectrum after 300 years

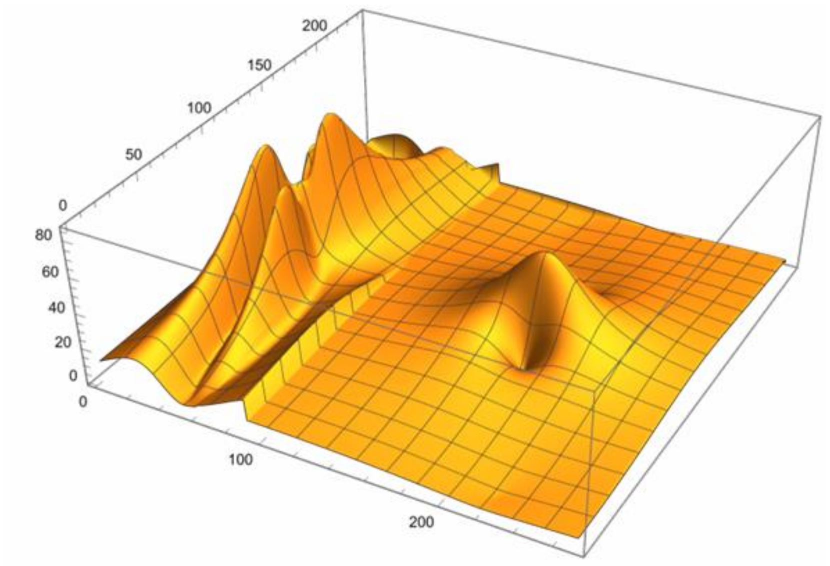




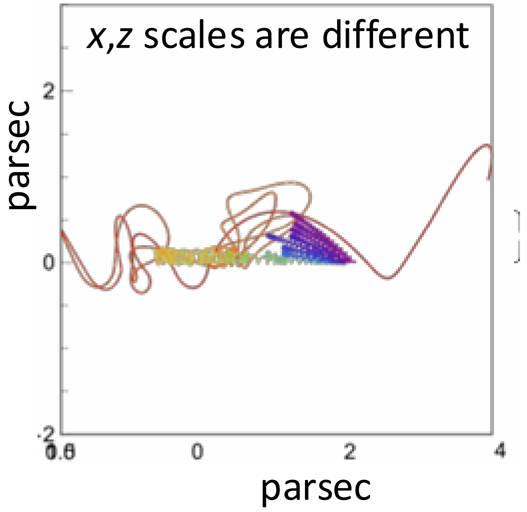
300,000 particles injected at random positions with energy 100TeV, followed for 300year

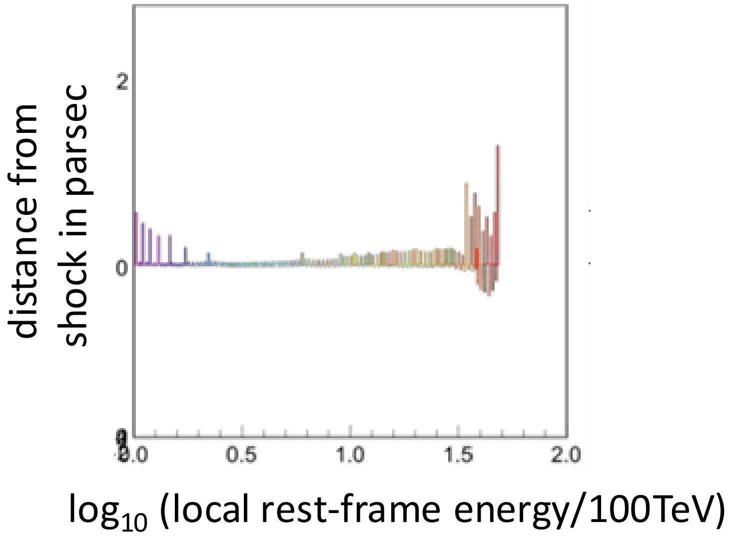
50μG mirror in parallel 5μG uniform background tilted at 30degrees ($\theta=30^o$)





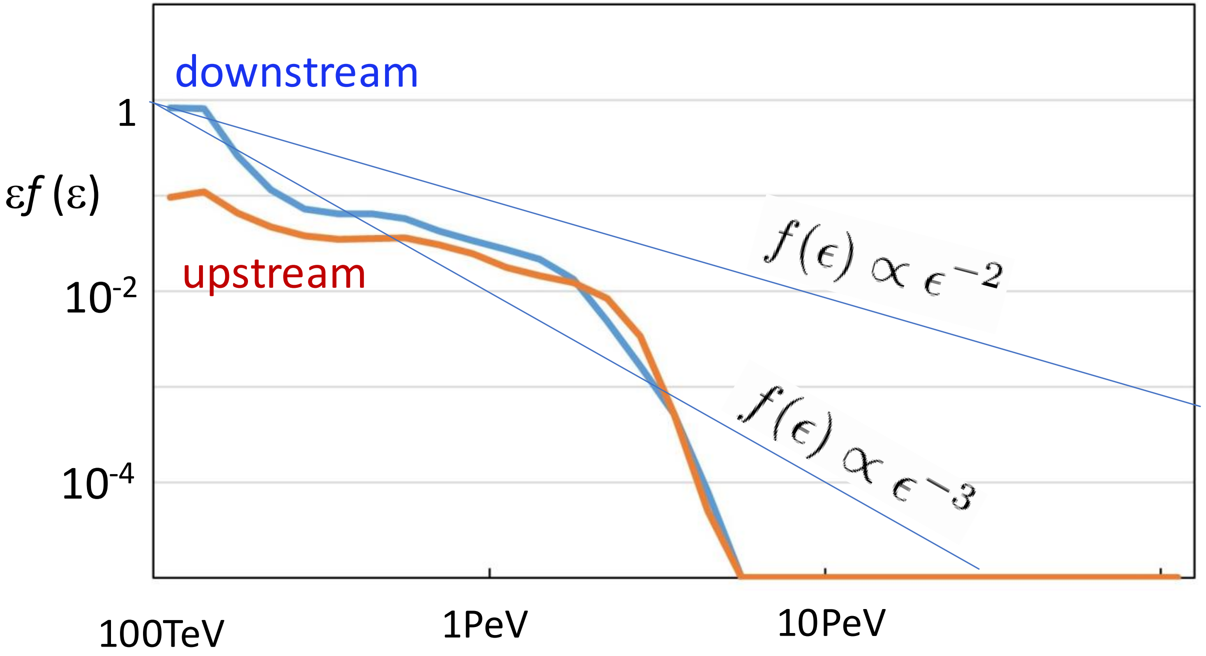
Trajectory of CR accelerated reaching 5PeV, colour-coded by energy

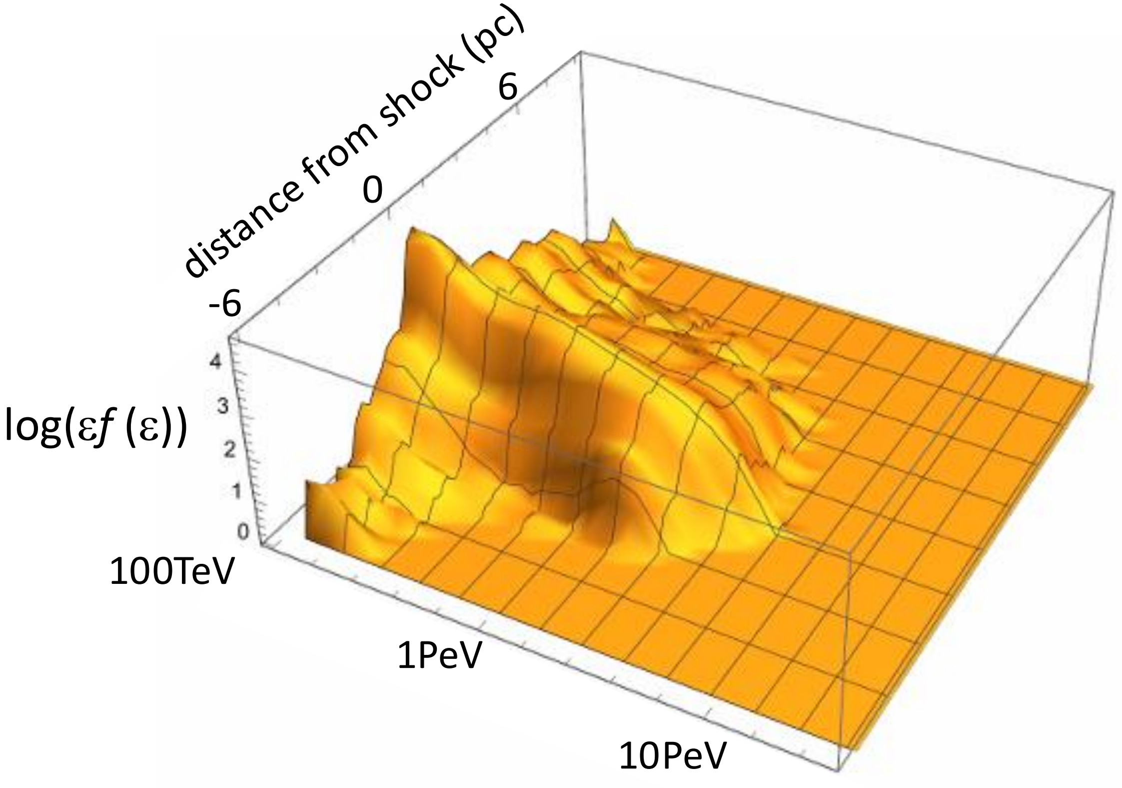




Spectrum after 300 years

50μG mirror





Increase radius to 10pc, time to 1000 year (like SN1006)

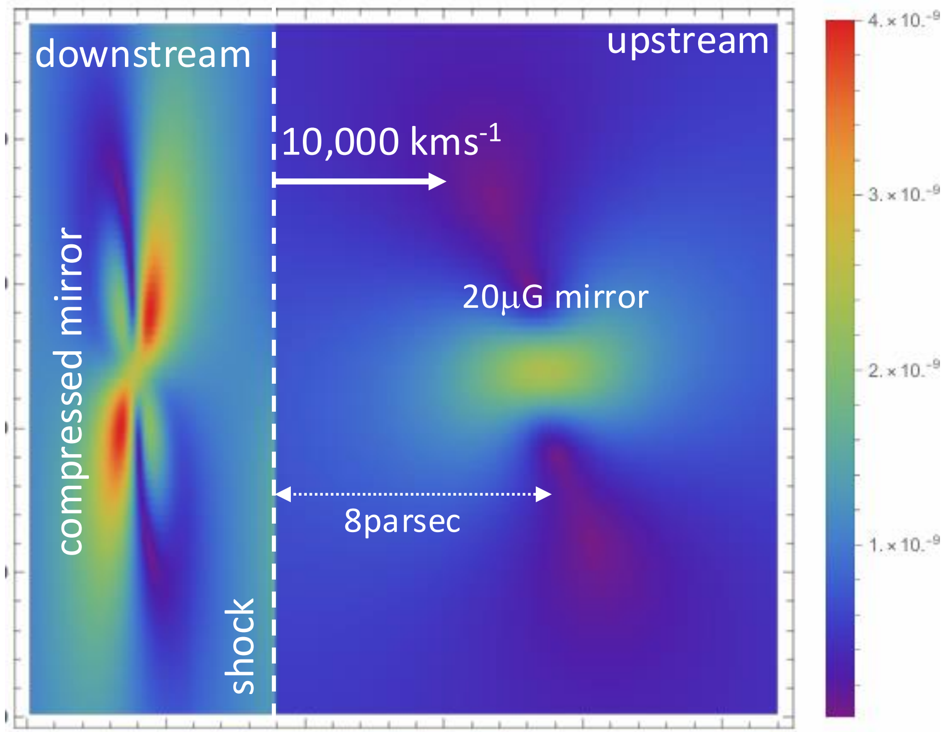
Average expansion velocity = 10,000 kms^{-1}

Today: closer to 5000 kms^{-1}

Same geometry but expanded by 3.3x

uniform background at 30°

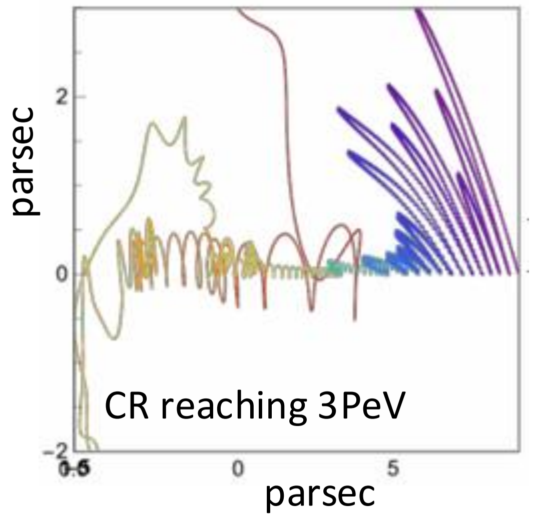
(θ=30°)

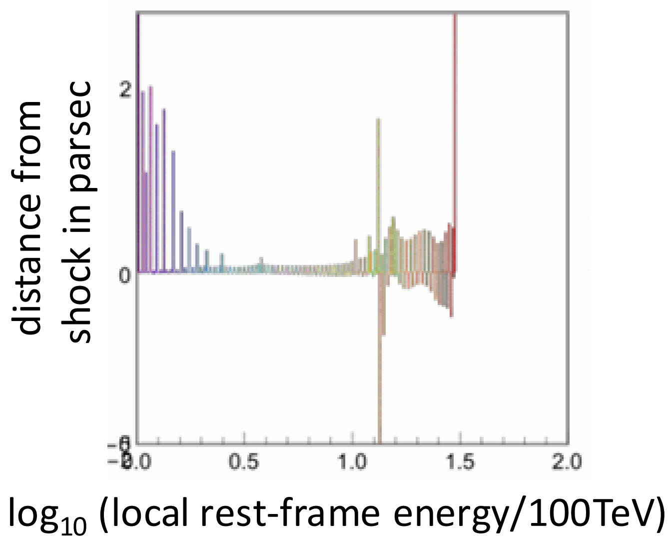


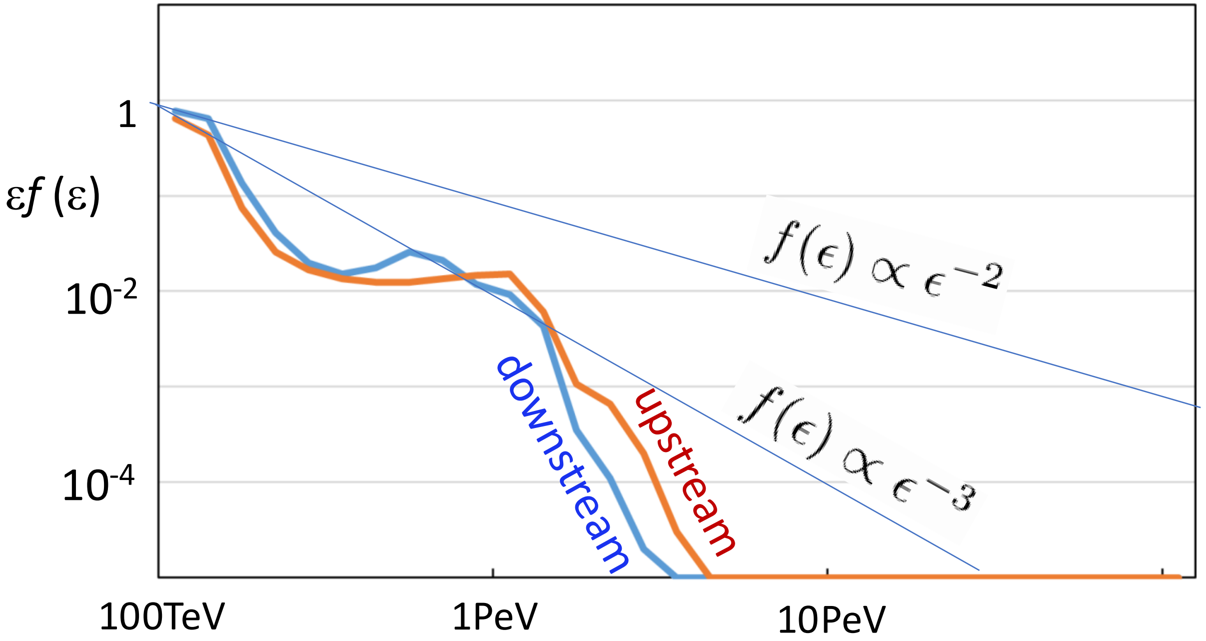
mirror at 8pc, follow for 1000 year

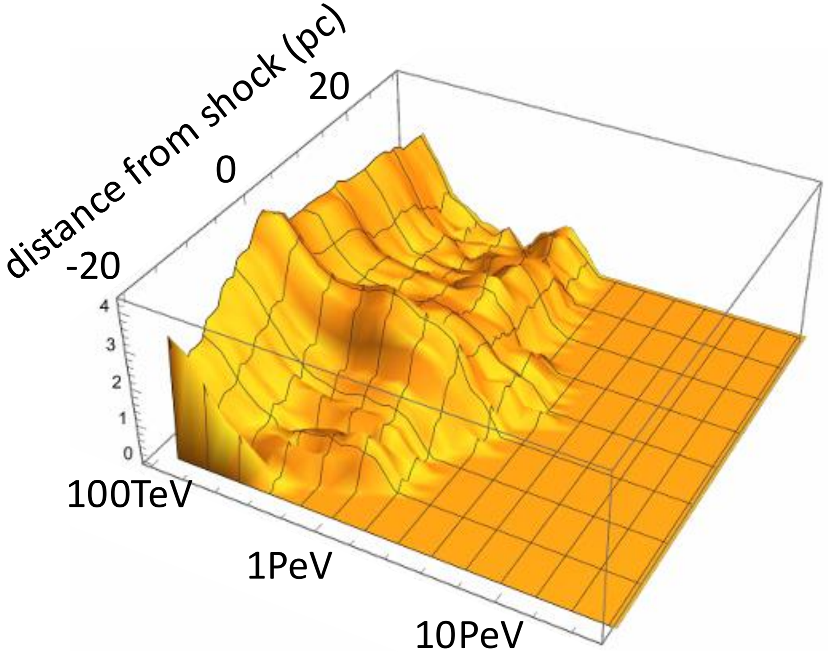
background field tilted at 30degrees ($\theta=30^o$)

20μG mirror





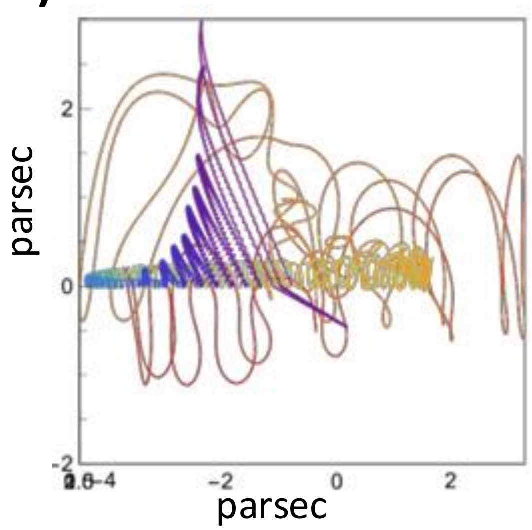


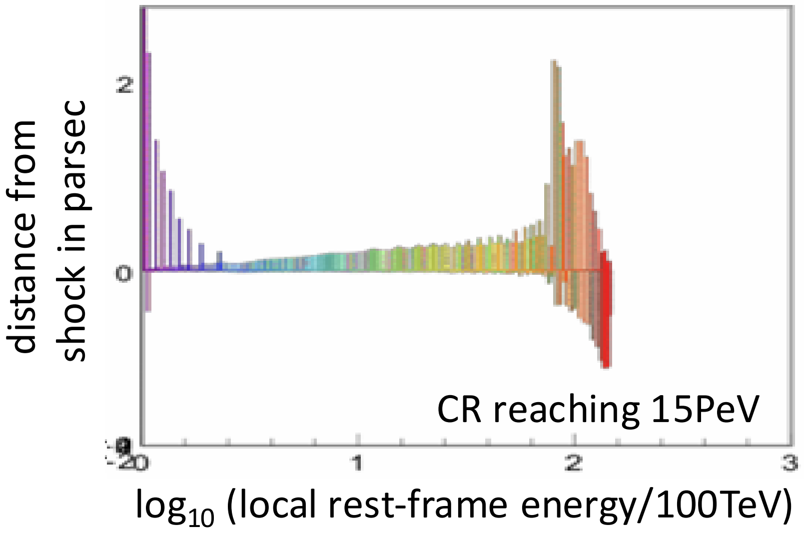


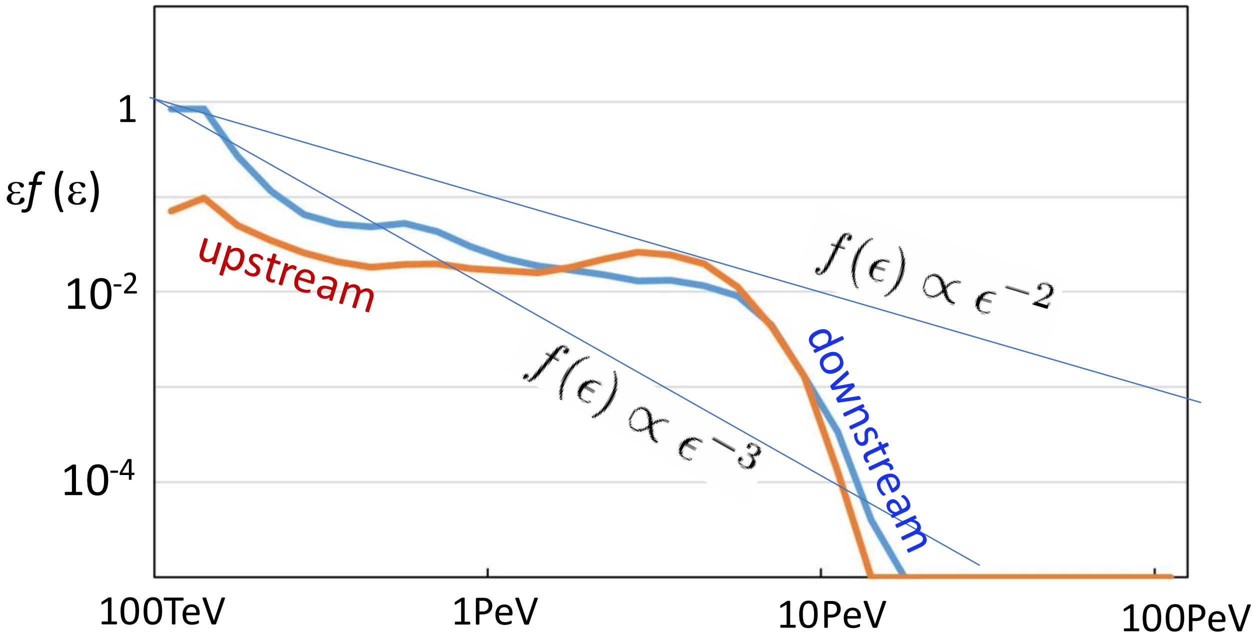
mirror at 8pc, follow for 1000 year

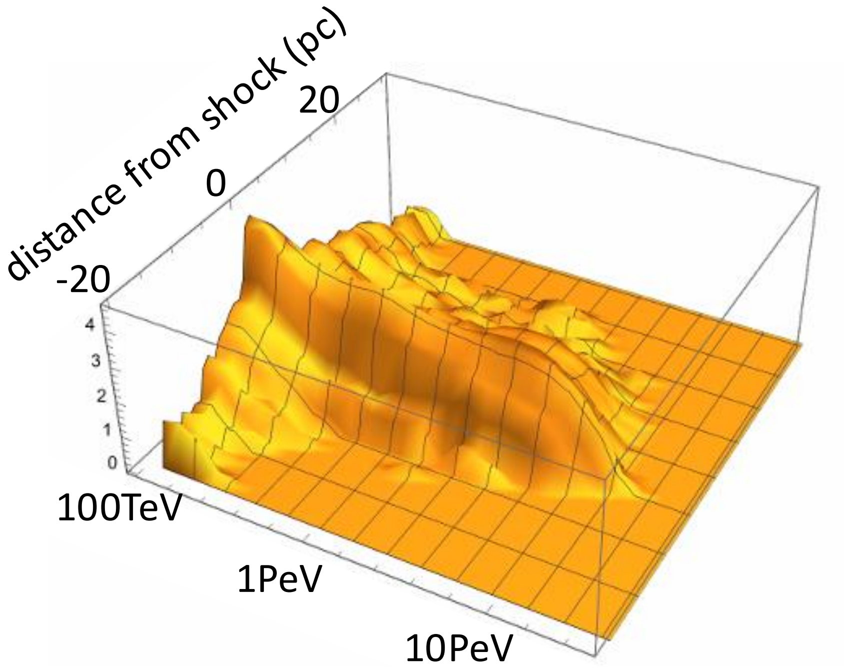
background field tilted at 30degrees (θ=30°)

50μG mirror



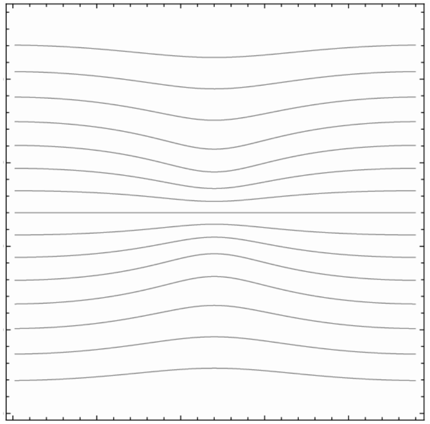




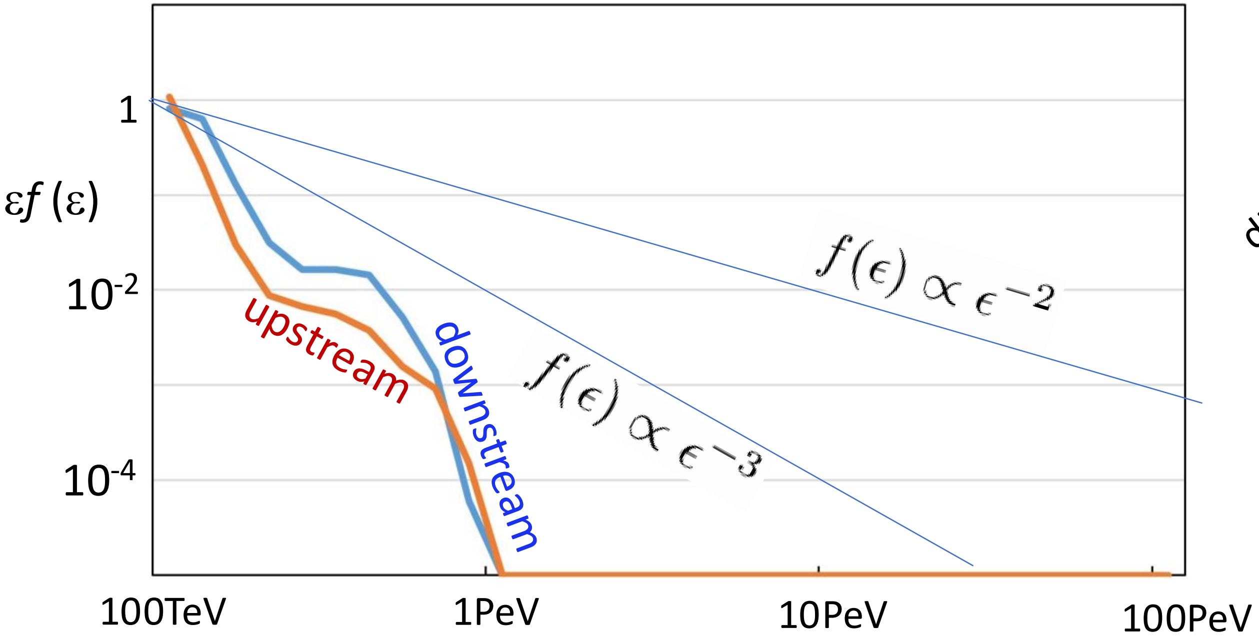


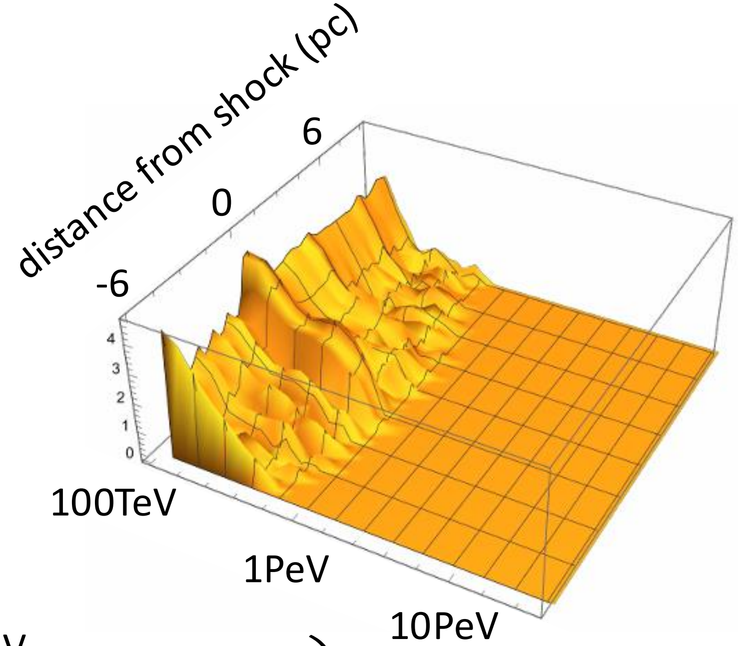
Parallel shock (θ=0°) after 300year

20muG mirror

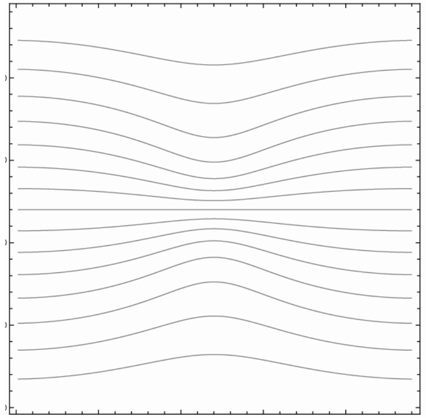


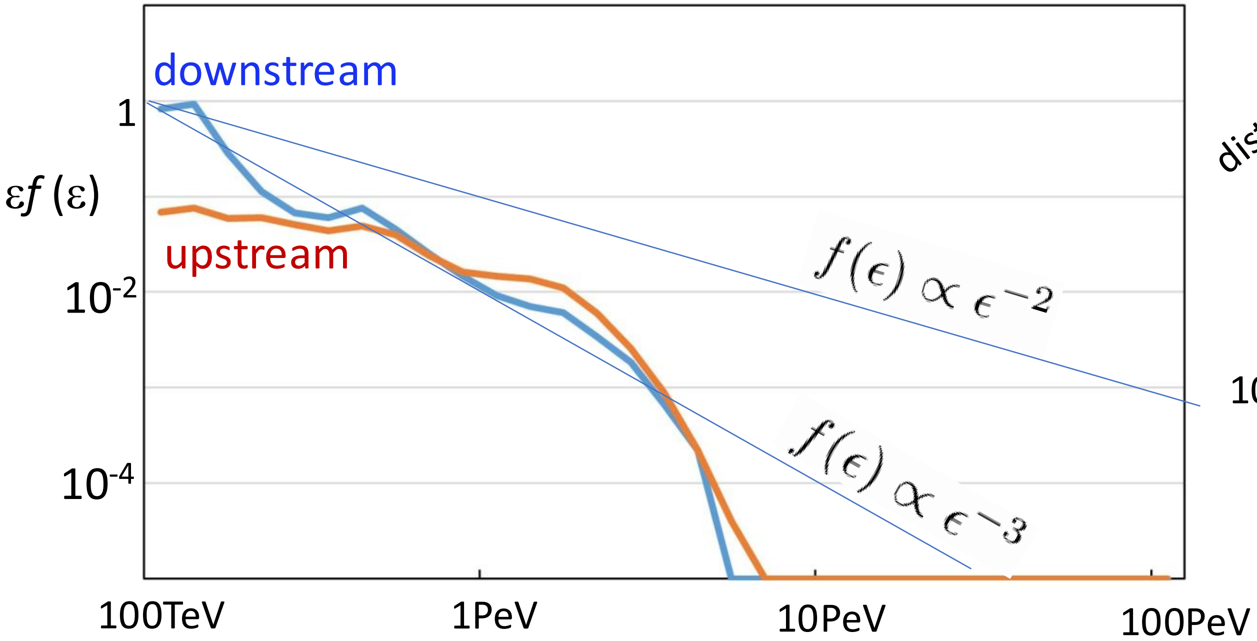
Magnetic field lines
In cylindrical geometry

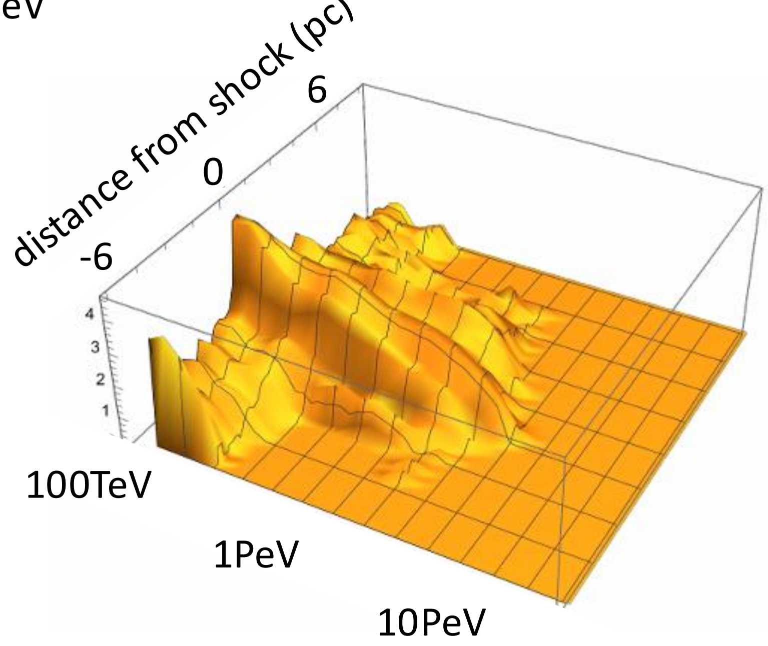




50muG mirror

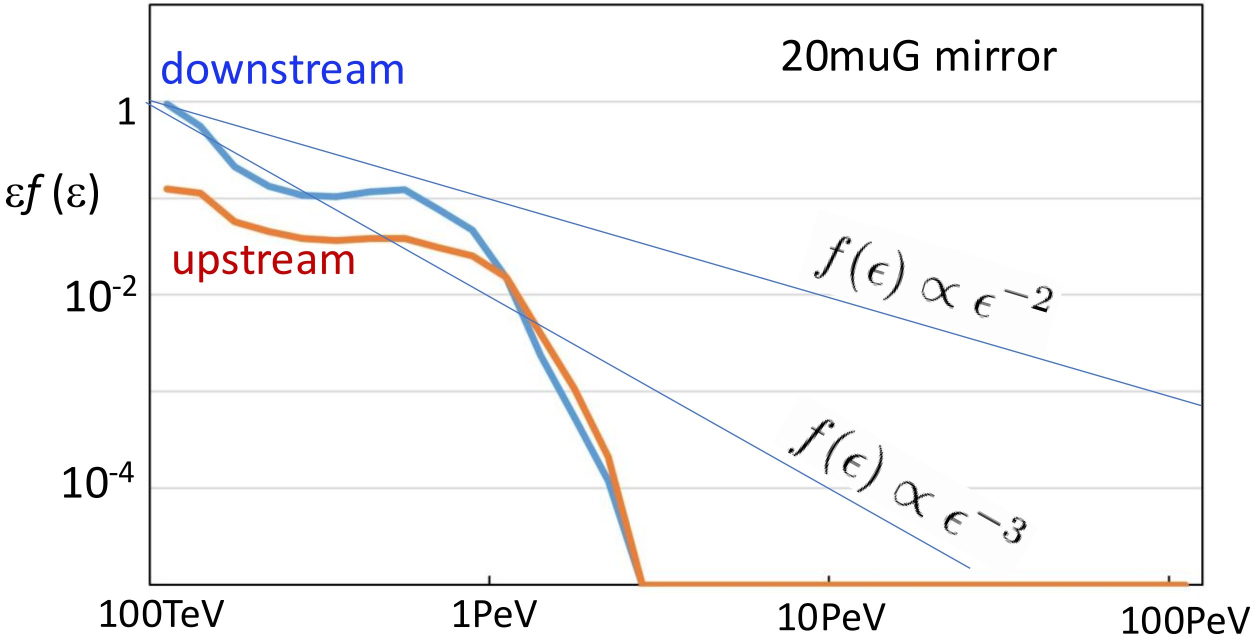


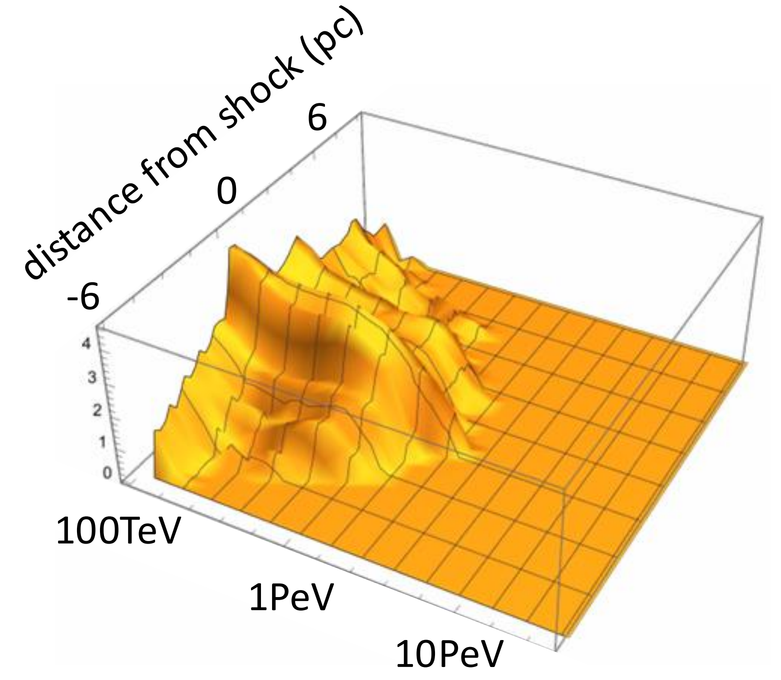




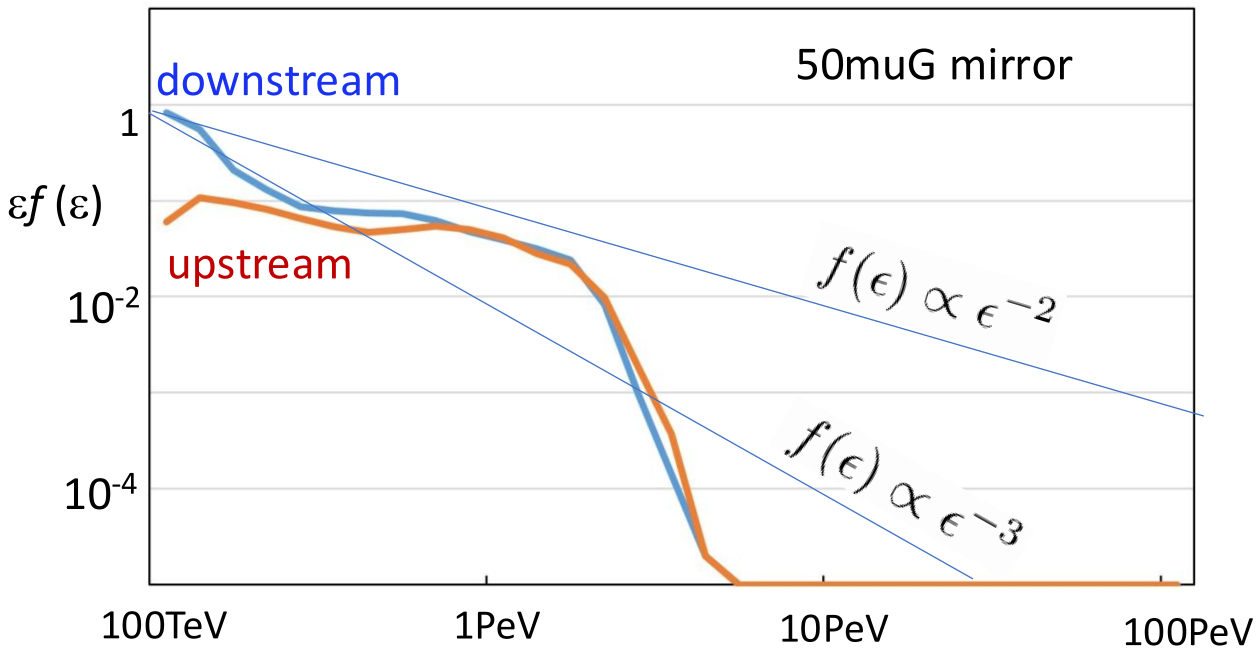
Oblique shock after 300 year
(θ=60°)

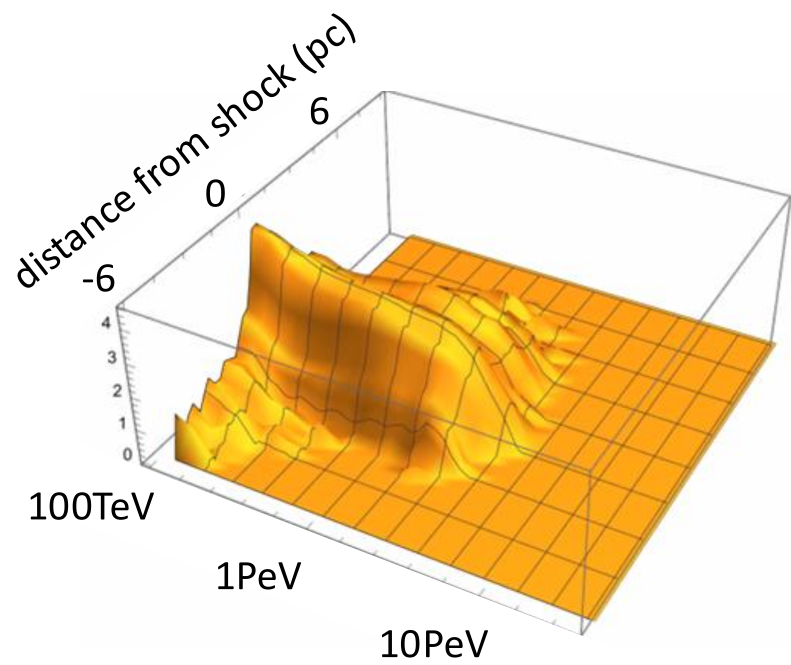
20muG mirror





50muG mirror





Compendium of spectra

The red line is a fiducial energy representing the Hillas limit

$$\epsilon_{Hillas} = u^2 t B_{mirror}$$

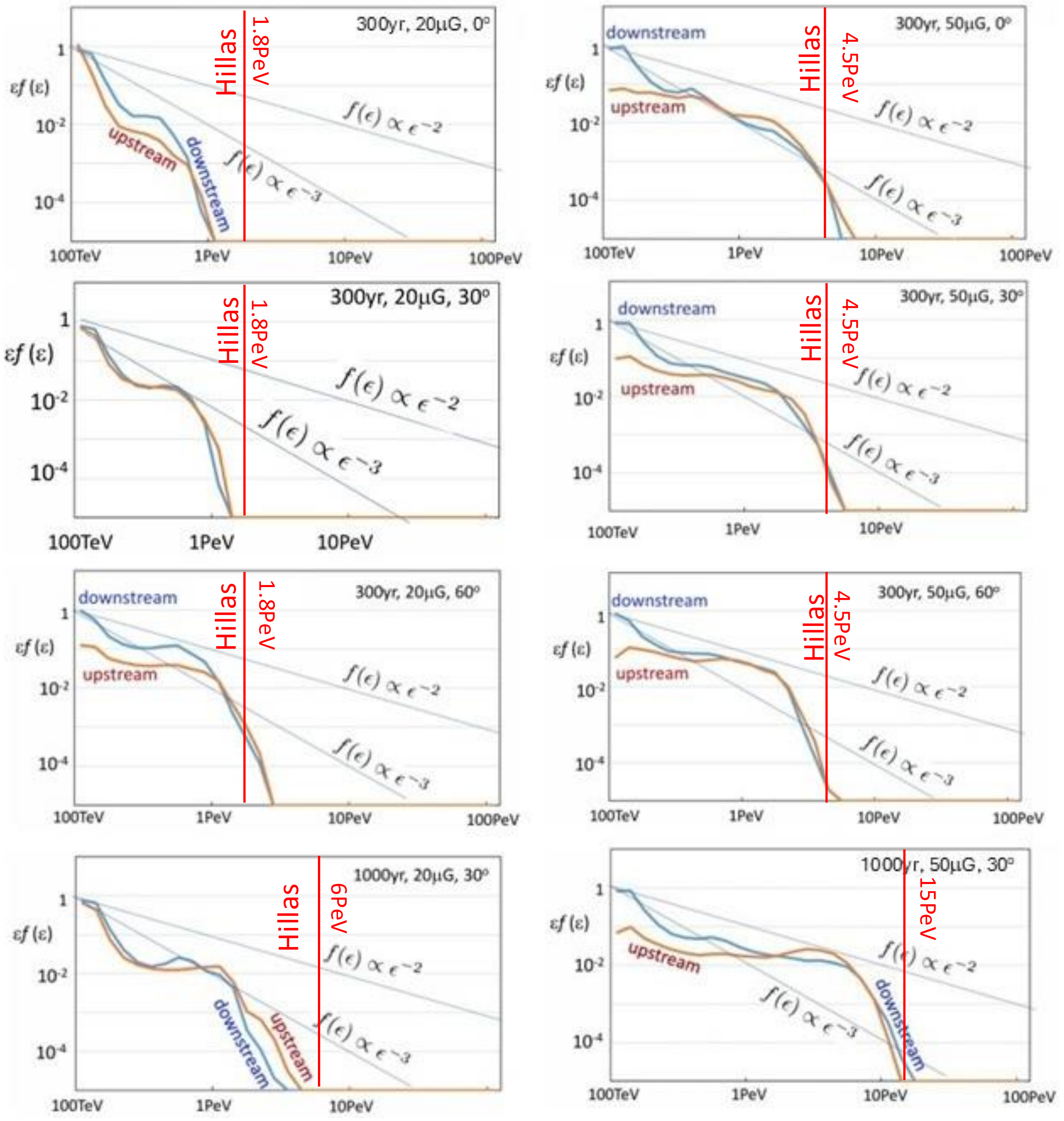
where $R = ut$

Energy spectrum turns over at

$$\sim 0.3 u^2 t B_{mirror}$$

20μG

50μG



300yr, 0°

300yr, 30°

300yr, 60°

1000yr, 30°

Conclusions

Mirror-dominated shock acceleration (MDSA) can accelerate CR to

$$\sim 0.3u^2tB_{mirror}$$

MDSA energy determined by peaks in magnetic field

DSA energy determined by average magnetic field

Advantage of MDSA: don't need magnetic field amplification

Disadvantage of MDSA: need pre-existing mirror(s) in surrounding medium

DSA with magnetic field amplification works well up 100TeV

MDSA probably better at PeV and beyond

MDSA, and therefore PeV acceleration, more likely where medium is already non-uniform on a parsec scale